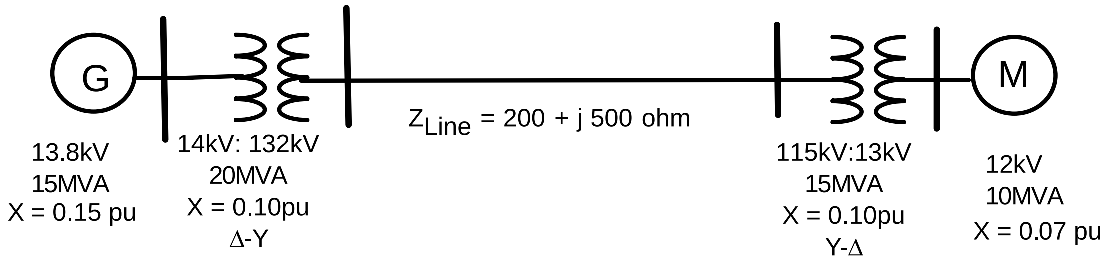
ECE 523: Homework #1

Solution

$MVA := 1000kW \qquad MVAR := MVA \qquad pu := 1$

1. Problem 1.1 in the text book

$S_b := 10MVA$



Divide the system into 3 zones. Zone I is the generator, Zone II is the transmission line, and Zone III is the motor. The transformers are the boundaries between zones.

Zone II Calculations:

$V_{b2} := 100kV$

$$Z_{b2} := \frac{V_{b2}^2}{S_b} \qquad Z_{b2} = 1000\,\Omega$$

$$Z_{line} := 200ohm + j \cdot 500ohm$$

$$Z_{line\_pu} := \frac{Z_{line}}{Z_{b2}} \qquad \boxed{Z_{line\_pu} = (0.2 + 0.5i) \cdot pu}$$

Zone I Calculations

$$V_{b1} := V_{b2} \cdot \left(\frac{14kV}{132kV}\right) \qquad V_{b1} = 10.61 \cdot kV$$

- Change of base calculation on the generator reactance:

$V_{gen\_rated} := 13.8kV \qquad S_{gen\_rated} := 15MVA$

$$X_{gen\_pu\_new} := 0.15pu\left(\frac{V_{gen\_rated}}{V_{b1}}\right)^2 \cdot \left(\frac{S_b}{S_{gen\_rated}}\right)$$

$$\boxed{X_{gen\_pu\_new} = 0.1693 \cdot pu}$$

- Transformer T1 change of base:

$V_{low\_T1\_rated} := 14kV \qquad S_{T1\_rated} := 20MVA$

$$X_{T1\_pu\_new} := 0.10pu\left(\frac{V_{low\_T1\_rated}}{V_{b1}}\right)^2 \cdot \left(\frac{S_b}{S_{T1\_rated}}\right)$$

$$\boxed{X_{T1\_pu\_new} = 0.0871 \cdot pu}$$

<u>Zone III Calculations:</u>

$$V_{b3} := V_{b2} \cdot \left(\frac{13kV}{115kV}\right) \qquad V_{b3} = 11.3 \cdot kV$$

- Transformer T2 change of base:

$V_{low\_T2\_rated} := 13kV \qquad S_{T2\_rated} := 15MVA$

$$X_{T2\_pu\_new} := 0.10pu\left(\frac{V_{low\_T2\_rated}}{V_{b3}}\right)^2 \cdot \left(\frac{S_b}{S_{T2\_rated}}\right)$$

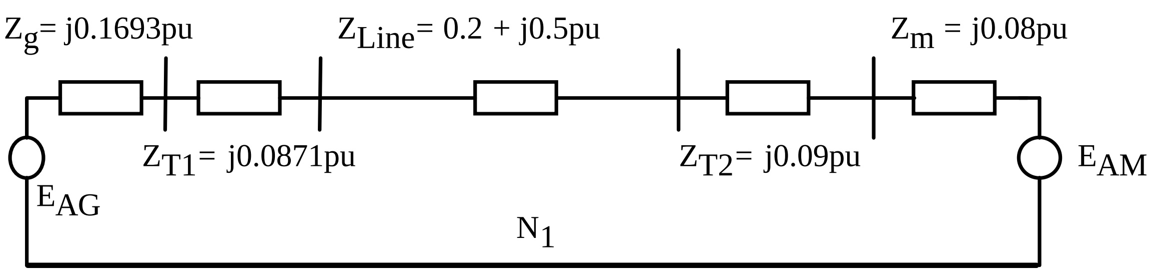
$$\boxed{X_{T2\_pu\_new} = 0.09 \cdot pu}$$

- Motor impedance change of base:

