Recap

$$c^* \equiv \frac{P_0 A^*}{\dot m}$$

$$C_J \equiv \frac{J}{P_0 A^*}$$

$$(C_J)_{ideal} = \underbrace{\sqrt{\frac{2\gamma^2}{(\gamma-1)}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}\left[1-\left(\frac{P_e}{P_0}\right)^{\frac{\gamma-1}{\gamma}}\right]}}_{\frac{\dot m u_e}{P_0 A^*}} + \left(\frac{P_e}{P_0} - \frac{P_a}{P_0}\right)\frac{A_e}{A^*}$$

$$(C_J)_{conv} = \sqrt{\gamma^2\left(\frac{2}{\gamma+1}\right)^{\frac{2\gamma}{\gamma-1}}} + \left[\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} - \frac{P_a}{P_0}\right]$$

$\rightarrow$ plot $\left.\dfrac{C_J}{C_{J\,conv}}\right]_{ideal}$

To understand Fig. 8

$$- C_J = \sqrt{\frac{2\gamma^2}{(\gamma-1)}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}\left[1-\left(\frac{P_e}{P_0}\right)^{\frac{\gamma-1}{\gamma}}\right]} + \left(\frac{P_e}{P_0} - \frac{P_a}{P_0}\right)\frac{A_e}{A^*}$$

Appears w/ negative sign

Positive sign, want P_e/P_0 as high as possible. physically, extra thrust directly from pressure

$\rightarrow$ want $\frac{P_e}{P_0}$ as small as possible,

physically the lower $\frac{P_e}{P_0}$ the higher u_e

- Also refer to fig. 3 in Liepmann
- take for granted that throat <u>is</u> sonic

  $\Rightarrow$ flow in divergent is supersonic

  so that as $\frac{A_e}{A^*}\uparrow$, $\frac{P_e}{P_0}\downarrow$

- note in fig. 8, $\frac{A_e}{A^*}$ (i.e. $\frac{P_e}{P_0}$) is indep. var.

  in fig. 3, P_a " " "

→ in fig. 8, pick any value of p_a/p_o and stay on that line

→ start @ $\frac{A_e}{A^*}=1$ (no divergent) → $p_e = p^*$

$p_a < p_e$ under expanded

As $\frac{A_e}{A^*}\uparrow$ case k in fig. 3 $\quad p_e < p^*$

$p_a < p_e$ still under expanded

As $\frac{A_e}{A^*}\uparrow$, $\frac{p_e}{p_o}\downarrow$, reach case j, $p_e = p_a$ perfect, isentropic complete expansion

As $\frac{A_e}{A^*}\uparrow$, past $\left(\frac{A_e}{A^*}\right)_{opt}$, $\frac{p_e}{p_o}\downarrow$ $\quad p_e < p_a$

overexpanded

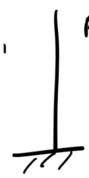
p_adjustments outside nozzle (shocks)

cases g & h in fig. 3

$C_J \downarrow$ from max value

→ @ some value of $\frac{A_e}{A^*}$, normal shock first appears on exit plane, case f in fig. 3

p_{o_1} | $p_{o_2} = p_{shex} = p_a$, dashed line in fig. 8



→ As $\frac{A_e}{A^*}\uparrow$ $\frac{p_e}{p_o}\downarrow$ shock moves into nozzle

case d in fig. 3

Again $p_e = p_a$ w/ discontinuity somewhere in nozzle

Remarks

i) while the presence of a shock in the nozzle causes $\frac{J}{J_{conv}}$ to decrease, the effect is less pronounced than in the absence of a shock. Before shock line, curves drop much more steeply than after shock.

ii) plots of $\frac{J}{J_{conv}}$ vs. $\frac{A_e}{A^*}$ for cases w/ shock in nozzle do <u>not</u> tell where shock is

iii) The lower $\frac{P_a}{P_0}$, the larger (heavier) the $\frac{A_e}{A^*}$ necessary to achieve a perfect expansion. (max thrust)

iv) If $P_a = 0$, there is <u>no</u> max thrust

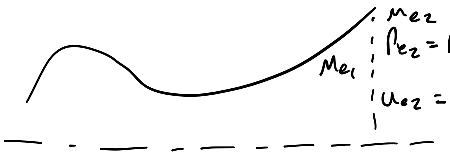
As $\frac{A_e}{A^*} \uparrow$, J increases unbounded

Project 2 (portions of curves w/ shocks)

- in expression for C_J, $P_0 = P_{01}$, A_1^*
- If shock present, $P_e = P_a$

To determine thrust

i) shock on exit plane



$P_{e2} = P_a$, T_{e2}, ρ_{e2}

$u_{e2} = M_{e2} a_2 = M_{e2}\sqrt{\gamma R T_{e2}}$

$$J = \dot{m} u_e + \cancel{(P_e - P_a)}^{\,0} A_e$$

$$J = \dot{m} u_e = \rho_{e2} u_{e2} A_e u_{e2} = A_{e2} u_{e2}^2 \rho_{e2} =$$

$$= A_e M_{e2}^2 \gamma \underbrace{R T_{e2} \rho_{e2}}_{P_{e2}} = \gamma A_e M_{e2}^2 P_{e2}$$

$$C_J = \frac{J}{p_0 A^*} = \gamma M_e^2 \frac{p_{e2}}{p_{01}} \frac{A_e}{A^*_1} = \gamma M_e^2 \underbrace{\frac{p_a}{p_{01}}}_{\text{curve param}} \frac{A_e}{A^*_1}$$

Operationally

- A_e/A^*_1 sets M_{e1}
- M_{e1} sets $\frac{p_{e1}}{p_{01}}$ (isentropic)

- M_{e1} sets $\frac{p_{e2}}{p_{e1}}$ (shock jump)
- $\frac{p_{e2}}{p_0} = \frac{p_{e1}}{p_{01}} \frac{p_{e2}}{p_{e1}} \stackrel{?}{=} \frac{p_a}{p_{01}}$

check true:
- yes → M_{e1} sets M_{e2} (shock jump) → calculate J
- no → iterate $\frac{A_e}{A^*_1}$