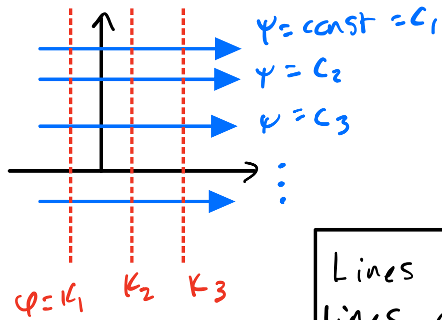
Lecture 14: Uniform flow

Simple potential flow solutions

1. Uniform flow in x-direction

(U_∞ is velocity of flow)



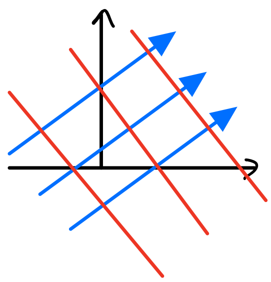
$$\psi(x,y) = U_\infty y$$

$$u = U_\infty, \quad v = 0$$

$$\varphi(x,y) = U_\infty x$$

$\boxed{\text{Lines of constant } \varphi \text{ are orthogonal to lines of constant } \psi \text{ (streamlines)}}$

1a) uniform flow



$\psi = Ay - Bx$, where $A, B = const$

$\varphi = Ax + By$ $\quad$ * compute vorticity

$$\nabla \times \vec{V} = ?$$

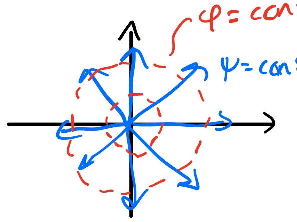
show vorticity is zero!

Flow irrotational

Circulation, $\Gamma = 0$

Lecture 15: Source & Sink

2. Flow in radial direction: source or sink



$V_\theta = 0$; $V_r = ?$

* A 2D line source $(+V_r)$ or sink $(-V_r)$
* Consider an injection of mass @ origin ; $\dot{m}$ = mass flow rate

Let's connect $\dot{m}$ to $\vec{V}$:

continuity $\nabla \cdot \vec{V} = 0$, expand

$$\frac{1}{r}\underbrace{\frac{\partial}{\partial r}(r V_r)}_{=0} + \frac{1}{r}\cancel{\frac{\partial}{\partial \theta}}(V_\theta)^{\to 0} = 0$$

$= 0$, thus $r V_r = \text{constant}$

$\rightarrow V_r = \frac{C_1}{r}$

$$\dot{m} = \ell \int_0^{2\pi} \rho V_r \cdot r\, d\theta = 2\pi r \rho V_r \ell$$

