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Class section (circle one): 301 302 303 304 305 306

ME 364

Exam 1

February 17, 2020, 3:30 – 4:20 pm

Necessary:

- Calculator
- Pencil or pen for working the exam

Allowed:

- The textbook
- Lecture notes
- Other notes that you have personally written (e.g. formula sheet)

Prohibited:

- Communicative devices (laptop, tablet, phone, etc…)
- Solved problems other than problems you've solved personally, examples in your book(s) or that were given to you by the instructor as part of this class.

Instructions:

- There are three problems:
    - Problem 1: 30 pts
    - Problem 2: 40 pts
    - Problem 3: 30 pts
- Show all your work clearly. Correct answers without work will not receive credit.
- Any mistakes should be neatly crossed out but not erased (might help you get more partial credit).
- Put a box around your final answers and only your final answers.
- Do not open or start the exam before you are instructed to do so.

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Problem 1

Figure 1 illustrates a solid cylindrical rod with diameter D, length L, and **constant** conductivity k. One end of the rod (at $x = L$) is joined to a refrigerator that is maintained at temperature T_C. The joint causes a contact resistance, R''_C, between the refrigerator and the end of the rod. The other end of the rod (at $x = 0$) is maintained at room temperature, T_H. There is no contact resistance at this end. At its surface, the rod experiences convection, which has heat transfer coefficient $\bar{h}$ and fluid temperature T_C. You may neglect radiation heat transfer for this problem. In addition, you may assume the temperature distribution in the rod is 1-D.

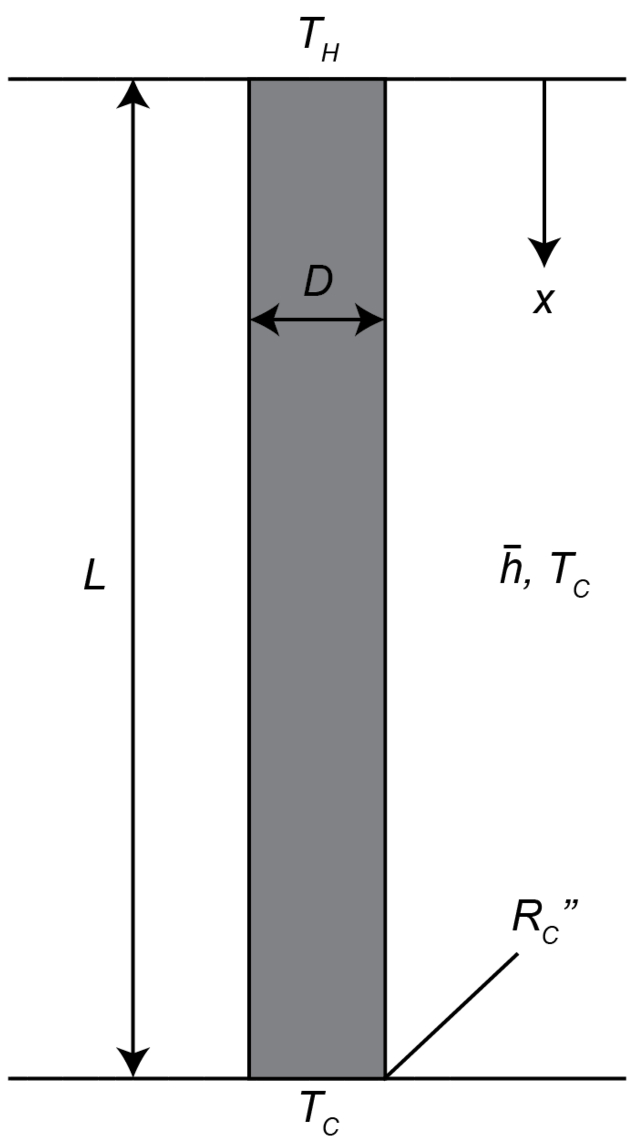

Figure 1: Solid cylindrical rod.

