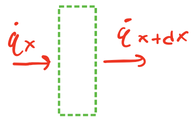
Kyle Adler ME 364

STEADY STATE 1-D CONDUCTION W/O GEN.

Fourier's: $q = -KA\nabla T = -KA_c \frac{dT}{dx}$, $q'' = \frac{q}{A}$

- CV Analysis:



Area:

Planar: $HW / \pi R^2$ Cyl: $2\pi r L$ Sph: $4\pi r^2$

1) Draw CV
2) Write E-bal: $q_x = q_{x+dx}$
3) Taylor expand: $q_x = q_x + \frac{dq_x}{dx} dx$, $\therefore \frac{dq}{dx} = 0$
4) Sub rate laws into $\frac{dq}{dx}$
5) Solve ODE (usually separable)

RESISTANCE CONCEPT

- only use when q is constant (no gen, stored, $K = const$)
  - ok to approx. solid cyl. w/ planar resistance ($L_{eff} = \frac{Vol}{A_c}$)

$q = \frac{\Delta T}{R_{total}}$

Series: Big R >>, Parallel: Small R >>

Plane wall: $R_{pw} = \frac{L}{KA_c}$

Sphere: $R_{sph} = \frac{1}{4\pi K}\left[\frac{1}{r_{in}} - \frac{1}{r_{out}}\right]$

contact: $R_c = \frac{R_c''}{A_s}$

$R_{rad} \approx \frac{1}{A_s \sigma \varepsilon 4 \bar{T}^3}$

$\bar{T}$ = avg. abs. temp

Cylinder: $R_{cyl} = \frac{\ln\left(\frac{r_{out}}{r_{in}}\right)}{2\pi L K}$

Convection: $R_{conv} = \frac{1}{h A_s}$

Rad.: $R_{rad} = \frac{1}{A_s \sigma \varepsilon (T_s^2 + T_\infty^2)(T_s + T_\infty)}$

$\sigma = 5.67 \times 10^{-8}$ [$W/m^2 \cdot K^4$]

- Drawing slopes: mark BC for reference
  - Contact has step ΔT over $\Delta x = 0$
  - Same res. = Same slope

1-D CONDUCTION W/ GENERATION

$g = \dot{g}'''V$, $g = \dot{g}''' A_c dx$

	Plane wall
Governing diff. eq.	$\frac{d^2T}{dx^2} = -\frac{\dot{g}'''}{K}$
Temp. gradient	$\frac{dT}{dx} = -\frac{\dot{g}'''}{K} x + C_1$
General Sol'n	$T = \frac{-\dot{g}'''}{2K} x^2 + C_1 x + C_2$

- Temp. profile w/ generation will depend on BC, often parabolic

EXTENDED SURFACE APPROXIMATION

- Biot number used to approximate 2D solid as 1D

$$Biot = \frac{R_{cond}\text{(in dir. to neglect)}}{R_{surr}\text{ (in same direction)}} \cong \frac{\Delta T_{cond}}{\Delta T_{surr}}$$

- If Biot << 0.1, 1D approx. is valid
- use area in direction to neglect for both numerator & denominator
  - E.g. neglect r: $A_c = 2\pi RL$, length = R.
    neglect x: $A_c = \pi R^2$, length = L
- R_{surr} can include both convection & radiation

ANALYTICAL SOLUTIONS FOR EXTENDED SURFACES

- Use CV analysis to determine governing ODE for fin

$$\frac{d^2T}{dx^2} - m^2 T = -m^2 T_\infty + \dots \quad \text{where } m = \sqrt{\frac{h \cdot per}{k A_c}}$$

- use method of undetermined coefficients to solve ODE

$$T = T_h + T_p, \quad T_h = C_1 \exp(mx) + C_2 \exp(-mx), \quad T_p = ?$$

- T_p depends on RHS of ODE:

RHS	GUESS
const (A)	C_3
Ax	$C_3 x + C_4$
Ax^2	$C_3x^2 + C_4 x + C_5$
$A\sin(x)$ or $A\cos(x)$	$C_3 \sin(x) + C_4 \cos(x)$
$A\exp(x)$	$C_3 \exp(x)$

Constants C_3, C_4 found by subbing T_p guess into ODE.

