Pricing Electricity Futures Options under Enlarged Filtrations

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Abstract: We derive risk-neutral option price formulas for plain-vanilla and exotic electricity futures derivatives on the basis of diverse arithmetic multi-factor Ornstein-Uhlenbeck spot price models admitting seasonality. In these setups, we take additional forward-looking knowledge on future price behavior into account via multiple initially enlargements of the underlying information filtration. In this insider trading context, we also correlate electricity prices with outdoor temperature and treat a related pricing problem under supplementary temperature forecasts. Meanwhile, we use Fourier transform techniques and results from complex analysis to handle the emerging anticipating conditional expectations. As a by-product we derive related risk and information premia. The paper can be regarded as an extension of [4] and [5], since in [4] no future information is involved while in [5] no option pricing is considered.

Keywords and phrases: Electricity futures, option pricing, mean-reverting multi-factor model, Itô-Lévy process, enlargement of filtration, forward-looking information, insider trading, information premium, Fourier transform

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1 Introduction

The creation of competitive energy markets such as the European Energy Exchange (EEX), where electrical energy is traded as a commodity, has brought up new mathematical challenges concerning adequate pricing approaches for available energy derivatives. Briefly summing up the most relevant findings in [2], [3], [4], [5], [7], [8], [9], [13], [15], [18], [20], [22] and [23], we claim that electrical energy exchanges/prices exhibit a seasonal spiky price behavior (due to the non-storability of the underlying *flow commodity* leading to inelastic demand) along with a strong mean-reversion to a periodic trend line showing slow stochastic variation itself, a lack of arbitrage opportunities, high price volatilities, heavy-tailed empirical return distributions, incompleteness and a nearly monopolistic structure with only a few *big players* as market participants (acting on separated regional markets) whose individual trading activities may shift prices essentially. In addition, electricity prices are strongly correlated with outdoor temperature and other commodity prices such as of gas, oil or coal, for example.
With reference to [5] and [7] we now present some of the most convincing arguments that count in favor for an incorporation of anticipative information into mathematical (option) pricing approaches particularly in electricity markets. First of all, electricity depicts a commodity which is non-storable or has at least very limited storage possibilities (except from indirect ones like in water reservoirs). This lack of storability causes a collapse of conventional buy-and-hold strategies and is responsible for calling electricity a *flow commodity*. Since consumers cannot buy for storage, there is no reason why *today's* prices (and the corresponding backward-looking filtration solely generated by the history of the

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involved spot price noises up to the present) should reflect public knowledge about *future* events like, for example, the introduction of carbon dioxide emission costs next year, the planned building of new connecting cables to other electricity markets, a future apportionment concerning the promotion of renewable/green electricity (e.g. "*Ökostrom-Umlage 2013*" in Germany) or weather forecasts (as dealt with in [16]). Evidently, the *present* electricity (spot) price is unaffected by such *future* market information while it is a result of *today's* supply and demand situation only. In conclusion, in markets for non-storable commodities forward-looking information about *future* market conditions is *not* incorporated in *today's* prices, but in *forward* or *futures* prices.

Another plausible example in [5] deals with the outage of a major power plant (e.g. during the next quarter) which indisputably constitutes some worthy anticipating information being available at least to well-informed market *insiders*. In this case, the energy supply side will be reduced significantly so that one expects an increase of the *future* (but not of the present) price level, since traders/consumers cannot buy for storage and thus, *today's* prices should not be affected. As an immediate consequence, the mathematical pricing mechanisms for options written on electricity futures with a delivery period hitting the outage interval should adequately take this additional knowledge into account what may culminate in an enlargement of the underlying information filtration. For this reason, throughout this article we will be confronted with a persistent discussion concerning adequate pricing approaches for electricity futures contracts based upon (initially) enlarged filtrations – particularly to avoid "*information miss-specification*" [5] in our models.

From a mathematical point of view, we may distinguish two cases: Whenever we assume future information on the driving *jump* noises to be available, we apply tailor-made approximation procedures involving techniques from complex analysis in order to derive the corresponding option price estimates. Contrarily, whenever the historical filtration is enlarged with respect to *Brownian* noise, we obtain more explicit *analytical* anticipating option price formulas. As a basis for our investigations we use different (slightly extended) versions of the mean-reverting arithmetic multi-factor pure-jump approach presented in [4] which seems to be extremely suitable to derive an accurate description of empirical electricity price behavior; cf. Fig. 1 and 2 in [4]. Actually, the main motivation of the present work is to combine [4] and [5] while providing appropriate derivation methods for electricity futures option prices under additional forward-looking information modeled by enlarged filtrations.

The paper is organized as follows: In Chapter 2 our initial pure-jump electricity spot price model is introduced. Next, we deduce an electricity futures price representation under a risk-neutral probability measure[2] and obtain a related backward-looking call option price formula. In Chapter 3 we define the *information premium* and innovatively focus on the pricing of electricity futures contracts under additional future information modeled by initially enlarged filtrations. This procedure culminates in the provision of electricity futures and connected option prices under complementary knowledge on future price behavior or outdoor temperature. Ultimately, we propose a forward-looking jump-diffusion spot/option price model including both Brownian motion (BM) and pure-jump components.

2 Modeling Electricity Spot and Futures Prices

Let $\left(\Omega, \mathbb{F}, (\mathcal{F}_t)_{t\in[0,T]}, \mathbb{P}\right)$, $T < \infty$, be a filtered probability space obeying the *usual hypotheses* of Chapter I.1 in [24].

2.1 An arithmetic pure-jump multi-factor electricity spot price model

Parallel to [4], we model the electricity spot price $S \coloneqq (S_t)_{t\in[0,T]}$ via

[2] Since electrical energy is non-storable and thus, the spot neither is a traded asset, it does not make sense to require the (discounted) electricity spot price to form a martingale under an equivalent *martingale* measure (EMM). Note that *any* measure, equivalent to $\mathbb{P}$, constitutes an admissible risk-neutral candidate – cf. Section 4.1.1 in [7].

(2.1)
$$S_t \coloneqq \mu(t) + Y_t$$

wherein $\mu(t)$ represents a deterministic and periodic seasonality function (indicating the *floor* or *lower bound* of the spot price) while $Y_t \coloneqq \sum_{k=1}^{n} w_k X_t^k$ with constant weights $w_k \geq 0$ and zero-reverting pure-jump Ornstein-Uhlenbeck (OU) components

(2.2)
$$dX_t^k = -\lambda_k\, X_t^k\, dt + \sigma_k(t)\, dL_t^k.$$

Herein, $\lambda_k > 0$ denote constant mean-reversion speeds and $\sigma_k(t) > 0$ are bounded and deterministic volatility functions controlling the seasonal variation of the jump amplitudes. Further, for all $k = 1, \dots, n$ the stochastic processes L_t^k intersperse random price fluctuations including both small (daily) variations and large-amplitude spikes. Slightly deviating from [4], we announce the concrete form of the independent, càdlàg, integrable, monotone-increasing, finite-variation, $\mathcal{F}_t$-adapted pure-jump Lévy-type/additive/Sato processes (so-called *subordinators*; see [25]) L_t^k due to

(2.3)
$$L_t^k \coloneqq \int_0^t \int_{D_k} z\, dN_k(s,z)$$

where $D_k \subseteq \,]0,\infty[\, \subset \mathbb{R}$. In the latter equation N_k depicts a one-dimensional Poisson random measure (PRM) on $[0,T] \times]0,\infty[$ with previsible (time-inhomogeneous) $\mathbb{P}$-compensator $\rho_k(s) d\nu_k(z) ds$ such that the objects

(2.4)
$$d\widetilde{N}_k^{\mathbb{P}}(s,z) \coloneqq dN_k(s,z) - \rho_k(s)\, d\nu_k(z)\, ds$$

indicate $\mathbb{P}$-martingale integrators for every k. Herein, the deterministic functions $\rho_k(s) \geq 0$ control the seasonal variation of the jump intensities and the one-dimensional Lévy measures ν_k declare positive and finite Borel measures on D_k obeying

$$\int_{D_k} (1 \wedge z^2)\, d\nu_k(z) < \infty.$$

The deterministic initial values are given by $X_0^k \coloneqq x_k$.
Moreover, we use the components $X_t^1, \dots, X_t^l$ $(1 \leq l \leq n)$ to model the long-term level of the spot price, i.e. the daily volatile stochastic variations of the deterministic trend line $\mu(t)$. Thus, the processes $X_t^1, \dots, X_t^l$ are supposed to permit (Brownian-motion-like) small fluctuations with jump sizes in a set $D_k \coloneqq \,]0, \varepsilon_k[$ for a small number $\varepsilon_k > 0$ along with a slow mean-reversion velocity λ_k for $k = 1, \dots, l$. On the opposite, the members $X_t^{l+1}, \dots, X_t^n$ model the (seasonal) short-term spiky variations, i.e. the large price jumps. Consequently, we may choose $D_k \coloneqq [\varepsilon_k, \infty[$ for an arbitrary (maybe large) number $\varepsilon_k > 0$ together with a high mean-reversion speed λ_k for $k = l+1, \dots, n$. In this context, a high mean-reversion speed ensures that after a large-amplitude jump the spot price rapidly turns back to about the former level. Since in (2.3) we have permitted *positive* jump amplitudes only, the mean-reverting nature of the components $X_t^1, \dots, X_t^n$ guarantees that the spot price (2.1) is not only increasing but also (randomly) decreasing.
However, for $0 \leq t \leq u \leq T$ the solution of (2.2) reads as

(2.5)
$$X_u^k = X_t^k e^{-\lambda_k(u-t)} + \int_t^u \sigma_k(s)\, e^{-\lambda_k(u-s)}\, dL_s^k.$$

Furthermore, the randomized trend line of the spot price

(2.6)

$$\eta_t \coloneqq S_t - \sum_{k=l+1}^{n} w_k X_t^k = \mu(t) + \sum_{k=1}^{l} w_k X_t^k$$

is given through the periodic floor function μ which is randomly perturbed by a sum of slowly varying weighted noises $X_t^1, \ldots, X_t^l$. Thus, in order to get an idea about the shape of η_t one first has to filter out the large-amplitude spot price spikes being generated by $X_t^{l+1}, \ldots, X_t^n$. Section 8.4.4 in [17] and Chapter 4 in [23] are dedicated to this topic.

2.2 Switching to a risk-neutral measure

We now introduce an (with respect to $\mathbb{P}$) equivalent risk-neutral measure $\mathbb{Q}$. To this end, for deterministic and integrable functions $h_k(s,z)$ we define the Radon-Nikodym derivative

(2.7)

$$\left.\frac{d\mathbb{Q}}{d\mathbb{P}}\right|_{\mathcal{F}_t} \coloneqq \prod_{k=1}^{n} \mathfrak{E}(M^k)_t > 0$$

with local $\mathbb{P}$-martingales

$$M_t^k \coloneqq \int_0^t \int_{D_k} \left[e^{h_k(s,z)} - 1\right] d\widetilde{N}_k^{\mathbb{P}}(s,z)$$

and Doléans-Dade exponentials

$$\mathfrak{E}(M^k)_t \coloneqq \exp\left\{M_t^k - \frac{1}{2}[(M^k)^c]_t\right\} \times \prod_{0 \le s \le t} (1 + \Delta M_s^k)\, e^{-\Delta M_s^k}$$

for $k = 1, \ldots, n$ and $t \in [0,T]$. We instantly receive

$$\mathfrak{E}(M^k)_t = \exp\left\{\int_0^t \int_{D_k} h_k(s,z)\, d\widetilde{N}_k^{\mathbb{P}}(s,z) - \int_0^t \int_{D_k} \left[e^{h_k(s,z)} - 1 - h_k(s,z)\right] \rho_k(s)\, d\nu_k(z)\, ds\right\}$$

whereas Itô's formula yields the local $\mathbb{P}$-martingale representation

$$\mathfrak{E}(M^k)_t = 1 + \int_0^t \int_{D_k} \mathfrak{E}(M^k)_{s-} \left[e^{h_k(s,z)} - 1\right] d\widetilde{N}_k^{\mathbb{P}}(s,z).$$

We further impose a Novikov condition (cf. Theorem 12.21 in [12]) reading in our setup as

$$\mathbb{E}_{\mathbb{P}}\left[\exp\left\{\int_0^t \int_{D_k} \left[1 - e^{h_k(s,z)} + h_k(s,z)\, e^{h_k(s,z)}\right] \rho_k(s)\, d\nu_k(z)\, ds\right\}\right] < \infty$$

for all $k = 1, \ldots, n$ and $t \in [0,T]$. Consequently, it holds $\mathbb{E}_{\mathbb{P}}[\mathfrak{E}(M^k)_t] = 1$ for all $t \in [0,T]$ and $k = 1, \ldots, n$ such that the exponentials $\mathfrak{E}(M^k)$ are true $\mathbb{P}$-martingales. Using Girsanov's theorem, we state that the (time-inhomogeneous) $\mathbb{Q}$-compensated PRMs

(2.8)

$$d\widetilde{N}_k^{\mathcal{F},\mathbb{Q}}(s,z) \coloneqq d\widetilde{N}_k^{\mathbb{Q}}(s,z) \coloneqq dN_k(s,z) - e^{h_k(s,z)} \rho_k(s)\, d\nu_k(z)\, ds$$

depict (non-stationary) $\mathcal{F}$-adapted $\mathbb{Q}$-martingale integrators for all $k = 1, \ldots, n$.

2.3 Electricity futures prices under the multi-factor approach

In what follows, we focus on a futures contract promising its holder the delivery of one unit of electrical energy, say 1 MWh, over the future delivery period $[\tau_1, \tau_2]$. To attain this right, the trader has to pay the electricity futures price (cf. Section 4.1 in [7])

(2.9)
$$F_t(\tau_1,\tau_2) \coloneqq F_t^{\mathcal{F},\mathbb{Q}}(\tau_1,\tau_2) \coloneqq \frac{1}{\tau_2-\tau_1}\int_{\tau_1}^{\tau_2} \mathbb{E}_{\mathbb{Q}}(S_u|\mathcal{F}_t)\, du$$

at time t prior to the start of the delivery period, i.e. $0 \leq t \leq \tau_1 < \tau_2 \leq T$.

Proposition 2.1. (Proposition 3.1 in [4]) *The price of an electricity futures $F_t(\tau_1,\tau_2)$ at time $t \in [0,\tau_1]$ with delivery period $[\tau_1,\tau_2]$ is given by the $\mathcal{F}_t$-adapted (non-stationary, independent increment) $\mathbb{Q}$-Sato-martingale (cf. [25])*

(2.10)
$$F_t(\tau_1,\tau_2) = F_0(\tau_1,\tau_2) + \sum_{k=1}^{n}\int_0^t\int_{D_k} z\, \Lambda_k(s,\tau_1,\tau_2)\, d\widetilde{N}_k^{\mathcal{F},\mathbb{Q}}(s,z)$$

with a deterministic volatility function

(2.11)
$$\Lambda_k(s,\tau_1,\tau_2) \coloneqq \frac{w_k\,\sigma_k(s)}{(\tau_2-\tau_1)\,\lambda_k}\left[e^{-\lambda_k(\tau_1-s)} - e^{-\lambda_k(\tau_2-s)}\right] \geq 0$$

and a deterministic initial value

(2.12)
$$F_0(\tau_1,\tau_2) \coloneqq \frac{1}{\tau_2-\tau_1}\int_{\tau_1}^{\tau_2}\left[\mu(u) + \sum_{k=1}^{n} w_k\left(x_k\, e^{-\lambda_k u} + \int_0^u\int_{D_k} z\,\sigma_k(s)\, e^{-\lambda_k(u-s)}\, e^{h_k(s,z)}\,\rho_k(s)\, d\nu_k(z)\, ds\right)\right] du.$$

Remark 2.2. *It holds $\Lambda_k > 0$ whenever $w_k \neq 0$. Further note that Λ_k is decreasing in τ_1. Hence, if the delivery period starts far in the future, the present arrival of new information has a more or less negligible influence on the futures price (2.10). In [7] it is argued that in this case the market has much time left to adjust before delivery takes place. Consequently, the electricity futures price is less sensitive to changes in the spot and thus, Λ_k decreases with an increasing time to maturity. This economically reasonable feature frequently is called (averaged) Samuelson effect.*

2.4 Electricity futures multi-factor call option prices

Before turning to electricity derivatives pricing under forward-looking information, in this preparatory section we extend the put option result of Proposition 4.1 in [4] yet to its call option counterpart in order to have some benchmark model available later. Firstly, we introduce a risk-free bond

(2.13)
$$\beta_t \coloneqq \beta_0\, e^{rt}$$

with $\beta_0 > 0$ and a constant interest rate $r > 0$. In addition, we define the call option payoff with strike price $K > 0$ via

(2.14)
$$C_T \coloneqq C_T(K,\tau_1,\tau_2) \coloneqq [F_T(\tau_1,\tau_2) - K]^+ \coloneqq \max\{0, F_T(\tau_1,\tau_2) - K\}.$$

As usual, we assume that the market uses the measure $\mathbb{Q}$ introduced in (2.7) for both futures and related option pricing (see e.g. [7] or the beginning of Chapter 4 in [4]). Consequently, no-arbitrage theory yields the call option price representation

$$C_t = e^{-r(T-t)}\,\mathbb{E}_{\mathbb{Q}}([F_T(\tau_1,\tau_2) - K]^+ | \mathcal{F}_t). \tag{2.15}$$

Let us define the one-dimensional Fourier transform associated to a real function $f \in \mathcal{L}^1(\mathbb{R})$ due to

$$\hat{f}(y) \coloneqq \int_{\mathbb{R}} f(x)\, e^{-iyx} dx \tag{2.16}$$

whereas its inverse is given by

$$f(x) = \frac{1}{2\pi} \int_{\mathbb{R}} \hat{f}(y)\, e^{iyx} dy \tag{2.17}$$

(cf. e.g. [7], [10]). Note that $g(x) \coloneqq [x - K]^+ \notin \mathcal{L}^1(\mathbb{R}^+)$ while $q(x) \coloneqq e^{-ax} g(x) \in \mathcal{L}^1(\mathbb{R})$ with real dampening parameter $0 < a < \infty$. A straightforward calculation delivers

$$\hat{q}(y) = \int_K^\infty e^{-(a+iy)x}\,[x - K]\, dx = \frac{e^{-(a+iy)K}}{(a+iy)^2}. \tag{2.18}$$

For the remainder of this article we assume that adjusted versions of Condition G and Condition A given on p. 72 respectively p. 74 in [7] hold such that $F_t(\tau_1,\tau_2) < \infty\ \forall\, t \in [0,\tau_1]$; also see Proposition 9.4 in [7]. Further remind that the spot price (2.1) reverts to a deterministic and *bounded* seasonality function and thus $F_t(\tau_1,\tau_2) < \infty\ \forall\, t \in [0,\tau_1]$ appears natural not only from a practitioner's point of view. We get the following result:

Proposition 2.3. *Denote the risk-free interest rate by r. Then the price C_t at time $t \in [0,\tau_1]$ of a European call option with strike $K > 0$ written on the electricity futures (2.10) is given by*

$$C_t \coloneqq C_t(K,\tau_1,\tau_2) = \frac{e^{-r(T-t)}}{2\pi} \int_{\mathbb{R}^+} \frac{e^{(a+iy)\,\{F_t(\tau_1,\tau_2)-K\}}}{(a+iy)^2} \prod_{k=1}^{n} e^{\psi_k(y,t,T)}\, dy$$

with characteristic exponents

$$\psi_k(y,t,T) \coloneqq \int_t^T \int_{D_k} \left[e^{(a+iy)\, z\, \Lambda_k(s,\tau_1,\tau_2)} - 1 - (a+iy)\, z\, \Lambda_k(s,\tau_1,\tau_2)\right] e^{h_k(s,z)} \rho_k(s)\, d\nu_k(z)\, ds.$$

Proof. Identify $q(\cdot)$ inside (2.15) first, then apply (2.17) and finally use the Lévy-Khinchin formula for additive processes (see Prop. 2.1 in [7], respectively Prop. 1.9 in [19]). ■

3 Electricity Derivatives and Future Information

In this chapter we aim at developing anticipating pricing mechanisms for diverse types of electricity futures contracts. To begin with, let us recall that the historical filtration

$$\mathcal{F}_t \coloneqq \sigma\{L_s^1, \dots, L_s^n : 0 \le s \le t\} \tag{3.1}$$

does only 'look into the past' while all available information coming from market observation up to time t is stored in this retro sigma-algebra. Actually, this traditional financial approach hardly reflects the case at hand in markets for non-storable commodities such as electricity. Hence, inspired by [5], we propose to model the flow of forward-looking information by an (initially) enlarged filtration $\mathcal{G}_t \supset \mathcal{F}_t$ in the following.

