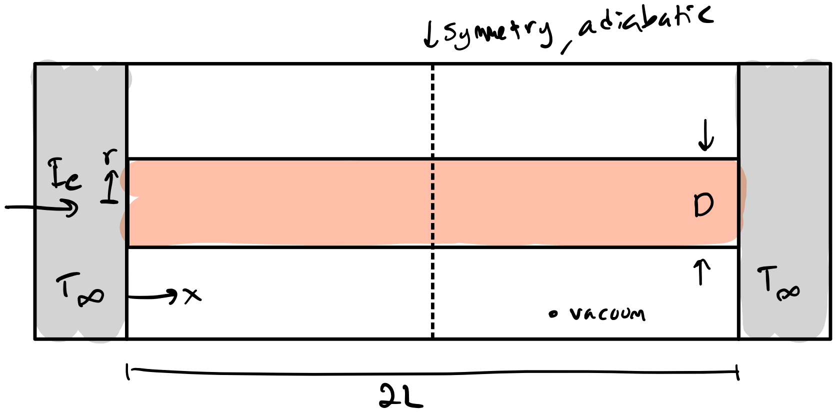
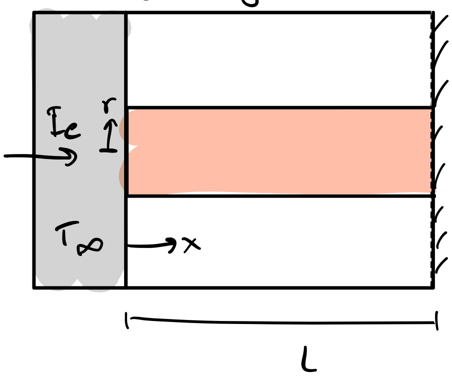


$$\dot{q}''' = \frac{I_e^2 \rho_e}{A_c^2}$$

Half - Symmetry:



$M = 11 \quad N = 301, \quad L = 1mm, \quad t_{sim} = 0.01(s)$

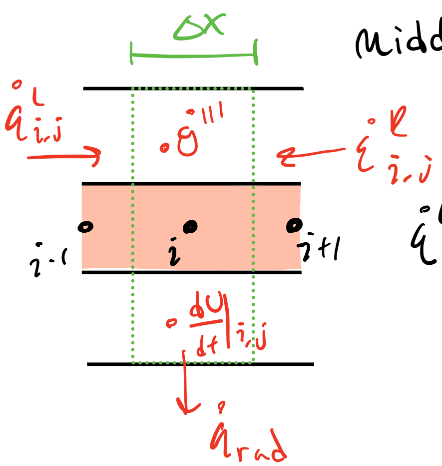
$$\Delta x = \frac{L}{M-1}, \quad \Delta t = \frac{t_{sim}}{N-1}$$

↳ $x_i, \; t_j$

Left Boundary:

$T_{1,j} = T_\infty$ → base has $T = const = T_\infty$

$$\forall \; j \in (2:N)$$



Middle Nodes:

IN + GEN − OUT + STORED

$$\dot{q}^L_{i,j} + \dot{q}^R_{i,j} + \dot{g}'''(A_c \Delta x) = \frac{dU}{dt}\Big|_{i,j} + \dot{q}_{rad}$$

$$\frac{dU}{dt}\Big|_{i,j} = \underbrace{\rho A_c \Delta x}_{mass} \, c \, \frac{dT}{dt}\Big|_{i,j}$$

$q^L_{i,j} = \frac{kA_c}{\Delta x}(T_{i-1,j} - T_{i,j})$ $\qquad \dot q_{rad} = \varepsilon A_s \sigma (T_s^4 - T_\infty^4)$

