

## Agenda

- 1) Revision quizzes
- 2) unique element (rest all coming thrice)
- 3) every element will occur twice except two de

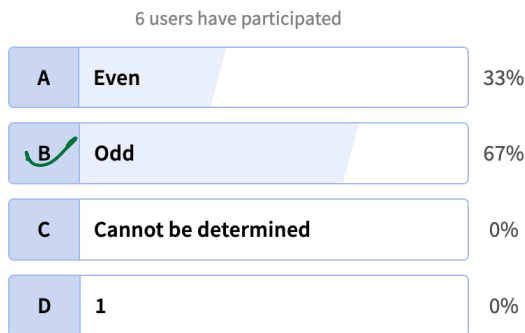
} very important

2.5 hrs

class notes  
revision → 25 mins step

solving assign

If the last bit(0<sup>th</sup> bit) of a number is 1 (rest can be anything), then the number is?



25 → 11001 =

24 → 11000 =

What is the time complexity of the operation to check whether the  $i$ -th bit of a number with  $N$  bits is set or not?

no. A

→ check  $i$ th bit :  $A \& (1 \ll i)$    
  $\swarrow$  0 (odd)   
  $\searrow$  non-zero (on)

→ set  $i$ th bit :  $A | (1 \ll i)$

→ toggle  $i$ th bit :  $A \wedge (1 \ll i)$

What is the time complexity to count the total number of set bits of an integer with  $N$  bits?

T.C:  $O(\text{no. of bits})$

↓

$O(n)$

(Based on last class ques logic)

↓

no. bits = 32

32 iterations  $\Rightarrow O(1)$

Q. Given  $A[]$ , all elements occurs thrice except one element. Find the unique element.

$A = [4, 5, 5, 4, 1, 6, 6, 4, 5, 6]$

ans = 1

$A = [5, 7, 5, 9, 7, 11, 11, 7, 5, 11]$

ans = 9

→ Brute-force : two loops

for (go to every element)

TC:  $O(n^2)$

for (find freq of ele by travelling complete)

Demand : TC  $\rightarrow O(n)$

SC  $\rightarrow O(1)$

$$A = [5 \ 7 \ 5 \ 9 \ 7 \ 11 \ 11 \ 7 \ 5 \ 11]$$

0    1    2    3    4    5    6    7    8    9

$$ans = \overset{2}{\cancel{0}} \overset{1}{\cancel{0}} \overset{1}{\cancel{0}} \overset{1}{\cancel{0}}$$

$$= \underline{9}$$

$$5 \rightarrow \overset{3}{0} \overset{2}{1} \overset{1}{0} \overset{0}{1}$$

$$7 \rightarrow 0 \ 1 \ 1 \ 1$$

$$5 \rightarrow 0 \ 1 \ 0 \ 1$$

$$9 \rightarrow 1 \ 0 \ 0 \ 1$$

$$7 \rightarrow 0 \ 1 \ 1 \ 1$$

$$11 \rightarrow 1 \ 0 \ 1 \ 1$$

$$11 \rightarrow 1 \ 0 \ 1 \ 1$$

$$7 \rightarrow 0 \ 1 \ 1 \ 1$$

$$5 \rightarrow 0 \ 1 \ 0 \ 1$$

$$11 \rightarrow 1 \ 0 \ 1 \ 1$$

---


$$4 \ 6 \ 6 \ 10$$

$$1 \ 0 \ 0 \ 1$$

$$\hookrightarrow 9$$

0 to 3)

$$\underline{\underline{0^{th} \text{ bit}}} \Rightarrow 10 \ (10 \% 3 = 1)$$

$$\underline{\underline{1^{st} \text{ bit}}} \Rightarrow 6 \ (6 \% 3 = 0)$$

$$\underline{\underline{2^{nd} \text{ bit}}} \rightarrow 6 \ (6 \% 3 = 0)$$

$$\underline{\underline{3^{rd} \text{ bit}}} \rightarrow 4 \ (4 \% 3 = 1)$$

↖ N

function uniqueNum (A[])

ans = 0;

for (i → 0 to 31) {

count = 0;

// in how many elements of A[], i<sup>th</sup> bit is "on"

for (j → 0 to N-1) {

if ((A[j] & (1 << i)) != 0) {

count++;

}

}

if (count % 3 == 1) {

// set i<sup>th</sup> bit in ans

ans = ans | (1 << i);

}

}

return ans;

TC:  $O(N)$

SC:  $O(1)$

}

function uniqueNum (A[]) <sup>N</sup>

ans = 0;

for (i → 0 to 31) {

count = 0;

// in how many elements of A[], i<sup>th</sup> bit is "on"

for (j → 0 to N-1) {

if ((A[j] & (1 << i)) != 0) {

count++;

}

}

if (count % 3 == 1) {

// set i<sup>th</sup> bit in ans

ans = ans | (1 << i);

}

}

return ans;

A = [ 8 5 8 8 6 5 5 ]

8 → <sup>3 2 1 0</sup> 1 0 0 0

5 → 0 1 0 1

8 → 1 0 0 0

8 → 1 0 0 0

6 → 0 1 1 0

5 → 0 1 0 1

5 → 0 1 0 1

count : 3 4 1 3

ans = <sup>1 1</sup> 0 0 0 0

= 6

to do : dry run this  
eg. using code

A = [ 5 7 5 9 7 11 11 7 5 11 ]

Q. Given an  $Arr$  in which all elements will occur twice except two elements. Find those two elements.

$$A = \begin{bmatrix} 4 & 5 & 4 & 1 & 6 & 6 & 5 & 3 \end{bmatrix} \Rightarrow 1 \quad 3$$

