PI control example

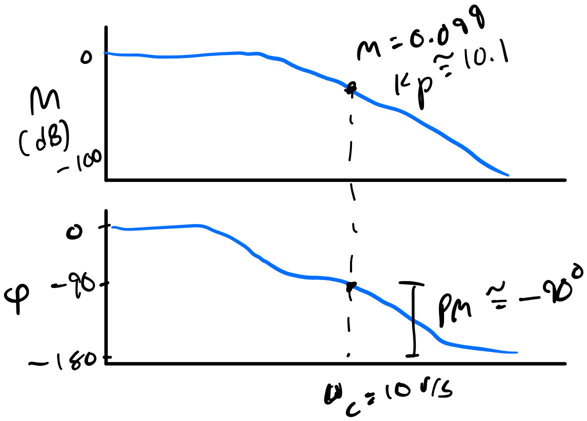
$I_m \dot{\omega} + b\omega = \tau$

$\tau = K_t i \qquad L\frac{di}{dt} + Ri = V \quad \rightarrow \quad \frac{\Omega(s)}{V(s)} = \frac{100}{(s+1)(s+100)}$

specs:

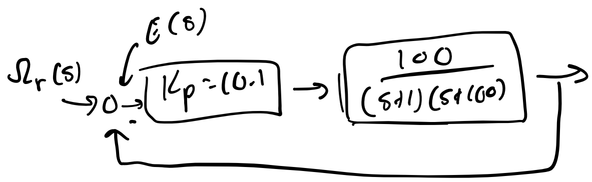
- PM > 70°
- CL bandwidth $\omega_{BW} > 2\,Hz$
- $e_{ss} < 0.01$ rad due to unit step

OL freq. response:



Set $\omega_c = 10\,r/s$ (~1.5 Hz),

will likely set $\omega_{BW} > 2\,Hz$



$$e_{ss} = \lim_{s \to 0}\left[s \frac{E(s)}{\Omega_r(s)} \cdot \Omega_r(s) \right]$$

unit step (1/s)

$$e_{ss} = \lim_{s \to 0} \frac{E(s)}{\Omega_r(s)}$$

$$\frac{E(s)}{\Omega_r(s)} = \frac{1}{1 + \frac{1010}{(s+1)(s+100)}} = \frac{(s+1)(s+100)}{s^2 + 101s + 100(1+K_p)} \qquad 1010 = K_p 100$$

$$\rightarrow e_{ss} = \frac{1}{1+K_p} \cong 0.09 \geq 0.01$$

To reduce ss error:

option 1: increase gain → this will increase ω_c (more control effort) & decrease PM

option 2: lag compensator

option 3: PI control

$$D(s) = \frac{K_p\left(s + \frac{K_I}{K_p}\right)}{s}$$

PI zero placement:

- speed of error response increases w/ larger z_{p1}
- Zero $\left(z_{p1} = \frac{K_I}{K_p}\right)$ should be set lower than ω_c to avoid affecting PM

$\rightarrow$ set z_{p1} well below $\omega_c = 10$ rad/s

$$z_{p1} = \frac{K_I}{K_p} = 1$$

Recall $\angle G(s) = -90^\circ$ @ $\omega_c = 10$ rad/s

$\rightarrow$ can lose up to 20° & satisfy $PM = 70^\circ$

set z_{p1} such that phase from PI is -20° @ $\omega_c = 10$ rad/s

$$\angle\left(K_p \cdot \frac{j\omega_c + z_{p1}}{j\omega_c}\right) = -20^\circ$$

$$= \operatorname{atan}\left(\frac{\omega_c}{z_{p1}}\right) - \operatorname{atan}\left(\frac{\omega_c}{0}\right) = -20^\circ$$

$$\rightarrow \operatorname{atan}\left(\frac{\omega_c}{z_{p1}}\right) = 70^\circ \rightarrow \boxed{z_{p1} = 3.6}$$

$\rightarrow$ we'll use $z_{p1} = 1$, but can use any $z_{p1} < 3.6$

$\boxed{z_{p1} = 1}$, $K_p = 10$, $\rightarrow K_I = 10$

$M(\omega = \omega_c) = 0.1 \rightarrow K_p = \frac{1}{M} = 10$

Error response is slower for smaller z_{p1} values order (k)

unit step: $r(t) = 1(t) \rightarrow \frac{1}{s}$ (0 order)

unit ramp: $r(t) = t \rightarrow \frac{1}{s^2}$ (1^{st} order)

unit accel.: $r(t) = \frac{1}{2}t^2 \rightarrow \frac{1}{s^3}$ (2^{nd} order)

$$\rightarrow e_{ss} = \lim_{s \to 0} \left[T(s) \cdot \frac{1}{s^k} \right]$$

Separate poles of $D(s)$ & $G(s)$ at $s=0$

$$D(s)\, G(s) = \underbrace{DG_0(s)}_{\text{D·G w/ poles @ 0 removed}} \frac{1}{s^n}$$

$\rightarrow e_{ss} = \lim$