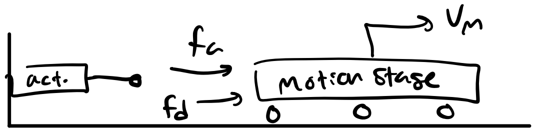
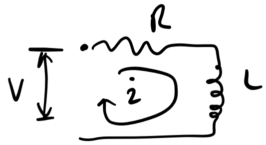
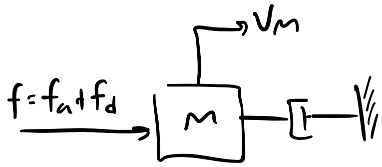


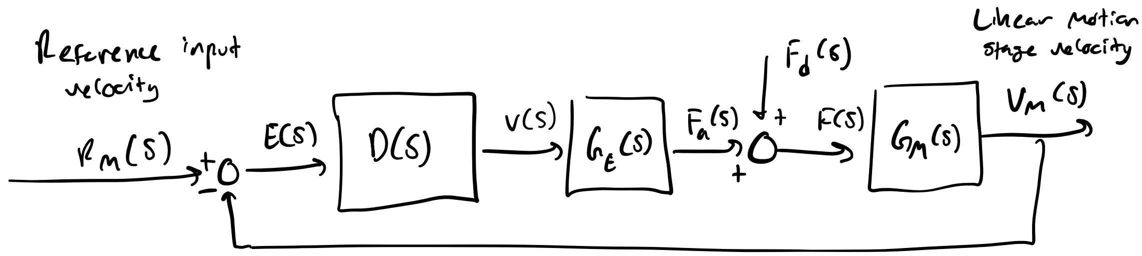
electrical dynamics





$f_a = K_t i \,, \quad L\frac{di}{dt} + Ri = V$

$M\dot{v}_m + cv_m = f$



$G_m(s) = \frac{V(s)}{F(s)} \,, \quad G_e(s) = \frac{F_a(s)}{V(s)} = \frac{F_a(s)}{I(s)} \cdot \frac{I(s)}{V(s)}$

Problem 1

Evaluate transfer functions

a) $G_m(s) = \frac{V(s)}{F(s)}$ $\quad \mathcal{L}[EOM] = \mathcal{L}[M\dot{v}_m + cv_m = f]$

$\rightarrow Ms V_m(s) + cV_m(s) = F(s)$

$(Ms + c) V_m(s) = F(s) \rightarrow \boxed{\frac{V_m(s)}{F(s)} = G_m(s) = \frac{1}{Ms + c}}$

b) $G_E(s) = \frac{F_a(s)}{V(s)}$ $\quad \mathcal{L}[f_a = K_t i]$

$\rightarrow F_a(s) = K_t I(s) \qquad (1)$

$\mathcal{L}[L\frac{di}{dt} + Ri = V]$

$\rightarrow LsI(s) + RI(s) = V(s)$

$I(s) = \frac{V(s)}{Ls + R} \qquad (2)$

plug (2) into (1): $F_a(s) = K_t \dfrac{V(s)}{Ls+R}$

$$\rightarrow \boxed{\frac{F_a(s)}{V(s)} = G_E(s) = \frac{K_t}{Ls+R}}$$

Problem 2 Design $D(s)$ for motion stage

a) Assume $G_E(s) = 1.0$, Find K_p s.t. $D(s) = K_p$ results in $\tau_{CL} = 0.1$

$$\frac{E(s)}{R_m(s)} = \frac{1}{1 + D(s)G_E(s)G_m(s)}, \quad \text{For } G_E(s) = 1.0: \quad \frac{E(s)}{R_m(s)} = \frac{Ms+C}{\underbrace{Ms+(C+K_p)}_{\Delta_{CL}}}$$

$\Delta_{CL} = Ms + (C+K_p)$

$\text{Root}(\Delta_{CL}) = \left[s = -\dfrac{C+K_p}{M}\right] \rightarrow \tau = \dfrac{M}{C+K_p}$

$\tau_{CL} = 0.1 = \dfrac{M}{C+K_p} \rightarrow C+K_p = 10M$

$\hookrightarrow \boxed{K_p = 10M - C}$

matlab: $V_m(t)$ due to $R_m(t) = 1(t)$, verify τ

As anticipated, $X(t=\tau) = 37\%$ away from X_{ss}

b) K_p s.t. $\tau = 0.01 \rightarrow K_p = 100M - C$

$\rightarrow$ verify w/ Matlab – same 37% away

c) – matlab, K_p for both a & b

$\rightarrow G_E \neq 1$, $G_E(s) = \dfrac{K_t}{Ls+R}$

d) open loop trans. functions

$G_{full}(s) = G_E(s)\, G_M(s)$

$G_{approx}(s) = G_M(s)$ rlocus

$K_{p_a} = 4.75$ $K_{p_b} = 49.75$

Looking at these gain values on the root locus plot for G_{full} & G_{approx}, we notice that for $K_p = 4.75$, both CL roots are on the real axis. However, for $K_p = 49.75$, the approx. root locus stays on the real axis while the full system ascends the imaginary axis, resulting in oscillations.

