Problem 1

$$G(s) = \frac{X(s)}{F(s)} = \frac{100\,(20s+1)^2(s+2)^2}{s^3(s+10)(10s^2+20s+100{,}000)}$$

a) $f(t) = 5\sin(0.1t)$

$\rightarrow$ MATLAB calc M & φ

Input: $A\sin(\omega t)$

$\Rightarrow X_{ss}(t) = M[A\sin(\omega t+\varphi)]$ where $\begin{cases} M = |G(j\omega)| \\ \varphi = \angle G(j\omega) \end{cases}$

input magnitude

magnitude ratio

phase angle

For input frequency ω_0:

$$M = |G(j\omega_0)| = |G(s)|_{s=j\omega_0}$$

$$= \sqrt{Re(G(j\omega_0))^2 + Im(G(j\omega_0))^2}$$

$$\varphi = \text{atan2}\left[\frac{Im(G(j\omega_0))}{Re(G(j\omega_0))}\right]$$

$$\boxed{X_{ss}(t) = 2.005[5\sin(0.1t - 138°)]}$$

b) $f(t) = 2\sin(10t)$

$\rightarrow$ MATLAB

$$\boxed{X_{ss}(t) = 0.297[2\sin(10t + 21.69°)]}$$

c) $f(t) = 100\sin(100t)$

$\rightarrow$ MATLAB

$$\boxed{X_{ss}(t) = 19.9[100\sin(100t - 86.64°)]}$$

Problem 2

$$G(s) = \frac{X(s)}{F(s)} = \frac{100\,(20s+1)^2(s+2)^2}{s^3(s+10)(10s^2+20s+100{,}000)}$$

Bode form:

$$\frac{(1/10)\quad 1}{(1/10)\ 10s^2+20s+100{,}000} \rightarrow \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$$

$$= \frac{1/10}{s^2+2s+10{,}000} = \frac{1}{100{,}000} \cdot \frac{10{,}000}{s^2+2s+10{,}000}$$

$$G(s) = 100\left(\frac{1}{s^2}\right)(20s+1)^2\left[2(0.5s+1)\cdot 2(0.5s+1)\right]\frac{1}{10}\,\frac{1}{(0.1s+1)}\,\frac{1}{100{,}000}\cdot\frac{10{,}000}{s^2+2s+10{,}000}$$

$$G(s) = \frac{4}{10{,}000}\cdot\left(\frac{1}{s^2}\right)\cdot(20s+1)^2\cdot(0.5s+1)^2\cdot\left(\frac{1}{0.1s+1}\right)\cdot\left(\frac{10{,}000}{s^2+2s+10{,}000}\right)$$

Term $\frac{4}{10{,}000} = \frac{1}{2500}$: $M = \frac{1}{2500}$, $\varphi = 0 = 4 \cdot 10^{-4}$

Term $\frac{1}{s^2}$: $M = \left| \frac{1}{(j\omega)^2} \right| = \frac{1}{\omega^2}$, $\varphi = -(2)(90^\circ) = -180^\circ$

$\log M = -2 \log \omega$ (-2 slope)

Term $(20s+1)^2$: $\omega_b = \frac{1}{20} \frac{r}{s}$ $\omega \ll \omega_b$: $\log M = 0$, $\varphi \approx 0^\circ$

$\omega \gg \omega_b$: $\log M = 2$ slope, $\varphi \approx 180^\circ$

Term $(0.5s+1)^2$: $\omega_b = 2 \frac{r}{s}$ $\omega \ll \omega_b$: $\log M = 0$, $\varphi = 0^\circ$

$\omega \gg \omega_b$: $\log M = 2$ slope , $\varphi \approx 180^\circ$

Term $\left(\frac{1}{0.1s+1} \right)$: $\omega_b = 10 \frac{r}{s}$ $\omega \ll \omega_b$: $\log M = 0$, $\varphi \approx 0^\circ$

$\omega \gg \omega_b$: $\log M = -1$, $\varphi = -90^\circ$

Term $\left(\frac{10{,}000}{s^2+2s+10{,}000} \right)$: $\omega_n^2 = 10{,}000$, $2\zeta\omega_n = 2 \rightarrow \omega_n = 100$, $\zeta = 0.01$

