Real nozzles:

- Must assess how "bad" our theory is
  - How much does the actual nozzle flow depart from 1-D theory?
  - For fixed A_e/A^*, how much does the nozzle shape affect thrust / performance?
  - Is combustion really at $P_0 = \text{const}$?
  - Is the composition of combustion products (propellant) fixed or varying in nozzle?

Departure from 1-D theory

In perfect 1-D case, all quantities only depend on x

Only important variable is $\frac{A(x)}{A^*}$

since that sets $M(x)$ which sets everything else.

In real flow

- Quantities depend on at least 2 variables (e.g. $P = P(x, r)$)

  $\underline{u} = u_x \hat{i} + u_r \hat{e}_r$
- Nozzle shape is very important

Rules of thumb
- Shape of converging section of nozzle is <u>not</u> very important
- Shape of divergent " " is <u>extremely</u> important
- Try to avoid shock waves

Conical nozzle

purpose: show effects from non 1-D physics

Conical: simplest departure from 1-D

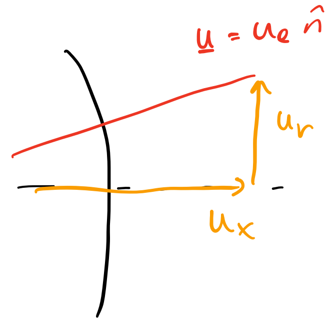
P, ρ, T = const on <u>any</u> spherical surface

$|\underline{u}|$ = " " " " "

Direction of $\underline{u}$ varies over " "

$\underline{u} \perp A_e'$

P_e acts $\perp A_e'$



Physically: If $\underline{u}$ has radial comp. u_r @ exit, wasting momentum in dir. $\perp$ to motion of rocket

Force balance in x-direction

In 1-D we had $\mathcal{J} = \dot{m} u_e + (P_e - P_a) A_e$

Here start from mom-eq

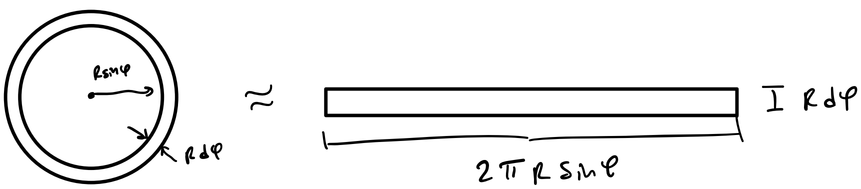
$$\mathcal{J} = \int_{cs} u_x \rho \underline{u} \cdot d\underline{A} - \int_{cs} p\, d\underline{A} \cdot \hat{i} = \underbrace{\int_{cs} u_x \rho \underline{u} \cdot dA\, \hat{n}}_{1} - \underbrace{\int_{cs} p\, dA\, \hat{n} \cdot \hat{i}}_{2}$$

About 2: only contribution from $p = P_a - P_e$ = const on A_e'

$$= -\int_{cs} p\, dA\, \hat{n} \cdot \hat{i} = (P_e - P_a) A_e$$

↑ proj. of A_e' onto plane $\perp$ to $\hat{i}$

About 1: Mass crosses CS on A_e'



$$dA = 2\pi R^2 \sin\varphi\, d\varphi$$

$\underline{u} \cdot \hat{n} = u_e \hat{n} \cdot \hat{n} = u_e$

$u_x = u_e \cos\varphi$

$$\rightarrow \quad \mathcal{J} = \int_0^\alpha \underbrace{u_e \cos\varphi}_{u_x}\, \rho_e \underbrace{u_e}_{\underline{u} \cdot \hat{n}}\, \underbrace{2\pi R^2 \sin\varphi\, d\varphi}_{dA} + (P_e - P_a) A_e$$

$$= \rho_e u_e^2\, 2\pi R^2 \underbrace{\int_0^{\alpha} \sin\varphi \cos\varphi \, d\varphi}_{= -\frac{\cos^2\varphi}{2}\Big|_0^{\alpha}} + (p_e - p_a) A_e$$

$$J = 2\pi R^2 \rho_e u_e^2 \cdot \left(\frac{1-\cos^2\alpha}{2}\right) + (p_e - p_a) A_e$$

Now relate A_e' & A_e

$$\underbrace{A_e'}_{\text{on sphere}} = \int_0^{\alpha} dA = \int_0^{\alpha} 2\pi R^2 \sin\varphi \, d\varphi = -2\pi R^2 \cos\varphi \Big|_0^{\alpha}$$
$$= 2\pi R^2 (1 - \cos\alpha)$$

$$A_e = \pi r^2 \Big|_{\varphi = \alpha} = \pi R^2 \sin^2\alpha = \pi R^2 (1 - \cos^2\alpha)$$
$$= \pi R^2 (1 - \cos\alpha)(1 + \cos\alpha)$$

$$A_e = \frac{1+\cos\alpha}{2} A_e'$$

Rewrite J

$$J = \underbrace{2\pi R^2 \rho_e u_e (1-\cos\alpha)}_{\rho_e u_e A_e' = \dot{m}} \left(\frac{1+\cos\alpha}{2}\right) u_e + (p_e - p_a)\left(\frac{1+\cos\alpha}{2}\right) A_e'$$

$$J = \frac{1+\cos\alpha}{2} \left[\dot{m} u_e + (p_e - p_a) A_e'\right]$$

But since $\begin{cases} A_e \approx A_e' \\ (p_e - p_a) A_e' << \dot{m} u_e \end{cases}$

Can approximate

$$J = \frac{1+\cos\alpha}{2} \Big[\underbrace{\dot{m} u_e + (p_e - p_a) A_e}_{J_{1\text{-}D}}\Big]$$