Problem 1 OL Freq $G(s)$ plot given

a) Using experimental response, design a proportional controller K_p.

Requirements: $PM \geq 45^\circ$

ω_{BW} as high as possible

$PM = 45^\circ$ @ $\omega_c = 10^1$ $\quad \nearrow K_p = \frac{1}{M|_{PM}}$

$M_{PM=45^\circ} = 10^{-2} \rightarrow \boxed{K_p = 100}$

b) Estimate CL GM

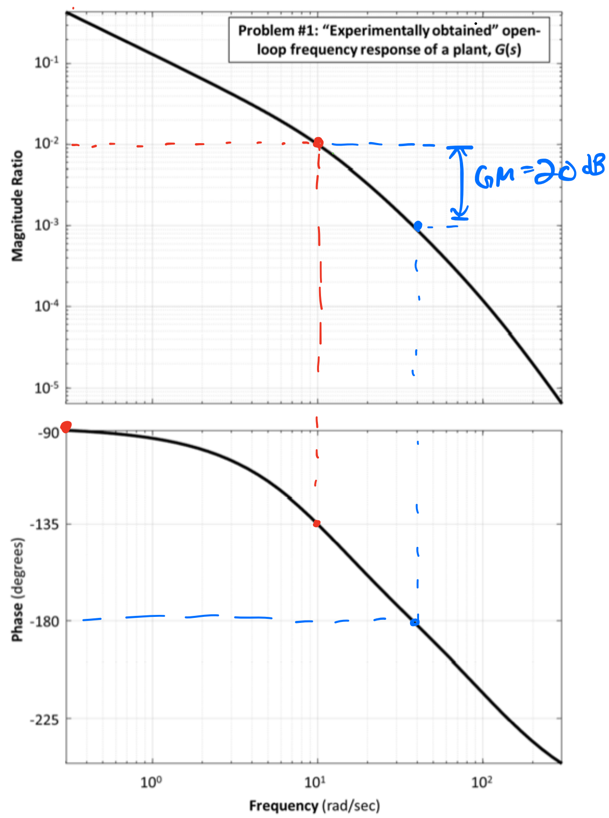
$\boxed{GM = 20\,dB}$

c) Estimate CL ω_{BW}

$\omega_c \leq \omega_{BW} \leq 2\omega_c$

$\omega_c = 10$ r/s

$\rightarrow \boxed{\omega_{BW} \approx 15 \text{ r/s}}$



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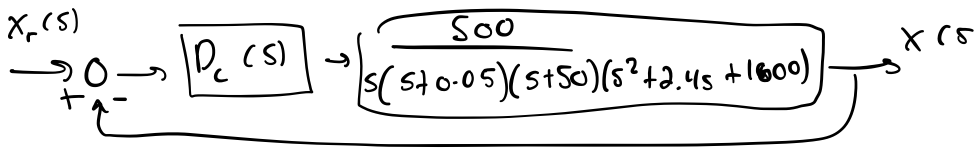
Problem 2



Requirements: $PM = 60^\circ$, $\omega_c = 2$

a) Design a PD compensator satisfying requirements

$$\boxed{D_c(s) = K_d s + K_p}$$

For $PM = 60^\circ$:

$$\angle D_c(j\omega)G(j\omega)\Big|_{\omega=\omega_c} = -120^\circ$$

$$= \left[\angle (K_d j\omega_c + K_p) + \angle\left(\frac{500}{s(s+0.05)(s+50)(s^2+2.4s+1600)}\right)\right]_{\omega_c=2}$$

$$-120^\circ = \text{atan}\left(\frac{2K_d}{K_p}\right) + \text{atan}\left(\frac{0}{500}\right) - \text{atan}\left(\frac{2}{0}\right) - \text{atan}\left(\frac{2}{0.05}\right) - \text{atan}\left(\frac{2}{50}\right) - \text{atan}\left(\frac{4.8}{1604}\right)$$

$$-120^\circ = \text{atan}\left(\frac{2K_d}{K_p}\right) + 0 - 90^\circ - 88.568^\circ - 2.2906^\circ - 0.1714$$

$$\rightarrow \text{atan}\left(\frac{2K_d}{K_p}\right) = 61.03^\circ \rightarrow \boxed{\frac{K_d}{K_p} = 0.90314}$$

$$M: \left|D_c(j\omega_c)G(j\omega_c)\right| = 1 = \sqrt{(2K_d)^2 + K_p^2} \cdot \frac{500}{2\sqrt{2^2+0.05^2}\sqrt{2^2+50^2}\sqrt{4.8^2+1604^2}}$$

$$\boxed{\sqrt{4K_d^2 + K_p^2} = \frac{1}{0.001557} = 642.316}$$

→ Solve system in Matlab

$$\rightarrow \boxed{K_d = 280.9728,\ K_p = 311.1073}$$

$$\boxed{\omega_{BW} = 2.82 \text{ r/s}}$$

GM = 6.61 dB
PM = 58.8°

b) Lead compensator

$$D_c(s) = K\frac{(s+z)}{(s+p)}$$

$PM = 60^\circ$
$\omega_c = 2$

→ still need 60° PM @ $\omega_c = 2$ → Need 61.03° from compensator

$$\varphi_{max} = 61.03^\circ,\ \omega_{max} = \omega_c = 2$$

$$\alpha = \frac{1-\sin\varphi_{max}}{1+\sin\varphi_{max}} = 0.06674 \quad \rightarrow LR = 14.98 \quad \text{(reasonable)}$$

