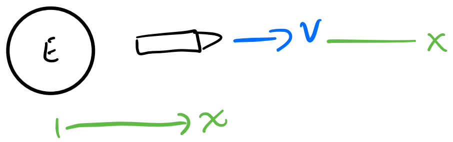
<u>Example</u> Rocket escape FBD = KD



(Booster off)

"EOM" ≡ Equation of motion

2^{nd} order $\bar{x}$, $\dot{\bar{x}}$, $\ddot{\bar{x}}$

Differential equation

$$\left.\begin{aligned} \Sigma\bar{F} &= m\bar{a} \\ \Sigma\bar{M} &= \dot{\bar{H}}_g \end{aligned}\right\}$$ use to get EOM

$$\bar{F}_g = m\bar{a}$$

EOM

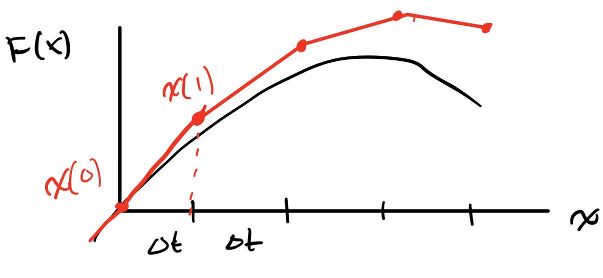
$$-\frac{G M_e M}{x^2} = m\ddot{x} \rightarrow \boxed{m\ddot{x} + \frac{G M_e M}{x^2} = 0}$$

<u>To solve numerically:</u>

Euler integration → First order method

$$\dot{z} = f(z) \quad (\text{e.g. } \dot{x} = x) \quad (x = e^t)$$

$$x_{n+1} = x_n + \Delta t\, f(t, x_n)$$ 1^{st} order deriv. expansion



e.g. $\dot{x} = x$

$i=0$ $x(0) = 1$

$i=1$ $x(1) =$

$i=2$

to reduce error: - reduce Δt
- higher order method

-> 4^{th}-5^{th} order R-K method

some methods adaptive -> optimizes based on order of time

Prep rocket for ode45 (1^{st} method)

$$m\ddot{x} + G\frac{M_E m}{x^2} = 0$$

Apply change of variables:

let $z = \begin{Bmatrix} x \\ \dot{x} \end{Bmatrix}$ then $\dot{z} = \begin{Bmatrix} \dot{x} \\ \ddot{x} \end{Bmatrix} = \begin{Bmatrix} z(2) \\ \end{Bmatrix}$

-> Rearrange: $\ddot{x} = \dfrac{-GM_E}{x^2}$

$$\dot{z} = \begin{Bmatrix} \dot{x} \\ \dfrac{-GM_E}{x^2} \end{Bmatrix} = \begin{Bmatrix} z(2) \\ \dfrac{-GM_E}{z(1)^2} \end{Bmatrix} = f(z)$$

ODE45 recipe

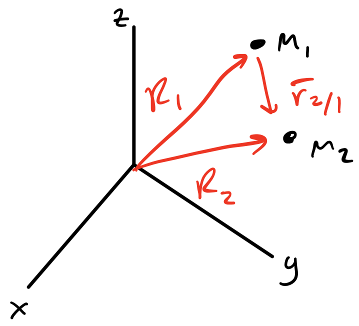
1) Need IC's
2) function file / handle for $\dot{z} = f(z)$
3) Call ode45 $[t, z_{out}] =$ ode45 (odefun, tspan, z_0)
4) Need to extract / plot

Orbits → 3D

Two body:

$$m_1 \ddot{\bar{R}}_1 + m_2 \ddot{\bar{R}}_2 = 0$$

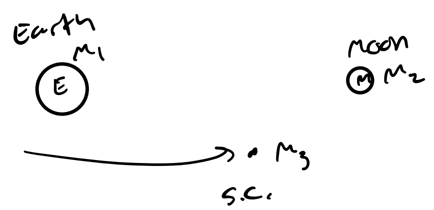
$$\frac{G m_1 m_2}{r^3} \bar{r}_{2/1} = m_1 \ddot{\bar{r}}_1$$



