Recap

Momentum eq. for fixed CV

$$\frac{d}{dt}\int_{CV} \rho \underline{u}\, d\forall + \int_{CS} \rho \underline{u}\, \underline{u} \cdot d\underline{A} = \int_{CV} \rho \underline{g}\, d\forall - \int_{CS} p\, d\underline{A} + \underline{F_E}$$

2.3 CV moving @ constant velocity (rectilinear)

→ equation stays the same, but $\underline{u}$ must be measured relative to CV

Ground-fixed reference frame : $\underline{X}\ \underline{Y}\ \underline{Z}$

CV-fixed " " : $x\ y\ z$

mom. eq. rewrites:

$$\frac{d}{dt}\int_{CV} \rho \underline{u}_{xyz}\, d\forall + \int_{CS} \rho \underline{u}_{xyz} \cdot d\underline{A} = \int_{CV} \rho \underline{g}\, d\forall - \int_{CS} p\, d\underline{A} + \underline{F_E}$$

2.4 CV moving w/ rectilinear acceleration

mom. rewrites:

$$\frac{d}{dt}\int_{CV} \rho \underline{u}_{xyz}\, d\forall + \int_{CS} \rho \underline{u}_{xyz}\, \underline{u}_{xyz} \cdot d\underline{A} = \int_{CV} \rho \underline{g}\, d\forall - \int_{CS} p\, d\underline{A} + \underline{F_E} - \int_{CV} \rho \underbrace{\underline{a}_{rel}}_{\text{accel. of CV rel. to } \underline{XYZ}}\, d\forall$$

3. cons. of energy

Recall 1st law:

$de = dq - dw$ ↑ performed by fluid on surroundings

$$\rightarrow dE = dQ - dW \rightarrow \frac{dE}{dt} = \frac{dQ}{dt} - \frac{dW}{dt} = \dot{Q} - \dot{W}$$

Recall:

$$\frac{d}{dt}(\text{Property w/in CV}) = \frac{d}{dt}\int_{CV} \frac{\text{Prop}}{\text{unit mass}} \rho\, d\forall + \int_{CS} \frac{\text{Prop}}{\text{unit mass}} \rho\, \underline{u}_{rel} \cdot d\underline{A}$$

↑ rel. to CV

Here: Property is total energy E

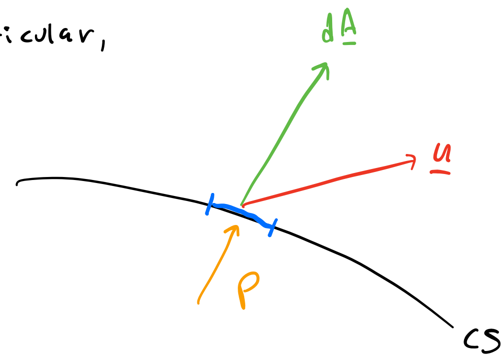
$$\frac{\text{Total energy}}{\text{unit mass}} = \overset{\text{int. energy}}{e} + \frac{u^2}{2} + gz = e_0$$

$$\rightarrow \frac{dE}{dt} = \frac{d}{dt}\int \left(e + \frac{u^2}{2} + gz\right)\rho \, d\forall + \int_{CS}\left(e + \frac{u^2}{2} + gz\right)\rho \, \underline{u}_{rel} \cdot d\underline{A} = \dot{Q} - \dot{W} \quad (1)$$

Distinguish among different "types" of $\dot{W}$

$$\dot{W} = \underbrace{\dot{W}_{press}}_{\text{moving piston or in/out flow}} + \underbrace{\dot{W}_{shear}}_{\text{friction}} + \underbrace{\dot{W}_{shaft}}_{\text{turbine/comp/fan}}$$

In particular,



$$d\dot{W}_{press} = d\underline{F}_{press} \cdot \underline{u}$$ ← velocity of fluid rel. to $\underline{XYZ}$

w/ $d\underline{F}_{press} = p \, d\underline{A}$

Rewrite $\underline{u}$ as $\underline{u} = \underbrace{\underline{u} - \underline{u}_b}_{\underline{u}_{rel}\text{, of fluid relative to CV}} + \underline{u}_b$ ← vel. of boundary of CV relative to $\underline{XYZ}$

$$\rightarrow d\dot{W}_{press} = p \, \underline{u}_{rel} \cdot d\underline{A} + p \, \underline{u}_b \cdot d\underline{A}$$

$$= \frac{p}{\rho} \cdot \rho \underline{u}_{rel} \cdot d\underline{A} + p \, \underline{u}_b \cdot d\underline{A}$$

$$\rightarrow \dot{W}_{press} = \int_{CS} \frac{p}{\rho} \rho \underline{u}_{rel} \cdot d\underline{A} + \int_{CS} p \, \underline{u}_b \cdot d\underline{A}$$

