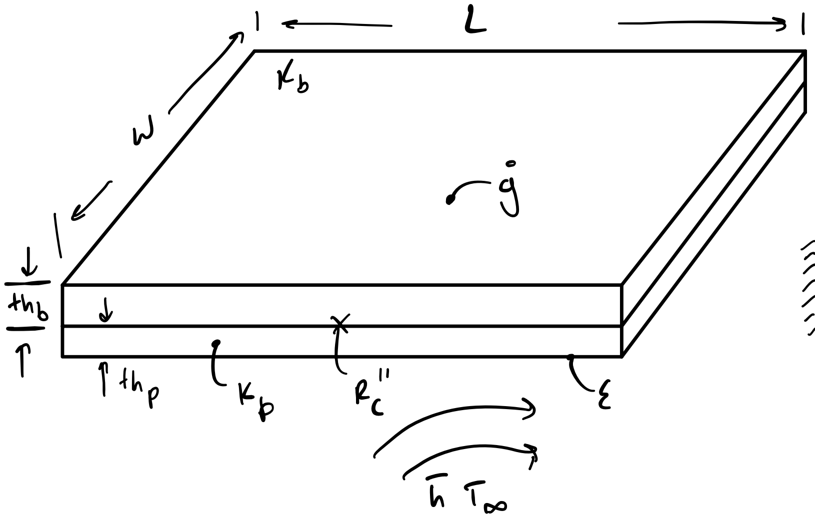

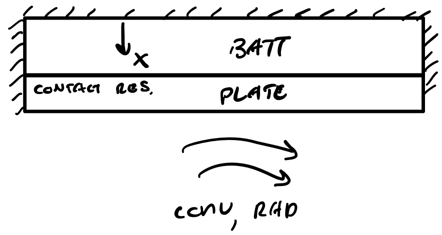

$T_{max} = 100$ [C] $\quad \dot{g} = 35$ [W] $\quad W = 10$ [cm], $L = 10$ [cm], $th_b = 7$ [mm]

$th_p = 5$ [mm]

$R_c'' = 2 \times 10^{-3}$ (K-m²/W) $\quad k_b = k_p = 2$ [W/m·K] $\quad T_\infty = 15$ [C]

$\bar{h} = 100$ (W/m²·K] $\quad \varepsilon = 1$

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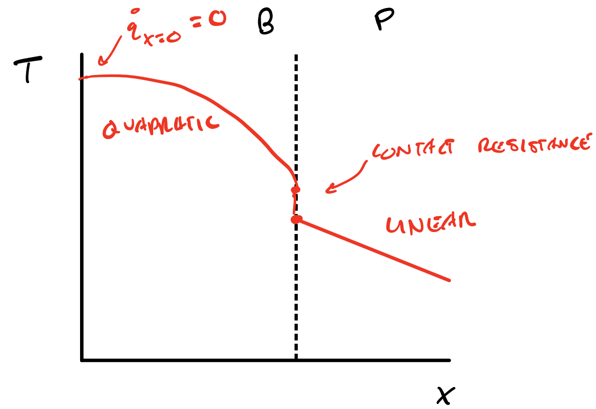
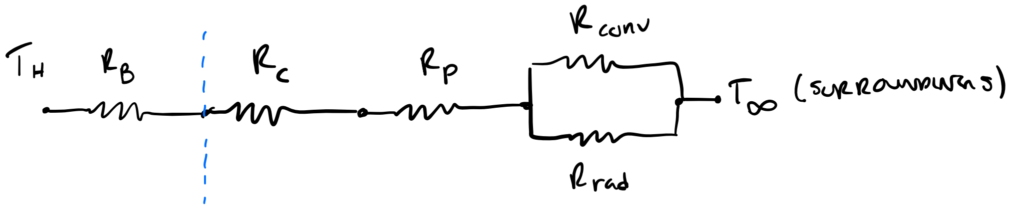
a) TEMP PROFILE



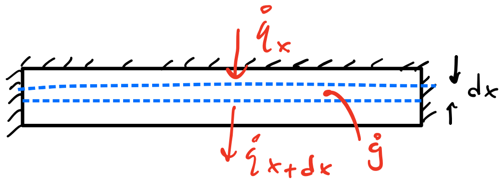
b) RESISTANCE BETWEEN $x = th_b$ & SURR



$$R_{total} = R_p + R_c + \left(\frac{1}{R_{conv}} + \frac{1}{R_{rad}}\right)^{-1}$$