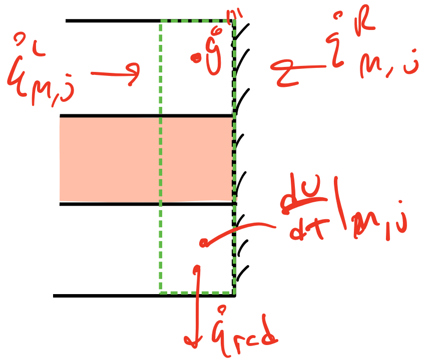
$\dot q^R_{i,j} = \frac{kA_c}{\Delta x}(T_{i+1,j} - T_{i,j})$ $\qquad = \varepsilon\, p_{er} \cdot dx \cdot \sigma (T_{ij}^4 - T_\infty^4)$

$$\left.\frac{dT}{dt}\right|_{i,j} = \underbrace{\frac{\frac{kA_c}{\Delta x}\left(T_{i-1,j} + T_{i+1,j} - 2T_{ij}\right) + \dot g''' A_c \Delta x - \dot q_{rad}}{\rho A_c \Delta x\, c}}$$

$$T_{i,j+1} = T_{ij} + \left.\frac{dT}{dt}\right|_{ij} \cdot \Delta t$$

$$\forall\, i \in (2:M-1)\,;\ \forall\, j \in (2:N-1)$$

Right boundary:



IN + GEN = OUT + STORED

$$\dot q^L_{M,j} + \dot q^R_{M,j} + \dot g'''\left(A_c \frac{\Delta x}{2}\right) = \left.\frac{dU}{dt}\right|_{M,j} + \dot q_{rad}$$

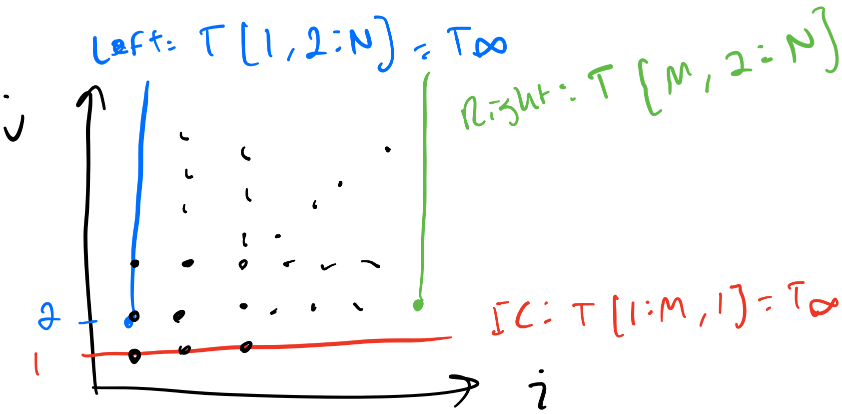
$$\left.\frac{dU}{dt}\right|_{M,j} = \underbrace{\rho A_c \frac{\Delta x}{2}}_{mass} c \left.\frac{dT}{dt}\right|_{M,j}$$

$q^L_{M,j} = \frac{kA_c}{\Delta x}(T_{M-1,j} - T_{M,j})$ $\qquad \dot q_{rad} = \varepsilon A_s \sigma (T_s^4 - T_\infty^4)$

$\dot q^R_{M,j} = 0$, adiabatic $\qquad = \varepsilon\, p_{er} \cdot \frac{dx}{2} \cdot \sigma (T_{ij}^4 - T_\infty^4)$

$$\left.\frac{dT}{dt}\right|_{M,j} = \underbrace{\frac{\frac{kA_c}{\Delta x}\left(T_{M-1,j} - T_{M,j}\right) + \dot g''' A_c \frac{\Delta x}{2} - \dot q_{rad}}{\rho A_c \frac{\Delta x}{2}\, c}}$$

$$T_{m,j+1} = T_{m,j} + \left.\frac{dT}{dt}\right|_{m,j} \cdot \Delta t \quad \forall j \in (2:N-1)$$



a) Plot of $T(x)$ for t: 1, 100, 200, 301

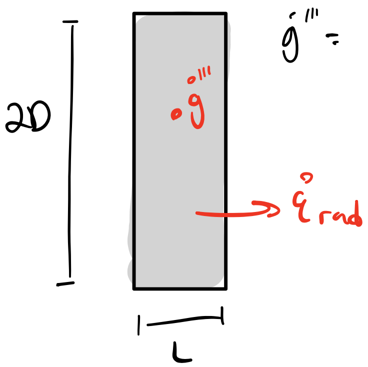
b) Position x, time t that $T > T_{melt} = 1358\,K$