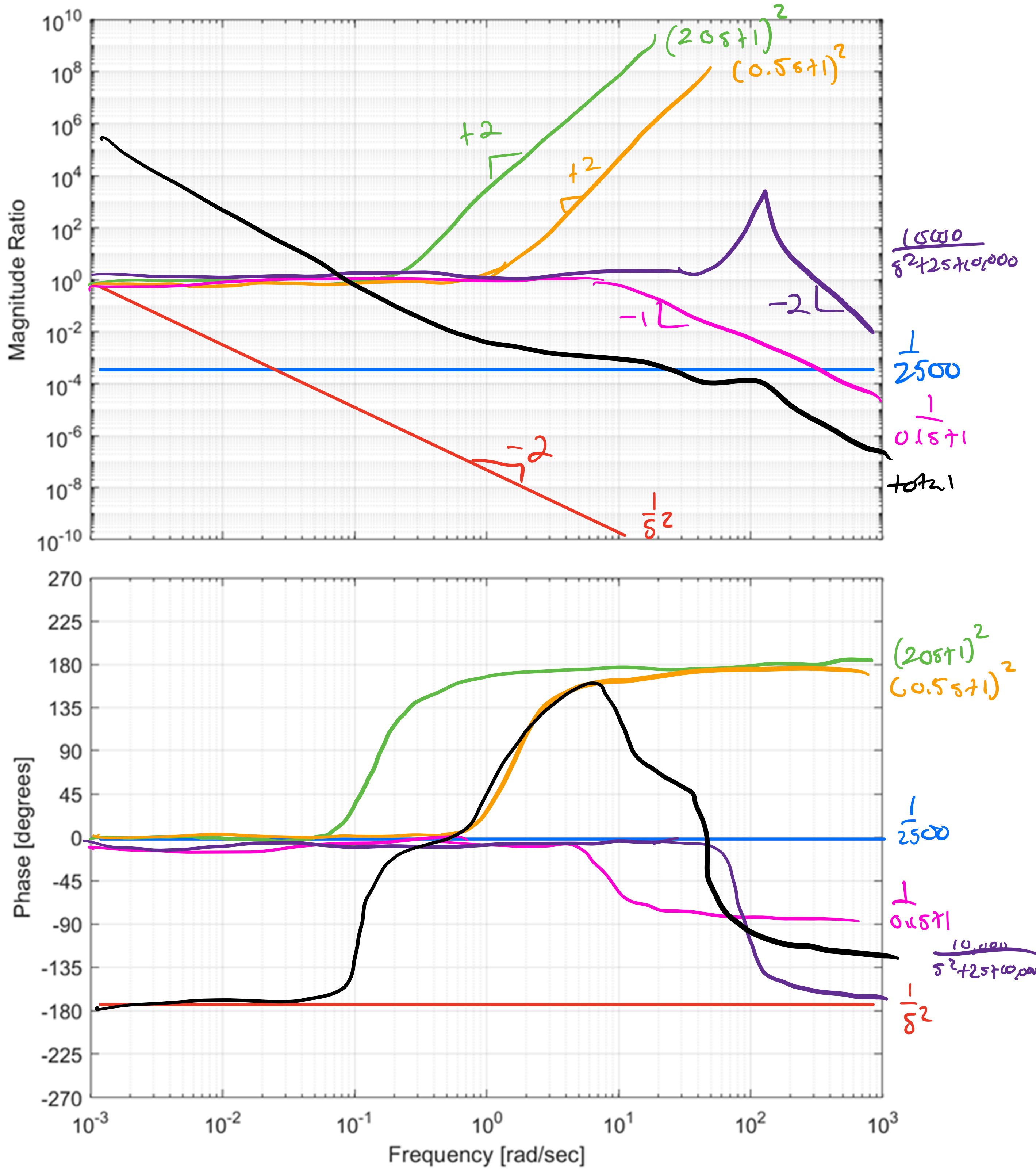
if $\omega \ll \omega_n$ $\log M = 0$, $\varphi = 0$

if $\omega \gg \omega_n$ $\log M = -2$ $\varphi = -180^\circ$

$\omega_r = \omega_n \sqrt{1 - 2\zeta^2}$, $M_p = \frac{1}{2\zeta\sqrt{1-\zeta^2}}$

ME 446 Fall 2023 **Homework #5**

Please submit your answers to the questions and all supporting work including your Matlab scripts, and, where appropriate, program results (plots, explanations). Your Matlab scripts should be readable, with comments, sensible variable names, indentation of code-block, etc.



```
% Written by Kyle Adler for ME446
```