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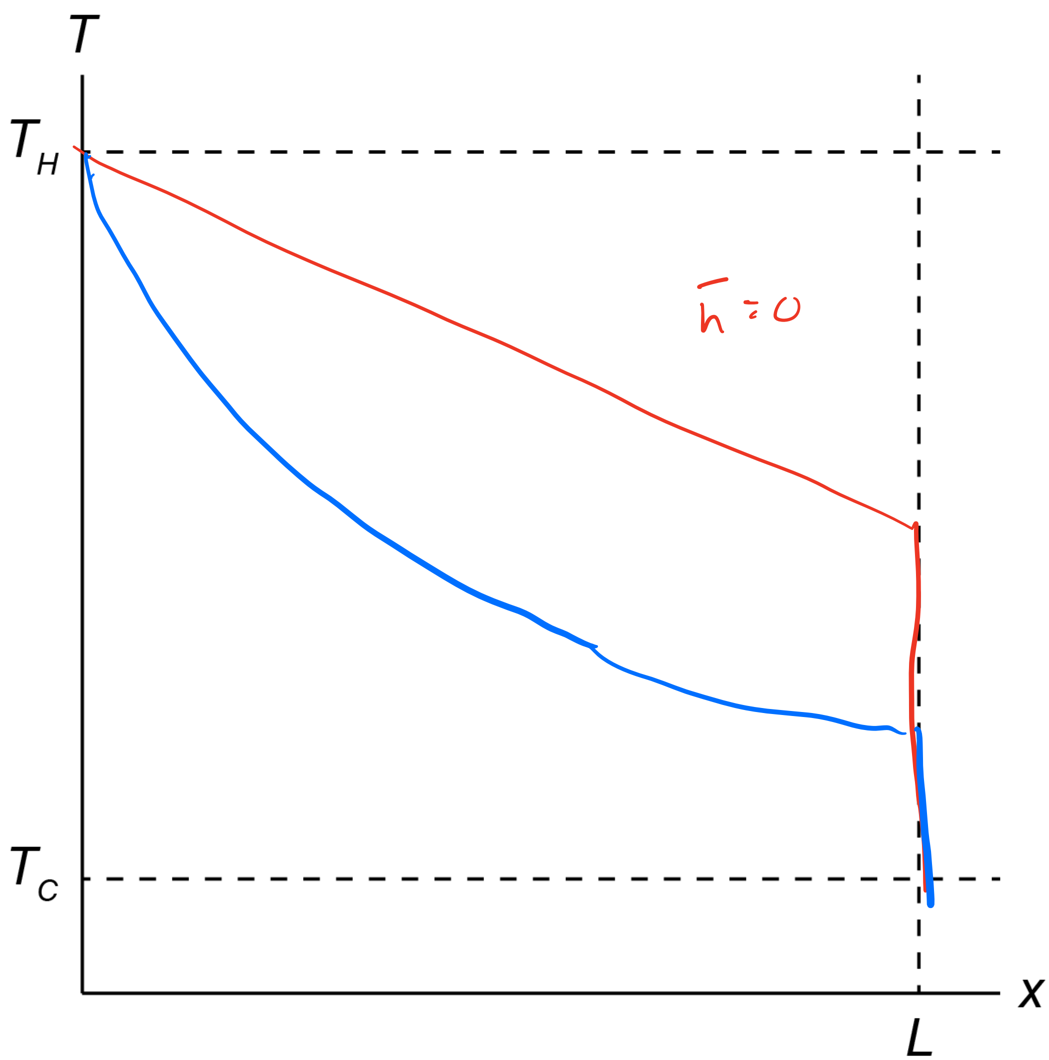
a) On the axes below, sketch the temperature distribution in the rod as a function of x for the case when the rod experiences **no convection** at its surface ($\bar{h} = 0$). Note that the temperatures T_H and T_C are both labeled for you.

b) On the same axes as part (a), sketch the temperature distribution in the rod as a function of x for the case when the rod experiences convection at its surface ($\bar{h} > 0$). Be sure to clearly label which sketch corresponds to part (a) and which sketch corresponds to part (b).

