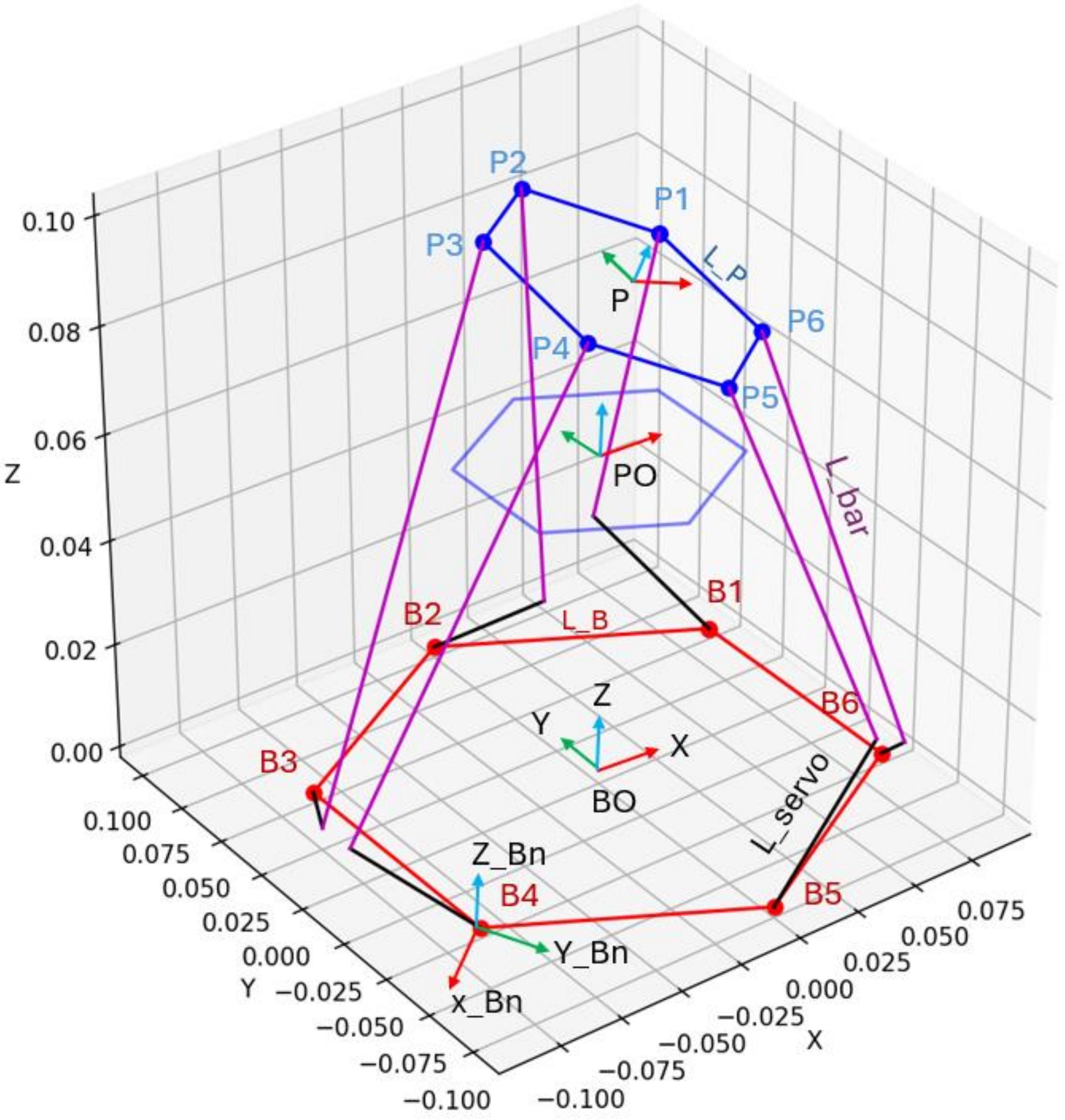
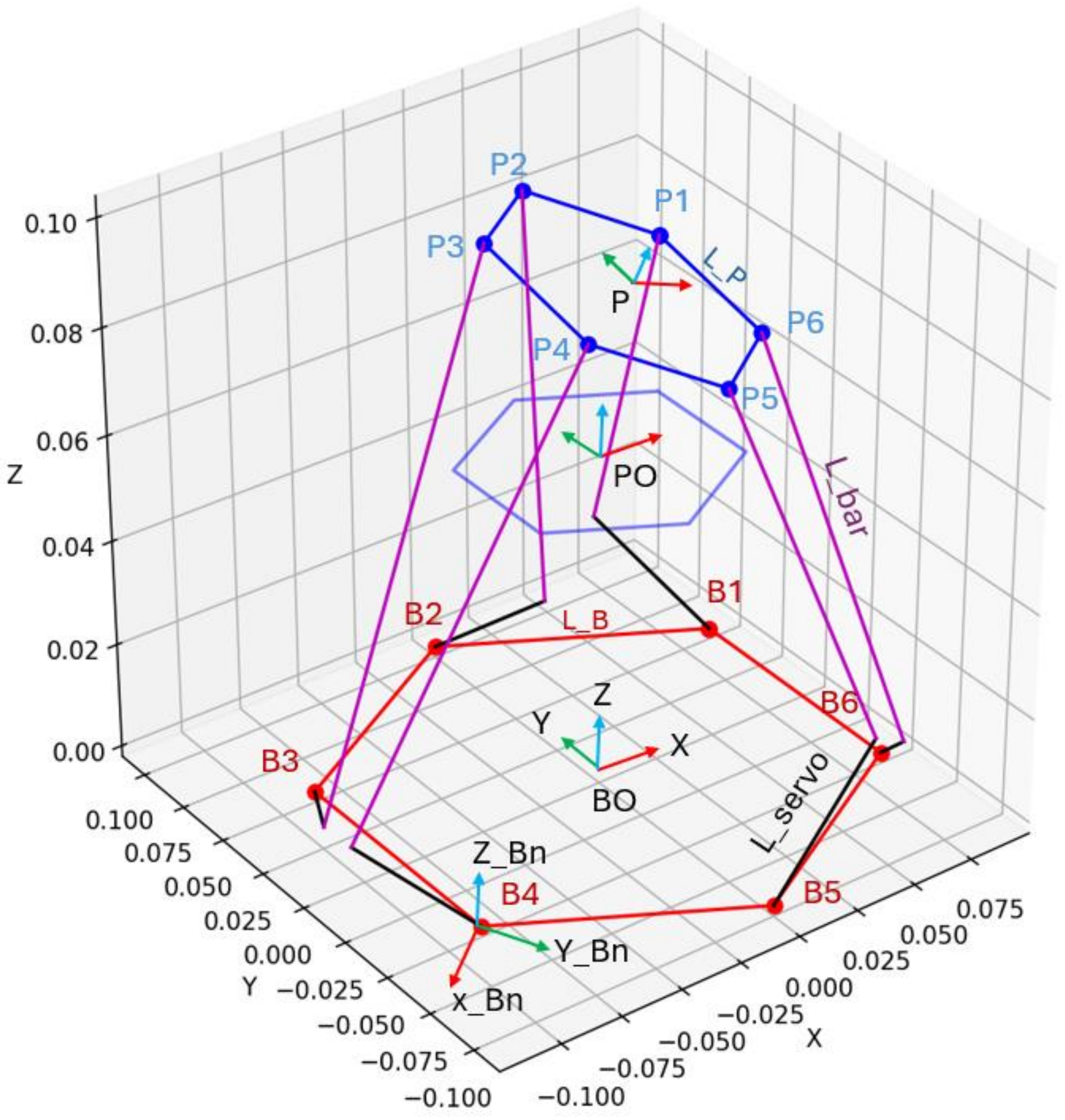


