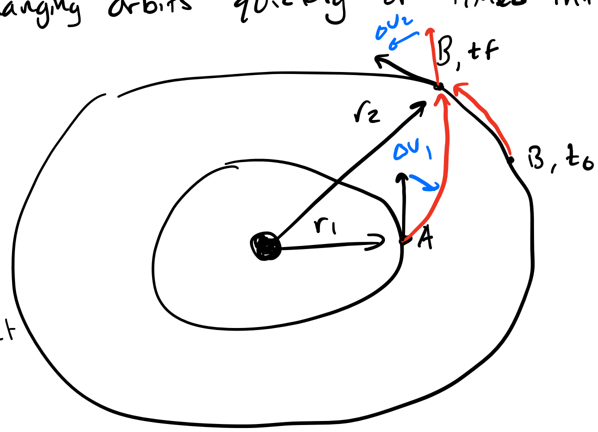
Project: 4-5 people - detailed presentation

→ Ranked list of 5 topics

Lambert's problem - changing orbits quickly or timed interception

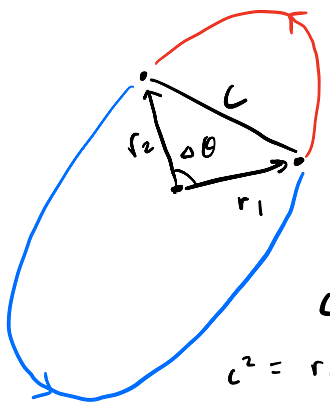
Requirements

- goal: $r_1 \to r_2$ orbit
- Need: target orbit
  interceptor orbit
- need target position @ impact
- need timing goal
- need transfer orbit



Application! use Lambert to build orbit geometry

Assume elliptical orbit to start



Known: $r_1, r_2, \Delta\theta$

2 possible ellipse paths

short path

long path

Focus 1 @ celestial body

Focus 2 → unknown

$c \equiv$ chord

$$c^2 = r_1^2 + r_2^2 - 2 r_1 r_2 \cos \Delta\theta$$

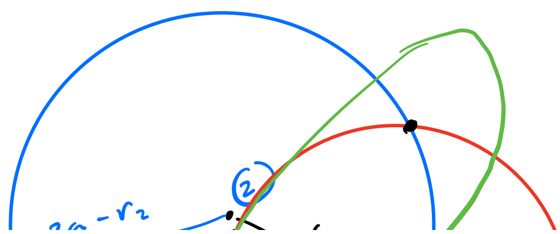
→ Finding a

$$s \equiv \text{semi-perimeter} = \frac{r_1 + r_2 + c}{2}$$

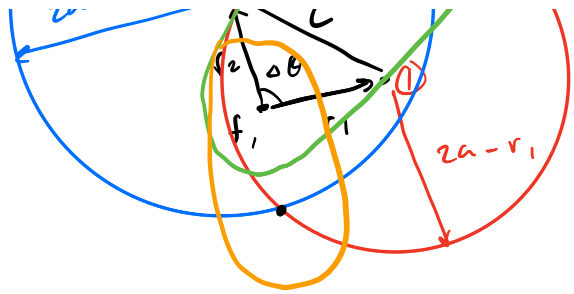
Next: Find all possible ellipses

1) Define $2a - r_1$
   $2a - r_2$

2) Draw circles



3) Find intersections



To pick one option, we need bounds: time or a

<u>Specify based on time:</u> Kepler's:

$$M = \frac{2\pi}{T}(t - t_p) = E - e\sin E$$

θ vs E $\quad \tan\frac{\theta}{2} = \sqrt{\frac{1+e}{1-e}}\tan\left(\frac{E}{2}\right)$

$\therefore$ between r_1 & r_2

$$M_2 - M_1 = \underbrace{\frac{2\pi}{T}}_{\sqrt{\frac{\mu}{a^3}}} \underbrace{\Delta t}_{\text{time of flight}} = E_2 - E_1 - e(\sin E_2 - \sin E_1)$$

