雪兰莪暨吉隆坡福建会馆
新 纪 元 学 院

联合主办

ANJURAN BERSAMA
PERSATUAN HOKKIEN SELANGOR DAN KUALA LUMPUR
&
KOLEJ NEW ERA

第二十八届（2013 年度）
雪隆中学华罗庚杯数学比赛
PERTANDINGAN MATEMATIK PIALA HUA LO-GENG
ANTARA SEKOLAH-SEKOLAH MENENGAH
DI NEGERI SELANGOR DAN KUALA LUMPUR
YANG KE-28(2013)

~~初中组~~

BAHAGIAN MENENGAH RENDAH

日期　：2013 年 8 月 25 日 （星期日）
Tarikh　: 25 Ogos 2013 (Hari Ahad)

时间　：10:00→12:00 （两小时）
Masa　: 10:00→12:00 （2 jam）

地点　：新纪元学院 UG 活动中心
Tempat : UG Hall Kolej New Era
　　　　　Block C, Lot 5, Seksyen 10, Jalan Bukit,
　　　　　43000 Kajang, Selangor

INSTRUTIONS
1. Calculators not allowed.
2. Logarithm table is not to be used.
3. 4 marks will be awarded for each correct answer and 1 mark will be deducted for each wrong answer.
4. (E)***indicates "none of the above".

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1. $(-1)^1 - (-1)^2 - (-1)^3 - \ldots - (-1)^{2013} =$

   A. -2013 B. -2012 C. -1 D. 1 E. ***

2. 求 $\dfrac{135}{2013^2 - 2010 \times 2016}$ 之值。

   Find the value of $\dfrac{135}{2013^2 - 2010 \times 2016}$.

   A. 13   B. 15   C. 31   D. 35   E. ***

3. $\left(2^{2^r}\right)^2 =$

   A. $2^{2^{2r}}$ B. 2^{4^r} C. $2^{2^{r+1}}$ D. 4^{2^r} E. ***

4. 求 $\log\dfrac{1}{2} + \log\dfrac{2}{3} + \log\dfrac{3}{4} + \ldots + \log\dfrac{999}{1000}$ 之值。

   Find the value of $\log\dfrac{1}{2} + \log\dfrac{2}{3} + \log\dfrac{3}{4} + \ldots + \log\dfrac{999}{1000}$.

   A. -10 B. -3 C. 10   D. 3   E. ***

5. 若 $4^x 6^y 9^z = 4^2 6^3 9^4$，则 $x+y$ 之值一定是

   If $4^x 6^y 9^z = 4^2 6^3 9^4$, then the value of $x+y$ must be

   A. 5   B. 6   C. 7   D. 8   E. ***

6. 若 $5 \le p, q \le 10$ ，那么 $\dfrac{p}{q}$ 的最大值与最小值之差为

   If $5 \le p, q \le 10$, then the difference between the largest and the smallest possible values of $\dfrac{p}{q}$ is

   A. $\dfrac{5}{4}$ B. $\dfrac{1}{2}$ C. $\dfrac{3}{2}$ D. 2   E. ***

7. 在 2008, 2010, 2012, 2014, 2016, 2018, 2020 及 2022 这八个号码当中,其中四个号码的平均值为 2013。那么剩余的四个号码的平均值是多少?

   Among the eight numbers 2008, 2010, 2012, 2014, 2016, 2018, 2020 and 2022, some four numbers have an average of 2013. What is the average of the other four numbers?

   A. 2015   B. 2016   C. 2017   D. 2018   E. ***

8. 在某一年的八月，只有 4 个星期日与 4 个星期四。那么那年的九月一号是_____。

   For a particular year, there are exactly 4 Sundays and 4 Thursday in August. Then 1 September of the year is a _____.

   A. 星期一   B. 星期二   C. 星期三   D. 星期四   E. ***
        Monday   Tuesday   Wednesday   Thursday   ***

9. 已知 $\dfrac{9 + 99 + 999 + \ldots + 999999999}{9}$ 是一个 9 位数的号码，这号码不包含数字 a ，求 a 之值。

   Given that $\dfrac{9 + 99 + 999 + \ldots + 999999999}{9}$ is a 9-digit number containing no digit a, find the value of a.

   A. 0   B. 2   C. 4   D. 6   E. ***

10. 分数 $\dfrac{17}{10}$ 可以写成 $1 + \dfrac{1}{a + \dfrac{1}{b + \dfrac{1}{c}}}$ 的形式，其中 a, b 及 c 是正整数。求 $a + b + c$ 之值。

   The fraction $\dfrac{17}{10}$ can be written in the form $1 + \dfrac{1}{a + \dfrac{1}{b + \dfrac{1}{c}}}$, where a, b and c are positive integers. Find the value of $a + b + c$.

