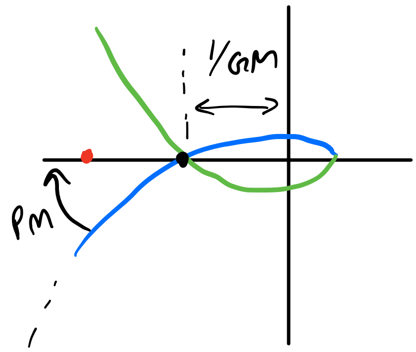
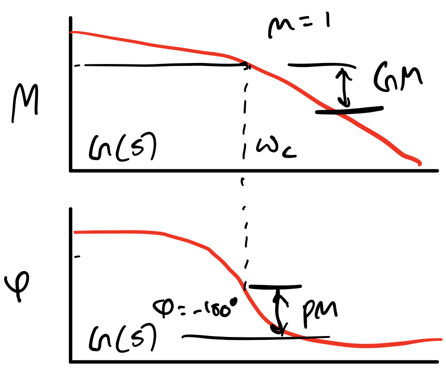
Stability → Nyquist

- Gain margin, GM
- phase margin, PM



OL Freq. response



Usually look at freq. response plot, can be useful to look @ Nyquist plot especially if complex system

What can closed loop system characteristics be evaluated by open-loop response?

- stability? ✓
- speed of response?
- Damping ratio?
- steady state error?

Bandwidth & speed of response

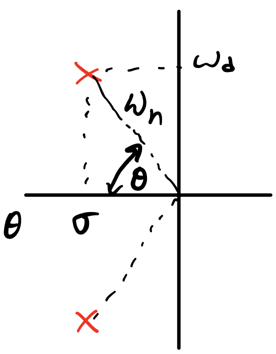
2nd order sys approximation

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Roots of a(s):

$$s = \zeta\omega_n \pm j\underbrace{\left(\omega_n\sqrt{1-\zeta^2}\right)}_{\omega_d}$$

$\zeta = \cos\theta$



Impulse response:

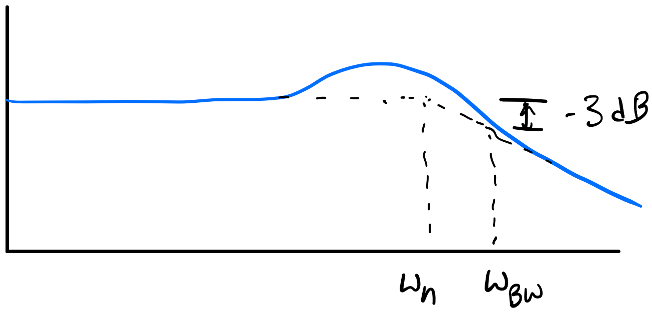
$$x(t) = e^{-\sigma t}\sin(\omega_d t + \varphi)$$

speed of response:

$$\sigma = \zeta\omega_n \rightarrow \tau = -\frac{1}{\sigma} = \frac{1}{\zeta\omega_n}$$

Speed of response proportional to ω_n

$\omega_n \propto |\sigma| = \frac{1}{\tau}$



$$\left.\begin{array}{l} \omega_{BW} \propto \omega_n \\ \omega_n \propto |\sigma| = \frac{1}{\tau} \end{array}\right\} \omega_{BW} \propto \frac{1}{\tau}$$

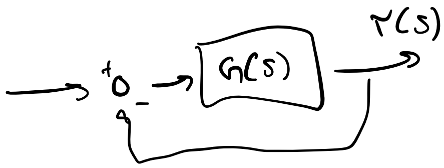
speed of response proportional to ω_{BW}

Stability Margins & Transient Response

Approx:

$\omega_c \longleftrightarrow \omega_{BW}$

crossover bandwidth



$$T(s) = \frac{r(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$\rightarrow G(s) = \frac{\omega_n^2}{s(s + 2\zeta\omega_n)}$$

closed loop: $T(s)$ open loop: $G(s)$

For most systems: $\boxed{\omega_c \le \omega_{BW} \le 2\omega_c}$

- useful approx. for $\zeta < 1$
- accurate only for assumed 2^{nd} order sys
- Reasonable rule of thumb

Summary: $\boxed{\tau = -\frac{1}{\sigma} = \frac{1}{\zeta\omega_n} \propto \frac{1}{\omega_n}}$

$\omega_{BW} \propto$ speed of response

$\boxed{\omega_c \propto \omega_{BW} \propto \text{speed of response}}$

Approximate relationship:

$PM \leftrightarrow \zeta$

phase margin damping ratio

evaluate phase at $\omega = \omega_c$

$$PM = \angle G(j\omega_c) + 180^\circ$$

⋮

$$PM = \tan^{-1}\left[\frac{2\zeta}{\sqrt{\sqrt{1+4\zeta^4} - 2\zeta^2}}\right]$$

For most systems:

$$\boxed{\zeta \cong PM/100}$$

- useful for $PM < 70^\circ$
- rule of thumb