Problem 3

a) PD compensator

1) $\tau_{CL} = 0.01\ [s]$

2) $\zeta = 0.7$ (damping ratio)

- Evaluate K_p, K_d s.t. specs satisfied

→ complete system model (G_{full})

$$\frac{E(s)}{R_m(s)} = \frac{1}{1 + D(s) G_E(s) G_m(s)}, \quad G_E(s) = \frac{K_t}{Ls+R}, \quad G_m(s) = \frac{1}{Ms+C}$$

$$D(s) = K_p + K_d s$$

$$\frac{E(s)}{R_m(s)} = \frac{1}{1 + \frac{K_t}{Ls+R} \cdot \frac{K_p + K_d s}{Ms+C}} = \frac{(Ls+R)(Ms+C)}{\underbrace{LMs^2 + (RM + CL + K_t K_d)s + RC + K_t K_p}_{\Delta(s)}}$$

$$\Delta(s) = s^2 + 2\zeta\omega_n s + \omega_n^2 \rightarrow 2\zeta\omega_n = \frac{RM + CL + K_t K_d}{LM} \quad (1)$$

$$\rightarrow \omega_n^2 = \frac{RC + K_t K_p}{LM} \quad (2)$$

$$\tau = \frac{1}{\xi \omega_n} \rightarrow \omega_n = \frac{1}{\tau \xi} = \frac{1}{(0.01)(0.7)} = \frac{100}{0.7}$$

(1): $K_d = \dfrac{2\xi\omega_n LM - (RM + CL)}{K_t}$

(2) $K_p = \dfrac{\omega_n^2 LM - RC}{K_t}$

– Response is as expected

b) Disturbance rejection

→ limit SS errors due to $f_d(t)$, minimize K_p, K_d

1) $\tau_{cL} \leq 0.01$

2) $\xi = 0.7$

3) $e(\infty) \leq 0.025$ due to $f_d(t) = 10 \cdot 1(t)$

$$\frac{E(s)}{F_d(s)} = \frac{G_m(s)}{1 + D(s) G_E(s) G_m(s)} = \frac{1}{1/G_m(s) + D(s) G_E(s)}$$

$$= \frac{1}{(ms+c) + (K_p + K_d s)\frac{K_t}{Ls+R}} = \frac{Ls+R}{(Ls+R)(ms+c) + K_p K_t + K_d K_t s} \leftarrow \Delta(s)$$

FVT: $F_d(s) = 10 \cdot \frac{1}{s}$

$$e(\infty) = \lim_{s \to 0} \left[\cancel{s} \cdot \frac{\cancel{L}s + R}{(L\cancel{ms^2} + (RM + CL + \cancel{K_t K_d})s + RC + K_t K_p)} \cdot \frac{10}{\cancel{s}} \right]$$

0 0 0

$$e(\infty) = \frac{10R}{(RC + K_t K_p)} \leq 0.025 \quad (3) \qquad \frac{1}{s} < 3$$

SAME CHAR. EQN AS PART a):

$$\Delta(s) = s^2 + 2\xi\omega_n s + \omega_n^2 \rightarrow 2\xi\omega_n = \frac{RM + CL + K_t K_d}{LM} \quad (1)$$

$$\rightarrow \omega_n^2 = \frac{RC + K_t K_p}{LM} \quad (2)$$

From (3): $\dfrac{10R}{0.025} \le RC + K_t K_p$

$$K_p \ge \left(\frac{10R}{0.025} - RC\right)/K_t = 399.75$$

$\rightarrow \boxed{K_p \ge 399.75}$

From (2):

$\tau \le 0.01$, $\tau\xi \le 0.007$, $\omega_n = \dfrac{1}{\tau\xi} \ge \dfrac{1}{0.007}$

$$\omega_n^2 = \frac{RC + K_t K_p}{LM} \ge \left(\frac{1}{0.007}\right)^2$$

$$\rightarrow K_p \ge \frac{\left(\frac{1}{0.007}\right)^2 LM - RC}{K_t} \rightarrow \boxed{K_p \ge 203.8316}$$

$\boxed{\text{Select tighter requirement, } \therefore K_p \ge 399.75}$

From (1): $\omega_n \ge \dfrac{1}{0.007}$, $2\zeta\omega_n \ge \dfrac{1.4}{0.007}$

$$\rightarrow \frac{RM + CL + K_t K_d}{LM} \ge \frac{1.4}{0.007}$$

$$\rightarrow K_d \ge \left(\frac{1.4}{0.007} \cdot LM - RM - CL\right)/K_t$$

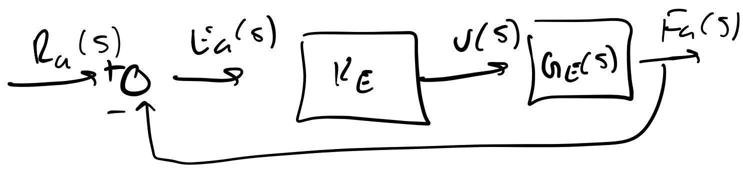
$$\rightarrow \boxed{K_d \ge 1.4950}$$

To minimize K_d, K_p, select smallest value that satisfies specifications: $K_p = 399.75$, $K_d = 1.4950$

$\rightarrow$ matlab plot is as expected with $e(ss) = 0.025$

Problem 4

a)