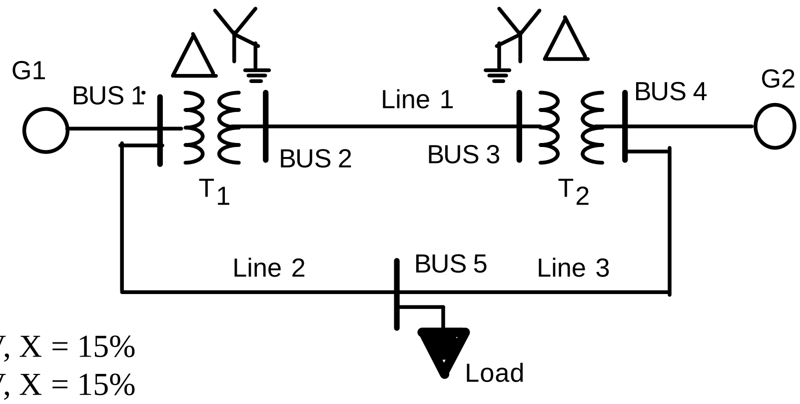
$V_{motor\_rated} := 12kV \qquad S_{motor\_rated} := 10MVA$

$$X_{motor\_pu\_new} := 0.07pu\left(\frac{V_{motor\_rated}}{V_{b3}}\right)^2 \cdot \left(\frac{S_b}{S_{motor\_rated}}\right)$$

$$\boxed{X_{motor\_pu\_new} = 0.08 \cdot pu}$$



2. Sketch a per phase, per unit equivalent circuit for the system below. Use a system MVA base of 100MVA, and a voltage base of 220 kV on the high voltage transmission line section.



Using the following equipment nameplate data:

G1:	50 MVA, 13.8 kV, X = 15%
G2:	25 MVA, 14.4 kV, X = 15%
T1:	60 MVA, 13.8 : 230 kV, X = 10%
T2:	30 MVA, 230 : 13.8 kV, X = 10%
Line 1:	10 + j100 Ohm
Line 2:	0.05 + j0.5 Ohm
Line 3:	0.05 + j0.5 Ohm
Load:	25 MVA, 0.9pf lagging

- System MVA Base

$S_B := 100\text{MVA}$

- Voltage base for the high voltage part of the system (Line 1):

$V_{B2} := 220\text{kV}$

- Voltage base for the rest of the system

$$V_{B1} := V_{B2}\cdot\left(\frac{13.8\text{kV}}{230\text{kV}}\right) \qquad V_{B1} = 13.2\cdot\text{kV}$$

- Impedance Bases

$$Z_{B1} := \frac{{V_{B1}}^2}{S_B} \qquad Z_{B1} = 1.74\,\Omega$$

$$Z_{B2} := \frac{{V_{B2}}^2}{S_B} \qquad Z_{B2} = 484\,\Omega$$

- Change of Base Calculations:

Generator G1 $\quad X_{G1} := 0.15\text{pu}\cdot\left(\frac{13.8\text{kV}}{V_{B1}}\right)^2\cdot\left(\frac{S_B}{50\text{MVA}}\right) \qquad \boxed{X_{G1} = 0.33\cdot\text{pu}}$

Generator G2 $X_{G2} := 0.15\text{pu}\cdot\left(\frac{14.4\text{kV}}{V_{B1}}\right)^2\cdot\left(\frac{S_B}{25\text{MVA}}\right)$ $\boxed{X_{G2} = 0.71\cdot\text{pu}}$

Transformer T1: $X_{t1} := 0.10\text{pu}\cdot\left(\frac{13.8\text{kV}}{V_{B1}}\right)^2\cdot\left(\frac{S_B}{60\text{MVA}}\right)$ $\boxed{X_{t1} = 0.18\cdot\text{pu}}$

Transformer T2: $X_{t2} := 0.10\text{pu}\cdot\left(\frac{13.8\text{kV}}{V_{B1}}\right)^2\cdot\left(\frac{S_B}{30\text{MVA}}\right)$ $\boxed{X_{t2} = 0.36\cdot\text{pu}}$

- Convert Line Impedances to Per Unit

$Z_{L1} := 10\text{ohm} + j\cdot 100\text{ohm}$

$Z_{L1pu} := \frac{Z_{L1}}{Z_{B2}}$ $\boxed{Z_{L1pu} = (0.02 + 0.21i)\cdot\text{pu}}$

$Z_{L2} := 0.05\text{ohm} + j\cdot 0.5\text{ohm}$

$Z_{L2pu} := \frac{Z_{L2}}{Z_{B1}}$ $\boxed{Z_{L2pu} = (0.03 + 0.29i)\cdot\text{pu}}$

$Z_{L3} := 0.05\text{ohm} + j\cdot 0.5\text{ohm}$

$Z_{L3pu} := \frac{Z_{L3}}{Z_{B1}}$ $\boxed{Z_{L3pu} = (0.03 + 0.29i)\cdot\text{pu}}$

- Eqvuialent Load Impedance (can consider it parallel or series connected RL circuit)

$\text{MagSload} := 25\text{MVA}$ $\text{pfload} := 0.9$ lagging

$P_{load} := \text{MagSload}\cdot\text{pfload}$ $P_{load} = 22.5\cdot\text{MW}$

$\theta_{load} := \text{acos}(\text{pfload})$ $\theta_{load} = 25.84\cdot\text{deg}$

$Q_{load} := \text{MagSload}\cdot\sin\left(\theta_{load}\right)$ $Q_{load} = 10.9\cdot\text{MVAR}$

$S_{load} := P_{load} + j\cdot Q_{load}$ $S_{load} = (22.5 + 10.9i)\cdot\text{MVA}$

