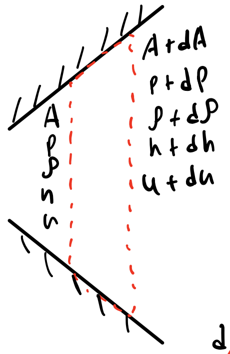
Recap



Mass: $\rho u A = \text{const}.$

Can be adiabatic, not isentropic

Can <u>NOT</u> be isentropic w/o adiabatic

$$\frac{d\rho}{\rho} + \frac{du}{u} + \frac{dA}{A} = 0$$

Momentum: $\rho u \frac{du}{dx} = -\frac{dp}{dx}$

Energy:

$$\underbrace{\cancel{\frac{d}{dt}}}_{S.S.}\int_{CV}\left(e+\frac{u^2}{2}\right)\rho\, dV + \int_{CS}\left(h+\frac{u^2}{2}\right)\underbrace{(\rho \underline{u}_{rel}\cdot d\underline{A})}_{d\dot m} = \overset{\text{Adiabatic}}{\cancel{\dot Q}} - \int_{CS}\rho \underline{u}_b\cdot d\underline{A} - \dot{W}_{shaft}$$

$$-\left(h+\frac{u^2}{2}\right)\cancel{\dot m} + \left((h+dh)+\frac{(u+du)^2}{2}\right)\cancel{\dot m} = 0$$

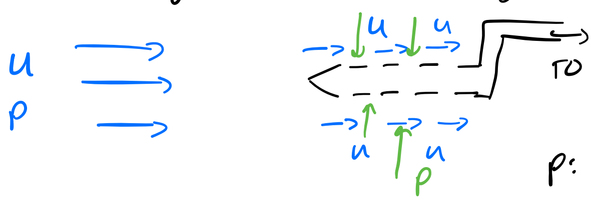
$$\to \quad -\cancel{h} - \cancel{\frac{u^2}{2}} + \cancel{h} + dh + \cancel{\frac{u^2}{2}} + \underbrace{\cancel{\frac{(du)^2}{2}}}_{H.O.T} + \cancel{2}\frac{u\,du}{\cancel{2}}$$

$$\to \quad \boxed{dh + u\,du = 0} \quad \text{energy eqn.}$$

state equation: $p = \rho R T$

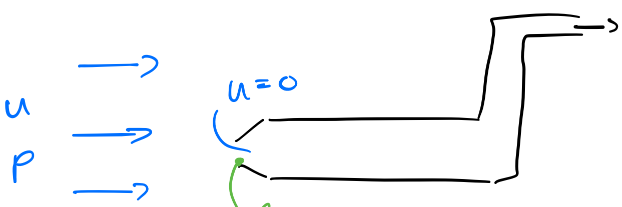
Stagnation state

- Distinguish between 2 types of pressure



TO Pressure gauge

p: static pressure



p_0 : stagnation pressure

From $dh + u\,du = 0$

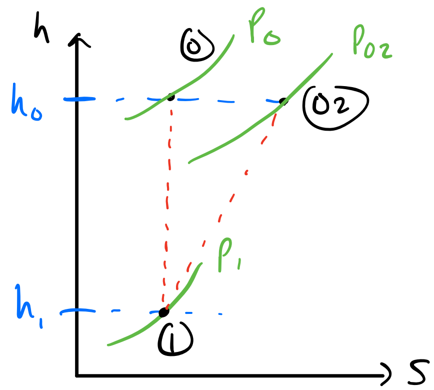
$$h + \frac{u^2}{2} = \text{const} = h_0$$

requires adiabatic not isentropic

① : $h_1, u_1 \neq 0, P_1$: static

⓪ : $h_0, u_0 = 0, P_0$: stagnation

① → ⓪ isentropically



(02) : $h_0, u_{02} = 0, \underbrace{P_{02} < P_0}$

KE → enthalpy flow lost some of ability to perform work

Suppose ⓪ → ① isentropic

If ① → ⓪ is isentropic, we recover same h_0, P_0

If ① → (02) is not isentropic, we recover the same h_0, but $P_{02} < P_0$

Mach number

i) speed of sound: velocity at which an isentropic, infinitesimal disturbance in pressure (sound wave) propagates thru flow $\equiv a$

Can show: $\underbrace{a^2 = \left(\frac{\partial P}{\partial \rho}\right)_s}_{\text{For any gas}}$

For perfect gas, isentropic process $\frac{P}{P_0} = \left(\frac{\rho}{\rho_0}\right)^\gamma$

$$\rightarrow \left(\frac{\partial P}{\partial \rho}\right)_s = \gamma \frac{P}{\rho} = \gamma R T$$

$$\rightarrow \boxed{a = \sqrt{\gamma P/\rho} = \sqrt{\gamma R T}}$$ γ, R gas properties

→ T, a, h all related → only depend on each other

ii) Mach number

$$M \equiv \frac{u}{a} \begin{matrix} \rightarrow \\ \rightarrow \end{matrix} \text{local properties}, \quad M \text{ varies}$$

<u>Qualitative analysis of Mach number effects</u>

For incompressible flow ($\rho = const$)

$$\left\{ \begin{array}{ll} \rho u A = const & \text{(mass)} \\ p + \frac{1}{2}\rho u^2 = const & \text{(Bernoulli)} \end{array} \right\} \text{ if } A\uparrow,\ u\downarrow,\ P\uparrow$$

For compressible ($\rho \neq const$), still look for A, u, P relationship

mom: $\rho u \frac{du}{dx} = -\frac{dP}{dx}$

$$\rho u \, du = -dP \rightarrow du = -\frac{dP}{\rho u} \qquad (*)$$

since $\left\{ \begin{array}{l} \rho \geq 0 \\ u \geq 0 \end{array} \right.$ then $\left\{ \begin{array}{l} du < 0 \rightarrow dP > 0 \\ du > 0 \rightarrow dP < 0 \end{array} \right.$ analogous to bernoulli

cont: $\frac{dA}{A} = -\frac{d\rho}{\rho} - \frac{du}{u}$

For isentropic flow

$$= -\frac{1}{\rho}\underbrace{\left[\left(\frac{\partial \rho}{\partial P}\right)_s dp + \left(\frac{\partial \rho}{\partial s}\right)_p ds\right]}_{d\rho} - \frac{1}{u}\underbrace{\left(-\frac{dP}{\rho u}\right)}_{du \text{ from } (*)} = -\frac{1}{\rho}\underbrace{\left(\frac{\partial \rho}{\partial P}\right)_s}_{1/a^2} dP + \frac{1}{\rho u^2} dP$$

$$= -\frac{1}{\rho}\frac{1}{a^2} dP + \frac{1}{\rho u^2} dP = \frac{dP}{\rho}\left(\frac{1}{u^2} - \frac{1}{a^2}\right)$$

$$\boxed{\frac{dA}{A} = \frac{dP}{\rho u^2}\left(1 - M^2\right)}$$

$\rho, u^2, a \geq 0$

Sign of dP depends on $\left\{ \begin{array}{l} \text{sign of } (1 - M^2) \\ \text{sign of } dA \end{array} \right.$