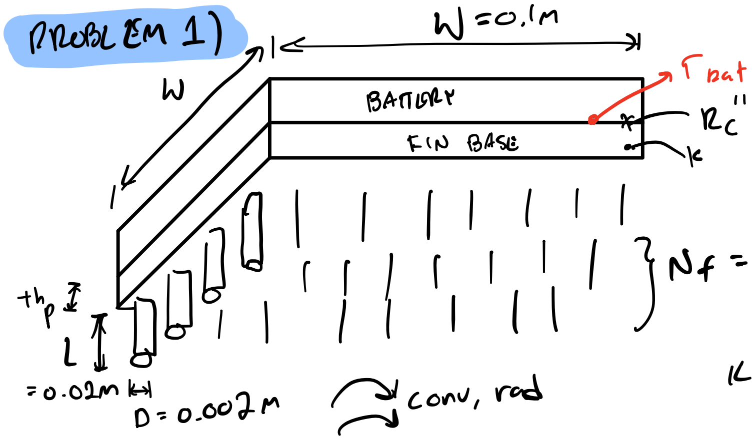
PROBLEM 1)



$\dot{q} = 35\,[W]$

$R_c'' = 2\times10^{-3}\,[K\cdot m^2/W]$

$th_p = 0.005\ m$

$T_\infty = 288.15\ K$

$\bar{h} = 100\,[W/m^2K]$ $\quad \varepsilon = 1$

$k = 2\,[W/m\text{-}K]$

1a) single fin, find Biot number & decide 1D or NOT

$$Bi = \frac{R_{cond,xverse}}{R_{surr}}, \quad R_{cond,xverse} = \frac{D/2}{k\cdot\frac{\pi}{4}D^2}$$

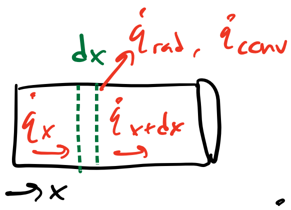
$$R_{surr} = \left[\frac{1}{R_{conv}} + \frac{1}{R_{rad}}\right]^{-1}$$

$$R_{conv} = \frac{1}{\bar{h}\,A_s}, \quad R_{rad} = \frac{1}{\sigma\varepsilon A_s 4\bar{T}^3}$$

$$A_s = \frac{\pi}{4}D^2 + \pi D L, \quad \bar{T} = 300\,K \ (\text{GUESS})$$

$\rightarrow$ EES

1b) ASSUME 1D APPROX. VALID, FIND T(x), ASSUME $\bar{T} = 300\,K$



$$IN + \cancel{GEN} = OUT + \cancel{STORED}$$

$$\dot{q}_x = \dot{q}_{x+dx} + \dot{q}_{rad} + \dot{q}_{conv} \qquad \dot{q}_x = -kA_c\frac{dT}{dx}$$

$$\dot{q}_{x+dx} = \dot{q}_x + \frac{d\dot{q}_x}{dx}dx$$

$$\hookrightarrow\ 0 = \frac{d\dot{q}_x}{dx}dx + \overbrace{\bar{h}\,dA_s(T(x)-T_\infty)}^{\dot{q}_{conv}} + \overbrace{\sigma\varepsilon\,dA_s 4\bar{T}^3(T(x)-T_\infty)}^{\dot{q}_{rad}}$$

$$0 = \frac{d\dot{q}_x}{dx}\cancel{dx} + \bar{h}(per\cdot\cancel{dx})(T(x)-T_\infty) + \sigma\varepsilon(per\cdot\cancel{dx})4\bar{T}^3(T(x)-T_\infty)$$

$$0 = \frac{d}{dx}\left[-kA_c \frac{dT}{dx}\right] + \left(\bar{h} + \sigma \varepsilon 4 \bar{T}^3\right) \cdot per \cdot \left(T(x) - T_\infty\right)$$

$$0 = -kA_c \frac{d^2T}{dx} + \quad '' \quad\quad ''$$

$$\rightarrow \quad \frac{d^2T}{dx} = \frac{\left(\bar{h} + \sigma \varepsilon 4 \bar{T}^3\right) \cdot per \cdot \left(T(x) - T_\infty\right)}{kA_c}$$

$$\boxed{\frac{d^2T(x)}{dx^2} - \frac{\left(\bar{h} + \sigma \varepsilon 4 \bar{T}^3\right) \cdot per}{kA_c} T(x) = -\frac{\left(\bar{h} + \sigma \varepsilon 4 \bar{T}^3\right) \cdot per}{kA_c} T_\infty}$$

1c) $\quad T = T_h + T_p$

HOMOGENEOUS SOLUTION:

$$\frac{d^2T(x)}{dx^2} - \frac{\left(\bar{h} + \sigma \varepsilon 4 \bar{T}^3\right) \cdot per}{kA_c} T(x) = 0$$

