Exam: 2-side 8.5" x 11" Note sheet

Last time: given $\bar{r}$ & $\bar{v}$ in cartesian, get orbital elements

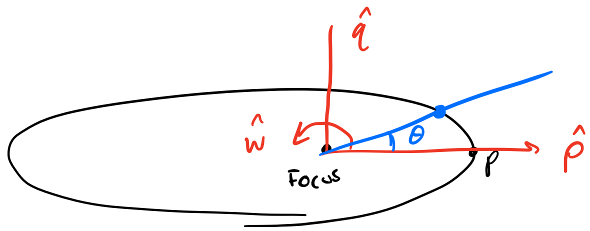
→ classical modified set

From lecture 10 ($\bar{r}, \bar{v} \rightarrow a, e, i, \theta, \omega, \Omega$)

we want to invert it.

$$(a, e, \theta, i, \omega, \Omega) \rightarrow (x, y, z, v_x, v_y, v_z)$$

① get into "perifocal frame" $\equiv$ $\bar{r}_{pqw} = r_p \hat{p} + r_q \hat{q} + r_w \hat{w}$



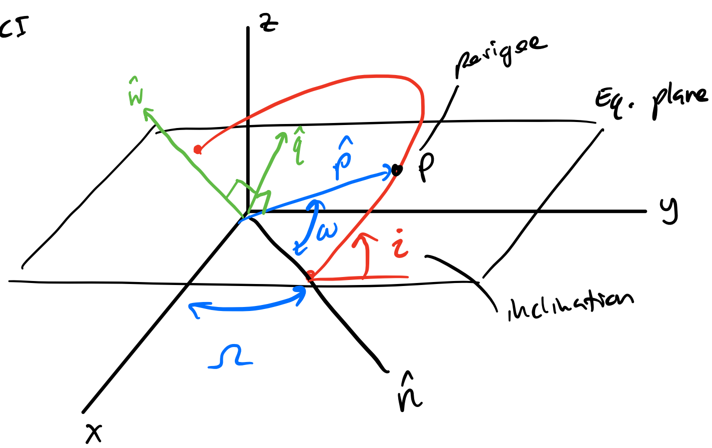
Easy to show $\bar{r}_{pqw} = \cos\theta \, \hat{p} + r\sin\theta \, \hat{q} + 0 \, \hat{w}$

take d/dt → slide 21

$$\boxed{\bar{v}_{pqw} = -\sqrt{\frac{\mu}{a(1-e^2)}} \sin\theta \, \hat{p} + \sqrt{\frac{\mu}{a(1-e^2)}} (e + \cos\theta) \, \hat{q}}$$

② Transform from $(\hat{p}, \hat{q}, \hat{w}) \rightarrow (x, y, z)$

xyz → ECI



many ways to align but do this:

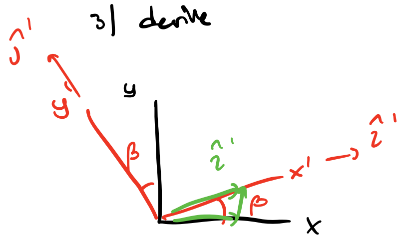
1) rotate about $\hat{w}$ by $-\omega$, aligns $\hat{p}$ w/ $\hat{n}$

Review: transformation matrix $\longrightarrow$ above is like a "z" rotation

derive w/ 1) look down axis

2) draw old & new in 2D

3) derive



$xyz \rightarrow x'y'z'$

$\hat{i}' = \cos\beta\,\hat{i} + \sin\beta\,\hat{j}$

$\hat{j}' = -\sin\beta\,\hat{i} + \cos\beta\,\hat{j}$

$\hat{k}' = \hat{k}$

$$\therefore \begin{Bmatrix} \hat{i}' \\ \hat{j}' \\ \hat{k}' \end{Bmatrix} \underbrace{\begin{bmatrix} \cos\beta & \sin\beta & 0 \\ -\sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{[T_z(\beta)]} \begin{Bmatrix} \hat{i} \\ \hat{j} \\ \hat{k} \end{Bmatrix}$$

3 axis transform

slide 24: $[T_y(\beta)]$, 25 $[T_x(\beta)]$

<u>use as a sequence</u>: e.g. $\bar{r}_{pqw} \rightarrow \bar{r}_{xyz}$: 3-1-3

called "Euler sequence"

$$\bar{r}_{xyz} = \underset{\text{about } \hat{k}}{\overset{\text{3rd}}{[T_z(-\Omega)]}} \underset{\text{about } \hat{n}}{\overset{\text{2nd}}{[T_x(-i)]}} \underset{\text{about } \hat{w}}{\overset{\text{1st}}{[T_z(-\omega)]}} \bar{r}_{pqw}$$

use reverse order (matrix alg.)