Objective

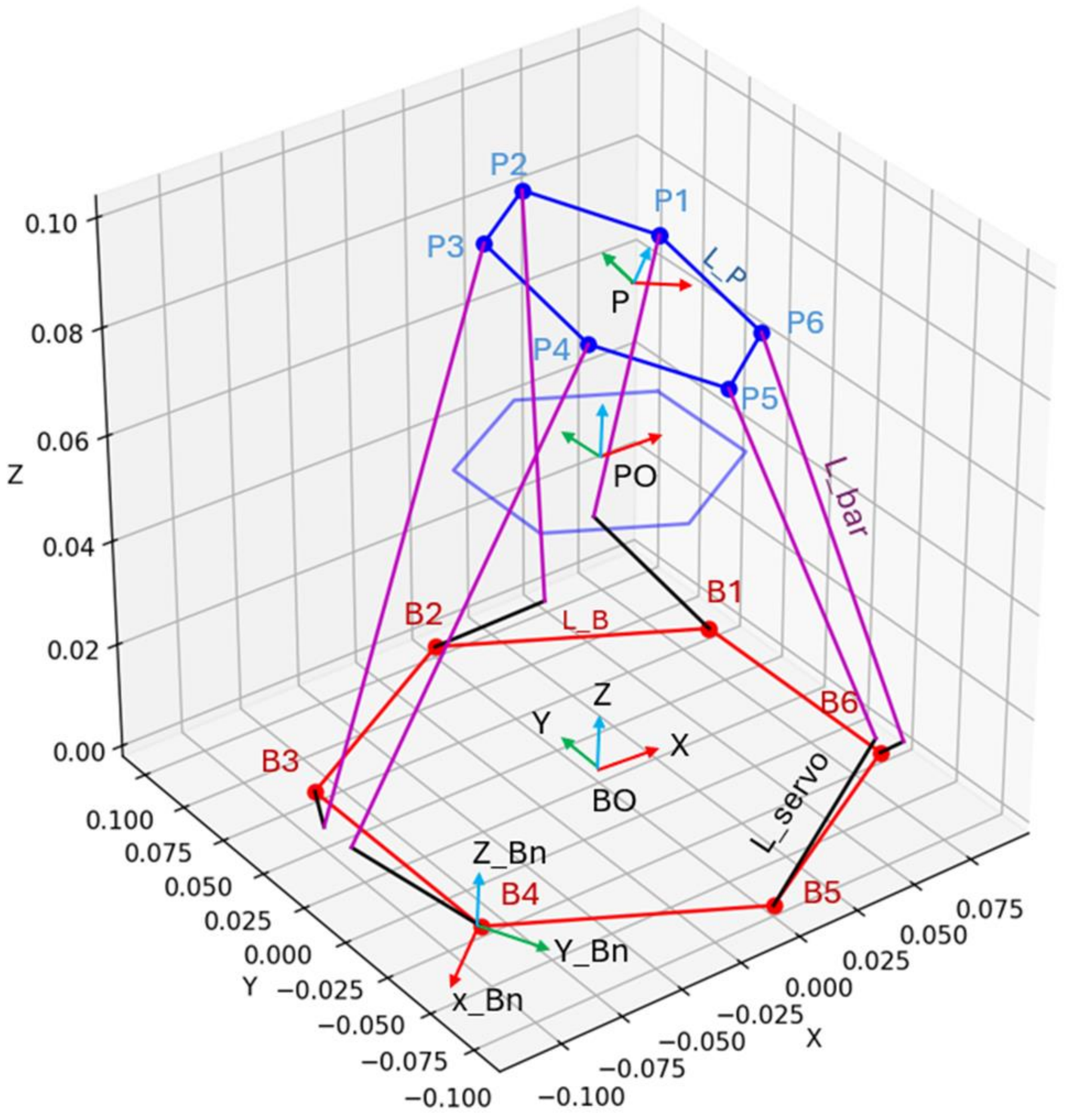
<u>Find the actuator inputs</u> (servo angle)

- I.e., find the rotation of servos (which are located at Bn) required, such that the platform moves from the original orientation (PO) to the final orientation P.



Method

1. Find the distance between base joint Bn and platform joint Pn for all joint pairs based on the final desired platform orientation
2. Find the servo angle that achieves the distance between Bn and Pn while satisfying all required constraints



Key Concepts

<u>Position Vector</u> (e.g., position of Bn rel. BO)

$$^{Bn}\boldsymbol{p}_{BO} = [X_{Bn}, Y_{Bn}, Z_{Bn}]^T,$$
$$for\ n \in \{1 \ldots 6\}$$

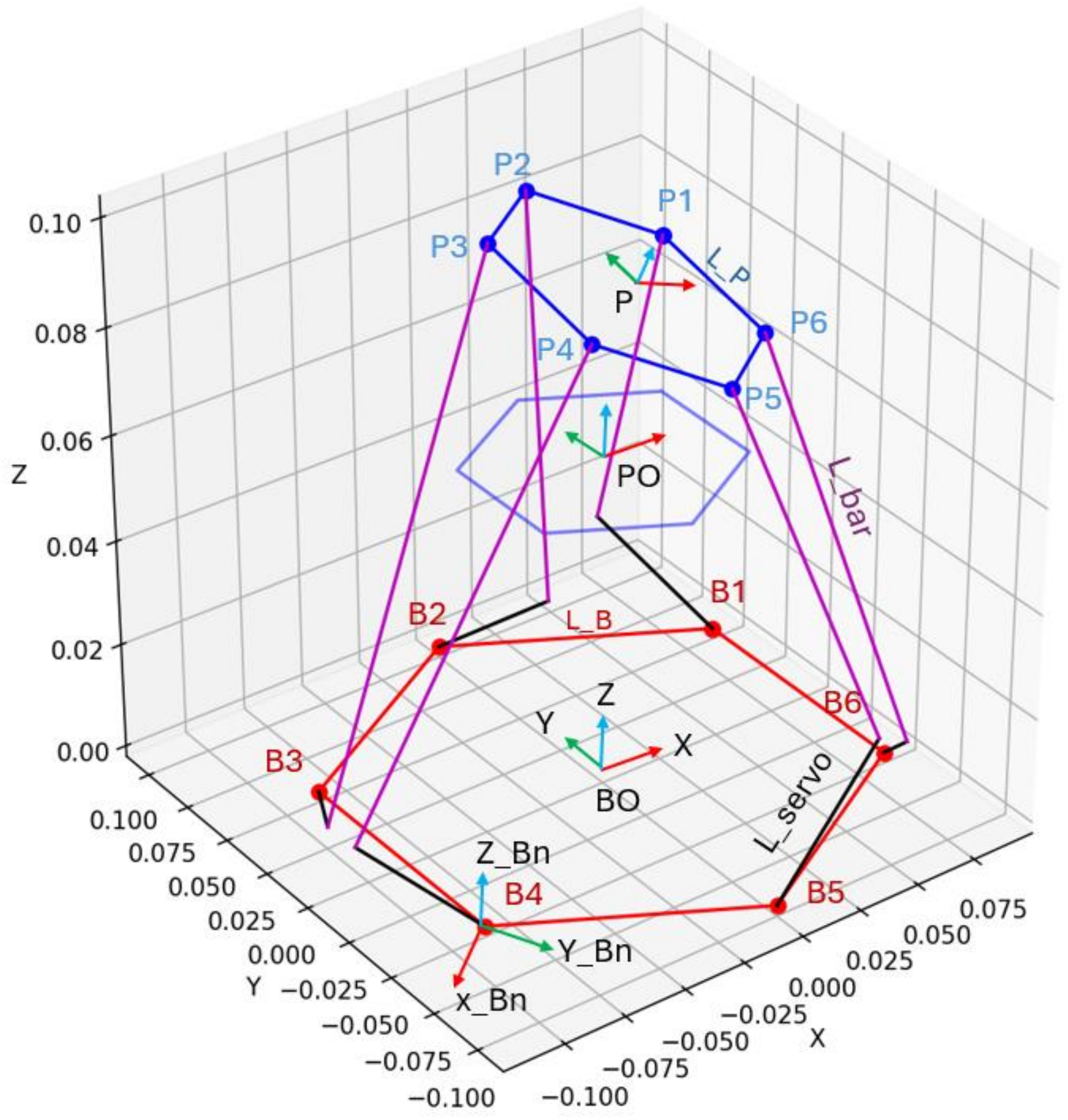
<u>Rotation Matrix</u> (e.g., rotation of Bn rel. BO)

$$^{Bn}_{BO}\boldsymbol{R} = Rot(roll_{Bn}, pitch_{Bn}, yaw_{Bn}), for\ n \in \{1 \ldots 6\}$$

$$R = R_z(\alpha)\, R_y(\beta)\, R_x(\gamma) = \overbrace{\begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}}^{\text{yaw}} \overbrace{\begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix}}^{\text{pitch}} \overbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\gamma & -\sin\gamma \\ 0 & \sin\gamma & \cos\gamma \end{bmatrix}}^{\text{roll}}$$
$$= \begin{bmatrix} \cos\alpha\cos\beta & \cos\alpha\sin\beta\sin\gamma - \sin\alpha\cos\gamma & \cos\alpha\sin\beta\cos\gamma + \sin\alpha\sin\gamma \\ \sin\alpha\cos\beta & \sin\alpha\sin\beta\sin\gamma + \cos\alpha\cos\gamma & \sin\alpha\sin\beta\cos\gamma - \cos\alpha\sin\gamma \\ -\sin\beta & \cos\beta\sin\gamma & \cos\beta\cos\gamma \end{bmatrix}$$

