Problem 1: For each of the four flows, answer the following:

(1) compute vorticity field. Is flow irrotational?
(2) Does a stream function exist? If yes, compute. If no, explain
(3) Sketch a few stream lines in Matlab
(4) Briefly describe flow

[A]: $(u, v, w) = (Kx, -Ky, 0)$

A1: $\vec{\xi} = \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\hat{k} = \left(K - (-K)\right)\hat{k} = 0$

$\xi = 0 \rightarrow$ Irrotational flow

A2: Yes, $\psi = Kxy$

A3: Matlab figure 1

A4: Flow along axes from ∞ and $-\infty$ towards origin, diverging along other axis to ∞ / $-\infty$

[B]: $(u_r, u_\theta, u_z) = (0, -\frac{\Gamma}{2\pi r}, 0)$

B1: $\vec{\xi} = \frac{1}{r}\left(\frac{\partial(-\Gamma/2\pi)}{\partial r} - \frac{\partial(0)}{\partial\theta}\right) = 0 \rightarrow$ Irrotational

B2: Yes, $\psi = \frac{\Gamma}{2\pi}\ln(r)$

$u_\theta = -\frac{\partial\psi}{\partial r}$

B3: Matlab figure 2

B4: Uniform circular flow around a point, slowing down as $r\uparrow$

[C]: $(u_r, u_\theta, u_z) = (0, \Omega r, 0)$

C1: $\vec{\xi} = \frac{1}{r}\left(\frac{\partial(\Omega r^2)}{\partial r} - \frac{\partial(0)}{\partial\theta}\right) = \frac{2\Omega r}{r} = 2\Omega\,\hat{u}_z$

C2: Stream function exists for 2D incompressible; check incomp. continuity:

$$\underline{\nabla}\cdot\underline{u} = \frac{1}{r}\frac{\partial(\cancel{0})}{\partial r} + \frac{1}{r}\frac{\partial(\cancel{\Omega r})}{\partial\theta} + \frac{\partial(\cancel{0})}{\partial z} = 0 \checkmark$$

0 0 0

$V_\theta = \Omega r = -\frac{\partial\psi}{\partial r}$

$\psi = -\frac{1}{2}\Omega r^2$

C3: matlab figure 3

C4: uniform circular flow around a point, speeding up as $r\uparrow$

[D]: $(u,v,w) = (cx/(x^2+y^2), cy/(x^2+y^2), 0)$

D1: $\vec{\xi} = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = -\frac{2Cxy}{(x^2+y^2)^2} - \frac{2cxy}{(x^2+y^2)^2} = 0$

$\vec{\xi} = 0 \rightarrow$ IRROTational

D2: Check incomp: $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = \frac{c\cancel{(y^2-x^2)}}{\cancel{(x^2+y^2)^2}} + \frac{c\cancel{(x^2-y^2)}}{\cancel{(x^2+y^2)^2}} = 0$$

$\rightarrow$ stream function exists:

$u = \frac{\partial \psi}{\partial y} \rightarrow \psi = c \cdot \arctan\frac{y}{x}$

D3: matlab figure 4

D4: Source flow out from single point

Problem 2 For each flow:

1) Does a potential flow function exist?
2) Find pot. flow fn. & plot potential lines.

[A]: $(u,v,w) = (Kx, -Ky, 0)$

A1: Potential flow fn valid for irrotational flows.

Check vorticity: $\vec{\xi} = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0 - 0 = 0$

$\therefore\ \varphi$ exists

A2: $u = \frac{\partial \varphi}{\partial x},\ v = \frac{\partial \varphi}{\partial y} \rightarrow \varphi = \frac{Kx^2}{2} - \frac{Ky^2}{2} = \frac{K}{2}(x^2-y^2)$

See matlab figure 5

[B]: $(u_r, u_\theta, u_z) = (0, -\frac{\Gamma}{2\pi r}, 0)$

B1: $\vec{\xi} = 0$ by P1.B.1, so φ exists

B2: $u_r = \frac{\partial \varphi}{\partial r},\ u_\theta = \frac{1}{r}\frac{\partial \varphi}{\partial \theta} \rightarrow \varphi = -\frac{\Gamma}{2\pi}\theta$

See figure 6

[C]: $(u_r, u_\theta, u_z) = (0, \Omega r, 0)$

C1: From P1.C.1, $\vec{\xi} \neq 0$, $\therefore \varphi$ is not defined

C2: N/A.