3.1 The information premium in electricity markets

In accordance to (2.9) we define the $\mathcal{G}$-anticipating electricity futures price associated to a delivery of a certain amount of electrical energy over the period $[\tau_1, \tau_2]$ via

(3.2)
$$F_t^{\mathcal{G},\mathbb{Q}}(\tau_1,\tau_2) := \frac{1}{\tau_2-\tau_1}\int_{\tau_1}^{\tau_2} \mathbb{E}_{\mathbb{Q}}(S_u|\mathcal{G}_t)\,du$$

where $t \in [0,\tau_1]$. Moreover, recall that in [5] the *information premium* was defined 1.) under $\mathbb{P}$ and 2.) for forwards. We deviate from this scenario and introduce the information premium for $\mathbb{Q}$-futures via

$$\mathfrak{I}_t^{\mathcal{G},\mathcal{F},\mathbb{Q}}(\tau_1,\tau_2) := \mathfrak{I}_t^{\mathbb{Q}}(\tau_1,\tau_2) := F_t^{\mathcal{G},\mathbb{Q}}(\tau_1,\tau_2) - F_t^{\mathcal{F},\mathbb{Q}}(\tau_1,\tau_2). \tag{3.3}$$

Verbalizing, $\mathfrak{I}$ measures the information asymmetry between two sorts of electricity futures price processes, one associated to $\mathcal{G}$ and one to $\mathcal{F}$, but both under $\mathbb{Q}$. Parallel to [5], we suppose the market participants to have some insider information about the spot price behavior at a fixed future time τ. For example, it appears reasonable to expect the introduction of carbon dioxide emission costs at time τ to influence the spot price mean level η_τ (see Section 2.2 in [5] for empirical evidences and a precise justification of this statement). Due to (2.6), this kind of additional information evidently affects the noises $L_\tau^1, \dots, L_\tau^l$. In this regard, we introduce the overall filtration

$$\mathcal{H}_t := \mathcal{F}_t \vee \sigma\{X_\tau^1,\dots,X_\tau^l\} = \mathcal{F}_t \vee \sigma\{L_\tau^1,\dots,L_\tau^l\} \tag{3.4}$$

which may be associated with complete/exhaustive information at time t about the electricity spot price trend line value η_τ [see (2.6)]. Similar to [5], we next assume the inclusions

$$\mathcal{F}_t \subset \mathcal{G}_t \subset \mathcal{H}_t \tag{3.5}$$

for $0 \le t < \tau$ whereas $\mathcal{F}_t = \mathcal{G}_t$ holds for all $t \ge \tau$.

Condition A. *Combining (2.3) with (2.8), we see that L_t^k is (not necessarily a Lévy but) a Sato process under $\mathbb{Q}$, admitting independent but non-stationary increments (cf. [10], [25]). As Lemma 3.1 below holds for* Lévy *processes only [see Remark 3.2 (d)], $L^1,\dots,L^l$ must also permit* stationary *increments, if we want to work with (3.4)-(3.5). Thus, for $k=1,\dots,l$ we assume $\rho_k(s) := \rho_k \ge 0$ and $h_k(s,z) := h_k(z)$ from now on (which is not necessary for $k=l+1,\dots,n$), such that $L^1,\dots,L^l$ become* Lévy *processes under $\mathbb{Q}$. Fortunately, the choice of* constant *long-term jump intensities $\rho_1,\dots,\rho_l$ is empirically reasonable and does not stand in contradiction to the economical practice (see p. 4 in [4]). We further suppose $\mathfrak{L} := (L^1,\dots,L^l)$ to be an l-dimensional Lévy process.*

Lemma 3.1. *(a) For an intermediate filtration $\mathcal{G}_t$ such as given in (3.5) the stochastic processes*

(3.6)
$$\hat{L}_t^k := L_t^k - \int_0^t \frac{\mathbb{E}_{\mathbb{Q}}(L_\tau^k - L_s^k|\mathcal{G}_s)}{\tau - s}\,ds$$

are $(\mathcal{G}_t,\mathbb{Q})$-martingales for all $k=1,\dots,l$ and $t \in [0,\tau[$.

(b) For an overall filtration $\mathcal{H}_t$ such as implemented in (3.4) the stochastic processes

(3.7)

$$\tilde{L}_t^k \coloneqq L_t^k - \int_0^t \frac{L_\tau^k - L_s^k}{\tau - s} ds$$

are $(\mathcal{H}_t, \mathbb{Q})$-martingales for all $k = 1, \dots, l$ and $t \in [0, \tau[$.

(c) For all $k = 1, \dots, l$ and time indices $0 \le t \le s < \tau$ we have the equality

(3.8)

$$\mathbb{E}_{\mathbb{Q}}(L_\tau^k - L_s^k | \mathcal{G}_t) = \frac{\tau - s}{\tau - t} \, \mathbb{E}_{\mathbb{Q}}(L_\tau^k - L_t^k | \mathcal{G}_t).$$

(d) For all $k = 1, \dots, l$ and $s \in [0, \tau[$ the $(\mathcal{G}, \mathbb{Q})$-compensator of $dN_k(s, z)$ is given by

(3.9)

$$d\nu_k^{\mathcal{G},\mathbb{Q}}(s, z) \coloneqq \frac{1}{\tau - s} \, \mathbb{E}_{\mathbb{Q}}\left(\int_{u=s}^{u=\tau} dN_k(u, z) \middle| \mathcal{G}_s \right) ds$$

while the $(\mathcal{G}, \mathbb{Q})$-compensated random measure is of the form

(3.10) $$d\tilde{N}_k^{\mathcal{G},\mathbb{Q}}(s, z) \coloneqq dN_k(s, z) - d\nu_k^{\mathcal{G},\mathbb{Q}}(s, z).$$

Proof. (a) See Lemma 3.3 in [5], p. 16 in [11], resp. Proposition 16.52 in [12].

(b) This follows from (a) and the *taking out what is known rule* for conditional expectations. (Also see p. 16 in [11].)

(c) Combine Proposition A.3 in [5] (with $g(u) \coloneqq \frac{1}{\tau - u}$ and $f(u) \equiv 1$) and Remark A.4 in [5] (with $L \coloneqq L^k$) to verify this.

(d) For each $k = 1, \dots, l$ the proof of Proposition 5.2 in [11], resp. Proposition 16.53 in [12], with z replaced by an arbitrary bounded and deterministic function $f_k(z)$ which vanishes in any small neighborhood of zero and that determines a measure on D_k with weight zero whenever $z \longrightarrow 0^+$, resp. with z replaced by any $f_k(z)$ invertible and integrable with respect to $\nu_k^{\mathcal{G},\mathbb{Q}}$, here applies equally. The representation (3.9) then follows from (2.3) and (3.6). ■

Remark 3.2. *(a) Note that for $k = l + 1, \dots, n$ the PRMs $\tilde{N}_k^{\mathcal{F},\mathbb{Q}}(t, z)$ in (2.8) are (not only $\mathcal{F}_t$-adapted but also) $\mathcal{G}_t$-adapted $\mathbb{Q}$-martingale integrators, since $\mathcal{F}_t \subset \mathcal{G}_t$ holds for all $0 \le t < \tau$.*

(b) Since (3.9) is stochastic, (3.10) is not a (compensated) Poisson random measure in the classical sense – compare the sequel of Example 16.38 in [12]. In this context, also recall that a semimartingale possesses independent increments if and only if its characteristics are non-random.

(c) Recall that for a fixed index $k^* \in \{1, \dots, l\}$ and a filtration $\hat{\mathcal{G}}_t$ obeying $\mathcal{F}_t \subset \hat{\mathcal{G}}_t \subset \mathcal{F}_t \vee \sigma\{L_\tau^{k^*}\}$ the process $\hat{L}_t^{k^*}$ [see (3.6)] even constitutes a $(\hat{\mathcal{G}}_t, \mathbb{Q})$-martingale, being this a stronger conclusion [as long as we reasonably presume $\sigma\{L_\tau^{k^*}\} \subset \sigma\{L_\tau^1, \dots, L_\tau^l\}$ and particularly $\hat{\mathcal{G}}_t \subset \mathcal{G}_t \subset \mathcal{H}_t$] than the one actually given in Lemma 3.1 (a). Similarly, the process $\tilde{L}_t^{k^*}$ [see (3.7)] not only is a $\mathcal{H}_t$-martingale then, but even a $(\mathcal{H}_t \supset) \, \mathcal{F}_t \vee \sigma\{L_\tau^{k^*}\}$-martingale under $\mathbb{Q}$.

(d) [The following argumentation is due to J. Jacod (private communication).] *We show that the* Lévy *process property of $L^1, \dots, L^l$ is obligatory for Lemma 3.1 (recall Condition A). Assume L_t to be an arbitrary, càdlàg, increasing, (in general non-stationary) deterministic* function *with $L_0 = 0$. Then L_t a*

priori possesses $\mathbb{Q}$*-independent increments and hence, indicates a (trivial) Sato process under* $\mathbb{Q}$*. In this case, we observe* $\mathcal{F}_t = \sigma\{L_s : 0 \le s \le t\} = \{\emptyset, \Omega\} = \mathcal{H}_t \ (= \mathcal{G}_t)$ *for* $0 \le t < \tau$*. Consequently, if* $\tilde{L}_t := L_t - \int_0^t \frac{L_\tau - L_s}{\tau - s} ds$*,* $0 \le t < \tau$*, still were a* $(\mathcal{H}_t, \mathbb{Q})$*-martingale [cf. (3.7)], then* $\tilde{L}_t$ *would be identically zero, since for all* $0 \le t < u < \tau$ *we would receive* $\tilde{L}_t = \mathbb{E}_\mathbb{Q}(\tilde{L}_u | \mathcal{H}_t) = \mathbb{E}_\mathbb{Q}(\tilde{L}_u | \{\emptyset, \Omega\}) = \mathbb{E}_\mathbb{Q}[\tilde{L}_u] \equiv \mathbb{E}_\mathbb{Q}[\tilde{L}_0] = \mathbb{E}_\mathbb{Q}[L_0] = 0$*. But* $\tilde{L} \equiv 0$ *is not valid in general, particularly not when* L *is monotone increasing (recall our assumption above). Interestingly, the only case where* $\tilde{L} \equiv 0$ *holds is obtained for* $L_t := \gamma t$ *with an arbitrary constant* γ *(in our framework* γ *must be strictly positive), that is, when* L_t *also possesses* stationary *increments and thus, is a* Lévy *process, yet. In conclusion,* Itô's enlarged filtration result *(yielding the basis for Lemma 3.1 (a) and (b); see the proof of Proposition 16.52 in [12]) does finally not hold for Sato processes, so that the Lévy process property of* $L^1, \dots, L^l$ *in the context of (3.6)-(3.10) is indeed necessary.*

Let us return to the information premium now. Since $X_u^{l+1}, \dots, X_u^n$ are $\mathbb{Q}$-independent of $L_u^1, \dots, L_u^l$, conditioning the sum $\sum_{k=l+1}^n w_k X_u^k$ on $\mathcal{G}_t$ coincides with conditioning the latter on $\mathcal{F}_t$. Moreover, as every X_t^k is $\mathcal{F}_t$-adapted, each X_t^k simultaneously is $\mathcal{G}_t$-adapted. Thus, for $0 \le t \le \tau_1 < \tau \le \tau_2$ we receive

$$\mathfrak{I}_t^\mathbb{Q}(\tau_1, \tau_2) = \int_{\tau_1}^{\tau_2} \sum_{k=1}^l \frac{w_k}{\tau_2 - \tau_1} \left\{ \mathbb{E}_\mathbb{Q}\left(\int_t^u \sigma_k(s)\, e^{-\lambda_k(u-s)}\, dL_s^k \middle| \mathcal{G}_t \right) - \mathbb{E}_\mathbb{Q}\left[\int_t^u \sigma_k(s)\, e^{-\lambda_k(u-s)}\, dL_s^k \right] \right\} du.$$

From now on, we presume $\tau_1 \le u < \tau \le \tau_2$ in the latter equation which induces $t \le s \le u < \tau$. Consequently, we may apply Lemma 3.1 (a) and (c) along with the tower property what leads us to

$$\mathbb{E}_\mathbb{Q}\left(\int_t^u \sigma_k(s)\, e^{-\lambda_k(u-s)}\, dL_s^k \middle| \mathcal{G}_t \right) = \frac{\mathbb{E}_\mathbb{Q}(L_\tau^k - L_t^k | \mathcal{G}_t)}{\tau - t} \int_t^u \sigma_k(s)\, e^{-\lambda_k(u-s)}\, ds.$$

Moreover, with respect to (2.3), (2.8) and Condition A, the above usual expectation becomes

$$\mathbb{E}_\mathbb{Q}\left[\int_t^u \sigma_k(s)\, e^{-\lambda_k(u-s)}\, dL_s^k \right] = \int_t^u \int_{D_k} z\, \sigma_k(s)\, e^{-\lambda_k(u-s)}\, e^{h_k(z)}\, \rho_k\, d\nu_k(z)\, ds.$$

Therewith, for $0 \le t \le \tau_1 < \tau \le \tau_2$ the information premium points out as

(3.11)
$$\mathfrak{I}_t^\mathbb{Q}(\tau_1, \tau_2) = \sum_{k=1}^l \frac{w_k}{\tau_2 - \tau_1} \int_{\tau_1}^{\tau_2} \int_t^u \sigma_k(s)\, e^{-\lambda_k(u-s)} \left[\frac{\mathbb{E}_\mathbb{Q}(L_\tau^k - L_t^k | \mathcal{G}_t)}{\tau - t} - \rho_k \int_{D_k} z\, e^{h_k(z)} d\nu_k(z) \right] ds\, du$$

which extends equality "(3.3) in [5]" to the $\mathbb{Q}$-*risk-neutral* electricity *futures* case. Oppositely, for a partition $0 \le t \le \tau \le \tau_1 \le u \le \tau_2$ we observe $\mathcal{F}_t \subseteq \mathcal{G}_t \subseteq \mathcal{G}_\tau = \mathcal{F}_\tau$ and thus, iterated conditioning yields

$$\begin{aligned}\mathbb{E}_\mathbb{Q}(X_u^k | \mathcal{G}_t) - \mathbb{E}_\mathbb{Q}(X_u^k | \mathcal{F}_t) &= \mathbb{E}_\mathbb{Q}(X_\tau^k | \mathcal{G}_t) + \mathbb{E}_\mathbb{Q}\big(\mathbb{E}_\mathbb{Q}(X_u^k - X_\tau^k | \mathcal{F}_\tau) \big| \mathcal{G}_t\big) - \mathbb{E}_\mathbb{Q}(X_\tau^k | \mathcal{F}_t) - \mathbb{E}_\mathbb{Q}\big(\mathbb{E}_\mathbb{Q}(X_u^k - X_\tau^k | \mathcal{F}_\tau) \big| \mathcal{F}_t\big) \\ &= e^{-\lambda_k(u-\tau)} \big[\mathbb{E}_\mathbb{Q}(X_\tau^k | \mathcal{G}_t) - \mathbb{E}_\mathbb{Q}(X_\tau^k | \mathcal{F}_t) \big].\end{aligned}$$

In conclusion, the information premium for $\tau \le \tau_1$ reads as

(3.12)
$$\mathfrak{I}_t^\mathbb{Q}(\tau_1, \tau_2) = \sum_{k=1}^l \frac{\Lambda_k(\tau, \tau_1, \tau_2)}{\sigma_k(\tau)} \big[\mathbb{E}_\mathbb{Q}(X_\tau^k | \mathcal{G}_t) - \mathbb{E}_\mathbb{Q}(X_\tau^k | \mathcal{F}_t) \big]$$

which extends equation "(3.4) in [5]" to the $\mathbb{Q}$-*risk-neutral* electricity *futures* case.

Remark 3.3. *Since $\mathcal{F}_t = \mathcal{G}_t$ for $t \geq \tau$, the information premium vanishes in this instance. The latter fact seems quite natural from an economical point of view, as in this case the additional information either consists of* present $(t = \tau)$ *or of* past $(t > \tau)$ *information, none being any longer relevant in the context of forward-looking insider trading.*
Moreover, parallel to [5], the information premium (3.12) tends to zero as τ_2 approaches infinity (for fixed τ and τ_1). Hence, for $\tau \leq \tau_1$ the supplementary information impact given through (3.12) approximately vanishes for contracts with long delivery periods ending far in the future, i.e. $\tau_2 \to \infty$.

In addition, we briefly study the *risk premium* for our current model. Slightly extending Definition 2.2 in [5], for $0 \leq t \leq \tau_1 < \tau \leq \tau_2$ we define the risk premium via

$$\mathfrak{R}_t^{\mathcal{G}}(\tau_1, \tau_2) \coloneqq \mathfrak{R}_t^{\mathcal{G},\mathbb{P},\mathbb{Q}}(\tau_1, \tau_2) \coloneqq F_t^{\mathcal{G},\mathbb{Q}}(\tau_1, \tau_2) - F_t^{\mathcal{G},\mathbb{P}}(\tau_1, \tau_2).$$

Comparing the risk premium with the information premium, one might declare the former to measure a certain kind of ($\mathcal{G}$-forward-looking) $\mathbb{P}$-$\mathbb{Q}$-difference whereas the latter captures a ($\mathbb{Q}$-risk-neutral) $\mathcal{F}$-$\mathcal{G}$-difference in the underlying futures prices. Similar computations as in the derivation of (3.11) yield

$$\begin{aligned}\mathfrak{R}_t^{\mathcal{G}}(\tau_1, \tau_2) = \frac{1}{\tau_2 - \tau_1} \int_{\tau_1}^{\tau_2} \int_t^u \Bigg\{ &\sum_{k=1}^{l} w_k\, \sigma_k(s)\, e^{-\lambda_k(u-s)}\, \frac{\mathbb{E}_{\mathbb{Q}}(L_\tau^k | \mathcal{G}_t) - \mathbb{E}_{\mathbb{P}}(L_\tau^k | \mathcal{G}_t)}{\tau - t} \\ &+ \sum_{k=l+1}^{n} \int_{D_k} w_k\, z\, \sigma_k(s)\, e^{-\lambda_k(u-s)} \left[e^{h_k(s,z)} - 1\right] \rho_k(s)\, d\nu_k(z) \Bigg\} ds\, du.\end{aligned}$$

Note that if the involved random components L_τ^k $(k = 1, \dots, l)$ were $\mathcal{G}_t$-measurable, then the risk premium would become deterministic. For $\mathcal{G}_t$ replaced by $\mathcal{H}_t$ this would be the case.

