ECE 523: Homework #2

Solution

Define Units and Transformations:

$MVA := 1000kW \qquad pu := 1$

Phase A symmetrical components transform — Phase B symmetrical components transform — Phase C symmetrical components transform

$$a := e^{j\cdot 120deg} \qquad A_{012} := \begin{pmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{pmatrix} \qquad B_{012} := \begin{pmatrix} 1 & a & a^2 \\ 1 & 1 & 1 \\ 1 & a^2 & a \end{pmatrix} \qquad C_{012} := \begin{pmatrix} 1 & a^2 & a \\ 1 & a & a^2 \\ 1 & 1 & 1 \end{pmatrix}$$

1. Determine the symmetrical components for the following currents and sketch a phasor diagram, based on

(a) Phase "a" referenced components
(b) Phase "b" referenced components
(c) Phase "c" referenced components

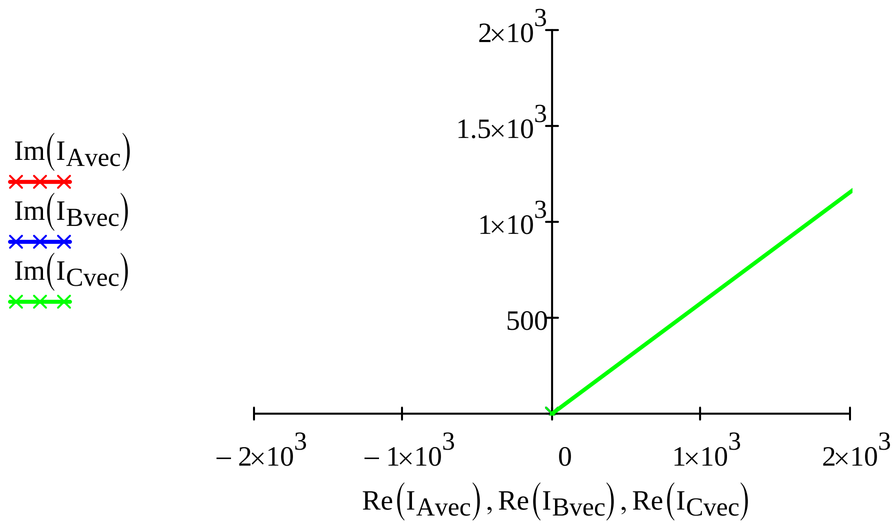
$$I_A := 0A\cdot e^{0deg}$$

$$I_B := 0A\cdot e^{-j\cdot 120deg}$$

$$I_C := 2500A\cdot e^{j\cdot 30deg}$$

$$I_{Avec} := \begin{pmatrix} 0 \\ I_A \end{pmatrix} \qquad I_{Bvec} := \begin{pmatrix} 0 \\ I_B \end{pmatrix} \qquad I_{Cvec} := \begin{pmatrix} 0 \\ I_C \end{pmatrix}$$

- Initial Phasor Diagram



(a) Phase "a" referenced components

$$\begin{pmatrix} I_{A0} \\ I_{A1} \\ I_{A2} \end{pmatrix} := A_{012}^{-1} \cdot \begin{pmatrix} I_A \\ I_B \\ I_C \end{pmatrix}$$

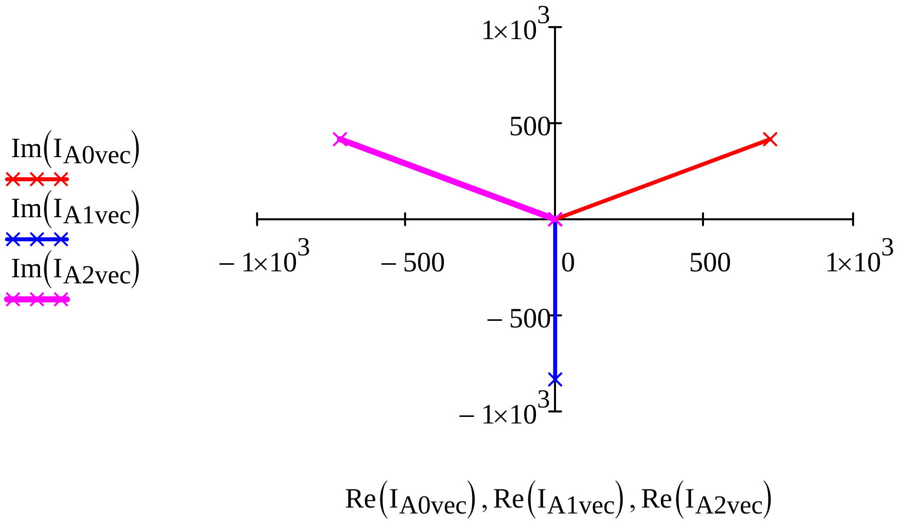
$\left|I_{A0}\right| = 833.333\,A$ $\qquad$ $\arg(I_{A0}) = 30\cdot deg$

$\left|I_{A1}\right| = 833.333\,A$ $\qquad$ $\arg(I_{A1}) = -90\cdot deg$

$\left|I_{A2}\right| = 833.333\,A$ $\qquad$ $\arg(I_{A2}) = 150\cdot deg$

- Phase A symmetrical components phasor diagram

$$I_{A0vec} := \begin{pmatrix} 0 \\ I_{A0} \end{pmatrix} \qquad I_{A1vec} := \begin{pmatrix} 0 \\ I_{A1} \end{pmatrix} \qquad I_{A2vec} := \begin{pmatrix} 0 \\ I_{A2} \end{pmatrix}$$



- Note that the phase A components sum to zero.