$$R_{cond} = \frac{L}{k \cdot A_c} \qquad R_{both} = \left[\left(\frac{L}{kA_c} \right)^{-1} + \right.$$



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c) The general solution for the temperature within the rod is:

$$T = C_1 \exp(mx) + C_2 \exp(-mx) + T_c$$

where $m = \sqrt{\frac{4\bar{h}}{kD}}$. Develop two equations that can be solved to provide the two unknown constants C_1 and C_2. Your equations should be written in terms of C_1 and C_2 as well as the other symbols defined in the problem statement. DO NOT ATTEMPT TO SOLVE THESE EQUATIONS.

$T(x=0) = T_H = C_1 e^0 + C_2 e^0 + T_c \rightarrow$

$$\boxed{C_1 + C_2 = T_H - T_c}$$

AT $x = L$: $\dot{q} = \frac{\Delta T}{R}$

$$\dot{q}_{cond}(x=L) = \frac{T(x=L) - T_c}{R_c}$$

$-kA_c \frac{d}{dx}($

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Problem 2

A human cell is comprised of a nucleus surrounded by a layer of material called cytoplasm, which is encased inside of a **thin** cell membrane. The cell can be modelled as a composite sphere as shown in Figure 2, where the outer diameters of the nucleus and cytoplasm are $d_n = 20\times10^{-6}$ m and $d_c = 100\times10^{-6}$ m, respectively. The thickness of the cell membrane can be neglected. As shown in Figure 2, there is a uniform generation of heat $\dot{g}'''$ within the nucleus, whereas the outer surface of the cell is uniformly exposed to a fluid with temperature $T_\infty = 30$ ºC and heat transfer coefficient $\bar{h} = 200$ W/(m^2-K). The temperature of the outer surface of the cell is measured to be $T_s = 34$ ºC. Assume that the conductivity of the nucleus and cytoplasm is $k_{cell} = 0.1$ W/(m-K) and there is no contact resistance between layers. You may neglect radiation heat transfer for this problem.