Problem 1

part a

```
A = 5;
w = 0.1;
syms s;
sysG = ( 100*(20*s+1)^2*(s+2)^2 ) / ( s^3*(s+10)*(10*s^2+20*s+100000) );
M = vpa(abs(subs(sysG,s,j*w)))
phi = vpa(atan2(imag(subs(sysG,s,j*w)),real(subs(sysG,s,j*w)))*180/pi)

% part b
A = 2;
w = 10;
M = vpa(abs(subs(sysG,s,j*w)))
phi = vpa(atan2(imag(subs(sysG,s,j*w)),real(subs(sysG,s,j*w)))*180/pi)

% part c
A = 100;
w = 100;
M = vpa(abs(subs(sysG,s,j*w)))
phi = vpa(atan2(imag(subs(sysG,s,j*w)),real(subs(sysG,s,j*w)))*180/pi)
```

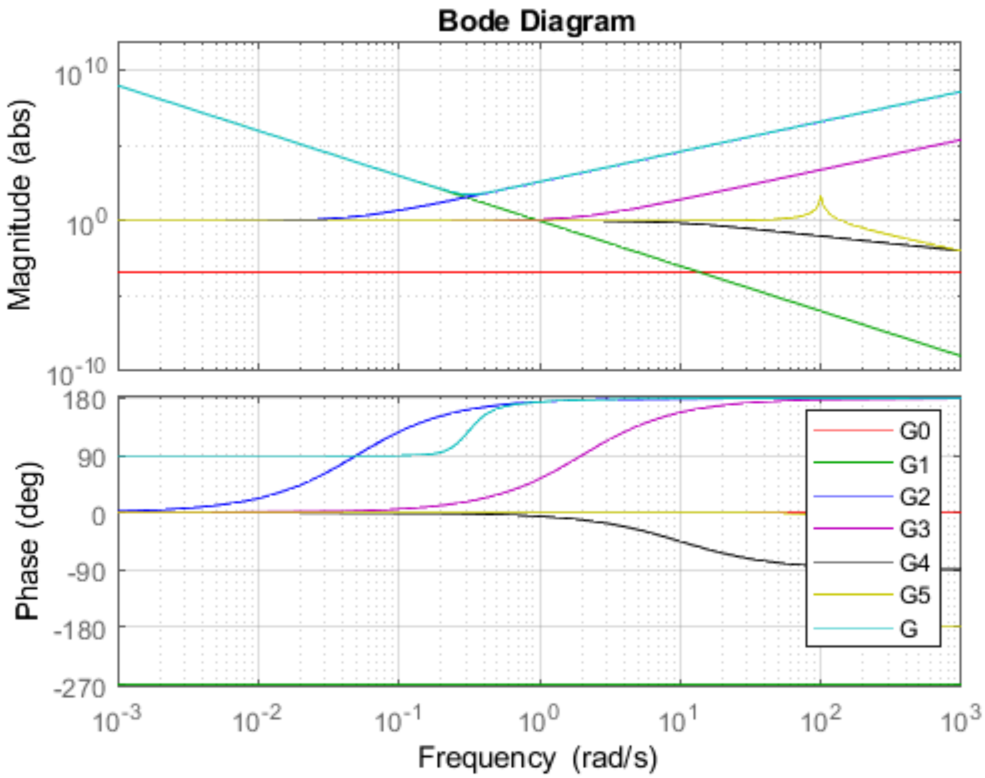
```
M =

2.0049017620189047578796067215935


phi =

-137.97937651635199515799134031912


M =

0.29713451981109431774809427399743


phi =

21.691433139349293021430057361222


M =

19.908709078897488056756453862501
```

```
phi =

-86.6382283135889990361684277722952
```

Problem 2b

```
s = tf('s')

G0 = tf([1/2500],[1]);
G1 = 1/s^3;
G2 = (20*s+1)^2;
G3 = (0.5*s+1)^2;
G4 = 1/(0.1*s+1);
G5 = 10000/(s^2+2*s+10000);
G = G0+G1+G2+G3+G4+G5

figure(1)
h = bodeplot(G0,'r',G1,'g',G2,'b',G3,'m',G4,'k',G5,'y',G,'c');
p = getoptions(h);
p.MagUnits = 'abs';
p.MagScale = 'log';
setoptions(h,p);
grid on;
legend('G0','G1','G2','G3','G4','G5','G')
```

```
s =

  s

Continuous-time transfer function.


G =

  40.03 s^8 + 484.4 s^7 + 4.011e05 s^6 + 4.044e06 s^5 + 4.13e05 s^4

                                   + 4e04 s^3 + 1.2 s^2 + 1002 s + 10000

  ----------------------------------------------------------------------

                0.1 s^6 + 1.2 s^5 + 1002 s^4 + 10000 s^3


Continuous-time transfer function.
```



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