EMA 550 HW2 Kyle Adler

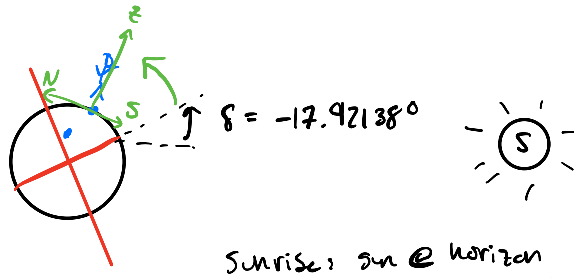
Problem 1 Calculate sunrise & sunset time in Madison, WI on Jan. 30 2012. CST, use actual declination of sun, lat & Long.

Plan: Declination → Hour angle w/ trig. → Convert time

From JPL Horizons:

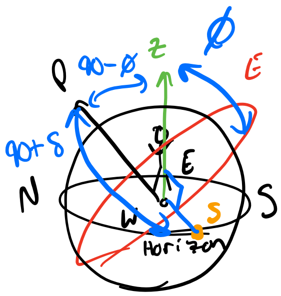
Input: Sun, Geocentric, 2012-01-30, decimal deg.

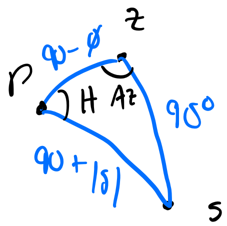
Output: DEC: $-17.92138°$



Sunrise: sun @ horizon
noon: sun due south

$\phi = 43.0722°N$ Long $89.4008°W$





→ Find H w/ spherical law of cosines:

$$\cos(90°) = \cos(90-\phi)\cos(90+|\delta|) + \sin(90-\phi)\sin(90+|\delta|)\cos H$$

$$0 = \cos(46.9278°)\cos(107.92138°) + \sin(46.9278°)\sin(107.92138°)\cos H$$

$$\rightarrow \cos H = \frac{-(0.6829)(-0.3077)}{(0.73049)(0.95147)} \rightarrow H = \operatorname{acos}(0.3023)$$

$$\rightarrow H = 72.4017° \cdot \frac{24\,hr}{360°} = 4.826\,hr$$

$$\boxed{H = 4\,hr\ 49.6\,min}$$

Shift from GMT with longitude: $89.4008^\circ \cdot \frac{24hr}{360^\circ} = 5hr\ 57.6\ min$

$\rightarrow$ Noon time $= [\text{Noon GMT} - \text{CST shift}] + H_{long}$

$= [12:00 - 6] + [5:57.6] = 11:57:36$ am

Noon time $\pm$ H :

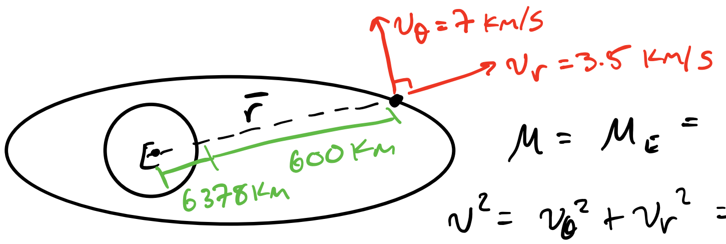
$$\boxed{\begin{array}{l}\text{Sunrise at } 7:08:00 \text{ AM} \\ \text{Sunset at } 4:47:12 \text{ PM}\end{array}}$$

Actual data from time and date.com:

Sunrise at 7:15 AM (+ 7:00)
Sunset at 5:06 PM (+19:00)

Both times earlier than actual. One possible explanation is that the data takes refraction into account, which could explain why sunset is later, but not why sunrise is later.

