UNIVERSITY OF BRISTOL

Fall 2024 Topics in CS Question

SEMT30003/4
Computational Neuroscience

TIME ALLOWED:
1 hour

This paper contains one in-depth question with three sections that contain multiple parts. **All** components must be attempted and will be used for assessment. The maximum for this section is 50 marks.

Word counts are not mandatory, but rather hints to help you avoid spending more time than needed. You may exceed the advised word count if you wish. You may also use diagrams and mathematics as appropriate in short answer questions.

Please write your student number and/or username on the line below.
(To ensure blinded marking, do not write your name)

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Other instructions:

- Only approved non-programmable calculators are permitted
- Write your answers in the question paper
- Blank pages are provided at the end if you need extra space. Clearly label which answer you are writing.

Addendum: Answers should be written clearly enough that markers can distinguish between a half-remembered memorised response, and a clear response drafted anew from deep understanding. Poor wording or incomplete sentences that blur the lines between these two may be cause for a half-mark $\frac{1}{2}$ deduction (that is, 1.5 marks are awarded where the clarity or grammar create some ambigity), but you do not need to be too verbose. In all cases at least 1.5 marks should be awarded when the marker can glean that understanding is essentially correct.

(a) [6 marks] General knowledge: (3–9 minutes, 1–3 minutes per question)

i. What do neuroscientists mean when they say "rate coding"? Describe a scenario where rate coding might be a *poor* model of neuronal activity (hint: what information in spiking codes does rate coding ignore?). (≈20–90 words) **[2]**

The general idea behind rate coding is that only the average number of spikes matter. This average might be taken over time, or across a population. In either case, the exact timing and identity of each spike is not used to encode information, only the mean rate.

If these details contain relevant information for biophysical computation, rate coding will not be a good model. Expect students to provide the calculation of precise inter-aural time delays, phase coding, or binary models of neural activity, as instances where rate coding is an inappropriate abstraction.

Addendum: Markers should consider "rate" and "frequency" interchangeable here. The distinction is not necessary for this question, but for those interested: The word frequency is sometimes used when neural spiking is roughly periodic (not homogeneous Poisson or random). The standard Wilson-Cowan-Amari neural field reduction used to justify rate models typically thinks in terms of the average fraction of a neural population that is active, and this does not require periodicity. However, the f–I curve for the LIF model is an example of a rate reduction where the rate truly is a frequency of periodic spiking. There are also other reductions, such as the Byrne-Coombes model, that derive a continuous rate-like quantity from a population of periodically spiking neurons.

ii. Where in the human brain can one find neurons that respond to visual input, with receptive fields that detect oriented edges and exhibit phase invariance ("complex cells")? (≈1–5 words) **[2]**

Primary visual cortex, striate cortex, V1, or Broadmann area 17 (these are synonyms). Higher-order visual areas that retain texture information are also acceptable, such as V2, V3, Broadmann areas 18, 19, peristriate cortex, extrastriate cortex, visual cortex, or the occipital lobe. No marks for "retina" or "LGN" or earlier structures in visual processing.

iii. Briefly define *sparsity* in the context of population coding, and discuss its role either in encoding visual information in the retina, or encoding memories in hippocampus. (≈30–90 words) **[2]**

Full marks for explicitly or implicitly providing a valid description of what sparsity *is*, and at least one benefit of sparsity. For example:

Sparsity means that, at any given instant, only a small fraction of the neurons in a population are currently spiking. Sparsity can decrease energy costs, and help separate representations. Sparsity is important for example in encoding memories in CA3, or efficiently sending a compressed representation of visual input from the retina to the brain. In the visual system, sparse overcomplete representations use many neurons selective for specific features to represent visual input (e.g. V1 simple or complex cells sensitive to oriented edges). From a linear perspective, there are more features than strictly required to encode the visual stimuli (overcomplete); Activation is, however, sparse: Only those neurons best matching a particular input are used.

Most reasonable attempts should get full marks here. It would be unusual for a student to discuss structural sparsity and not activity sparsity, since this was not emphasised in the course. But, as usual, all scientifically correct, deep, and earnest answers get full marks.

(b) [22 marks] Leaky integrate-and-fire dynamics: (10–20 minutes)

Consider a Leaky Integrate-and-Fire (LIF) neuron obeying the following dynamics:

$$\begin{aligned} &C\dot{v} = \tfrac{1}{R}(E_\ell - v) + I \\ &\text{if } v(t) > v_{\text{threshold}} \text{ then } v(t) \leftarrow v_{\text{reset}} \text{ and emit a spike} \end{aligned} \tag{1}$$

Assume that $v_{\text{reset}} = E_\ell$.

i. Discuss the physiological interpretation of C, R, E_ℓ, I, and $v_{\text{threshold}}$, and how this model approximates the ionic currents across the neuronal membrane during the action potential. (address all 5 variables ