Recap

Thrust for CV fixed or moving @ const $\underline{u}$

$$\mathcal{T} = \dot m u_e + (P_e - P_a) A_e$$

Equivalent exhaust velocity $u_{eq} \equiv u_e + \frac{P_e - P_a}{\dot m} A_e$

such that $\mathcal{T} = \dot m u_{eq}$

Impulse $I = \int_0^{t_b} \mathcal{T}\, dt$ ← burn time $= u_{eq} \int_0^{t_b} \dot m\, dt = u_{eq} M_p$

Specific impulse $I_{sp} = \frac{I}{M_p g_e} = \frac{u_{eq}}{g_e}$ earth gravity @ SL

Typical values for chemical rockets

$200\text{s} \leq I_{sp} \leq 500\text{s}$ → 2000 m/s → "cookin'"

3. Acceleration

Recall momentum eq. for accelerating CV

$$\frac{d}{dt}\int_{CV} \rho \underline{u}_{xyz}\, d\forall + \int_{CS} \rho \underline{u}_{xyz} \underline{u}_{xyz} \cdot d\underline{A} = -\int_{CS} P\, d\underline{A} + \int_{CV} \rho \underline{g}\, d\forall - \int_{CV} \rho \underbrace{\underline{a}_{rel}}_{\text{accel. of CV rel. to ground } \underline{XYZ}} d\forall + \underline{F}_E$$

→ Rectilinear trajectory

→ Project in 1 direction, drop xyz subscript

i) $\frac{d}{dt}\int_{CV} \rho \underline{u}_{xyz}\, d\forall = 0$ b/c in tanks, $\rho \neq$ const, $\underline{u}_{xyz} \approx 0$

in TC, ρ / u_{xyz} vary in space but steady

ii) $\int_{CS} \rho \underline{u}_{xyz} \underline{u}_{xyz} \cdot d\underline{A} = \dot m u_e (-\hat{i})$

on exit, $P = P_e + P_a - P_a = P_a + (P_e - P_a)$

iii) $\int_{CS} -P\, d\underline{A} = -(P_e - P_a) A_e (+\hat{i})$

iv) Project $\underline{g}$ in $\hat{i}$: $\int_{CV} \rho \underline{g}\, d\forall \rightarrow g\cos\theta M(t) (-\hat{i})$

v) $\underline{F}_E = D(-\hat{i})$

vi) $-\int_{CV} \rho \underline{a}_{rel}\, d\forall = -\underbrace{\underline{a}_{rel}}_{\frac{du}{dt}(\hat{i})} \underbrace{\int_{CV} \rho\, d\forall}_{M(t)} = -\frac{du}{dt}(\hat{i})\, M(t)$

Reassemble:

$$-u_e \dot{m} = (P_e - P_a)A_e - M(t)\, g\cos\theta - D - M(t)\frac{du}{dt}$$

$$\boxed{\underbrace{u_e \dot{m} + (P_e - P_a)A_e}_{T} = M(t)\, g\cos\theta + D + M(t)\frac{du}{dt}}$$

(Full form)

(A) Simplified version

$g = D = 0$?

$$\rightarrow \underbrace{u_e \dot{m} + (P_e - P_a)A_e}_{} = M(t)\frac{du}{dt}$$

$$\dot{m}\, u_{eq} = M(t)\frac{du}{dt}$$

Relate $\dot{m}$ & $m(t)$

$$\underbrace{\dot{m}}_{>0} = -\frac{dM(t)}{\underbrace{dt}_{<0}}$$

b/c exiting

b/c M decreases w/ time

$$\rightarrow -\frac{dM(t)}{dt} u_{eq} = M(t)\frac{du}{dt}$$

assume constant

$$-\frac{dM(t)}{M(t)} u_{eq} = du$$

$$\rightarrow -u_{eq}\int_0^t \frac{dM(t)}{M(t)} = \int du$$

$$\Delta u = -u_{eq} \ln\frac{M(t)}{M(t{=}0)} = \boxed{u_{eq} \ln\frac{M_0}{M(t)} = \Delta u}$$ Rocket equation

constant thrust accelerates more as mass ↓

Can expand $M(t) = M_0 - \dot{m}t$ assume $\dot{m} = \text{const.}$

$$\Delta u = u_{eq} \ln\frac{M_0}{M_0 - \dot{m}t}$$

If Δu is attained over entire burn, $t \to t_b$, $M(t_b) = M_s + M_L$

$= M_0 - M_p$

Recall $R = \frac{M_0}{M_b} = \frac{M_0}{M_0 - M_p} = \frac{M_0}{M_s + M_L}$

$$\boxed{\Delta u(t_b) = u_{eq} \ln R}$$

Remarks

i) It looks like if M_p is large enough, can get $\Delta u(t_b)$ larger than u_{eq} by "any" factor desired (we'll rectify this)

ii) Since Δu scales w/ u_{eq}, we want u_{eq} as large as possible
for fixed $\dot{m}$, since $\dot{m} = \rho_e u_e A_e$, large u_e requires low ρ_e