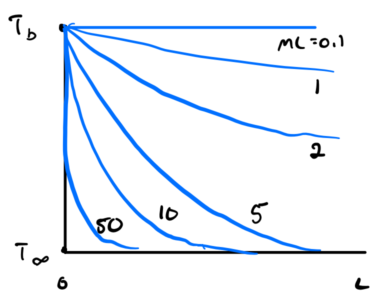
C_1, C_2 found by applying BC's to T

FIN BEHAVIOR

- Parameter mL can tell us if fin temp will be very close to T_b or T_∞

$$(mL)^2 = \left[\sqrt{\frac{h \cdot per}{kA_c}}\right]^2 \cdot \frac{R_{cond,x}}{R_{conv}} \cong \frac{\Delta T_{cond,x}}{\Delta T_{conv}}$$

- Small mL → small ΔT_{cond}
  ↳ $T_{fin} \cong T_b$
- Large mL → large ΔT_{cond}
  ↳ $T_{fin} \cong T_\infty$



FIN EFFICIENCY / RESISTANCE

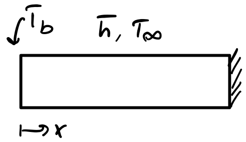
- η_{fin} is ratio of heat transfer to fin (q_{fin}) to the heat transfer to ideal fin where $T_{fin} = T_b$ everywhere

$$\eta_{fin} = \frac{q_{fin}}{\bar{h} A_{s,fin}(T_b - T_\infty)}$$

η_{fin} close to 1 : $T_{fin} \cong T_b$ (small mL)

η_{fin} close to 0 : $T_{fin} \cong T_\infty$ (large mL)

Adiabatic tip:



$$\frac{T - T_\infty}{T_b - T_\infty} = \frac{\cosh(m(L-x))}{\cosh(mL)}$$

$$\dot{q}_{fin} = (T_b - T_\infty)\sqrt{\bar{h} \cdot per\, k A_c}\, \tanh(mL)$$

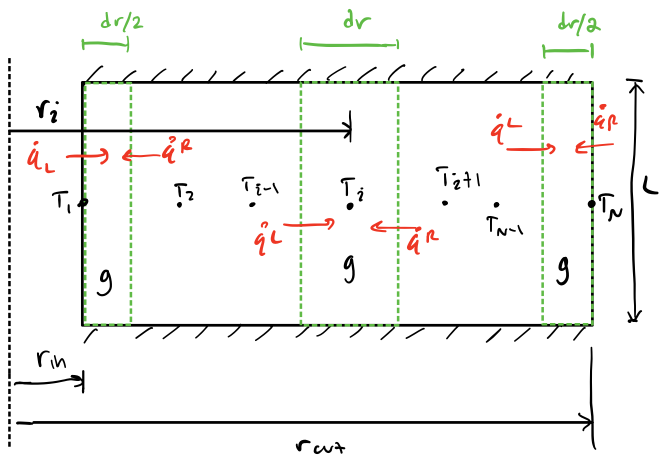
$$\eta_{fin} = \tanh(mL)/(mL)$$

- $R_{fin} = \dfrac{1}{\bar{h} A_s \eta_{fin}}$

- Multiple fins: multiply A_s by N_{fins}
- Base can have conv/rad in parallel w/ fins (note to subtract fin area)
- When T_∞ not given, try approximating with sketch using η or mL

NUMERICAL SOLUTIONS: 1D SS CONDUCTION

- Nodally based w/ 1 node on each boundary
- uses CV analysis
- Length to conduct always Δx or Δr
- volume: $2\pi\, r[i]\, L\, dr$ (cyl) or $4\pi\, r[i]^2 dr$ (sph)



0D Transient Conduction

- Is lumped capacitance justified?
  - → Check Biot: $Bi = \frac{R_{cond,int}}{R_{ext}}$
  - → Justified if $Bi \ll 1$

$R_{cond,int} = \frac{L_{cond}}{k A_s}$, $L_{cond} = \frac{V}{A_s}$

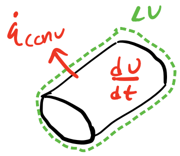
Note: R_{ext} related to direction of heat flow (from object to surr), series or parallel

