Name

A

Problem 1

A thin plate of length L separates two fluids flowing in opposite directions. The fluid flowing along the top of the plate from left to right is hot, with a free stream temperature T_H. The fluid flowing along the bottom of the plate is cold, with a free stream temperature T_C. Assume that the separating plate is infinitely thin; therefore, it offers no resistance to heat transfer across it (in the stream-to-stream direction) and infinite resistance to heat transfer along it (in the direction parallel to the flow). Also, assume that the flow remains laminar on both sides of the plate and that the hot and cold fluids have approximately the same properties and same free stream velocity.

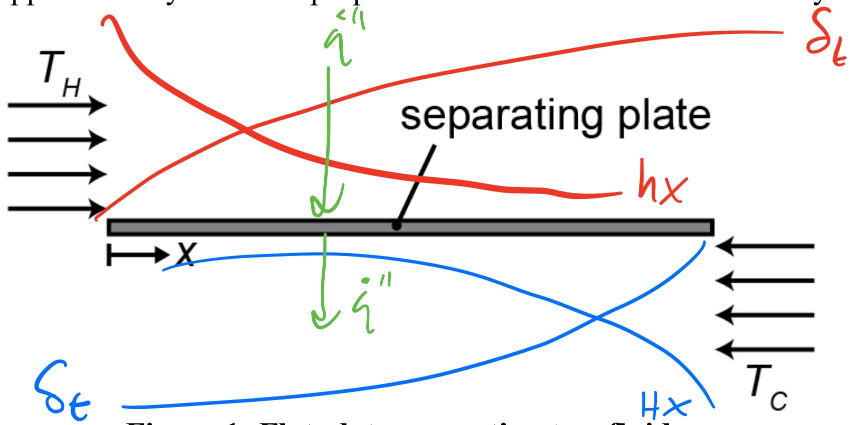
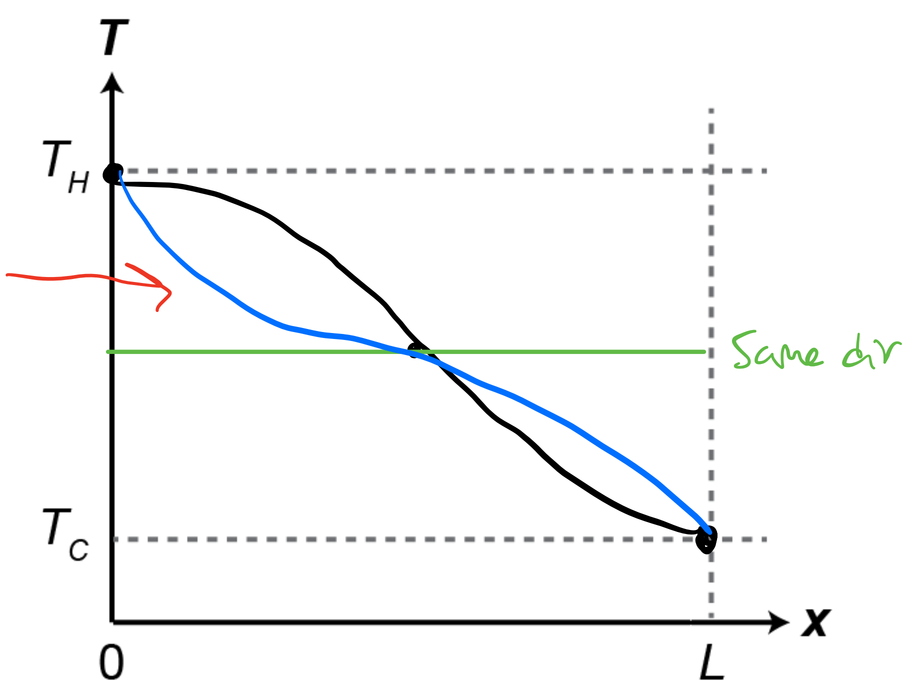


Figure 1: Flat plate separating two fluids.

a) On the axes below, qualitatively sketch the steady-state temperature of the plate as a function of position x. Here, $x = 0$ is at the left edge of the plate and $x = L$ is at the right edge.



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b) At what x position is the local heat flux $\dot{q}''$ from the hot side fluid to the cold side fluid maximized? Justify with calculations.

$$H: \quad \dot{q}''_{lam} = k \frac{T_S - T_H}{\delta_{t,lam}} = k \frac{(T_S - T_H) Re_x^{0.5} Pr^{1/3}}{4.916x}$$

?

