Coursework

SEMT30003/4

This lecture covers:

- Inhomogeneous Poisson process as a statistical model of a spike train
- Spike train statistics (Inter-spike interval, Fano factor, coefficient of variation)
- Spike-Triggered-Average (STA)
- Peri-Stimulus Time Histogram (PSTH)

Learning goals:

1. Be able to write computer code that explores the statistical properties of a neuron's spiking output, and how this spiking output relates to external stimuli.

Building on:

Neural recordings

Leaky Integrate-and-Fire (LIF)

Building up to:

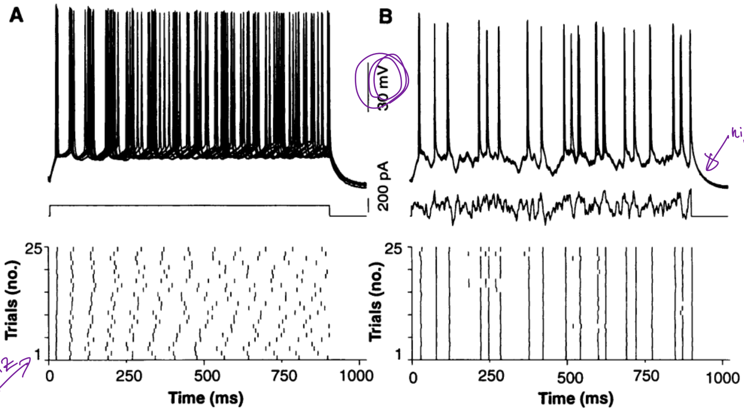
Coursework (70% marks) for 20-credit course version

RAW VOLTAGE TRACE

SPIKE TRAIN

bins 1 0 0 0 1 0 0 1 1 0 1 1 0 2 0 0 0 & maybe

list of times {1, 13, 21, 27, 50, ... }



1-2 KHZ

high sample rate

Fig. 1. Reliability of firing patterns of cortical neurons evoked by constant and fluctuating current. (**A**) In this example, a superthreshold dc current pulse (150 pA, 900 ms; middle) evoked trains of action potentials (approximately 14 Hz) in a regular-firing layer-5 neuron. Responses are shown superimposed (first 10 trials, top) and as a raster plot of spike times over spike times (25 consecutive trials, bottom). (**B**) The same cell as in (A) was again stimulated repeatedly, but this time with a fluctuating stimulus [Gaussian white noise, $\mu_s = 150$ pA, $\sigma_s = 100$ pA, $\tau_s = 3$ ms; see (*14*)].

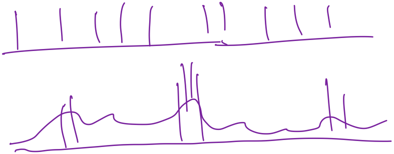
Mainen and Sejnowski 1995

Spike Train

- Electrical recordings at e.g. 30 KHz detect action potentials
- Find the action potentials in the voltage trace (spike sorting)
- Count action potentials in e.g. $\Delta t = 1$ ms bins
- 1 KHz binary or non-negative integer timeseries

homogeneous: constant rate X random

5Hz

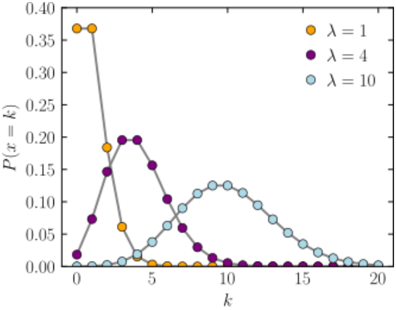


REAL NEURONS TIME VARYING RATES

Poisson process

$$k_t = \# \text{ of spikes in } [t, t + \Delta t) \in \mathbb{Z}_{\geq 0}$$

$$k_t \sim \text{Poisson}(\lambda \cdot \Delta t)$$



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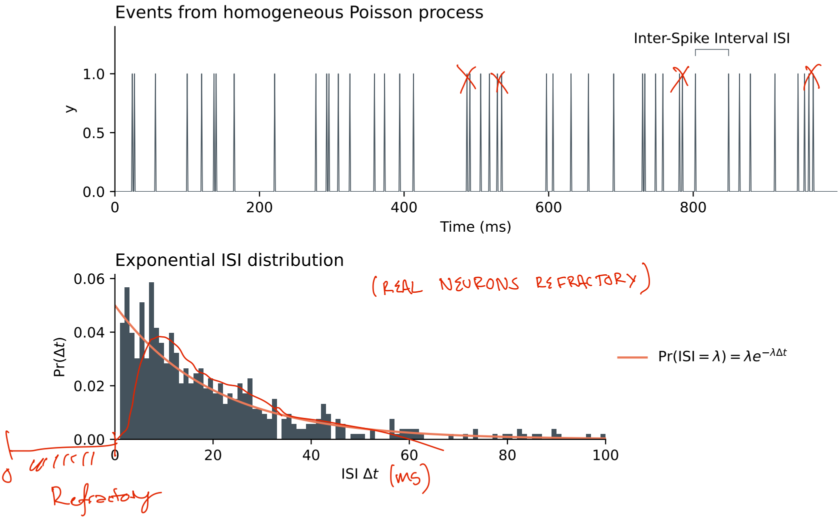
Homogeneous Poisson process



Homogeneous vs. inomogeneous Poisson processes

BIN TO CALC. FANO

Poisson process: $\Pr(\text{spike at time } t)$ depends only on intensity $\lambda(t)$.

Homogeneous: $\lambda(t)$ is constant, no history dependence (memoryless).

Inhomogeneous: $\lambda(t)$ changes (possibly depending on history) but spikes at time t still independent conditioned on $\lambda(t)$.

Fano factor $F = \dfrac{\sigma_k^2}{\mu_k}$

Coefficient of variation $\mathrm{cv} = \dfrac{\sigma_{\mathrm{ISI}}}{\mu_{\mathrm{ISI}}}$

CV CALCULATED FROM ISI

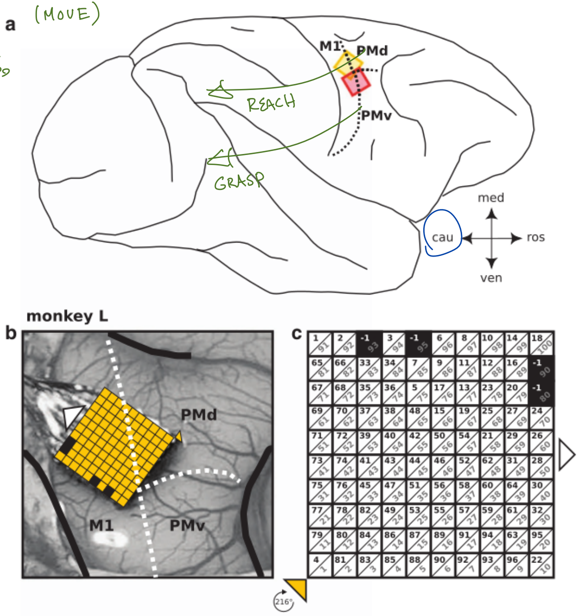
For homogeneous, $F = \mathrm{cv} = 1$

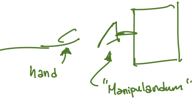
- Less random? $\mathrm{cv} < 1$ $F < 1$
- More random? $\mathrm{cv} > 1$ $F > 1$

TRIAL STRUCTURE

TIME →

START | Cue ? | Go (MOVE)

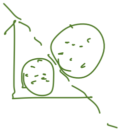


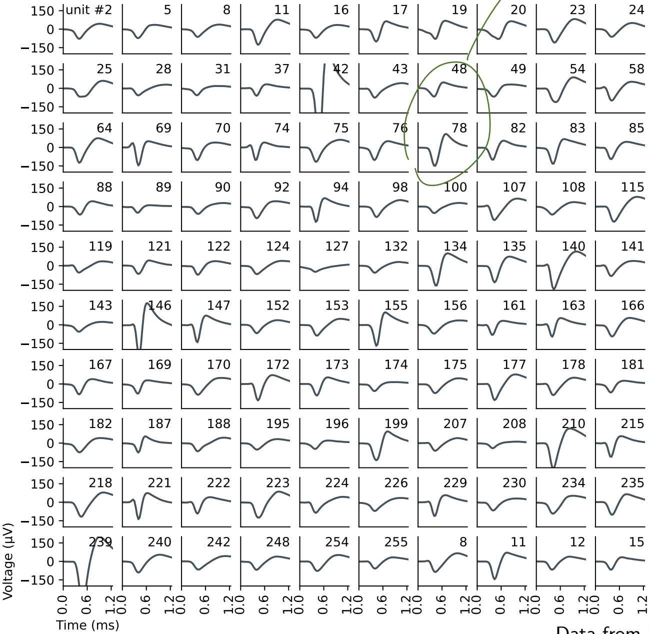


Brochier et al. (2018)

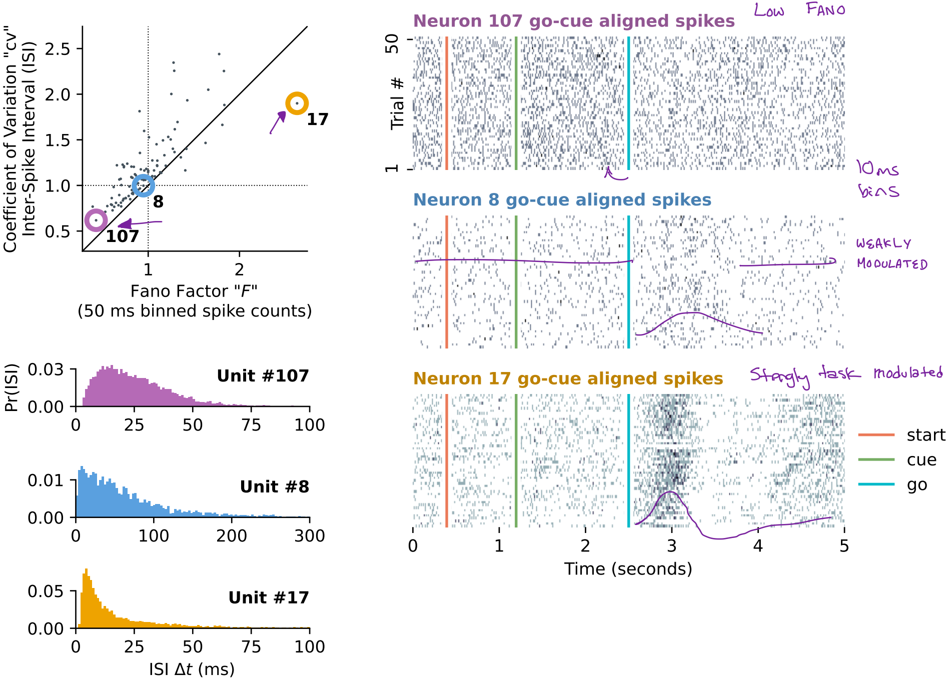
Extracellular voltage ~30kHz (usually)

SPIKES SEPARATED FROM SINGLE VOLTAGE TRACE VIA SPIKE SORTING





Data from Brochier et al. (2018)



Data from Brochier et al. (2018)

Inhomogeneous Poisson

"Real" neurons?

- Refractory period → less random than the homogeneous Poisson
- Varying external inputs → more random than the homogeneous Poisson

Inhomogeneous Poisson process

Model as Poisson with rate varying in time $\lambda(t)$

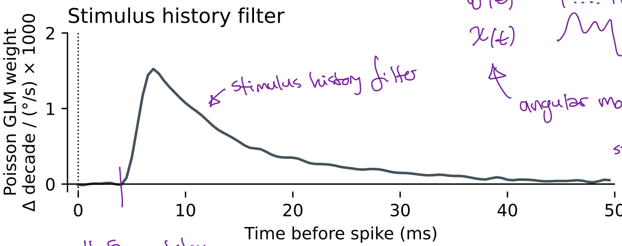
Assume $\lambda(t)$ changes slowly relative to timescale Δt

