雪 兰 莪 暨 吉 隆 坡 福 建 会 馆
新 纪 元 学 院
联合主办

ANJURAN BERSAMA
PERSATUAN HOKKIEN SELANGOR DAN KUALA LUMPUR
&
KOLEJ NEW ERA

第三十一届（2016 年度）
雪隆中学华罗庚杯数学比赛
PERTANDINGAN MATEMATIK PIALA HUA LO-GENG ANTARA SEKOLAH-SEKOLAH MENENGAH DI NEGERI SELANGOR DAN KUALA LUMPUR YANG KE-31(2016)

~~高中组~~
BAHAGIAN MENENGAH ATAS

日期 ： 2016 年 8 月 21 日 （星期日）
Tarikh : 21 Ogos 2016 (Hari Ahad)

时间 ： 10:00→12:00（两小时）
Masa : 10:00→12:00（2 jam）

地点 ： 新纪元学院黄迓莱活动中心
Tempat : Ng Ah Choo Multipurpose Hall, Kolej New Era
UG, Block C, Lot 5, Seksyen 10, Jalan Bukit,
43000 Kajang, Selangor

*****说明*****

1. 不准使用计算机。
2. 不必使用对数表。
3. 对一题得 4 分，错一题倒扣 1 分。
4. 答案 E：若是“以上皆非”或“不能确定”，一律以“***”代替之。

*****INSTRUTIONS*****

1. Calculators not allowed.
2. Logarithm table is not to be used.
3. 4 marks will be awarded for each correct answer and 1 mark will be deducted for each wrong answer.
4. (E)***indicates “none of the above”.

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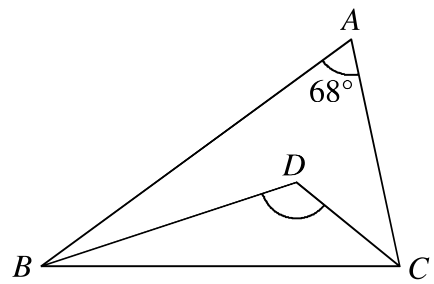
1. 求 $2013^{2013} \times 2017^{2017}$ 的个位数。

   Find the units digit of $2013^{2013} \times 2017^{2017}$.

   A. 1 B. 3 C. 7 D. 9 E. ****

2. 右图中，BD 及 CD 分别是 $\angle ABC$ 及 $\angle ACB$ 的平分线。若 $\angle A = 68°$，求 $\angle D$。

   In the figure, BD and CD are respectively the bisectors of $\angle ABC$ and $\angle ACB$. If $\angle A = 68°$, find $\angle D$.

   A. 136° B. 132° C. 128°
   D. 124° E. ****

3. 若 $a = 8^9$，$b = 16^6$，$c = 32^5$，则下列何者是**对**的？

   If $a = 8^9$, $b = 16^6$, $c = 32^5$, which of the following is **true**?

   A. $a < b < c$ B. $a < c < b$ C. $b < c < a$ D. $c < a < b$ E. ****

4. 求 $\dfrac{2+3+4+.......+99+100}{3+6+9+......+96+99}$ 的值。

   Find the value of $\dfrac{2+3+4+.......+99+100}{3+6+9+......+96+99}$.

   A. $\dfrac{100}{33}$ B. $\dfrac{101}{33}$ C. 3 D. $\dfrac{34}{11}$ E. ****

5. 已知 $x^2 + y^2 + z^2 = 20$ 及 $xy + yz - xz = 6$，求 $(x - y + z)^2$。

   Given that $x^2 + y^2 + z^2 = 20$ and $xy + yz - xz = 6$, find $(x - y + z)^2$.