$x_{crit} = x[11]$

$t_{crit} = t[174]$

c) $D_{base} = 2D$, generation. Length $= L$

Find $T(t)$ w/ lumped capacitance



$$\dot{g}''' = \frac{I_e^2 \rho_e}{A_c^2} \qquad \cancel{IN}^{0} + Gen = OUT + STORED$$

$$\dot{g}'''\left(\frac{\pi}{4}(2D)^2 \cdot L\right) = \dot{q}_{rad} + \frac{dU}{dt}$$

$$\frac{dU}{dt} = \frac{d(mu)}{dt} = m\frac{du}{dT} \cdot \frac{dT}{dt} = mc\frac{dT}{dt}$$

$$mc\frac{dT}{dt} = \dot{g}'''(\pi D^2 L) - A_s \sigma \varepsilon 4 \bar{T}^3 (T - T_\infty)$$

$$\frac{dT}{dt} + \frac{A_s \sigma \varepsilon 4 \bar{T}^3}{mc} T = \frac{A_s \sigma \varepsilon 4 \bar{T}^3}{mc} T_\infty + \frac{\dot{g}''' \pi D^2 L}{mc}$$

$$T = T_h + T_p : \quad T_h : \frac{dT_h}{dt} + \frac{A_s \sigma \varepsilon 4 \bar{T}^3}{mc} T_h = 0$$

$$\int \frac{dT_h}{dt} dt = \int -\frac{A_s \sigma \varepsilon 4 \bar{T}^3}{mc} T_h \, dt$$

$$\rightarrow T_h = -\frac{A_s \sigma \varepsilon 4 \bar{T}^3}{mc} T_h \, t + C_1$$

$$T_h \left(1 + \frac{A_s \sigma \varepsilon 4 \bar{T}^3}{mc} t\right) = C_1$$

$$T_h = \frac{C_1}{\left(1 + \frac{A_s \sigma \varepsilon 4 \bar{T}^3}{mc} t\right)}$$

$$T_p : \cancel{\frac{dT_p}{dt}}^{0} + \frac{A_s \sigma \varepsilon 4 \bar{T}^3}{mc} T_p = \frac{A_s \sigma \varepsilon 4 \bar{T}^3}{mc} T_\infty + \frac{\dot{g}''' \pi D^2 L}{mc}$$

Guess $T_p = \text{const}$

$$\rightarrow T_p = T_\infty + \frac{\dot{g}''' \pi D^2 L}{A_s \sigma \varepsilon 4 \bar{T}^3}$$

$$T = T_h + T_p = \frac{C_1}{\left(1 + \frac{A_s \sigma \varepsilon 4 \bar{T}^3}{mc} t\right)} + T_\infty + \frac{\dot{g}''' \pi D^2 L}{A_s \sigma \varepsilon 4 \bar{T}^3}$$

$$IC: \; T(t=0) = T_\infty : \quad \frac{C_1}{1} + T_\infty + \frac{\dot{g}''' \pi D^2 L}{A_s \sigma \varepsilon 4 \bar{T}^3} = T_\infty$$

$$\therefore C_1 = \frac{-\dot{g}''' \pi D^2 L}{A_s \sigma \varepsilon 4 \bar{T}^3}$$