$S_{load\_pu} := \frac{S_{load}}{S_B}$ $\qquad |S_{load\_pu}| = 0.25 \cdot pu$

Parallel Load

$R_{load} := \frac{V_{B1}^2}{P_{load}}$ $\qquad R_{load} = 7.74\,\Omega$

$R_{load\_pu} := \frac{R_{load}}{Z_{B1}}$ $\qquad \boxed{R_{load\_pu} = 4.44 \cdot pu}$

$X_{load} := \frac{V_{B1}^2}{Q_{load}}$ $\qquad X_{load} = 15.99\,\Omega$

$X_{load\_pu} := \frac{X_{load}}{Z_{B1}}$ $\qquad \boxed{X_{load\_pu} = 9.18 \cdot pu}$

- As a check: $V_{rated} := 1.0pu$

$S_{check} := \frac{V_{rated}^2}{R_{load\_pu}} + \frac{jV_{rated}^2}{X_{load\_pu}}$ $\qquad S_{check} = 0.23 + 0.11i$

$|S_{check}| = 0.25 \cdot pu$ $\qquad$ Correct magnitude

$\frac{Re(S_{check})}{|S_{check}|} = 0.9$ $\qquad$ Correct power factor, and since imaginary part is positive, it is lagging

- Series Connected Load

Recall: $S = V \cdot \bar{I}$ (Complex conjugate of I)

$S = V \cdot \overline{\left(\frac{V}{Z}\right)} = \frac{(|V|)^2}{\bar{Z}}$ $\qquad$ So: $Z := \overline{\left[\frac{(|V|)^2}{S}\right]}$ $\qquad$ Take complex conjugate of both sides...

$Z_{load\_series} := \overline{\left[\frac{(V_{rated})(V_{B1})^2}{S_{load}}\right]}$ $\qquad Z_{load\_series} = (6.27 + 3.04i)\,\Omega$

$Z_{load\_series\_pu} := \frac{Z_{load\_series}}{Z_{B1}}$ $\qquad \boxed{Z_{load\_series\_pu} = (3.6 + 1.74i) \cdot pu}$

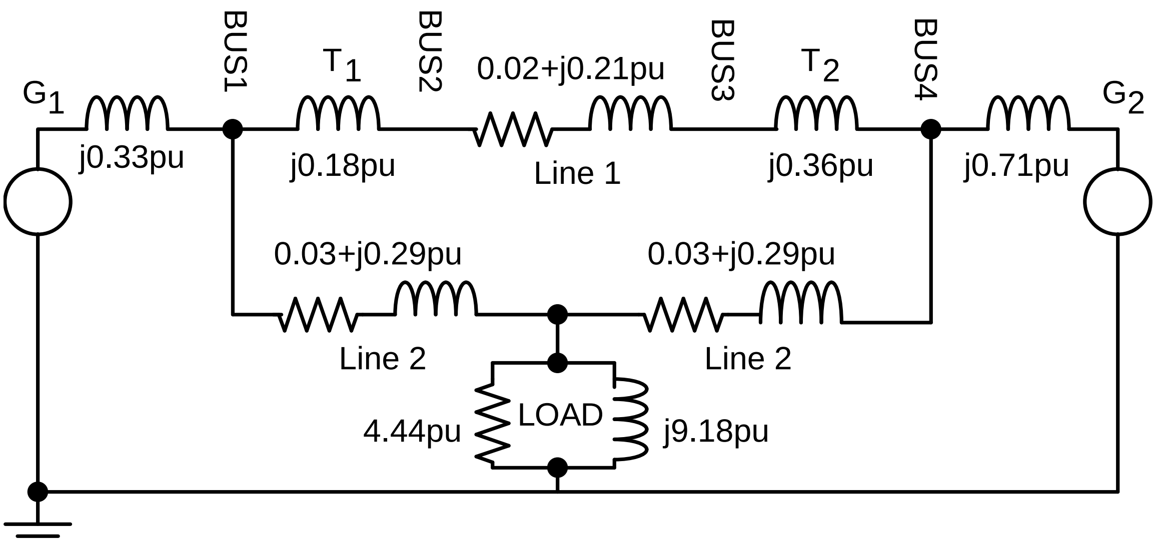
- Check result:

$$S_{check\_ser} := \frac{\overline{V_{rated}^{\ 2}}}{Z_{load\_series\_pu}} \qquad S_{check\_ser} = 0.23 + 0.11i$$

$$\left|S_{check\_ser}\right| = 0.25 \cdot pu \qquad \arg\left(S_{check\_ser}\right) = 25.84 \cdot deg$$ positive angle, lagging pf

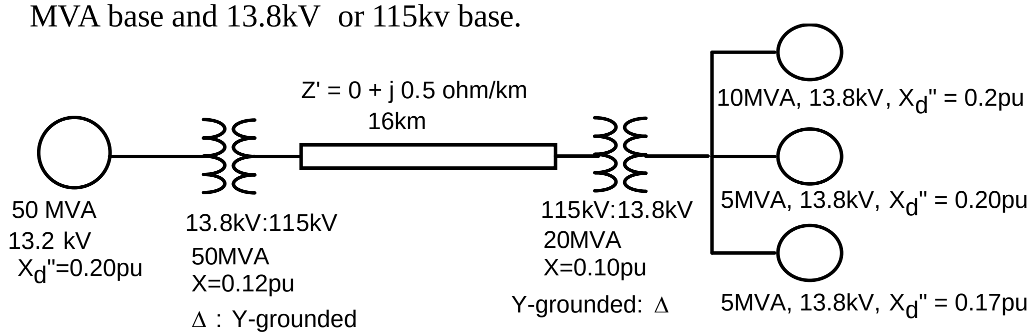
$$\frac{Re\left(S_{check\_ser}\right)}{\left|S_{check\_ser}\right|} = 0.9$$ correct power factor.

