Problem 1

1a) $\frac{P_2}{P_1}$ vs M_1 (left) $\frac{v_2}{v_1}$ vs. M_1 (right)

$$\frac{P_2}{P_1} = 1 + \frac{2\gamma}{\gamma+1}(M_1^2 - 1)$$

$$\frac{\rho_2}{\rho_1} = \frac{(\gamma+1)M_1^2}{2+(\gamma-1)M_1^2} \qquad \frac{\frac{1}{\rho_2}}{\frac{1}{\rho_1}} = \frac{v_2}{v_1} = \frac{2+(\gamma-1)M_1^2}{(\gamma+1)M_1^2}$$

-> See matlab figure

1b) Compressibility ($\sim \frac{v_2}{v_1}$) increases with increasing γ.

1c) Similarly, pressure ratio also increases with increasing γ

1d) As M_1 increases, the effects of increasing γ become more pronounced.

1e) Find $\frac{\Delta S}{R} = \frac{\Delta S}{R}(M_1, \gamma)$

$$\Delta S = S_2 - S_1 = c_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1} \to \frac{\gamma R}{\gamma-1} \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}$$

$$\frac{\Delta S}{R} = \frac{\gamma}{\gamma-1} \ln \frac{T_2}{T_1} - \ln \frac{P_2}{P_1}$$

$$\boxed{\frac{\Delta S}{R} = \frac{\gamma}{\gamma-1} \ln \frac{[2\gamma M_1^2 - (\gamma-1)]\cdot[(\gamma-1)M_1^2 + 2]}{(\gamma+1)^2 M_1^2} - \ln\left[1 + \frac{2\gamma}{\gamma+1}(M_1^2 - 1)\right]}$$

1f) Plot

$$\frac{P_{02}}{P_{01}} = \left[\frac{(\gamma+1)M_1^2}{2+(\gamma-1)M_1^2}\right]^{\frac{\gamma}{\gamma-1}} \cdot \left[\frac{\gamma+1}{2\gamma M_1^2 - (\gamma-1)}\right]^{\frac{1}{\gamma-1}}$$

1g) As γ increases:

- $\frac{\Delta S}{R}$ decreases for a given Mach number.
- stag. pressure ratio increases for a given Mach.

1h) For $M<1$, entropy change & pressure ratio are 0 and 1 respectively, therefore a shock does not exist.

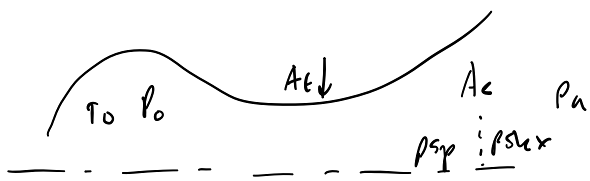
For $M>1$, $\frac{\Delta S}{R} > 0$ indicating the possibility of a shock.

1i) plot

1j) γ does not affect $\frac{\Delta S}{R}$ / $\frac{P_{02}}{P_{01}}$.

Increasing gamma increases M_1 / $\frac{P_{02}}{P_{01}}$ however.

Problem 2



$T_0 = 500K$

$A_t = 0.0001\ m^2$

$A_e = 0.00027\ m^2$

$P_0 = 279.4\ kPa$

$P_a = 240.1\ kPa$

$\gamma_{air} = 1.4$

2a) Normal shock in nozzle?

$$\frac{A_e}{A_t} = 2.7 \rightarrow \frac{P_e}{P_0} = 0.056 \text{ (sup)} \rightarrow P_{sup} = 15.64\ kPa$$

$\hookrightarrow M = 2.53$

$$\rightarrow \frac{P_e}{P_0} = 0.966 \text{ (sub)}, \rightarrow P_{sub} = 268.9\ kPa$$

$$\frac{P_{shk}}{P_{sup}} = 1 + \frac{2\gamma}{\gamma+1}(M_{sup}^2 - 1) = 7.26$$

$$\rightarrow P_{shk} = 7.26\ P_{sup} = (7.26)(15.64) = 113.59\ kPa$$

$P_{shk} < P_a < P_{sub} \rightarrow$ shock in nozzle

$$M_e^2 = -\frac{1}{\gamma-1} + \sqrt{\frac{1}{(\gamma-1)^2} + \frac{2}{(\gamma-1)}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}\left(\frac{P_{01}A_1^*}{P_e A_e}\right)^2}$$

$P_e = P_a$, known

$P_{01} = P_0$

A_e, $A^* = A_t$ known

$\rightarrow M_e = 0.248 \rightarrow \dfrac{P_e}{P_{02}} = 0.958$ (online calc)

$\rightarrow P_{02} = \dfrac{P_e}{0.958} = 250.6 \text{ kPa}$

$\rightarrow \dfrac{P_{02}}{P_{01}} = \dfrac{250.6 \text{ kPa}}{279.4 \text{ kPa}} \rightarrow M_1 = 1.595$

$\boxed{M_2 = 0.67}$

$\dfrac{A_1}{A_1^*} = f(M_1) \rightarrow 1.246$

$\rightarrow A_1 = 1.246 A_t = \boxed{1.246 \text{ cm}^2}$

2b) $u_e = M_e a_e = M_e \sqrt{\gamma R T_e}$

$T_{01} = T_{02} = T_{0e}$

$\dfrac{T_e}{T_0} = f(M_e) = 0.988 \rightarrow T_e = 494 \text{ K}$

$u_e = (0.248)\sqrt{(1.4)(287)(494)} = \boxed{110.5 \text{ m/s} = u_e}$

2c) $\dot{m} = \rho_e u_e A_e \qquad \rho_e = \dfrac{P_e}{R T_e}$

$\rightarrow \dot{m} = \dfrac{240\,100}{287 \cdot 494} \cdot 110.5 \cdot 0.00027 = \boxed{0.051 \text{ kg/s} = \dot{m}}$

2d) $J = \dot{m} u_e + \overset{0}{\cancel{(P_e - P_a)}} A_e = \boxed{5.58 \text{ N} = J}$

2e) P_0 s.t. no shock, same P_e

$\rightarrow$ Need $P_a = P_{sup}$

$\dfrac{P_e}{P_0} = 0.056 \rightarrow P_0 = \dfrac{240.1}{0.056} = \boxed{4287 \text{ kPa} = P_0}$

2f)

$$J = \dot{m} u_e + (\cancel{p_e - p_a}) A_e \quad 0, \; p_e = p_a$$

$$= \left(\frac{p_e}{RT_e} \cdot M_e \sqrt{\gamma R T_e}\, A_e \right) \cdot \left(M_e \sqrt{\gamma R T_e} \right)$$

$$J = \frac{p_e}{\cancel{RT_e}} M_e^2 A_e \cdot \gamma \cancel{RT_e} = \boxed{p_e M_e^2 A_e \gamma = J}$$

New M_e^2:

$$M_e^2 = -\frac{1}{\gamma-1} + \sqrt{\frac{1}{(\gamma-1)^2} + \frac{2}{(\gamma-1)} \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}} \left(\frac{p_{01} A^*_1}{p_e A_e} \right)^2} = 6.42$$

$$\rightarrow J = (240100)(6.42)(0.00027)(1.4) = \boxed{582.7 \text{ N} = J}$$