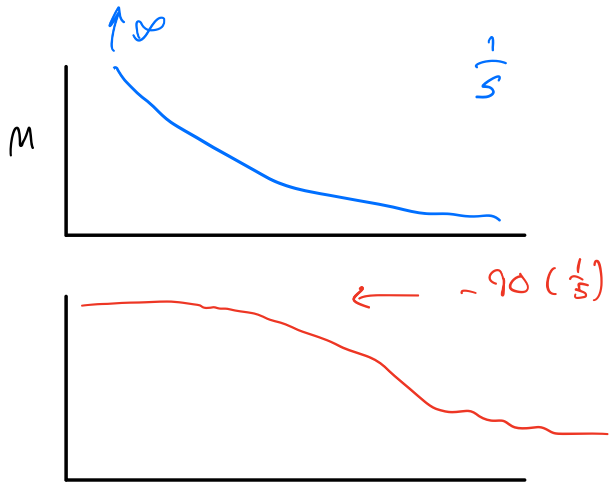
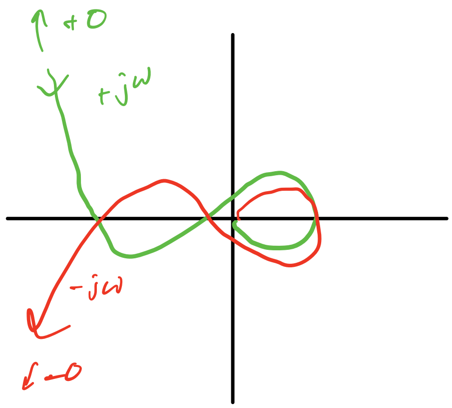


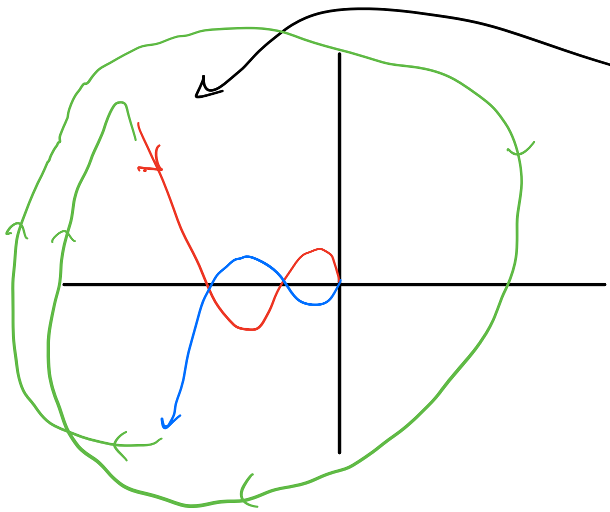
$-90\ (\frac{1}{s})$ $\quad -180\ (1/s^2)$ $\quad -270\ (1/s^3)$



3 OL → $3 \times 180°$

CW $\quad M = \infty$

$\omega = -0 \rightarrow \omega = +0$

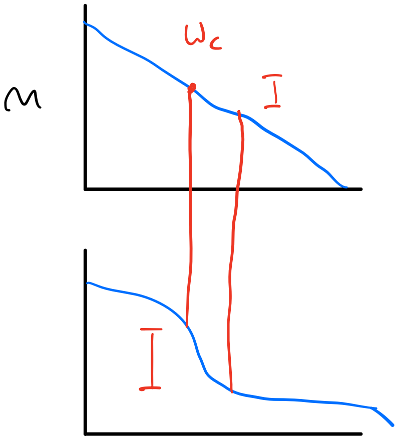


$\frac{1}{s^3} \rightarrow 3 \cdot (-90°)$

$-270°$

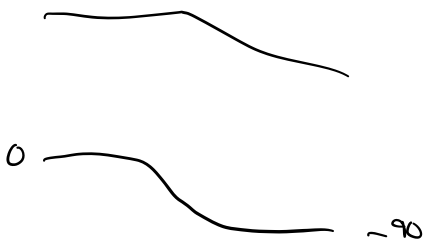
phase at 0^+

→ $3 \cdot 180°$ CW contour



PM → 5

Low pass filter



Example cont'd

$PM = 4.5^\circ \rightarrow \xi \approxeq 0.05 \rightarrow$

oscillations

→ lower ω_c w/ gain

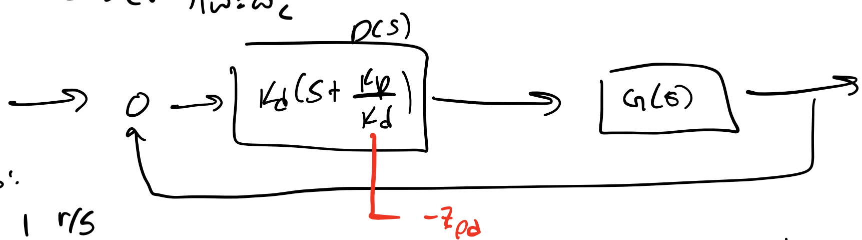
↳ slows response

→ add 50° of phase @ $\omega = 1$

PD compensator:

→ want PD phase $= 50^\circ$ at desired ω_c

$$\angle D(j\omega)\Big|_{\omega=\omega_c} = 0.8727$$



for this:

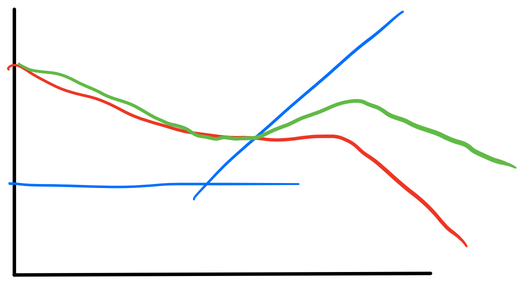
$\omega_c = 1$ r/s

Want $\angle D(j\omega_c) = 50^\circ = 0.8727 = \text{atan}\left(\frac{\omega_c}{z_{pd}}\right)$

(φ_{des}) ; ↙ 1

$$\tan(\varphi_{des}) = \frac{\omega_c}{z_{pd}}$$

$$z_{pd} = \frac{\omega_c}{\tan(\varphi_{des})} = \frac{1}{\tan(0.8727)} \cong 0.839$$



$K_d \rightarrow M = D \cdot G = 1$ @ $\omega = \omega_c$

$$D(s) = K_d(s + 0.839)$$

$$|D(j\omega_c)G(j\omega_c)| = 1 = K_d \frac{100\sqrt{0.839^2 + 1^2}}{1 \cdot \sqrt{0.1^2+1^2} \cdot \sqrt{99^2+2^2}}$$

$$= 1.31\, K_d = 1 \rightarrow K_d \cong 0.76$$

$$\Rightarrow D(s) = 0.76(s + 0.839)$$

→ system now has higher crossover & good PM

→ gain margin too low now due to amplification

Reduce high freq. amplification: low pass filter

$$D(s) = K_d\left(s + \frac{K_p}{K_d}\right)\left(\frac{\omega_b}{s+\omega_b}\right)$$

z_{pd} ↗   ↑ filter break freq.

Lead compensator: combined PD & low pass

$$D_c(s) = K_d\,\omega_b\left(\frac{s+z}{s+\omega_b}\right) = K\left(\frac{s+z}{s+p}\right)$$

where $z < p$