<u>Homogeneous Transformation Matrix</u> (e.g., translation and rotation of Bn rel. BO)

$$^{Bn}_{BO}\boldsymbol{T} = \begin{bmatrix} ^{Bn}_{BO}\boldsymbol{R} & ^{Bn}\boldsymbol{p}_{BO} \\ 000 & 1 \end{bmatrix}$$



Calculations

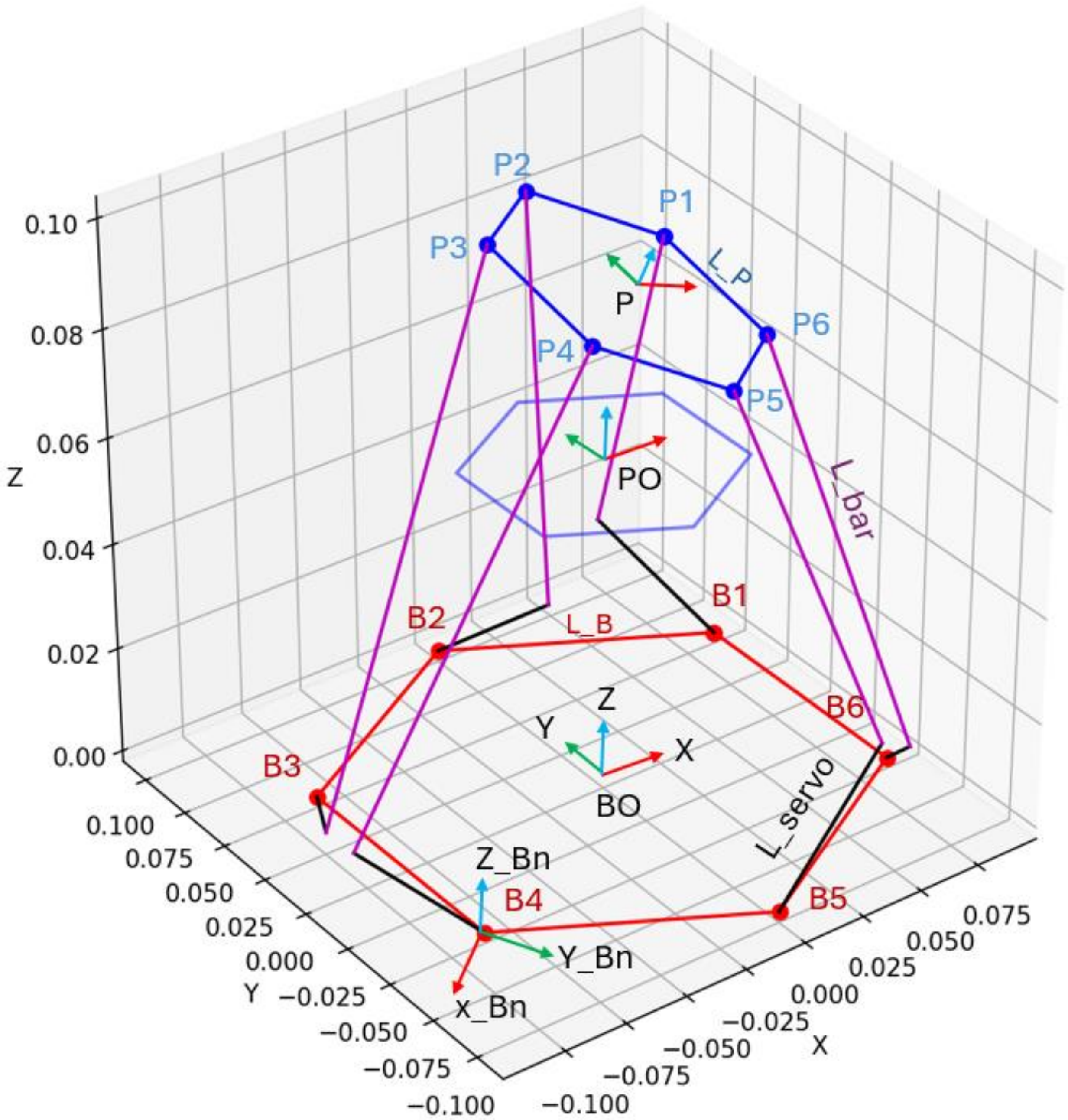
1. Find the distance between base joint Bn and platform joint Pn for all joint pairs based on the final desired platform orientation P

$$^{PO}\boldsymbol{p}_P = [X_P, Y_P, Z_P]^T$$

$$^{PO}_{P}\boldsymbol{R} = Rot(roll_P, pitch_P, yaw_P)$$

To find the distance, we first describe the platform joints relative to base origin BO, since having all points in a single coordinate system makes calculation easier.

$$^{P}\boldsymbol{p}_{Pn} \rightarrow {}^{BO}\boldsymbol{p}_{Pn}$$



Key parameters

Position and rotation of base joints (Bn) relative to the base origin (BO)

$$^{BO}\boldsymbol{p}_{Bn} = [X_{Bn}, Y_{Bn}, Z_{Bn}]^T, \qquad for\ n \in \{1 \dots 6\}$$

$$^{BO}_{Bn}\boldsymbol{R} = Rot(roll_{Bn}, pitch_{Bn}, yaw_{Bn}), for\ n \in \{1 \dots 6\}$$