- Lumped capacitance time constant (external)

$$\tau_{LC} = R_{ext} C, \quad C = \rho V c$$

$\boxed{EQ @ t = 5\tau_{LC}}$

- Numerical model:
  1) Draw CV
  2) Define energy terms:



IN + GEN = OUT + STORED

$$0 = \dot q_{conv} + \frac{dU}{dt}$$

$$0 = \bar h A_s (T - T_\infty) + \rho V c \frac{dT}{dt}$$

$$\rightarrow \frac{dT}{dt} + \frac{T}{\tau_{LC}} = \frac{T_\infty}{\tau_{LC}}$$

3) Rearrange for $\frac{dT}{dt} = \frac{1}{\tau_{LC}}(T_\infty - T)$
4) Use required integration scheme
5) IC: $T(t=0)$

- Numerical integration:
  - Euler: $T_{j+1} = T_j + \left.\frac{dT}{dt}\right|_{T_j, t_j} \Delta t$ (explicit)
  - Implicit: $T_{j+1} = T_j + \left.\frac{dT}{dt}\right|_{T_{j+1}, t_{j+1}} \Delta t$
  - Crank-Nicolson: $T_{j+1} = T_j + \left[\left.\frac{dT}{dt}\right|_{T_j, t_j} + \left.\frac{dT}{dt}\right|_{T_{j+1}, t_{j+1}}\right] \frac{\Delta t}{2}$ (implicit)
- Stability & Accuracy:
  - Explicit methods stable until crit. time step (τ)
  - Implicit always stable
  - Higher order (CN) more accurate than lower (implicit)
- Analytical: CV analysis for gov. ODE

$$\frac{dT}{dt} + \frac{T}{\tau_{LC}} = \ldots$$

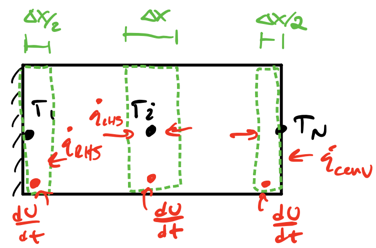
1) Split $T = T_h + T_p$
2) Solve separable for T_h & sub T_p guess into ODE for T_p

RHS	GUESS
c const	c_3
Ax	$c_3 x + c_4$
Ax^2	$c_3 x^2 + c_4 x + c_5$
$A\sin(x)/\cos(x)$	$c_3 \sin(x) + c_4 \cos(x)$
$A\exp(x)$	$c_3 \exp(x)$

1-D Transient Conduction

- τ_{diff}: (internal) – time for thermal wave to travel thru object

$$\tau_{diff} = \frac{(L_{cond})^2}{4\alpha}, \quad \alpha = \frac{k}{\rho c}$$



Each node:
1) Draw CV
2) Define Energy terms
3) Make $\frac{dT}{dt}$ subject
4) Use req'd integration model
5) IC @ all nodes: $T(t=0)$

Boundary Layer Growth

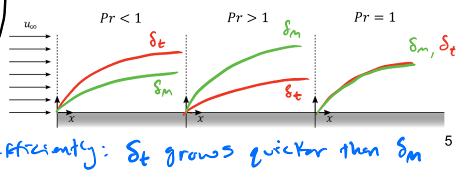
- Thermal BL thickness: $\delta_t \approx 2\sqrt{\alpha t}$
- Momentum " " : $\delta_m \approx 2\sqrt{\nu t}$
- Transport time: $t \approx \frac{x}{u_\infty}$

$$\frac{\delta_m}{x} \approx \frac{2}{\sqrt{Re_x}}$$

Re_x: flow over flat plate

$$\frac{\delta_t}{x} \approx \frac{2}{\sqrt{Re_x Pr}}$$

$\rightarrow \frac{\delta_t}{\delta_m} \approx \frac{1}{\sqrt{Pr}}, \quad Pr = \nu/\alpha = \frac{\mu c}{k}$



e.g. $Pr < 1$, fluid transports momentum more efficiently: δ_t grows quicker than δ_m

Laminar Boundary Layer

- Conduction problem w/ $L_{cond} = \delta_t$

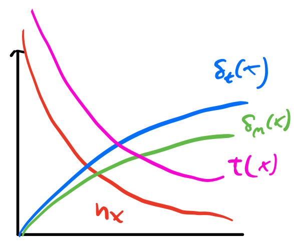
$$q''_{lam} = h_{lam}(T_s - T_\infty) \approx k\frac{T_s - T_\infty}{\delta_{t,lam}} \therefore h_{lam} \approx \frac{k}{\delta_{t,lam}}$$