$$p = \frac{\omega_{max}}{\sqrt{\alpha}} = 7.7417 \qquad z = \omega_{max}\sqrt{\alpha} = 0.5167$$

$$\rightarrow D_c(s) = K\left(\frac{s+0.5167}{s+7.7417}\right)$$

$$M: \left|D_c(j\omega_c)G(j\omega_c)\right| = 1 = K\frac{\sqrt{2^2+0.51^2}}{\sqrt{2^2+7.7417^2}} \cdot \frac{500}{2\sqrt{2^2+0.05^2}\sqrt{2^2+50^2}\sqrt{4.8^2+1604^2}}$$

$$K\frac{\sqrt{2^2+0.51^2}}{\sqrt{2^2+29.9^2}} = 642.316 \rightarrow \boxed{K = 2486.339}$$

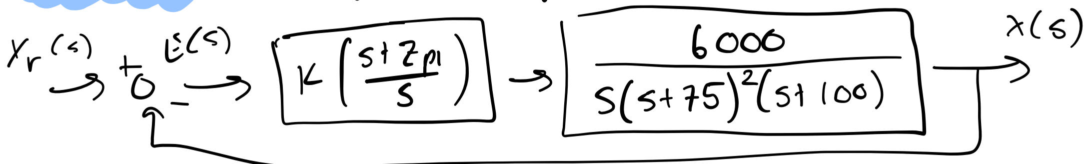
$$\rightarrow \boxed{\omega_{BW} = 2.7 \text{ r/s}} \quad GM = 25.7, \; PM = 58.7^\circ$$

c) $$\varphi_{req'd} = PM_{des} - \left(\angle G(j\omega)\big|_{\omega=\omega_{des}} + 180^\circ\right)$$

$$\boxed{\omega_{BW} = 5.5 \text{ rad/s}} \quad GM = 20\text{dB}, \; PM = 60^\circ$$

D) comparing the graphs, the PD & lead compensator have a slower settle time, due to their lower ω_{BW}. The 2nd compensator is less oscillatory than the PD & iterated lead due to its larger Gain margin.

Problem 3 Design PI compensator



$PM = 60^\circ$, $\omega_c = 10$ r/s $e(\infty) = 0$ from $X_r(t) = t$ (ramp)

e_{ss} reaches 0 asap larger z_{PI} $K = K_p$ $z_{PI} = \frac{K_I}{K_p}$

a) $$\frac{E(s)}{X_r(s)} = \frac{1}{1 + K\left(\frac{s+z_{PI}}{s}\right)\left(\frac{6000}{s(s+75)^2(s+100)}\right)}$$

$$\angle\left[D_c(s)\right]_{s=j\omega_c} = \operatorname{atan}\left(\frac{\omega_c}{z_{PI}}\right) - 90^\circ$$