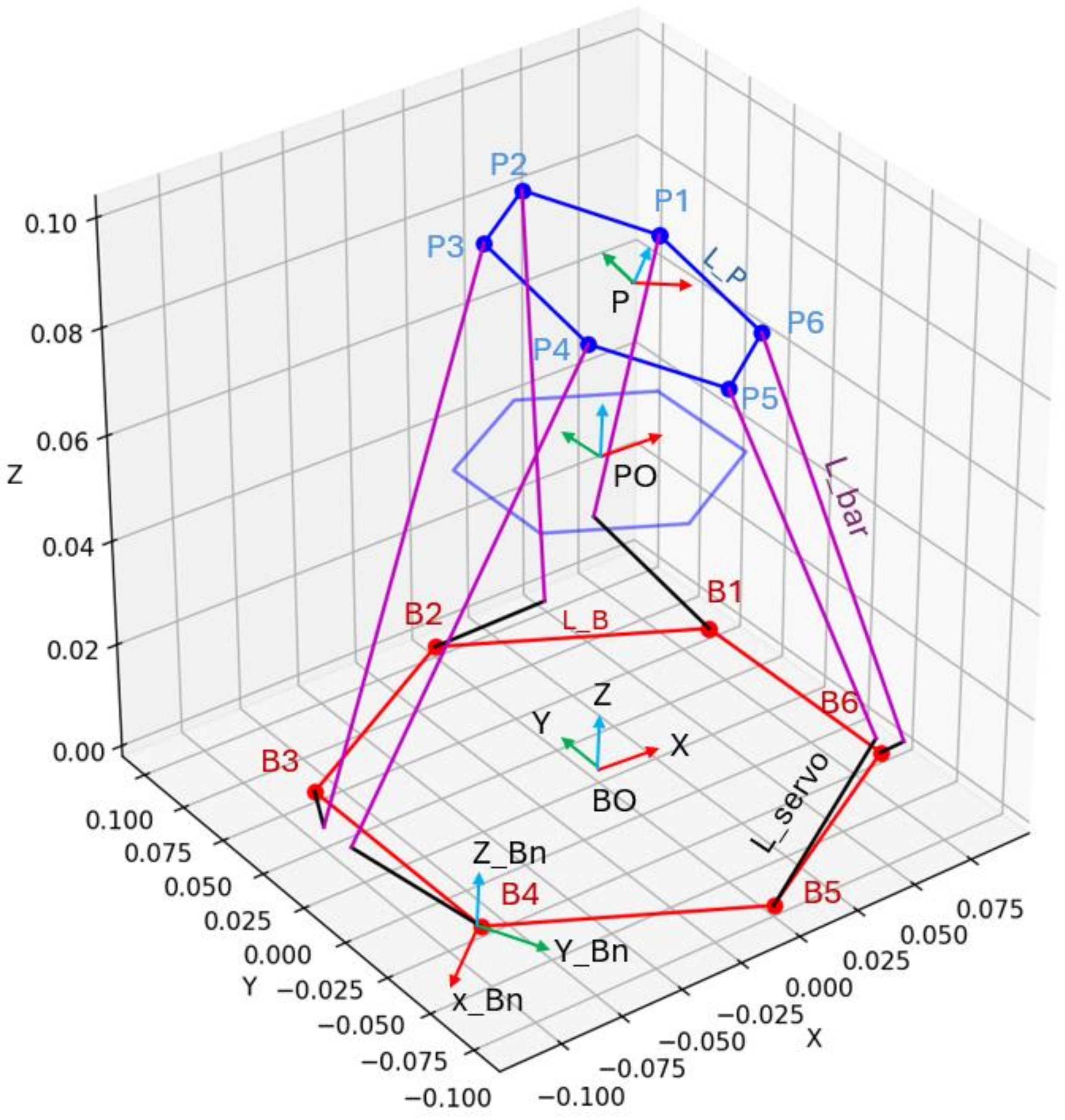
Position and rotation of platform joints (Bn) relative to the base origin (BO)

$$^{BO}\boldsymbol{p}_{PO} = [X_{PO}, Y_{PO}, Z_{PO}]^T = [0, 0, Z_{PO}]^T$$

$$^{BO}_{PO}\boldsymbol{R} = Rot(roll_{PO}, pitch_{PO}, yaw_{PO})$$

Position and rotation of platform joints (Bn) relative to the base origin (BO)

$$^{BO}\boldsymbol{p}_{Pn} = [x_{Pn}, y_{Pn}, z_{Pn}]^T, \qquad for\ n = \{1 \dots 6\}$$



Calculations

To find the distance, we first describe the platform joints relative to base origin BO, since having all points in a single coordinate system makes calculation easier.

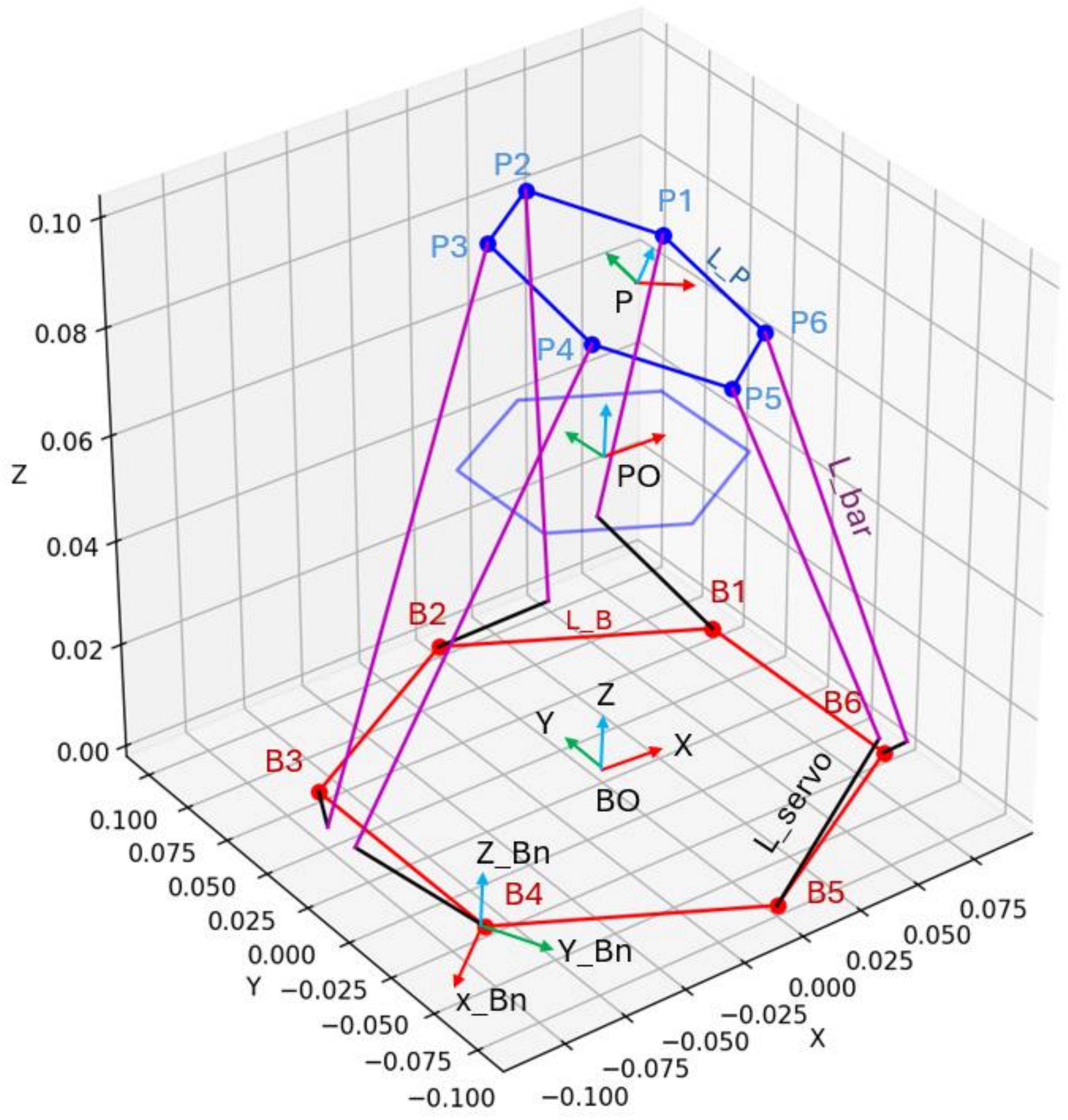
$$^{P}\boldsymbol{p}_{Pn} \rightarrow {}^{BO}\boldsymbol{p}_{Pn}$$

This can be achieved by multiplying $^{P}\boldsymbol{p}_{Pn}$ with a homogeneous transformation matrix $^{BO}_{P}\boldsymbol{T}$.

$$^{BO}\boldsymbol{p}_{Pn} = {}^{BO}_{P}\boldsymbol{T} \cdot {}^{P}\boldsymbol{p}_{Pn}$$

Using the property of homogeneous transformation matrix, $^{BO}_{P}\boldsymbol{T}$ can be further decomposed to,

$$^{BO}_{P}\boldsymbol{T} = {}^{BO}_{PO}\boldsymbol{T} \cdot {}^{PO}_{P}\boldsymbol{T}$$



Calculations

Using the property of homogeneous transformation matrix, ${}^{BO}_{P}\boldsymbol{T}$ can be further decomposed to,

$$ {}^{BO}_{P}\boldsymbol{T} = {}^{BO}_{PO}\boldsymbol{T} \cdot {}^{PO}_{P}\boldsymbol{T} $$

