Orbital transfers

- LEO → HEO, change r
- interplanetary earth → moon
- Last year: "crew 8" → ISS
  Dragon capsule
  28 hr "orbital chase"
  How to go A → B same r "Rendezvous"
  (change θ)
- change planes, i

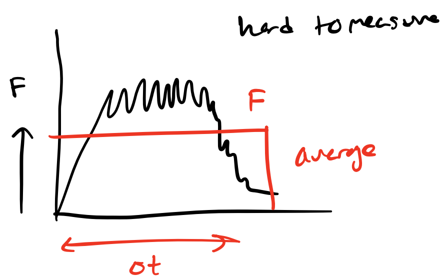
Assumptions

Transfers by $\dot{\vec{v}}$, of course ($\Delta\vec{v}$)

Reality $m\frac{d\vec{v}}{dt} = F$ Propulsion
- short duration
- high thrust
- Impulsive?

Fundamental assumption
"propulsion is impulsive"

Impulsive ≡ 1) Really short Δt
2) Large force



Impulse = $\bar{F}\Delta t$

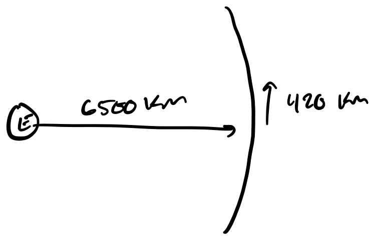
Apply $m\,d\vec{v} = F\,dt$

$$mv_2 - mv_1 = \int \bar{F}\,dt \approx \bar{F}\Delta t$$

Evaluate reality! Example: $v_{initial} = 7$ Km/s, rocket $\Delta t = 1$ min
$r_{initial} = 6500$ Km

Find "apparent straight line motion" of this burn

$$\Delta x \approx v_i \Delta t = 7\text{ Km/s }(60\text{ s}) = 420\text{ Km}$$



arc length $\theta = \frac{s}{r} = \frac{420}{6500}$

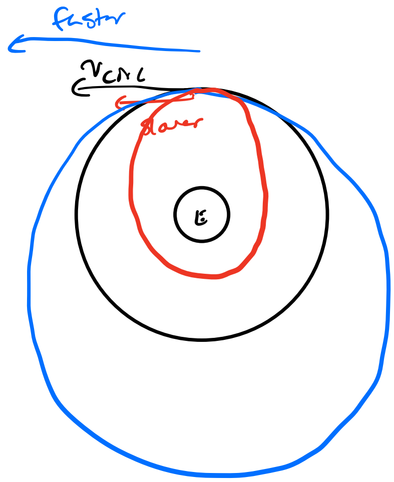
≈ 0.06 rad $= \Delta\theta$

$\Delta\theta$ small on orbit

even a few minutes of burn time, $\Delta\theta$ is small

$\therefore$ short $\Delta t \rightarrow$ craft has not changed orientation

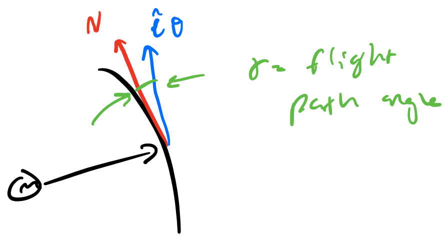
Planning ΔV



- depends on mag. & dir. of $\Delta\bar{V}$
- many parameters can change:
  p, i, Ω
- Δv changes but $\bar{r}$ does not

Start w/ coplanar transfers

1) Tangential $\Delta v \equiv \gamma = 0$
2) non-tangential $\Delta v \equiv \gamma \neq 0$



Tangential burns $\rightarrow$ more efficient

Review of transfer orbits : start w/ circular = v_c

$$v_{esc} = \sqrt{\frac{2\mu}{r_c}} = \sqrt{2}\, v_c \quad \therefore \text{ consider } \Delta v$$

$\Delta v < v_c(\sqrt{2}-1) \rightarrow$ elliptical

$\Delta v = v_c(\sqrt{2}-1) \rightarrow$ parabolic

$\Delta v > v_c(\sqrt{2}-1) \rightarrow$ hyperbolic

Build transfers

1) leave "initial orbit"
2) move on "transfer orbit"
3) Hit a "target orbit"
4) do w/ certain timing
5) cost

