

雪兰莪暨吉隆坡福建会馆  
新 纪 元 学 院

联合主办

**ANJURAN BERSAMA  
PERSATUAN HOKKIEN SELANGOR DAN KUALA LUMPUR  
&  
KOLEJ NEW ERA**

**第二十九届（2014 年度）  
雪隆中学华罗庚杯数学比赛**

**PERTANDINGAN MATEMATIK PIALA HUA LO-GENG  
ANTARA SEKOLAH-SEKOLAH MENENGAH  
DI NEGERI SELANGOR DAN KUALA LUMPUR  
YANG KE-29 (2014)**

**~~ 初 中 组 ~~  
BAHAGIAN MENENGAH RENDAH**

日期 : 2014 年 8 月 10 日（星期日）  
Tarikh : 10 Ogos 2014 (Hari Ahad)

时间 : 10:00 → 12:00 （两小时）  
Masa : 10:00 → 12:00 （2 jam）

地点 : 新纪元学院 UG 活动中心  
Tempat : UG Hall Kolej New Era  
Blok C, Lot 5, Seksyen 10, Jalan Bukit,  
43000 Kajang, Selangor.

**\*\*\* 说明 \*\*\***

1. 不准使用计算机。
2. 不必使用对数表。
3. 对一题得 4 分，错一题倒扣 1 分。
4. 答案 E: 若是“以上皆非”或“不能确定”，一律以“\*\*\*”代替之。

**\*\*\* INSTRUCTIONS \*\*\***

1. Calculators not allowed.
2. Logarithm table is not to be used.
3. 4 marks will be awarded for each correct answer and 1 mark will be deducted for each wrong answer.
4. (E) \*\*\* indicates “none of the above”.

1.  $3^2 + 4^2 + 12^2 =$

- A  $13^2$                       B  $15^2$                       C  $17^2$                       D  $19^2$                       E  $21^2$

2. 某数学运算  $\Delta$  被定义为  $a \Delta b = a^b$ 。那么  $2 \Delta (3 \Delta 2) =$

An operation  $\Delta$  is defined as  $a \Delta b = a^b$ . Then  $2 \Delta (3 \Delta 2) =$

- A 64                      B 128                      C 256                      D 512                      E 1024

3. 若  $2048 \times 2^{2014} = 2^k$ ，那么  $k =$

If  $2048 \times 2^{2014} = 2^k$ , then  $k =$

- A 2025                      B 2024                      C 2023                      D 2022                      E 2021

4. 已知  $a$  及  $b$  是正整数，且  $a^2 + b^2 - 13 = ab$ 。求  $ab$  的最大值。

Given that  $a$  and  $b$  are positive integers such that  $a^2 + b^2 - 13 = ab$ , find the largest possible value of  $ab$ .

- A 4                      B 8                      C 12                      D 13                      E \*\*\*

5. 求  $19\frac{17}{18} \times 35\frac{19}{20}$  的个位近似值。

Find the value of  $19\frac{17}{18} \times 35\frac{19}{20}$ , to the nearest integer.

- A 720                      B 719                      C 718                      D 717                      E 716

6. 有多少不同的  $n$  能满足以下条件？

“数据 3, 6, 7, 8,  $n$  的中位数等于其平均数”

How many distinct  $n$  satisfy the following condition?

“The median of the data 3, 6, 7, 8,  $n$  is equal to its mean”

- A 0                      B 1                      C 2                      D 3                      E 4

7. 求  $2014 \left( \frac{1}{19 \times 20} + \frac{1}{20 \times 21} + \frac{1}{21 \times 22} + \cdots + \frac{1}{37 \times 38} \right)$  之值。

Find the value of  $2014 \left( \frac{1}{19 \times 20} + \frac{1}{20 \times 21} + \frac{1}{21 \times 22} + \cdots + \frac{1}{37 \times 38} \right)$ .

- A 38                      B 53                      C 54                      D 63                      E 68

8. 设  $a > b > 1$ , 求  $\frac{1}{1 - \log_{a^2b}(a)} + \frac{1}{1 - \log_{ab^2}(b)}$  之值。

Let  $a > b > 1$ . Find the value of  $\frac{1}{1 - \log_{a^2b}(a)} + \frac{1}{1 - \log_{ab^2}(b)}$ .