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```
% Written by Kyle Adler for ME446
```

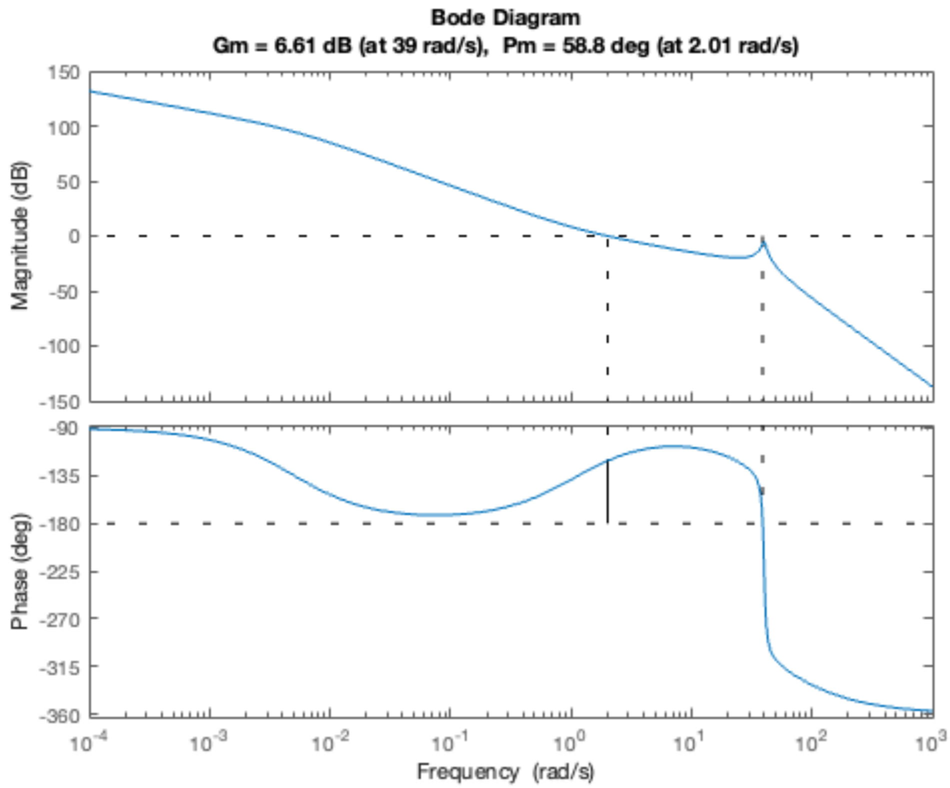
Problem 2

2a

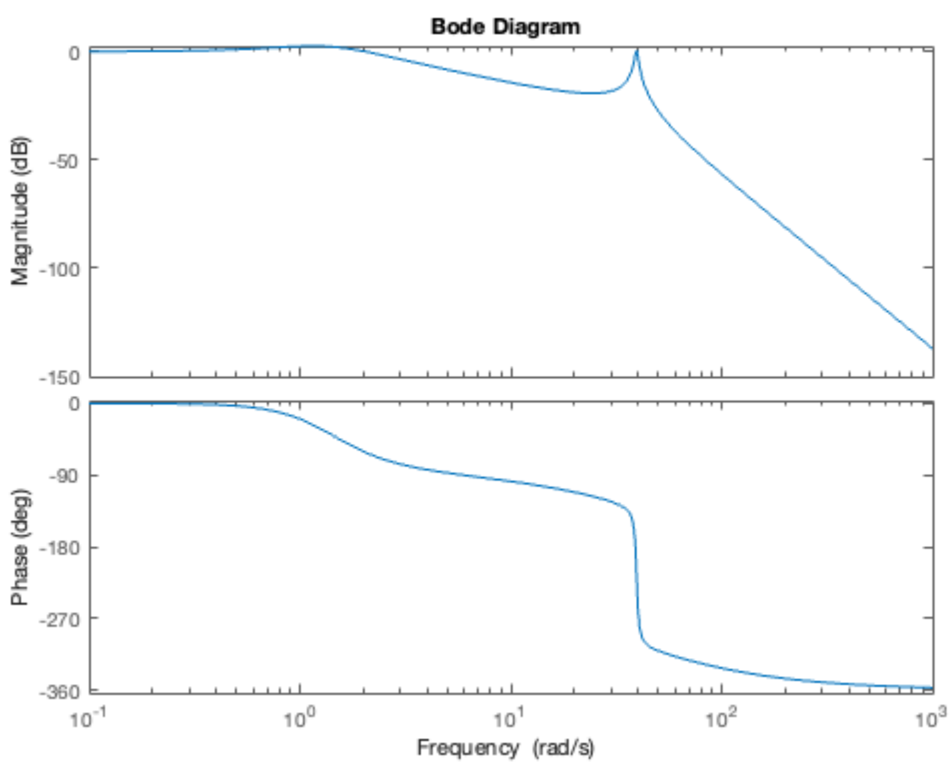
```
syms kd kp
e1 = 2*kd/kp == tan(61.029972*pi/180)
e2 = sqrt(4*kd^2+kp^2) == 642.3165738
[kd,kp] = solve([e1,e2],[kd,kp])
kd = double(kd(2))
kp = double(kp(2))

% evaluate system
s = tf('s')
Gs = 500 /( s*(s+0.005)*(s+50)*(s^2+2.4*s+1600) );
Dcs = kd*s + kp;
figure
margin(Dcs*Gs)

% closed loop response
figure
sysCL = feedback(Dcs*Gs,1);
bode(sysCL)
figure
step(sysCL)
```

```
e1 =

(2*kd)/kp == 4067370779986483/2251799813685248


e2 =

(4*kd^2 + kp^2)^(1/2) == 2824938166425365/4398046511104


kd =
```

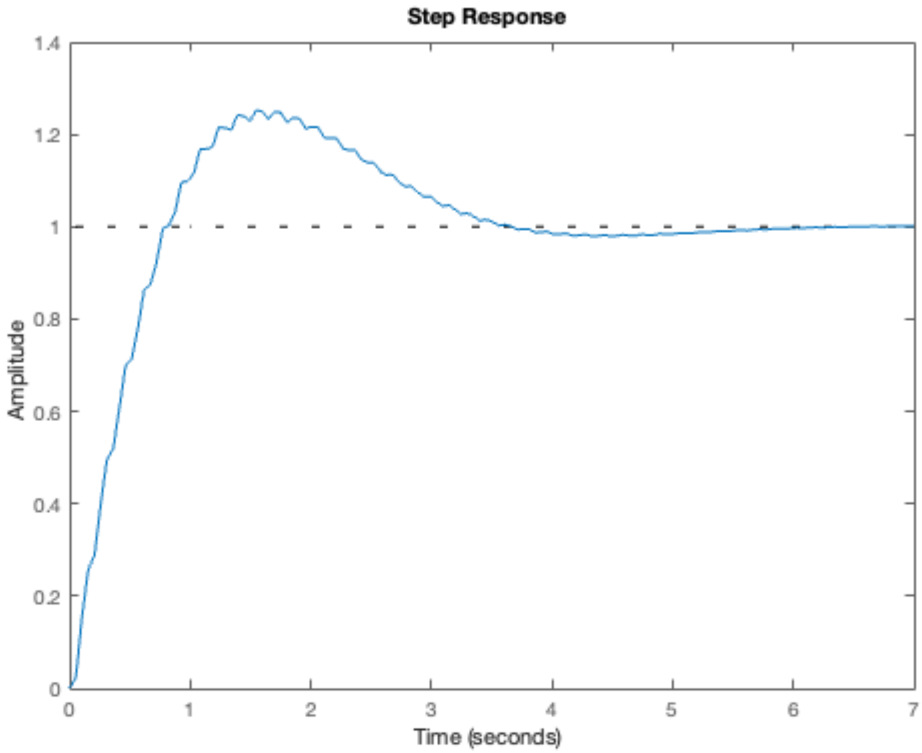
```
-(20336853899932415*6899461414197047485701871155877056362633808551698355299325
897^(1/2))/190119699834795700528571875958628712336850944

