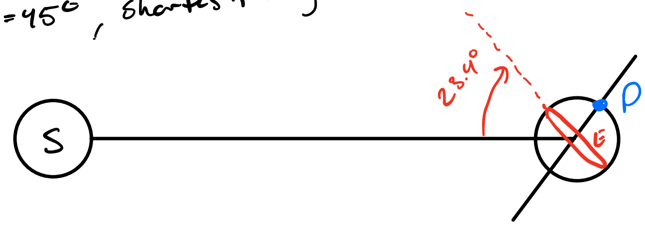
Example: Find sunrise

minneapolis $\phi = 45^\circ$, shortest day

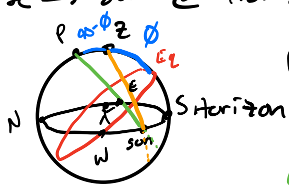


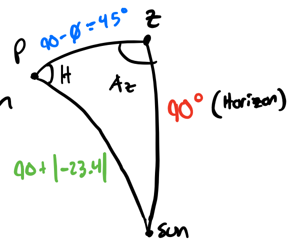
$\delta \equiv$ declination $= i_c = 23.4^\circ$

Find sunrise time (local) and direction

↓ Hour angle H ↓ Azimuth Az

Sunrise → Sun @ Horizon





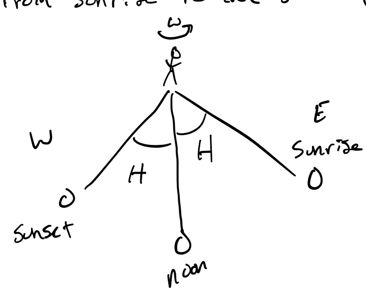
Know all sides → compute H w/ cosines

$$\cos(90^\circ) = \cos(45^\circ)\cos(90+23.4) + \sin(45^\circ)\sin(90+23.4)\cos H$$

$$\therefore \cos H = 0.433 \Rightarrow H = 64.3^\circ$$

$$H = 64.3^\circ \text{ deg}\left(\frac{24\text{hr}}{360^\circ}\right) = 4.29 \text{ hrs} = 4\text{ hr } 17\text{ min}$$

Time from sunrise to "due south" (noon)



General reference → Greenwich

Need to shift to minneapolis

$$-\ 93^\circ 16' \text{ from Greenwich}$$

$$= 93.27^\circ\left(\frac{24\text{hr}}{360^\circ}\right) = 6.217\text{hr}$$

→ 6hr 13 minutes after in M

$\therefore$ sunrise time $= \left[\overset{UT}{12:00} - \underset{\text{delay}}{\text{local time zone}} \right]$

$+ \underset{\text{local time}}{H_{long}} - H \text{ sunrise}$

$\therefore$ CST is 6 hrs behind UT, so sun is due South of Minn. at

$[12:00 \text{ UT} - 6 \text{ hrs}] + 6\text{ hr } 13 \text{ min} = 12:13 \text{ PM}$

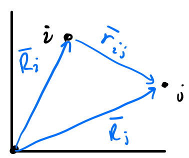
From $12:13 \text{ PM} \pm H \rightarrow$ sunrise $= 12:13 \text{ PM} - 4\text{hr } 17\text{m}$
$= 7:56 \text{ AM CST}$

Try to get $Az = 124.2^\circ$ from cosine law

Two body problem

- see slides for transformation matrices

start w/ n-body problem i^{th} & j^{th} bodies of n-total



Inertial Frame

$\bar{R}_i, \bar{R}_j$ - pos. vectors

$\bar{r}_{ij}$ - relative

$\bar{r}_{ij} = \bar{R}_j - \bar{R}_i \qquad i, j = 1, 2, \ldots, n$

Tools: Newton's 2^{nd}, 3^{rd}, 2^{nd}: $\Sigma \bar{F} = m\bar{a}$, $\bar{F} = G \dfrac{M_1 M_2}{r_{12}^2} \hat{u}_{12}$

$G = 6.67430\,(15) \times 10^{-11}\ m^3/kg\text{-}s^2$

decimal uncertainty

EOM: $\bar{F} = m\bar{a} \qquad m_i \ddot{\bar{R}}_i = G \sum_{\substack{j=1 \\ j \neq i}}^{n} \dfrac{m_i m_j}{r_{ij}^2} \hat{u}_{ij}$

becaus $\bar{r}_{ij} = -\bar{r}_{ji} \rightarrow$ use to sum over all bodies

$\sum_{i=1}^{n} m_i \ddot{\bar{R}}_i = \bar{0}$ due to Newton's 3^{rd} law

integrate twice, introduce $\underset{\text{C.O.M.}}{\bar{R}_g} = \dfrac{\Sigma m_i \bar{R}_i}{\Sigma m_i}$

$\therefore \bar{R}_{com}(t) = \overset{(3)}{\bar{C}_1} t + \overset{(3)}{\bar{C}_2}$

constants of int.

$\boxed{\text{Center of Sys. moves in a straight line}} \rightarrow$

think about rotations for rel. motion: think $\bar{R} \times \bar{F} = \bar{R} \times m\bar{a}$

- vector product

$$\bar{R}_i \times m_i \ddot{\bar{R}}_i = G \sum \frac{m_i m_j}{r_{ij}^2} \left(\bar{R}_i \times \underbrace{\bar{r}_{ij}}_{-\bar{R}_i \times \bar{R}_j} \right) \quad \bar{r}_{ij} = \bar{R}_j - \bar{R}_i$$

& All these cancel !

use: $\bar{R} \times m\bar{a} = \bar{R} \times m \frac{d\bar{v}}{dt}$

use: $\frac{d}{dt}\left(\bar{R}_i \times m \dot{\bar{R}}_i \right) = \bar{R}_i \times m_i \ddot{\bar{R}}_i = 0$ when summed over all bodies

$$\therefore \underbrace{\sum_{i=1}^{n} \left(R_i \times m \underbrace{\dot{\bar{R}}_i}_{\text{linear momentum}} \right)}_{\text{Angular momentum}} = \bar{C}_3$$

$$\boxed{\text{Thus angular momentum is conserved}}$$

Gravity conservative $\exists$ an energy function V

$$\bar{F}_i = m_i \ddot{\bar{R}}_i = -\underbrace{\bar{\nabla}}_{\text{gradient}} V = -\frac{\partial V}{\partial \bar{R}_i}$$

e.g. spring $V = \frac{1}{2} k x^2$

$\rightarrow$ Force $= \frac{dV}{dx} = kx$

$F = -\bar{\nabla} V$

take scalar product w/ $\dot{\bar{R}}_i$

$$\underbrace{\sum_{i=1}^{n} m_i \ddot{\bar{R}}_i \cdot \dot{\bar{R}}_i}_{\frac{\partial T}{\partial t}} = \sum_{i=1}^{n} -\frac{\partial V}{\partial \bar{R}_i} \cdot \frac{\partial \bar{R}_i}{\partial t}$$

$$= -\frac{\partial V}{\partial t}$$

Recall KE: $T = \frac{1}{2} \sum_{i=1}^{n} m_i \dot{\bar{R}}_i \cdot \dot{\bar{R}}_i$

integrate: $\rightarrow \boxed{T + V = C_4}$

$$V = -\frac{G}{2} \sum_{\substack{i=1 \\ i \neq j}}^{n} \sum_{\substack{j=1 \\ j \neq i}}^{n} \frac{m_i m_j}{r_{ij}}$$

$\rightarrow$ $\boxed{\text{Energy is conserved}}$

Need relative dynamics $\rightarrow$ for how each body acts