- Per Phase/Per Unit Equivalent Circut (using parallel R-L load):



3. A three-phase generator feeds three large synchronous motors over a 16km, 115kV transmission line, through a 115kV:13.8kV transformer bank, as shown below.

(a) Draw a per unit, per phasequivalent circuit with all reactances indicated in per unit on a 100 MVA base and 13.8kV or 115kv base.



(a) Draw a per unit, per phasequivalent circuit with all reactances indicated in per unit on a 100 MVA base and 13.8kV or 115kv base.

$$S_B := 100MVA$$

$$V_{B1} := 13.8kV$$

$$V_{B2} := V_{B1} \cdot \left(\frac{115}{13.8}\right) \qquad V_{B2} = 115 \cdot kV$$

$$V_{B3} := V_{B2} \cdot \left(\frac{13.8}{115}\right) \qquad V_{B3} = 13.8 \cdot kV$$

Generator $$X_{dpp} := 0.20pu \cdot \left(\frac{13.2kV}{V_{B1}}\right)^2 \cdot \left(\frac{S_B}{50MVA}\right) \qquad X_{dpp} = 0.37 \cdot pu$$

Transformer 1: $$X_{t1} := 0.12pu \cdot \left(\frac{13.8kV}{V_{B1}}\right)^2 \cdot \left(\frac{S_B}{50MVA}\right) \qquad X_{t1} = 0.24 \cdot pu$$

$$Z_{bII} := \frac{(V_{B2})^2}{S_B} \qquad Z_{bII} = 132.25\,\Omega$$

Per unit line impedance $$X_{line\_pu} := \frac{\left(0.5\frac{ohm}{km}\right) \cdot 16km}{Z_{bII}} \qquad X_{line\_pu} = 0.06 \cdot pu$$

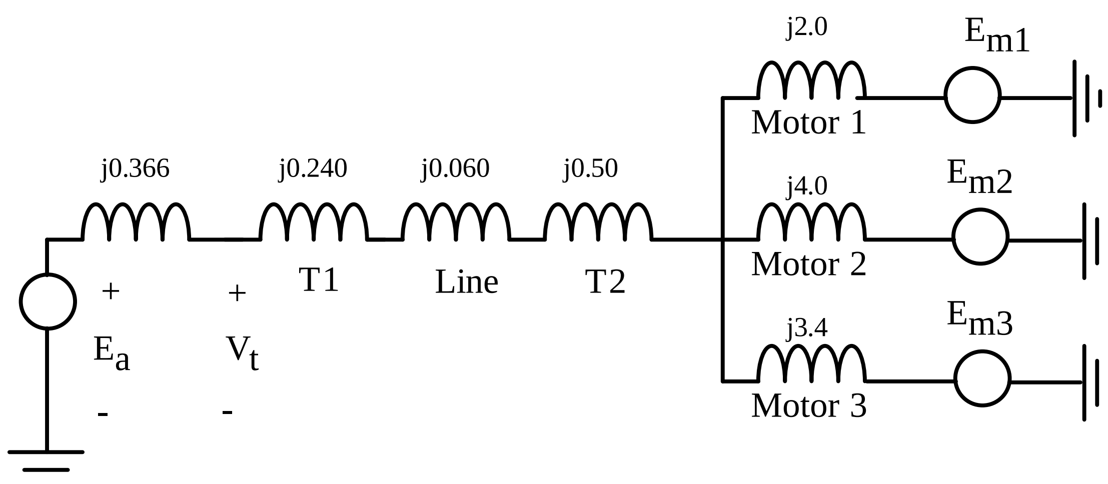
Transformer 2: $$X_{t2} := 0.10 \cdot \left(\frac{115kV}{V_{B2}}\right)^2 \cdot \left(\frac{S_B}{20MVA}\right) \qquad X_{t2} = 0.5$$

Motor 1: $X_{m1} := 0.2\cdot\left(\frac{13.8kV}{V_{B3}}\right)^2\cdot\left(\frac{S_B}{10MVA}\right)$ $X_{m1} = 2\cdot pu$

Motor 2: $X_{m2} := 0.2\cdot\left(\frac{13.8kV}{V_{B3}}\right)^2\cdot\left(\frac{S_B}{5MVA}\right)$ $X_{m2} = 4\cdot pu$

Motor 3: $X_{m3} := 0.17\cdot\left(\frac{13.8kV}{V_{B3}}\right)^2\cdot\left(\frac{S_B}{5MVA}\right)$ $X_{m3} = 3.4\cdot pu$

Per unit equivalent circuit diagram:



(b) The generator is controlled to maintain the voltage at the motor bus at 1.0pu at an angle of 0 degrees. The three motors are operating at full rating and 90% PF lagging. Determine the voltage required at the generator terminals assuming that there is no voltage regulating taps or similar equipment in this system.

