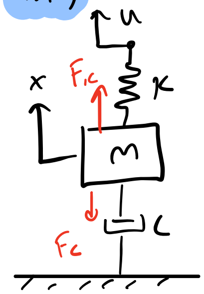
PROBLEM 1)

1.1) FIND EOM FOR OUTPUT x(t), INPUT u(t)



$m = 3$ kg

$c = 45$ N/m/s

$k = 150$ N/m

$\Sigma F = ma$

$k(u - x) - c\dot{x} = m\ddot{x}$

$$\boxed{m\ddot{x} + c\dot{x} + kx = ku}$$ EOM

1.2) FIND x(t) FOR $u(t) = e^{-3t}$, $x(0) = 2$, $\dot{x}(0) = 0$

$$\mathcal{L}[m\ddot{x}] = m\left(s^2 X(s) - s\,x(0) - \dot{x}(0)\right) = m\left(s^2 X(s) - 2s\right)$$

$$\mathcal{L}[c\dot{x}] = c\left(s\,X(s) - x(0)\right) = c\left(s\,X(s) - 2\right)$$

$$\mathcal{L}[kx] = k\,X(s)$$

$$\mathcal{L}[u(t)] = \int_0^\infty e^{-3t} e^{-st}\,dt = \int_0^\infty e^{-3t-st}\,dt$$

$$= \left[\frac{1}{-3-s} e^{-3t-st}\right]_{t=0}^{t=\infty} = 0 - \frac{1}{-3-s} = \frac{1}{s+3} = U(s)$$

$$\hookrightarrow \frac{X(s)}{U(s)} = \frac{1}{\frac{m}{k}s^2 + \frac{c}{k}s + 1} \leftarrow \Delta(s)$$

$$\Rightarrow \mathcal{L}[EOM] = m\left(s^2 X(s) - 2s\right) + c\left(sX(s) - 2\right) + kX(s) = \frac{1}{s+3}$$

$$= X(s)\left[ms^2 + cs + k\right] - 2ms - 2c = \frac{1}{s+3}$$

$$\rightarrow X(s) = \frac{\frac{1}{s+3} + 2ms + 2c}{ms^2 + cs + k} = \frac{\frac{1}{s+3} + 6s + 90}{3s^2 + 45s + 150}$$

$$(6s+90)(s+3) = 6s^2 + 108s + 270$$

$$x(s) = \frac{6s^2+108s+271}{(s+3)(3s^2+45s+150)}$$

$$r = \frac{-45 \pm \sqrt{45^2-4(3\cdot150)}}{2(3)} = -10,\ -5$$

$$\rightarrow x(s) = \frac{6s^2+108s+271}{(s+3)(s+10)(s+5)} = \frac{c_1}{s+3} + \frac{c_2}{s+10} + \frac{c_3}{s+5}$$

$$6s^2+108s+271 = c_1(s+10)(s+5) + c_2(s+3)(s+5) + c_3(s+3)(s+10)$$

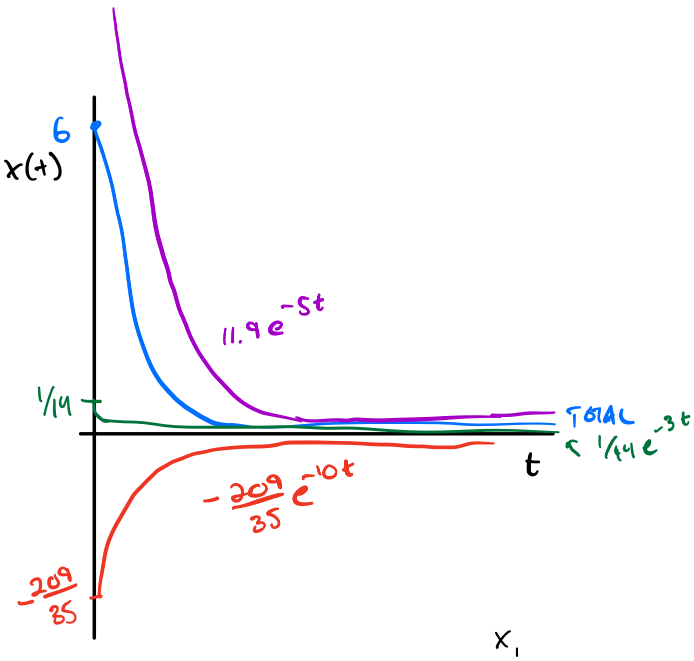
$$\begin{array}{ccc} c_1 & c_2 & c_3 \end{array}$$

$$\left[\begin{array}{ccc|c} s^2 & s^2 & s^2 & 6 \\ 15s & 8s & 13s & 108 \\ 50 & 15 & 30 & 271 \end{array}\right] \rightarrow \begin{array}{l} c_1 = \frac{1}{14} \\ c_2 = -\frac{209}{35} \\ c_3 = 11.9 \end{array}$$

