RECAP

cons. of mass (continuity)

$$\underbrace{\frac{d}{dt}\int_{CV} \rho \, d\forall}_{\text{rate of change of CV's mass content}} + \underbrace{\int_{CS} \rho \underline{u} \cdot d\underline{A}}_{\text{flow rate of mass thru CS}} = 0$$

Now write $\sum \underline{F} = \frac{d\underline{P}}{dt}$ for a moving fluid

use same formalism of continuity eq. to express $\frac{d\underline{P}}{dt}$ for fixed CV

Generally

$$\frac{d \text{ Property w/in CV}}{dt} = \frac{d}{dt}\int_{CV} \frac{\text{Property}}{\text{unit mass}} \cdot \rho \, d\forall + \int_{CS} \frac{\text{Property}}{\text{unit mass}} \rho \, \underline{u} \cdot d\underline{A}$$

If property $\equiv$ mass, $\frac{\text{mass}}{\text{unit mass}} = 1$

If property $\equiv$ momentum, $m\underline{u}$, $\frac{\text{momentum}}{\text{unit mass}} = \underline{u}$

$$\Rightarrow \frac{d\underline{P}}{dt} = \int_{CV} \underline{u} \, \rho \, d\forall + \int_{CS} \underline{u} \, \rho \underline{u} \cdot d\underline{A}$$

$$= \int_{CV} \rho \, \underline{u} \, d\forall + \int_{CS} \rho \, \underline{u} \, \underline{u} \cdot d\underline{A}$$

$\underline{P}$ is a vector

$\rho \underline{u}$ " "

$\underline{u} \cdot d\underline{A}$ " SCALAR

$\rho \underline{u} \, \underline{u} \cdot d\underline{A}$ " VECTOR

CV's content of momentum may change in time b/c:

1) portion of $\rho \neq 0$, $\underline{u} \neq 0$ inside cv varies in time

2) ρ inside cv varies

3) $\underline{u}$ inside cv varies

$\left.\right\}$ $\frac{d}{dt}\int_{cv} \rho \underline{u}\, d\forall$

4) non-zero net momentum flow rate through cs $\left.\right\}$ $\int_{cs} \rho \underline{u}\, \underline{u} \cdot d\underline{A}$

Newton's law rewrites

$$\frac{d}{dt}\int_{cv} \rho \underline{u}\, d\forall + \int_{cs} \rho \underline{u}\, \underline{u} \cdot d\underline{A} = \Sigma \underline{F}$$

About $\underline{F}$:

- Body - $\underline{F}_B$ (Gravity, inertia)
- Surface - $\underline{F}_S$ (Pressure, shear)
- External - $\underline{F}_E$ (From objects like wings, blades, supports, etc)

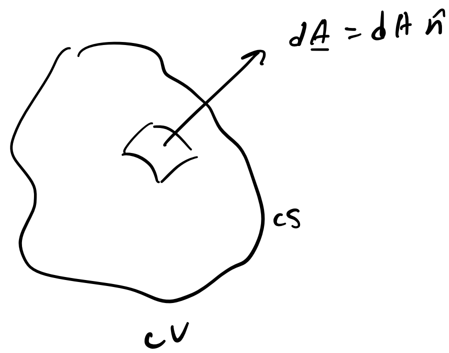
physical origin of F_E is a surface force
- F_E helps seprate them

$$\underline{F}_B = \int_{cv} \rho \underline{g}\, d\forall$$

subjects: fluid in CV

$$\underline{F}_S = -\int p\, d\underline{A}$$

p directed inward,
$d\underline{A}$ directed outward



Assume inviscid flow, $\mu = 0 \Rightarrow \tau = 0$

only need to include pressure forces

Reassemble:

$$\underbrace{\frac{d}{dt}\int_{CV} \rho \underline{u}\, d\forall + \int_{CS} \rho \underline{u}\, \underline{u}\cdot d\underline{A}}_{\frac{d\underline{P}}{dt}} = \underbrace{\int_{CV} \rho \underline{g}\, d\forall - \int_{CS} p\, d\underline{A} + \underline{F}_E}_{\Sigma \underline{F}}$$

can decompose into x, y, z directions

$$\frac{d}{dt}\int_{CV} \rho u\, d\forall + \int_{CS} \rho u\, \underline{u}\cdot d\underline{A} = \int_{CV} \rho g_x\, d\forall - \int_{CS} p\, \underbrace{d\underline{A}\cdot \hat{i}}_{dA_x} + F_{Ex}$$

$$\frac{d}{dt}\int_{CV} \rho v\, d\forall + \int_{CS} \rho v\, \underline{u}\cdot d\underline{A} = \int_{CV} \rho g_y\, d\forall - \int_{CS} p\, d\underline{A}\cdot \hat{j} + F_{Ey}$$

$\vdots$

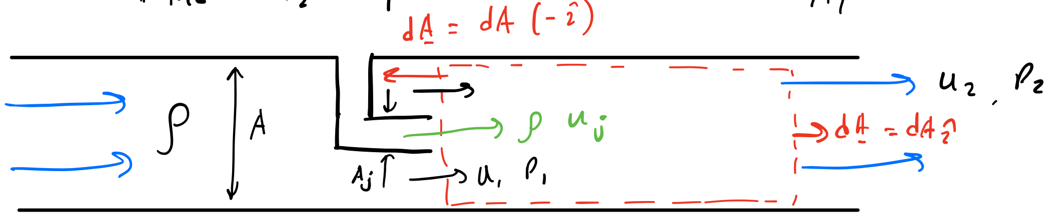
About $-\int_{CS} p\, d\underline{A}$

From hydrostatics, if $p = \text{const}$ over CS:

$$-\int_{CS} p\, d\underline{A} = -p\int d\underline{A} = 0$$

Ex. Jet pump - device used to increase pressure of fluid
steady-state flowing through pipe

Find $P_2 - P_1$ $\qquad\qquad$ $A_1 << A$



High-speed fluid (u_j) injected inside low-speed fluid (u_1, P_1). Downstream, (u_2, P_2).

Assume $\mu = 0$.

P_1 = const over A $\qquad$ Find $P_2 - P_1$.
P_2 = " "

Continuity: $-\dot{m}_{in} + \dot{m}_{out} = 0$ (steady-state)

$$-\rho(A - A_j)u_1 - \rho A_j u_j + \rho A u_2 = 0$$

$$\rightarrow u_2 = \frac{(A - A_j)\,u_1 + A_j u_j}{A} \qquad (1)$$

Momentum in $\hat{x}$:

$$\underbrace{\cancel{\frac{d}{dt}\int_{cv} \rho u \, d\forall}}_{0,\ \text{steady-state}} + \int_{cs} \rho u \, \underline{u} \cdot d\underline{A} = \int_{cv} \rho g_x \, d\forall - \int_{cs} P \, d\underline{A} \cdot \hat{\imath} + \underbrace{\cancel{F_{ex}}}_{0}$$

Could include viscous effects, jet arm in cv, etc.

$$\underbrace{-\rho(A - A_j)u_1^2 - \rho A_j u_j^2}_{\text{entering}} + \underbrace{\rho A u_2^2}_{\text{exiting}} = P_1 A - P_2 A$$

$$P_2 - P_1 = \rho\left[\left(1 - \frac{A_j}{A}\right)u_1^2 + \frac{A_j}{A}u_j^2 - u_2^2\right] \qquad (2)$$

$\rightarrow$ Replace u_2 from (1)

$$P_2 - P_1 = \rho \frac{A_j}{A}\left(1 - \frac{A_j}{A}\right)(u_1 - u_j)^2$$