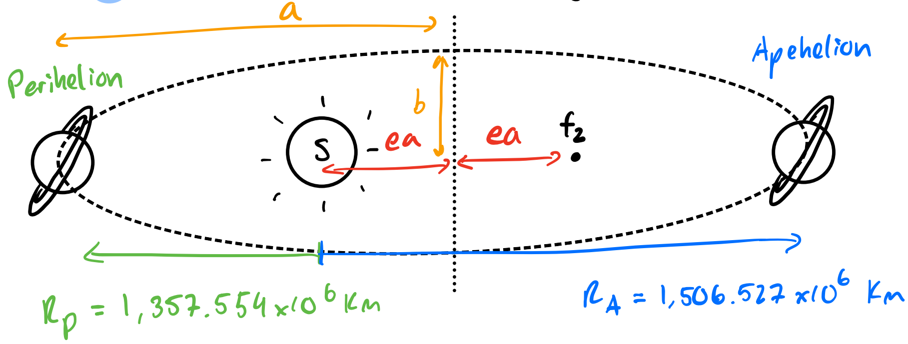
EMA 550 HW1 Kyle Adler

Problem 1 Saturn perihelion = 1,357.554 $\times 10^6$ Km, aphelion = 1,506.527 $\times 10^6$ Km

1a) Calculate eccentricity & semi-major axis & compare.



$$R_A = a(1+e) \quad , \quad R_p = a(1-e)$$

$$\frac{R_A}{R_p} = \frac{\cancel{a}(1+e)}{\cancel{a}(1-e)} \rightarrow \left(\frac{R_A}{R_p}\right)(1-e) = 1+e$$

$$\rightarrow \frac{R_A}{R_p} - \frac{R_A}{R_p}e = 1+e \rightarrow \frac{R_A}{R_p} - 1 = e + \frac{R_A}{R_p}e$$

$$\rightarrow \frac{R_A}{R_p} - 1 = e\left(1 + \frac{R_A}{R_p}\right) \rightarrow e = \frac{\left(\frac{R_A}{R_p} - 1\right)}{\left(1 + \frac{R_A}{R_p}\right)}$$

Plug in radii & solve for e:

$$\boxed{e = 0.05201}$$

$$\rightarrow a = \frac{R_A}{(1+e)} = \boxed{1432.04 \times 10^6 \text{ Km} = a}$$

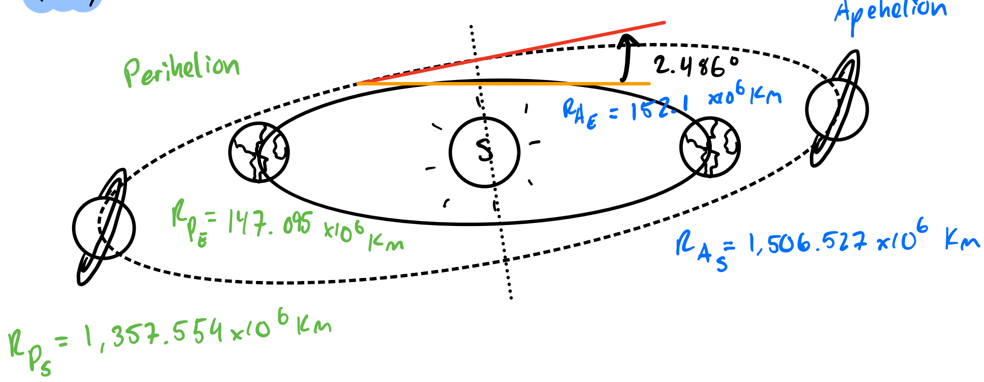
1b) Published values:

From NSSDC.GSFC.NASA.GOV : $e = 0.0520$ ✓

$a = 1{,}432 \times 10^6$ Km ✓

Inclination: 2.486°

1c) sketch



Problem 2 Titan $r_m = 1.22 \times 10^9 \, m$, $T = 15.95$ days.