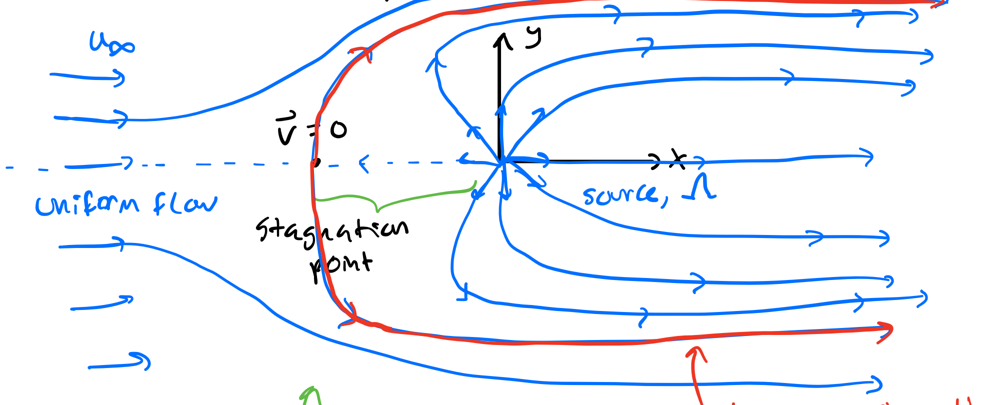
length into board

$\Lambda = \frac{\dot{m}}{\rho \ell}$ = volume flow rate $= 2\pi r V_r$

$$\boxed{V_r = \frac{\Lambda}{2\pi r}}$$

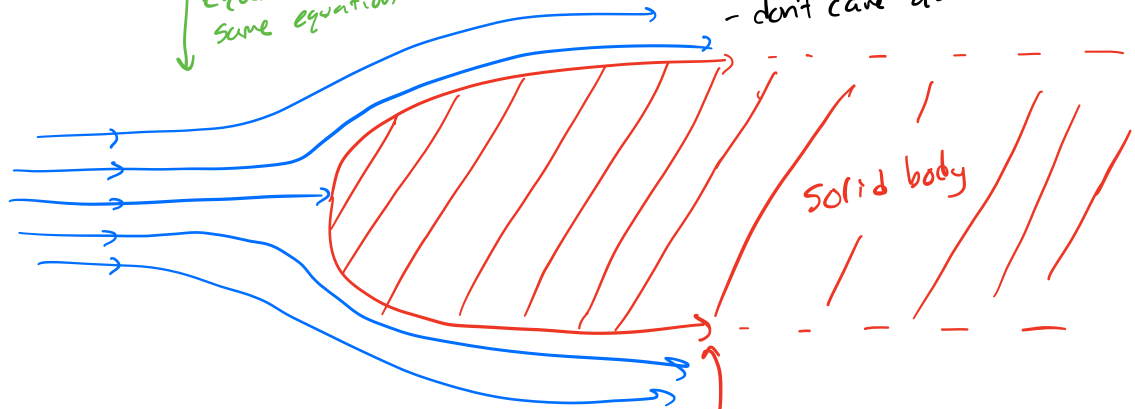
*
$$\boxed{\begin{aligned} \varphi &= \frac{\Lambda}{2\pi} \ln r \\ \psi &= \frac{\Lambda}{2\pi} \theta \end{aligned}}$$

Lecture 16: super positioning:

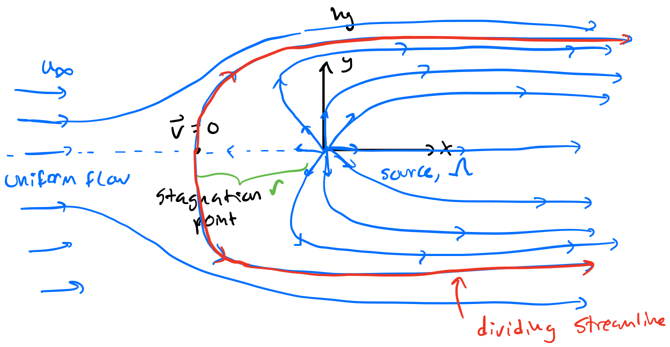


equivalent
same equation

- don't care about flow inside



dividing streamline determines shape of body



$$\varphi = U_\infty x + \frac{\Lambda}{2\pi} \ln r$$

$$\psi = U_\infty y + \frac{\Lambda}{2\pi} \theta$$

$$x = r\cos\theta$$

$$y = r\sin\theta$$

$$\varphi = U_\infty r\cos\theta + \frac{\Lambda}{2\pi} \ln r$$

$$\psi = U_\infty r\sin\theta + \frac{\Lambda}{2\pi} \varphi$$

Goal: Determine eqn. for dividing streamline

$$U_\infty r\sin\theta + \frac{\Lambda}{2\pi}\theta = \text{const}$$

→ Determine V_r & V_θ: $V_r = U_\infty \cos\theta + \frac{\Lambda}{2\pi r}$; $V_\theta = -U_\infty \sin\theta$

→ want to find stagnation point, it will occur on $-x$ axis

→ $V_r, V_\theta = 0$

$$U_\infty \cos\theta + \frac{\Lambda}{2\pi r} = 0 \quad \text{and} \quad U_\infty \sin\theta = 0$$

$\hookrightarrow \theta = -\pi, 0, \pi \ldots$

sub $\theta = \pi$ & solve for r

$$\rightarrow r = \frac{\Lambda}{2\pi U_\infty}$$

$$\Psi = U_\infty \left(\frac{r}{2\pi U_\infty}\right) \sin\overset{=\pi}{\theta} + \frac{\Lambda}{2\pi}\theta = \frac{\Lambda}{2} = \text{constant}$$

$$\rightarrow U_\infty r \sin\theta + \frac{\Lambda}{2\pi}\theta = \text{const} = \frac{\Lambda}{2}$$