- A 1                      B 2                      C 3                      D 4                      E 5

9. 已知正整数  $m$  共有 6 个正因数, 那么  $m^2$  的正因数的个数可能是

(例: 正整数 4 共有 3 个正因数, 分别为 1, 2 及 4。那么  $4^2$  共有五个正因数, 分别为 1, 2, 4, 8 及 16)

It is known that the positive integer  $m$  has 6 positive factors, then the number of positive factors of  $m^2$  can possibly be

(Example: the positive integer 4 has 3 positive factors, namely 1, 2 and 4. Then the number  $4^2$  has 5 positive factors, namely 1, 2, 4, 8 and 16)

- I 11                      II 12                      III 15

- A I                      B II                      C III                      D I, III                      E I, II, III

10. 若  $u$  及  $v$  是  $|x - \sqrt{3}| = |2x + \sqrt{5}|$  的解, 求  $uv$  之值。

If  $u$  and  $v$  are the roots of  $|x - \sqrt{3}| = |2x + \sqrt{5}|$ , find the value of  $uv$ .

- A 3                      B  $\frac{3}{4}$                       C  $\frac{2}{3}$                       D  $-\frac{2}{3}$                       E  $-\frac{3}{4}$

11. 已知  $x$  及  $y$  是正整数, 且  $5x + 17y \leq 2014$ 。求  $x$  的最大可能值。

Given that  $x$  and  $y$  are positive integers such that  $5x + 17y \leq 2014$ , find the largest possible value of  $x$ .

- A 402                      B 401                      C 400                      D 399                      E 398

12. 已知  $u$  及  $v$  为实数, 且

$$u = \frac{\sqrt{1-v^2} + \sqrt{v^2-1}}{1-v} + 13 - v,$$

求  $u$  之值。

Given that  $u$  and  $v$  are real numbers, and

$$u = \frac{\sqrt{1-v^2} + \sqrt{v^2-1}}{1-v} + 13 - v,$$

find the value of  $u$ .

- A 14                      B 13                      C 12                      D 11                      E 10

13. 已知  $(a+b):c=1:2$  及  $a:(b+c)=1:4$ , 那么  $a:c=$

Given that  $(a+b):c=1:2$  and  $a:(b+c)=1:4$ , then  $a:c=$

**A** 1:3                      **B** 2:5                      **C** 2:7                      **D** 3:10                      **E** 4:11

14. 设  $f(x)=x^2-7x+11$ 。若  $f(a)=a+2$  及  $f(b)=b+2$ , 求  $a+b$  之值。

Let  $f(x)=x^2-7x+11$ . Given that  $f(a)=a+2$  and  $f(b)=b+2$ , find the value of  $a+b$ .

**A** 9                      **B** 8                      **C** 7                      **D** 6                      **E** 5

15. 已知  $a$  及  $b$  为正整数, 且  $b>a$  及  $\frac{1}{a}+\frac{1}{b}=\frac{1}{10}$ 。

(例: 当  $a=11$  及  $b=110$ , 那么  $\frac{1}{a}+\frac{1}{b}=\frac{1}{11}+\frac{1}{110}=\frac{1}{10}$ )

求  $a$  的最大可能值。

Given that  $a$  and  $b$  are positive integers such that  $b>a$  and  $\frac{1}{a}+\frac{1}{b}=\frac{1}{10}$ .

(Example: when  $a=11$  and  $b=110$ , then  $\frac{1}{a}+\frac{1}{b}=\frac{1}{11}+\frac{1}{110}=\frac{1}{10}$ )

Find the largest possible value of  $a$ .

**A** 12                      **B** 14                      **C** 15                      **D** 18                      **E** 20

16. 已知  $x$  及  $y$  为正实数使得  $\frac{1}{x}+\frac{4}{y}=1$ 。求  $x+y$  的最小值。

Given that  $x$  and  $y$  are positive real numbers such that  $\frac{1}{x}+\frac{4}{y}=1$ . Find the minimum value of  $x+y$ .

**A** 8                      **B** 9                      **C** 10                      **D** 11                      **E** 12

17. 有多少个整数  $x$  满足以下条件?

“ $y$ 是整数, 且  $2x+8xy=2014$ ”

How many integers  $x$  satisfy the following condition?

“ $y$  is an integer and  $2x+8xy=2014$ ”

**A** 4                      **B** 6                      **C** 8                      **D** 10                      **E** 12

18. 有多少个正整数  $n$  使得  $\frac{n^2+45}{n+3}$  也是个正整数?

How many positive integer  $n$  are there such that  $\frac{n^2+45}{n+3}$  is also a positive integer?

**A** 4                      **B** 5                      **C** 6                      **D** 7                      **E** 8

19. 设  $p$  及  $q$  为质数, 且  $p<q$ 。已知  $p^2+q^2=6266$ , 求  $q$  之值。

Let  $p$  and  $q$  be prime numbers and  $p<q$ . Given that  $p^2+q^2=6266$ , find the value of  $q$ .

**A** 79                      **B** 71                      **C** 61                      **D** 59                      **E** 41

20. 已知 4 位数  $\overline{abbb}$  是个完全平方, 即  $\overline{abbb} = k^2$ ,  $k$  是正整数。求  $b$  之值。

Given that the 4-digit number  $\overline{abbb}$  is a perfect square, namely  $\overline{abbb} = k^2$  for some positive integer  $k$ . Find the value of  $b$ .

