Same example as lec 34 but PID instead

lead lag vs. PID ? filter?

Assumptions
- use PD to add phase
- Limit phase added to 75°
- Design for PM = 55° to compensate for PI loss
- $\omega_c = 40$ r/s

$$\text{PID}: \quad K_p + \frac{K_I}{s} + K_d s = \frac{K_d s^2 + K_p s + K_I}{s}$$

$$\text{PD}: \quad K_{PD}(s + z_{PD}) \quad \begin{cases} K_{PD} = K_d \\ z_{PD} = \frac{K_{P_1}}{K_d} \end{cases}$$

$$\text{PI}: \quad K_{PI}\frac{(s + z_{PI})}{s} \quad \begin{cases} K_{PI} = K_{P_2} \\ z_{PI} = \frac{K_I}{K_p} \end{cases}$$

$$\text{PD} \cdot \text{PI}: \quad K(s + z_{PD})\frac{(s + z_{PI})}{s} \quad \Big\} \quad K = K_{PD} K_{PI}$$

Relate

→ design PD for system, then add PI w/ no gain ($\omega_c \gg z_{PI}$ s.t. $M = 1$)

→ Note that w/ lead lag, gain changes error, but w/ PID with true integration, a const. error → zero

$$\text{PID}: \quad K_p + \frac{K_I}{s} + K_d s = \frac{K_d s^2 + K_p s + K_I}{s}$$

- (PD)(PI) is equivalent to PID

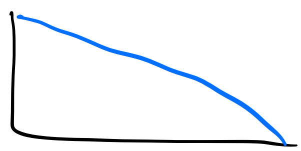
$$\rightarrow \begin{cases} K_p = K(z_{PD} + z_{PI}) \\ K_d = K \\ K_I = K z_{PD} z_{PI} \end{cases}$$

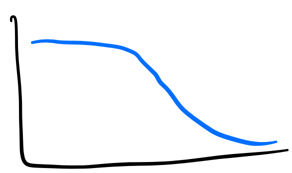
1) Design PI & PD
2) convert to PID

Example

$$G(s) = \frac{200}{s^2(s+10)}$$

- PM $\cong$ 50°, GM > 15 dB
- ω_{BW} as high as possible
- $e_{ss} < 0.001$ when $r(t)$ is:
  - $1(t)$
  - t
  - $\frac{1}{2}t^2$





$$G(s) = \frac{200}{s^2(s+110)}$$

Assume: PD can get 75° of phase

PM: $50^\circ \to$ design for 55°

55° PM, add $75^\circ \to -20^\circ \; (-180^\circ)$

$\hookrightarrow \angle G(s)\big|_{\omega_c} = -200$

$\omega_c = 40$ rad/s

Can you solve this exactly in MATLAB?

Need 75° @ 40 r/s

$$\underset{(\omega_c = 40)}{G(j\omega_c)} = \frac{200}{(-1600)(110+40j)}$$

$\to$ use atan2(y,x)

$$\angle G(j\omega_c) = -\operatorname{atan}\left(\frac{0}{-1600}\right) - \operatorname{atan}\left(\frac{40}{100}\right) = -199.98^\circ \quad (\sim 200^\circ)$$

Req'd phase (exact)

$$\varphi_{des} = -180^\circ + 55^\circ = -125^\circ$$

$$\varphi_{PD} = \varphi_{des} - \angle G$$

$$= -125 - (-199.98^\circ) = 74.98^\circ$$

$$\to \varphi_{PD} \cong 75^\circ$$

at $\omega_c = 40^d$

PD: $K(s + z_{pd})$ $\leftarrow$ design PD to get φ_{PD}

$$D_{PD}(j\omega_c) = K(j\omega_c + z_{pd})$$

$$\angle D_{PD}(j\omega_c) = \operatorname{atan}\left(\frac{\omega_c}{z_{pd}}\right) = 75^\circ$$

$$\to z_{pd} = \frac{\omega_c}{\tan 75^\circ} \cong 10.72$$

$$D_{PD}(s)\, G(s) = \underline{K}\,(s+10.72)\,\frac{200}{s^2(s+110)}$$

Need K s.t. $|M| = 1$ @ ω_c

$$\left| D_{PD} G \right|_{\omega=40} = \left| K(40j + 10.72) \cdot \frac{200}{(40j)^2(40j+110)} \right|$$

