2nd order linear ODE's

$$\frac{d^2y}{dx^2} + P(x)\frac{dy}{dx} + Q(x)\, y = 0$$

Assume $\quad y = e^{rx} \implies \frac{dy}{dx} = re^{rx}, \quad \frac{d^2y}{dx^2} = r^2 e^{rx}$

Sub in: $\left(r^2 + Pr + Q\right)e^{rx} = 0$

Find roots $\quad r_1 \, \& \, r_2$

Real: $\quad y = A e^{r_1 x} + B e^{r_2 x}$

Repeated: $\quad y = A e^{rx} + B x e^{rx}$

complex: $\quad y = e^{ax}\left(A \sin bx + B \cos bx\right)$

$$a, b = a \pm bi$$