

## McCulloch–Pitts neurons

The McCulloch–Pitts neuron model, or Threshold Logic Unit, was introduced in 1943 by Warren McCulloch and Walter Pitts<sup>1</sup> as a computational model of a neuronal network [1]. Their thinking was that neurons are joined to each other with connections of variable strength; in the soma the inputs from other neurons are added up and they determine the activity of the neuron in a non-linear way. They also knew that neurons tend to ignore input up to some threshold value before responding strongly. These properties they tried to include in their model neurons.

Artificial neurons, of the sort used in artificial intelligence, are described by a single dynamical variable,  $x_i$  say for a neuron labelled  $i$ ; the value of  $x_i$  is determined by the weighted input from the other neurons:

$$x_i = \phi \left( \sum_j w_{ij} x_j - \theta_i \right) \quad (1)$$

$\phi$  is an activation function,  $\theta$  is a threshold and the  $w_{ij}$  are the connection strengths weighting the inputs from the other neurons. The McCulloch–Pitts neuron was the first example of an artificial neuron and had a step function for  $\phi$ :

$$x_i = \begin{cases} 1 & \sum_j w_{ij} x_j > \theta_i \\ -1 & \text{otherwise} \end{cases} \quad (2)$$

Thus, the neuron has two states, it is in the on state,  $x_i = 1$  if the weighted input exceeds a threshold  $\theta_i$  and an off state,  $x_i = -1$  if it doesn't; the picture you might have of how this corresponds to the brain is that 'on' corresponds to rapid spiking and 'off' to spiking at a much lower rate. The  $w_{ij}$ , the connection strengths, are like the synapse strengths, a positive  $w_{ij}$  is an excitatory synapse and negative, an inhibitory; a given neurons have both negative and positive out-going synapses, that is there is no restriction that says  $w_{ij}$  always has the same sign for a given  $j$ , this is different from real neurons where all the outgoing synapses from a given neuron are either excitatory or inhibitory.

While it should be clear that this network has some of the properties, very abstracted, of a neuronal network, it might not be so clear what can be done

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<sup>1</sup>Walter Pitts was an interesting and odd man, a genius in the old-fashioned self-destructive and brilliant sense.

with the neurons. When they were working, at the dawn of the age of electronic computers, McCulloch and Pitts believed that their neurons might form the natural unit in computer circuits. In other words, they thought they might perform the role actually played by logical circuits. In fact, it is still not clear if the artificial neuron is or isn't the natural unit of computation since they are a component in, for example, deep learning networks. In fact, there are two major applications of McCulloch-Pitts neurons: the perceptron and the Hopfield network. These two applications add a rule for changing the connection strength to the original McCulloch-Pitts neuron.

## References

- [1] McCulloch, W and Pitts, W. (1943). A logical calculus of the ideas immanent in nervous activity. *Bulletin of Mathematical Biophysics*, 5:115–133.