Mixed models workshop

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Getting files

`github.com/mbjoseph/hierarchical_models`

Download as zipped folder

or

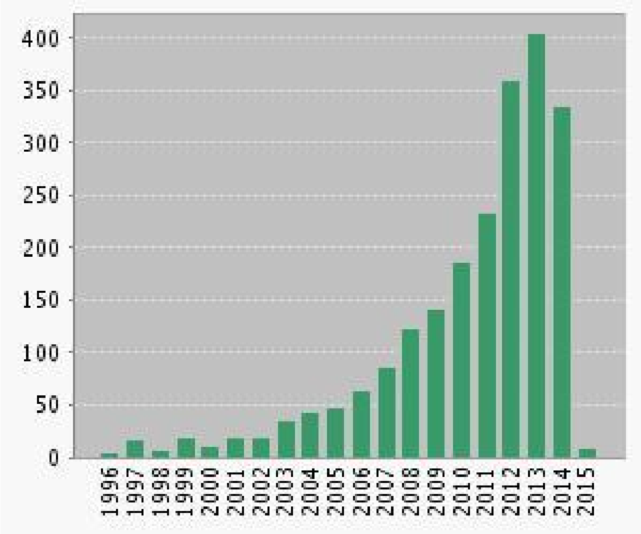
```
git clone git@github.com/mbjoseph/hierarchical_models
```

Overview

1. Why bother?
2. Random intercept models
3. Random slope and intercept models
4. Other resources

Why bother?

{"mixed model" OR
"mixed models" OR
"mixed modeling"}
AND {ecolog* OR evol*}



Why bother?

- increasing use
- broader scope of inference

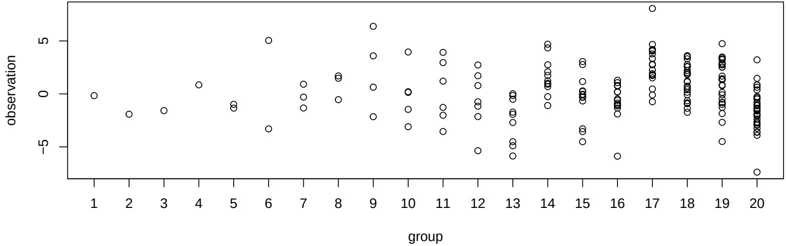
Why bother?

