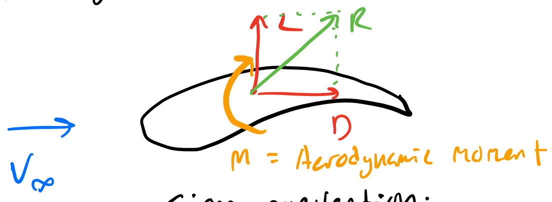
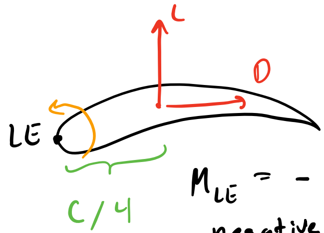
Lecture 4

Aerodynamic moments





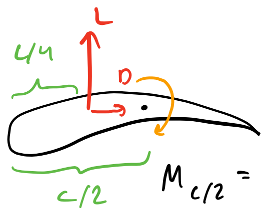
$M_{LE} = -L\frac{c}{4}$

negative sign!

Sign convention:

↻ + positive moment, wing tends to pitch up

↺ − negative moment, wing tends to pitch down



$M_{c/2} = L\frac{c}{4}$

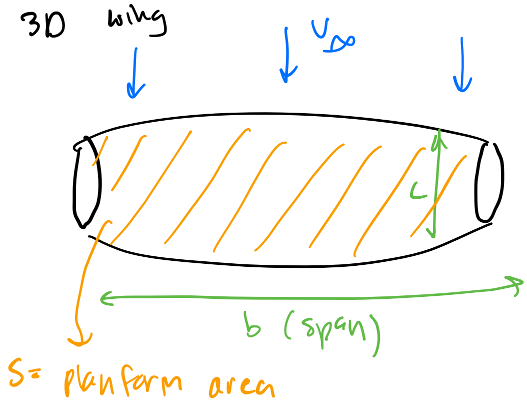
positive moment!

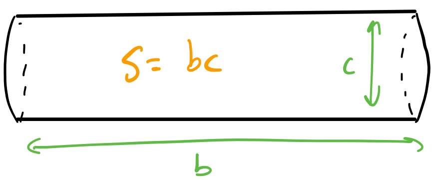
– neglect moment along chord line (Drag)

Lecture 5

Non-dimensional forces

3D wing





If no taper, rectangular wing

Lift, Drag : Newtons = N = $kg \cdot \frac{m}{s^2}$

Moment : Newton-meter = $N \cdot m$

What affects aerodynamic forces & moments?

- Surface area, S
  as S ↑, Forces ↑
- Velocity, V_∞
  as V_∞ ↑, Forces ↑
- Density, ρ_∞
  as ρ_∞ ↑, Forces ↑

3D wing Coefficients:

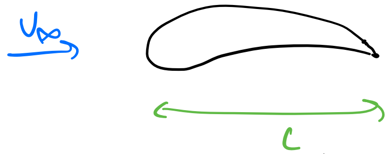
$$C_L = \frac{L}{\frac{1}{2}\rho_\infty V_\infty^2 S}$$ lift coefficient

$$C_D = \frac{D}{\frac{1}{2}\rho_\infty V_\infty^2 S}$$ drag coefficient

$$C_M = \frac{M}{\frac{1}{2}\rho_\infty V_\infty^2 S c}$$ moment coefficient

$q_\infty = \frac{1}{2}\rho_\infty V_\infty^2$ = dynamic pressure

2D wing



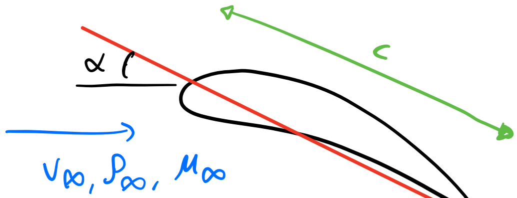
no planform area!
no span dimension
Forces defined "per unit span"

Lift / Drag: N/m

Moment: Nm/m = N   lowercase subscripts for 2D

$$C_l = \frac{L}{\frac{1}{2}\rho_\infty V_\infty^2 c} \; ; \; C_d = \frac{D}{\frac{1}{2}\rho_\infty V_\infty^2 c}$$

$$C_m = \frac{M}{\frac{1}{2}\rho_\infty V_\infty^2 c^2}$$



$$C_l = \frac{L}{\frac{1}{2}\rho_\infty V_\infty^2 c} \qquad C_d = \frac{D}{\frac{1}{2}\rho_\infty V_\infty^2 c}$$

$$C_l \,(\text{or } C_d) = f\Big(\underbrace{\begin{smallmatrix}\text{airfoil}\\ \text{shape}\end{smallmatrix}, \alpha, Re, M}_{\text{All non-dimensional}}\Big)$$

What parameters influence C_l & C_d?

