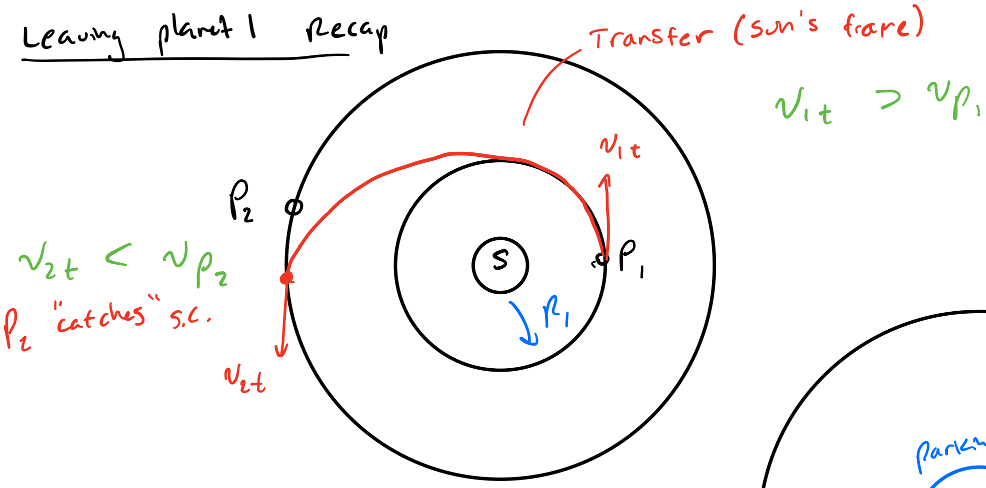
Exam 2: up to L20 (Rocketry) 4 sides note sheet

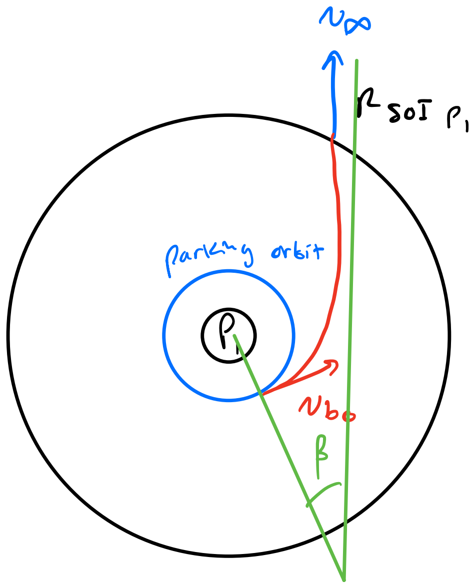
## Leaving planet 1 Recap




1) Calculate $v_\infty$ needed for transfer

$$v_\infty = v_{1t} - v_{P_1} \rightarrow \sqrt{\frac{\mu_{sun}}{R_1}}$$

↑ speed of s.c. rel. to $P_1$




2) use energy to find $v_{bo}$

$$\varepsilon_\infty = \varepsilon_{bo}$$

$$v_{bo} = \sqrt{v_\infty^2 + \frac{2\mu_1}{r_{bo}} - \frac{2\mu_1}{r_{SOI}}}$$

small!

3) calc. $\Delta v$ from parking orbit $\Delta v = v_{bo} - v_{c_1}$

Example: Earth to Mars

Assume LEO parking alt = 300 km

$$r_{bo} = 300\text{ km} + 6378\text{ km} = 6678\text{ km}$$

M = Mars
E = earth
S = sun

$$v_{c_1} = \sqrt{\frac{\mu_E}{r_{bo}}} = 7.726\text{ km/s}$$

$$v_{\infty,\,earth} = \sqrt{\frac{\mu_{sun}}{R_{E/S}}}\left(\sqrt{\frac{2R_{M/S}}{(R_{E/S} + R_{M/S})}} - 1\right) = 2.95\text{ km/s}$$

