Previously:

$$\rightarrow \gamma(\theta) = 2\alpha V_\infty \frac{(1+\cos\theta)}{\sin\theta} \qquad \text{(solution)}$$

Kutta – Joukowski: $L = \rho V_\infty \Gamma$

Net circulation: $\Gamma = \int_0^c \gamma(\xi)\,d\xi = \frac{c}{2}\int_0^\pi \gamma(\theta)\sin\theta\,d\theta$

$$\Gamma = \frac{c}{\cancel{2}}\int_0^\pi \cancel{2}\alpha V_\infty \frac{(1+\cos\theta)}{\cancel{\sin\theta}}\cancel{\sin\theta}\,d\theta = \alpha c V_\infty \underbrace{\int_0^\pi (1+\cos\theta)\,d\theta}_{\pi}$$

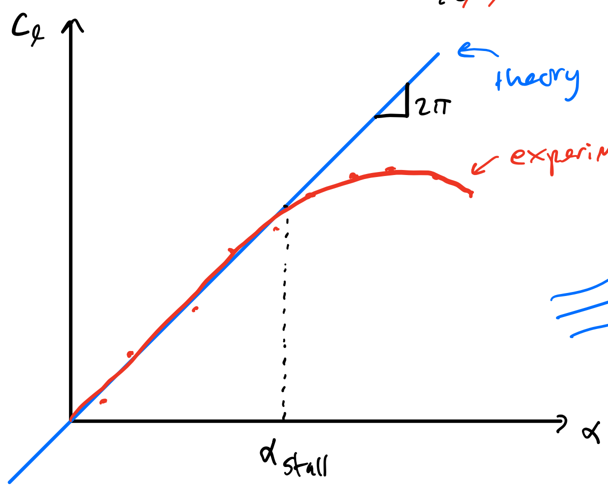
$$\boxed{\Gamma = \pi\alpha c V_\infty} \qquad \boxed{L = \pi\alpha c \rho V_\infty^2}$$

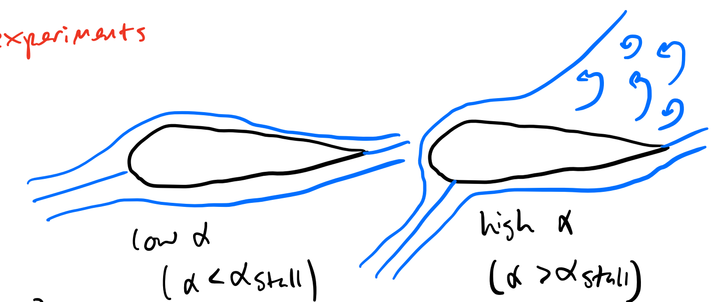
$C_\ell = \frac{L}{\frac{1}{2}\rho V_\infty^2 c}$ $\qquad q_\infty$ = dyn. pressure $= \frac{1}{2}\rho V_\infty^2$

$$\rightarrow C_\ell = \frac{\pi\alpha \cancel{c}\cancel{\rho}\cancel{V_\infty^2}}{\frac{1}{2}\cancel{\rho}\cancel{V_\infty^2}\cancel{c}} \Rightarrow \boxed{C_\ell = 2\pi\alpha}$$

* very famous result of T.A.T.

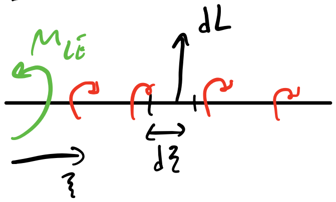
radians!





T.A.T. holds well for thin symmetric airfoils ("thin", $t/c < 0.1$) at low α ($\alpha \sim < 10°$)

Moments



$$M_{LE} = -\int_0^c \xi \cdot dL = -\int_0^c \xi \cdot \underbrace{\rho V_\infty \gamma(\xi)\,d\xi}_{dL}$$

Next: change variables, substitute sol'n in terms of $\gamma(\theta)$, solve for moment

$\rightarrow \quad M_{LE} = -\frac{1}{4}\rho V_\infty^2 c^2 \pi \alpha \Rightarrow C_{M_{LE}} = \frac{M_{LE}}{\frac{1}{2}\rho V_\infty^2 c^2} = \frac{M_{LE}}{q_\infty c^2}$

$\rightarrow \boxed{C_{M_{LE}} = -\frac{1}{2}\pi\alpha}$

$C_{M_{LE}} = -\frac{1}{4}C_\ell + C_{m,c/4}$

Can show $\Rightarrow \boxed{C_{m,c/4} = 0}$

center of pressure is at $c/4$

Symmetric foils

Cambered foil (general solution)