Where,

$$ {}^{PO}_{P}\boldsymbol{T} = \begin{bmatrix} {}^{PO}_{P}\boldsymbol{R} & {}^{PO}\boldsymbol{p}_P \\ 000 & 1 \end{bmatrix}, {}^{BO}_{PO}\boldsymbol{T} = \begin{bmatrix} {}^{BO}_{PO}\boldsymbol{R} & {}^{BO}\boldsymbol{p}_{PO} \\ 000 & 1 \end{bmatrix} $$

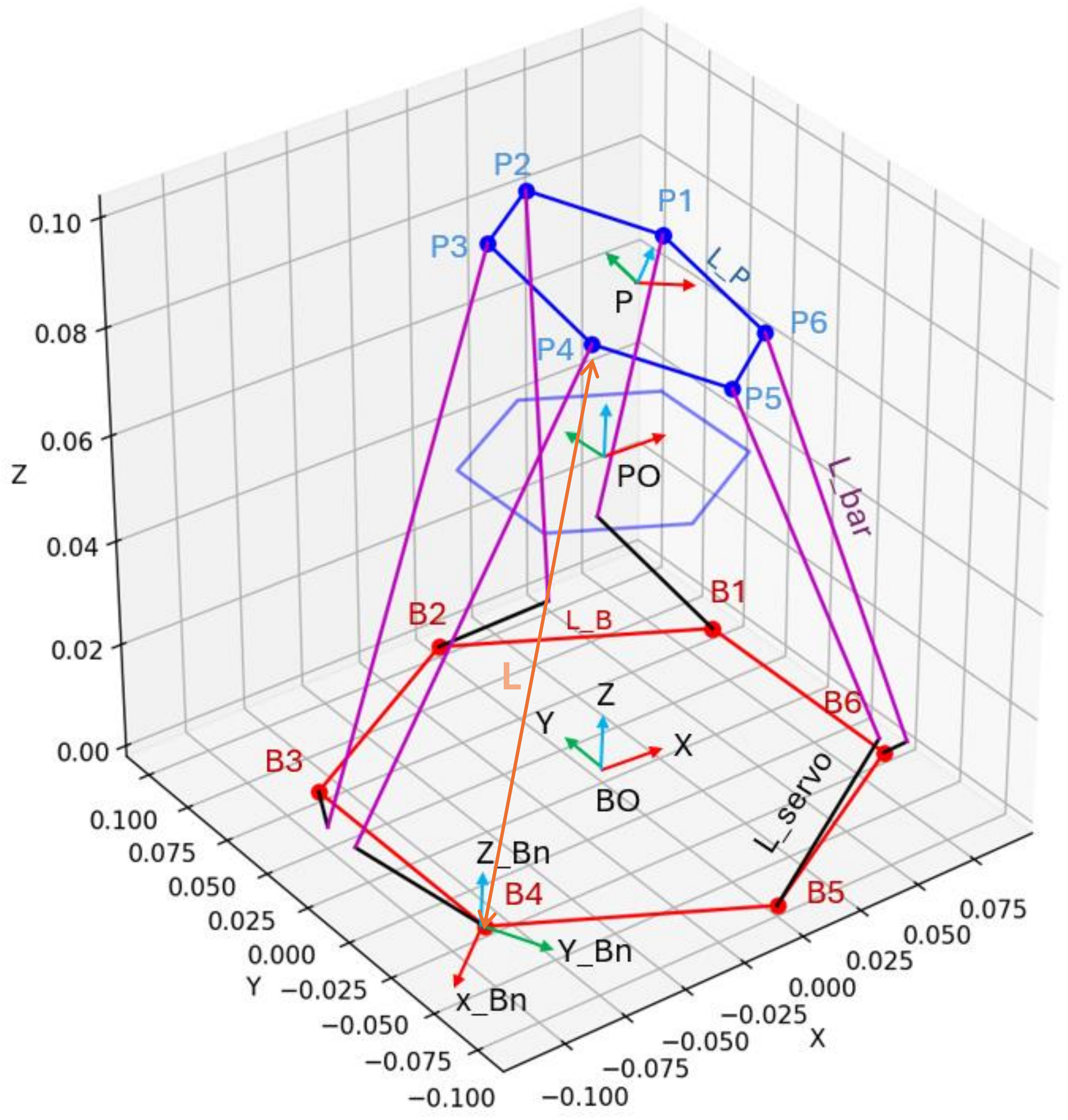
and,

$$ {}^{PO}\boldsymbol{p}_P = [X_P, Y_P, Z_P]^T = Func.Input $$

$$ {}^{PO}_{P}\boldsymbol{R} = Rot(roll_P, pitch_P, yaw_P) = Func.Input $$

$$ {}^{BO}\boldsymbol{p}_{PO} = [X_{PO}, Y_{PO}, Z_{PO}]^T = [0, 0, Z_{PO}]^T $$

$$ {}^{BO}_{PO}\boldsymbol{R} = Rot(roll_{PO}, pitch_{PO}, yaw_{PO}) = \mathrm{Rot}(0,0,0) $$



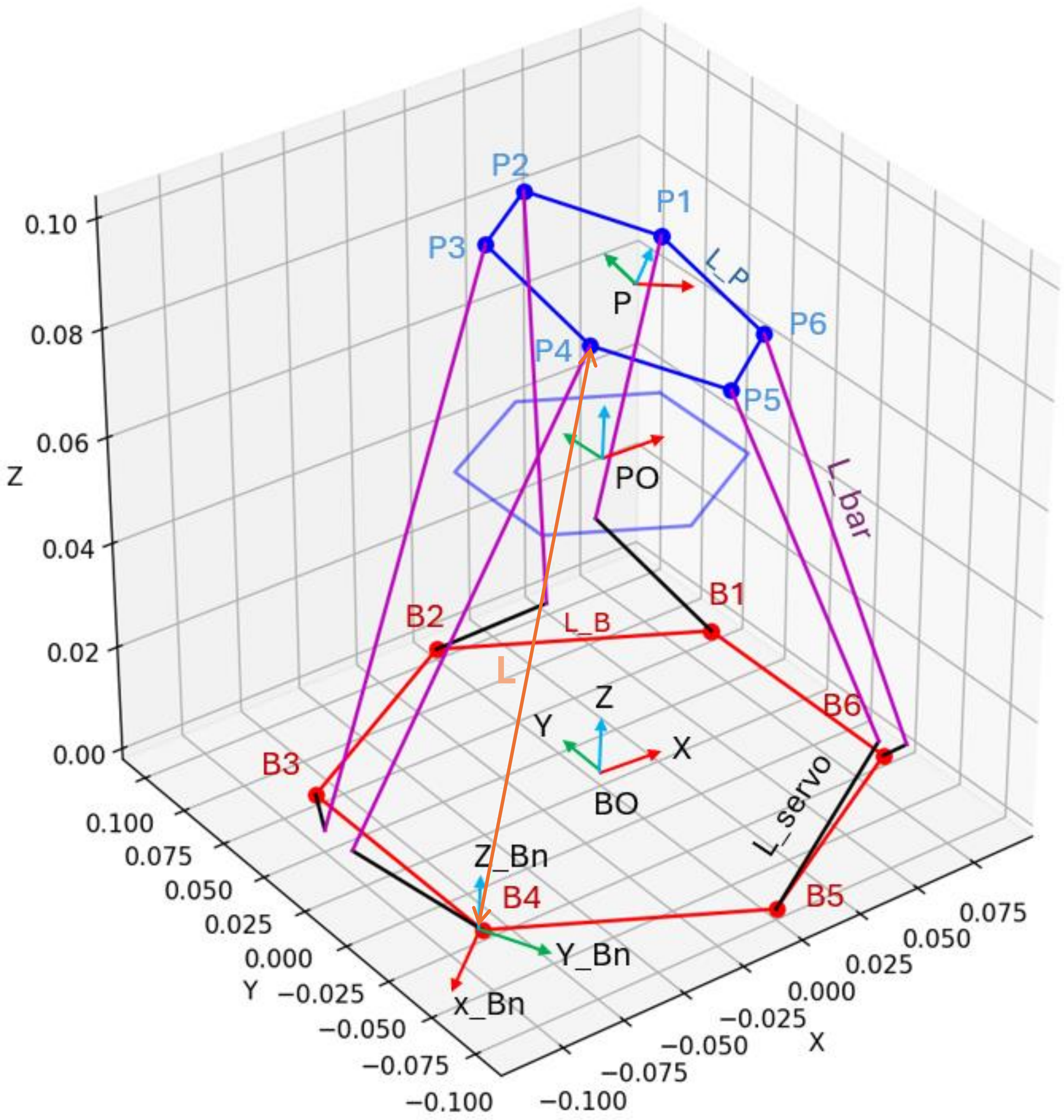
Calculations

Once ${}^{BO}_{P}\boldsymbol{T}$ is found, we can find platform position $\boldsymbol{p}_{Pn}$ described in the coordinate system of the base origin BO.