Problem 2 Satellite @ 600 km, $v_\theta = 7$ km/s, $v_r = 3.5$ km/s. Find e & θ at that instant. $R_E = 6378$ km



$r = 6378 + 600 = 6978$ km

$\mu = \mu_E = GM_E = 3.986 \times 10^5\ km^3/s^2$

$v^2 = v_\theta^2 + v_r^2 = (7^2 + 3.5^2) = 61.25$

vis viva: $v^2 = \mu\left(\frac{2}{r} - \frac{1}{a}\right) \rightarrow \frac{v^2}{\mu} = \frac{2}{r} - \frac{1}{a}$

$\rightarrow \frac{1}{a} = \frac{2}{r} - \frac{v^2}{\mu} \rightarrow a = \frac{1}{\frac{2}{r} - \frac{v^2}{\mu}} = \frac{1}{\frac{2}{6978} - \frac{61.25}{3.986\times10^5}}$

$\rightarrow a = 7521.497$ km

$\frac{h^2}{\mu} = a(1-e^2) \rightarrow \frac{h^2}{\mu a} = 1 - e^2 \rightarrow e = \sqrt{1 - \frac{h^2}{\mu a}}$

Need h: $|\bar{h}| = r^2\dot{\theta} = r v_\theta = (6978)(7\ km/s) = 48{,}846$

$$e = \sqrt{1 - \frac{(48{,}846)^2}{(3.986e5)(7521.5)}} = \boxed{0.45186 = e}$$

trajectory: $r(\theta) = \dfrac{a(1-e^2)}{1 - e\cos\theta} \rightarrow 1 - e\cos\theta = \dfrac{a(1-e^2)}{r}$

$$\rightarrow \cos\theta = \left[1 - \frac{a(1-e^2)}{r}\right] / e$$

$$\rightarrow \theta = a\cos\left(\left[1 - \frac{7521(1-0.45^2)}{6978}\right] / 0.45\right)$$

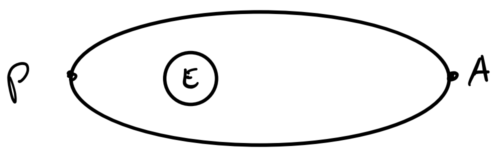
$$= a\cos(0.314) = \boxed{71.658^\circ = \theta}$$

Problem 3 Satellite, $T = 205$ min, $e = 0.4$ about Earth.

3a) At $\theta = 60^\circ$ find time τ since perigee

plan: $e, \theta \rightarrow E \rightarrow M \rightarrow t$

Eccentric anomaly:



$$\cos E = \frac{e + \cos\theta}{1 + e\cos\theta} = \frac{0.4 + 0.5}{1 + 0.4 \cdot 0.5}$$

$$\rightarrow E = a\cos(.75) = 41.41^\circ = 0.722 \text{ rad}$$

Mean anomaly:

$$M = E - e\sin E = 0.722 - 0.4 \cdot 0.661 = 0.4581 \text{ rad} = M$$

$$M = \frac{2\pi}{T}\tau \rightarrow \tau = \frac{TM}{2\pi} = \frac{205 \cdot 0.4581}{2\pi} = \boxed{14.95 \text{ min} = \tau}$$

3b) Find true anomaly θ at 45 mins past perigee

$\rightarrow$ Same thing but backwards.

$$M = \frac{2\pi}{205}(45) = 1.3792 \text{ rad}$$

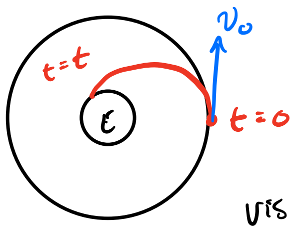
$M = 1.3792 = E - 0.4\sin E$ $\rightarrow$ iteratively solve in MATLAB fsolve:

$$E = 1.771228$$

$$\tan\frac{\theta}{2} = \left(\frac{1+e}{1-e}\right)^{1/2}\tan\left(\frac{E}{2}\right) = \sqrt{\frac{1.4}{0.6}}\,(1.22358) = 1.86906$$

$$\to \theta = 2.159 \text{ rad} = \boxed{123.7^\circ = \theta \quad \text{after } 45 \text{ mins}}$$