   A. 8 B. 14 C. 26 D. 32 E. ****

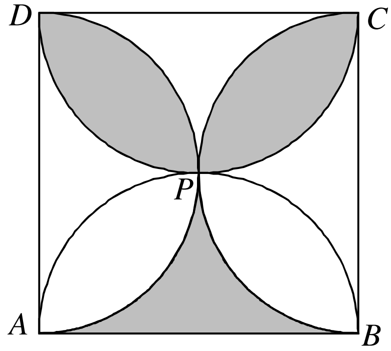
6. 已知$n=\underbrace{888...888}_{2016个8}-\underbrace{555...555}_{2016个5}$。求$n$除以 11 的余数。

Given that $n=\underbrace{888...888}_{2016\ \ 8's}-\underbrace{555...555}_{2016\ \ 5's}$, find the remainder when n is divided by 11.

A. 0 B. 2 C. 3 D. 7 E. ****

7. 右图中，$ABCD$是一边长为 2 的正方形，弧APB，BPC，CPD及DPA都是半圆。求阴影部分的面积。

In the figure, $ABCD$ is a square with side length 2, the arcs APB, BPC, CPD and DPA are semicircles. Find the area of the shaded region.



A. π B. 2 C. $\frac{3}{2}$

D. $\frac{\pi}{2}$ E. ****

8. 有三个正整数，它们任二者最多相差 3。若它们的积是 1080，求他们的和。

Three positive integers are such that they differ from each other by at most 3. If their product is 1080, find their sum.

A. 31 B. 29 C. 25 D. 21 E. ****

9. 若$S=1+\frac{1}{2^3}+\frac{1}{3^3}+\frac{1}{4^3}+\ldots+\frac{1}{n^3}+\ldots$，求$1+\frac{1}{3^3}+\frac{1}{5^3}+\ldots+\frac{1}{(2n-1)^3}+\ldots$的值。

If $S=1+\frac{1}{2^3}+\frac{1}{3^3}+\frac{1}{4^3}+\ldots+\frac{1}{n^3}+\ldots$, find the value of $1+\frac{1}{3^3}+\frac{1}{5^3}+\ldots+\frac{1}{(2n-1)^3}+\ldots$.

A. $\frac{S}{2}$ B. $\frac{2S}{3}$ C. $\frac{3S}{4}$ D. $\frac{7S}{8}$ E. ****

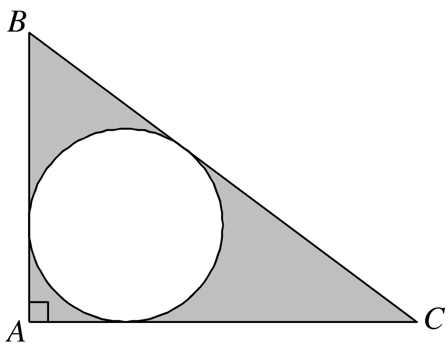
10. 求所有满足不等式$|3n+1|\le|2n-3|$的整数n之和。

Find the sum of all the integers n that satisfy the inequality $|3n+1|\le|2n-3|$.

A. 6 B. 3 C. -6 D. -10 E. ****

11. 右图所示是一直角三角形ΔABC及其内切圆。已知$AB=6$，$AC=8$，求阴影部分的面积。

The figure shows a right-angled triangle ΔABC and its inscribed circle. Given that $AB=6$, $AC=8$, find the area of the shaded region.



A. $48-9\pi$ B. $16-\pi$ C. $24-4\pi$

D. $20-3\pi$ E. ****

12. 求540的正因数的个数。

Find the number of positive factors of 540.

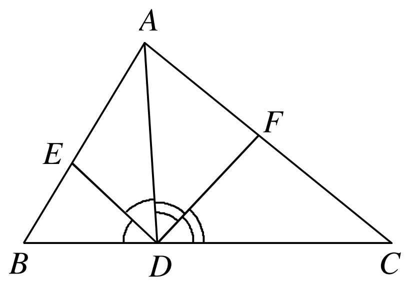
A. 18 B. 20 C. 24 D. 25 E. ****

13. 计算 $\dfrac{99\times101^2\times\left(100^2-99\right)\times\left(100^2+101\right)}{\left(100^3-1\right)\left(100^3+1\right)}$。

Compute $\dfrac{99\times101^2\times\left(100^2-99\right)\times\left(100^2+101\right)}{\left(100^3-1\right)\left(100^3+1\right)}$.