$x = 0.5$?

Suppose that the flow direction of the cold fluid is reversed so that it now flows in the same direction as the hot fluid (i.e., from left to right). All other parameters remain the same.

c) Re-sketch the steady-state temperature of the plate as a function of position x for these new flow conditions. Overlay this plot on the same axes from part (a).

Name________________________________

B

Problem 2

As shown in Figure 2, liquid water is injected deep underground through a smooth, thin-walled pipe with diameter $D = 0.1$ m. The outer surface of the pipe is in perfect contact with the ground, which has a fixed temperature of $T_g = -10$ ºC. The water is injected with a uniform inlet temperature $T_i = 23$ ºC and a mass flow rate $\dot{m} = 1$ kg/s. The water has the following properties which are constant: $\rho = 1000$ kg/m^3, $k = 0.6$ W/(m·K), $c_p = 4000$ J/(kg·K), $\mu = 8\times10^{-4}$ kg/(m·s), $Pr = 5$. Assume the water is fully developed everywhere in the pipe.

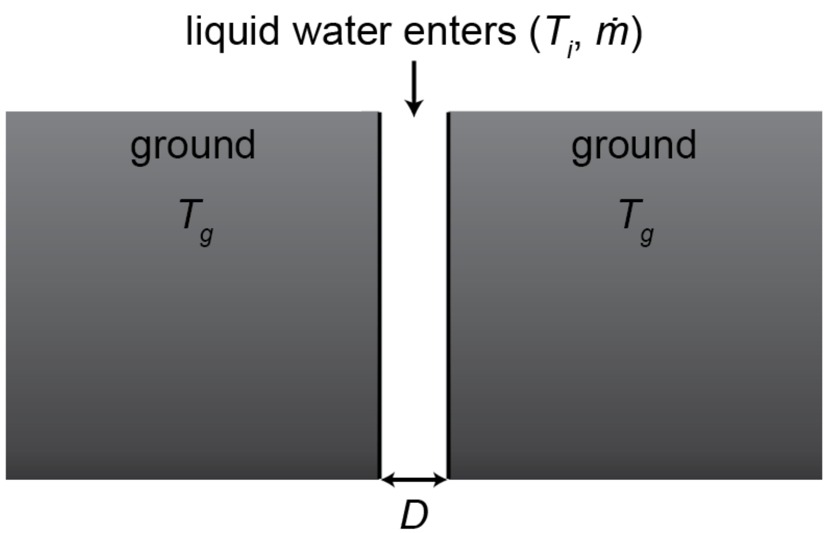
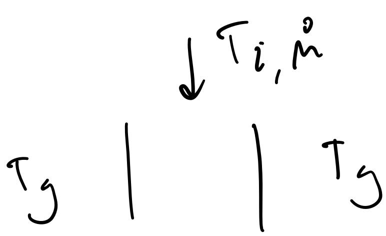


Figure 2. Buried pipe transporting liquid water.

a) Compute the average heat transfer coefficient due to convection in the pipe $\bar{h}$.



$$D_h = \frac{4A_c}{per} = \frac{\cancel{4}\left(\frac{\pi}{\cancel{4}}(0.1)^2\right)}{\cancel{\pi}(0.1)} = D = 0.1$$

$$\bar{u} = \frac{\dot{V}}{A_c} = \frac{\dot{m}/\rho}{A_c} = 0.1273 \text{ m/s}$$

$$Re_{D_h} = \frac{\rho \bar{u} D_h}{\mu} = 15915.49 \rightarrow \text{Turb}$$

$\rightarrow$ turb, internal, rel rough, FD value

$$Nu_{D_h} = \frac{\bar{h} D_h}{k} = \frac{\frac{f_{fd}}{8}(Re - 1000)Pr}{1 + 12.7(Pr^{2/3} - 1)\sqrt{\frac{f_{fd}}{8}}}, \quad f_{fd} = (0.79\ln(Re) - 1.64)^{-2}$$

$\rightarrow$ etc.

