雪 兰 莪 暨 吉 隆 坡 福 建 会 馆
新　纪　元　大　学　学　院　　联合主办

ANJURAN BERSAMA
PERSATUAN HOKKIEN SELANGOR DAN KUALA LUMPUR
&
KOLEJ UNIVERSITI NEW ERA

第三十二届（2017 年度）
雪隆中学华罗庚杯数学比赛
PERTANDINGAN MATEMATIK PIALA HUA LO-GENG ANTARA SEKOLAH-SEKOLAH MENENGAH DI NEGERI SELANGOR DAN KUALA LUMPUR YANG KE-32(2017)

~~高中组~~
BAHAGIAN MENENGAH ATAS

日期　：2017 年 8 月 13 日 （星期日）
Tarikh　：13 Ogos 2017 (Hari Ahad)

时间　：10:00→12:00（两小时）
Masa　：10:00→12:00（2 jam）

地点　：新纪元大学学院黄迓莱活动中心
Tempat　：Ng Ah Choo Multipurpose Hall, Kolej Universiti New Era
UG, Block C, Lot 5, Seksyen 10, Jalan Bukit,
43000 Kajang, Selangor

*****说明*****

1. 不准使用计算机。
2. 不必使用对数表。
3. 对一题得 4 分，错一题倒扣 1 分。
4. 答案 E：若是“以上皆非”或“不能确定”，一律以“***”代替之。

*****INSTRUTIONS*****

1. Calculators not allowed.
2. Logarithm table is not to be used.
3. 4 marks will be awarded for each correct answer and 1 mark will be deducted for each wrong answer.
4. (E)***indicates “none of the above”.

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1. 定义$k\,! = 1\times 2\times 3\times\cdots\times k$。若$n = 2017\,! - 9$，求$n$的个位数字。

   Define $k\,! = 1\times 2\times 3\times\cdots\times k$. Given that $n = 2017\,! - 9$. Find the units digit of n.

   A. 1　　B. 3　　C. 5　　D. 7　　E. ***

2. 已知ω是一复数，$\omega^7 = 1$，$\omega \neq 1$，求$\omega + \omega^2 + \omega^3 + \omega^4 + \omega^5 + \omega^6$的值。

   Given that ω is a complex number, $\omega^7 = 1$, $\omega \neq 1$, find the value of $\omega + \omega^2 + \omega^3 + \omega^4 + \omega^5 + \omega^6$.

   A. i　　B. -1　　C. 0　　D. 1　　E. ***

3. 求 840 的所有正的因数之和。

   Find the sum of all the positive factors of 840.

   A. 1440　　B. 2520　　C. 2880　　D. 3360　　E. ***

4. 若x是实数且$x^2 - 8x + 1 = 0$，求$x^4 + \dfrac{1}{x^4}$的值。

   If x is a real number such that $x^2 - 8x + 1 = 0$, find the value of $x^4 + \dfrac{1}{x^4}$.

   A. 3840　　B. 3842　　C. 3844　　D. 3846　　E. ***

5. 一粒梨的价钱是 RM 2，一粒苹果的价钱是 RM 1.50。慧玲付了 RM 100 买到的梨与苹果总共有n粒，求n的最大可能值。

   A pear costs RM 2 and an apple costs RM 1.50. Hui Ling paid RM 100 to buy a total of n pears and apples. What is the largest possible value of n?

   A. 64　　B. 65　　C. 66　　D. 67　　E. ***

6. 今天的中午 12 时到明天的中午 12 时之间，时钟上的时针与分针会有多少次形成$120°$角？

   Between 12 p.m. today and 12 p.m. tomorrow, how many times do the hour hand and the minute hand on a clock form an angle of $120°$?

   A. 24　　B. 44　　C. 46　　D. 48　　E. ***

7. 求满足方程式 $(2x^2+5x+1)^{2x-3}=1$ 的所有实数 x 之和。

Find the sum of all the real numbers x that satisfy the equation $(2x^2+5x+1)^{2x-3}=1$.