Find K_E s.t. $\tau_E = \frac{\tau_{cl}}{10} = 0.001\ s$

$$\frac{F_a(s)}{R_a(s)} = \frac{K_E G_E(s)}{1 + K_E G_E(s)} \qquad G_E(s) = \frac{K_t}{Ls+R}$$

$$= \frac{K_E K_t}{Ls + R + K_E K_t} \leftarrow \Delta(s)$$

$$\text{roots}(s) = s = -\frac{R + K_E K_t}{L}$$

$$\tau_E = -\frac{1}{\text{root}(s)} = \frac{L}{R + K_E K_t} = \tau_E$$

$$K_E = \left(\frac{L}{\tau_E} - R\right)/K_t = \boxed{19 = K_E}$$

b) with $\tau_E << \tau_{cl}$, $K_E G_E(s) \approx 1$

$$\frac{V_m(s)}{R_m(s)} = \frac{K_m \frac{1}{ms+c}}{1 + \frac{K_m}{ms+c}} = \frac{K_m}{ms + c + K_m}$$

$$S_{root} = -\frac{c + K_m}{m}$$

$$\tau_{cl} = \frac{m}{c + K_m} \rightarrow \boxed{K_m = \frac{m}{\tau_{cl}} - c = 49.75}$$

c) From matlab sys 3,

$$S_{root} = -893.675,\ -106.3250$$

$$\tau_{full} = -\frac{1}{S_{root}} = 0.0011,\ 0.0094$$

Time constant 0.0011 is very similar to the τ_E of 0.001. Similarly, 0.0094 is very close to τ_{cl} of 0.01.

Table of Contents


```
% Written by Kyle Adler for ME446
```

Problem 2

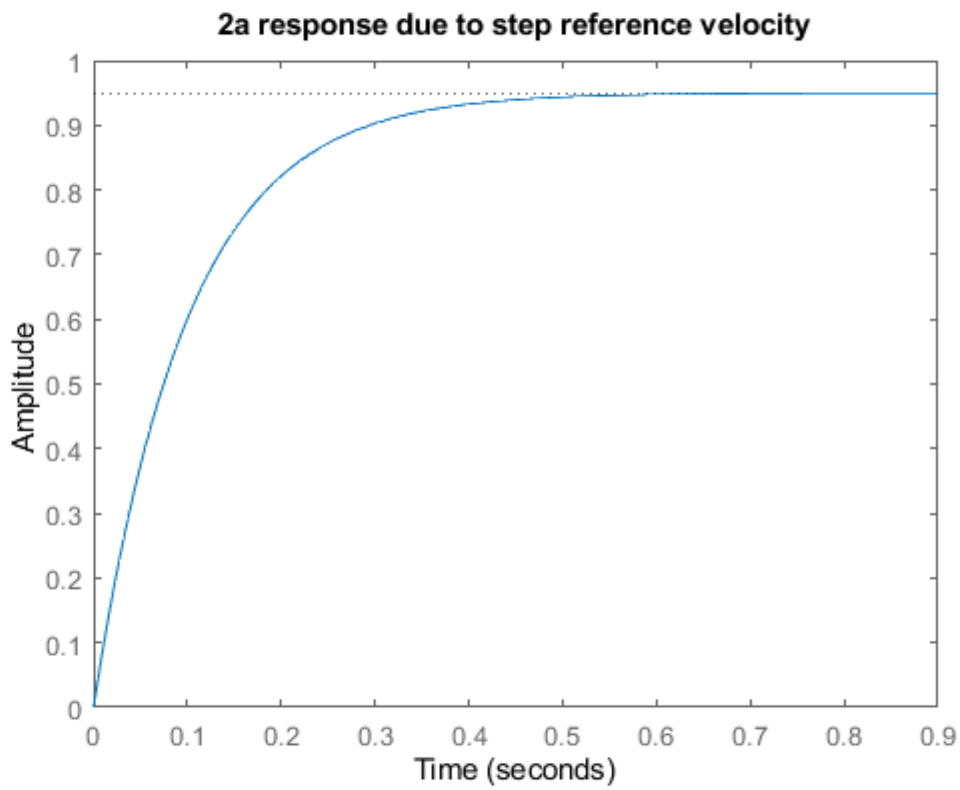
```
% sys params
    m = 0.5
    c = 0.25
    L = 0.2
    R = 10
    Kt = 10

% part a

    % plant
    Kp = 10*m-c % calculated for time const = 0.1
    D = Kp
    Ge = 1
    s = tf('s');
    Gm = 1/(m*s+c)
    sysP1a = feedback(D*Ge*Gm,1) % system output velocity to ref input
    figure(1);
    step(sysP1a) % simulate with step input
    title("2a response due to step reference velocity");

% part b
    Kp = 100*m-c % calculated for time const = 0.01
    D = Kp
    Ge = 1
    s = tf('s');
    Gm = 1/(m*s+c)
    sysP1b = feedback(D*Ge*Gm,1) % system output velocity to ref input
    figure(2);
    step(sysP1b) % simulate with step input
    title("2b higher gain response due to step reference velocity");

% part c
    Kp = 10*m/c % calculated for time const = 0.1
    D = Kp
    s = tf('s');
    Ge = Kt/(L*s+R)
    Gm = 1/(m*s+c)
    sysP1ca = feedback(D*Ge*Gm,1) % system output velocity to ref input
    figure(3);
    step(sysP1ca) % simulate with step input
```

