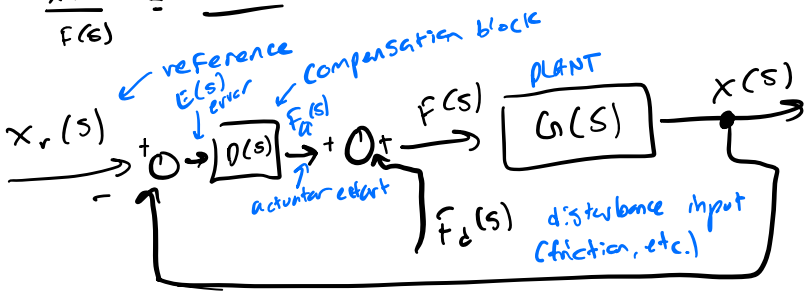
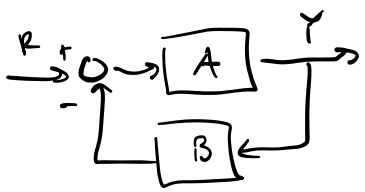
UNITY GAIN FEEDBACK

$$\frac{X(s)}{F(s)} = \frac{G(s)}{D_c(s)}$$




$$\frac{X(s)}{X_r(s)} \quad , \quad \frac{E(s)}{X_r(s)} \quad , \quad \frac{E(s)}{F_d(s)}$$

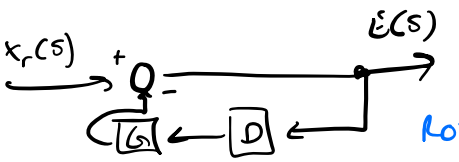



$$\Rightarrow \quad \frac{Y}{R} = \frac{A}{1+AB} \quad (-\text{ feedback})$$

$$\frac{A}{1-AB} \quad (+\text{ feedback})$$

$$\frac{X(s)}{X_r(s)} = \frac{D \cdot G}{1 + DG}$$

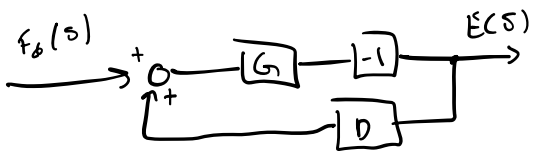
$$\frac{E(s)}{X_r(s)}:$$




ROTATE LOOP AROUND

$$\Rightarrow \quad \frac{E(s)}{X_r(s)} = \frac{1}{1 + DG}$$

$$\frac{E(s)}{F_d(s)}:$$




$$\Rightarrow \quad \frac{E(s)}{F_d(s)} = \frac{-G}{1-(G \cdot -1 \cdot D)} = \frac{-G}{1+GD}$$

→ GENERALLY DON'T CARE ABOUT SIGN, ONLY MAGN. OF DISTURBANCE

TIME DOMAIN PERFORMANCE SPECS

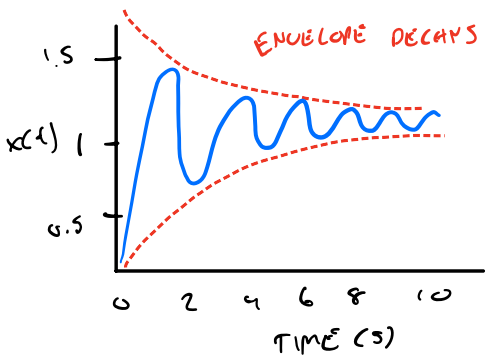
2ND ORDER SYSTEM: $\ddot{x} + \frac{c}{m}\dot{x} + \frac{k}{m}x = \frac{k}{m}f$

↳ $\left(s^2 + \frac{c}{m}s + \frac{k}{m}\right)X(s) = \frac{k}{m}F(s)$

NORMALIZED: $\frac{c}{m} = 2\zeta\omega_n$, $\frac{k}{m} = \omega_n^2$ ASSUME $0 \le \zeta \le 1$

.... → $x(t) = 1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin(\omega_d t + \varphi)$

$\omega_d = \omega_n\sqrt{1-\zeta^2}$ & $\varphi = \cos^{-1}(\zeta)$



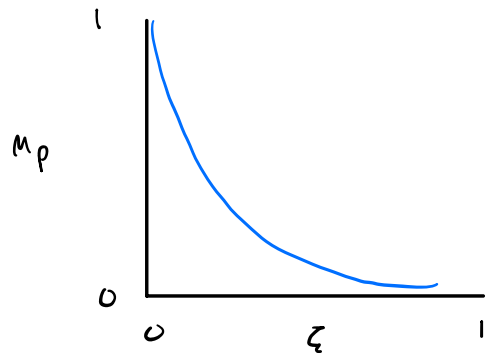

OVERSHOOT:

$M_p = \frac{x_{peak} - x_{ss}}{x_{ss}}$

$M_p = e^{-\pi\zeta/\sqrt{1-\zeta^2}}$

MAX OVERSHOOT VS. DAMPING RATIO

$M_p$ is $f(\zeta)$ ONLY




→ CAN ESTIMATE $\zeta$ FROM $M_p$

$\zeta = \sqrt{\frac{(\ln M_p)^2}{\pi^2 + (\ln M_p)^2}}$

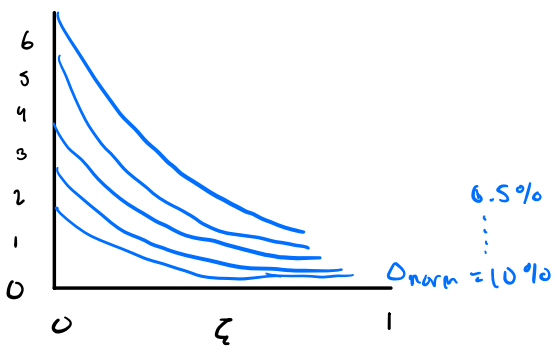
SETTLING TIME $T$: TIME FOR RESPONSE WITHIN $\pm 1\%$

$e^{-\zeta\omega_n t_s} = \Delta$

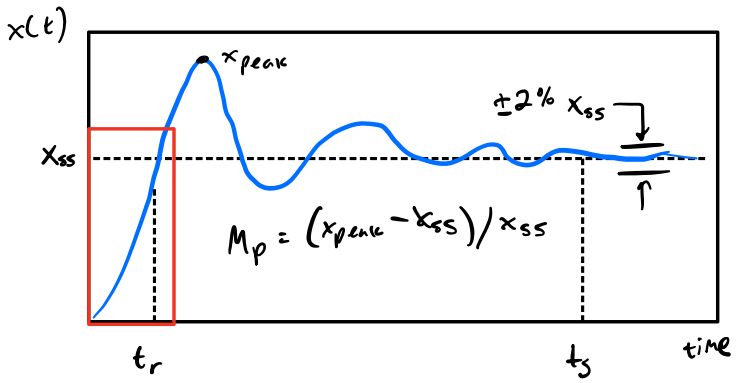
$-\zeta\omega_n t_s = \ln(\Delta)$

→ $t_s = -\frac{\ln\Delta}{\zeta\omega_n} = \frac{\ln\Delta}{\sigma}$

SETTLING TIME VS. $\zeta$

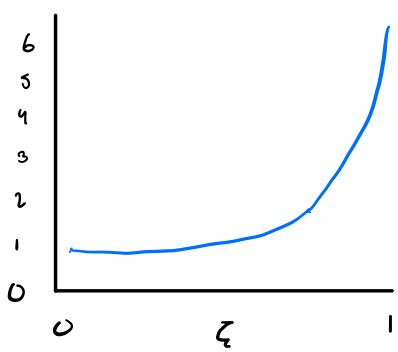



RISE TIME




$$t_r = \frac{\pi - \varphi}{\omega_d} = \frac{\pi - \varphi}{\omega_n \sqrt{1-\zeta^2}}$$

RISE TIME VS. $\zeta$




$t_r \cong 2.4/\omega_n$

($\zeta = 0.5$, $0 < x(1) < 1$)

EXAMPLE:

SPECS: $t_s \le 1.0$ settling time

$\Delta = 1\%$ steady state definition

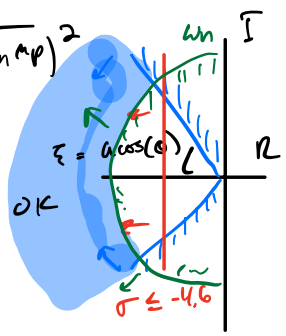
$t_r \le 0.3$ rise time

$M_p \le 25\%$ max overshoot

overshoot

$$M_p \cong e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}} \Rightarrow \zeta = \sqrt{\frac{(\ln M_p)^2}{\pi^2 + (\ln M_p)^2}}$$

$M_p \le 0.25 \Rightarrow \zeta \ge 0.4$




$$t_s \cong \frac{\ln\Delta}{\sigma} \rightarrow \sigma \cong \frac{\ln\Delta}{t_s}$$

$|\sigma| \ge 4.6$, $\sigma \le -4.6$

Rise time ($\zeta = 0.5$)

$$t_r \cong \frac{2.4}{\omega_n} \rightarrow \omega_n \cong \frac{2.4}{t_r} \quad \uparrow \le 0.3$$

$\rightarrow \omega_n \ge 8.0$