$1.327 \times 10^{11}\ \frac{km^3}{s^2}$

$2.279 \times 10^8$ km

$1.496 \times 10^8$ km
1 AU

$$v_{bo} = \sqrt{v_{\infty,Earth}^2 + \frac{2\mu_E}{r_{bo}} - \frac{2\mu_E}{\overset{9.24\times10^5\ km}{r_{SOI,E}}}} = 11.28\ km/s$$

$$\therefore\ \Delta v_1 = v_{bo} - v_{c1} = 11.28 - 7.73 = 3.55\ km/s$$

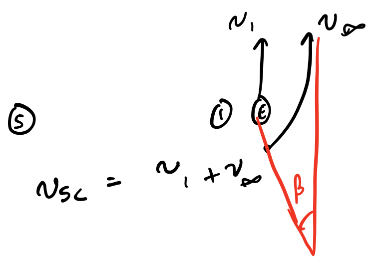
occurs at $\beta = \cos^{-1}\left[\dfrac{1}{1 + \dfrac{\overset{6678}{r_{bo}}\ \overset{2.95}{v_{\infty,E}^2}}{\underset{3.486\times10^5}{\mu_E}}}\right]$

$$\beta = 0.5099\ rad = 29.2^\circ$$

## Other concepts

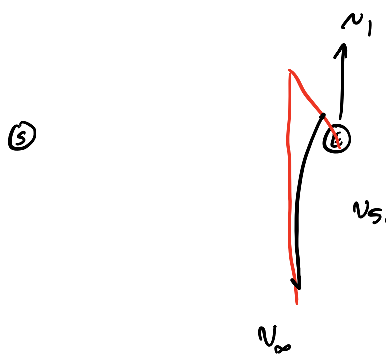
To go to outer planets

For prograde parking




$v_{sc} = v_1 + v_\infty$

To go to inner planets




$v_{sc} = v_1 - v_\infty$

concept of timing -> planet must "be there"

Burn time: consider phase $\phi(t)$ between planets

$$\phi(t) = \theta_2(t) - \theta_1(t) \qquad n_1 = \frac{2\pi}{T_1}$$

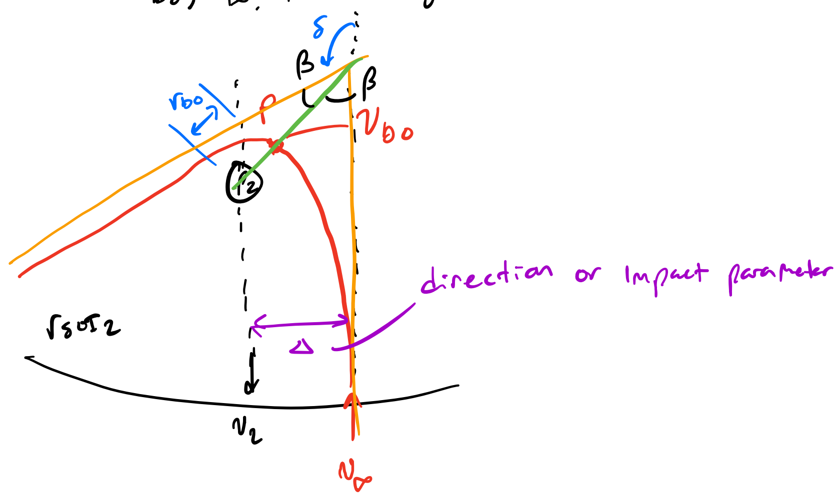
$$= \phi(0) + (n_2 - n_1)t \qquad n_2 = \frac{2\pi}{T_2}$$

$$\phi(0) = \pi - n_2 t_{12} \qquad \text{(slide 26)}$$

$$t_{12} = T_{tr,1/2} = \frac{\pi}{\sqrt{\mu_{Sun}}}\left(\frac{R_1 + R_2}{2}\right)^{3/2}$$

Let's develop arrival process

→ need $v_{bo}, v_\infty, \beta$ → right direction




$$v_\infty = v_{A,2} = v_2 - \tilde{v}_A = \sqrt{\frac{\mu_{sun}}{R_2}}\left(1 - \sqrt{\left(\frac{2R_1}{R_1+R_2}\right)}\right)$$

P2 speed

Speed of S.C. @ apoapse of transfer

energy gives $v_{bo} = \sqrt{v_\infty^2 + \frac{2\mu_2}{r_{bo}} - \frac{2\mu_2}{r_{SOI,2}}}$

