EMAT10007 Introduction to Computer Programming
Week 09 – Consolidation Exercise

Problem statement

In this exercise, you will use Python to simulate how a grain of sand sinks in water under gravity. Due to the smallness of the grain, its motion is affected by random collisions with water molecules.

Requirements and tips

Your code should be contained in a single Python (.py) file. When designing your code, you should think about:

- Whether the data types you are using are appropriate and optimal.
- Using if/else/elif statements to control the flow of your code.
- Using loops to carry out repetitive tasks rather than copying-and-pasting code.
- Using functions to break your code into smaller modules and eliminate code repetition.
- The guidelines on functional programming.
- Giving variables and functions concise and descriptive names.
- Documenting your code so that it is easy to understand how it works.
- Ensuring plots are easy to read and contain all of the necessary information.

A solution to the consolidation exercises will be made available on Friday at 5 pm.

Exercises

1. The radius and density of the grain can be found in the file `grain.csv`. Import this data into Python to compute the mass of the grain. The mass can be computed using the equation

$$m = \frac{4}{3}\rho R^3 \tag{1}$$

   where ρ is the density in units of kg/m^3 and R is the radius in units of m.

2. The motion of the sand grain will be calculated by first dividing time into N equally spaced points starting at $t = 0$ and ending at $t = t_{\text{end}}$. If $\Delta t = t_{\text{end}}/(N-1)$ represents the spacing between grid points, then we can write any time point t_n as $t_n = n\Delta t$, where $n = 0, 1, \ldots, N-1$. Assuming that $N = 1000$ and $t_{\text{end}} = 10^{-3}$ s, compute the times t_n and store them in a suitable data structure.

3. At each time point t_n, the force which acts on the grain must be calculated. The total vertical force acting on the grain at any time is given by

$$F = mg + c \tag{2}$$

   where m is the mass of the grain (calculated in Question 1), $g = 9.8$ m/s is the gravitational acceleration, and c is a collision force from the water molecules. The collision force at any

time point t_n can be set equal to a random number between -2×10^{-4} and 2×10^{-4}. Calculate the total vertical force at all of the time points t_n; note that the collision force should be different random number at each time point. **Hint**: You will need to import the `random` package by adding the code `import random` to the top of your Python file. A random number between 0 and 1 can be generated using the command `random.random()`.

4. Now that the force acting on the grain is known, the vertical speed v can be calculated by applying Newton's laws of motion. Let v_n denote the downwards speed of the sand grain at time t_n. Then, the speed of the sand grain at time $t_{n+1} = t_n + \Delta t$ is given by

$$v_{n+1} = v_n + \frac{F\Delta t}{m}, \tag{3}$$

where F is the force at time t_n that was calculated in Question 3. Compute v_n at each time point t_n up to and including $t_{N-1} = t_{\text{end}} = 10^{-3}$ s. You can assume that the initial speed of the grain is zero, that is, $v_0 = 0$ m/s. Plot the velocity as a function of time. Save this figure as `question4.png`. Ensure that your figure is appropriately labelled.

5. Due to the randomness in the model, each time you run your code, you will see a different result. However, the average results should be more or less the same. You will now extend your code to calculate the average speed V. To determine the average speed V, you must repeatedly calculate the speed v and then average the results.

   Run 5 simulations by generating new forces F at each time point t_n using Equation (2) and calculate new speeds using Equation (3). Then, for each time t_n, compute the average speed V_n by averaging over the 5 values of v_n that are obtained from your 5 simulations. Create a new figure and plot all 5 velocities v and the average speed V as a function of time on the same figure.

6. If there were no collisions, then the speed of the grain should obey the equation

$$v_e(t) = gt. \tag{4}$$

The mean speed V should converge to the speed v_e predicted by (4) as more and more simulations are used to compute the mean. Compute the mean speed V using 20, 40, and 80 simulations. Plot all of the average speeds along with the v_e in a new figure.