Review Board 2024.10.15

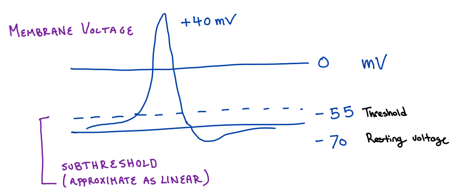
Today 15th Oct.

① Why were we covering O.D.Es, Euler's Method, Etc. ?

② Hippocampus, Hopfield, Delta Rule: Reprise
└ Tabled for tomorrow

③ Exam review (continued tomorrow)

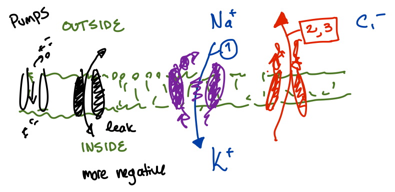
Action Potential



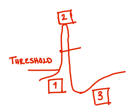
Subthreshold
(approximate as linear)

$\tau \dot{v} = -v + c$

¡This is why 1st order linear O.D.E.s!



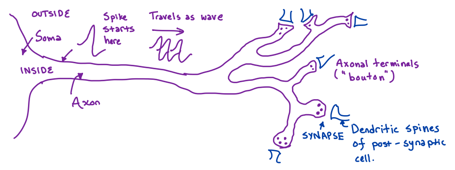
more negative
~ -70 mV



1: Initiate: V goes above voltage-gated Na^+ channel threshold, usually ≈ -55 mV → runaway excitation

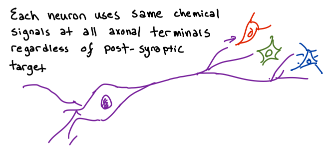
2: Stops rising: inward K^+ start opening ≈ -20 mv
Na^+ channels inactivate above ≈ 0 mV

3: Refractory Period: Open K^+ channels continue to conduct, hyperpolarize membrane. Na^+ channels stay inactive a bit, preventing spike.

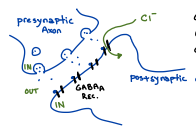


Dale's Principle: (Approx. truth)

Each neuron uses same chemical signals at all axonal terminals regardless of post-synaptic target



Example Synapse: Ionotropic GABA



GABA neurotransmitter
Opens chloride (−) channel
Cl^- enters, lowering voltage
(less likely to spike, "Inhibitory")

Glutamate
Ionotropic
- AMPA → +V (excitatory)
- NMDA → Ca^+ (signal for learning)

(Metabotropic Glu. R "MGluR" not on exam)

GABA
- $GABA_A$ – ionotropic
- $GABA_B$ – metabotropic

Linear O.D.E.s → membrane voltage

how to simulate membrane?

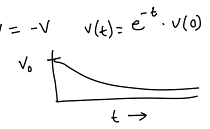
$x_0 \leftarrow$ somethin
for $t = 1 \,..\, 1000$
$\quad x \leftarrow f(x)$

in a loop
integrate O.D.E.
for example by forward Euler

$\dot{v} = -v + f$

$v \leftarrow \Delta t \cdot [f - v]$

$\dot{v} = -v \qquad v(t) = e^{-t} \cdot v(0)$



linear 1st-order O.D.E Soln. (scalar, stable case)

- decay from where we start

$\dot{v} = \underline{a} - v$

- toward where we are driven ("equilibrium value")

$v(t) = a + e^{-t}(v_0 - a)$

$\tau \dot{v} = -v \qquad V(t) = e^{-t/\tau} \cdot V_0$

- with time-constant τ "tau" (larger → slower)

$\tau \dot{v} = -v + c$

$V(t) = c + e^{-t/\tau}(v - c)$

- generic form

$\tau \, \cancel{RC} \, \dot{v} = \mathcal{E}_e - v + I \cdot R$

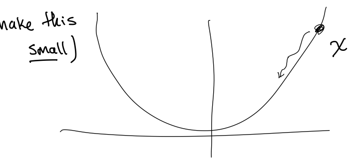
$\underbrace{\mathcal{E}_e + IR}_{c}$

- specific constants come from electronic properties of membrane, but we will cover this post-midterm.

Delta Rule

An optimization perspective

Loss Function (make this small)



$$\text{Loss} = \frac{1}{2}x^2 \longrightarrow \underbrace{\frac{d\,\text{Loss}}{dx}}_{\text{GRADIENT}} = x$$

for epoch in 1 ... 1000
$\quad x \leftarrow \eta(-x)$

gradient
Learning Rate
"descent"

TBC Wed. 16 Oct.