Practical aspects of Orbits

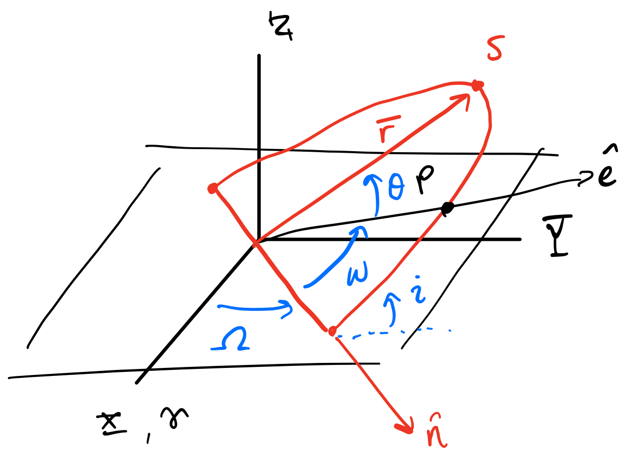
① Earth's oblateness

$$r_E - r_N \cong 22\,km$$

→ central force off

→ gravitational variation

Recall ECI



Perturbation theory : $\Omega \neq$ constant, $\omega \neq$ constant

→ Precession of ascending node $\dfrac{d\Omega}{dt} = \dot{\Omega} = \dfrac{-3 J_2 n r_e^2 \cos i}{2a^2 (1-e^2)^2}$

→ Precession of periapse $\dfrac{d\omega}{dt} = \dot{\omega} = \dfrac{3 J_2 n r_e^2}{4a^2 (1-e^2)^2}(4 - 5\sin^2 i)$

where $n = \dfrac{2\pi}{T}$; r_e = Earth's equatorial radius

a = semi-major axis; i = angle of inclination,

$J_2 = 1.082626 \times 10^{-3}$ (for Earth) "Legendre polynomial coeff"

Remarks

1) Ω → moves westward if $i < 90°$
→ " eastward if $i > 90°$

2) $\omega \rightarrow$ moves in orbit dir. if $i < 63.4^\circ$

$\rightarrow$ " opposite orbit dir. if $i > 63.4^\circ$

3) $0 < \dot{\Omega} < \sim 8$ deg/day, depends on i

$-5 < \dot{\omega} < \sim 17$ deg/day " "

Example 1: Find $\dot{\Omega}$, $\dot{\omega}$ for:

$$\left.\begin{aligned} Alt_P &= 280 \text{ km} \\ Alt_A &= 400 \text{ km} \end{aligned}\right\} \text{ near equator}$$

$i = 51.43^\circ$

Need orbit params:

$r_E = 6378$ km

$r_a = 6778$ km

$r_p = 6658$ km

Find a & e

$$e = \frac{r_a - r_p}{r_a + r_p} = 0.008931$$

$$a = \frac{r_a + r_p}{2} = 6718 \text{ km}$$

$$T = 2\pi\sqrt{\frac{a^3}{\mu}}$$

$$\dot{\Omega} = \frac{-3(1.08\times10^{-3})\left(\frac{2\pi}{T}\right)(6378)^2\cos(51.43^\circ)}{2(6718 \text{ km})^2(1-(.008931)^2)^2} = -1.046\times10^{-6}\ \frac{\text{rad}}{\text{s}}$$

= a few deg/day

$\rightarrow \dot{\omega} = 7.9143\times10^{-7}$ rad/s $\rightarrow$ westward precession

Example 2: Find i for periapse fixed

enforce $\dot{\omega} = 0 = (4 - 5\sin^2 i) \rightarrow i = 63.4°$

Example 3 Track orbital drift

given: $\bar{r} = -3670\hat{i} - 3870\hat{j} + 4400\hat{k}$ km

$\bar{v} = 4.7\hat{i} - 7.4\hat{j} + 1\hat{k}$ km/s

Find: $\bar{r}$ & $\bar{v}$ after 4 days

First: Lecture 10 slide 17 - Get mcS in ECI

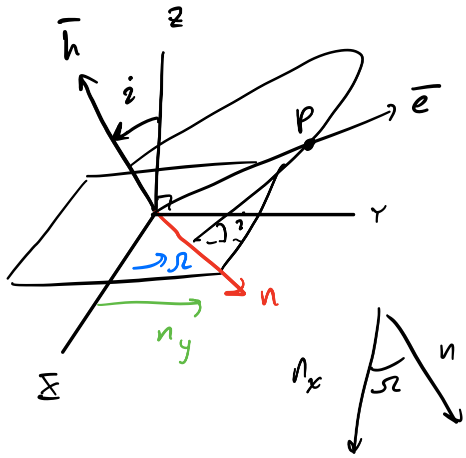
recap: $r = \sqrt{\bar{r}\cdot\bar{r}}$ $\quad v_r = \dfrac{\bar{r}\cdot\bar{v}}{r} \rightarrow v_r > 0$ sat moving away