[D]: $(u, v, w) = (cx/(x^2+y^2), cy/(x^2+y^2), 0)$

D1: By P.1. D.1, $\vec{\xi} = 0$, so φ exists

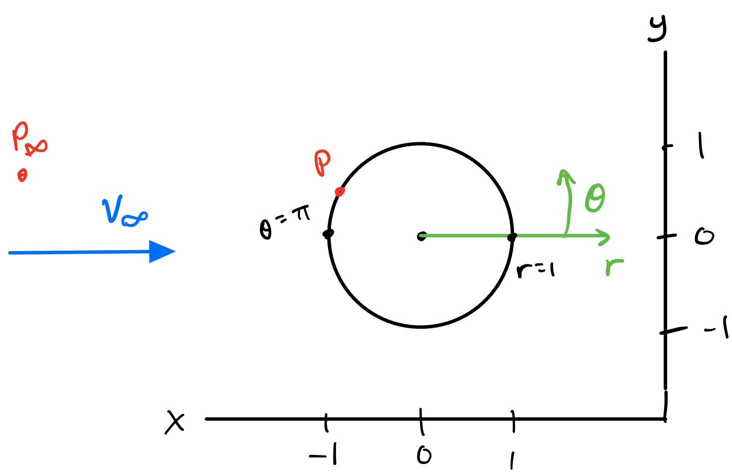
D2: $u = \frac{\partial \varphi}{\partial x}$, $v = \frac{\partial \varphi}{\partial y}$ $\rightarrow \varphi = \frac{c}{2} \ln(x^2+y^2)$

see figure 7

Problem 3 Velocity along surface of cylinder in V_∞ is

$$u_r = 0 \; ; \; u_\theta = -2V_\infty \sin\theta$$

3a) sketch diagram, $R = 1$



3b) use bernoulli to derive pressure coefficient @ surface as function of θ

$$C_p = \frac{P - P_\infty}{0.5 \rho u_\infty^2}$$

neglect

$$P_1 + \cancel{\rho_1 g h_1} + 0.5 \rho v_1^2 = P_2 + \cancel{\rho_2 g h_2} + 0.5 \rho v_2^2$$

let $P_1 = P_\infty$, $P_2 = P$

$$P - P_\infty = \frac{1}{2} \rho V_\infty^2 - \frac{1}{2} \rho V^2$$

along surface, $V = u_\theta = -2V_\infty \sin\theta$

$\rightarrow \; P - P_\infty = \frac{1}{2}\rho V_\infty^2 - \frac{1}{2}\rho(4V_\infty^2 \sin^2\theta)$

$P - P_\infty = \frac{1}{2}\rho U_\infty^2 \left(1 - 4\sin^2\theta\right)$ → plug into C_p

$$\rightarrow C_p = \frac{\cancel{\frac{1}{2}\rho U_\infty^2}(1 - 4\sin^2\theta)}{\cancel{\frac{1}{2}\rho u_\infty^2}}$$

$$\rightarrow \boxed{C_p = 1 - 4\sin^2\theta}$$

3c) Plot C_p as $f(\frac{x}{R})$ on upper & lower surface

$x = r\cos\theta \rightarrow \frac{x}{r} = \cos\theta$

Pressure distribution identical on upper & lower:

→ net lift = 0

3d) Plot C_p on front & back as $f(y/R)$

Pressure distribution identical

→ net drag = 0

3e) Derive an integral expression for C_d & C_ℓ along cylinder as $f(V_\infty, \theta)$

$c = 2R$

$0:\pi \;,\; \pi : 2\pi$

$$C_n = \frac{1}{c}\int_0^c (C_{p\ell} - C_{pu})\,dx = C_\ell$$

$$C_a = \frac{1}{c}\int_{LE}^{TE} (C_{pu} - C_{p\ell})\,dy = C_d$$

$x = R\cos\theta$

$dx = -R\sin\theta\,d\theta$

$y = R\sin\theta$

$dy = R\cos\theta\,d\theta$

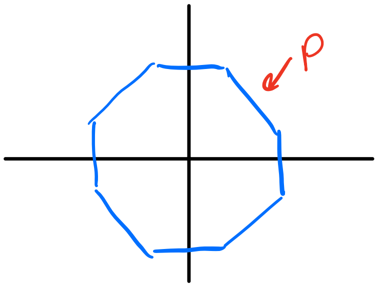
$$\rightarrow C_\ell = \frac{1}{2R}\int_0^{\pi}(1 - 4\sin^2\theta)(-R\sin\theta\,d\theta) - \frac{1}{2R}\int_\pi^{2\pi}(1 - 4\sin^2\theta)(-R\sin\theta\,d\theta)$$

$$\boxed{C_\ell = \frac{1}{2}\left[-\int_0^{\pi}(1 - 4\sin^2\theta)\sin\theta\,d\theta + \int_\pi^{2\pi}(1 - 4\sin^2\theta)\sin\theta\,d\theta\right] = 0}$$

$$C_d = \frac{1}{2R}\int_0^{\pi}(1 - 4\sin^2\theta)R\cos\theta\,d\theta - \int_\pi^{2\pi}(1 - 4\sin^2\theta)R\cos\theta\,d\theta$$

$$\boxed{C_d = \frac{1}{2}\left[\int_0^{\pi}(1 - 4\sin^2\theta)\cos\theta\,d\theta - \int_\pi^{2\pi}(1 - 4\sin^2\theta)\cos\theta\,d\theta\right] = 0}$$

Problem 4



$C_d =$