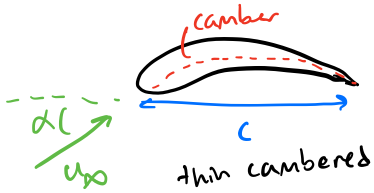
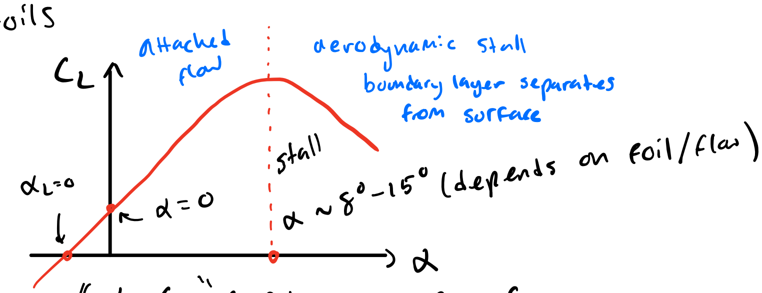
Potential flow over airfoils

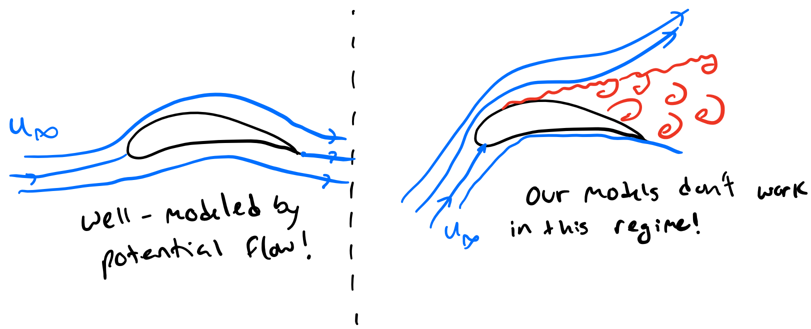


thin cambered airfoil



As $\alpha \uparrow$, $C_L \uparrow$, until aerodyn. stall

"$\alpha - C_L$" curve specific for airfoil shape, Re, M, experimental? simulation? theory?



Techniques for flow models:

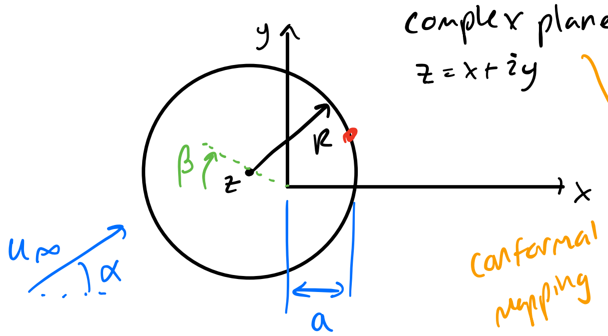
1. Joukowsky Transformation
   - complex analysis

2. Thin airfoil theory
   - use potential flow elements to construct arbitrary airfoil shapes

3. Computational Resources
   - Input any airfoil geometry & compute flow fields & forces

* 4. Wind tunnel testing
   ← next semester

L23: Joukowsky Transform



complex plane

$z = x + iy$

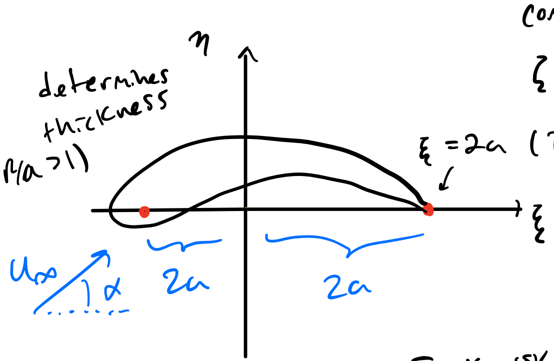
$F(z) = \phi + i\psi$

$$F(z) = U_\infty e^{-i\alpha}\left\{(z - z_0) + \frac{R^2 e^{2i\alpha}}{(z - z_0)}\right\} - \frac{i\Gamma}{2\pi}\ln\left\{\frac{(z - z_0)e^{-i\alpha}}{R}\right\}$$

conformal mapping

β determines camber

$\frac{R}{a}$ determines thickness ($R/a > 1$)



(z-plane)

complex plane

$\zeta = \xi + i\eta$

$$\boxed{\frac{\zeta}{a} = \frac{z}{a} + \frac{a}{z}}$$

$$\boxed{\zeta = z + \frac{a^2}{z}}$$

surface of airfoil:

$$\frac{\zeta}{a} = 1 + \frac{R}{a}\left\{e^{i\theta} - e^{i\beta}\right\} + \left[1 + \frac{R}{a}\left\{e^{i\theta} - e^{-i\beta}\right\}\right]^{-1}$$

Joukowsky class of airfoils!

$\frac{dF}{dz} = u - iv$ (Cauchy-Riemann eqns)

$$\frac{dF}{d\zeta} = U_\xi - iU_\eta = \frac{dF}{dz}\cdot\frac{dz}{d\zeta}$$

cylinder solution

mapping

$C_p \rightarrow$ forces!

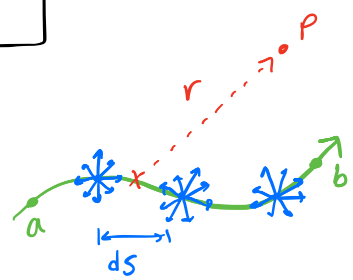
Lecture 24: Source panel method

Current method: given ψ or φ $\longrightarrow$ determine geometry $\rightarrow \vec{V}, C_p, C_D, C_L$

Panel methods: given geometry $\longrightarrow$ compute φ or ψ

preliminary concept: source sheet

Imagine arbitrary point P, distance r from element ds



$$d\varphi = \frac{\lambda\, ds}{2\pi} \ln r$$

contribution of 1 element to potential function at P

each element ds has strength equal to λds

source sheet has strength per unit length $\lambda = \lambda(s)$

$$\varphi_P = \int_a^b \frac{\lambda\, ds}{2\pi} \ln r$$

flow potential at P (contribution from all elements)

Strategy

1) Define geometry (airfoil), divide into N "panels"
2) Sum contribution from each panel
3) Enforce B.C.; No flow through walls; $\vec{V}_n = 0$
4) Results yield a system of N equations
5) Solve

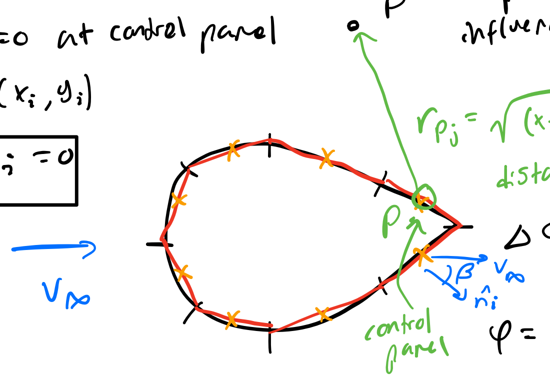
e.g. N = 8 panels

point P has a potential function influenced by all 8 panels

enforce $\vec{V}_n = 0$ at control panel

$$V_n = \frac{\partial}{\partial n_i}\left[\varphi(x_i, y_i)\right]$$

$$\boxed{V_n + V_\infty \cos\beta_i = 0}$$



$$r_{Pj} = \sqrt{(x - x_j)^2 + (y - y_j)^2}$$

distance to P from panel j

$$\Delta\varphi_j = \frac{\lambda_j}{2\pi} \int \ln r_{Pj}\, ds_j$$

$$\varphi = \sum_{j=1}^{N} \Delta\varphi_j$$

Next, to apply B.C., we move P to 1 midpoint & we call this the <u>control panel</u>

I_{ij} is a function of geometry only (not flow)

$$V_n = \frac{\lambda_i}{2} + \sum_{j=1}^{N} \frac{\lambda_i}{2\pi} \int_j \frac{\partial}{\partial n}\overbrace{(\ln r_{ij})\, ds_j}$$

contrib. from self

contrib. from other panels

$$\boxed{\frac{\lambda_i}{2} + \sum_{j=1}^{N} \frac{\lambda_i}{2\pi} I_{ij} + V_\infty \cos\beta_i = 0}$$

source panel method $(i \neq j)$

$\rightarrow$ Solve for $\lambda_1, \lambda_2, \lambda_3$, etc.

$$V_s = \frac{\partial \varphi}{\partial s} = \sum_{j=1}^{N} \frac{\lambda_j}{2\pi} \frac{\partial}{\partial s}(\ln r_{ij})\, ds_j$$

$V_i = V_s + V_{\infty,s}$ = Velocity @ panel surface

$$C_{p,i} = 1 - (V_i/V_\infty)^2$$

Notes

- input any geometry ✓
- non-lifting bodies, drag estimation