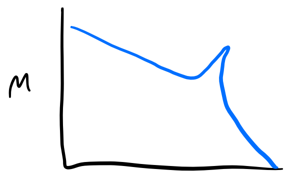
Resonance & Gain margin

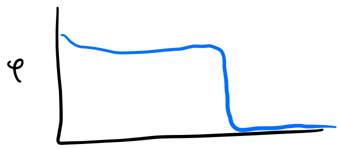
Ex. non-collocated

$$G(s) = \frac{c_1 s + c_2}{s(s^3 \sim\sim\sim)} \qquad PM = 70^\circ$$

$$G(s) = \frac{0.1(s+1000)}{s(s+0.05)(s^2+0.25s+200)}$$



- Set ω_c lower than flexible mode resonance (14 rad/s)
  - → design lead compensator



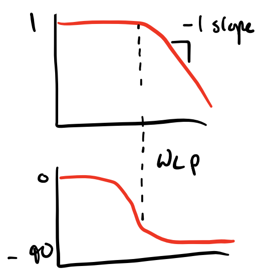
→ lead w/ $\omega_{c\,des} = 1$

→ $GM \approx 3\,dB \ll 20\,dB$

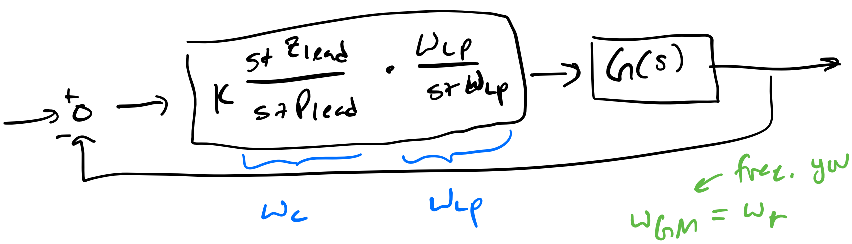
→ lower crossover frequency to increase Gain margin
- slows response

Other option - lower the magnitude of resonance peak

1st order low pass filter: $\frac{\omega_{LP}}{s+\omega_{LP}}$ → many other types of filters



→ adversely affects phase margin



← freq. you want to reduce magnitude (resonance)

$\omega_{GM} = \omega_r$

LP filter $D_{LP}(s) = \frac{\omega_{LP}}{s+\omega_{LP}}$ → $D_{LP}(j\omega_{GM}) = \frac{\omega_{LP}}{j\omega_{GM}+\omega_{LP}}$

$$\left|D_{lp}(j\underset{\omega_r}{\omega_{GM}})\right| = \frac{\omega_{lp}}{\sqrt{\omega_{GM}^2+\omega_{lp}^2}} = M_a < 1$$

set M_a & solve for ω_{lp}

$$\omega_{lp} = \frac{M_a\,\omega_{GM}}{\sqrt{1-M_a^2}}$$

$\omega_{GM} = 14$ r/s

iterate both

$M_a = 0.5 \rightarrow \omega_{lp} \cong 8.1$

$\omega_c \overset{?}{=} 0.8 - 0.9 \rightarrow D_{lp}(s) = \dfrac{8.1}{s+8.1}$

- larger M_a reduces resonance magnitude but induces more phase loss
- more phase loss will require larger lead ratio, increasing high-freq. amplification

modified design: $\omega_c = 1 \rightarrow \boxed{0.9}$

Notch filter $\quad G_{mode}(s) = \dfrac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$