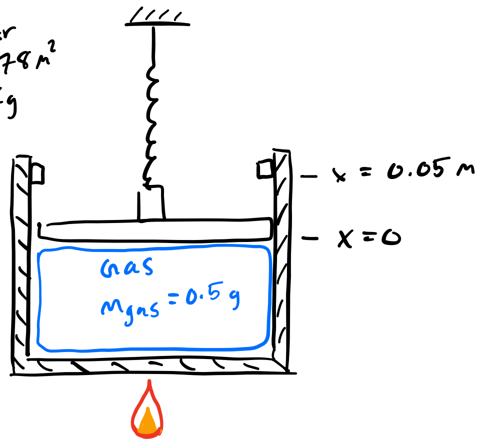
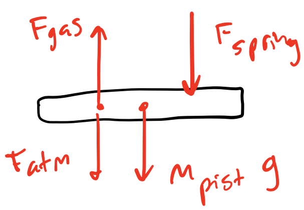
1) Piston-cylinder assembly. Gas expands, raising piston at constant speed until it hits the stops. $F_{spring} = Kx$, $K = 10{,}000$ N/m. Neglect friction. $e_i = 215$ KJ/Kg, $e_f = 337$ KJ/Kg

$P_{atm} = 1$ bar
$A_{pist} = 0.0078\,m^2$
$m_{pist} = 10$ Kg



1a) Determine initial absolute pressure of the gas

FBD Piston



$$\Sigma F = m\cancel{a}^{\,0}$$

$$F_{gas} = F_{atm} + m_{pist}\,g + K\cancel{x}^{\,x=0}$$

$$P_{gas} \cdot A_{pist} = P_{atm} \cdot A_{pist} + m_{pist} \cdot g$$

$$P_{gas} = P_{atm} + \frac{m_{pist} \cdot g}{A_{pist}}$$

$$\rightarrow P_{gas} = 100{,}000\,[Pa] + \frac{(10\,Kg)(9.81\,m/s^2)}{(0.0078\,m^2)}$$

$$\rightarrow \boxed{P_{gas} = 112.6\ kPa}$$

1b) Determine the work performed on the piston by gas

$$W = \int_{x_1}^{x_2} F_{gas}\,dx \qquad F_{gas} = F_{atm} + m_{pist}\,g + Kx$$

$$\rightarrow W_{gas} = \int_{x=0}^{x=0.05} \left[P_{atm} A_{pist} + m_{pist}\,g + Kx\right] dx$$

$$= \left[P_{atm} A_{pist} \cdot x + m_{pist}\,g \cdot x + \frac{K}{2} x^2\right]_{x=0}^{x=0.05}$$

$$W_{gas} = (100{,}000)(0.0078)(0.05) + (10)(9.81)(0.05) + \frac{(10{,}000)}{2}(0.05)^2$$

$$\rightarrow \boxed{W_{gas} = 56.4 \text{ J}}$$

1c) Determine the total heat transferred to the gas

$$de = dq - dw \rightarrow \Delta E = Q - W$$

$$Q = W + \Delta E = W_{gas} + m(e_2 - e_1)$$

$$= (56.4 \text{ J}) + (0.0005 \text{ kg})(337000 - 215000)$$

$$\rightarrow \boxed{Q = 117.4 \text{ J}}$$

2) CH 2 P3. Hint: look in T-S plane.

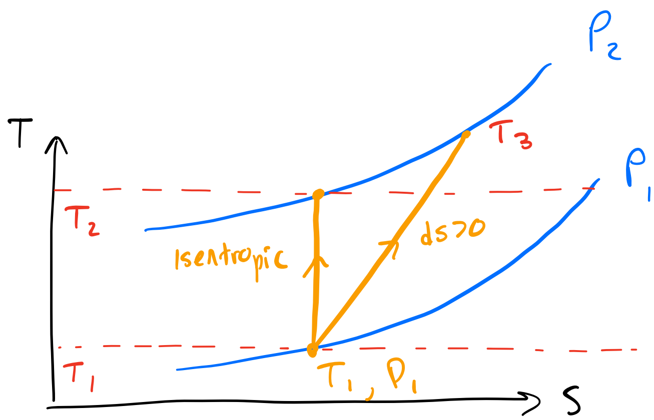
For adiabatic steady-flow compression, show from laws of thermo that the min. possible specific work for given initial T & p and final p requires an isentropic process.

Show that based on 2nd law, "adiabatic compression efficiency" η_c is:

$$\eta_c = \frac{W_{isen}}{W_{actual}} = \frac{\left(\frac{P_2}{P_1}\right)^{(\gamma-1)/\gamma} - 1}{\frac{T_2}{T_1} - 1}$$

Similarly, how should the adiabatic expansion efficiency be defined for flow of perfect gas with const. spec. heat?

