Problem 1 Satellite in orbit, ECI @ t_0:

$x = -3381.5$ km, $y = -5885.6$ km, $z = 4374.1$ km

$v_x = 6.1106$ km/s, $v_y = -1.1721$ km/s, $v_z = -3.1914$ km/s

1a) Calculate cartesian → MCS @ t_0

→ Can use code from HW4

Based on L14 example 1 code:

- get $r = \sqrt{\bar{r}\cdot\bar{r}}$, $v = \sqrt{\bar{v}\cdot\bar{v}}$, $v_r = \frac{\bar{r}\cdot\bar{v}}{r}$
- sign v_r indicates direction
- $\bar{h} = \bar{r} \times \bar{v}$, $\bar{n} = \hat{z} \times \bar{h}$

→ get e from vis-viva eqn

- Solve i, Ω, ω using $\bar{h}, \bar{n}, \bar{e}$

→ get θ from $\bar{r}, \bar{e}$ & a from $\dfrac{h^2}{\mu(1-e^2)}$

Plug into script: $\bar{r} = [-3381.5 \quad -5885.6 \quad 4374.1]$

$\bar{v} = [6.1106 \quad -1.1721 \quad -3.1914]$

→ $\boxed{e = 0.49, \ \Omega = 123.68°, \ i = 35.74°, \ \omega = 232.34°, \ \theta = 239.64°, \ a = 8000.05 \text{ km}}$

1b) Find position & velocity in cartesian ECI after 7 days of flight, including oblateness effects.

Apply perturbation theory:

$$\frac{d\Omega}{dt} = \frac{-3 J_2 n r_e^2 \cos i}{2a^2(1-e^2)^2}, \quad \frac{d\omega}{dt} = \frac{3 J_2 n r_e^2}{4a^2(1-e^2)^2}(4 - 5\sin^2 i)$$

$$n = \frac{2\pi}{T}, \quad J_{2,E} = 1.082626 \times 10^{-3}, \quad r_e = 6378 \text{ km}$$

$T = 2\pi\sqrt{\frac{a^3}{\mu}}$, $\mu = 3.986 \times 10^5$ $\rightarrow$ $T = 7121.15$ s

First find τ_0 from $\theta_0 \rightarrow E_0 \rightarrow M_0$

$\tan\frac{E_0}{2} = \sqrt{\frac{1-e}{1+e}}\tan\frac{\theta_0}{2} = -1.02 \rightarrow E_0 = -1.59$

$\rightarrow E_0 = 2\pi - 1.59 = 4.692$ rad

$M_0 = E_0 - e\sin E_0 \rightarrow 5.182$ rad

$\rightarrow \tau_0 = \frac{M_0}{\frac{2\pi}{T}} = 5872.85$ s

$\tau_{final} = 5872.85\text{ s} + \underbrace{7(24)(3600)}_{7\text{ days}} = 610672.85$ s

$\frac{T_{final}}{T} \rightarrow$ remainder $T_r = 0.75478\, T = 5374.9$ s

$\rightarrow M_{final} = \frac{2\pi}{T}\cdot T_r = 4.7424$ rad

$M_{final} = E_f - e\sin E_f$ $\rightarrow$ solve w/ MATLAB $= E_f = 4.29$

$\rightarrow \tan\frac{\theta_f}{2} = \sqrt{\frac{1+e}{1-e}}\tan\frac{E}{2} \rightarrow \theta_f = -2.414$ rad

$\rightarrow \theta_f = 3.8685$ rad

Now account for change in Ω, ω:

$$\frac{d\Omega}{dt} = \frac{-3 J_2 n r_e^2 \cos i}{2a^2(1-e^2)^2} = -1.284 \times 10^{-6} \text{ rad/s}$$

$$\frac{d\omega}{dt} = \frac{3 J_2 n r_e^2}{4a^2(1-e^2)^2}(4 - 5\sin^2 i) = 1.814 \times 10^{-6} \text{ rad/s}$$

$\Delta t = 7\cdot 24\cdot 3600$

$\Delta\Omega = \frac{d\Omega}{dt}\cdot\Delta t = -0.7765 \rightarrow \Omega_f = 1.382$ rad

