a) $Bi_x = \dfrac{R_{cond\,r}}{R_{surr}}$, $R_{cond_r} = \dfrac{r_o}{K \cdot A_{cyl}}$, $A_{cyl} = 2\pi r L$

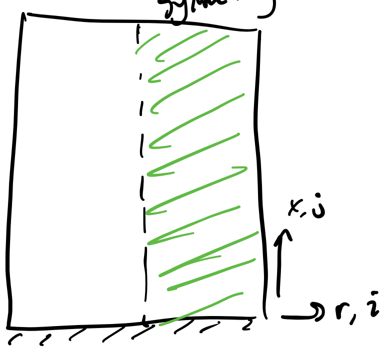
$R_{surr} = \dfrac{1}{\bar{h}\, A_s}$, $A_s = 2\pi r_o L + \pi r_o^2$

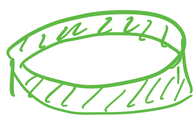
worst case Bi : $r = r_o$

→ Plug into EES

$Bi_r = \dfrac{R_{cond,x}}{R_{surr}}$, $R_{cond,x} = \dfrac{L}{K \cdot A_c}$, $A_c = \pi r_o^2$

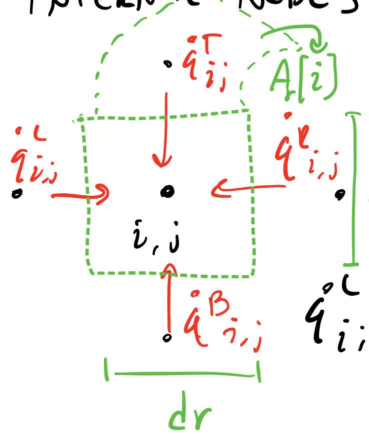
b)





$$\boxed{r = i, \quad x = j}$$

INTERNAL NODES



$\dot{q}^T_{ij} = K A_r[i] \left[\dfrac{T(i,j+1) - T(i,j)}{dx}\right]$

$\dot{q}^B_{ij} = \text{ ''} \left[\dfrac{T(i,j-1) - T(i,j)}{dx}\right]$

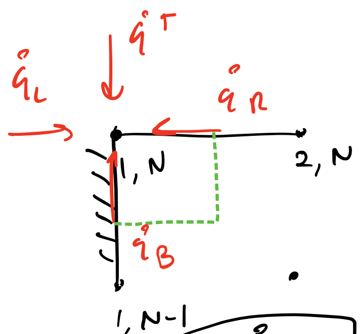
$\dot{q}^L_{ij} = K\left(dx \cdot 2\pi\left(r[i] - \frac{dr}{2}\right)\right) \cdot \left[\dfrac{T(i-1,j) - T(i,j)}{dr}\right]$

IN + ~~GEN~~ = ~~OUT~~ + ~~STORED~~

$\dot{q}^R_{ij} = K\left(dx \cdot 2\pi\left(r[i] + \frac{dr}{2}\right)\right) \cdot \left[\dfrac{T(i+1,j) - T(i,j)}{dr}\right]$

$\Sigma \dot{q}_{ij} = 0$

corners - upper left



$\dot{q}_T = \dot{q}_{T,candle} + \bar{h} A_r[1]\left[T_\infty - T(1,N)\right]$

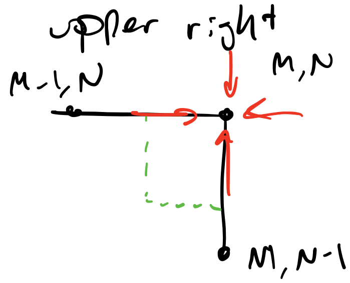
$\dot{q}_B = K A_r[1] \left[\dfrac{T(1,N-1) - T(1,N)}{dx}\right]$