Problem 4 Spaceship, 200 km circ. orbit about earth. $t=0$ engine fires, reducing velocity by 600 m/s. Find time in mins to impact.



initially: $e = 0 \to r = a = 6378 + 200 = 6578$ km

vis viva circular: $v_0 = \sqrt{\frac{\mu}{a}} = 7.78$ km/s

$v = v_0 - 600 \text{ m/s} = 7.184$ km/s

vis viva: $a = \dfrac{1}{\frac{2}{r} - \frac{v^2}{\mu}} = \dfrac{1}{\frac{2}{6578} - \frac{7.184^2}{3.986E5}}$

$\to a = 5728.9$ km

$e = \sqrt{1 - \frac{h^2}{\mu a}}$, $h = rv = (6578)(7.184) = 47256.35$

$\to e = \sqrt{1 - \dfrac{(47256.35)^2}{3.986E5 \cdot 5728.9}} = 0.1485 = e$

$R_p = a(1-e) = 4878$ km $\to$ impact before periapsis

Find θ @ impact when $r = R_E$

trajectory: $\cos\theta = \left[1 - \dfrac{a(1-e^2)}{r}\right] / e$

$\to \theta = \text{acos}\left[1 - \dfrac{5728(1-.1485^2)}{6378}\right] / .1485 = 0.61177 \text{ rad} = \theta$

Eccentric anomaly:

$\cos E = \dfrac{e + \cos\theta}{1 + e\cos\theta} = \dfrac{.1485 + \cos(.61172)}{1 + .1485\cos(.61172)} \Rightarrow 0.531 = E$

$M = E - e\sin E = 0.4558$

$M = 0.4558 = \frac{2\pi}{T} t$ $\qquad T = 2\pi \left(\frac{a^3}{\mu}\right)^{1/2} = 4315\ s = 71.923\ \text{mins}$

$\rightarrow t = \frac{MT}{2\pi} = \boxed{5.217\ \text{mins} = t_{impact}}$

Problem 5 molnija orbit, $A_p = 500\text{km}$, $T = 12\text{hrs}$

5a) Find R_A & e

$R_p = 6378\ \text{Km} + 500\ \text{Km} = 6878\ \text{Km}$

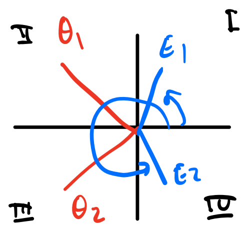
$T = 2\pi \left(\frac{a^3}{\mu}\right)^{1/2} \qquad \left(\frac{T}{2\pi}\right)^2 \mu = a^3 = \left(\frac{12 \cdot 3600}{2\pi}\right)^2 3.986E5$

$\rightarrow a = 26{,}610.21\ \text{Km}$

$R_p = a(1-e) \rightarrow e = 1 - \frac{R_p}{a} = 1 - \frac{6878}{26610} = \boxed{0.7415 = e}$

$\rightarrow R_A = a(1+e) = 26{,}610(1 + .7415) = \boxed{46{,}342.4\ \text{Km} = R_A}$

5b) time between $\theta = 135°$ & $\theta = 225°$



Eccentric anomaly:

$\cos E_1 = \frac{e + \cos\theta_1}{1 + e\cos\theta_1} \rightarrow E_1 = 1.498\ \text{rad}$ $\approx 86°$

$\cos E_2 = \frac{e + \cos\theta_2}{1 + e\cos\theta_2} \rightarrow E_2 = 2\pi - 1.498 = 4.7848\ \text{rad}$

Kepler: $M_1 = E_1 - e\sin E_1 = 2.238\ \text{rad}$

$M_2 = E_2 - e\sin E_2 = 4.045\ \text{rad}$

$\Delta t = t_2 - t_1 = \frac{T}{2\pi}(M_2 - M_1) = \frac{12\text{hrs}}{2\pi}(1.807)$

$\boxed{= 3.4516\ \text{hrs between } \theta = 135° \text{ \& } \theta = 225°}$