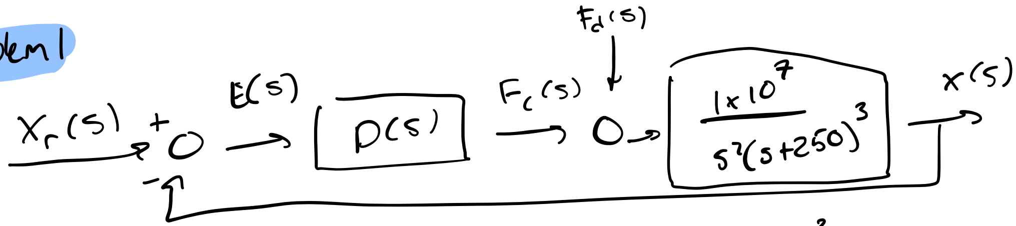
Problem 1



Req:

- PM $\geq 60°$
- GM ≥ 15 dB
- ω_{bw} as high as possible
- e_{ss} from $x_r(t) = 0.25t^2 \leq 0.01$
- e_{ss} from $f_d(t) = 3\cdot 1(t) \leq 0.03$
- error response reaches steady as fast as possible

Part A

Assumptions

$$D(s) = K\left(\frac{s+z_{lead}}{s+p_{lead}}\right)\left(\frac{s+z_{lag}}{s+p_{lag}}\right)$$

$\varphi_{lead,\,max} = 70°$

→ want PM = 60°, design to 65° for lag

$\varphi_{des} = -115°$

$$\varphi_{lead,max} = \varphi_{des} - \angle G(j\omega_c) = 70° \rightarrow \angle G(j\omega_c) = \underbrace{\overbrace{(-180°+65)}^{\varphi_{des}=-115°} - 70}_{-185°}$$

→ Find ω_c @ $\varphi \cong -185°$ from Bode

→ $\omega_c = 7.4$ rad/s

$$\angle G(j\omega_c) = \underbrace{-\tan^{-1}\left(\frac{7.28}{0}\right)\cdot 2}_{-180°} \underbrace{-3\tan^{-1}\left(\frac{7.28}{250}\right)}_{-5°}$$

$$\varphi_{lead} = \underbrace{-115° - \angle G(j\omega_c)}_{\text{Iterate to get}}$$

Iterate to get $\omega_c = 7.28$, $\varphi_{lead} \cong 70°$

$$\alpha = \frac{1-\sin\varphi_{lead}}{1+\sin\varphi_{lead}} = 0.0311 \rightarrow LR \cong 32.16$$

(reasonable)

→ $z = \omega_{max}\sqrt{\alpha} \cong 1.284$

→ $p = \dfrac{\omega_{max}}{\sqrt{\alpha}} \cong 41.287$

→ $D_{lead}(s) = K\left(\dfrac{s+1.284}{s+41.287}\right)$

$$M\Big|_{\omega=7.28} = |D_{lead}(j\omega)\,G(j\omega)| = K\cdot\frac{\sqrt{7.28^2+1.284^2}}{\sqrt{7.28^2+1.284^2}}\cdot\frac{1\times10^7}{7.28\cdot\left(\sqrt{7.28^2+250^2}\right)^3}$$

$1 = K\cdot 0.0021 \rightarrow K = 470.23$

→ $D_{lead}(s) = 470.23\left(\dfrac{s+1.284}{s+41.287}\right)$

$$\frac{E(s)}{X_r(s)} = \frac{1}{1 + 470.23\left(\frac{s+1.284}{s+41.287}\right)\frac{1\times10^7}{s^2(s+250)^3}}$$

matlab →

$$\lim_{s\to0}\left[s\cdot\frac{E(s)}{X_r(s)}\cdot\mathcal{L}[x_r(t)]\right] = 6.0534$$

→ lag ratio >6.84, set to 7

→ Set zero to be 10x smaller

$$z = \frac{w_c}{10} \qquad p = \frac{z}{\alpha}$$

$$\rightarrow D_{lag} = \frac{s+0.728}{s+0.104}$$

$$\rightarrow D_{leadlag}(s) = 470.23\left(\frac{s+1.284}{s+41.287}\right)\left(\frac{s+0.728}{s+0.104}\right)$$