A. -5　　B. $-\frac{7}{2}$　　C. $-\frac{5}{2}$　　D. -2　　E. ***

8. 一班内有 15 位男同学与 10 位女同学。男同学中 9 位有戴眼镜，女同学中 8 位有戴眼镜。从这班里任意选出一位同学。若这位被选出来的同学没戴眼镜，求他是男同学的概率。

In a class, there are 15 boys and 10 girls. 9 of the boys wear glasses and 8 of the girls wear glasses. A student is chosen randomly from the class. If this chosen student does not wear glasses, find the probability that this chosen student is a boy.

A. $\frac{3}{4}$　　B. $\frac{1}{2}$　　C. $\frac{2}{3}$　　D. $\frac{3}{5}$　　E. ***

9. 已知 $a=3\times10^{2017}$，$b=2\times10^{2017}+1$。求 a 与 b 的最大公因数。

Given that $a=3\times10^{2017}$, $b=2\times10^{2017}+1$. Find the greatest common divisor of a and b.

A. 1　　B. 3　　C. 21　　D. 39　　E. ***

10. 如图 1 所示，ABC 为一等腰三角形，G，H 两点在 BC 上，$EFGH$ 是一以 HG 为直径的半圆，它与 ΔABC 相切于点 E 及点 F。已知 $AB=AC=13$，$BC=10$，求半圆的直径 HG 的长。

As shown in the Figure 1, ABC is an isosceles triangle. G, H are two points on BC, and $EFGH$ is a semicircle with HG as diameter. The semicircle is tangent to ΔABC at the points E and F. Given that $AB=AC=13$, $BC=10$, find the diameter of the semicircle.

A. $\frac{60}{13}$　　B. $\frac{120}{13}$　　C. $\frac{124}{13}$

D. $\frac{62}{13}$　　E. ***

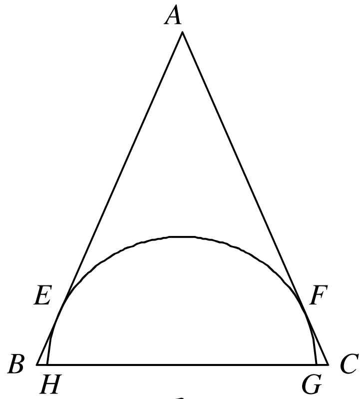

图 1
Figure 1

11. 求大于 $(2+\sqrt{7})^5$ 的最小整数。

Find the smallest integer larger than $(2+\sqrt{7})^5$.

A. 2163　　B. 2164　　C. 2165　　D. 2166　　E. ***

12. 如图 2 所示，圆 O_1 与圆 O_2 相交于 A，D 两点。AB 与 AC 分别为圆 O_1 与圆 O_2 的直径。已知 AB 垂直于 AC，$AB=12$，$AC=16$，$AD=x$。若 $y=10x$，求 y 的值。

As shown in the Figure 2, the circle O_1 and the circle O_2 intersect at two points A and D. AB and AC are respectively the diameters of the circle O_1 and the circle O_2. Given that AB is perpendicular to AC, $AB=12$, $AC=16$, $AD=x$. If $y=10x$, find the value of y.

A. 108　　B. 100　　C. 96

D. 90　　E. ***

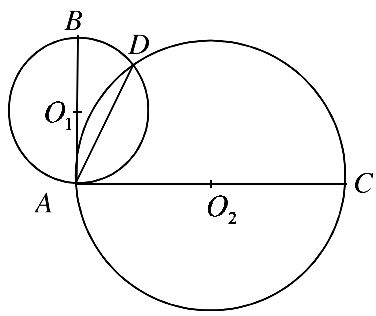

图 2
Figure 2

13. 如图 3 中，纵横交错的直线将 70 个点连接起来。若要沿着这些直线由点 A 走到点 B，每次只能向右走或向上走，有多少种走法？

In the Figure 3, the horizontal lines and vertical lines interweave with each other to connect the 70 points. How many ways can one goes from the point A to the point B along these lines, if he is only allowed to move to the right or move upward?