$R_p = \frac{th}{kA_c}$, $R_c = \frac{R_c''}{A_s}$, $R_{conv} = \frac{1}{hA_s}$, $R_{rad} = \frac{1}{\sigma \varepsilon A_s (T_H^2+T_c^2)(T_H+T_c)}$

d) $x = 0 \rightarrow th_b$ FIND ODE



IN + GEN = OUT + ~~STORED~~

$$\cancel{\dot q_x} + \dot g = \cancel{\dot q_x} + \frac{d\dot q_x}{dx}dx$$

$$\dot g''' A_c \cancel{dx} = \frac{d\dot q_x}{dx}\cancel{dx} \qquad \dot q = -kA_c \frac{dT}{dx}$$

$$\dot g''' \cancel{A_c} = \frac{d}{dx}\left(-k\cancel{A_c}\frac{dT}{dx}\right) \rightarrow$$

d) $$\boxed{-\frac{\dot g'''}{k} = \frac{d^2T}{dx^2}}$$

$$\rightarrow \int \frac{d}{\cancel{dx}}\left(\frac{dT}{dx}\right)\cancel{dx} = \int -\frac{\dot g'''}{k}\,dx$$

$$\frac{dT}{dx} = -\frac{\dot g'''}{k}x + C_1$$

e) $$\rightarrow \boxed{T(x) = -\frac{\dot g'''}{2k}x^2 + C_1 x + C_2}$$ } WITHIN BATT

f) BC1) $\left.\frac{dT}{dx}\right|_{x=0} = 0$

$$\therefore \frac{dT}{dx} = 0 = -\frac{\dot g'''}{k}(0) + C_1 \rightarrow \boxed{C_1 = 0}$$

BC2) $\dot q_{cond} = \dot q_{rad} + \dot q_{conv}$

$$R_{eq} = \left(\frac{1}{R_{rad}} + \frac{1}{R_{conv}}\right)^{-1}$$

BATT

PLATE

$\rightarrow$ EES

$$\dot{q}_{\substack{\text{THRU}\\ \text{PLATE}}} = \dot{q}_{cond} \text{ @ BATT EDGE}$$

$$\left[\frac{T_I - T_S}{R_{eq}} = -k A_c \frac{dT}{dx}\right|_{x=th_b}$$

$\uparrow$ $R_{eq} = R_{TOTAL}$

PLATE: $\frac{d\dot{q}}{dx} = 0 \qquad \dot{q} = -k A_c \frac{dT}{dx}$

```
$UNITSYS SI K Pa J mass rad

// ME364 HW02 Kyle Adler

// Problem 1

// b)

// givens
T_max = converttemp('C','K',100[C])          // given tmax
g_dot = 35 [W]                               // given total generation
w = 10[cm]*convert(cm,m)
L = 10[cm]*convert(cm,m)
th_b = 7[mm]*convert(mm,m)
th_p = 5[mm]*convert(mm,m)
R_dprime_c = 2e-3 [K-m^2/W]
k_p = 2 [W/m-K]
k_b = k_p
T_infinity = converttemp('C','K',15[C])
h_bar = 100 [W/m^2-K]
epsilon = 1
A_c = w*L
A_s = A_c

T_S = T_infinity + 15[K]                     // initial guess, rule of thumb to guess around 15K above surroundings
T_C = T_infinity

// thermal resistances
R_p = th_p/(k_p*A_c)
R_c = R_dprime_c/A_s
R_conv = 1/(h_bar*A_s)
R_rad = 1/(sigma#*epsilon*A_s*(T_S^2+T_C^2)*(T_S+T_C))

R_total = R_p + R_c + (1/R_conv + 1/R_rad)^(-1)

// c) The radiation resistance has a very small contribution that can be neglected, since it is in parallel with a small resistance of convection. This can be shown by leaving it out and noting the difference:

R_total_norad_ratio = (R_p + R_c + (1/R_conv)^(-1))/R_total                // results in 4% change

// The other resistances have a significant contribution.

// f) solve for unknown constants of integration
g_dot_tprime = g_dot/(w*L*(th_p+th_b))
x = th_b                                     // x at edge of battery

// BC2 equation
q_dot_cond = -k_b*A_c*(-g_dot_tprime/k_b*x)
q_dot_cond = (T_interface - T_S)/R_total     // equate q_dot_cond with T_I-T_S/Req to get T_interface, q_dot_cond at interface is equal to q_dot in plate

T_interface = -g_dot_tprime/(2*k_b)*x^2+C_2 // use T_interface in temp equation at x = th_b to get C_2

// g) plot temp with x
//T_x = -g_dot_tprime/(2*k_b)*x_var^2+C_2

"Yes, the temperature remains below the maximum limit of T_max."
```

// h) multiple batteries stacked

T_wall = C_2

*"Comment out th_b and inspect table of T_wall vs th_b to find T_max, resulting in th_b_max = 0.035 = 5*th_b. 5 batteries can be stacked before overheating occurs."*