$$ {}^{BO}\boldsymbol{p}_{Pn} = {}^{BO}_{P}\boldsymbol{T} \cdot {}^{P}\boldsymbol{p}_{Pn}, \qquad for\ n \in \{1 \ldots 6\} $$

The position of base joints Bn relative to the base origin BO is predefined (i.e., dependent on design itself).

$$ {}^{BO}\boldsymbol{p}_{Bn} = [X_{Bn}, Y_{Bn}, Z_{Bn}]^{T}, \qquad for\ n \in \{1 \ldots 6\} $$



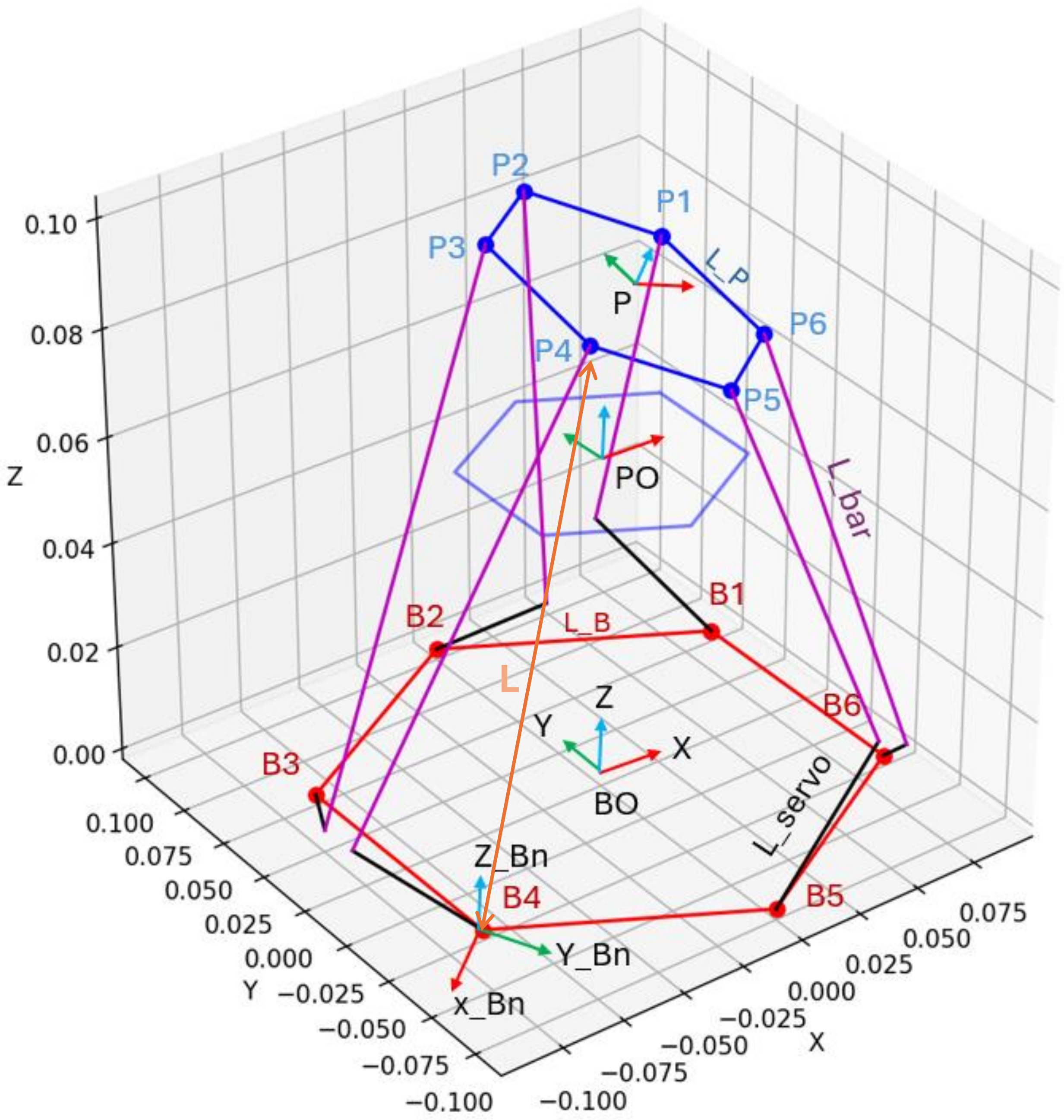
Calculations

2. Find the servo angle that achieves the distance between Bn and Pn while satisfying all required constraints

For linear actuators, the derived lengths between the platform joints Pn and the base joints Bn can be used.

$$L_n = |{}^{BO}\boldsymbol{p}_{Pn} - {}^{BO}\boldsymbol{p}_{Bn}| = norm({}^{BO}\boldsymbol{p}_{Pn} - {}^{BO}\boldsymbol{p}_{Bn})$$

However, for a Stewart Platform with servo motors, the offset due to the servo rotation must be considered.



Calculations

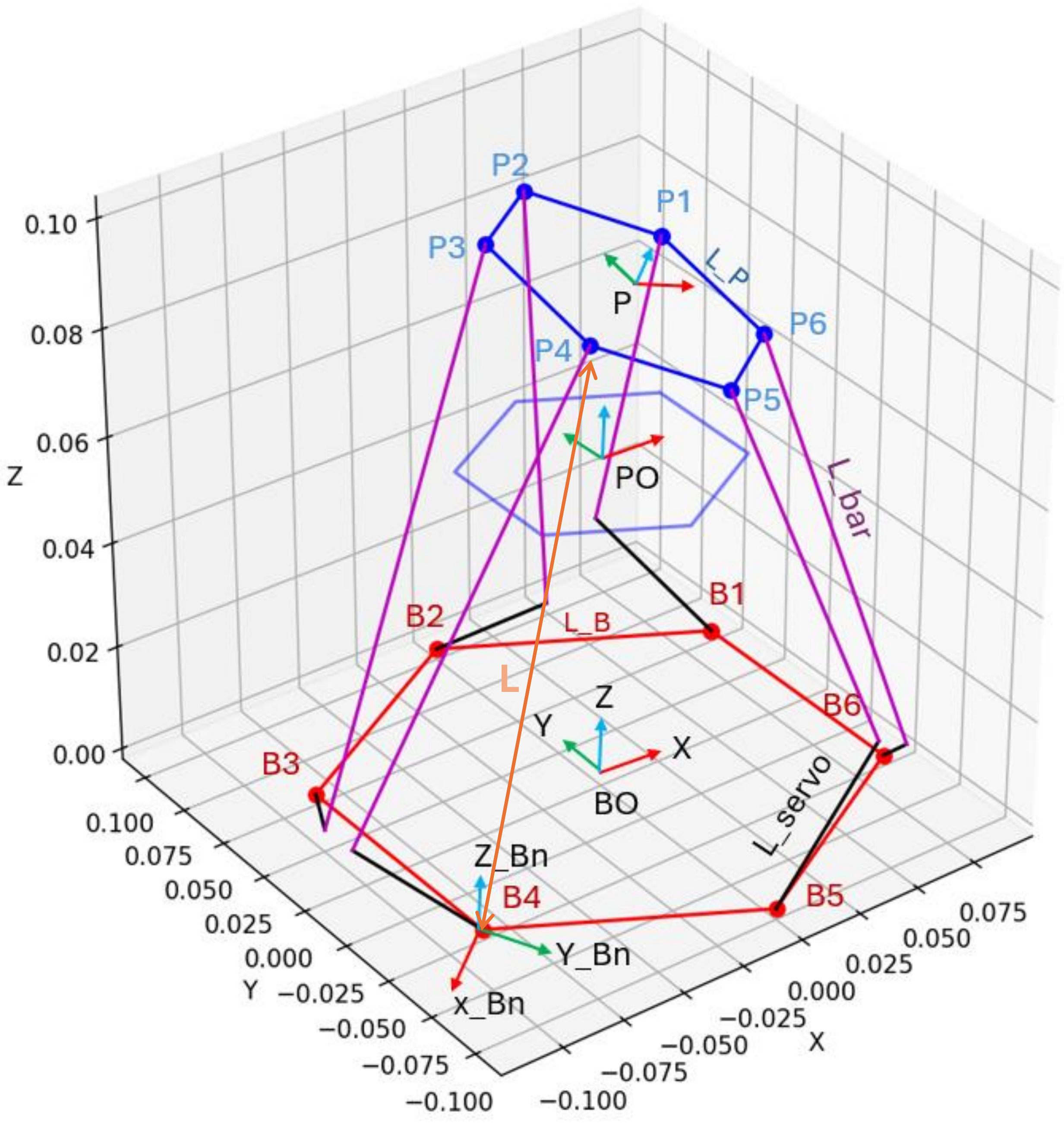
To find the required servo rotations, multiple physical constraints must be satisfied.

