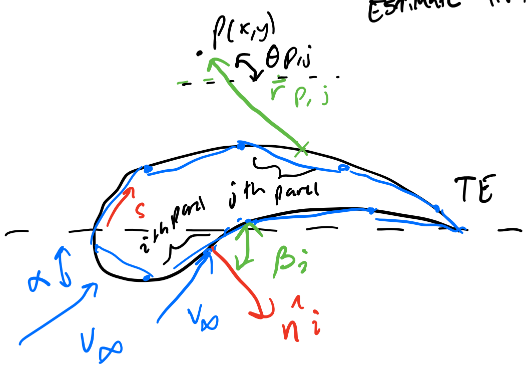
Recap:

Thin airfoil theory: ($C_l = 2\pi\alpha + \ldots$) Analytical expression for C_l, C_m
valid for thin airfoils, small α

Source panel method: Numerical method, discretize into N panels
distribution of sources / sinks
Non-lifting bodies of arbitrary geometry

Vortex panel method: Numerical panel method, distribution
of vortices / circulation / lift
~~Estimate~~ lift on arbitrary geometry.



- Split into N panels
  - vortex sheet on each panel

$\gamma(s) =$ constant on each panel

$\gamma_1, \gamma_2, \gamma_3 \ldots \gamma_N$ unknowns

- Seek a sys. of N equations
  solve for γ_i

- Impose that surf. is streamline
  $\Rightarrow$ B.C. on each panel, no flow thru panel

- Kutta; $\gamma_{TE} = 0$

- move P to midpoint of ith panel then ith panel becomes the control point

- Velocity potential at P due to jth panel

$$\Delta\varphi_j = \frac{-1}{2\pi}\int_j \theta_{ij}\gamma_j \, ds_j$$

$$\varphi(P(x_i, y_i)) = \sum_{j=1}^{N} \varphi_j \longrightarrow \varphi(x_i, y_i) = -\sum_{j=1}^{N} \frac{\gamma_j}{2\pi}\int_j \theta_{ij}\, ds_j$$

$$\theta_{ij} = \operatorname{atan}\left(\frac{y_i - y_j}{x_i - x_j}\right)$$

→ Impose B.C.:

$$V_n = \frac{\partial}{\partial n}\left(\varphi(x_i, y_i)\right)$$

$$\underbrace{V_{\infty,n}}_{V_\infty \cos\beta_i} + \underbrace{\frac{\partial}{\partial n}\left(\varphi(x_i, y_i)\right)}_{-\sum_{j=1}^{N} \frac{\gamma_j}{2\pi} \int_j \frac{\partial}{\partial n}(\theta_{ij})\, ds_j} = 0$$

$$\rightarrow \boxed{V_\infty \cos\beta_i - \sum_{j=1}^{N-1} \frac{\gamma_j}{2\pi} \underbrace{\int_j \frac{\partial \theta_{ij}}{\partial n}\, ds_j}_{= J_{ij} = \text{function of geometry}} = 0}$$

N eqns, N unknown (γ's)

Kutta condition:

(i-1)th panel

TE ($\gamma_{TE} = 0$)

ith panel

$$\gamma_i + \gamma_{i-1} = 0$$

$$\boxed{\gamma_i = -\gamma_{i-1}}$$

N+1 eqs

N unknowns

→ over-constrained!

→ sol'n: sum over N-1 panels, skip 1 panel

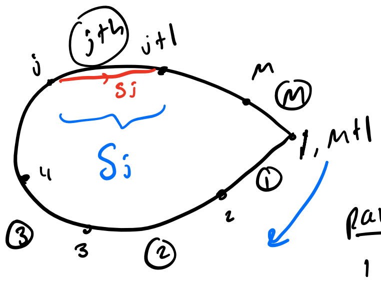
Arbitrary which one to skip

Details

Linear distribution of $\gamma(s_j)$

M+1 unknowns

→ M+1 eqs

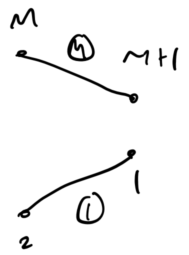


$$\gamma(s_j) = \gamma_j + (\gamma_{j+1} - \gamma_j)\frac{s_j}{S_j}$$

s_j → local c/s along panel

S_j → length of panel (const)

panel	endpts
1	1,2
2	2, 3
j	j, j+1



$\gamma_1 + \gamma_{M+1} = 0$

Previously $\gamma_i = \text{const}$

$$V_\infty \cos\beta_i - \sum_{j=1}^{N-1} \frac{\gamma_j}{2\pi} \int_j \frac{\partial \theta_{ij}}{\partial n_i} ds_j = 0$$

New expression

$$V_\infty(-\sin\varphi_i \cos\alpha + \cos\varphi_i \sin\alpha)$$

expanding

$$= \frac{1}{2\pi} \sum_{j=1}^{M} \int_{a_j}^{b_j} [\quad] ds_j$$

$$\left[\gamma_j + (\gamma_{j+1} + \gamma_j)\frac{s_j}{S_j}\right] \frac{1}{1+\left(\frac{y_i - y_j}{x_i - x_j}\right)^2} \cdot \frac{\frac{dy_i}{dn_i}(x_i - x_j) - (y_i - y_j)\frac{dx_i}{dn_i}}{(x_i - x_j)^2}$$

Next steps:

- mathematical manipulation of integral to get discrete variables for our vortex panel code.
  - make sure conventions match

$$\frac{\partial x_i}{\partial n_i} = \cos\beta_i \qquad \beta_i = \varphi_i + \pi/2$$

$$\frac{\partial y_i}{\partial n_i} = \sin\beta_i$$

$$\frac{1}{2\pi V_\infty} \sum_{j=1}^{M} H_{ij}\gamma_j + K_{ij}\gamma_{j+1} = \sin(\alpha - \varphi_i)$$

@ each C.P.

$\rightarrow$ compute H_{ij} & K_{ij}

$$\sum_{j=1}^{M} L_{ij}\gamma_j = N_i \quad \text{for } i = 1, 2, \ldots, M+1$$

$$[L]\{\gamma\} = \{N\}$$

solve for γ in matlab

Next compute tangential velocity

$$V_{t_i} = \frac{\partial \phi}{\partial s_i}$$

→ Bernoulli ⇒ $C_{p_i} = 1 - \left(\frac{V_{t_i}}{V_\infty}\right)$

PS11

$\varphi_1 = >\pi$