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Assume for the remainder of the problem the average heat transfer coefficient $\bar{h}$ = 250 W/(m^2·K); note that this may or may not be the correct answer to part (a).

b) How deep can the water be injected before it starts to freeze (i.e., its mean temperature drops below 0 ºC)?

const. wall temp: $T_m(x) = T_s - (T_s - T_{in}) \exp\left(-\frac{P \cdot x \cdot \bar{h}}{\dot{m} c}\right)$

$$0°C = -10°C - (-10°C - 23°C) e^{\left(-\frac{\pi (0.1) \times 250}{1 \cdot 4000}\right)}$$

$$\ln\left(\frac{-10}{-33}\right) = -\frac{\pi \cdot 0.1 \cdot x \cdot 250}{4000}$$

$$= 60.8 \text{ [m]}$$

Now suppose that the pipe is not thin-walled, but has an inner diameter D_{in} = 0.1 m, an outer diameter D_{out} = 0.12 m, and a conductivity k_p = 200 W/(m·K). All other parameters of the problem remain the same.

c) Accounting for this additional thermal resistance due to conduction through the tube wall, do you expect that your answer to part (b) will change significantly? Justify with calculations.

$q''_{turb} \approx \bar{h}(T_s - T_\infty) \rightarrow R_{turb} = \frac{1}{\bar{h}} = 0.004$

$$R_{cyl} = \frac{\ln\left(\frac{0.12/2}{0.1/2}\right)}{2\pi L k} = 2.4 \times 10^{-6}$$

No sig. change

$R_{total} = R_{turb} + R_{cyl}$ (series)

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C

Problem 1

Figure 1 illustrates flow through a rectangular duct. The height of the duct is $H = 0.01$ m and the width of the duct is $W = 0.05$ m (into the page). The total length of the duct in the flow direction is $L_{total} = 1$ m. The duct has two sections of equal length, $L = 0.5$ m. The surface temperature of the first section of the duct is maintained at a constant temperature, $T_s = 200$ K. The second section of the duct is insulated and therefore you may assume that the surface of the second section is adiabatic. Fluid enters the duct with a uniform temperature, $T_{in} = 100$ K. The mass flow rate is $\dot{m} = 0.25$ kg/s and the properties of the fluid are $\rho = 1000$ kg/m^3, $c_p = 100$ J/(kg-K), $\mu = 0.01$ kg/(m-s), and $k = 1.0$ W/(m-K).

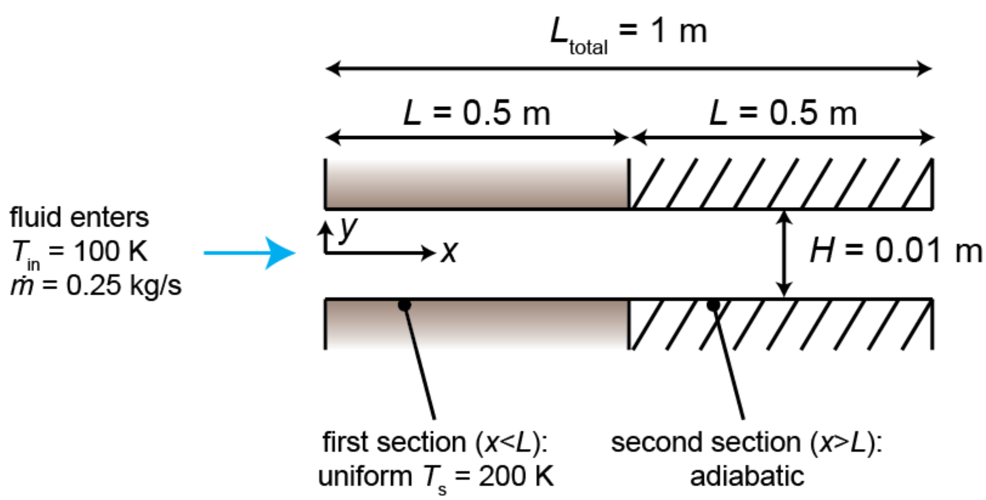

Figure 1: Rectangular duct.

a) Is the flow in the duct laminar or turbulent? Justify your answer.
b) Estimate the hydrodynamic and thermal entry lengths ($x_{fd,h}$ and $x_{fd,t}$) for the flow.

For the remaining questions, assume that the flow becomes hydrodynamically and thermally fully developed at exactly half-way through the heated portion of the duct (i.e., $x_{fd,h} \approx x_{fd,t} \approx L/2$); note that this may or may not be the correct answer to part (b).

c) The average Nusselt number in the first section of the duct (i.e., from x=0 to x= L) is $\overline{Nu} = 5.2$. Use this value to estimate the average heat transfer coefficient in the first section of the duct.
d) What is the mean temperature of the fluid leaving the first section of the duct?