(20336853899932415*68994614141970474857018711558770563626338085516983552993258
97^(1/2))/190119699834795700528571875958628712336850944


kp =

-(2560*6899461414197047485701871155877056362633808551698355299325897^(1/2))/
21614107462800768704320475530793
 (2560*6899461414197047485701871155877056362633808551698355299325897^(1/2))/
21614107462800768704320475530793


kd =

  280.9728


kp =

  311.1073


s =

  s

Continuous-time transfer function.
```



Bode Diagram
Gm = 6.61 dB (at 39 rad/s), Pm = 58.8 deg (at 2.01 rad/s)
Magnitude (dB)
Phase (deg)
Frequency (rad/s)



Bode Diagram
Magnitude (dB)
Phase (deg)
Frequency (rad/s)

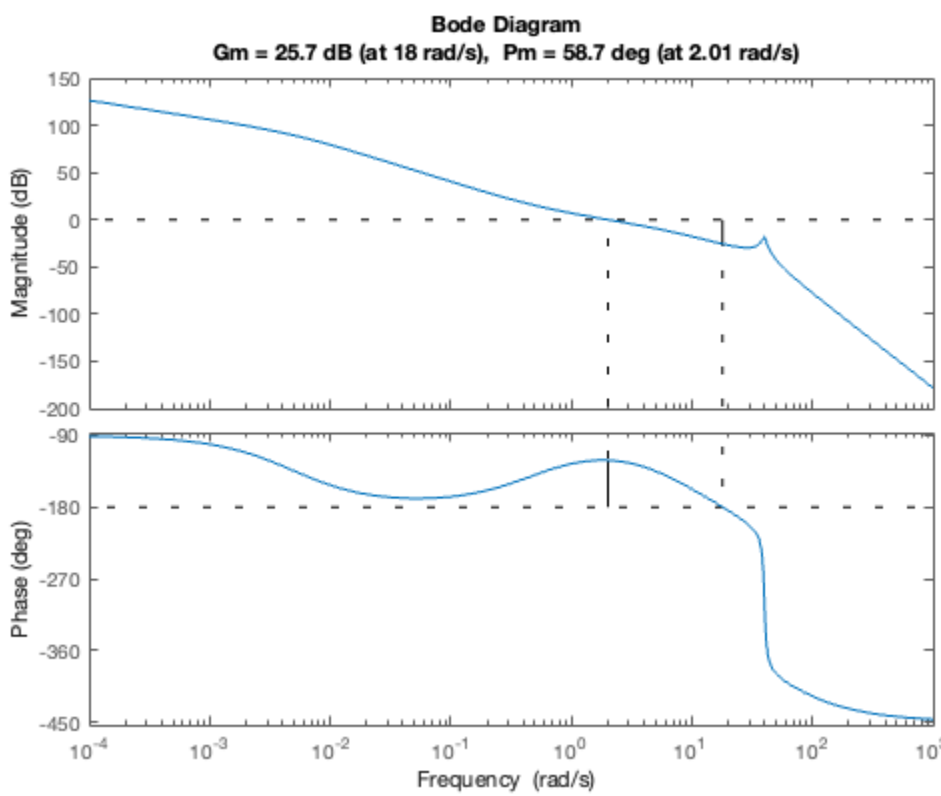


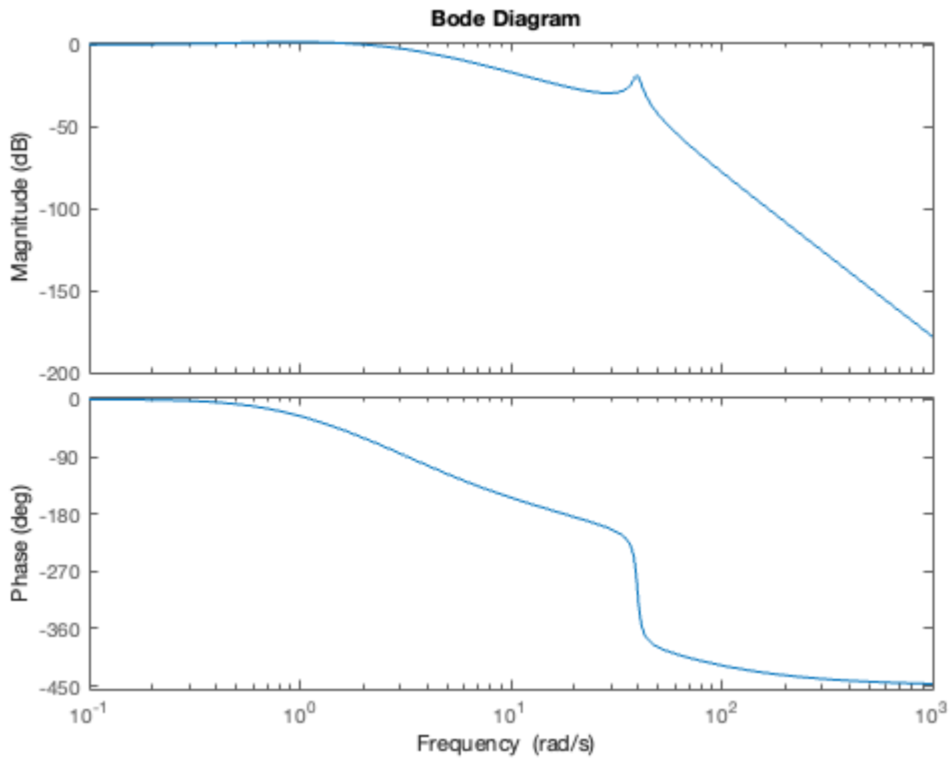
2b

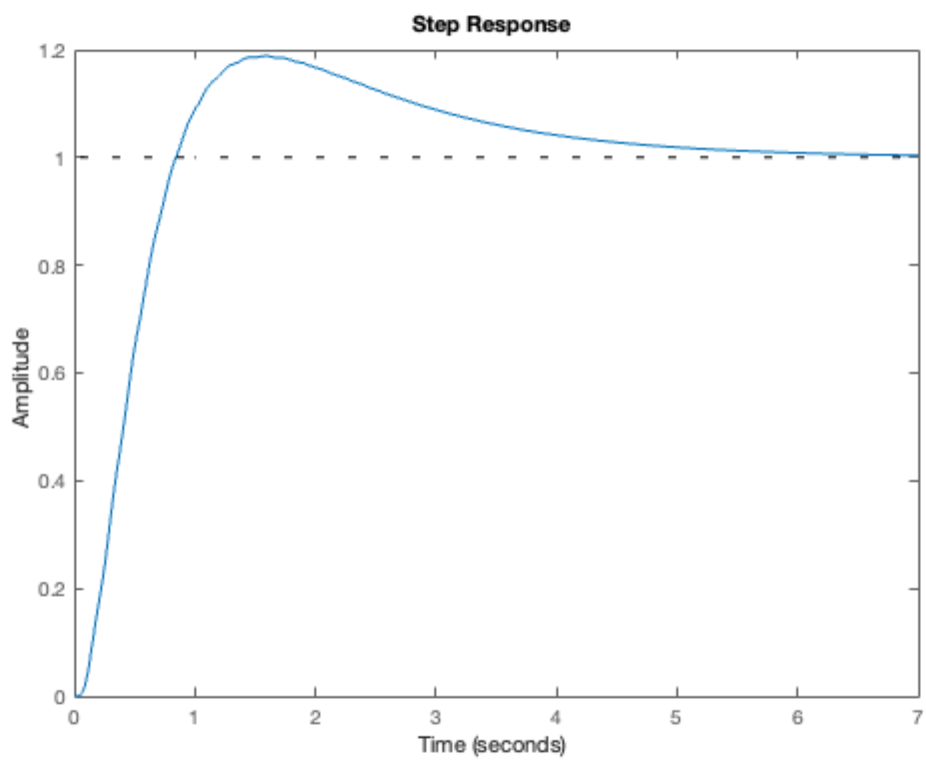
```
s = tf('s')
Gs = 500 /( s*(s+0.005)*(s+50)*(s^2+2.4*s+1600) );
z = 0.51667658
p = 7.7417868
k = 2486.338994
Dcs = k*(s+z)/(s+p)
figure
margin(Dcs*Gs)
sysCL = feedback(Dcs*Gs,1);
figure
bode(sysCL)
figure
step(sysCL)
```

```
s =

  s

Continuous-time transfer function.