(b) Phase "b" referenced components

$$\begin{pmatrix} I_{B0} \\ I_{B1} \\ I_{B2} \end{pmatrix} := B_{012}^{-1} \cdot \begin{pmatrix} I_A \\ I_B \\ I_C \end{pmatrix}$$

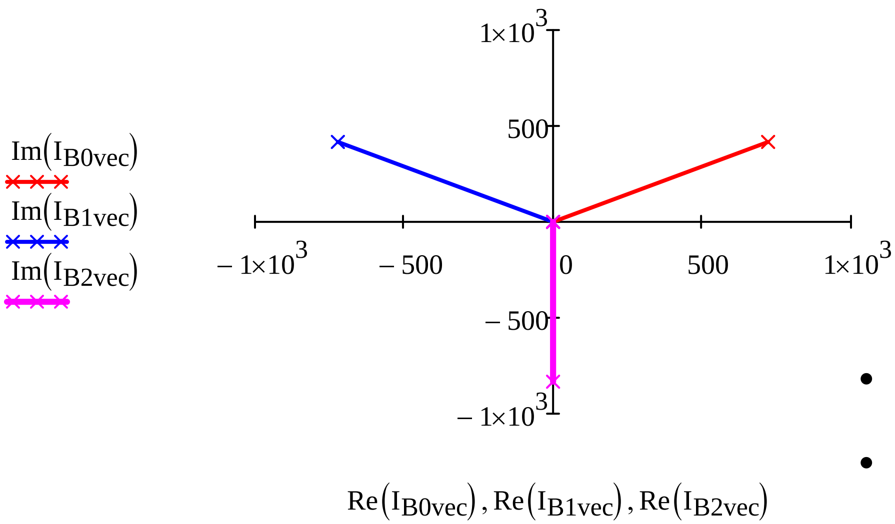
$\left| I_{B0} \right| = 833.333\,A$ $\qquad$ $\arg(I_{B0}) = 30 \cdot deg$ $\qquad$ same as IA0

$\left|I_{B1}\right| = 833.333\,A$ $\qquad$ $\arg(I_{B1}) = 150\cdot deg$

$\left|I_{B2}\right| = 833.333\,A$ $\qquad$ $\arg(I_{B2}) = -90\cdot deg$

- Phase B symmetrical components phasor diagram

$$I_{B0vec} := \begin{pmatrix} 0 \\ I_{B0} \end{pmatrix} \qquad I_{B1vec} := \begin{pmatrix} 0 \\ I_{B1} \end{pmatrix} \qquad I_{B2vec} := \begin{pmatrix} 0 \\ I_{B2} \end{pmatrix}$$



- Note that the phase B components sum to zero.
- And $I_{B0} = I_{A0}$

(c) Phase "c" referenced components $\begin{pmatrix} I_{C0} \\ I_{C1} \\ I_{C2} \end{pmatrix} := C_{012}^{-1}\cdot\begin{pmatrix} I_A \\ I_B \\ I_C \end{pmatrix}$

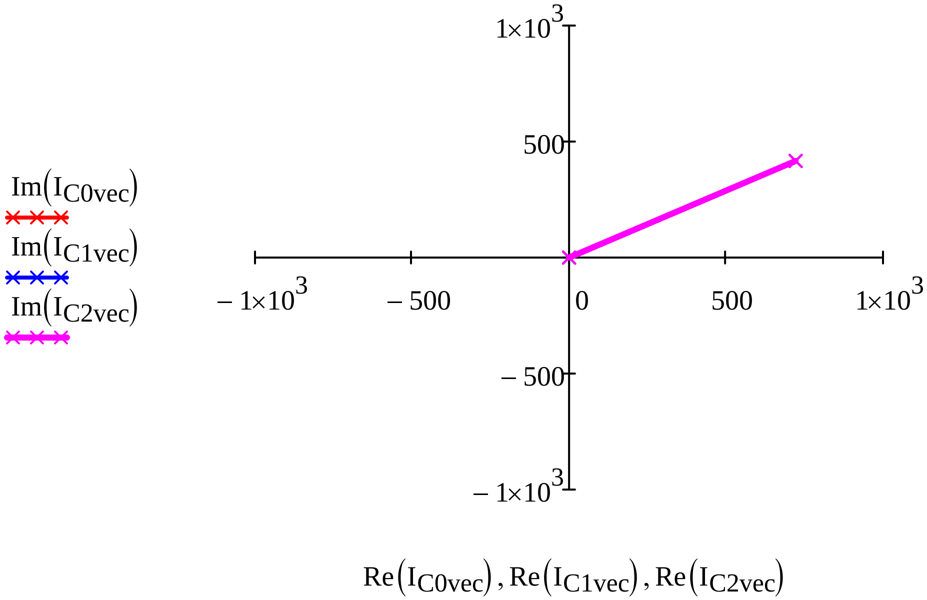
$\left|I_{C0}\right| = 833.333\,A$ $\qquad$ $\arg\left(I_{C0}\right) = 30\cdot deg$ $\qquad$ same as IA0

$\left|I_{C1}\right| = 833.333\,A$ $\qquad$ $\arg\left(I_{C1}\right) = 30\cdot deg$

$\left|I_{C2}\right| = 833.333\,A$ $\qquad$ $\arg\left(I_{C2}\right) = 30\cdot deg$

- Phase C symmetrical components phasor diagram

$$I_{C0vec} := \begin{pmatrix} 0 \\ I_{C0} \end{pmatrix} \qquad I_{C1vec} := \begin{pmatrix} 0 \\ I_{C1} \end{pmatrix} \qquad I_{C2vec} := \begin{pmatrix} 0 \\ I_{C2} \end{pmatrix}$$