$\Delta\omega = \frac{d\omega}{dt}\cdot\Delta t = 1.097 \rightarrow \omega_f = 5.152$ rad

→ Plug into MCS → ECI cartesian code from HW4

→ $\boxed{\begin{aligned} \bar{r} &= [-4662.6 \quad -8086.4 \quad 2203.6] \text{ km} \\ \bar{v} &= [4.665 \quad -6.478 \quad -3.362] \text{ km/s} \end{aligned}}$

Problem 2 Two circular orbits:

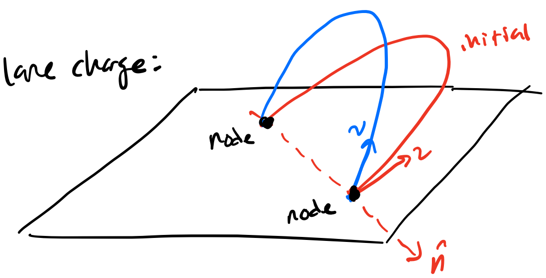
Initial $r = 5000$ km, $i = 60°$

Final $r = 150{,}000$ km, $i = 0°$

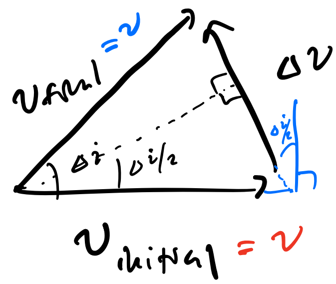
calculate Δv for each impulse + total Δv for each transfer:

2a) plane change + Hohmann

Simple plane change:



Isosceles

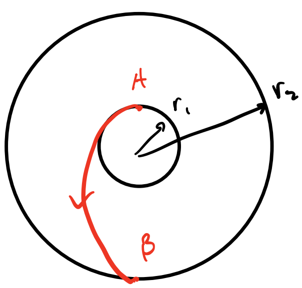


$\therefore \frac{\Delta v}{2} = v \sin\left(\frac{\Delta i}{2}\right)$ $\quad \boxed{\Delta v = 2 v \sin\left(\frac{\Delta i}{2}\right)}$ For a tangential burn $(\gamma = 0)$

$v_{circ} = \sqrt{\frac{\mu}{r}} = 8.928$ km/s, $\Delta i = 60°$

→ $\Delta v = 2(8.928)\sin(30°) = \boxed{8.9286 \text{ km/s} = \Delta v_1}$

Hohmann $r = 5000 \to r = 150000$, $a = 77500$



$\Delta v_A = v_{tA} - v_{c1} = \sqrt{\frac{\mu}{r_1}}\left(\sqrt{\frac{r_2}{a}} - 1\right) = \boxed{3.493 \text{ km/s} = \Delta v_2}$

$\Delta v_B = v_{c2} - v_{tb} = \sqrt{\frac{\mu}{r_2}}\left(1 - \sqrt{\frac{r_1}{a}}\right) = \boxed{1.216 \text{ km/s} = \Delta v_3}$

→

$\boxed{\begin{aligned} &\text{plane change + hohmann total } \Delta v: \\ &\Delta v_1 + \Delta v_2 + \Delta v_3 = 13.6377 \text{ km/s} \end{aligned}}$

2b) Hohmann + plane change

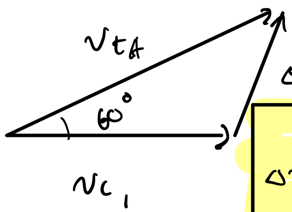
$$\Delta v_1 = v_{tA} - v_{c1} = 3.493 \text{ km/s}$$

$$\Delta v_2 = v_{c2} - v_{tB} = 1.216 \text{ km/s}$$

$$\Delta v_3 = 2 v_{c2} \sin(30°) = 1.63 \text{ km/s}$$

$$\text{total } \Delta v = 6.34 \text{ km/s}$$

2c) Hohmann combined w/ plane change at perigee



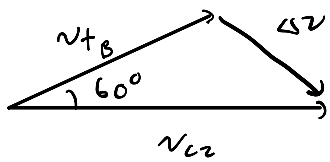
Law of cosines:

$$\Delta v_1 = \sqrt{v_{c1}^2 + v_{tA}^2 - 2 v_{c1} v_{tA} \cos 60°} = 11.095 \text{ km/s}$$

$$\Delta v_2 = v_{c2} - v_{tB} = 1.216 \text{ km/s}$$

$$\text{total } \Delta v = 12.31 \text{ km/s}$$

2d) Hohmann w/ plane change @ apogee



$$\Delta v_1 = v_{tA} - v_{c1} = 3.493 \text{ km/s}$$

$$\Delta v_2 = \sqrt{v_{c2}^2 + v_{tB}^2 - 2 v_{c2} v_{tB} \cos 60°} = 1.47 \text{ km/s}$$

$$\text{total } \Delta v = 4.97 \text{ km/s}$$

2e) optimally split

$$\Delta i = 60° = \alpha_1 + \alpha_2$$

$$\Delta v_1^2 = v_{tA}^2 + v_{c1}^2 - 2 v_{tA} v_{c1} \cos \alpha_1$$

$$\Delta v_2^2 = v_{tB}^2 + v_{c2}^2 - 2 v_{tB} v_{c2} \cos \alpha_2$$

$$\rightarrow \alpha_2 = 60° - \alpha_1$$

$\rightarrow$ minimize Δv_{tot} w.r.t. α_1 in MATLAB

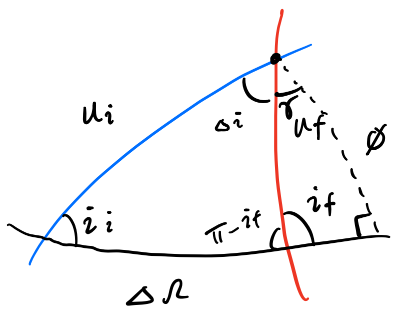
$\rightarrow$ $\boxed{\Delta v_1 = 3.495, \; \Delta v_2 = 1.463, \; \Delta v_{tot} = 4.958 \text{ km/s}}$

$\rightarrow$ Optimally split plane change is most efficient.
Hohmann w/ PC @ apogee is really close to optimal.

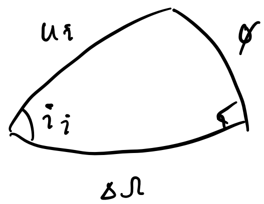
Problem 3

Space shuttle @ $i = 28.5°$ $\rightarrow$ $i = 35°$

If transfer at 20° latitude, what angle should $\vec{v}$ be rotated, what is change in Ω?



$u_i = \omega + \theta$



$$\frac{\sin u_i}{\sin 90°} = \frac{\sin x}{\sin i_i} \rightarrow u_i = \sin^{-1}\left(\frac{\sin 20°}{\sin 28.5°}\right) = 45.79°$$

$\rightarrow$ $\cos 35° = \cos 28° \cos \Delta\theta - \sin 28° \sin \Delta\theta \cos 45.78°$

$\rightarrow$ $\boxed{\Delta\theta = 8.6°}$

$$\Delta\Omega = \cos^{-1}\left(\frac{\cos\Delta\theta - \cos i_f \cos i_i}{\sin i_f \sin i_i}\right) = \boxed{10.8° = \Delta\Omega}$$

Problem 4

Satellites A & B in circular orbit $r = 26610$ km.
B ahead of A by 180°. Calculate a of phasing orbit to rendezvous after 1/2 revolution of target. Find total Δv

$\rightarrow$ phasing orbit period $T_{ph} = \frac{1}{2} T_{target} = \frac{1}{2} \frac{2\pi}{\sqrt{\mu}} a^{3/2}$

$\rightarrow$ $T_{ph} = 21600$ s $\rightarrow$ $a_{ph} = \left(\frac{T\sqrt{\mu}}{2\pi}\right)^{2/3} = 16763$ km

$v_0 = \sqrt{\frac{\mu}{26610}} = 3.87$ km/s, $N = 0.5$

$$\Delta v = \frac{\Delta\theta}{2\pi}\,\frac{\mu}{3 v_0 a_0 N} = \frac{\pi}{2\pi}\,\frac{3.986E5}{3(3.87)(26610)(0.5)} = \boxed{1.29 \text{ km/s} = \Delta v \text{ to rendezuous}}$$