- $\lambda(t) \approx$ constant for $t \in [t_0, t_0 + \Delta t)$
- k_{t_0} = Poisson$(\lambda(t_0) \cdot \Delta t)$

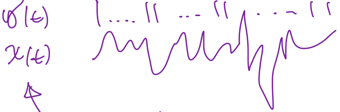
Δt

"constant" rate is small bins

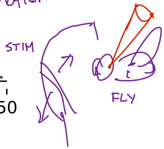
ok apx.



4-5 ms delay



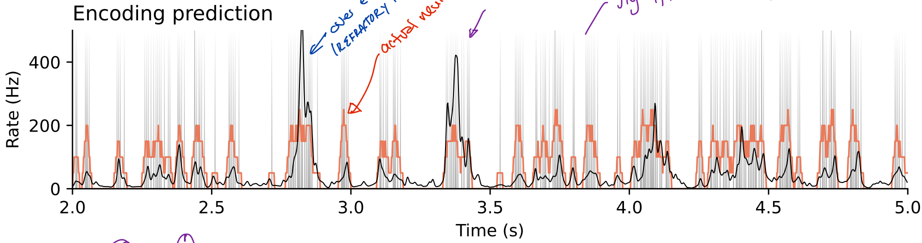
angular motion



over estimate (REFRACTORY MISSING)

actual neuron

fly H1 neuron spikes



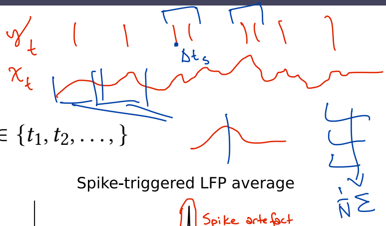
CAN BUILD FANCIER ENCODING MODEL W DEEP NETWORK

STA and PSTH

Record

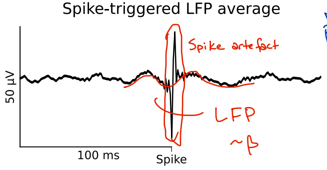
Stimulus or other signal (e.g. movement) $x(t)$

Spike train from a neuron with spike times $t_s \in \{t_1, t_2, \ldots, \}$



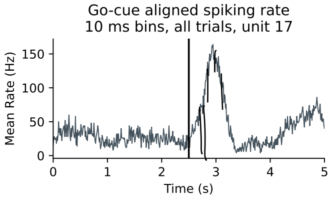
Spike Triggered Average (STA)

- For each spike time time t_s, get $x(t)$ surrounding spike time $x(t_s + \tau)$, $\tau \in [-\Delta, \Delta]$
- Average these together



Peristimulus Time Histogram (PSTH)

- Align spikes to stimulus onset
- Combine trials
- Take histogram



Event-triggered average

- Align & average continuous signal[a]

[a] e.g. LFP

Aside: Bernoulli Process

Since neurons have a maximum firing rate, we can choose Δt small enough so that all time bins contain either 0 or 1 spikes. Binary spike trains can be modelled as coin flips (Bernoulli) process. For small Δt the Poisson and Bernoulli models behave similarly. The math for the Poisson process is slightly simpler, but the maximum rate limit of the Bernoulli process can be useful. Bernoulli–process models can be fit as logistic regression, where the probability p is a linear-nonlinear function $p = f(\boldsymbol{w}^\top \boldsymbol{x} - \vartheta)$ of regression features $\boldsymbol{x}$, and $f(\cdot)$ is the sigmoidal logistic nonlinearity $f(a) = [1 + \exp(-a)]^{-1}$. You may also see binary models fit using "probit" regression, which is similar to logistic regression but uses the sigmoidal nonlinearity taken from the cumulative distribution function of a standard normal distribution. There are certain applications where logistic vs. probit regression are more convenient. Probit regression of binary spike trains is related to a class of models called the "dichotomized Gaussian", which can be used to model correlations in spiking population activity.