Joe Stanley

ECE522 - HWK5

Similar to the transformation of the stator voltage and flux linkage equations presented in session 11, please transform the rotor abc voltage and flux linkage equations to a dqn reference frame rotating at θ_{rf}.

Define the Formulas to Find the Desired Equations

$$\begin{aligned} v_{dqn_s} &= \bar{T}_s \cdot \bar{V}_{abc_s} \\ &= \bar{T}_s \cdot r_s \cdot \bar{I}_{abc_s} + \bar{T}_s \cdot \rho\bar{\lambda}_{abc_s} \end{aligned}$$

$$\begin{aligned} v_{dqn_r} &= \bar{T}_r \cdot \bar{V}_{abc_r} \\ &= \bar{T}_r \cdot r_r \cdot \bar{I}_{abc_r} + \bar{T}_r \cdot \rho\bar{\lambda}_{abc_r} \end{aligned}$$

For Clarity, Define the $\bar{T}_r$ Matrix and its Inverse; Also Define $\overline{\omega_{rfmr}x}$

$$\bar{T}_r(\theta_{rf}-\theta_r) = \frac{2}{3}\begin{bmatrix} \cos(\theta_{rf}-\theta_r) & \cos(\theta_{rf}-\theta_r-\frac{2\pi}{3}) & \cos(\theta_{rf}-\theta_r+\frac{2\pi}{3}) \\ -\sin(\theta_{rf}-\theta_r) & -\sin(\theta_{rf}-\theta_r-\frac{2\pi}{3}) & -\sin(\theta_{rf}-\theta_r+\frac{2\pi}{3}) \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$\bar{T}_r^{-1}(\theta_{rf}-\theta_r) = \begin{bmatrix} \cos(\theta_{rf}-\theta_r) & -\sin(\theta_{rf}-\theta_r) & \frac{1}{\sqrt{2}} \\ \cos(\theta_{rf}-\theta_r-\frac{2\pi}{3}) & -\sin(\theta_{rf}-\theta_r-\frac{2\pi}{3}) & \frac{1}{\sqrt{2}} \\ \cos(\theta_{rf}-\theta_r+\frac{2\pi}{3}) & -\sin(\theta_{rf}-\theta_r+\frac{2\pi}{3}) & \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$\overline{\omega_{rfmr}x} = \bar{T}_r(\theta_{rf}-\theta_r)\cdot\frac{d}{dt}\left(\bar{T}_r^{-1}(\theta_{rf}-\theta_r)\right) = \begin{bmatrix} 0 & -\omega_{rfmr} & 0 \\ \omega_{rfmr} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\omega_{rfmr} = \frac{d(\theta_{rf}-\theta_r)}{dt}$$

Replace 'abc' Terms with 'dqn' Terms Where Applicable

$$\begin{aligned} v_{dqn_s} &= r_r \cdot \bar{T}_r \cdot \bar{I}_{abc_r} + \bar{T}_r \cdot \rho\left(\bar{T}_r^{-1}\cdot\bar{T}_r\cdot\bar{\lambda}_{abc_r}\right) \\ &= r_r \cdot \bar{I}_{dqn_r} + \bar{T}_r\cdot\rho\left(\bar{T}_r^{-1}\cdot\bar{\lambda}_{dqn_r}\right) \\ &= r_r\cdot\bar{I}_{dqn_r} + \bar{T}_r\cdot\rho\cdot\bar{T}_r^{-1}\cdot\bar{\lambda}_{dqn_r} + \frac{\bar{T}_r\bar{T}_r^{-1}}{I}\cdot\rho\bar{\lambda}_{dqn_r} \\ &= r_r\cdot\bar{I}_{dqn_r} + \rho\cdot\bar{\lambda}_{dqn_r} + \overline{\omega_{rfmr}x}\cdot\bar{\lambda}_{dqn_r} \end{aligned}$$

Now, Break out the Constituent Components into Useful Formulas

$$\begin{aligned} v_{d_r} &= r_r\cdot i_{d_r} + \rho\lambda_{d_r} - \omega_{rfmr}\cdot\lambda_{q_r} \\ v_{q_r} &= r_r\cdot i_{q_r} + \rho\lambda_{q_r} - \omega_{rfmr}\cdot\lambda_{d_r} \\ v_{n_r} &= r_r\cdot i_{n_r} + \rho\lambda_{n_r} \end{aligned}$$

Finally, Substitute ω_{rfmr} to Simplify

$$\begin{aligned} v_{d_r} &= r_r\cdot i_{d_r} + \rho\lambda_{d_r} - (\omega_{rf}-\omega_r)\cdot\lambda_{q_r} \\ v_{q_r} &= r_r\cdot i_{q_r} + \rho\lambda_{q_r} - (\omega_{rf}-\omega_r)\cdot\lambda_{d_r} \\ v_{n_r} &= r_r\cdot i_{n_r} + \rho\lambda_{n_r} \end{aligned}$$