Isotropic smoothing of image via Heat equation

import library

In [ ]:
```
import numpy as np
import matplotlib.image as img
import matplotlib.pyplot as plt
from matplotlib import cm
import matplotlib.colors as colors
from skimage import color
from skimage import io
```

load input image

- filename for the input image is 'barbara_color.jpeg'

In [ ]:
```
I0 = io.imread('barbara_color.jpeg')
```

check the size of the input image

In [ ]:
```
# ++++++++++++++++++++++++++++++++++++++++++++++++++
# complete the blanks
#
num_row     = (I0.shape)[0]
num_column  = (I0.shape)[1]
num_channel = (I0.shape)[2]
#
# ++++++++++++++++++++++++++++++++++++++++++++++++++

print('number of rows of I0 = ', num_row)
print('number of columns of I0 = ', num_column)
print('number of channels of I0 = ', num_channel)
```

```
number of rows of I0 =  512
number of columns of I0 =  512
number of channels of I0 =  3
```

convert the color image into a grey image

In [ ]:
```
# ++++++++++++++++++++++++++++++++++++++++++++++++++
# complete the blanks
#
I = color.rgb2gray(I0)

num_row     = (I0.shape)[0]
num_column  = (I0.shape)[1]
#
# ++++++++++++++++++++++++++++++++++++++++++++++++++
```

```
print('number of rows of I = ', num_row)
print('number of columns of I = ', num_column)
```

```
number of rows of I =  512
number of columns of I =  512
```

normalize the converted image

- normalize the converted grey scale image so that its maximum value is 1 and its minimum value is 0

In [ ]:
```
# ++++++++++++++++++++++++++++++++++++++++++++++++++
# complete the blanks
#
I = (I-np.min(I))/(np.max(I)-np.min(I))
#
# ++++++++++++++++++++++++++++++++++++++++++++++++++

print('maximum value of I = ', np.max(I))
print('minimum value of I = ', np.min(I))
```

```
maximum value of I =  1.0
minimum value of I =  0.0
```

define a function to compute the derivative of input matrix in x(row)-direction

- forward difference : $I[x+1,y]-I[x,y]$

In [ ]:
```
def compute_derivative_x_forward(I):

    D = np.zeros(I.shape)

    # ++++++++++++++++++++++++++++++++++++++++++++++++++
    # complete the blanks
    #
    padded_image = np.pad(I,
    (1, 1),
    mode='edge',
    )
    rolled_image=np.roll(padded_image, 1)
    D=(rolled_image-padded_image)


    #
    # ++++++++++++++++++++++++++++++++++++++++++++++++++

    return D
```

- backward difference : $I[x,y]-I[x-1,y]$

In [ ]:
```
def compute_derivative_x_backward(I):
```

```
    D = np.zeros(I.shape)

    # ++++++++++++++++++++++++++++++++++++++++++++++++++
    # complete the blanks
    #
    padded_image = np.pad(I,
    (1, 1),
    mode='edge',
    )
    rolled_image=np.roll(padded_image, -1)
    D=(padded_image-rolled_image)


    #
    # ++++++++++++++++++++++++++++++++++++++++++++++++++

    return D
```

define a function to compute the derivative of input matrix in y(column)-direction

- forward difference : $I[x, y+1] - I[x, y]$

In [ ]:

```
def compute_derivative_y_forward(I):

    D = np.zeros(I.shape)

    # ++++++++++++++++++++++++++++++++++++++++++++++++++
    # complete the blanks
    #
    padded_image = np.pad(I,
        (1, 1),
        mode='edge',
        )
    rolled_image=np.roll(padded_image, 1,axis=0)
    D=(rolled_image-padded_image)



    #
    # ++++++++++++++++++++++++++++++++++++++++++++++++++

    return D
```

