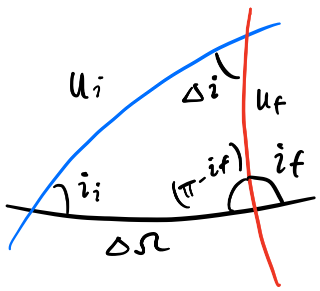
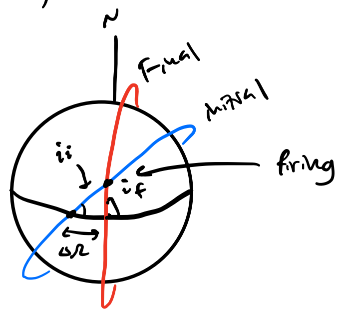
Example 4 suppose after launch, need plane change

General rotations (change both i & Ω)



let $u_i = \omega + \theta$



Steps for this

1) Find Δi, spherical triangle $(i_i, i_f, \Delta\Omega)$

2) Find θ at firing w/ $u_i = \omega + \theta$ & triangle (i_i, i_f, u_i)

3) Find ΔV knowing Δi, a, e

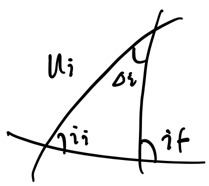
e.g. $\cos A = -\cos B \cos C + \sin B \sin C \cos a$

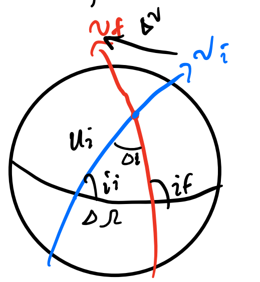
E.g. circ. orbit, $i = 55°$, $\Omega = 0°$, $r = 7500$ km

Find timing & Δv to change to $i = 40°$, $\Omega = 45°$

1) Find Δi

$$\cos \Delta i = -\cos(i_i)\underbrace{\cancel{\cos}(\pi - i_f)}_{-\cos(i_f)} + \sin(i_i)\underbrace{\sin(\cancel{\pi} - i_f)}_{\sin(i_f)}\cos(\Delta\Omega)$$





$$\cos \Delta i = \cos \underbrace{\cancel{i_i}}_{55°} \cos \underbrace{\cancel{i_f}}_{40°} + \sin \underbrace{\cancel{i_i}}_{55°} \sin \underbrace{\cancel{i_f}}_{40°} \cos \underbrace{\cancel{\Delta\Omega}}_{45°}$$

$\Delta\Omega = 55° - 0°$

$\rightarrow$ $= 0.8117$

$\rightarrow \Delta i = 35.74°$

2) Find v_i $\rightarrow$ orbit params: r, a, e $\rightarrow$ vis viva

circular: $v = \sqrt{\frac{\mu}{r}}$ $\rightarrow v = 7.28$ km/s

3) Find $\Delta v = 2 v_i \sin\left(\frac{\Delta i}{2}\right)$ $(e = 0)$ circular

$$\Delta v = 2(7.29)\sin\left(\frac{35.74}{2}\right) = 4.47\ \text{km/s}$$

4) To find timing → use $u_i = \cancel{\omega}^{\,0,\ \text{circular}} + \theta$ → $u_i = \theta$

(circ. triangle)

$$\cos(\cancel{\pi} - i_f) = -\cos(i_i)\cos(\Omega i) + \sin(i_i)\sin(\Omega i)\,\underline{\cos(u_i)}$$ isolate

$-\cos(i_f)$

$$\therefore \cos u_i = \frac{-\cos \overset{40^\circ}{\cancel{i_f}} + \cos \overset{55^\circ}{\cancel{i_i}} \cos \overset{35.74^\circ}{\cancel{\Omega i}}}{\sin \underset{55^\circ}{\cancel{i_i}} \sin \underset{35.74^\circ}{\cancel{\Omega i}}}$$

→ $u_i = 128.9°$

→ $\theta = 128.9°$

Lec 18: Bi-elliptic w/ plane change

→ goal: reduce Δv @ perigee of transfer

$$\Delta v = 2 v_i \sin \frac{\Delta i}{2}$$

Steps

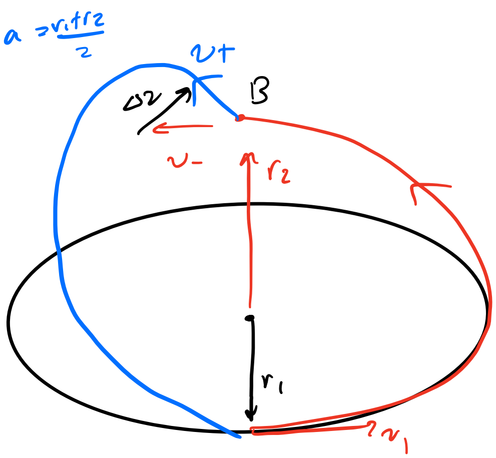
1) Assume circular to start
2) ΔV_1 → large ellipse
3) ΔV_2 → rotates large ellipse
4) ΔV_3 → back to circular orbit

See L18 S2

1) initial orbit $v_{c_1} = \sqrt{\frac{\mu}{r_1}}$ pick large $a \geq \frac{r_1 + r_2}{2}$

2) choose r_2 $\Delta v_1 = v_{1t_A} - v_{c_1}$

$$v_{1t_A} = \sqrt{\frac{2\mu r_2}{r_1(r_1+r_2)}}$$



3) plane change

$$\Delta v = 2 v_{1t_B} \sin\left(\frac{\Delta i}{2}\right)$$

$$v_{1t_B} = \sqrt{\frac{2\mu r_1}{r_2(r_1+r_2)}}$$

4) Final orbit $v_{c_2} = v_{c_1}$

$$\Delta v_3 = v_{c_2} - v_{1t_A}$$

$\Delta v_{tot} = \underbrace{|\Delta v_1| + |\Delta v_2| + |\Delta v_3|}_{\text{same}}$

show for $\Delta i = 90$ Bi-elliptic gives $\Delta v = 0.82\, v_{c_1}$

simple plane change

$\Delta v = 1.4\, v_{c_1}$

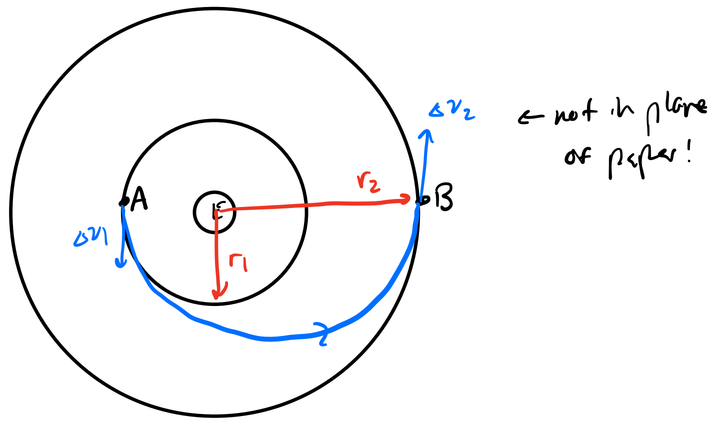
inclination & radius

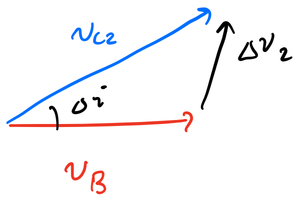
Hohmann w/ plane change - plane @ A or B? = slower, make Δi @ B

=> one plane

$v_{c_1} = \sqrt{\frac{\mu}{r_1}}$ $v_{c_2} = \sqrt{\frac{\mu}{r_2}}$

$a = \frac{r_1 + r_2}{2}$ $v_B = \sqrt{\mu\left(\frac{2}{r_2} - \frac{1}{a}\right)}$





