雪兰莪暨吉隆坡福建会馆新 纪 元 学 院

联合主办

ANJURAN BERSAMA PERSATUAN HOKKIEN SELANGOR DAN KUALA LUMPUR & KOLEJ NEW ERA

第三十届 (2015年度)

雪隆中学华罗庚杯数学比赛

PERTANDINGAN MATEMATIK PIALA HUA LO-GENG ANTARA SEKOLAH-SEKOLAH MENENGAH DI NEGERI SELANGOR DAN KUALA LUMPUR YANG KE-30(2015)

~~初中组~~

BAHAGIAN MENENGAH RENDAH

日期 : 2015 年 8 月 9 日 (星期日) Tarikh : 09 Ogos 2015 (Hari Ahad)

时间 : 10:00→12:00 (两小时) Masa : 10:00→12:00 (2 jam)

地点 :新纪元学院黄迓菜活动中心

Tempat: NG AH CHOO MULTIPURPOSE HALL Kolej New Era

Block C, Lot 5, Seksyen 10, Jalan Bukit,

43000 Kajang, Selangor

- 1. 不准使用计算机。
- 2. 不必使用对数表。
- 3. 对一题得4分, 错一题倒扣1分。

INSTRUTIONS

- 1. Calculators not allowed.
- 2. Logarithm table is not to be used.
- 3. 4 marks will be awarded for each correct answer and 1 mark will be deducted for each wrong answer.

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1. 以数字来更换以下的字母,使到任何三个相邻的数字之和为2015。

字母 E 应当以_____来更换。

Replace the letters by numbers so that the sum of any three consecutive numbers is of sum 2015.

The letter *E* should be replaced by _____.

- A. 607
- B. 671
- C. 672
- D. 673
- E. 674

2. 已知
$$\frac{1}{60} + \frac{1}{140} = \frac{1}{k}$$
, 求 k 之值。

Given that $\frac{1}{60} + \frac{1}{140} = \frac{1}{k}$, find the value of k.

- A. 21
- B. 24
- C. 35
- D. 42
- E. 45

3. 某店的所有商品都需要附上 6%的消费税 (GST)。比如,顾客需要付 RM1060 来买售价 RM1000 的商品。若王先生付了 RM2597 来买电视机,求该电视机的售价。

The Goods and Services Tax (GST) of 6% is charged for all appliances at a shop. For example, customers need to pay RM1060 to buy an appliance with selling price RM1000. If Mr. Heng paid RM2597 for a TV set, find the selling price of the TV set.

- A. RM2400
- B. RM2450
- C. RM2460
- D. RM2480
- E. RM2500

4. 已知
$$2a+7b=5(3a-b)$$
,求 $\frac{a}{b}$ 之值。

Given that 2a+7b=5(3a-b), find the value of $\frac{a}{b}$.

- A. $\frac{13}{8}$ B. $\frac{13}{12}$ C. $\frac{12}{13}$

5. 求奇数
$$n$$
 的最小值使到 $1+3+5+7+9+\cdots+n>300$ 。

Find the smallest odd integer *n* such that $1+3+5+7+9+\cdots+n > 300$.

- A. 29
- B. 31
- C. 33
- D. 35
- E. 37

The average of 2015 numbers is 2015. After one of the numbers is removed, the new average is 2014. Find the value of that removed number.

- A. 4029
- B. 4027
- C. 2015
- D. 2014
- E. 1008

7.
$$2015 - 2014 + 2013 - 2012 + \dots + 3 - 2 + 1 =$$

- A. 1005
- B. 1006
- C. 1007
- D. 1008
- E. 1009

The last non-zero digit of $10! = 1 \times 2 \times 3 \times \cdots \times 10 = 3628800$ is 8. Find the last non-zero digit of $20! = 1 \times 2 \times 3 \times \cdots \times 20$.

- A. 2
- B. 4
- C. 6
- D. 8
- E. 9

As shown in Figure 2, ABCD is a rectangle where the diagonal BD = 10. Find the greatest value of the area of *ABCD*.

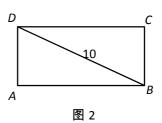
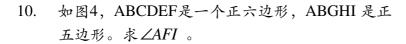


Figure 2

- A. 48
- B. 49
- C. 50
- D. 51
- E. 52



As shown in Figure 4, ABCDEF is a regular hexagon and ABGHI is a regular pentagon. Find ∠AFI .



B. 84°

C. 86°



E. 90°

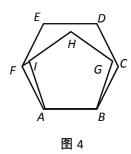


Figure 4

如图5,两个直角三角形ABC及BCD相连,线 11. 段BC是它们的公共边。 以AB, BD, DC及 CA, 画出四个大小不一的正方形, 其中三个正 方形的面积分别为5,24及39。求剩下的一个正 方形的面积。

> As shown in Figure 5, two right triangles ABC and BCD are joined by the common segment BC. Using the sides AB, BD, DC and CA, four different squares are drawn. The areas of the three squares are 5, 24 and 39. Find the area of the remaining square.



