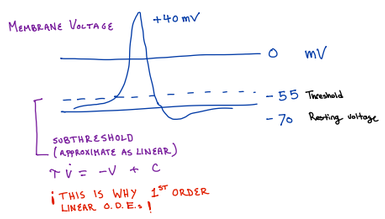
Review Board 2024.10.15

Today 15th Oct.

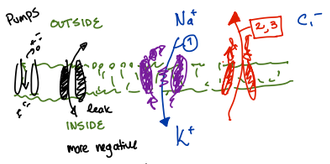
1. Why were we covering O.D.Es, Euler's Method, etc.?
2. Hippocampus, Hopfield, Delta Rule: Reprise
   - Tabled for tomorrow
3. Exam review (continued tomorrow)

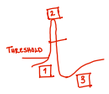
Action Potential



$\tau \dot{v} = -v + c$

¡ This is why 1st order linear O.D.Es !

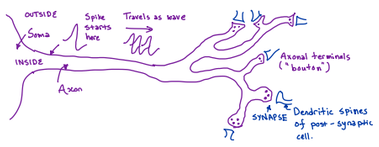




1: Initiate: V goes above voltage-gated Na^+ channel threshold, usually ≈ -55 mV → runaway excitation

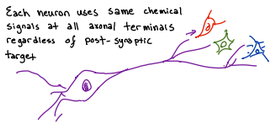
2: Stops rising: inward K^+ start opening ≈ -20 mV; Na^+ channels inactivate above ≈ 0 mV

3: Refractory Period: Open K^+ channels continue to conduct, hyperpolarize membrane. Na^+ channels stay inactive a bit, preventing spike.

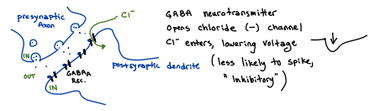


Dale's Principle: (Approx. truth)

Each neuron uses same chemical signals at all axonal terminals regardless of post-synaptic target.



Example Synapse: Ionotropic GABA



GABA neurotransmitter opens chloride (−) channel; Cl^- enters, lowering voltage (less likely to spike, "inhibitory")

Glutamate (ionotropic)
- AMPA → +V (excitatory)
- NMDA → Ca^+ (signal for learning)

GABA (metabotropic Glu. R "MGluR" not on exam)
- $GABA_A$ – ionotropic
- $GABA_B$ – metabotropic

Linear O.D.E.s → membrane voltage

$x_0 \leftarrow$ something

for $t = 1 \dots 1000$

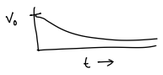
$x \leftarrow f(x)$ — in a loop

$\dot{v} = -v + f$ — integrate O.D.E.

$v \leftarrow \Delta t \cdot [f - v]$ — for example by forward Euler

How to simulate membrane?

$\dot{v} = -v \quad v(t) = e^{-t} \cdot v(0)$



linear 1st-order O.D.E soln. (scalar, stable case)

$\dot{v} = a - v$

- decay from where we start
- toward where we are driven ("equilibrium value")

$v(t) = a + e^{-t}(v_0 - a)$

$\tau \dot{v} = -v \quad v(t) = e^{-t/\tau} \cdot v_0$

- with time-constant τ "tau" (larger → slower)

$\tau \dot{v} = -v + c$

$v(t) = c + e^{-t/\tau}(v - c)$

- generic form

$\tau \dot{v} = \mathcal{E}_e - v + I \cdot R$

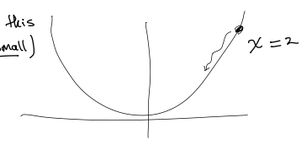
- specific constants come from electronic properties of membrane, but we will cover this post-midterm.

Delta Rule

An optimization perspective

Loss Function

(make this small)



GRADIENT

$\text{Loss} = \frac{1}{2}x^2 \rightarrow \frac{d\text{Loss}}{dx} = x$

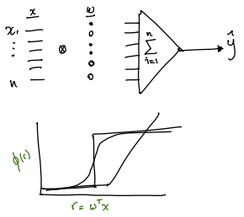
for epoch in 1 ... 1000

$x \leftarrow \eta(-x)$ (gradient)

Learning Rate; "descent"

w x

$\text{Loss}(w_1, w_2 \cdots w_n ; x_1 \cdots x_n, y^*)$



Activation function $\phi(\cdot)$
- Many different monotonic operators depending on model.
- CW2 used a hard threshold.