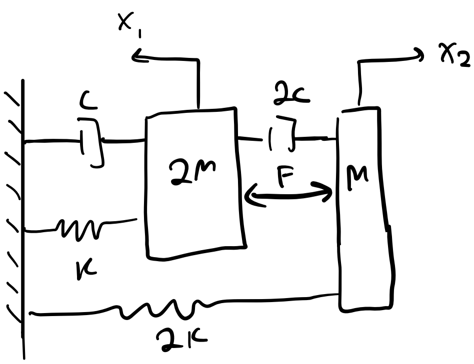
FROM TABLE:

$$\boxed{x(t) = \frac{1}{14}e^{-3t} - \frac{209}{35}e^{-10t} + 11.9e^{-5t}}$$

1.3)

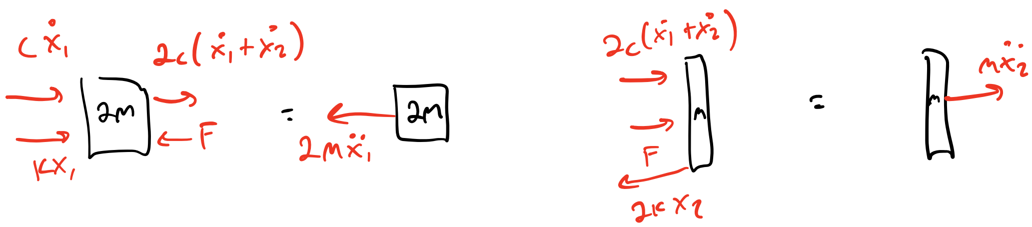


PROBLEM 2)



2.1) DERIVE EOM

FBD/KD:



$$M\ddot{x}_1 = F - c\dot{x}_1 - kx_1 - 2c(\dot{x}_1+\dot{x}_2)$$

$$m\ddot{x}_2 = F + 2c(\dot{x}_1+\dot{x}_2) - 2kx_2$$

$$\boxed{\begin{aligned} 2m\ddot{x}_1 + 3c\dot{x}_1 + kx_1 &= F(t) - 2c\dot{x}_2 \quad (1) \\ m\ddot{x}_2 - 2c\dot{x}_2 + 2kx_2 &= F(t) + 2c\dot{x}_1 \quad (2) \end{aligned}}$$

2.2)

→ TAKE LT

$$(1):\ (2ms^2 + 3cs + k)\,X_1(s) = F(s) - 2cs\,X_2(s) \quad (3)$$

$$(2):\ (ms^2 - 2cs + 2k)\,X_2(s) = F(s) + 2cs\,X_1(s) \quad (4)$$

↓ USE MATLAB

$$(3):\ (2ms^2 + 3cs + k)\,X_1(s) = F(s) - 2cs\,X_2(s)$$

$$\hookrightarrow \boxed{\frac{X_1(s)}{F(s)} = \frac{2(s^2 - 2s + 6)}{(4s^4 - s^3 + 29s^2 + 12s + 36)}} = T_1(s)$$

$$(4):\ (ms^2 - 2cs + 2k)\,X_2(s) = F(s) + 2cs\,X_1(s)$$

$$\hookrightarrow \boxed{\frac{X_2(s)}{F(s)} = \frac{4s^2 + 5s + 6}{(4s^4 - s^3 + 29s^2 + 12s + 36)}} = T_2(s)$$

2.3) MATLAB TO GET $x_1(t)$, $x_2(t)$ FROM $t = 0$ to $t = 30$ s

$m = 1\,kg \quad c = 0.5 \quad k = 3.0$

PROBLEM 3)

3.1) → MATLAB

3.2) FVT: $x(\infty) = \lim_{s \to 0} [s\,X(s)]$

$$F(s) = \frac{10}{s}$$

$$X_1(\infty) = \lim_{s \to 0} \left[\cancel{s} \left(\frac{10}{\cancel{s}}\right) \frac{2\cancel{s^2} + 12}{4\cancel{s^4} + 7\cancel{s^3} + 31\cancel{s^2} + 24\cancel{s} + 36} \right] = \boxed{\frac{120}{36}}$$

$$x_2(\infty) = \lim_{s \to 0} \left[\cancel{s} \left(\frac{10}{\cancel{s}}\right) \frac{4\cancel{s^2} + 6}{4\cancel{s^4} + 7\cancel{s^3} + 31\cancel{s^2} + 24\cancel{s} + 36} \right] = \frac{10}{6} = \boxed{\frac{5}{3}}$$

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% Written by Kyle Adler for ME446
```

Problem 2

```
clear
% part 2

% params
m = 1; %kg
c = 0.5; %N/m-s
k = 3; %N/m
syms x1 x2 F s
eq1 = (2*m*s^2+3*c*s+k)*x1 == F-2*c*s*x2
eq2 = (m*s^2-2*c*s+2*k)*x2 == F+2*c*s*x1
sol = solve([eq1,eq2],[x1,x2])
T1 = simplify(sol.x1/F)
T2 = simplify(sol.x2/F)

% part 3

% params
m = 1; %kg
c = 0.5; %N/m-s
k = 3; %N/m

% transfer function
s = tf('s');
T_1 = (2*s^2+12)/(4*s^4+7*s^3+31*s^2+24*s+36)
T_2 = (4*s^2+6)/(4*s^4+7*s^3+31*s^2+24*s+36)