- Phase C components do not sum to 0
- Again $I_{C0}=I_{A0}$

2. Repeat problem 1. with the following currents:

$$I_A := 4500A\, e^{-j\cdot 25.84deg}$$

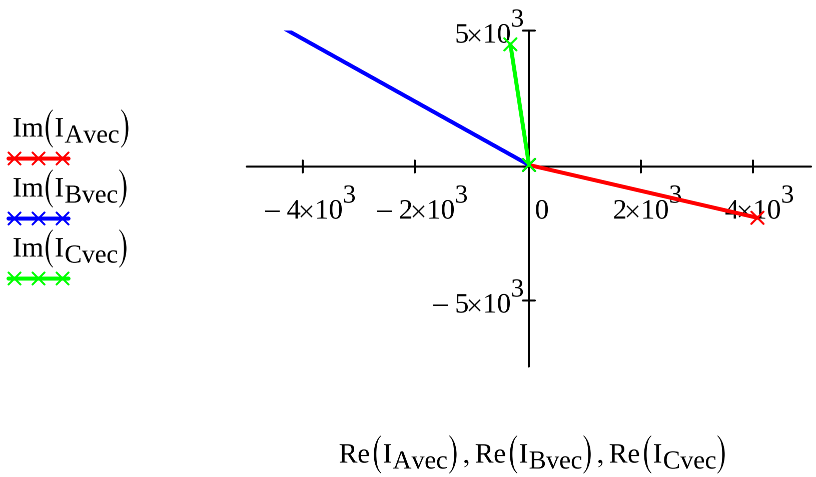
$$I_B := 8503A\cdot e^{-j\cdot 229.5deg}$$

$$I_C := 4500A\cdot e^{j\cdot 94.16deg}$$

Now have significant load current on top of a fault current

- Initial Phasor Diagram

$$I_{Avec} := \begin{pmatrix} 0 \\ I_A \end{pmatrix} \qquad I_{Bvec} := \begin{pmatrix} 0 \\ I_B \end{pmatrix} \qquad I_{Cvec} := \begin{pmatrix} 0 \\ I_C \end{pmatrix}$$



(a) Phase "a" referenced components

$$\begin{pmatrix} I_{A0} \\ I_{A1} \\ I_{A2} \end{pmatrix} := A_{012}^{-1} \cdot \begin{pmatrix} I_A \\ I_B \\ I_C \end{pmatrix}$$

$\left| I_{A0} \right| = 3056.873\,A$ $\quad \arg\left(I_{A0}\right) = 101.311 \cdot deg$

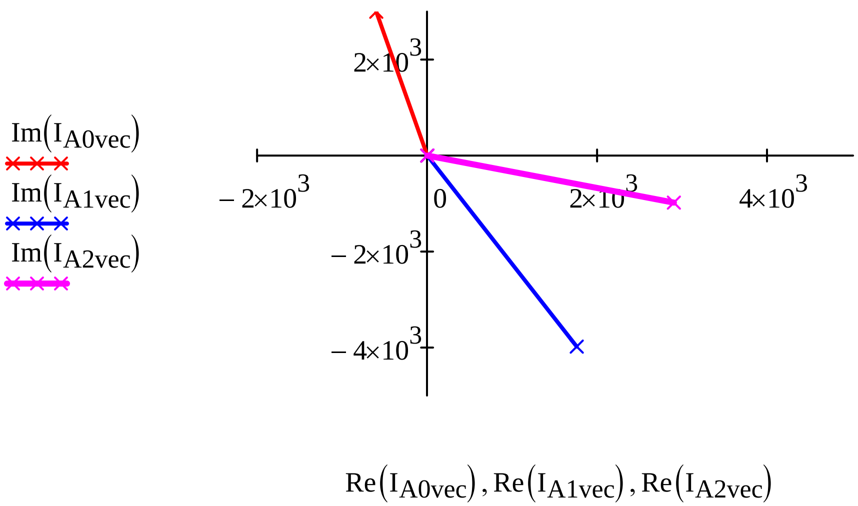
$\left| I_{A1} \right| = 4348.723\,A$ $\quad \arg\left(I_{A1}\right) = -66.214 \cdot deg$

$\left| I_{A2} \right| = 3056.873\,A$ $\quad \arg\left(I_{A2}\right) = -18.689 \cdot deg$

- Note that $|I_{A2}| = |I_{A0}|$, but angles differ by 120 degrees
- I_{A1} differs since it includes load current

- Phase A symmetrical components phasor diagram

$$I_{A0vec} := \begin{pmatrix} 0 \\ I_{A0} \end{pmatrix} \qquad I_{A1vec} := \begin{pmatrix} 0 \\ I_{A1} \end{pmatrix} \qquad I_{A2vec} := \begin{pmatrix} 0 \\ I_{A2} \end{pmatrix}$$



Now I_{A1} is very different from aligning with having a balanced 3 phase set due to the load current

(b) Phase "b" referenced components $\begin{pmatrix} I_{B0} \\ I_{B1} \\ I_{B2} \end{pmatrix} := B_{012}{}^{-1} \cdot \begin{pmatrix} I_A \\ I_B \\ I_C \end{pmatrix}$

