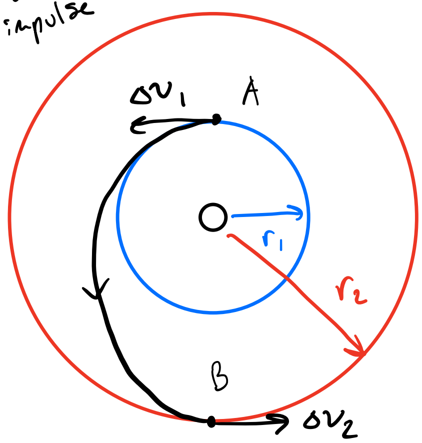
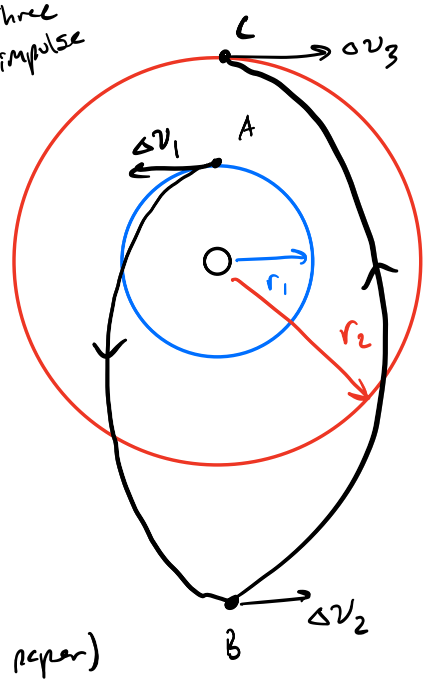
Recap: Hohmann vs. Bi-elliptic

Hohmann — Two impulse



Bi-elliptic — Three impulse

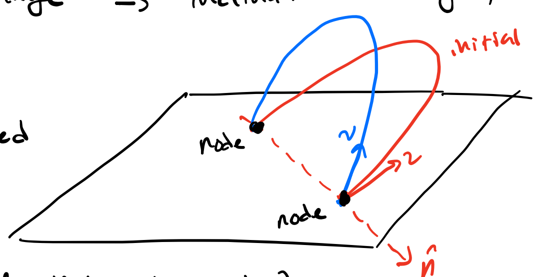


Comparison Summary (pressing paper)

- $\forall$ 2-Δv maneuvers, Hohmann is most optimal (lowest Δv)
  for all
- for $r_C/r_A \gtrsim 15.58$, Bi-elliptic has lower Δv_{tot} (if $r_B > r_A$)
  See slide 18 in L16
- the tradeoff for bi-elliptic is much higher transfer time

Next: Plane change

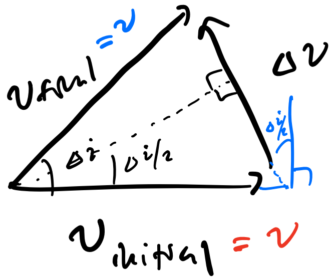
"Simple plane change" → inclination change, i



Apply plane change on line of nodes like axis of rotation

in this case

- no change in speed
- " " " γ (constant e, θ, ω)
- no change in Ω (must be at a node)



Isosceles

$\therefore \frac{\Delta v}{2} = v \sin\left(\frac{\Delta i}{2}\right)$

$$\boxed{\Delta v = 2v\sin\left(\frac{\Delta i}{2}\right)}$$

For a tangential burn ($\gamma = 0$)

Note: thrust applied at $90° + \Delta i/2$ from original dir.

For non-tangential burn ($\gamma \neq 0$) need proj. of v into θ dir.

$$\therefore \boxed{\Delta v = 2v\sin\left(\frac{\Delta i}{2}\right)\cos\gamma}$$

simple plane change for non-tangential

Remarks

1) Expensive!
if $\Delta i = 60°$, then $\Delta v = v$! same as initial v!