- increasing use
- broader scope of inference
- *improved estimates*

Scenario

Estimate group means α_j with data y_{ij} from J groups

Tragically unequal sample sizes

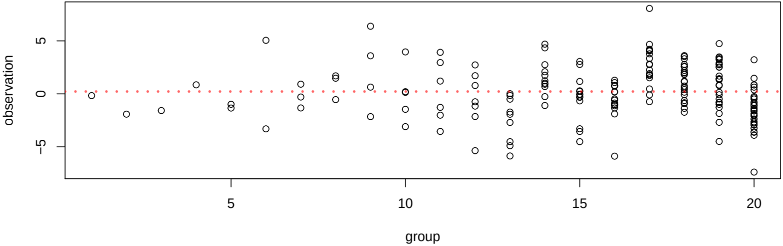


Overly optimistic ANOVA

Choose between two models

1. Grand mean/total pooling: $\bar{Y}_{..}$

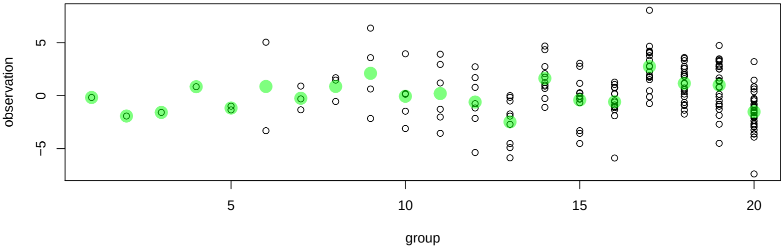
$$\mu_1 = \mu_2 = \ldots = \mu_K$$



Overly optimistic ANOVA

Choose between two models

1. Grand mean: $\bar{Y}_{..}$
2. Indep. means/no pooling: $\bar{Y}_{j.}$

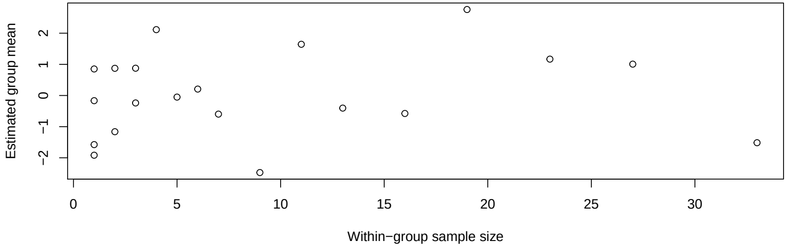


Overly optimistic ANOVA

```r
anova(mod1, mod2)
```

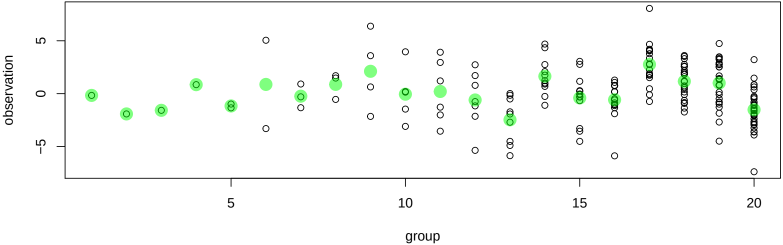
```
## Analysis of Variance Table
##
## Model 1: Y ~ 1
## Model 2: Y ~ 1 + factor(id)
##   Res.Df     RSS Df Sum of Sq      F    Pr(>F)
## 1    186 1134.44
## 2    167  738.02 19    396.42 4.7212 1.191e-08 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1
```

What's the deal with small sample sizes n_j?



Overfitting much?

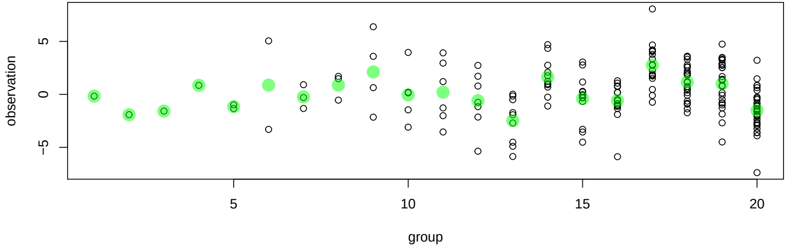
High parameter to data ratio for small n_j



What else?

Is there an option better than $\bar{y}_{j.}$?

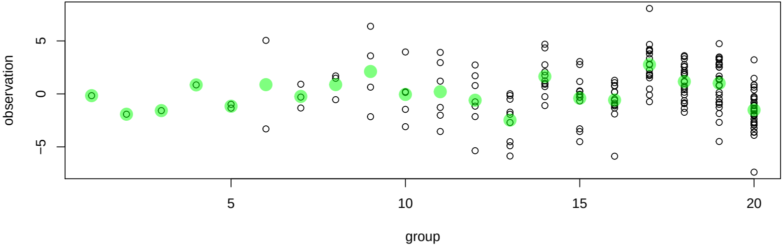
Oddly, yes! When we have > 2 groups (see Stein's paradox)



Conceptualizing a better estimate

Which estimates do we trust least?

What information can improve those estimates?



A better estimate

Mixture of sample and grand mean:

$$\hat{\alpha}_j = \lambda_j \bar{y}_{j.} + (1 - \lambda_j)\bar{y}_{..}$$

$$0 < \lambda < 1$$

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$$0 < \lambda < 1$$

Compromise b/t:

total pooling ($H_0 : \lambda = 0$) & no pooling ($H_A : \lambda = 1$)

Hierarchical models

Random effects impose shrinkage!

$$y_{ij} \sim Normal(\alpha_j, \sigma_y)$$

$$\alpha_j \sim Normal(\mu_\alpha, \sigma_\alpha)$$

Hierarchical models

$$y_{ij} \sim Normal(\alpha_j, \sigma_y)$$

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Amt shrinkage:

- information in group j (e.g. n_j)
- variance attributable to groups

$$\frac{\sigma_\alpha}{\sigma_\alpha + \sigma_y}$$

Connection to ANOVA

$$y_{ij} \sim Normal(\alpha_j, \sigma_y)$$

$$\alpha_j \sim Normal(\mu_\alpha, \sigma_\alpha)$$

$$0 < \sigma_\alpha < \infty$$

Compromise b/t

- Total pooling: $\sigma_\alpha = 0$
- No pooling: $\sigma_\alpha = \infty$

Synonyms

- "partial pooling"
- "semi-pooling"
- "hierarchical pooling"
- "shrinkage"
- "borrowing information"
- "borrowing strength (of information)"

Demo

shrinkage.R

Recap

Random effects impose partial pooling

$$y_{ij} \sim Normal(\alpha_j, \sigma_y)$$

$$\alpha_j \sim Normal(\mu_\alpha, \sigma_\alpha)$$

*see nba_freethrows.R for a real-world example

Aside

Scope of inference:

Observed sites or groups $j \in 1, ..., J$

and

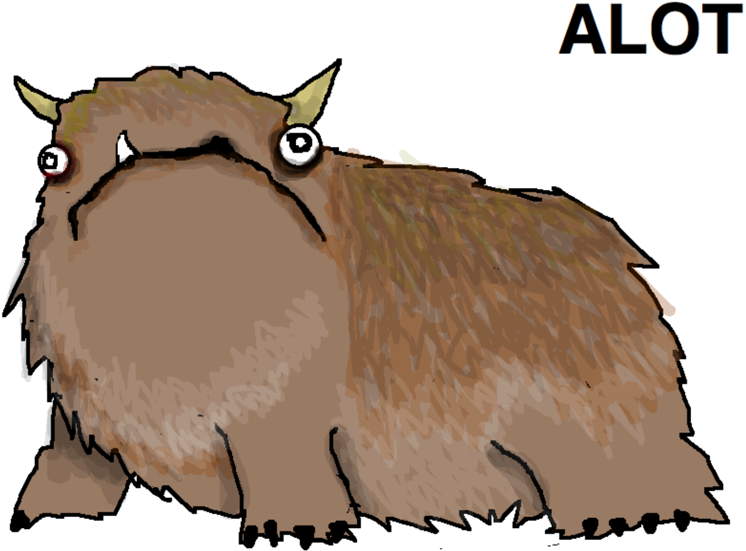
Unobserved sites or groups $j \in J + 1, ...$

see prediction.R for more

Mixed effects

Combination of fixed *and* random effects

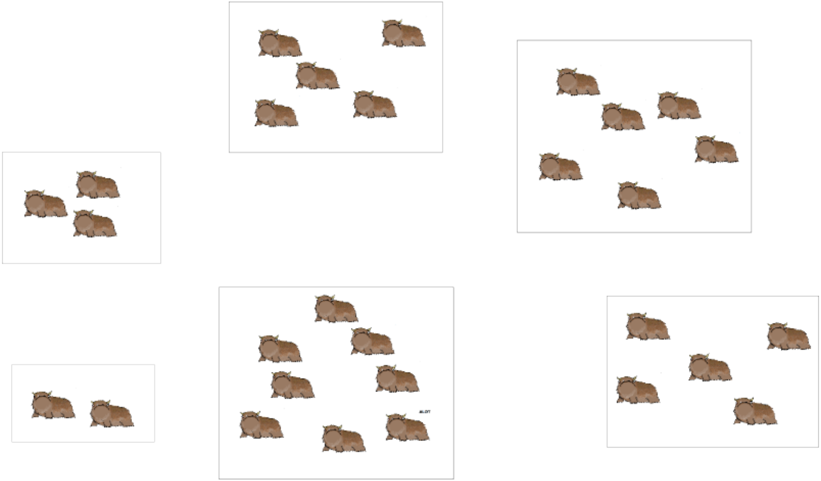
e.g. let's say we study Alot blood parasites



Questions & sampling scenario

Do large-bodied Alots have more blood parasites?

Random sample of n_j individuals at each of J sites.



Demo

Alot example

Other resources

Mixed Effects Models and Extensions in Ecology with R (2009). Zuur, Ieno, Walker, Saveliev and Smith. Springer.

lme4: Mixed-effects modeling with R (2010). Bates, Douglas. Springer.

Generalized linear mixed models: a practical guide for ecology and evolution (2009). Benjamin M. Bolker, et al. TREE.