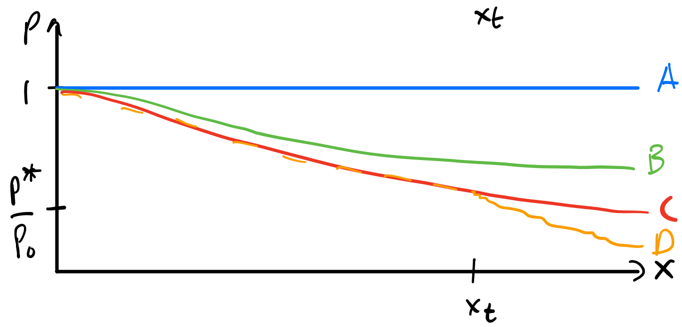
Recap

$$\frac{p}{p_0} = \left(1+\frac{\gamma-1}{2}M^2\right)^{\frac{\gamma}{1-\gamma}} \qquad (*)$$

$$\frac{A}{A_*} = \frac{1}{M}\left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M^2\right)\right]^{\frac{\gamma+1}{2(\gamma-1)}} \qquad (**)$$

$$\frac{p^*}{p} = \frac{p(M=1)}{p_0} = \left(1+\frac{\gamma-1}{2}\right)^{\frac{\gamma}{1-\gamma}} \qquad \left(\text{For } \gamma=1.4 \quad \frac{p^*}{p} = 0.5283\right)$$

Convergent nozzle



A: $p_a = p_0$ no flow

B: $p^* < p_a$ isentropic, subsonic throughout

$p_t = p_a$ as $p_a\downarrow$ $\dot{m}\uparrow$

C: $p_a = p^*$ " "

sonic @ throat $p_t = p^* = p_a$

$\dot{m}$ reached its maximum

D: $p_a < p^*$ exactly same as C, except after throat

p-adjustment after throat

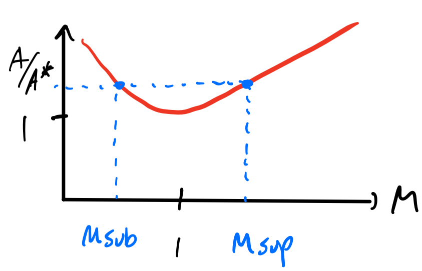
$p_t = p^*$

Since $J = \underbrace{\dot{m}u_e}_{\text{dominant term}} + (p_e - p_a)A_e$

to increase thrust for fixed $\dot{m}$, must increase u_e.

to do this, add divergent portion to nozzle

Recall



equation for $\frac{A}{A^*}$ exists two solutions for a given A_e/A^*

$M_{sub} < 1$, $M_{sup} > 1$

To these two values of M, there correspond two different values of exit pressure ratio

$$\frac{P_{sub}}{P_0} = \left(1 + \frac{\gamma-1}{2} M_{sub}^2\right)^{\frac{\gamma}{1-\gamma}}$$

→ See P graph handout

$$\frac{P_{sup}}{P_0} = \left(1 + \frac{\gamma-1}{2} M_{sup}^2\right)^{\frac{\gamma}{1-\gamma}}$$

Keep P_0 fixed, start w/ $P_a = P_0$ & decrease P_a

A: $P_a = P_0$ no flow

B: $P_{sub} < P_a$ isentropic, subsonic throughout
$A_t \neq A^*$ $P_t > P^*$, $P_e = P_a$

C: $P_a = P_{sub}$ " , " "
"sonic" @ throat, $A_t = A^*$, $P_t = P^*$, $P_e = P_a = P_{sub}$

D: $P_a = P_{sup}$ isentropic, subsonic/sonic/supersonic
$A_t = A^*$, $P_t = P^*$, $P_e = P_a = P_{sup}$

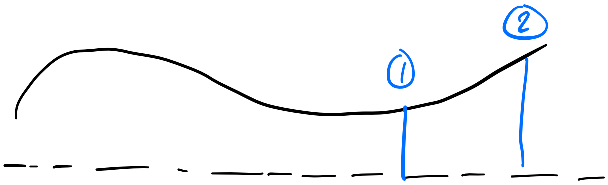
D': If $P_a < P_{sup}$, same as case D
$A_t = A^*$, $P_t = P^*$, $P_e = P_{sup} \neq P_a$
P- adjustment past exit area

Ex. Isentropic flow

G: air; Section 1: $\gamma = 1.4$, $A_1 = 0.04\ m^2$, $M_1 = 1.5$, $T_1 = 511\ K$, $P_1 = 151\ kPa$

Section 2: $T_2 = 213\ K$

F: u_2, A_2



Strategy:

$u_2 = M_2 a_2 = M_2\sqrt{\gamma R T_2}$

→ Find T_2/T_0 then get M_2

Find A_2/A^*, then A_2

$$\frac{T_0}{T_1} = \left(1 + \frac{\gamma-1}{2} M_1^2\right) = 1.450$$

given = 1.5

$$\frac{T_0}{T_2} = \frac{T_0}{T_1}\cdot\frac{T_1}{T_2} = 1.450 \cdot \frac{511}{213} = 2.798$$

$$\frac{T_0}{T_2} = \left(1 + \frac{\gamma-1}{2} M_2^2\right) = 2.798 \rightarrow M_2 = \sqrt{\left(\frac{T_0}{T_2} - 1\right)\frac{2}{\gamma-1}} = 3 = M_2$$

$$u_2 = M_2\sqrt{\gamma R T_2} = \boxed{878 \text{ m/s} = u_2}$$

$$\frac{A}{A_*} = \frac{1}{M}\left[\frac{2}{\gamma-1}\left(1 + \frac{\gamma-1}{2}M^2\right)\right]^{\frac{\gamma+1}{2(\gamma-1)}}$$

@ $M_1 = 1.5$, $\frac{A_1}{A_*} = 1.176$

@ $M_2 = 3$, $\frac{A_2}{A_*} = 4.235$

$$\rightarrow \frac{A_2}{A_1} = \frac{A_2}{A_*}\,\frac{A_*}{A_1} = \frac{4.235}{1.176} = 3.601$$

$$\rightarrow A_2 = 3.601 A_1 \boxed{= 2.305 = A_2}$$