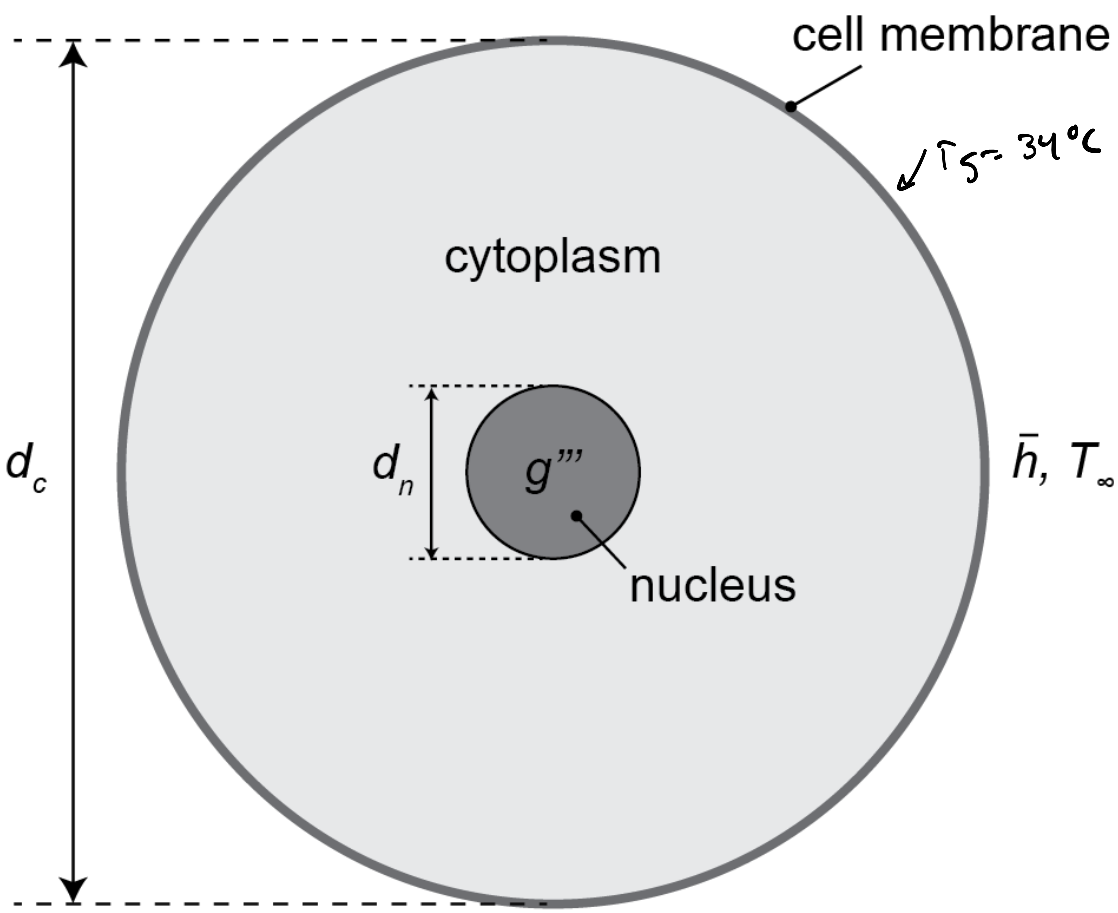

Figure 2: Human cell modelled as a composite sphere.

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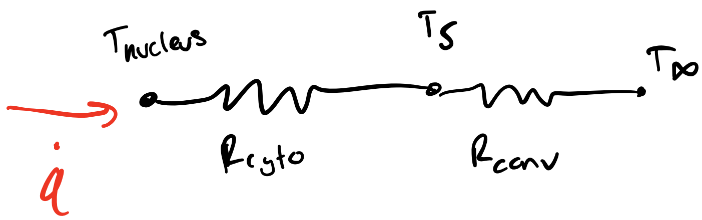
a) Draw the relevant thermal resistance network for this problem. Clearly indicate what each resistance represents and calculate the value of each resistance.



$$R_{cyto} = \frac{1}{4\pi k}\left[\frac{1}{20\times10^{-6}} - \frac{1}{100\times10^{-6}}\right]$$

$$= 31831$$

b) Calculate the volumetric heat generation rate within the nucleus ($\dot{g}'''$).

$$\dot{q} = \dot{g}''' V \quad \rightarrow \dot{g}'''$$

c) Calculate the temperature at the interface between the nucleus and the cytoplasm (T_n).

Suppose that there is an area-specific contact resistance between the cytoplasm and the cell membrane which is $R_c'' = 1.4\times10^{-5}$ m^2-K/W.

d) Do you expect there to be a large error in the calculation from part (c) because this contact resistance was neglected? Show your calculations and explain.

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Problem 3

A thin and conductive heater which generates a constant heat output per unit area $\dot{q}''$ is sandwiched between two identical heat sinks as shown in Figure 3. Each heat sink has a square base with side width W and thickness th_b. Each heat sink has an array of N fins where each fin has square cross section of side width a and length L. The heat sinks are exposed to a coolant at temperature T_∞ and convection coefficient $\bar{h}$ on the left and right sides. In this problem, the fins can be treated as 1D extended surfaces with **adiabatic tips**. You may neglect contact resistances and radiation heat transfer, and you may assume the problem to be 1D in the x-direction.

