$F(s)=\int_0^\infty f(t)e^{-st}dt$

Add phase

Q: what does adding 2 much phase do? eg. 90° from PD

Lead

$D_{lead}(s)=\dfrac{(s+z)}{(s+p)}$, $z<p$

lead ratio: $\frac{1}{\alpha}$

- Assume φ_{max}
- Pad PM des
- Find ω_c where

$\varphi_{lead}=PM_{des}-(\angle G(j\omega)+180°)$

$\rightarrow \angle G(j\omega) = PM_{des}-\varphi_{max}-180$

- Calc α, p, z, K for $|G(j\omega)|=1$

PD

$D_{PD}(s)=K(s+z_{pd})$

- Assume φ_{max}
- Pad PM des
- Find ω_c where

$\varphi_{PD}=\varphi_{des}-\angle G$ ($\varphi_{des}=PM-180°$)

Solve $\angle D_{PD}(j\omega)=\varphi_{PD}$

e.g. $z_{pd}=\dfrac{\omega_c}{\tan 75°}\approx 10.72$

$\rightarrow$ calc K for $|D_{PD}(j\omega)|=1$

lead-lag

$D_c(s)=$

$K\left(\frac{s+p}{s+z}\right)\left(\frac{s+p}{s+z}\right)$

lead lag

lead-PI

$=K\left(\frac{s+z}{s+p}\right)\left(\frac{s+z_{pI}}{s}\right)$

PD-lag

$=K(s+z_{pd})\left(\frac{s+z}{s+p}\right)$

PID

$=K_p+\frac{K_I}{s}+K_d s$

$K_p=K(z_{pd}+z_{pI})$

$K_d=K$

$K_I=K z_{pd} z_{pI}$

Decrease error, lose phase

Lag

lag ratio $\alpha=\frac{z}{p}>1$

- $D_{lag}(s)=\dfrac{s+z}{s+p}$, $p<z$
- moves pole away from origin (PI @ origin)
  - reduce φ_{loss} below z
  - increase low freq. gain by α

$\rightarrow$ To reduce e_{ss} by $15\times$: $\alpha=15$

- Set $z<\omega_c$

$\rightarrow p=z/\alpha$ NO K

PI

$D_{PI}(s)=K_p\left(s+\frac{K_I}{K_p}\right)/s=\dfrac{s+z_{PI}}{s}$

- error response speed ↑ as $z_{PI}=\frac{K_I}{K_p}$ ↑
- Set $z_{PI}=\frac{\omega_c}{10}$

or exactly: $\operatorname{atan}\left(\frac{\omega_c}{z_{PI}}\right)=PM$

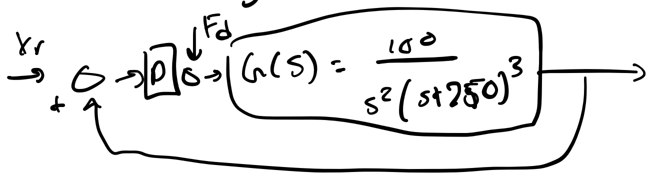
$(\angle D_{PI}(s)=\varphi_{loss,max})$

↑ $z_{PI,max}$

K_p from $|D_{PI}(j\omega_c)|=1$?

Ex

PD - lag



$PM \geq 60°$
$GM \geq 15 dB$
ω_{BW} wsh

e_{ss} :
Y_r $0.25t^2$: < 0.01
f_d $\mathfrak{z}(t)$ ≤ 0.03

PD : $D_{PD}(s) = K(s + z_{pd})$

- Assume $\varphi_{max} = 70°$ (given)

$\varphi_{des} = -180° + PM(\text{padded}) = -115°$

$\angle G(j\omega) = \varphi_{ds} - \varphi_{max} = -185°$ $\rightarrow$ bode plot : $\angle G(j\omega) = -185$ @ $\underline{\omega_c = 7 \text{ rad/s}}$

$\angle D_{pd}(j\omega) = \text{atan}\left(\frac{Im}{Re}\right) = \text{atan}\left(\frac{7}{z_{pd}}\right) = 70°$

$\rightarrow z_{pd} = \frac{7}{\tan 70°} = 0.0785$

$|D_{pd}(j\omega_c) \cdot G(j\omega_c)| = 1 = K\sqrt{7^2 + 0.0785^2} \cdot \frac{K\,10^7}{7^2\sqrt{7^2+250^2}^{\,3}}$

$\rightarrow K = 10.95$

$D_{pd} = 10.95(s + 0.0785)$

Lag: $D_{lag} = \frac{s+z}{s+p}$ from FVT: $\alpha = 7$

$\lim_{s \to 0} \left(s\frac{X(s)}{R(s)}R(s)\right)$

$\alpha = 7$, set $z_{lag} < \omega_c$

$z = 0.7$

$p = \frac{z}{\alpha} = 0.1$ $\rightarrow D_{lag} = \frac{s+0.7}{s+0.1}$

$\rightarrow D(s) = 10.95(s + 0.0785)\left(\frac{s+0.7}{s+0.1}\right)$

$\rightarrow K$

$D_{lead} = K\left(\frac{s+z}{s+p}\right)$

→ φ_{max} → α → z, p

→ find ω_c ↗

NOT

$D_{lag} = \cancel{K}\left(\frac{s+z}{s+p}\right)$

Set α for error

Set $z < \omega_c$

→ calc p

PD

$= K(s + z_{pd})$

→ φ_{max} → φ_{bs} → find ω_c

→ $\angle D_{pd}(j\omega_c) = \varphi_{max}$

→ solve for z_{pd}

$|D_{pd} G(j\omega)| = 1$ → solve K

PI

$\frac{(s + z_{pi})}{s}$

either guess $z_{pi} = \frac{\omega_c}{10}$

or calc exactly