CONICAL NOZZLE

$\mathcal{T} = 3.66 \cdot 10^6$ N $\qquad C_{\mathcal{T}} = 1.66$

$p_a = 10^5$ Pa $\qquad p_0 = 7 \cdot 10^6$ Pa

$\frac{A_c}{A_t} = 1.5 \qquad \frac{A_e}{A_t} = 15$

$L^* = 1.2$ m $\qquad \theta_c = 20° \qquad \theta_e = \theta_N = 15°$

FIND: R_t, R_e, V_c, L_{CONV}, L_{CYL}, L_n

i) $C_{\mathcal{T}} \equiv \frac{\mathcal{T}}{p_0 A^*} \Rightarrow A^* = \frac{\mathcal{T}}{p_0 C_{\mathcal{T}}} = \frac{3.66 \cdot 10^6}{7 \cdot 10^6 \cdot 1.66} =$

$= 0.3214 \ m^2$

$A^* = A_t = \pi R_t^2 \Rightarrow R_t = \sqrt{\frac{A^*}{\pi}} = \sqrt{\frac{0.3214}{\pi}} = 0.3199$ m

ii) $A_e = 15 A_t = 4.8214 \ m^2 \Rightarrow R_e = \sqrt{\frac{A_e}{\pi}} = 1.2388$ m

iii) FROM $L^* = \frac{V_c}{A_t} \qquad V_c = L^* A_t = 1.2 \cdot 0.3199 = 0.3587 \ m^3$

iv) $\frac{A_c}{A_t} = \frac{R_c^2}{R_t^2} = 1.5 \rightarrow R_c = \sqrt{1.5}\, R_t = 0.3918\ m$

v) $V_c = V_{CYL} + V_{CONV} = \left(\pi R_c^2 L_{CYL}\right) + \frac{\pi}{3} L_{CONV}\left(R_c^2 + R_t^2 + R_c R_t\right)$

FROM CLASS NOTES

$$L_{CONV} = \frac{R_t\left(\sqrt{\frac{A_c}{A_t}} - 1\right) + 1.5 R_t\left(\sec\theta_c - 1\right)}{\tan\theta_c} =$$

$$= \frac{0.3199\left(\sqrt{1.5} - 1\right) + 1.5 \cdot 0.3199\left(\sec 20^\circ - 1\right)}{\tan 20^\circ} = 0.2821\ m$$

$$V_{CONV} = \frac{\pi}{3} \cdot 0.2821\left(0.3918^2 + 0.3199^2 + 0.3918 \cdot 0.3199\right) = 0.1126\ m^3$$

$$V_{CYL} = V_c - V_{CONV} = 0.3857 - 0.1126 = 0.2731\ m^3$$

$$V_{CYL} = \pi R_c^2 L_{CYL} \rightarrow L_{CYL} = \frac{V_{CYL}}{\pi R_c^2} = \frac{0.2731}{\pi\ 0.3918^2} = 0.5665\ m$$

vi) $$L_n = \frac{R_t\left(\sqrt{\frac{A_e}{A_t}} - 1\right) + 0.382 R_t\left(\sec\theta_N - 1\right)}{\tan\theta_N} =$$

$$X_N = 0.382\, R_t \sin\theta_N = 0.382 \; 0.3199 \; \sin 30^\circ =$$

$$= 0.0611 \text{ m}$$

$$R_N = R_t + 0.382\, R_t (1 - \cos\theta_N) =$$

$$= 0.3199 + 0.382 \; 0.3199 (1 - \cos 30^\circ) = 0.3362 \text{ m}$$

$$X_E = (L_N)_{PAR} = 2.4120 \text{ m}$$

$R_E = 1.2755$ (IDEAL VALUE: SAME AS FOR CONICAL NOZZLE TO ENSURE $\varepsilon = 15$)

$$② - ③ \rightarrow 2C(X_N - X_E) = \tan\theta_N - \tan\theta_E$$

$$C = \frac{\tan\theta_N - \tan\theta_E}{2(X_N - X_E)} = \frac{\tan 30^\circ - \tan 12.5^\circ}{2(0.0611 - 2.4120)} =$$

$$= -0.0756$$

$$② + ③ \rightarrow 2b + 2C(X_N + X_E) = \tan\theta_N + \tan\theta_E$$

$$b = \frac{\tan\theta_N + \tan\theta_E - 2C(X_N + X_E)}{2} =$$

$$= \frac{0.3199\left(\sqrt{15}-1\right)+0.382\cdot 0.3199\left(\text{SEC}\,15^{\circ}-1\right)}{\text{TAN}\,15^{\circ}} =$$

$$= 3.4457\ \text{M}$$

PARABOLIC NOZZLE

70% PARABOLIC NOZZLE MEANS

$$\left(L_N\right)_{PAR} = 0.7\left(L_N\right)_{CONV}$$

$$\left(L_N\right)_{PAR} = 0.7\ 3.4457 = 2.4120\ \text{M}$$

FROM FIG. 4.16, FOR $\varepsilon = 15$ AND $L_f = 70\%$

$\theta_N = 30^{\circ}$ $\qquad\qquad$ $\theta_E = 12.5^{\circ}$

PARABOLA DESCRIBED BY

$$R(x) = a + bx + cx^2$$

ENFORCE

$$\begin{cases} R(x_N) = R_N \rightarrow a + bx_N + cx_N^2 = R_N & (1) \\ \left.\dfrac{dR}{dx}\right|_{x_N} = \text{TAN}\,\theta_N \rightarrow b + 2cx_N = \text{TAN}\,\theta_N & (2) \\ \left.\dfrac{dR}{dx}\right|_{x_E} = \text{TAN}\,\theta_E \rightarrow b + 2cx_E = \text{TAN}\,\theta_E & (3) \end{cases}$$

$$c = \frac{\tan 30^\circ - \tan 12.5^\circ - 2(-0.0756)(0.0611 + 2.4120)}{2} = 0.5866$$

$$① \rightarrow a = R_N - b x_N - c x_N^2 =$$

$$= 0.3362 - 0.5866 \cdot 0.0611 - (-0.0756)\, 0.0611^2 =$$

$$= 0.3007$$

VERIFY EXIT RADIUS

$$R(x_E) = a + b x_E + c x_E^2 =$$

$$= 0.30076 + 0.5866 \cdot 2.4120 + (-0.0756)\, 2.4120^2 =$$

$$= 1.2755 \text{ m}$$

CLOSE ENOUGH TO $(R_E)_{CON} = 1.2388 \text{ m}$