z =

    0.5167
```

```
p =

    7.7418


k =

   2.4863e+03


Dcs =

  2486 s + 1285
  -------------
    s + 7.742

Continuous-time transfer function.
```




Bode Diagram
Magnitude (dB)
0
-50
-100
-150
-200
Phase (deg)
0
-90
-180
-270
-360
-450
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
Frequency (rad/s)


Step Response
Amplitude
1.2
1
0.8
0.6
0.4
0.2
0
0
1
2
3
4
5
6
7
Time (seconds)

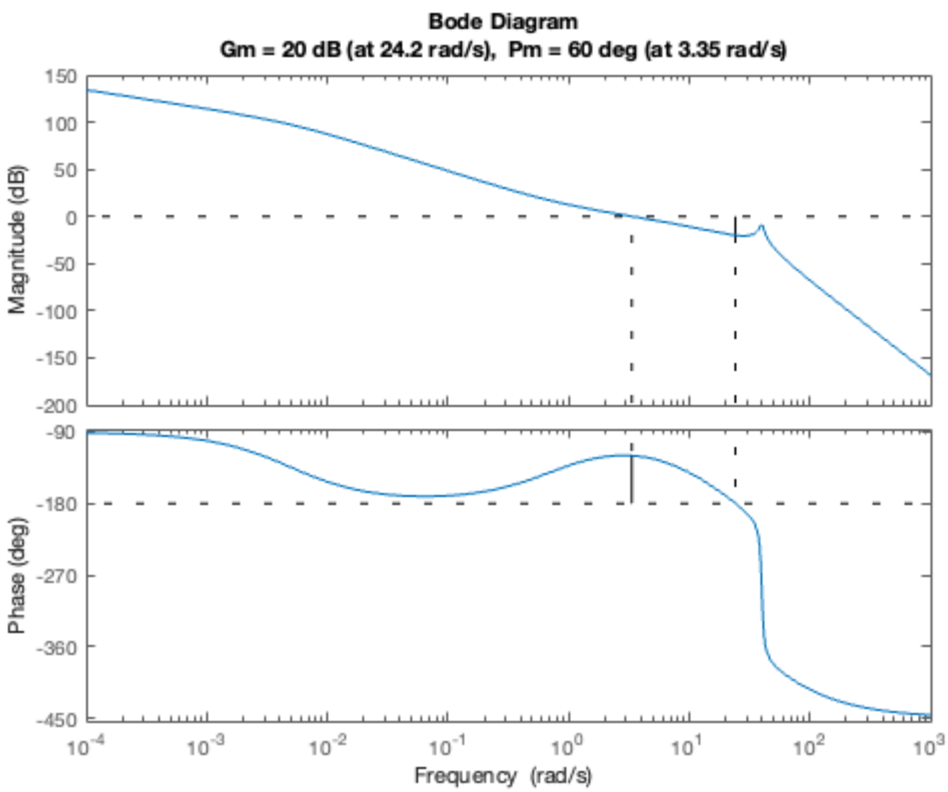
2c

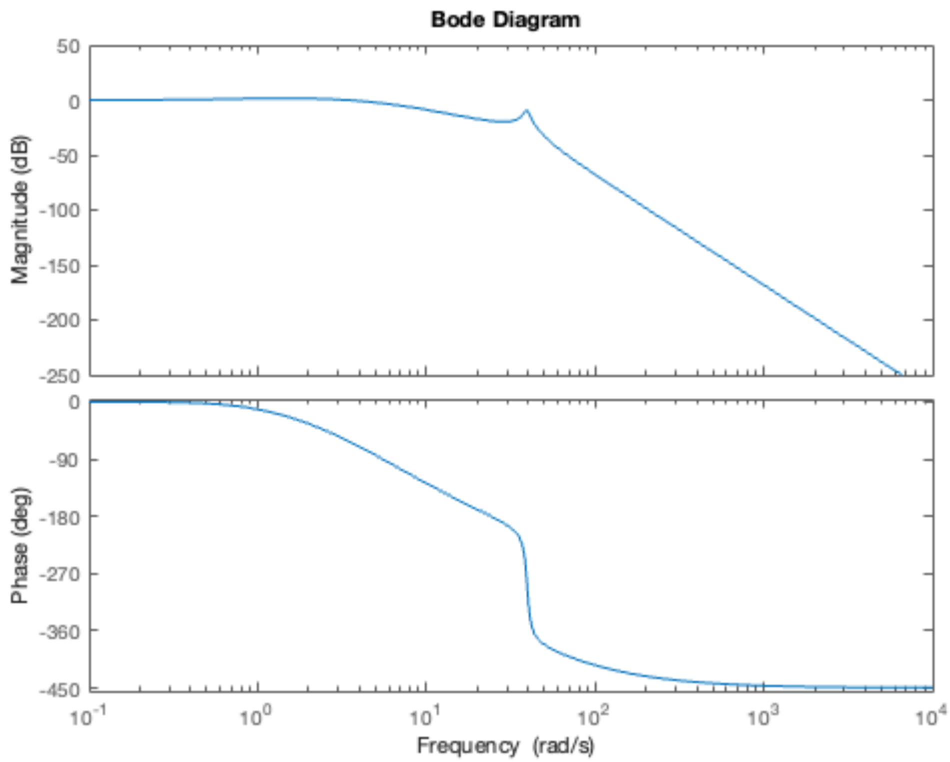
```
s = tf('s')
Gs = 500 /( s*(s+0.005)*(s+50)*(s^2+2.4*s+1600) );
wc = 3.35 % guess, iterated to increase wc but keeping GM>20
[m,p] = bode(Gs,wc)
phi = 60 - (p+180)
phi = phi*pi/180
alpha = (1-sin(phi))/(1+sin(phi))
p = wc/sqrt(alpha)
z = wc*sqrt(alpha)
Dcs = (s+z)/(s+p)
[m,p] = bode(Dcs*Gs,wc)
k = 1/m
Dcs = k*Dcs

figure
margin(Dcs*Gs)