   A. 6   B. 7   C. 8   D. 9   E. ***

11. 有多少整数 a 满足 $\left| a - \dfrac{4}{a} \right| = \dfrac{4}{a} - a$?

How many such integers a satisfy $\left| a - \dfrac{4}{a} \right| = \dfrac{4}{a} - a$?

A. 9   B. 8   C. 5   D. 4   E. ***

12. 设 n 是一个正整数，且 $\dfrac{1}{2} + \dfrac{1}{4} + \dfrac{1}{6} + \dfrac{1}{n}$ 是个整数。若 k 整除 n，以下哪一项不可能是 k 的值？

Let n be a positive integer such that $\dfrac{1}{2} + \dfrac{1}{4} + \dfrac{1}{6} + \dfrac{1}{n}$ is an integer. If k divides n, which is the following is not a possible value of k ?

A. 2   B. 3   C. 4   D. 5   E. ***

13. 已知 n 是整数，且联立方程

$$\begin{cases} nx + y = 13 \\ 7x - 2y = 0 \end{cases}$$

有正整数解，求 y 的最大值。

Given that n is an integer and the system of equation

$$\begin{cases} nx + y = 13 \\ 7x - 2y = 0 \end{cases}$$

has positive integer solutions, find the largest possible value of y.

A. 112   B. 105   C. 91   D. 42   E. ***

14. 127 及 433 都是 3 位数号码，而且其数字之和皆为 10（即 $1 + 2 + 7 = 4 + 3 + 3 = 10$）。共有多少 3 位数号码 $\overline{abc}$ 满足 $a + b + c = 10$ ？

Note that 127 and 433 are examples of 3-digit integers with the sum of digits equal to 10 (That is $1 + 2 + 7 = 4 + 3 + 3 = 10$). How many 3-digit integers $\overline{abc}$ altogether satisfying $a + b + c = 10$?

A. 45   B. 48   C. 54   D. 55   E. ***

15. 在某个聚会，每一个男生跟 4 个女生握手，而每一个女生跟 3 个男生握手。若该聚会共有 24 个男生，求女生的人数。

In a party, each boy shakes hand with 4 girls while each girl shakes hand with 3 boys. If there are 24 boys in the party, find the number of girls in the party.

A. 32   B. 28   C. 22   D. 18   E. ***

16. 某长方形的长增加了 $p\%$，其宽减少了 $q\%$。若该长方形的面积保持不变，那么 $q =$

The length of a rectangle is increased by $p\%$, and its width is reduced by $q\%$. If the area of the rectangle is remain unchanged, then $q =$

A. p B. $\dfrac{p}{1+p}$ C. $\dfrac{50p}{50+p}$ D. $\dfrac{100p}{100+p}$ E. ***

17. 已知 N 是最小的整数使到 N 与 207 之积的最后四个号码为 2013，即 $207N = **2013$，那么 N 的首两个号码是什么？

Given that N is the smallest integer such that the last four digits of the product of N and 207 is 2013, that is, $207N = **2013$, what is the first 2 digits of N?

A. 14   B. 15   C. 16   D. 17   E. ***

18. 若 $x^2 + x - 1 = 0$，求 $x^3 + 2x^2 + 2013$ 之值。

If $x^2 + x - 1 = 0$, find the value of $x^3 + 2x^2 + 2013$.

A. 2011   B. 2012   C. 2013   D. 2014   E. ***

19. 已知 k 及 n 是整数，且 $\dfrac{n}{10 - n} = k^2$。有多少这样的整数 k？

Given that k and n are integers for which $\dfrac{n}{10 - n} = k^2$. How many such integers k?

A. 6   B. 7   C. 8   D. 9   E. ***

20. 已知 $(a - 2b + 3c)^2 + \sqrt{\dfrac{a - 2c}{b - c}} + \sqrt{\dfrac{2c - a}{b + c}} = 0$，求 $a:b$ 之比。

Given that $(a - 2b + 3c)^2 + \sqrt{\dfrac{a - 2c}{b - c}} + \sqrt{\dfrac{2c - a}{b + c}} = 0$, find the ratio $a:b$.