Eqn to solve for γ: $\frac{1}{2\pi}\int_0^c \frac{\gamma(\xi)d\xi}{x-\xi} = V_\infty\left(\alpha - \frac{dz}{dx}\right)$

slope of camberline

change of variables: $\frac{1}{2\pi}\int_0^\pi \frac{\gamma(\theta)\sin\theta\, d\theta}{\cos\theta - \cos\theta_0} = V_\infty\left(\alpha - \frac{dz}{dx}\right)$

sol'n strategy: seek a correction term that accounts for camber (dz/dx term), in form of fourier series

$\gamma(\theta) = 2\alpha V_\infty\left(\frac{1+\cos\theta}{\sin\theta}\right)$ uncambered solution

$\gamma(\theta) = 2V_\infty\left[A_0\frac{1+\cos\theta}{\sin\theta} + \sum_{n=1}^{\infty} A_n \cdot \sin(n\theta)\right]$ cambered solution

need A_0, A_n!

$A_0 = f(\alpha, dz/dx)$, $A_n = f(dz/dx)$

$\rightarrow$ Substitute proposed solution back into governing eqn:

$$\frac{1}{\pi}\underbrace{\int_0^\pi \frac{A_0(1+\cos\theta)\,d\theta}{\cos\theta - \cos\theta_0}}_{=\pi} + \frac{1}{\pi}\sum_{n=1}^{\infty}\underbrace{\int_0^\pi \frac{A_n \sin(n\theta)\sin\theta\, d\theta}{\cos\theta - \cos\theta_0}}_{=-\pi\cos(n\theta_0)} = \alpha - \frac{dz}{dx}$$

$$\frac{1}{\cancel{\pi}}A_0\cancel{\pi} + \frac{1}{\cancel{\pi}}\sum_{n=1}^{\infty}A_n\left(-\cancel{\pi}\cos n\theta_0\right) = \alpha - dz/dx$$

$A_0 - \sum_{n=1}^{\infty} A_n \cos n\theta_0 = \alpha - dz/dx \Rightarrow$ rearrange terms

* $x = c/2(1-\cos\theta_0)$

$$f(\theta_0) = \frac{dz}{dx} = (\alpha - A_0) + \sum_{n=1}^{\infty} A_n \cos n\theta_0$$

Recall fourier cosine expansion

$$f(\theta) = B_0 + \sum_{n=1}^{\infty} B_n \cos n\theta \qquad \text{with } B_0 = \frac{1}{\pi}\int_0^{\pi} f(\theta)\, d\theta$$

$$B_n = \frac{2}{\pi}\int_0^{\pi} f(\theta)\cos n\theta\, d\theta$$

can match terms to reveal expressions for A_0 & A_n

$$\rightarrow \quad \alpha - A_0 = \frac{1}{\pi}\int_0^{\pi} \frac{dz}{dx}\, d\theta_0$$

$$A_n = \frac{2}{\pi}\int_0^{\pi} \frac{dz}{dx}\cos n\theta_0\, d\theta_0$$

1) Need geometry dz/dx
2) perform integrals, get A_0, A_n
3) use A_0 & A_n to get γ

$$\gamma(\theta) = 2V_\infty\left[A_0 \frac{1+\cos\theta}{\sin\theta} + \sum_{n=1}^{\infty} A_n \sin(n\theta)\right]$$

Next, compute total $\Gamma = \int_0^c \gamma(\xi)\, d\xi = \frac{c}{2}\int_0^{\pi} \gamma(\theta)\sin\theta\, d\theta$

$$\Gamma = cV_\infty\left[A_0 \underbrace{\int_0^{\pi} 1+\cos\theta\, d\theta}_{=\pi} + \sum_{n=1}^{\infty} A_n \underbrace{\int_0^{\pi} \sin(n\theta)\sin\theta\, d\theta}_{= \begin{cases} \pi/2 & \text{if } n=1 \\ 0 & \text{if } n\neq 1\end{cases}}\right]$$

$$\boxed{\Gamma = cV_\infty \pi \left(A_0 + \frac{A_1}{2}\right)} \rightarrow L = \rho V_\infty \Gamma$$