- Shear stress @ surface related to velocity grad.

$$\tau_{lam} = \mu \frac{\partial u}{\partial y}\Big|_{y=0} \approx \mu \frac{u_\infty}{\delta_{m,lam}}$$

$$\rightarrow \downarrow h_x \approx \frac{k}{\delta_t(x)\uparrow}$$

relationship between fluid properties & BL thickness

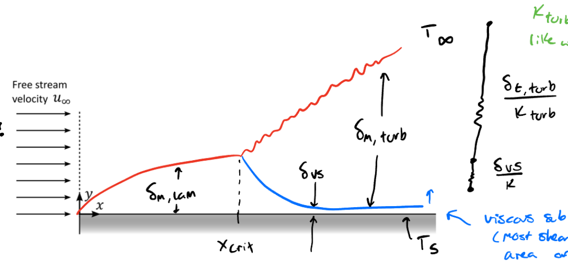


Turbulent Boundary Layer

- Turbulent BL to free stream:

$$\dot{q}''_{conv} = h_{turb}(T_s - T_\infty)$$

$$\approx \frac{T_s - T_\infty}{\left(\frac{\delta_{vs}}{k} + \frac{\delta_{t,turb}}{k_{turb}}\right)}$$



k_{turb} (eff. conductivity) >> k

likewise $\mu_{turb} >> \mu$

$\delta_{vs} << \delta_{lam} << \delta_{turb}$

viscous sublayer (most shear occurs, small area of laminar flow)

$$R''_{conv} = R''_{vs} + R''_{turb} = \frac{\delta_{vs}}{k} + \cancel{\frac{\delta_{t,turb}}{k_{turb}}} \text{ eff.} \rightarrow 0 \rightarrow \dot{q}''_{turb} \approx \frac{k(T_s - T_\infty)}{\delta_{vs}}$$

External Flow correlations

Self-similar model

$\delta_t = \dfrac{4.916\, x}{Re_x^{0.5}\, Pr^{1/3}}$, $\delta_m = \dfrac{4.916\, x}{Re_x^{0.5}}$

$Nu_x \approx \frac{1}{2}\sqrt{Re \cdot Pr}$

approx

$Nu_x = 0.332\, Re^{0.5} \cdot Pr^{1/3}$

exact

approx.

$C_f \approx \dfrac{1}{\sqrt{Re_x}}$, $C_f = \dfrac{0.664}{\sqrt{Re_x}}$ exact

$Nu_x \approx \dfrac{C_f Re}{2} \xrightarrow[\text{better}]{} Nu_x \approx \dfrac{Pr^{1/3} C_f Re_x}{2}$ "Chilton-colburn"

- Dimensionless correlations:

$Re = \dfrac{\rho u_m L_{char}}{\mu}$, $C_f = \dfrac{2\tau_s}{\rho u_\infty^2}$, $Nu = \dfrac{h L_{char}}{k}$