A. 1:2   B. 2:3   C. 3:4   D. 4:5   E. ***

21. 设 $f(x) = \dfrac{7}{x^2 + 5x + 1}$ 及 $a < b < 1$ 使到 $f(a) = a$ 及 $f(b) = b$。求 $a + b$ 之值。

Let $f(x) = \dfrac{7}{x^2 + 5x + 1}$ and $a < b < 1$ such that $f(a) = a$ and $f(b) = b$. Find the value of $a + b$.

A. -6 B. -5 C. 5   D. 6   E. ***

22. 当 4×2011^3 被 2013 除时，其余数为

Find the remainder when 4×2011^3 is divided by 2013.

    A. 1981   B. 1988   C. 2005   D 2011   E. ***

23. $11\left(\dfrac{1}{1} + \dfrac{1}{1+2} + \dfrac{1}{1+2+3} + \dfrac{1}{1+2+3+4} + \cdots + \dfrac{1}{1+2+3+\ldots 10}\right) =$

    A. 20   B. 21   C. 22   D. 23   E. ***

24. 若 $\overline{abc} = 123$，那么 $\overline{abc} \times \overline{cba} = 39483$。若 $\overline{xyz} \times \overline{zyx} = 396396$，求 y 之值。

If $\overline{abc} = 123$, then $\overline{abc} \times \overline{cba} = 39483$. If $\overline{xyz} \times \overline{zyx} = 396396$, find the value of y.

    A. 1   B. 2   C. 4   D. 5   E. ***

25. 号码 1543608792 能被 66 整除，而且它没有重复的数字。在所有能被 66 整除，而且没有重复的数字的号码当中，x 是最大的。求 x 的最后 4 位数字。

Note that the number 1543608792 is divisible by 66 and it has no repeated digits. Let x be the largest integer divisible by 66 for which x has no repeated digits. What is the last 4 digits of x?

    A. 3210   B. 4130   C. 1240   D. 4032   E. ***

26. 将数字 1 至 8 填进立方体的角（图 1），使到每个面的四个数字之和都一样。求 a 之值。

The numbers 1 to 8 are to filled into the corners of a cube (Figure 1) so that four numbers of each face have a common sum. What is the value of a?
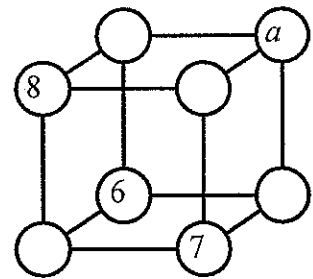
    A. 1   B. 2   C. 3

    D. 4   E. 5

图 1

Figure 1

27. 图 2 共有几个长方形？

（注：正方形也是长方形）

How many rectangles are in Figure 2?
(Note: a square counts as a rectangle)

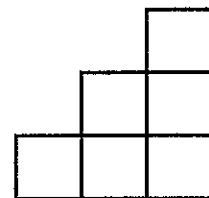   A. 15   B. 14   C. 13

    D. 12   E. ***

图 2

Figure 2

28. 如图 3，已知 $\angle A = \angle B = 119°$ 及

$\angle C = \angle D + 1° = \angle E + 2° = \angle F + 3°$，求 $\angle F$。

As shown in Figure 3, it is known that $\angle A = \angle B = 119°$ and $\angle C = \angle D + 1° = \angle E + 2° = \angle F + 3°$, find $\angle F$.
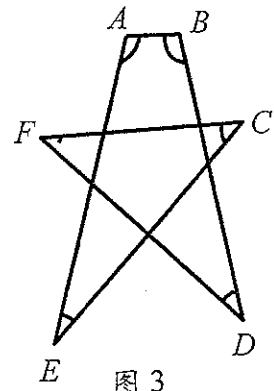
A. 28° B. 29° C. 30°

D. 31° E. ***

图 3

Figure 3

29. 如图 4，ABC 是一个直角三角形。已知 $\angle A = 90°$，$AB = 3$，$BC = 4$ 及三角形 ABC 的面积为 x，那么

As shown in Figure 4, the triangle ABC is a right angled triangle. Given that $\angle A = 90°$, $AB = 3$, $BC = 4$ and the area of triangle ABC is x, then

A. $x = 6$ B. $4 < x < 6$ C. $3 < x < 3.5$

D. $3.5 < x < 4$ E. ***

图 4

Figure 4

30. 如图 5，ACD 是直角三角形，$AD = 750$，$BD = 350$ and $CD = 210$。求 $AB : BC$ 之比。

As shown in Figure 5, ACD is a right angled triangle, $AD = 750$, $BD = 350$ and $CD = 210$ .Find the ratio of $AB : BC$.
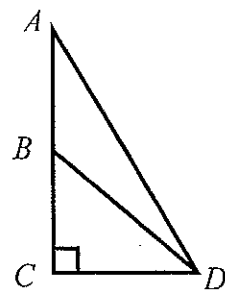
A. 11:7 B. 6:5 C. 5:4

D. 3:2 E. ***

图 5

Figure 5

31. 如图 6，$ABCD$ 是边长为 6 的正方形。点 O 是正方形内部的一点。已知 $AS = AP = CQ = CR = 4$，求阴影部分的面积。

As shown in Figure 6, $ABCD$ is a square with side length 6. The point O is an interior point in the square. It is known that $AS = AP = CQ = CR = 4$, find the area of shaded region.