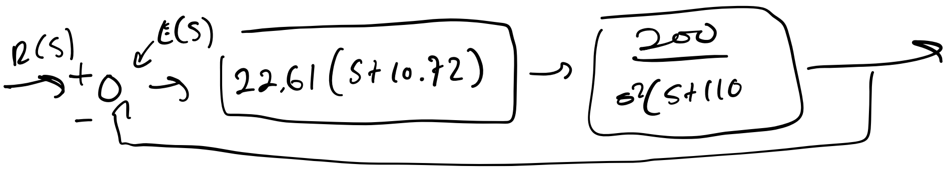
$$= K\sqrt{40^2 + 10.72^2} \cdot \frac{200}{\sqrt{40^2+110^2}\,(1600)}$$

$$1 = K \cdot 0.0442 \to K \cong 22.61$$

$$\rightarrow D_{PD}(s) = \underbrace{22.61}_{K_{PD}}\,(s + \underbrace{10.72}_{z_{PD}})$$

NOW PI

$e_{ss} \le 0.001$, $r(t) = 1(t), t, \frac{1}{2}t^2$



$$\frac{E(s)}{R(s)} = \frac{1}{1 + 22.61(s+10.72) \cdot \frac{200}{s^2(s+110)}}$$

$$\frac{E(s)}{R(s)} = \frac{s^3 + 110s^2}{s^3 + 110s^2 + 4522s + 4.847\times10^4} \qquad (= s^2(s+110))$$

$$e_{ss} = \lim_{s\to0}\left[s \cdot \frac{E(s)}{R(s)}\, R(s)\right]$$

$r(t)$	$R(s)$	
$1(t)$	$1/s$	$\rightarrow e_{ss} = 0$
t	$1/s^2$	$\rightarrow e_{ss} = 0$
$\frac{1}{2}t^2$	$1/s^3$	$\rightarrow e_{ss} = \frac{110}{4.847\times10^4} \approx 0.0023 > 0.001$

integral control turns all const. e_{ss} to $\emptyset$ (if lag, $\alpha > 2.3$)

$$D_{PI}(s) = \frac{s + z_{PI}}{s}$$

adds slow pole, slower than lead bc @ 0

Lose $5°$ from PI

Guess 10x lower $z_{PI} = \frac{\omega_c}{10} = 4 \rightarrow D_{PI} = \frac{s+4}{s}$

OR find exactly:

$$D_{PI}(j\omega_c) = \frac{j\omega_c + z_{PI}}{j\omega_c}, \quad \angle D_{PI}(j\omega_c) = \operatorname{atan}\left(\frac{\omega_c}{z_{PI}}\right) - \underbrace{\operatorname{atan}\left(\frac{\omega_c}{0}\right)}_{-90°} = -5°$$

$$\rightarrow z_{PI} = \frac{40}{\tan 85°} \cong 3.5$$

(4 would have lost a bit of phase)

$$\rightarrow D_{PI} = \frac{s+3.5}{s} \qquad D_{PD} = 22.61(s+10.72)$$

$$D(s) = \frac{22.61(s+10.72)(s+3.5)}{s}$$

Convert to PID:

$$K_p = K(z_{PD} + z_{PI}) \approx 321.5$$

$$K_d = K = 22.61$$

$$K_I = K z_{PD} z_{PI} \approx 848.3$$

$$\boxed{\rightarrow D(s) = 321.2 + \frac{848.3}{s} + 22.61 s}$$