(1) rewrites

$$\frac{d}{dt}\int_{CV}\left(e+\frac{u^2}{2}+gz\right)\rho\, d\forall + \int_{CS}\left(e+\frac{u^2}{2}+gz\right)\rho\, \underline{u}_{rel}\cdot d\underline{A} = \dot{Q} - \overbrace{\int_{CS}\frac{p}{\rho}\rho\,\underline{u}_{rel}\cdot d\underline{A}}^{\text{term 1}} - \int_{CS} p\,\underline{u}_b\cdot d\underline{A} - \dot{W}_{shear} - \dot{W}_{shaft}$$

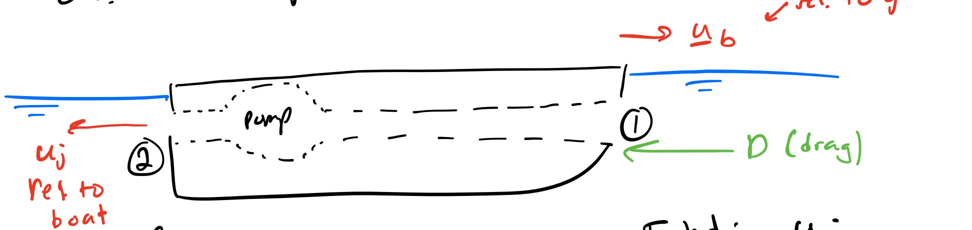
→ move term 1 to LHS

$$\rightarrow \frac{d}{dt}\int_{CV}\left(e+\frac{u^2}{2}+gz\right)\rho\, d\forall + \int_{CS}\Big(\underbrace{e+\frac{p}{\rho}}_{h\,=\,e+\frac{p}{\rho}}+\frac{u^2}{2}+gz\Big)\rho\,\underline{u}_{rel}\cdot d\underline{A} = \dot{Q} - \int_{CS} p\,\underline{u}_b\cdot d\underline{A} - \cancel{\dot{W}_{shear}} - \dot{W}_{shaft}$$

neglect friction

$$\rightarrow \boxed{\frac{d}{dt}\int_{CV}\left(e+\frac{u^2}{2}+gz\right)\rho\, dV + \int_{CS}\left(h+\frac{u^2}{2}+gz\right)\rho\,\underline{u}_{rel}\cdot d\underline{A} = \dot{Q} - \underbrace{\int_{CS} p\,\underline{u}_b\cdot d\underline{A}}_{\text{e.g. fluid pushing onto piston}} - \dot{W}_{shaft}}$$

Ex. water jet boat



Given

$\underline{u}_b = u_b\hat{i}$, $u_b = 20$ km/h

$D = 1000$ N

$\dot{\forall} = 3600$ ℓ/min

$T_1 = T_2$

Find: u_j

T

pump power

prop. efficiency

→ Work in boat-fixed co-ords, steady state → $\frac{d}{dt} = 0$

mass: $\left.\begin{array}{l}\dot{m}_{in} = \dot{m}_{out} \\ \rho = \text{const}\end{array}\right\}\ \dot{\forall}_{in} = \dot{\forall}_{out} = \dot{\forall}$

unknown

$$\text{momentum: } \int_{CS}\rho\,\underline{u}_{xyz}\,\underline{u}_{xyz}\cdot d\underline{A} = \underline{F}_B \underbrace{- \int_{CS} p\, d\underline{A} + \underline{F}_{shear}}_{\text{Drag, } = D \text{ (same as } F_E)}$$

in x-dir: in: negative

$$-D = -\underbrace{\rho\dot{\forall}}_{\dot{m}} u_1 + \rho\dot{\forall} u_2$$

$u_1 = -u_b$

$u_2 = -u_j$

$$-D = \rho\dot{\forall}(u_b - u_j)$$

$$u_j = \frac{D}{\rho \dot{V}} + u_b = 80\ km/h \quad \text{(rel. to boat)}$$

Thrust, $T = D$ → SS!

→ now power

Energy

$$\int_{cv} \left(\cancel{h} + \frac{u^2}{2} + g\cancel{z}\right) \rho u_{rel} \cdot dA = \underbrace{\cancel{\dot{Q}}}_{=0} - \dot{W}_{shaft}$$

$\Delta z = 0$

$\Delta h = c_p \cancel{\Delta T}$ → 0, $\Delta T = 0$

means pump performs work on fluid

$$\rightarrow \dot{W}_{shaft} = \underbrace{-\underbrace{\dot{m}}_{\rho u_{rel} A} \underbrace{\left(\frac{u_j^2}{2} - \frac{u_b^2}{2}\right)}_{>0}}_{<0} = -\dot{W}_{pump}$$

→ $\dot{W}_{pump} > 0$