$\dot{q}_L = 0$ By symmetry

$\dot{q}_R = K\left(\frac{dx}{2} \cdot 2\pi\left(r[1] + \frac{dr}{2}\right)\right)\left[\dfrac{T(2,N) - T(1,N)}{dr}\right]$

$$\boxed{\Sigma \dot{q} = 0}$$

upper right



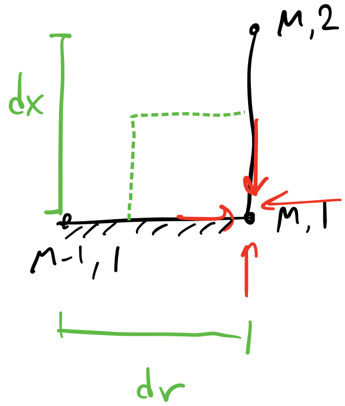
$$\dot{q}_T = \dot{q}_{T,\text{cond'n}} + \bar{h} A_r[M] \left[T_\infty - T(M,N) \right]$$

$$\dot{q}_B = k A_r[M] \left[\frac{T(M,N-1) - T(M,N)}{dx} \right]$$

$$\dot{q}_L = k\left(\frac{dx}{2} \cdot 2\pi \left(r[M] - \frac{dr}{2} \right) \right) \left(\frac{T(M-1,N) - T(M,N)}{dr} \right)$$

$$\dot{q}_R = \bar{h} \left(\frac{dx}{2} \cdot 2\pi r[M] \left[T_\infty - T(M,N) \right] \right)$$

Bottom right



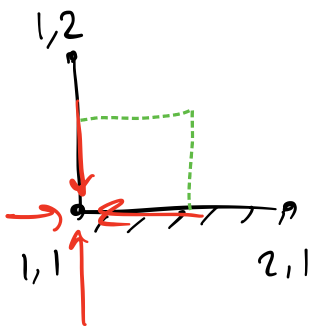
$$\dot{q}_T = k A_r[M] \left[T(M,2) - T(M,1) \right] / dx$$

$$\dot{q}_B = 0, \text{ adiabatic}$$

$$\dot{q}_L = k\left(\frac{dx}{2} \cdot 2\pi \left(r[M] - \frac{dr}{2} \right) \right) \left(\frac{T(M-1,1) - T(M,1)}{dr} \right)$$

$$\dot{q}_R = \bar{h} \left(\frac{dx}{2} \cdot 2\pi r[M] \right) \left[T_\infty - T(M,1) \right]$$

Bottom Left



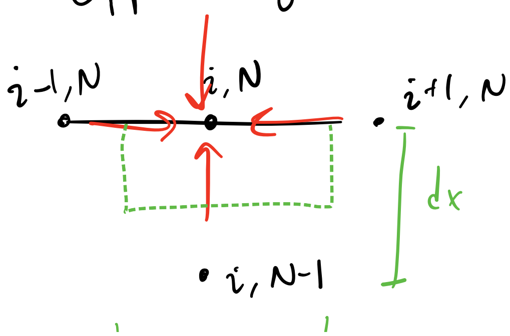
$$\dot{q}_T = k A_r[1] \left(T(1,2) - T(1,1) \right) / dx$$

$$\dot{q}_B = 0, \text{ adiabatic}$$

$$\dot{q}_L = \bar{h} \left(\frac{dx}{2} \cdot 2\pi \cdot r[1] \right) \left[T_\infty - T(1,1) \right]$$

$$\dot{q}_R = k\left(\frac{dx}{2} \cdot 2\pi r\left([1] + \frac{dr}{2} \right) \right) \left(\frac{T(2,1) - T(1,1)}{dr} \right)$$

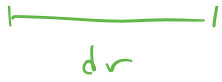
upper edge



$$\dot{q}_T = \dot{q}_{T,\text{cond'n}}[i] + \bar{h} A_r[i] \left[T_\infty - T(i,N) \right]$$

