Recap 3-body problem

Recall: Assume system w/ All bodies rotate w/ Ω

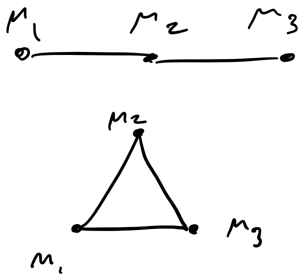
EOM's:

$$(\ddot{x}_k - 2\dot{y}_k\omega - x_k\omega^2) = G\sum_{j\neq k}\frac{m_j}{r_{jk}^3}(x_j - x_k)$$

$$(\ddot{y}_k + 2\dot{x}_k\omega - y_k\omega^2) = G\sum_{j\neq k}\frac{m_j}{r_{jk}^3}(y_j - y_k)$$

$$\ddot{z}_k = G\sum_{j\neq k}\frac{m_j}{r_{jk}^3}(z_j - z_k)$$

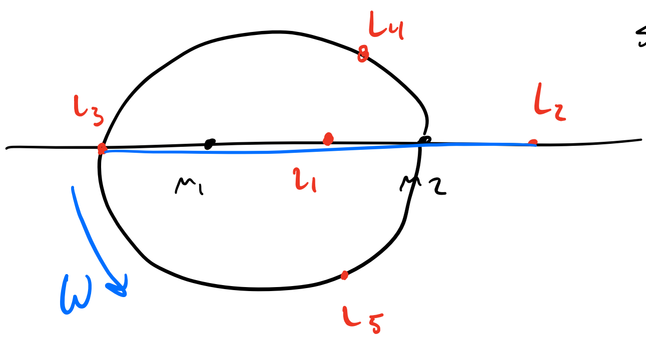
2 solutions:



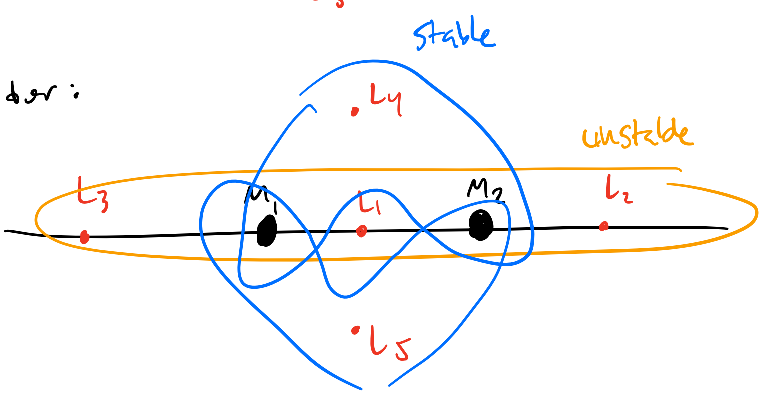
Quintic equation $\rightarrow$ 5 "lagrange points" L1 - L5

$M_1 >> M_2 >> M_3$

Stable positions for m_3?



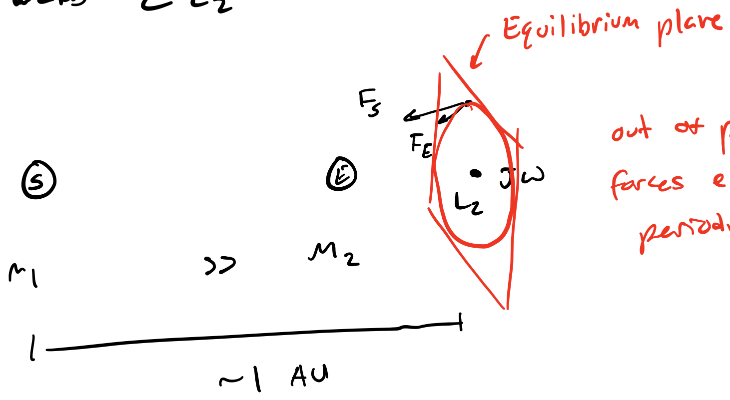
Consider:



→ L_1, L_2, L_3 → unstable

→ L_4, L_5 → stable if $\frac{m_1}{m_2} > 24.96$ (Sun-Earth ✓)