A. 1 B. 99 C. 100 D. 101 E. ****

14. 如图所示，D、E、F 分别是 ΔABC 中 BC、AB、AC 边上的点，DE 及 DF 分别是 $\angle ADB$ 及 $\angle ADC$ 的角平分线。已知 $AE : EB = 9 : 7$， $AF : FC = 6 : 7$， 求 $BD : DC$。



As shown in the figure, D, E, F are respectively points on the sides BC, AB and AC of ΔABC, DE and DF are respectively the angle bisectors of $\angle ADB$ and $\angle ADC$. Given that $AE : EB = 9 : 7$, $AF : FC = 6 : 7$, find $BD : DC$.

A. $2:3$ B. $3:2$ C. $3:4$ D. $49:54$ E. ****

15. 已知 x 满足方程式 $x^2-4x-3=0$，求 x^3-19x 的值。

Given that x satisfies the equation $x^2-4x-3=0$, find the value of x^3-19x.

A. 12 B. 2 C. -9 D. -38 E. ****

16. 若 $0<x<1$， $y=x^x$， $z=x^y$，则下列何者是**对**的?

If $0<x<1$, $y=x^x$, $z=x^y$, which of the following is **true**?

A. $x<y<z$ B. $x<z<y$ C. $y<x<z$ D. $z<y<x$ E. ****

17. 在一个箱子中有 n 张分别写上数字 1、2、3、…、n 的卡片。从箱中抽出一张写上数字 m 的卡片后，剩下的 $n-1$ 张卡片上的数字之和为 203， 求 m 的值。

In a box there are n pieces of cards which are numbered 1, 2, 3, ….., n respectively. After a card with number m written on it is withdrawn from the box, the sum of the numbers on the remaining $n-1$ pieces of cards is 203. Find the value of m.

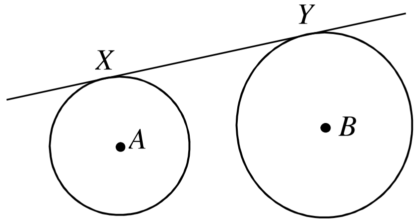
A. 1 B. 3 C. 5 D. 7 E. ****

18. 求满足方程式$3x+4y=101$的正整数对(x, y)的个数。

Find the number of ordered pairs of positive integers (x, y) that satisfy the equation $3x+4y=101$.

A. 7 B. 8 C. 9 D. 10 E. ****

19. 如图所示，以 A、B 两点为圆心的二圆分别与直线 XY 外切于 X、Y 两点。若 $XY=40\,\text{cm}$，$AB=41\,\text{cm}$，四边形 $ABYX$ 的面积为$660\,\text{cm}^2$，圆 A 与圆 B 的半径分别为r_A及r_B，且$r_A<r_B$，求$\frac{r_B}{r_A}$。



As shown in the figure, the two circles with centres at A and B are tangent to the line XY at X and Y respectively. If $XY=40\,\text{cm}$, $AB=41\,\text{cm}$, the area of the quadrilateral $ABYX$ is $660\,\text{cm}^2$, the radii of circle A and circle B are respectively r_A and r_B, with $r_A<r_B$, find $\frac{r_B}{r_A}$.

A. $\frac{5}{2}$ B. $\frac{9}{4}$ C. $\frac{7}{4}$ D. 2 E. ****

20. 有多少个锐角三角形其每个角的度数是不相等的整数，且最大的角是最小的角的两倍?

How many acute-angled triangles are there such that all of the angles are mutually distinct integers, and the largest angle is two times the smallest angle?

A. 6 B. 7 C. 8 D. 9 E. ****

21. 求最小的整数n使得$n\left(\sqrt{65}-8\right)>1$。

Find the smallest integer n such that $n\left(\sqrt{65}-8\right)>1$.