- A 0                      B 1                      C 4                      D 5                      E 6

21. 若  $x = 2 + \sqrt{3}$ , 求  $x^3 - 2x^2 - 7x + 2014$  之值。

If  $x = 2 + \sqrt{3}$ , find the value of  $x^3 - 2x^2 - 7x + 2014$ .

- A 2011                      B 2012                      C 2013                      D 2014                      E 2015

22. 已知 4 位数  $\overline{abcd}$  是 4 的倍数。求  $a + b + c + d$  的最大可能值。

Given that the four digit integer  $\overline{abcd}$  is a multiple of 4. Find the largest possible value of  $a + b + c + d$ .

- A 32                      B 33                      C 34                      D 35                      E 36

23. 求  $2012^5$  被 2014 除的余数。

Find the remainder when  $2012^5$  is divided by 2014.

- A 2012                      B 2011                      C 1992                      D 1982                      E 1906

24. 在 3 点之后, 时针与分针何时 (准确至分钟) 第一次形成  $35^\circ$ ?

What is the first time, to the nearest minutes, after 3 o'clock that the clock hand and the minute hand of the clock make an angle of  $35^\circ$ ?

- A 3:13                      B 3:12                      C 3:11                      D 3:10                      E 3:11

25. 利用 0 至 9 十个数字组成两个五位数  $M$  及  $N$  使得  $\frac{M}{N} = \frac{1}{2}$  且没有号码被重复使用。(例:  $M = 48651$ ,  $N = 97302$ )

求  $M + N$  的最小可能值。

The 10 digits 0 to 9 are used to form two 5-digit numbers  $M$  and  $N$  such that  $\frac{M}{N} = \frac{1}{2}$  and no digit is repeatedly used.

(Example:  $M = 48651$ ,  $N = 97302$ )

Find the smallest possible value of  $M + N$ .

- A 30702                      B 40425                      C 40455                      D 40644                      E 44595

26. 点  $O$  是四边形  $ABCD$  内的一点, 它与四个顶点的距离为 2, 3, 4 及 8 (次序不一)。求  $ABCD$  的面积的最大可能值。

The point  $O$  is a point inside a quadrilateral  $ABCD$  such that its distances from the four vertices are 2, 3, 4 and 8 in some order. Find the largest possible value for the area of  $ABCD$ ?

- A 32                      B 33                      C 34                      D 35                      E 36

27. 图 1 中共有多少个三角形?

How many triangles are there in the following Figure 1?

- A 15                      B 14                      C 13  
D 12                      E 11

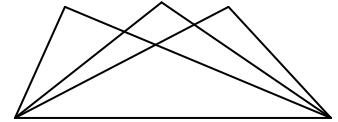


图 1  
Figure 1

28. 如图 2, 已知长方形  $ABCD$  的面积为 130, 点  $E$  及  $F$  在对角线  $BD$  上使得  $DE : EF : FB = 1 : 2 : 2$   
求三角形  $CEF$  的面积。

As shown in Figure 2, it is known that the area of rectangle  $ABCD$  is 130. The points  $E$  and  $F$  are on the diagonal  $BD$  such that

$$DE : EF : FB = 1 : 2 : 2$$

Find the area of triangle  $CEF$ .

- A 26                      B 27                      C 28  
D 29                      E 30

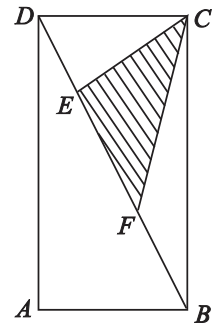


图 2  
Figure 2

29. 如图 3,  $BCE$  是条直线,  $2\angle ABD = 3\angle EBD$  及  $2\angle ACD = 3\angle ECD$ 。已知  $\angle BAC = 80^\circ$ , 求  $\angle BDC$ 。

As shown in Figure 3,  $BCE$  is a straight line,  $2\angle ABD = 3\angle EBD$  and  $2\angle ACD = 3\angle ECD$ . Given that  $\angle BAC = 80^\circ$ , find  $\angle BDC$ .

- A  $30^\circ$                       B  $32^\circ$                       C  $35^\circ$   
D  $38^\circ$                       E  $42^\circ$

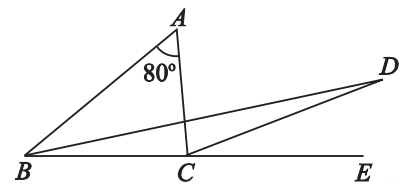


图 3  
Figure 3

30. 如图 4, 每一个圆的面积为 6。每一个圆的中心正好是星形的顶点。求阴影部分的总面积。

As shown in Figure 4, the area of each circle is 6. The centers of these five circles are the vertices of the star. Find the total area of shaded regions.

