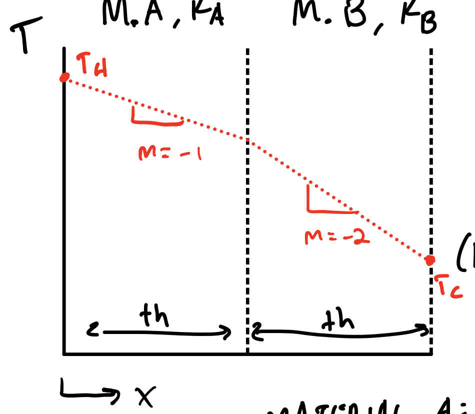
# 1) SKETCH TEMP PROFILES

## a) PLANE WALL




$T(x=0) = T_h$ , $T(x=2th) = T_c$

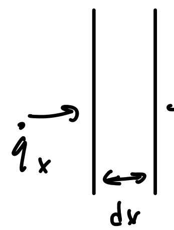
$K_A = 2K_B$

FOURIER'S: $\dot{q}'' = -K \nabla T$ ) 1D, AREA

(1) $\dot{q} = -K \cdot A_c \cdot \dfrac{dT}{dx}$

MATERIAL A:

$\text{IN} + \cancel{\text{GEN}} = \text{OUT} + \cancel{\text{STORED}}$ ($\dot{g} = 0$, $\dfrac{dU}{dt} = 0$)




$\rightarrow \dot{q}_{x+dx}$ $\therefore \dot{q}_x = \dot{q}_{x+dx}$

$\dot{q}_{x+dx} = \underbrace{\dot{q}_x + \dfrac{d\dot{q}_x}{dx}\,dx}_{\text{TAYLOR EXPANSION}}$

$\therefore \dfrac{d\dot{q}_x}{dx} = 0 \rightarrow \dot{q}_x = \text{CONST}$

$\rightarrow$ FROM EQ (1): IF $\dot{q}$ = CONST, $A_c$ CONST, $\rightarrow \dfrac{dT}{dx} \propto \dfrac{1}{K}$

SINCE $K_A = 2 \cdot K_B$,

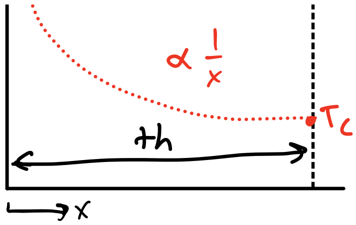
$$\boxed{\therefore \left.\frac{dT}{dx}\right|_A = \frac{1}{2} \cdot \left.\frac{dT}{dx}\right|_B}$$

$\dfrac{dT}{dx} = \text{CONST}$

## b) PLANE WALL

COND. VARIES, $K(x) = C_1 x + C_2$ | $C_1, C_2 > 0$


FROM FOURIER'S LAW:




$$\dot{q} = -K(x) A_c \frac{dT}{dx} \quad (1)$$

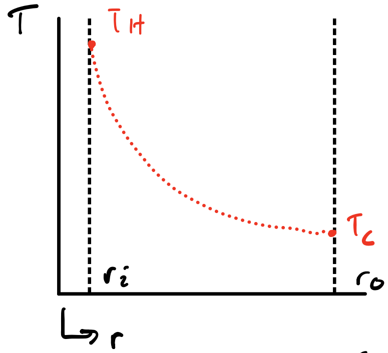
BY EARLIER PROOF, $\frac{d\dot{q}_x}{dx} = 0 \rightarrow \dot{q}_x = \text{CONST}$