- shape of airfoil: t/c, camber, LE shape, TE shape
- Angle of attack, α
- viscosity of fluid = μ [kg/m·s]

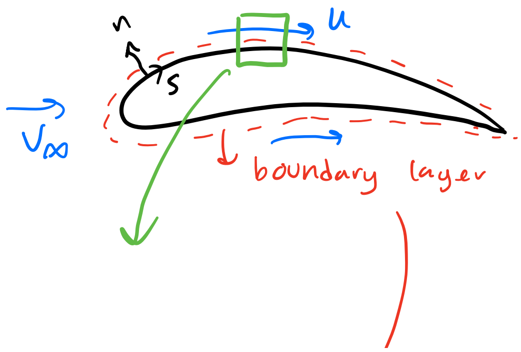
ND: Reynolds Number = $Re = \dfrac{\rho_\infty V_\infty c}{\mu_\infty}$

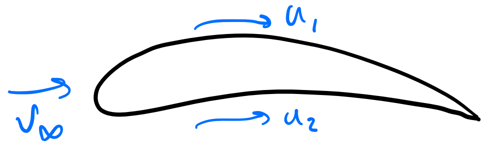
- Speed of sound → a_∞

ND: Mach number = $M = \dfrac{V_\infty}{a_\infty}$

Lecture 6: Pressure / Shear

what creates aerodynamic forces?

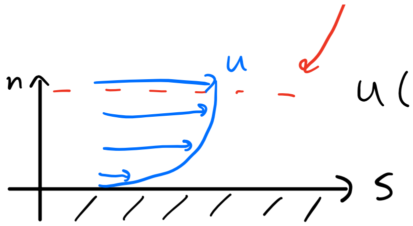




$u_1 > u_2$

Bernoulli: as $u \uparrow$ $P \downarrow$

$\rightarrow P_1 < P_2$



$u(n)$ = boundary layer velocity profile

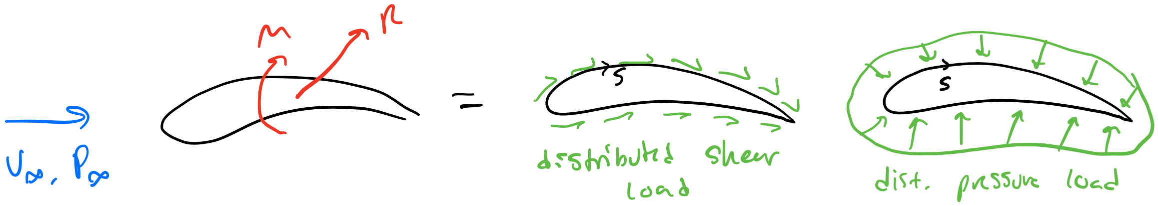
same units as shear stress:
pressure · area = Force
shear · area = Force

No-slip condition: $u(n=0) = 0$

Boundary layer (BL) has vorticity

BL has friction, fluid friction = shear

$$\tau_{wall} = \text{shear stress at wall} = \mu \left.\frac{\partial u}{\partial n}\right|_{wall}$$



distributed shear load

$\tau(s)$

- always tangent to surface

dist. pressure load

$p(s)$

- always acts normal to surface

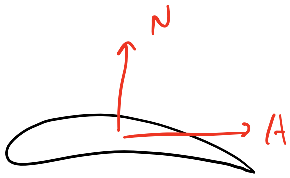
Skin friction coeff.

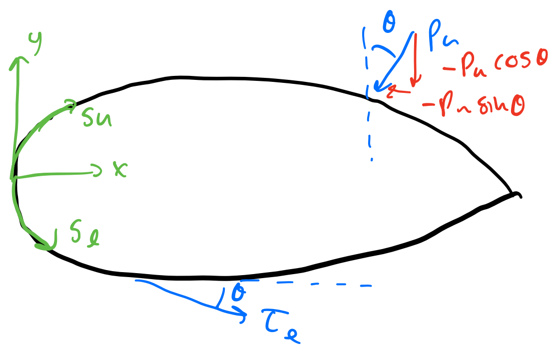
$$C_f = \frac{\tau}{q_\infty}$$

Pressure coefficient

$$C_p = \frac{p - p_\infty}{q_\infty}$$

$$q_\infty = \tfrac{1}{2} \rho_\infty V_\infty^2$$





$$N = \int_{LE}^{TE} \left(P_u \cos\theta + \tau_u \sin\theta\right) dS_u + \int_{LE}^{TE} \left(P_l \cos\theta - \tau_l \sin\theta\right) dS_l$$

$$A = \int_{LE}^{TE} \left(-P_u \sin\theta + \tau_u \cos\theta\right) dS_u + \int_{LE}^{TE} \left(P_l \sin\theta + \tau_l \cos\theta\right) dS_l$$