The motors have a total rating of 20 MVA

$S_{load\_mag} := 20MVA$ $pfload := 0.90$ $\phi_{load} := acos(0.90)$ $\phi_{load} = 25.84\cdot deg$

$$S_{load} := S_{load\_mag}\cdot e^{j\cdot\phi_{load}}$$

$S_{load\_pu} := \frac{S_{load}}{100MVA}$ $S_{load\_pu} = 0.18 + 0.09i$ $\left|S_{load\_pu}\right| = 0.2$

$$V_{load\_pu} := 1.0e^{j\cdot 0}$$

$$I_{load\_pu} := \frac{\overline{S_{load\_pu}}}{V_{load\_pu}} \qquad I_{load\_pu} = 0.18 - 0.09i$$

$$\left|I_{load\_pu}\right| = 0.2 \qquad \arg\left(I_{load\_pu}\right) = -25.84 \cdot deg$$

Generator Terminal Voltage:

$$V_t := V_{load\_pu} + I_{load\_pu} \cdot \left(j \cdot X_{t1} + j \cdot X_{line\_pu} + j \cdot X_{t2}\right)$$

$\boxed{\left|V_t\right| = 1.08}$ $\boxed{\arg\left(V_t\right) = 7.67 \cdot deg}$ $\boxed{\left|V_t\right| \cdot V_{B1} = 14.9 \cdot kV}$ line to line

(c) Calculate the voltage required behind the subtransient reactance for the generator and each of the motors

Generator Internal Voltage:

$$E_a := V_t + I_{load\_pu} \cdot \left(j \cdot X_{dpp}\right)$$

$\boxed{\left|E_a\right| = 1.12}$ $\boxed{\arg\left(E_a\right) = 10.79 \cdot deg}$ $\boxed{\left|E_a\right| \cdot \frac{V_{B3}}{\sqrt{3}} = 8.94 \cdot kV}$ line to neutral

Motor 1:

$$I_{motor1} := I_{load\_pu} \cdot \left(\frac{10MVA}{\left|S_{load}\right|}\right) \qquad I_{motor1} = (0.09 - 0.04i) \cdot pu$$

$$\left|I_{motor1}\right| = 0.1 \cdot pu \qquad \arg\left(I_{motor1}\right) = -25.84 \cdot deg$$

$$E_{am1} := V_{load\_pu} - I_{motor1} \cdot \left(j \cdot X_{m1}\right)$$

$\boxed{\left|E_{am1}\right| = 0.93}$ $\boxed{\arg\left(E_{am1}\right) = -11.16 \cdot deg}$ $\boxed{\left|E_{am1}\right| \cdot \frac{V_{B3}}{\sqrt{3}} = 7.41 \cdot kV}$

line to neutral

Motor 2:

$$I_{motor2} := I_{load\_pu} \cdot \left(\frac{5MVA}{\left|S_{load}\right|}\right) \qquad I_{motor2} = (0.05 - 0.02i) \cdot pu$$

$$\left|I_{motor2}\right| = 0.05 \cdot pu \qquad \arg\left(I_{motor2}\right) = -25.84 \cdot deg$$

$E_{am2} := V_{load\_pu} - I_{motor2} \cdot (j \cdot X_{m2})$

$\left| E_{am2} \right| = 0.93$ $\quad$ $\arg(E_{am2}) = -11.16 \cdot deg$ $\quad$ $\left| E_{am2} \right| \cdot \frac{V_{B3}}{\sqrt{3}} = 7.41 \cdot kV$

line to neutral

Motor 3:

$I_{motor3} := I_{load\_pu} \cdot \left(\frac{5MVA}{\left| S_{load} \right|} \right)$ $\quad$ $I_{motor3} = (0.05 - 0.02i) \cdot pu$

$\left| I_{motor3} \right| = 0.05 \cdot pu$ $\quad$ $\arg(I_{motor3}) = -25.84 \cdot deg$

$E_{am3} := V_{load\_pu} - I_{motor3} \cdot (j \cdot X_{m3})$

$\left| E_{am3} \right| = 0.94$ $\quad$ $\arg(E_{am3}) = -9.38 \cdot deg$ $\quad$ $\left| E_{am3} \right| \cdot \frac{V_{B3}}{\sqrt{3}} = 7.48 \cdot kV$

line to neutral

(d) Calculate the line current in Amperes

Base current on the transmission line segment:

$I_{B2} := \frac{S_B}{\sqrt{3} \cdot V_{B2}}$ $\quad$ $I_{B2} = 502.04\,A$

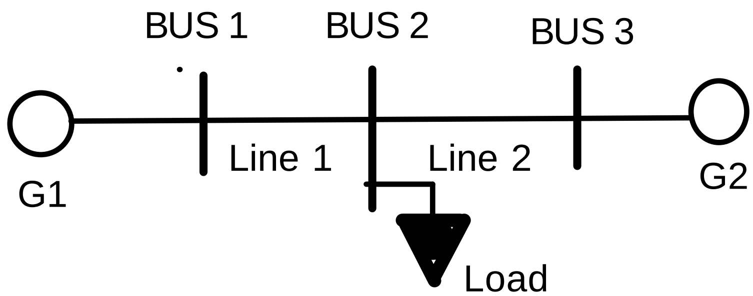
$I_{line} := I_{load\_pu} \cdot I_{B2}$ $\quad$ $\left| I_{line} \right| = 100.41\,A$