→ Evaluate: $\omega = 52.8$ @ $\phi = -180°$

$\boxed{GM \cong 22\,dB}$

$\boxed{\omega_{BW} \cong 10\ rad/s}$

$\boxed{PM = 60.1°}$

$\boxed{e_{ss\ x_r} = 0.0076 < 0.01}$

$\boxed{e_{ss\ f_d} = 0.0293 < 0.03}$

Part B Design PID $D(s) = K_p + \frac{K_I}{s} + K_d s = D_{PI}(s)\cdot D_{PD}(s) = \frac{K_d s^2 + K_p s + K_I}{s}$

Req:

- PM $\geq 60°$
- GM ≥ 15dB
- W_{bw} as high as possible
- e_{ss} from $x_r(t) = 0.25t^2 \leq 0.01$
- e_{ss} from $f_d(t) = 3\cdot1(t) \leq 0.03$
- error response reaches steady as fast as possible

→ Design PD for phase Then PI

Assume PD can get up to 70°

→ same process, $\Phi_{PD} \cong 70°$ @ $\omega_c = 7.28$ rad/s

PD: $K(s+z_{pd})$

$$D_{PD}(j\omega_c) = K(j\omega_c + z_{pd})$$

$$\angle D_{PD}(j\omega_c) = \operatorname{atan}\left(\frac{\omega_c}{z_{pd}}\right) = 70°$$

$$\rightarrow z_{pd} = 2.65$$

K: $$\left|D_{PD}G\right|_{\omega_c} = \left| K(j\omega_c + 2.65) \cdot \frac{1\times10^7}{7.28 \cdot \left(\sqrt{7.28^2+250^2}\right)^3} \right|$$

$$\rightarrow K = 10.7$$

PI: integral term zeroes error

$$\rightarrow e_{ss} = \lim_{s\to0} \left[s \cdot \frac{\sim\sim s^2}{\sim + c} \cdot X_r(s) \right]$$ ← non zero for $X_r(t) = 0.25t^2$

$$D_{PI}(s) = \frac{s+z_{PI}}{s}$$

can lose 5° → $\angle D_{PI}(j\omega_c) = \tan^{-1}\left(\frac{\omega_c}{z_{PI}}\right) - \tan^{-1}\left(\frac{\omega_c}{0}\right) = -5°$

$$\rightarrow z_{PI} = \frac{\omega_c}{\tan 85°} = 0.637$$

$$\rightarrow D_{PI}(s) = \frac{s+0.637}{s}$$

$$K_p = K(z_{PD} + z_{PI}) = 35.17$$

$$K_d = K = 10.7$$

$$K_I = K\, z_{PD}\, z_{PI} = 18.06$$

→ $\boxed{D_{PID}(s) = 35.17 + 10.7s + \frac{18.06}{s}}$

→ $\boxed{GM \cong 30\text{ dB}}$ → $\boxed{PM = 60°}$

→ $\boxed{\omega_{BW} \cong 10 \text{ r/s}}$ $\boxed{e_{ss} = 0 \text{ for both inputs}}$

Table of Contents


```
% Written by Kyle Adler for ME446
```

Problem A

```
s = tf('s');
Gsys = 1e7/(s^2*(s+250)^3);
```

Part A

```
bode(Gsys,{0.1,10e4}) % estimate crossover frequency
wc = 7.28 % rad/s
[m,p] = bode(Gsys,wc)
phi_lead = -115-p

Dlead = (s+1.284)/(s+41.287)
[m,p] = bode(Gsys*Dlead,wc)
K = 1/m

Dlead = K*(s+1.284)/(s+41.287)

EoverXr = feedback(1,Gsys*Dlead)
syms t
XrS = laplace(0.25*t^2)
XrS = 1/(2*s^3)
insideLimit = s*EoverXr*(XrS) %0/0, lhopital
ess_xr = 6.451e08/1.208e10 % lag ratio ~5.34

EoverFd = feedback(Gsys,Dlead)
FdS = 3/s
insideLimit = s*EoverFd*FdS
ess_fd = 1.239e09/6.038e09 % lag ratio ~6.84

alpha = 7
z = wc/10
p = z/alpha
Dlag = (s+z)/(s+p)

% evaluate system
Dleadlag = Dlead*Dlag
bode(Gsys*Dleadlag)
[m,p] = bode(Gsys*Dleadlag,52.8);
GM = 20*log10(1/m)
```

```
margin(Gsys*Dleadlag)

EoverXr = feedback(1,Gsys*Dleadlag)
XrS = 1/(2*s^3)
insideLimit = s*EoverXr*(XrS) %0/0, lhopital
ess_xr = 6.709e07/8.791e09

EoverFd = feedback(Gsys,Dleadlag)
FdS = 3/s
insideLimit = s*EoverFd*FdS
ess_fd = 1.288e08/4.396e09
nyquist(Gsys*Dleadlag)
```

```
wc =

    7.2800


m =

    0.0121


p =

 -185.0039


phi_lead =

   70.0039


Dlead =

  s + 1.284
  ---------
  s + 41.29

Continuous-time transfer function.

m =

    0.0021


p =

 -115.0065


K =
```