- backward difference : $I[x, y] - I[x, y-1]$

In [ ]:

```
def compute_derivative_y_backward(I):

    D = np.zeros(I.shape)

    # ++++++++++++++++++++++++++++++++++++++++++++++++++
    # complete the blanks
    #
```

```
    padded_image = np.pad(I,
    (1, 1),
    mode='edge',
    )
    rolled_image=np.roll(padded_image, -1,axis=0)
    D=(padded_image-rolled_image)


    #
    # ++++++++++++++++++++++++++++++++++++++++++++++++++

    return D
```

define a function to compute the laplacian of input matrix

- $\Delta I = \nabla^2 I = \frac{\partial^2 I}{\partial x^2} + \frac{\partial^2 I}{\partial y^2}$
- $\Delta I = I[x+1, y] + I[x-1, y] + I[x, y+1] + I[x, y-1] - 4 * I[x, y]$
- ΔI = derivative_x_forward - derivative_x_backward + derivative_y_forward - derivative_y_backward

In [ ]:
```
def boundry_condition(I):
   I = np.pad(I,
    (1, 1),
    mode='edge',
    )
   return I
```

In [ ]:
```
def compute_laplace(I):

    laplace = np.zeros(I.shape)

    # ++++++++++++++++++++++++++++++++++++++++++++++++++
    # complete the blanks
    #
    laplace=compute_derivative_x_forward(I)-compute_derivative_x_backward(I)
    +compute_derivative_y_forward(I)-compute_derivative_y_backward(I)
    laplace=np.delete(laplace,[0,-1],axis=0)
    laplace=np.delete(laplace,[0,-1],axis=1)


    #
    # ++++++++++++++++++++++++++++++++++++++++++++++++++

    return laplace
```

define a function to compute the heat equation of data I with a time step

- $I = I + \delta t * \Delta I$

In [ ]:
```
def heat_equation(I, time_step):
```

```
    I_update = np.zeros(I.shape)

    # ++++++++++++++++++++++++++++++++++++++++++++++++++
    # complete the blanks
    #
    I=I+(time_step*compute_laplace(I))
    I_update=I


    #
    # ++++++++++++++++++++++++++++++++++++++++++++++++++

    return I_update
```

run the heat equation over iterations

In [ ]:

```
def run_heat_equation(I, time_step, number_iteration):

    I_update = np.zeros(I.shape)

    for t in range(number_iteration):
        # ++++++++++++++++++++++++++++++++++++++++++++++++++
        # complete the blanks
        #
        I=heat_equation(I,time_step)


        #
        # ++++++++++++++++++++++++++++++++++++++++++++++++++
    I_update=I
    return I_update
```

---

functions for presenting the results

In [ ]:

```
def function_result_01():

    plt.figure(figsize=(8,6))
    plt.imshow(I0)
    plt.show()
```

In [ ]:

```
def function_result_02():

    plt.figure(figsize=(8,6))
    plt.imshow(I, cmap='gray', vmin=0, vmax=1, interpolation='none')
    plt.show()
```

In [ ]:
```
def function_result_03():

    L = compute_laplace(I)

    plt.figure(figsize=(8,6))
    plt.imshow(L, cmap='gray')
    plt.show()
```

In [ ]:
```
def function_result_04():

    time_step   = 0.25
    I_update    = heat_equation(I, time_step)

    plt.figure(figsize=(8,6))
    plt.imshow(I_update, vmin=0, vmax=1, cmap='gray')
    plt.show()
```

In [ ]:
```
def function_result_05():

    time_step           = 0.25
    number_iteration    = 128

    I_update = run_heat_equation(I, time_step, number_iteration)

    plt.figure(figsize=(8,6))
    plt.imshow(I_update, vmin=0, vmax=1, cmap='gray')
    plt.show()
```

In [ ]:
```
def function_result_06():

    time_step           = 0.25
    number_iteration    = 512

    I_update = run_heat_equation(I, time_step, number_iteration)

    plt.figure(figsize=(8,6))
    plt.imshow(I_update, vmin=0, vmax=1, cmap='gray')
    plt.show()
```

In [ ]:
```
def function_result_07():

    L = compute_laplace(I)

    value1 = L[0, 0]
    value2 = L[-1, -1]
    value3 = L[100, 100]
    value4 = L[200, 200]

    print('value1 = ', value1)
    print('value2 = ', value2)
    print('value3 = ', value3)
    print('value4 = ', value4)
```

In [ ]:
```
```

```
def function_result_08():

    time_step   = 0.25
    I_update    = heat_equation(I, time_step)

    value1 = I_update[0, 0]
    value2 = I_update[-1, -1]
    value3 = I_update[100, 100]
    value4 = I_update[200, 200]

    print('value1 = ', value1)
    print('value2 = ', value2)
    print('value3 = ', value3)
    print('value4 = ', value4)
```

