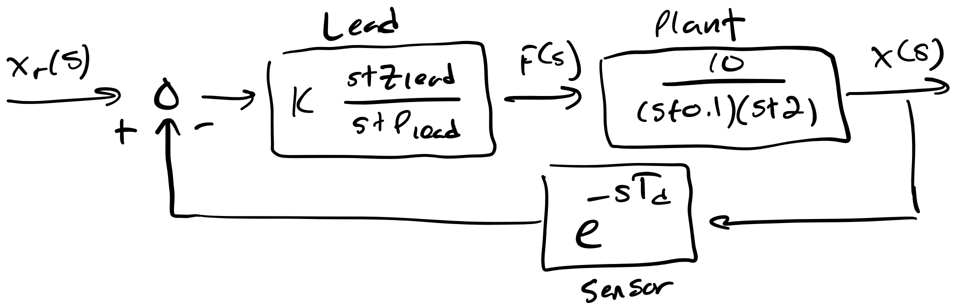
# Problem 1

$$G(s) = \frac{X(s)}{F(s)} = \frac{10}{(s+0.1)(s+2)}$$

→ lead compensated feedback, $T_d = 0.01\,s$

$PM > 60°$, $GM > 15\,dB$

$\omega_{BW}$ as high as possible




→ Assume 75° possible from lead

$$\varphi_{des} = -180° + PM = -180° + 60° = -120°$$

$$\varphi_{des} = \angle G(s) + \varphi_{lead} = -120° = \angle G(s) + 75°$$

$$\rightarrow \angle G(s)\big|_{\omega_{des}} = -195° \rightarrow \text{MATLAB}: \omega_{des} \cong 32\ r/s$$

$$\varphi_{lead} \cong 75°: \quad \alpha = \frac{1-\sin 75°}{1+\sin 75°} = 0.01733, \quad \frac{1}{\alpha} = 57.7,$$ high but OK

$$z = \omega_{max}\sqrt{\alpha} \cong 4.213 \qquad p = \frac{\omega_c}{\sqrt{\alpha}} \cong 243.064$$

$$\rightarrow D_{lead} = K\left(\frac{s+4.213}{s+243.064}\right)$$

$$|G(j\omega_c)D(j\omega_c)H(j\omega_c)| = 1 \rightarrow \text{MATLAB} \rightarrow K = 779.33$$

$$\rightarrow D_{lead} = 779.33\left(\frac{s+4.213}{s+243.064}\right)$$

Check in MATLAB:

$PM = 60.2°$ ✓ $GM = 11.7\,dB$ ✗

→ Iterate, lower $\omega_c$ until GM met

w/ $\omega_c = 22\ r/s$, $PM = 66.8°$ ✓, $GM = 15.1\,dB$ ✓

## Problem 2

EOM: $\begin{cases} I_1\ddot{\theta}_1 + b_s\dot{\theta}_1 - b_s\dot{\theta}_2 + K_s\theta_1 - K_s\theta_2 = \tau_1 \\ I_2\ddot{\theta}_2 - b_s\dot{\theta}_1 + b_s\dot{\theta}_2 - K_s\theta_1 + K_s\theta_2 = 0 \end{cases}$ , $G(s) = \dfrac{\theta_2(s)}{\tau_1(s)}$

### a) G(s)

$$I_2 s^2\theta_2(s) - b_s s\,\theta_1(s) + b_s s\theta_2(s) - K_s\theta_1(s) + K_s\theta_2(s) = 0$$

$$(I_2 s^2 + b_s s + K_s)\,\theta_2(s) = (b_s s + K_s)\,\theta_1(s) \rightarrow \theta_1(s) = \theta_2(s)\,\frac{I_2 s^2 + b_s s + K_s}{b_s s + K_s} \quad (1)$$

$$I_1 s^2\theta_1(s) + b_s s\,\theta_1(s) - b_s s\theta_2(s) + K_s\theta_1(s) - K_s\theta_2(s) = \tau_1(s)$$

$$(I_1 s^2 + b_s s + K_s)\theta_1(s) + (-b_s s - K_s)\theta_2(s) = \tau_1(s) \quad (2)$$

Sub (1) into (2):

$$\frac{(I_1 s^2 + b_s s + K_s)(I_2 s^2 + b_s s + K_s)}{(b_s s + K_s)}\theta_2(s) - (b_s s + K_s)\theta_2(s) = \tau_1(s)$$

$$\rightarrow \frac{\theta_2(s)}{\tau_1(s)} = G(s) = \left[\frac{(I_1 s^2 + b_s s + K_s)(I_2 s^2 + b_s s + K_s)}{(b_s s + K_s)} - (b_s s + K_s)\right]^{-1}$$

$$\rightarrow G(s) = \frac{(b_s s + K_s)}{(I_1 s^2 + b_s s + K_s)(I_2 s^2 + b_s s + K_s) - (b_s s + K_s)^2}$$

$$\rightarrow \boxed{G(s) = \frac{0.002\,s + 36}{s^2(0.01 s^2 + 0.00202\,s + 36.36)}}$$