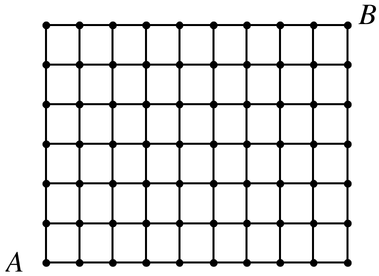

图 3
Figure 3

A. 5005　　B. 3003　　C. 2002

D. 1287　　E. ***

14. 有多少组正整数 (x_1, x_2, x_3, x_4) 满足 $x_1 + x_2 + x_3 + x_4 = 12$？

How many four-tuples of positive integers (x_1, x_2, x_3, x_4) satisfy $x_1 + x_2 + x_3 + x_4 = 12$?

A. 220　　B. 495　　C. 165　　D. 330　　E. ***

15. 如图 4 所示，D 是线段 BC 上的一点使得 $BD : CD = 3 : 2$，E 是线段 AD 上的一点使得 $AE = 3ED$。BE 的延长线与 AC 相交于点 F。已知 ΔABC 的面积为 $350\ \text{cm}^2$，求 ΔADF 的面积。

As shown in the Figure 4, D is a point on the line segment BC such that $BD : CD = 3 : 2$, E is a point on the line segment AD such that $AE = 3ED$. The extension of the line BE meets AC at the point F. Given that the area of ΔABC is $350\ \text{cm}^2$, find the area of ΔADF.

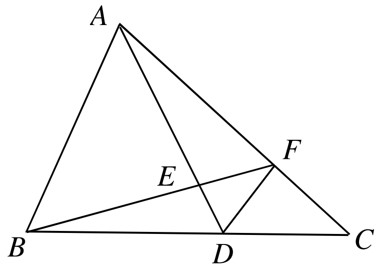

图 4
Figure 4

A. $80\,\text{cm}^2$　　B. $84\,\text{cm}^2$　　C. $90\,\text{cm}^2$　　D. $98\,\text{cm}^2$　　E. ***

16. 若 N 是自然数使得对于任意一个集合 $A = \{N-1, N, N+1\}$ 里的元素 n，$2^n > n^3$，求 N 的最小可能值。

If N is a natural number such that for any element n in the set $A = \{N-1, N, N+1\}$, $2^n > n^3$, find the smallest possible value of N.

A. 8　　B. 9　　C. 10　　D. 11　　E. ***

17. 已知 $\cos 2\theta = -\dfrac{1}{3}$，求 $\sin^6\theta - \cos^6\theta$ 的值。

Given that $\cos 2\theta = -\dfrac{1}{3}$, find the value of $\sin^6\theta - \cos^6\theta$.

A. $-\dfrac{1}{3}$　　B. $\dfrac{1}{3}$　　C. $-\dfrac{7}{27}$　　D. $\dfrac{7}{27}$　　E. ***

18. 已知 $n = \left(19^3 - 3\times18\times19 - 1\right)^2$。求 n 的正因数的个数。

Given that $n = \left(19^3 - 3\times18\times19 - 1\right)^2$. Find the number of positive factors of n.

A. 72　　B. 78　　C. 91　　D. 216　　E. ***

19. 求$\frac{1}{2}+4\times\frac{1}{4}+9\times\frac{1}{8}+16\times\frac{1}{16}+25\times\frac{1}{32}+\ldots+\frac{n^2}{2^n}+\ldots$。

Find $\frac{1}{2}+4\times\frac{1}{4}+9\times\frac{1}{8}+16\times\frac{1}{16}+25\times\frac{1}{32}+\ldots+\frac{n^2}{2^n}+\ldots$.