Minimize $\frac{1}{2}(\hat{y} - y^*)^2 \quad \hat{y} = \phi(\sum_i w_i x_i)$

"activation" r

$\frac{d}{dw_k}\text{Loss}(\cdots) = (\hat{y} - y^*)\phi'(\cdots)x_k$ "waves hands"

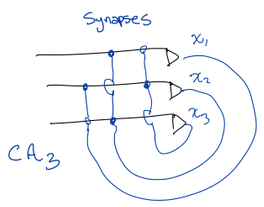
$\Delta w_k = \eta \cdot (y^* - \hat{y})\, x_k$

learning rate $\eta = 0.001$; prediction error

$w_k \leftarrow w_k + \Delta w_k$

Hippocampal Formation & Hopfield

Autoassociative: many axons recurrent in CA3



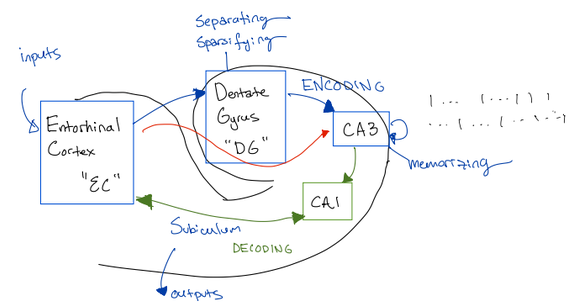
Weights for $x = (1, -1, -1)$

$x_1 = 1$

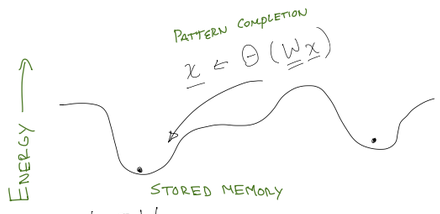
$x_2 = -1$

$x_3 = -1$

$\Delta W = \begin{bmatrix} 0 & -1 & -1 \\ -1 & 0 & 1 \\ -1 & 1 & 0 \end{bmatrix}$



Hopfield-Net Energy $-\frac{1}{2}x^T W x = -\frac{1}{2}\sum_{i,j} x_i x_j w_{ij}$



$W_{ij} = \sum_{\text{patterns "a"}} x_i^a x_j^a \quad i \neq j$

$W_{i=j} = 0$

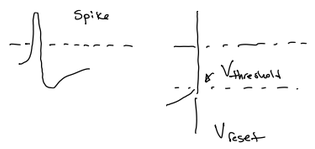
$-\frac{1}{2}x^T W x$ we want low energy

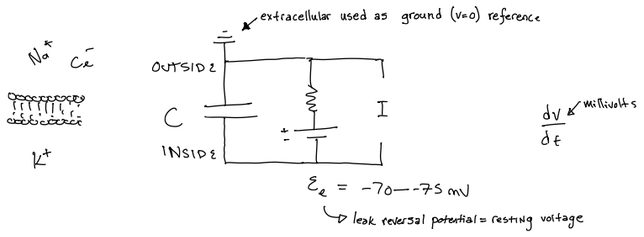
Want to remember	Pattern energy	
$x_i = x_j$	$x_i \cdot x_j \cdot w_{ij}$	
We learned	$-1 \cdot -1 \cdot 1 = 1$	$\mathcal{E} = -1/2$ ← better
$w_{ij} = +1$	$1 \cdot -1 \cdot 1 = -1$	$\mathcal{E} = +1/2$

Computational Neuroscience Topics in CS Review

LIF Simple model of spiking + membrane voltage

Key approximations in LIF:
- Face The Spike
- ≈ subthreshold voltage as linear

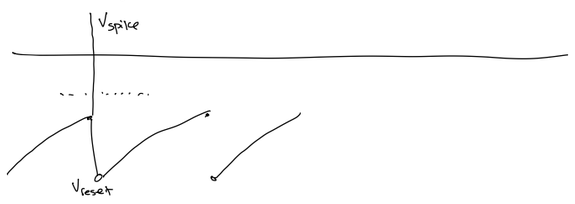




$C\dot{v} = \frac{1}{R}(\mathcal{E}_e - v) + I$ (applied current)

