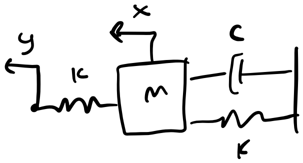
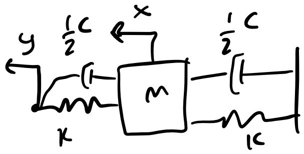
Effects of zeros & addit'l poles

→ 2 real poles

→ RHS has real zero





$$m\ddot{x} + c\dot{x} + 2Kx = Ky \qquad m\ddot{x} + c\dot{x} + 2Kx = Ky + \tfrac{1}{2}c\dot{y}$$

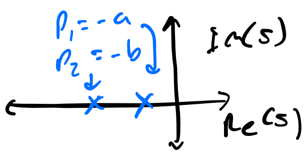
$$\frac{X(s)}{Y(s)} = \frac{K}{ms^2 + cs + 2K} \leftarrow D(s) \qquad \frac{X(s)}{Y(s)} = \frac{\frac{1}{2}cs + K}{ms^2 + cs + 2K}$$

← real zero $s = -\frac{2K}{c}$

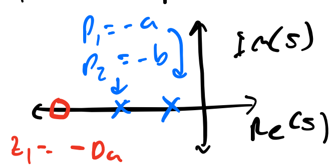
- Poles are roots of ∆s, denom: qualitatively determine response
- Zeros are roots of numerator: can affect the transient response

Step response of 2nd order w/ 2 real poles, effect of adding real zero

Two real poles:



Two real poles + real zero



$$\frac{X(s)}{F(s)} = \frac{ab}{(s+a)(s+b)} \qquad \frac{X(s)}{F(s)} = \frac{\left(\frac{b}{D}\right)(s + Da)}{(s+a)(s+b)}$$

$$\ddot{x} + (a+b)\dot{x} + (ab)x = (ab)f \qquad \ddot{x} + (a+b)\dot{x} + (ab)x = (ab)f + \frac{b}{D}\dot{f}$$

UNIT STEP: LAPLACE

$$x(s) = \frac{ab}{s(s+a)(s+b)} \qquad x(s) = \frac{\frac{b}{D}(s+Da)}{s(s+a)(s+b)}$$

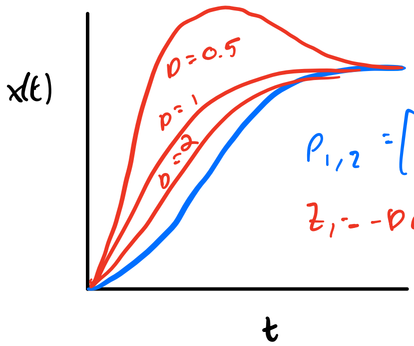
↓

PARTIAL FRACTIONS, DIFF. CONSTANTS FOR EACH

$$x(t) = 1 - \left(\frac{b}{b-a}\right)e^{-at} + \left(\frac{a}{b-a}\right)e^{-bt} \quad \Bigg| \quad x(t) = 1 - \left(\frac{b \boxed{- b/D}}{b-a}\right)e^{-at} + \left(\frac{a \boxed{- b/D}}{b-a}\right)e^{-bt}$$

NUMERATOR DYNAMICS
AFFECT TRANSIENT

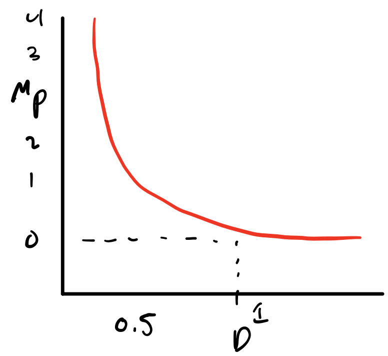
STEP RESPONSE



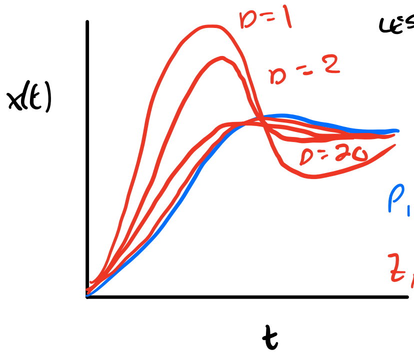
$$P_{1,2} = \begin{bmatrix} -2 \\ -4 \end{bmatrix}$$