Cost of transfer

$h = \bar{r} \times \bar{v}$

specific orbit energy $\mathcal{E} = \frac{1}{2} v^2$

change in energy for transfer:

$$\Delta \mathcal{E} = -\frac{\mu}{2a_2} + \frac{\mu}{2a_1}$$

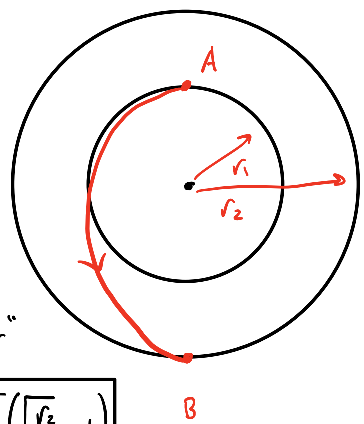
a_1 = initial orbit

a_2 = target orbit

$$\mathcal{E}_{cost} \cong \frac{\Delta v^2}{2}$$

Transfer 1: Hohmann transfer

circular → elliptical → circular



can show:

ellipse $r_p = r_1$, $r_a = r_2$, $a = \frac{r_1 + r_2}{2}$

the most energy efficient transfer

Analysis orbit equations: 2 Δv's

t - "transfer"

$$\Delta v_A = v_{t_A} - v_{c_1}$$

$$= \sqrt{\mu\left(\frac{2}{r_1} - \frac{1}{a}\right)} - \sqrt{\frac{\mu}{r_1}} = \boxed{\sqrt{\frac{\mu}{r_1}}\left(\sqrt{\frac{r_2}{a}} - 1\right)}$$

algebra

$$\Delta v_B = v_{c_2} - v_{t_B}$$

$$= \sqrt{\frac{\mu}{r_2}} - \sqrt{\mu\left(\frac{2}{r_2} - \frac{1}{a}\right)} = \boxed{\sqrt{\frac{\mu}{r_2}}\left(1 - \sqrt{\frac{r_1}{a}}\right)}$$

$$\Delta v_{total} = \Delta v = |\Delta v_A| + |\Delta v_B|$$

Remarks

1) Δv +/− , Δv mag } direction gives speeding up or slowing down

e.g. $r_2 > r_1$ Δv_A same dir. as motion

$r_2 < r_1$ Δv_A opposite dir. as motion

2) Use logic to check ΔV mag,

3) $r_2 > r_1 \rightarrow$ A is perigee & B is apogee

$r_2 < r_1 \rightarrow$ A is apogee & B is perigee

4) Time w/ transfer = half elliptical orbit T

$$\therefore \; T_t = \left(\tfrac{1}{2}\right) 2\pi \sqrt{\frac{a^3}{\mu}}$$

5) Slow transfer!

if long burn Δt, multiple hohmann?
inefficient or save?

Example 2 Hohmann LEO to GEO

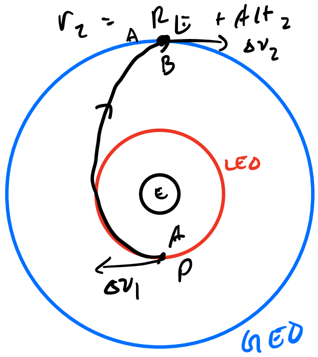
LEO: $Alt_1 = 322$ Km $\quad R_E = 6378$ Km

GEO: $Alt_2 =$ something Km

Find total ΔV & orbit energies

$r_1 = R_E + Alt_1 = 6700$ Km , $v_1 = \sqrt{\frac{\mu}{r_1}} = 7.713$ Km/s

$r_2 = R_E + Alt_2 = 42238$ Km $\quad v_2 = \sqrt{\frac{\mu}{r_2}} = 3.072$ Km/s



$$\Delta v_1 = v_{tp} - v_1 = \sqrt{\frac{\mu}{r_1}}\left(\sqrt{\frac{r_2}{a}} - 1\right)$$

$\Delta v_1 = 2.42$ Km/s w/ $a = \frac{r_1 + r_2}{2}$

$$\Delta v_2 = v_2 - v_{tA} = \sqrt{\frac{\mu}{r_2}}\left(1 - \sqrt{\frac{r_1}{a}}\right)$$

$\Delta v_2 = 1.46$ Km/s

$\Delta V_{tot} = |\Delta v_1| \ldots = \boxed{3.885 \text{ Km/s}}$

Aside: $T_t = \left(\frac{1}{2}\right) 2\pi \sqrt{\frac{a^3}{\mu}} = 5.29$ hrs ✓ between 1.5 - 24

Energies $\quad \varepsilon = \frac{1}{2} v^2 \quad \Delta\varepsilon = \underbrace{-\frac{\mu}{2a_2}}_{\text{final}} + \underbrace{\frac{\mu}{2a_1}}_{\text{initial}}, \quad \varepsilon_{cost} \cong \frac{\Delta v^2}{2}$

$\varepsilon_{initial} = -\frac{\mu}{2r_1} = -29.75 \text{ Km}^2/\text{s}^2 \quad \Big\} \quad \Delta\varepsilon = \varepsilon_{final} - \varepsilon_{initial} = 25.03 \text{ Km}^2/\text{s}^2$

$$\varepsilon_{final} = -\frac{\mu}{2r_2} = -4.7185 \ km^2/s^2$$

$$\varepsilon_t = -\frac{\mu}{2a_t} = -8.145 \ km^2/s^2$$

now since $\varepsilon_{initial} < \varepsilon_t < \varepsilon_{final}$

Δv applied in same dir. as orbit

Next time: what if we aim faster?