James Webb @ L_2



out of plane forces establish periodic halo orbit

We need 3D orbits

$$r \to \bar{r} = r_x \hat{i} + r_y \hat{j} + r_z \hat{k}$$

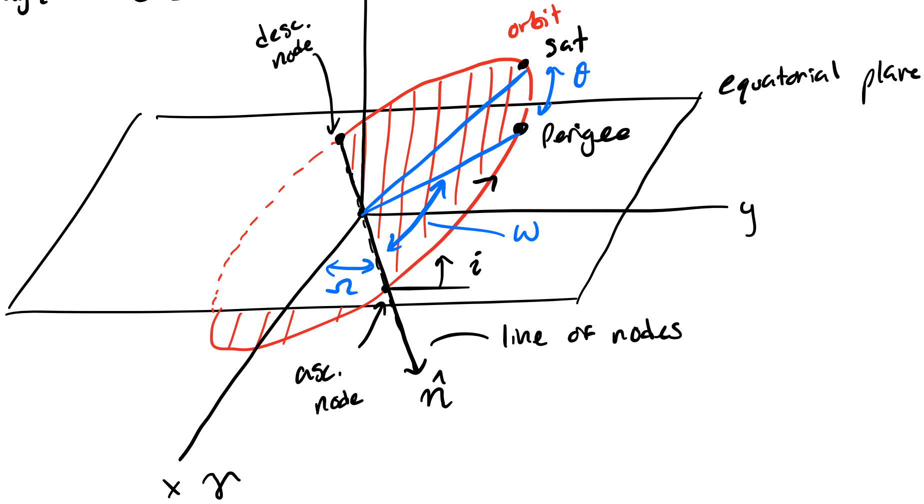
$$\bar{v} = v_x \hat{i} + v_y \hat{j} + v_z \hat{k}$$

but no orbital elements

→ define orbits w/ 6 component "state" (3D)

<u>First set:</u> "modified classic set"

Let xyz be ECI



Set 1

a - semi-major axis

e - eccentricity

i - inclination

θ - true anomaly

Ω - right ascension ($\measuredangle$ from x axis on eq. plane, ccw)

ω - argument of perigee (p from $\hat{n}$)

Set 2

cartesian x, y, z, v_x, v_y, v_z

Set 3

ADBARV

α = right ascension

δ = declination

β = flight path angle

A = angle of $\bar{v}$ from true north

r = radius

v = speed

Set 4

LDBARV

$\alpha \to \lambda$ = longitude

$\lambda = \alpha - \alpha_G$

G = Greenwich

Vectors

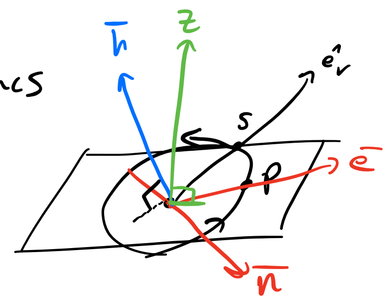
$|\bar{r}| = \sqrt{\bar{r} \cdot \bar{r}}$

$v_r = \dfrac{\bar{r} \cdot \bar{v}}{r}$

Example 1: construct MCS from $\bar{r}$ & $\bar{v}$

$\bar{r}, \bar{v} \to (a, e, i, \theta, \Omega, \omega$

pro: easy to track / predict orbit in MCS



step 1: approaching or leaving perigee

perigee = closest approach

use radial speed $v_r = \dfrac{\bar{r} \cdot \bar{v}}{r}$ $\to$ $r = \sqrt{\bar{r} \cdot \bar{r}}$

$\bar{v}_r > 0$ moving away

$\bar{v}_r < 0$ moving towards p

Step 2: need $\bar{h}$, $\bar{n}$, $\bar{e}$ where $\bar{n}$ is dir. of nodeline
$\bar{e}$ locates periapse

recall $\bar{h} = \bar{r} \times \bar{v}$

$$= \begin{vmatrix} i & j & k \\ x & y & z \\ v_x & v_y & v_z \end{vmatrix} \qquad h = \sqrt{\bar{h} \cdot \bar{h}}$$

then, $\bar{n} = \hat{k} \times \bar{h} \Rightarrow n = \sqrt{\bar{n} \cdot \bar{n}}$

then use trajectory eqns to show

$$\bar{e} = \left(\frac{v^2}{\mu} - \frac{1}{r}\right)\bar{r} - \frac{\bar{r} \cdot \bar{v}}{\mu}\bar{v} \qquad e = \sqrt{\bar{e} \cdot \bar{e}}$$

Step 3: i, Ω, ω

i from $\bar{h}$ & $\hat{k}$ $\boxed{i = \cos^{-1}\left(\frac{h_z}{h}\right)}$

Ω based on $\bar{n}$ $\boxed{\Omega = \cos^{-1}\left(\frac{n_x}{n}\right)}$

ω " " $\bar{n}$, $\bar{e}$ $\boxed{\omega = \cos^{-1}\left(\frac{\bar{n} \cdot \bar{e}}{|\bar{n}||\bar{e}|}\right)}$

Step 4: a, semi-major axis

trajectory: $a = \dfrac{h^2}{\mu(1-e^2)}$

step 5: find θ

example w/ #s

given $\bar{r} = -6045\,\hat{i} - 3490\,\hat{j} + 2500\,\hat{k}$ km

$\bar{v} = -3.457\,\hat{i} + 6.618\,\hat{j} + 2.533\,\hat{k}$ km/s

Find location/dir in MCS