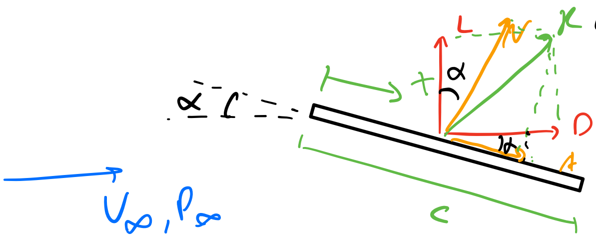
Problem 1 Thin airfoil at $\alpha = 5°$ (flat plate) has pressure distribution:

$$\text{upper} \begin{cases} C_{p,u} = 1 - 300\left(\frac{x}{c}\right)^2 & \text{for } \frac{x}{c} < 0.1 \rightarrow \\ C_{p,u} = -2.2277 + 2.2277\,\frac{x}{c} & \text{for } \frac{x}{c} > 0.1 \end{cases}$$

$$\text{Lower} \left\{ C_{p,l} = 1 - 0.95\,\frac{x}{c} \right.$$

1a) compute lift & drag coefficients



$$C_p = \frac{p - p_\infty}{q_\infty} = \underbrace{\frac{p - p_\infty}{\frac{1}{2}\rho_\infty V_\infty^2}}_{\text{Keep } q_\infty \text{ through}}$$

$$C_l = \frac{L}{\frac{1}{2}\rho_\infty V_\infty^2 c}$$

- solve for N, A & transform to L & D

$$C_p = \frac{p - p_\infty}{q_\infty} \qquad C_l = \frac{L}{q_\infty c}$$

$$p(x) = C_p(x) \cdot q_\infty$$

$$\left[\tfrac{N}{m}\right] \quad N = -\int_0^{0.1} C_{p_u}(x) \cdot q_\infty \, d\left(\tfrac{x}{c}\right) - \int_{0.1}^{1} C_{p_u}(x) \cdot q_\infty \, d\left(\tfrac{x}{c}\right) + \int_0^1 C_{p_l}(x) \cdot q_\infty \, d\left(\tfrac{x}{c}\right)$$

$$C_N = \frac{N}{q_\infty} = -\left[\frac{x}{c} - 100\left(\frac{x}{c}\right)^3\right]_0^{0.1} - \left[-2.2277\,\frac{x}{c} + \frac{2.2277}{2}\left(\frac{x}{c}\right)^2\right]_{0.1}^{1} + \left[\frac{x}{c} - 0.475\left(\frac{x}{c}\right)^2\right]_0^1$$

$$C_N = -\left[0.1 \cancel{- 100(0.001)}\right] - \left[\left(-2.2277 + \frac{2.2277}{2}\right)\left(-0.22277 + \frac{0.022277}{2}\right)\right]$$

0

$$\frac{N}{q_\infty} = +\left[1 - 0.475\right] = 1.427 = C_N = \frac{N}{q_\infty}$$

$$C_l = C_N \cos\alpha \,, \quad C_d = C_N \sin\alpha$$

$$\rightarrow \boxed{\begin{aligned} C_l &= 1.427\cos(5°) = 1.422 \\ C_d &= 1.427\sin(5°) = 0.124 \end{aligned}}$$

1b) $\tau_w(x) = 0.332\,\rho u_\infty^2 \sqrt{\dfrac{\mu}{\rho u_\infty x}}$ ← $\dfrac{1}{Re_x}$

Rewrite in terms of non-dimensional params.

$q = \frac{1}{2}\rho_\infty V_\infty^2$

$$\frac{\tau_w(x)}{\frac{1}{2}\rho_\infty u_\infty^2} = \boxed{0.664\sqrt{\frac{1}{Re_x}} = C_f}$$

1c) use C_f to get C_d & C_ℓ due to shear

$Re_c = 100{,}000$

$$C_f^{(x)} = 0.664\sqrt{\frac{\mu}{\rho u_\infty x}} \quad \left(\sqrt{\tfrac{1}{c}}\right)$$

$$= 0.664\sqrt{\frac{\mu}{\rho u_\infty}}\sqrt{\frac{1}{x}} \quad \left(\sqrt{\tfrac{c}{1}}\right)$$

$$C_f^{(x)} = 0.664\sqrt{\frac{\mu}{\rho u_\infty c}}\sqrt{\frac{c}{x}}$$

$$\rightarrow C_f^{(x/c)} = 0.664\sqrt{\frac{1}{100{,}000}}\ \frac{1}{\sqrt{x/c}}$$

$$C_f = \int_0^1 0.664\sqrt{\frac{1}{100{,}000}}\cdot (x/c)^{-1/2}\, d\left(\frac{x}{c}\right)$$

$$C_f = \Big[0.0021\,(2)\,(x/c)^{1/2}\Big]_0^1 = 0.0042$$

$$\rightarrow \boxed{\begin{aligned} C_\ell &= -0.0042\sin(5) = -0.00037 \\ C_d &= 0.0042\cos(5) = 0.00418 \end{aligned}}$$