EQ (1): $\frac{dT}{dx} = -\frac{\dot{q}}{K(x) A_c}$

SINCE $\dot{q}$, $A_c$ CONST, $\frac{dT}{dx} \propto -\frac{1}{K(x)}$

$$\rightarrow \frac{dT}{dx} \propto -\frac{1}{c_1 x + c_2}$$

c) LONG ROD




$K = \text{CONST}$

FOURIER'S:

$$\dot{q} = -K A_c \frac{dT}{dr} \quad (1)$$

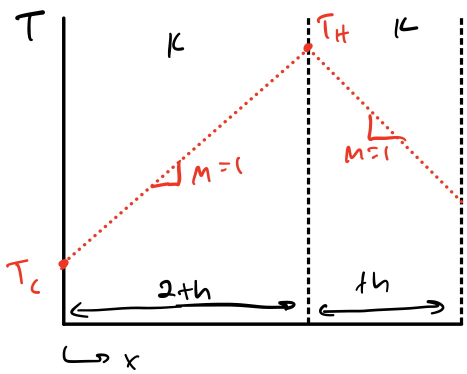
BY EARLIER PROOF, $\frac{d\dot{q}_r}{dr} = 0 \rightarrow \dot{q}_r = \text{CONST}$

EQ (1): $\frac{dT}{dr} = -\frac{\dot{q}_r}{K A_c}$ $\quad A_c = 2\pi r L$

$\dot{q}_r, K, L$ CONST

$$\rightarrow \frac{dT}{dr} \propto -\frac{1}{r}$$

d) PLANE WALL




FOURIER'S:

$$\dot{q} = -K A_c \frac{dT}{dx}$$

EARLIER PROOF: $\frac{d\dot{q}}{dx} = 0$

$$0 = \frac{d}{dx}\left(-K A_c \frac{dT}{dx}\right)$$

$$\int 0 \, dx = -K A_c \int \frac{d^2T}{dx^2} \, dx$$

$\int -\, C_1 = \int dT \, dx$ $\quad$ BC 1) $T = T_c$

$$\int -\frac{dx}{kA_c} \int \frac{}{dx} \, dx \qquad BC1)\ T_{x=0} = T_c$$

$$BC2)\ T_{x=2th} = T_H$$

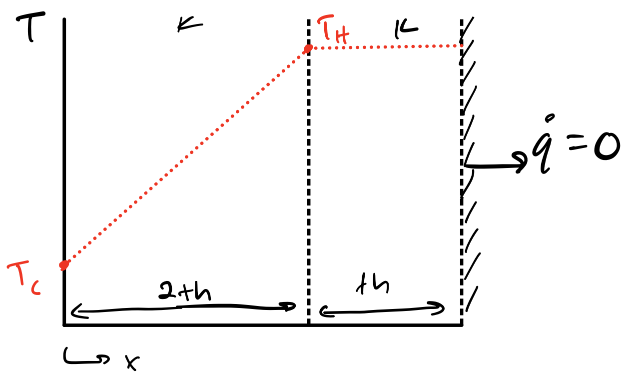
$$T(x) = -\frac{C_1 x}{kA_c} + C_2$$

$$T(x=0) = C_2 = T_c$$

$$T(x=2th) = \frac{-2C_1 th}{kA_c} + T_c = T_H \rightarrow C_1 = \frac{(T_c - T_H)kA_c}{2th}$$

