Combustion

Thrust $J = J\left(r, p_0, A^*, \frac{A_e}{A^*}\right)$

Exit velocity $u_e = u_e(T_0)$

velocity increment $\Delta u = \Delta u\,(u_e, R)$

→ combustion products depend on T_0!

Objective: Determine what T_0 combustion process delivers

1. Foundations (back to Ch. 2)

Mixtures of gasses

In comb. process, reactants & products consist of gaseous mixtures of chemical compounds

Treat them using Dalton's law

i) Assume each constituent occupies the entire volume available to the mixture, @ the mixture's temperature,
⇒ each component will have its own partial pressure

ii) Total pressure is sum of partial pressures
" int. energy is sum of internal energies.

Subscripts $\begin{cases} 1, 2, \ldots, n & \text{for constituents} \\ m & \text{" mixture} \end{cases}$

Mass: $M_m = M_1 + M_2 + \ldots + M_n$

mass fractions: $\frac{M_i}{M_m} = Mf_i$

Temperature: $T_m = T_1 = T_2 = T_n$

Pressure: $P_m = P_1 + P_2 + \ldots + P_n$

volume: $\forall_m = \forall_1 = \forall_2 = \forall_n$

$$M_m \underset{\text{sp. vol}}{v_m} = M_1 V_1 = M_2 v_2 = M_n v_n$$

For each gas, $P_i v_i = \dfrac{\bar{R}}{\bar{M}_i} T_i = \dfrac{\bar{R}}{\bar{M}_i} T_m$

$$v_i = \frac{\forall_i}{M_i} = \frac{\forall_m}{M_i}$$

$$\rightarrow P_i = \frac{1}{v_i} \frac{\bar{R}}{\bar{M}_i} T_i = \frac{M_i}{\forall_m} \frac{\bar{R}}{\bar{M}_i} T_m = \underbrace{\frac{M_i}{\bar{M}_i}}_{n_i = \#\text{ of moles}} \bar{R} \frac{T_m}{\forall_m}$$

Then $\dfrac{P_i}{P_m} = \dfrac{P_i}{\Sigma P_i} = \dfrac{n_i}{\underbrace{n_1 + n_2 + \ldots + n_n}_{x_i = \text{mole fraction}}} \dfrac{\cancel{\bar{R}}\, \cancel{T_m}}{\cancel{\forall_m}} \Big/ \dfrac{\cancel{\bar{R}}\, \cancel{T_m}}{\cancel{\forall_m}}$

$$\rightarrow \frac{P_i}{P_m} = x_i$$

Molecular mass of mixture

$$\bar{M}_m = \sum_{i=1}^{n} x_i \bar{M}_i$$

Specific heats

in thermo, use $\left| \begin{array}{l} \left.\dfrac{de}{dT}\right|_v = c_v \\ \left.\dfrac{dh}{dT}\right|_p = c_p \end{array} \right.$ w/ $\left\{ \begin{array}{l} [e] = [h] = \dfrac{J}{kg} \\ [c_v] = [c_p] = \dfrac{J}{kg\text{-}K} \end{array} \right.$

In combustion, use $\left\{ \begin{array}{l} \left.\dfrac{d\bar{e}}{dT}\right|_v = \bar{c}_v \\ \left.\dfrac{d\bar{h}}{dT}\right|_p = \bar{c}_p \end{array} \right.$ w/ $\left\{ \begin{array}{l} [\bar{e}] = [\bar{h}] = \dfrac{J}{kmol} \\ [\bar{c}_v] = [\bar{c}_p] = \dfrac{J}{kmol\text{-}K} \end{array} \right.$

Safest to use $[\bar{M}] = kg/kmol$

$$\bar{R} = 8314 \ \frac{J}{kmol\text{-}K}$$

To find $\bar{c}_{v_m}$ & $\bar{c}_{p_m}$, start from

$$E_m = E_1 + E_2 + \ldots + E_n$$

$$m_m e_m = m_1 e_1 + m_2 e_2 + \ldots + m_n e_n$$

→ spec. int. energy of mixture $e_m = \dfrac{m_1 e_1 + m_2 e_2 + \ldots + m_n e_n}{m_m}$

$$\left.\frac{de_m}{dT}\right|_v \equiv c_{v_m} = \frac{m_1 c_{v_1} + m_2 c_{v_2} + \ldots + m_n c_{v_n}}{m_m} = \sum_{i=1}^{n} \frac{m_i}{m_m} c_{v_i} = \boxed{\sum_{i=1}^{n} mf_i \, c_{v_i} = c_{v_m}}$$

Similarly,

$$\boxed{c_{p_m} = \sum_{i=1}^{n} mf_i \, c_{p_i} \ , \quad \bar{c}_{v_m} = \sum_{i=1}^{n} x_i \bar{c}_{v_i} \ , \quad \bar{c}_{p_m} = \sum_{i=1}^{n} x_i \bar{c}_{p_i}}$$

then $\gamma_m = \dfrac{c_{p_m}}{c_{v_m}} = \dfrac{\bar{c}_{p_m}}{\bar{c}_{v_m}}$

<u>Problem statement</u>

Given $\begin{cases} \text{mixture of reactants} \\ \text{Reaction conditions (injection temp, pressure)} \end{cases}$

Find $\begin{cases} \text{chemical composition of products} \\ \text{Temperature} \quad \text{"} \quad \text{"} \end{cases}$

<u>Challenge:</u>

composition of products affects temp. of products we'll use $\left\{\begin{array}{l}\text{cons. energy} \\ \text{chemical equilibrium}\end{array}\right.$