ω_c - OL characteristic

→ locate bump in ^(Lead compensator) phase at ω_c to increase PM

Lead compensation

- what is freq. where phase is maximized? $\quad D_c(s) = K\left(\frac{s+z}{s+p}\right)$

  i.e. ω_{max} as function of p & z

$$\varphi = \angle D_c(j\omega) = \tan^{-1}\left(\frac{\omega}{z}\right) - \tan^{-1}\left(\frac{\omega}{p}\right)$$

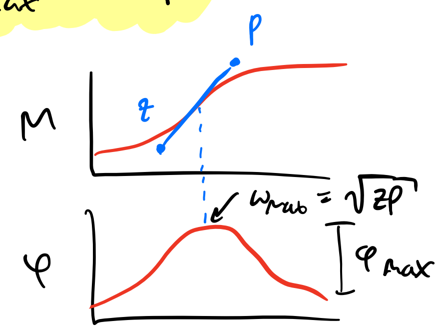
$$\tan^{-1}x - \tan^{-1}y = \tan^{-1}\left(\frac{x-y}{1+xy}\right)$$

$$x = \frac{\omega}{z}, \; y = \frac{\omega}{p}$$

$$\rightarrow \varphi = \tan^{-1}\left(\frac{\omega(p-z)}{\omega^2 - zp}\right) \Rightarrow \tan\varphi = \frac{\frac{\omega}{z} - \frac{\omega}{p}}{1 + \frac{\omega^2}{zp}}$$

$\frac{d}{dt}$ to get ω_{max} → $\omega_{max} = \sqrt{zp}$

$$\log \omega_{max} = \frac{1}{2}(\log z + \log p)$$



How do we adjust φ_{max}?

  i.e. φ_{max} as $f(z, p)$ $\quad$ Lead ratio: $\frac{p}{z}$

Define: $\begin{cases} T_D = \frac{1}{z} \rightarrow z = \frac{1}{T_D} \\ \alpha = \frac{z}{p} \rightarrow p = \frac{1}{T_D \alpha} \end{cases}$ $\quad \varphi_{max}$ @ $\omega_{max} = \sqrt{zp}$

$$\rightarrow \omega_{max} = \sqrt{zp} = \frac{1}{T_D\sqrt{\alpha}}$$

Sub into $\tan\varphi_{max} = \frac{1-\alpha}{2\sqrt{\alpha}}$ $\quad$ max phase as func of Lead ratio $\left(\frac{1}{\alpha}\right)$

$$\rightarrow \sin\varphi_{max} = \frac{1-\alpha}{1+\alpha} \rightarrow \alpha = \frac{1-\sin\varphi_{max}}{1+\sin\varphi_{max}}$$

Lead ratio $\left(\frac{1}{\alpha}\right)$ as function of required phase

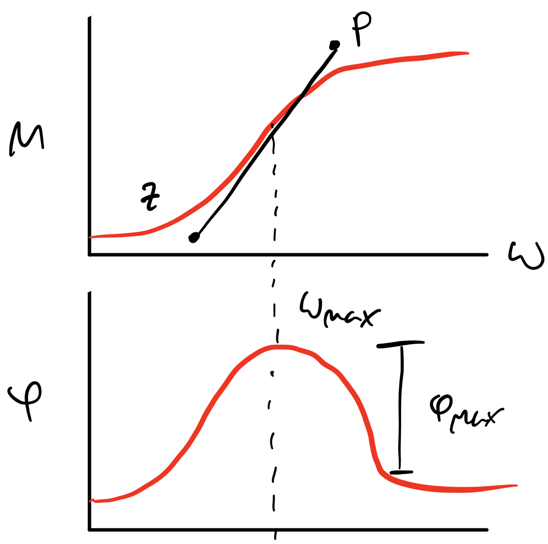
From φ_{max} and ω_{max}:

$$\alpha = \frac{1-\sin\varphi_{max}}{1+\sin\varphi_{max}}$$

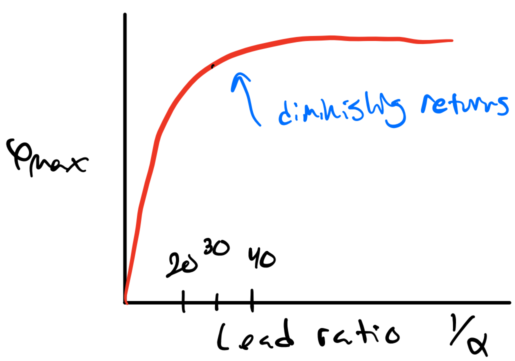
Pole:

$$\omega_{max} = \sqrt{zp} = p\sqrt{\alpha} \rightarrow \boxed{p = \frac{\omega_{max}}{\alpha}}$$

Zero: $\omega_{max} = \sqrt{zp} = \frac{z}{\sqrt{\alpha}} \rightarrow \boxed{z = \omega_{max}\sqrt{\alpha}}$