First, the length between the platform joint Pn and base joints Bn must satisfy the following as derived earlier.

$$L_n = |{}^{BO}\boldsymbol{p}_{Pn} - {}^{BO}\boldsymbol{p}_{Bn}| = |{}^{Bn}\boldsymbol{p}_{Pn} - {}^{Bn}\boldsymbol{p}_{Bn}|$$

Second, the servo arm can only be on the YZ plane of the base joints' coordinate systems. I.e., the servo rotates around the x-axis of base joint's x-axis.

$$y_{Bn}^2 + z_{Bn}^2 = L_{servo}^2$$



Calculations

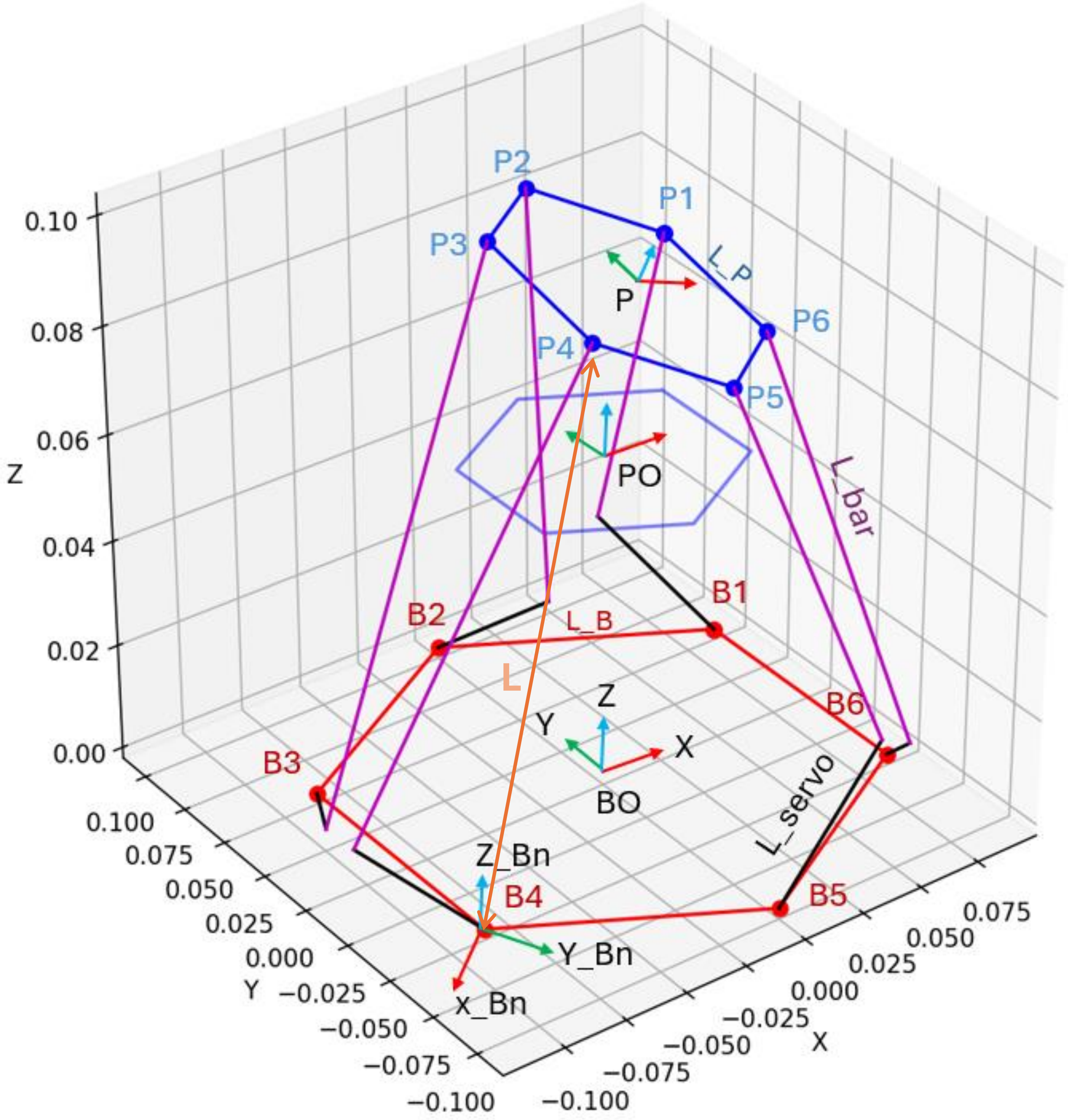
Third, since the length from the end-effector (tip of the servo arm) and the platform joint Pn is connected by a bar, this length must be constant.

$$(x_{Pn} - x_{Bn})^2 + (y_{Pn} - y_{Bn})^2 + (z_{Pn} - z_{Bn})^2 = L_{\text{bar}}^2$$

Where all coordinates are described relative to base joint Bn.

Finally, since the joint between the servo arm and bar can only be on the YZ plane of Bn,

