Leaky Integrate-and-Fire, Rate Neurons

SEMT30003/4

This lecture covers:

- Leaky Integrate and Fire (LIF)
- Rate Neurons

Learning goals:

1. What simplifications does the Leaky Integrate and Fire (LIF) model make?
2. How to solve for the firing firing rate of a model LIF neuron given constant current?
3. Connect the f–I curve to the idea of a rate neuron (rate coding)

Building on:

First-order ordinary differential equations

Membrane voltage dynamics

Building up to:

Visual system (+ rate neuron models)

Hodgkin Huxley model of the action potential

Topics in Computer Science Final exam question

Modelling Scales

Physiological, Quantitative

Biological Realism, Data needed to identify parameters

⋮

Chemistry
Biochemistry

Conductance Models:

- Hodgkin–Huxley

Leaky Integrate and Fire:

- Simplify, fake the spikes

Rate Neurons:

- Average rate of spikes only

Neural Mass/Field Models
Cognitive Neuroscience

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Computational Efficiency, Mathematical Tractability

Phenomenological, Qualitative

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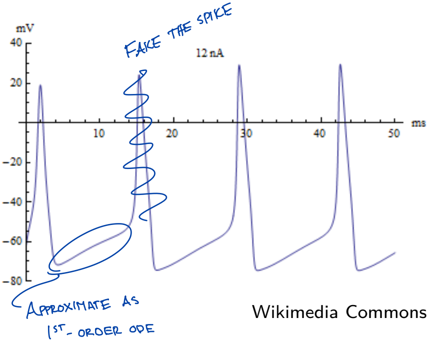
Full Conductance Model e.g. Hodgkin–Huxley

Nonlinear, action potential costly to simulate

Study role of ion channels, conductances, dendritic morphology (shape), etc.



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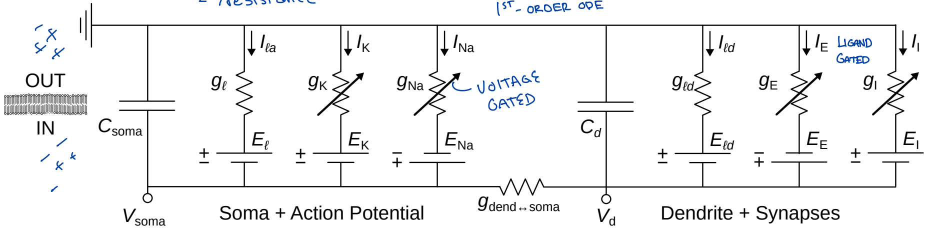
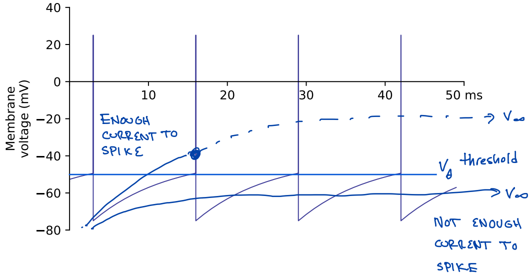
g = conductance
= 1/resistance

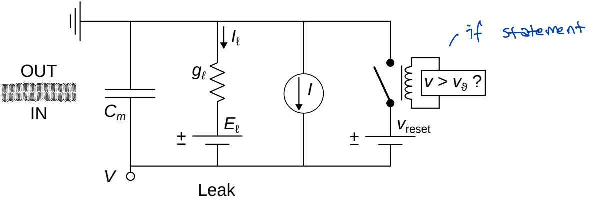


Leaky Integrate-and-Fire (LIF)

Departs from physiology, but sufficient to build intuition

Easy to integrate





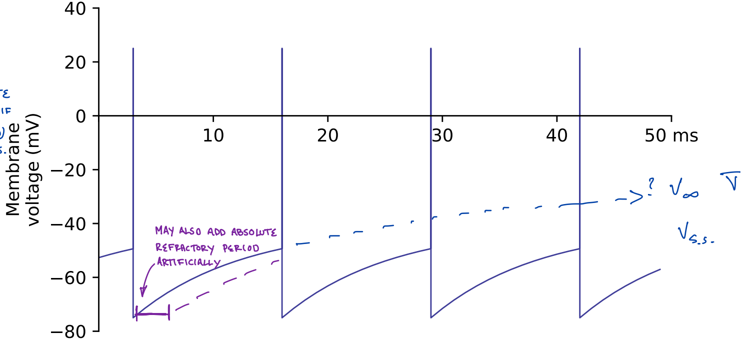
Leaky Integrate-and-Fire (LIF)

SOLVE STEADY-STATE VOLTAGE, & CHECK IF $V_\infty > V_\vartheta$ (THRESHOLD) TO SEE IF IT SPIKES.