$$\Rightarrow \boxed{T(x) = \frac{(T_H - T_c)}{2th} x + T_c}$$

**e)** PLANE WALL



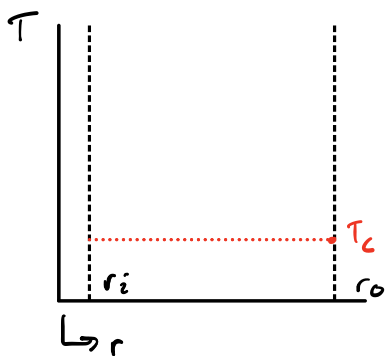

ALONG WALL, $\dot{q}_{IN} = \dot{q}_{OUT} = 0$

$\hookrightarrow \dot{q} = 0$

$$\dot{q} = -kA_c \frac{dT}{dx} = 0$$

$$\therefore \frac{dT}{dx} = 0 \rightarrow T = \text{CONST}$$

**f)** ROD




$$\dot{q}_{IN} - \dot{q}_{out} = 0$$

$$\dot{q} = -kA_c \frac{dT}{dr} = 0$$

$$\rightarrow \frac{dT}{dr} = 0$$

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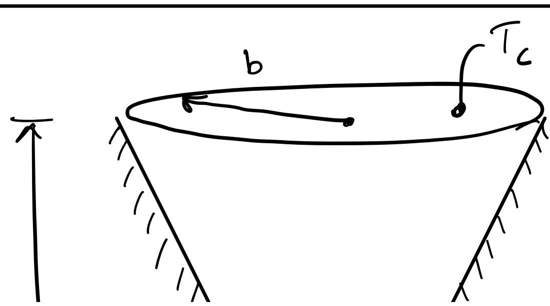
**2)**

$b = 5\,cm = 0.05\ [m]$

$x_a = 1\,cm = 0.01\ [m]$

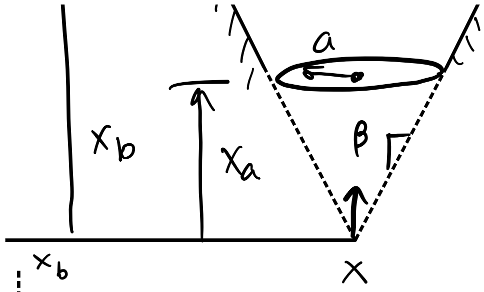
$x_b = 4\,cm = 0.04\ [m]$

$a = b/$

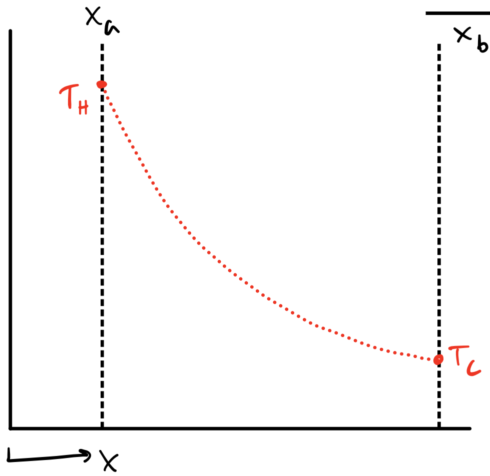


$\beta = \ /x_b$

$T_c = 80\ [K] \qquad T_H = 285\ [K]$

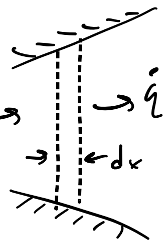



a) T




FOURIER'S: $q'' = -k\nabla T$

$\rightarrow \dot{q} = -kA_c \dfrac{dT}{dx} \quad (1)$




$\dot{q}_x \rightarrow \quad \rightarrow \dot{q}_{x+dx} = \dot{q}_x + \dfrac{d\dot{q}_x}{dx}dx$

$\dot{q}_x = \dot{q}_{x+dx}, \therefore \dfrac{d\dot{q}_x}{dx} = 0 \quad (2)$

(1) & (2): $0 = \dfrac{d}{dx}\left(-kA_c\dfrac{dT}{dx}\right)$

$A_c = \pi (r)^2 \qquad r = x\beta$

$A_c = \pi \beta^2 x^2$

b)

$\rightarrow \boxed{0 = -k\pi\beta^2 \dfrac{d}{dx}\left(x^2 \dfrac{dT}{dx}\right)} = x^{\cancel{2}}\dfrac{d^2T}{dx^2} + 2\cancel{x}\dfrac{dT}{dx} = 0$

LOSE $x=0$ AS SOLUTION
$\rightarrow$ NOT NEEDED

$0 = x\dfrac{d^2T}{dx^2} + 2\dfrac{dT}{dx} \rightarrow \int 0\, dx = \int \dfrac{d}{\cancel{dx}}\left(x^2\dfrac{dT}{dx}\right)\cancel{dx}$

