Last time: Kepler -> prove these w/ Newton

Newton Laws:
1) objects in motion stay in motion unless force
2) $\Sigma \bar{F} = m \bar{a}$
3) Equal & opposite forces

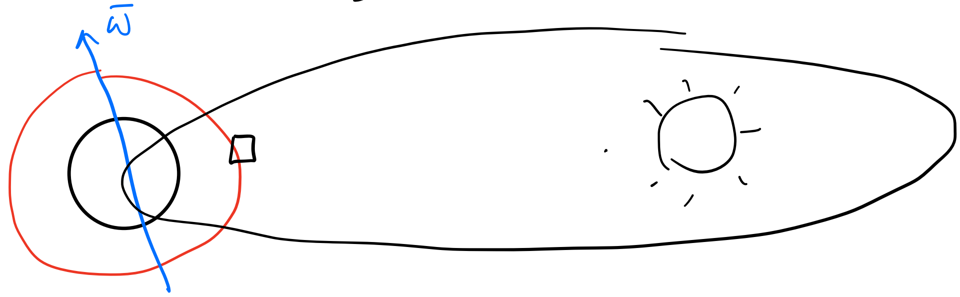
objects in universe exert forces on eachother
-> Force proportional to the mass & inversely proportional ($\propto \frac{1}{r^2}$) to radius

Law of gravitation

$$\bar{F} = G \frac{m_1 m_2}{|\bar{r}|^2}$$

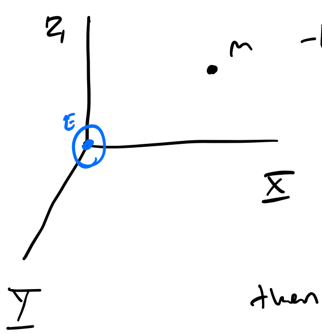
Dynamics review

Sat. orbits earth, earth orbits sun, etc.




Layers of motion

Requires nomenclature




- Let XYZ be "inertial" Ref. frame
  -> fixed / not accelerating
  centered on earth
- Let $\bar{r}$ be vector to mass m

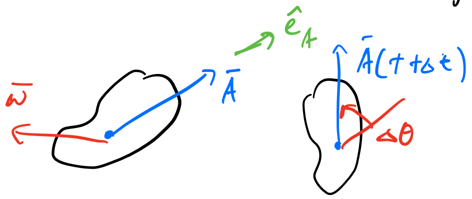
then $\bar{r} = x\,\bar{I} + y\,\bar{J} + z\,\bar{K}$, $\bar{I}\ \bar{J}\ \bar{K}$ unit vectors

dist $\overline{EM} = |\bar{r}| = \sqrt{x^2 + y^2 + z^2} = \sqrt{\bar{r} \cdot \bar{r}}$

vel $\bar{v} = \frac{dx}{dt}\bar{I} + \frac{dy}{dt}\bar{J} + \frac{dz}{dt}\bar{K}$

accel $\bar{a} = \frac{d^2x}{dt^2}\bar{I} + \frac{d^2y}{dt^2}\bar{J} + \frac{d^2z}{dz^2}\bar{K} = \ddot{x}\bar{I} + \ddot{y}\bar{J} + \ddot{z}\bar{K}$

introduce rotation: e.g. Earth's orbit




Let $\bar{\omega}$ = angular vel. of A

body rotates by $\Delta\theta$

$|\bar{\omega}| = \lim \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$

to describe $\dot{\bar{A}}$

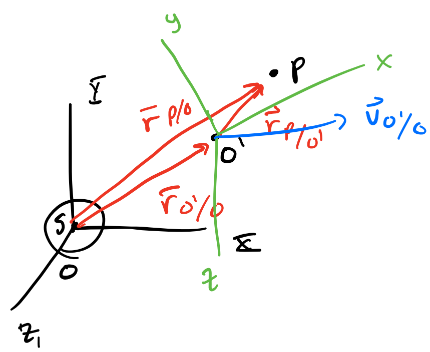
- how $\bar{A}$ changes length ⟶
- how $\bar{A}$ changes dir ⟶

$\bar{A} = A\hat{e}_A$

$\dot{\bar{A}} = \dot{A}\hat{e}_A + \bar{\omega}\times\bar{A}$ } From dynamics

note $\dot{A}\hat{e}_A = \dot{A}_x\bar{I} + \dot{A}_y\bar{J} + \dot{A}_z\bar{K}$ ⟶

Now, due to rotations → let c-sys move




O - Sun

O' - Earth

P - sat.

Let c-sys xyz have motion

$\left(\bar{\omega}, \frac{d\bar{r}_{O'/O}}{dt}, \frac{d^2\bar{r}_{O'/O}}{dt}\right.$

to get $\bar{v}_P, \bar{a}_P$:

$\bar{v}_P = \bar{v}_{O'} + (\bar{v}_P)_{xyz} + \bar{\omega}\times\bar{r}_{P/O'}$

vel. origin xyz   vel. vel. P   rot. of c-sys

$$\bar{a}_p = \bar{a}_{o'} + (\bar{a}_p)_{xyz} + \underbrace{\bar{\alpha} \times \bar{r}_{p/o'}}_{\text{tangential}} + \underbrace{\bar{\omega} \times (\bar{\omega} \times \bar{r}_{p/o'})}_{\text{normal}} + \underbrace{2\bar{\omega} \times (\bar{v}_p)}_{\text{coriolis}}{}_{xyz}$$

$$\bar{\alpha} = \frac{d\bar{\omega}}{dt}$$

**Remarks**

1) $(\bar{v}_p)_{xyz}$, $(\bar{a}_p)_{xyz}$ - rel. as seen when observing from xyz