Setup for ode 45

$$\text{Let } \bar{y} = \begin{Bmatrix} \bar{R}_1 \\ \bar{R}_2 \\ \dot{\bar{R}}_1 \\ \dot{\bar{R}}_2 \end{Bmatrix} = \underbrace{\begin{matrix} R_{1x} \\ R_{1y} \\ R_{1z} \\ R_{2x} \\ \vdots \\ \dot{R}_{2z} \end{matrix}}_{12\times 1}, \quad \dot{\bar{y}} = \begin{Bmatrix} \dot{\bar{R}}_1 \\ \dot{\bar{R}}_2 \\ \ddot{\bar{R}}_1 \\ \ddot{\bar{R}}_2 \end{Bmatrix}$$

3-body problem

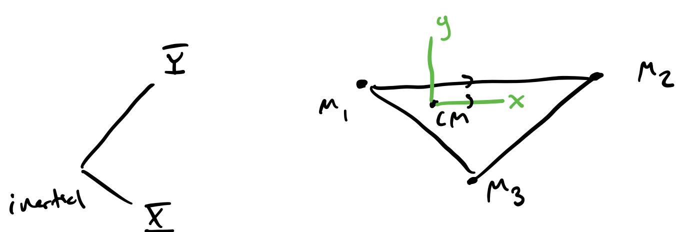


EOM solutions to 3-body

- no closed form
- chotic (IC dependent)
- see youtube

Chotic solutions not useful

→ study to find "stable" solution



try to study w/ dynamic

let xyz be attached to CM of system

Assumption

- bodies rotate @ constant angular rate in same plane (like earth, moon about sun)

- Let xyz rotate w/ ω and x axis // to $\bar{r}_{12}$
- seek stable equilibrium

Use EOM based on $\bar{F} = m\bar{a}$, $\bar{F} = \frac{G m_1 m_2}{r^2}\bar{r}_{12}$

- use $\underline{\bar{XYZ}}$ inertial

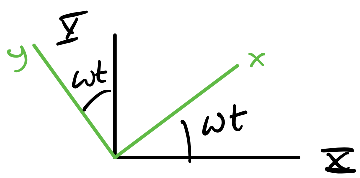
$$m_i \ddot{\bar{R}}_i = G \sum_{j=1,\, j\neq i}^{3} \frac{m_i \cdot m_j}{r_{ij}^3} \bar{r}_{ij}$$

→ scalar form:

$$\ddot{X}_k = G \sum_{j\neq k} \frac{m_j}{r_{jk}^3}(X_j - X_k)$$ similar for Y, Z

Relate absolute $X \to x$, etc.

Recall $\omega = \text{const}$



$X_k = x_k \cos\omega t - y_k \sin\omega t$

$Y_k = x_k \sin\omega t + y_k \cos\omega t$

Sub into EOM to get in $\underline{\bar{XYZ}}$

→

$$\boxed{\begin{aligned}
(\ddot{x}_k - 2\dot{y}_k\omega - x_k\omega^2) &= G\sum_{j\neq k}\frac{m_j}{r_{jk}^3}(x_j - x_k)\\
(\ddot{y}_k + 2\dot{x}_k\omega - y_k\omega^2) &= G\sum_{j\neq k}\frac{m_j}{r_{jk}^3}(y_j - y_k)\\
\ddot{z}_k &= G\sum_{j\neq k}\frac{m_j}{r_{jk}^3}(z_j - z_k)
\end{aligned}}$$

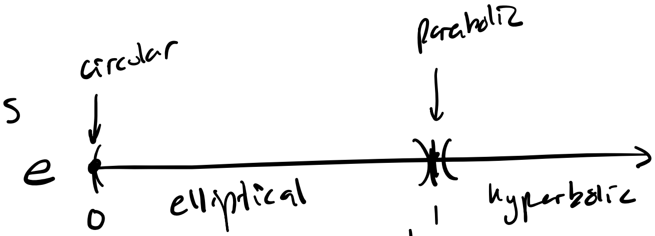
Solve to find stable points

Solutions -> next time

Quick quiz

Speed of orbits

consider:



$v^2 = \frac{\mu}{r}$ | $v^2 = \frac{2\mu}{r} - \frac{\mu}{a}$ | $v^2 = \frac{2\mu}{r}$ | $v^2 = \frac{2\mu}{r} + \frac{\mu}{a}$

try in matlab

lec 9 slide 2