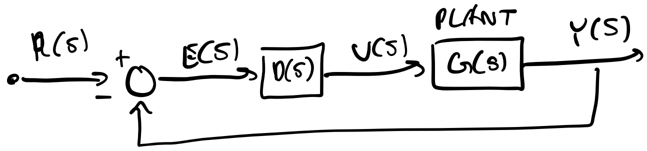
LAST TIME:

PID CONTROL: Proportional - Integral - Derivative

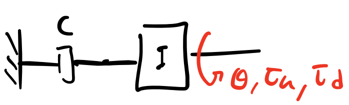
- most common controller form
- standard to which other control laws are compared
- Developed heuristically - prior to control theory



	effect	Analogous to
Proportional: $u(t) = K_p e(t) \rightarrow D(s) = K_p$	effect $\propto$ error	(SPRING)
Integral: $u(t) = K_I \int e(t)\,dt \rightarrow D(s) = \frac{K_I}{s}$	effect $\propto \int$ error	Helps error but can make it worse if a much
Derivative: $u(t) = K_d \frac{d}{dt} e(t) \rightarrow D(s) = K_D s$	effect $\propto \frac{d}{dt}$ error	(DAMPER)

TYPE	TRANSFER FUNCTION	COMMONLY USED IN SYSTEMS
P	K_p	1st & 2nd order
PI	$K_p + \frac{K_I}{s} = \frac{K_p s + K_I}{s}$	1st & some 2nd order
PD	$K_p + K_d s$	2nd order
PID	$K_p + \frac{K_I}{s} + K_d s = \frac{K_d s^2 + K_p s + K_I}{s}$	2nd order

1^{st} order System: PID CONTROL:



torque (input) $\tau = \tau_a + \tau_d$

applied disturbance

EOM: $I\ddot{\theta} + C\dot{\theta} = \tau$

CONTROL $\omega = \dot{\theta}$

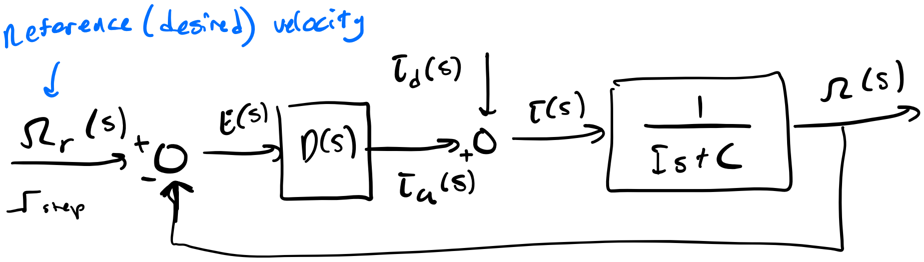
$$\frac{\theta(s)}{\tau(s)} = \frac{1}{Is^2 + Cs}$$

$\omega = \dot{\theta}$

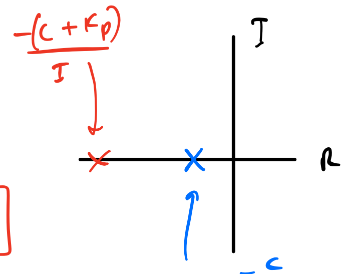
$\mathcal{L}$: $\Omega(s) = s\,\theta(s)$

$$\frac{s\theta(s)}{\tau(s)} = \frac{\Omega(s)}{\tau(s)} = \frac{1}{Is + C} \rightarrow I\dot{\omega} + C\omega = \tau \quad \text{SAME THING}$$

$$G(s) = \frac{\Omega(s)}{\tau(s)} = \frac{1}{Is + C}$$



$$D(s) = K_p$$



$$\frac{\Omega(s)}{\Omega_r(s)} = \frac{K_p\left(\frac{1}{Is+c}\right)}{1 + \frac{K_p}{Is+c}} = \frac{K_p}{\boxed{Is + (c+K_p)}}$$

$\Delta(s)$

Root of $\Delta(s)$ $\rightarrow$ $\left[s = \frac{-(c+K_p)}{I} \right]$

$s = -\frac{c}{I}$, $\tau = \frac{I}{c}$

nominal time const.

$$\tau = \frac{I}{c+K_p}$$

Time const. decreases, speeds sys. up $\rightarrow$ proportional control speeds up 1st order sys

Look at error:

$$\frac{E(s)}{\Omega_r(s)} = \frac{1}{1 + \frac{K_p}{Is+c}} = \frac{Is + c}{Is + c + K_p}$$

$T(s) \cdot I(s)$ $\quad 1/s$

$$E_{ss} = E(\infty) = \lim_{s\to 0}\left[s \, \frac{E(s)}{\Omega_r(s)} \cdot \Omega_r(s) \right]$$

$$= \lim_{s\to 0}\left[\cancel{s} \cdot \frac{\cancel{Is}^{\,0} + c}{{}_0\cancel{Is} + c + K_p} \cdot \frac{1}{\cancel{s}} \right]$$

$$\rightarrow E(\infty) = \frac{c}{c + K_p}$$

reduces SS error

e.g. $\Omega_r(s) = 3 \cdot \frac{1}{s}$ step of mag. 3

FIND DISTURBANCE REJECTION:

$$\frac{E(s)}{\tau_d(s)} = \frac{\frac{1}{Is+C}}{1 + \frac{K_p}{Is+C}} = \frac{1}{Is+C+K_p}$$

e.g. $\tau_d = 1(t)$

$$E(\infty) = \lim_{s\to 0}\left[\cancel{s} \cdot \frac{1}{I\cancel{s}+C+K_p} \cdot \frac{1}{\cancel{s}}\right]$$

$$E(\infty) = \frac{1}{K_p+C}$$

← still making error smaller

can't always crank up K_p though (vibrations from sensor noise, higher order)