**Problem 1** Use Dalton's law to show mol. mass of mixture of n gases

$$\bar{M}_m = \sum_{i=1}^{n} x_i \bar{M}_i$$

Dalton's law: i) $\forall_i = \forall_m = \forall$, $T_i = T_m = T$

ii) $P_m = \sum_{i=1}^{n} P_i$

$\forall_i = \forall \rightarrow m_1 v_1 = m_2 v_2 \ldots$ $\quad R_i = \frac{\bar{R}}{\bar{M}_i}$

$\bar{M}_m = \frac{m_m}{n_m}$, $\quad \bar{M}_i = \frac{m_i}{n_i} \rightarrow m_i = n_i \bar{M}_i$

$m_m = \sum_{i=1}^{n} m_i = \sum_{i=1}^{n} n_i \bar{M}_i$, $\quad x_i = \frac{n_i}{n_m} \rightarrow n_i = x_i n_m$

$$\rightarrow \bar{M}_m = \frac{m_m}{n_m} = \frac{\sum_{i=1}^{n} (x_i \cancel{n_m} \bar{M}_i)}{\cancel{n_m}} = \boxed{\sum_{i=1}^{n} x_i \bar{M}_i = \bar{M}_m}$$

**Problem 2** Mixture of gasses, 10 Kg $N_2$, 10 Kg $H_2$, 15 Kg He.

$P_m = 6.7\,MPa$, $T_m = 300\,K$ Find $\bar{M}_m$ & $\gamma_m$

| | $\bar{M}$ | $\gamma$ |
|---|---|---|
| $N_2$ | 28 | 1.4 |
| $H_2$ | 2 | 1.4 |
| He | 4 | 1.67 |

$$\bar{M}_m = \sum_{i=1}^{n} x_i \bar{M}_i$$

$n_{N_2} = \frac{10\,Kg}{28\,Kg/Kmol} = 0.357\,Kmol \rightarrow x_{N_2} = 0.0392$

$n_{H_2} = \frac{10\,Kg}{2\,Kg/Kmol} = 5\,Kmol \rightarrow x_{H_2} = 0.549$

$n_{He} = \frac{15\,Kg}{4\,Kg/Kmol} = 3.75\,Kmol \rightarrow x_{He} = 0.412$

$n_m = 9.107\,Kmol$

$$\rightarrow \bar{M}_m = x_{N_2}\bar{M}_{N_2} + x_{H_2}\bar{M}_{H_2} + x_{He}\bar{M}_{He}$$

$$= (0.0392)(28) + (0.549)(2) + (0.412)(4)$$

$$\rightarrow \boxed{\bar{M}_M = 3.843 \ \ kg/kmol}$$

$$\gamma_m = \frac{\overline{c_{p_m}}}{\overline{c_{v_m}}} = \frac{\sum_{i=1}^{n} x_i \overline{c_{p_i}}}{\sum_{i=1}^{n} x_i \overline{c_{v_i}}}$$

$$c_p = c_v + R \ , \quad \gamma = \frac{c_p}{c_v}$$

$$\frac{c_p}{c_v} = 1 + \frac{R}{c_v} = \gamma$$

$$\rightarrow \frac{\bar{R}}{\gamma_i - 1} = \overline{c_{v_i}} \ ; \quad \overline{c_{p_i}} = \overline{c_{v_i}} + \bar{R}$$

$$\overline{c_v}_{N_2} = \frac{8314}{1.4-1} = 20785 \ , \quad \overline{c_p}_{N_2} = 29099$$

$$\overline{c_v}_{H_2} = \ '' \quad '' \ , \quad \overline{c_p}_{H_2} = \ ''$$

$$\overline{c_v}_{He} = \frac{8314}{1.67-1} = 12409 \ , \quad \overline{c_p}_{He} = 20723$$

$$\gamma_m = \frac{(0.0392)(29099) + (0.549)(29099) + (0.412)(20723)}{(0.0392)(20785) + (0.549)(20785) + (0.412)(12409)}$$

$$\rightarrow \boxed{\gamma_m = 1.48}$$

**Problem 3** Hydrocarbon $CH_2$ burns w/ 30% excess air to reach $T_{prod} = 2200$ K. Dissociation products OH, O, H, NO, N.

Estimate equilibrium concentration of NO @ T = 2200K

Assuming concentrations of $N_2$, $O_2$, $H_2O$, $CO_2$ are as if no dissociation.

$$N_2 + O_2 \rightleftarrows 2NO \qquad @\ 2200\,K \qquad K_p = e^{-6.866}$$

Air: 79% $N_2$, 21% $O_2$

Combustion:

$$1\ CH_2 + 1.5\ O_2 \overset{+\text{ excess}}{} + x\ N_2 \longrightarrow 1\ CO_2 + 1\ H_2O + \overset{\text{excess}}{\wedge}\ O_2 + x\ N_2$$

