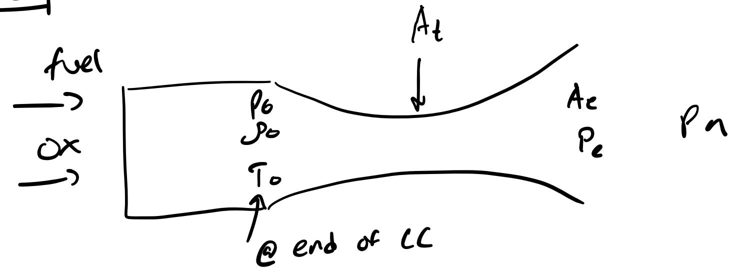
Recap



$$T_0 = T_0(\bar{M}, Q_R, P_0)$$

$\bar{M}$: Molar mass

Q_R: $\dfrac{\text{Heat released}}{\text{unit mass}}$

Express J in terms of $\begin{cases} P_0 \ \rho_0 \ T_0 \\ \bar{M} \ Q_R \ \gamma \\ \text{Geometry} \end{cases}$

Remarks

1) H_2/O_2 rockets w/ very low $\bar{M}$
   → very high I_{sp} (~450 s)
   u_e (~4500 m/s)

   rockets w/ higher $\bar{M}$, lower I_{sp}/u_e

2) Low $\bar{M}$, high $\dfrac{M_{tank}}{M_p}$

   b/c:
   i) ρ_{H_2} is low, fixed mass H_2 needs large (heavy) tank
   ii) H_2 liquid @ 77 K, need massive insulation

   Alternative: solid rocket, $\underbrace{\text{no need for insulation or turbopumps}}_{\text{much lighter}}$

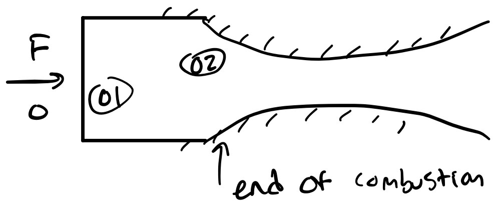
We'll study combustion in chapter 6

For now, link J to T_0 (end of CC)

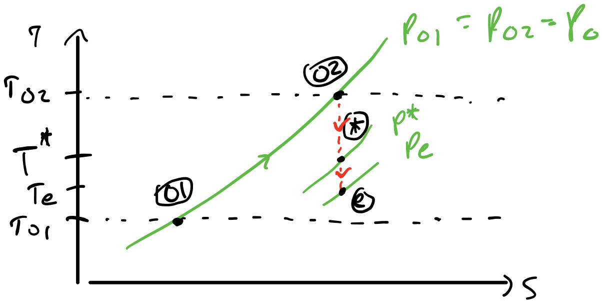
1. <u>Performance</u>

simplifications:

- combustion products = perfect gas, const. composition
- combustion = const. pressure process
- expansion : steady, 1D, isentropic (No shock)



Book uses ① & ②, prefer (01) & (02) to indicate stagnation,



From energy w/in CC

$$\cancel{\frac{d}{dt}\int_{cv} \rho\left(e + \frac{u^2}{2}\right) d\forall} + \int_{cs} \rho\left(h + \frac{u^2}{2}\right) \underline{u}_{rel} \cdot d\underline{A} = \dot{Q} - \cancel{\dot{W}_{shaft}} - \cancel{\dot{W}_{shear}} - \cancel{\int_{cs} p \underline{u}_b \cdot d\underline{A}}$$

$$\cancel{\dot{m}}(h_{02} - h_{01}) = \dot{Q} = Q_R \cancel{\dot{M}}$$ Always true (e-bal)

heat released / unit mass

$Q_R = c_p(T_{02} - T_{01})$ only true if $c_p = const$ (Not a good assumption for $\Delta T \sim 2000K$)

From $\dot{Q} = Q_R \dot{m}$

$$\frac{\dot{Q}}{\dot{m}} = Q_R \rightarrow \frac{Q_R}{c_p} = T_{02} - T_{01} \rightarrow \boxed{T_{02} = T_{01} + \frac{Q_R}{c_p}} \quad (A)$$

In process (02) → (e), $\dot{Q}=0$ ⟹ $h_{02} = h_{0e} = h_e + \frac{u_e^2}{2}$

$$\frac{u_e^2}{2} = h_{02} - h_e \quad \text{(energy balance)}$$

$$\frac{u_e^2}{2} = c_p(T_{02} - T_e) \quad (\text{only if } c_p = \text{const})$$

$$u_e = \sqrt{2c_p(T_{02}-T_e)} = \sqrt{2c_p T_{02}\left(1-\frac{T_e}{T_{02}}\right)}$$ → depends on M

↳ depends on A

If (02) to (e) is also isentropic

$$\frac{T_e}{T_{02}} = \left(\frac{p_e}{p_{02}}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\Rightarrow u_e = \sqrt{2c_p T_{02}\left(1-\left(\frac{p_e}{p_{02}}\right)^{\frac{\gamma-1}{\gamma}}\right)} \quad \text{(B)}$$

Recall $\begin{cases} c_p = \frac{\gamma}{\gamma-1} R \\ R = \frac{\bar{R}}{\bar{M}} \end{cases}$

$$u_e = \sqrt{2\frac{\gamma \bar{R}}{(\gamma-1)\bar{M}} T_{02}\left[1-\left(\frac{p_e}{p_{02}}\right)^{\frac{\gamma-1}{\gamma}}\right]} \quad \text{(C)}$$

For fixed $\begin{cases} \gamma \\ T_{02} \\ \frac{p_e}{p_{02}} \end{cases}$ $u_e \uparrow$ ($I_{sp} \uparrow$) as $\bar{M} \downarrow$

About Q_R → plug (A) into (B)

$$u_e = \sqrt{2c_p\left(T_{01} + \frac{Q_R}{c_p}\right)\left[1-\left(\frac{p_e}{p_{02}}\right)^{\frac{\gamma-1}{\gamma}}\right]}$$

Let's note $T_{01} \ll Q_R/c_p$

$$\rightarrow u_e \approx \sqrt{2Q_R\left[1-\left(\frac{p_e}{p_{02}}\right)^{\frac{\gamma-1}{\gamma}}\right]}$$

introduce $\bar{Q}_R \equiv Q_R \cdot \bar{M}$

↑ heat released per mole propellant

$$\mathcal{Q}_R = \frac{\bar{Q}_R}{\bar{M}} \Rightarrow u_e \approx \sqrt{2\frac{\bar{Q}_R}{\bar{M}}\left[1-\left(\frac{p_e}{p_{o2}}\right)^{\frac{\gamma-1}{\gamma}}\right]}$$

propellant properties ↑   ↑ nozzle property

For high u_e, look for high $\bar{Q}_R$ & low $\bar{M}$

Ratio $\frac{\bar{Q}_R}{\bar{M}}$ depends upon $\frac{\text{Fuel}}{\text{oxidizer}}$ ratio (lean or rich compared to stoichiometric)

using a high fuel : oxidizer ratio (rich)

$\rightarrow \left\{ \begin{array}{l} \text{low } \bar{M} \\ \text{low } \bar{Q}_R \end{array} \right.$ b/c not all fuel burns

$\rightarrow$ Gain in low $\bar{M}$ offsets loss in low $\bar{Q}_R$