### b) Lead compensator, PM = 60°, $\omega_c = 3$ r/s, Find GM

$\varphi_{des} = -120°$

$\angle G(s)\big|_{\omega = 3\,r/s} = -180° \rightarrow \varphi_{lead} = 60°$

$\rightarrow \alpha = \dfrac{1 - \sin 60°}{1 + \sin 60°} = 0.0718$

$z = \omega_c\sqrt{\alpha} \cong 0.804$ , $p = \dfrac{\omega_c}{\sqrt{\alpha}} \cong 11.2$

$\rightarrow$ MATLAB: $D_{lead} = 33.85\,\dfrac{(s + 0.804)}{(s + 11.2)}$

Nyquist / Rlocus: $GM \cong 2.06$

c) lead & low pass

$\rightarrow \boxed{K \frac{s + z_{lead}}{s + p_{lead}} \left( \frac{\omega_{lp}}{s + \omega_{lp}} \right)} \rightarrow$ matlab

## Table of Contents



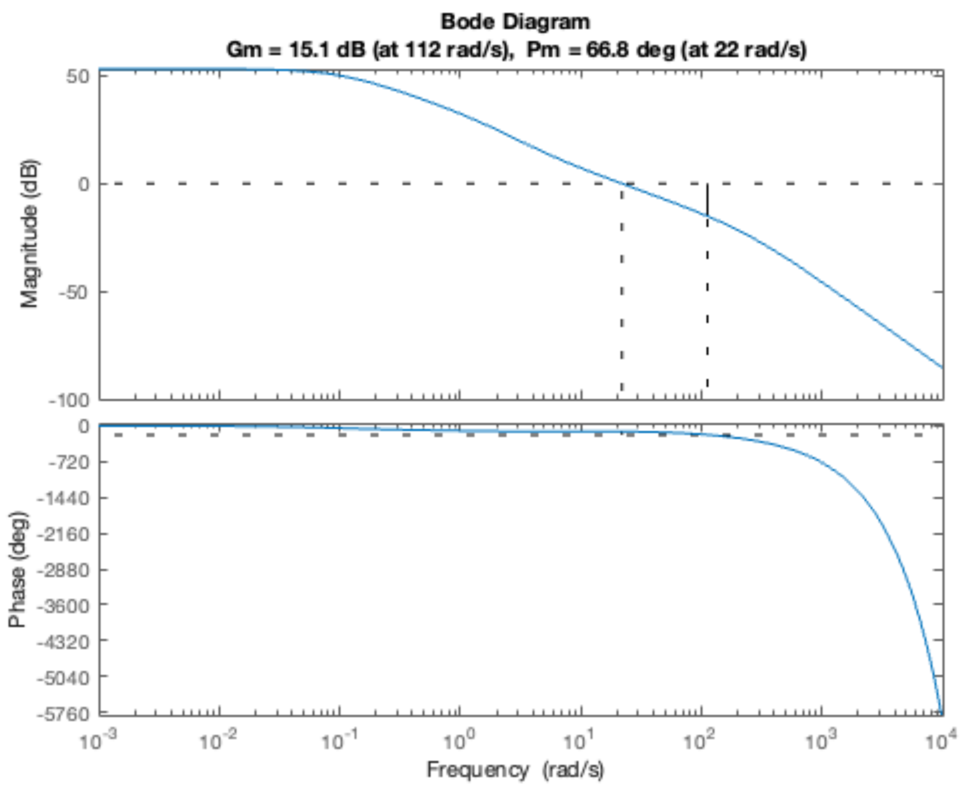
```
% Written by Kyle Adler for ME446
```

# Problem 1

find wdes where angle G(s) = -195deg

```
s = tf('s');
Gsys = 10/( (s+0.1)*(s+2) );
Td = 0.01; % seconds, time delay
H = exp(-s*Td); %time delay function
bode(Gsys*H)
wc = 22; % wdes, initially 32, iterated down to 22 to achieve GM requirement

% find lead gain
K=1; Dlead = K*(s+4.213)/(s+243.064); % lead with K=1 to solve for K
[m,p] = bode(Gsys*H*Dlead,wc)
K = 1/m
Dlead = K*(s+4.213)/(s+243.064);

% evalutate system
margin(Gsys*H*Dlead)
```

```
m =

    0.0019


p =

 -113.1629


K =

  529.5244
```