Adiabatic compression efficiency:



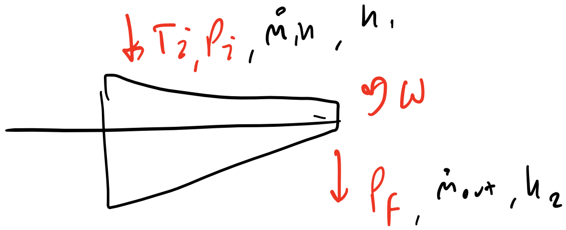
← Shows that for $\Delta s > 0$, ΔP results in larger ΔT

$$de = \cancel{dq}^{\,adb} + dw$$

Ideal gas: $de = C_v dT$

$\therefore dw \propto dT$

$\Rightarrow$ minimum work when $ds = 0$ (isentropic)



$\dot m_{in} = \dot m_{out} = \dot m$

neglect

$$\frac{\dot m \left[\Delta h + \frac{1}{2}\cancel{\Delta V^2}\right] = \dot w_{shaft}}{\dot m}$$

$\rightarrow dh = dw, \quad \Delta h = \Delta w$

$W_{isen} = h_{2i} - h_1$

$W_{actual} = h_2 - h_1$

$$\rightarrow \frac{W_{isen}}{W_{actual}} = \frac{h_{2i} - h_1}{h_2 - h_1} = \frac{\Delta h_i}{\Delta h}$$

$\Delta h = (C_v + R)\Delta T = C_p \Delta T$

$$\rightarrow \frac{W_{isen}}{W_{actual}} = \frac{\cancel{C_p}\Delta T_i}{\cancel{C_p}\Delta T} = \frac{T_{2i} - T_1}{T_2 - T_1} = \frac{T_1 (T_{2i}/T_1 - 1)}{T_2 - T_1}$$

For isentropic T_{2i}: $\frac{T_{2i}}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$

$$\rightarrow \frac{W_{isen}}{W_{actual}} = \frac{T_1\left[\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]}{T_2 - T_1} = \boxed{\frac{\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} - 1}{\frac{T_2}{T_1} - 1} = \eta_c}$$

For turbine:

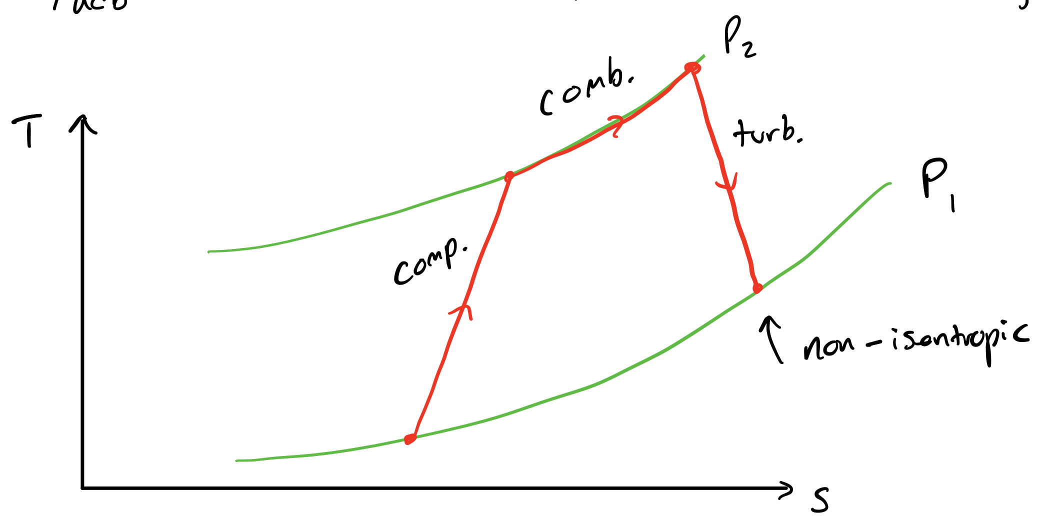
$$\frac{W_{actual}}{W_{isen}} = \frac{C_p \Delta T}{C_p \Delta T_i} = \frac{T_1 - T_2}{T_1 - T_{2i}}$$

$$\rightarrow \boxed{\eta_T = \frac{1 - \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}}{1 - \frac{T_2}{T_1}}}$$

3) Turbojet engine, compressor → combustion chamber (isobaric) → turbine → atmosphere.