$0 = \frac{1}{R}(\mathcal{E}_\ell - V) + I$

$IR + \mathcal{E}_\ell - V = 0$

$V_\infty = \mathcal{E}_\ell + IR$



LIF model:

g

$$C\dot{v} = \frac{1}{R}(v_\mathrm{r} - v) + I$$

if $v(t) > v_\vartheta$ then

$v(t) \leftarrow v_\mathrm{r}$

Emit a spike

MISTAKE: EQUILIBRIUM (RESTING) POTENTIAL NEED NOT BE SAME AS RESET V_r, BUT IS SOMETIMES SET THE SAME FOR SIMPLICITY. SHOULD READ "$\mathcal{E}_\ell$".

$v(t)$: membrane voltage

v_ϑ: Threshold (spike when $v(t) > v_\vartheta$)

v_r: Reset voltage ($v(t) \leftarrow v_\mathrm{r}$ after spike)

R: Membrane resistance

C: Membrane capacitance

I: Current through the membrane

ASIDE (NOT IN COURSE NOTES)

- there are <u>many</u> models that simplify voltage dynamics & "fake" the spike
- LIF not necessarily always "most" natural
- Exponential Integrate & Fire
- Izhikevich Neuron
- Quadratic Integrate & Fire
- Multi - (timescale) - Quadratic Integrate & Fire

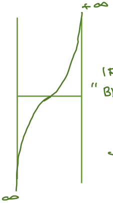
QIF

STATE ABSTACT, NOT QUITE VOLTAGE

$$\dot{x} = x^2 + I$$

$$x(t) = \sqrt{I} \cdot \tan\left(c_0 + \sqrt{I} \cdot t\right)$$

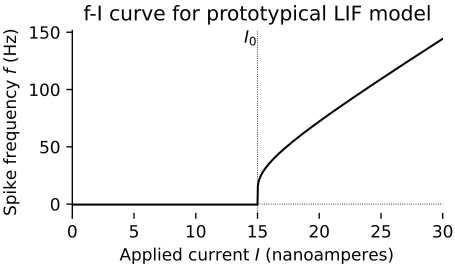
constant from initial condition



IF SPIKES, "BLOWS UP" IN FINITE TIME

CHANGE COORDINATES & JOIN AT ±∞ TO GET THETA NEURON

"f–I" curve



- F–I curve maps applied current "I" → spiking frequency "F"
- Shown today in the case of the Leaky Integrate-and-Fire (LIF) neuron

$$C\dot{v} = \frac{1}{R}(v_{\mathrm{r}} - v) + I$$

if $v(t) > v_\vartheta$ then $v(t) \leftarrow v_{\mathrm{r}}$ and emit a spike

Q: Solve for the spiking frequency f as a function of applied current I in the LIF model

$$C\dot{v} = \frac{1}{R}(v_r - v) + I$$

if $v(t) > v_\vartheta$ then $v(t) \leftarrow v_r$ and emit a spike

Membrane voltage ODE:

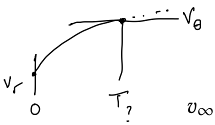
$$C\dot{v} = \frac{1}{R}(v_r - v) + I$$

KNOWN & CONSTANT

$$\underbrace{RC}_{\tau}\,\dot{v} = \underbrace{v_r + IR}_{v_\infty} - v$$

$$\tau\dot{v} = v_\infty - v$$

Spikes if $v_\infty \geq v_\vartheta$



$V(T) = V_\theta$

First-order ODE solution:

$$\underbrace{v(T)}_{v_\vartheta} = v_\infty + e^{-T/\tau}(\underbrace{v(0)}_{v_r} - v_\infty)$$