% closed loop
figure
sysCL = feedback(Dcs*Gs,1)
bode(sysCL)
figure
step(sysCL)
```

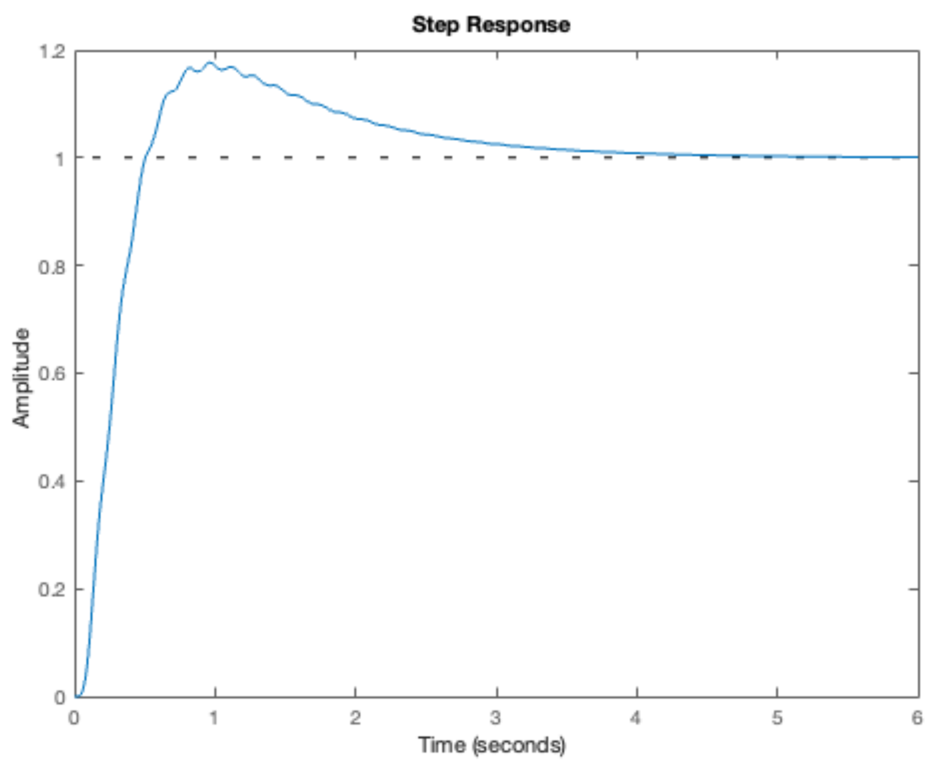
```
s =

  s

Continuous-time transfer function.

wc =

    3.3500


m =

   5.5959e-04


p =

 -184.0375


phi =

   64.0375
```

```
phi =

    1.1177


alpha =

    0.0531


p =

   14.5321


z =

    0.7723


Dcs =

  s + 0.7723
  ----------
  s + 14.53

Continuous-time transfer function.

m =

   1.2900e-04


p =

 -120.0000


k =

   7.7520e+03


Dcs =

  7752 s + 5987
  -------------
    s + 14.53

Continuous-time transfer function.