$$z_1 = -Da = -2D$$

MAX OVERSHOOT



STEP RESPONSE



LESS DESIRABLE, NOT NECESSARILY NEEDS MORE DAMPING

$$P_{1,2} = \begin{bmatrix} -0.5 + 0.866j \\ -0.5 - 0.866j \end{bmatrix}$$

$$z_1 = -0.5D$$

WHEN ZERO IN RHP (+Re)

$$\ddot{x} + (a+b)\dot{x} + (ab)x = (ab)f + \frac{b}{D}\dot{f}$$

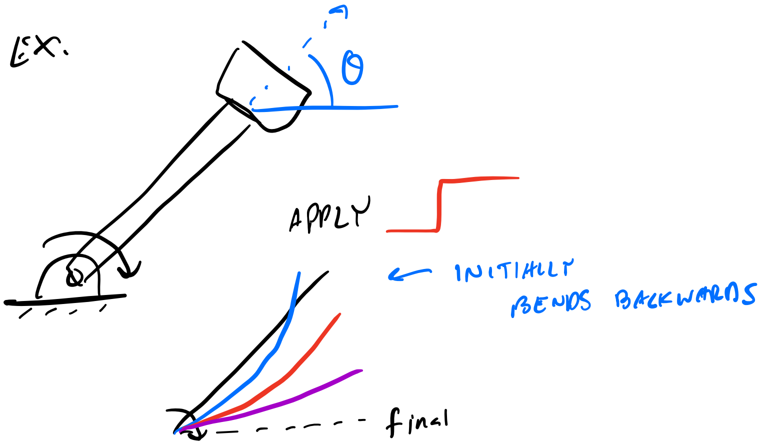
D is negative

$$\dots (ab)f - \frac{b}{D}\dot{f}$$

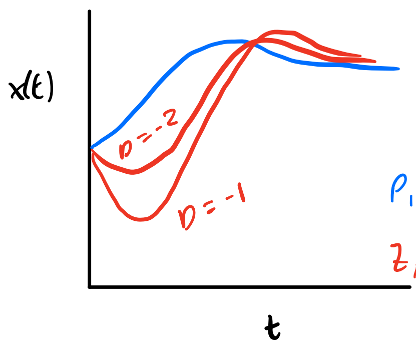
$$\hookrightarrow \frac{d}{dt}\left(1(t)\right) = \delta(t)$$

IMPULSE IN OPPOSITE DIR.
DUE TO $-D$

Ex.



STEP RESPONSE



HARD TO EXPECT THESE BEHAVIORS
→ DON'T ASSUME YOU ALWAYS JUST NEED MORE DAMPING

$$p_{1,2} = \begin{bmatrix} -0.5 + 0.866j \\ -0.5 - 0.866j \end{bmatrix}$$

$$z_1 = -0.5D$$

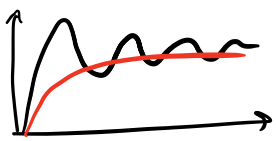
TRANSIENT RESPONSE AFFECTED BY:

- DOMINANT ROOTS OF CHAR. EQN. (DOMINANT POLES)
- HIGHER ORDER ROOTS (i.e. poles)
- NUMERATOR ROOTS (i.e. zeros)

FEEDBACK ANALYSIS

- TRACKING/REGULATION

- MODIFY SYS. DYNAMICS:
  e.g. control rocket



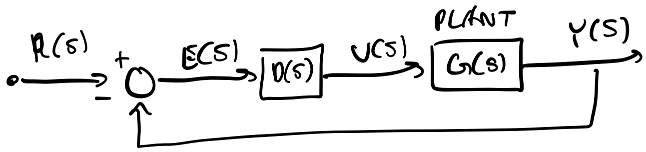
- ROBUSTNESS TO DISTURBANCES & UNCERTAINTY
- STABILIZE UNSTABLE SYSTEMS

CLOSED LOOP CONTROL

- Sensor & feedback to alter sys. behavior

PID CONTROL: Proportional - Integral - Derivative

- most common controller form
- standard to which other control laws are compared
- Developed heuristically - prior to control theory



Proportional: $u(t) = K_p e(t) \rightarrow D(s) = K_p$

Integral: $u(t) = K_I \int e(t)\,dt \rightarrow D(s) = \frac{K_I}{s}$

Derivative: $u(t) = K_d \frac{d}{dt} e(t) \rightarrow D(s) = K_D s$