$$J_{con} = \frac{1+\cos\alpha}{2}\left(\dot{m}u_e + (p_e - p_a)A_e'\right)$$

$$\simeq \frac{1+\cos\alpha}{2}\underbrace{\left(\dot{m}u_e + (p_e - p_a)A_e\right)}_{J_{1D}}$$

Line in fig. 11.5 is $\frac{1+\cos\alpha}{2} \cdot \underbrace{0.995}_{\text{viscous losses}}$

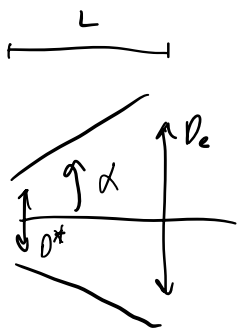
small losses compared to ideal

Temptation: make α very small

But: must keep $\frac{A_e}{A^*}$ necessary to achieve desired $\begin{cases} M_e \\ p_e/p_0 \rightarrow \frac{J}{p_0 A^*} \end{cases}$

Then we need a long (heavy) nozzle

For given $\begin{cases} \text{throat} & (A^*, D^*) \\ \text{exit} & (A_e, D_e) \end{cases}$



Relate length to semi-angle α

$$\rightarrow \frac{A_e}{A^*} = \left(\frac{D_e}{D}\right)^2 = \left(\frac{D^* + 2L\tan\alpha}{D^*}\right)^2 \qquad D_e = D^* + 2L\tan\alpha$$

$$\frac{L}{D^*} = \frac{\sqrt{A_e/A^* - 1}}{2\tan\alpha}$$

$A_e/A^* = 100$
$\alpha = 30° \rightarrow L/D^* = 7.8$
$\alpha = 15° \rightarrow L/D^* = 16.8$

To mitigate thrust loss by making α smaller, huge price in weight

Better approach: Shape the nozzle so as to have "as purely axial as possible" velocity @ exit

Nozzle design criteria

- Prevent formation of normal & oblique shocks inside nozzle (2-D Gas dynamics)
- Have purely axial velocity on as large a fraction of exit area as possible

Approaches:

Best "Bell" shape
- method of characteristics (applied math)
- calculus of variations (paper by Rao)

Good
- parabola (paper by Allman & Hoffman)

Poor
- conical nozzle (typical $\alpha = 15^\circ$)

common to all of these:

throat section is described by circular arcs

Improvement from conical to parabolic nozzle:

→ class figures

Combustion chambers

→ present some sizing criteria

suppose J, P_0, C_J, C^*, I_{sp} selected based on performance/requirements/past designs

Then A^* also determined → can be used as starting point of CC design

In general, CC's are surfaces of revolution (pressure vessels) b/c they yield the minimum thickness (weight) for an assigned pressure

convention: CC volume, V_c, is space between injector & plane of throat area

Requirement: cc volume must be large (long) enough to allow complete fuel/oxidizer
- atomization
- evaporization
- mixing
- reaction