<u>Define:</u> $E_p = \frac{1}{2}(E_1 + E_2)$

$E_m = \frac{1}{2}(E_2 - E_1) > 0$

$\therefore \quad r_1 + r_2 = a(1 - e\cos E_1) + a(1 - e\cos E_2)$

$= 2a(1 - e\cos E_p \cos E_m)$ $\quad$ trig id & subs

<u>Define:</u> $\cos \underset{\uparrow}{\overset{xi}{\xi}} = e\cos E_p$ $\qquad$ <u>Relates:</u> chord, a, s in Lambert's Eq

$\alpha = \xi + E_m$

$\beta = \xi - E_m$

combine to get <u>Lambert's Equation</u>

$$\boxed{\begin{aligned} &\sqrt{\mu}\,\Delta t = a^{3/2}\left[\alpha - \beta - (\sin\alpha - \sin\beta)\right] \\ &\text{with} \quad \sin\left(\frac{\alpha}{2}\right) = \sqrt{\frac{s}{2a}}, \quad \sin\left(\frac{\beta}{2}\right) = \sqrt{\frac{s-c}{2a}} \\ &c = \sqrt{r_1^2 + r_2^2 - 2r_1r_2\cos\Delta\theta}, \quad s = \frac{r_1 + r_2 + c}{2} \end{aligned}}$$

Remarks

1) usually know $\Delta t, r_1, r_2, \Delta\theta$

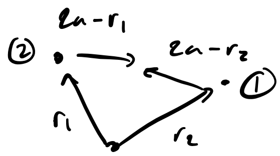
$\rightarrow$ Find a w/ numerical solution

2) How could we bound our Δt or a?

Bound 1: ellipse goes to parabola $\rightarrow$ $a \rightarrow \infty$

Bound 2: $a \rightarrow a_{min}$

min. a case



see slide 12

Think Energy $\mathcal{E} = -\frac{\mu}{2a}$

At $\mathcal{E} = \mathcal{E}_{max}$, $a = a_{min}$

$c = 2a_m - r_1 + 2a_m - r_2$

$c = 4a_m - (r_1 + r_2)$

$\frac{c + r_1 + r_2}{2} = 2a \quad \therefore \quad s = 2a_m, \quad \boxed{a_m = \frac{s}{2}}$

3) $a_m < a < \infty \rightarrow$ Bounds for ellipses

at max energy $a_m = \frac{s}{2}$, what is Δt_m?

α_m: $\therefore \sin\left(\frac{\alpha_m}{2}\right) = \sqrt{\frac{s}{2a_m}} = \sqrt{\frac{s}{2(s/2)}} = 1$

$\therefore \boxed{\alpha_m = \pi}$

β_m: $\therefore \sin\left(\frac{\beta_m}{2}\right) = \sqrt{\frac{s-c}{2}}$ plug into Lambert's

$\therefore \boxed{\Delta t_m = \sqrt{\frac{s^3}{8\mu}}\left(\pi - \beta_m + \sin\beta_m\right)}$ w/ $\mathcal{E}_{max}$, a_{min}

As we approach parabolic $a \rightarrow \infty$

$$\underline{\Delta t_p} = \frac{1}{3}\sqrt{\frac{2}{\mu}}\left[s^{3/2} - \underbrace{\text{sgn}(\sin\Delta\theta)}_{\text{gives } 0,1,-1 \text{ based on sign of } \sin\Delta\theta}(s-c)^{3/2}\right]$$

$\downarrow$

min transfer Δt

<u>Also need</u> $\Delta\theta$ & Δt to uniquely determine.

<u>Procedure</u>

1) calc Δt_p (min Δt)

   If desired $\Delta t > \Delta t_p$ -> elliptical transfer

2) calc Δt_m, determine quadrant of α

   if $\Delta t \leq \Delta t_m \rightarrow \alpha = \alpha_0$; else $\alpha = 2\pi - \alpha_0$

   primary

3) Determine quadrant of β

   primary

   if $0 \leq \Delta\theta < \pi$, $\rightarrow \beta = \beta_0$; else $\beta = -\beta_0$

4) Numerically solve Lambert's for a