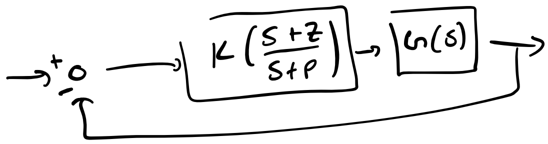
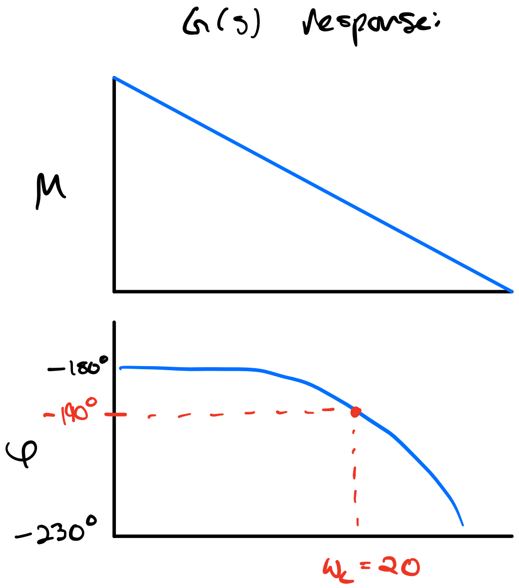
Faster response: $\omega_c \uparrow$

More damping: $PM \uparrow$

Lead compensation





Design for:

- $PM = 60^\circ$
- CL ω_{bw} as high as possible

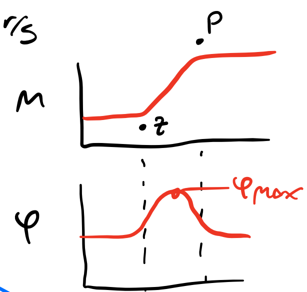
For lead compensator:

- Max phase is 90° but has too much high-freq. amplification
- Assume we can get 70° of phase w/o excessive high-freq. amp.

→ can set ω_c as high as 20 r/s & maintain $PM = 60^\circ$

For 70° from lead @ $\omega_c = 20$ r/s

$\alpha = \frac{z}{p}$ → lead ratio: $\frac{1}{\alpha}$ (20-30)

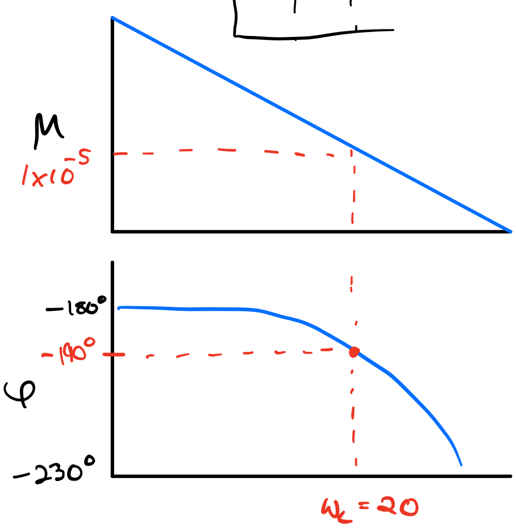


$$\alpha = \frac{1 - \sin(\varphi_{max})}{1 + \sin(\varphi_{max})}, \quad \varphi_{max} = 70^\circ$$

$\alpha \cong 0.031$, $LR(\frac{1}{\alpha}) \cong 30$

$z = \omega_{max}\sqrt{\alpha} \cong 3.53$

$p = \omega_{max}/\sqrt{\alpha} \cong 113.4$



$$D_{lead}(s) = K\frac{s + 3.53}{s + 113.4}$$

$$|D \cdot G|_{\omega=\omega_c} = 1$$

↓

$$|D(j\omega_c)G(j\omega_c)|_{\omega_c=20} = \left| K \frac{20j+3.53}{20j+113.4} \right| \cdot \underbrace{|G(20j)|}_{1\times10^{-5}}$$

$$\rightarrow \quad 1 = K \frac{\sqrt{20^2+3.53^2}}{\sqrt{20^2+113.4^2}} \cdot 1\times10^{-5} \quad \rightarrow \quad K \cong 5.67\times10^5$$

$$\rightarrow D_{lead}(s) = 5.67\times10^5 \, \frac{s+3.53}{s+113.4}$$

Calculate phase:

$$\angle G(j\omega_c) = -190^\circ$$

$$\angle \left[D_{lead}(j\omega_c) \cdot G(j\omega_c) \right] =$$

70° as designed

$$= \angle D + (-190) = \left[\operatorname{atan} \frac{20}{3.53} - \operatorname{atan} \frac{20}{113.4} \right] + (-190^\circ)$$

$$= -120^\circ$$

Steady state errors

$$\omega_c < \omega_{BW} < 2\omega_c$$

$\omega_c \sim \omega_{bw} \sim$ speed of response

$$\zeta \cong PM/100$$

We've looked at:

- stability ✓
- speed of response ✓
- Damping ratio ✓
- Steady state error ?

Integral control