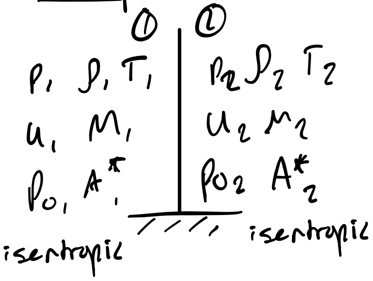
Recap



$$\frac{p_{02}}{p_{01}} = \dots$$

$$p_{01} A_1^* = p_{02} A_2^*$$

3 important pressures,

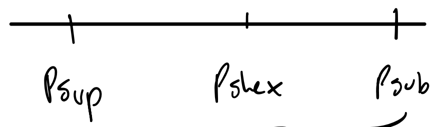
p_{sub}, p_{sup}, purely isentropic flow

$p_{sh,ex}$ just downstream of shock <u>on</u> exit plane

$$\frac{p_{sup}}{p_0} = \left(1 + \frac{\gamma-1}{2} M_{sup}^2\right)^{\gamma/1-\gamma}$$ isentropic relation

$$\frac{p_{sh,ex}}{p_{sup}} = \frac{p_2}{p_1} = 1 + \frac{2\gamma}{\gamma+1}\left(M_{sup}^2 - 1\right)$$ shock jump

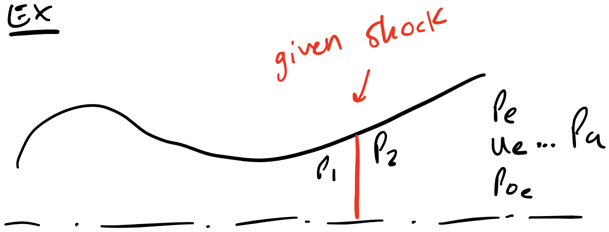
$$\frac{p_{sh,ex}}{p_0} = \frac{p_{sh,ex}}{p_{sup}} \frac{p_{sup}}{p_0}$$



If $p_{sh,ex} < p_a < p_{sub}$, there is a shock somewhere in nozzle

In plots of p/p_0, $p_0 = p_{01}$ for all cases (even if $p_{02} \neq p_{01}$)
(for plotting convenience)

Ex



G $\begin{cases} \gamma = 1.4 \\ P_o = 250\text{ kPa} \\ T_o = 300\text{K} \\ P_1 = 68.1\text{ kPa} \end{cases}$

shock @ P_1

$P_a = 180\text{ kPa}$

F $\begin{cases} M_1 \\ P_2 \\ P_{oe} \\ u_e \end{cases}$

Find M_1: $\frac{P_1}{P_o} = \left(1 + \frac{\gamma-1}{2} M_1^2\right)^{\frac{\gamma}{1-\gamma}}$

$\rightarrow M_1 = 1.5$ from calc

Find P_2: $\frac{P_2}{P_1} = 1 + \frac{2\gamma}{\gamma+1}(M_1^2 - 1)$ shock jump

$\rightarrow P_2 = 167$ kPa

Find P_{oe}: $P_{oe} = P_{o2}$ isentropic after shock

$$\frac{P_{o2}}{P_{o1}} = \left[\frac{(\gamma+1)M_1^2}{2+(\gamma-1)M_1^2}\right]^{\gamma/\gamma-1} \left[\frac{\gamma+1}{2\gamma M_1^2 - (\gamma-1)}\right]^{1/\gamma-1}$$

$= 0.929$

$\rightarrow P_{o2} = 233$ kPa $= P_{oe}$

Find u_e: $u_e = M_e a_e = M_e\sqrt{\gamma R T_e}$

isen. relations: need P_e $\rightarrow$ $P_e = P_a$ b/c shock in nozzle

$\rightarrow \frac{P_e}{P_{oe}} = \left(1 + \frac{\gamma-1}{2} M_e^2\right)^{\frac{\gamma}{1-\gamma}}$ $\rightarrow$ $M_e = 0.618$ from calc

T_e: $T_{oe} = T_o$ b/c T_o stays const across shock

$\frac{T_e}{T_{oe}} = \frac{T_e}{T_o} = \left(1 + \frac{\gamma-1}{2} M_e^2\right)^{-1} \rightarrow 279\text{ K}$

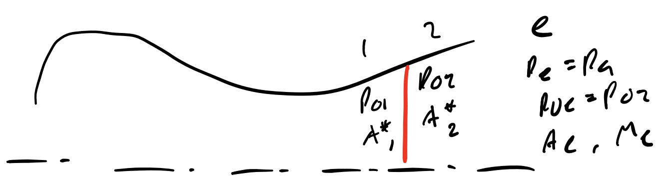
$\rightarrow u_e = M_e a_e = M_e\sqrt{\gamma R T_e} = 0.618\sqrt{1.4 \cdot 287 \cdot 279} = 207$ m/s

