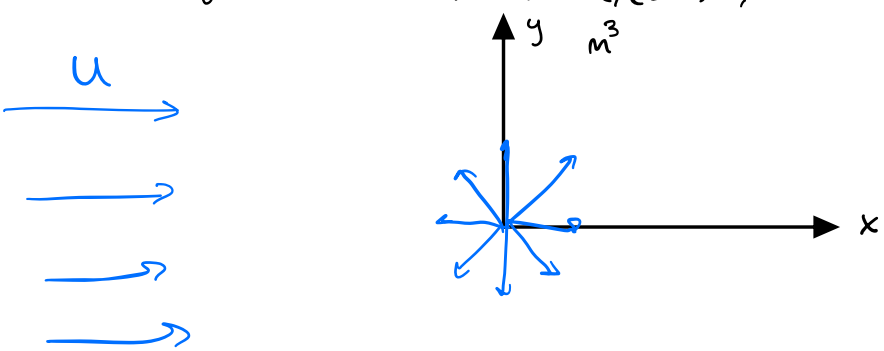
Problem 1 Potential flow of uniform flow in $+x$ U, line source @ origin strength m, $m = Q/(2\pi b)$, b = width into page



(a) streamfunction $\psi(r,\theta)$ as $f(m,U)$

uniform flow: $u = U$, $v = 0$

$$\psi(x,y) = Uy$$

$$\psi(r,\theta) = Ur\sin\theta$$

source flow: $v_\theta = 0$

Textbook: uniform flow: $v_r = \frac{\Lambda}{2\pi r}$

$$v_r = \frac{\Lambda}{2\pi r} = \frac{m \cdot 2\pi b}{2\pi r} = \frac{mb}{r} = v_r$$

$$v_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta}$$

$$\psi(r,\theta) = mb\theta$$

→ Superposition: $\boxed{\psi(r,\theta) = Ur\sin\theta + mb\theta}$

(b) potential function

uniform: $u = U$, $v = 0$

→ $\varphi(x,y) = Ux$

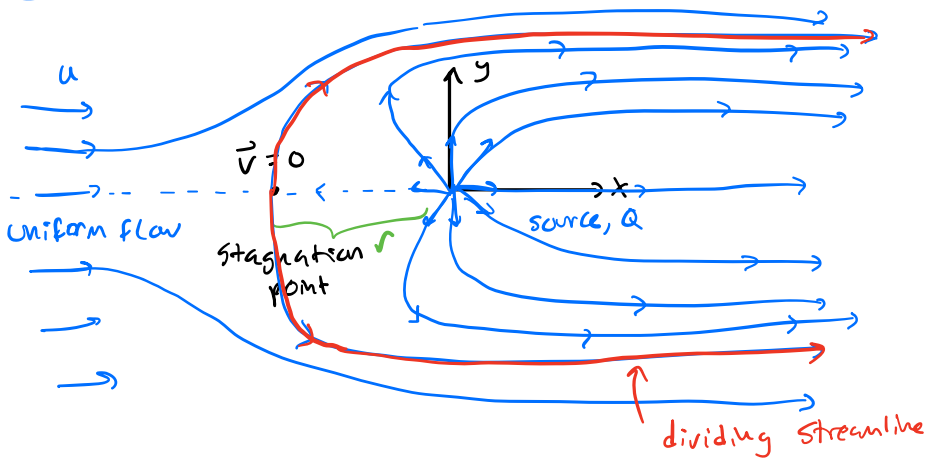
→ $\varphi(r,\theta) = Ur\cos\theta$

source: $v_r = \frac{mb}{r}$, $v_\theta = 0$

$$\varphi(r,\theta) = mb\ln r$$

→ $\boxed{\varphi(r,\theta) = Ur\cos\theta + mb\ln r}$

(1c) sketch streamlines



(1d) Bernoulli: highest pressure ↔ lowest velocity → Stag. point

Stag. point: $V_r = V_\theta = 0$

$v_r = u\cos\theta + \frac{m}{r} = 0$ (-x axis)

$V_\theta = u\sin\theta = 0 \rightarrow$ @ $\theta = -\pi, 0, \pi \ldots$

$\rightarrow \theta = \pi$

$\rightarrow v_r = u(-1) + \frac{m}{r} = 0 \rightarrow r = \frac{m}{u}, \theta = \pi$

$\rightarrow$ $\boxed{x = r\cos\theta = -\frac{m}{u}, \; y = 0}$

(1e) Find values of Ψ of dividing streamline

$\psi(r,\theta) = u r\sin\theta + m\theta \rightarrow$ plug in stag. point coords

$\rightarrow$ $\boxed{\psi = \pi m = \text{const.}}$

(1f) Find velocity magnitude along div. streamline as $f(m,u)$

$\pi m = u r\sin\theta + m\theta \rightarrow r = \frac{m(\pi-\theta)}{u\sin\theta}$

$\rightarrow v_r = u\cos\theta + \frac{m}{r} = u\cos\theta + \frac{u\sin\theta}{(\pi-\theta)}$

$v_r = u\left(\cos\theta + \sin\theta/(\pi-\theta)\right)$

$\rightarrow V_\theta = U \sin\theta$

$$\rightarrow |V(r,\theta)| = \sqrt{U^2\left[\cos\theta + \sin\theta/(\pi-\theta)\right]^2 + U^2\sin^2\theta}$$

$$= U\sqrt{\cos^2\theta + \frac{2\sin\theta\cos\theta}{\pi-\theta} + \frac{\sin^2\theta}{(\pi-\theta)^2} + \sin^2\theta}$$

$$\boxed{|V(r,\theta)| = U\sqrt{1 + \frac{\sin^2\theta}{(\pi-\theta)^2} + \frac{2\sin\theta\cos\theta}{\pi-\theta}}}$$

1g) Find point along streamline of min. pressure
→ max velocity

$$\frac{\partial V(r,\theta)}{\partial\theta} = 0$$

→ matlab vpa solve & fplot

→ V_{max} @ $\theta \cong 1.1$, $\sim 63^\circ$
And @ $\theta = 2\pi - 1.1$

$$\frac{V}{U} \text{ @ } \theta = 1.1 = 1.26$$

$$r = \frac{mb(\pi-\theta)}{U\sin\theta}$$

$$\rightarrow r = 2.29\,\frac{mb}{U}$$

→ point $(r,\theta) = \left(2.29\,\frac{mb}{U},\ 1.1\right)$