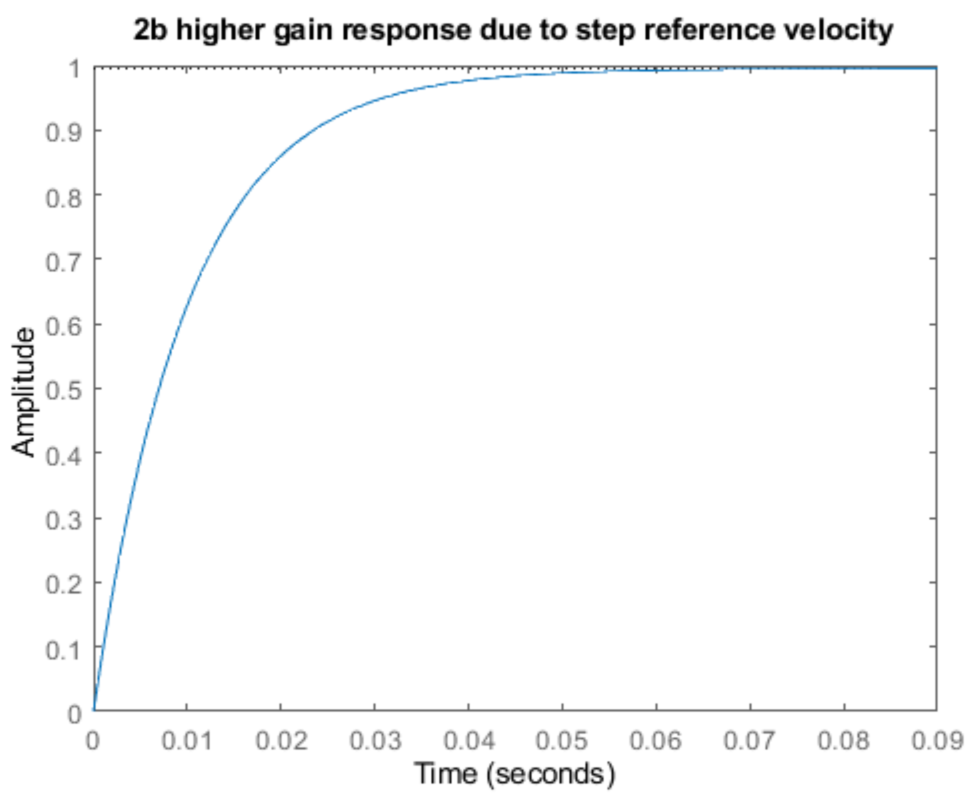
```
    title("2c full system response due to step reference velocity");


    Kp = 100*m/c % calculated for time const = 0.01
    D = Kp
    s = tf('s');
    Ge = Kt/(L*s+R)
    Gm = 1/(m*s+c)
    sysP1cb = feedback(D*Ge*Gm,1) % system output velocity to ref input
    figure(4);
    step(sysP1cb) % simulate with step input
    title("2c full system high gain response due to step reference velocity");

% part d
    Gfull = Ge*Gm
    Gapprox = Gm
    figure(5); rlocus(Gfull)
    title("2d root locus full sys")
    xlim([-50 50])
    ylim([-50 50])
    figure(6); rlocus(Gapprox) % can use second arg with k values to get
 visual range
    title("2d root locus approx sys")
    xlim([-50 50])
    ylim([-50 50])


m =

    0.5000


c =

    0.2500


L =

    0.2000


R =

    10


Kt =

    10


Kp =
```

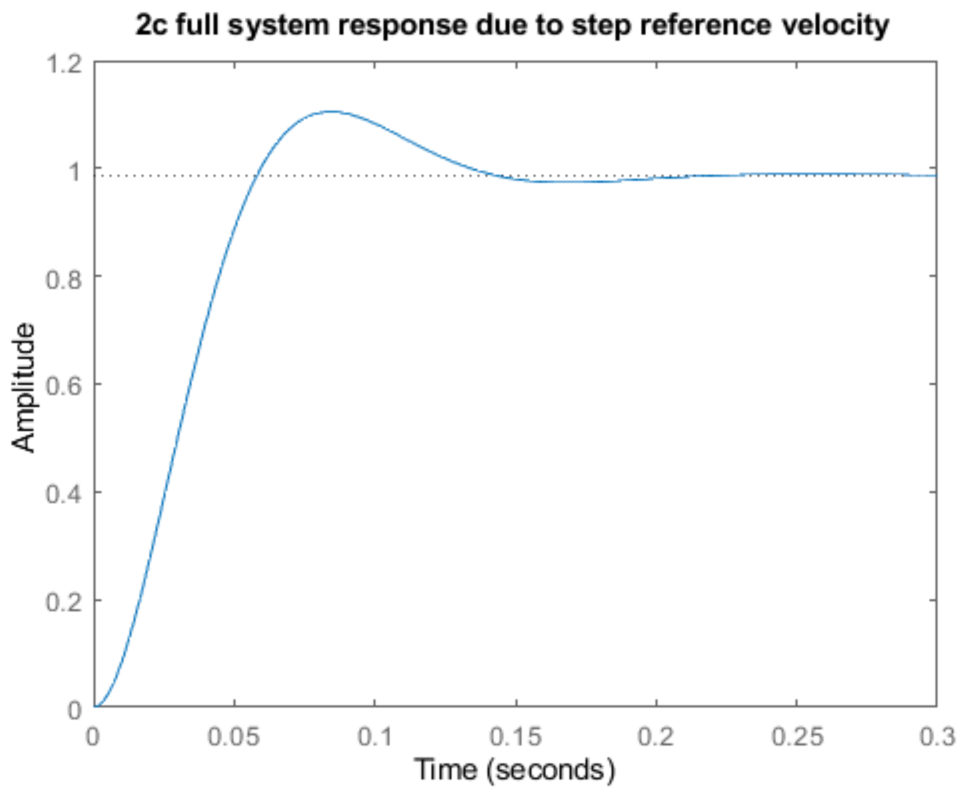
```
    4.7500


D =

    4.7500


Ge =

     1


Gm =

       1
  ------------
  0.5 s + 0.25

Continuous-time transfer function.


sysP1a =

    4.75
  ---------
  0.5 s + 5

Continuous-time transfer function.


Kp =

   49.7500


D =

   49.7500


Ge =

     1


Gm =

       1
  ------------
  0.5 s + 0.25

Continuous-time transfer function.
```