SOL: $\frac{d^2T_h}{dx^2} - m^2 T_h = 0 \;\therefore\; T_h = C_1 \exp(mx) + C_2 \exp(-mx)$

$$\rightarrow \quad \boxed{m = \sqrt{\frac{\left(\bar{h} + \sigma \varepsilon 4 \bar{T}^3\right) \pi D}{kA_c}}}$$

$$\therefore \quad \boxed{T_h = C_1 \exp(mx) + C_2 \exp(-mx)}$$

PARTICULAR SOLUTION:

GUESS SAME FORM AS RHS: $\quad T_p = C_3$

$\rightarrow$ PLUG IN C_3 FOR $T(x)$:

$$\cancelto{0}{\frac{d^2C_3}{dx^2}} - \frac{\cancel{\left(\bar{h} + \sigma \varepsilon 4 \bar{T}^3\right) \cdot per}}{\cancel{kA_c}} C_3 = -\frac{\cancel{\left(\bar{h} + \sigma \varepsilon 4 \bar{T}^3\right) \cdot per}}{\cancel{kA_c}} T_\infty$$

$$\therefore \quad C_3 = T_\infty = T_p$$

$$\boxed{T = T_h + T_p = C_1 \exp(mx) + C_2 \exp(-mx) + T_\infty}$$

1d) → EES

BC 1) $T(x=0) = T_0$

BC 2) $\dot{q}\big|_{x=L} = \dot{q}_{conv} + \dot{q}_{rad}$

$$-kA_c \frac{dT}{dx}\bigg|_{x=L} = \bar{h} A_c (T_s - T_\infty) + \sigma \varepsilon A_c 4 \bar{T}^3 (T_s - T_\infty)$$

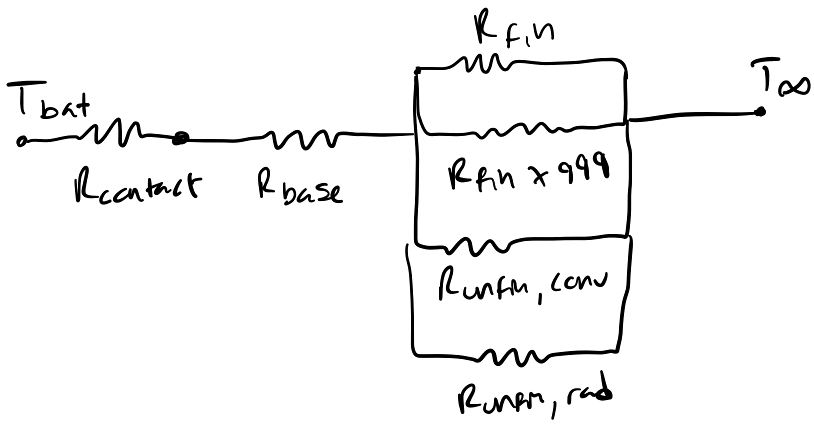
$$-k\cancel{A_c}\left[C_1 m e^{mL} - C_2 m e^{-mL}\right] = \left(\bar{h} + \sigma \varepsilon 4 \bar{T}^3\right) \cancel{A_c} (T_s - T_\infty)$$

1e)
$$\eta_f = \frac{\tanh(mL) + m \frac{A_c}{per}}{mL\left(1 + \frac{A_c}{per \cdot L}\right) \cdot \left(1 + mL \frac{A_c}{per \cdot L} \tanh(mL)\right)}$$

$$m = 2\sqrt{\frac{\bar{h} + \sigma \varepsilon 4 \bar{T}^3}{Dk}}$$

A_f = total fin area

A_c = cross section



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$UNITSYS SI K Pa J mass rad

// Problem 1)

// Givens
W = 0.1 [m]
g_dot = 35 [W]
th_p = 0.005 [m]
R_dprime_C = 2e-3 [K*m^2/W]
T_infinity = 288.15 [K]
N_f = 1000 [-]
h_bar = 100 [W/m^2-K]
epsilon = 1 [-]
D = 0.002 [m]
//L = 0.02 [m]                          // comment for part f,
k = 2 [W/m-K]

// 1a)
T_bar = 300 [K]                         // guess value above T_infinity
A_S = pi#/4*D^2 + pi#*D*L               // total surface area including tip
A_C = pi#/4*D^2                         // cross sectional area
A_S_xverse = pi#*D*L                    // surface area excluding tip

//The course textbook (5.1.1) indicates it is appropriate to use the ratio of the volume to the surface area as the "conduction length",
 and to use the total surface area as the "cross-sectional area" normal to the conduction heat flow

R_cond_xverse = (pi#*(D/2)^2*L)/(A_S_xverse)/(k*A_S_xverse)          // resistance in transverse direction to see if it is signifi
number

R_conv = 1/(h_bar*A_S)
R_rad = 1/(sigma#*epsilon*A_S*4*T_bar^3)
R_surr = (1/R_conv + 1/R_rad)^(-1)

