Problem 1: subsonic linear theory

Subsonic, upstream mach M. wavy wall, $y_w = h\cos\left(2\pi \frac{x}{\ell}\right)$. h = amp, ℓ = wavelength. $\frac{h}{\ell} << 1$. Use small perturbation theory to find φ & c_p.

Elliptic, linear PDE, separation of variables.

Governing equation: $(1-M_\infty^2)\dfrac{\partial^2\hat{\varphi}}{\partial x^2} + \dfrac{\partial^2\hat{\varphi}}{\partial y^2} = 0$

1.) Assume $\hat{\varphi}(x,y) = F(x)\,G(y)$.

$$\to (1-M_\infty^2)\frac{d^2F(x)}{dx^2}G(y) + \frac{d^2G(y)}{dy^2}F(x) = 0$$

$$\to \frac{d^2F(x)}{dx^2}G(y) + \frac{1}{1-M_\infty^2}\frac{d^2G(y)}{dy^2}F(x) = 0$$

$$\to \underbrace{\frac{1}{F(x)}\frac{d^2F(x)}{dx^2}}_{A(x)} + \frac{1}{1-M_\infty^2}\underbrace{\frac{1}{G(y)}\frac{d^2G(y)}{dy^2}}_{B(y)} = 0$$

2) $A(x) = -K^2$, $\frac{1}{1-M_\infty^2}B(y) = K^2$

$$x:\ \frac{1}{F(x)}\frac{d^2F(x)}{dx^2} = -K^2, \qquad y:\ \frac{1}{G(y)}\frac{d^2G(y)}{dy^2} = K^2(1-M_\infty^2)$$

wolfram:

$$\begin{cases} F(x) = C_2\sin(Kx) + C_1\cos(K\ ) \\ G(y) = C_3 e^{yK\sqrt{1-M_\infty^2}} + C_4 e^{-yK\sqrt{1-M_\infty^2}} \end{cases}$$

BC1: As $y \to \infty$, V, $\nabla\varphi$ finite

$$\nabla\varphi = f(G(y)) \neq \infty \quad \therefore\ C_3 = 0$$

$$F(x)\,G(y) = C_4 e^{-yK\sqrt{1-M_\infty^2}}\left(C_2\sin(Kx) + C_1\cos(Kx)\right)$$

$$\varphi = F(x)\,G(y) = e^{-yK\sqrt{1-M_\infty^2}}\left(C_2C_4\sin(Kx) + C_1C_4\cos(Kx)\right)$$