For the remainder of this problem, assume that the mean temperature of the fluid leaving the first section of the duct is $T_{out} = 175$ K; note that this may or may not be the correct answer to part (d).

e) On the plot below, sketch the mean temperature as a function of position x in the duct. Make sure that your sketch extends all the way to the outlet of the duct (i.e., from $x = 0$ to $x = L_{total}$).

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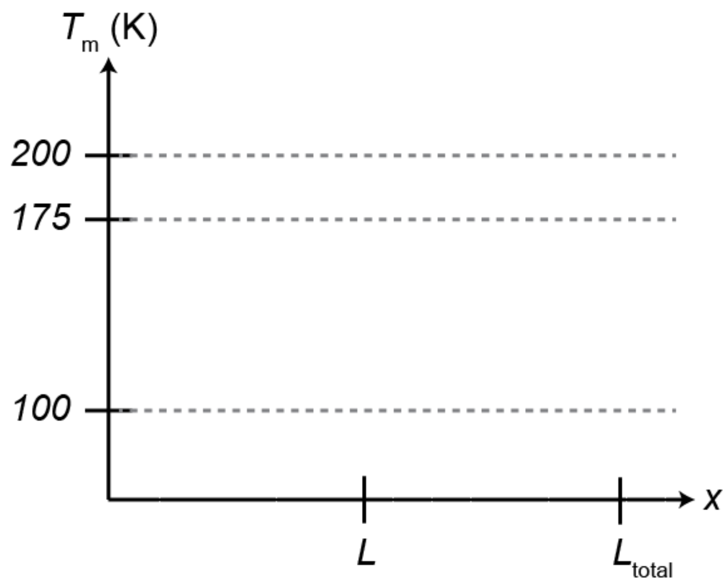
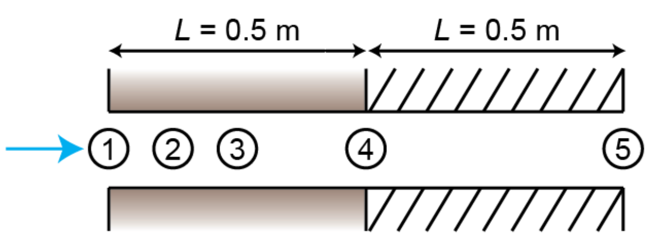
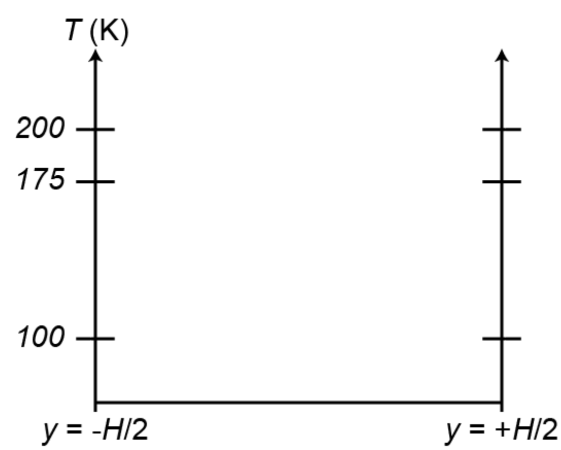


f) On the plot below, sketch the fluid temperature as a function of y at five x-locations in the duct that correspond to the following axial positions:

1: inlet to the duct
2: half-way through the thermally developing region (i.e., $x_2 = x_{fd,t}/2$)
3: at the thermal entry length (i.e., $x_3 = x_{fd,t}$)
4: at the end of the first section (i.e., $x_4 = L$)
5: at the outlet of the duct (i.e., $x_5 = L_{total}$)





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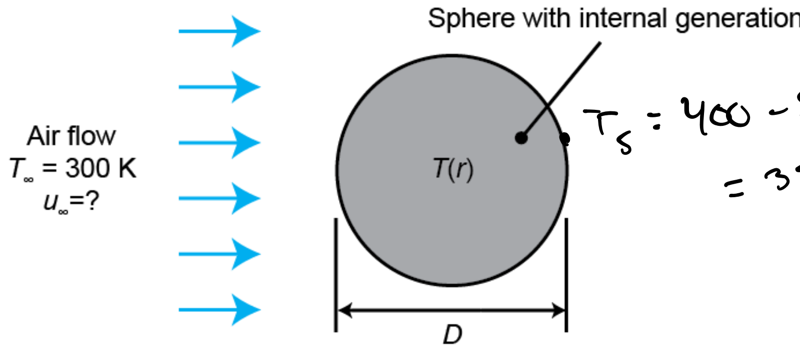
D

