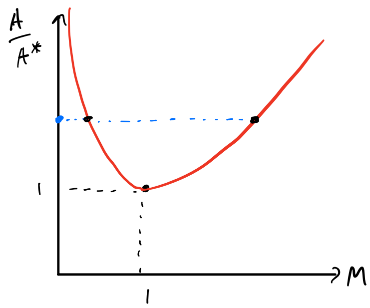
Recap

$$\frac{T}{T_0} = \left(1 + \frac{\gamma-1}{2}M^2\right)^{-1}$$

$$\frac{P}{P_0} = \left(1 + \frac{\gamma-1}{2}M^2\right)^{\frac{\gamma}{1-\gamma}}$$

$$\frac{\rho}{\rho_0} = \left(1 + \frac{\gamma-1}{2}M^2\right)^{\frac{1}{1-\gamma}}$$

$$\frac{A}{A^*} = \frac{1}{M}\left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M^2\right)\right]^{\frac{\gamma+1}{2(\gamma-1)}}$$



If $M=1$ occurs, it can only happen at an area minimum

in that case, $A_{min} = A_t = A^*$

Existence of A_{min} does not guarantee $M=1$ is ever reached

For each value of A/A^*, 2 possible values of M

iii) values of T/T_0, P/P_0, ρ/ρ_0, A/A^*

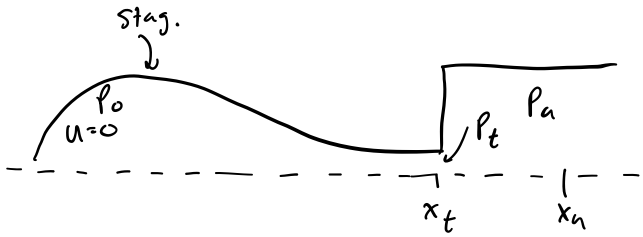
- ✓ online calc.
- ✓ tabulated
- ✓ plotted → From plots:
  - changes of T, ρ, P very small for $M<1$
  - throughout M range, T, ρ, P decrease monotonically

useful to set aside sonic value of P

$$\frac{P^*}{P} = \left(1 + \frac{\gamma-1}{2}\underset{\uparrow\, 1}{M^2}\right)^{\frac{\gamma}{1-\gamma}} = \left(1 + \frac{\gamma-1}{2}\right)^{\frac{\gamma}{1-\gamma}}$$

For $\gamma = 1.4$, $\frac{P^*}{P_0} = 0.5283$

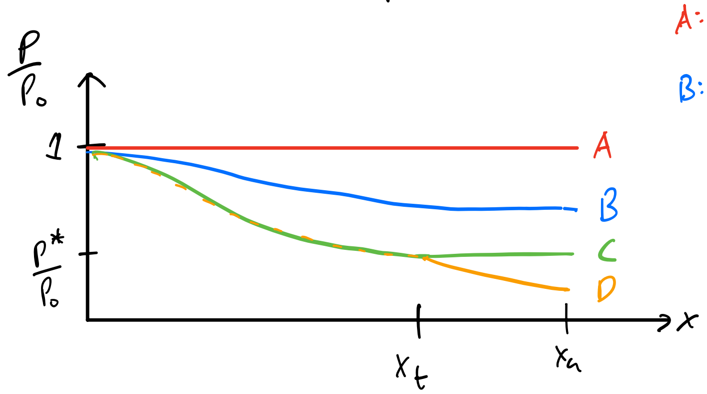
4.3 Converging nozzle



P_a = ambient pressure
generally $P_a \neq 1$ atm

Keeping P_0 fixed, start w/ $P_a = P_0$ and progressively decrease P_a

Flow governed by
$$\begin{cases} A/A^* = \dfrac{1}{M}\left[\dfrac{2}{\gamma+1}\left(1+\dfrac{\gamma-1}{2}M^2\right)\right]^{\frac{\gamma+1}{2(\gamma-1)}} \\ \dfrac{P}{P_0} = \left(1+\dfrac{\gamma-1}{2}M^2\right)^{\frac{\gamma}{1-\gamma}} \end{cases}$$