$$\rightarrow \boxed{T(t) = \frac{-\dot{g}''' \pi D^2 L / (A_s \sigma \varepsilon 4 \bar{T}^3)}{\left(1 + \frac{A_s \sigma \varepsilon 4 \bar{T}^3}{mc} t\right)} + \frac{\dot{g}''' \pi D^2 L}{A_s \sigma \varepsilon 4 \bar{T}^3} + T_\infty}$$

```
$Load Incompressible
$unitsystem SI K J Pa
$tabstops 0.2 0.4 0.6 2.5

$varinfo T[] units='K'
$varinfo x[] units='m'
$varinfo time[] units='s'

"ME364 Homework 5
Fall, 2023"

"Numerical solution to 1D non-steady-state conduction"

D = 0.12 [mm]*convert(mm,m)                    "Fuse diameter"
L = 1 [mm]*convert(mm,m)                       "Domain length"
epsilon = 0.3 [-]                              "Surface emissivity"

t_sim = 0.01 [s]                               "Simulation duration"

T_infty = 300 [K]                              "Surroundings temperature (also initial temperature"
T_melt = 1358 [K]                              "Fuse melting temperature"

T_prop = (T_infty + T_melt)/2                  "Temperature used for property evaluation"
rho_e=electricalresistivity(Copper, T=T_prop)  "Electrical resistivity"
c = cp(Copper, T=T_prop)                       "Specific heat capacity"
k=conductivity(Copper, T=T_prop)               "Thermal conductivity"
rho=density(Copper, T=T_prop)                  "Density"

I_e = 50 [amp]                                 "Electrical current through the fuse"
g_dot_tprime = I_e^2 * rho_e / (pi#*D^2/4)^2   "Thermal generation in the wire"

Ac=pi/4*D^2
As=2*pi*D/2*DELTAx


"Domain discretization"
M = 11     "Nodes in x direction"
N = 301                                        "Nodes in t domain"

"Step size"
DELTAx = L / (M-1)
DELTAt = t_sim / (N-1)

"Node locations in space and time"
Duplicate i=1,M
    x[i] = (i-1)*DELTAx
End
Duplicate j=1,N
    time[j] = (j-1)*DELTAt
End

"Initial condition"
Duplicate i=1,M
    T[i,1] = T_infty
End

"Left Node"
```

```
Duplicate j=2,N
    T[1,j]=T_infty
End
"Middle Node"
Duplicate i=2,M-1
    Duplicate j=1,N-1
        T[i,j+1]=T[i,j] + ( (k*Ac/DELTAx) * (T[i-1,j]+T[i+1,j]-2*T[i,j]) + (g_dot_tprime*Ac*DELTAx) - (sigma#*epsilon*As)*(T[i,j]^4-T_infty^4
) ) / (rho*Ac*DELTAx*c) * DELTAt
    End
End
"Right Node"
Duplicate j=1,N-1
    T[M,j+1]=T[M,j] + ( (k*Ac/DELTAx) * (T[M-1,j]-T[M,j]) + (g_dot_tprime*Ac*DELTAx/2) - (sigma#*epsilon*As/2)*(T[M,j]^4-T_infty^4) ) /
(rho*Ac*DELTAx/2*c) * DELTAt
End

"Part B"
x_crit = x[11]
t_crit = time[174]
```

