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ECE522 - HWK4

Similar to the development for the stator voltage equation presented in sections 11 through 14 of s06b, please develop the rotor voltage equation presented in section 15 starting from the following equations.

Starting Rotor Voltage Equations

$$v_{ar} = r_r i_{ar} + \frac{d\lambda_{ar}}{dt}$$

$$v_{ar} = r_r i_{ar} + \frac{d\lambda_{ar,as}}{dt} + \frac{d\lambda_{ar,ar}}{dt}$$

Target Rotor Voltage Equation

$$0 = (\frac{r_r}{s} + jX_{lr})I_r + jX_m(I_r - I_s)$$

Phasor Equations

$$\sqrt{2}V_{rms_r}\cos(\omega_e t) = \frac{r_r}{s}\sqrt{2}I_{rms_r}\cos(\omega_e t - \theta_{vir}) + \omega_e\left(L_{lr} + \frac{3}{2}L_{mr}\right)\sqrt{2}I_{rms_r}\cos\left(\omega_e t - \theta_{vir} + \frac{\pi}{2}\right) + \omega_e\frac{3}{2}L_{rs}\sqrt{2}I_{rms_s}\cos\left(\omega_e t - \theta_{vis} + \frac{\pi}{2} + \theta_{s0}\right)$$
$$V_{rms_r}\cos(\omega_e t) = \frac{r_r}{s}I_{rms_r}\cos(\omega_e t - \theta_{vir}) + \omega_e\left(L_{lr} + \frac{3}{2}L_{mr}\right)I_{rms_r}\cos\left(\omega_e t - \theta_{vir} + \frac{\pi}{2}\right) + \omega_e\frac{3}{2}L_{rs}I_{rms_s}\cos\left(\omega_e t - \theta_{vis} + \frac{\pi}{2} + \theta_{s0}\right)$$

$$V_{rms_r}e^{j\omega_e t} = \frac{r_r}{s}I_{rms_r}e^{j(\omega_e t - \theta_{vir})} + \omega_e\left(L_{lr} + \frac{3}{2}L_{mr}\right)I_{rms_r}e^{j(\omega_e t - \theta_{vir} + \frac{pi}{2})} + \omega_e\frac{3}{2}L_{rs}I_{rms_s}e^{j(\omega_e t - \theta_{vis} + \frac{pi}{2} + \theta_{s0})}$$
$$V_{rms_r}e^{j\omega_e t} = \frac{r_r}{s}I_{rms_r}e^{j(\omega_e t - \theta_{vir})} + \omega_e\left(L_{lr} + \frac{3}{2}L_{mr}\right)I_{rms_r}e^{j(\omega_e t - \theta_{vir})} + \omega_e\frac{3}{2}L_{rs}I_{rms_s}e^{j(\omega_e t - \theta_{vis} + \theta_{s0})}$$
$$V_{rms_r}e^{j0} = \frac{r_r}{s}I_{rms_r}e^{-j\theta_{vir}} + j\omega_e\left(L_{lr} + \frac{3}{2}L_{mr}\right)I_{rms_r}e^{-j\theta_{vir}} + j\omega_e\frac{3}{2}L_{rs}I_{rms_s}e^{-j\theta_{vis}}e^{j\theta_{s0}}$$

Define Phasor Variables

$$\bar{V}_{ar} = V_{rms_r}e^{j0}$$
$$\bar{I}_{ar} = I_{rms_r}e^{-j\theta_{vir}}$$
$$\bar{I}_{as} = I_{rms_s}e^{-j\theta_{vis}}$$

Insert New Variable Notation

$$\bar{V}_{ar} = \frac{r_r}{s}\bar{I}_{ar} + j\omega_e\left(L_{lr} + \frac{3}{2}L_{mr}\right)\bar{I}_{ar} + j\omega_e\frac{3}{2}L_{rs}\bar{I}_{as}e^{j\theta_{s0}}$$

Turns Ratio Transformation

$$\bar{V}_{ar} = \frac{r_r}{s}\bar{I}_{ar} + j\omega_e\left(L_{lr} + \frac{3}{2}kN_{re}^2\right)\bar{I}_{ar} + j\omega_e\frac{3}{2}kN_{re}^2N_{se}\left(\frac{N_{re}e^{j\theta_{s0}}}{N_{se}}\frac{N_{se}}{N_{re}e^{j\theta_{s0}}}\right)\bar{I}_{as}e^{j\theta_{s0}}$$
$$\bar{V}_{ar} = \frac{r_r}{s}\bar{I}_{ar} + j\omega_e\left(L_{lr} + \frac{3}{2}kN_{re}^2\right)\bar{I}_{ar} + j\omega_e\left(\frac{3}{2}kN_{re}^2N_{se}\frac{N_{re}}{N_{se}}\right)\left(\frac{N_{se}}{N_{re}}\bar{I}_{as}e^{j\theta_{s0}}\right)$$
$$\bar{V}_{ar} = \frac{r_r}{s}\bar{I}_{ar} + j\omega_e\left(L_{lr} + L_m\right)\bar{I}_{ar} + j\omega_e L_m\bar{I'}_{as}$$
$$\bar{V}_{ar} = \left(\frac{r_r}{s} + j\omega_e L_{lr}\right)\bar{I}_{ar} + j\omega_e L_m(\bar{I}_{ar} + \bar{I'}_{as})$$

Simplify Notation

$$V_r \equiv \bar{V}_{ar} = 0 \qquad I_r \equiv \bar{I}_{ar} \qquad I_s \equiv -\bar{I'}_{as}$$
$$X_{lr} \equiv \omega_e L_{lr} \qquad X_m \equiv \omega_e L_m \qquad X_{ls} \equiv \omega_e L'_{ls}$$

Simplify Equation

$$V_r = \left(\frac{r_r}{s} + jX_{lr}\right)I_r + jX_m\left(I_r - I_s\right)$$

$$0 = \left(\frac{r_r}{s} + jX_{lr}\right)I_r + jX_m\left(I_r - I_s\right)$$