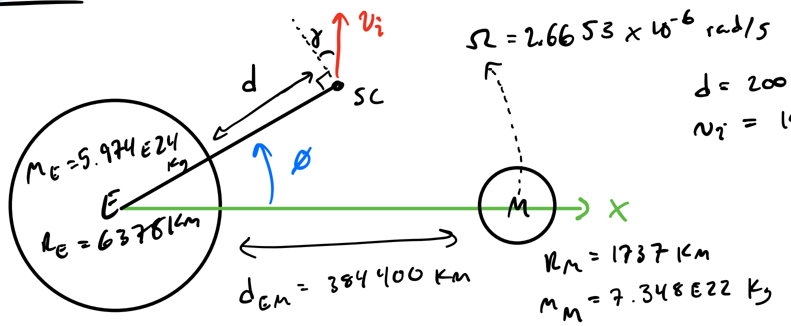
EMA 550 HW4 Kyle Heller

Problem 1 Earth, moon, satellite. Initial LEO → launched w/ v_i.

$\Omega = 2.6653 \times 10^{-6}$ rad/s



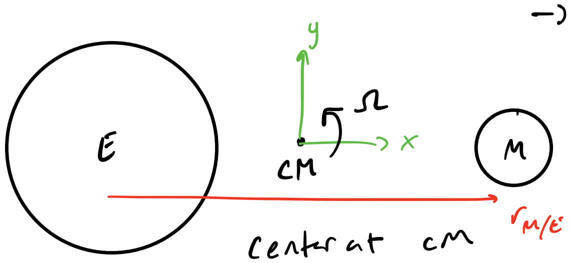
$d = 200$ km

$v_i = 10.9148$ km/s @ $\gamma = 20°$

@ $\phi = -90°$

1a) Define cartesian reference frame & determine position & velocity @ t=0

→ rotating frame w/ Ω



$$\pi_E = \frac{m_E}{m_E + m_M} = 0.98785$$

$$\pi_M = \frac{m_M}{m_E + m_M} = 0.1215$$

$$r_{M/E} = 384400 + 6378 + 1737$$
$$= 392515$$

→ $x_E = -\pi_M r_{M/E} = -4769.26 \text{ km } \hat{i}$, $y_E = z_E = 0$

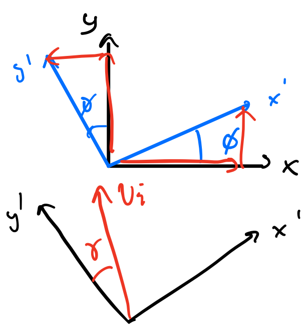
$x_M = \pi_E r_{M/E} = 387745.74 \text{ km } \hat{i}$, $y_M = z_M = 0$

Space craft:

$x_S = x_E + (R_E + d)\cos\phi = -4769.26 + (6578)\cancel{\cos(-90°)}^{0}$ $\boxed{= -4769.26 \text{ km} = x_s}$

$y_S = y_E + (R_E + d)\sin\phi = 0 + (6578)\cancel{\sin(-90°)}^{-1}$ $\boxed{= -6578 \text{ km} = y_s}$

$z_S = 0$



$x = x'\cos\phi - y'\sin\phi$

$y = x'\sin\phi + y'\cos\phi$

$$\rightarrow \begin{Bmatrix} x \\ y \end{Bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix} \begin{Bmatrix} x' \\ y' \end{Bmatrix}$$

$$\overline{v_i} = v_i \sin\gamma\, x' + v_i \cos\gamma\, y' \rightarrow \begin{Bmatrix} x' \\ y' \end{Bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{bmatrix} \begin{Bmatrix} x \\ y \end{Bmatrix}$$

$$= v_i\left(\sin\gamma\cos\phi\, \hat{x} + \sin\gamma\sin\phi\, \hat{y} - \cos\gamma\sin\phi\, \hat{x} + \cos\gamma\cos\phi\, \hat{y}\right)$$

$$\boxed{\begin{aligned} v_{sx} &= v_i(\sin\gamma\cos\phi - \cos\gamma\sin\phi) = 10.256 \text{ km/s} \\ v_{sy} &= v_i(\sin\gamma\sin\phi + \cos\gamma\cos\phi) = -3.733 \text{ km/s} \\ v_{sz} &= 0 \end{aligned}}$$

1b) Numerical simulation of restricted 3-body problem

EOM's:

$$\ddot{x} - 2\Omega\dot{y} - \Omega^2 x = -\frac{\mu_1}{r_1^3}(x + \pi_2 r_{12}) - \frac{\mu_2}{r_2^3}(x - \pi_1 r_{12})$$

$$\ddot{y} + 2\Omega\dot{x} - \Omega^2 y = -\frac{\mu_1}{r_1^3}y - \frac{\mu_2}{r_2^3}y$$

$$\ddot{z} = -\frac{\mu_1}{r_1^3}z - \frac{\mu_2}{r_2^3}z \quad \to 0$$

$r_1 = r_s - r_E \qquad r_2 = r_s - r_M$

→ MATLAB plot at end of solutions

1c) time & distance at closest approach:

- set t_f >> (14 days)

→ Closest approach @ $\boxed{t = 3.18 \text{ days}, \ d = -554.32 \text{ km}}$

well something is wrong!

I can't figure it out though :(

Problem 2 $\bar{r}, \bar{v}$ in ECI to MCS

outline: – used L14 example 1 code

- get $r = \sqrt{\bar{r}\cdot\bar{r}}$, $v = \sqrt{\bar{v}\cdot\bar{v}}$, $v_r = \frac{\bar{r}\cdot\bar{v}}{r}$
- sign v_r indicates direction
- $\bar{h} = \bar{r}\times\bar{v}$, $\bar{n} = \hat{z}\times\bar{h}$

→ get e from vis-viva eqn

- solve i, Ω, ω using $\bar{h}, \bar{n}, \bar{e}$

→ get θ from $\bar{r}, \bar{e}$ & a from $\dfrac{h^2}{\mu(1-e^2)}$

→ MATLAB Script:

for $r = [2500, 16000, 4000]$, $v = [-3, -1, 5]$

$$\boxed{\begin{array}{ll} e = 0.47 & \omega = 22.080 \\ \Omega = 73.74^\circ & \theta = 353.6^\circ \\ i = 62.530 & a = 31161.6 \text{ km} \end{array}}$$

Problem 3 MCS to ECF $\bar{r}$ & $\bar{v}$

perifocal frame pqw

$$\bar{r}_{pqw} = (r\cos\theta)\hat{p} + (r\sin\theta)\hat{q}$$

$$\bar{v}_{pqw} = -\sqrt{\frac{\mu}{a(1-e^2)}}\sin\theta\,\hat{p} + \sqrt{\frac{\mu}{a(1-e^2)}}(e+\cos\theta)\hat{q}$$

→ rotation matrices like in L11 example 1 code

$$T3_{RA} \rightarrow T1_{inc} \rightarrow T3_{\omega}$$

→ $$\boxed{\begin{array}{ll} \bar{r} = [-4806.21,\ 5245.54,\ 3568.86] & \text{km} \\ \bar{v} = [-5.572,\ -4.243,\ -1.13] & \text{km/s} \end{array}}$$

Printout order: P1, P2, P3, Referenced code