SOLUTION

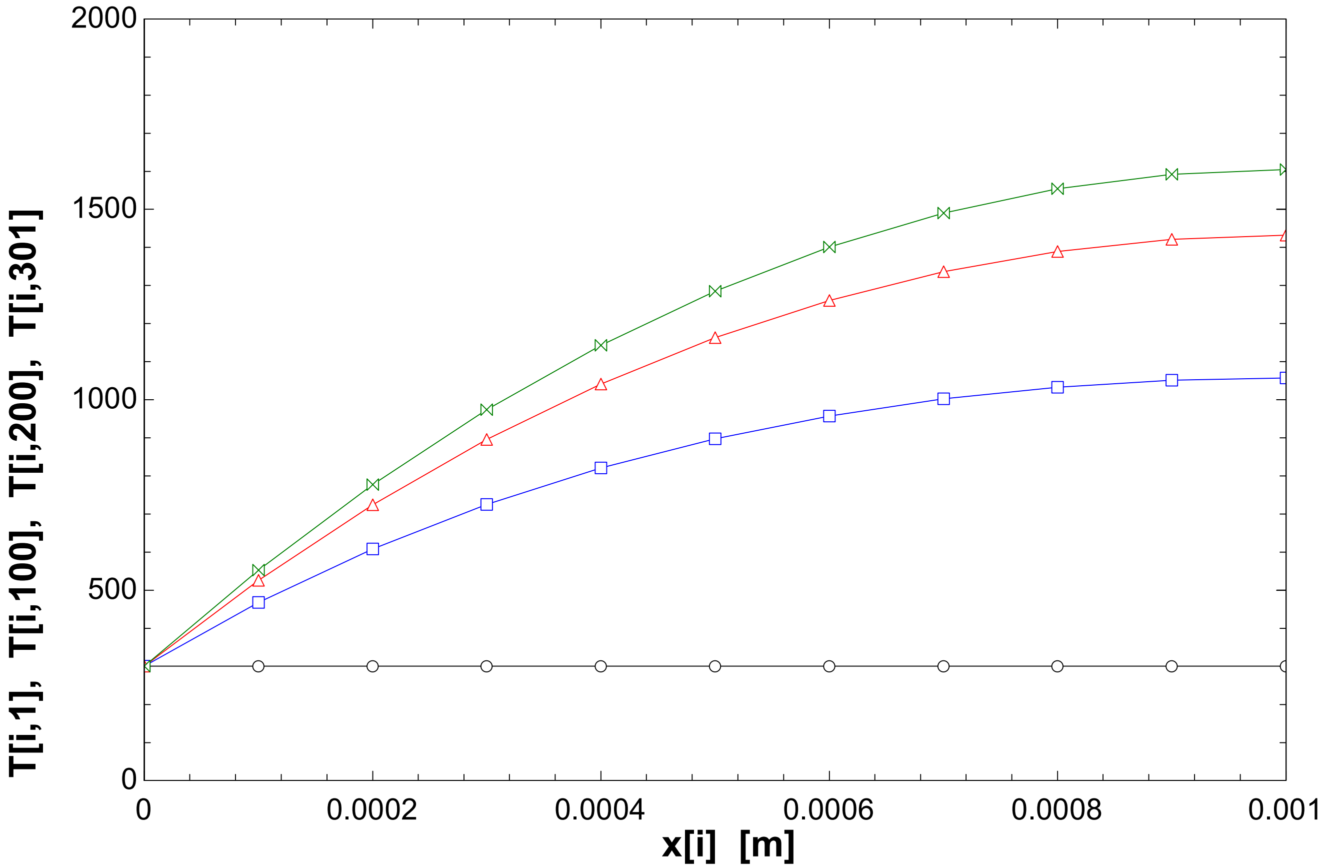
Unit Settings: SI K Pa J mass deg

Ac = 1.131E-08 [m^2]	As = 3.770E-08 [m^2]
c = 450.9 [J/kg-K]	D = 0.00012 [m]
Δt = 0.00003333 [s]	Δx = 0.0001 [m]
ε = 0.3 [-]	$\dot{g}'''$ = 1.073E+12 [W/m^3]
I_e = 50 [amp]	k = 369 [W/m-K]
L = 0.001 [m]	M = 11 [-]
N = 301 [-]	ρ = 8697 [kg/m^3]
ρ_e = 5.488E-08 [Ω-m]	t_{crit} = 0.005767 [s]
T_{infty} = 300 [K]	T_{melt} = 1358 [K]
T_{prop} = 829 [K]	t_{sim} = 0.01 [s]
x_{crit} = 0.001 [m]	

No unit problems were detected.

KEY VARIABLES

t_{crit} = 0.005767 [s] *b) time that T>Tmelt*
x_{crit} = 0.001 [m] *b) position that T>Tmelt*


2000
1500
1000
500
0
0
0.0002
0.0004
0.0006
0.0008
0.001
x[i] [m]
T[i,1], T[i,100], T[i,200], T[i,301]