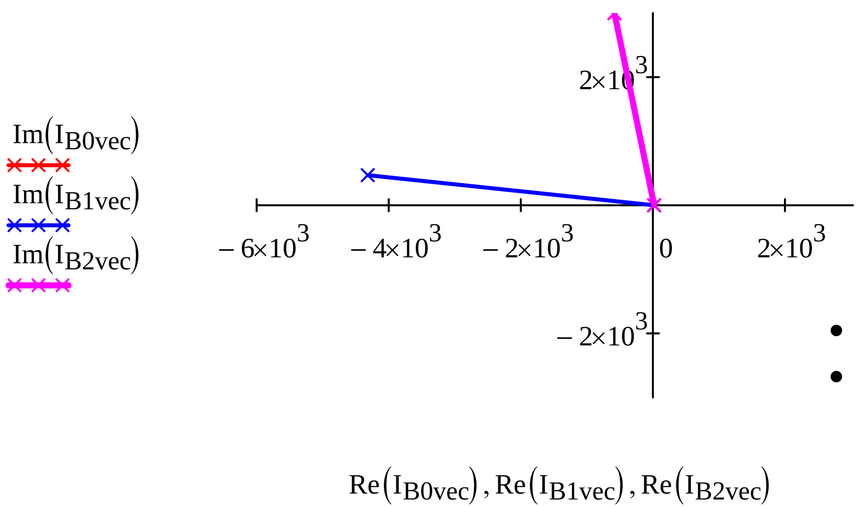
$\left| I_{B0} \right| = 3056.873\,A$ $\quad$ $\arg\left(I_{B0}\right) = 101.311 \cdot deg$ $\quad$ same as I_{A0}

$\left| I_{B1} \right| = 4348.723\,A$ $\quad$ $\arg\left(I_{B1}\right) = 173.786 \cdot deg$

$\left| I_{B2} \right| = 3056.873\,A$ $\quad$ $\arg\left(I_{B2}\right) = 101.311 \cdot deg$ $\quad$ Now $I_{B0} = I_{B2}$, shows BG fault

- Phase B symmetrical components phasor diagram

$I_{B0vec} := \begin{pmatrix} 0 \\ I_{B0} \end{pmatrix}$ $\qquad$ $I_{B1vec} := \begin{pmatrix} 0 \\ I_{B1} \end{pmatrix}$ $\qquad$ $I_{B2vec} := \begin{pmatrix} 0 \\ I_{B2} \end{pmatrix}$



- I_{B2} and I_{B0} equal to each other
- Note that I_{B1} not equal to the other two.

(c) Phase "c" referenced components $\begin{pmatrix} I_{C0} \\ I_{C1} \\ I_{C2} \end{pmatrix} := C_{012}^{-1} \cdot \begin{pmatrix} I_A \\ I_B \\ I_C \end{pmatrix}$

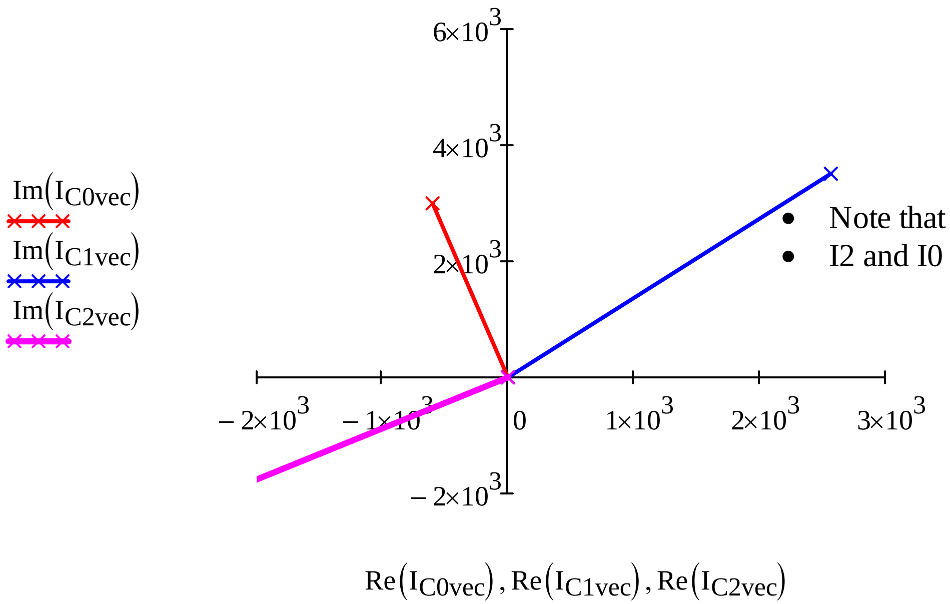
$\boxed{|I_{C0}| = 3056.873\,A}$ $\boxed{\arg(I_{C0}) = 101.311 \cdot deg}$ same as I_{A0}

$\boxed{|I_{C1}| = 4348.723\,A}$ $\boxed{\arg(I_{C1}) = 53.786 \cdot deg}$

$\boxed{|I_{C2}| = 3056.873\,A}$ $\boxed{\arg(I_{C2}) = -138.689 \cdot deg}$ Note that $|I_{C2}| = |I_{C0}|$, but angles differ by 120 degrees

- Phase C symmetrical components phasor diagram

$I_{C0vec} := \begin{pmatrix} 0 \\ I_{C0} \end{pmatrix}$ $I_{C1vec} := \begin{pmatrix} 0 \\ I_{C1} \end{pmatrix}$ $I_{C2vec} := \begin{pmatrix} 0 \\ I_{C2} \end{pmatrix}$



- Note that the phase C compoents no longer sum to zero.
- I2 and I0 offset by 120 degrees

3. Determine the phase voltage given the following phase "a" referenced symmetrical components. Repeat assuming they are instead phase "b" and then phase "c" referenced symmetrical components

