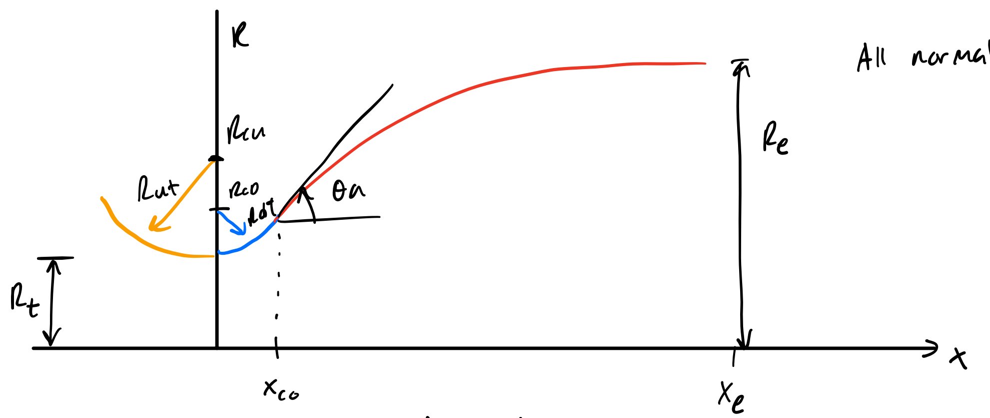
EMA 524 Project 1 Logic & equations

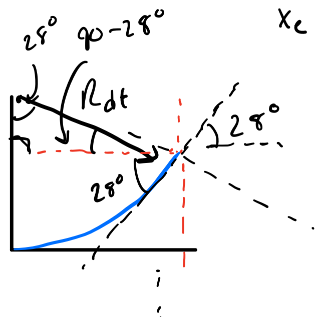
Geometry:

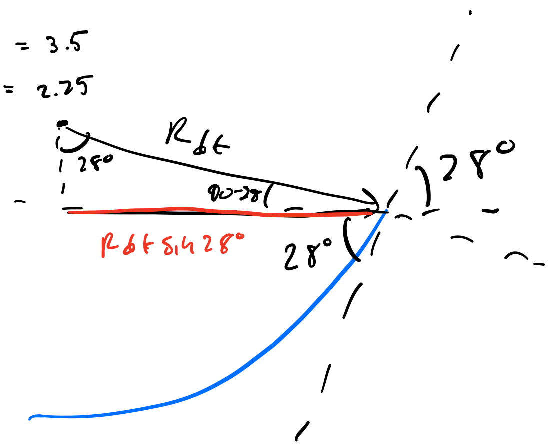


All normalized by R_t

solve: X_{co}
R_{co}
$R_{cu} = R_t + R_{ut} = 3.5$
$R_{cd} = R_t + R_{cD} = 2.25$
X_e

$R_{ut} = 2.5$
$R_{dt} = 1.25$
$R_t = 1$
$\theta_a = 28°$
$R_e = 4.5$





$X_{co} = 0.58684$

$R_{co} = 1.1463$

$x_{co} = R_{dt}\sin 28°$

$R(x) = a + bx + cx^2$

$R_{co} = R_{cD} - R_{dt}\cos 28°$

$R(x = x_{co}) = R_{co}$

$\frac{dR(x)}{dx} = b + 2cx$

$\frac{dR(x)}{dx}(x = x_{co}) = \tan(28°) \approx 0.53171$

$R_{co} = a + bX_{co} + cX_{co}^2$

$\tan 28° = b + 2c\,X_{co}$

$R_e = b + 2c\,X_e$

$\rightarrow c = -0.02107$

$b = 0.5564$

$a = 0.827$

Logic:

Input: P_e case & x vector

Case $P_e = P_{sup}$:

$$\left.\begin{array}{ll} x < 0: & M_{sub}(x) \\ x = 0: & M = 1 \\ x > 0: & M_{sup}(x) \end{array}\right\} \quad \frac{A}{A^*} = \frac{1}{M}\left[\frac{2}{\gamma-1}\left(1+\frac{\gamma-1}{2}M^2\right)\right]^{\frac{\gamma+1}{2(\gamma-1)}}$$

