Contents:

- Pre-Mid -

Date:

$e = x - x^n$ (Error)

$\bar{e} = |x - x^n|$ (Absolute)

$\tilde{e} = \frac{\bar{e}}{|x|} = \frac{|x - x^n|}{|x|}$ (Relative)

- Newton Raphson Method ⟷

- Bisection Method
- Regular Falsi Method
- Fixed Point Iterative Method
- Newton's Forward Interpolation.
- Newton's Backward Interpolation
- Divided Difference.
- Interpolation.
- Lagrange's Method.
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- Newton's Forward Differenciation
- Newton's Backward Differenciation.

Bisection Method

Example 1: $x^3 - x - 11 = 0$

Sol. $f(x) = x^3 - x - 11$

$\boxed{f(0) = -11}$ -ve

$f(1) = 1 - 1 - 11$

$\boxed{f(1) = -11}$ -ve

$f(2) = 8 - 2 - 11$

$\boxed{f(2) = -5}$ -ve

$f(3) = 27 - 3 - 11$

$\boxed{f(3) = 13}$ +ve

∴ The roots lies b/w $[2, 3]$

First Approximation

$c = \frac{2+3}{2} \Rightarrow \boxed{c = 2.5}$

$f(2.5) = (2.5)^3 + 2.5 - 11$

$\boxed{f(2.5) = 2.125}$ +ve first Approx.

The roots lie b/w $[\overset{-ve}{2}, \overset{+ve}{2.5}]$

Second Approx.

$c = \frac{2 + 2.5}{2}$

$c = 2.25$

$f(2.25) = (2.25)^3 + 2.25 + 11$

$f(2.25) = -1.8893$

roots lie b/w $[\overset{-ve}{2.25}, \overset{+ve}{2.5}]$

3rd Approx.

$c = \frac{2.25 + 2.5}{2}$

$c = \cancel{0.021}\ 2.375$

$f(2.375) = (2.375)^3 - 2.375 - 11$

$= \boxed{0.02148}$ +ve

roots lie

$[2.25, 2.375]$

4th Approx.

$c = \frac{2.25 + 2.375}{2}$

$c = 2.3125$

$f(2.3125) = 12.367 + 2.3125 - 11$

$= \boxed{-0.946}$ -ve

roots lie $[2.313, 2.375]$

Date: Numerical Analysis

5th Approx

$$c = \frac{2.313 + 2.375}{2}$$

$$c = \cancel{0.469} \quad 2.344$$

$$f(2.344) = (2.344)^3 - 2.344 - 1$$
$$= -0.469$$

roots lie b/w $[2.344, 2.375]$

Date:

Regular Falsi Method

$$\cancel{x_2 = f(x}$$

$$x_2 = x_0 - f(x_0) \cdot \frac{x_1 - x_0}{f(x_1) - f(x_0)}$$

$f(x) = x\log_{10} x - 1.2 \cancel{= 0}$

$f(1) = (1)\log_{10}(1) - 1.2 \cancel{= 0}$

$f(1) = -1.2$ (-ve)

$f(2) = (2)\log_{10}(2) - 1.2$

$f(2) = \boxed{-0.5979}$ (-ve)

$f(3) = 3\log_{10}(3) - 1.2$

$= \boxed{0.2313}$ x_0 (+ve)

$\begin{array}{cc} x_0 & x_1 \end{array}$

$[2, 3]$

roots lie b/w $[\cancel{-0.5979}, \cancel{0.2313}]$

First Appr -

$$x_2 = 2 - (-0.5979) \cdot \frac{3-2}{0.2313 + 0.5979}$$

$x_2 = 2 + 0.72105$

$x_2 = \cancel{+1.2789}$ $\boxed{+2.72105}$

Date:

Second Approx.

$$f(2.72105) = 2.72105 \log_{10}(2.72105) - 1.2$$

$$= \boxed{-0.017} \quad -ve$$

Updated intervals : $[\overset{x_0}{2.72105}, \overset{x_1}{3}]$

$$x_2 = x_0 - f(x_0) \cdot \frac{x_1 - x_0}{f(x_1) - f(x_0)}$$

$$x_2 = 2.72105 + 0.017 \cdot \frac{3 - 2.72105}{0.2313 + 0.017}$$

$$x_2 = 2.7401$$

Third Approx.

$$f(2.7401) = 2.7401 \log_{10} 2.7401 - 1.2$$

$$= \boxed{-0.0004} \quad -ve$$

Updated Interval : $[2.7401, 3]$

Date:

$$\overset{x_0}{2.7401}, \overset{x_1}{3}$$ [2.7401, 3]

$$x_2 = x_0 - f(x_0) \cdot \frac{x_1 - x_0}{f(x_1) - f(x_0)}$$

$$= 2.7401 + (-0.0004) \cdot \frac{3 - 2.7401}{0.2313 + 0.0004}$$

$$\cancel{\Rightarrow 1.1217 + 2.7401 \; (0.0004)}$$

$$\boxed{x_2 \Rightarrow 2.7405}$$

$$f(2.7405) = 2.7405 \log_{10} 2.7405 - 1.2$$

$$= -0.0001$$

Updated Intervals = [2.7405 , 3]

Roots lie b/w

Date:

Newton's Raphson Method

$$\cancel{x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}}$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

Q.

