Problem 1

$$\frac{J}{P_0 A^*} = \underbrace{\sqrt{\frac{2\gamma^2}{\gamma-1}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}\left[1-\left(\frac{P_e}{P_0}\right)^{\frac{\gamma-1}{\gamma}}\right]}}_{\text{part 1}} + \underbrace{\left(\frac{P_e}{P_0}-\frac{P_a}{P_0}\right)\frac{A_e}{A^*}}_{\text{part 2}}$$

No shock, $1.1 \le M_e \le 10.0$

$$\frac{P}{P_0} = \left(1+\frac{\gamma-1}{2}M^2\right)^{\frac{\gamma}{1-\gamma}} \qquad (*)$$

$$\frac{A}{A_*} = \frac{1}{M}\left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M^2\right)\right]^{\frac{\gamma+1}{2(\gamma-1)}} \qquad (**)$$

Problem 2

$J = 5E6$ N, $P_0 = 7E6$ Pa, $T_0 = 2800$ K. Perfect gas, $\gamma = 1.4$

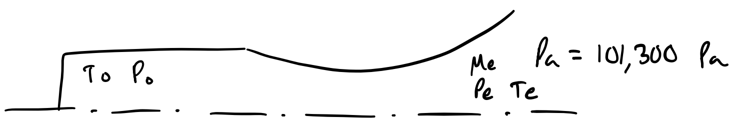
Find a) I_{sp}, b) $\dot{m}$, c) D_t, d) D_e, e) $J(30\text{ km})$

f) $J(P_0 = 21\text{ MPa})$ g) $J(H_2)$ h) $J(T_0 = 3600\text{K})$

a-d: assume $P_e = P_a$

f: $P_a = 101.3$ kPa

g-h) $P_0 = 7E6$ Pa, $P_e = P_a$



$$J = \dot{m}U_e + (\cancel{P_e - P_a})^{0} A_e$$

$$I_{sp} = \frac{J}{\dot{m}g_e} = \frac{U_{eq}}{g_e} = \frac{U_e}{g_e} \qquad U_e = M_e a_e = M_e\sqrt{\gamma R T_e}$$

$R = 287$

$\gamma = 1.4$

P_e/P_0 known, find M_e & get exit conditions

$$\frac{P}{P_0} = \left(1-\frac{\gamma-1}{2}M^2\right)^{\frac{\gamma}{1-\gamma}} \rightarrow \frac{P_e}{P_0} = \left(1-\frac{\gamma-1}{2}M_e^2\right)^{\frac{\gamma}{1-\gamma}}$$

online calc: $M_e = 3.43 \rightarrow \frac{T_e}{T_0} = 0.298 \rightarrow T_e = 834.8144$

$$\rightarrow U_e = (3.43)\sqrt{1.4 \cdot 287 \cdot 834} = 1986.97 \text{ m/s}$$

$$I_{sp} = \frac{U_e}{g_e} = \frac{1986}{9.81} = \boxed{202.5 \text{ s} = I_{sp}} \quad \text{a)}$$

$$\dot{m} = \frac{J}{U_{eq}} = \frac{5E6}{1987} = \boxed{2516.4 \text{ Kg/s} = \dot{m}} \quad \text{b)}$$

$$\frac{A_e}{A^*} = \left(\frac{D_e}{D_t}\right)^2 = 6.364 \qquad \dot{m} = \rho_e u_e A_e = \frac{p_e}{RT_e} u_e A_e \rightarrow A_e = \frac{\dot{m} R T_e}{p_e u_e}$$

$$\rightarrow A_e = 2.995 \ m^2 = \frac{\pi}{4} D_e^2 \rightarrow \boxed{D_e = 1.953 \ m} \quad c)$$

$$A^* = \frac{A_e}{6.364} = 0.4707 = \frac{\pi}{4} D_t^2 \rightarrow \boxed{D_t = 0.774 \ m} \quad d)$$

