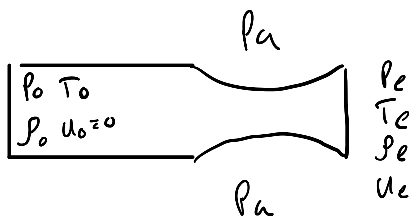
Important



combustion chamber
≡
stagnation conditions
≡
Reservoir

stagnation: $u_o = 0$

In reality, $u_o \neq 0$ just enough for combustion products to enter the nozzle

P_a

$$\frac{u_o^2}{2} << h_o$$

P_a ≡ ambient pressure

in general:

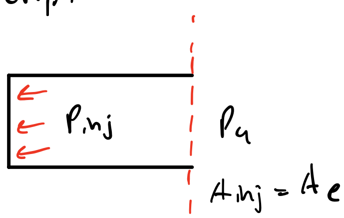
i) $P_e \neq P_a$

ii) $P_a \neq 1\,atm$

Qualitative physical description

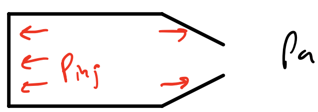
i) No nozzle

→ Thrust due to pressure against CC end wall



ii) Purely convergent nozzle

- assume A_e is the same as no nozzle.
- area of end wall is larger than before



- P_{inj} same as before

→ Pressure against inside convergent walls acts against thrust

→ Pressure against outside of convergent wall acts with thrust

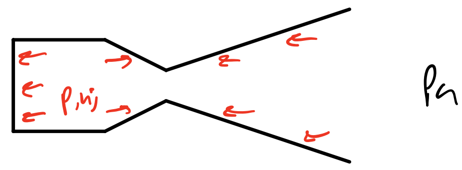
→ $P_{inj} - P_a$ same as before but A_{inj} larger than before

→ Net result: higher thrust than before

iii) Convergent - divergent nozzle

- P inside divergent wall is with thrust

- P outside divergent wall is against thrust



- P(x) inside divergent decreases monotonically
  - in "first part" of div: $p(x) > p_a$
  - in "second part" of div: $p(x) < p_a$

→ Net effect: higher thrust than i) & ii)

<u>All</u> this is still described by

$$J = \dot{m} u_e + (p_e - p_a) A_e$$

The case $p_e = p_a$ is called "optimum expansion"

Must develop the math to predict

$\dot{m}$, p_e, u_e } Based on { Chemical properties, Nozzle geometry, Stagnation conditions

In general, for assigned chemical species & stag. conds

$p = p(x)$, $\rho = \rho(x)$, $T = T(x)$, $u = u(x)$ } Because { $A = A(x)$ ✓, Shock wave ✓, $\dot{Q} \neq 0$, Heat transfer, $\mu \neq 0$, viscous effects } ignore

Start w/ $A = A(x)$, No shocks, $\dot{Q} = \mu = 0$

Strategy:

0) Introduce Mach #

i) Develop $p = p(M)$, $\rho = \rho(M)$, $T = T(M)$, $u = u(M)$

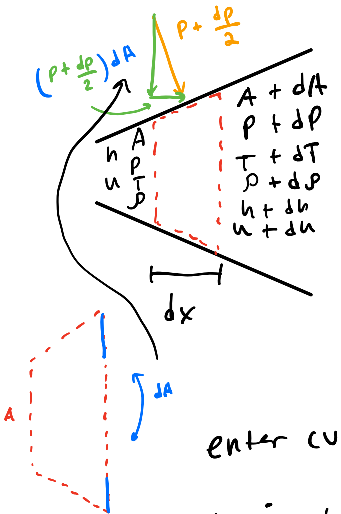
ii) Develop $M = M(A(x))$ Relation

iii) Reassemble

→ 4 unknowns
↳ 4 equations $\left\{ \begin{array}{l} \text{Cons. mass} \\ \text{Cons. momentum} \\ \text{Cons. energy} \\ \text{Eq. of state} \end{array} \right.$ ← we'll use ideal gas law → can use more complex eq.s

1. Governing equations

Write in form for infinitesimally small volume



Mass

$\rho u A = \text{const.}$

$d(\rho u A) = 0 \qquad u A\, d\rho + \rho A\, du + \rho u\, dA = 0$

$\div \rho u A$:

$$\boxed{\frac{d\rho}{\rho} + \frac{du}{u} + \frac{dA}{A} = 0}$$

Momentum

Recall

$$\frac{d}{dt}\cancel{\int_{CV} \rho u\, d\forall}^{\,S.S.} + \int_{CS} u\, \rho \underline{u} \cdot d\underline{A} = -\int_{CS} p\, d\underline{A} + \cancel{\underline{F}_{shear}}^{\,\mu = 0}$$

enter CV: negative

$$-u\dot{m} + (u + du)\dot{m} = pA - (p + dp)(A + dA) + (p + \tfrac{dp}{2})\, dA$$

$$du\, \underbrace{\dot{m}}_{\rho u A} = \cancel{pA} - \cancel{pA} - \cancel{p\,dA} - dp\,A - \underbrace{dp\,dA}_{H.O.T.} + \cancel{p\,dA} + \underbrace{\frac{dp}{2}\, dA}_{H.O.T.}$$

→ divide by dx

$$\rho u \cancel{A} \frac{du}{dx} = -\cancel{A} \frac{dp}{dx}$$

$$\rightarrow \boxed{\rho u \frac{du}{dx} = -\frac{dp}{dx}} \qquad \text{Euler's Eqn}$$