Locating shock in nozzle

if $\frac{P_{shex}}{P_o} < \frac{P_a}{P_o} < \frac{P_{sub}}{P_o}$ there is a shock in nozzle

If $P_a = P_{shex}$, shock is on exit plane

If $P_{shex} < P_a$, " " between throat & exit



$G \begin{cases} P_{01} \\ A_t \\ A_e \\ P_e = P_a \end{cases}$ $F \begin{cases} M_e \\ P_{0e} = P_{02} \\ M_1 \text{ just before shock} \\ A_1 \text{ @ shock} \end{cases}$ could iterate by guessing Area!

i) isentropic right of shock

@ Exit: $$\frac{P_e}{P_{0e}} = \frac{P_e}{P_{02}} = \left(1 + \frac{\gamma-1}{2} M_e^2\right)^{\gamma/1-\gamma}$$

@ Exit: $$\frac{A_e}{A_2^*} = \frac{1}{M_e}\left[\frac{2}{\gamma+1}\left(1 + \frac{\gamma-1}{2} M_e^2\right)\right]^{\frac{\gamma+1}{2(\gamma-1)}}$$

unknown P_{0e}, M_e, A_2^*

Multiply: $$\frac{P_e}{P_{02}} \cdot \frac{A_e}{A_2^*} = \left(1 + \frac{\gamma-1}{2} M_e^2\right)^{\gamma/1-\gamma} \cdot \frac{1}{M_e}\left[\frac{2}{\gamma+1}\left(1 + \frac{\gamma-1}{2} M_e^2\right)\right]^{\frac{\gamma+1}{2(\gamma-1)}}$$

$$= \frac{1}{M_e}\left[\frac{2}{\gamma+1}\right]^{\frac{\gamma+1}{2(\gamma-1)}}\left[1 + \frac{\gamma-1}{2} M_e^2\right]^{\underbrace{\frac{\gamma}{1-\gamma} + \frac{\gamma+1}{2(\gamma-1)}}_{= -1/2}}$$

Raise both sides to -2 → get EQ:

$$M_e^4 + b M_e^2 + c = 0$$

$$M_e^2 = \frac{-1}{\gamma-1} + \sqrt{\frac{1}{(\gamma-1)^2} + \frac{2}{(\gamma-1)}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}\left(\frac{P_{02} A_2^*}{P_e A_e}\right)^2}$$

$P_{02} A_2^*$ unknown, but $P_{02} A_2^* = P_{01} A_1^*$

$$\rightarrow \boxed{M_e^2 = -\frac{1}{\gamma-1} + \sqrt{\frac{1}{(\gamma-1)^2} + \frac{2}{(\gamma-1)}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}\left(\frac{P_{01} A_1^*}{P_e A_e}\right)^2}}$$

Find $M_e < 1$

ii) $$\frac{P_e}{P_{02}} = \frac{P_e}{P_{0e}} = \left(1 + \frac{\gamma-1}{2} M_e^2\right)^{\frac{\gamma}{1-\gamma}}$$ → Find $P_{0e} = P_{02}$

iii) Across shock: $$\frac{P_{02}}{P_{01}} = \left[\frac{(\gamma+1) M_1^2}{2 + (\gamma-1) M_1^2}\right]^{\frac{\gamma}{\gamma-1}}\left[\frac{\gamma+1}{2\gamma M_1^2 - (\gamma-1)}\right]^{\frac{1}{\gamma-1}}$$

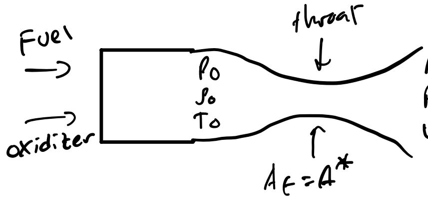
→ Find $M_1 > 1$ (root solver)

iv) $\frac{A_1}{A^*_1} = \frac{1}{M_1}\left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M_1^2\right)\right]^{\frac{\gamma+1}{2(\gamma-1)}}$

$\rightarrow$ Find A_1

§5 - Thrust chambers

$\equiv$ combustion chamber + nozzle



P_0, ρ_0, T_0 = const, known

A_t, A_e, P_a known

Thrust $\mathcal{T} = \dot{m}u_e + (P_e - P_a)A_e$

$= \rho_e u_e^2 A_e + (P_e - P_a)A_e$

From §4, $\begin{cases} M_e = M_e\left(\frac{A_e}{A^*}, \gamma\right) \\ P_e = P_e(M_e, P_0, \gamma) \\ \rho_e \quad \cdot \quad \cdot \\ T_e = T_e(M_e, T_0, \gamma) \end{cases}$

$T_0 = T_0(\bar{M}, Q_R, P_0)$

P_0 set by pumps

Task: Study thrust dependence upon:

$\begin{cases} \text{stag. conditions} \\ \text{prop. properties} \\ \text{geometry} \end{cases}$