$C_1 = x^2 \dfrac{dT}{dx} \rightarrow \int \dfrac{dT}{\cancel{dx}}\cancel{dx} = \int \dfrac{C_1}{x^2}dx$

c) $\boxed{T(x) = -\dfrac{C_1}{x} + C_2}$

d) BC1: $T(x = x_a) = T_h$

BC2: $T(x = x_b) = T_c$

FROM BC1: $T(x_a) = -\dfrac{C_1}{x_a} + C_2 = T_h \quad (1)$

FROM BC2: $T(x_b) = -\frac{C_1}{x_b} + C_2 = T_c \quad (2)$

FROM (1): $285\,[K] = -\frac{C_1}{0.01\,[m]} + C_2 = -100\,C_1 + C_2 = 285 \quad (3)$

FROM (2): $80\,[K] = -\frac{C_1}{0.04\,[m]} + C_2 = -25\,C_1 + C_2 = 80 \quad (4)$

FROM (3): $C_2 = 285 + 100\,C_1$

SUB INTO (4): $-25\,C_1 + (285 + 100\,C_1) = 80$

$\hookrightarrow \quad 75\,C_1 = -205$

$\rightarrow C_1 = -\frac{41}{15} \approx -2.733$

SUB KNOWN $C_1$ INTO

$\rightarrow \quad C_2 = 285 + 100\left(-\frac{41}{15}\right) = \frac{35}{3} \approx 11.67$

$$\boxed{\therefore \quad T(x) = \frac{2.733}{x} + 11.667}$$

**e)** $\dot{q} = -k A_c \frac{dT}{dx}$

$k$: EES LOOKUP

$A_c$: $\pi \beta^2 x^2$

$\frac{dT}{dx} = -\frac{C_1}{x^2}$

**f)** IN EES

**g)** $x = x_a$ CONVECTION $\rightarrow \bar{h},\ T_h$

BC EQUATION: $\dot{q}_{conv} = \bar{h} \cdot A_s \cdot (T_s - T_\infty)$

$\Rightarrow \quad \dot{q}_{conv} = \bar{h} \cdot (\pi \beta^2 a^2) \cdot (T_s - T_h)$