$pu := 1$

$$V_0 := 0.274pu \cdot e^{-j \cdot 90deg}$$

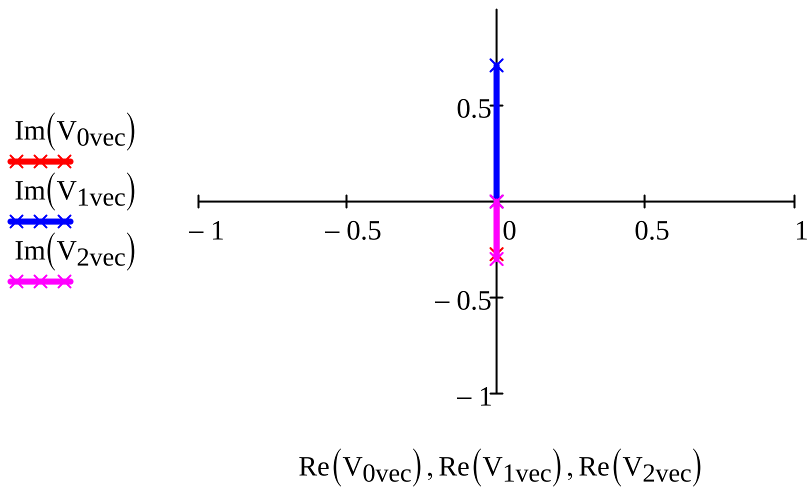
$$V_1 := 0.709pu \cdot e^{j \cdot 90deg}$$

$$V_2 := 0.299pu \cdot e^{-j \cdot 90deg}$$

$$V_{0vec} := \begin{pmatrix} 0 \\ V_0 \end{pmatrix} \qquad V_{1vec} := \begin{pmatrix} 0 \\ V_1 \end{pmatrix} \qquad V_{2vec} := \begin{pmatrix} 0 \\ V_2 \end{pmatrix}$$

- Initial Phasor Diagram

Note: phasor diagrams not required for this problem, but are included for illustration.



(a) Phase "a" referenced components

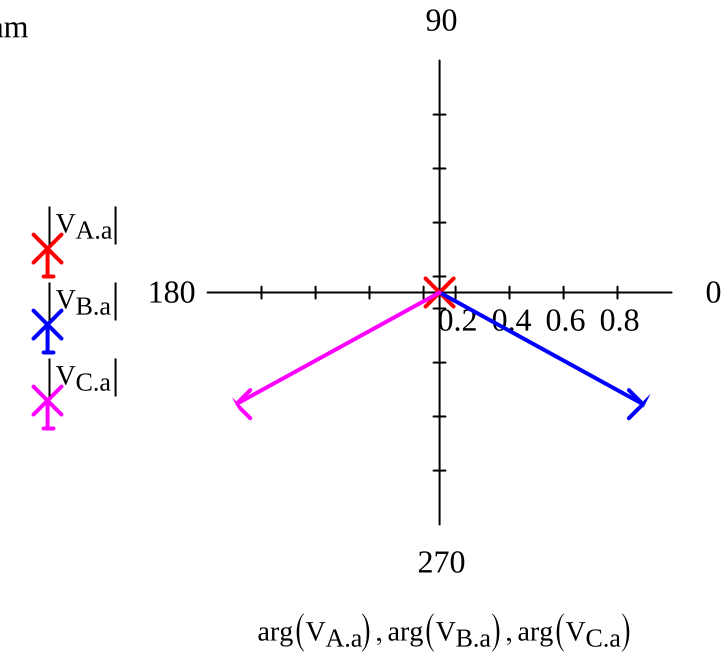
$$\begin{pmatrix} V_{A.a} \\ V_{B.a} \\ V_{C.a} \end{pmatrix} := A_{012} \cdot \begin{pmatrix} V_0 \\ V_1 \\ V_2 \end{pmatrix}$$

$\left|V_{A.a}\right| = 0.136$ $\quad$ $\arg\left(V_{A.a}\right) = 90 \cdot deg$

$\left|V_{B.a}\right| = 0.996$ $\quad$ $\arg\left(V_{B.a}\right) = -28.754 \cdot deg$

$\left|V_{C.a}\right| = 0.996$ $\quad$ $\arg\left(V_{C.a}\right) = -151.246 \cdot deg$

- Part (a) phasor diagram



(b) Phase "b" referenced components

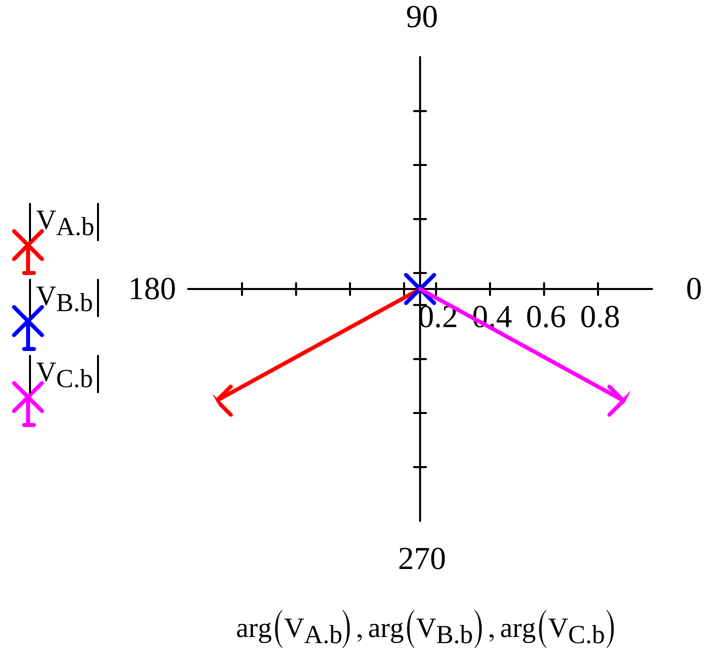
$$\begin{pmatrix} V_{A.b} \\ V_{B.b} \\ V_{C.b} \end{pmatrix} := B_{012} \cdot \begin{pmatrix} V_0 \\ V_1 \\ V_2 \end{pmatrix}$$

$\left|V_{A.b}\right| = 0.996$ $\quad$ $\arg\left(V_{A.b}\right) = -151.246 \cdot deg$

$\left|V_{B.b}\right| = 0.136$ $\quad$ $\arg\left(V_{B.b}\right) = 90 \cdot deg$

$\left|V_{C.b}\right| = 0.996$ $\quad$ $\arg\left(V_{C.b}\right) = -28.754 \cdot deg$ $\quad$ Just rotates the phasors...