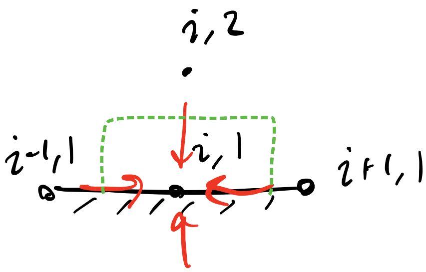
$$\dot{q}_B = k A_r[i] \left[T(i,N-1) - T(i,N) \right] / dx$$

$$\dot{q}_L = k \frac{dx}{2} \cdot 2\pi \left(r[i] - \frac{dr}{2} \right) \left[\frac{T(i-1,N) - T(i,N)}{dr} \right]$$



$$\dot{q}_R = k \frac{dx}{2} 2\pi \left(r[i] + \frac{dr}{2}\right)\left[\frac{T(i+1,N) - T(i,N)}{dr}\right]$$

Lower edge



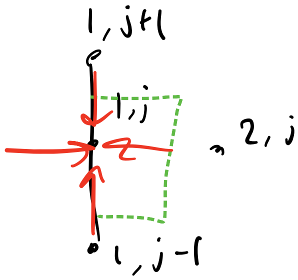
$$\dot{q}^T = k \cdot A_r[i]\left[T(i,2) - T(i,1)\right]/dx$$

$\dot{q}_B = 0$, adiabatic

$$\dot{q}^L = k \cdot \frac{dx}{2} \cdot 2\pi\left(r[i] - \frac{dr}{2}\right)\left[\frac{T(i-1,1) - T(i,1)}{dr}\right]$$

$$\dot{q}^R = k \cdot \frac{dx}{2} \cdot 2\pi\left(r[i] + \frac{dr}{2}\right)\left[\frac{T(i+1,1) - T(i,1)}{dr}\right]$$

Left edge



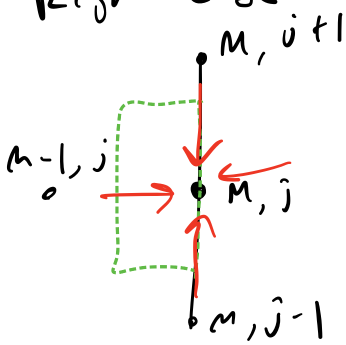
$$\dot{q}_T = k \cdot A_r[1]\left[T(1,j+1) - T(1,j)\right]/dx$$

$$\dot{q}_B = k \cdot A_r[1]\left[T(1,j-1) - T(1,j)\right]/dx$$

$$\dot{q}_L = \bar{h} \cdot dx \cdot 2\pi r[1]\left[T_\infty - T(1,j)\right]$$

$$\dot{q}_R = k \cdot dx \cdot 2\pi\left(r(1) + \frac{dr}{2}\right)\left[T(2,j) - T(1,j)\right]/dr$$

Right edge



$$\dot{q}^T = k \cdot A_r(M)\left[T(M,j+1) - T(M,j)\right]/dx$$

$$\dot{q}_B = k \cdot A_r(M)\left[T(M,j-1) - T(M,j)\right]/dx$$

$$\dot{q}_L = k \cdot dx \cdot 2\pi\left(r[M] - \frac{dr}{2}\right)\left[T(M-1,j) - T(M,j)\right]/dr$$

$$\dot{q}_R = \bar{h}\, dx \cdot 2\pi r(M)\left[T_\infty - T(M,j)\right]$$

```
$tabstops 0.2 0.4 0.6 2.5
$unitsystem SI C Pa J

"Given problem information"
q_dot_dprime_max = 2500 [W/m^2]
r_o  = 3 [cm]*convert(cm,m)
$LOAD Incompressible
k=conductivity(Paraffin_Wax, T=20[C])
T_inf = 18 [C]
h_bar = 7 [W/m^2-K]
L = 6 [cm]*convert(cm,m)

// a)
A_cyl = 2*pi#*r_o*L
A_c = pi#*r_o^2
A_s = A_cyl+A_c

L_eff_x = A_c*L/A_c // = L
L_eff_r = A_c*L/A_cyl

// Biot number in x direction
R_cond_r = L_eff_r/(k*A_cyl)          // Resistance of conduction in direction of interest
R_surr_r = 1/(h_bar*A_cyl)            // Resistance of surroundings (convection) in direction of interest
Bi_x = R_cond_r/R_surr_r

// Biot number in r direction
R_cond_x = L_eff_x/(k*A_c)            // Resistance of conduction in direction of interest
R_surr_x = 1/(h_bar*A_c)              // Resistance of surroundings (convection) in direction of interest
Bi_r = R_cond_x/R_surr_x

// b)

M = 51  "Nodes in radial direction"
N = 21      "Nodes in axial direction"

"Radial domain"
$varinfo r[] units='m'
r[1] = 1e-9 [m]
Duplicate i=2,M
    r[i] = (i-1)/(M-1)*r_o
End
DELTAr = r_o / (M-1)                  "Radial direction step size"
dr = DELTAr

"Calculation of energy absorbed by flux from the candle at each top-surface radial position"
$varinfo q_dot_dprime[] units='W/m^2'
Duplicate i=1,M
    q_dot_dprime[i] = q_dot_dprime_max*exp(-5*r[i] / r_o)
"Function giving flux as a function of radial position"
End

"Axial domain"
$varinfo x[] units='m'
Duplicate j=1,N
    x[j] = (j-1)/(N-1)*L
End
DELTAx = L/(N-1)                      "Axial direction step size"
```

```
dx = DELTAx

"Create array for top surface area"
$varinfo Ar[] units='m^2'
Duplicate i=2,M-1
    Ar[i] = pi*( (r[i]+DELTAr/2)^2 - (r[i]-DELTAr/2)^2)
End
Ar[1] = pi*(DELTAr/2)^2
Ar[M] = pi*( r[M]^2 - (r[M]-DELTAr/2)^2 )

"calculate heat flux on top surface from candle"
$varinfo q_dot_t_candle[] units='W'
Duplicate i=1,M
    q_dot_t_candle[i] = Ar[i] * q_dot_dprime[i]
End

"!---------------- solution --------------------"


"! NOTE: I did not realize the x index started from the top left and went down. My indices start from the bottom left. This should not affect the final temperatures but my equations will look slightly different."


"--------------------- Internal nodes"

Duplicate i=2,M-1
    Duplicate j=2,N-1
        {top + bottom + left + right = 0}
        k*Ar[i]*(T[i,j+1]-T[i,j])/dx + k*Ar[i]*(T[i,j-1]-T[i,j])/dx + k*dx*2*pi#*(r[i]-dr/2)*(T[i-1,j]-T[i,j])/dr + k*dx*2*pi#*(r[i]-dr/2)*(T[i+1,j]-T[i,j])/dr
= 0
    End
End

"--------------------- Corner nodes"

{top + bottom + left + right = 0}

"upper left"
 q_dot_t_candle[1] + h_bar*Ar[1]*(T_inf-T[1,N]) + k*Ar[1]*(T[1,N-1]-T[1,N])/dx + 0 +  k*dx/2*2*pi#*(r[1]+dr/2)*(T[2,N]-T[1,N])/dr = 0
"upper right"
 q_dot_t_candle[M] + h_bar*Ar[M]*(T_inf-T[M,N]) + k*Ar[M]*(T[M,N-1]-T[M,N])/dx + k*dx/2*2*pi#*(r[M]+dr/2)*(T[M-1,N]-T[M,N])/dr +
h_bar*dx/2*2*pi#*r[M]*(T_inf-T[M,N])  = 0
"lower right"
k*Ar[M]*(T[M,2]-T[M,1])/dx + k*dx/2*2*pi#*(r[M]-dr/2)*(T[M-1,1]-T[M,1])/dr + h_bar*dx/2*2*pi#*r[M]*(T_inf-T[M,1])  = 0
"lower left"
k*Ar[1]*(T[1,2]-T[1,1])/dx + h_bar*dx/2*2*pi#*r[1]*(T_inf-T[1,1]) + k*dx/2*2*pi#*(r[1]+dr/2)*(T[2,1]-T[1,1])/dr  = 0


"--------------------- Edge (non corner) nodes"

"Upper"
Duplicate i=2,M-1
    q_dot_t_candle[i] + h_bar*Ar[i]*(T_inf-T[i,N]) + k*Ar[i]*(T[i,N-1]-T[i,N])/dx + k*dx/2*2*pi#*(r[i]-dr/2)*(T[i-1,N]-T[i,N])/dr +  k*dx/2*2*pi#
*(r[i]+dr/2)*(T[i+1,N]-T[i,N])/dr = 0
End
"Lower"
```

```
Duplicate i=2,M-1
    k*Ar[i]*(T[i,2]-T[i,1])/dx + 0 + k*dx/2*2*pi#*(r[i]-dr/2)*(T[i-1,1]-T[i,1])/dr +  k*dx/2*2*pi#*(r[i]+dr/2)*(T[i+1,1]-T[i,1])/dr = 0
End
"Left"
Duplicate j=2,N-1
    k*Ar[1]*(T[1,j+1]-T[1,j])/dx + k*Ar[1]*(T[1,j-1]-T[1,j])/dx + h_bar*dx*2*pi#*r[1]*(T_inf-T[1,j]) + k*dx*2*pi#*(r[1]+dr/2)*(T[2,j]-T[1,j])/dr = 0
End
"Right"
Duplicate j=2,N-1
    k*Ar[M]*(T[M,j+1]-T[M,j])/dx + k*Ar[M]*(T[M,j-1]-T[M,j])/dx + k*dx*2*pi#*(r[M]-dr/2)*(T[M-1,j]-T[M,j])/dr + h_bar*dx*2*pi#*r[M]*(T_inf-T[M,j]) = 0
End


"Keep track of min and max temperatures in the domain"
Tmin = min(T[1..M,1..N])
Tmax = max(T[1..M,1..N])

T_underflame = T[1,N] // Node under flame is 1,N for me because my j index starts at bottom left
```

SOLUTION

Unit Settings: SI C Pa J mass deg

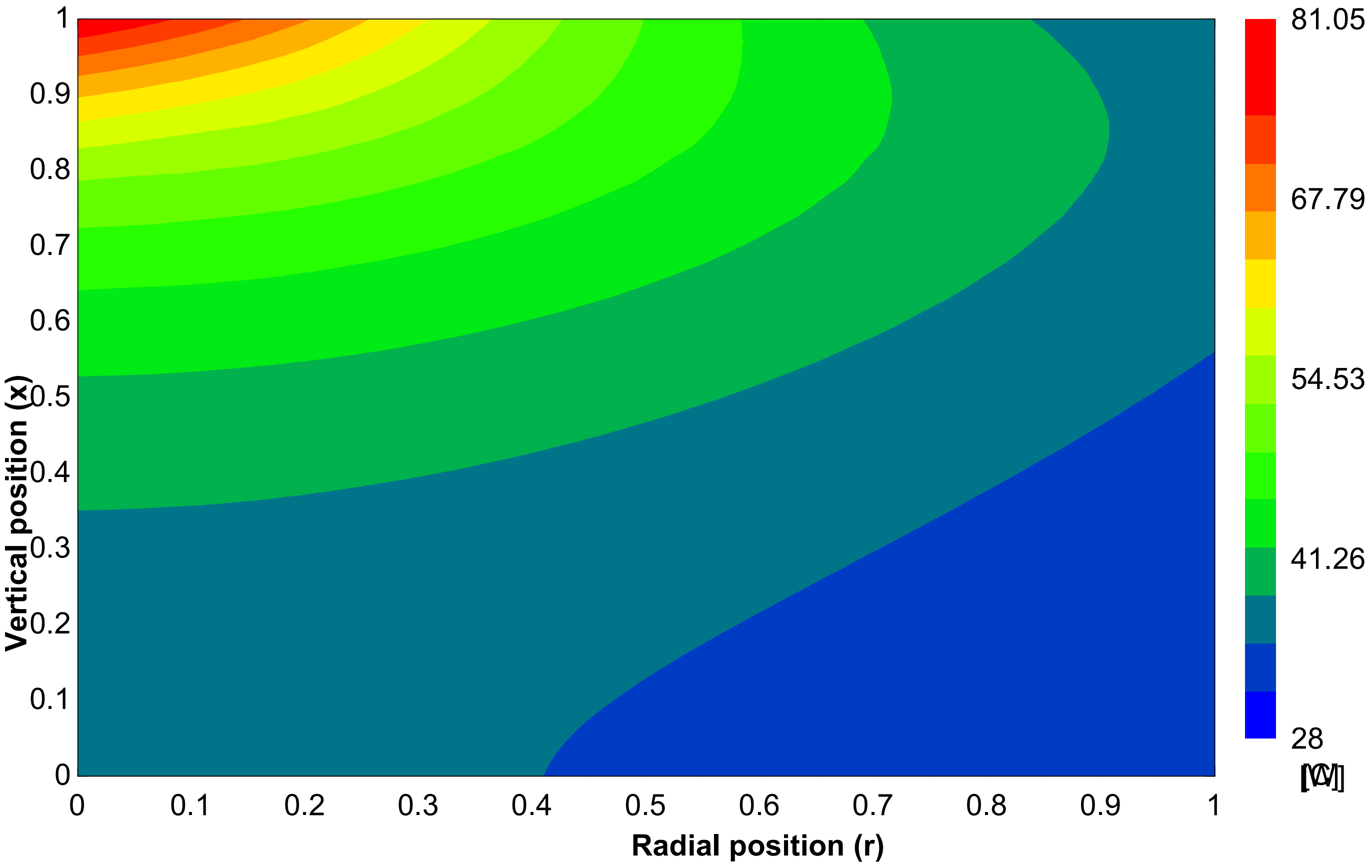
A_c = 0.002827 [m^2]
A_s = 0.01414 [m^2]
Bi_x = 0.4375 [-]
Δx = 0.003 [m]
dx = 0.003 [m]
k = 0.24 [W/m-K]
$L_{eff,r}$ = 0.015 [m]
M = 51 [-]
$\dot{q}''_{max}$ = 2500 [W/m^2]
$R_{cond,x}$ = 88.42 [K/W]
$R_{surr,r}$ = 12.63 [K/W]
Tmax = 81.05 [C]
T_{inf} = 18 [C]

A_{cyl} = 0.01131 [m^2]
Bi_r = 1.75 [-]
Δr = 0.0006 [m]
dr = 0.0006 [m]
$\bar{h}$ = 7 [W/m^2-K]
L = 0.06 [m] {2.362 [in]}
$L_{eff,x}$ = 0.06 [m]
N = 21 [-]
$R_{cond,r}$ = 5.526 [K/W]
r_o = 0.03 [m] {1.181 [in]}
$R_{surr,x}$ = 50.53 [K/W]
Tmin = 28.01 [C]
$T_{underflame}$ = 81.05 [C]

No unit problems were detected.

KEY VARIABLES

Bi_x = 0.4375 [-]	*a) Biot number for approximation in x direction, r direction as transverse. Bi_x >> 0.1, 1D not justified.*
Bi_r = 1.75 [-]	*a) Biot number in r direction. Bi_r >> 0.1, 1D not justified.*
$T_{underflame}$ = 81.05 [C]	*b) Temperature under the flame. Node under flame is 1,N for me because my j index starts at bottom left.*


Vertical position (x)
Radial position (r)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
81.05
67.79
54.53
41.26
28