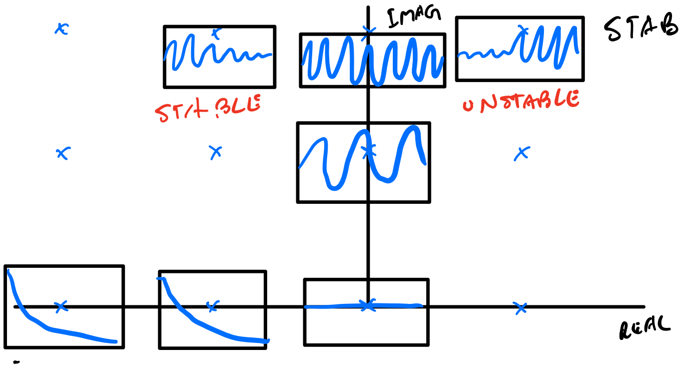
STABILITY & RESPONSE CHARACTERISTICS

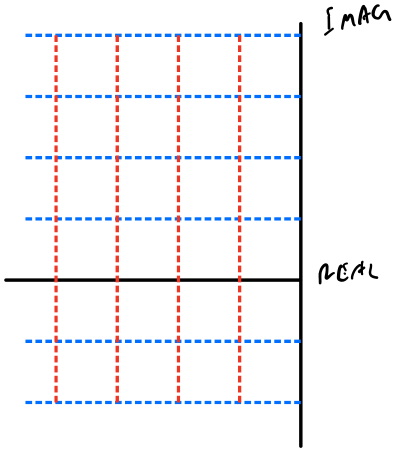


↑ OSCILLATION

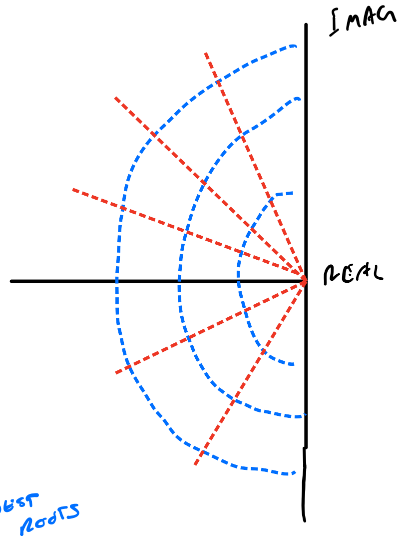
→ SPEED OF RESPONSE

REAL ROOT - GROWS WITHOUT BOUND

LINES OF CONSTANT ω & σ



LINES OF CONSTANT ω_n & ζ



DOMINANT ROOT APPROXIMATION

GENERALLY THE SLOWEST ROOTS

- SYSTEMS OF ORDER >2: COMBINATION OF ROOTS THAT ARE:
  - DISTINCT REAL (i.e. first order)
  - DISTINCT COMPLEX CONJUGATE PAIRS (i.e. 2nd order w/ oscillations)

APPROXIMATION

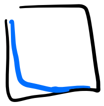
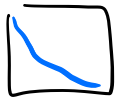
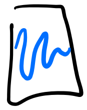
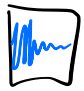
- USE THE ROOTS HAVING THE LARGEST TIME CONSTANT (SMALLEST REAL PART) TO ESTIMATE RESPONSE
- APPROX. IS VERY SOUND AS LONG AS DOM. ROOTS IS INDEED VERY DOMINANT

$$T(s) = \frac{x(s)}{F(s)} = \frac{(s+z_1)(s+z_2)\ldots(s+z_m)}{(s+p_1)(s+p_2)\ldots(s+p_n)}$$

RESPONSE TO STEP INPUT: $u_s(t) \rightarrow F(s) = \frac{1}{s}$

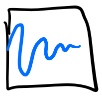
$$X(s) = \frac{C_0}{s} + \underbrace{\frac{C_1}{(s+p_1)} + \dots + \frac{C_n}{(s+p_n)}}$$

x_{ss}

$x(t) = x_{ss}$ +  +  +  + ... + 

SLOWEST ROOT
AS MEASURED BY REAL PART

APPROX:

$x(t) \cong x_{ss}$ +  + ...

EX.

$$\frac{d^4y}{dt^4} + 15\frac{d^3y}{dt^3} + 75\frac{d^2y}{dt^2} + 145\frac{dy}{dt} + 84y = F$$

$$\frac{Y(s)}{F(s)} = \frac{1}{s^4 + 15s^3 + 75s^2 + 145s + 84} \leftarrow \Delta(s)$$

ROOTS([1 15 75 145 84])

$r = \dots\dots + -1$

E.G. $r = \left.\begin{matrix} -100 \\ -30 \\ -20 \pm 20i \\ -1 \end{matrix}\right\}$

↑ SMALLEST REAL PART

$$\boxed{\frac{Y(s)}{F(s)} = \frac{C}{s+1}}$$

$1/\cancel{s}$

FINAL VALUE THEOREM

$$y(\infty) = \lim_{s\to 0}\left[\cancel{s}\,\frac{Y(s)}{F(s)} \cdot F(s)\right] = \frac{1}{84}$$

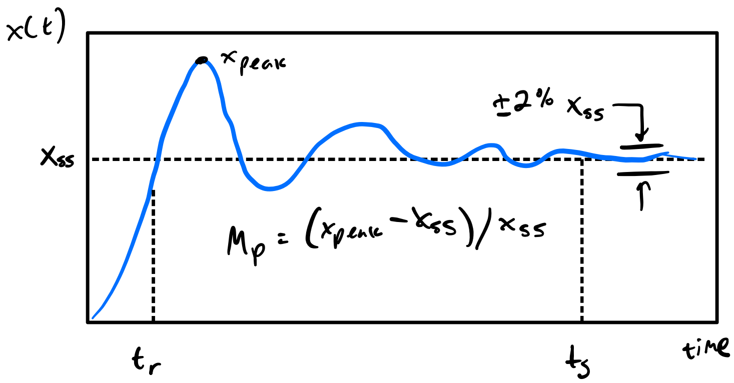
$$y(\infty) = \lim_{s\to 0}\left[\cancel{s}\,\frac{C}{s+1}\,\frac{1}{\cancel{s}}\right] = C$$

$$C = \frac{1}{84}$$

$$\frac{Y(s)}{F(s)} = \frac{\frac{1}{84}}{s+1}, \qquad Y(s)(s+1) = F(s)\,\frac{1}{84}$$

$$\dot{y} + y = \frac{1}{84} f$$

TIME DOMAIN PERFORMANCE SPECIFICATIONS



t_r : Rise time

t_s : Settle time

M_p : overshoot

$f = KF$

$$\ddot{x} + \frac{c}{m}\dot{x} + \frac{k}{m}x = \frac{k}{m} f$$

$$\left(s^2 + \frac{c}{m}s + \frac{k}{m}\right)X(s) = \frac{k}{m}F(s)$$

NORMALIZED : $\frac{c}{m} = 2\zeta\omega_n$, $\frac{k}{m} = \omega_n^2$

Assume $0 \le \zeta \le 1$

$$\ldots \longrightarrow x(t) = 1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin(\omega_d t + \varphi)$$

$$\omega_d = \omega_n\sqrt{1-\zeta^2} \quad \& \quad \varphi = \cos^{-1}(\zeta)$$