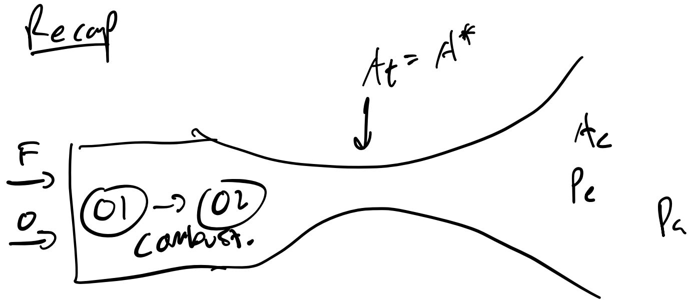
Recap



$$\dot{Q} = \dot{m}(h_{02} - h_{01})$$

$$\to T_{02} = T_{01} + \frac{Q_R}{c_p} \qquad (A)$$

$$h_{02} = h_{0e} = h_e + \frac{u_e^2}{2} \qquad \text{Always true}$$

$$u_e = \sqrt{2 c_p T_{02}\left(1 - \frac{T_e}{T_{02}}\right)}$$

$\vdots$

If no shock (fully isen):

$$u_e = \sqrt{\frac{2\gamma \bar{R}}{(\gamma-1)\bar{M}} T_{02}\left[1 - \left(\frac{P_e}{P_{02}}\right)^{\frac{\gamma-1}{\gamma}}\right]} \qquad (C)$$

From here on, $P_{02} = P_0$
$T_{02} = T_0$
$\rho_{02} = \rho_0$ nozzle entrance

CC end

$$u_e = \sqrt{\frac{2\gamma \bar{R}}{(\gamma-1)\bar{M}} T_0\left[1 - \left(\frac{P_e}{P_0}\right)^{\frac{\gamma-1}{\gamma}}\right]} \qquad (C')$$

About thrust

$$J = \dot{m} u_e + (P_e - P_a) A_e$$

Recall (10/16): $\dot{m} = \rho u A = \rho^* u^* A^* = \rho_0 A^* \sqrt{\frac{\gamma}{R T_0}} \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}}$

$$\to \quad \dot{m} = P_0 A^* \sqrt{\frac{\bar{M}}{\bar{R} T_0}} \left[\gamma \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}\right]^{1/2} \qquad (D) \qquad \text{(only isentropic)}$$

Rewrite thrust w/ (C') + (D)

$$\boxed{\frac{J}{P_0 A^*} = \sqrt{\frac{2\gamma^2}{(\gamma-1)}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}\left[1 - \left(\frac{P_e}{P_0}\right)^{\frac{\gamma-1}{\gamma}}\right]} + \left(\frac{P_e}{P_0} - \frac{P_a}{P_0}\right)\frac{A_e}{A^*}} \qquad (E)$$

- only for isentropic (no shocks!)
- useful b/c solely in terms of pressure ratios

- T_0 & $\bar{M}$ do not appear b/c they affect u_e & $\dot{m}$ in reciprocal ways.

1.1 Characteristic velocity

Defined as $c^* \equiv \dfrac{P_0 A^*}{\dot{m}}$ ← actual value (F)

If we use (D) for $\dot{m}$:

$$c^*_{ideal} = \sqrt{\frac{1}{\gamma}\left(\frac{\gamma+1}{2}\right)^{\frac{\gamma+1}{\gamma-1}} \frac{\bar{R} T_0}{\bar{M}}}$$ ← ideal value

Actual & ideal values may be different due to non-isentropic effects & Real-gas effects

c^* useful as a term of reference

Ex. values in table 11.1

(book's T_{0_1} is our T_0)

- oxidizer : fuel mass ratios chosen to maximize u_e
- all u_e's much higher than c^*, provided A_e/A^* large enough

1.2 Thrust coefficient

Defined as $\underbrace{C_J \equiv \dfrac{J}{P_0 A^*}}_{\text{Actual value}}$ (G)

This assumes $P_a > 0$

If $P_a = 0$ (expans. into vacuum) use:

$$C_{J_v} = \frac{J_v}{P_0 A^*}$$

↑ vacuum

If use (E)

$$C_{J\,ideal} = \sqrt{\frac{2\gamma^2}{(\gamma-1)}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}\left[1-\left(\frac{p_e}{p_0}\right)^{\frac{\gamma-1}{\gamma}}\right]} + \left(\frac{p_e}{p_0}-\frac{p_a}{p_0}\right)\frac{A_e}{A^*}$$

ideal value of C_J (only if no shocks)

For given p_0, p_a, γ : $C_{J\,ideal}$ only depends on $\frac{A_e}{A^*}$

b/c $\frac{p_e}{p_0}$ is set by $\frac{A_e}{A^*}$ (sets M, sets $\frac{p_e}{p_0}$)

→ Can't write $\frac{p_e}{p_0}$ in terms of $\frac{A_e}{A^*}$ (not possible)

Comparing real & ideal values of c^* & C_J tells how well the combustion chamber & the nozzle perform

To demonstrate how important the divergent part of the nozzle is, take a purely convergent nozzle :

- evaluate $\frac{p_e}{p_0} = \frac{p^*}{p_0} = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}$ (take for granted sonic throat)

- plug into (E) and get :

$$C_{J\,conv} = \sqrt{\gamma^2\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}} + \left[\left(\frac{2}{\gamma-1}\right)^{\frac{\gamma}{\gamma-1}} - \frac{p_a}{p_0}\right]\underbrace{\frac{A_e}{A^*}}_{=1} \quad (H)$$

b/c purely conv.

To assess advantage of adding divergent portion, for fixed $\dot{m}$, c^* plot $\frac{C_{J\,ideal}}{C_{J\,conv,\,ideal}} = \frac{(E)}{(H)} = \frac{J_{ideal}}{J_{conv}}$

→ Fig. 11.3