→ review example on slide 29!

Example 1 : Given $(a, e, i, \theta, \Omega, \omega)$ find $\bar{r}, \bar{v}$

$a = 10\,000$ km

$e = 0.5$

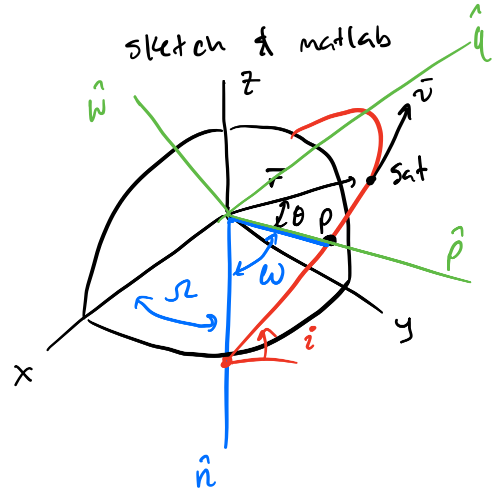
$\theta = 20°$

$i = 10°$

$\Omega = 35°$

$\omega = 50°$

sketch & matlab



Make calculator in Matlab:

see slides for eqns

When using transformations → rotating "backwards" by $-\omega$ about $\hat{w}$ first

$$\left[T_z(-\omega)\right] = \left[T_z(\omega)\right]^{-1} = \left[T_z(\omega)\right]^T$$

Can plug in $-\omega$ given R, $R^T = R^{-1} = R(-\theta)$

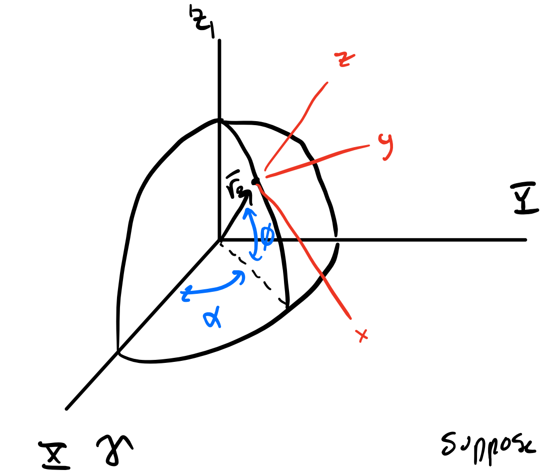
Answer:

$\bar{r} = -2387.9\,\hat{i} + 9091.3\,\hat{j} + 1554.6\,\hat{k}$ km

$\bar{v} = -6.4\,\hat{i} - 1.638\,\hat{j} + 0.41\,\hat{k}$ km/s

code: modified classic s2t to RV.m

Example 2 Given topocentric coords (launch), transform to ECI coords (orbit)



Topocentric ≡ observer's C-sys

$z \to$ radial outward

$x \to$ southward

$y \to$ eastward

Given:

α = Right ascension

ϕ = latitude

Suppose we have $(xyz) \to (XYZ)$

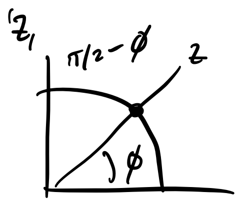
note: need to translate by r_0

We have two angles to rotate & 1 translation

1) Move origin of (xyz) to (XYZ)

2) Align z w/ Z_1, rotate about y : $\frac{\pi}{2} - \phi$

3) Align x w/ X, rotate about Z_1 : $-\alpha$



$$\begin{Bmatrix} X \\ Y \\ Z_1 \end{Bmatrix} = \left[T_3(-\alpha)\right]\left[T_2(-(\pi/2-\phi))\right]\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & [\ \ ] \end{bmatrix}\begin{Bmatrix} x \\ y \\ z \end{Bmatrix}$$

$1 + \frac{r_0}{z}$

$r_z = z + r_0$

$r_z = z + r_0$

$r_z = z\left(1 + \frac{r_0}{z}\right)$