$T_s = T(x = x_a)$

```
$Load Incompressible
$UNITSYS SI K Pa J mass rad

"Kyle Adler ME364 HW01"

"givens"
b = 0.05 [m]
x_b=0.04 [m]
x_a=0.01[m]
beta = b/x_b
T_h = 285 [K]
T_c = 80 [K]
k=conductivity(Stainless_AISI310, T=average(T_h,T_c))

"calculated by hand"
C_1 = -41/15  [1/K-m]
C_2 = 35/3  [1/K]

"area and dTdx equations"
A = pi#*beta^2*x^2
dTdx = -C_1/x^2

"value for x, qdot does not change with respect to x so any nonzero real value yields same answer"
x = x_a                                        "commented for part f"

"e) heat transfer through support"
q_dot = -k*A*dTdx

"f) plot of temp as function of x between x=x_a and x=x_b"
T=-C_1/x+C_2
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SOLUTION

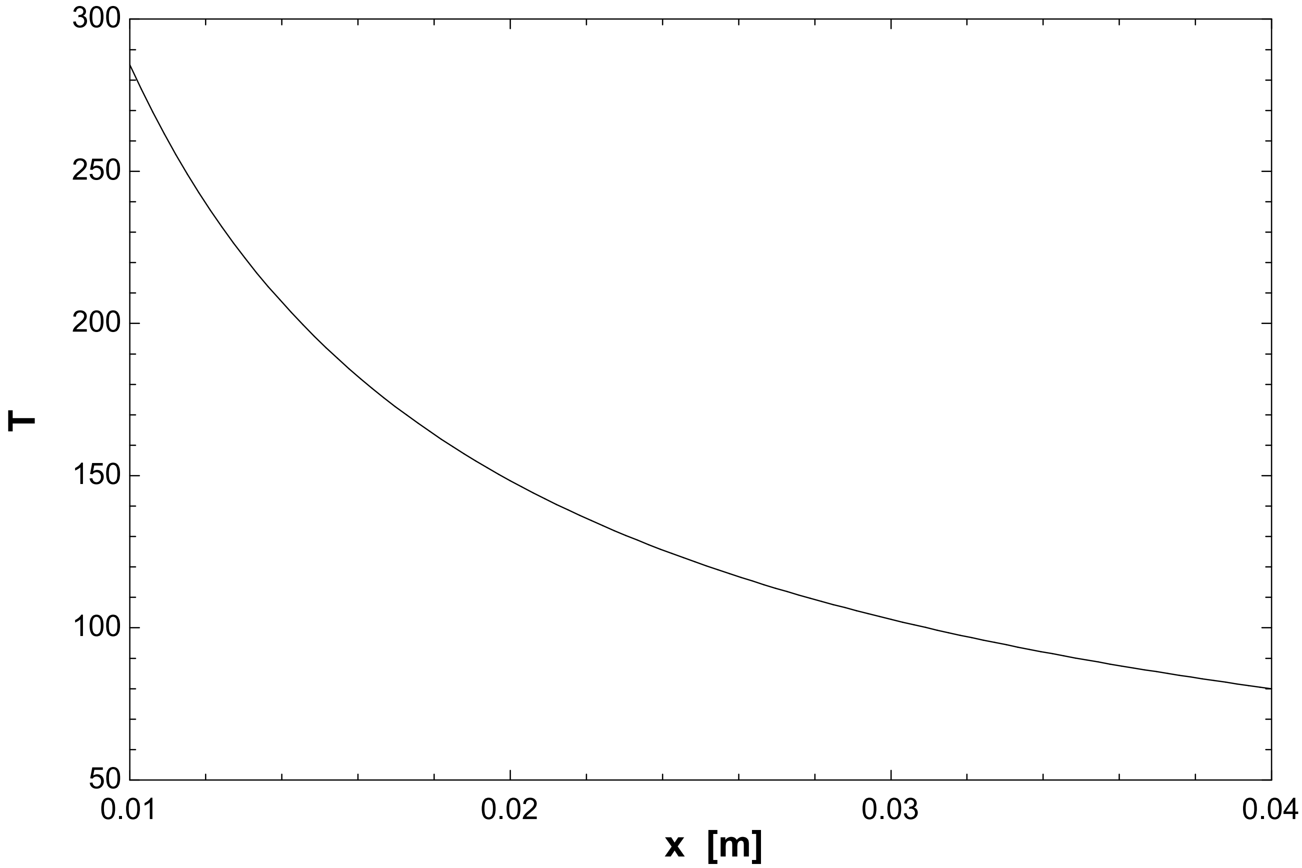
**Unit Settings: SI K Pa J mass rad**

| | | | | |
|---|---|---|---|---|
| $A$ = 0.0004909 [$m^2$] | $b$ = 0.05 [m] | $\beta$ = 1.25 [-] | $C_1$ = -2.733 [K-m] | $C_2$ = 11.67 [K] |
| dTdx = 27333 [K/m] | $k$ = 9.073 [W/m-K] | $\dot{q}$ = -121.7 [W] | $T$ = 285 [K] | $T_c$ = 80 [K] |
| $T_h$ = 285 [K] | $x$ = 0.01 [m] | $x_a$ = 0.01 [m] | $x_b$ = 0.04 [m] | |

No unit problems were detected.

KEY VARIABLES

$\dot{q}$ = -121.7 [W] *e) heat transfer through support*


300
250
200
150
100
50
T
0.01
0.02
0.03
0.04
x [m]