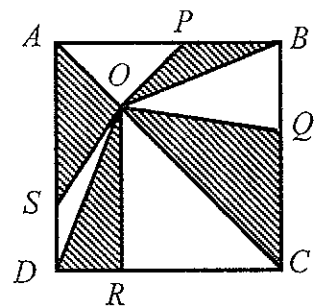A. 15 B. 18 C. 21

D. 22 E. ***

图 6

Figure 6

32. 如图 7，*ABC* 是个三角形，*AB* = 6，*BC* = 9 及
∠*BAC* = 120°。点 *D* 在线段 *BC* 上，且 ∠*ADC* = 60°。
求 *CD* 的长度。

As shown in Figure 7, *ABC* is a triangle with *AB* = 6,
BC = 9 and ∠*BAC* = 120°. The point *D* is on *BC* and
∠*ADC* = 60°. Find the length of *CD*.
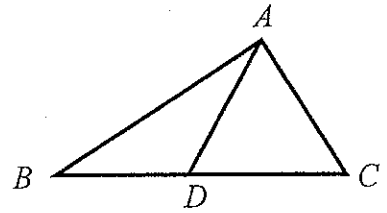
   A. 4   B. 4.5   C. 5

   D. 5.5   E. ***

图 7

Figure 7

33. 如图 8，较小的圆形的半径为 1，其余的圆形的半径为
r。此 4 圆形相互相切，求 *r* 之值。

As shown in Figure 8, the radius of the smallest circle is 1
and the rest are of radius *r*. These 4 circles are mutually
tangent, find the value of *r*.

   A. $3-\sqrt{3}$ B. $5-\sqrt{3}$ C. $3+2\sqrt{3}$
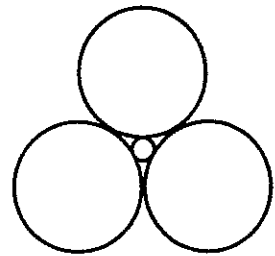
   D. $5+\sqrt{3}$ E. ***

图 8

Figure 8

34. 如图 9，*ABCD* 是个平行四边形。点 *E* 在线段 *AB* 上，
且 ∠*ADE* = ∠*EDC*。*DE* 及 *CB* 相遇在点 *F*。若
AE = 2*EB* = 5，求 *CF* 的长度。

As shown in Figure 9, *ABCD* is a parallelogram. The point
E is on *AB* and ∠*ADE* = ∠*EDC*. *DE* and *CB* meet at *F*.
If *AE* = 2*EB* = 5, find the length of *CF*.

   A. 6.5   B. 7   C. 7.5
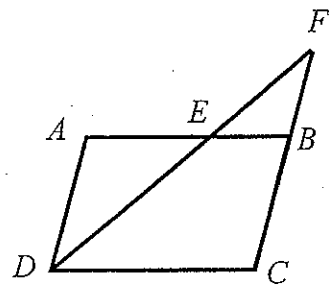
   D. 8   E. ***

图 9

Figure 9

35. 如图 10，三角形 *ABC* 的面积为 60。点 *P*，*Q* 及 *R* 分
别在线段 *AB*，*BC* 及 *CA* 上，且 3*AP* = *PB*，2*BQ* = *QC*
及 *CR* = *RA*。求三角形 *PQR* 的面积。

As shown in Figure 10, *ABC* is a triangle with area 60. *P*,
Q and *R* is on *AB*, *BC* and *CA* respectively, where
3*AP* = *PB*，2*BQ* = *QC* and *CR* = *RA*. Find the area of
triangle *PQR*.
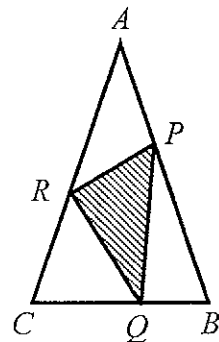
   A. 17.5   B. 17   C. 16.5

   D. 15.5   E. ***

图 10

Figure 10