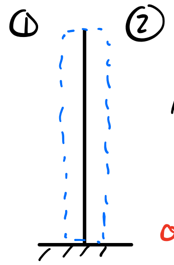
Task: Find $\frac{p_2}{p_1}, \frac{T_2}{T_1}, \frac{\rho_2}{\rho_1}, \frac{u_2}{u_1}, M_2, \frac{p_{02}}{p_{01}}, \frac{A^*_2}{A^*_1}$ in terms of M_1 (immediately before shock) & γ

Use conservation eqns w/ "pill box" CV



CV small:
$A_1 = A_2 = A$

mass: $\rho_1 u_1 A_1 = \rho_2 u_2 A_2$

→ $\rho_1 u_1 = \rho_2 u_2$ (1)

Momentum:

$$\underbrace{\frac{d}{dt}\cancel{\int \rho \mathbf{u}\, d\forall}}_{0,\ \text{steady}} + \int_{CS} \rho \mathbf{u}\, \mathbf{u}_{rel}\cdot d\mathbf{A} = \int_{CV} p\, d\mathbf{A} + \cancel{\mathbf{F_B}}^{0}$$

$$-\rho u_1^2 A + \rho_2 u_2^2 A = (p_1 - p_2)A$$

$$p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2 \quad (2)$$

Rearrange: $p_1\left(1+\frac{\rho_1 u_1^2}{p_1}\right) = p_2\left(1+\frac{\rho_2 u_2^2}{p_2}\right)$ Recall $a^2 = \gamma \frac{p}{\rho}$

$$p_1(1+\gamma M_1^2) = p_2(1+\gamma M_2^2) \rightarrow \frac{p_2}{p_1} = \frac{1+\gamma M_1^2}{1+\gamma M_2^2} \quad (3)$$

Energy:

$$\frac{d}{dt}\cancel{\int \rho\left(e+\frac{u^2}{2}\right)d\forall}^{0} + \int_{CS}\rho\left(h+\frac{u^2}{2}\right)\mathbf{u}_{rel}\cdot d\mathbf{A} = \cancel{\dot Q}^{0} - \cancel{\dot W_{shaft}}^{0} - \cancel{\dot W_{shear}}^{0} - \cancel{\int p\, \mathbf{u}_b \cdot d\mathbf{A}}^{0}$$

$$\dot m\left(h_2 + \frac{u_2^2}{2} - h_1 - \frac{u_1^2}{2}\right) = 0 \rightarrow h_2 + \frac{u_2^2}{2} = h_1 + \frac{u_1^2}{2} = h_0 = c_p T_0$$

total enthalpy / stag. enth.

Stagnation enthalpy & stag. temp stay unchanged across shock

using $h = c_p T$

$$c_p T_1\left(1+\frac{u_1^2}{c_pT_1}\right) = c_pT_2\left(1+\frac{u_2^2}{c_pT_2}\right)$$

$\left\{ \begin{array}{l} c_p = \frac{\gamma}{\gamma-1}R \\ a^2 = \gamma RT = \gamma\frac{p}{\rho} \end{array} \right.$ $u = Ma$ ★

$$\frac{T_2}{T_1} = \frac{1+\frac{\gamma-1}{2}M_1^2}{1+\frac{\gamma-1}{2}M_2^2} \quad (4)$$

$\frac{(2)}{(1)}$: $u_1\left(\frac{p_1}{\rho_1 u_1^2}+1\right) = u_2\left(\frac{p_2}{\rho_2 u_2^2}+1\right)$

$$M_1 a_1\left(\frac{a_1^2}{\gamma u_1^2}+1\right) = M_2 a_2\left(\frac{a_2^2}{\gamma u_2^2}+1\right) \rightarrow M_1\sqrt{\gamma R T_1}\left(\frac{1}{\gamma M_1^2}+1\right) = M_2\sqrt{\gamma R T_2}\left(\frac{1}{\gamma M_2^2}+1\right)$$

$$\rightarrow \sqrt{\frac{T_1}{T_2}} = \frac{1+\gamma M_2^2}{1+\gamma M_1^2} \frac{M_1}{M_2} \quad (5)$$

$$(5)^2 \cdot (4) \rightarrow A M_2^4 + B M_2^3 + C = 0 \qquad A, B, C = \text{constant}$$

$$M_2^2 = \frac{1+\frac{\gamma-1}{2}M_1^2}{\gamma M_1^2 - \frac{\gamma-1}{2}} \quad (6) \quad \rightarrow \text{plot handout}$$

Remarks:

i) No shock possible if $M_1 < 1 \rightarrow$ shock can only form in supersonic flow

ii) $M_2 \leq 1$ flow after shock is always <u>subsonic</u>

iii) if $M_1 = 1 \rightarrow M_2 = 1$ infinitely weak shock $\equiv$ acoustic wave

Plug (4) into (3) and get

$$\frac{P_2}{P_1} = 1 + \frac{2\gamma}{\gamma+1}\left(M_1^2 - 1\right) \geq 1$$

plug (4) into (6):

$$\frac{T_2}{T_1} = \frac{\left[2\gamma M_1^2 - (\gamma-1)\right]\cdot\left[(\gamma-1)M_1^2 + 2\right]}{(\gamma+1)^2 M_1^2} \geq 1$$

From (1) : $\frac{u_1}{u_2} = \frac{\rho_2}{\rho_1} = \frac{P_2}{\cancel{R}T_2} \cdot \frac{\cancel{R}T_1}{P_1} = \frac{P_2}{P_1} \cdot \frac{T_1}{T_2}$

$$\frac{u_1}{u_2} = \frac{\rho_2}{\rho_1} = \frac{(\gamma+1)M_1^2}{2+(\gamma-1)M_1^2} \geq 1 \qquad P, T, \rho \uparrow \text{ across shock}$$

$$u \downarrow \text{ across shock}$$

<u>About P_0</u>

- flow is isentropic before & after shock

$$\frac{P_1}{P_{01}} = \left(1 + \frac{\gamma-1}{2}M_1^2\right)^{\gamma/1-\gamma}$$

after shock: $\frac{P_2}{P_{02}} = \left(1 + \frac{\gamma-1}{2}M_2^2\right)^{\frac{\gamma}{1-\gamma}}$

across shock: $M_2 = \left(\frac{1+\frac{\gamma-1}{2}M_1^2}{\gamma M_1^2 - \frac{\gamma-1}{2}}\right)^{1/2}$

$$\frac{p_2}{p_1} = 1 + \frac{2\gamma}{\gamma+1}\left(M_1^2 - 1\right)$$

then $\frac{p_{02}}{p_{01}} = \frac{p_{02}}{p_2}\,\frac{p_2}{p_1}\,\frac{p_1}{p_{01}} = \cdots$

$$\rightarrow \frac{p_{02}}{p_{01}} = \left[\frac{(\gamma+1)M_1^2}{2+(\gamma-1)M_1^2}\right]^{\frac{\gamma}{\gamma-1}} \cdot \left[\frac{\gamma+1}{2\gamma M_1^2-(\gamma-1)}\right]^{\frac{1}{\gamma-1}} \leq 1$$

flow degradation
loss of ability to perform mechanical work

<u>about a^*</u>

- $a^* = \sqrt{\gamma R T^*}$ $T^* = T_0\left(1 + \frac{\gamma-1}{2} 1^2\right)^{-1}$

But T_0 const across shock $\rightarrow$ $a^* =$ const across shock

<u>about A^*</u>