In [ ]:

```
def function_result_09():

    time_step           = 0.25
    number_iteration    = 128

    I_update = run_heat_equation(I, time_step, number_iteration)

    value1 = I_update[0, 0]
    value2 = I_update[-1, -1]
    value3 = I_update[100, 100]
    value4 = I_update[200, 200]

    print('value1 = ', value1)
    print('value2 = ', value2)
    print('value3 = ', value3)
    print('value4 = ', value4)
```

In [ ]:

```
def function_result_10():

    time_step           = 0.25
    number_iteration    = 512

    I_update = run_heat_equation(I, time_step, number_iteration)

    value1 = I_update[0, 0]
    value2 = I_update[-1, -1]
    value3 = I_update[100, 100]
    value4 = I_update[200, 200]

    print('value1 = ', value1)
    print('value2 = ', value2)
    print('value3 = ', value3)
    print('value4 = ', value4)
```

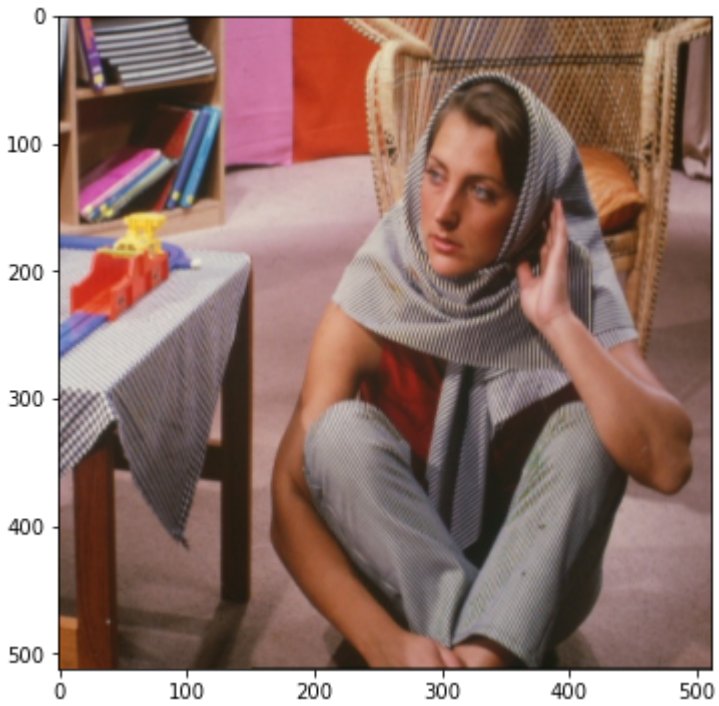
---

results

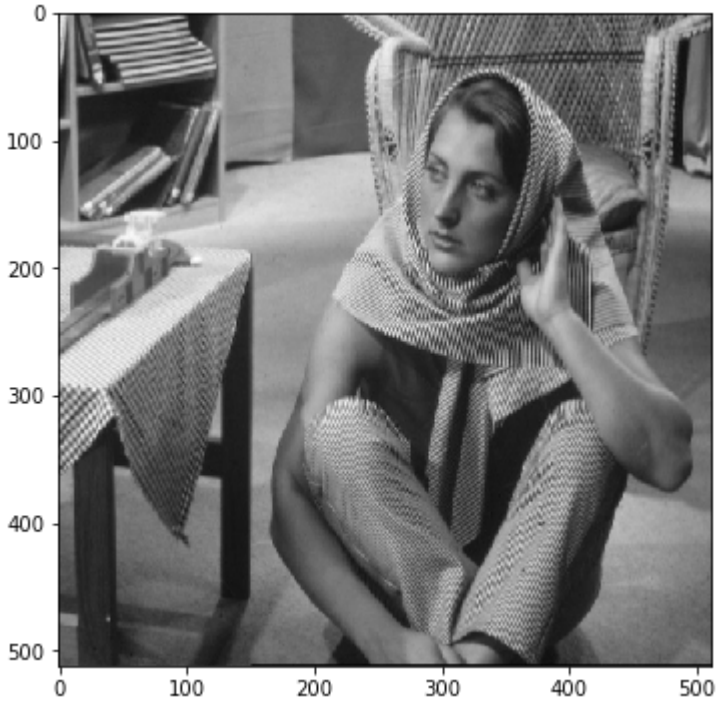
In [ ]:

```
number_result = 10

for i in range(number_result):
    title = '## [RESULT {:02d}]'.format(i+1)
    name_function = 'function_result_{:02d}()'.format(i+1)

    print('**************************************************')
    print(title)
    print('**************************************************')
    eval(name_function)
```

