EMA 550 HW3 Kyle Adler

Problem 1 Satellite orbiting sun, $a = 10$ AU, $e = 2.4$, passed periapsis 1 yr ago. Find: 1a – 1e

1a) mean angular speed around sun

mean angular rate $n \equiv \frac{2\pi}{T} = \left(\frac{\mu}{a^3}\right)^{1/2}$ $\mu = GM$

$\mu = (6.67\times10^{-11}\ m^3/kg\text{-}s^2)(1.989\times10^{30}\ kg) = 1.3275\times10^{20}\ m^3/s^2$

$a = 1.496\times10^{12} \rightarrow \left(\frac{\mu}{a^3}\right)^{1/2} = \boxed{n = 6.296\times10^{-9}\ \frac{1}{s}}$

1b) mean anomaly $M_H = M$

$M = nt$ (t = time since t_p (perigee))

$\rightarrow M = (6.296\times10^{-9}\ 1/s)(31536000\ s) = \boxed{0.1986\ \text{rad} = M}$

1c) Eccentric anomaly

$M_H = e\sinh H - H$ $\rightarrow$ iterate w/ MATLAB

$\rightarrow \boxed{H = 0.14104}$

1d) Distance from sun

$\cosh H = \frac{r+a}{ae} \rightarrow ae\cosh H - a = r$

$\rightarrow r = (10\ AU)(2.4)\cosh(0.14104) - (10\ AU)$

$\boxed{r = 2.130\times10^{12}\ m}$

1e) True anomaly

$\tan\frac{\theta}{2} = \sqrt{\frac{e+1}{e-1}}\tanh\left(\frac{H}{2}\right) = \sqrt{\frac{3.4}{1.4}}\tanh\left(\frac{0.14104}{2}\right) = 0.10971$

$\rightarrow \theta = 2\,\text{atan}(0.10971) = \boxed{0.21856\ \text{rad} = \theta}$

Problem 2 Comet parabolic around Sun. $R_p = 5E6$ km

2a) Speed @ p

parabolic: $e=1$ $\quad v = \sqrt{\frac{2\mu}{r}}$, same μ as P1: 1.3275×10^{20} m^3/s^2

$$\rightarrow v = \sqrt{\frac{2(1.3275\times10^{20})}{5E9}} = \boxed{230.4 \text{ km/s} = v \text{ @ periapsis}}$$

2b) time spent within 150×10^6 km $\quad p = 2R_p$

$$r = \frac{p}{1+\cos\theta} \rightarrow \cos\theta = \frac{p}{r} - 1 = \frac{2R_p}{r} - 1$$

$$\rightarrow \theta = a\cos(-0.9333) = 2.774 \text{ rad}$$

$$2\sqrt{\frac{\mu}{p^3}}(t - \cancel{t_p}) = \tan\frac{\theta}{2} + \frac{1}{3}\tan^3\left(\frac{\theta}{2}\right)$$

$$\rightarrow t = 2.493\times10^6 \text{ s}$$

Symmetric orbit: $\Delta t = 2\cdot t = \boxed{4.98\times10^6 \text{ s} = 57.7 \text{ days} = \Delta t}$

Problem 3 Spacecraft in circular orbit around Earth, alt. 500 km

boost → hyperbolic, $v + 75\%$

3a) Find speed increase

Circular: $v = \sqrt{\frac{\mu}{r}}$, $\mu_E = 3.986\times10^5$ km^3/s^2

$r = R_E + alt = 6878$ km

$$\rightarrow v_1 = \sqrt{\frac{3.986\times10^5}{6878}} = 7.61 \text{ km/s} = v_1$$

$$\rightarrow v_2 = 1.75\,v_1 = 13.322 \text{ km/s}$$

$$\rightarrow \boxed{\Delta v = v_2 - v_1 = 5.71 \text{ km/s}}$$

3b) Find a, e of hyperbolic

vis viva: $v = \sqrt{\mu\left(\frac{2}{r} + \frac{1}{a}\right)}$

$\rightarrow \quad v^2 = \frac{2\mu}{r} + \frac{\mu}{a} \quad \rightarrow \quad \frac{\mu}{a} = v^2 - \frac{2\mu}{r}$

$\rightarrow \quad a = \mu \cdot \frac{1}{v^2 - \frac{2\mu}{r}} \quad \rightarrow \quad = \frac{1}{\frac{v^2}{\mu} - \frac{2}{r}} = \boxed{6473 \text{ km} = a}$

For $\theta = 0$, $\quad r = \frac{a(e^2-1)}{1+e} = \frac{a\cancel{(e+1)}(e-1)}{\cancel{(1+e)}} = a(e-1)$

$\rightarrow \quad e = \frac{r}{a} + 1 = \frac{6878}{6473} + 1 = \boxed{2.062 = e}$

3c) How long to reach $r = 384{,}000$ km

plan: $H \rightarrow M_H \rightarrow t$

$\cosh H = \frac{r+a}{ae} = \frac{384000 + 6473}{6473 \cdot 2.062}$

$\rightarrow H = \operatorname{acosh}(29.25) = 4.069$ rad

$M_H = e\sinh H - H = 56.22 = \sqrt{\frac{\mu}{a^3}}\, t$

$\rightarrow t = \sqrt{\frac{a^3}{\mu}} \cdot 56.22 = 46374 \text{ s} = \boxed{12.88 \text{ hrs} = t}$

Problem 4 @ t_0, 1000 kg sat orbiting Earth w/ $\bar{r}_0, \bar{v}_0$.

Solve numerically to find max altitude & plot.

EOM: $\ddot{x} + \frac{\mu x}{x^3} = 0 \rightarrow \ddot{\bar{r}} + \frac{\mu}{r^3}\bar{r} = 0$

$\rightarrow$ max alt. $= 9686$ km @ $t = 1.695$ hrs