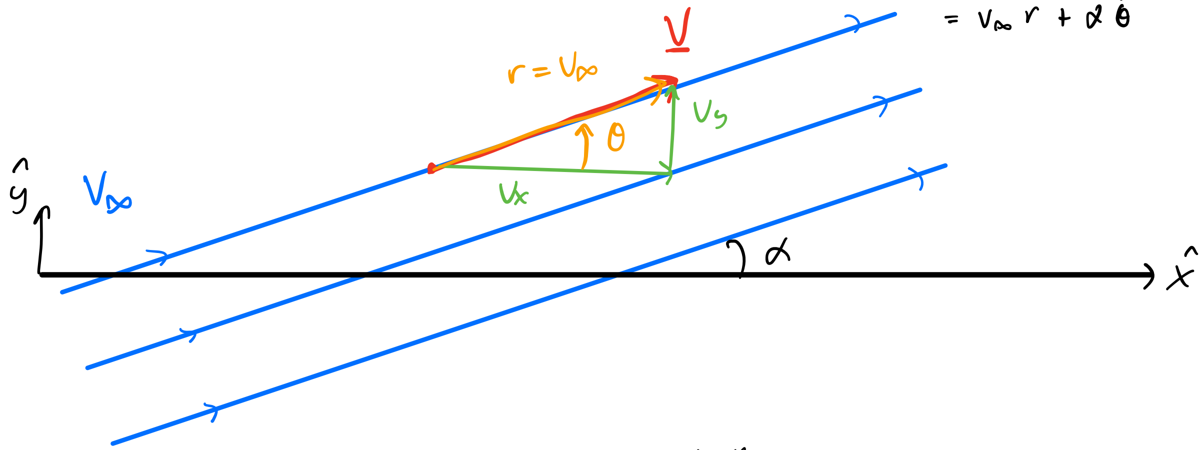
1d) $C_{\ell\ \text{total}} = 1.4216 \rightarrow -0.03\%$ lift from shear

$C_{d\ \text{total}} = 0.1282 \rightarrow 3.26\%$ drag from shear

Problem 2 Uniform flow of velocity V_∞ w/ angle α w/ $\hat{x}$

2a) Draw velocity vector at arb. point in space, Show its cart. & polar coords.

$\underline{V} = V_\infty \cos\alpha\, \hat{i} + V_\infty \sin\alpha\, \hat{j}$
$= V_\infty \hat{r} + \alpha \hat{\theta}$



2b) Determine stream function in both cart. & polar

$[\psi = \psi(x,y)$ & $\psi = \psi(r,\theta)]$

Cartesian: $u = \frac{\partial \psi}{\partial y}$, $v = -\frac{\partial \psi}{\partial x}$ | $\underbrace{V_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta},\ V_\theta = -\frac{\partial \psi}{\partial r}}_{\text{from textbook}}$

$$\boxed{\psi(x,y) = V_\infty \cos\alpha\, y - V_\infty \sin\alpha\, x}$$

Polar:
$v_r = U_x \cos\theta + U_y \sin\theta$
$v_\theta = -U_x \sin\theta + U_y \cos\theta$

Substitute $\rightarrow$ $U_x = V_\infty \cos\alpha$, $V_y = V_\infty \sin\alpha$

$v_r = V_\infty \cos\alpha \cos\theta + V_\infty \sin\alpha \sin\theta$

$v_\theta = -V_\infty \cos\alpha \sin\theta + V_\infty \sin\alpha \cos\theta$

$\cos(A-B) = \cos A \cos B + \sin A \sin B$, $\sin(A-B) = \sin A \cos B - \cos A \sin B$

$\rightarrow v_r = V_\infty \cos(\theta - \alpha)$
$v_\theta = -V_\infty \sin(\theta - \alpha)$

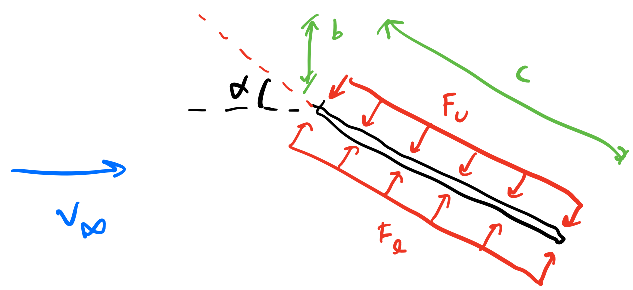
$\rightarrow$ integrate V_θ w.r.t. r $\rightarrow -\int -V_\infty \sin(\theta - \alpha)\, dr$

$\rightarrow$ $$\boxed{\psi(r,\theta) = V_\infty r \sin(\theta - \alpha)}$$

Problem 3 Infinitely thin flat plate, chord c & width b, angle α

$P_u(s) = c_1$ $P_\ell(s) = c_2$, $c_2 > c_1$

3a) Figure w/ distributed forces on plate



3b) compute M_{LE} $M = \underline{r} \times \underline{F}$

$$M_{LE} = \int_0^c s \cdot F(s)\, ds$$

$$= \int_0^c \left[+ P_u(s)(sb) - P_\ell(s)(sb) \right] ds$$ $\leftarrow$ prove $\int = 0$

$$M_{LE} = \int_0^c c_1 b s \, ds - \int_0^c c_2 b s \, ds$$

$$M_{LE} = \frac{c_1 b}{2} c^2 - \frac{c_2 b}{2} c^2$$

$$\boxed{M_{LE} = \frac{c^2 b}{2}(c_1 - c_2)}$$

3c) Compute $M_{c/2}$

$$M_{c/2} = +\int_{-c/2}^{0} c_1 b s \, ds + \int_{0}^{c/2} c_1 b s \, ds$$

$$+\int_{-c/2}^{0} c_2 b s \, ds + \int_{0}^{c/2} c_2 b s \, ds$$

$$M_{c/2} = \frac{c_1 b}{2}\left[\left(0 - \cancel{\left(-\frac{c}{2}\right)^2}\right) + \left(\cancel{\left(\frac{c}{2}\right)^2} - 0\right)\right] \rightarrow \boxed{M_{c/2} = 0}$$

$$+ \frac{c_2 b}{2}\left[\left(0 - \cancel{\left(-\frac{c}{2}\right)^2}\right) + \left(\cancel{\left(\frac{c}{2}\right)^2} - 0\right)\right]$$