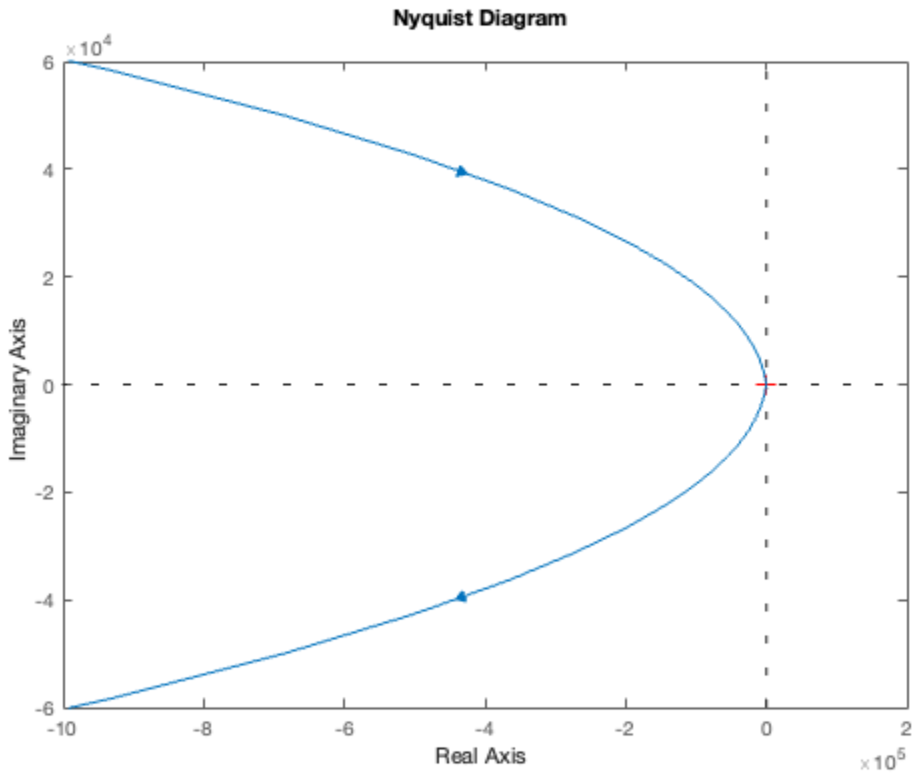
```
  470.2333


Dlead =

  470.2 s + 603.8
  ---------------
     s + 41.29

Continuous-time transfer function.

EoverXr =

         s^6 + 791.3 s^5 + 2.185e05 s^4 + 2.337e07 s^3 + 6.451e08 s^2

  ---------------------------------------------------------------------------

  s^6 + 791.3 s^5 + 2.185e05 s^4 + 2.337e07 s^3 + 6.451e08 s^2 + 4.702e09 s

                                                              + 6.038e09


Continuous-time transfer function.

XrS =

1/(2*s^3)


XrS =

    1
  -----
  2 s^3

Continuous-time transfer function.

insideLimit =

          s^7 + 791.3 s^6 + 2.185e05 s^5 + 2.337e07 s^4 + 6.451e08 s^3

  -----------------------------------------------------------------------------

  2 s^9 + 1583 s^8 + 4.369e05 s^7 + 4.673e07 s^6 + 1.29e09 s^5 + 9.405e09 s^4

                                                          + 1.208e10 s^3


Continuous-time transfer function.

ess_xr =
```

```
    0.0534


EoverFd =

                                    1e07 s + 4.129e08
  -------------------------------------------------------------------------
  s^6 + 791.3 s^5 + 2.185e05 s^4 + 2.337e07 s^3 + 6.451e08 s^2 + 4.702e09 s
                                                             + 6.038e09

Continuous-time transfer function.

FdS =

  3
  -
  s

Continuous-time transfer function.

insideLimit =

                                 3e07 s^2 + 1.239e09 s
  ---------------------------------------------------------------------------
  s^7 + 791.3 s^6 + 2.185e05 s^5 + 2.337e07 s^4 + 6.451e08 s^3 + 4.702e09 s^2
                                                           + 6.038e09 s

Continuous-time transfer function.

ess_fd =

    0.2052


alpha =

     7


z =

    0.7280
```

```
p =

    0.1040


Dlag =