A: $P_a = P_0$; no flow

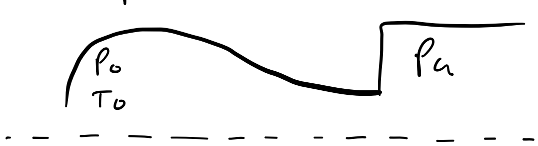
B: $P^* < P_a < P_0$ subsonic flow
$P_t = P_a$
$M(x=x_t) < 1$
As $P_a \downarrow$, $\dot{m} \uparrow$ subs. throughout

C: $P_a = P^*$ sonic at throat
$P_t = P^* = P_a$

D: $P_a < P^*$ subsonic throughout
sonic @ throat
pressure decrease after nozzle

Since between stagnation & throat P-distribution for cases for C & D is exactly the same, $\dot{m}$ reaches its maximum value in case C (sonic throat). Lowering back pressure below P^* does <u>not</u> increase $\dot{m}$: the flow is <u>"choked"</u>

<u>Example</u>



$$G \begin{cases} \gamma = 1.4 \\ P_0 = 406 \text{ kPa} \\ T_0 = 2368 \text{ K} \\ A_t = 0.01 \text{ m}^2 \\ P_a = 240 \text{ kPa} \end{cases} \qquad F \{ \dot{m}$$

→ Need P_t, u_t for $\dot{m}$

1) verify if $M_t = 1$ or < 1

if $P_a \leq P^* \Rightarrow M_t = 1$

if $P_a > P^* \Rightarrow M_t < 1$

$$\frac{P^*}{P_0} = \left(1 - \frac{\gamma-1}{2}\right)^{\frac{\gamma}{1-\gamma}} = 0.5283 \rightarrow P^* = 0.5283\, P_0 = 214.5 \text{ kPa}$$

$P_a > P^* \rightarrow M_t < 1$

→ this is case B ($P_t = P_a$)

$$\frac{P_t}{P_0} = \frac{P_a}{P_0} = \left(1+\frac{\gamma-1}{2}M_t^2\right)^{\frac{\gamma}{1-\gamma}}$$

$$M_t = \left[\frac{2}{\gamma-1}\left[\left(\frac{P_a}{P_0}\right)^{\frac{1-\gamma}{\gamma}} - 1\right]\right]^{1/2} = 0.9$$

$$\dot m = \rho_t u_t A_t$$

$$u_t = M_t a_t$$

$$a_t = \sqrt{\gamma R T_t}$$

$$\frac{T_t}{T_0} = \left(1+\frac{\gamma-1}{2}M^2\right)^{-1} \Rightarrow T_t = 317\ K$$

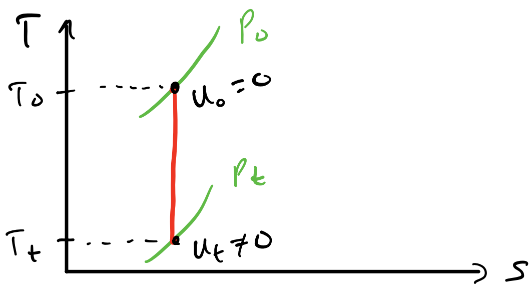
$\rightarrow a_t = \sqrt{1.4 \cdot 287 \cdot 317} = 358\ m/s$

$u_t = 0.9 \cdot 358 = 322\ m/s$

ideal gas

$$\boxed{\rho_t = \frac{P_t}{RT_t}} = 2.64\ \frac{kg}{m^3}$$

$\rightarrow \dot m = 8.5\ kg/s$



For fixed P_0, ρ_0, T_0

once $M(x_t) = 1$, $\dot m$ can no longer increase

Recall $J = \underbrace{\dot m u_e}_{\text{usually dominant}} + (P_e - P_a)A_e$

→ can increase thrust by increasing u_e

→ use divergent bell following throat to increase exit velocity