3.2 Electricity futures prices under enlarged filtrations

In order to value options written on the futures price (3.2) we need a more explicit representation for $F_t^{\mathcal{G},\mathbb{Q}}(\tau_1, \tau_2)$. To this end, the following result will be helpful.

Lemma 3.4. *The stochastic processes*

$$\left(\frac{\mathbb{E}_{\mathbb{Q}}(L_\tau^k - L_t^k | \mathcal{G}_t)}{\tau - t} \right)_{t \in [0, \tau[}$$

embody $(\mathcal{G}_t, \mathbb{Q})$-martingales for all $k = 1, \dots, l$ and $t \in [0, \tau[$.

Proof. Obviously, $\mathbb{E}_{\mathbb{Q}}(L_\tau^k - L_t^k | \mathcal{G}_t)/(\tau - t)$ is $\mathbb{Q}$-integrable and $\mathcal{G}_t$-adapted for all $t \in [0, \tau[$. Secondly, for $0 \leq t \leq s < \tau$ we apply the tower property and (3.8) yielding

$$\mathbb{E}_{\mathbb{Q}}\left(\frac{\mathbb{E}_{\mathbb{Q}}(L_\tau^k - L_s^k | \mathcal{G}_s)}{\tau - s} \middle| \mathcal{G}_t \right) = \frac{\mathbb{E}_{\mathbb{Q}}(L_\tau^k - L_s^k | \mathcal{G}_{s \wedge t})}{\tau - s} = \frac{\mathbb{E}_{\mathbb{Q}}(L_\tau^k - L_t^k | \mathcal{G}_t)}{\tau - t}. \blacksquare$$

Theorem 3.5. *For $0 \leq t \leq \tau_1 < \tau \leq \tau_2$ the $\mathcal{G}$-anticipative electricity futures price obeys*

$$\begin{aligned} dF_t^{\mathcal{G},\mathbb{Q}}(\tau_1, \tau_2) = &\sum_{k=1}^{l} [\Lambda_k(t, \tau_1, \tau_2) - \Phi_k(t)] \left(dL_t^k - \frac{\mathbb{E}_{\mathbb{Q}}(L_\tau^k - L_t^k | \mathcal{G}_t)}{\tau - t} dt \right) + \sum_{k=1}^{l} \Phi_k(t)\, d\mathbb{E}_{\mathbb{Q}}(L_\tau^k | \mathcal{G}_t) \\ &+ \sum_{k=l+1}^{n} \Lambda_k(t, \tau_1, \tau_2) \int_{D_k} z\, d\widetilde{N}_k^{\mathcal{F},\mathbb{Q}}(t, z) \end{aligned}$$

where $\Lambda_k(t, \tau_1, \tau_2)$ is as in (2.11) and $\Phi_k(t)$ is defined in (3.17) below. Hence, $F_t^{\mathcal{G},\mathbb{Q}}(\tau_1, \tau_2)$ is a $(\mathcal{G}, \mathbb{Q})$-martingale.

Proof. At first, we put (2.1) into (3.2) yielding

(3.13)
$$F_t^{\mathcal{G},\mathbb{Q}}(\tau_1,\tau_2)=\frac{1}{\tau_2-\tau_1}\int_{\tau_1}^{\tau_2}\left\{\mu(u)+\mathbb{E}_{\mathbb{Q}}\left(\sum_{k=1}^{l}w_kX_u^k\,\middle|\,\mathcal{G}_t\right)+\mathbb{E}_{\mathbb{Q}}\left(\sum_{k=l+1}^{n}w_kX_u^k\,\middle|\,\mathcal{F}_t\right)\right\}du.$$

From now on, we suppose $0\le t\le\tau_1\le u<\tau\le\tau_2$. Then, successively applying (2.5), Lemma 3.1 (a) and (c) [note that $0\le t\le s\le u<\tau$ holds], Fubini's theorem and the tower property, we receive

(3.14)
$$\mathbb{E}_{\mathbb{Q}}\left(\sum_{k=1}^{l}w_kX_u^k\,\middle|\,\mathcal{G}_t\right)=\sum_{k=1}^{l}w_k\left\{X_t^k\,e^{-\lambda_k(u-t)}+\frac{\mathbb{E}_{\mathbb{Q}}(L_\tau^k-L_t^k|\mathcal{G}_t)}{\tau-t}\int_t^u\sigma_k(s)\,e^{-\lambda_k(u-s)}\,ds\right\}.$$

Appealing to (2.3), (2.5), (2.8) and Condition A, the second conditional expectation in (3.13) becomes

(3.15)
$$\mathbb{E}_{\mathbb{Q}}\left(\sum_{k=l+1}^{n}w_kX_u^k\,\middle|\,\mathcal{F}_t\right)=\sum_{k=l+1}^{n}w_k\left\{X_t^ke^{-\lambda_k(u-t)}+\int_t^u\int_{D_k}z\,\sigma_k(s)\,e^{-\lambda_k(u-s)}e^{h_k(s,z)}\rho_k(s)\,d\nu_k(z)\,ds\right\}.$$

Substituting (3.14) and (3.15) into (3.13) while identifying (2.11), we end up with

(3.16)
$$F_t^{\mathcal{G},\mathbb{Q}}(\tau_1,\tau_2)=\Gamma(t)+\sum_{k=1}^{n}\Psi_k(t)\,X_t^k+\sum_{k=1}^{l}\Phi_k(t)\,Z_t^k$$

wherein we have just introduced the deterministic functions

(3.17)
$$\Psi_k(t)\coloneqq\frac{\Lambda_k(t,\tau_1,\tau_2)}{\sigma_k(t)},\qquad\Phi_k(t)\coloneqq\int_{\tau_1}^{\tau_2}\int_u^t\frac{w_k\,\sigma_k(s)\,e^{-\lambda_k(u-s)}}{(t-\tau)(\tau_2-\tau_1)}ds\,du,$$

$$\Gamma(t)\coloneqq\int_{\tau_1}^{\tau_2}\frac{\mu(u)}{\tau_2-\tau_1}du-\sum_{k=l+1}^{n}\int_{\tau_1}^{\tau_2}\int_u^t\int_{D_k}\frac{w_k\,\sigma_k(s)}{\tau_2-\tau_1}z\,e^{-\lambda_k(u-s)}\,e^{h_k(s,z)}\rho_k(s)\,d\nu_k(z)\,ds\,du$$

and the random variables

(3.18)
$$Z_t^k\coloneqq\mathbb{E}_{\mathbb{Q}}(L_\tau^k-L_t^k|\mathcal{G}_t)=\mathbb{E}_{\mathbb{Q}}(L_\tau^k|\mathcal{G}_t)-L_t^k.$$

Denoting the derivative with respect to t by an inverted comma, Itô's product rule leads us to

(3.19)
$$dF_t^{\mathcal{G},\mathbb{Q}}(\tau_1,\tau_2)=\left[\Gamma'(t)+\sum_{k=1}^{l}\Phi_k'(t)\,Z_t^k\right]dt+\sum_{k=1}^{n}\Lambda_k(t,\tau_1,\tau_2)\,dL_t^k+\sum_{k=1}^{l}\Phi_k(t)\,dZ_t^k.$$

In the next step, we aim to express the dynamics (3.19) in terms of $(\mathcal{G}_t,\mathbb{Q})$-martingale integrators. To this end, verify that for $t\in[0,\tau[$ it holds

(3.20)
$$dZ_t^k=(\tau-t)\,d\left(\frac{Z_t^k}{\tau-t}\right)-\frac{Z_t^k}{\tau-t}dt.$$

Substituting (2.3), (2.8), (3.6) and (3.20) into (3.19), with $t \in [0, \tau_1]$, $\tau_1 < \tau$, we eventually obtain the $\mathcal{G}$-forward-looking electricity futures price $\mathbb{Q}$-representation [associated to the case $u < \tau$] reading

(3.21)
$$dF_t^{\mathcal{G},\mathbb{Q}}(\tau_1, \tau_2) =$$
$$\sum_{k=1}^{l} \Lambda_k(t, \tau_1, \tau_2) \left(dL_t^k - \frac{Z_t^k}{\tau - t} dt \right) + \sum_{k=l+1}^{n} \Lambda_k(t, \tau_1, \tau_2) \int_{D_k} z \, d\widetilde{N}_k^{\mathcal{F},\mathbb{Q}}(t, z) + \sum_{k=1}^{l} \Phi_k(t) \, (\tau - t) \, d\left(\frac{Z_t^k}{\tau - t} \right)$$

with vanishing drift. Hence, with respect to (3.18), Lemma 3.1 (a), Remark 3.2 (a) and Lemma 3.4, the futures in (3.21) constitutes a $(\mathcal{G}_t, \mathbb{Q})$-martingale. In the light of Equality (3.2), this fact is not a surprising result. In conclusion, (3.21) essentially extends our former representation (2.10), respectively Proposition 3.1 in [4]. Finally, verify that

$$d\left(\frac{Z_t^k}{\tau - t} \right) = \frac{d\mathbb{E}_{\mathbb{Q}}(L_\tau^k | \mathcal{G}_t)}{\tau - t} - \frac{1}{\tau - t} \left(dL_t^k - \frac{Z_t^k}{\tau - t} dt \right).$$

Merging the latter equality into (3.21), we get the claimed representation. ∎

Remark 3.6. *In the sequel of (3.13) we have presumed $u < \tau$ in order to be able to apply Lemma 3.1 (c). Complementarily, we now examine the case $u \geq \tau$. To claim our findings right at the beginning, we announce that also for $u \geq \tau$ we receive a futures price representation that very closely resembles (3.21). Yet, for $0 \leq t \leq \tau_1 < \tau \leq u \leq \tau_2$ we decompose the conditional expectation appearing in the derivation of (3.14) via*

$$\mathbb{E}_{\mathbb{Q}}\left(\int_t^u \sigma_k(s) e^{-\lambda_k(u-s)} dL_s^k \,\middle|\, \mathcal{G}_t \right) = \mathbb{E}_{\mathbb{Q}}\left(\int_t^{\tau-} \sigma_k(s) e^{-\lambda_k(u-s)} dL_s^k \,\middle|\, \mathcal{G}_t \right) + \mathbb{E}_{\mathbb{Q}}\left(\int_\tau^u \sigma_k(s) e^{-\lambda_k(u-s)} dL_s^k \,\middle|\, \mathcal{G}_t \right)$$
$$=: \mathfrak{S}_1 + \mathfrak{S}_2.$$

Note that in $\mathfrak{S}_1$ it holds $t \leq s < \tau$ so that we can apply Lemma 3.1 (a) and (c) here while in $\mathfrak{S}_2$ we observe $t < \tau \leq s \leq u$ inducing $\mathcal{F}_t \subset \mathcal{G}_t \subset \mathcal{G}_\tau = \mathcal{F}_\tau$. As a consequence, we get the equalities

$$\mathfrak{S}_1 = \frac{\mathbb{E}_{\mathbb{Q}}(L_\tau^k - L_t^k | \mathcal{G}_t)}{\tau - t} \int_t^\tau \sigma_k(s) \, e^{-\lambda_k(u-s)} \, ds,$$

$$\mathfrak{S}_2 = \mathbb{E}_{\mathbb{Q}}\left(\mathbb{E}_{\mathbb{Q}}\left(\int_\tau^u \sigma_k(s) \, e^{-\lambda_k(u-s)} \, dL_s^k \,\middle|\, \mathcal{F}_\tau \right) \middle|\, \mathcal{G}_t \right) = \mathbb{E}_{\mathbb{Q}}\left[\int_\tau^u \sigma_k(s) \, e^{-\lambda_k(u-s)} \, dL_s^k \right].$$

Referring to (2.3), (2.8) and Condition A, for $k = 1, \dots, l$ we further obtain

$$\mathbb{E}_{\mathbb{Q}}\left(\int_t^u \sigma_k(s) \, e^{-\lambda_k(u-s)} \, dL_s^k \,\middle|\, \mathcal{G}_t \right) =$$
$$\frac{\mathbb{E}_{\mathbb{Q}}(L_\tau^k - L_t^k | \mathcal{G}_t)}{t - \tau} \int_\tau^t \sigma_k(s) \, e^{-\lambda_k(u-s)} \, ds + \int_\tau^u \int_{D_k} z \, \sigma_k(s) \, e^{-\lambda_k(u-s)} \, e^{h_k(z)} \, \rho_k \, d\nu_k(z) \, ds$$

wherein the last summand is deterministic and independent of t.
Moreover, similar computations as in (3.13)-(3.21) currently lead us to exactly the same futures price dynamics [but yet associated to the case $u \geq \tau$] as originally given in (3.21), respectively in Theorem 3.5, except from a new function

(3.22)

$$\widetilde{\Phi}_k(t) := \int_{\tau_1}^{\tau_2} \int_{\tau}^{t} \frac{w_k\, \sigma_k(s)\, e^{-\lambda_k(u-s)}}{(t-\tau)(\tau_2-\tau_1)} ds\, du \left(= \int_{\tau}^{t} \frac{\Lambda_k(s,\tau_1,\tau_2)}{t-\tau} ds \right)$$

instead of $\Phi_k(t)$. Evidently, the only difference between $\widetilde{\Phi}_k(t)$ and $\Phi_k(t)$ can be detected in the lower integration bound of the inner integral. Thus, with respect to our upcoming option pricing purposes we conclude that it is not really necessary to differ between the cases $u < \tau$ and $u \geq \tau$, since both instances induce (essentially) the same futures price dynamics. For the sake of notational simplicity, we will always assume $u < \tau$ in our proceedings, unless stated otherwise.

Unfortunately, the representation found in Theorem 3.5 is not really suitable for electricity derivatives pricing, as it lacks any independent increment property with respect to $\mathcal{G}$ [recall Remark 3.2 (b)], nor is its distribution/characteristic function known. In addition, from (3.5) we merely know that $\mathcal{G}$ contains more information than $\mathcal{F}$ and less than $\mathcal{H}$ which might cause some difficulties in the *practical* modeling of available future information. For these reasons, we now propose to replace the non-explicit filtration $\mathcal{G}$ by the *explicitly* enlarged intermediate filtration

(3.23)
$$\mathcal{G}_t^* := \mathcal{F}_t \vee \sigma\{L_\tau^1, \dots, L_\tau^p\}$$

with $1 \leq p \leq l < n$. Then $\mathcal{F}_t \subset \mathcal{G}_t^* \subset \mathcal{H}_t$ for $t < \tau$ and $\mathcal{G}_t^* = \mathcal{F}_t$ for $t \geq \tau$ hold true. Putting $p = l$ yet corresponds to $\mathcal{G}_t^* = \mathcal{H}_t$ and thus to complete knowledge of the spot price mean level at the future time τ. In contrast, the case $p < l$ represents a scenario wherein the market participants merely have access to some restricted additional information about the future spot price behavior, sounding more realistic. More importantly, from Lemma 3.1 (b) [and Condition A adapted to (3.23)] we deduce that for all $k = 1, \dots, p$ and $t \in [0, \tau[$ the stochastic processes

(3.24)
$$L_t^k - \int_0^t \frac{L_\tau^k - L_s^k}{\tau - s} ds$$

constitute $(\mathcal{G}_t^*, \mathbb{Q})$-martingales. The following result provides the futures price dynamics for an informed agent with information flow modeled by $\mathcal{G}^*$. It is interesting to compare Theorem 3.7 with Proposition 2.1 above.

Theorem 3.7. *For $t \in [0, \tau_1]$, $\tau_1 < \tau$, the futures price $F_t^{\mathcal{G}^*,\mathbb{Q}}(\tau_1,\tau_2)$ obeys the $\mathcal{G}_t^*$-adapted $\mathbb{Q}$-martingale dynamics*

(3.25)

$$dF_t^{\mathcal{G}^*,\mathbb{Q}}(\tau_1,\tau_2) = \sum_{k=1}^{p} [\Lambda_k(t) - \Phi_k(t)] \left(dL_t^k - \frac{L_\tau^k - L_t^k}{\tau - t} dt \right) + \sum_{k=p+1}^{n} \Lambda_k(t) \int_{D_k} z\, d\widetilde{N}_k^{\mathcal{F},\mathbb{Q}}(t,z)$$

where $\Lambda_k(t) := \Lambda_k(t,\tau_1,\tau_2)$ and $\Phi_k(t)$ are such as defined in (2.11), respectively (3.17).