SOLUTION

Unit Settings: SI K Pa J mass rad

A_c = 0.01 [m^2]	A_s = 0.01 [m^2]
C_2 = 335.2 [K]	ε = 1 [-]
$\dot{g}$ = 35 [W]	$\dot{g}'''$ = 291667 [W/m^3]
$\bar{h}$ = 100 [W/m^2-K]	k_b = 2 [W/m-K]
k_p = 2 [W/m-K]	L = 0.1 [m]
$\dot{q}_{cond}$ = 20.42 [W]	R_c = 0.2 [K/W]
R_{conv} = 1 [K/W]	R'_c = 0.002 [K-m^2/W]
R_p = 0.25 [K/W]	R_{rad} = 17.05 [K/W]
R_{total} = 1.395 [K/W]	$R_{total,norad,ratio}$ = 1.04 [-]
th_b = 0.007 [m]	th_p = 0.005 [m]
T_C = 288.2 [K]	T_∞ = 288.2 [K]
$T_{interface}$ = 331.6 [K]	T_{max} = 373.2 [K]
T_S = 303.2 [K]	T_{wall} = 335.2 [K]
w = 0.1 [m]	x = 0.007 [m]

No unit problems were detected.

KEY VARIABLES

R_{total} = 1.395 [K/W]	*b) total thermal resistance from edge of battery*
R_c = 0.2 [K/W]	*c)*
R_{rad} = 17.05 [K/W]	*c)*
R_{conv} = 1 [K/W]	*c) convection resistance*
R_p = 0.25 [K/W]	*c)*
$R_{total,norad,ratio}$ = 1.04 [-]	*c) ratio of R_total without radiation to R_total, 4% difference*
C_2 = 335.2 [K]	*f) 2nd constant of integration, C_1 found by hand*

Parametric Table: T_wall vs th_b

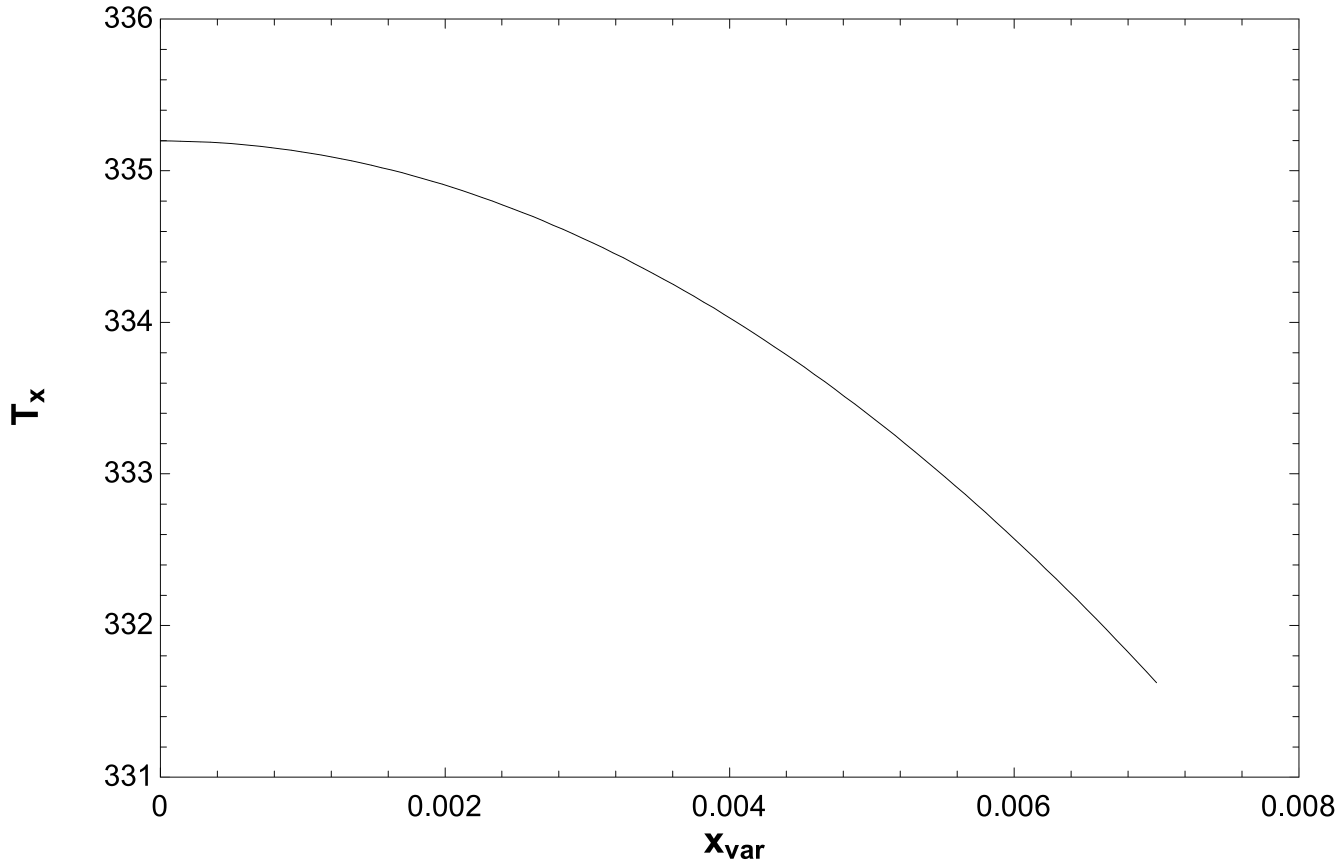
	T_{wall} [K]	th_b [m]
Run 1	*335.2*	0.007
Run 2	*336.7*	0.007636
Run 3	*338.1*	0.008273
Run 4	*339.4*	0.008909
Run 5	*340.7*	0.009545
Run 6	*341.9*	0.01018
Run 7	*343*	0.01082
Run 8	*344.1*	0.01145
Run 9	*345.2*	0.01209
Run 10	*346.2*	0.01273
Run 11	*347.2*	0.01336