Problem 3

As shown in Figure 3, air at a temperature of $T_\infty = 300$ K flows over a solid sphere of diameter $D = 0.1$ m which experiences a uniform rate of volumetric generation. The sphere has a thermal conductivity $k_s = 2$ W/(m-K), and the properties of the air are the following: $c_{p,a} = 875$ J/(kg-K), $k_a = 0.025$ W/(m-K), $\mu_a = 20\times10^{-6}$ kg/(m-s), $\rho_a = 1$ kg/m^3, and $Pr_a = 0.7$. Assuming 1-D, steady state conduction in the radial direction, the temperature profile in the sphere is given by $T(r) = C_1 - C_2 r^2$, where $C_1 = 400$ K and $C_2 = 5000$ K/m^2.



$T_s = 400 - 5000(0.05)^2$

$= 387.5$ K

Figure 3. Air flow over a sphere.

a) Compute the free stream velocity of the air. For this calculation, use the following correlation for the Nusselt number: $\overline{Nu}_D = 2 + 0.6 Re_D^{1/2} Pr^{1/3}$

$$\overline{Nu}_D = \frac{\bar{h} D}{k} \rightarrow \bar{h} = \frac{k}{D}\left[2 + 0.6\sqrt{\frac{\rho u_\infty D}{\mu}}\,(0.7)^{1/3}\right]$$

$$q = \bar{h} A_s (T_s - T_\infty)$$

$$q = -k A_c \frac{dT}{dx}$$

E

Name________________________________

Problem 1

As shown in Fig. 1, a cylindrical rod is initially perfectly insulated on all surfaces except for its right-side face ($x = L$) which is exposed to a fluid at $T_H = 100$ ºC and heat transfer coefficient $h_H = 10$ W/(m^2·K). At a time $t = 0$, the insulation at the left-side face ($x = 0$) is removed and replaced with a fluid at $T_C = 0$ ºC and heat transfer coefficient $h_C = 200$ W/(m^2·K). The rod has a length $L = 100$ mm, a diameter $D = 1$ mm, thermal conductivity $k = 1$ W/(m·K), density ρ = 1000 kg/m^3, and specific heat capacity $c_p = 100$ J/(kg·K).

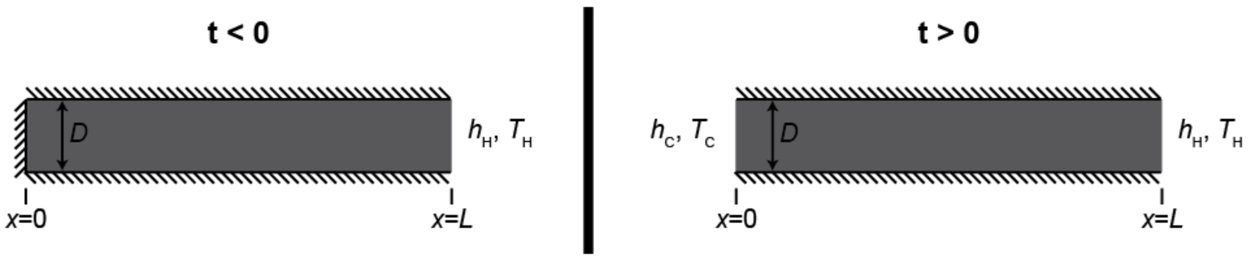
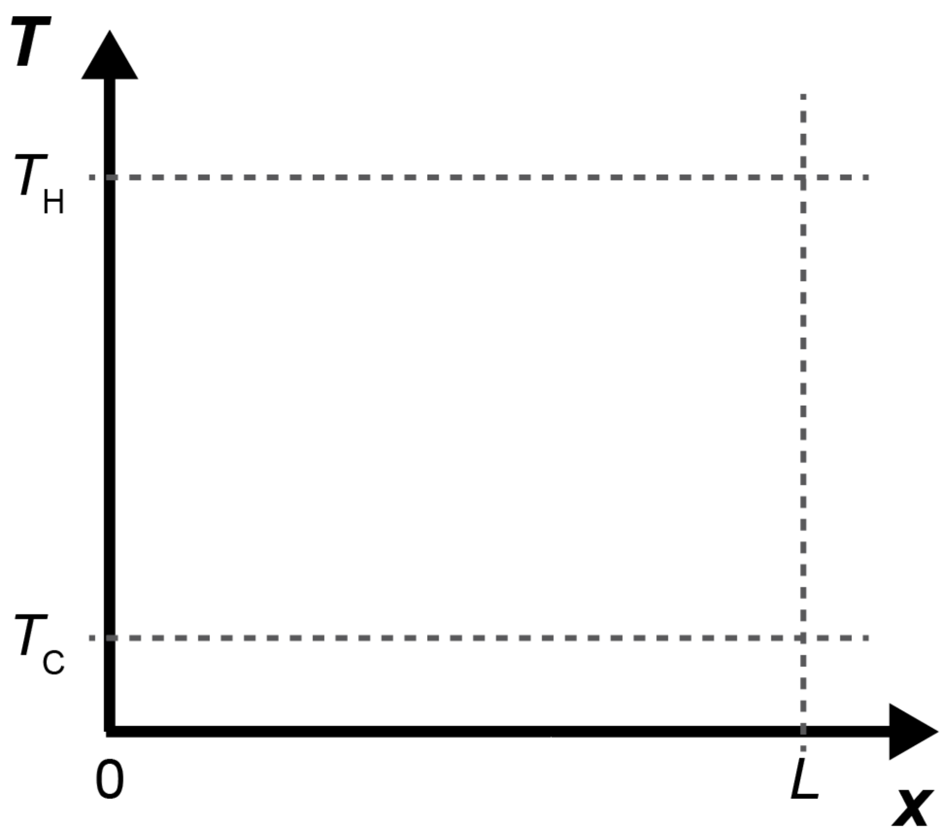


