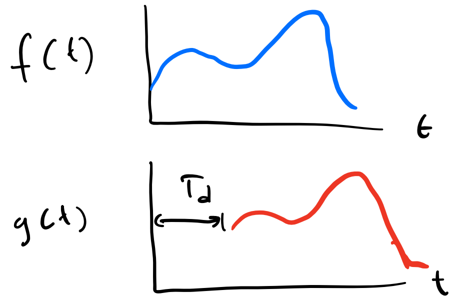
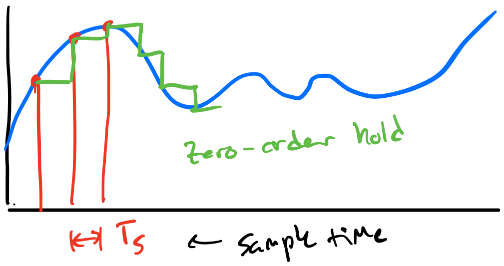
Pure time delay

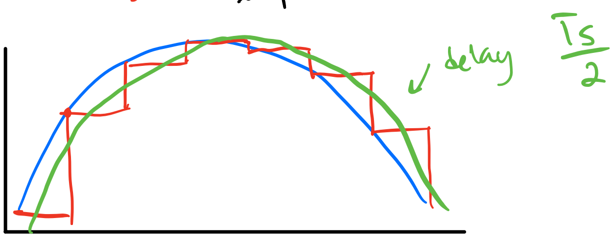
- Process transport delay
- sample & hold delay
- communication delay

$$g(t) = \begin{cases} 0 & 0 \le t < T_d \\ f(t - T_d) & t \ge T_d \end{cases}$$



Sample rate - not continuous like analog circuits





Laplace of delay:

$$G(s) = \mathcal{L}[g(t)] = \int_0^\infty e^{-st} g(t)\, dt$$

$$= \int_0^\infty e^{-st} f(t - T_d)\, dt$$

$\tau = t - T_d$

$t = \tau + T_d \rightarrow dt = d\tau + \cancel{dT_d}^{\,0}$

$dt = d\tau$

$$= \int_0^\infty e^{-s(\tau - T_d)} f(\tau)\, d\tau$$

$$= e^{-sT_d} \underbrace{\int_0^\infty e^{-s\tau} f(\tau)\, d\tau}_{= F(s)}$$

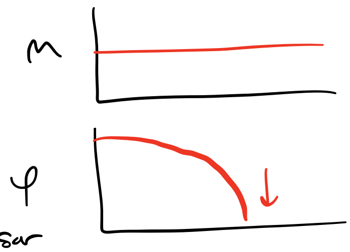
$$\rightarrow G(s) = e^{-sT_d} F(s)$$ ← f w/o delay

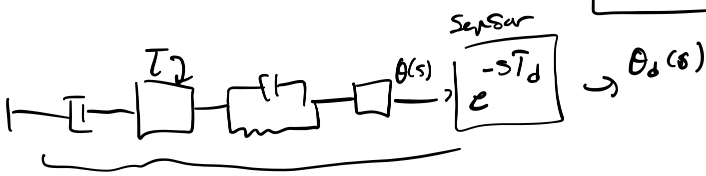
$$e^{-sT_d}\Big|_{s=j\omega} = e^{-j\omega T_d}$$

$$\left| e^{-j\omega T_d} \right| = 1$$

→ phase function of ω

$$\angle e^{-j\omega T_d} = -\omega T_d$$





$$\frac{\theta(s)}{T(s)} = \frac{10}{s(s+10)(s^2+80s+10^4)}$$