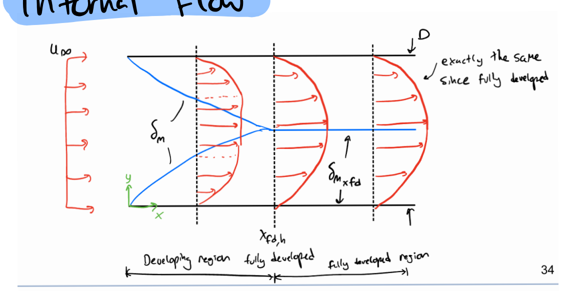
Transistion to turbulence

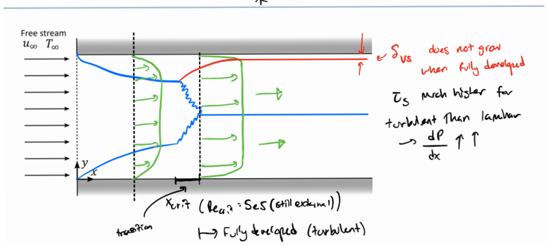
$Re \geq Re_{crit} \approx 5e5$

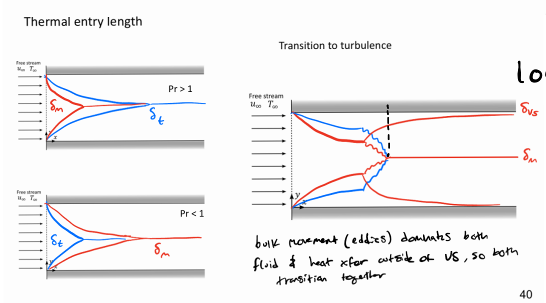
$Re_{crit} = \dfrac{\rho u_\infty \cdot x_{crit}}{\mu}$

inertial forces overcome viscous damping forces at Re_{crit}

Internal Flow







Beyond the hydrodynamic entry length:

x_{fd}, $\delta_m = \dfrac{D}{2}$

Recall Re # over flat plate

$Re = \dfrac{\rho u_\infty x}{\mu} \to \dfrac{\text{inertial}}{\text{viscous}}$ | $Re_{D_h} = \dfrac{\rho \bar{u} D_h}{\mu}$

$D_h = \dfrac{4 \cdot A_c}{Per}$ cross-sectional area, wetted perimeter

$\bar{u} = \int_{A_c} u \, dA_c$

$\bar{u} = \dfrac{\dot{V}}{A_c} = \dfrac{\dot{m}}{\rho A_c}$

$x_{fd,h} \approx \dfrac{Re_{D_h} \cdot D_h}{16}$

$\dfrac{x_{fd,t,lam}}{x_{fd,h,lam}} \approx Pr$

friction factor

local: "Moody" → Darcy chart

$f = \left(-\dfrac{dP}{dx}\right)\dfrac{2 D_h}{\rho u_m^2}$ u_m = mean velocity

Average → "Apparent"

$\bar{f} = \Delta P \cdot \dfrac{2 D_h}{L \rho u_m^2}$, $\Delta P = P_{x=0} - P_{x=L}$

Internal flow correlations

- Dimensionless: $Re_{D_h} = \frac{\rho u_m D_h}{\mu}$, $f = \frac{\Delta P}{L} \frac{2D_h}{\rho u_m^2}$, $Nu_{D_h} = \frac{h D_h}{k}$

Characteristics: $D_h = 4A_c/per$

FD laminar
- T & u grad. over entire A_c
- → f(duct shape) but not roughness e
- → Nu(shape, BC) but not Re, Pr, e

FD turb
- T & u grad over viscous sublayer
- → f(e) but not shape
- → Nu(Re, Pr, e) but not shape or BC

Internal flow: Energy

const heat flux q'': $T_m(x) = T_{in} + \frac{per \cdot q''}{\dot{m} c} x$

const wall temp: $T_m(x) = T_s - (T_s - T_{in}) \exp\left(- \frac{per \cdot x \cdot \bar{h}}{\dot{m} c}\right)$

const ext. temp: $T_m(x) = T_\infty - (T_\infty - T_{in}) \exp\left(\frac{-U A(x)}{\dot{m} c}\right)$

