TRANSFER FUNCTION → $T(s) = \frac{X(s)}{F(s)}$ DEFINED AS RATIO OF OUTPUT TO INPUT

GIVEN A SYSTEM: $\ddot{x} + a\dot{x} + bx = c\dot{f} + df$, $x(0) = 0$, $\dot{x}(0) = 0$

[IC'S SET TO ZERO, ONLY INTERESTED W/ RESPONSE FROM INPUT

TAKE LAPLACE:

$(s^2 + as + b)X(s) = (cs + d)F(s)$

CHAR. EQN

$T(s) = \frac{X(s)}{F(s)}$

TRANSFER FN CAN BE USED TO FIND RESPONSE OF SYSTEM TO ARB. FORCING $f(t)$

$X(s) = T(s)\,F(s)$

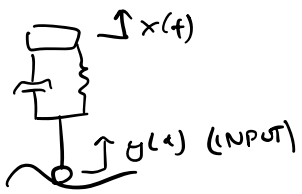
E.G. $T(s) = \frac{10s + 5}{s^2 + 4s + 5} = \frac{X(s)}{F(s)}$

$(s^2 + 4s + 5)X(s) = (10s + 5)F(s)$

↓ ↓ ↓

$\ddot{x} + 4\dot{x} + 5x$ etc

EVALUATE $T(s) = \frac{X(s)}{Y(s)}$



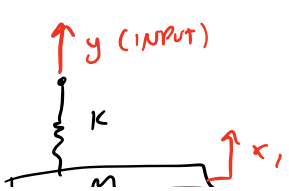
$m\ddot{x} + c\dot{x} + kx = c\dot{y} + ky$

L.T. (SET IC = 0)

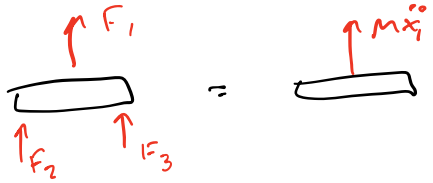
$ms^2X(s) + csX(s) + kX(s) = csY(s) + kY(s)$

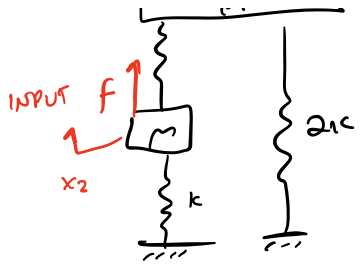
$(ms^2 + cs + k)X(s) = (cs + k)Y(s)$

$\frac{X(s)}{Y(s)} = \frac{cs + k}{ms^2 + cs + k}$ ← $\Delta(s)$



$\bar{F} = m\bar{a}$





$k(y - x_1) + k(x_2 - x_1) + 2k(-x_1) = m\ddot{x}_1$

$F_1 \qquad F_2 \qquad F_3$

$\rightarrow \; m\ddot{x}_1 + 4kx_1 - kx_2 = ky$

$(\ldots) \rightarrow \; m\ddot{x}_2 - kx_1 + 2kx_2 = f$

2 EQN, 2 DOF

EVALUATE: $T_1(s) =$

L.T. BOTH SIDES

$(ms^2 + 4k)\,X_1(s) - kX_2(s) = kY(s) \quad (1)$

$(ms^2 + 2k)\,X_2(s) - kX_1(s) = F(s) \quad (2)$

$T_1(s) = \dfrac{X_1(s)}{F(s)} \rightarrow$ SET TO ZERO TO FIND RESPONSE FROM F(s) ONLY $\Rightarrow Y(s) = 0$

FROM (1): $X_2(s) = \dfrac{(ms^2 + 4k)}{k} X_1(s)$

SUB INTO (2): $(ms^2 + 2k)\left(\dfrac{ms^2 + 4k}{k}\right) X_1(s) - kX_1(s) = F(s)$

$\Rightarrow \; \dfrac{m^2s^4 + 6mks^2 + 7k^2}{k} X_1(s) = F(s)$

$\dfrac{X_1(s)}{F(s)} = \dfrac{k}{m^2s^4 + 6mks^2 + 7k^2} \leftarrow \Delta(s)$ (CHAR. EQN.)

$T_2(s) = \dfrac{X_2(s)}{F(s)} \rightarrow Y(s) = 0$

FROM (1): $X_1(s) = \frac{K}{ms^2+4K} X_2(s)$

SUB INTO (2): $(ms^2+2K) X_2(s) - K \frac{K}{ms^2+4K} X_2(s) = F(s)$

$\rightarrow \quad \frac{X_2(s)}{F(s)} = \frac{ms^2+4K}{m^2s^4+6mKs^2+7K^2}$ ← SAME $\Delta(s)$,

SAME FOR $\frac{X_1(s)}{Y(s)}$, ETC

DYNAMIC SYSTEMS IN MATLAB : TRANSFER FUNCTION DEFINITIONS

$$G(s) = \frac{s(s+3)}{(s+2)(s^2+s+1)}$$

tf = Transfer function

```
s = tf('s');
sysG = s*(s+3) / ((s+2)*(s^2+s+1))
```

ALT:

```
num = [1 3 0]
den = [1 3 3 2]
sysG = tf(num, den)
```

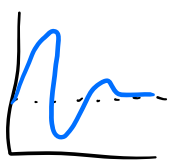
OR

```
num = conv([1 0], [1 3])
```

"CONV" TO MULTIPLY POLYNOMIALS

TIME DOMAIN SIMULATION

```
step(sysG)
tFinal = 10
t = [0:0.001:tFinal]
step(sysG, t)
```

→ 

OR `t = linspace(0, tFinal, 1000)`

OR STORE: `[y, t] = step(sysG, t)`

→ `plot(t,y)`

`xlabel`
`ylabel` ...

EX. SINUSOID $u = \sin(3t)$

t = linspace

$u = \sin(3t)$

$y = \text{lsim}(sys, u, t)$

OR CONSTRUCT PULSE

w/ FOR / IF LOOP