  s + 0.728
  ---------
  s + 0.104

Continuous-time transfer function.

Dleadlag =

  470.2 s^2 + 946.1 s + 439.6
  ---------------------------
     s^2 + 41.39 s + 4.294

Continuous-time transfer function.

GM =

   21.9728


EoverXr =


  s^7 + 791.4 s^6 + 2.185e05 s^5 + 2.339e07 s^4 + 6.475e08 s^3 + 6.709e07 s^2


  ---------------------------------------------------------------------------

  s^7 + 791.4 s^6 + 2.185e05 s^5 + 2.339e07 s^4 + 6.475e08 s^3 + 4.769e09 s^2

                                                  + 9.461e09 s + 4.396e09


Continuous-time transfer function.

XrS =

    1
  -----
  2 s^3

Continuous-time transfer function.

insideLimit =


  s^8 + 791.4 s^7 + 2.185e05 s^6 + 2.339e07 s^5 + 6.475e08 s^4 + 6.709e07 s^3
```

```
  ----------------------------------------------------------------------------
   2 s^10 + 1583 s^9 + 4.371e05 s^8 + 4.678e07 s^7 + 1.295e09 s^6

                            + 9.539e09 s^5 + 1.892e10 s^4 + 8.791e09 s^3

Continuous-time transfer function.

ess_xr =

    0.0076

EoverFd =

                        1e07 s^2 + 4.139e08 s + 4.294e07

  -----------------------------------------------------------------------------
  s^7 + 791.4 s^6 + 2.185e05 s^5 + 2.339e07 s^4 + 6.475e08 s^3 + 4.769e09 s^2

                                                    + 9.461e09 s + 4.396e09

Continuous-time transfer function.

FdS =

  3
  -
  s

Continuous-time transfer function.

insideLimit =

                      3e07 s^3 + 1.242e09 s^2 + 1.288e08 s

  -----------------------------------------------------------------------------
  s^8 + 791.4 s^7 + 2.185e05 s^6 + 2.339e07 s^5 + 6.475e08 s^4 + 4.769e09 s^3

                                                + 9.461e09 s^2 + 4.396e09 s

Continuous-time transfer function.

ess_fd =
```