Assume 1 mol $CH_2$ → Need 1.5 mol $O_2$ to react

30 % extra : 1.5 mol · 0.3 → 0.45 mol excess $O_2$

$$N_2 : O_2 = 79 : 21 \rightarrow N_2 = \frac{79}{21} \cdot 1.95\ O_2 = 7.336 \text{ mol } N_2$$

$$N_2 + O_2 \rightleftharpoons 2NO \qquad K_p = \frac{P_{NO}^2}{P_{N_2}^1 P_{O_2}^1} = e^{-6.866}$$

$$K_n = \frac{x_{NO}^2}{x_{N_2} x_{O_2}} = P_M^{1+1-2} K_p = K_p$$

$$x_{NO} = ? \qquad x_{N_2} = \frac{7.336}{1+1+0.45+7.336} \qquad x_{O_2} = \frac{0.45}{1+1+0.45+7.336}$$

$$\rightarrow K_n = K_p = \frac{x_{NO}^2}{\frac{7.336 \cdot 0.45}{(1+1+0.45+7.336)^2}} = \frac{x_{NO}^2}{0.0345} = e^{-6.866} = 3.59E{-5}$$

$$\rightarrow \boxed{x_{NO} = 0.005995 = 6000 \text{ ppm}}$$

**Problem 4** $N_2H_4$ Monopropellant

$$3N_2H_4 \rightarrow 4NH_3 + N_2$$

**4a)** Assuming $T_f = 298K$, Find $T_{\text{adiabatic flame}}$

$$\underset{@T_f}{N_2H_4} \longrightarrow \boxed{RXN} \xrightarrow{NH_3+N_2} \underset{@T_f}{} \boxed{Hx} \xrightarrow{NH_3+N_2} @T_2$$

$$\uparrow Q_R = H_{Rpf} \qquad \uparrow Q_2 = H_{P2} - H_{Pf}$$

$$Q_R = H_{RPF} = \sum_j (n_j Qf_j)_{prods} - \sum_i (n_i Qf_i)_{reacts}$$

$$= \left[4 \cdot (-45.94E6) + 1 \cdot (0)\right] - \left[3 \cdot (97.42E6)\right] \quad [J]$$

$$Q_R = -476.02E6 \ J$$

$$Q_2 = H_{P2} - H_{Pf} = \sum_j n_j (\bar{h} - \bar{h}_0) \xrightarrow[\text{assume } c_p = \text{const}]{} \sum_j n_j \bar{c}_p (T_2 - T_f)$$

$$Q_2 = (T_2 - T_f)\left[4 \cdot (63.5E3) + 1 \cdot (32E3)\right]$$

$$Q_2 = (T_2 - 298)(286E3)$$

$$Q = Q_R + Q_2 = 0 \rightarrow Q_2 = -Q_R$$

$$\rightarrow (T_2 - 298)(286E3) = -(-476.02E6)$$

$$T_2 - 298 = \frac{476.02E6}{286E3}$$

$$\rightarrow T_2 = 1664.4 + 298 = \boxed{1962.4 \ K = T_{adb}}$$

4b) $3 N_2H_4 \rightarrow \mu NH_3 + \nu N_2 + \xi H_2$

$NH_3 \rightleftharpoons \frac{1}{2} N_2 + \frac{3}{2} H_2$ , $K_p(T)$ given [atm]

Assume incomplete decomp. @ $P_m = 75$ atm.

Find $\mu, \nu, \xi$ , $x_{NH_3}, x_{N_2}, x_{H_2}$ , $T_{adb}$

cons. mass:

N: $6 = \mu + 2\nu \rightarrow \nu = \frac{6-\mu}{2}$

H: $12 = 3\mu + 2\xi \rightarrow \xi = \frac{12 - 3\mu}{2}$

Chem. EQ:

$$\chi_{NH_3} = \frac{\mu}{\mu+\nu+\xi} \qquad \chi_{N_2} = \frac{\nu}{\mu+\nu+\xi} \qquad \chi_{H_2} = \frac{\xi}{\mu+\nu+\xi}$$

$$K_n = \frac{\chi_{N_2}^{1/2}\,\chi_{H_2}^{3/2}}{\chi_{NH_3}} = P_m^{1-1/2-3/2}\,K_p = P_m^{-1}\,K_p$$

$$P_m^{-1}\cdot\left[10^{-(AT^4+BT^3+CT^2+DT+E)}\right] = \frac{\left(\frac{\nu}{\mu+\nu+\xi}\right)^{1/2}\left(\frac{\xi}{\mu+\nu+\xi}\right)^{3/2}}{\left(\frac{\mu}{\mu+\nu+\xi}\right)}$$

→ Solve for T w/ root solver

Thermo:

$$Q = Q_R + Q_2 = 0$$

$$Q_R = H_{RPf} = \Sigma\left(n_j Q_{fj}\right)_{prod} - \Sigma\left(n_i Q_{fi}\right)_{React}$$

$$= \mu(-45.94E6) + \nu(0) + \xi(0) - 3\cdot(97.42E6)$$

$$Q_2 = H_{P_2} - H_{Pf} = \Sigma\left[n_j \bar{C}_{p_j}\,(T_2 - T_f)\right]_{prod}$$

$$Q = 0 \rightarrow Q_R = -Q_2$$

$$\rightarrow \quad -H_{RPF} = \left[\mu(63.5E3) + \nu(32E3) + \xi(30E3)\right](T_2 - T_f)$$

solve for $T_2$ in matlab w/ thermo

$$\boxed{\begin{aligned} &\rightarrow T_2 = 1360.08\ K, \quad \mu = 0.02018 \\ &\rightarrow \nu = 2.990 \\ &\rightarrow \xi = 5.970 \end{aligned}}$$

$$\boxed{\rightarrow \chi_{NH_3} = 0.00225, \quad \chi_{N_2} = 0.333, \quad \chi_{H_2} = 0.665}$$

4c) $T_{adb}$ from a: 1962.4K; $T_{adb}$ from b: 1360.08K

The adiabatic flame temperature is lower for part b because part b includes decomposition, which takes energy that would otherwise be used to raise the temperature of the products.

4d) $P_m = [150, 300, 600]$ atm

& $\mu_{start} = [0.03, 0.05, 0.05]$

4e) As pressure increases, adiabatic flame temperature increases. This is because less dissociation is occuring (visible in the increasing $X_{NH_3}$ values) due to the increase in pressure.