EMA 550 HW6 KYLE ADLER

Problem 1 At instant, $alt_{sat} = 400$ km. 30 mins later, $alt = 1000$ km, $\theta + 120°$. what is alt perigee?

$\rightarrow$ Lambert's

$r_E = 6378$ km $\quad r_1 = 6778$ km $\quad r_2 = 7378$ km $\quad t = 30\,min = 1800s$

using lambert_ bisection code in matlab: $a = 7560.6$ km

$$\alpha = 2\sin^{-1}\left(\sqrt{\frac{s}{2a}}\right) \quad \beta = 2\sin^{-1}\left(\sqrt{\frac{s-c}{2a}}\right)$$

$$a(1-e^2) = \frac{4a(s-r_1)(s-r_2)}{c^2}\sin^2\left(\frac{\alpha+\beta}{2}\right) = 7450.5\text{ km}$$

$$\rightarrow e = 0.1206$$

$$r_p = a(1-e) = 6648.8\text{ km} \rightarrow \boxed{alt_p = 270.8\text{ km}}$$

Problem 2 Rocket constant gravity g. $\dot{m}$ const, u_e const.

Find $h(\tau)$, $\tau = t_{burnout}$

$$m\dot{v} = \dot{m}c + F_{ext}$$

axial: $$\int m\frac{dv}{\cancel{dt}} = \int \frac{dm}{\cancel{dt}}\,c + \int \frac{mg}{m}\cdot dt$$

$$v = \int \frac{1}{m}\,dm\cdot c + gt$$

$$\int \frac{dh}{dt} = \int c\ln\left(\frac{m_f}{m_0}\right) \int gt$$

$$h = \boxed{c\ln\left(\frac{m_f}{m_0}\right)t + \frac{1}{2}gt^2 = h(\tau)}$$

Problem 3

3- stage rocket 1000 kg payload Earth -> moon

$\Delta V = 14$ km/s. Each stage 90% fuel, 10% payload/structure.

$u_e = 4500$ m/s. Find min. mass of rocket @ launch.

$\Delta V = 14$ km/s $\quad m_3 = 1000$ kg $\quad m_{f3} = 9000$ kg

$u_e = 4.5$ km/s $\quad m_2 = ? \quad m_{f2} = 9 \cdot m_2$

$m_1 = ? \quad m_{f1} = 9 \cdot m_1$

$$\Delta v_3 = c \ln\left(\frac{10 m_3}{m_3}\right)$$

$$\Delta v_2 = c \ln\left(\frac{10 m_3 + 10 m_2}{10 m_3 + m_2}\right)$$

$$\Delta v_1 = c \ln\left(\frac{10 m_3 + 10 m_2 + 10 m_1}{10 m_3 + 10 m_2 + m_1}\right)$$

$$\Delta v_{tot} = \Delta v_1 + \Delta v_2 + \Delta v_3$$

$m_{tot} = 10 \cdot (m_1 + m_2 + m_3)$ $\leftarrow$ minimize in matlab: $\boxed{m_0 = 25153.7 \text{ kg}}$