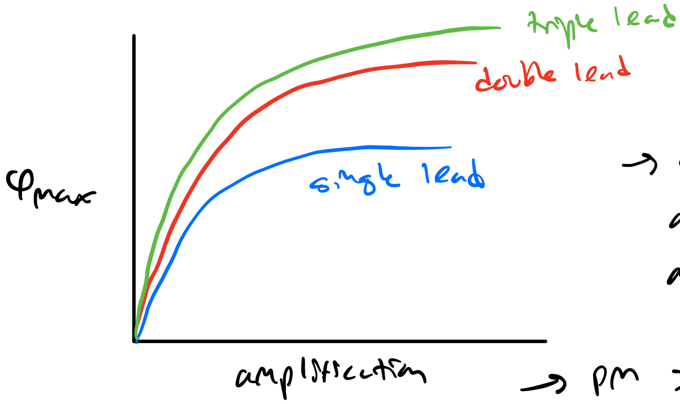
Lead compensator: $D_c(s) = K\left(\frac{s+z}{s+p}\right)$



→ max lead ratio ~35°

→ max φ ~ 70°

Need more: cascaded lead compensators



→ adds complexity & added poles - could add weird transient behavior

→ PM > 150° is just not gonna happen

Ex.

ω_{BW} between ω_c & $2\omega_c$

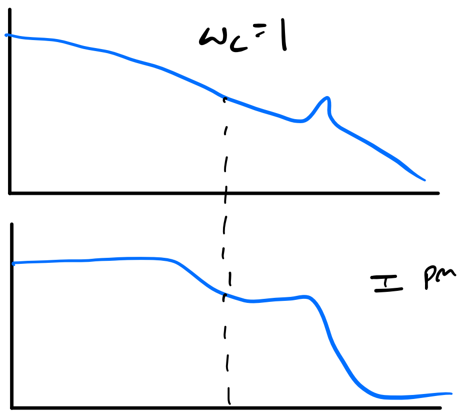
PM 50° GM 10dB > 1 r/s CL bandwidth

want control & feedback on same side for no delay
→ not TSS example

w/ compliance for this sys: $G(s) = \dfrac{10}{s(s+0.1)(s^2+2s+100)}$

Bode

Set ω_c hoping to set desired ω_{BW}



I PM = 5° → Need phase req'd at $\omega_c = 1$

$$G(s) = \frac{10}{s(s+0.1)(s^2+2s+100)}$$

$$\omega_c = 1, \quad \angle G(j\omega_c) = -\tan^{-1}\left(\frac{1}{0}\right) - \tan^{-1}\left(\frac{1}{0.1}\right) - \tan^{-1}\left(\frac{2}{99}\right)$$

↳ $\cong -175.5°$

$$\varphi_{req'd} = PM_{desired} - \left(\angle G(jw)\Big|_{\omega = \omega_{des}} + 180^\circ \right)$$

$PM = 50^\circ \rightarrow$ $\boxed{\text{Need } 45.5^\circ}$

$\omega_{max} = \omega_c = 1$

$$\alpha = \frac{z}{p} = \frac{1 - \sin\varphi_{max}}{1 + \sin\varphi_{max}} \cong 0.168$$

$$\rightarrow \frac{1}{\alpha} \cong 6.0$$

Check Lead ratio
not too large (>30)
6 is fine ✓

$$P_{lead} = \frac{\omega_{max}}{\sqrt{\alpha}} \cong 2.44$$

$$z_{lead} = \omega_{max}\sqrt{\alpha} \cong 0.41$$

$$D_{lead}(s) = K\left(\frac{s+0.41}{s+2.44}\right)$$

Calculate gain:

→ set gain to set magnitude = 1

$$\left| D(j\omega_c)\, G(j\omega_c) \right| = 1$$

$\omega_c = 1$

$$= K \underbrace{\frac{j+0.41}{j+2.44}}_{D} \cdot \underbrace{\frac{10}{j(j+0.1)(j^2+2j+100)}}_{G}$$

$$|DG|_{\omega=\omega_c} = K \frac{\sqrt{1^2+0.41^2}}{\sqrt{1^2+2.44^2}} \left(\frac{10}{\sqrt{1^2}\sqrt{1^2+0.1^2}\sqrt{2^2+99^2}} \right)$$

$$|DG|_{\omega=\omega_c} = 0.041\,K = 1 \implies \boxed{K = \frac{1}{0.041} \cong 24.3}$$

$$\boxed{\rightarrow D(s) = 24.3\left(\frac{s+0.41}{s+2.44}\right)}$$

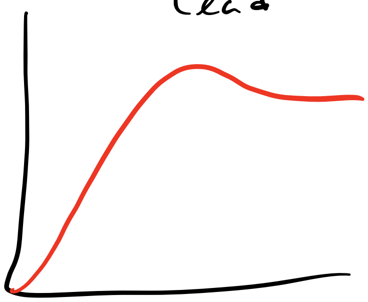
Check PM, GM, ω_{BW}

$PM = 50^\circ$ (exactly as needed)

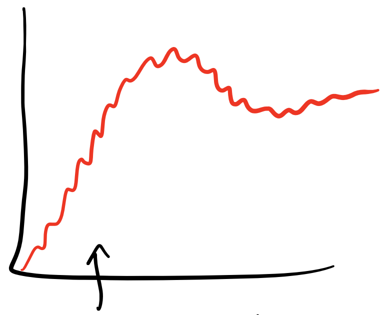
$GM = 21.1\,dB$

$\rightarrow \omega_{BW} \approxeq 1.7 \frac{rad}{s}$

Lead



PD



high freq. gain

no low pass filter