Proof. Similar arguments as in (3.13)-(3.18) yield

(3.26)

$$F_t^{\mathcal{G}^*,\mathbb{Q}}(\tau_1,\tau_2) = \tilde{\Gamma}(t) + \sum_{k=1}^{n} \Psi_k(t)\, X_t^k + \sum_{k=1}^{p} \Phi_k(t)\, \mathbb{E}_{\mathbb{Q}}(L_\tau^k - L_t^k | \mathcal{G}_t^*)$$

wherein we have just introduced the shorthand notation

$$\tilde{\Gamma}(t) := \int_{\tau_1}^{\tau_2} \frac{\mu(u)}{\tau_2 - \tau_1} du - \sum_{k=p+1}^{n} \int_{\tau_1}^{\tau_2} \int_{u}^{t} \int_{D_k} \frac{w_k\, z\, \sigma_k(s)\, e^{-\lambda_k(u-s)}}{\tau_2 - \tau_1} e^{h_k(s,z)}\, \rho_k(s)\, d\nu_k(z)\, ds\, du.$$

Recall that L_t^k is $\mathcal{G}_t^*$-adapted for each $k = 1, \dots, p$, as $\mathcal{F}_t \subset \mathcal{G}_t^*$. Similarly, every L_τ^k is $\mathcal{G}_t^*$-adapted for $k = 1, \dots, p$, since L_τ^k trivially is $\sigma\{L_\tau^k\}$-adapted and, in addition, for a certain (fixed) index $k \in \{1, \dots, p\}$ the inclusions $\sigma\{L_\tau^k\} \subset \sigma\{L_\tau^1, \dots, L_\tau^p\} \subset \mathcal{F}_t \vee \sigma\{L_\tau^1, \dots, L_\tau^p\} = \mathcal{G}_t^*$ are valid. Hence, we obtain

(3.27)

$$\mathbb{E}_{\mathbb{Q}}(L_\tau^k - L_t^k | \mathcal{G}_t^*) = L_\tau^k - L_t^k = \int_t^\tau \int_{D_k} z\, dN_k(s, z)$$

for all $k = 1, \dots, p$. With respect to the derivation methodology of property (3.21) – but now using (3.24), (3.26) and (3.27) – for $t \in [0, \tau_1]$, $\tau_1 < \tau$, we eventually get (3.25). ■

Comparing Theorem 3.5 with Theorem 3.7, we appreciate that the latter not only lacks the differential $d\mathbb{E}_{\mathbb{Q}}(L_\tau^k | \mathcal{G}_t)$, but also that $\mathcal{G}^*$ should be more appropriate than $\mathcal{G}$ for practical applications.
In accordance to (3.24) and Lemma 3.1 (d), the $(\mathcal{G}^*, \mathbb{Q})$-compensator of $dN_k(s, z)$ for $k = 1, \dots, p$ reads

(3.28)

$$d\nu_k^{\mathcal{G}^*, \mathbb{Q}}(s, z) := \frac{1}{\tau - s} \int_{u=s}^{u=\tau} dN_k(u, z)\, ds$$

whereas the $(\mathcal{G}^*, \mathbb{Q})$-compensated random measure (RM) (remind Remark 3.2 (b) and verify that we are in a similar setting as in Example 16.38 in [12]) is of the form

(3.29)
$$d\tilde{N}_k^{\mathcal{G}^*, \mathbb{Q}}(s, z) := dN_k(s, z) - d\nu_k^{\mathcal{G}^*, \mathbb{Q}}(s, z).$$

Combining (3.24) with (3.28)-(3.29), we obtain the linking equality

(3.30)

$$L_t^k - \int_0^t \frac{L_\tau^k - L_s^k}{\tau - s} ds = \int_0^t \int_{D_k} z\, d\tilde{N}_k^{\mathcal{G}^*, \mathbb{Q}}(s, z).$$

Anyway, instead of introducing the filtration $\mathcal{G}^*$ we could also work with the *martingale representation theorem* (see p. 267 in [1]) to get rid of the last differential in (3.21). More precisely, instead of working with (3.20), we may directly take t-differentials in (3.18) leading us to $dZ_t^k = dM_t^k - dL_t^k$ wherein $M_t^k := \mathbb{E}_{\mathbb{Q}}(L_\tau^k | \mathcal{G}_t)$ constitutes a $(\mathcal{G}_t, \mathbb{Q})$-martingale in $t \in [0, \tau[$ for each $k \in \{1, \dots, l\}$. That is, in accordance to p. 8 in [24], L_τ^k "*closes*" the martingale M_t^k for each $k \in \{1, \dots, l\}$. Due to the martingale representation theorem we further deduce that for every $k \in \{1, \dots, l\}$ there exists a $\mathcal{G}_t$-predictable and $\mathbb{Q}$-square-integrable stochastic process $\zeta_k(t)$ such that

(3.31)

$$d\mathbb{E}_{\mathbb{Q}}(L_\tau^k | \mathcal{G}_t) = \zeta_k(t) \left\{ dL_t^k - \frac{Z_t^k}{\tau - t} dt \right\}$$

holds for all $t \in [0, \tau[$. Merging (3.20) and (3.31) into (3.21), [instead of (3.21)] we presently derive

$$dF_t^{\mathcal{G}, \mathbb{Q}}(\tau_1, \tau_2) =$$

$$\sum_{k=1}^{l} [\Lambda_k(t, \tau_1, \tau_2) - \{1 - \zeta_k(t)\}\, \Phi_k(t)] \left(dL_t^k - \frac{\mathbb{E}_{\mathbb{Q}}(L_\tau^k - L_t^k | \mathcal{G}_t)}{\tau - t} dt \right) + \sum_{k=l+1}^{n} \Lambda_k(t, \tau_1, \tau_2) \int_{D_k} z\, d\tilde{N}_k^{\mathcal{F}, \mathbb{Q}}(t, z)$$

standing in close analogy to (3.25). The recently found futures price decomposition neither possesses $\mathcal{G}$-independent increments nor are the involved processes $\zeta_k(t)$ known explicitly – both being disadvantages for option pricing purposes.

3.3 Forward-looking electricity futures option prices

3.3.1 Call option price. We now concentrate on the evaluation of a European call option written on the futures (3.25). Referring to (2.14), we define $C_T^{\mathcal{G}^*} := \left[F_T^{\mathcal{G}^*,\mathbb{Q}}(\tau_1,\tau_2) - K\right]^+$. Then, similarly to (2.15), for $0 \le t \le T$ the risk-neutral pricing formula yields

$$C_t^{\mathcal{G}^*} = e^{-r(T-t)}\,\mathbb{E}_{\mathbb{Q}}\left(\left[F_T^{\mathcal{G}^*,\mathbb{Q}}(\tau_1,\tau_2) - K\right]^+ \middle| \mathcal{G}_t^*\right). \tag{3.32}$$

The call option price (3.32) can be seen as some kind of *subjective price* for an informed agent with information flow $\mathcal{G}^*$. All other market participants (who do not have access to the additional information modeled by $\mathcal{G}^*$) still price options under $\mathcal{F}$.
Setting $q(x) := e^{-ax}\,[x - K]^+ \in \mathcal{L}^1(\mathbb{R}^+)$, $0 < a < \infty$, and $F_t^* := F_t^{\mathcal{G}^*,\mathbb{Q}} := F_t^{\mathcal{G}^*,\mathbb{Q}}(\tau_1,\tau_2)$ we deduce

$$C_t^{\mathcal{G}^*} = e^{-r(T-t)}\,\mathbb{E}_{\mathbb{Q}}\left(e^{aF_T^*}\,q(F_T^*)\middle|\mathcal{G}_t^*\right) \tag{3.33}$$

(with $e^{aF_T^*} < \infty$; recall our remark previous to Prop. 2.3), whereas an application of (2.17) leads us to

$$C_t^{\mathcal{G}^*} = \frac{e^{-r(T-t)}}{2\pi}\int_{\mathbb{R}^+} \hat{q}(y)\, e^{(a+iy)F_t^*}\,\mathbb{E}_{\mathbb{Q}}\left(e^{(a+iy)[F_T^*-F_t^*]}\middle|\mathcal{G}_t^*\right) dy \tag{3.34}$$

with $\hat{q}(y)$ as given in (2.18). Putting $\Lambda_k(s) := \Lambda_k(s,\tau_1,\tau_2)$ and $\Xi_k(s,z) := z\,[\Lambda_k(s) - \Phi_k(s)]$ while using the decomposition (3.25) along with (3.30), we further obtain

$$\mathbb{E}_{\mathbb{Q}}(\exp\{(a+iy)[F_T^*-F_t^*]\}|\mathcal{G}_t^*) = \tag{3.35}$$

$$\mathbb{E}_{\mathbb{Q}}\left(\exp\left\{(a+iy)\left[\sum_{k=1}^{p}\int_t^T\int_{D_k}\Xi_k(s,z)\,d\widetilde{N}_k^{\mathcal{G}^*,\mathbb{Q}}(s,z) + \sum_{k=p+1}^{n}\int_t^T\int_{D_k} z\,\Lambda_k(s)\,d\widetilde{N}_k^{\mathcal{F},\mathbb{Q}}(s,z)\right]\right\}\middle|\mathcal{G}_t^*\right).$$

Yet, (3.35) does not reduce to a usual expectation, as the involved price process F^* does not possess $\mathbb{Q}$-independent increments w.r.t. $\mathcal{G}^*$. In fact, the presence of $\widetilde{N}^{\mathcal{G}^*,\mathbb{Q}}$ in (3.35) avoids this property, because for each $k \in \{1,\dots,p\}$ both $\widetilde{N}_k^{\mathcal{G}^*,\mathbb{Q}}$ and $\mathcal{G}^*$ contain L_τ^k – see Remark 3.2 (b). As a consequence, F^* neither is a Lévy- nor an additive process (w.r.t. $\mathcal{G}^*$) and thus, (3.35) can neither be handled by the Lévy-Khinchin formula (similarly to Proposition 2.3). Since F^* neither is Markovian w.r.t. $\mathcal{G}^*$, we cannot derive any *partial integro differential equation* (PIDE) for the underlying call price value function (as presented in e.g. Proposition 12.1 and Chapter 14 in [10]). As the distribution of F^* is neither known, it is also impossible to apply any measure transformation arguments. Presently, the *analytical* treatment (if there is any appropriate at all) of (3.35) is a standing problem. However, in the sequel we propose two approximation methods involving results from complex analysis to treat (3.35):

1. To begin with, we recall that the challenge in (3.33) consists in finding a proper handling of the anticipating conditional expectation

$$\mathbb{E}_{\mathbb{Q}}\left(e^{aF_T^*}\,q(F_T^*)\middle|\mathcal{G}_t^*\right) \tag{3.36}$$

with $q(x) := e^{-ax}\,[x - K]^+ \in \mathcal{L}^1(\mathbb{R}^+)$, $0 < a < \infty$. Moreover, in accordance to (3.25) and (3.30), for times $0 \le t \le T$ the futures price F_t^* satisfies the $(\mathcal{G}_t^*,\mathbb{Q})$-martingale decomposition

(3.37)

$$F_t^* = F_0^* + \sum_{k=1}^{p} \int_0^t \int_{D_k} \Xi_k(s,z)\, d\widetilde{N}_k^{\mathcal{G}^*,\mathbb{Q}}(s,z) + \sum_{k=p+1}^{n} \int_0^t \int_{D_k} \Lambda_k(s)\, z\, d\widetilde{N}_k^{\mathcal{F},\mathbb{Q}}(s,z)$$

wherein $\Lambda_k(s)$ and $\Xi_k(s,z)$ are such as defined in the context of (3.35). Appealing to (2.12), we further assume the initial value F_0^* to be deterministic. Next, applying (2.17) on (3.36), we derive

(3.38)

$$\mathbb{E}_{\mathbb{Q}}\big(e^{aF_T^*}\, q(F_T^*) \big| \mathcal{G}_t^*\big) = \frac{1}{2\pi} \int_{\mathbb{R}^+} \hat{q}(y)\, \mathbb{E}_{\mathbb{Q}}\big(e^{(a+iy)F_T^*} \big| \mathcal{G}_t^*\big)\, dy.$$

Hence, instead of (3.36) we may equivalently examine the conditional expectation on the right hand side of (3.38) in the following. Since a, y and F_T^* are real-valued, the object $z_T \coloneqq a_T + iy_T \coloneqq aF_T^* + iyF_T^*$ designates a stochastic complex number. Further on, we introduce the *holomorphic* (see p. 118 in [14]) function $f\colon \mathbb{C} \longrightarrow \mathbb{C}$ via $f(\zeta) \coloneqq e^{\zeta}$ which can be developed into a power series due to

(3.39)

$$f(\zeta) = \sum_{\nu=0}^{\infty} \frac{\zeta^\nu}{\nu!}$$

(for further reading see e.g. Chapter V in [14] or Chapter II in [21]) with convergence radius

$$\Re = \Big(\lim_{\nu\to\infty} \sup 1/\sqrt[\nu]{\nu!}\Big)^{-1} = \infty.$$

Thus, (3.39) is valid on the entire complex plane. Using (3.39), we obtain the approximation

(3.40)

$$\mathcal{C}_{\mathbb{Q}}^{\mathcal{G}^*}(F_T^*; t, a, y) \coloneqq \mathbb{E}_{\mathbb{Q}}\big(e^{(a+iy)F_T^*} \big| \mathcal{G}_t^*\big) = \sum_{\nu=0}^{\infty} \frac{(a+iy)^\nu}{\nu!} \mathbb{E}_{\mathbb{Q}}((F_T^*)^\nu | \mathcal{G}_t^*) \approx \mathcal{T}_d + \mathcal{R}_d$$

wherein the (stochastic) d-th order Taylor polynomial is given by

(3.41)

$$\mathcal{T}_d \coloneqq \mathcal{T}_d(F_T^*; a, y, \mathcal{G}_t^*, \mathbb{Q}) \coloneqq 1 + (a+iy)\, F_t^* + \sum_{\nu=2}^{d} \frac{(a+iy)^\nu}{\nu!} \mathbb{E}_{\mathbb{Q}}((F_T^*)^\nu | \mathcal{G}_t^*)$$

and the (stochastic) Lagrange-type approximation error (the *remainder term*) possesses the structure

(3.42)

$$\mathcal{R}_d \coloneqq \mathcal{R}_d(F_T^*; a, y, \xi) \coloneqq \frac{e^{\xi}}{(d+1)!} (a+iy)^{d+1} (F_T^*)^{d+1}.$$

Herein, $\xi \in \mathbb{C}$ is such that $|\xi| \le F_T^* \sqrt{a^2+y^2}$ holds $\mathbb{Q}$-a.s. Additionally, we presume $0 < F_T^* \le M < \infty$ $\mathbb{Q}$-a.s. from now on with a constant $M > 0$.[3] Hence, we observe $|\xi| \le M \sqrt{a^2+y^2}$ and $|\mathcal{R}_d| \to 0$ for $d \to \infty$. Regarding (3.41), we should devote our attention towards the computation of $\mathbb{E}_{\mathbb{Q}}((F_T^*)^\nu | \mathcal{G}_t^*)$ for indices $\nu = 2, \dots, d$ in the following. For this purpose, we introduce a family of real-valued polynomials $\{g_\nu(x) \coloneqq x^\nu \mid x \in [0, M] \subset \mathbb{R}^+;\ \nu = 0,1,\dots,d\}$. To treat the objects $\mathbb{E}_{\mathbb{Q}}((F_T^*)^\nu | \mathcal{G}_t^*)$ appearing in (3.41), one might propose to apply Itô's formula on $g_\nu(F_T^*)$ $(\nu = 2, \dots, d)$ in order to derive a representation of

[3] Having simulated various futures price trajectories $(F_t^*: 0 \le t \le T)$ in practice, it should not cause any further trouble to implement a reasonable upper bound M such that $F_T^* \in\,]0, M]$ is met with a $\mathbb{Q}$-probability close to one.

the latter in terms of stochastic integrals. Unfortunately, neither the incoming infinite sum nor the remaining conditional expectation seems to be analytically tractable, as the underlying dynamics (3.37) is rather demanding.

Nevertheless, since F^* designates a $(\mathcal{G}^*,\mathbb{Q})$-martingale, we have $\mathbb{E}_{\mathbb{Q}}(F_T^*|\mathcal{G}_t^*) = F_t^*$ for all $0 \le t \le T$. Inspired by this observation, we yet propose a *linear* interpolation scheme to approximate $\mathbb{E}_{\mathbb{Q}}((F_T^*)^\nu|\mathcal{G}_t^*)$ for each $\nu = 2,\dots,d$. To this end, we implement a (not necessarily equidistant) partition of the real interval $[0,M]$ due to $\mathfrak{P} \coloneqq \{0 = x_0 < x_1 < \dots < x_m = M\}$ whereas we define the mesh $\Delta \coloneqq \Delta(\mathfrak{P}) \coloneqq \max_{0\le j\le m-1}|x_{j+1} - x_j|$. Our key idea is to approximate the convex polynomial functions $g_\nu(x)$ $(\nu = 2,\dots,d)$ in each interval $[x_j, x_{j+1}]$ for $j = 0,\dots,m-1$ by its particular secants

(3.43)

$$s_j^\nu(x) = \frac{x_{j+1}^\nu - x_j^\nu}{x_{j+1} - x_j}\left(x - x_j\right) + x_j^\nu.$$

Then, for $\nu = 2,\dots,d$ and $j = 0,\dots,m-1$ the approximation error ε_j^ν in each interval $[x_j, x_{j+1}]$ is given by the difference $\varepsilon_j^\nu \coloneqq \varepsilon_j^\nu(x) \coloneqq s_j^\nu(x) - g_\nu(x)$ which is bounded through

(3.44)

$$0 \le \varepsilon_j^\nu \le \frac{\nu\,(\nu-1)}{8}\left(x_{j+1} - x_j\right)^2 x_{j+1}^{\nu-2}.$$

Obviously, the right hand side of (3.44) vanishes as the mesh becomes finer, i.e. as Δ approaches zero. Consequently, we likewise deduce $\varepsilon_j^\nu \to 0$ whenever $\Delta \to 0$. In conclusion, we observe $s_j^\nu(x) \to g_\nu(x)$ for $\Delta \to 0$ whenever $x \in [x_j, x_{j+1}]$. These observations lead us to the approximation

(3.45)

$$g_\nu(x) \approx \sum_{j=0}^{m-1} s_j^\nu(x)\,\mathbb{1}_{]x_j,x_{j+1}]}(x)$$

wherein $x \in\,]0,M]$ and $\nu = 2,\dots,d$ while $\mathbb{1}$ depicts the indicator function. Thus, taking (3.43), (3.45) and the $(\mathcal{G}^*,\mathbb{Q})$-martingale property of F^* into account, for $\nu = 2,\dots,d$ and $j = 0,\dots,m-1$ we may *estimate* [with vanishing approximation error ε_j^ν, as long as $\Delta \to 0$; recall (3.44)] the conditional expectations appearing on the right hand side of (3.41) via

(3.46)

$$\mathbb{E}_{\mathbb{Q}}((F_T^*)^\nu|\mathcal{G}_t^*) \approx \frac{x_{j+1}^\nu - x_j^\nu}{x_{j+1} - x_j}\left(F_t^* - x_j\right) + x_j^\nu$$

whenever $x_j < F_T^* \le x_{j+1}$ $\mathbb{Q}$-a.s. Collecting (3.40), (3.41) and (3.46), we get the approximation

(3.47)

$$\mathcal{C}_{\mathbb{Q}}^{\mathcal{G}^*}(F_T^*;t,a,y) \approx 1 + (a+iy)F_t^* + \sum_{\nu=2}^{d}\frac{(a+iy)^\nu}{\nu!}\left[\frac{x_{j+1}^\nu - x_j^\nu}{x_{j+1} - x_j}\left(F_t^* - x_j\right) + x_j^\nu\right] =: \mathcal{A}_j^d(y;a,F_t^*)$$

whenever $x_j < F_T^* \le x_{j+1}$ $\mathbb{Q}$-a.s. In conclusion, referring to (3.33), (3.38), (3.40) and (3.47), we end up with the $\mathcal{G}^*$-forward-looking electricity futures call option price estimate

(3.48)

$$C_t^{\mathcal{G}^*} \approx \frac{e^{-r(T-t)}}{2\pi}\int_{\mathbb{R}^+} \hat{q}(y)\,\mathcal{A}_j^d(y;a,F_t^*)\,dy$$

whenever $x_j < F_T^* \leq x_{j+1}$ $(j = 0, \ldots, m-1)$ holds $\mathbb{Q}$-a.s. Herein, $\hat{q}(y)$ and $\mathcal{A}_j^d(y; a, F_t^*)$ are such as defined in (2.18) resp. (3.47), while the futures price F_t^* has to be simulated numerically by using (3.37).

