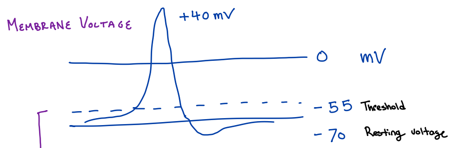
Review Board 2024.10.15

Today 15th Oct.

1. Why were we covering O.D.E.s, Euler's Method, Etc.?
2. Hippocampus, Hopfield, Delta Rule: Reprise — Tabled for tomorrow
3. Exam review (continued tomorrow)

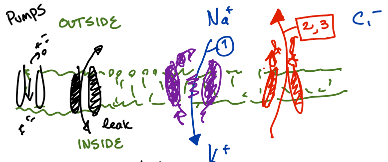
Action Potential



Subthreshold (approximate as linear)

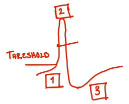
$\tau \dot{V} = -V + C$

¡This is why 1st order linear O.D.E.s!



more negative

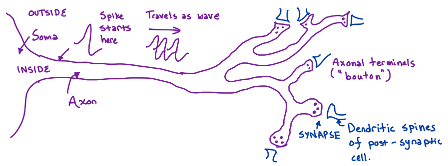
~ -70 mV



1: Initiate: V goes above voltage-gated Na^+ channel threshold, usually ≈ -55 mV → runaway excitation

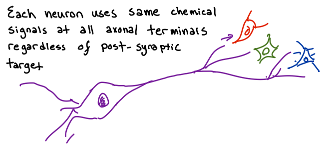
2: Stops rising: inward K^+ start opening ≈ -20 mv. Na^+ channels inactivate above ≈ 0 mV

3: Refractory Period: Open K^+ channels continue to conduct, hyperpolarize membrane. Na^+ channels stay inactive a bit, preventing spike.

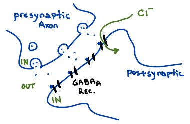


Dale's Principle: (Approx. truth)

Each neuron uses same chemical signals at all axonal terminals regardless of post-synaptic target



Example Synapse: Ionotropic GABA



GABA neurotransmitter opens chloride (-) channel

Cl^- enters, lowering voltage

(less likely to spike, "inhibitory")

Glutamate

Ionotropic:

- AMPA → +V (excitatory)
- NMDA → Ca^+ (signal for learning)

GABA (Metabotropic Glu. R "MGluR" not on exam)

- $GABA_A$ – ionotropic
- $GABA_B$ – metabotropic

Linear O.D.E.s → membrane voltage

$x_0 \leftarrow$ something

for $t = 1 \ldots 1000$

$x \leftarrow f(x)$

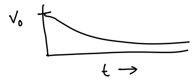
how to simulate membrane?

in a loop integrate O.D.E. for example by forward Euler

$\dot{v} = -v + f$

$v \leftarrow \Delta t \cdot [f - v]$

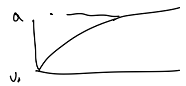
$\dot{v} = -v \quad v(t) = e^{-t} \cdot v(0)$



linear 1st-order O.D.E. Soln. (scalar, stable case)

- decay from where we start

$\dot{v} = a - v$



- toward where we are driven ("equilibrium value")

$v(t) = a + e^{-t}(v_0 - a)$

$\tau \dot{v} = -v \quad v(t) = e^{-t/\tau} \cdot v_0$

- with time-constant τ "tau" (larger → slower)

$\tau \dot{v} = -v + c$

$v(t) = c + e^{-t/\tau}(v - c)$

- generic form

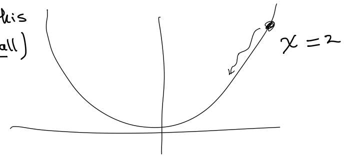
$\tau \, \cancel{RL} \, \dot{v} = \varepsilon_e - v + I \cdot R$

- specific constants come from electronic properties of membrane, but we will cover this post-midterm.