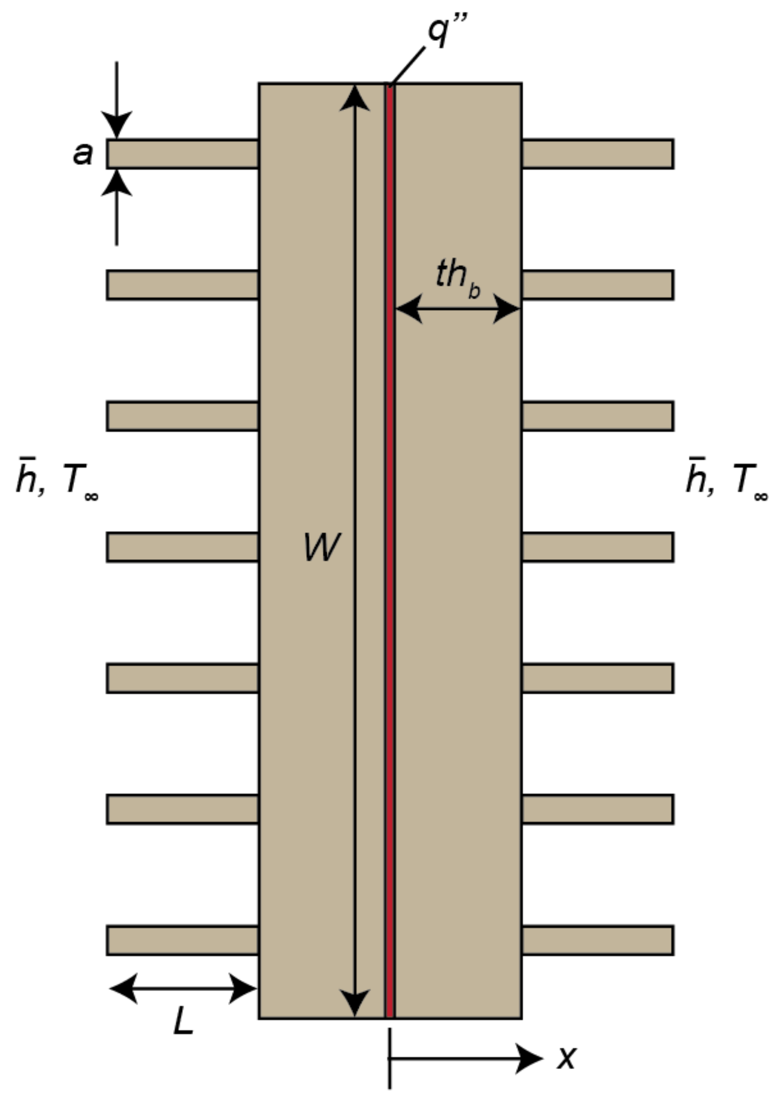

Figure 3: Thin heater sandwiched between two heat sinks.