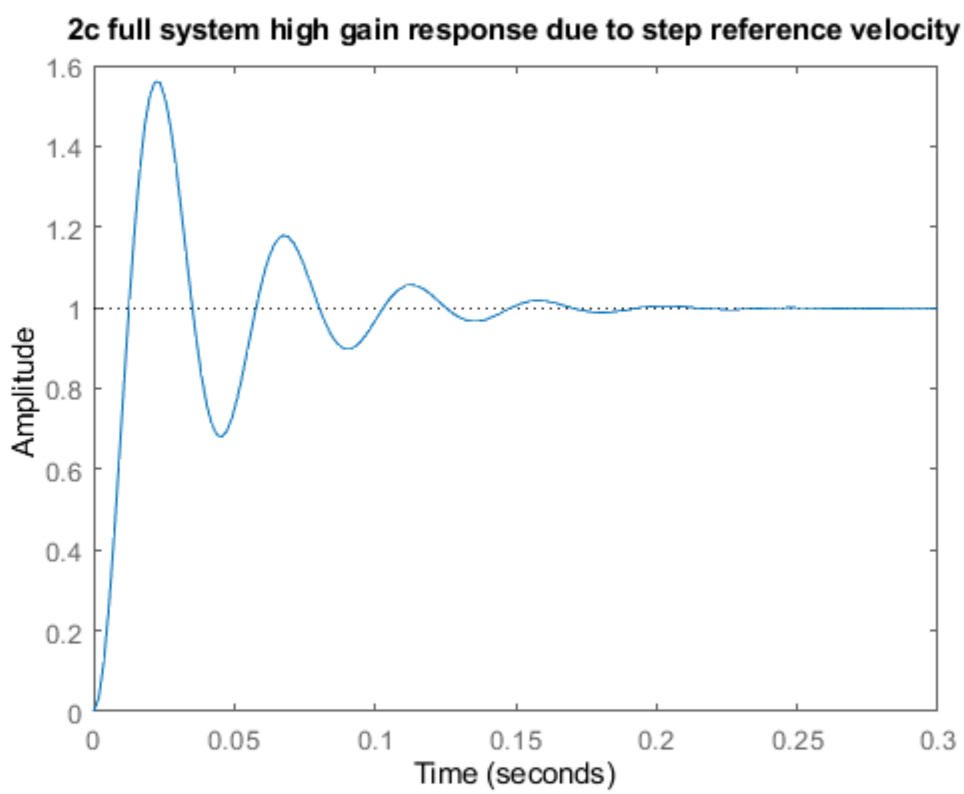
```
sysP1b =

    49.75
  ----------
  0.5 s + 50

Continuous-time transfer function.


Kp =

    20


D =

    20


Ge =

      10
  ----------
  0.2 s + 10

Continuous-time transfer function.


Gm =

       1
  ------------
  0.5 s + 0.25

Continuous-time transfer function.


sysP1ca =

            200
  ------------------------
  0.1 s^2 + 5.05 s + 202.5

Continuous-time transfer function.


Kp =

   200


D =
```