A. 15 B. 16 C. 17 D. 18 E. ****

22. 求$\sqrt{12+\sqrt{12+\sqrt{12+\sqrt{12+\dots}}}}$。

Find $\sqrt{12+\sqrt{12+\sqrt{12+\sqrt{12+\dots}}}}$.

A. $\sqrt{15}$ B. 4 C. $\sqrt{17}$ D. $\sqrt{18}$ E. ****

23. 一袋中有 10 粒红球及 5 粒黄球。A 先从袋中任意取出一球，不放回；B 再从袋中任意取出一球。求 B 取出的是黄球的概率。

There are 10 red balls and 5 yellow balls in a bag. A draws a ball randomly from the bag first, without placing the ball back, then B draws a ball randomly from the bag. Find the probability that the ball drawn by B is a yellow ball.

A. $\frac{2}{7}$ B. $\frac{1}{3}$ C. $\frac{2}{5}$ D. $\frac{1}{4}$ E. ****

24. 大卫在星期二、星期三、星期四、星期五都说真话，在其他天都说谎。伊丽莎白在星期一、星期二、星期六、星期天都说真话，在其他天都说谎。某天他们两人都说“昨天我说谎”。他们说这话当天是______。

David tells the truth on Tuesday, Wednesday, Thursday and Friday. He lies on all other days. Elizabeth tells the truth on Monday, Tuesday, Saturday and Sunday. She lies on all other days. One day they both said, “Yesterday I lied”. The day they said that was ______ .

A. 星期一 Monday B. 星期三 Wednesday C. 星期五 Friday D. 星期六 Saturday E. ****

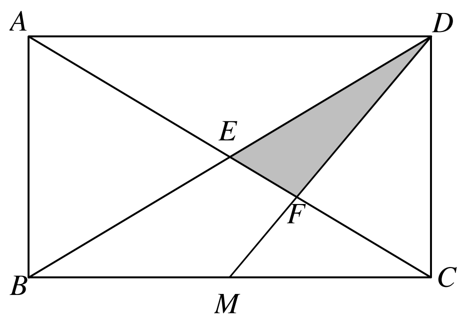
25. 有多少个整数 n 使得 $f(n) = \frac{3n+9}{2n+1}$ 也是整数?

How many integers n are there such that $f(n) = \frac{3n+9}{2n+1}$ is also an integer?

A. 4 B. 6 C. 8 D. 10 E. ****

26. 右图中，$ABCD$ 是长方形，$AB = 6$，$BC = 10$，M 是 BC 的中点。求 ΔDEF 的面积。

In the figure, $ABCD$ is a rectangle. $AB = 6$, $BC = 10$, M is the midpoint of BC. Find the area of ΔDEF .



A. 5 B. 6 C. 8 D. 9 E. ****

27. 求满足方程式 $(2^x - 32)^2 + (3^x - 9)^2 = (3^x - 2^x + 23)^2$ 的所有实数 x 之和。

Find the sum of all the real numbers x that satisfy the equation

$(2^x - 32)^2 + (3^x - 9)^2 = (3^x - 2^x + 23)^2$.

A. 5 B. 6 C. 7 D. 8 E. ****

28. 对于任意的正整数n，令

$(1+x)(1+2x)(1+3x)......(1+nx) = a_0 + a_1x + a_2x^2 + ... + a_nx^n$。

求最小的正整数n使得$a_0 + a_1 + a_2 + ... + a_n$能被 7 整除。

For any positive integer n, let

$(1+x)(1+2x)(1+3x)......(1+nx) = a_0 + a_1x + a_2x^2 + ... + a_nx^n$.

Find the smallest positive integer n such that $a_0 + a_1 + a_2 + ... + a_n$ is divisible by 7.