% create input function for t <10
t_1 = transpose(0:0.1:9.9);
F_1 = t_1.*10; % F = 10t
% 2. when t>=10
t_2 = transpose(10:0.1:30);
F_2 = zeros(size(t_2));
% 3. combine t's
t = [t_1
    t_2];
% 4. combine F's
F = [F_1
    F_2];

% plot
figure(1);
hold on
[x1,t] = lsim(T_1,F,t);
[x2,t] = lsim(T_2,F,t);
plot(t,x1, 'DisplayName','X1')
plot(t,x2, 'DisplayName','X2')
xlabel('time (seconds)')
```

```
title('Problem 2.3')
ylabel('output [m]')
grid on
legend
```

```
eq1 =

x1*(2*s^2 + (3*s)/2 + 3) == F - s*x2


eq2 =

x2*(s^2 - s + 6) == F + s*x1


sol =

  struct with fields:

    x1: (2*F*(s^2 - 2*s + 6))/(4*s^4 - s^3 + 29*s^2 + 12*s + 36)
    x2: (F*(4*s^2 + 5*s + 6))/(4*s^4 - s^3 + 29*s^2 + 12*s + 36)


T1 =

(2*(s^2 - 2*s + 6))/(4*s^4 - s^3 + 29*s^2 + 12*s + 36)


T2 =

(4*s^2 + 5*s + 6)/(4*s^4 - s^3 + 29*s^2 + 12*s + 36)


T_1 =

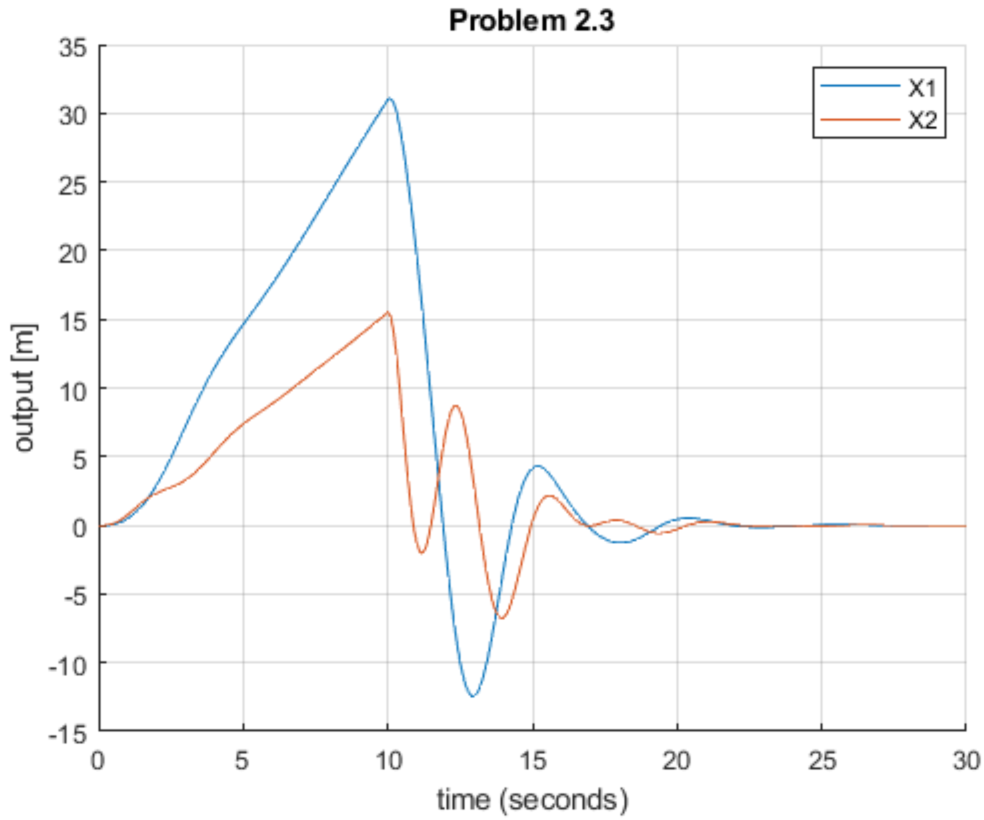
            2 s^2 + 12
  ----------------------------------
  4 s^4 + 7 s^3 + 31 s^2 + 24 s + 36

Continuous-time transfer function.


T_2 =

            4 s^2 + 6
  ----------------------------------
  4 s^4 + 7 s^3 + 31 s^2 + 24 s + 36

Continuous-time transfer function.
```



Problem 3

```
% step function
t_f = 50
% create linspace from  0 to t_f, transpose to be 1000x1
t = linspace(0,t_f,1000).';
% construct step function
F = ones(size(t)).*10;

% plot
figure(2);
hold on
[x1,t] = lsim(T_1,F,t);
[x2,t] = lsim(T_2,F,t);
plot(t,x1, 'DisplayName','X1')
plot(t,x2, 'DisplayName','X2')
xlabel('time (seconds)')
title('Problem 3.1')
ylabel('output [m]')
grid on
legend
```

```
t_f =

    50
```

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