4. Draw the per unit, Thevenin equivalent circuit for the system below looking out from the load bus if:

(a) The generator internal voltages are equal in magnitude and angle (label both as E_1 and present your results as a function of E_1)
(b) The generator internal voltages are not equal (label one as E_1 and the other E_2 in your solution, and present your results as a function of E_1 and E_2)

Impedance values (all on consistent bases, no change of base needed):

G1: X = 0.1 pu
G2: X = 0.1 pu
Line 1: X = 0.1 pu
Line 2: X = 0.1 pu
Load: Z = j 0.1 pu



Answer part (b) first to get a general approach, and then apply that result to part (a)

(b) The generator internal voltages are not equal (label one as E_1 and the other E_2 in your solution, and present your results as a function of E_1 and E_2)

$Z_{g1} := j\cdot 0.1pu$ $\quad$ $Z_{Line1} := j\cdot 0.1pu$ $\quad$ $Z_{load} := j\cdot 0.1pu$

$Z_{g2} := j\cdot 0.1pu$ $\quad$ $Z_{Line2} := j\cdot 0.1pu$

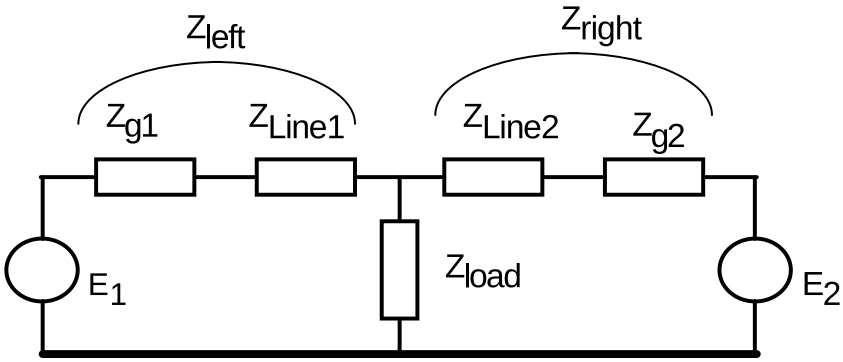
- Impedance to the left of the load:

$Z_{left} := Z_{g1} + Z_{Line1}$ $\qquad$ $Z_{left} = 0.2i\cdot pu$

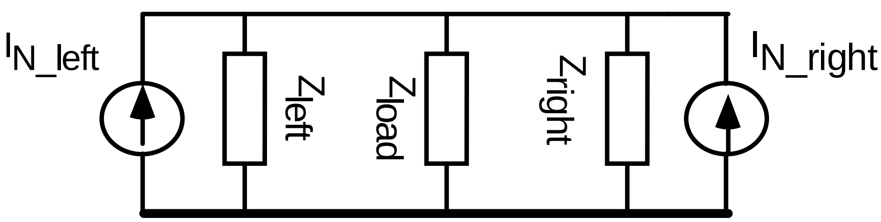
- Impedance to the right of the load:

$Z_{right} := Z_{g2} + Z_{Line2}$ $\qquad$ $Z_{right} = 0.2i\cdot pu$

- Resulting equivalent circuit

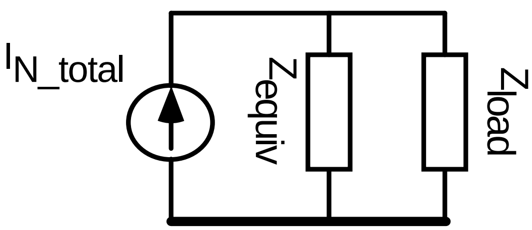


- Now take Norton Equivalent to left and the right, leading to this circuit:



$$I_{N\_left} = \frac{E_1}{Z_{left}} \qquad I_{N\_right} = \frac{E_2}{Z_{right}}$$

- Now combine the two Norton equivalents in parallel



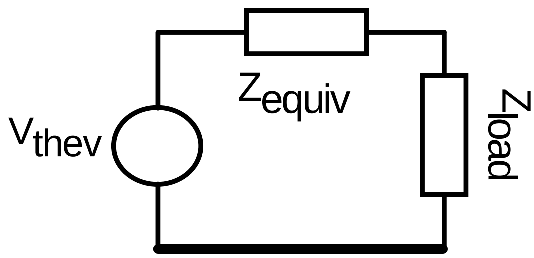
$$I_{N\_total} = I_{N\_left} + I_{N\_right} = \frac{E_1}{Z_{left}} + \frac{E_2}{Z_{right}}$$

- Equivalent impedance

$$Z_{equiv} := \left(\frac{1}{Z_{left}} + \frac{1}{Z_{right}}\right)^{-1} \qquad \boxed{Z_{equiv} = 0.1i \cdot pu}$$

- Convert Norton Equivalent Back to Thevenin' Equivalent

$$V_{thev} = I_{N\_total} \cdot Z_{equiv} = E_1 \cdot \frac{Z_{equiv}}{Z_{left}} + E_2 \cdot \frac{Z_{equiv}}{Z_{right}}$$



- In this case:

$$V_{thev} = E_1 \cdot \left(\frac{j \cdot 0.1}{j \cdot 0.2}\right) + E_2 \cdot \left(\frac{j \cdot 0.1}{j \cdot 0.2}\right) = 0.5 \cdot E_1 + 0.5 \cdot E_2$$

(a) The generator internal voltages are equal in magnitude and angle (label both as E_1 and present your results as a function of E_1)

Now we have identical voltages, so:

$V_{thev} = 0.5 \cdot E_1 + 0.5 \cdot E_1 = E_1$ As one would expect.....