2) Need to check both nodes (they are not necessarily at equatorial plane)
(the speeds not necessarily the same)

→ make plane change @ low speed node

3) suppose $\Delta i = 90°$ large!
$\Delta v = 2v\sin 45° = 2v\frac{\sqrt{2}}{2} = \sqrt{2}\,v$
$= v_{esc}$

Example 1: simple plane change for circular orbit

givens: $v = 5.89$ km/s, $e = 0$, $\Delta i = 15°$

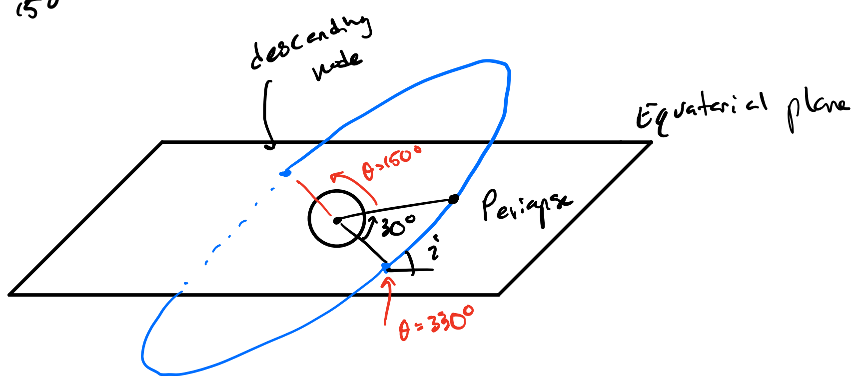
then: $e = 0$, circular $\gamma = 0$ everywhere, tangential

$\therefore \Delta v = 2v\cos\gamma\sin\frac{\Delta i}{2} = 2(5.89)\sin(7.5°)$

$$\boxed{\Delta v = 1.54 \text{ km/s}}$$

Example 2 elliptical orbit

given: $e = 0.3$ $p = 17{,}858.8$ km, $\omega = 30°$, target $\theta = 330°$ (change here)
$\Delta i = 15°$



Determine Δv:

→ need v → requires r, a, elliptical equations & γ

$$\therefore \quad p = a(1-e^2) \rightarrow a = \frac{p}{1-e^2} = \frac{17{,}858.8 \text{ km}}{1-0.3^2} = 19625 \text{ km}$$

$$\therefore \quad r = \frac{p}{1+e\cos\theta} = \frac{17858.8}{1+0.3\cos(330°)} = 14{,}175.8 \text{ km}$$

around earth:

$$v = \sqrt{\mu\left(\frac{2}{r}-\frac{1}{a}\right)} = 5.99 \text{ km/s} \quad \text{vis viva}$$

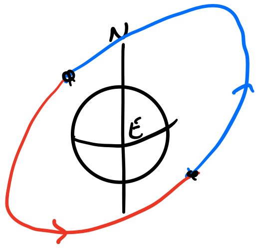
$$\gamma = \tan^{-1}\left(\frac{e\sin\theta}{1+e\cos\theta}\right) = -0.118 \text{ rad} = -6.79°$$

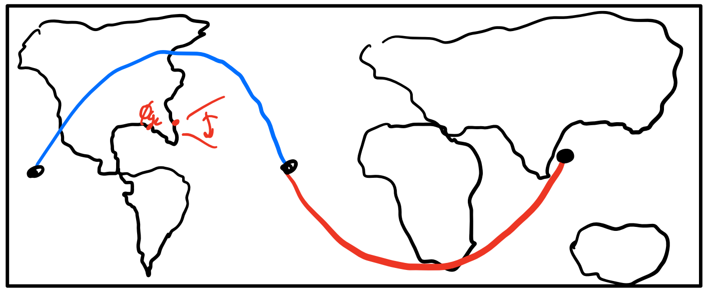
Finally: $\Delta v = 2v\sin\left(\frac{\Delta i}{2}\right)\cos(\gamma) = 2(5.99)\sin\left(\frac{15°}{2}\right)\cos(-6.79°)$

$$= \boxed{1.55 \text{ km/s}}$$

Note try descending node instead (farther from perigee i.e. slower)
→ $\Delta v = 0.912$ km/s (better @ $\theta = 150°$)