Free convection

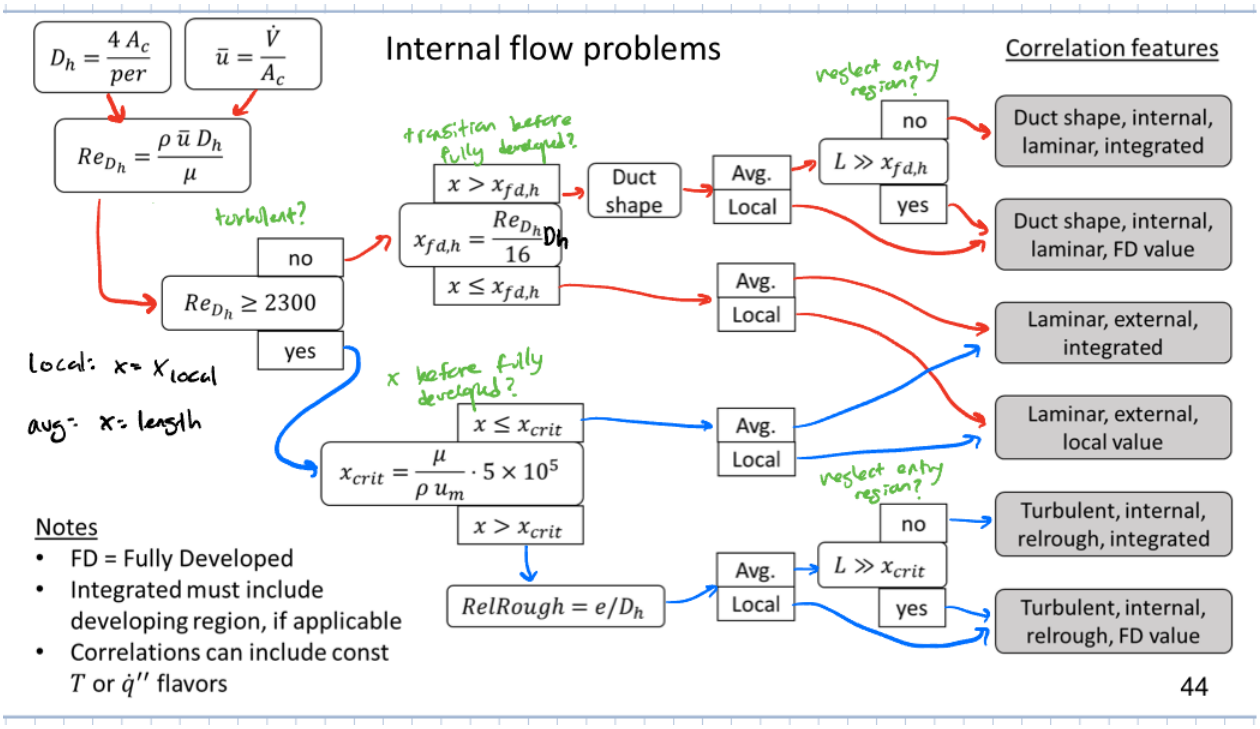
Dimensionless: $u_{char,nc} = \sqrt{g L_{char} \beta (T_s - T_\infty)}$, $Re = \frac{\rho u_{char,nc} L_{char}}{\mu}$

$Gr = Re^2$, $Ra = Gr\, Pr = \frac{g \beta L_{char}^3 (T_s - T_\infty)}{\nu \alpha}$, $\overline{Nu} = \frac{\bar{h} L_{char}}{k}$

- Combined free & forced:

same dir: $\bar{h}_{comb} = \left(\bar{h}_{forced}^m + \bar{h}_{free}^m\right)^{1/m}$, $m \approx 3$

opposite: $\bar{h}_{comb} = \left(\max[\bar{h}_{forced}, \bar{h}_{free}]^m - \min[\bar{h}_{forced}, \bar{h}_{free}]^m\right)^{1/m}$



Heat exchangers

$NTU = \frac{UA}{\dot{c}_{min}}$, $UA = \frac{1}{R_{tot}}$, $C_R = \frac{\dot{c}_{min}}{\dot{c}_{max}}$, $\dot{q}_{max} = \dot{c}_{min}(T_{H,in} - T_{c,in})$

- Relate $\dot{q}$:

$\dot{q} = \dot{c}_H (T_{H,in} - T_{H,out})$

$\dot{q} = \dot{c}_c (T_{c,out} - T_{c,in})$

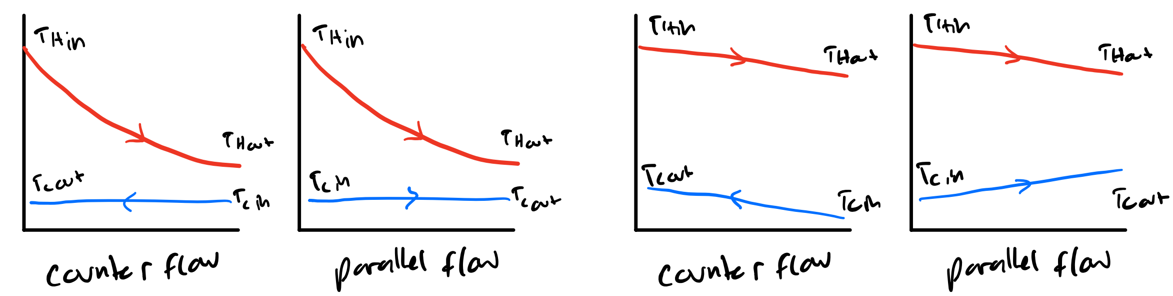
$\varepsilon = \frac{\dot{q}}{\dot{q}_{max}}$