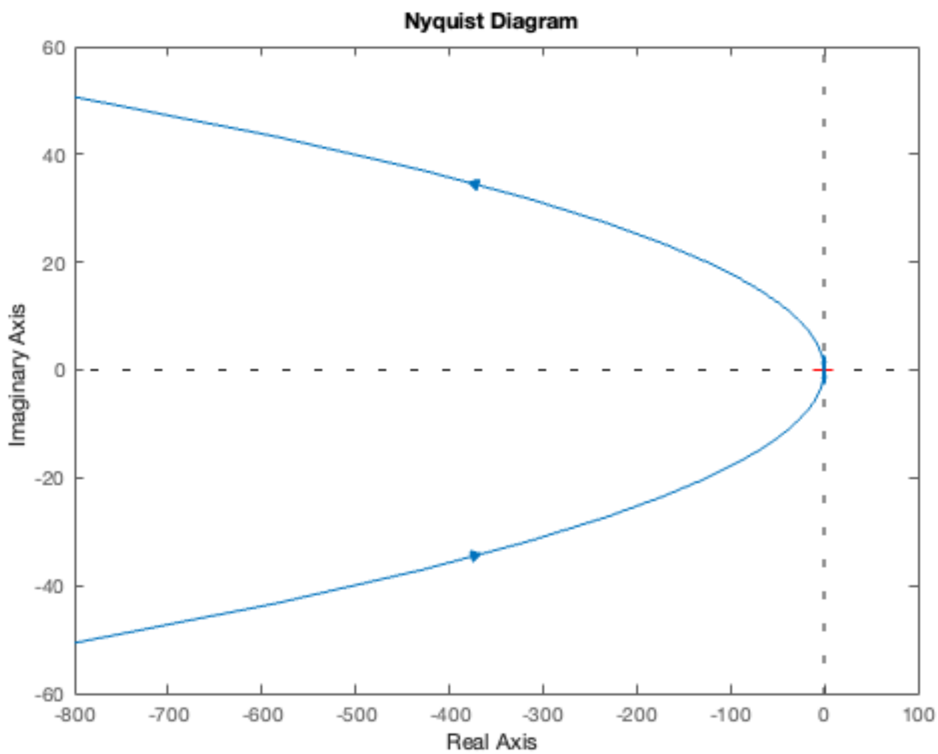


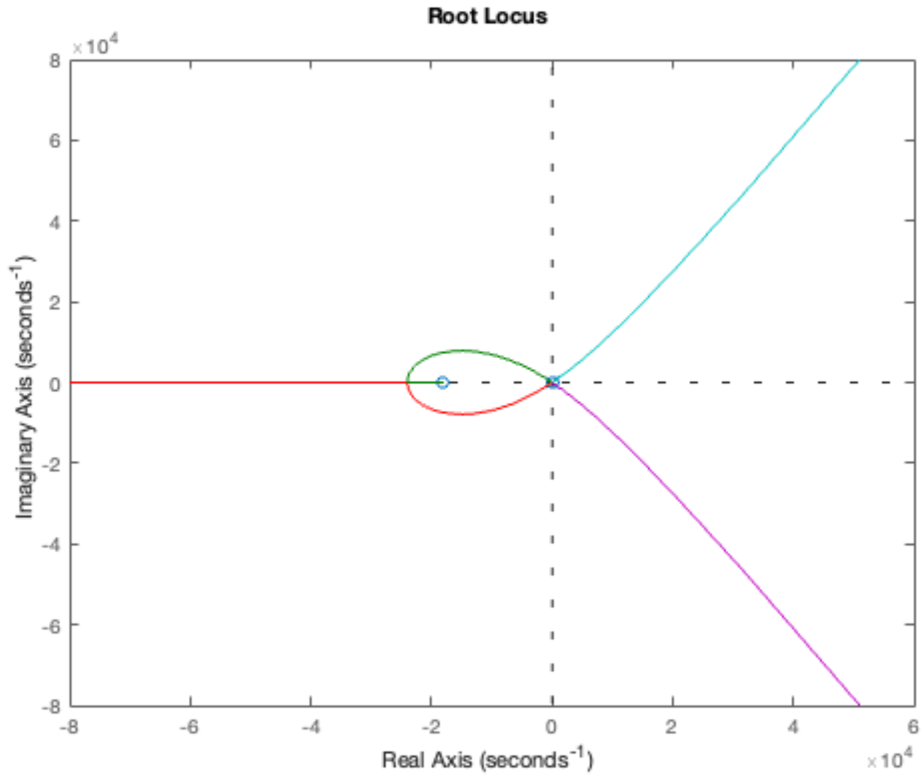

# Problem 2b

```
bs = 0.002; ks = 36; I1 = 1.0; I2 = 0.01; % constants
Gsys = (bs*s+ks) / ( (I1*s^2+bs*s+ks)*(I2*s^2+bs*s+ks)-(bs*s+ks)^2 );
wc = 3; % given
[m,p] = bode(Gsys,wc)

% design lead
K=1; Dlead = K*(s+0.804)/(s+11.2);
[m,p] = bode(Gsys*Dlead,wc)
K=1/m
Dlead = K*(s+0.804)/(s+11.2);
sysCL = feedback(Gsys*Dlead,1);
sysOL = Gsys*Dlead
figure; nyquist(sysOL)
figure; rlocus(sysOL)
%step(sysCL)
```

```
m =

    0.1103


p =
```

```
 -180.0000


m =

    0.0295


p =

 -119.9978


K =

   33.8508


sysOL =

          0.0677 s^2 + 1219 s + 979.8
  --------------------------------------------
  0.01 s^5 + 0.114 s^4 + 36.38 s^3 + 407.2 s^2

Continuous-time transfer function.
```



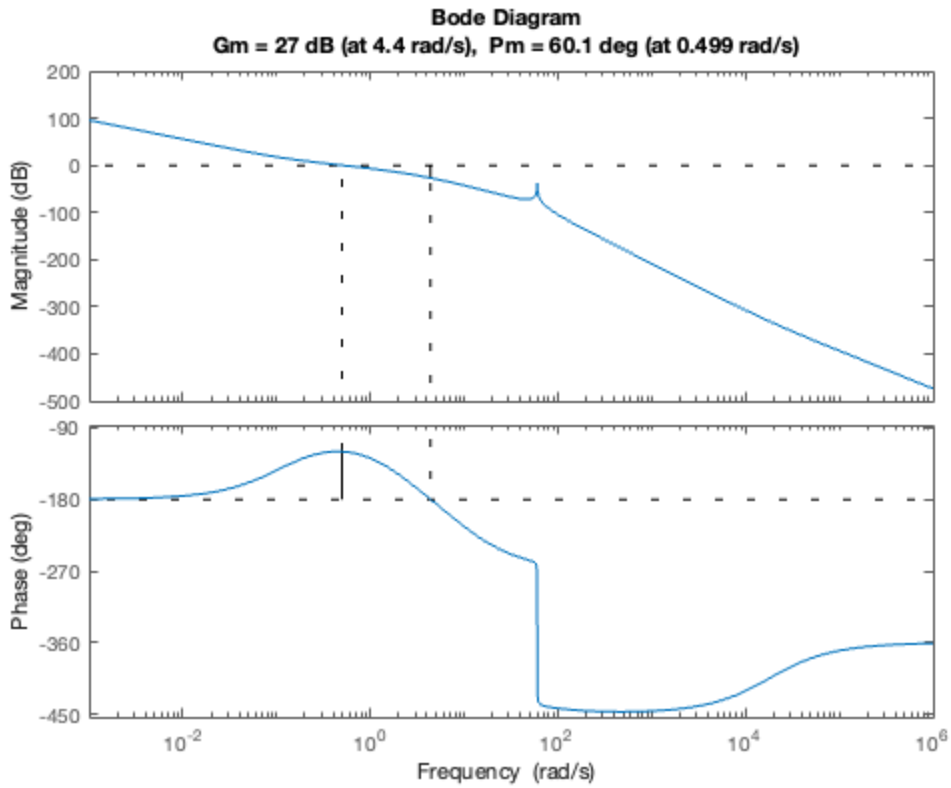



# 2c

```
bs = 0.002; ks = 36; I1 = 1.0; I2 = 0.01; % constants
Gsys = (bs*s+ks) / ( (I1*s^2+bs*s+ks)*(I2*s^2+bs*s+ks)-(bs*s+ks)^2 );
wc = 0.5; % given
[m,p] = bode(Gsys,wc)

% design lead
phi = 63
alpha = (1-sind(phi))/(1+sind(phi));
p = wc*sqrt(alpha);
z = wc/sqrt(alpha);
Dlead = (s+p)/(s+z);
[m,p] = bode(Gsys*Dlead,wc)
K=1/m
Dlead = K*Dlead

% low pass
wlp = 10;
lp = wlp/(s+wlp);
sysOL = Gsys*Dlead*lp
margin(sysOL)
```

*m* =

```
    3.9607


p =

 -180.0000


phi =

    63


m =

    0.9509


p =

 -117.0000


K =

    1.0517


Dlead =

  1.052 s + 0.1262
  ----------------
     s + 2.083

Continuous-time transfer function.

sysOL =

               0.02103 s^2 + 378.6 s + 45.45
  ---------------------------------------------------------
  0.01 s^6 + 0.1228 s^5 + 36.59 s^4 + 439.4 s^3 + 757.3 s^2

Continuous-time transfer function.
```

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