PI control

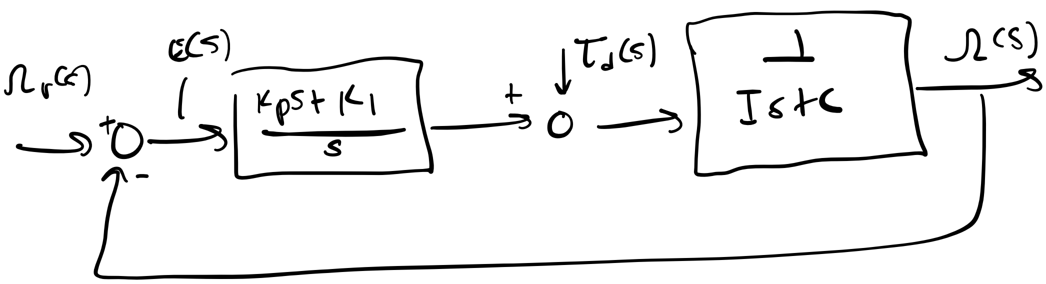
$$K_p + \frac{K_I}{s} = \frac{K_p s + K_1}{s}$$

$$\frac{\Omega(s)}{\Omega_r(s)} = \frac{sK_p + K_1}{Is^2 + (c+K_p)s + K_1}$$ ← zero

← 2nd order now

→ allows for arbitrary placement of CL poles

(i.e. two control gains, two roots



$$\Delta_{CL}(s) = s^2 + \left(\frac{c+K_p}{I}\right)s + \frac{K_I}{I}$$

$$\omega_r(t) = 1(t) \rightarrow e(\infty) = \frac{c}{K_p + c}$$ Proportional only, steady error

$$\frac{E(s)}{\Omega_r(s)} = \frac{1}{1 + \frac{K_p s + K_1}{(Is+c)s}} = \frac{Is^2 + cs}{Is^2 + (c+K_p)s + K_1}$$

$$e(\infty) = \lim_{s \to 0}\left[s\,\frac{E(s)}{\Omega_r(s)} \cdot \frac{1}{s}\right] = \frac{0}{K_I} = 0$$ Error goes to 0, faster as $K_I \uparrow$

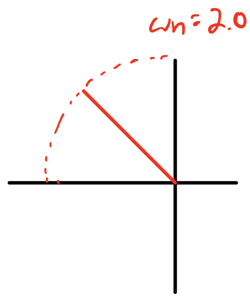
Ramp: $\omega_r(t) = t$ → Prop: $e = \infty$

$$e(\infty) = \lim_{s \to 0}\left[\cancel{s}\,\frac{Is^{\cancel{2}} + c\cancel{s}}{Is^2 + (c+K_p)s + K_1}\,\frac{1}{\cancel{s^2}}\right] = \frac{c}{K_1}$$ No longer infinity, constant

integration increases order of system, but also changes error due to higher order input

only need to change K_1, not K_P

$\omega_r(t) = \frac{1}{2}t^2 \rightarrow \frac{1}{s^3}$



Want: $\begin{cases} \zeta \geq 0.5, \quad \omega_n > 2.0 \\ e(\infty) \leq 0.8 \quad \omega_r(t) = 10t \end{cases}$

$I = 0.05 \qquad C = 0.025$

$$\delta_{CL}(s): \quad s^2 + \underbrace{2\zeta\omega_n}_{=} s + \underbrace{\omega_n^2}_{=} = 0$$

$$s^2 + \frac{C+K_P}{I} s + \frac{K_1}{I} = 0$$

$K_P = 2\zeta\omega_n I - C \quad = 0.075$

$K_1 = \omega_n^2 I \quad = 0.2$

$e(\infty) < 0.8 \quad \omega_r(t) = 10t$

$$\frac{E(s)}{\Omega_r(s)} = \frac{Is^2 + Cs}{Is^2 + (C+K_P)s + K_1} \qquad \Omega_r(s) = \frac{10}{s^2}$$

$$e(\infty) = \lim_{s \to 0} \left[\cancel{s} \frac{\cancel{Is^2} + C\cancel{s}}{\cancel{Is^2} + \cancel{(C+K_P)s} + K_1} \cdot \frac{10}{\cancel{s^2}} \right] = 10 \frac{C}{K_1} \leq 0.8$$

$\therefore K_1 \geq 0.3125$

$$D(s) = K_P + \frac{K_1}{s} = 0.1 + \frac{0.3125}{s}$$

1st order systems:

- Proportional speeds up response
  - Integral reduces errors, elim. unit step error
  - Derivative not helpful, makes it slower
  - comb. of PI allows for arbitrary placement of closed-loop poles

- Root locus: how do poles change as gain changes