Hyperion $r_m = 1.48 \times 10^9 \, m$, Find T_H

Law 3: the square of the period T of the orbit is proportional to the cube of the semi-major axis a $T^2 \propto a^3$

$$T_T^2 = k r_T^3 \quad , \quad T_H^2 = k r_H^3$$

$$\hookrightarrow k = \frac{T_T^2}{r_T^3} \quad \rightarrow T_H = \sqrt{\frac{T_T^2}{r_T^3} r_H^3} = \sqrt{\frac{15.95^2}{1.22E9^3} (1.48E9)^3}$$

$$= \sqrt{454.18} \rightarrow \boxed{T_H \cong 21.31 \text{ days}}$$

Actual: 21.28 days (NASA Horizons)

Problem 3 Find normal accel. of Earth around sun & sun around MW.

Around sun: $r = 93E6$ mi, $T = 1$ yr

" MW: $r = 1.9356E17$ mi, $V = 180$ mi/s

$$a_{normal} (2D) = -\omega^2 r$$

Sun: $\omega_{sun} = \frac{2\pi}{T} = \frac{2\pi \text{ rad}}{3.154E7 \text{ sec}} = 1.9921E{-7} \text{ rad/s}$

$r_{sun} = 1.4966E11 \, m$

$\rightarrow \left| a_{n,sun} \right| = (1.9921 E{-7})^2 (1.4966 E11) = \dfrac{0.00594 \ m/s^2}{9.81 \ m/s^2}$

$\rightarrow$ $\boxed{a_{n,sun} = 0.06 \,\% \text{ Earth gravity}}$

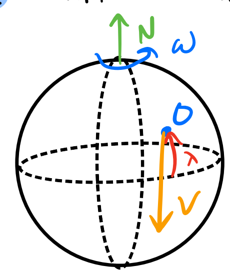
MW: $T_{MW} = \dfrac{2\pi r_{MW}}{V_{MW}} = \dfrac{2\pi (1.9356 E17 \ mi)}{180 \ mi/s} = 6.756 E15 \ sec$

$\omega_{MW} = \dfrac{2\pi}{T_{MW}} = 9.3 E{-16} \ rad/s$

$a_{n,MW} = \omega_{MW}^2 r_{MW} = (9.3 E{-16})^2 (3.115 E20) = \dfrac{2.694 E{-10} \ m/s^2}{9.81 \ m/s^2}$

$\rightarrow$ $\boxed{a_{n,MW} = 0.00000000275 \,\% \text{ Earth gravity}}$

Problem 4 Particle launched in rotating frame

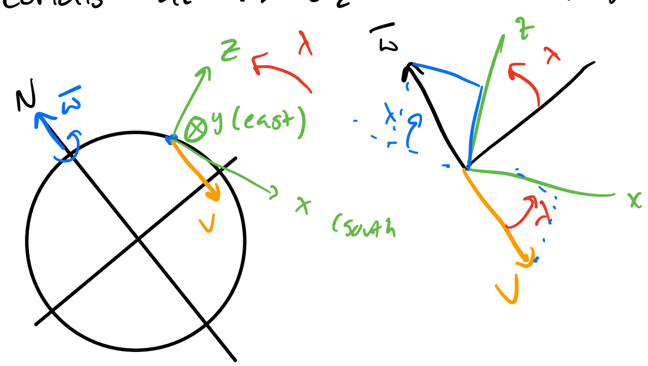


λ = latitude

V = velocity launched at $t=0$

4a) Find magnitude & direction of coriolis force in terms of V, ω, λ

Coriolis accel.: $\bar{a}_c = 2\bar{\omega} \times (\bar{v}_p)_{xyz}$



$\bar{\omega} = \omega \sin\lambda \, z - \omega\cos\lambda \, x$

$\bar{V} = V\sin\lambda \, z + V\cos\lambda \, x$

$$\bar{a}_c = 2\omega(\sin\lambda\, z - \cos\lambda\, x) \times v(\sin\lambda\, z + \cos\lambda\, x)$$

0 $\quad \sin\lambda\cos\lambda(+y)$

$(-\cos\lambda\sin\lambda)(-y)$ $\quad 0$

$$\rightarrow \boxed{\bar{a}_c = 4\omega V\cos\lambda\sin\lambda\,(\hat{y}) \rightarrow \text{Eastward}}$$

4b) The hemisphere will change the sign of λ. This will not affect the $\cos\lambda$ term, but it will affect the $\sin\lambda$, since $\sin(-\lambda) = -\sin(\lambda)$, flipping the sign & by extension the direction 180°, changing it in this case to westward.