$f(x) = x^4 - x - 10 = 0$

$f'(x) = 4x^3 - 1 = 0$

Lets Assume $x_i = 0$ initially,

$f(0) = (0)^4 - (0) - 10$
$= -10$

$f(1) = (1)^4 - 1 - 10$
$= -10 \rightarrow -ve$

$f(2) = (2)^4 - 2 - 10$
$= 16 - 2 - 10$
$= +4$ +ve

$\cancel{f(3) = (3)^4 - 3 - 10}$
$\cancel{= 27 - 3 - 10}$
$\cancel{= 14 \rightarrow +ve}$

roots = $[\overset{-ve}{1}, \overset{+ve}{2}]$

$\boxed{x_0 = 2}$

~~First Approx.~~

$\cancel{x_1 = 2 - \frac{4}{256 - 1}}$

$\cancel{x_1 = 2 - \frac{4}{255}}$

$\cancel{\frac{510 - 4}{255}}$

$\cancel{x_1 = \frac{506}{255}}$

$\cancel{x_1 = 1.98431}$

closer to zero will be taken as x_0.

First Approx.

$x_1 = 2 - \frac{4}{32 - 1}$

$x_1 = \frac{62 - 4}{31}$

$\boxed{x_1 = 1.8717}$

Date:

$f(1.871) = (1.871)^4 - (1.871) - 10$

$= 0.3834$

$f'(x_1) = 4(1.871)^3 - 1$

$= 25.198$

$f'(x_2) = 4(1.856)^3 - 1$

$= 24.5737$

$f(1.85\cancel{57}) = (1.856)^4 - (1.856) - 10$

$= 0.0102$

M. Zain محمد زین

STAEDTLER

Second Approx.

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$x_2 = 1.871 - \frac{0.3834}{25.198}$

$\cancel{x_2} = \cancel{\frac{47.1454 - 0.3834}{25.198}}$

$x_2 = 1.871 - 0.0152$

$\boxed{x_2 = 1.8557}$

Third Approximation

$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$

$x_3 = \cancel{1.871}\ 1.856 - \frac{(+0.010)}{24.573}$

$\boxed{x_3 = \cancel{1.8705}}$

1.8555

If 4 decimal places, the round off until 5 decimal places.

Fixed Point Iterative Method

$$\sin x = 5x - 2$$

Example 2

~~$x_0 = 0.5$~~

Let,

$$x = g(x) \longrightarrow (1)$$

$$x_{n+1} = g(x_n) \longrightarrow (2)$$

Given,

$$x_0 = 0.5$$

Q. → $\sin x = 5x - 2$

$$5x = \sin x + 2$$

$$x = \frac{\sin x + 2}{5} \longrightarrow (A)$$

1st Approx:

$$x_{n+1} = \frac{\sin x_n + 2}{5} \quad (\text{From } (2))$$

$$x_{0+1} = \frac{\sin x_0 + 2}{5}$$

In Radian only →

$$x_1 = \frac{\sin(0.5) + 2}{5} = \boxed{0.49588}$$

2nd Approx.

$$x_2 = \frac{\sin(0.49588) + 2}{5}$$

$$x_2 = \boxed{0.49516}$$

3rd Approx.

$$x_3 = \frac{\sin(0.49516) + 2}{5}$$

$$\boxed{x_3 = 0.49503}$$

4th Approx.

$$x_4 = \frac{\sin(0.49503) + 2}{5}$$

$$x_4 = \boxed{0.49501}$$

So,

$x_4 = 0.4950$ is the root of equation

$\sin x = 5x - 2$

INTERPOLATION - Newton's forward & Backward Interpolation formula.

Date:

Example 1

	0	1	2	3	4
x	1891	1901	1911	1921	1931
y	46	66	81	93	101

$x = 1895$

Newton's forward difference table

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
1891	46				
		$66-46 = 20$			
1901	66		-5		
		$81-66 = 15$		2	
1911	81		-3		$-1-2 = -3$
		$93-81 = 12$		-1	
1921	93		-4		
		$101-93 = 8$			
1931	101				

$h = x_1 - x_0 = 1901 - 1891 = \boxed{10}$ ✓

→ See from slide ahead

Date:

Example 2

$\boxed{x = 4}$?