Biot = R_cond_xverse/R_surr

// 1c)
m = sqrt((h_bar+sigma#*epsilon*4*T_bar^3)*pi#*D/(k*A_C))              // calculated m formula

// T = C_1*exp(m*x) + C_2*exp(-m*x) + T_infinity

// 1d)
T_S = 200 [K] //guess T_S
T_0 = 200 [K] //guess T_0

T_0 = C_1*exp(m*0[m]) + C_2*exp(-m*0[m]) + T_infinity                  // BC1

T_L = C_1*exp(m*L) + C_2*exp(-m*L) + T_infinity                        // T(x=L) for BC2

-k*(C_1*m*exp(m*L)-C_2*m*exp(-m*L)) = (h_bar+sigma#*epsilon*4*T_bar^3)*(T_L-T_infinity)                  // BC2

// 1e)
R_base = th_p/(k*W^2)                   // conduction resistance through plate
R_contact = R_dprime_C/W^2              // given contact resistance

per = pi#*D                             // fin perimeter
m_n = 2*sqrt( (h_bar+sigma#*epsilon*4*T_bar^3) / (D*k) )               // m_new
eta_fin = (tanh(m_n*L)+m_n*A_C/(per)) / ( m_n*L*(1+A_C/(per*L)) * (1+m_n*L*A_C/(per*L)*tanh(m_n*L))) // given
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R_fin = 1/(h_bar*A_S*eta_fin)              // formula with eta value
A_unf = W^2-N_f*A_C                        // unfinned area
R_unfinned_conv = 1/(h_bar*(A_unf))        // convection of unfinned area
R_unfinned_rad = 1/(sigma#*epsilon*A_unf*4*T_bar^3)        // radiation of unfinned area

R_total = R_base + R_contact + 1/( N_f/R_fin + 1/R_unfinned_conv + 1/R_unfinned_rad )        //series an

T_bat = g_dot*R_total + T_infinity         // rearranged q = (T_H - T_C)/R

// 1f)

rho_fin = 3000 [kg/m^3]                    // given fin density
vol_fin = A_C*L                            // fin volume
m_f = rho_fin*vol_fin

t_flight = 3600[s] - (T_bat - 300[K])*(60[s/K]) - N_f*m_f*(1800[s/kg])
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SOLUTION

Unit Settings: SI K Pa J mass rad

Maximization of t_flight(L) 48 iterations: Quadratic Approximations method (Rel. Tol=1.0E-04)

A_C = 0.000003142 [m^2]
A_S = 0.00003668 [m^2]
$A_{S,xverse}$ = 0.00003354 [m^2]
A_{unf} = 0.006858 [m^2]
Biot = 0.02902 [-]
C_1 = -1.916 [K]
C_2 = -86.23 [K]
D = 0.002 [m]
ε = 1 [-]
η_{fin} = 0.5029 [-]
$\dot{g}$ = 35 [W]
$\bar{h}$ = 100 [W/m^2-K]
k = 2 [W/m-K]
L = 0.005338 [m]
m = 325.8 [1/m]
m_f = 0.00005031 [kg]
m_n = 325.8 [1/m]
N_f = 1000 [-]
per = 0.006283 [m]
ρ_{fin} = 3000 [kg/m^3]
R_{base} = 0.25 [K/W]
$R_{cond,xverse}$ = 7.454 [K/W]
$R_{contact}$ = 0.2 [K/W]
R_{conv} = 272.6 [K/W]
R'_C = 0.002 [K*m^2/W]
R_{fin} = 542 [K/W]
R_{rad} = 4452 [K/W]
R_{surr} = 256.9 [K/W]
R_{total} = 0.8387 [K/W]
$R_{unfinned,conv}$ = 1.458 [K/W]
$R_{unfinned,rad}$ = 23.81 [K/W]
th_p = 0.005 [m]
T_0 = 200 [K]
$\bar{T}$ = 300 [K]
T_{bat} = 317.5 [K]
t_{flight} = 2459 [s]
T_∞ = 288.2 [K]
T_L = 262.1 [K]
T_S = 200 [K]
vol_{fin} = 1.677E-08 [m^3]
W = 0.1 [m]

No unit problems were detected.

KEY VARIABLES

Maximization of t_flight(L) 48 iterations: Quadratic Approximations method (Rel. Tol=1.0E-04)

Biot = 0.02902 [-] *1a) Biot number is sufficiently less than 1, indicating that the conduction resistance through the fins is low enough compared to the resistance to the surroundings, and can be neglected.*
C_1 = -1.916 [K] *1d) C1 ~= 0*
C_2 = -86.23 [K] *1d) C2*

L = 0.005338 [m] *1f) optimized length via min/max for flight time*
R_{total} = 0.8387 [K/W] *1e) total thermal resistance*
T_{bat} = 317.5 [K] *1e) battery temperature*