Middle East Technical University
Department of Computer Engineering

# CENG 111

Fall '07

Worksheet 2

## REGULATIONS

**Due date:** 17 December, Monday *(Not subject to postpone)*

**Submission:** Electronically. You will be submitting your program source code through a file which you will name as `hw2.scm` and submit through the cow system. The procedure of submission will be announced on the course's news (tin) group. Resubmission is allowed (till the last moment of the due date), the last will replace the previous, provided you answer the interactive question positively.

**Team:** There is **no** teaming up. The homework has to be done/turned in individually.

**Cheating:** All parts involved (source(s) and receiver(s)) get zero.

## PROBLEM

All of us know what matrix algebra is. This homework is about implementing matrix algebra over the rational number domain. You will be writing a function that you will name `mateval`. `mateval` will take an sexpr very similar to mathematical expressions that you are used to. The difference is that now matrices and rational numbers are also accepted in the mathematical operations. The mathematical operations allowed are `+`, `-`, `*`. It is possible to multiply matrices by rationals or integers. Addition/subtraction of rationals or integers to/from matrices is forbidden. Rules for matrix algebra has to hold (like the requirement of dimensional equality in addition/subtraction, etc.).

- The representation of rational numbers is a list of two members: The integer value of the numerator, followed by a positive integer value, different, than 1 of the denominator. Our rational numbers do not include integers. Also, per se, our rationals have any common multiplier removed from their numerator and denominator. In other word, the greates common divisor of the numerator and the denominator is 1. Examples:

  `(4 13)` is representing $\frac{4}{13}$

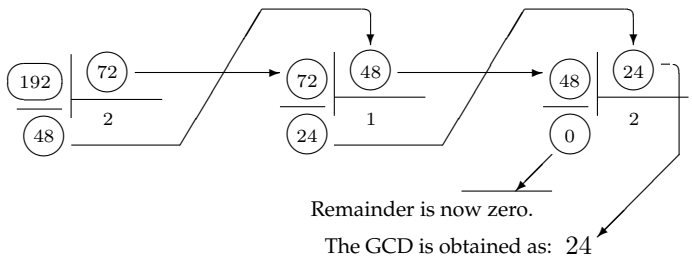
  `(-1 2)` is representing $-\frac{1}{13}$

  `(1 -2)` is illegal.

  `(6 1)` is illegal.

  `(12 36)` is illegal.

  You shall use the following algorithm for finding the GCD *(greatest common divisior)*. Let us assume we want to find GCD(192,72):




- The representation for matrices is a list of lists (each of which represent a row).
  The element of a matrix can be any one of an integer or rational. A $1 \times 1$ matrix is invalid.

Example:

| | |
|---|---|
| `((2 -3) (1 5))` | $\begin{bmatrix} 2 & -3 \\ 1 & 5 \end{bmatrix}$ |
| `((2 (-1 2)) ((7 3) 5))` | $\begin{bmatrix} 2 & -\frac{1}{2} \\ \frac{7}{3} & 5 \end{bmatrix}$ |
| `((-8 (-1 2) (3 25)) ((7 3) (1 5) (-9 23)))` | $\begin{bmatrix} -8 & -\frac{1}{2} & \frac{3}{25} \\ \frac{7}{3} & \frac{1}{5} & -\frac{9}{23} \end{bmatrix}$ |
| `((2 3))` | $\begin{bmatrix} 2 & 3 \end{bmatrix}$ |
| `((2) (3))` | $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ |
| `(((2 3) 1))` | $\begin{bmatrix} \frac{2}{3} & 1 \end{bmatrix}$ |

# SPECIFICATION

- You will name the function that does the evaluation as `mateval`.
- $+$, $-$, $*$ operations among rationals and/or integers will be implemented.
- Matrix operation of $+$, $-$, $*$ will be implemented. $+$, $*$ can be nary except unary (unary usage of $+$, $*$ is forbidden). $-$ is strictly binary. In addition to operations among matrices, multiplication of a matrix by a rational or an integer is allowed.
- There is no restriction on the size of matrices except that $1 \times 1$ matrices are invalid.
- You shall try to write short and concise functions. This will influence the grade you will get.
- The rules that define a rational (in the previous section) has to hold for the value returned by `mateval`. In other words any rational returned as a value or sub part of the value of a `mateval` call, has to comply fully with the definition of a rational.
- The input, your functions will be tested with, will be error-free. You do not have to perform any error check. Also no $1 \times 1$ matrix will be a result.
- Do not use floating points ever.

# BONUS

If you implement the inverse operation for a non singular square matrix which has integer or rational elements you will get and additional 50% bonus. If you decide to do so you have to name the function as `matinv`. You are free to chose among matrix inversion algorithms.

# EXAMPLE

$$(1-\frac{1}{3}) \times \begin{bmatrix} 2 & -\frac{1}{2} \\ \frac{7}{3} & 5 \end{bmatrix} \times \begin{bmatrix} -8 & -\frac{1}{2} & \frac{3}{25} \\ \frac{7}{3} & \frac{1}{5} & -\frac{9}{23} \end{bmatrix} + \begin{bmatrix} 2 & 3 & -1 \\ \frac{1}{2} & \frac{1}{3} & -1 \end{bmatrix} = \begin{bmatrix} -\frac{85}{9} & -\frac{34}{15} & -\frac{408}{575} \\ -\frac{25}{6} & \frac{2}{9} & -\frac{3653}{1725} \end{bmatrix}$$

```
> (mateval '(+ (* (- 1 (1 3))
                  ((2 (-1 2)) ((7 3) 5))
                  ((-8 (-1 2) (3 25)) ((7 3) (1 5) (-9 23))))
               ((2 3 -1) ((1 2) (1 3) -1)) ))
(((-85 9) (34 15) (-408 575)) ((-25 6) (2 9) (-3653 1725)))
```