Recap

Case 1

$$G\begin{cases} u_e = \text{const} \\ \varepsilon \quad '' \\ M_{01}, M_L, n \end{cases} \rightarrow \lambda = \frac{1}{\left(\frac{M_{01}}{M_L}\right)^{1/n} - 1}$$

no need to calculate α

Case 2

$$G\begin{cases} u_e = \text{const} \\ \varepsilon_i \neq \text{const} \\ M_{01}, M_L, n \end{cases} \rightarrow \frac{M_L}{M_{01}} = \prod_{i=1}^{n} \frac{\alpha \varepsilon_i}{1-\alpha-\varepsilon_i+\alpha\varepsilon_i} \rightarrow \alpha$$

$$\hookrightarrow \lambda_i = \frac{\alpha \varepsilon_i}{1-\alpha-\varepsilon_i}$$

Case 3

$$\begin{cases} u_{ei} \neq \text{const} \\ \varepsilon_i \neq \text{const} \\ u_n, n \end{cases} \quad u_n = \sum_{i=1}^{n} u_{ei} \ln \frac{1+\alpha u_{ei}}{\alpha u_{ei} \varepsilon_i} \rightarrow \alpha$$

$$R_i = \frac{1+\alpha u_{ei}}{\alpha u_{ei} \varepsilon_i}$$

$$\lambda_i = \frac{1-\varepsilon_i R_i}{R_i - 1}$$

General Rule

LOX – LH2 engines are best for first stage because:

i) If H_2 leaks, it can be replenished

ii) Mechanical malfunctions (more likely than for hydrocarbon engine) can be found & fixed

§5 Propulsion efficiency

NOT describing: - frictional losses
- power losses to turbo pumps
- heat losses
- drag

Rather Intrinsic demand of Newton's third law:
To get momentum forward, we must eject mass with backwards momentum

Following refers to each individual stage

For now, assume u_e is const. in time.

$$\rightarrow \quad \mathcal{J} = \dot{m} u_e \quad (\text{Neglect } P_e - P_a \text{ term})$$

Define propulsion efficiency $\eta = \dfrac{\text{Power to accelerate rocket}}{\text{power to accel. rocket + power wasted within ejected gases}}$

Instantaneously

-power to accelerate rocket is equal to

$$= \mathcal{J} u = \dot{m} \underset{\text{w.r.t. vehicle}}{u_e} \underset{\text{w.r.t. ground}}{u}$$

$$\text{power in ejected gases} = \frac{1}{2} \dot{m} \left(u_{\text{gas w.r.t. ground}} \right)^2 = \frac{1}{2} \dot{m} \overbrace{\left(\underset{\text{wrt rocket}}{u_e} - u \right)^2}^{\text{wrt ground}}$$

$\div u_e^2$

$$\eta = \frac{\cancel{\dot{m}} u_e u}{\cancel{\dot{m}} u_e u + \frac{1}{2} \cancel{\dot{m}} (u_e - u)^2} = \frac{\frac{u}{u_e}}{\frac{u}{u_e} + \frac{1}{2}\left(1 - \frac{u}{u_e}\right)^2} = \frac{2 \frac{u}{u_e}}{1 + \left(\frac{u}{u_e}\right)^2}$$

Expression always the same

value varies as u varies

Can also introduce "mean propulsion efficiency"

using an averaging procedure

$$\eta_M \equiv \frac{1}{\text{range}\left(\frac{u}{u_e}\right)} \cdot \int_0^{\text{range}(u/u_e)} \eta \, d\left(\frac{u}{u_e}\right)$$

$$= \frac{1}{\Delta u / u_e} \int_0^{\Delta u / u_e} \frac{2 \, u/u_e}{1 + \left(\frac{u}{u_e}\right)^2} d\left(\frac{u}{u_e}\right) = \frac{1}{\Delta u / u_e} \ln\left(1 + \left(\frac{\Delta u}{u_e}\right)^2\right)$$

1-D Steady, compressible flow

1-D: All variables (P, T, ρ, u, etc) only depend upon x

$\underline{u} = u_x \hat{i}$ → only one velocity component

(unphysical, @ best $v \ll u$)

steady: $\frac{\partial}{\partial t} = 0$, @ any point, quantities are constant in time

compressible: $\rho \neq$ const