One might finally wonder why we have not directly applied secant approximation techniques on the argument process $e^{(a+iy)F_T^*}$ appearing on the right hand side of (3.38) – instead of developing the latter into a complex power series initially. The point here is that $e^{(a+iy)F_T^*}$ constitutes a *complex* function involving F_T^* whereas the Taylor-polynomial in (3.41) merely contains *real* polynomials involving F_T^* which may be approximated by a *real* linear interpolation approach, like presented above. We recall that, in the absence of any *analytical* computation methods for $\mathbb{E}_{\mathbb{Q}}\left(e^{(a+iy)F_T^*} \middle| \mathcal{G}_t^*\right)$, our goal has been to establish a *linear* estimation scheme in order to exploit the $(\mathcal{G}^*, \mathbb{Q})$-martingale property of F^*. Ultimately, we refer to (2.18), (3.43), (3.47) as well as (3.48) and establish the following approximation for the anticipative call option price (3.33).

Proposition 3.8. *The $\mathcal{G}^*$-anticipative call option price can be approximated via*

(3.49)

$$C_t^{\mathcal{G}^*} \approx \frac{e^{-r(T-t)}}{2\pi} \int_{\mathbb{R}^+} \frac{e^{-(a+iy)K}}{(a+iy)^2} \left[1 + (a+iy)\, F_t^* + \sum_{\nu=2}^{d} \frac{(a+iy)^\nu}{\nu!}\, s_j^\nu(F_t^*)\right] dy$$

whenever $x_j < F_T^* \leq x_{j+1}$ $(j = 0, \ldots, m-1)$ holds $\mathbb{Q}$-a.s. Herein, $s_j^\nu(\cdot)$ is such as defined in (3.43).

2. Catching up Equation (3.38) again, we now propose to treat the object $\mathbb{E}_{\mathbb{Q}}\left(e^{(a+iy)F_T^*} \middle| \mathcal{G}_t^*\right)$ with Cauchy's integral formula, alternatively. Sticking to similar notions as in (3.40) and the sequel of (3.38), we remind

(3.50)
$$C_{\mathbb{Q}}^{\mathcal{G}^*}(F_T^*; t, a, y) = \mathbb{E}_{\mathbb{Q}}\left(e^{(a+iy)F_T^*} \middle| \mathcal{G}_t^*\right) = \mathbb{E}_{\mathbb{Q}}\left(f(z_T) \middle| \mathcal{G}_t^*\right)$$

with an *entire* function $f(\zeta) \coloneqq e^\zeta$. To proceed in our arguing we recall the following result.

Theorem 3.9. (Cauchy's integral formula; CIF) *Suppose G is an (arbitrary) open subset of the complex plane $\mathbb{C}$ and $f: G \longrightarrow \mathbb{C}$ constitutes a holomorphic function on G. Additionally, let $\mathcal{K} \subset G$ be a closed, rectifiable, positive-oriented Jordan curve with winding number equal to* one *about $z_0 \in I(\mathcal{K})$ whereby $I(\mathcal{K}) \subset G$ with $\mathcal{K} \cap I(\mathcal{K}) = \emptyset$ denotes the interior of $\mathcal{K}$. Then for all $z_0 \in I(\mathcal{K})$ we have the representation*

(3.51)

$$f(z_0) = \frac{1}{2\pi i} \int_{\mathcal{K}} \frac{f(\zeta)}{\zeta - z_0}\, d\zeta$$

while we obtain $f(z_0) = 0$ whenever $z_0 \in \mathbb{C} \setminus \overline{I(\mathcal{K})} = \mathbb{C} \setminus \{\mathcal{K} \cup I(\mathcal{K})\}$.

Proof. See e.g. Chapter IV.4 in [14]. ∎

With view on (3.50), we aim to express $f(z_T)$ due to (3.51). For this purpose, we choose $G \coloneqq \mathbb{C}$ and $\mathcal{K} \coloneqq \mathcal{K}_\varrho(0) \coloneqq \{\zeta \in \mathbb{C}: |\zeta| = \varrho\}$. Yet, we presume ϱ to be *sufficiently large* in the sense of obeying $\varrho > M\sqrt{a^2 + y^2}$ with a constant $M > 0$. Then $z_T \in I\left(\mathcal{K}_\varrho(0)\right) \coloneqq \{\zeta \in \mathbb{C}: |\zeta| < \varrho\}$ holds $\mathbb{Q}$-a.s., as long as we assume $0 < F_T^* \leq M$ $\mathbb{Q}$-a.s. [parallel to the sequel of (3.42)]. Particularly, the representation (3.51) is valid for an *arbitrary* curve $\mathcal{K}$ [as long as $\mathcal{K}$ fulfills the requirements of Theorem 3.9] and thus, the actual value in (3.51) does neither depend on the length of $\mathcal{K}$ nor on the radius ϱ (this property is called "*path independency*" [14]). Hence, it is indeed possible to choose ϱ *sufficiently large* in the above sense. Consequently, we may apply Theorem 3.9 on (3.50) what leads us to

(3.52)

$$C_{\mathbb{Q}}^{\mathcal{G}^*}(F_T^*;t,a,y) = \frac{1}{2\pi i} \int_{|\zeta|=\varrho} e^{\zeta}\, \mathbb{E}_{\mathbb{Q}}\left(\frac{1}{\zeta - z_T}\middle|\mathcal{G}_t^*\right) d\zeta$$

wherein $|z_T| < \varrho$ holds $\mathbb{Q}$-a.s. Furthermore, we deduce

(3.53)

$$\mathbb{E}_{\mathbb{Q}}\left(\frac{1}{\zeta - z_T}\middle|\mathcal{G}_t^*\right) = \frac{1}{\zeta}\, \mathbb{E}_{\mathbb{Q}}\left(\frac{1}{1 - \frac{a+iy}{\zeta} F_T^*}\middle|\mathcal{G}_t^*\right).$$

(Alternatively, one might apply Itô's formula on $f(X_T) \coloneqq 1/X_T$ with $X_T \coloneqq \zeta - (a+iy)F_T^*$ where F_T^* obeys (3.37). Unfortunately, the incoming infinite sum is difficult to handle and not very useful for further computations.)
Since for $w_T \coloneqq \frac{a+iy}{\zeta} F_T^* \in \mathbb{C}$ with $\zeta \in \mathcal{K}_\varrho(0)$ we observe $|w_T| \le \frac{M}{\varrho}\sqrt{a^2+y^2} < 1$ $\mathbb{Q}$-a.s., we may develop the holomorphic function $A(w_T) \coloneqq 1/(1-w_T)$ into a geometric power series (with compact convergence in the open disk $\{w_T \in \mathbb{C}: |w_T| < 1\}$). As a consequence, (3.53) may be transformed into

(3.54)

$$\mathbb{E}_{\mathbb{Q}}\left(\frac{1}{\zeta - z_T}\middle|\mathcal{G}_t^*\right) = \frac{1}{\zeta}\, \mathbb{E}_{\mathbb{Q}}(A(w_T)|\mathcal{G}_t^*) = \frac{1}{\zeta}\sum_{\nu=0}^{\infty} \mathbb{E}_{\mathbb{Q}}((w_T)^\nu|\mathcal{G}_t^*) = \sum_{\nu=0}^{\infty} \frac{(a+iy)^\nu}{\zeta^{\nu+1}} \mathbb{E}_{\mathbb{Q}}((F_T^*)^\nu|\mathcal{G}_t^*).$$

Hence, merging (3.50), (3.52) and (3.54) into (3.38), we receive

(3.55)

$$\mathbb{E}_{\mathbb{Q}}\left(e^{aF_T^*}\, q(F_T^*)\middle|\mathcal{G}_t^*\right) = \frac{1}{4\pi^2 i} \sum_{\nu=0}^{\infty} \mathbb{E}_{\mathbb{Q}}((F_T^*)^\nu|\mathcal{G}_t^*) \int_{\mathbb{R}^+} \hat{q}(y)\,(a+iy)^\nu \int_{\mathcal{K}_\varrho(0)} \frac{e^\zeta}{\zeta^{\nu+1}} d\zeta\, dy$$

whereby standard arguments from complex analysis declare the remaining $d\zeta$-integral to equal $2\pi i/(\nu!)$. Next, substituting (2.18) and (3.55) into (3.33), we deduce the call option price formula

(3.56)

$$C_t^{\mathcal{G}^*} = \frac{e^{-r(T-t)}}{2\pi} \sum_{\nu=0}^{\infty} \mathbb{E}_{\mathbb{Q}}((F_T^*)^\nu|\mathcal{G}_t^*) \int_{0+}^{\infty} \frac{(a+iy)^{\nu-2}}{\nu!}\, e^{-(a+iy)K}\, dy$$

wherein F_T^* is given through (3.37). The terms $\mathbb{E}_{\mathbb{Q}}((F_T^*)^\nu|\mathcal{G}_t^*)$ appearing inside (3.56) may be approximated similarly to (3.46) whereas one ought to use Taylor polynomial estimates again and thus, choose finitely many summands in (3.54), respectively in (3.56), i.e. $\nu = 0, \dots, d$ with $d < \infty$ [as in (3.40)-(3.41)]. In this case, the resulting approximation stemming from (3.56) possesses a similar structure as found in (3.49).

3.3.2 Put option price. According to (3.32)-(3.34), with a real function $p(x) \coloneqq [K-x]^+ \in \mathcal{L}^1(\mathbb{R}^+)$ the forward-looking put option price at time $t \le T$ with strike $K > 0$ written on (3.25) is given by

$$P_t^{\mathcal{G}^*} = e^{-r(T-t)}\, \mathbb{E}_{\mathbb{Q}}\left(p\left(F_T^{\mathcal{G}^*,\mathbb{Q}}\right)\middle|\mathcal{G}_t^*\right) = \frac{e^{-r(T-t)}}{2\pi} \int_{\mathbb{R}^+} \frac{1 - iyK - e^{-iyK}}{y^2}\, \mathbb{E}_{\mathbb{Q}}\left(e^{iyF_T^{\mathcal{G}^*,\mathbb{Q}}}\middle|\mathcal{G}_t^*\right) dy$$

wherein the conditional expectation can be treated similar to the one in (3.33), respectively to $C_{\mathbb{Q}}^{\mathcal{G}^*}(F_T^*;t,0,y)$ in (3.40). Alternatively, the *Put-Call-Parity* applied on (3.32) delivers

$$P_t^{\mathcal{G}^*} = C_t^{\mathcal{G}^*} + e^{-r(T-t)}\left[K - F_t^{\mathcal{G}^*,\mathbb{Q}}(\tau_1,\tau_2)\right].$$

3.3.3 Asian option price. With reference to Section 9.2.2 in [7] we define the price of an Asian option at time $t \in [0, \tau_1]$ paying $f\left(\int_{\tau_1}^{\tau_2} S_u\, du\right) \in \mathcal{L}^1(\mathcal{G}^*, \mathbb{Q})$ with a real function $f \in \mathcal{L}^1(\mathbb{R}^+)$ at maturity $\tau_2\ (> \tau_1 \geq t)$ via

(3.57)

$$A_t^{\mathcal{G}^*} := A_t^{\mathcal{G}^*}(\tau_1, \tau_2) := e^{-r(\tau_2 - t)}\, \mathbb{E}_{\mathbb{Q}}\left(f\left(\int_{\tau_1}^{\tau_2} S_u\, du\right)\middle|\mathcal{G}_t^*\right).$$

An application of (2.17) delivers

(3.58)

$$A_t^{\mathcal{G}^*} = \frac{e^{-r(\tau_2 - t)}}{2\pi} \int_{\mathbb{R}^+} \hat{f}(y)\, \mathbb{E}_{\mathbb{Q}}\left(e^{iy \int_{\tau_1}^{\tau_2} S_u\, du}\middle|\mathcal{G}_t^*\right) dy$$

where $\hat{f}$ depicts the Fourier transform of $f \in \mathcal{L}^1(\mathbb{R}^+)$. Next, using (2.1), (2.5), (2.11) and (3.17), we derive

(3.59)

$$\int_{\tau_1}^{\tau_2} S_u\, du = \int_{\tau_1}^{\tau_2} \mu(u)\, du + \sum_{k=1}^{n} (\tau_2 - \tau_1)\, \Psi_k(t)\, X_t^k + \sum_{k=1}^{n} \int_t^{\tau_2} (\tau_2 - \mathfrak{m})\, \Lambda_k(s, \mathfrak{m}, \tau_2)\, dL_s^k$$

with $\mathfrak{m} := \mathfrak{m}(s) := \max\{s, \tau_1\}\ (\leq \tau_2)$. Hence, the conditional expectation in (3.58) factors into

(3.60)

$$\mathbb{E}_{\mathbb{Q}}\left(e^{iy \int_{\tau_1}^{\tau_2} S_u\, du}\middle|\mathcal{G}_t^*\right) = \exp\left\{iy\left[\int_{\tau_1}^{\tau_2} \mu(u)\, du + \sum_{k=1}^{n} (\tau_2 - \tau_1)\, \Psi_k(t)\, X_t^k\right]\right\} \times \mathbb{E}_{\mathbb{Q}}\left(\exp\left\{\sum_{k=1}^{n} \int_t^{\tau_2} \eta_k(s)\, dL_s^k\right\}\middle|\mathcal{G}_t^*\right)$$

wherein we have set $\eta_k(s) := iy\, \pi_k(s)$ and $\pi_k(s) := (\tau_2 - \mathfrak{m})\, \Lambda_k(s, \mathfrak{m}, \tau_2) \geq 0$. Note that, similarly to (3.35), for $\tau \leq \tau_2$ the conditional expectation on the right hand side of (3.60) does not reduce to a usual one.[4] For this reason, we now apply approximation techniques as invented in (3.40)-(3.48). To ease the notation, for $0 \leq t < \tau_2$ we first establish the (real-valued) stochastic process

$$0 \leq H_{t,\tau_2} := \sum_{k=1}^{n} \int_t^{\tau_2} \pi_k(s)\, dL_s^k$$

and the complex function $h(\zeta) := e^{\zeta}$. Parallel to our previous assumption on F_T^* given in the sequel of (3.42), we now presume $0 \leq H_{t,\tau_2} \leq M$ to be valid $\mathbb{Q}$-a.s. for all $0 \leq t < \tau_2$ with a constant $M > 0$. Consequently, the conditional expectation on the right hand side of (3.60) can be expressed as

$$\mathbb{E}_{\mathbb{Q}}\left(\exp\left\{\sum_{k=1}^{n} \int_t^{\tau_2} \eta_k(s)\, dL_s^k\right\}\middle|\mathcal{G}_t^*\right) = \mathbb{E}_{\mathbb{Q}}\left(h\left(iy\, H_{t,\tau_2}\right)\middle|\mathcal{G}_t^*\right).$$

Recalling (3.40), (3.41) and (3.43), we approximate the holomorphic function $h(\cdot)$ appearing inside the latter equation by its (complex) d-th order Taylor polynomial what leads us to

[4] There is a slight difference between (3.35) and (3.60), as the former equation contains $\widetilde{N}^{\mathcal{G}^*,\mathbb{Q}}$ at the place of L, respectively of N, inside the latter. Hence, for $(t <)\ \tau \leq \tau_2$ the conditional expectation in (3.60) does not reduce to a usual one, since $\int_t^{\tau_2} \eta_k(s)\, dL_s^k$ (for $k = 1, \dots, p$) is not $\mathbb{Q}$-independent of $\mathcal{G}_t^*$.

(3.61)

$$\mathbb{E}_{\mathbb{Q}}\left(\exp\left\{ \sum_{k=1}^{n} \int_{t}^{\tau_2} \eta_k(s)\, dL_s^k \right\} \middle| \mathcal{G}_t^* \right) \approx$$

$$\sum_{\nu=0}^{d} \frac{(iy)^\nu}{\nu!} \mathbb{E}_{\mathbb{Q}}\left(s_j^\nu(H_{t,\tau_2}) \middle| \mathcal{G}_t^*\right) = \sum_{\nu=0}^{d} \frac{(iy)^\nu}{\nu!} \left(\frac{x_{j+1}^\nu - x_j^\nu}{x_{j+1} - x_j} \left\{ \mathbb{E}_{\mathbb{Q}}\left(H_{t,\tau_2} \middle| \mathcal{G}_t^*\right) - x_j \right\} + x_j^\nu \right)$$

whenever $x_j < H_{t,\tau_2} \le x_{j+1}$ $(j = 0, \dots, m-1)$ is valid $\mathbb{Q}$-a.s.[5] With respect to (3.23) and the definition of H, the last conditional expectation in (3.61) may be decomposed as

$$\mathbb{E}_{\mathbb{Q}}\left(H_{t,\tau_2} \middle| \mathcal{G}_t^*\right) = \sum_{k=1}^{p} \mathbb{E}_{\mathbb{Q}}\left(\int_{t}^{\tau-} \pi_k(s)\, dL_s^k \middle| \mathcal{G}_t^* \right) + \sum_{k=1}^{p} \mathbb{E}_{\mathbb{Q}}\left(\int_{\tau}^{\tau_2} \pi_k(s)\, dL_s^k \middle| \mathcal{G}_t^* \right) + \sum_{k=p+1}^{n} \mathbb{E}_{\mathbb{Q}}\left(\int_{t}^{\tau_2} \pi_k(s)\, dL_s^k \middle| \mathcal{F}_t \right).$$

Meanwhile, we recall that $\mathcal{F}_t \subset \mathcal{G}_t^* \subset \mathcal{G}_\tau^* = \mathcal{F}_\tau$ is valid for all $t < \tau\ (\le \tau_2)$. Hence, taking (3.8) [adjusted to $\mathcal{G}^*$], (3.24), (3.27) and the tower property into account, we get

$$\mathbb{E}_{\mathbb{Q}}\left(H_{t,\tau_2} \middle| \mathcal{G}_t^*\right) = \sum_{k=1}^{p} \frac{L_\tau^k - L_t^k}{\tau - t} \int_{t}^{\tau} \pi_k(s)\, ds + \sum_{k=1}^{p} \mathbb{E}_{\mathbb{Q}}\left(\mathbb{E}_{\mathbb{Q}}\left(\int_{\tau}^{\tau_2} \pi_k(s)\, dL_s^k \middle| \mathcal{F}_\tau \right) \middle| \mathcal{G}_t^* \right) + \sum_{k=p+1}^{n} \mathbb{E}_{\mathbb{Q}}\left[\int_{t}^{\tau_2} \pi_k(s)\, dL_s^k \right].$$

Referring to (2.3) and (2.8), the latter equation points out as

$$\begin{aligned} \mathbb{E}_{\mathbb{Q}}\left(H_{t,\tau_2} \middle| \mathcal{G}_t^*\right) &= \sum_{k=1}^{p} \frac{L_\tau^k - L_t^k}{\tau - t} \int_{t}^{\tau} \pi_k(s)\, ds + \sum_{k=1}^{p} \mathbb{E}_{\mathbb{Q}}\left(\mathbb{E}_{\mathbb{Q}}\left[\int_{\tau}^{\tau_2} \pi_k(s)\, dL_s^k \right] \middle| \mathcal{G}_t^* \right) \\ &\quad + \sum_{k=p+1}^{n} \int_{t}^{\tau_2} \int_{D_k} z\, \pi_k(s)\, e^{h_k(s,z)}\, \rho_k(s)\, d\nu_k(z)\, ds. \end{aligned}$$

With view on Condition A [adjusted to $\mathcal{G}^*$], we introduce the deterministic abbreviations

$$\beta_k(t,\tau) := \int_{t}^{\tau} \frac{\pi_k(s)}{\tau - t} ds, \qquad \tilde{\xi}_k(t,v) := \int_{t}^{v} \int_{D_k} z\, \pi_k(s)\, e^{h_k(z)}\, \rho_k\, d\nu_k(z)\, ds,$$

$$\xi_k(t,v) := \int_{t}^{v} \int_{D_k} z\, \pi_k(s)\, e^{h_k(s,z)}\, \rho_k(s)\, d\nu_k(z)\, ds$$

and therewith receive the decomposition

$$\mathbb{E}_{\mathbb{Q}}\left(H_{t,\tau_2} \middle| \mathcal{G}_t^*\right) = \sum_{k=1}^{p} \beta_k(t,\tau) \{L_\tau^k - L_t^k\} + \sum_{k=1}^{p} \tilde{\xi}_k(\tau,\tau_2) + \sum_{k=p+1}^{n} \xi_k(t,\tau_2).$$

In conclusion, the estimation in (3.61) can be rewritten as

[5] Note that the secants in (3.43) have originally been defined for $\nu = 2, \dots, d$. Nevertheless, we may extend this setting here to $\nu = 0, \dots, d$ without any further restrictions. We recall that H_{t,τ_2} [playing the role of F_T^* in (3.46)] may become zero yet while F_T^* has been strictly positive by definition. Hence, we currently have to respect the additional (but trivial) instance $H_{t,\tau_2} = x_0 \equiv 0$ while $\mathcal{T}_d \equiv 1$ holds in this case. We conclude that it is indeed possible to extend our former presumption $0 < F_T^* \le M$ yet to $0 \le H_{t,\tau_2} \le M$.