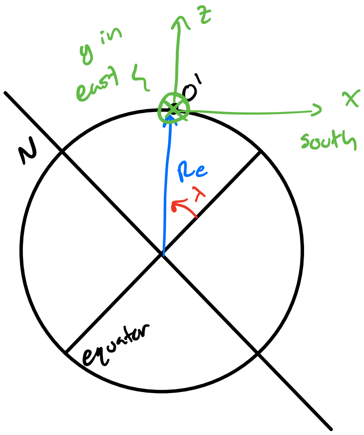
2) $\bar{\omega}$, $\bar{\alpha}$ is of the c-sys

3) tangential and normal accel → physical

4) Coriolis accel → important for rockets

**Example: Projectile**

Find ~~relative~~ acceleration of projectile launched on earth's surface relative to launch site (at instant of launch)




$\lambda \equiv$ lat. angle above equator (madison ~43.07°)

att. xyz to launch site, let it rotate w/ $\omega_e$ = earth rot.

Topocentric → z "up" outward

given: $(\bar{v}_p)_{xyz}$ rel. to launch

what is $\bar{a}$ absolute?

$(\bar{a}_p)_{xyz}$ relative to launch site

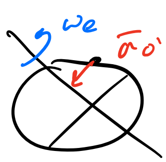
let's neglect drag: know that $\bar{a}$ absolute $= \bar{g}$, $|\bar{g}| = 9.81\ m/s^2$

$$\bar{g} = \bar{a}_p = \underbrace{\cancel{\bar{a}_{o'}}}_{}{}^{0} + (\bar{a}_p)_{xyz} + \cancel{\bar{\alpha}}^{0} \times \bar{r}_{p/o'} + \bar{\omega} \times \bar{\omega} \times \bar{r}_{p/o'} + 2\bar{\omega} \times (\bar{v}_p)_{xyz}$$

- c-sys rotates with $\omega_e$ (neglect earth rotation about sun - Hw)

- $\omega_e \cong$ const mag, and dir → $\bar{\alpha} \approx 0$

- let's check $\bar{a}_{o'} = 0$




$\bar{a}_{o'} = \bar{\omega}_e \times \bar{\omega}_e \times \bar{R}_e$

$= \omega_e^2 R_e \cos\lambda \; \hat{e}_n$

↖ normal dir

$\boxed{R_e = 6378 \text{ km}}$

$$\omega_e = \frac{1 \text{ rev}}{\text{day}} = 7.27 \times 10^{-5} \text{ rad/s}$$

$$|\bar{a}_{o'}| = (7.27 \times 10^{-5} \text{ rad/s})^2 (6378 \times 10^3) \cos(43.07^\circ) \approx 0$$

$$= 0.025 \text{ m/s}^2 \ll 9.81 \text{ m/s}^2$$

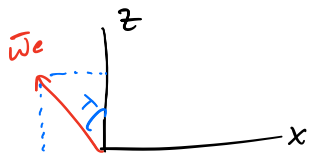
By similar argument, $\bar{\omega}_e \times \bar{\omega}_e \times \bar{R}_e \approx 0$

$$\therefore \bar{g} = (\bar{a}_p)_{xyz} + 2\bar{\omega} \times (\bar{v}_p)_{xyz}$$

$$\boxed{(\bar{a}_p)_{xyz} = \bar{g} - 2\bar{\omega} \times (\bar{v}_p)_{xyz}}$$ relative accel. of launch

points north          in xyz

Note: $\bar{\omega}_e$ needs to be transformed




---

Reference frames & c-systems

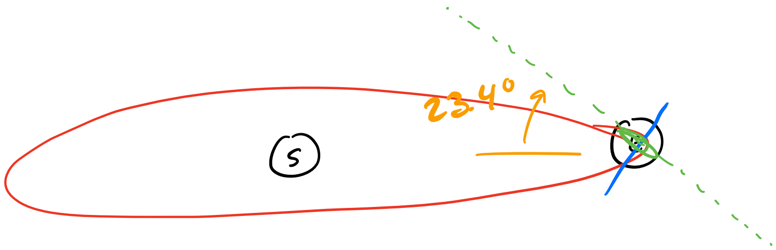
See slides for definitions

- vast distances → use AU

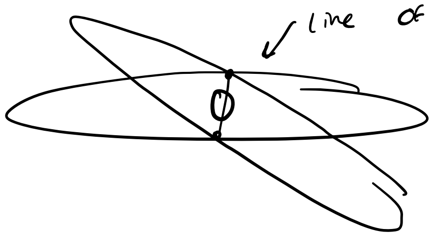
1 AU ≡ mean dist. Earth to Sun.

$\boxed{1 \text{ AU} = 1.5 \times 10^8 \text{ km}}$

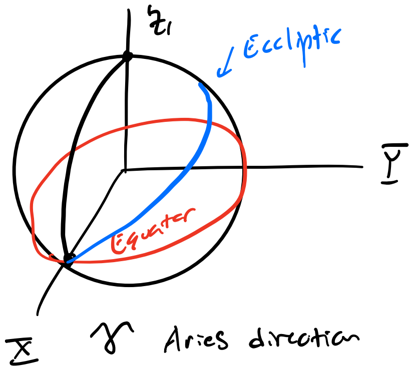
- Planets orbit "closely" in the same plane, not exactly
  - → use Earth's orbit plane ≡ "Ecliptic plane"

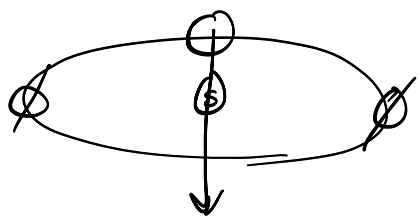



  - → use Earth's "Equatorial plane" as well




First c-sys "Earth Centered Inertial" ECI




♈ Aries direction

points to sun on 1st day of spring