B. 12

C. 14

D. 16

E. 18

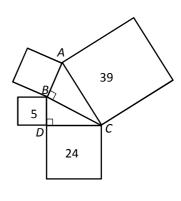


图 5

Figure 5

已知四个正整数a, b, a+b及a-b是质数, 那么这四个质数之和是___。 12.

Given that the four positive integers a, b, a+b and a-b are prime numbers, the sum of these four prime numbers is ____.

A. 质数 prime B. 偶数 even

C. 3的倍数

D. 5的倍数

E. 7的倍数

multiple of 3

multiple of 5

multiple of 7

13. 已知 a 及 b 是正数,且2a+3b=60。求 ab 的最大值。

Given that a and b are positive numbers such that 2a+3b=60, find the largest possible value of ab.

A. 124

B. 126

C. 144

D. 150

E. 152

14. 若 $a=\sqrt{2015^2-4030}$, $b=\sqrt{2013^2+4026}$ 及 c=2014 。那么以下哪项正确? If $a=\sqrt{2015^2-4030}$, $b=\sqrt{2013^2+4026}$ and c=2014, then which of the following statements is true?

A. a < b < c

B. a = b < c

C. b < a < c

D. c < b = a

E. a=b=c

15. 在一个数学比赛,共有30题。每答对一题得5分,答错一题扣2分,没有作答的题目没有分。某个学生没有作答一些题目,共得87分。有多少题目他/她没有作答呢?

At a math contest, 30 problems were given. Each correct answer earned 5 points, and 2 points were deducted for each incorrect answer. No point if a problem is not answered. A particular student did not answer some problems, receiving a score of 87. How many questions did he/she not answer?

A. 7

B. 6

C. 5

D. 4

E. 3

16. 在2015年,爷爷的岁数是他孙子的岁数的7倍。在2017年,爷爷的岁数将是他孙子的岁数的6倍,那么在____年,爷爷的岁数将是他孙子的岁数的5倍。

In year 2015, grandpa's age is 7 times of his grandson's age. In year 2017, grandpa's age will be 6 times of his grandson's age then; in year _____, grandpa's age will be 5 times of his grandson's age then.

A. 2018

B. 2019

C. 2020

D. 2021

E. 2022

17. 某个直角三角形的边长分别为3x, 4x-8及4x+1。求该三角形的面积。

A right angled triangle has sides measuring 3x, 4x-8 and 4x+1. Find the area of that triangle.

A. 220

B. 210

C. 190

D. 180

E. 170

18. 已知 $x^2 + 2y = 7 = y^2 + 2x$, 且x > y。求x - y之值。

Given that $x^2 + 2y = 7 = y^2 + 2x$ where x > y, find the value of x - y.

A. 5

B. 4

C. 3

D. 2

E. 1

将整数1至2015排在一起,形成一个新的号码,如下 19.

当这个号码除以9时,余数是多少?

A number is formed by writing the integers from 1 to 2015 in a connected way, as follows:

1234567...201320142015

What is the remainder when the above number is divided by 9?

- A. 0
- B. 1
- C. 3
- D. 6
- E. 8

已知 $u^2 + 5u + 3 = 0$ 及 $3v^2 + 5v + 1 = 0$,且 $uv \neq 1$ 。求 $\frac{uv + 3u + 1}{v}$ 之值。 20.

Given that $u^2 + 5u + 3 = 0$ and $3v^2 + 5v + 1 = 0$ where $uv \ne 1$, find the value of

- A. 4

- B. 5 C. 6 D. 7
- E. 8

21. 已知 $a_1 \times a_2 \times a_3 \times \cdots \times a_{100} = 100$, a_i 是正整数。求 $a_1 + a_2 + a_3 + \cdots + a_{100}$ 的最小值。

Given that $a_1 \times a_2 \times a_3 \times \cdots \times a_{100} = 100$ where each a_i is a positive integer, find the least possible value of $a_1 + a_2 + a_3 + \cdots + a_{100}$.

- A. 100
- B. 105
- C. 110
- D. 118
- E. 199

$$22. \quad 2 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2 + \cdots}}}} =$$

- A. $1+\sqrt{2}$ B. $\sqrt{2}-1$ C. $1-\sqrt{2}$ D. $2-\sqrt{2}$ E. $2+\sqrt{2}$

23. 如图1,正方形DEFG内接于直角三角形ABC。已知 AD=20及EB=45,求DE的长度。

As shown in Figure 1, square DEFG is inscribed in the right triangle ABC. It is known that AD = 20 and EB = 45, find the length of DE.



