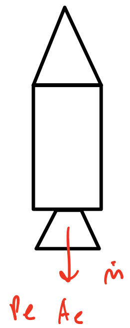
Problem 1

Rocket, init. mass M_0, $u_0 = 0$, u_e = const.
constant $\dot{m}$, p_e, A_e, $p_a = 0$, $g = D = 0$



1a) Determine expression for time t_e it takes for rocket velocity $u = u_e$

Recall rocket eqn for $g = D = 0$

$$u_{eq} \ln \frac{M_0}{M(t)} = \Delta u$$

$$u = \Delta u = u_e \quad @\, t = t_e$$

$$\rightarrow \left(u_e + \frac{p_e A_e}{\dot{m}}\right) \ln \frac{M_0}{M(t_e)} = u_e$$

$$\rightarrow 1 = \left(1 + \frac{p_e A_e}{\dot{m} u_e}\right) \ln \frac{M_0}{M(t_e)} \rightarrow \ln \frac{M_0}{M(t_e)} = \frac{1}{1 + \frac{p_e A_e}{\dot{m} u_e}} = \frac{\dot{m} u_e}{\dot{m} u_e + p_e A_e}$$

$$\frac{M_0}{M(t_e)} = e^{\left(\frac{\dot{m} u_e}{\dot{m} u_e + p_e A_e}\right)} \rightarrow M(t_e) = M_0 \, e^{\left(-\frac{\dot{m} u_e}{\dot{m} u_e + p_e A_e}\right)}$$

$M(t_e) = M_0 - \dot{m} t_e$

$$\rightarrow M_0 - \dot{m} t_e = M_0 \, e^{-\left(\frac{\dot{m} u_e}{\dot{m} u_e + p_e A_e}\right)}$$

$$\rightarrow \quad t_e = -\frac{M_0}{\dot{m}} \left(e^{-\left(\frac{\dot{m} u_e}{\dot{m} u_e + p_e A_e}\right)} - 1\right)$$

$$\boxed{t_e = \frac{M_0}{\dot{m}} \left(1 - e^{-\left(\frac{\dot{m} u_e}{\dot{m} u_e + p_e A_e}\right)}\right)}$$

1b) Plot t_e vs. u_e ; see plot 1

1c) Plot u vs. t ; see plot 2

$$u(t) = \left(u_e + \frac{p_e A_e}{\dot{m}}\right) \ln \frac{M_0}{M_0 - \dot{m} t}$$

Problem 2 $D = 0$, $g = \text{const.}$ $g_{cb} = \dfrac{G M_{cB}}{(R_{cb}+h)^2}$

Rocket initially at rest in space $(g=0)$

$u_{eq} = 4400 \text{ m/s}$, $\dfrac{M_p}{M_0} = 0.4$, $t_b = 12s$

2a) Find u_b

Recall rocket eqn for $g = D = 0$

$$u_{eq} \ln \frac{M_0}{M_b} = \Delta u_b$$

$$M_b = M_0 - M_p$$
$$= 1 - \frac{M_p}{M_0} \quad 0.4 \quad = 0.6$$

$$\rightarrow u_b = u_{eq} \ln \frac{M_0}{M_0 - M_p} = u_{eq} \ln \frac{1}{1 - \frac{M_p}{M_0}}$$

$$\rightarrow u_b = (4400) \ln\left(\frac{1}{1-0.4}\right) = \boxed{2247.63 \text{ m/s} = u_b}$$

2b) How far has it traveled @ burnout

Recall h_b eqn from lecture:

$$R = \frac{M_0}{M_b} = \frac{1}{0.6}$$

$$-\frac{u_e t_b}{R-1} \ln R + u_e t_b - \cancel{g_e t_b^2} = h_b$$

$g = 0$

$$\rightarrow h_b = (4400)(12) - \frac{(4400)(12)}{5/3 - 1} \ln\left(5/3\right)$$

$$\rightarrow \boxed{h_b = 12{,}343 \text{ m}}$$

2c) Now from earth's surface, find u_b

Recall: $u(t) = -u_e \ln \dfrac{M(t)}{M_0} - g_e t$

$$u_b = -u_e \ln \frac{M_b}{M_0} - g_e t_b$$

$$u_b = -(4400)\ln(0.6) - (9.81)(12) = \boxed{2130 \text{ m/s} = u_b}$$

2d) Find h_b

Recall: $-\frac{u_e t_b}{R-1} \ln R + u_e t_b - g_e t_b^2 = h_b$

$R = \frac{M_o}{M_b} = \frac{1}{0.6} = 5/3$

$$h_b = -\frac{(4400)(12)}{2/3} \ln(5/3) + (4400)(12) - (9.81)(12)^2/2$$

$\rightarrow \boxed{h_b = 11636 \text{ m}}$

2e) Max altitude it will reach?

$$KE = PE$$

$$\frac{1}{2} M_b u_b^2 = M_b g_e (h_{max} - h_b)$$

$$\rightarrow h_{max} = \frac{u_b^2}{2 g_e} + h_b$$

$$= \frac{(2130)^2}{2(9.81)} + 11636 = \boxed{h_{max} = 242,875 \text{ m}}$$

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2f) Now on moon, find u_b

$$g_m = \frac{G M_m}{R_m^2} = 1.61 \text{ m/s}^2$$

$$u_b = -u_e \ln \frac{M_b}{M_o} - g_m t_b$$

$$= -(4400) \ln(0.6) - 1.61(12) = \boxed{u_b = 2228 \text{ m/s}}$$

2g) Find h_b

$-\frac{u_e t_b}{R-1} \ln R + u_e t_b - g_e t_b^2 = h_b$

$R = \frac{M_o}{M_b} = \frac{1}{0.6} = 5/3$

$$h_b = -\frac{(4400)(12)}{2/3} \ln(5/3) + (4400)(12) - (1.61)(12)^2/2$$

$$= \boxed{h_b = 12,227 \text{ m}}$$

2h) Find h_{max}

$$KE = PE$$

$$\frac{1}{2} M_b u_b^2 = M_b g_m (h_{max} - h_b)$$

$$\Rightarrow h_{max} = \frac{u_b^2}{2 g_m} + h_b$$

$$= \frac{(2228)^2}{2(1.61)} + 12{,}227 = \boxed{h_{max} = 1{,}553{,}836 \ m}$$

2i) 1.5 Million meters is very far, so our constant gravity assumption is likely very unreasonable.

$$g_m (h = 1555944 \, m) = \frac{G M_m}{(R_m + h)^2} = 0.45 \ m/s \ << \ 1.61 \ m/s$$