# Frequency Domain control system design

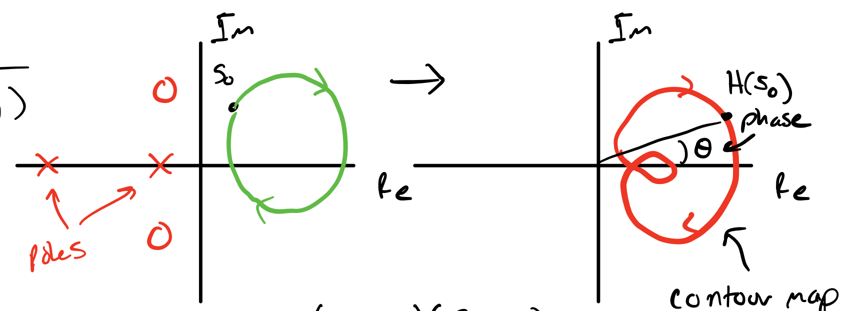
## Nyquist stability criterion

- used to evaluate stability of closed loop system
  - how close to being unstable, margin of stability, robustness
- used to evaluate time-domain characteristics of closed-loop system ($\zeta, \omega_n, e_{ss}$
- For real systems, Nyquist plot evaluated from system open-loop frequency response

## The Argument principle

- evaluate $H(s)$ for values corresponding to a closed, <u>clockwise</u> contour in complex plane

$$H(s) = \frac{s^2 + 2s + 5}{(s+1)(s+5)}$$




Evaluated at point $s_0$: $H(s_0) = \frac{b(s_0)}{a(s_0)} = \frac{(s_0 - z_1)(s_0 - z_2)\ldots}{(s_0 - p_1)(s_0 - p_2)\ldots}$

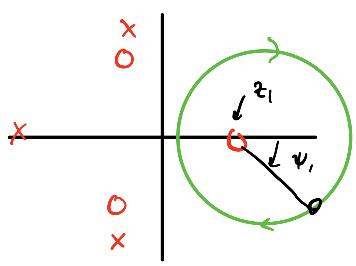
$$\text{phase: } \angle H(s_0) = (\psi_1 + \psi_2 + \ldots) - (\varphi_1 + \varphi_2 + \ldots)$$

$$\text{phase: } \psi_i = \angle(s_0 - z_i) \quad \text{or} \quad \varphi_i = \angle(s_0 - p_1)$$

phase: - each term will return to its initial value if the pole/zero is not inside contour

- each term inside the contour will change by $N \cdot 360°$

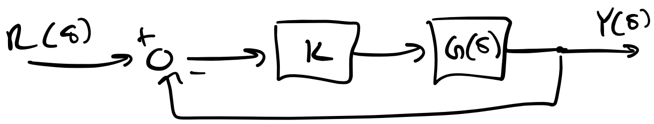
Ex:




<u>Zero $z_1$</u>:
phase of $z_1$ term will experience net $-360°$ change

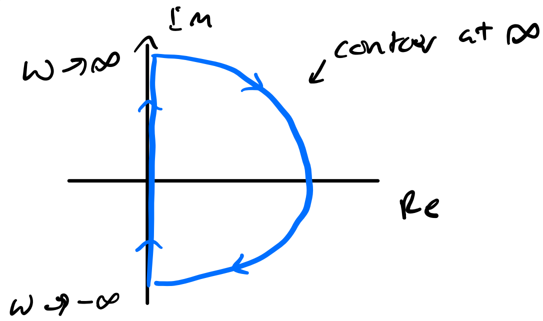
clockwise contour: clockwise encirclement

<u>For a pole</u>: counter clockwise encirclement




CL Transfer function:

$$\frac{Y(s)}{R(s)} = \frac{KG(s)}{1+KG(s)}$$




$1 + KG(s)$

$$1 + K\frac{b(s)}{a(s)} = \frac{a(s) + Kb(s)}{a(s)}$$

$a(s) + Kb(s)$ ← roots of $\Delta_{CL}(s)$ (zeros of $1+KG(s)$) $Z$

$a(s)$ ← open loop poles of $G(s)$ (poles of $1+KG(s)$) $P$

Evaluate CW contour around entire RH plane of $\left[1 + KG(s)\right]_{s=C}$

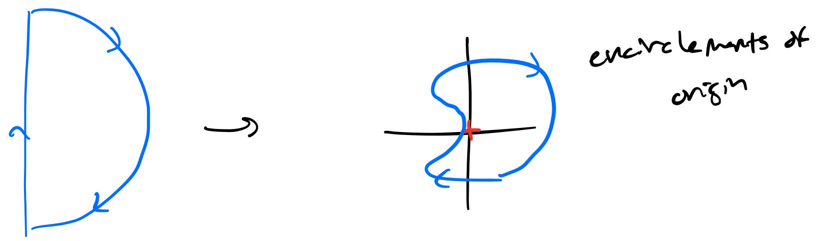
[Net # of CW encirclements of origin] = [# CW due to roots of $\Delta_{CL}(s)$ in RHP] − [# CCW due to poles of $G(s)$ in RHP]

i.e. unstable CL poles

i.e. unstable OL poles

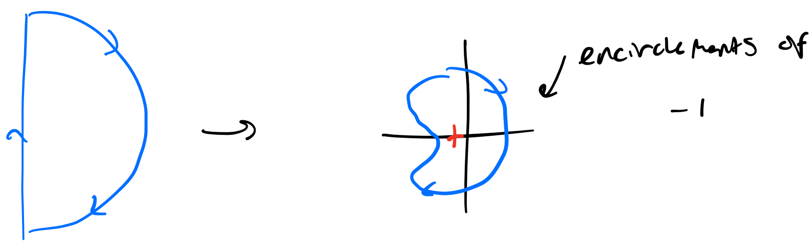
$$N = Z - P \rightarrow \boxed{Z = N + P}$$

$1 + KG(s)$:




Subtract 1

$KG(s)$



Evaluation of contour map $[KG(s)]_{s=C}$

1) Evaluate contour along positive imaginary, ($s = 0 \to j\infty$), using frequency response of $KG(s)$

2) Contour map at infinity: we can assume that $|KG(s)|$ is infinitesimally small, thus can ignore arc at $\infty$ because it can not encircle $-1$.

3) Evaluate contour along negative imaginary ($s = 0 \to -j\infty$), by reflecting over real axis because imaginary part changes sign when $s = -j\omega$ (real part does not)