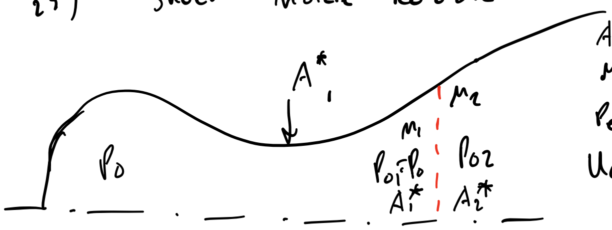
Portions of curves w/ shock

case i) Shock on exit plane

ii) shock inside nozzle




$A_e$

$M_e < 1$

$P_e = P_a$

$U_e = M_e a_e = M_e \sqrt{\gamma R T_e}$

$$J = \dot{m} U_e + (P_e - \cancel{P_a})A_e$$

$$= \rho_e U_e A_e U_e = \dots = \gamma A_e M_e^2 P_e$$

$$C_J \equiv \frac{J}{P_0 A^*} = \frac{J}{P_{01} A^*_1} = \gamma M_e^2 \underbrace{\frac{P_e}{P_{01}} \frac{A_e}{A^*_1}}$$

$$C_J = \gamma M_e^2 \frac{P_e A_e}{P_{02} A^*_2}$$

$\Leftrightarrow$ product is constant

(I)

Operationally, to plot $\frac{C_J}{C_{J\,conv}}$ vs $\frac{A_e}{A^*_1}$

a) pick parameter value of $P_e/P_{01}$

b) pick value of $0 < M_e < 1$

$\rightarrow M_e$ sets $A_e/A^*_2$ & $\frac{P_e}{P_{02}}$ (isentropic relations)

$\rightarrow$ Calculate $C_J$ using (I) ,

To find $A_e/A^*_1$ (shock relation requires $M_1$)

$$\frac{P_{02}}{P_{01}} = \frac{P_a}{P_{01}} \checkmark \frac{P_{02}}{P_e} \checkmark \quad (B/C\ P_e = P_a)$$

$$\rightarrow \frac{A^*_1}{A^*_2} = \frac{P_{02}}{P_{01}}$$

$\rightarrow \frac{A_e}{A^*_1} = \frac{A_e}{A^*_2} \cdot \frac{A^*_2}{A^*_1}$ $\rightarrow$ Store $\frac{A_e}{A^*_1}$, $C_J$, $\frac{P_a}{P_{01}}$ in 3 arrays

→ repeat to b), then repeat to a)

→ backwards to set $f(x)$ & solve for $(x)$, but it avoids root solver for $M$ given $\frac{A}{A^*}$

## Implications of nozzle size & proportions

Return to $(\epsilon)$:

$$\frac{J}{P_0 A^*} = \sqrt{\frac{2\gamma^2}{(\gamma-1)}\left(\frac{2}{(\gamma+1)}\right)^{\frac{\gamma+1}{\gamma-1}}\left[1-\left(\frac{P_e}{P_0}\right)^{\frac{\gamma-1}{\gamma}}\right]} + \left(\frac{P_e}{P_0} - \frac{P_a}{P_0}\right)\frac{A_e}{A^*}$$

i) For fixed $A^*$, $P_0$, $\gamma$, effect of $A_e/A^*$

- $\frac{A_e}{A^*}$ sets $\frac{P_e}{P_0}$ (isentropic)
- For rockets operating entirely @ high altitude, $P_a \approx 0$ at all times
- For rockets operating across a range of altitudes, $P_a$ varies
- For these latter, it's mandatory to know altitude vs. time history to select optimum compromise

consider rocket starting @ SL & going to high altitude.

- If nozzle designed to deliver $P_e = P_a$ @ SL (high $P_a$) as $P_a \downarrow$ w/ altitude, $P_e/P_0$ inside sqrt (w/ negative sign) adversely affects thrust. Nozzle is <u>under expanded</u>.

$\frac{A_e}{A^*}$ could be larger, gaining more exit velocity at the expense of exit pressure

- If $A_e/A^*$ designed to deliver very low $P_e$, then near-sea level: high $P_a$ adversely affects thrust.

- Also, structural problems
  - if $A_e/A^*$ is very large, large portions of internal nozzle surface are @ $p << p_a$ $\rightarrow$ severe structural loads

- Also, during startup transient flow may detach from nozzle walls and may enter a detach/reattach cycle
  $\rightarrow$ dangerous oscillating loads

$\Rightarrow$ Conclusion: Find "compromise" value of $p_e/p_0$
(i.e. $A_e/A^*$)

ii) Effect of $p_0$ (For fixed $J$, $\gamma$, $\frac{A_e}{A^*}$, $p_a$)

in Eq. (E), to keep $U_{ts}$ fixed,

If $p_0 \uparrow \rightarrow A^* \downarrow$

Since entire nozzle dimensions (weight) scale w/ $A^*$, lowering $A^*$ is attractive

$\rightarrow$ see fig. 3 Heister

But, if $p_0 \uparrow$ $\begin{cases} \text{CC thickness} \uparrow \\ \text{power/size/weight of turbopumps} \uparrow \end{cases}$

Seek optimization of opposing effects

## Real nozzles

In a real axisymmetric nozzle
- all properties ($M$, $P$, $T$, $\rho$, $\underline{V}$) vary in 2 directions
- velocity is 2D
  $\underline{u} = u\hat{i} + v_r\hat{e}_r$