(3.62)

$$\mathbb{E}_{\mathbb{Q}}\left(\exp\left\{\sum_{k=1}^{n}\int_{t}^{\tau_2}\eta_k(s)\,dL_s^k\right\}\middle|\mathcal{G}_t^*\right)\approx\mathfrak{M}_t^j(\tau,\tau_2;y,d;\eta):=$$

$$\sum_{\nu=0}^{d}\frac{(iy)^\nu}{\nu!}\left(\frac{x_{j+1}^\nu-x_j^\nu}{x_{j+1}-x_j}\left\{-x_j+\sum_{k=1}^{p}\beta_k(t,\tau)\,\{L_\tau^k-L_t^k\}+\sum_{k=1}^{p}\tilde{\xi}_k(\tau,\tau_2)+\sum_{k=p+1}^{n}\xi_k(t,\tau_2)\right\}+x_j^\nu\right).$$

We recall that the emerging components $L_\tau^1,\dots,L_\tau^p$ are already *known* from (3.23). In other words, a practitioner should have *guessed/established* these values (due to his available future information) previously so that a proper numerical evaluation of $\mathfrak{M}_t^j(\tau,\tau_2;y,d;\eta)$ yet should not cause any difficulties. Having simulated multiple paths of H_{t,τ_2} a practitioner should also be able to determine a reasonable upper bound M such that the above presumption $0\le H_{t,\tau_2}\le M$ for $0\le t<\tau_2$ is fulfilled with a probability close to one. Likewise, the constraint "*whenever* $x_j<H_{t,\tau_2}\le x_{j+1}$ $(j=0,\dots,m-1)$" ought to be implementable into a numerical simulation algorithm without any additional trouble, since H_{t,τ_2} has to be simulated anyway and thus, it is clear which particular intervals $]x_j,x_{j+1}]$ are hit by the actually realized trajectory of H_{t,τ_2}. Finally, we remark that our present reasoning about practical numerical application issues is valid for the evaluation of the call price formula (3.49), too.
For the sake of completeness, we claim that for $0\le t<\tau_2<\tau$ we receive the deterministic expression

$$\mathbb{E}_{\mathbb{Q}}\left(H_{t,\tau_2}\middle|\mathcal{G}_t^*\right)=\sum_{k=1}^{p}\tilde{\xi}_k(t,\tau_2)+\sum_{k=p+1}^{n}\xi_k(t,\tau_2)$$

showing that future information about the spot price driving noises $L_\tau^1,\dots,L_\tau^p$ evidently becomes irrelevant for an Asian option maturing at τ_2 where $\tau>\tau_2$. In conclusion, we have proven the following result which extends Proposition 9.8 in [7] to the anticipative information case.

Proposition 3.10. *The Asian option price (3.58) may be approximated via*

$$A_t^{\mathcal{G}^*}\approx\frac{e^{-r(\tau_2-t)}}{2\pi}\int_{\mathbb{R}^+}\hat{f}(y)\,\mathfrak{M}_t^j(\tau,\tau_2;y,d;\eta)\,\exp\left\{iy\left[\int_{\tau_1}^{\tau_2}\mu(u)\,du+\sum_{k=1}^{n}(\tau_2-\tau_1)\,\Psi_k(t)\,X_t^k\right]\right\}dy$$

where X_t^k, $\mathfrak{M}_t^j$ and $\Psi_k(t)$ are such as defined in (2.5), (3.62), respectively (3.17).

3.3.4 Floor option price. In order to protect against low electricity prices during a pre-specified time period $[\tau_1,\tau_2]$ a retailer may enter an electricity floor option being a European-type contract which ensures a cash flow at intensity $[K-S_u]^+$ with strike $K>0$ at *arbitrary* time $u\in[\tau_1,\tau_2]$. Thus, referring to Section 5.1 in [8], the arbitrage-free price of an electricity floor option at time $t\in[\tau_1,\tau_2]$ yet under $\mathcal{G}_t^*$ is given by

(3.63)

$$\mathrm{Floor}^*(t):=\mathrm{Floor}^{\mathcal{G}^*}(t;K,\tau_1,\tau_2):=\mathbb{E}_{\mathbb{Q}}\left(\int_{t\vee\tau_1}^{\tau_2}e^{-r(u-t)}\,[K-S_u]^+du\middle|\mathcal{G}_t^*\right).$$

Next, with $g(x):=[K-x]^+\in\mathcal{L}^1(\mathbb{R}^+)$ and (2.17), an application of Fubini's theorem yields

(3.64)

$$\mathrm{Floor}^*(t)=\frac{1}{2\pi}\int_{t\vee\tau_1}^{\tau_2}\int_{\mathbb{R}^+}e^{-r(u-t)}\hat{g}(y)\,\mathbb{E}_{\mathbb{Q}}\left(e^{iyS_u}\middle|\mathcal{G}_t^*\right)dy\,du.$$

Taking (2.1) and (2.5) into account, the conditional expectation in (3.64) factors into

(3.65)

$$\mathbb{E}_{\mathbb{Q}}(e^{iyS_u} \big| \mathcal{G}_t^*) = \exp\left\{iy\left[\mu(u) + \sum_{k=1}^{n} w_k\, X_t^k\, e^{-\lambda_k(u-t)}\right]\right\} \times \mathbb{E}_{\mathbb{Q}}\left(\exp\left\{\sum_{k=1}^{n}\int_t^u \varepsilon_k(s)\, dL_s^k\right\}\middle| \mathcal{G}_t^*\right)$$

wherein we have put $\varepsilon_k(s) \coloneqq iy\,\tilde{\pi}_k(s)$ with $\tilde{\pi}_k(s) \coloneqq \tilde{\pi}_k(s,u) \coloneqq w_k\,\sigma_k(s)\,e^{-\lambda_k(u-s)} \geq 0$. Obviously, the conditional expectation on the right hand side of (3.65) may be treated similarly to the one in (3.61) whereas η_k yet has to be replaced by ε_k, π_k by $\tilde{\pi}_k$ and τ_2 by u. Hence, parallel to (3.62), for $0 \leq t < \tau \leq u \leq \tau_2$ we deduce the approximation

$$\mathbb{E}_{\mathbb{Q}}\left(\exp\left\{\sum_{k=1}^{n}\int_t^u \varepsilon_k(s)\, dL_s^k\right\}\middle| \mathcal{G}_t^*\right) \approx \mathfrak{M}_t^j(\tau, u; y, d; \varepsilon).$$

In conclusion, we have proven the following result:

Proposition 3.11. *The anticipative floor option price (3.64) can be approximated via*

(3.66)

$$\text{Floor}^*(t) \approx \int_{t\vee\tau_1}^{\tau_2} \frac{e^{-r(u-t)}}{2\pi} \int_{\mathbb{R}^+} \hat{g}(y)\, \mathfrak{M}_t^j(\tau, u; y, d; \varepsilon)\, \exp\left\{iy\left[\mu(u) + \sum_{k=1}^{n} w_k\, X_t^k\, e^{-\lambda_k(u-t)}\right]\right\} dy\, du$$

wherein $\hat{g}(y) = \frac{1-iyK-e^{-iyK}}{y^2}$ *while* $\mathfrak{M}_t^j$ *and* X_t^k *are such as defined in (3.62), respectively (2.5).*

3.4 Pricing electricity contracts under temperature forecasts

In the Scandinavian energy market *Nord Pool* the main driver of electricity demand is outdoor temperature. Since low temperatures imply high prices (due to an increasing demand for heating), one usually observes a negative correlation between temperature and electricity prices [5]. (Of course, there also exist geographical areas where high temperatures lead to an increase of electricity demand due to air conditioning.)

Inspired by Section 3.2 in [5] we now assume electricity market participants to have access to weather forecasts including outdoor temperature. Appealing to [5], [6] and [16], we suppose the temperature process Θ to follow the OU-type dynamics

(3.67)

$$d\Theta_t = dm(t) + \vartheta\,[m(t) - \Theta_t]\,dt + \sum_{k=1}^{l} \xi_k\, dB_t^k.$$

Herein, the mean-reversion speed $\vartheta > 0$ is constant whereas the bounded, continuous and deterministic function $m(t)$ indicates the seasonal temperature mean level. Additionally, the volatilities ξ_k constitute positive constants and B_t^k embody standard BMs under $\mathbb{P}$ for all $k = 1, \dots, l$. The latter processes are assumed to be independent and independent of the jump noises $L_t^1, \dots, L_t^n$. The $\mathbb{P}$-solution of (3.67) reads as

(3.68)

$$\Theta_t = m(t) + e^{-\vartheta t}\,[\Theta_0 - m(0)] + \sum_{k=1}^{l}\int_0^t \xi_k\, e^{-\vartheta(t-s)} dB_s^k.$$

Moreover, we now take the base components $X_t^1, \dots, X_t^l$ (which formerly have been used to model the long-term level of the spot) to be *connected* with temperature via a constant $\zeta \in [-1,1]$. Hence, appealing to equality "(3.6) in [5]", for $k = 1, \dots, l$ we newly replace our former equation (2.2) through

$$dX_t^k = -\lambda_k\, X_t^k\, dt + \sigma_k(t)\left[\zeta\, dB_t^k + \sqrt{1-\zeta^2}\, dL_t^k\right] \tag{3.69}$$

while property (2.2) remains untouched for $k = l+1, \dots, n$. Obviously, in (3.69) we are not facing the classical setting with *correlated BMs* whereas $[B_\cdot^k, L_\cdot^k]_t = 0$ for all $k = 1, \dots, l$ and $0 \le t \le T$. Thus, ζ does not play the role of a common correlation parameter. However, for $k = 1, \dots, l$ and $0 \le t \le u \le T$ the solution of (3.69) is of the form

$$X_u^k = X_t^k\, e^{-\lambda_k(u-t)} + \zeta \int_t^u \sigma_k(s)\, e^{-\lambda_k(u-s)} dB_s^k + \sqrt{1-\zeta^2} \int_t^u \sigma_k(s)\, e^{-\lambda_k(u-s)} dL_s^k. \tag{3.70}$$

In order to price options written on the spot price (2.1) [but yet with extended base components such as given in (3.69)] we need to switch to a risk-neutral probability measure, say $\bar{\mathbb{Q}}$. For this purpose, we slightly modify the Radon-Nikodym derivative (2.7), instead defining

$$\left.\frac{d\bar{\mathbb{Q}}}{d\mathbb{P}}\right|_{\bar{\mathcal{F}}_t} := \prod_{k=1}^{l} \mathfrak{E}(G_k \circ B^k)_t \times \prod_{k=1}^{n} \mathfrak{E}(M^k)_t \tag{3.71}$$

with deterministic and time-dependent real functions $G_k(t)$, continuous Doléans-Dade exponentials

$$\mathfrak{E}(G_k \circ B^k)_t := \exp\left\{\int_0^t G_k(s)\, dB_s^k - \frac{1}{2}\int_0^t G_k(s)^2\, ds\right\} \tag{3.72}$$

and a (backward-looking) filtration

$$\bar{\mathcal{F}}_t := \sigma\{L_r^1, \dots, L_r^n, B_r^1, \dots, B_r^l \colon 0 \le r \le t\}. \tag{3.73}$$

From Girsanov's theorem (see Theorem 12.21 in [12]) we deduce that for all $k = 1, \dots, l$ the processes

$$\bar{B}_t^{k,\bar{\mathcal{F}},\bar{\mathbb{Q}}} := \bar{B}_t^k := B_t^k - \int_0^t G_k(s)\, ds \tag{3.74}$$

constitute $\bar{\mathcal{F}}_t$-adapted BMs under $\bar{\mathbb{Q}}$. Moreover, combining (3.68) and (3.74), we get the $\bar{\mathbb{Q}}$-dynamics

$$\Theta_t = m(t) + e^{-\vartheta t}\left[\Theta_0 - m(0)\right] + \sum_{k=1}^{l} \int_0^t \xi_k\, G_k(s)\, e^{-\vartheta(t-s)} ds + \sum_{k=1}^{l} \int_0^t \xi_k\, e^{-\vartheta(t-s)} d\bar{B}_s^k. \tag{3.75}$$

Similarly, putting (3.74) into (3.70), for $k = 1, \dots, l$ and $0 \le t \le u \le T$ we receive

$$X_u^k = X_t^k\, e^{-\lambda_k(u-t)} + \tag{3.76}$$

$$\zeta \int_t^u \sigma_k(s)\, G_k(s)\, e^{-\lambda_k(u-s)} ds + \zeta \int_t^u \sigma_k(s)\, e^{-\lambda_k(u-s)} d\bar{B}_s^k + \sqrt{1-\zeta^2} \int_t^u \sigma_k(s)\, e^{-\lambda_k(u-s)} dL_s^k.$$

Further on, regarding (3.68) and (3.75), we introduce the overall filtration

(3.77)
$$\bar{\mathcal{H}}_t \coloneqq \bar{\mathcal{F}}_t \vee \sigma\left\{\int_0^\tau e^{\vartheta r} dB_r^k : k = 1, \dots, l\right\} = \bar{\mathcal{F}}_t \vee \sigma\left\{\int_0^\tau e^{\vartheta r} d\bar{B}_r^k : k = 1, \dots, l\right\}$$

[recall (3.74) for the last equality] which – with view on (3.75) – might be associated with complete knowledge of the future temperature at time τ. As before, we come up with a non-explicit intermediate filtration $\breve{\mathcal{G}}_t$ obeying

(3.78)
$$\bar{\mathcal{F}}_t \subset \breve{\mathcal{G}}_t \subset \bar{\mathcal{H}}_t$$

for $0 \leq t < \tau$ whereas $\bar{\mathcal{F}}_t = \breve{\mathcal{G}}_t$ holds for all $t \geq \tau$. In this setting, $\breve{\mathcal{G}}_t$ represents the *effective* time-t information about temperature at the future time τ which the informed traders have knowledge of. Again, we implement an *explicitly enlarged* intermediate filtration $\bar{\mathcal{G}}_t$ consisting of a subfamily of the components in $\bar{\mathcal{H}}_t$, namely

(3.79)
$$\bar{\mathcal{G}}_t \coloneqq \bar{\mathcal{F}}_t \vee \sigma\left\{\int_0^\tau e^{\vartheta r} d\bar{B}_r^k : k = 1, \dots, d; (d \leq l)\right\}$$

which also satisfies $\bar{\mathcal{F}}_t \subset \bar{\mathcal{G}}_t \subset \bar{\mathcal{H}}_t$ for $0 \leq t < \tau$. To ease the notation, for all $0 \leq s < \tau$ we define

(3.80)
$$a(s) \coloneqq 2\vartheta e^{\vartheta s} / \left(e^{2\vartheta\tau} - e^{2\vartheta s}\right).$$

Lemma 3.12. *(a) Let* $\breve{\mathcal{G}}_t$ *as in (3.78) and* $a(s)$ *as in (3.80). Then the stochastic process*

(3.81)
$$\bar{B}_t^{k,\breve{\mathcal{G}},\bar{\mathbb{Q}}} \coloneqq \bar{B}_t^k - \int_0^t a(s)\, \mathbb{E}_{\bar{\mathbb{Q}}}\left(\int_s^\tau e^{\vartheta r} d\bar{B}_r^k \,\middle|\, \breve{\mathcal{G}}_s\right) ds$$

depicts a $\left(\breve{\mathcal{G}}_t, \bar{\mathbb{Q}}\right)$*-Brownian motion for all* $k = 1, \dots, l$ *and* $t \in [0, \tau[$.

(b) Let $\bar{\mathcal{G}}_t$ *as in (3.79) and* $a(s)$ *as in (3.80). Then the stochastic process*

(3.82)
$$\bar{B}_t^{k,\bar{\mathcal{G}},\bar{\mathbb{Q}}} \coloneqq \bar{B}_t^k - \int_0^t a(s) \int_s^\tau e^{\vartheta r} d\bar{B}_r^k \, ds$$

constitutes a $(\bar{\mathcal{G}}_t, \bar{\mathbb{Q}})$*-Brownian motion for all* $k = 1, \dots, d$ *and* $t \in [0, \tau[$.

(c) For all $k = 1, \dots, d$ *and time indices* $0 \leq t \leq s < \tau$ *we have the equality*

(3.83)
$$\mathbb{E}_{\bar{\mathbb{Q}}}\left(\int_s^\tau e^{\vartheta r} d\bar{B}_r^k \,\middle|\, \bar{\mathcal{G}}_t\right) = \mathbb{E}_{\bar{\mathbb{Q}}}\left(\int_t^\tau e^{\vartheta r} d\bar{B}_r^k \,\middle|\, \bar{\mathcal{G}}_t\right) \frac{e^{2\vartheta\tau} - e^{2\vartheta s}}{e^{2\vartheta\tau} - e^{2\vartheta t}}.$$

Proof. (a) This follows from Proposition 3.4 in [5].[6]

[6] In Proposition 3.4 in [5] the filtration $\mathcal{G}$ must be assumed to be such as defined in "(3.9) in [5]" and *not* as in "(3.2) in [5]".

(b) If we replace $\breve{\mathcal{G}}_s$ in (a) by $\bar{\mathcal{G}}_s$ and hereafter decompose (for $k = 1, \dots, d$)

$$\int_s^\tau e^{\vartheta r} d\bar{B}_r^k = \int_0^\tau e^{\vartheta r} d\bar{B}_r^k - \int_0^s e^{\vartheta r} d\bar{B}_r^k$$

we get the claimed result by *taking out what is known*, since $\int_0^\tau e^{\vartheta r} d\bar{B}_r^k$ is $\bar{\mathcal{G}}_s$-measurable [see (3.79)] and $\int_0^s e^{\vartheta r} d\bar{B}_r^k$ is $\bar{\mathcal{F}}_s$-measurable [see (3.73) and (3.74)] and thus, the latter is $\bar{\mathcal{G}}_s$-measurable, likewise.

