Recall boat

$u_j = \frac{D}{\rho \forall} + u_b$; $T = D$

Propulsion efficiency

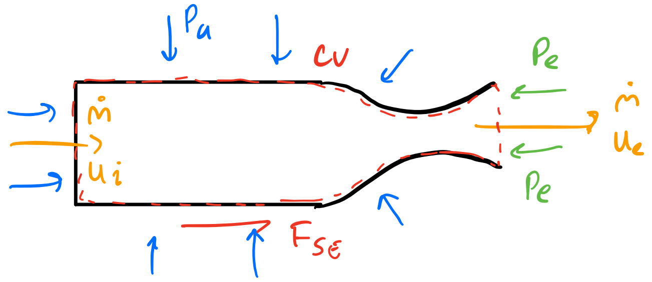
$$\eta_p = \frac{\text{what I want}}{\text{what I spend}} = \frac{\overbrace{T \cdot u_b}^{\text{power to keep } u_b \text{ const.}}}{\dot{W}_{pump}} = \frac{\dot{m}(u_j - u_b)\, u_b}{\frac{1}{2}\dot{m}(u_j^2 - u_b^2)} = \frac{2u_b}{u_j + u_b}$$

work or power

Conflicting needs | For high thrust, need high u_j
For high η_p need low u_j

III Rocket performance

cons. momentum for rocket: $p = P_a$ everywhere outside except exit



In general:

i) $P_e \neq P_a$

ii) $P_e \neq 1\ atm$

iii) $P_a \neq 1\ atm$

iv) $u_i << u_e$

v) Cannot use Bernoulli $(p + \frac{1}{2}\rho u^2 = const)$ b/c " only valid for $\rho = const.$

Apply

$$\frac{d}{dt}\int_{CV} \rho \underline{u}\, d\forall + \int_{CS} \rho \underline{u}\, \underline{u}\cdot d\underline{A} = -\int_{CS} p\, d\underline{A} + \underline{F_E}$$

$\frac{d}{dt}\int_{CV} \rho \underline{u}\, d\forall = 0$, steady state

neglect u_i, P_e, u_e uniform over A_e

$$\int_{CS} \rho \underline{u}\, \underline{u}\cdot d\underline{A} = u_e \hat{i}\,(\rho_e u_e A_e) = \dot{m} u_e \hat{i}$$

u_e relative to rocket

$p = P_a = const$, everywhere on CV except A_e, where $p = P_e$

$P_e = P_a + P_e - P_a$

$$-\int p\, d\underline{A} = -\int_{CS} \cancelto{0}{P_a\, d\underline{A}} - \int_{A_e} (P_e - P_a)\, d\underline{A} = (P_a - P_e) A_e \hat{i}$$

Reassemble

$$\dot{m} u_e \hat{i} + (P_e - P_a) A_e \hat{i} = F_{SE} \hat{i}$$ stand force balancing thrust

$\rightarrow$ $\mathcal{T} = F_{SE}$ in magnitude

$\rightarrow$ $\boxed{\mathcal{T} = \dot{m} V_e + (P_e - P_a) A_e}$ in $-\hat{x}$ direction

if engine moves at constant velocity

$\rightarrow$ velocities relative to CV

then: $\mathcal{T} = \dot{m} V_{e_{xyz}} + (P_e - P_a) A_e$

From now on, remember that u is relative to CV (drop subscript)

we'll have to learn how u_e & P_e relate to:

- nozzle shape
- CC conditions

Rocket parameters

$$\mathcal{T} = \dot{m} u_e + (P_e - P_a) A_e$$

$$= \underbrace{\rho_e u_e A_e}_{\dot{m}_e} u_e + (P_e - P_a) A_e$$

Prioritize u_e over ρ_e

$$= \rho_e A_e u_e^2 + (P_e - P_a) A_e$$

mission req's:

- payload
- altitude
- final velocity

affected by:
- thrust
- " duration
- limits imposed on thrust by gas dynamics & combustion
- limits imposed on acceleration by structural req's
- relative magnitude of $\begin{cases} M_e \\ M_p \\ M_s \end{cases}$

Useful to define equivalent exhaust velocity

$$u_{eq} \equiv u_e + \left(\frac{P_e - P_a}{\dot m}\right) A_e \quad \text{such that} \quad \mathcal{T} = \dot m \, u_{eq}$$

Recall impulse

$$I \equiv \int F \, dt \quad ; \quad \text{in present case, } F = \mathcal{T}$$

Call burn period t_b. Assume $u_{eq} = \text{const}$ w.r.t. time

$$I = \int_0^{t_b} \dot m \, u_{eq} \, dt = u_{eq} \int_0^{t_b} \dot m \, dt = u_{eq} M_p$$

$$\Rightarrow \frac{I}{M_p} = u_{eq} = \frac{\mathcal{T}}{\dot m}$$

Define "specific impulse"

$$I_{sp} \equiv \frac{I}{M_p g_e} = \frac{u_{eq}}{g_e}$$

total change in impulse per unit weight propellant

$$[I_{sp}] = s$$

Isp is another way to describe exhaust velocity

no matter where rocket is, g_e is always accel. of gravity on earth