0.0293



Part B

```
bode(Gsys,{0.1,10e4}) % estimate crossover frequency
wc = 7.28 % rad/s
[m,p] = bode(Gsys,wc)
phi_pd = -115-p

z_pd = wc/tan(phi_pd*pi/180)
Dpd = (s+z_pd)
[m,p] = bode(Gsys*Dpd,wc)
K = 1/m
Dpd = K*(s+z_pd)

z_pi = wc/tan(85*pi/180)

kp = K*(z_pd+z_pi)
kd = K
ki = K*z_pd*z_pi

Dpid = kp + kd*s + ki/s

% evaluate system
bode(Gsys*Dpid)
```

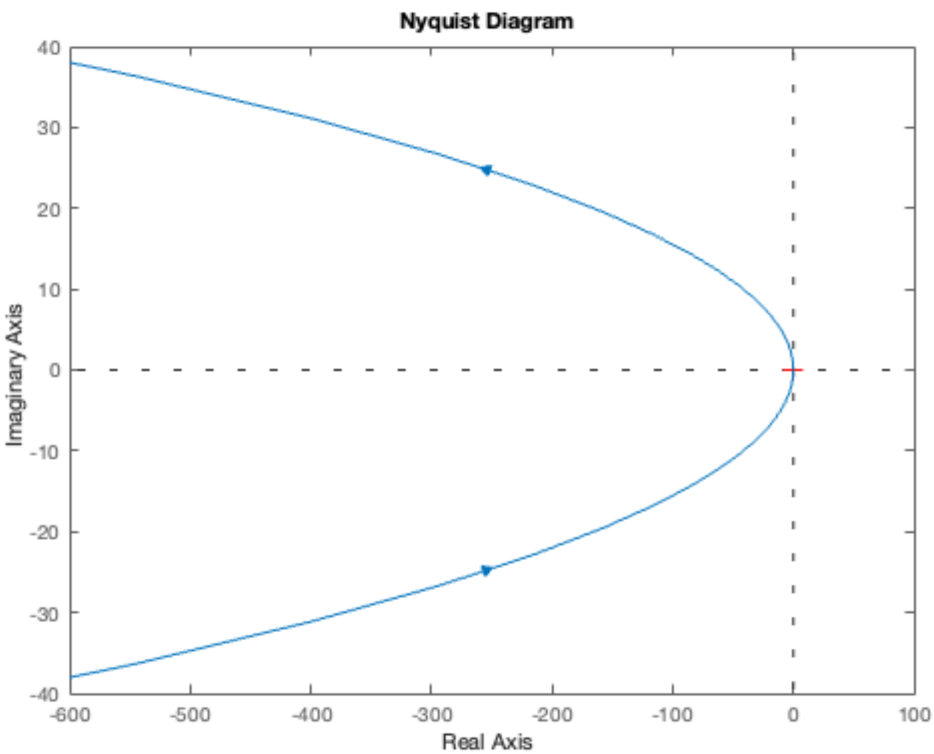
```
[m,p] = bode(Gsys*Dpid,142);
GM = 20*log10(1/m)

[m,p] = bode(Gsys*Dpid,wc);
PM = p+180

EoverXr = feedback(1,Gsys*Dpid)
XrS = 1/(2*s^3)
insideLimit = s*EoverXr*(XrS) %0/0, lhopital

EoverFd = feedback(Gsys,Dpid)
FdS = 3/s
insideLimit = s*EoverFd*FdS

nyquist(Gsys*Dpd)
```

```
wc =

    7.2800


m =

    0.0121


p =

 -185.0039


phi_pd =

   70.0039


z_pd =

    2.6491


Dpd =

  s + 2.649

Continuous-time transfer function.

m =

    0.0934


p =
```

```
 -115.0000


K =

   10.7029


Dpd =

  10.7 s + 28.35

Continuous-time transfer function.

z_pi =

    0.6369


kp =

   35.1702


kd =

   10.7029


ki =

   18.0587


Dpid =

  10.7 s^2 + 35.17 s + 18.06
  --------------------------
              s

Continuous-time transfer function.

GM =

   29.9737


PM =

   60.0000


EoverXr =
```

```
                s^6 + 750 s^5 + 187500 s^4 + 1.562e07 s^3
  -----------------------------------------------------------------------
  s^6 + 750 s^5 + 187500 s^4 + 1.562e07 s^3 + 1.07e08 s^2 + 3.517e08 s
                                                              + 1.806e08

Continuous-time transfer function.

XrS =

    1
  -----
  2 s^3

Continuous-time transfer function.

insideLimit =

                 s^7 + 750 s^6 + 187500 s^5 + 1.562e07 s^4
  --------------------------------------------------------------------------
  2 s^9 + 1500 s^8 + 375000 s^7 + 3.125e07 s^6 + 2.141e08 s^5 + 7.034e08 s^4
                                                           + 3.612e08 s^3

Continuous-time transfer function.

EoverFd =

                               1e07 s
  -----------------------------------------------------------------------
  s^6 + 750 s^5 + 187500 s^4 + 1.562e07 s^3 + 1.07e08 s^2 + 3.517e08 s
                                                              + 1.806e08

Continuous-time transfer function.

FdS =

  3
  -
  s
```

```
Continuous-time transfer function.

insideLimit =

                              3e07 s^2
  ------------------------------------------------------------------------
  s^7 + 750 s^6 + 187500 s^5 + 1.562e07 s^4 + 1.07e08 s^3 + 3.517e08 s^2
                                                          + 1.806e08 s

Continuous-time transfer function.
```



Published with MATLAB® R2023b