@ 30 km, $p_e \neq p_a$

$$J = \dot{m} u_e + (p_e - p_a) A_e = 5\text{E}6 + (101300 - 1172)(2.995)$$

$$\boxed{J = 5.299 \ MN} \quad e)$$

f): $P_0 = 21 \ MPa$, $p_a = 101300 \ Pa$

Same $A_e/A^* = 6.364 \rightarrow p_e/p_0 = 0.01447$,

$$J = p_0 A^* \left[\sqrt{\frac{2\gamma^2}{(\gamma-1)} \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}} \left[1 - \left(\frac{p_e}{p_0}\right)^{\frac{\gamma-1}{\gamma}}\right]} + \left(\frac{p_e}{p_0} - \frac{p_a}{p_0}\right) \frac{A_e}{A^*} \right]$$

$$\boxed{J = 15.6 MN}$$

g) Same inlet conditions & Hydrogen ($\gamma = 1.4$) $\rightarrow$ Same thrust

$$\rightarrow \boxed{J = 5 \ MN}$$

h) Thrust w/ $T_0 = 3600 \ K$

T_0 does not appear in thrust equation above,

$$\therefore \boxed{J = 5 \ MN}$$

Problem 3

Rocket thrust chamber, $\bar{m} = 25$ Kg/kmol, $\gamma = 1.2$, $p_0 = 10$ MPa, $T_0 = 3000$ K, $A_t = 0.1\ m^2$. Find thrust for scenarios 1-4.

3.1) $p_a = 0 \qquad A_e = 0.1\ m^2$

$\rightarrow$ purely convergent, $A_e = A_t = A^* \swarrow p_a < p^*$

$$J = p_0 A^* \left[\sqrt{\frac{2\gamma^2}{(\gamma-1)} \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}} \left[1 - \left(\frac{p_e}{p_0}\right)^{\frac{\gamma-1}{\gamma}}\right]} + \left(\frac{p_e}{p_0} - \cancelto{0}{\frac{p_a}{p_0}}\right) \cancelto{1}{\frac{A_e}{A^*}} \right]$$

$$\frac{p_e}{p_0} = \frac{p^*}{p_0} = 0.5645$$

$\rightarrow \boxed{J = 1.242\ MN}$

3.2) $p_a = 0, \quad A_e = 4.06\ m^2$

$\rightarrow$ C-D, sonic @ throat

$$J = p_0 A^* \left[\sqrt{\frac{2\gamma^2}{(\gamma-1)} \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}} \left[1 - \left(\frac{p_e}{p_0}\right)^{\frac{\gamma-1}{\gamma}}\right]} + \left(\frac{p_e}{p_0} - \cancelto{0}{\frac{p_a}{p_0}}\right) \frac{A_e}{A^*} \right]$$

$\frac{p_e}{p_0}$ set by $\frac{A_e}{A^*} = 40.6 \rightarrow \frac{p_e}{p_0} = 0.00205$

$\rightarrow \boxed{J = 1.886\ MN}$

3.3) $p_a = 0.101$ MPa, $A_e = 0.1\ m^2$

$$J = p_0 A^* \left[\sqrt{\frac{2\gamma^2}{(\gamma-1)} \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}} \left[1 - \left(\frac{p_e}{p_0}\right)^{\frac{\gamma-1}{\gamma}}\right]} + \left(\frac{p_e}{p_0} - \frac{p_a}{p_0}\right) \cancelto{1}{\frac{A_e}{A^*}} \right]$$

$\rightarrow \frac{p_e}{p_0} = 0.5645$

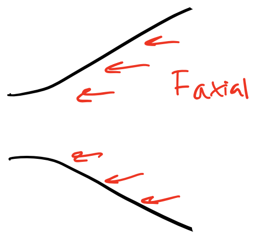
$\rightarrow \boxed{J = 1.232\ MN}$

3.4) $p_a = 0.101$ MPa, $A_e = 4.06\ m^2$

$$J = P_0 A^* \left[\sqrt{\frac{2\gamma^2}{(\gamma-1)} \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}} \left[1 - \left(\frac{P_e}{P_0}\right)^{\frac{\gamma-1}{\gamma}}\right]} + \left(\frac{P_e}{P_0} - \frac{P_a}{P_0}\right) \frac{A_e}{A^*} \right]$$

$\frac{P_e}{P_0} = 0.00205$ 40.6

$$\rightarrow \boxed{J = 1.475 \text{ MN}}$$



$$F_{axial} = F_{CD} - F_{conv\ only} = J_{case4} - J_{case3}$$

$$= 1.475 \text{ MN} - 1.232 \text{ MN} = \boxed{0.243 \text{ MN} = F_{axial}}$$

compared to overall thrust of 1.475 MN, only 0.243 or ~16.5 % of that comes from the diverging section.