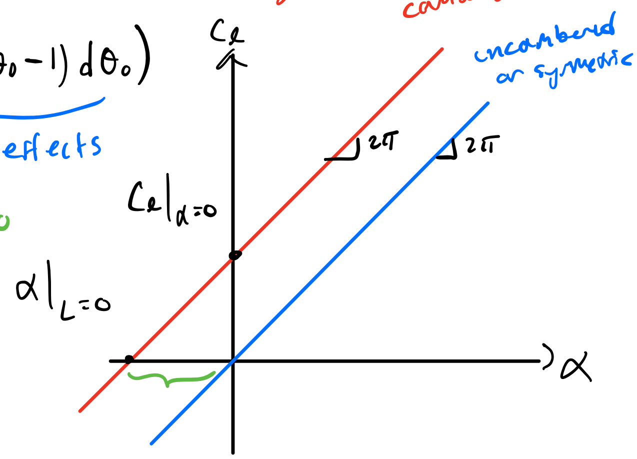
$$\boxed{C_\ell = \pi(2A_0 + A_1)}$$

$$\underbrace{\begin{aligned} A_0 &= \alpha - \frac{1}{\pi}\int_0^{\pi} \frac{dz}{dx}\, d\theta_0 \\ A_1 &= \frac{2}{\pi}\int_0^{\pi} dz/dx \cos\theta_0\, d\theta_0 \end{aligned}}_{\text{combine to 1 integral}} \Bigg\} \text{ same as earlier}$$

cambered sol'n

$$\rightarrow C_\ell = 2\pi\Big(\underbrace{\alpha}_{\text{uncambered sol'n}} + \underbrace{\frac{1}{\pi}\int_0^{\pi} \frac{dz}{dx}(\cos\theta_0 - 1)\, d\theta_0}_{\text{camber effects}}\Big)$$

$$\rightarrow C_\ell = 2\pi(\alpha - \alpha|_{L=0}) \qquad = -\alpha|_{L=0}$$



can compute $C_{M,LE}$; $C_{M,c/4}$

$$C_{M,LE} = -\left(\frac{C_\ell}{4} + \frac{\pi}{4}(A_1 - A_2)\right)$$

$$C_{M,c/4} = \frac{\pi}{4}(A_2 - A_1)$$

↑

Independent of α ; Still the aerodynamic center!

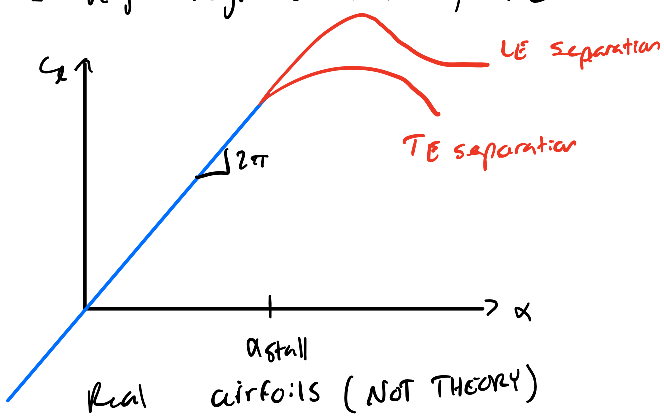
Discussion of thin airfoil theory

Assumptions

- small angles → foil must be "thin" (small changes in camber)
  → α must be small

$t/c < 0.12$; $\alpha < 12^\circ$

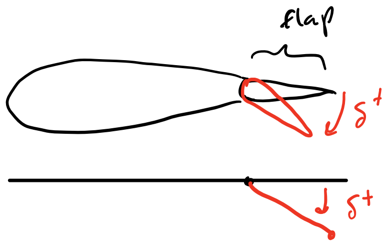
- inviscid flow → at high α, flow separation occurs, highly viscous!
- incompressible → when $M < 0.3$, good assumption
- high reynolds number, $Re \sim 10^6$

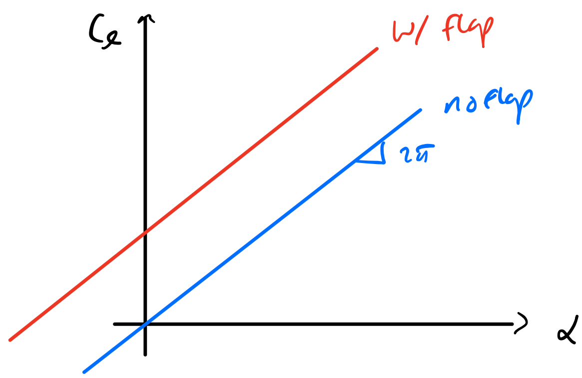


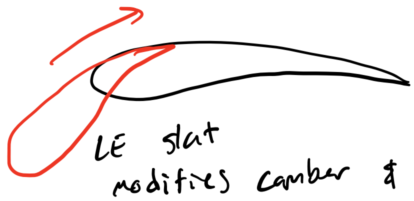
Real airfoils (NOT THEORY)

- have flow separation at high α
- Re #, surface roughness, turbulence strongly influence α_{stall} & separation dynamics

High-lift devices







LE slat
modifies camber & effective chord