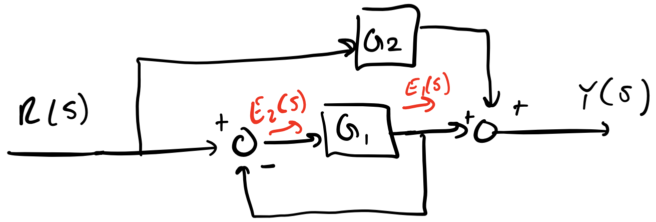
PROBLEM 1

EVAL. $T(s) = \frac{Y(s)}{R(s)}$

1.1)



$Y(s) = G_2 R(s) + E(s)$

$E_1(s) = G_1 E_2(s)$

$E_2(s) = R(s) - E_1(s)$

$\rightarrow E_1(s) = G_1(R(s) - E_1(s))$

$(1+G_1)E(s) = G_1 R(s)$

$\rightarrow E(s) = \frac{G_1 R(s)}{1+G_1}$

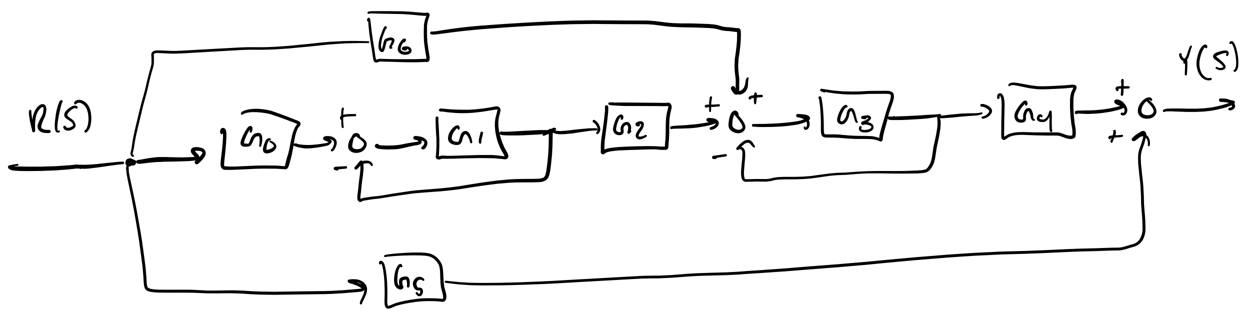
$\Rightarrow Y(s) = G_2 R(s) + \frac{G_1 R(s)}{1+G_1}$

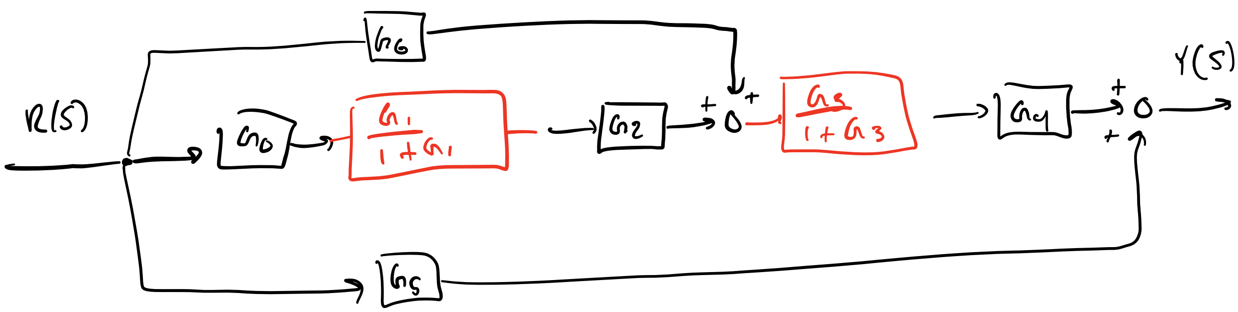
$\Rightarrow \boxed{\frac{Y(s)}{R(s)} = G_2 + \frac{G_1}{1+G_1}}$

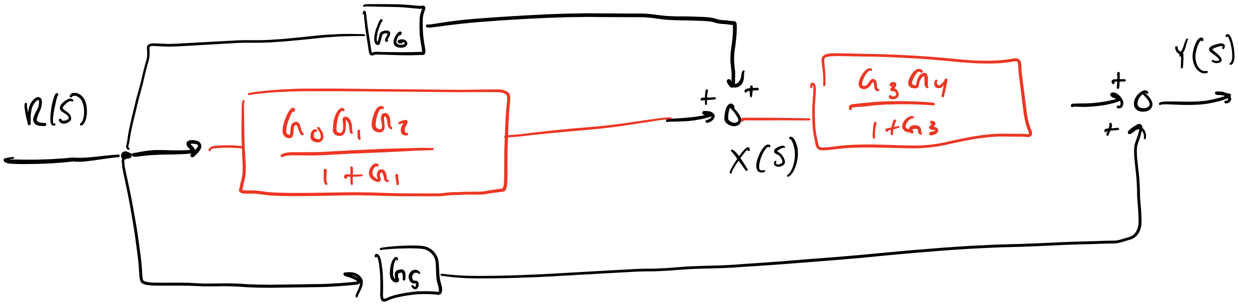
$\frac{G_2 + G_2 G_1 + G_1}{1+G_1}$

1.2)