(c) This follows from Proposition A.3 in [5] with $g(t) \coloneqq a(t)$, $f(u) \coloneqq e^{\vartheta u}$, $B \coloneqq \bar{B}^k$ and $T_1 \coloneqq \tau$. ∎

For notational reasons, let us introduce the [$\bar{\mathcal{G}}_s$-adapted] Brownian $(\bar{\mathcal{G}}, \bar{\mathbb{Q}})$-information yield

(3.84)

$$\bar{\theta}_s^k \coloneqq a(s) \int_s^\tau e^{\vartheta r} d\bar{B}_r^k.$$

Further on, appealing to (3.2), we define the temperature-forecast electricity futures price under $\bar{\mathcal{G}}$ by

(3.85)

$$\bar{F}_t \coloneqq F_t^{\Theta,\bar{\mathcal{G}},\bar{\mathbb{Q}}}(\tau_1, \tau_2) \coloneqq \frac{1}{\tau_2 - \tau_1} \int_{\tau_1}^{\tau_2} \mathbb{E}_{\bar{\mathbb{Q}}}(S_u | \bar{\mathcal{G}}_t)\, du.$$

Theorem 3.13. *The futures price (3.85) fulfills the* $(\bar{\mathcal{G}}, \bar{\mathbb{Q}})$*-Sato-martingale representation*[7]

(3.86)

$$d\bar{F}_t = \sum_{k=1}^d \left[\Lambda_k(t)\, \zeta - e^{\vartheta t}\, \Pi_k(t)\right] d\bar{B}_t^{k,\bar{\mathcal{G}},\bar{\mathbb{Q}}} + \zeta \sum_{k=d+1}^l \Lambda_k(t)\, d\bar{B}_t^{k,\bar{\mathcal{F}},\bar{\mathbb{Q}}} + \sqrt{1-\zeta^2} \sum_{k=1}^l \int_{D_k} z\, \Lambda_k(t)\, d\tilde{N}_k^{\bar{\mathcal{F}},\bar{\mathbb{Q}}}(t,z)$$
$$+ \sum_{k=l+1}^n \int_{D_k} z\, \Lambda_k(t)\, d\tilde{N}_k^{\bar{\mathcal{F}},\bar{\mathbb{Q}}}(t,z)$$

where $\Lambda_k(t) \coloneqq \Lambda_k(t, \tau_1, \tau_2)$ *is such as defined in (2.11) and*

$$\Pi_k(t) \coloneqq \frac{2\vartheta\zeta}{e^{2\vartheta\tau} - e^{2\vartheta t}} \int_{\tau_1}^{\tau_2} \int_t^u \frac{w_k \sigma_k(s)}{\tau_2 - \tau_1} e^{\vartheta s - \lambda_k(u-s)} ds\, du.$$

Proof. Merging (2.1) into (3.85) while recalling (3.79), we derive the decomposition

[7] $\bar{F}$ indeed possesses independent increments w.r.t. $\bar{\mathcal{G}}$; compare (3.79) and Lemma 3.12 (b) with (3.86) to verify this. Note that the dynamics (3.86) not necessarily is strictly positive, since the involved BM-terms may become negative and thus, drive the futures price $\bar{F}$ to negative values. In practice, one might choose μ and σ_k [implicitly contained in (3.85) via S] in such a way that negative values for the spot S and the futures $\bar{F}$ only appear with negligible probability. Likewise, the choice of an adequate jump size distribution (jumps are strictly positive here) may help to avoid negative spot/futures prices. On pp. 74-75 in [7] it is argued that *"an arithmetic model apparently allows for negative prices, a phenomenon which sounds odd in any normal market, since this means that the buyer [...] receives money rather than pays. However, in the electricity market [...] it can be more costly for a producer to switch off the generators than to pay someone to consume electricity in the case of more supply than demand. Thus, electricity is given away along with a payment. In fact, in almost all [...] electricity markets, negative prices occur from time to time, although very rarely."*. This citation defends our choice of an arithmetic BM-driven electricity spot/futures model which may generate negative prices (but possibly with a very small probability only).

(3.87)

$$\bar{F}_t = \frac{1}{\tau_2 - \tau_1} \int_{\tau_1}^{\tau_2} \left\{ \mu(u) + \sum_{k=1}^{d} w_k \, \mathbb{E}_{\bar{\mathbb{Q}}}(X_u^k | \bar{\mathcal{G}}_t) + \sum_{k=d+1}^{l} w_k \, \mathbb{E}_{\bar{\mathbb{Q}}}(X_u^k | \bar{\mathcal{F}}_t) + \sum_{k=l+1}^{n} w_k \, \mathbb{E}_{\bar{\mathbb{Q}}}(X_u^k | \bar{\mathcal{F}}_t) \right\} du.$$

In the following, we compute the three types of conditional expectations appearing inside (3.87) in their order of appearance: Substituting (3.76) into the first expectation, we get

(3.88)

$$\mathbb{E}_{\bar{\mathbb{Q}}}(X_u^k | \bar{\mathcal{G}}_t) = X_t^k \, e^{-\lambda_k(u-t)} + \zeta \int_t^u \sigma_k(s) \, G_k(s) \, e^{-\lambda_k(u-s)} ds + \zeta \, \mathbb{E}_{\bar{\mathbb{Q}}}\left(\int_t^u \sigma_k(s) \, e^{-\lambda_k(u-s)} d\bar{B}_s^k \middle| \bar{\mathcal{G}}_t \right)$$
$$+ \sqrt{1-\zeta^2} \, \mathbb{E}_{\bar{\mathbb{Q}}}\left(\int_t^u \sigma_k(s) \, e^{-\lambda_k(u-s)} dL_s^k \middle| \bar{\mathcal{F}}_t \right).$$

With respect to Lemma 3.12 (b), (2.3) and (3.84), [for $u < \tau$] the latter equation turns into

(3.89)

$$\mathbb{E}_{\bar{\mathbb{Q}}}(X_u^k | \bar{\mathcal{G}}_t) = X_t^k \, e^{-\lambda_k(u-t)} + \zeta \int_t^u \sigma_k(s) \, G_k(s) \, e^{-\lambda_k(u-s)} ds + \zeta \int_t^u \sigma_k(s) \, e^{-\lambda_k(u-s)} \mathbb{E}_{\bar{\mathbb{Q}}}(\bar{\theta}_s^k | \bar{\mathcal{G}}_t) ds$$
$$+ \sqrt{1-\zeta^2} \, \mathbb{E}_{\bar{\mathbb{Q}}}\left[\int_t^u \int_{D_k} z \, \sigma_k(s) \, e^{-\lambda_k(u-s)} dN_k(s,z) \right].$$

Taking (2.8), (3.80), (3.83) and (3.84) into account, Equality (3.89) finally becomes

(3.90)

$$\mathbb{E}_{\bar{\mathbb{Q}}}(X_u^k | \bar{\mathcal{G}}_t) = X_t^k \, e^{-\lambda_k(u-t)} + \zeta \int_t^u \sigma_k(s) \, G_k(s) \, e^{-\lambda_k(u-s)} ds + 2\vartheta\zeta \int_t^u \sigma_k(s) e^{\vartheta s} e^{-\lambda_k(u-s)} \frac{\int_t^\tau e^{\vartheta r} d\bar{B}_r^k}{e^{2\vartheta\tau} - e^{2\vartheta t}} ds$$
$$+ \sqrt{1-\zeta^2} \int_t^u \int_{D_k} z \, \sigma_k(s) e^{-\lambda_k(u-s)} e^{h_k(s,z)} \rho_k(s) d\nu_k(z) ds.$$

Merging (2.3), (2.8) and (3.76) into the second conditional expectation in (3.87), we obtain

(3.91)

$$\mathbb{E}_{\bar{\mathbb{Q}}}(X_u^k | \bar{\mathcal{F}}_t) = X_t^k \, e^{-\lambda_k(u-t)} + \zeta \int_t^u \sigma_k(s) \, G_k(s) \, e^{-\lambda_k(u-s)} ds$$
$$+ \sqrt{1-\zeta^2} \int_t^u \int_{D_k} z \, \sigma_k(s) \, e^{-\lambda_k(u-s)} e^{h_k(s,z)} \rho_k(s) \, d\nu_k(z) \, ds.$$

Implanting (2.3), (2.5) and (2.8) into the third conditional expectation of (3.87), we ultimately deduce

(3.92)

$$\mathbb{E}_{\bar{\mathbb{Q}}}(X_u^k | \bar{\mathcal{F}}_t) = X_t^k e^{-\lambda_k(u-t)} + \int_t^u \int_{D_k} z \, \sigma_k(s) \, e^{-\lambda_k(u-s)} e^{h_k(s,z)} \rho_k(s) \, d\nu_k(z) \, ds.$$

Collecting (3.90), (3.91) and (3.92), Equation (3.87) can be rearranged in shorthand notation as

(3.93)

$$\bar{F}_t = \bar{\Gamma}(t) + \sum_{k=1}^{n} \Psi_k(t)\, X_t^k + \sum_{k=1}^{d} \Pi_k(t)\, W_t^k$$

with $\Psi_k(t)$ as defined in (3.17) and new abbreviations

(3.94)

$$\Pi_k(t) := \frac{2\vartheta\zeta}{e^{2\vartheta\tau} - e^{2\vartheta t}} \int_{\tau_1}^{\tau_2} \int_t^u \frac{w_k \sigma_k(s)}{\tau_2 - \tau_1} e^{\vartheta s - \lambda_k(u-s)} ds\, du, \qquad W_t^k := \int_t^\tau e^{\vartheta r} d\bar{B}_r^k,$$

$$\begin{aligned}\bar{\Gamma}(t) := \int_{\tau_1}^{\tau_2} \frac{\mu(u)}{\tau_2 - \tau_1} du + \zeta \sum_{k=1}^{l} \int_{\tau_1}^{\tau_2} \int_t^u \frac{w_k \sigma_k(s)}{\tau_2 - \tau_1} e^{-\lambda_k(u-s)} G_k(s)\, ds\, du \\ + \sqrt{1-\zeta^2} \sum_{k=1}^{l} \int_{\tau_1}^{\tau_2} \int_t^u \int_{D_k} \frac{w_k \sigma_k(s)}{\tau_2 - \tau_1} z\, e^{-\lambda_k(u-s)} e^{h_k(s,z)} \rho_k(s)\, d\nu_k(z)\, ds\, du \\ + \sum_{k=l+1}^{n} \int_{\tau_1}^{\tau_2} \int_t^u \int_{D_k} \frac{w_k \sigma_k(s)}{\tau_2 - \tau_1} z\, e^{-\lambda_k(u-s)} e^{h_k(s,z)} \rho_k(s)\, d\nu_k(z)\, ds\, du.\end{aligned}$$

Applying Itô's product rule on (3.93) while recalling (2.2), (2.3), (2.8), (2.11), (3.17), (3.69), (3.74), (3.80), (3.82), (3.84) and (3.94), we get [similar to the derivation of (3.21)] the (local) $(\bar{\mathcal{G}}_t, \bar{\mathbb{Q}})$-Sato-martingale dynamics claimed in (3.86). ■

Theorem 3.14. *The price of a European call option written on the futures (3.86) is given by*

(3.95)

$$C_t^{\bar{\mathcal{G}}} = \frac{e^{-r(T-t)}}{2\pi} \int_{\mathbb{R}} \frac{e^{(a+iy)[\bar{F}_t - K]}}{(a+iy)^2} \times \prod_{k=1}^{d} e^{b_k(y,t,T)} \times \prod_{k=d+1}^{l} e^{c_k(y,t,T)} \times \prod_{k=1}^{n} e^{\chi_k(y,t,T)}\, dy$$

with deterministic functions

$$b_k(y,t,T) := \int_t^T \frac{(a+iy)^2}{2} \left[\Lambda_k(s,\tau_1,\tau_2)\,\zeta - e^{\vartheta s}\, \Pi_k(s)\right]^2 ds, \qquad c_k(y,t,T) := \int_t^T \frac{(a+iy)^2}{2} \Lambda_k(s,\tau_1,\tau_2)^2\, \zeta^2 ds,$$

$$\chi_k(y,t,T) := \int_t^T \int_{D_k} \left[e^{\Upsilon_k(s,z)} - 1 - \Upsilon_k(s,z)\right] e^{h_k(s,z)} \rho_k(s)\, d\nu_k(z)\, ds,$$

$$\Upsilon_k(s,z) := (a+iy)\, z\, \Lambda_k(s,\tau_1,\tau_2) \left[1 + \left(\sqrt{1-\zeta^2} - 1\right) \mathbb{1}_{\{1,\dots,l\}}(k)\right].$$

(Note that b_k and c_k merely differ by an additive information drift. Here, $\mathbb{1}$ denotes the indicator function.)

Proof. In accordance to (3.34), for $0 \le t \le T$ we currently get

(3.96)

$$C_t^{\bar{\mathcal{G}}} := C_t^{\bar{\mathcal{G}}}(K,\tau_1,\tau_2) = \frac{e^{-r(T-t)}}{2\pi} \int_{\mathbb{R}} \hat{q}(y)\, e^{(a+iy)\bar{F}_t}\, \mathbb{E}_{\bar{\mathbb{Q}}}\left(e^{(a+iy)[\bar{F}_T - \bar{F}_t]} \middle| \bar{\mathcal{G}}_t\right) dy.$$

Taking the $\bar{\mathbb{Q}}$-independent increment property of the $\bar{\mathcal{G}}$-adapted Sato-process $\bar{F}$ [such as given in (3.86)] into account, the conditional expectation inside (3.96) factors into

(3.97)

$$\mathbb{E}_{\bar{\mathbb{Q}}}\left(e^{(a+iy)[\bar{F}_T-\bar{F}_t]}\middle|\bar{\mathcal{G}}_t\right) =$$

$$\mathbb{E}_{\bar{\mathbb{Q}}}\left[\exp\left\{\sum_{k=1}^{d}\int_t^T (a+iy)\left[\Lambda_k(s,\tau_1,\tau_2)\,\zeta - e^{\vartheta s}\,\Pi_k(s)\right] d\bar{B}_s^{k,\bar{\mathcal{G}},\bar{\mathbb{Q}}} + \sum_{k=d+1}^{l}\int_t^T (a+iy)\,\Lambda_k(s,\tau_1,\tau_2)\,\zeta\, d\bar{B}_s^k \right.\right.$$
$$\left.\left. + \sum_{k=1}^{n}\int_t^T\int_{D_k} (a+iy)\, z\,\Lambda_k(s,\tau_1,\tau_2)\left[1+\left(\sqrt{1-\zeta^2}-1\right)\mathbb{1}_{\{1,\ldots,l\}}(k)\right] d\widetilde{N}_k^{\bar{\mathcal{F}},\bar{\mathbb{Q}}}(s,z)\right\}\right] =$$

$$\prod_{k=1}^{d} e^{b_k(y,t,T)} \times \prod_{k=d+1}^{l} e^{c_k(y,t,T)} \times \prod_{k=1}^{n} e^{\chi_k(y,t,T)}$$

with b_k, c_k and χ_k such as defined in Theorem 3.14. Herein, we have used Itô's isometry twice along with the extended Lévy-Khinchin formula (as in the proof of Proposition 2.3 above). Merging (2.18) and (3.97) into (3.96), we finally end up with the announced call price formula (3.95). ■

Essentially, (3.95) exhibits three different classes of product terms which may be interpreted economically: Firstly, $e^{b_k(y,t,T)}$ is closely linked with risk-reducing temperature forecasts. Secondly, $e^{c_k(y,t,T)}$ descends from the remaining uncertainty concerning future temperature behavior. Thirdly, $e^{\chi_k(y,t,T)}$ represents omnipresent electricity price (jump-) risk originating from other risk sources than temperature (like e.g. carbon dioxide emission permit prices). Summing up, we ultimately cherish that is has been possible to compute the conditional expectation in (3.97) explicitly which has not been the case in our former pure-jump setups; remind e.g. (3.35).

Remark 3.15. *It appears reasonable to assume (not only outdoor temperature but also) carbon emission allowance (EUA) prices to have a major impact on electricity prices as well. Intuitively, one suspects a positive correlation between EUA prices and electricity spot and futures prices. Indeed, convincing empirical evidences which manifest this proposition have been detected in Section 2.2 of [5]. Fortunately, our above modeling framework allows for an incorporation of such dependency structures between electricity and carbon permit prices, since we may correlate the EUA price, A^0 say, possibly obeying $dA_t^0 = A_t^0\left[\alpha\, dt + \sum_{k=1}^{l}\beta_k\, dW_t^k\right]$ (with constants $\alpha \in \mathbb{R}$, $\beta_k > 0$ and BMs W_t^k), with the electricity spot price (2.1) in an analogous way as just described for the temperature case by replacing the temperature noises B_t^k in (3.69) through the EUA price noises W_t^k for $k = 1, \ldots, l$. Moreover, the forward-looking machinery with enlarged filtrations then might be applied similarly whereas anticipating insider information about future EUA prices might not be as commonly available as temperature forecasts, admittedly. The pricing of carbon emission allowances under future information on the market zone net position is treated in Chapter 6 of [17].*

3.5 A mixed model for electricity spot, futures and option prices

As it seems to be hardly possible to compute expectations of the type (3.35) analytically, we now propose a *mixed* electricity spot price model including both Brownian motion (BM) and pure-jump terms. In accordance to our explanations in the sequel of (2.4) concerning a splitting of the spot price driving noises into small and large-amplitude jump components, we currently propose to replace equality (2.2) through

(3.98) $$dX_t^k = -\lambda_k\, X_t^k\, dt + \sigma_k\, dB_t^k \qquad (k = 1, \ldots, l)$$

with strictly positive and constant volatilities $\sigma_1, \ldots, \sigma_l$ along with standard $\mathbb{P}$-BMs $B_t^1, \ldots, B_t^l$ and

(3.99) $$dX_t^k = -\lambda_k\, X_t^k\, dt + \sigma_k(t)\, dL_t^k \qquad (k = l+1, \ldots, n)$$

with pure-jump Lévy type noises L_t^k such as defined in (2.3). Hence, the small-amplitude fluctuations of the long-term level of the spot price are reasonably modeled by BMs now whereas the short-term spiky components remain untouched. Similar to before, we assume all involved random processes such as $B_t^1, \dots, B_t^l, L_t^{l+1}, \dots, L_t^n$ to be $\mathbb{P}$-independent. The associated *mixed* spot price is given by

(3.100)

$$S_t = \mu(t) + \sum_{k=1}^{l} w_k X_t^k + \sum_{k=l+1}^{n} w_k X_t^k$$

which may become negative – remind Section 3.2.2 in [7] and our footnote dedicated to (3.86) in this context. Slightly deviating from (3.71), we next define the *mixed* Radon-Nikodym derivative by