Figure 1: Cylindrical rod for times *t*<0 and *t*>0.

(a) Sketch the temperature distribution $T(x)$ in the rod for times $t < 0$ on the axes below.



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(b) Sketch the steady-state temperature distribution $T(x)$ in the rod when $t \to \infty$. Overlay this sketch on the same axes for part (a).

(c) Sketch the temperature distribution $T(x)$ in the rod when $t = 250$ sec. Overlay this sketch on the same axes for part (a).

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F

Problem 2

Inside of a microwave oven is a cube of cheese that is sitting on a large plate with a fixed temperature T_{plate} = 300 K (see Fig. 2). There is a contact resistance between the bottom side of the cube and the plate with an area-specific value of R_c'' = 50 m^2·K/W. The cube has a side length a = 1 cm and thermal properties: k = 1 W/(m·K), c_p = 1 J/(kg·K), and ρ = 1 kg/m^3. The cheese is initially in thermal equilibrium with the plate, but at time t = 0 the microwave is turned on which produces uniform heat generation in the cheese at a rate $\dot{g}'''$= 10 W/m^3. Ignore thermal radiation and convection for this problem.

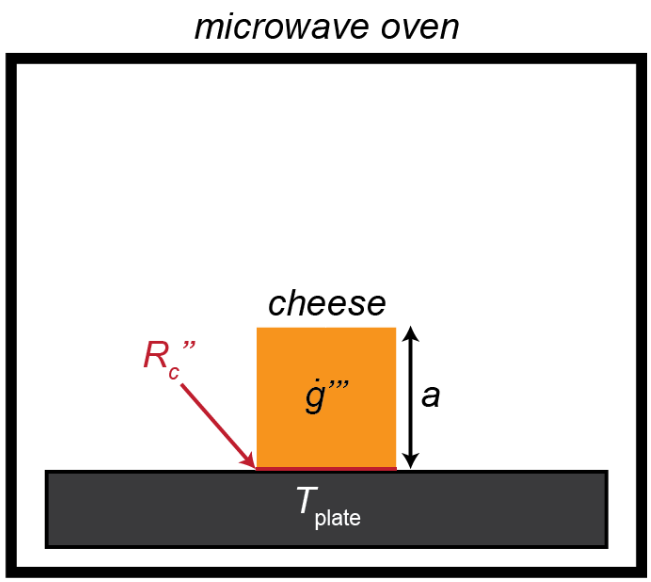

Figure 2: Cheese cube in a microwave oven.

a) Is a lumped capacitance model of the cheese valid? Justify with calculations.

b) Assume the answer to part (a) is yes. You want to use a numerical scheme with a time step of $\Delta t = 1.5$ sec to compute the temperature of the cheese as a function of time. Use Euler's method to compute the temperature of the cheese at the end of the first time step (at $t = \Delta t$).

c) Will the numerical solution from part (b) be stable? Justify with calculations.