Statistical Signal Processing
A.A. 2017/2018

Computer Lab 4 – Kalman filter

Duration: 3 hours

In this lab, you are provided with the set of observed coordinates (x,y – horizontal and vertical) of an object moving on a 2D plane at approximately constant velocity. Your task is to track the object using a Kalman filter, i.e. estimate the next (x,y) position of the object, from the observation of the previous positions.

Simulating the motion:

In order to be able to change the parameters of the motion generation (and particularly the observation noise), you are provided with the Matlab code for generating the motion trajectory. The code is based on the following model.

- The state vector contains coordinates and velocities: $\mathrm{z}_t^T = (\mathrm{z}_{1t}\ \mathrm{z}_{2t}\ \mathrm{v}_{1t}\ \mathrm{v}_{2t})$ (see slides). The object has initial coordinates (0,0) and velocity (Δ, Δ).
- Only the coordinates (but not the velocities) are observed

This leads to a linear dynamical system with:

$$\boldsymbol{A} = \begin{pmatrix} 1 & 0 & \Delta & 0 \\ 0 & 1 & 0 & \Delta \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\boldsymbol{C} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

and $\boldsymbol{B} = \boldsymbol{D} = 0$. In the simulation, the velocity is set to $\Delta = 1.5$, but you are encouraged to change it.

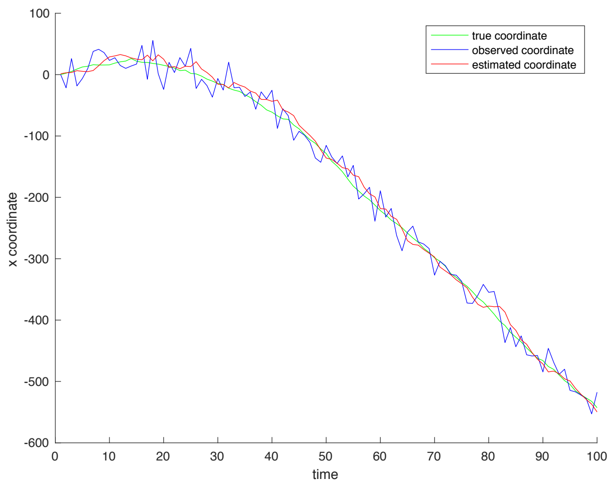
The system noise vector ϵ_t is Gaussian with diagonal covariance matrix; the noise on the two coordinates has standard deviation σ_{Qx}, while the noise on the two velocity components has standard deviation σ_{Qv} (typically smaller). Notice that the actual object velocity is not constant because of the noise term at each time instant; you will use a constant velocity model in the Kalman filter, and your system noise model will account for the mismatch.

The observation noise vector δ_t is Gaussian with diagonal covariance matrix, and standard deviation σ_R on each component. (Notice that we have two "delta's" – capital delta for the velocity and small delta for the observation noise, and they have different meanings.)

Designing the Kalman filter

Your task is to **design a Kalman filter** based on a constant velocity model, which tracks the next (x,y) position of the object, from the observation of the previous positions.

For setting up your Kalman filter, you can assume that parameters Δ, σ_{Qx}, σ_{Qv}, σ_R are known, although in practice they could hardly be known exactly and it is interesting to experiment with mismatched parameters. To see if your Kalman filter is working well, you should plot the estimated position of the object over time with respect to the true position (i.e., the first two entries of the state vector) and the observed position. Depending on the chosen parameters, for each coordinate the graph may look something like this:



Suggestion: when implementing your Kalman filter, you will have to choose initial values for $\boldsymbol{\mu}_t$ and $\boldsymbol{\Sigma}_t$. Provided that you do not make very unreasonable assumptions, the Kalman filter will update those estimates from observed data, so the initial choices are not very critical. Initially, you can use the true values, but you are encouraged to perturbed the initial conditions to see how the Kalman filter adapts over time.

Test your Kalman filter modifying the values of some of the parameters, including standard deviation of system noise and measurement noise, initial values for $\boldsymbol{\mu}_t$ and $\boldsymbol{\Sigma}_t$.