3a) Sketch turbojet on T-s diagram

$\eta_{adb} = 0.86$, $PR = 5.2$, alt = 10,000 m, $C_p = 1.0 \frac{kJ}{kg\text{-}K}$



3b) Calculate compression work per unit mass air (neglect ΔVelocity)

If adiabatic & reversible → isentropic

$$\frac{W_{isen}}{W_{actual}} = \eta_c \rightarrow W_{actual} = \frac{W_{isen}}{\eta_c}$$

neglect

$$\Delta W_{isen} = \Delta h + \frac{1}{2}\cancel{\Delta} V^2 = h_{2i} - h_1 = \boxed{\Delta h_i = C_p \Delta T_i}$$

$$\Delta T_i = T_{2_i} - T_1$$

isentropic: $\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$

$$\rightarrow = T_1\left[\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]$$

$$\gamma = \frac{c_p}{c_v} = \frac{c_p}{c_p - R}$$

$$\frac{\gamma-1}{\gamma} = \frac{R}{c_p}$$

$$\gamma - 1 = R/c_v$$

$$\Rightarrow W_{isen} = C_p \cdot T_1 \left[\left(\frac{P_2}{P_1}\right)^{R/C_p} - 1 \right]$$

$$\Rightarrow W_{actual} = C_p \cdot T_1 \left[\left(\frac{P_2}{P_1}\right)^{R/C_p} - 1 \right] / \eta_c$$

$$W_{actual} = (1.0)(223.25)\left[(5.2)^{287/1000} - 1 \right] / \eta_c$$

$$\rightarrow \boxed{W_{actual} = 157.1 \ \frac{KJ}{Kg}}$$

3c) Calculate exit temperature of compressor

$$W_{actual} = C_p \Delta T$$

$$\rightarrow T_2 = T_1 + \frac{W_{actual}}{C_p} = (223.25) + \left(\frac{157.1}{1.0}\right)$$

$$\rightarrow \boxed{T_2 = 380.3 \ K}$$

3d) Adiabatic turbine $\eta_T = 0.9$, $PR = 3.0$, $T_1 = 1000\ K$
alt = 10,000 m. $C_p = 1.142\ KJ/Kg\text{-}K$. $R = .287\ \frac{KJ}{Kg\text{-}K}$
Can it run the compressor?

$$\frac{W_{actual}}{W_{isen}} = \eta_T$$

Same as compressor equation:

$$W_{actual} = C_p \cdot T_1 \left[\left(\frac{P_2}{P_1}\right)^{R/C_p} - 1 \right] \cdot \eta_T$$

$$W_{actual} = (1.142)(1000)\left[\left(\frac{1}{3}\right)^{\frac{0.287}{1.142}} - 1 \right] \cdot 0.9$$

$$\boxed{W_{actual} = -248\ KJ/Kg\text{-}K \rightarrow 248 > 157 \rightarrow \text{Yes}}$$

3e) Calculate exit temperature

$$W_{actual} = C_p \Delta T$$

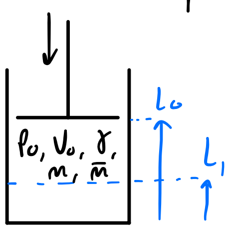
$$\rightarrow T_2 = T_1 + \frac{W_{actual}}{c_p} = (1000) + \left(\frac{-248}{1.142}\right)$$

$$\rightarrow \boxed{T_2 = 782.8 \text{ K}}$$

4) Piston & Cylinder. Neglect gravity. Diameter D, initial distance L_0, contains gas of mass m, mol. mass $\overline{M}$, spec. heat ratio γ. Initial pressure & temp P_0 & T_0.

Piston compresses gas isentropically to $L_1 < L_0$ from end wall. Separately, compresses isothermally to L_1.

4a) Determine work per unit mass by piston on gas for isentropic case. Answer in P_0, v_0, γ, L_0/L_1.



$$A = \pi D^2/4 \qquad V_0 = L_0 A \qquad V_1 = L_1 A$$

$$\rho_0 = \frac{m}{L_0 A}, \quad \rho_1 = \frac{m}{L_1 A}$$

isentropic: $\frac{P_2}{P_1} = \left(\frac{\rho_2}{\rho_1}\right)^\gamma$

$$\rightarrow \frac{P_1}{P_0} = \left(\frac{\rho_1}{\rho_0}\right)^\gamma = \left(\frac{L_0}{L_1}\right)^\gamma \rightarrow P_1 = P_0\left(\frac{L_0}{L_1}\right)^\gamma$$

$$\frac{P}{P_0} = \left(\frac{L_0}{L}\right)^\gamma \qquad dV = A\,dL$$

$$\int dw = \int P\,dV \rightarrow w = \int_{L_0}^{L_1} P_0\left(\frac{L_0}{L}\right)^\gamma \cdot A\,dL$$

