HW 5 - Need to watch recordings

Frequency domain

- Transfer Function for plant
  - → Can tell stability, oscillation, SS error

Roots at origin	Real roots	Complex roots
integration $\frac{1}{s^n}$	1st order denom $\frac{1}{\tau s+1}$	2nd order denom $\frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$
differentiation s^n	1st order num $\tau s+1$	2nd order num $\frac{s^2+2\xi\omega_n s+\omega_n^2}{\omega_n^2}$

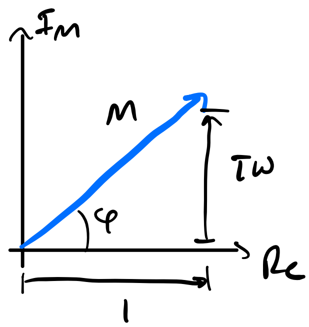
$G(s) = \tau s + 1$ (bode form)

$G(j\omega) = 1 + \tau\omega j$

$|G(j\omega)| = M = \sqrt{1+\tau^2\omega^2}$

$\log M = \frac{1}{2}\log(1+\tau^2\omega^2)$

$\varphi = \tan^{-1}(\omega\tau)$



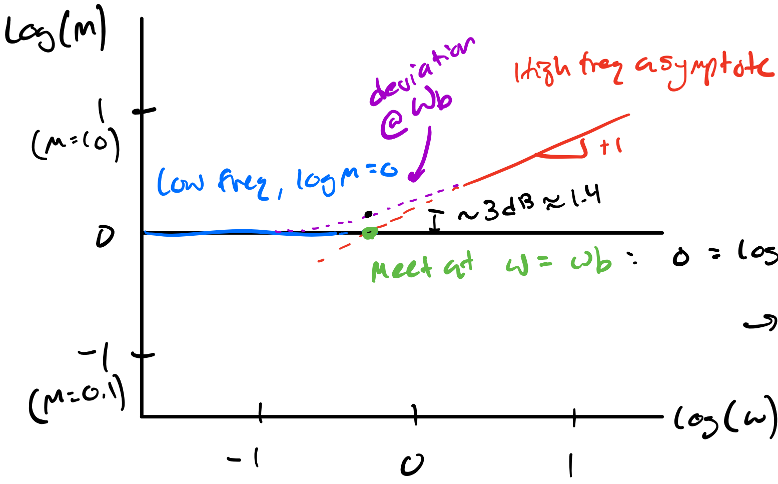
Frequency response $(\tau s+1)$

Low frequency response: $\omega << 1/\tau$: $\frac{1}{2}\log(1) = 0 \rightarrow M = 1$

$\varphi = 0$

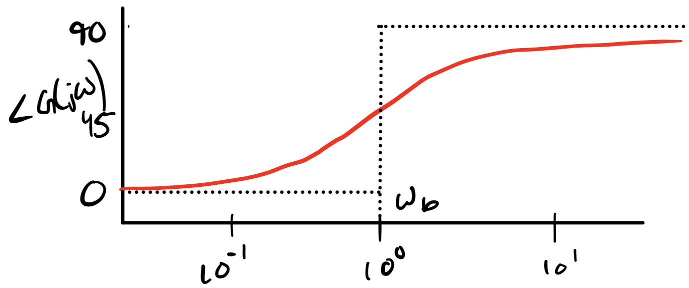
High freq.: $\omega >> 1/\tau$: $M \approx \underbrace{\log(\tau)}_{\text{const.}} + \underbrace{\log(\omega)}_{+1\text{ slope}}$

$\varphi = \tan^{-1}(\omega\tau) \approx \tan^{-1}(\infty) = 90^\circ$



$0 = \log(\tau) + \log(\omega_b)$

$\rightarrow \omega_b = \frac{1}{\tau}$

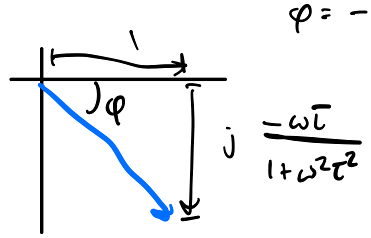


Denominator of real root: $G(s) = \frac{1}{\tau s+1}$

$G(j\omega) = \frac{1}{1+j\omega\tau}$

$M = \frac{1}{\sqrt{1+\omega^2\tau^2}}$, $\log M = -\frac{1}{2}\log(1+\tau^2\omega^2)$

$\varphi = -\tan^{-1}(j\omega)$



Low freq:

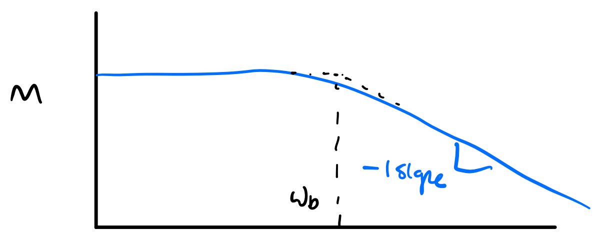
M: $-\frac{1}{2}\log(0) = 0$

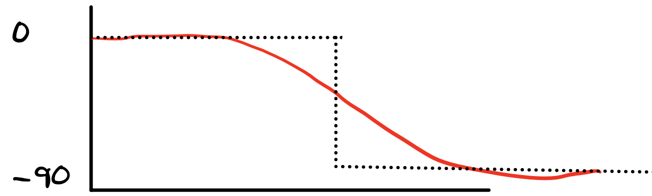
φ: $-\tan^{-1}(0) = 0$

High freq:

M: $\overbrace{-\log(\tau)}^{\text{const}}\overbrace{-\log(\omega)}^{-1\text{ slope}}$

φ: $-\tan^{-1}(\infty) = -90°$





Generally, transfer function is made up of these 1st (or 2nd) order terms

$$G(s) = G_1(s)\,G_2(s)\ldots G_q(s) \qquad q \text{ terms}$$

$$G(j\omega) = G_1(j\omega)\,G_2(j\omega)\ldots G_q(j\omega)$$

Polar:

$$G(j\omega) = (M_1 e^{j\theta_1})(M_2 e^{j\theta_2})\ldots$$

$$G(j\omega) = (M_1 M_2 \ldots)\,e^{j(\theta_1+\theta_2+\ldots)}$$

$$|G(j\omega)| = M_1 M_2 \ldots M_q$$

$$\angle G(j\omega) = \theta_1 + \theta_2 + \ldots + \theta_q$$

Ex. $$G(s) = \frac{2(4s+8)}{(s+1)^2}$$

real root

repeated real root

Bode form: e.g. $s+3 \rightarrow 3\underbrace{\left(\tfrac{1}{3}s+1\right)}_{\text{bode}}$

$$2 \cdot \frac{4s+8}{(s+1)^2} \rightarrow \frac{2 \cdot 8\left(\tfrac{1}{2}s+1\right)}{(s+1)(s+1)}$$

$$\rightarrow G(s) = \frac{16(0.5s+1)}{(s+1)(s+1)}$$

[$16 \rightarrow m = 16,\ \alpha = 0$

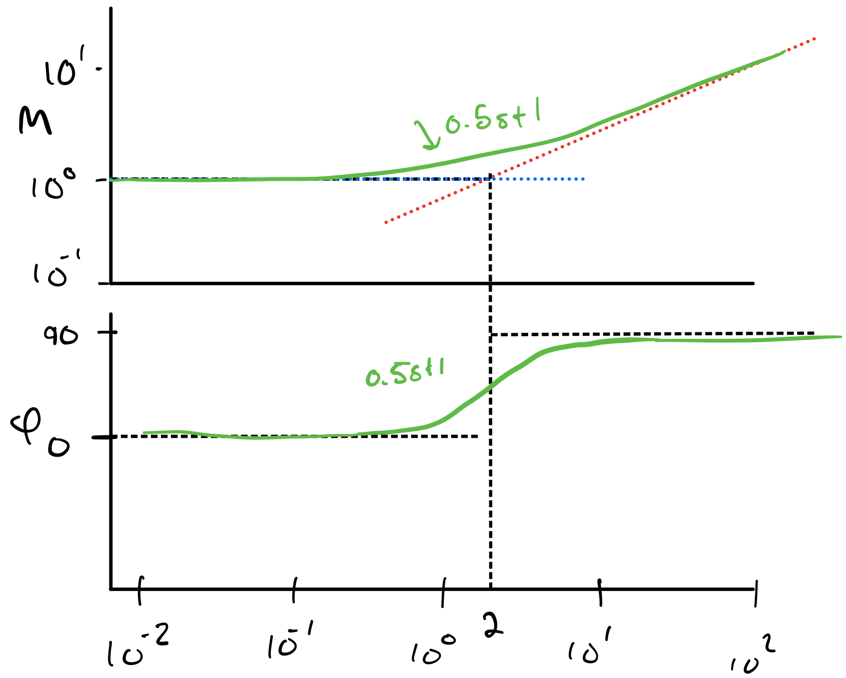
$0.5s+1$ term: $\omega_b = \frac{1}{0.5} = \tau$

$\omega \ll \omega_b$: $M \approx 1$, $\varphi \approx 0$

$\omega = \omega_b$: M: $+3\,dB\ (1.4)$ $\varphi = +45^\circ$

$\omega \gg \omega_b$: M: $+1$ "slope" $\varphi = +90^\circ$

dB: $20\log_{10}(\#)$ $\quad$ $+20\ dB/\text{decade}$



$\frac{1}{(s+1)^2}$ term:

$\frac{1}{s+1}$: $\omega \ll \omega_b$: $M \approx 1 \rightarrow \log M = 0$ $\quad \varphi \approx 0$

$\omega = \omega_b$: M: $-3\,dB$, $\varphi = -45$

$\omega \gg \omega_b$: M: -1 "slope" $\varphi = -90$

$\frac{1}{(s+1)^2}$: $\omega \ll \omega_b$: $\log M \approx 0$, $\varphi \approx 0$

$\omega = \omega_b$: $2 \cdot -3 = -6\,dB$, $\varphi = 2 \cdot -45 = -90^\circ$

$\omega \gg \omega_b$: -2 "slope" $\varphi \approx -180$