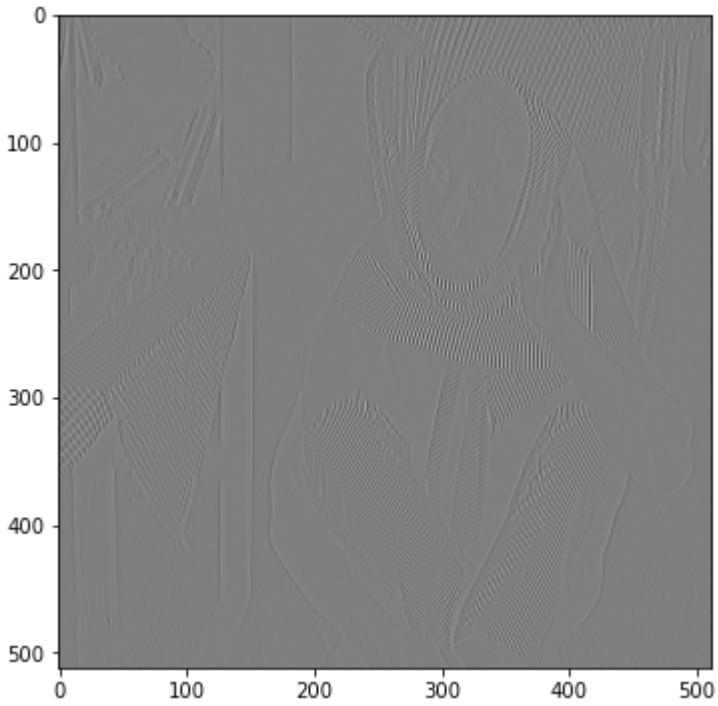
```
**************************************************
## [RESULT 01]
**************************************************
```



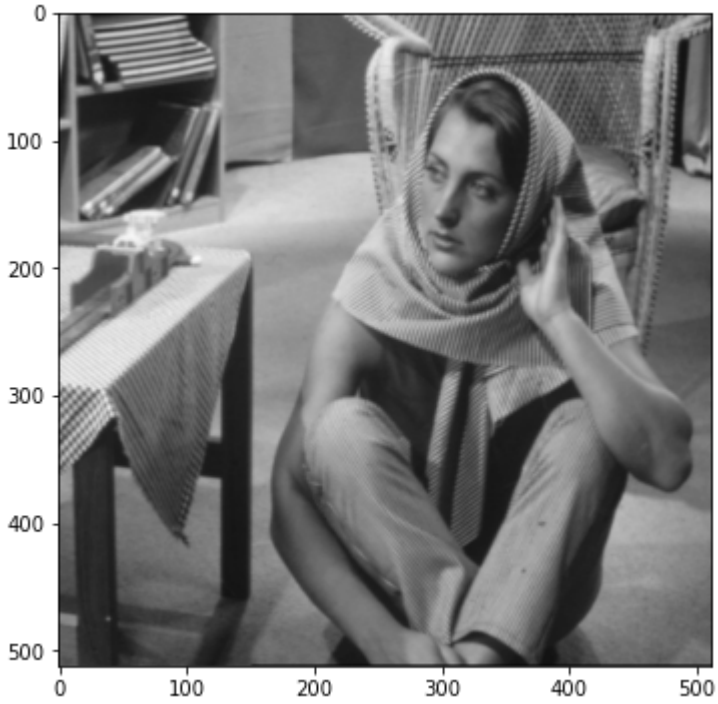

```
**************************************************
## [RESULT 02]
**************************************************
```