A. 4 B. 5 C. 6 D. 7 E. ****

29. 由 1、2、……、5 这五个数字所组成的数字不重复的五位数有$5!=120$个。设a_1，a_2，a_3，……，a_{120}是这些数按由小到大排列而成的序列，即$a_1=12345$，$a_2=12354$，$a_3=12435$，……..。求a_{50}。

There are $5!=120$ five-digit integers that are formed by the five digits 1, 2, ……, 5 with no repeated digits. Let a_1, a_2, a_3, …… , a_{120} be the sequence formed by these numbers in ascending order, i.e., $a_1=12345$, $a_2=12354$, $a_3=12435$, etc. Find a_{50}.

A. 31254 B. 31245 C. 25431 D. 31425 E. ****

30. 求无穷级数$1-\frac{2}{3}+\frac{3}{3^2}-\frac{4}{3^3}+\frac{5}{3^4}-\frac{6}{3^5}+\ldots$的和。

Find the sum of the infinite series $1-\frac{2}{3}+\frac{3}{3^2}-\frac{4}{3^3}+\frac{5}{3^4}-\frac{6}{3^5}+\ldots$.

A. $\frac{9}{8}$ B. $\frac{9}{16}$ C. $\frac{3}{4}$ D. $\frac{3}{5}$ E. ****

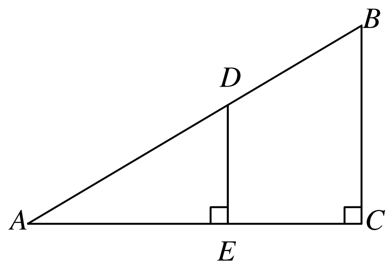
31. 已知ΔABC中，$\angle A=60°$，$BC=7\sqrt{3}$ cm，$AB-AC=2$ cm。若ΔABC的面积为$(\sqrt{3}\,x)\text{cm}^2$，求$4x$的值。

Given that in ΔABC, $\angle A=60°$, $BC=7\sqrt{3}$ cm, $AB-AC=2$ cm. If the area of ΔABC is $(\sqrt{3}\,x)\text{cm}^2$, find the value of $4x$.

A. 143 B. 147 C. 286 D. 294 E. ****

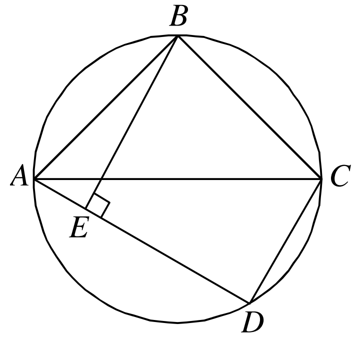
32. 如图所示，$\angle C=90°$，$DE\perp AC$。已知$AE=BC$，$AD=4$，$DE+AC=12$，求$\frac{AB}{BC}$。

As shown in the figure, $\angle C=90°$, $DE\perp AC$. Given that $AE=BC$, $AD=4$, $DE+AC=12$, find $\frac{AB}{BC}$.



A. 2 B. $\frac{5}{2}$ C. $\frac{8}{3}$ D. 3 E. ****

33. 如图所示，AC 是圆的直径，BE 垂直 AD 于 E。已知 $AB = BC$，四边形 $ABCD$ 的面积为 144cm^2，求 BE 的长。



As shown in the figure, AC is a diameter of the circle, BE is perpendicular to AD at E. Given that $AB = BC$, and the area of the quadrilateral $ABCD$ is 144cm^2, find the length of BE.

A. 12 cm B. 15 cm C. 16 cm D. 18 cm E. ****

34. 已知 x、y、z 是正实数且 $xy + 2yz + 3zx = 7$，求 $4x^2 + 3y^2 + 5z^2$ 的最小可能值。

Given that x, y, z are positive real numbers such that $xy + 2yz + 3zx = 7$, find the smallest possible value of $4x^2 + 3y^2 + 5z^2$.

A. 21 B. 14 C. $\frac{21}{2}$ D. 7 E. ****

35. 有多少个小于 2000 的正整数 n 使得 $11n+5$ 是平方数?

How many positive integers n less than 2000 is such that $11n+5$ is a perfect square?

A. 24 B. 25 C. 26 D. 27 E. ****