Law of cosines

$$\Delta v_2^2 = v_B^2 + v_{c2}^2 - 2 v_B v_{c2} \cos \Delta i$$ ✓

Can I split a plane change?

$$\Delta i = \alpha_1 + \alpha_2$$

α_1: at Δv_1 (A) α_2: at Δv_2 (B)

-> make split plane change

1) make α_1 change at perigee of transfer (A)

① $\Delta v_1^2 = v_A^2 + v_{c_1}^2 - 2 v_A v_{c_1} \cos \alpha_1$

↑ trans. @ A

2) make α_2 change @ apogee of transfer (β)

② $\Delta v_2^2 = v_B^2 + v_{c2}^2 - 2 v_B v_c \cos\alpha_2$

cost function $\Delta v_{tot} = |\Delta v_1| + |\Delta v_2|$ optimize!

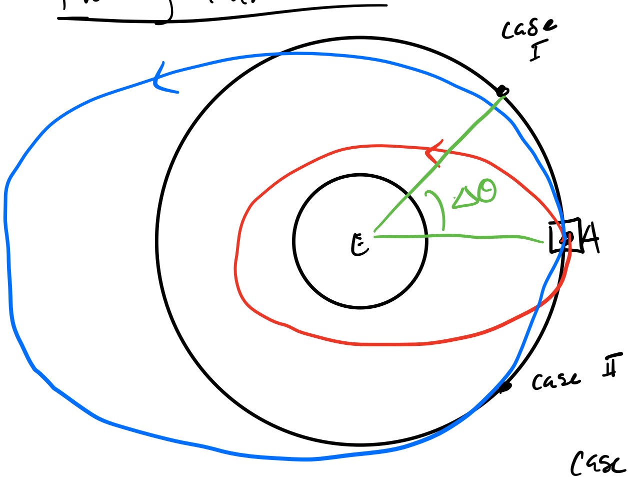
Find min, where slope = 0

use $\alpha_2 = \alpha_i - \alpha_1$

$\Delta v_{tot} = f(\alpha_1)$

Set $\frac{\partial \Delta v_{tot}}{\partial \alpha_1}$, find α_1 see slides for closed form

Phasing maneuvers



phasing ≡ changing relative θ

case I

- slow down, drop into "shallower" faster transfer orbit

maintaining final radius

rendesvous back at A

Case II

- speed up, "raise" into "deeper" but slower transfer orbit

let $\Delta\theta$ = phase difference of trasfer

Using: VTB viva, mean avg. velocity, small Δv vs. v

Show: $\Delta V = \frac{\Delta\theta}{2\pi} \frac{M}{3 v_0 a_0 N}$

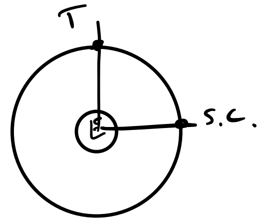
a_0, v_0 = current orbit & speed

N = # of orbits req'd for interception

Example 1 Advance a 12hr orbit by 90° in 117 hrs

Find Δv.

Assume circular $\rightarrow a_0 = r$



$\Delta\theta$ relative to target, $\Delta\theta = -90^\circ$

we are behind!

$\therefore T = 12\text{hrs} = 43200\text{ s} = 2\pi\sqrt{\frac{a^3}{\mu}} \Rightarrow a_0 = r = 26{,}610\text{ km}$

$\therefore v_0 = \sqrt{\frac{\mu}{a_0}} = 3.87\text{ km/s}$

For N, $N = \frac{117\text{hr}}{12\text{hr}} = 9.75$ orbits include decimal!

$$\Delta v = \frac{\Delta\theta}{2\pi}\frac{\mu}{3 v_0 a_0 N} = \frac{(-\pi/2)}{2\pi}\frac{3.986\times10^5\text{ km}^3/\text{s}^2}{3(3.87\text{ km/s})(26610\text{ km})9.75}$$

$\Delta v = -0.33$ km/s

Slow down to catch up