Parametric Table: T_wall vs th_b

	T_{wall} [K]	th_b [m]
Run 12	348.1	0.014
Run 13	349.1	0.01464
Run 14	350	0.01527
Run 15	350.9	0.01591
Run 16	351.8	0.01655
Run 17	352.6	0.01718
Run 18	353.4	0.01782
Run 19	354.3	0.01845
Run 20	355.1	0.01909
Run 21	355.9	0.01973
Run 22	356.6	0.02036
Run 23	357.4	0.021
Run 24	358.2	0.02164
Run 25	358.9	0.02227
Run 26	359.7	0.02291
Run 27	360.4	0.02355
Run 28	361.1	0.02418
Run 29	361.9	0.02482
Run 30	362.6	0.02545
Run 31	363.3	0.02609
Run 32	364	0.02673
Run 33	364.7	0.02736
Run 34	365.4	0.028
Run 35	366	0.02864
Run 36	366.7	0.02927
Run 37	367.4	0.02991
Run 38	368.1	0.03055
Run 39	368.7	0.03118
Run 40	369.4	0.03182
Run 41	370.1	0.03245
Run 42	370.7	0.03309
Run 43	371.4	0.03373
Run 44	372	0.03436
Run 45	372.7	0.035
Run 46	373.3	0.03564
Run 47	373.9	0.03627
Run 48	374.6	0.03691
Run 49	375.2	0.03755
Run 50	375.8	0.03818
Run 51	376.5	0.03882
Run 52	377.1	0.03945
Run 53	377.7	0.04009
Run 54	378.4	0.04073
Run 55	379	0.04136
Run 56	379.6	0.042
Run 57	380.2	0.04264
Run 58	380.8	0.04327
Run 59	381.5	0.04391
Run 60	382.1	0.04455
Run 61	382.7	0.04518
Run 62	383.3	0.04582
Run 63	383.9	0.04645

Parametric Table: T_wall vs th_b

	T_{wall} [K]	th_b [m]
Run 64	*384.5*	0.04709
Run 65	*385.1*	0.04773
Run 66	*385.7*	0.04836
Run 67	*386.3*	0.049
Run 68	*387*	0.04964
Run 69	*387.6*	0.05027
Run 70	*388.2*	0.05091
Run 71	*388.8*	0.05155
Run 72	*389.4*	0.05218
Run 73	*390*	0.05282
Run 74	*390.6*	0.05345
Run 75	*391.2*	0.05409
Run 76	*391.8*	0.05473
Run 77	*392.3*	0.05536
Run 78	*392.9*	0.056
Run 79	*393.5*	0.05664
Run 80	*394.1*	0.05727
Run 81	*394.7*	0.05791
Run 82	*395.3*	0.05855
Run 83	*395.9*	0.05918
Run 84	*396.5*	0.05982
Run 85	*397.1*	0.06045
Run 86	*397.7*	0.06109
Run 87	*398.3*	0.06173
Run 88	*398.9*	0.06236
Run 89	*399.4*	0.063
Run 90	*400*	0.06364
Run 91	*400.6*	0.06427
Run 92	*401.2*	0.06491
Run 93	*401.8*	0.06555
Run 94	*402.4*	0.06618
Run 95	*403*	0.06682
Run 96	*403.5*	0.06745
Run 97	*404.1*	0.06809
Run 98	*404.7*	0.06873
Run 99	*405.3*	0.06936
Run 100	*405.9*	0.07


336
335
334
333
332
331
T_x
0
0.002
0.004
0.006
0.008
x_{var}