FIND $\frac{Y(s)}{R(s)}$





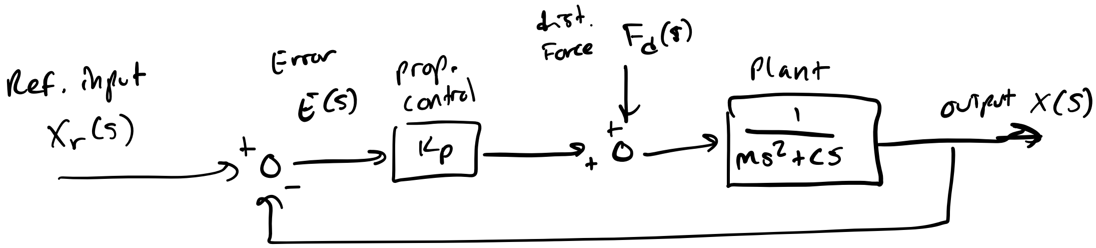


$$X(s) = \left(\frac{G_0 G_1 G_2}{1+G_1} + G_6\right) R(s)$$

$$Y(s) = \left(\left(\frac{G_0 G_1 G_2}{1+G_1} + G_6\right) \cdot \frac{G_3 G_4}{1+G_3} + G_5\right) R(s)$$

$$\boxed{\frac{Y(s)}{R(s)} = \left(\frac{G_0 G_1 G_2}{1+G_1} + G_6\right) \frac{G_3 G_4}{1+G_3} + G_5}$$

1.3) Evaluate : $\dfrac{X(s)}{X_r(s)}$, $\dfrac{E(s)}{X_r(s)}$, $\dfrac{E(s)}{F_d(s)}$



$$E(s) = X_r(s) - X(s)$$

$$X(s) = \frac{K_p}{ms^2+cs} E(s) \longrightarrow E(s) = \frac{ms^2+cs}{K_p} X(s)$$

$$\frac{ms^2+cs}{K_p} X(s) = X_r(s) - X(s) \longrightarrow \left(\frac{ms^2+cs}{K_p} + 1\right) X(s) = X_r(s)$$

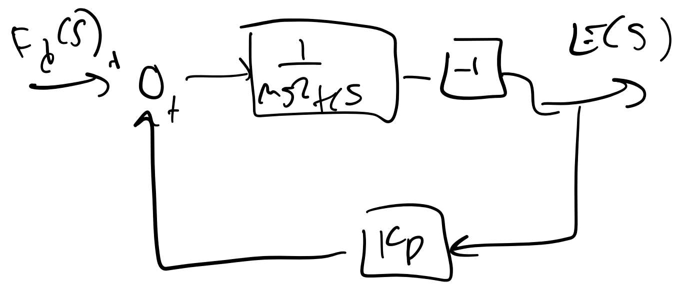
$$\boxed{\frac{X(s)}{X_r(s)} = \frac{K_p}{ms^2+cs+K_p}}$$

$\frac{E(s)}{X_r(s)}$: $X(s) = \frac{K_p}{ms^2+cs} E(s)$

$$X_r(s) = \underbrace{E(s) + \frac{K_p}{ms^2+cs} E(s)}_{(1+\dots)E(s)}$$

$$\frac{E(s)}{X_r(s)} = \frac{1}{1+\frac{K_p}{ms^2+cs}} = \frac{ms^2+cs}{ms^2+cs+K_p}$$

$\frac{E(s)}{F_d(s)}$:



$$\frac{E(s)}{F_d(s)} = \frac{\frac{-1}{ms^2+cs}}{1 - \frac{K_p}{ms^2+cs} \cdot -1}$$

+ feedback

PROBLEM 2

EOM's:

$$2m\ddot{x}_1 + 3c\dot{x}_1 + 2c\dot{x}_2 + kx_1 = F$$
$$m\ddot{x}_2 + 2c\dot{x}_2 + 2c\dot{x}_1 + 2kx_2 = F$$

