LAPLACE

$$\mathcal{L}[f(t)] = \int_{-\infty}^{\infty} f(t) \cdot e^{st}\, dt$$

t - domain — s - domain

$$\mathcal{L}[f(t)] = \int_{0^-}^{\infty} f(t) \cdot e^{-st}\, dt$$ unilateral Laplace

e.g. $$\int_{0^-}^{\infty} e^{-at} e^{-st}\, dt$$

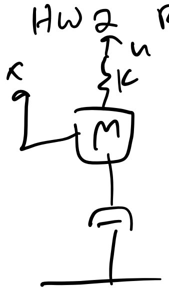
$$= \int_{0^-}^{\infty} e^{-(s+a)t}\, dt$$

$$= \left[\frac{1}{-(s+a)} e^{-(s+a)t} \right]_{0^-}^{\infty}$$

$$= -\frac{1}{s+a} \left[\cancel{e^{-(s+a)\infty}}^{\,0} - \underbrace{e^{-(s+a)\cdot 0}}_{1} \right]$$

$$= \frac{1}{s+a} = F(s)$$

HW 2 Review



assume $u > x$

$\Sigma F = ma$

$$k(u-x) - c\dot{x} = m\ddot{x}$$

$$\rightarrow \ddot{x} + 15\dot{x} + 150x = 1500u$$

$x(0) = 2$ $\dot{x}(0) = 0$ $u(t) = e^{-3t}$

$$s^2 X(s) - s(2) - 0$$

$$15\left(sX(s) - 2\right)$$

$$\mathcal{L}[u(t)] = \int_{-\infty}^{\infty} = \int_{0^-}^{\infty} e^{-3t} e^{-st}\, dt$$

$$= \int_{0^-}^{\infty} e^{(-3-s)t}\, dt$$

$$= \frac{1}{-3-s}\left[e^{(-3-s)t} \right]_{0^-}^{t=\infty} = \frac{1}{-3-s}\left[\overset{0}{\cancel{e^{(-3-s)\infty}}} - \underset{1}{\cancel{e^{(-3-s)\cdot 0}}} \right]$$

$$U(s) = \frac{1}{3+s}$$

$$\rightarrow \quad s^2 X(s) - 2s + 15sX(s) - 30 + 150X(s) = \frac{150}{3+s}$$

$$\left(3s^2 + 45s + 150\right) X(s) - 6s - 90 = \frac{150}{s+3}$$

$$X(s) = \frac{\frac{150}{s+3} + 6s + 90}{3s^2 + 45s + 150} = \frac{2s^2 + 36s + 140}{(s+3)(s+5)(s+10)}$$

$$\frac{2s^2 + 36s + 146}{(s+3)(s+5)(s+10)} = \frac{C_1}{s+3} + \frac{C_2}{s+5} + \frac{C_3}{s+10}$$

$$2s^2 + 36s + 140 = c_1(s+5)(s+10) + c_2(s+3)(s+10) + c_3(s+3)(s+5)$$

$$c_1 = c_1(s^2 + 15s + 50) + c_2(s^2 + 13s + 30) + c_3(s^2 + 8s + 15)$$

$$2s^2 = c_1 s^2 + c_2 s^2 + c_3 s^2$$

$$36s = c_1 15s + c_2 13s + c_3 8s$$

$$140 = c_1 50 + c_2 30 + c_3 15$$

$$x(t) = c_1 e^{-3t} + c_2 e^{-5t} + c_3 e^{-10t}$$