- A 29                      B 28                      C 27  
D 26                      E 25

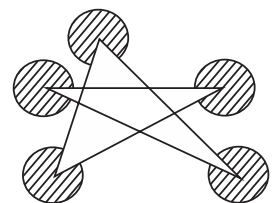


图 4  
Figure 4

31. 如图 5,  $AOB$  是半径为 14 的四分之一圆;  $PQRO$  是长方形。已知三角形  $PQR$  的周长为 35, 求阴影部分的周长。(取  $\pi = \frac{22}{7}$ )

As shown in Figure 5,  $AOB$  is a quarter circle with radius 14 and  $PQRO$  is a rectangle. Given that the perimeter of triangle  $PQR$  is 35, find the perimeter of the shaded region. (Take  $\pi = \frac{22}{7}$ )

- A 43                      B 44                      C 45  
D 46                      E 47

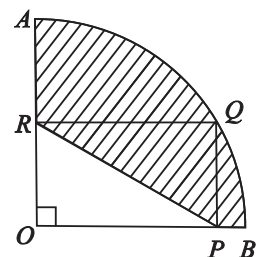


图 5  
Figure 5

32. 如图6, 点 $E$ 及 $F$ 分别在直线 $AB$ 及 $BC$ 上使得

$$AE:EB = 2:1 \text{ 及 } BF:FC = 2:1.$$

$AF$  及  $DE$  相交于  $G$ . 求  $AG:GF$ .

As shown in Figure 6, the points  $E$  and  $F$  are on the line  $AB$  and  $BC$  respectively such that

$$AE:EB = 2:1 \text{ and } BF:FC = 2:1.$$

$AF$  and  $DE$  intersect at  $G$ . Find  $AG:GF$ .

- A 3:4                      B 4:5                      C 6:7  
D 7:8                      E 7:9

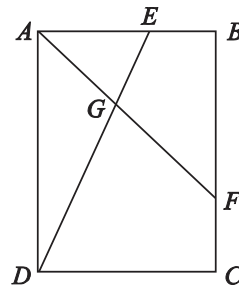


图 6  
Figure 6

33. 如图 7,  $AB=7$ ,  $BC=12$  及  $AC=11$ . 已知点  $D$  是  $BC$  上的点使得  $\angle ADC = 90^\circ$ , 求  $AD$  的长度。

As shown in Figure 7,  $AB=7$ ,  $BC=12$  and  $AC=11$ . Given that the point  $D$  is on  $BC$  such that  $\angle ADC = 90^\circ$ , find the length of  $AD$ .

- A  $\frac{77}{12}$                       B  $\frac{9}{\sqrt{2}}$                       C  $3\sqrt{5}$   
D  $2\sqrt{10}$                       E 6.5

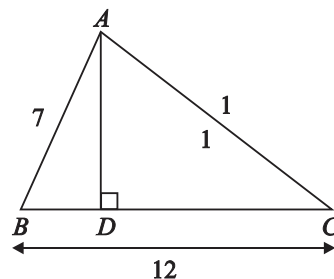


图 7  
Figure 7

34. 如图 8,  $AM = BN = AB$  及  $\angle C = 41^\circ$ . 求  $\angle APB$ .

As shown in Figure 8,  $AM = BN = AB$ , and  $\angle C = 41^\circ$ . Find  $\angle APB$ .

- A  $98^\circ$                       B  $94^\circ$                       C  $90^\circ$   
D  $86^\circ$                       E  $82^\circ$

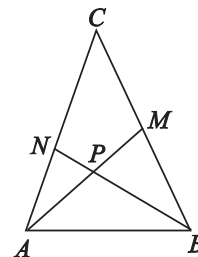


图 8  
Figure 8

35. 如图 9,  $AB$  是半圆的直径。点  $C$  及  $D$  在圆周上, 且  $\angle CBD = \angle DBA$ 。弦  $AC$  与  $BD$  相交于  $E$ 。已知  $AE=36$  及  $EC=28$ , 求  $DE$  的长度。

As shown in Figure 9,  $AB$  is the diameter of the semicircle. The points  $C$  and  $D$  are on the circle and  $\angle CBD = \angle DBA$ . The chords  $AC$  and  $BD$  intersect at  $E$ . Given that  $AE=36$  and  $EC=28$ . Find the length of  $DE$ .

- A 28                      B 22                      C 17  
D 12                      E 10

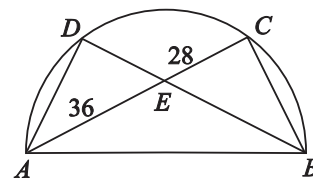


图 9  
Figure 9