Forward

	x	$f(x)$
0	3	180
1	5	150
2	7	120
3	9	90

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
3	180			
		-30		
5	150		0	
		-30		0
7	120		0	
		-30		
9	90			

$h = x_1 - x_0 = 5 - 3 = \boxed{2}$

$p = \frac{x - x_0}{h} = \frac{4-3}{2} = \boxed{0.5}$

$$y(x) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\cdot\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\cdot\Delta^3 y_0$$

$$+ \frac{p(p-1)(p-2)(p-3)}{4!}\cdot\Delta^4 y_0$$

$y(x) = 180 + 0.5(-30)$

$y =$

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Newton's Backward Difference formula

$$h = x_1 - x_0 \qquad\qquad p = \frac{x - x_n}{h}$$

$$y(x) = y_n + p\nabla y_n + \frac{p(p+1)}{2!}\cdot\nabla^2 y_n + \frac{p(p+1)(p+2)}{3!}\cdot\nabla^3 y_n$$

$$+ \frac{p(p+1)(p+2)(p+3)}{4!}\cdot\nabla^4 y_n$$

$$y(x) = y_n + p\nabla y_n$$

Newton's Divided Difference.

Date:

	x_0	x_1	x_2	x_3
x	300	304	305	307
$y = f(x)$	2.4771	2.4829	2.4843	2.4871
	y_0	y_1	y_2	y_3

$x = 301$

x	y	1st order	2nd order
300	2.4771		
		$\frac{2.4829 - 2.4771}{304 - 300} = 0.0014$	
304	2.4829		$\frac{0.0014 - 0.0014}{305 - 300} = \frac{0}{5} = 0$
		$\frac{2.4843 - 2.4829}{305 - 304} = 0.0014$	
305	2.4843		$\frac{0.0014 - 0.0014}{307 - 304} = \frac{0}{3} = 0$
		$\frac{2.4871 - 2.4843}{307 - 305} = 0.0014$	
307	2.4871		

Newton's Divided Difference Formula:

$$f(x) = y_0 + (x - x_0) f[x_0, x_1] + (x - x_0)(x - x_1) f[x_0, x_1, x_2]$$

$$f(301) = 2.4771 + (301 - 300) \times 0.0014 + (301 - 300)(300 - 304) \times 0$$

$$f(301) = 2.4771 + 1(0.0014) = 2.4785$$

$$y = 2.4785$$

HUAWEI

Langrage's Interpolation

Newton's Divided Difference Table

$$f[x_0, x_1] = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

*

$\boxed{\text{3rd order karna he}}$

Langrange's ~~Inverse~~ Interpolation

$$\cancel{f(y)}$$

$$f(x) = \frac{(x-x_1)(x-x_2)\cdots(x-x_n)}{(x_0-x_1)(x_0-x_2)\cdots(x_0-x_n)} y_0 +$$

$$\frac{(x-x_0)(x-x_2)(x-x_n)}{(x_1-x_0)(x_1-x_2)(x_1-x_n)} y_1 +$$

$$\frac{(x-x_0)(x-x_1)\cdots(x-x_n)}{(x_2-x_0)(x_2-x_1)\cdots(x_2-x_n)} y_2$$

Time (t) ⇒ Independent (x)
velocity (u) ⇒ Dependent (y)

Date:

t	0.0	1.0	1.5	2.0
u	2,5	3.8	4,6	5.3

Velocity = u = 2.75 (~~Independent~~ dependent)
y =

By Lagrange's inverse

x	y
t	u
0.0	2,5
1.0	3,8
1.5	4,6
2.0	5,3

Langrange's Inverse Interpolation (Same as Langrange's Interpolation)

↓

Formula:

$$f(y) = \frac{(y-y_1)(y-y_2)(y-y_3)}{(y_0-y_1)(y_0-y_2)(y_0-y_3)} \times x_0 + \frac{(y-y_0)(y-y_2)(y-y_3)}{(y_1-y_0)(y_1-y_2)(y_1-y_3)} \times x_1$$

$$+ \frac{(y-y_0)(y-y_1)(y-y_3)}{(y_2-y_0)(y_2-y_1)(y_2-y_3)} \times x_2 + \frac{(y-y_0)(y-y_1)(y-y_2)}{(y_3-y_0)(y_3-y_1)(y_3-y_2)} \times x_3$$

$$f(2.75) = \frac{(2.75-3.8)(2.75-4.6)(2.75-5.3)}{(2.5-3.8)(2.5-4.6)(2.5-5.3)} \times 0 + \frac{(2.75-2.5)(2.75-4.6)(2.75-5.3)}{(3.8-2.5)(3.8-4.6)(3.8-5.3)} \times 1$$

$$+ \frac{(2.75-2.5)(2.75-3.8)(2.75-5.3)}{(4.6-2.5)(4.6-3.8)(4.6-5.3)} \times \cancel{4.6}\,1.5 + \frac{(2.75-2.5)(2.75-3.8)(2.75-4.6)}{(5.3-2.5)(5.3-3.8)(5.3-4.6)} \times 2$$

Date:

$f(2.75) = 8 + 0.756 + 0.855 +$

Incomplete Sol.

$$f(y) = \frac{(y-y_1)(y-y_2)(y-y_3)}{(y_0-y_1)(y_0-y_2)(y_0-y_3)} \times x_0$$