Motivation: complication: earth spins
"Ground tracks" → observer's view of orbit path (ECEF)
→ Track path from node to node





ΔN° = angle ground track traverses for one period

timing related to Earth spin period

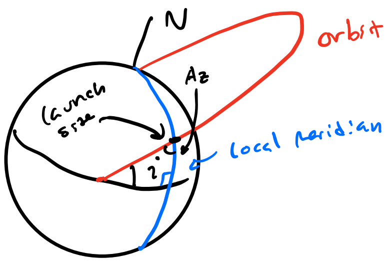
$$T_E = 24 \text{ hours}, \quad \omega_E = \frac{360^\circ}{24\,hr} = 15^\circ/hr$$

$$\therefore \text{ orbital period } T = \frac{\text{Node disp one orbit}}{\omega_E} = \frac{360^\circ - \Delta N}{15^\circ/hr}$$

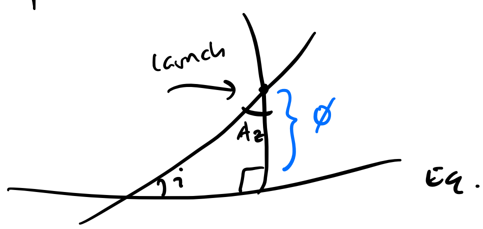
Ground tracks shaped by:

- inclination
- orbital period
- perigee location

<u>Implications</u> → launch geometry affects possible inclinations



Spherical triangle:



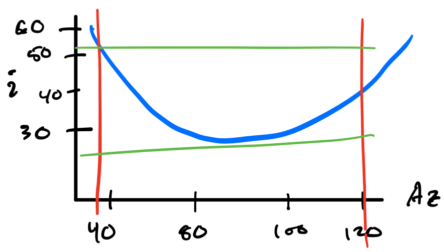
<u>triangle</u>

ϕ = latitude of launch

$$\cos i = -\cos Az \, \cancel{\cos 90^\circ}^{\,0} + \sin Az \, \cancel{\sin 90^\circ}^{\,1} \cos \phi$$

$$\rightarrow \cos i = \sin Az \cos \phi$$

plot $i = f(Az)$



Cape canaveral

$\phi_{gc} = 28.533°$

$A_z = 35° - 120°$

concerns

- airspace
- failure (debris)

→ can get orbits $i = 30° - 60°$

Remarks

- $\cos i = \sin A_z \cos \phi_{gc}$ → $\sin|A_z| \le 1$ → $i \ge \phi_{gc}$
- minimize i by maximizing A_z
- to target i & launch location, $A_z = \sin^{-1}\left(\frac{\cos i}{\cos \phi_{gc}}\right)$

Example 3 in US, want $i = 90°$

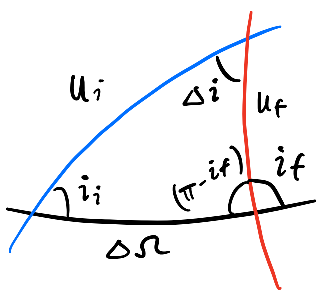
USAF East $\phi_{gc} = 28.533°$

USAF West $\phi_{gc} = 34.669°$

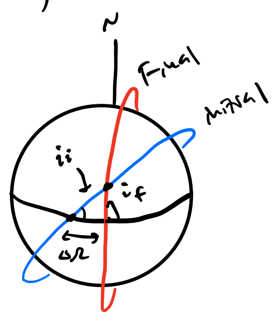
$A_z = \sin^{-1}\left(\cancelto{0}{\frac{\cos(90°)}{\cos(\phi_{gc})}}\right)$

Example 4 suppose after launch, need plane change

General rotations (change both i & Ω)



let $u_i = \omega + \theta$



Set up & Solve