(3.101)

$$\left.\frac{d\widetilde{\mathbb{Q}}}{d\mathbb{P}}\right|_{\tilde{\mathcal{F}}_t} \coloneqq \prod_{k=1}^{l} \mathfrak{E}(G_k \circ B^k)_t \times \prod_{k=l+1}^{n} \mathfrak{E}(M^k)_t$$

with multipliers $\mathfrak{E}(G_k \circ B^k)_t$ as in (3.72), $\mathfrak{E}(M^k)_t$ as defined in Section 2.2 above and a new initial filtration $\tilde{\mathcal{F}}_t \coloneqq \sigma\{B_r^1, \dots, B_r^l, L_r^{l+1}, \dots, L_r^n : 0 \le r \le t\}$. Similarly to (3.74), for all $k = 1, \dots, l$ we declare

(3.102)

$$\tilde{B}_t^k \coloneqq \tilde{B}_t^{k,\tilde{\mathcal{F}},\widetilde{\mathbb{Q}}} \coloneqq B_t^k - \int_0^t G_k(s)\, ds$$

to constitute $\tilde{\mathcal{F}}_t$-adapted BMs under $\widetilde{\mathbb{Q}}$. Yet, for $0 \le t \le u \le T$ the solution of (3.98) points out as

(3.103)

$$X_u^k = X_t^k e^{-\lambda_k(u-t)} + \sigma_k \int_t^u e^{-\lambda_k(u-s)}\, dB_s^k$$

$(k = 1, \dots, l)$ whereas (3.99) is solved by

(3.104)

$$X_u^k = X_t^k e^{-\lambda_k(u-t)} + \int_t^u \sigma_k(s)\, e^{-\lambda_k(u-s)}\, dL_s^k$$

$(k = l+1, \dots, n)$. With reference to (3.102), Equation (3.103) further yields the $\widetilde{\mathbb{Q}}$-representation

(3.105)

$$X_u^k = X_t^k e^{-\lambda_k(u-t)} + \sigma_k \int_t^u e^{-\lambda_k(u-s)}\, G_k(s)\, ds + \sigma_k \int_t^u e^{-\lambda_k(u-s)}\, d\tilde{B}_s^k$$

$(k = 1, \dots, l)$. In accordance to (3.77), we next introduce the overall/global filtration

$$\widetilde{\mathcal{H}}_t \coloneqq \tilde{\mathcal{F}}_t \vee \sigma\left\{\int_0^\tau e^{\lambda_k r} dB_r^k : k = 1, \dots, l\right\} = \tilde{\mathcal{F}}_t \vee \sigma\left\{\int_0^\tau e^{\lambda_k r} d\tilde{B}_r^k : k = 1, \dots, l\right\}$$

whereas, parallel to (3.79), we implement an associated explicit intermediate filtration via

(3.106)

$$\tilde{\mathcal{G}}_t \coloneqq \tilde{\mathcal{F}}_t \vee \sigma\left\{\int_0^\tau e^{\lambda_k r} d\tilde{B}_r^k : k = 1, \dots, d; (d \le l)\right\}.$$

Presently, we observe $\tilde{\mathcal{F}}_t \subset \tilde{\mathcal{G}}_t \subset \tilde{\mathcal{H}}_t$ for $0 \le t < \tau$ and $\tilde{\mathcal{F}}_t = \tilde{\mathcal{G}}_t$ for $t \ge \tau$. Furthermore, we put

(3.107)

$$a_k(s) := \frac{2\,\lambda_k\, e^{\lambda_k s}}{e^{2\lambda_k \tau} - e^{2\lambda_k s}}\,, \qquad \tilde{\theta}_s^k := a_k(s) \int_s^\tau e^{\lambda_k r} d\tilde{B}_r^k.$$

Then, with respect to Lemma 3.12 (b), we deduce that

(3.108)

$$\tilde{B}_t^{k,\tilde{\mathcal{G}},\tilde{\mathbb{Q}}} := \tilde{B}_t^k - \int_0^t \tilde{\theta}_s^k\, ds$$

constitutes a $(\tilde{\mathcal{G}}_t, \tilde{\mathbb{Q}})$-BM for all $k = 1, \dots, d$ and $t \in [0, \tau[$. Additionally, Lemma 3.12 (c) yields

(3.109)

$$\mathbb{E}_{\tilde{\mathbb{Q}}}\left(\int_s^\tau e^{\lambda_k r} d\tilde{B}_r^k \middle| \tilde{\mathcal{G}}_t \right) = \mathbb{E}_{\tilde{\mathbb{Q}}}\left(\int_t^\tau e^{\lambda_k r} d\tilde{B}_r^k \middle| \tilde{\mathcal{G}}_t \right) \frac{e^{2\lambda_k \tau} - e^{2\lambda_k s}}{e^{2\lambda_k \tau} - e^{2\lambda_k t}}$$

for all $k = 1, \dots, d$ and $0 \le t \le s < \tau$. In accordance to (3.2), for all $t \in [0, \tau_1]$ we define the $\tilde{\mathcal{G}}$-forward-looking *mixed* electricity futures price by

(3.110)

$$\tilde{F}_t := F_t^{\tilde{\mathcal{G}},\tilde{\mathbb{Q}}}(\tau_1, \tau_2) := \frac{1}{\tau_2 - \tau_1} \int_{\tau_1}^{\tau_2} \mathbb{E}_{\tilde{\mathbb{Q}}}\big(S_u \big| \tilde{\mathcal{G}}_t\big)\, du.$$

In this setup we get the following result:

Theorem 3.16. *The futures price (3.110) follows the $(\tilde{\mathcal{G}}, \tilde{\mathbb{Q}})$-martingale dynamics*

$$d\tilde{F}_t = \sum_{k=1}^{d} \left[\sigma_k\, \hat{\Psi}_k(t) - \tilde{\Pi}_k(t)\, e^{\lambda_k t}\right] d\tilde{B}_t^{k,\tilde{\mathcal{G}},\tilde{\mathbb{Q}}} + \sum_{k=d+1}^{l} \sigma_k\, \hat{\Psi}_k(t)\, d\tilde{B}_t^{k,\tilde{\mathcal{F}},\tilde{\mathbb{Q}}} + \sum_{k=l+1}^{n} \int_{D_k} z\, \Lambda_k(t)\, d\tilde{N}_k^{\tilde{\mathcal{F}},\tilde{\mathbb{Q}}}(t,z)$$

where $\Lambda_k(t) := \Lambda_k(t, \tau_1, \tau_2)$ is such as defined in (2.11) and

(3.111)

$$\hat{\Psi}_k(t) := \frac{w_k}{\tau_2 - \tau_1} \frac{e^{-\lambda_k(\tau_1 - t)} - e^{-\lambda_k(\tau_2 - t)}}{\lambda_k}, \qquad \tilde{\Pi}_k(t) := \frac{2\, w_k\, \lambda_k\, \sigma_k}{e^{2\lambda_k t} - e^{2\lambda_k \tau}} \int_{\tau_1}^{\tau_2} \int_u^t \frac{e^{\lambda_k(2s-u)}}{\tau_2 - \tau_1} ds\, du.$$

Proof. Merging (3.100) into (3.110), we deduce the decomposition

(3.112)

$$\tilde{F}_t = \frac{1}{\tau_2 - \tau_1} \int_{\tau_1}^{\tau_2} \left[\mu(u) + \sum_{k=1}^{d} w_k \mathbb{E}_{\tilde{\mathbb{Q}}}\big(X_u^k \big| \tilde{\mathcal{G}}_t\big) + \sum_{k=d+1}^{l} w_k \mathbb{E}_{\tilde{\mathbb{Q}}}\big(X_u^k \big| \tilde{\mathcal{F}}_t\big) + \sum_{k=l+1}^{n} w_k \mathbb{E}_{\tilde{\mathbb{Q}}}\big(X_u^k \big| \tilde{\mathcal{F}}_t\big) \right] du.$$

In what follows, we compute the three types of conditional expectations inside (3.112) in their order of appearance: Using (3.105) and (3.107)-(3.109), [for $u < \tau$] the first object therein becomes

(3.113)

$$\mathbb{E}_{\tilde{\mathbb{Q}}}\big(X_u^k \big| \tilde{\mathcal{G}}_t\big) = X_t^k e^{-\lambda_k(u-t)} + \sigma_k \int_t^u e^{-\lambda_k(u-s)}\, G_k(s)\, ds + 2\, \lambda_k\, \sigma_k\, \frac{\int_t^\tau e^{\lambda_k r} d\tilde{B}_r^k}{e^{2\lambda_k \tau} - e^{2\lambda_k t}} \int_t^u e^{\lambda_k(2s-u)} ds.$$

Utilizing (3.105) again, the second conditional expectation in (3.112) turns out as

(3.114)

$$\mathbb{E}_{\widetilde{\mathbb{Q}}}\left(X_u^k \middle| \tilde{\mathcal{F}}_t\right) = X_t^k e^{-\lambda_k(u-t)} + \sigma_k \int_t^u e^{-\lambda_k(u-s)}\, G_k(s)\, ds.$$

Finally, referring to (2.3), (2.8) and (3.104), we observe the third expectation to be of the form

(3.115)

$$\mathbb{E}_{\widetilde{\mathbb{Q}}}\left(X_u^k \middle| \tilde{\mathcal{F}}_t\right) = X_t^k e^{-\lambda_k(u-t)} + \int_t^u \int_{D_k} z\, \sigma_k(s)\, e^{-\lambda_k(u-s)} e^{h_k(s,z)} \rho_k(s)\, d\nu_k(z)\, ds.$$

Hence, substituting (3.113)-(3.115) into (3.112), we obtain the shorthand representation

(3.116)

$$\tilde{F}_t = \hat{\Gamma}(t) + \sum_{k=1}^{n} \hat{\Psi}_k(t)\, X_t^k + \sum_{k=1}^{d} \tilde{\Pi}_k(t)\, \widetilde{W}_t^k$$

with abbreviations

(3.117)

$$\hat{\Gamma}(t) := \int_{\tau_1}^{\tau_2} \frac{\mu(u)}{\tau_2 - \tau_1} du + \sum_{k=1}^{l} \frac{w_k\, \sigma_k}{\tau_2 - \tau_1} \int_{\tau_1}^{\tau_2} \int_t^u e^{-\lambda_k(u-s)}\, G_k(s)\, ds\, du$$
$$+ \sum_{k=l+1}^{n} \frac{w_k}{\tau_2 - \tau_1} \int_{\tau_1}^{\tau_2} \int_t^u \int_{D_k} z\, \sigma_k(s)\, e^{-\lambda_k(u-s)} e^{h_k(s,z)} \rho_k(s)\, d\nu_k(z)\, ds\, du,$$

$$\widetilde{W}_t^k := \int_t^{\tau} e^{\lambda_k r} d\tilde{B}_r^k$$

while $\hat{\Psi}_k(t)$ and $\tilde{\Pi}_k(t)$ are as defined in (3.111). Applying Itô's product rule on (3.116) while using (2.3), (2.8), (2.11), (3.98), (3.99), (3.102), (3.107), (3.108) and (3.117), we obtain the claimed $\widetilde{\mathbb{Q}}$-dynamics for $\tilde{F}$. ■

As described in the footnote dedicated to (3.86), the futures price $\tilde{F}$ may become negative as well. Comparing (3.106) with the dynamics found in Theorem 3.16, we recognize that $\tilde{F}$ designates a $(\tilde{\mathcal{G}}, \widetilde{\mathbb{Q}})$-Sato-martingale.
Further, we appeal to (3.34) and introduce the $\tilde{\mathcal{G}}$-forward-looking call option price at time $t \le T$ via

(3.118)

$$\tilde{C}_t := C_t^{\tilde{\mathcal{G}}}(K, \tau_1, \tau_2) = \frac{e^{-r(T-t)}}{2\pi} \int_{\mathbb{R}} \hat{q}(y)\, e^{(a+iy)\tilde{F}_t}\, \mathbb{E}_{\widetilde{\mathbb{Q}}}\left(e^{(a+iy)[\tilde{F}_T - \tilde{F}_t]} \middle| \tilde{\mathcal{G}}_t\right) dy$$

where $\hat{q}(y)$ is such as given in (2.18). Taking Theorem 3.16 into account while introducing the shorthand notation $\theta_k(s) := (a + iy)\, \Lambda_k(s, \tau_1, \tau_2)$, the conditional expectation in (3.118) factors into

(3.119)

$$\mathbb{E}_{\widetilde{\mathbb{Q}}}\left(e^{(a+iy)[\tilde{F}_T - \tilde{F}_t]} \middle| \tilde{\mathcal{G}}_t\right) = \prod_{k=1}^{d} P_1^k \times \prod_{k=d+1}^{l} P_2^k \times \prod_{k=l+1}^{n} P_3^k$$

with multipliers

(3.120)

$$P_1^k := \mathbb{E}_{\widetilde{\mathbb{Q}}}\left[\exp\left\{\int_t^T (a+iy)\left[\sigma_k \widehat{\Psi}_k(s) - \widetilde{\Pi}_k(s)e^{\lambda_k s}\right] d\widetilde{B}_s^{k,\tilde{\mathcal{G}},\widetilde{\mathbb{Q}}}\right\}\right]$$
$$= \exp\left\{\int_t^T \frac{(a+iy)^2}{2}\left[\sigma_k \widehat{\Psi}_k(s) - \widetilde{\Pi}_k(s)e^{\lambda_k s}\right]^2 ds\right\},$$

$$P_2^k := \mathbb{E}_{\widetilde{\mathbb{Q}}}\left[\exp\left\{\int_t^T (a+iy)\,\sigma_k\,\widehat{\Psi}_k(s)\,d\widetilde{B}_s^k\right\}\right] = \exp\left\{\int_t^T \frac{(a+iy)^2}{2}\,\sigma_k^2\,\widehat{\Psi}_k(s)^2\,ds\right\},$$

$$P_3^k := \mathbb{E}_{\widetilde{\mathbb{Q}}}\left[\exp\left\{i\int_t^T \int_{D_k} (y-ia)\,z\,\Lambda_k(s,\tau_1,\tau_2)\,d\widetilde{N}_k^{\tilde{\mathcal{F}},\widetilde{\mathbb{Q}}}(s,z)\right\}\right]$$
$$= \exp\left\{\int_t^T \int_{D_k} \left[e^{z\,\theta_k(s)} - 1 - z\,\theta_k(s)\right] e^{h_k(s,z)}\rho_k(s)\,d\nu_k(z)\,ds\right\}.$$

Herein, for the computation of P_1^k and P_2^k we have made use of Itô's isometry whereas for the treatment of P_3^k we have exploited (2.8) along with Proposition 2.1 in [7] and Proposition 1.9 in [19]. Finally, we put (2.18) and (3.119) into (3.118) and obtain the following result:

Theorem 3.17. *The electricity futures call option price in the jump-diffusion model (3.98)-(3.100) reads as*

(3.121)

$$\tilde{C}_t = \frac{e^{-r(T-t)}}{2\pi}\int_{\mathbb{R}} \frac{e^{(a+iy)[\tilde{F}_t - K]}}{(a+iy)^2} \times \prod_{k=1}^{d} P_1^k \times \prod_{k=d+1}^{l} P_2^k \times \prod_{k=l+1}^{n} P_3^k\,dy$$

where P_1^k, P_2^k and P_3^k are such as claimed in (3.120).

At this step, it is interesting to compare Theorem 3.17 with Proposition 2.3. Moreover, regarding (3.121), we recognize three different classes of risk terms: Firstly, P_1^k is closely connected with risk-reducing $\tilde{\mathcal{G}}$-forward-looking information on a selection of the Brownian noises driving the mean-level of the spot price. Secondly, the terms P_2^k can be associated to some kind of remaining risk w.r.t. the long-term level of the spot. Roughly speaking, the difference between P_1^k and P_2^k (which evidently consists in an additive information drift) describes to what extend the intermediate filtration $\tilde{\mathcal{G}}$ is smaller than the overall filtration $\widetilde{\mathcal{H}}$. Yet, we observe that if $\tilde{\mathcal{G}}_t = \widetilde{\mathcal{H}}_t$ (exhaustive knowledge) and hence, if $d = l$, then the factors P_2^k would equal P_1^k so that the product in (3.119) would simplify to

$$\prod_{k=1}^{l} P_1^k \times \prod_{k=l+1}^{n} P_3^k.$$

In other words, if the index d appearing in (3.106) is far from l (and thus, close to one), then there is not much supplementary information on the future behavior of the long-term level of the spot price available. As a consequence, the (actually *non*-forward-looking) members P_2^k in this case have a major impact on the resulting option price which sounds economically reasonable. Vice versa, if d is close to l, then there indeed is some worthy future information on the majority of the long-term level driving noises available what reasonably emphasizes the impact of the multipliers P_1^k (which themselves have been associated with additional insider information on the future electricity spot price mean level).

Thirdly, the terms P_3^k inside (3.121) originate from the omnipresent risk of an occurrence of electricity price spikes which periodically appear due to sudden imbalances in supply and demand.
Furthermore, recall that the price of a *put* option written on $\tilde{F}$ easily can be obtained from (3.121) by exploiting the Put-Call-Parity. Ultimately, we emphasize that in (3.120) it fortunately has been possible to compute the appearing expectations *analytically* which seemed to be impossible in our (forward-looking) pure-jump cases presented in Section 3.3.

3.5.1 The information premium in the mixed model. As a closing issue, we investigate the information premium also for our recent *mixed* spot price model. Taking (3.3), (3.100), (3.102), (3.105) and (3.107)-(3.110) into account, we get

(3.122)

$$\mathfrak{I}_t^{\tilde{\mathcal{G}},\tilde{\mathcal{F}},\tilde{\mathbb{Q}}}(\tau_1,\tau_2) := F_t^{\tilde{\mathcal{G}},\tilde{\mathbb{Q}}}(\tau_1,\tau_2) - F_t^{\tilde{\mathcal{F}},\tilde{\mathbb{Q}}}(\tau_1,\tau_2) = 2\sum_{k=1}^{d} \frac{\lambda_k\, w_k\, \sigma_k}{\tau_2-\tau_1}\, \frac{\int_t^\tau e^{\lambda_k r}\, d\tilde{B}_r^k}{e^{2\lambda_k \tau} - e^{2\lambda_k t}} \int_{\tau_1}^{\tau_2}\int_t^u e^{\lambda_k(2s-u)} ds\, du$$

whenever $\tau_1 \le u < \tau \le \tau_2$. Note that the double integral in (3.122) can be computed further by Fubini's theorem. To treat the opposite case $0 \le t \le \tau \le \tau_1 \le \tau_2$ we apply iterated-conditioning as presented in the context of (3.12) leading us to

$$\mathfrak{I}_t^{\tilde{\mathcal{G}},\tilde{\mathcal{F}},\tilde{\mathbb{Q}}}(\tau_1,\tau_2) = \sum_{k=1}^{d} \frac{\Lambda_k(\tau,\tau_1,\tau_2)}{\sigma_k(\tau)} \left[\mathbb{E}_{\tilde{\mathbb{Q}}}\left(X_\tau^k \middle| \tilde{\mathcal{G}}_t\right) - \mathbb{E}_{\tilde{\mathbb{Q}}}\left(X_\tau^k \middle| \tilde{\mathcal{F}}_t\right)\right].$$

Herein, Λ_k is such as defined in (2.11).

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