"turn angle" $\delta = 2\sin^{-1}\left(\frac{1}{1+\frac{r_{bo}v_\infty^2}{\mu_2}}\right)$

Finally "impact parameter" $b$

conserve momentum $r_{bo}v_{bo} = b v_\infty$

$$b = r_{bo}\frac{v_\infty}{v_{bo}}$$

$$b \approx r_{bo}\sqrt{1 + \frac{2\mu_2}{r_{bo}v_{bo}^2}}$$

WHAT CAN HAPPEN

1. $r_{bo} \le r_{planet}$ → impact
2. $r_{bo} = r_p$ → periapse $\Delta v$ for capture
3. $r_{bo} > r_{planet}$ → flyby

For capture set $r_p = r_{bo}$ ↙ enforce

$$v_{p,\,capture} = \sqrt{\frac{\mu_2(1+e_{capture})}{r_p}}$$

$$\Delta v = v_{p,\,capture} - v_{bo}$$

– Max for circular
→ decreases as
$e$ increases

Example: Mars injection

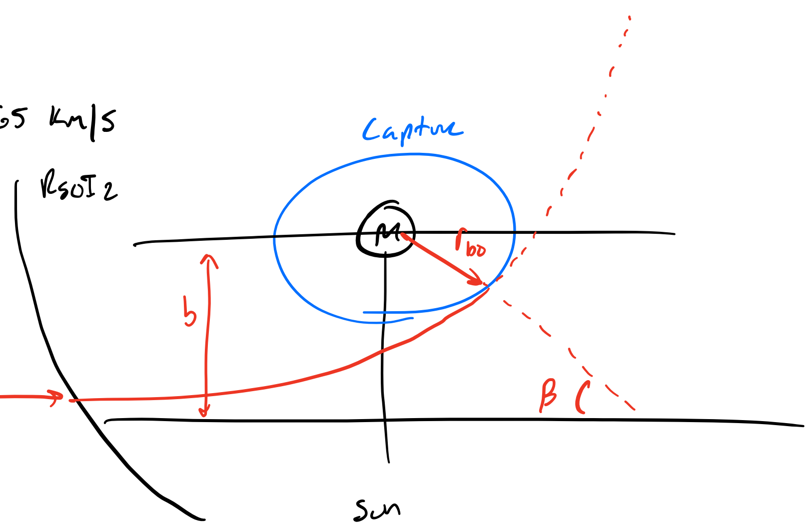
$$v_2 = \sqrt{\frac{\mu_{sun}}{R_{M/S}}} = 24.13 \text{ km/s}$$

$$v_{t2} = \sqrt{2\mu_{sun}\left(\frac{R_{E/S}}{R_{M/S}(R_{E/S}+R_{M/S})}\right)} = 21.48 \text{ km/s}$$

at apoapse

$$v_\infty = v_2 - v_a = 2.65 \text{ km/s}$$

(approach)




$$\boxed{\text{Find min } \Delta v \text{ for capture w/ } T = 7 \text{ hrs (multiple options)}}$$

$$a_{mars} = \left(\frac{T}{2\pi}\sqrt{\mu_{mars}}\right)^{2/3} = 8882 \text{ km} \qquad \mu_{mars} = 4.306\times10^4 \text{ km}^3/\text{s}^2$$

Find $r_p$ to go w/ $a$ s.t. min $\Delta v = v_{p\,capture} - v_{bo}$

Params:

For optimal capture ellipse (min $\Delta v$)

$$\boxed{r_{p\,cap} = \frac{2\mu_2}{v_\infty^2}\,\frac{1-e_{cap}}{1+e_{cap}} \qquad \Delta v = v_\infty\sqrt{\frac{1-e_{cap}}{2}} \qquad r_{a\,cap} = \frac{2\mu_2}{v_\infty^2} \qquad b = r_{p\,cap}\sqrt{\frac{2}{1-e_{cap}}}}$$

$\Delta V$ mano -> circular

mars example show: $e_{mars} = 0.3833$

$r_{p\,cap} = 5447$ km $= r_{bo}$

$r_{a\,cap} = 12263$ km

$\Delta V = 1.4715$ km/s

$b = 9809$ km

$\beta = 58°$