Delta Rule

An optimization perspective

Loss Function (make this small)

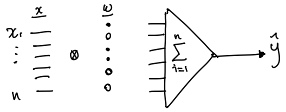


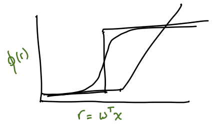
$\text{Loss} = \frac{1}{2}x^2 \longrightarrow \frac{d\,\text{Loss}}{dx} = x$ (gradient)

for epoch in 1 … 1000

$x \leftarrow \eta(-x)$ — gradient "descent"; η: Learning Rate

$\text{Loss}(\overbrace{w_1, w_2 \cdots w_n}^{w} ; \overbrace{x_1 \cdots x_n}^{x}, y^*)$





Activation function $\phi(\cdot)$

- Many different monotonic operators depending on model.
- cw2 used a hard threshold.

minimize $\frac{1}{2}(\hat{y} - y^*)^2 \qquad \hat{y} = \phi\Big(\underbrace{\sum_i w_i x_i}_{\text{"activation" } r}\Big)$

$\frac{d}{dw_k} \text{Loss}(\ldots) = (\hat{y} - y^*) \, \cancel{\phi'(\ldots)} \, x_k$ "waves hands"

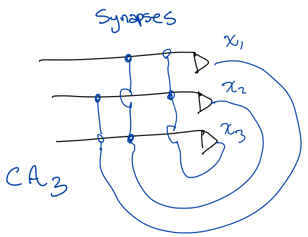
$\Delta w_k = \eta \cdot \underbrace{(y^* - \hat{y})}_{\text{prediction error}} x_k$

learning rate $\eta = 0.001$

$w_k \leftarrow w_k + \Delta w_k$

Hippocampal Formation & Hopfield

Autoassociative: many axons recurrent in CA3



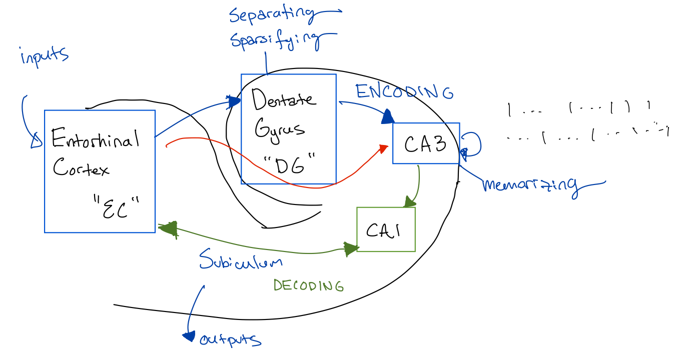
Weights for $x = (1, -1, -1)$

$x_1 = 1$

$x_2 = -1$

$x_3 = -1$

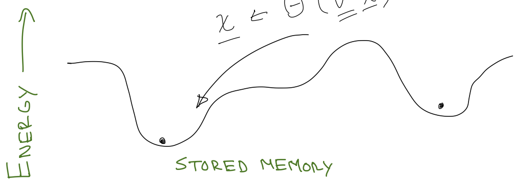
$$\Delta W = \begin{bmatrix} 0 & -1 & -1 \\ -1 & 0 & 1 \\ -1 & 1 & 0 \end{bmatrix}$$



Hopfield-Net Energy $-\frac{1}{2} x^T \underline{W} \underline{x} = \frac{-1}{2} \sum_{ij} x_i x_j w_{ij}$

Pattern completion

$x \leftarrow \Theta(Wx)$



$w_{ij} = \sum_{\text{patterns "a"}, i \neq j} x_i^a x_j^a$

$w_{i=j} = 0$

$-\frac{1}{2} x^T \underline{W} \underline{x}$ We want low energy

Want to remember

$x_i = x_j$

We learned

$w_{ij} = +1$

Pattern energy

$x_i \cdot x_j \cdot w_{ij}$

$-1 \cdot -1 \cdot 1 = 1 \qquad \varepsilon = -1/2$ ← better

$1 \cdot -1 \cdot 1 = -1 \qquad \varepsilon = +1/2$