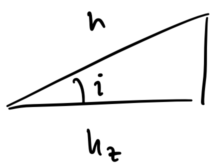
$v = \sqrt{\bar{v}\cdot\bar{v}}$

$\bar{h} = \bar{r} \times \bar{v}$, $h = \sqrt{\bar{h}\cdot\bar{h}}$

$$\bar{e} = \left(\frac{v^2}{\mu} - \frac{1}{r}\right)\bar{r} - \frac{\bar{r}\cdot\bar{v}}{\mu}\bar{v},$$

$e = \sqrt{\bar{e}\cdot\bar{e}}$





$h_z = h\cos i$

$h_z = \bar{h}\cdot\hat{z}$

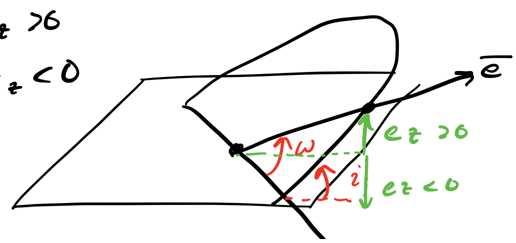
$$\Omega = \cos^{-1}\left(\frac{n_x}{n}\right)$$

$0 \le \Omega \le 180°$ for $n_y > 0$

$180 \le \Omega \le 360°$ for $n_y < 0$

$\omega = \cos^{-1}\left(\dfrac{\bar{n}\cdot\bar{e}}{ne}\right)$ $\quad 0 \le \omega \le 180 \quad e_z > 0$

$180 \le \omega \le 360° \quad e_z < 0$



θ: $\theta = \cos^{-1}\left(\frac{\bar{e}\cdot\bar{r}}{er}\right)$ $0 \le \theta < 180$ $v_r \ge 0$
$180 \le \theta < 360$ $v_r < 0$

a: $a = \frac{\bar{h}}{\mu(1-e^2)}$ See Lec 14 matlab file

Results @ $t=0$

$a = 10643.7$ KM
$i = 39.687°$
$e = 0.426$
$\Omega_0 = 130.32°$ at $t=0$
$\omega_0 = 42.37°$ "
$\theta_0 = 52.4°$ "

To account for $\omega, \Omega \neq$ constant

$\Omega(t) = \dot{\Omega}t + \Omega_0$
$\omega(t) = \dot{\omega}t + \omega_0$

Before accounting for this, propagate θ (Kepler3)

1) $\sqrt{\frac{\mu}{a^3}}(t - t_p) = E - e\sin E$:

$\rightarrow \boxed{t_0 - t_p = 681\text{ s}}$

Know θ_0
$\rightarrow E_0$
$\rightarrow$ solve for $t_0 - t_p$

2) Find final time since perigee passage

$t_f - t_p = (t_0 - t_p) + 4\text{ days}(24)(3600)\frac{\text{sec}}{\text{day}} = 346231\text{ sec}$

need to find remaining partial period

3) compute period $T = 2\pi\sqrt{\frac{a^3}{\mu}} = 10927\text{ s}$

4) $\frac{t_f - t_p}{T} = \frac{346231}{10927} = 31.685$

$\rightarrow$ Sat. is at $.685\,T = t_f' - t_p$
$= 7485\text{ s}$

5) Find $\theta(t_f)$

$$E_f - e\sin E_f = \sqrt{\frac{\mu}{a^3}}\,(t_f' - t_p) = 4.3 \text{ rad}$$

matlab $\rightarrow E_f = 3.98$ rad

relate $\theta \rightarrow E$ $\rightarrow \theta_f = 211.25^\circ$

Account for oblateness

$\rightarrow$ plug into $\dot{\Omega}, \dot{\omega}$ eqns

4 days later

node line
$$\begin{cases} \dot{\Omega} = -2.2067 \times 10^{-5} \text{ deg/s} \\ \Omega(t_f) = \Omega_0 + \dot{\Omega}\,(4(24)(3600)\text{ sec}) \\ \Omega(t_f) = 122.7^\circ \end{cases}$$

$\Delta\Omega = -7.62^\circ$ in 4 days

perigee
$$\begin{cases} \dot{\omega} = 2.811 \times 10^{-5} \text{ deg/s} \\ \omega(t_f) = \omega_0 + \dot{\omega}\,(4(24)(3600)) \\ \omega(t_f) = 52.09^\circ \end{cases}$$

$\Delta\omega = 9.72^\circ$ in 4 days

plug into solver to get $\bar{r}_f, \bar{v}_f$

$\bar{r}_f = 9683\hat{i} + 4325\hat{j} - 8702\hat{k}$ km

$\bar{v}_f = -3.05\hat{i} + 3.31\hat{j} + .64\hat{k}$ km/s