- Effectiveness - NTU:
  - use tables to relate

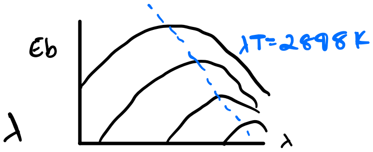
- counterflow HX perform better than parallel-flow except for at two limits: when $C_R \to 0$ and $NTU \to 0$:

$C_R \to 0$
(one fluid doesn't ΔT significantly)

$NTU \to 0$
(undersized, no significant ΔT)



Black body emission



- Radiation is $f(\lambda, T)$
- Blackbody will emit over large range of λ
- Distribution of E_b increases & shifts toward smaller λ as $T \uparrow$

$$E_b = \int_0^\infty E_{b,\lambda}\, d\lambda = \sigma T^4, \quad \sigma = 5.67 \times 10^{-8}\ [W/m^2\text{-}K^4]$$

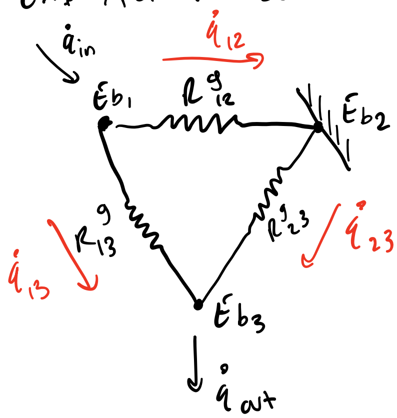
- External fraction function (Blackbody in EES)

$$F_{0-\lambda_1} = \int_0^{\lambda_1} \frac{c_1}{\sigma T^4 \lambda^5 \left(\exp\left(\frac{c_2}{\lambda T}\right) - 1\right)} d\lambda, \quad F_{\lambda_1-\lambda_2} = F_{0-\lambda_2} - F_{0-\lambda_1}$$

Black body Radiation exchange

- View Factors:
  1. Inspection
  2. Rules
     - enclosure: $\sum_{j=1}^{N} F_{i,j} = 1$
     - Reciprocity: $A_i F_{ij} = A_j F_{ji}$
     - consolidation: $F_{1,2+3} = F_{12} + F_{13}$
     - symmetry
  3. Analysis
     - Tables, EES view factors

Ex: Adiabatic surface



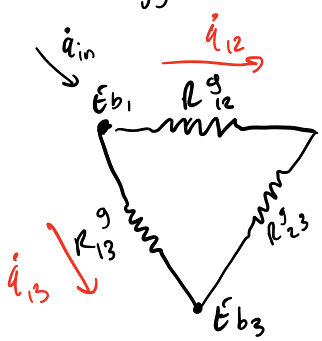
use equivalent resistance

$$q = \frac{E_{b_1} - E_{b_3}}{R^g_{eq}}$$

$$R_{eq} = \left[\frac{1}{R^g_{13}} + \frac{1}{R^g_{12} + R^g_{23}}\right]^{-1}$$

Ex: No adiabatic

→ energy balance on nodes



$$\dot{q}_{in} = \dot{q}_{13} + \dot{q}_{12}$$

$$\therefore \dot{q}_{in} = \frac{E_{b_1} - E_{b_3}}{R^g_{13}} + \frac{E_{b_1} - E_{b_2}}{R^g_{12}}$$