Case $P_e = P_{sub}$:

Always $M_{sub}(x)$

Case $\frac{P_e}{P_0} = \left(1+\frac{P_{sub}}{P_0}\right)/2$:

$A_t \neq A^*$ → Find M_e from $\frac{P_e}{P_0} = \left(1+\frac{\gamma-1}{2}M_e^2\right)^{\frac{\gamma}{1-\gamma}}$

→ Find $\frac{A_e}{A^{*\prime}} = \frac{1}{M_e}\left[\frac{2}{\gamma-1}\left(1-\frac{\gamma-1}{2}M_e^2\right)\right]^{\frac{\gamma+1}{2(\gamma-1)}}$

→ $\frac{A(x)}{A^{*\prime}} = \frac{A(x)}{A_t}\frac{A_t}{A_e}\frac{A_e}{A^{*\prime}}$ → Find M

Case $\frac{P_e}{P_0} = \frac{P_{sup}}{P_0} + 0.75\left(P_{sub}/P_0 + P_{sup} \text{ over } P_0\right)$

→ compute $\frac{P_{slex}}{P_{sup}} = 1+\frac{2\gamma}{\gamma+1}\left(M_{sup}^2-1\right)$

$\frac{P_{slex}}{P_0} = \frac{P_{slex}}{P_{sup}} \cdot \frac{P_{sup}}{P_0}$ → Shock in nozzle

→ calculate M_e w/ shock in nozzle:

$$M_e^2 = -\frac{1}{\gamma-1} + \sqrt{\frac{1}{(\gamma-1)^2} + \frac{2}{(\gamma-1)}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}\left(\frac{P_{0_1}A_1^*}{P_eA_e}\right)^2}$$

$$\frac{p_{eo}}{p_{02}} = \left(1 + \frac{(\gamma-1)}{2} M_e^2\right)^{\frac{\gamma}{1-\gamma}}$$

$$\frac{p_{02}}{p_{01}} = \frac{1}{\frac{p_{01}}{p_{0e}}} \cdot \frac{p_e}{p_{02}}$$

$$\frac{p_{02}}{p_{01}} = \left[\frac{(\gamma+1) M_1^2}{2+(\gamma-1)M_1^2}\right]^{\frac{\gamma}{\gamma-1}} \cdot \left[\frac{\gamma+1}{2\gamma M_1^2-(\gamma-1)}\right]^{\frac{1}{\gamma-1}}$$

↳ solve for $M1$ → solve for $\frac{A_1}{A_1^*}$ w/ area-mach relation

→ shock relations

→ $$\frac{T_2}{T_1} = \frac{[2\gamma M_1^2-(\gamma-1)] \cdot [(\gamma-1)M_1^2+2]}{(\gamma+1)^2 M_1^2}$$

→ $$\frac{u_1}{u_2} = \frac{\rho_2}{\rho_1} = \frac{(\gamma+1)M_1^2}{2+(\gamma-1)M_1^2}$$ → $$M_2 = \left(\frac{1+\frac{\gamma-1}{2}M_1^2}{\gamma M_1^2 - \frac{\gamma-1}{2}}\right)^{1/2}$$

$$\frac{A_2}{A_2^*} = $$ → mach-area relation

Before shock:

$$\frac{A}{A^*} = \frac{A}{A_t}$$

After shock:

$$\frac{A}{A^*} = \frac{A}{A_2^*}$$

→ solve machs for each area

Then, using mach numbers & γ for each case, calculate $\frac{p}{p_0}$, $\frac{T}{T_0}$, $\frac{\rho}{\rho_0}$, $\frac{u}{a_t}$

$$\frac{T_0}{T} = \left(1 + \frac{r-1}{2} M^2\right)$$

$$\frac{P_0}{P} = \left(1 + \frac{r-1}{2} M^2\right)^{\frac{r}{r-1}}$$

$$\frac{\rho_0}{\rho} = \left(1 + \frac{r-1}{2} M^2\right)^{\frac{1}{r-1}}$$

→ for case 3: mutiply by $\frac{P_{01}}{P_{02}}$ to account for post-shock

→ for case 3: using sonic throat area for A_2^*