$1/R = g$

if $v(t) > V_{threshold}$:
- $v(t) \leftarrow V_{reset}$
- (maybe save t in list of spike time)
- (updating postsynaptic conductances of target)



C: capacitance

v: membrane voltage (polarized = negative)

g: conductance = 1/resistance $V = IR$

$\mathcal{E}_e$ or V_e: leak reversal/resting potential: where voltage goes if $I = 0$

I: applied current

$V(t) = V_\infty + e^{-t/\tau}(V_0 - V_\infty)$ (longer = slower)

- V_∞: steady state, aka V_{ss}, $\bar{v}$
- V_0: start here

How do we check if spike for given "I", V_{thresh}?

$RC\dot{v} = V_r - v + RI$

$\tau = RC \qquad V_\infty = V_r + RI \overset{?}{>} V_{thresh}$ (negative; less negative)

$C\dot{v} = g(\mathcal{E}_e - v) + I$ note $g = 1/R$ set $\tau = RC$

$\tau \dot{v} = V_\infty - v$ mult. by R; $V_\infty = \mathcal{E}_e + IR$

$V(t) = V_\infty + e^{-\frac{t}{\tau}}(V_0 - V_\infty)$

$RC\dot{v} = \mathcal{E}_e - v + IR$

$\tau \dot{v} = V_\infty - v$

$C\dot{v} = \frac{1}{R}(\mathcal{E}_e - v) + I$

$RC\dot{v} = \mathcal{E}_e - v + IR$

$\tau = RC \qquad V_\infty = \mathcal{E}_e$

Find spike period given constant I applied current

Assume $v(0) = V_r \qquad v(t) = V_{threshold}$

$v(t) = V_\infty + e^{-t/\tau}(V_0 - V_\infty)$ (unknown)

$V_\infty = V_e + RI$

$V_{th} = V_\infty + e^{-t/\tau}(V_{reset} - V_\infty)$ (solve)

$\frac{V_{th} - V_\infty}{V_{reset} - V_\infty} = e^{-t/\tau}$

$\ln\left(\frac{V_{th} - V_\infty}{V_{reset} - V_\infty}\right) = -\frac{t}{\tau}$

$t = -\tau \cdot \ln\left(\frac{V_{th} - V_\infty}{V_{reset} - V_\infty}\right) \qquad V_\infty = V_r + IR$

$t = -\tau \cdot \ln\left(\frac{V_{th} - V_r - IR}{V_{reset} - V_r - IR}\right)$

Rate f ($Hz = \frac{1}{s}$) $\frac{1}{t}$

Hodgkin–Huxley

$C\dot{v} = g_e(\mathcal{E}_e - v) + g_{Na^+}(\mathcal{E}_{Na^+} - v) + g_{K^+}(\mathcal{E}_{K^+} - v) + I$

Maximal conductance

$g_{Na^+} = \bar{g}_{Na^+} m^3 h$ — inactivation stop spike; gating term, "gate open" start spike

$g_{K^+} = \bar{g}_{K^+} n^4$ ← start repolarizing

Conductances		
Leak	Na^+	out
Na^+		
K^+	K^+	in

$\dot{m} = \alpha_m(v)\cdot(1-m) + \beta_m(v)\cdot(0-m)$

$\dot{h} =$ " " "

$\dot{n} =$ " " "

Given this model of time dependence of K^+ gate "n"

$\tau_n(v)\cdot\dot{n} = n_\infty(v) - n$

↳ express in form $\dot{n} = \alpha\cdot(1-n) - \beta\cdot n$

(Take τ_n & n_∞ as known)

Solve for α, β in terms of τ_n & n_∞

Assume $v(t)$ is constant

$\tau\dot{n} = n_\infty - n$

$\dot{n} = \frac{n_\infty}{\tau} - \frac{1}{\tau}\cdot n$

$\dot{n} = \alpha\cdot(1-n) - \beta\cdot n$

$= \alpha - \alpha n - \beta n$

$= \alpha - (\alpha+\beta)n$

match coefficients

$\alpha = \frac{n_\infty}{\tau}$

$\alpha + \beta = \frac{1}{\tau}$

$\beta = \frac{1}{\tau} - \alpha = \frac{1}{\tau} - \frac{n_\infty}{\tau} = \frac{1-n_\infty}{\tau}$