Z.equiv stays the same, it does not depend on the voltage output of the source

5. Problem 1.4 in the text book

The following table of values has been prepared for various line sections in a small electric system. Find the total p.u. impedance and shunt susceptance of each line on a 10 MVA base, using the nominal line voltage as a base.

$S_{b4} := 10MVA$

The table has two different voltage bases:

$V_{b13\_8} := 13.8kV$ $V_{b69} := 69kV$

Impedance and admittance bases

$Z_{b13\_8} := \frac{V_{b13\_8}^{2}}{S_{b4}}$ $\quad Z_{b13\_8} = 19.04\,\Omega$

$Y_{b13\_8} := \frac{1}{Z_{b13\_8}}$ $\quad Y_{b13\_8} = 0.05 \cdot \text{mho}$

$Z_{b69} := \frac{V_{b69}^{2}}{S_{b4}}$ $\quad Z_{b69} = 476.1\,\Omega$

$Y_{b69} := \frac{1}{Z_{b69}}$ $\quad Y_{b69} = 0.0021 \frac{1}{\Omega}$

Line 1:

$\text{Length1} := 5\text{mi}$ $\quad R_1 := 0.278 \frac{\text{ohm}}{\text{mi}}$ $\quad X_1 := 0.690 \frac{\text{ohm}}{\text{mi}}$ $\quad X_{c1} := 0.16\text{M}\Omega \cdot \text{mi}$

$Z_{total1} := \left(R_1 + j \cdot X_1\right) \cdot \text{Length1}$ $\quad Z_{total1} = (1.39 + 3.45i)\,\Omega$

$B_{c1} := \frac{\text{Length1}}{X_{c1}}$ $\quad B_{c1} = 3.13 \times 10^{-5} \cdot \text{mho}$

Per Unit results:

$Z_{1\_pu} := \frac{Z_{total1}}{Z_{b13\_8}}$ $\quad \boxed{Z_{1\_pu} = (0.07 + 0.18i) \cdot \text{pu}}$

$B_{c1\_pu} := \frac{B_{c1}}{Y_{b13\_8}}$ $\quad \boxed{B_{c1\_pu} = 0.000595 \cdot \text{pu}}$ Admittance

It is ok to express Bc as a capactive reactance instead:

$X_{c1\_pu} := \frac{1}{B_{c1\_pu}}$ $\quad \boxed{X_{c1\_pu} = 1680.32 \cdot \text{pu}}$

Line 2:

$\text{Length2} := 2\text{mi}$ $\quad R_2 := 1.374 \frac{\text{ohm}}{\text{mi}}$ $\quad X_2 := 0.816 \frac{\text{ohm}}{\text{mi}}$ $\quad X_{c2} := 0.193\text{M}\Omega \cdot \text{mi}$

$Z_{total2} := \left(R_2 + j \cdot X_2\right) \cdot \text{Length2}$ $\quad Z_{total2} = (2.75 + 1.63i)\,\Omega$

$B_{c2} := \frac{\text{Length2}}{X_{c2}}$ $\quad B_{c2} = 1.04 \times 10^{-5} \cdot \text{mho}$

Per Unit results:

$$Z_{2\_pu} := \frac{Z_{total2}}{Z_{b13\_8}} \qquad \boxed{Z_{2\_pu} = 0.14 + 0.09i}$$

$$B_{c2\_pu} := \frac{B_{c2}}{Y_{b13\_8}} \qquad \boxed{B_{c2\_pu} = 0.000197}$$

It is ok to express Bc as a capactive reactance instead:

$$X_{c2\_pu} := \frac{1}{B_{c2\_pu}} \qquad \boxed{X_{c2\_pu} = 5067.21 \cdot pu}$$

Line 3:

$$Length3 := 3.9mi \qquad R_3 := 0.445\frac{ohm}{mi} \qquad X_3 := 0.711\frac{ohm}{mi} \qquad X_{c3} := 0.157M\Omega \cdot mi$$

$$Z_{total3} := \left(R_3 + j \cdot X_3\right) \cdot Length3 \qquad Z_{total3} = (1.74 + 2.77i)\,\Omega$$

$$B_{c3} := \frac{Length3}{X_{c3}} \qquad B_{c3} = 2.48 \times 10^{-5} \cdot mho$$

Per Unit results:

$$Z_{3\_pu} := \frac{Z_{total3}}{Z_{b13\_8}} \qquad \boxed{Z_{3\_pu} = 0.09 + 0.15i}$$

$$B_{c3\_pu} := \frac{B_{c3}}{Y_{b13\_8}} \qquad \boxed{B_{c3\_pu} = 0.000473}$$

It is ok to express Bc as a capactive reactance instead:

$$X_{c3\_pu} := \frac{1}{B_{c3\_pu}} \qquad \boxed{X_{c3\_pu} = 2113.86 \cdot pu}$$