<sub>0    1    2    3    4    5    6    7</sub>

$$A = \begin{bmatrix} 9 & 5 & 6 & 7 & 6 & 2 & 7 & 5 \end{bmatrix} \Rightarrow 2 \quad 9$$

<sub>0    1    2    3    4    5    6    7</sub>

Brute force :  $O(n^2)$  {two loops}

$\downarrow$  better  
 $O(n \log n)$  {sorting}

$$A = \begin{bmatrix} 9 & 5 & 6 & 7 & 6 & 2 & 7 & 5 \end{bmatrix}$$

<sub>0    1    2    3    4    5    6    7</sub>

$$\begin{aligned} \text{xorAll} &= 9 \wedge \cancel{5} \wedge \cancel{6} \wedge \cancel{7} \wedge \cancel{6} \wedge 2 \wedge \cancel{7} \wedge \cancel{5} \\ &= 9 \wedge 2 \\ &= 11 \end{aligned}$$

xor of two numbers, find those numbers.

|     |         |      |
|-----|---------|------|
| xor | 1001    | (a)  |
|     | 0010    | (2)  |
|     | 1011    | (11) |
|     | 3 2 1 0 |      |

$\rightarrow$  0<sup>th</sup> bit in one of the no. is on and in other no. it off.

$$9 \wedge 5 \wedge 7 \wedge 7 \wedge 5 = 9$$

A = [9 5 6 7 6 2 7 5]

0    1    2    3    4    5    6    7

green  $\rightarrow$  0<sup>th</sup> bit is on

$$6 \wedge 6 \wedge 2 = 2$$

red  $\rightarrow$  0<sup>th</sup> bit is off

$\rightarrow$  A = [9 5 6 7 6 11 7 5]

0    1    2    3    4    5    6    7

i) xorAll = xor of entire array =  $9 \wedge 5 \wedge 6 \wedge 7 \wedge 6 \wedge 11 \wedge 7 \wedge 5$

$= 9 \wedge 11$

$= 2$

$$\begin{array}{r} 1001 \\ \text{xor } 1011 \\ \hline 0010 \\ \text{3 2 1 0} \end{array}$$

ii) In variable "xorAll" find the first bit which is on

0 to 31  $\Rightarrow$  ans: 1

iii) categorising the array in two parts

ele in which  
1<sup>st</sup> bit is on

ele in which  
1<sup>st</sup> bit is off



$$A = \begin{bmatrix} \textcircled{9} & \textcircled{5} & \textcircled{6} & \textcircled{7} & \textcircled{6} & \textcircled{11} & \textcircled{7} & \textcircled{5} \\ 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{bmatrix} \quad \dots \quad \begin{array}{c} \text{---} \\ 3 \end{array} \quad \begin{array}{c} \text{---} \\ 2 \end{array} \quad \begin{array}{c} \text{---} \\ 1 \end{array} \quad \begin{array}{c} \text{---} \\ 0 \end{array}$$

$$\text{num1} = 6^7 \cdot 6^{11} \cdot 7 = 14$$

(1<sup>st</sup> bit is on)

$$\text{num2} = 9^5 \cdot 5 = 9$$

(1<sup>st</sup> bit is off)

function solve (A[]) {  
 ↗ N

1) find xor of all elements

xorAll = 0;

for (i → 0 to N-1) {

| xorAll = xorAll ^ A[i];  
 }  
3

1) in xorAll, find the pos<sup>n</sup> of rightmost set bit

pos = -1;

for (i → 0 to 31) {

| if ( (xorAll & (1 << i)) != 0 ) {  
 | pos = i;  
 | break;  
 }  
 }  
3 3

1) categorise array into 2 parts based of bit at pos

num1 = 0; // contribution of ele in which bit at pos is on

num2 = 0; // contribution of ele in which bit at pos is off

for (i → 0 to N-1) {

| if ( (A[i] & (1 << pos)) != 0 ) {  
 | num1 = num1 ^ A[i];  
 }  
 | else {  
 | num2 = num2 ^ A[i];  
 }  
 }  
3 3

print (num1);

print (num2);

T.C :  $O(N)$

S.C :  $O(1)$

}

function solve (A[]) {

1) find xor of all elements

xorAll = 0;

for (i → 0 to N-1) {

    xorAll = xorAll ^ A[i];

2) in xorAll, find the pos<sup>n</sup> of rightmost set bit

pos = -1;

for (i → 0 to 31) {

    if ((xorAll & (1 << i)) != 0) {

        pos = i;

        break;

    }

3) categorise array into 2 parts based of bit at pos

num1 = 0; // contribution of ele in which bit at pos is on

num2 = 0; // contribution of ele in which bit at pos is off

for (i → 0 to N-1) {

    if ((A[i] & (1 << pos)) != 0) {

        num1 = num1 ^ A[i];

    }

    else {

        num2 = num2 ^ A[i];

    }

print (num1);

print (num2);

}

A = [ 5 9 5 6 6 2 13 2 ]

0 1 2 3 4 5 6 7

$$2) \text{ xorAll} = 5 \oplus 9 \oplus 5 \oplus 6 \oplus 6 \oplus 2 \oplus 13 \oplus 2$$

$$= 9 \oplus 13 = 4$$

$$9 \rightarrow 1001$$

$$13 \rightarrow 1101$$


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$$4 \rightarrow 0100$$

3 2 1 0

$$2) \text{ pos} = 2$$

3) A = [ 5 9 5 6 6 2 13 2 ]

0 1 2 3 4 5 6 7

$$\text{num1} = 5 \oplus 9 \oplus 5 \oplus 6 \oplus 6 \oplus 2 \oplus 13 \oplus 2 = 13$$

$$\text{num2} = 9 \oplus 2 \oplus 2 = 9$$

$$3 \oplus 2 \oplus 0$$

to do: dry run on one eg.