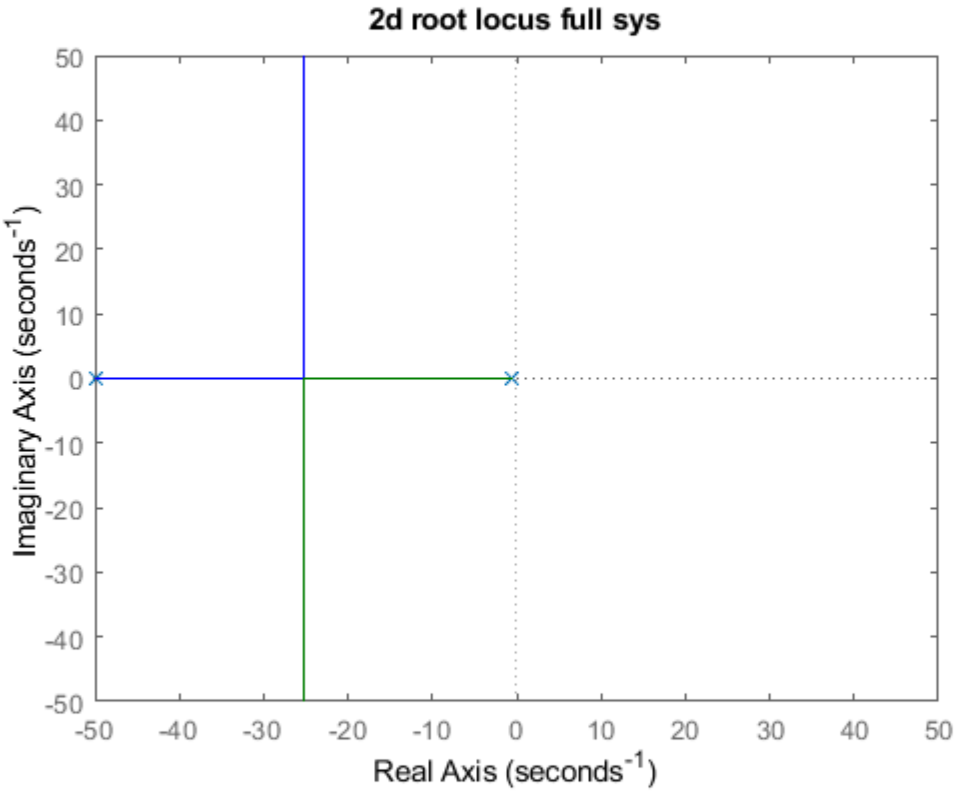
```
   200


Ge =

      10
  ----------
  0.2 s + 10

Continuous-time transfer function.


Gm =

       1
  ------------
  0.5 s + 0.25

Continuous-time transfer function.


sysP1cb =

           2000
  -----------------------
  0.1 s^2 + 5.05 s + 2002

Continuous-time transfer function.


Gfull =

            10
  ----------------------
  0.1 s^2 + 5.05 s + 2.5

Continuous-time transfer function.


Gapprox =

       1
  ------------
  0.5 s + 0.25

Continuous-time transfer function.
```

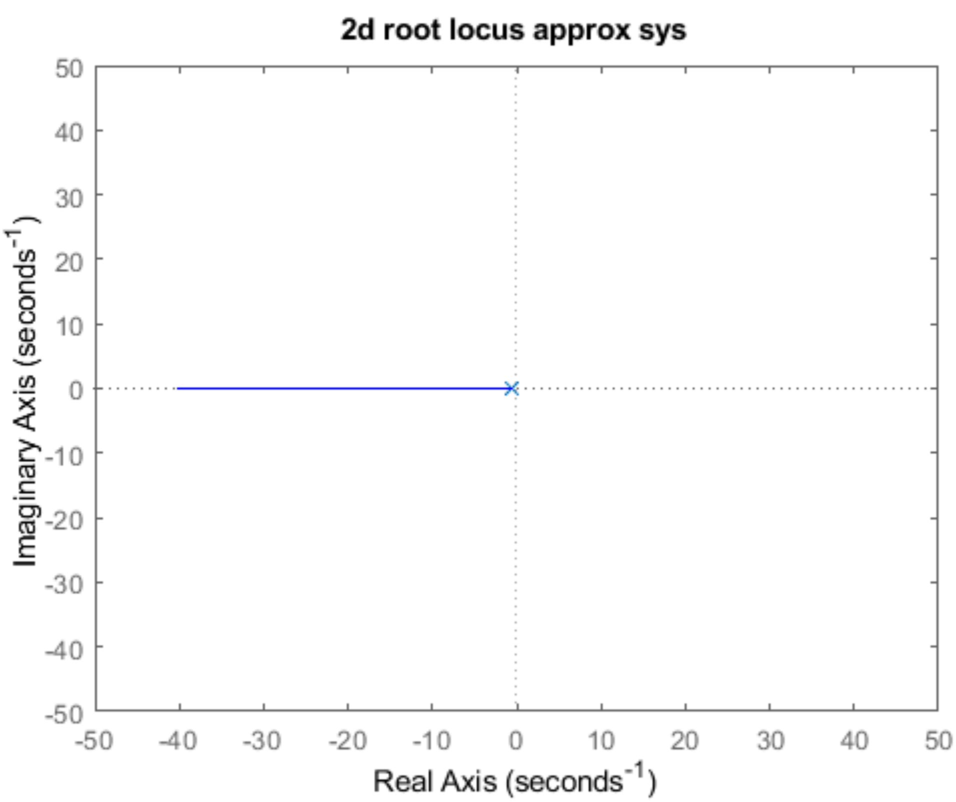

2a response due to step reference velocity
Amplitude
Time (seconds)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1


2b higher gain response due to step reference velocity
Amplitude
Time (seconds)
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1


2c full system response due to step reference velocity
Amplitude
Time (seconds)
0
0.2
0.4
0.6
0.8
1
1.2
0.05
0.1
0.15
0.2
0.25
0.3


2c full system high gain response due to step reference velocity
Amplitude
Time (seconds)
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
0.05
0.1
0.15
0.2
0.25
0.3


2d root locus full sys
Imaginary Axis (seconds^{-1})
Real Axis (seconds^{-1})


2d root locus approx sys
Imaginary Axis (seconds^{-1})
Real Axis (seconds^{-1})

