LAPLACE XFORM

$$\mathcal{L}[f(t)] = \int_{-\infty}^{+\infty} f(t)e^{-st}\,dt$$

- SOLVE IVP FOR LIN. DIFF. EQ.
- BASIS FOR MAJORITY OF CONTROLS TECHNIQUES

S: LAPLACE VARIABLE

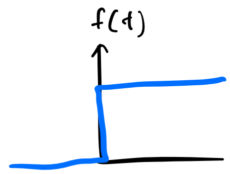
ONE-SIDED (UNILATERAL)

$$\mathcal{L}_-[f(t)] = F(s) = \int_{0^-}^{\infty} f(t)e^{-st}\,dt$$

$\mathcal{L}[f(t)]$ $\begin{cases} \text{- FUNCTION OF } S \\ \text{- NOT ALL FUNCTIONS HAVE LAPLACE, INT. MUST CONVERGE. (e.g. } f(t) \neq e^{t^2}) \end{cases}$

- TAKES FUNCTION OF TIME & PRODUCES FUNCTION OF S
- WORKS ON MOST FUNCTIONS OF TIME, INCLUDING DIFF. EQ.

EX. $f(t) = 1(t)$ FOR $t \geq 0$



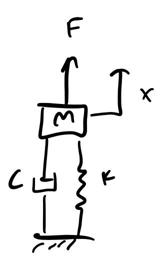
$$\boxed{F(s) = \int_{0^-}^{\infty} f(t)e^{-st}\,dt}$$

$$F(s) = \int_{0^-}^{\infty} e^{-st}\,dt = \left[-\frac{1}{s}e^{-st}\right]_{t=0}^{t=\infty}$$

$$= -\frac{1}{s}\left[\cancel{e^{-s\infty}}^{\,0} - \cancel{e^{-s\cdot 0}}^{\,1}\right] = \boxed{\frac{1}{s} = F(s), \quad f(t) = 1(t)}$$

LT PAIR

- USE LT TO SOLVE LIN. CONST. COEFF. DIFF. EQ.



$$m\ddot{x} + c\dot{x} + kx = F(t) \qquad \text{IC: } \begin{matrix} x(0) = x_0 \\ \dot{x}(0) = \dot{x}_0 \end{matrix}$$

$$\mathcal{L}[m\ddot{x} + c\dot{x} + kx] = \mathcal{L}[F(t)] = F(s)$$

$\mathcal{L}[x(t)]$

WE'LL SHOW THAT $\mathcal{L}[m\ddot{x} + c\dot{x} + kx] = (ms^2 + cs + k)\,X(s) + g(s)$

PROPERTIES

LINEARITY $\mathcal{L}[a\,f(t) + b\,g(t)] = a\mathcal{L}[f(t)] + b\mathcal{L}[g(t)]$

DERIVATIVE $\mathcal{L}(\dot{f}(t)) = \int_{0^-}^{\infty} \frac{df}{dt} e^{-st}\,dt \Rightarrow \mathcal{L}[\dot{f}(t)] = sF(s) - f(0)$

$$\mathcal{L}[\ddot{f}(t)] = s^2F(s) \underbrace{- sf(0) - \dot{f}(0)}$$

IC's

$$\mathcal{L}[\dddot{f}(t)] = s^3F(s) \underbrace{- s^2f(0) - s\dot{f}(0) - \ddot{f}(0)}$$

$$\boxed{\mathcal{L}\left[\frac{d^nf}{dt^n}\right] = s^nF(s)} + \text{IC TERMS}$$

NOT NEEDED FOR GENERAL SOLUTION, ONLY PARTICULAR

INTEGRATION $\mathcal{L}\left[\int f(t)\,dt\right] = \frac{F(s)}{s} + \text{IC TERMS}$

$$\mathcal{L}\left[\int\int\dots\int f(t)\,dt\right] = \frac{F(s)}{s^n} + \text{IC's}$$