Exam 12/6 : HW7 / HW8 , lead-lag , PID

ASK: PI increases order of sys, does lag do the same? What difference do the pole/roots make

Also: HW 7 PM not exactly equal is that wrong

Ex: lead-lag compensation

$$G(s) = \frac{200}{s^2(s+110)}$$

- PM $\geq$ 50°, GM $\geq$ 15 dB
- ω_{BW} as high as possible
- $e_{ss} < 0.001$ when $r(t)$ is:
  - $1(t)$
  - t
  - $\frac{1}{2}t^2$

Assumptions

- Use lead compensator to add phase
- Limit lead phase to max of 75° (Typically 60°-80°)
- design for 55° PM to add extra (to compensate for lag)
- ω_c of 40 rad/s where $\angle G(s) \approx -200°$ is highest possible given PM = 55°

$$\angle G(j\omega_c) = -199.98°$$

$$\varphi_{des} = -180° + 55° = -125°$$

$$\varphi_{lead} = \varphi_{des} - \angle G(j\omega_c)$$

$$\varphi_{lead} = -125 - (-199.98°) \cong 75°$$

$$\alpha = \frac{1-\sin\varphi_{max}}{1+\sin\varphi_{max}} = 0.0173 \rightarrow \text{lead ratio} \cong 57.7$$

(a bit high but ok)

$$\rightarrow z = \omega_{max}\sqrt{\alpha} \cong 5.266$$

$$p = \frac{\omega_{max}}{\sqrt{\alpha}} \cong 303.8$$

$$D_{lead}(s) = K\frac{s+z}{s+p} = K\left(\frac{s+5.266}{s+303.8}\right)$$

$$M\Big|_{\omega=40} = \left|D_{lead}(40j)\, G(40j)\right|$$

$$M\Big|_{\omega=40} = (1.41\times10^{-4})\, K = 1$$

$$\rightarrow K \cong 7.11\times10^{3}$$

$$\rightarrow D_{lead}(s) = 7.11\times10^{3}\left(\frac{s+5.266}{s+303.8}\right)$$

$\rightarrow \dfrac{E(s)}{R(s)} = \dfrac{s^4 + \cdots + s^2}{s^4 + \cdots + 7.458 \times 10^6}$

$1(t) \rightarrow e_{ss} = 0$ ✓

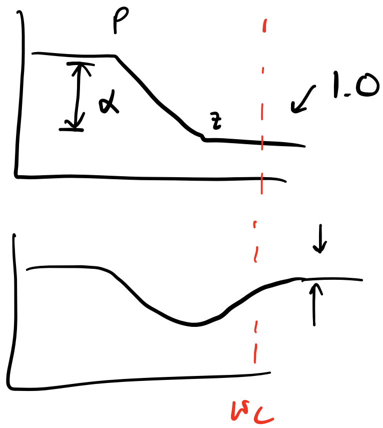
$t \rightarrow e_{ss} = 0$ ✓

$\frac{1}{2}t^2 \rightarrow e_{ss} \approx 0.0045$ ✗

exceeds spec of 0.001 by 4.5x

→ set lag ratio to 5

→ set lag zero to be 10x smaller than W_c to avoid too much phase loss (iterate in practice)



$D_{lag}(s) = \dfrac{s+z}{s+p}$

$z = \dfrac{w_c}{10} = 4$

$p = \dfrac{z}{\alpha} = 0.8$

$\rightarrow D_{lag}(s) = \dfrac{s+4}{s+0.8}$

$D_{leadlag}(s) = 7.11 \times 10^3 \left(\dfrac{s+5.266}{s+303.8}\right)\left(\dfrac{s+4}{s+0.8}\right)$

PID can be approached similarly to lead-lag

PID: $K_p + \dfrac{K_I}{s} + K_d s = \dfrac{K_d s^2 + K_p s + K_I}{s}$

— (PD)(PI) is equivalent to PID

$\rightarrow \left\{ \begin{array}{l} K_p = K(z_{PD} + z_{PI}) \\ K_d = K \\ K_I = K z_{PD} z_{PI} \end{array} \right\}$