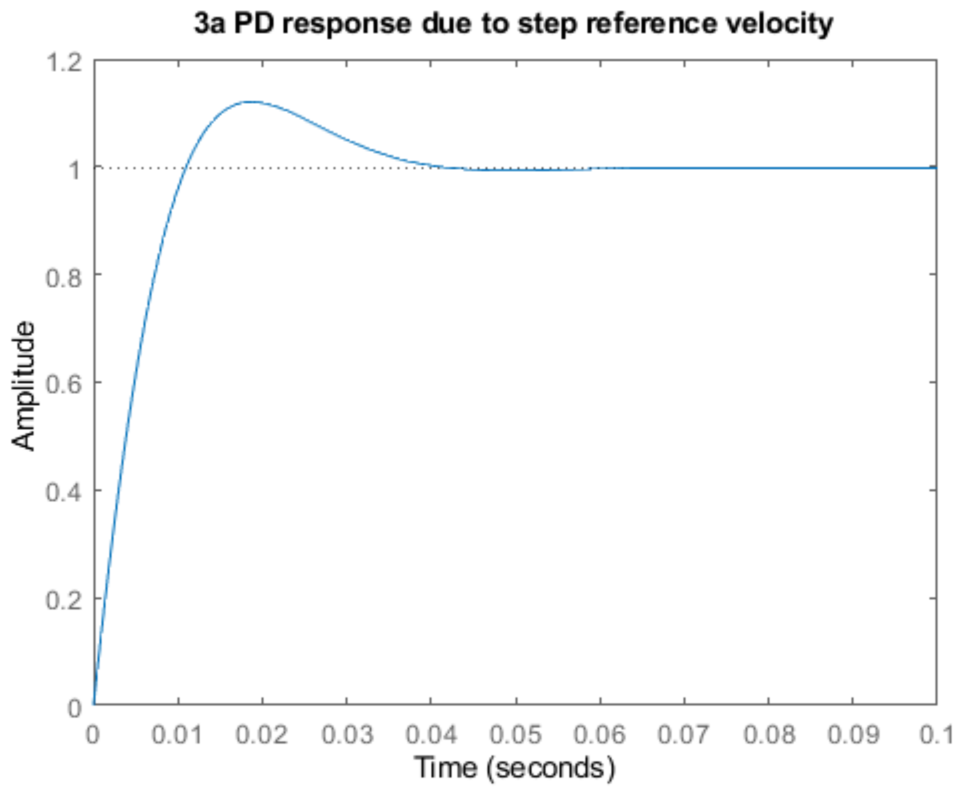
Problem 3

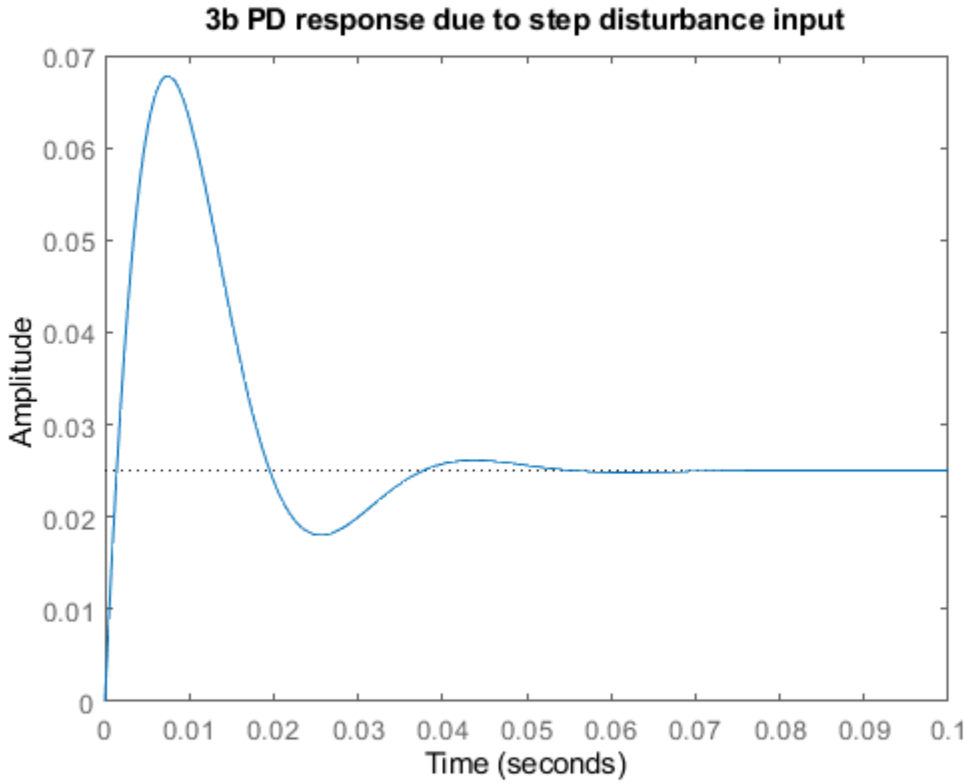
```
clear

% sys params
    m = 0.5
    c = 0.25
    L = 0.2
    R = 10
    Kt = 10
% part a
    z = 0.7; tau = 0.01; % given specs
    wn = 1/(tau*z);
    Kd = (2*z*wn*L*m - (R*m+c*L))/Kt
    Kp = (wn^2*L*m - R*c)/Kt
    s = tf('s');
    D = Kp+Kd*s;
    Ge = Kt/(L*s+R);
    Gm = 1/(m*s+c);
    sys = feedback(D*Ge*Gm,1)
    figure(7); step(sys)
    title("3a PD response due to step reference velocity");
    xlim([0,0.1])
% part b
    Kp_b1 = (10*R/(0.025)-R*c)/Kt
    Kp_b2 = ((1/0.007)^2*L*m-R*c)/Kt
    Kd_b1 = (1.4/0.007*L*m-R*m-c*L)/Kt

    Kp = 399.75
    Kd = 1.4950
    D = Kp+Kd*s;
    Ge = Kt/(L*s+R);
    Gm = 1/(m*s+c);
    sys = feedback(Gm,D*Ge)
    figure(8); step(10*sys) % linearity: 10x input = 10x response
    title("3b PD response due to step disturbance input");
    xlim([0,0.1])
```

```
m =

    0.5000


c =

    0.2500


L =

    0.2000
```

```
R =

    10


Kt =

    10


Kd =

    1.4950


Kp =

  203.8316


sys =

     14.95 s + 2038
  ---------------------
  0.1 s^2 + 20 s + 2041

Continuous-time transfer function.


Kp_b1 =

  399.7500


Kp_b2 =

  203.8316


Kd_b1 =

    1.4950


Kp =

  399.7500


Kd =

    1.4950
```

```
sys =

        0.2 s + 10
  ---------------------
  0.1 s^2 + 20 s + 4000

Continuous-time transfer function.
```