$$= P_0 A \int_{L_0}^{L_1} \left(\frac{L_0}{L}\right)^\gamma dL$$

→ wolfram: $P_0 A \left[\frac{L\left(\frac{L_0}{L}\right)^{\gamma}}{1-\gamma} \right]_{L_0}^{L_1} = P_0 A \left[\frac{L_1 \left(\frac{L_0}{L_1}\right)^{\gamma}}{1-\gamma} - \frac{L_0 (1)^{\gamma}}{1-\gamma} \right]$

$$= P_0 A \left(\frac{L_1 \left(\frac{L_0}{L_1}\right)^{\gamma} - L_0}{1-\gamma} \right)$$

$$\omega = \frac{P_0 A \left[\left(\frac{L_0}{L_1}\right)^{\gamma} - L_0/L_1 \right]}{(1-\gamma) L_1}$$ divide by L_0/L_1

$$\boxed{\frac{w}{m} = \frac{P_0 V_0}{(1-\gamma)} \left[\left(\frac{L_0}{L_1}\right)^{\gamma-1} - 1 \right]}$$

4b) Determine final temperature of gas, using 1.) isentropic process laws and 2) 1st law of thermo

1) $T_2 = T_1 \left(\frac{P_2}{P_1} \right)^{\frac{\gamma-1}{\gamma}}$

$$\rightarrow T_1 = T_0 \left(\frac{P_1}{P_0} \right)^{\frac{\gamma-1}{\gamma}}$$

$$= T_0 \left(\left(\frac{L_0}{L_1} \right)^{\gamma} \right)^{\frac{\gamma-1}{\gamma}}$$

$$\boxed{T_1 = T_0 \left(\frac{L_0}{L_1} \right)^{\gamma-1}}$$

2) 1st law of thermo

$de = \cancel{dq}^{\text{adb}} - P\,dV$

ideal gas: $de = C_v\, dT$

$C_v = C_p - R$

$\gamma = \frac{C_p}{C_v}$ $\quad C_v = \gamma C_v - R$

$C_v = \frac{R}{\gamma - 1}$

from 4a:

$$C_v\, dT = -\frac{P_0 V_0}{(1-\gamma)} \left[\left(\frac{L_0}{L_1} \right)^{\gamma-1} - 1 \right]$$

$$P v = RT$$

$$P_o V_o = mRT$$

$$R\Delta T = P_o v_o\left[\left(\frac{L_o}{L_1}\right)^{\gamma-1} - 1\right]$$

$$\cancel{R}\Delta T = \cancel{R}T_o\left[\left(\frac{L_o}{L_1}\right)^{\gamma-1} - 1\right]$$

$$T_1 - \cancel{T_o} = T_o\left(\frac{L_o}{L_1}\right)^{\gamma-1} - \cancel{T_o}$$

$$\rightarrow \boxed{T_1 = T_o\left(\frac{L_o}{L_1}\right)^{\gamma-1}}$$

4c) Find work per unit mass for isothermal in terms of p_o, V_o, L_o/L_1

$$Pv = RT$$

$$P = \frac{RT}{v}$$

$$dw = P\,dV$$

$$\int dw = \int \frac{RT}{v}\,dV$$

$$w = \int_{V_o}^{V_1} \frac{RTm}{V}\,dV = RTm\int_{V_o}^{V_1} \frac{1}{V}\,dV$$

$$W = RTm \ln\frac{V_1}{V_o} \rightarrow w = P_o V_o m \ln\frac{V_1}{V_o}$$

$$\frac{V_1}{V_o} = \frac{L_1}{L_o}$$

$$\boxed{\frac{W}{m} = -P_o V_o \ln\frac{L_o}{L_1}}$$

4d) Develop an expression for ratio $\frac{\Delta w_s}{\Delta w_t}$

$$\frac{\Delta w_s}{\Delta w_T} = \frac{\frac{\cancel{P_o}\cancel{V_o}}{(1-\gamma)}\left[\left(\frac{L_o}{L_1}\right)^{\gamma} - 1\right]}{\cancel{P_o}\cancel{V_o}\ln\frac{L_1}{L_o}}$$

$$\boxed{\frac{\Delta w_s}{\Delta w_T} = -\frac{\left(\frac{L_o}{L_1}\right)^{\gamma-1} - 1}{\ln\frac{L_o}{L_1}(1-\gamma)}}$$

4e) See plot

4f) $d\cancel{e} = dq - dw \rightarrow dq = dw$

isothermal

$\rightarrow \Delta Q = \Delta W$

$\rightarrow \boxed{\Delta Q = -M P_0 v_0 \ln \frac{L_0}{L_1}}$