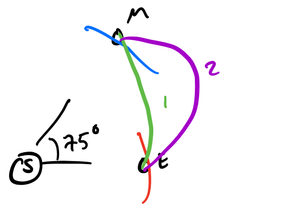
Recap Mars Example

$\Delta\theta_{Earth/Mars} = 75°$ "at a moment"

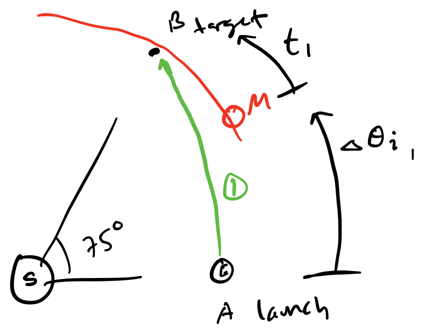
Find all times for "all paths"

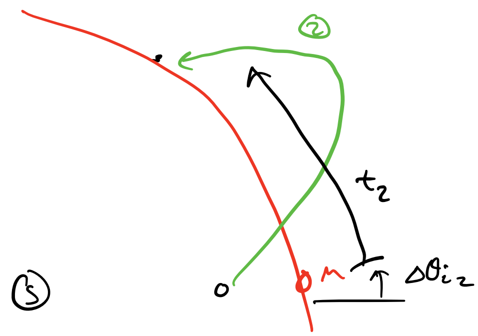
Doesn't address timing



To address timing where is mars initially

$\theta_{M/S} \neq 75°$ to intercept

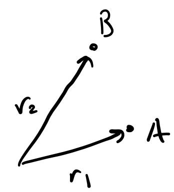




$t_2 > t_1$

$\Delta\theta_{i_1} \neq \Delta\theta_{i_2}$

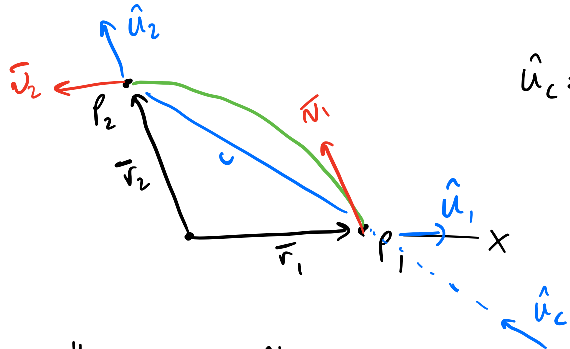
Additional elements needed:



Eccentricity: $p = a(1-e^2) = \dfrac{4a(s-r_1)(s-r_2)}{c^2}\sin^2\left(\dfrac{\alpha+\beta}{2}\right)$

Solve for e

Need $\bar{v}_1$ & $\bar{v}_2$



$\hat{u}_1 = \dfrac{\bar{r}_1}{|\bar{r}_1|}$ $\quad$ $\hat{u}_2 = \dfrac{\bar{r}_2}{|\bar{r}_2|}$

$\hat{u}_c = \dfrac{\bar{r}_2 - \bar{r}_1}{c}$

$A = \sqrt{\dfrac{\mu}{4a}}\cot\left(\dfrac{\alpha}{2}\right)$

$B = \sqrt{\dfrac{\mu}{4a}}\cot\left(\dfrac{\beta}{2}\right)$

$\bar{v}_1 = (B+A)\hat{u}_c + (B-A)\hat{u}_1$

$\bar{v}_2 = (B+A)\hat{u}_c - (B-A)\hat{u}_2$

Finally, Find Δv

(local x axis to P_1)

ex. if P_1 is circular orbit, $\bar{v}_{init} = 0\hat{i} + v_{c_1}\hat{j}$

$\therefore \Delta\bar{v}_1 = \bar{v}_1 - \bar{v}_{init}$, $|\Delta v_1| = \sqrt{\Delta\bar{v}_1 \cdot \Delta\bar{v}_1}$

$\Delta v_1 \rightarrow$ gets you on transfer

Example 2 object seen in 3D

$\bar{r}_1 = 5000\hat{i} + 1000\hat{j} + 2100\hat{k}$ km

1 hour later $\bar{r}_2 = -14600\hat{i} + 2500\hat{j} + 7000\hat{k}$ km

Determine perigee (does it hit?)

Find time to perigee

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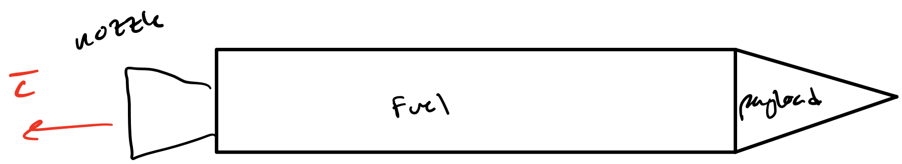
Basic Rocketry

so far → "impulsive thrust" → some burns need more time

(short time, high load) ↳ continuous thrust transfers

→ requires more consideration

consider rocket schematic mass = payload + structure + propellant



$\bar{c}$ ≡ "effective" relevant exhaust velocity

↳ average

as fuel is burned & exhausted, "mass flow rate" $b = -\dot{m} \geq 0$ indicate loss

Start w/ Newton

$$\Sigma \bar{F} = m \frac{dv}{dt}$$

$$\int \Sigma \bar{F} = \int m \frac{dv}{dt} \rightarrow \bar{F}_{ext} \Delta t = \Delta(m\bar{v}) \qquad \bar{P} = m\bar{v}$$

lin. momentum

Consider: t, Δt: $= P_2 - P_1$

$t = \bar{P}_1 = m\bar{v}$

$$t + \Delta t = \bar{P}_2 = \underbrace{(m - b\Delta t)(\bar{v} + \Delta\bar{v})} + \underbrace{b\Delta t(\bar{v} + \bar{c})}$$

rocket exhaust

$$\therefore \quad \bar{F}_{ext}\Delta t = \cancel{m\bar{v}} + m\Delta\bar{v} - \cancel{b\Delta t\bar{v}} - b\Delta t\Delta\bar{v} + \cancel{b\Delta t\bar{v}} + b\Delta t\bar{c} - \cancel{\underline{m\bar{v}}}_{\bar{P}_1}$$

divide by Δt, let $\Delta t \to 0$

$$\bar{F}_{ext} = m\dot{\bar{v}} + b\bar{c}$$

or $$\boxed{m\dot{\bar{v}} = -b\bar{c} + \bar{F}_{ext}}$$

Two cases to consider: $\bar{F}_{ext}$ is gravity or other effect

- High thrust - solid / liquid / nuclear

  Impulsive → $\bar{F}_{ext} \approx 0$

- Low thrust - ion / plasma

  non-impulsive → $\bar{F}_{ext} \neq 0$

Solving: High thrust assumption → $\bar{F}_{ext} = 0$

$$m\dot{\bar{v}} = -b\bar{c}, \quad \text{use } b = -\dot{m}$$

$$m\dot{\bar{v}} = \dot{m}\bar{c}$$

$$m\frac{dv}{\cancel{dt}} = \frac{dm}{\cancel{dt}}\bar{c}$$

$$\int_{v_0}^{v} dv = \int_{m_0}^{m} \frac{1}{m}\,dm\,\bar{c}$$

$$\therefore \quad \Delta\bar{v} = \bar{v} - \bar{v}_0 = \bar{c}\ln\left(\frac{m}{m_0}\right) = -c\ln\left(\frac{m_0}{m}\right)$$

let $M_0 = m - \Delta M$ ↗ prop. consumed

change to scalar $$-\frac{\Delta v}{c} = \ln\left(\frac{m_0}{m}\right) = \ln\left(1 - \frac{\Delta m}{m}\right)$$

$$\therefore \boxed{\frac{\Delta m}{m_0} = 1 - e^{(-\Delta v / c)}}$$ mass consumption eqn

Remarks

1) gives Δm for given Δv

2) use total Δv – don't need to account for individual impulses

3) need value for c (Isp)