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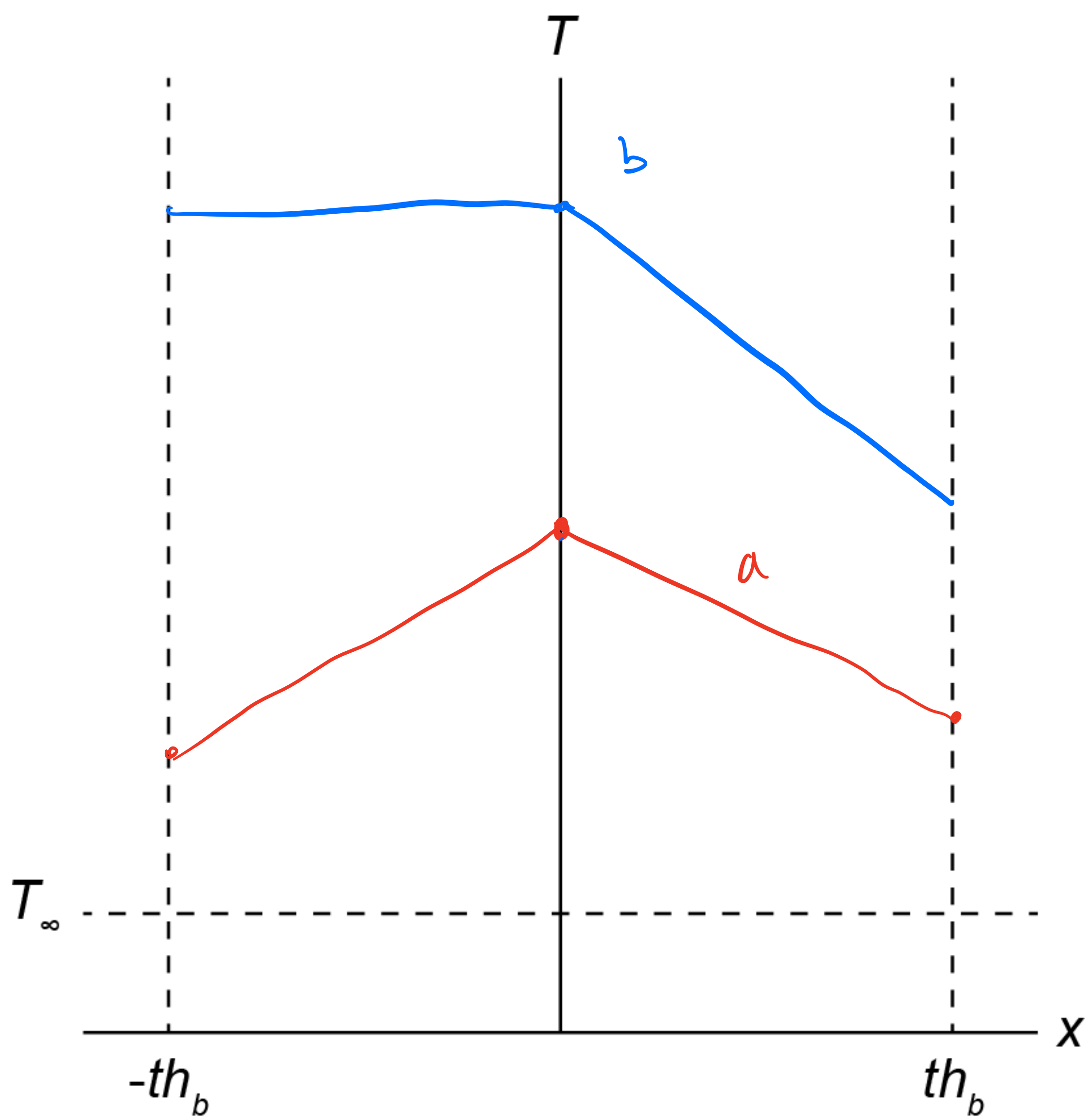
a) On the axes below, sketch the steady-state temperature distribution in the composite base structure as a function of x between $-th_b$ *and* th_b. Note that the temperature T_∞ is labeled for you.

b) On the same axes as part (a), sketch the steady-state temperature distribution in the composite base structure as a function of x for the case when there is a loss of coolant on the left side ($\bar{h} = 0$ on that side). Be sure to clearly label which sketch corresponds to part (a) and which sketch corresponds to part (b).



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The heater is now feedback controlled to maintain the surface temperature of the base on the coolant side ($x = th_b$) at a constant temperature T_b. It is later discovered that **exactly** the same heater power is required to maintain a fixed surface temperature T_b when the fins on the heat sink are removed and convection only occurs at the surface of the base exposed to the coolant.

c) Develop an expression for the fin efficiency η_f. The expression should only be written in terms of the variables provided in the problem statement.

$$\eta_{fin} = \frac{\dot{q}_{fin}}{\bar{h}\, 4aL\, (T_b - T_\infty)} \qquad \dot{q}_{fin} = \dot{q}_{conv,\, nofin}$$

$$\dot{q}_{conv,\, nofin} = (T_b - T_\infty)\, \bar{h}\, a^2$$

$$\eta_{fin} = \frac{\cancel{(T_b - T_\infty)} \cdot \cancel{\bar{h}}\, a^2}{\cancel{\bar{h}}\, 4aL\, \cancel{(T_b - T_\infty)}} = \frac{\cancel{a^2}}{\cancel{W^2}} \quad \frac{a}{4L}$$