$m=1,\ c=0.5$
$k=3$

$$\frac{X_1(s)}{F(s)} = \frac{ms^2 + 2k}{2m^2s^4 + 7cms^3 + (10c^2 + 5km)s^2 + 8cks + 2k^2}$$

$$\frac{X_2(s)}{F(s)} = \frac{2ms^2 + cs + k}{2m^2s^4 + 7cms^3 + (10c^2 + 5km)s^2 + 8cks + 2k^2}$$

2.1) DOMINANT ROOT APPROX. ON TRANSFER FUNCTIONS

$\frac{X_1(s)}{F(s)}$: Plug Denom. into Matlab to get roots :

Dominant roots: $-0.3418 \pm 1.1100i$
(smallest real magnitude)

$$\frac{X_1(s)}{F(s)} \approx \frac{c_1}{(s+0.3418-1.11i)(s+0.3418+1.11i)}$$

Final value theorem:

$F(s)$

$$x_1(\infty) = \lim_{s\to 0}\left[\cancel{s}\,\frac{ms^2 + 2k}{2m^2s^4 + 7cms^3 + (10c^2 + 5km)s^2 + 8cks + 2k^2} \cdot \frac{1}{\cancel{s}}\right]$$

$\rightarrow$ MATLAB : $x_1(\infty) = \frac{1}{3}$

$$\frac{1}{3} = \lim_{s\to 0}\left[\cancel{s} \cdot \frac{c_1}{(s+0.3418-1.11i)(s+0.3418+1.11i)} \cdot \frac{1}{\cancel{s}}\right]$$

$\rightarrow$ MATLAB: $c_1 = 0.4496$

$$\boxed{\therefore\ \frac{X_1(s)}{F(s)} \approx \frac{0.4496}{(s+0.3418-1.11i)(s+0.3418+1.11i)}}$$

$\rightarrow$ SAME STEPS FOR $\frac{X_2(s)}{F(s)}$, C_2

$C_2 = 0.2248$

$$\boxed{\frac{X_2(s)}{F(s)} \approx \frac{0.2248}{(s+0.3418-1.11i)(s+0.3418+1.11i)}}$$

Problem 3)

3.1) Sys 1: $\frac{X_1(s)}{X_r(s)}$

$$E_1(s) = X_r(s) - X(s)$$

$$X(s) = \frac{K_p}{ms^2+Cs+K} E(s)$$

$$\rightarrow \frac{ms^2+Cs+K}{K_p} X(s) = X_r(s) - X(s)$$

$$\left(\frac{ms^2+Cs+K}{K_p} + 1\right) X(s) = X_r(s)$$

$$\frac{1}{1+\frac{ms^2+Cs+K}{K_p}}$$

$$\boxed{\frac{X(s)}{X_r(s)} = \frac{K_p}{ms^2+Cs+K+K_p}}$$

$$ms^2+Cs+K+K_p$$

$$E_2(s) = X_r(s) - X(s)$$

$$X(s) = K_p \frac{Cs+K}{ms^2+Cs+K} E_2(s) \rightarrow E_2(s) = \frac{ms^2+Cs+K}{K_p(Cs+K)} X(s)$$

$$\left(\frac{ms^2+cs+k}{k_p(cs+k)}+1\right)x_2(s)=x_r(s)$$

$$\frac{x_2(s)}{x_r(s)}=\frac{1}{\frac{ms^2+cs+k}{k_p(cs+k)}+1}$$

$$\boxed{\frac{x_2(s)}{x_r(s)}=\frac{k_p(cs+k)}{ms^2+(c+k_pc)s+(k+k_pk)}}$$

Sys 1: $k_p=200$, $m=10$, $c=50$, $k=250$

$$\frac{x_1(s)}{x_r(s)}=\frac{k_p}{ms^2+cs+(k+k_p)}$$

poles: $10s^2+50s+450$

roots $=\frac{-50\pm\sqrt{2500-18000}}{20}$

Sys 1: $-2.5+6.225i$, $-2.5-6.225$

a) $M_p=e^{-\zeta\pi/\sqrt{1-\zeta^2}}$ $\omega_d=6.225$

$\sigma=-2.5=-\zeta\omega_n$

$$\omega_n=\frac{\omega_d}{\sqrt{1-\zeta^2}}$$

$$-2.5=-\frac{\zeta\,\omega_d}{\sqrt{1-\zeta^2}}$$

$$\omega_d^2=\omega_n^2(1-\zeta^2)$$

$$\omega_d^2 = \omega_n^2 - (\xi\omega_n)^2$$

$$\xi\omega_n = -\sigma$$

$$\omega_d^2 = \omega_n^2 - (-\sigma)^2$$

$$\omega_d^2 = \omega_n^2 + \sigma^2$$