A. 12 B. 10 C. 8 D. 6 E. ***

20. 若a，b，c为正的实数，求$(a+b+c)\left(\frac{1}{a}+\frac{9}{b}+\frac{25}{c}\right)$的最小可能值。

If a, b, c are positive real numbers, find the minimum possible value of $(a+b+c)\left(\frac{1}{a}+\frac{9}{b}+\frac{25}{c}\right)$.

A. 75 B. 81 C. 100 D. 105 E. ***

21. 若x，y是正数使得$2x+3y=2016$且xy的值最大，求$x-y$的值。

If x, y are positive numbers such that $2x+3y=2016$ and xy has the maximum value, find the value of $x-y$.

A. 168 B. 224 C. 336 D. 448 E. ***

22. 令 $S=\sqrt{1+\frac{1}{1^2}+\frac{1}{2^2}}+\sqrt{1+\frac{1}{2^2}+\frac{1}{3^2}}+\sqrt{1+\frac{1}{3^2}+\frac{1}{4^2}}+\ldots\ldots+\sqrt{1+\frac{1}{98^2}+\frac{1}{99^2}}+\sqrt{1+\frac{1}{99^2}+\frac{1}{100^2}}$。

已知$S=m-\frac{1}{n}$，其中m，n为正整数，求$m+n$的值。

Let $S=\sqrt{1+\frac{1}{1^2}+\frac{1}{2^2}}+\sqrt{1+\frac{1}{2^2}+\frac{1}{3^2}}+\sqrt{1+\frac{1}{3^2}+\frac{1}{4^2}}+\ldots\ldots+\sqrt{1+\frac{1}{98^2}+\frac{1}{99^2}}+\sqrt{1+\frac{1}{99^2}+\frac{1}{100^2}}$.

Given that $S=m-\frac{1}{n}$, where m, n are positive integers. Find the value of $m+n$.

A. 100 B. 150 C. 198 D. 200 E. ***

23. 已知$n=\sqrt{\underbrace{111\ldots1}_{200\text{个}1}-\underbrace{222\ldots2}_{100\text{个}2}}$是一个整数，求$n$的各位数字之和。

Given that $n=\sqrt{\underbrace{111\ldots1}_{200\text{ digits of }1}-\underbrace{222\ldots2}_{100\text{ digits of }2}}$ is an integer, find the sum of the digits of n.

A. 200 B. 300 C. 400 D. 900 E. ***

24. 已知$S=\{1, 2, 3, \ldots, 199, 200\}$及$H=\{(a, b, c) \mid a, b, c\in S, a<c, a+c=2b\}$。求 H 的元素的个数。

Given that $S=\{1, 2, 3, \ldots, 199, 200\}$ and $H=\{(a, b, c) \mid a, b, c\in S, a<c, a+c=2b\}$. Find the number of elements of H.

A. 9900 B. 4950 C. 10100 D. 5050 E. ***

25. 有多少对正整数(m, n)满足下列的条件：

(a) m 与 n 是二位数；

(b) $m-n=16$；

(c) m^2 与 n^2 的最后两位数字相同。

How many pairs of positive integers (m, n) satisfy the following conditions?

(a) m and n are two-digit numbers;

(b) $m-n=16$;

(c) The last two digits of m^2 and n^2 are the same.

A. 0 B. 1 C. 2 D. 3 E. ***

26. 设 $\lfloor x \rfloor$ 为不大于 x 的最大整数（如：$\lfloor 3.5 \rfloor = 3$，$\lfloor 3 \rfloor = 3$）。已知 $a = \lfloor \log_3 x \rfloor$，$b = \left\lfloor \log_3 \frac{81}{x} \right\rfloor$，其中 x 是正实数，求 $a^2 - 2b^2$ 的最大可能值。

Let $\lfloor x \rfloor$ denotes the largest integer that is not larger than x. (For example: $\lfloor 3.5 \rfloor = 3$, $\lfloor 3 \rfloor = 3$.) Given that $a = \lfloor \log_3 x \rfloor$, $b = \left\lfloor \log_3 \frac{81}{x} \right\rfloor$, where x is a positive real number, find the largest possible value of $a^2 - 2b^2$.