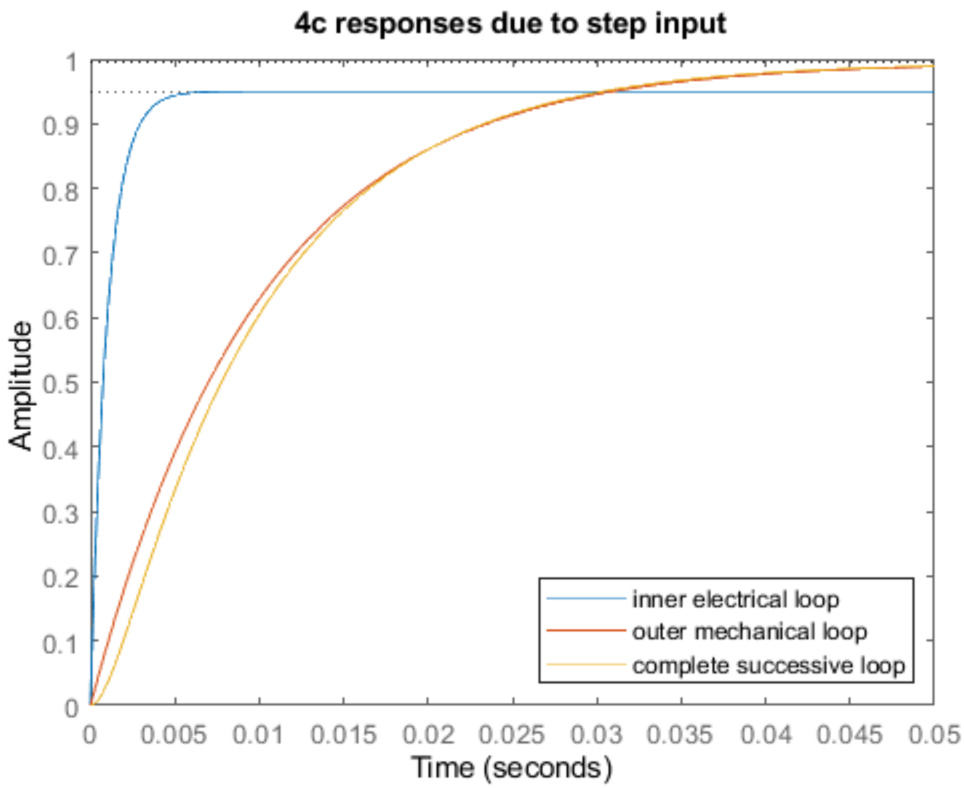
Problem 4

```
% part a
    tcl = 0.01
    te = tcl/10
    Ke = (L/te-R)/Kt

% part b
    Km = m/tcl-c

% part c
    figure(12);hold on
    sys1 = feedback(Ke*Ge,1)
    step(sys1)
    sys2 = feedback(Km*Gm,1)
    step(sys2)
    sys3 = feedback(Km*feedback(Ke*Ge,1)*Gm,1)
    step(sys3)
    title("4c responses due to step input");
    xlim([0,0.05])
    legend('inner electrical loop','outer mechanical loop','complete
 successive loop',Location='southeast')
    hold off

    s = roots([0.1 100 9502])
```

```
    tau_s1 = -1.0/s(1)
    tau_s2 = -1.0/s(2)
```

```
tcl =

    0.0100


te =

   1.0000e-03


Ke =

    19


Km =

   49.7500


sys1 =

      190
  -----------
  0.2 s + 200

Continuous-time transfer function.


sys2 =

    49.75
  ----------
  0.5 s + 50

Continuous-time transfer function.


sys3 =

           9452
  ----------------------
  0.1 s^2 + 100 s + 9502

Continuous-time transfer function.


s =

 -893.6750
```

```
-106.3250


tau_s1 =

    0.0011


tau_s2 =

    0.0094
```



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