Line 4:

$$Length4 := 6.2mi \qquad R_4 := 0.278\frac{ohm}{mi} \qquad X_4 := 0.730\frac{ohm}{mi} \qquad X_{c4} := 0.172M\Omega \cdot mi$$

$$Z_{total4} := \left(R_4 + j \cdot X_4\right) \cdot Length4 \qquad Z_{total4} = (1.72 + 4.53i)\,\Omega$$

$$B_{c4} := \frac{Length4}{X_{c4}} \qquad B_{c4} = 3.6 \times 10^{-5} \cdot mho$$

Per Unit results:

$$Z_{4\_pu} := \frac{Z_{total4}}{Z_{b13\_8}}$$ $$\boxed{Z_{4\_pu} = 0.09 + 0.24i}$$

$$B_{c4\_pu} := \frac{B_{c4}}{Y_{b13\_8}}$$ $$\boxed{B_{c4\_pu} = 0.000686}$$

It is ok to express Bc as a capactive reactance instead:

$$X_{c4\_pu} := \frac{1}{B_{c4\_pu}}$$ $$\boxed{X_{c4\_pu} = 1456.73 \cdot pu}$$

Line 5:

$$Length5 := 7.3mi \qquad R_5 := 0.088 \frac{ohm}{mi} \qquad X_5 := 0.330 \frac{ohm}{mi} \qquad X_{c5} := 0.142M\Omega \cdot mi$$

$$Z_{total5} := (R_5 + j \cdot X_5) \cdot Length5 \qquad Z_{total5} = (0.64 + 2.41i)\,\Omega$$

$$B_{c5} := \frac{Length5}{X_{c5}} \qquad B_{c5} = 5.14 \times 10^{-5} \cdot mho$$

Per Unit results:

$$Z_{5\_pu} := \frac{Z_{total5}}{Z_{b13\_8}}$$ $$\boxed{Z_{5\_pu} = 0.03 + 0.13i}$$

$$B_{c5\_pu} := \frac{B_{c5}}{Y_{b13\_8}}$$ $$\boxed{B_{c5\_pu} = 0.000979}$$

It is ok to express Bc as a capactive reactance instead:

$$X_{c5\_pu} := \frac{1}{B_{c5\_pu}}$$ $$\boxed{X_{c5\_pu} = 1021.43 \cdot pu}$$

Line 6:

$$Length6 := 10.0mi \qquad R_6 := 0.445 \frac{ohm}{mi} \qquad X_6 := 0.711 \frac{ohm}{mi} \qquad X_{c6} := 0.157M\Omega \cdot mi$$

$$Z_{total6} := (R_6 + j \cdot X_6) \cdot Length6 \qquad Z_{total6} = (4.45 + 7.11i)\,\Omega$$

$$B_{c6} := \frac{Length6}{X_{c6}} \qquad B_{c6} = 6.37 \times 10^{-5} \cdot mho$$

Per Unit results:

$$Z_{6\_pu} := \frac{Z_{total6}}{Z_{b69}}$$ $$\boxed{Z_{6\_pu} = 0.01 + 0.01i}$$

$$B_{c6\_pu} := \frac{B_{c6}}{Y_{b69}} \qquad \boxed{B_{c6\_pu} = 0.030325}$$

It is ok to express Bc as a capactive reactance instead:

$$X_{c6\_pu} := \frac{1}{B_{c6\_pu}} \qquad \boxed{X_{c6\_pu} = 32.98 \cdot pu}$$

Line 7:

$$Length7 := 25mi \qquad R_7 := 0.278\frac{ohm}{mi} \qquad X_7 := 0.730\frac{ohm}{mi} \qquad X_{c7} := 0.172M\Omega \cdot mi$$

$$Z_{total7} := \left(R_7 + j \cdot X_7\right) \cdot Length7 \qquad Z_{total7} = (6.95 + 18.25i)\,\Omega$$

$$B_{c7} := \frac{Length7}{X_{c7}} \qquad B_{c7} = 1.45 \times 10^{-4} \cdot mho$$

Per Unit results:

$$Z_{7\_pu} := \frac{Z_{total7}}{Z_{b69}} \qquad \boxed{Z_{7\_pu} = 0.01 + 0.04i}$$

$$B_{c7\_pu} := \frac{B_{c7}}{Y_{b69}} \qquad \boxed{B_{c7\_pu} = 0.069201}$$

It is ok to express Bc as a capactive reactance instead:

$$X_{c7\_pu} := \frac{1}{B_{c7\_pu}} \qquad \boxed{X_{c7\_pu} = 14.45 \cdot pu}$$

Summary =

Line	Rpu	Xpu	Bcpu	Xcpu
1	0.073	0.1812	0.000595	1680.32
2	0.1443	0.0857	0.000197	5067.21
3	0.0911	0.1456	0.000473	2113.86
4	0.0905	0.2377	0.000686	1456.73
5	0.0337	0.1265	0.000979	1021.43
6	0.0094	0.0149	0.030325	32.98
7	0.0146	0.0383	0.06920	14.45