A. 36 B. 32 C. 18 D. 12 E. ***

27. 考虑以下的条件：

n 是一个有理数且二次方程式 $3x^2 - \left(6 + 2\sqrt{5}\right)x + \sqrt{5}n - 45 = 0$ 的其中一个根是有理数。

求满足这条件的所有 n 之和。

Consider the following condition:

n is a rational number and one of the roots of the equation $3x^2 - \left(6 + 2\sqrt{5}\right)x + \sqrt{5}n - 45 = 0$ is rational.

Find the sum of all n that satisfies this condition.

A. -4 B. -2 C. 2 D. 4 E. ***

28. 有多少组正整数(x, y, z)满足方程式$19x + 20y + 21z = 399$？

How many triples of positive integers (x, y, z) satisfy the equation $19x + 20y + 21z = 399$?

A. 5 B. 7 C. 9 D. 11 E. ***

29. 如图 5 所示，O 是圆心，$AB \,//\, CD$。已知 $\angle COD = 3\angle AOB$，$AB : CD = 5 : 12$，求 ΔAOB 的面积与 ΔBOC 的面积之比。

As shown in the Figure 5, O is the center of the circle, $AB \,//\, CD$. Given that $\angle COD = 3\angle AOB$, $AB : CD = 5 : 12$, find the ratio of the area of ΔAOB to the area of ΔBOC.

A. $5:6$ B. $5:7$ C. $6:7$

D. $7:9$ E. ***

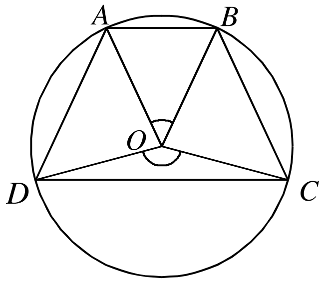

图 5

Figure 5

30. 已知一数列$\{a_n\}$的首项为$a_1=5$，且对于所有的$n\geq 1$，$a_{n+1}=\dfrac{1+\sqrt{3}a_n}{\sqrt{3}-a_n}$。求$a_{100}$的值。

Given that the first term of a sequence $\{a_n\}$ is $a_1=5$, and $a_{n+1}=\dfrac{1+\sqrt{3}a_n}{\sqrt{3}-a_n}$ for all $n\geq 1$. Find the value of a_{100}.

A. $-\dfrac{13\sqrt{3}}{11}-\dfrac{10}{11}$　B. $-\dfrac{13\sqrt{3}}{37}-\dfrac{10}{37}$　C. $-\dfrac{1}{5}$　D. 5　E. ***

31. 如图 6 所示，ΔABC是一等边三角形纸板，边长等于 12。将此纸板沿直线DE折，使得其顶点A落在BC边上的点F。若$BF=4$，$DE=x$，求x^2。

As shown in the Figure 6, ΔABC is a cardboard of the shape an equilateral triangle with side length 12. The cardboard is folded along the line DE such that the vertex A falls on the point F on BC. If $BF=4$, $DE=x$, find x^2.

