SS Error

$e_{ss} = \lim_{s\to 0} [sE(s)]$

$\frac{E(s)}{R(s)} = \frac{1}{1+D(s)G(s)}$

$\rightarrow e_{ss} = \lim_{s\to 0} \left[s \frac{1}{1+D(s)G(s)} R(s) \right]$

consider polynomial inputs:

$k=0$ step: $\rightarrow \frac{1}{s}$

$k=1$ ramp: $\rightarrow 1/s^2$

$k=2$ parabolic: $\rightarrow 1/s^3$

$\left.\right\} R(s) = \frac{1}{s^{k+1}}$

$\rightarrow e_{ss} = \lim_{s\to 0} \left[\frac{1}{1+D(s)G(s)} \cdot \frac{1}{s^k} \right]$

separate poles @ origin

$D(s)G(s) = DG_0(s) \cdot \frac{1}{s^n}$

e.g. $DG_0(s) = \frac{K(s+z_{p1})}{ms+c}$

why do only poles at origin matter?
why not $(s + 0.0000001)$

DG_0 is const when $s=0$

$\rightarrow e_{ss} = \lim_{s\to 0} \frac{1}{(s^n + DG_0(s))} \cdot \frac{s^n}{s^k}$

n = # poles @ origin of $D(s)G(s)$ (or 2)
k = degree of polynomial input, $R(s)$
$DG_0(0)$ is const.

if $n > k$: $e_{ss} = 0$

if $n = k$: $e_{ss} = \lim_{s\to 0} \frac{1}{s^n + DG_0(s)} = \begin{cases} \frac{1}{DG_0(0)} & \text{if } n \neq 0 \\ \frac{1}{1+DG_0(0)} & \text{if } n = 0 \end{cases}$ (no poles @ origin)

if $n < k$: $e_{ss} = \infty$

$\downarrow$

$\frac{1}{1+DG_0(0)} \approx \frac{1}{DG_0(0)}$ when $DG_0(0) \gg 1$

what is $DG_0(0)$?

Magnitude ratio of $D(s)G(s) = DG_0(s)\frac{1}{s^n}$

$M = |DG_0(j\omega)| \cdot \left|\frac{1}{(j\omega)^n}\right| = |DG_0(j\omega)| \cdot \frac{1}{\omega^n}$

As $\omega \to 0$: $M \to \underbrace{|DG_0(0)|}_{\text{gain}} \frac{1}{\omega^n}$ $\leftarrow$ $-n$ slope from $\frac{1}{s^n}$

$M|_{\omega\to 0} \propto |DG_0(0)|$ AND $e_{ss} \propto \frac{1}{DG_0(0)}$ when $e_{ss} \neq 0$, $e_{ss} \neq \infty$

$$\boxed{\therefore\ e_{ss} \propto \frac{1}{M|_{\omega \to 0}}}$$

SS error is inversely proportional to low frequency gain of $D(s)G(s)$

PI/Lag compensation

PI control: $D_c(s) = K_p + \frac{K_I}{s} = \frac{K_p(s + z_{PI})}{s}$ ← zero @ $\frac{K_I}{K_p}$ & pole @ origin

$z_{PI} = \frac{K_I}{K_p}$

- PI compensation increases low freq. gain
- But PI comp. has:
  - phase loss below z_{PI}
  - suffers from integrator windup

Lag compensation: $D_c(s) = \frac{s+z}{s+p}$ where $p < z$

- moves pole away from origin
  - Reduce phase loss below z
  - Reduce integrator windup
  - low freq. gain is increased by z/p

Lag ratio $\alpha = z/p > 1$

e.g. Reduce e_{ss} from 0.14 to 0.01

$\alpha = 15$

$z = 0.1$ ← a bit lower than $\omega_c = 0.63$ r/s

$\rightarrow p = z/\alpha$

$\therefore\ e_{ss} \cong 0.01$