ENTROPY

RECAP:

REVERSIBLE PROCESS $\qquad ds = \dfrac{dq}{T}$

$$dq = T\,ds$$

SPECIFIC VOLUME

FIRST LAW OF THERMO: $\qquad de = dq - p\,dv$

$$de = T\,ds - p\,dv$$

CALCULATE Δs FOR AN IDEAL GAS:

$$\left.\begin{aligned} T\,ds &= de + p\,dv \\ de &= c_v\,dT \\ dv &= d\left(\frac{1}{\rho}\right) = -\frac{d\rho}{\rho^2} \\ p &= \rho R T \end{aligned}\right\} \quad ds = c_v \frac{dT}{T} - R\frac{d\rho}{\rho}$$

INTEGRATE

SPECIALIZE TO PERFECT GAS (c_v = CONST)

$$\boxed{s_2 - s_1 = c_v \ln\frac{T_2}{T_1} - R\ln\frac{\rho_2}{\rho_1}}$$

ALSO RECALL

$$dh = de + p\,dv + v\,dp$$

$$T\,ds = de + p\,dv = dh - v\,dp$$

$$= c_p\,dT - v\,dp$$

$$v = \frac{RT}{p}$$

$\rightarrow \quad ds = c_p \dfrac{dT}{T} - R\dfrac{dp}{p}$ $\Big]$ SIMILAR TO LAST EXPRESSION

$\rightarrow \quad s_2 - s_1 = c_p \ln\dfrac{T_2}{T_1} - R\ln\dfrac{p_2}{p_1}$

ISENTROPIC PROCESS (REVERSIBLE & ADIABATIC)

$ds = 0 \Rightarrow s_2 - s_1 = 0$

$$c_p \ln \frac{T_2}{T_1} = R \ln \frac{P_2}{P_1} \longrightarrow \frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{R/c_p}$$

RECALL $c_p = c_v + R$ $\qquad \gamma = \frac{c_p}{c_v}$ = RATIO OF SPECIFIC HEATS

$$\rightarrow \frac{R}{c_p} = \frac{c_p - c_v}{c_p} = \frac{\gamma - 1}{\gamma}$$

$$\boxed{\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}} \qquad c_p = \frac{\gamma}{\gamma - 1} R = \frac{\gamma}{\gamma - 1} \frac{\bar{R}}{M}$$

WILL USE LATER

$P = \rho R T$

$$\boxed{\frac{P_2}{P_1} = \left(\frac{\rho_2}{\rho_1}\right)^{\gamma} \qquad \frac{T_2}{T_1} = \left(\frac{\rho_2}{\rho_1}\right)^{\gamma - 1}}$$

P IS ABSOLUTE

1) SOME PEOPLE USE

$$\frac{P}{\rho^{\gamma}} = \text{CONST}$$

MEANINGLESS?? UNITS?

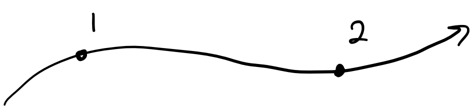
- WE LIKE DIMENSIONLESS VALUES

2) $\frac{P_2}{P_1} = \left(\frac{\rho_2}{\rho_1}\right)^{\gamma}$ IS <u>NOT</u> AN EQUATION OF STATE

$P = \rho R T$ IS AN EQ OF STATE

→ RELATES THERMODYNAMIC PROPERTIES

PROCESS EQN: TAKES US FROM ONE KNOWN STATE TO ANOTHER



EACH STATE IS UNIQUELY SATISFIED BY 2 INDEPENDENT THERMO. VARIABLES

ABOUT γ

$\gamma = C_p / C_v$ & $C_p = C_v + R$

$C_v = \frac{n}{2} R$ WHERE n = # OF QUADRATIC TERMS IN QUANT. MECH. HAMILTONIAN

$\gamma = 1 + \frac{2}{n}$

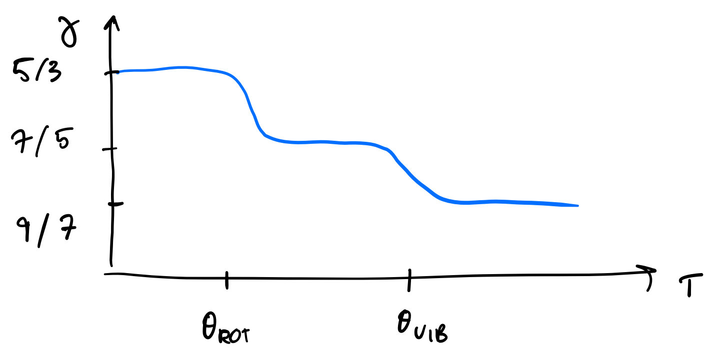
MONATOMIC MOLECULE: $n = 3$ @ ALL TEMP. $\rightarrow \gamma = 1 + \frac{2}{3} = 5/3$
(E.G. HELIUM, NOBLE GASSES)

DIATOMIC MOLECULE:

$T < \theta_{ROT}$ $\quad n = 3 \rightarrow \gamma \approx 1.6$

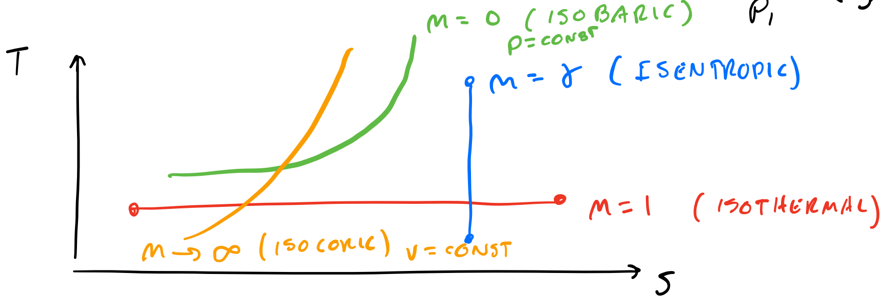
$\theta_{ROT} < T < \theta_{VIB}$ $\quad n = 5 \rightarrow \underline{\gamma \approx 1.4}$ AIR

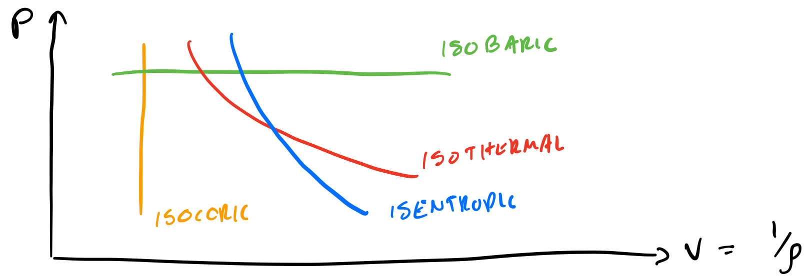
$\theta_{VIB} < T$ $\quad n = 7 \rightarrow \gamma \approx 1.29$



POLYTROPIC PROCESSES

- IF $ds \neq 0$, CAN DESCRIBE PROCESS AS $\frac{P_2}{P_1} = \left(\frac{\rho_2}{\rho_1}\right)^m$





II Conservation Equations In Integral Form

Notation

Velocity Field

$$\underline{u} = u(x,y,z,t)\,\hat{i} + v(x,y,z,t)\,\hat{j} + w(x,y,z,t)\,\hat{k}$$