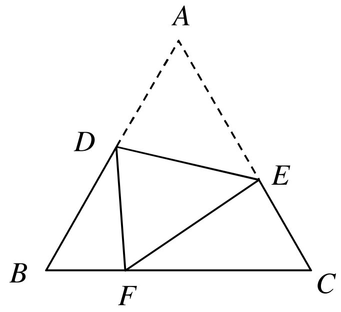

图 6
Figure 6

A. $\dfrac{1019}{25}$　B. $\dfrac{1029}{25}$　C. $\dfrac{1039}{25}$

D. $\dfrac{1049}{25}$　E. ***

32. 已知x_0，x_1，x_2，…是一数列，$x_0=1$且对于所有$n\geq 1$，

$$x_n=-\frac{225}{n}(x_0+x_1+x_2+\ldots+x_{n-1})$$

求$x_0+2x_1+2^2x_2+\ldots+2^{224}x_{224}+2^{225}x_{225}$的值。

Given that x_0, x_1, x_2, … is a sequence of numbers such that $x_0=1$ and for all $n\geq 1$,

$$x_n=-\frac{225}{n}(x_0+x_1+x_2+\ldots+x_{n-1})$$

Find the value of $x_0+2x_1+2^2x_2+\ldots+2^{224}x_{224}+2^{225}x_{225}$

A. -2　B. -1　C. 1　D. 2　E. ***

33. 如图 7 所示，ΔABC为等边三角形，直线DE，FG，KL分别平行于AC，BC，AB；直线FL，DK，GE分别平行于AC，BC，AB。这六条直线将ΔABC分为 10 个区域，其面积由小到大顺序为S_1，S_2，…，S_{10}。当S_1的值最大时，设$x=\dfrac{S_{10}}{S_1}$。求$2x$的值。

As shown in the Figure 7, ΔABC is an equilateral triangle. The lines DE, FG, KL are parallel respectively to AC, BC, AB, and the lines FL, DK, GE are parallel respectively to AC, BC, AB. These six lines divide ΔABC into 10 regions. The areas of these 10 regions in ascending order are S_1, S_2, …, S_{10}. Let $x=\dfrac{S_{10}}{S_1}$, when S_1 has the largest value, find the value of $2x$.

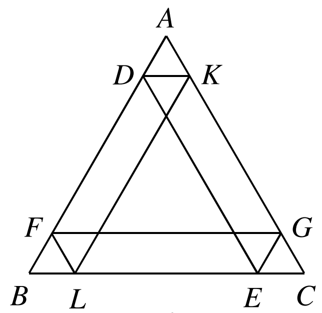

图 7
Figure 7

A. 4　B. 5　C. 6　D. 7　E. ***

34. 如图 8 所示，圆C_1与圆C_2内切于点A，它们的半径分别为 7 及 9，AB与AD分别是两圆的直径。点P及点T分别在圆C_1与圆C_2上使得直线PT与圆C_1相切。若$PT=6$，$\angle TAB=x$，求$\cos^2 x$。

As shown in the Figure 8, the circle C_1 is tangent internally to the circle C_2 at the point A. The radii of the circles are 7 and 9 respectively. AB and AD are diameters of the circles. The points P and T are on the circles C_1 and C_2 respectively such that the line PT is tangent to the circle C_1. If $PT=6$, $\angle TAB=x$, find $\cos^2 x$.

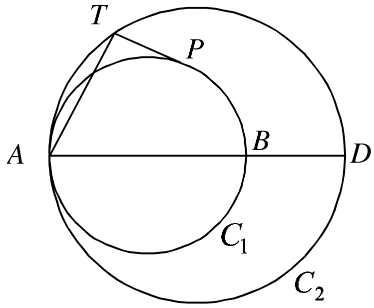

图 8
Figure 8

A. $\frac{3}{4}$　　B. $\frac{2}{3}$　　C. $\frac{3}{5}$　　D. $\frac{1}{2}$　　E. ***

35. 一个袋子中装有 20 粒分别写上数字 1 至 20 的球。从袋中任意取出二球，求这两粒球上面的数字的和可以被 4 整除的概率。

A bag contains 20 balls that are numbered 1 to 20. Two balls are drawn randomly from the bag. Find the probability that the sum of the numbers on the two balls is divisible by 4.

A. $\frac{1}{4}$　　B. $\frac{4}{19}$　　C. $\frac{9}{38}$　　D. $\frac{5}{19}$　　E. ***

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