Find time T where voltage reaches threshold v_ϑ

$$v_\vartheta = v_\infty + e^{-T/\tau}[v_r - (v_r + IR)]$$

$$v_\vartheta = v_\infty - e^{-T/\tau} IR$$

$$v_\infty - v_\vartheta = e^{-T/\tau} IR$$

$$e^{-T/\tau} S = \frac{v_\infty - v_\vartheta}{IR}$$

you can add fixed absolute refractory time here if you like

$$T = \tau \ln\left(\frac{IR}{v_\infty - v_\vartheta}\right) + T_{\text{ABS. REFRACT}}$$

period

Frequency is reciprocal of period $f = \dfrac{1}{T}$

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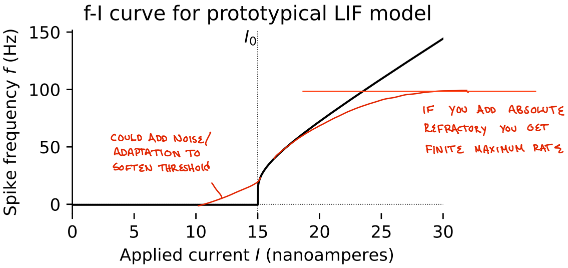
What if we only care about the average number of spikes per unit time?
Do we need to model spikes at all?



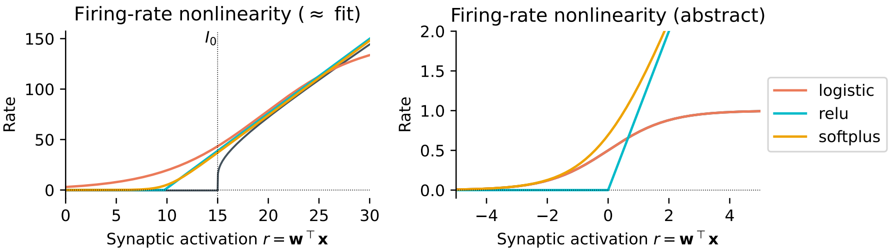
Rate Coding

- Only average # of spikes matters (over time **and/or** population).
- Exact timing of each spike is not used to encode information.

OK $\approx$ for signals that **change slowly** (e.g. musculoskeletal dynamics low-pass filter spikes from α-motor neurons) or information represented in **average population activity of many neurons**.

Nice math: Continuously varying rates are differentiable: Calculus, ODEs, etc.

Nice machine learning: Differentiable nonlinearity emits "rate" rather than $\{0, 1\}$



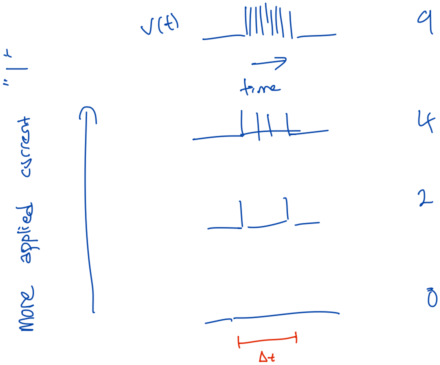
Rate Coding

- Only average # of spikes matters (over time **and/or** population).
- Exact timing of each spike is not used to encode information.

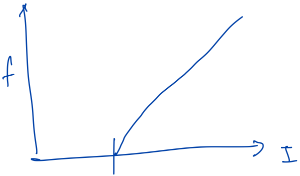
NOT rate coding:

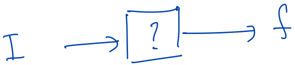
- Phase code: Spike time relative to rhythmic activity carries information
- Timing code: Medial superior olive (or in owls: "nucleus laminaris") detects 10–700 μs Δt in sound arrival at left/right ears.
- Anything that models single spikes or restricts neuronal outputs to $\{0, 1\}$

SPIKES PER UNIT TIME



IF ONLY SPIKE RATE MATTERS
SIMPLER TO TREAT NEURON AS
BLACK BOX FUNCTION FROM
ACTIVATION → RATE (FREQUENCY)





Exam

Leaky Integrate and Fire (LIF)

What physical phenomena do R, C, $v(t)$, I, τ, $v_{\text{threshold}}$, v_{reset} approximate?

Determine whether neuron will cross threshold for a given input

What is the minimum current I_0 to elicit a spike?

Solve the voltage equation forward in time from some initial condition

F–I Curve

Solve for the spiking frequency f as a function of applied current I

What current I elicits spiking frequency f?

How does the F-I curve change when R, C, I, v_ϑ, v_{r} increase/decrease?

Rate coding

What is rate coding; its pros/cons; what information does it capture/ignore?

Are models that average over *populations* of neurons still rate coding?

State example (from biology) where rate coding is a good/bad approximation.