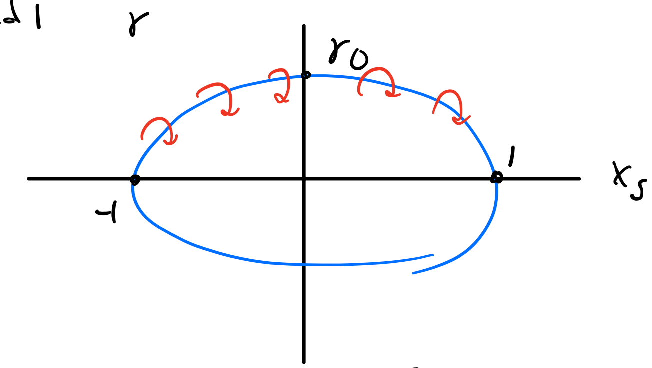
Problem 1 vortex sheet in shape of parabola, x, z dimensionless

1(a) write expression, $z_s(x_s)$

$z_s(x_s) =$ $(-1, 0)$, $(0, 2)$

$$\boxed{z_s(x_s) = -2x_s^2 + 2}$$

1(b) clockwise vorticity distribution per unit length $\gamma = \gamma(x_s)$
symmetric about x-axis, elliptic, peak γ_0 at $x_s = 0$, 0 at $x_s = -1$ and 1



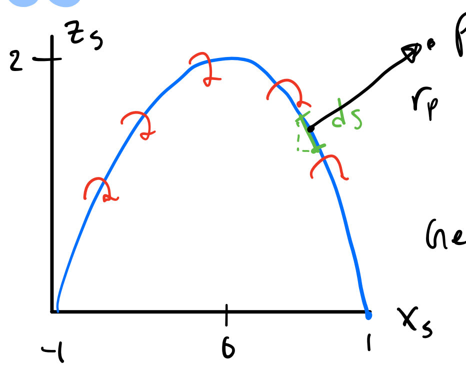
ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \rightarrow \quad \frac{x_s^2}{1} + \frac{\gamma^2}{\gamma_0^2} = 1$$

$$\gamma^2 / \gamma_0^2 = 1 - x_s^2$$

$$\boxed{\gamma(x_s) = \gamma_0 \sqrt{1 - x_s^2}}$$

1(c) $\overset{\text{segment}}{d\Gamma} = \gamma(s)\, ds$. Find φ at $P(x, z)$, r_p away from segment.



$$ds = \sqrt{dx_s^2 + dz_s^2}$$

$$r_p = \sqrt{(x_p - x_s)^2 + (z_p - z_s)^2}$$

General: $\Psi = \frac{\Gamma}{2\pi} \ln r$

$$d\Psi_p = \frac{d\Gamma}{2\pi} \ln r_p$$

$$\boxed{d\Psi_p = \frac{\gamma(s)\,ds}{2\pi}\ln r_p}$$

1d) $$\boxed{\Psi_p = \int_S \frac{\gamma(s)}{2\pi}\ln r_p\,ds}$$

1e) $$\int_S \frac{\gamma}{2\pi}\ln r_p\,ds = \int \frac{\gamma}{4\pi}\ln\left[(x_p-x_s)^2+(z_p-z_s)^2\right]ds$$

$$ds^2 = dx_s^2 + dz_s^2 \quad \rightarrow \quad ds = \sqrt{dx^2+dz^2}$$

$$dz_s = -4x_s\,dx_s \quad \rightarrow \quad ds = \sqrt{dx_s^2 + 16x_s^2 dx_s^2}$$

$$\rightarrow \quad ds = dx_s\sqrt{1+16x_s^2}$$

$$\gamma(x_s) = \gamma_0\sqrt{1-x_s^2} \qquad 2$$

$$\rightarrow \boxed{\Psi(x_s) = \int_a^b \frac{\gamma_0\sqrt{1-x_s^2}}{4\pi}\ln\left[(x_p-x_s)^2+(z_p-(-2x_s^2+2))^2\right]\cdot\sqrt{1+16x_s^2}\;dx_s}$$

1f) → Matlab figures 1 & 2

1g) $$\Gamma = \int_a^b \gamma\,ds \quad \rightarrow \quad \Gamma = \int_{-1}^{1}\gamma(x_s)\,dx_s$$

$$\rightarrow \Gamma = \int_{-1}^{1}\gamma_0\sqrt{1-x_s^2}\,dx_s = \gamma_0\cdot\frac{\pi}{2}$$

$$\rightarrow \boxed{\Gamma = \frac{\pi\gamma_0}{2}}$$

Problem 2 NACA Profiles

2a) camber line in terms of z/c, p, m, t

$$\begin{cases} z_c(x) = \frac{m}{p^2}(2px - x^2) & 0 \le x \le p \\ z_c(x) = \frac{m}{(1-p)^2}\left[1 - 2p + 2px - x^2\right] & p < x \le 1 \end{cases}$$

2b)

$$\frac{dz_c}{dx_c} = \begin{cases} \frac{2m}{p^2}(p - x) & 0 \le x \le p \\ \frac{2m}{(1-p)^2}(p - x) & p < x \le 1 \end{cases}$$

2c) → see matlab plot