B. 29

C. 30

D. 31

E. 32

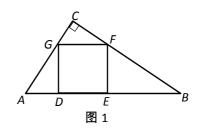


Figure 1

24. 如图3,一个直径为24的半圆覆盖着另一个直径 为12的半圆。求阴影部分的周长。

As shown in Figure 3, a semicircle with diameter 24 overlaps another semicircle with diameter 12. Find the perimeter of the shaded region.



B. 8π

C. 9π

D. 10π

E. 11π

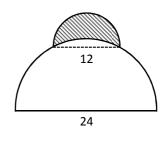


图 3

Figure 3

25. 完全平方数是指可以写成某个整数的平方,如0,1,4,9等等。

共有多少个整数 n 使得 $\frac{n}{100-n}$ 是个完全平方数?

A perfect square can be written as square of some integer, for example 0, 1, 4, 9 etc. How many integers n are there such that $\frac{n}{100-n}$ is a perfect square?

A. 3

B. 4

C. 5

D. 6

E. 7

26. 如图6,O 是圆的中心,AC是圆的切线,B是AO与圆的交点。已知 AC = 24 及 AB = 16,求BO的长度。

As shown in Figure 6, O is the center of the circle, AC is a tangent to the circle at C while B is the intersection point of the circle and AO. Given that AC = 24 and AB = 16, find the length of BO.



B. 11

C. 12

Figure 6

图 6

D. 13

E. 14

27. 如图7, ABC 是等腰三角形, AB = AC = 7。点 D在线BC上使到AD=6。求 $BD\times DC$ 之值。

> As shown in Figure 7, ABC is an isosceles triangle with AB = AC = 7, D is on BC such that AD = 6. Find the value of $BD \times DC$.



B. 13

C. 14

D. 15

E. 16

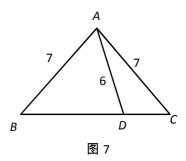


Figure 7

28. 如图8, ABCD 是平行四边形, 点P在ABD内使 到三角形APB的面积为12、三角形PBC的面积为 20。求三角形BDP的面积。

> As shown in Figure 8, ABCD is a parallelogram and P is a point in ABD such that the area of triangle APB is 12 while the area of the triangle PBC is 20. Find the area of triangle BDP.



B. 11

C. 9

D. 8

E. 7

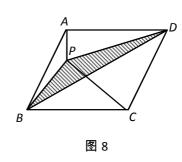


Figure 8

29. 以下哪些是对的?

Which of the following is/are true?

I.
$$1^3 + 2^3 + 3^3 + 4^3 = (1 + 2 + 3 + 4)^2$$

II.
$$1^3 + 2^3 + 2^3 + 4^3 = (1 + 2 + 2 + 4)^2$$

III.
$$1^3 + 3^3 + 3^3 + 4^3 = (1 + 3 + 3 + 4)^2$$

A. I, II, III

B. I, II

C. I, III

D. I

E. II, III

$$30. \qquad \frac{\left(x^x y^y\right)^2}{x^y y^x} =$$

A.
$$\left(\frac{x}{y}\right)^{2x-y}$$
 B. $\left(\frac{x}{y}\right)^{x^2-y}$ C. $\frac{x^{2x-y}}{y^{x-2y}}$ D. $\frac{x^{x^2-y}}{y^{x-y^2}}$ E. $\frac{x^{2x-y}}{y^{2y-x}}$

若 $\overline{abcd} = 9 \times \overline{dcba}$,其中a, b, c 及 d 代表不同的号码,求 c 之值。 31.

If $abcd = 9 \times dcba$, where a, b, c and d represent different digits, find the value of c.

- B. 2
- C. 4
- D. 6

32. $\frac{2}{5} < \frac{x}{v} < \frac{1}{2}$, 其中 x 及 y 是正整数, 求 y 的最小值。

If $\frac{2}{5} < \frac{x}{v} < \frac{1}{2}$, where x and y are positive integers, find the smallest possible value of y.

- A. 6
- B. 7
- C. 10
- D. 20
- E. 25

33.

If $2\log_n(n-2) + \log_{n-2}(n) = 3$, find the value of n.

- A. 1
- B. 3
- C. 4
- D. 5
- E. 10

号码1,2,3,4,5,6,7及9被用来组成四个2位数质数,每个号码只用一 34. 次。求这四个质数的和。

The digits 1, 2, 3, 4, 5, 6, 7, and 9 are used to form four 2-digit prime numbers, with each digit used exactly once. Find the sum of these four primes.

- A. 190
- B. 180
- C. 170
- D. 160
- E. 150

0.201515151515...= 35.

- A. $\frac{2015}{9900}$ B. $\frac{1995}{9990}$ C. $\frac{1995}{9900}$ D. $\frac{2015}{9990}$ E. $\frac{2015}{9999}$