Problem 4 $\bar{M} = 15$ kg/kmol, $T_0 = 3300$ K, Find A_t & A_e req'd for $J = 500$ kN & $I_{sp} = 300$ s. $P_a = 0.1$ MPa, $\gamma = 1.4$

part 1: $u_{eq} = g_e I_{sp} = u_e$ (ideal) $= 2943$ m/s

$$J = \dot{m} u_e \rightarrow \dot{m} = \frac{J}{u_e} = \frac{500{,}000}{2943} = 169.9 \text{ kg/s}$$

$$u_e = \sqrt{\frac{2\gamma \bar{R}}{(\gamma-1)\bar{M}} T_0 \left[1 - \left(\frac{P_e}{P_0}\right)^{\frac{\gamma-1}{\gamma}}\right]} \rightarrow u_e^2 \cdot \frac{(\gamma-1)\bar{M}}{2\gamma \bar{R} T_0} = 1 - \left(\frac{P_e}{P_0}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\rightarrow \left(\frac{P_e}{P_0}\right)^{\frac{\gamma-1}{\gamma}} = 1 - u_e^2 \frac{(\gamma-1)\bar{M}}{2\gamma \bar{R} T_0} \rightarrow \frac{P_e}{P_0} = \left(1 - u_e^2 \frac{(\gamma-1)\bar{M}}{2\gamma \bar{R} T_0}\right)^{\frac{\gamma}{\gamma-1}}$$

$$\rightarrow P_0 = P_e \left(1 - u_e^2 \frac{(\gamma-1)\bar{M}}{2\gamma \bar{R} T_0}\right)^{\frac{\gamma}{1-\gamma}} = \boxed{5.19 \text{ MPa} = P_0}$$

$$\dot{m} = \rho^* u^* A^* \rightarrow A^* = \frac{\dot{m}}{\frac{p^*}{RT^*} \sqrt{\gamma R T^*}}$$

$$T^* = \left(1 + \frac{\gamma-1}{2}\right)^{-1} \cdot T_0 = 2750 \text{ K}$$

$$p^* = P_0 \left(1 + \frac{\gamma-1}{2}\right)^{\frac{\gamma}{1-\gamma}} = 2.74 \text{ MPa}$$

$\rightarrow \boxed{A^* = 0.0646\ m^2}$

From $\frac{P_e}{P_0} \rightarrow M_e = 3.23 \rightarrow \frac{A_e}{A^*} = 5.285$

$\rightarrow \boxed{A_e = 0.3416\ m^2}$

Part 2:

$\rightarrow$ Find thrust if $P_a = 0.03$ MPa

$$J = P_0 A^* \left[\underbrace{\sqrt{\frac{2\gamma^2}{(\gamma-1)} \left(\frac{2}{(\gamma+1)}\right)^{\frac{\gamma+1}{\gamma-1}} \left[1 - \left(\frac{P_e}{P_0}\right)^{\frac{\gamma-1}{\gamma}}\right]}}_{\text{same}} + \left(\frac{P_e}{P_0} - \frac{P_a}{P_0}\right)\frac{A_e}{A^*} \right]$$

$$J = 500{,}000\ N + (P_e - P_a) A_e$$

$\rightarrow \boxed{J = 524\ kN}$

what about $P_e = P_a = 0.03$ MPa

$$J = P_0 A^* \left[\sqrt{\frac{2\gamma^2}{(\gamma-1)} \left(\frac{2}{(\gamma+1)}\right)^{\frac{\gamma+1}{\gamma-1}} \left[1 - \left(\frac{P_e}{P_0}\right)^{\frac{\gamma-1}{\gamma}}\right]} + \cancel{\left(\frac{P_e}{P_0} - \frac{P_a}{P_0}\right)}^{0,\ P_e = P_a} \frac{A_e}{A^*} \right]$$

$$= (5.19E6)(0.065 m^2)\sqrt{\quad \ldots}$$

$\rightarrow \boxed{J = 533\ kN}$