Real surface radiation

- Emit less radiation than BB & do not absorb incident
- Hemispherical emissivity: ratio of radiation rate emitted in all directions to rate of blackbody

$$\varepsilon_\lambda = \frac{E_\lambda}{E_{b,\lambda}}$$

- Total hemispherical emissivity: ratio of total radiation to BB

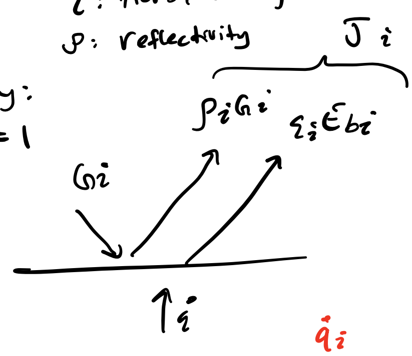
$$\varepsilon = \frac{E}{E_b} = \frac{\int_0^\infty E_\lambda \, d\lambda}{\sigma T^4} = \frac{\int_0^\infty \varepsilon_\lambda E_{b\lambda} \, d\lambda}{\sigma T^4}$$

- Kirchoff's law: $\alpha_{\lambda,\theta,\varphi} = \varepsilon_{\lambda,\theta,\varphi}$ (absorptivity = emissivity)

  Diffuse surface: $\alpha_\lambda = \varepsilon_\lambda$ Diffuse gray: $\alpha_{\lambda\theta\varphi} = \varepsilon_{\lambda\theta\varphi}$

- Surface E-bal: $\alpha_{\lambda\theta\varphi} + \tau_{\lambda\theta\varphi} + \rho_{\lambda\theta\varphi} = 1$ τ: transmissivity, ρ: reflectivity

  Diffuse surface: $\alpha_\lambda + \tau_\lambda + \rho_\lambda = 1$ Diffuse gray: $\alpha + \tau + \rho = 1$

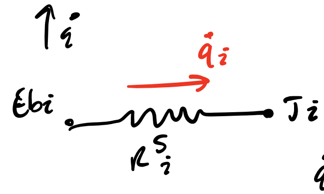


Diffuse gray radiation exchange

- Radiosity (J) - radiation leaving per area

  $J_i = \rho_i G_i + \varepsilon_i E_{bi}$

- Surface resistance (R_i^S) - between E_b & J



$$R_i^S = \frac{1-\varepsilon_i}{A_i \varepsilon_i}$$

- if black surface, $\varepsilon_i = 1$:

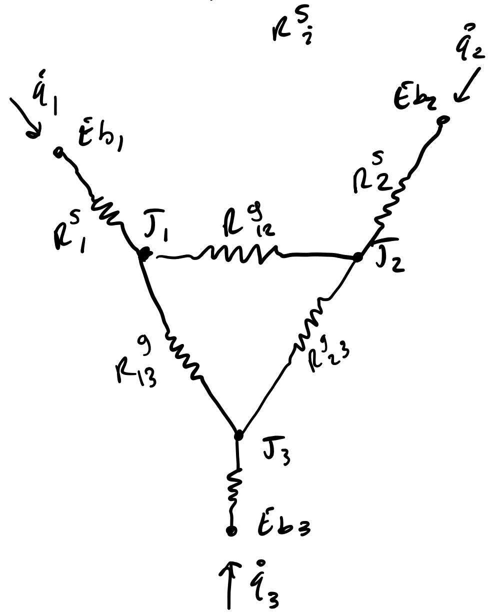
  $\rightarrow R_i^S = 0 \rightarrow E_{bi} = J_i$

- Radiation heat transfer between E & J:

$$\dot{q}_i = \left(\frac{\varepsilon_i A_i}{1-\varepsilon_i}\right)(E_{bi} - J_i)$$

- Between surfaces i & j:

$$\dot{q}_{ij} = \frac{J_i - J_j}{R_{ij}}, \quad R_{ij} = \frac{1}{A_i F_{ij}} = \frac{1}{A_j F_{ji}}$$



- If adiabatic, $\dot{q}_i = 0 \therefore J_i = E_{bi}$

  $\rightarrow \varepsilon$ doesn't matter, but surface is not a blackbody

HW 11
Equations: $R^g_{ij} = \frac{1}{F_{ij} A_i}$, $R^s_i = \frac{1-\varepsilon_i}{A_i \varepsilon_i}$

$\dot{q}_{11} + \dot{q}_{1,2} + \dot{q}_{13} + \dot{q}_{14} = 0$, $\dot{q}_{1,j} = \frac{J_1 - J_j}{R^g_{1,j}}$ $\forall j \in (2:4)$

BB: $E_b = J$ BC's: $\dot{q}_1 = I_c V_c$, $\dot{q}_2 = \frac{T_2 - T_b}{R_{cond}}$