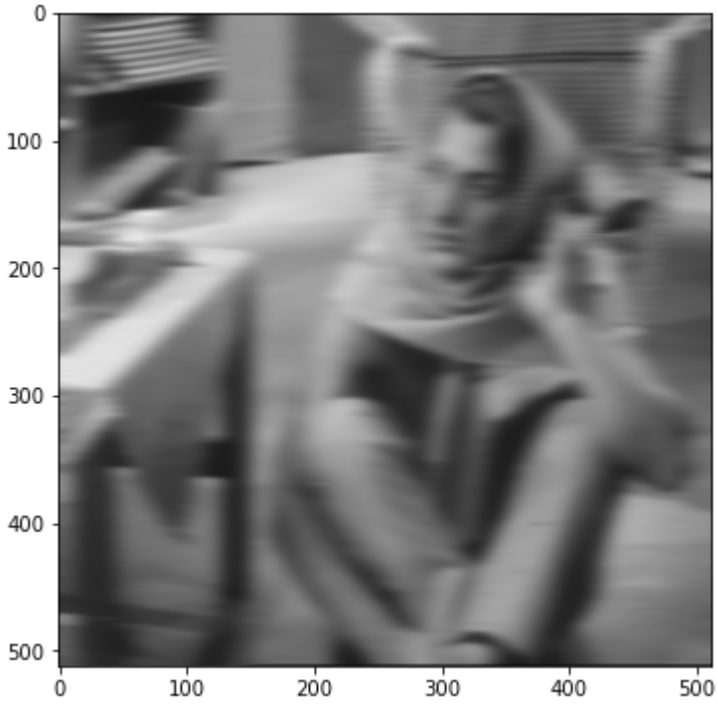
****
[RESULT 03]
****



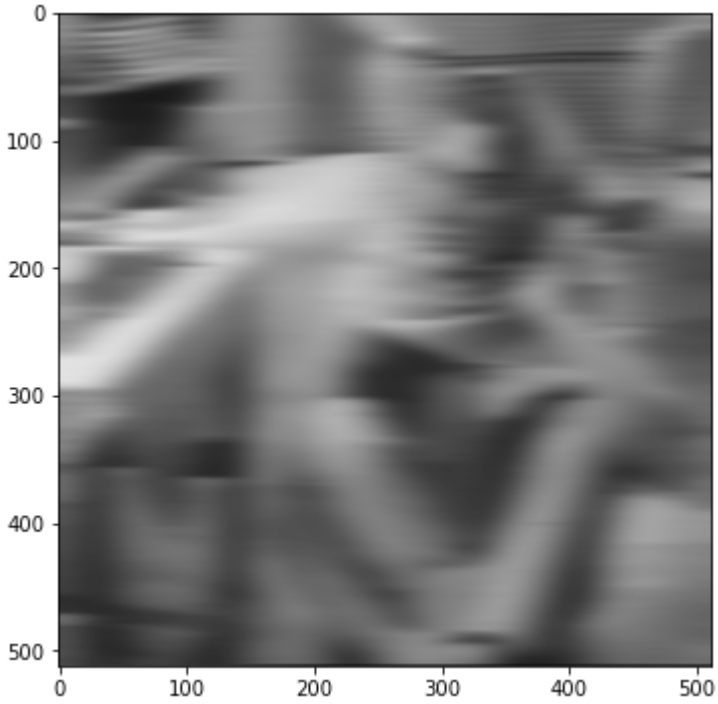
****
[RESULT 04]
****



****
[RESULT 05]
****



****
[RESULT 06]
****




```
**************************************************
## [RESULT 07]
**************************************************
value1 =  0.0047143874650160955
value2 =  -0.008825961742986221
value3 =  -0.11698238602814942
value4 =  -0.039716827843437885
**************************************************
## [RESULT 08]
**************************************************
value1 =  0.025667551616430768
value2 =  0.07162797512137903
value3 =  0.5111424457269547
value4 =  0.5973786011027158
**************************************************
## [RESULT 09]
**************************************************
value1 =  0.022454411720807345
value2 =  0.0696946701006636
value3 =  0.3808823842304887
value4 =  0.5960132897368672
**************************************************
## [RESULT 10]
**************************************************
value1 =  0.02095128223860235
value2 =  0.06828228470749188
value3 =  0.34507570700236034
value4 =  0.6105959951801931
```

In [ ]:

In [ ]: