a) $\dot{q}_r + \dot{g}''' \cdot V = \dot{q}_{r+dr}$

$V = A \cdot dr = 2\pi r L dr$

$\cancel{\dot{q}_r} + \dot{g}''' 2\pi r L \cancel{dr} = \cancel{\dot{q}_r} + \frac{d\dot{q}_r}{dr} \cancel{dr}$

$\rightarrow \frac{d\dot{q}_r}{dr} = \dot{g}''' 2\pi r L$

b) $\dot{q}_r = -k A_c \frac{dT}{dr}$

$\frac{d\dot{q}_r}{dr} = \dot{g}''' \cancel{2\pi} r \cancel{L} = \frac{d}{dr}\left[-k \cdot \cancel{2\pi} r \cancel{L} \frac{dT}{dr}\right]$

$\int -\frac{\dot{g}'''}{k} r \, dr = \int \frac{d}{\cancel{dr}}\left[r \frac{dT}{dr}\right] \cancel{dr}$

$\int \frac{-\dot{g}'''}{2k} r^{\cancel{2}} + \frac{C_1}{r} \, dr = \int \cancel{r} \frac{dT}{\cancel{dr}} \cancel{dr}$

$\underbrace{\frac{-\dot{g}'''}{4k}}_{\text{From T.G.}} r^2 + C_1 \cancel{\ln r} + C_2 = T(r)$

0

c) BC's:

1) $-k A_c \frac{dT}{dr}\Big|_{r=r_{out}} = \bar{h} A_c (T - T_\infty)_{T=T_{out}}$

2) $\frac{dT}{dr} = 0 \Big|_{r=0}$

d) BC2:

$\frac{dT}{dr} = \frac{-\dot{g}'''}{2k} r + \frac{C_1}{r}$ $\quad 0 = \frac{C_1}{0}$

$$r\cancel{\frac{dT}{dr}} = -\frac{\dot{g}'''}{2k}\cancel{r^2} + C_1 \qquad \rightarrow C_1 = 0$$

$$\rightarrow T = -\frac{\dot{g}'''}{4k} r^2 + C_2$$

$$-k\,\cancel{2\pi r_{out} L}\,\frac{dT}{dr}\Big|_{r=r_{out}} = \bar{h}\,\cancel{2\pi r L}\,(T - T_\infty) \quad T = T_{out}$$

$$\cancel{-}k\left(\cancel{-}\frac{\dot{g}'''}{2\cancel{k}} r\right) = \bar{h}\left(-\frac{\dot{g}'''}{4k} r^2 + C_2 - T_\infty\right)$$

$$\frac{\dot{g}''' r}{2\bar{h}} = -\frac{\dot{g}'''}{4k} r^2 + C_2 - T_\infty$$

$$C_2 = \frac{\dot{g}''' r}{2\bar{h}} + \frac{\dot{g}'''}{4k} r^2 + T_\infty$$

$$C_2 = \frac{\dot{g}''' r}{2}\left(\frac{1}{h} + \frac{r}{2k}\right) + T_\infty \Big|_{r=r_{out}}$$