sysCL =
```

```
                              3.876e06 s + 2.993e06

  ----------------------------------------------------------------------

  s^6 + 66.94 s^5 + 2482 s^4 + 1.05e05 s^3 + 1.163e06 s^2 + 3.882e06 s

                                                            + 2.993e06

Continuous-time transfer function.
```




Bode Diagram
Magnitude (dB)
50
0
-50
-100
-150
-200
-250
Phase (deg)
0
-90
-180
-270
-360
-450
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
10^{4}
Frequency (rad/s)

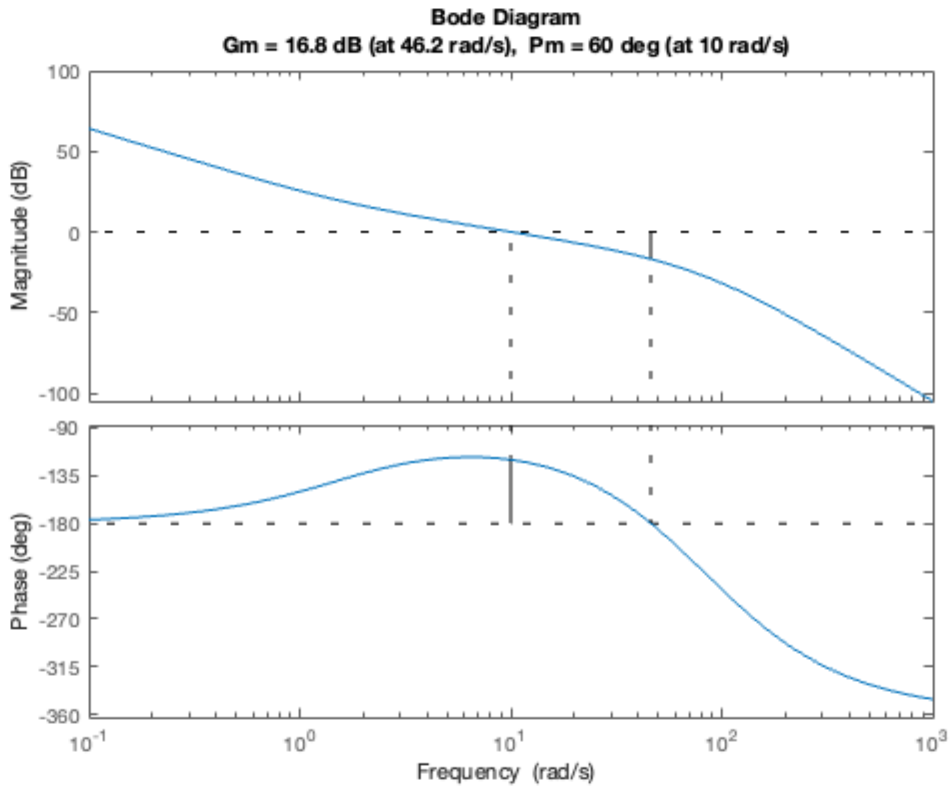

Step Response
Amplitude
1.2
1
0.8
0.6
0.4
0.2
0
0
1
2
3
4
5
6
Time (seconds)

Problem 3

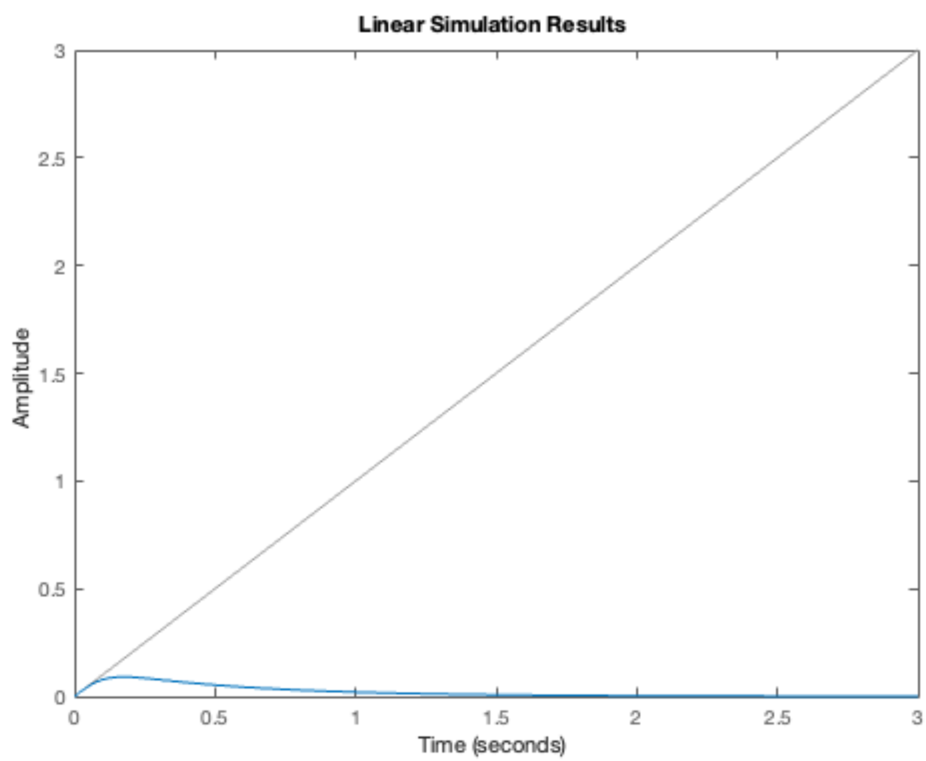
```
s = tf('s')
wc = 10
G = 6000 /( s*(s+75)^2*(s+100) )
[m,p] = bode(G,wc)
phi = 60 - (p+180)
syms zpi
kp = 946.86
zpi = wc/tand(phi+90)
D = kp*(s+zpi)/s

figure
margin(D*G)
figure
sysCL = feedback(1,D*G)
t=linspace(0,3,300);
xr = t;
lsim(sysCL,xr,t)
```

```
s =

  s

Continuous-time transfer function.

wc =

    10


G =

                  6000
  ------------------------------------
  s^4 + 250 s^3 + 20625 s^2 + 562500 s

Continuous-time transfer function.

m =

    0.0010


p =

 -110.8999


phi =

   -9.1001
```

```
kp =

  946.8600


zpi =

    1.6018


D =

  946.9 s + 1517
  --------------
        s

Continuous-time transfer function.

sysCL =

             s^5 + 250 s^4 + 20625 s^3 + 562500 s^2
  ------------------------------------------------------------
  s^5 + 250 s^4 + 20625 s^3 + 562500 s^2 + 5.681e06 s + 9.1e06

Continuous-time transfer function.
```



Bode Diagram
Gm = 16.8 dB (at 46.2 rad/s), Pm = 60 deg (at 10 rad/s)
Magnitude (dB)
Phase (deg)
Frequency (rad/s)



Linear Simulation Results
Amplitude
Time (seconds)

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