$$x_{Bn} = 0$$



Calculations

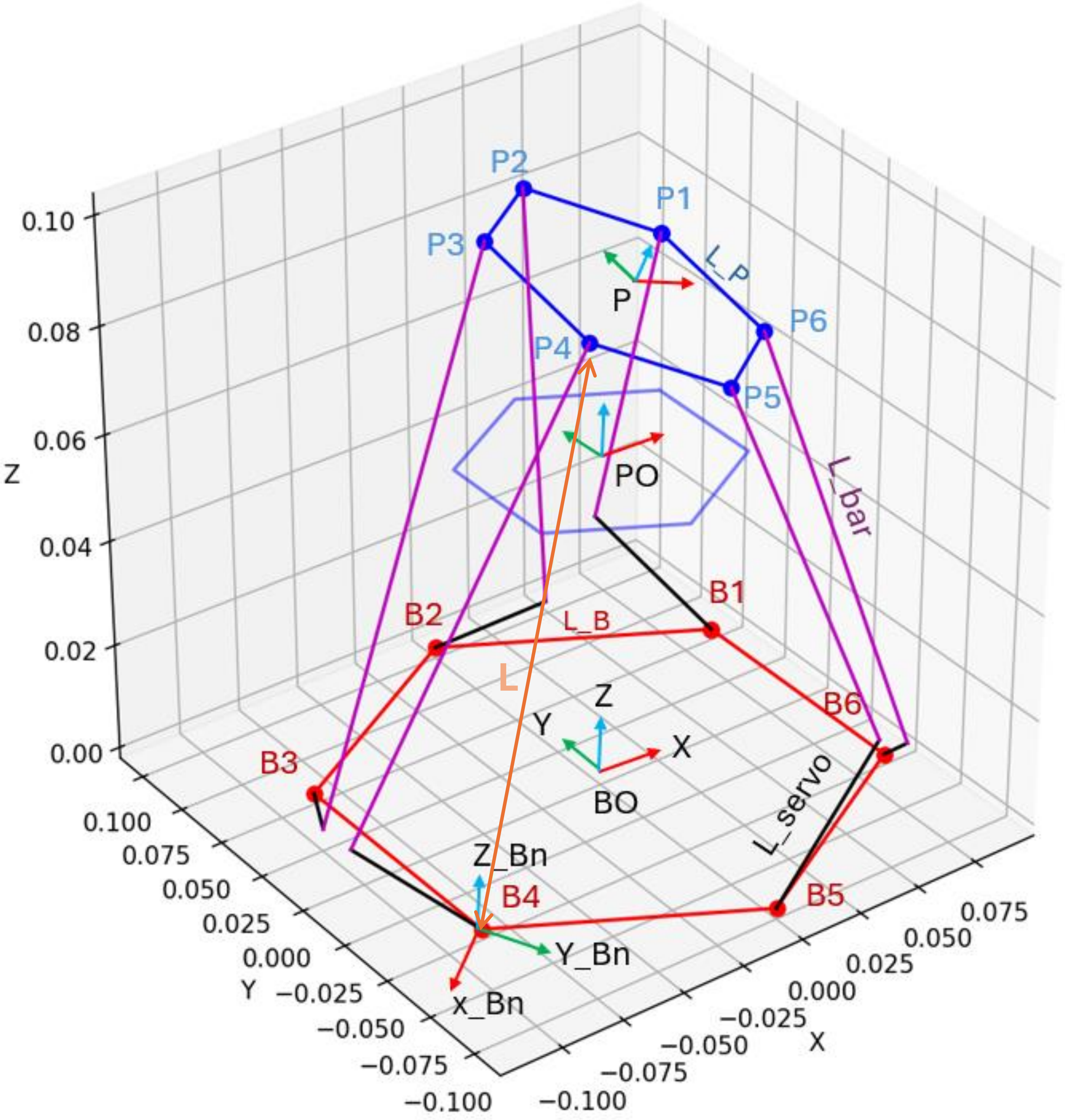
All constraints:

$$L_n = |{}^{BO}\boldsymbol{p}_{Pn} - {}^{BO}\boldsymbol{p}_{Bn}| = |{}^{Bn}\boldsymbol{p}_{Pn} - {}^{Bn}\boldsymbol{p}_{Bn}|$$

$$y_{Bn}^2 + z_{Bn}^2 = L_{servo}^2$$

$$(x_{Pn} - x_{Bn})^2 + (y_{Pn} - y_{Bn})^2 + (z_{Pn} - z_{Bn})^2 = \mathrm{L}_{\mathrm{bar}}^2$$

$$x_{Bn} = 0$$



Calculations

By substitution, following equations can be found.

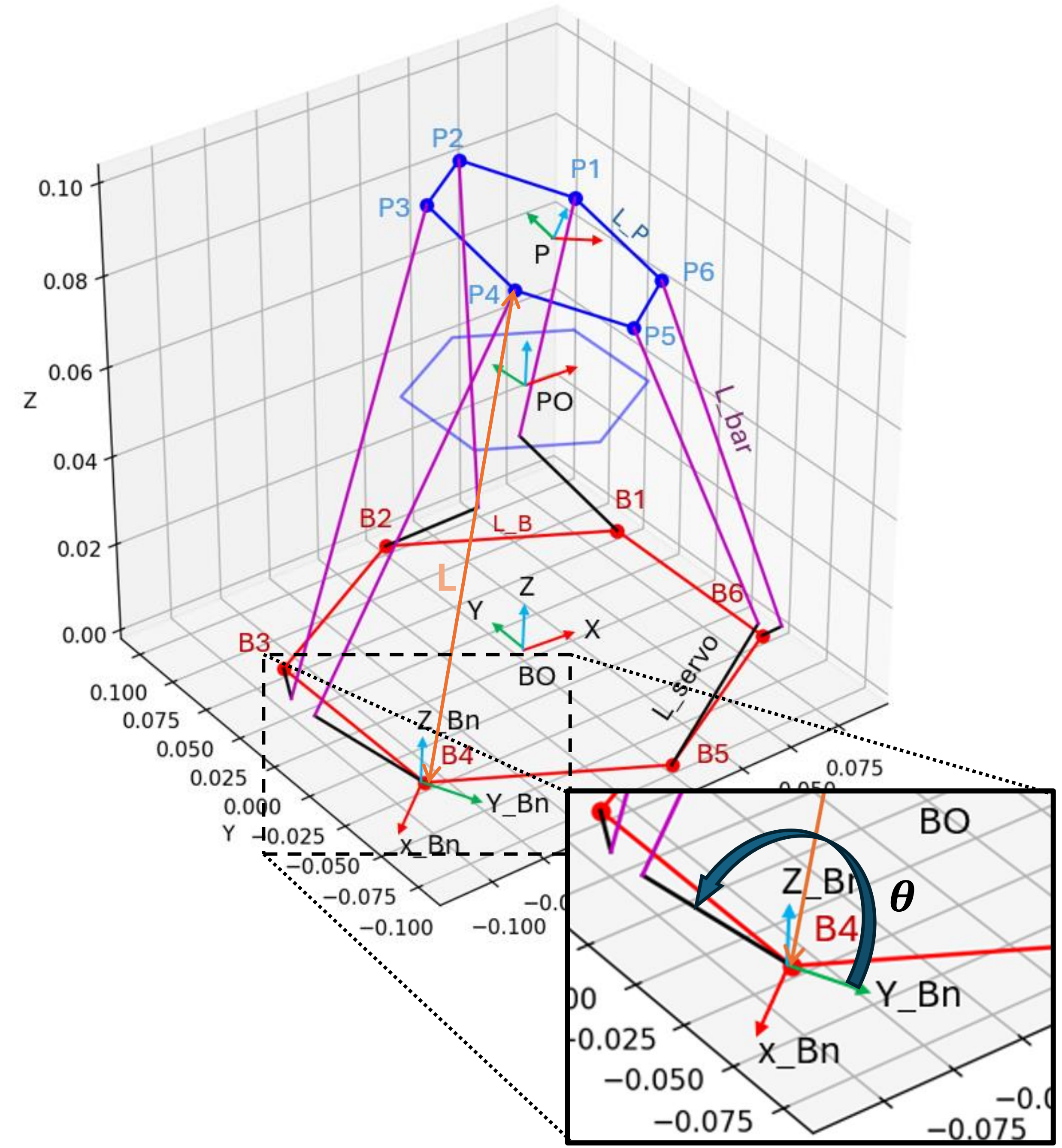
$$(4y_{Pn}^2 + 4z_{Pn}^2) \cdot y_{Bn}^2 + 4y_{Pn}D \cdot y_{Bn} + D^2 - 4z_{Pn}^2 L_{servo}^2 = 0$$

$$z_{Bn} = \sqrt{L_{servo}^2 - y_{Bn}^2}$$

Where,

$$D = L_{bar}^2 - L_{servo}^2 - x_{Pn}^2 - y_{Pn}^2 - z_{Pn}^2$$

Solving the quadratic equation, it is possible to find y_{Bn}, and by substituting , z_{Bn} can be found.



Calculations

Once y_{Bn} and z_{Bn} are found, check that the solution satisfy the final constraint,

$$L_n = |^{BO}\boldsymbol{p}_{Pn} - {}^{BO}\boldsymbol{p}_{Bn}| = |^{Bn}\boldsymbol{p}_{Pn} - {}^{Bn}\boldsymbol{p}_{Bn}|$$

Where,

$$^{Bn}\boldsymbol{p}_{Pn} = [x_{Pn}, y_{Pn}, z_{Pn}]^T$$

$$^{Bn}\boldsymbol{p}_{Bn} = [x_{Bn}, y_{Bn}, z_{Bn}]^T$$

Finally, the servo angles can be found by,

$$\theta = arctan\left(\frac{z_{Bn}}{y_{Bn}}\right)$$