- Part (b) phasor diagram



(c) Phase "c" referenced components

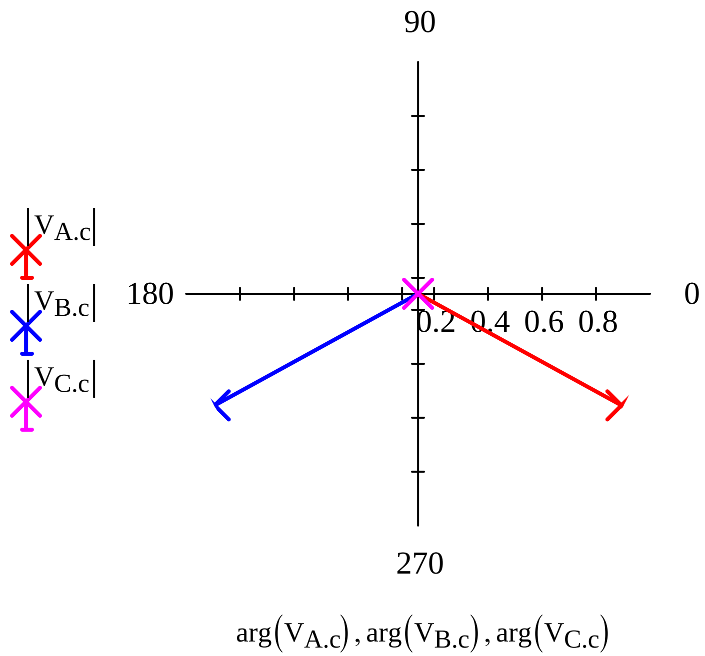
$$\begin{pmatrix} V_{A.c} \\ V_{B.c} \\ V_{C.c} \end{pmatrix} := C_{012} \cdot \begin{pmatrix} V_0 \\ V_1 \\ V_2 \end{pmatrix}$$

$\left|V_{A.c}\right| = 0.996$ $\quad\quad$ $\arg\left(V_{A.c}\right) = -28.754 \cdot deg$

$\left|V_{B.c}\right| = 0.996$ $\quad\quad$ $\arg\left(V_{B.c}\right) = -151.246 \cdot deg$ $\quad\quad$ Just rotates the phasors...

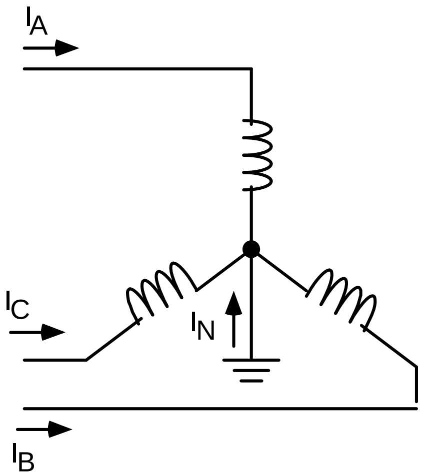
$\left|V_{C.c}\right| = 0.136$ $\quad\quad$ $\arg\left(V_{C.c}\right) = 90 \cdot deg$

- Part (c) phasor diagram



3. Problem 2.10 in Anderson:
   If the load is unbalanced, neutral current will exist. Find the relationship between the neutral current I_n and the phase a zero sequence current I_{a0}

- Redraw the figure:



Using Kirchhoff's Current Law:

$$In = -(Ia + Ib + Ic)$$

Using the Symmetrical Components Transformation

$$Ia0 = \frac{1}{3}\cdot(Ia + Ib + Ic)$$

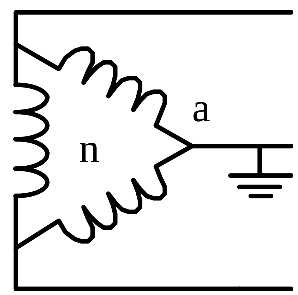
Therefore $$\boxed{Ia0 = \frac{-1}{3}\cdot In}$$

4. Problem 2.13 in Anderson

The ungrounded system below has a phase to ground fault on phase "a". Assume that the line to ground (and line to neutral voltages) were balanced three phase set before the fault occured. Do the following:

(a) Find the symmetrical components of the phase a line-to-neutral voltages when a ground fault is applied (V_{an0}, V_{an1}, V_{an}).

(b) Repeat part (a) using line to ground voltages instead of the line to neutral voltages and find (V_{ag0}, V_{ag1}, V_{ag2}).



(a) Find the symmetrical components of the phase a line-to-neutral voltages when a ground fault is applied (V_{an0}, V_{an1}, V_{an}).

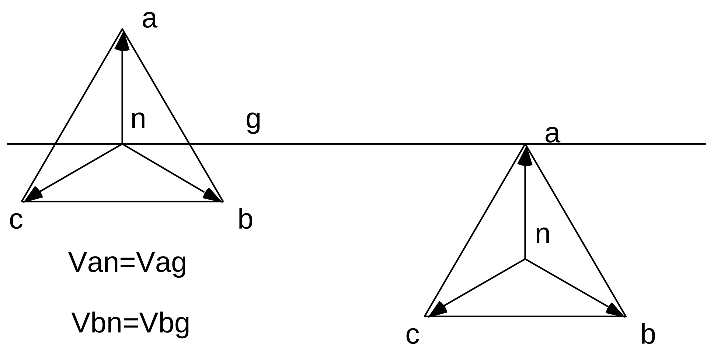
- Note that the problem statement is asking for the phase to neutral voltages, not the phase to ground voltages.
- The line to line voltages are a balanced three phase set, even with the ground fault present.

$$Vab = Van - Vbn$$

$$Vbc = Vbn - Vcn$$

$$Vca = Vcn - Van \qquad Vab + Vbc + Vca = 0$$

The line to neutral voltages also form a balanced three phase set, since the neutral point will shift with them (since its free to move).