Lecture 7: Fluid flow governing equations

Navier - stokes eqn. govern all fluid flow.

we want: $\underbrace{\vec{V}(t,\vec{x}) \; \& \; P(t,\vec{x})}$

Field variables

$\vec{x}$ = position ; t = time

contour plot

Our goal:

modeling tools

airfoil shape $\rightarrow$ numerical solver (CFD) $\rightarrow$ $P(t,\vec{x})$ & $\vec{V}(t,\vec{x})$ $\rightarrow$ integrate P & $\vec{V}$ $\rightarrow$ Forces: Lift, Drag, moments

α, Re, M

solve gov. eqns.

Governing eqn. 1: cons. mass (continuity)

$$\frac{d\rho}{dt} + \nabla \cdot (\rho \vec{V}) = 0$$ full eqn w/ no assumptions

$\frac{d\rho}{dt} = 0$, if flow incompressible: $\nabla \cdot \vec{V} = 0$

If flow is 2D & planar, $\vec{V} = u\hat{i} + v\hat{j}$

$$\boxed{\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0}$$

If flow is 2D & cylindrical, $\vec{V} = u_r \hat{r} + u_\theta \hat{\theta}$

$$\boxed{\frac{\partial (r u_r)}{\partial r} + \frac{\partial u_\theta}{\partial \theta} = 0}$$

Governing eqn. 2: cons. momentum (Navier-stokes)

$\Sigma \vec{F} = m\vec{a}$ applied to a fluid particle

skip derivation

$$\Sigma \vec{F} = m \frac{d\vec{v}}{dt} = \frac{d(m\vec{v})}{dt} = \text{rate of change of linear momentum}$$

$$\boxed{\rho \left\{ \frac{\partial u}{\partial t} + \nabla \cdot (u \vec{v}) \right\} = -\frac{\partial P}{\partial x} + \mu \nabla^2 u}$$ $\Rightarrow$ momentum in x - dir

$$\boxed{\rho \left\{ \frac{\partial v}{\partial t} + \nabla \cdot (v \vec{V}) \right\} = -\frac{\partial P}{\partial y} + \mu \nabla^2 v} \Rightarrow \text{momentum in } y\text{-dir}$$

Newtonian fluid, incompressible, 2D, planar

→ solve with continuity eqn → 3 eq, 3 unknowns (u, v, P)

Lecture 8: Stream function

Stream function, ψ "psi" only defined for steady 2D flows

steady ↓ $\frac{\partial [\,]}{\partial t} = 0$; 2D ↓ planar or cylindrical

For an incompressible flow, stream function defined as:

$$\boxed{u = \frac{\partial \psi}{\partial y} \quad \text{and} \quad v = -\frac{\partial \psi}{\partial x}}$$

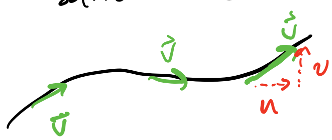
starting w/ continuity:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad \text{substitute } \psi \text{ definition} \rightarrow \frac{\partial}{\partial x}\left(\frac{\partial \psi}{\partial y}\right) - \frac{\partial}{\partial y}\left(\frac{\partial \psi}{\partial x}\right) = 0$$

$$\frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial^2 \psi}{\partial x \partial y} = 0$$

Stream function satisfies continuity!

Stream line:



tangent to velocity vectors

$$\frac{\partial y}{\partial x} = \frac{v}{u}$$

Expand $$d\psi = \underbrace{\frac{\partial \psi}{\partial x}}_{-v} dx + \underbrace{\frac{\partial \psi}{\partial y}}_{u} dy = -v\,dx + u\,dy = 0$$

If $d\psi = 0$, then $\psi = \text{const}$, and $v\,dx = u\,dy$

or $\frac{dy}{dx} = \frac{v}{u}$

$$\boxed{\text{Lines of constant } \psi \text{ are stream lines}}$$

Stream function exists for 2D incompressible flow

– check incompressible continuity $= 0$