Thermo Fundamentals

Variables of state ≡ Process-independent

Internal energy, $e = e(v, T)$

Ideal gas: $de = C_v\, dT$

$P = \rho R T$, $R = \frac{\bar{R}}{\bar{m}} = 8314\ \frac{J}{Kmol\text{-}K}$

Isentropic: $ds = 0$:

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\frac{P_2}{P_1} = \left(\frac{\rho_2}{\rho_1}\right)^{\gamma}$$ ← Not eqn. of state

$$\frac{T_2}{T_1} = \left(\frac{\rho_2}{\rho_1}\right)^{\gamma-1}$$

Sign convention
work by fluid: negative
entering CV: negative

Work / Energy

$de = dq - dw = dq - P dv$

↖ by fluid

$h \equiv e + Pv = e + \frac{P}{\rho}$

$dh = C_p\, dT$, $C_p = C_v + R$, $\gamma = \frac{C_p}{C_v}$

Reversible: $ds \equiv \frac{dq}{T} \geq 0$

$\rightarrow dq = T ds \rightarrow s_2 - s_1 = C_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}$

Conservation of mass (continuity)

$$\frac{d}{dt}\int_{CV} \rho\, d\forall + \int_{CS} \rho \underline{u}_{rel} \cdot d\underline{A} = 0$$

$$\rightarrow \int_{CV} \frac{\partial \rho}{\partial t} d\forall + \int_{CS} \rho \underline{u}_b \cdot d\underline{A} + \int_{CS} \rho \underline{u}_{rel} \cdot d\underline{A} = 0$$

Conservation of momentum

$$\frac{d}{dt}\int_{CV} \rho \underline{u}\, d\forall + \int_{CS} \rho\, \underline{u}\, \underline{u} \cdot d\underline{A} = \int_{CV} \rho \underline{g}\, d\forall - \int_{CS} P d\underline{A} + \underline{F_E}$$

Conservation of energy

$$\frac{d}{dt}\int_{CV}\left(e + \frac{u^2}{2} + gz\right)\rho\, d\forall + \int_{CS}\left(h + \frac{u^2}{2} + gz\right)\rho \underline{u}_{rel} \cdot d\underline{A} = \dot{Q} - \int_{CS} P\, \underline{u}_b \cdot dA - \dot{W}_{shaft}$$

Rocket engines

$J = \dot{m} u_e + (P_e - P_a) A_e = M(t)\, g \cos\theta + D + M(t) \frac{du}{dt}$ - Full form 1-D steady state

$J = \dot{m} u_e + (P_e - P_a) A_e = \dot{m} u_{eq}$ - 1-D, steady-state, $D = g = 0$

$I = \int F dt = u_{eq} M_p \rightarrow I / M_p = u_{eq} = \frac{J}{\dot{m}}$

$I_{sp} = \frac{I}{M_p g_e} = \frac{u_{eq}}{g_e}$

$\dot{m} = \rho u A$

Δu & stuff

$\Delta u = u_{eq} \ln \frac{M_0}{M(t)}$

$\Delta u(t_b) = u_{eq} \ln R \rightarrow R = \frac{M_0}{M_b} = \frac{M_0}{M_0 - M_p} = \frac{M_0}{M_s + M_e}$

Homework 1:

- Expanding gas/piston: FBD → work → $\Delta E = Q - W$
- compressor efficiency: mass bal → e-bal → compare isen & actual
- turbojet: T-S diagram between & along isobars, isentropic gas laws

HW2:

- air/water tank: density as $f(\forall)$, $\forall(t)$

Rocket launched vertically, $I_{sp} = 343$ s $\bar{M} = 30$ Kg/Kmol, $\gamma = 1.3$

CC $\to A_e$ isentropic. $T_0 = 4714$ K, $A_e = 0.1 m^2$, $P_e = 0.1$ bar

$D = 0$

At 9000 m, $P_a = 0.3$ bar, $M = 2500$ Kg $a_R = 7.53 \ m/s^2$

Find a) $\dot{m}$ b) h_e c) P_0

$$\text{Given} \begin{cases} I_{sp} = 343 \text{ s} & A_e = 0.1 \ m^2 & a_R = 7.53 \ m/s^2 \\ \bar{M} = 30 \text{ Kg/Kmol} & P_e = 0.1 \text{ bar} \\ \gamma = 1.3 & P_a = 0.3 \text{ bar} \\ T_0 = 4714 \text{ K} & M = 2500 \text{ Kg} \end{cases}$$

Find $\dot{m}$:

$$\dot{m} = \rho_e u_e A_e$$

$$I_{sp} = \frac{u_{eq}}{g_e} \to u_{eq} = 3365 = u_e + \frac{(P_e - P_a)A_e}{\dot{m}}$$

$$a_r = \frac{T}{M} - g_e = \frac{\dot{m} u_{eq}}{M} - g_e \to \dot{m} = \frac{(a_r + g_e)M}{u_{eq}}$$

$$\to \dot{m} = \frac{(a_r + g_e)M}{g_e I_{sp}}$$

b) h_e : perfect gas : $h_e = C_p T_e$

$$C_p = C_v + R = \frac{C_p}{\gamma} + R \to C_p(1 + \tfrac{1}{\gamma}) = R$$

$$\to C_p = \frac{R}{1 + \frac{1}{\gamma}}$$

isentropic: $\dfrac{T}{T_0} = \left(\dfrac{P}{P_0}\right)^{\frac{\gamma - 1}{\gamma}}$

3-stage rocket, $u_f = 12345$ m/s, $M_L = 1025$ kg $0 = g = 0$

$M_{0_1} = 100{,}000$ kg, $M_{0_2} = 18000$ kg, $M_{0_3} = 4000$ kg

$M_{p_1} = 75000$ kg, $M_{p_2} = 12000$ kg

$I_{sp} = 400$ s

Find a) R_1 & R_2

b) $R_3, \lambda_3, \varepsilon_3$

c) M_{p3}, M_{s3}

$$R_1 = \frac{M_{0_1}}{M_{b_1}} = \frac{M_{0_1}}{M_{0_1} - M_{p_1}} = 4 \qquad R_2 = \frac{M_{02}}{M_{02} - M_{p2}} = 3$$

$$u_f = u_{eq}(\ln R_1 + \ln R_2 + \ln R_3) \qquad u_{eq} = I_{sp} g_e$$

$$\frac{u_f}{I_{sp} g_e} - \ln R_1 - \ln R_2 = \ln R_3 \rightarrow R_3 = 1.937 = \frac{M_{03}}{M_{03} - M_{p3}}$$

$$\frac{(1.937 - 1) M_{03} = 1.937 M_{p3}}{1.937}$$

$$\boxed{\rightarrow M_{p3} = 1934.9}$$

$$\lambda_3 = \frac{M_L}{M_{03} - M_L} = 0.344$$

$$\varepsilon_3 = \frac{M_{b3} - M_L}{M_{03} - M_L} = \frac{M_{03} - M_{p3} - M_L}{M_{03} - M_L}$$

$\rightarrow \varepsilon_3 = 0.35$

$$M_{s3} = M_{03} - M_{p3} - M_L = 1040.1$$

NOW $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = \varepsilon_{opt}$. M_{01}, M_L stay the same

$\Delta u = 12345$ Find optimum masses $\rightarrow M_{03}$

$\rightarrow$ case 1 $\rightarrow$ calculus of variations

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