Van=Vag

Vbn=Vbg

Vcn=Vcg

Vag=0

Vng= -Van

Vbg= - Vab

Vcg= Vca

So calculating the symmetrical components:

$$V0 = \frac{1}{3}(Van + Vbn + Vcn) = 0$$

$$V1 = \frac{1}{3}\left(Van + a\cdot Vbn + a^2\cdot Vcn\right) = \left(\frac{3}{3}\cdot Van = Van\right)$$

$$V2 = \frac{1}{3}\left(Van + a^2 Vbn + a\cdot Vcn\right) = 0$$

As a check, set up a per unit case:

$$Van := 1V\cdot e^{j\cdot 0}$$

$$Vbn := 1V\cdot e^{-j\cdot 120deg}$$

$$Vcn := 1V\cdot e^{j\cdot 120deg}$$

$$V_{012} := A_{012}^{-1}\cdot\begin{pmatrix}Van\\Vbn\\Vcn\end{pmatrix} \qquad \boxed{V_{012} = \begin{pmatrix}0\\1\\0\end{pmatrix}V}$$

(b) Repeat part (a) using line to ground voltages instead of the line to neutral voltages and find (V_{ag0}, V_{ag1}, V_{ag2}).

$$Vng := -Van$$

$$Vag := Van + Vng$$

$$Vbg := Vbn + Vng$$

$$Vcg := Vcn + Vng$$

So calculating the symmetrical components:

$$V0 = \frac{1}{3}(Vag + Vbg + Vcg) = (Van + Vbn + Vcn + 3Vng) = Vng$$

$$V1 = \frac{1}{3}\left(Vag + a\cdot Vbg + a^2\cdot Vcg\right) = \frac{1}{3}\left[Van + a\cdot Vbn + a^2\cdot Vcn + Vng\cdot\left(1 + a + a^2\right)\right] = \frac{3}{3}\cdot Van = Van$$

$$V2 = \frac{1}{3}\left(Vag + a^2 Vbg + a\cdot Vcg\right) = \frac{1}{3}\left[Van + a^2\cdot Vbn + a\cdot Vcn + Vng\cdot\left(1 + a^2 + a\right)\right] = 0$$

Again, as a check set up the per unit case:

As a check, set up a per unit case:

$$Van := 1V\cdot e^{j\cdot 0}$$

$$Vbn := 1V\cdot e^{-j\cdot 120deg}$$

$$Vcn := 1V\cdot e^{j\cdot 120deg}$$

$Vng := -Van \qquad Vng = -1\,V$

$Vag := Van + Vng \qquad Vag = 0\,V$

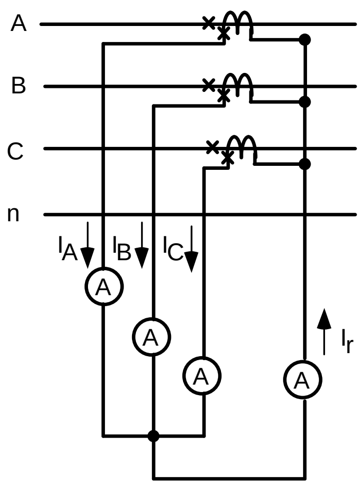
$Vbg := Vbn + Vng \qquad Vbg = (-1.5 - 0.866i)\,V$

$Vcg := Vcn + Vng \qquad Vcg = (-1.5 + 0.866i)\,V$

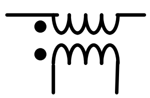
$$V_{012\_ag} := A_{012}^{-1} \cdot \begin{pmatrix} Vag \\ Vbg \\ Vcg \end{pmatrix} \qquad \boxed{V_{012\_ag} = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} V}$$

6. Do the following (modified version of problem 2.12)

(a) A set of current transformers reads the following currents (in Amperes). If the current transformers each have a turns ratio of 5:500 (usually referred to as a current transformation ratio or CTR of 500:5) calculate the primary currents in amps.



Note that the symbol: 

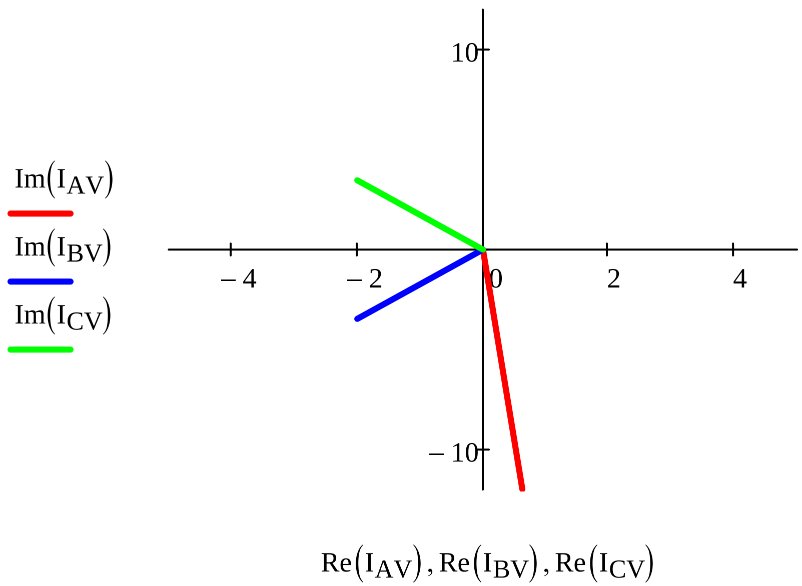
is equivalent to: 

$$I_A := 12e^{-j\cdot 87deg}$$

$$I_B := 4\cdot e^{-j\cdot 120deg}$$

$$I_C := 4\cdot e^{j\cdot 120deg}$$

$$I_{AV} := \begin{pmatrix} I_A \\ 0 \end{pmatrix} \quad I_{BV} := \begin{pmatrix} I_B \\ 0 \end{pmatrix} \quad I_{CV} := \begin{pmatrix} I_C \\ 0 \end{pmatrix}$$



$$CTR := \frac{500}{5}$$

- Based on the polarity marks on the CT's:

$I_a := I_A \cdot CTR$ $\quad \left|I_a\right| = 1200$ $\quad \arg(I_a) = -87 \cdot deg$

$I_b := I_B \cdot CTR$ $\quad \left|I_b\right| = 400$ $\quad \arg(I_b) = -120 \cdot deg$

$I_c := I_C \cdot CTR$ $\quad \left|I_c\right| = 400$ $\quad \arg(I_c) = 120 \cdot deg$

(b) Calculate the symmtrical components of the secondary currents (I_{a0}, I_{a1}, I_{a2}).

$$I_{012\_secondary} := A_{012}^{-1} \cdot \begin{pmatrix} I_A \\ I_B \\ I_C \end{pmatrix}$$

$I_{A0} := I_{012\_secondary_0}$ $\quad \left|I_{A0}\right| = 4.15$ $\quad \arg(I_{A0}) = -105.716 \cdot deg$

$I_{A1} := I_{012\_secondary_1}$ $\quad \left|I_{A1}\right| = 4.922$ $\quad \arg(I_{A1}) = -54.247 \cdot deg$

$I_{A2} := I_{012\_secondary_2}$ $\quad \left|I_{A2}\right| = 4.15$ $\quad \arg(I_{A2}) = -105.716 \cdot deg$

(c) Calculate the current measured by the fourth ammeter (I_r) and compare it to the zero sequence current calculated in part (b). How do they compare?

- Doing a KCL node equation where the currents meet:

$$I_r := I_A + I_B + I_C \qquad \boxed{\left|I_r\right| = 12.449} \qquad \boxed{\arg(I_r) = -105.716 \cdot deg}$$

From above:

$$\left|I_{A0}\right| = 4.15 \qquad \arg(I_{A0}) = -105.716 \cdot deg$$

$$\boxed{I_r - 3 \cdot I_{A0} = 0} \qquad \boxed{\text{So } 3I0 = Ir}$$

(d) Using the primary current calculated in part (a), repeat part (b) if the CTs are connected in delta

- First, find the secondary currents on the CTs inside the delta

$$I_{AB\_sec\_delta} := \frac{I_a}{CTR}$$

$$I_{BC\_sec\_delta} := \frac{I_b}{CTR}$$

$$I_{CA\_sec\_delta} := \frac{I_c}{CTR}$$

- Now we need convert these to the line currents that would actually go to the relays.
- The easiest approach is to do conversion in the symmetrical components domain.

$$\begin{pmatrix} I_{A0\_D} \\ I_{A1\_D} \\ I_{A2\_D} \end{pmatrix} := A_{012}^{-1} \cdot \begin{pmatrix} I_{AB\_sec\_delta} \\ I_{BC\_sec\_delta} \\ I_{CA\_sec\_delta} \end{pmatrix}$$

$$\left|I_{A1\_D}\right| = 4.922 \qquad \arg\left(I_{A1\_D}\right) = -54.247\cdot deg$$

$$\left|I_{A2\_D}\right| = 4.15 \qquad \arg\left(I_{A2\_D}\right) = -105.716\cdot deg$$

$$\left|I_{A0\_D}\right| = 4.15 \qquad \arg\left(I_{A0\_D}\right) = -105.716\cdot deg$$

- Now find line currents (secondary is treated as the low side)

$$I_{A1sec\_Y} := e^{-j\cdot 30deg}\cdot I_{A1\_D} \qquad \boxed{\left|I_{A1sec\_Y}\right| = 4.922} \qquad \boxed{\arg\left(I_{A1sec\_Y}\right) = -84.247\cdot deg}$$

$$I_{A2sec\_Y} := e^{j\cdot 30deg}\cdot I_{A2\_D} \qquad \boxed{\left|I_{A2sec\_Y}\right| = 4.15} \qquad \boxed{\arg\left(I_{A2sec\_Y}\right) = -75.716\cdot deg}$$

$$\boxed{I_{A0sec\_Y} := 0}$$

$$\begin{pmatrix} I_{Asec\_relay} \\ I_{Bsec\_relay} \\ I_{Csec\_relay} \end{pmatrix} := A_{012}\cdot\begin{pmatrix} I_{A0sec\_Y} \\ I_{A1sec\_Y} \\ I_{A2sec\_Y} \end{pmatrix}$$

$$\boxed{\left|I_{Asec\_relay}\right| = 9.047} \qquad \boxed{\arg\left(I_{Asec\_relay}\right) = -80.345\cdot deg}$$

$$\boxed{\left|I_{Bsec\_relay}\right| = 5.147} \qquad \boxed{\arg\left(I_{Bsec\_relay}\right) = 107.144\cdot deg}$$

$$\boxed{\left|I_{Csec\_relay}\right| = 4} \qquad \boxed{\arg\left(I_{Csec\_relay}\right) = 90\cdot deg}$$

Recall that with the Y connected CTs we saw:

$$I_A := 12e^{-j\cdot 87deg}$$

$$I_B := 4\cdot e^{-j\cdot 120deg}$$

$$I_C := 4\cdot e^{j\cdot 120deg}$$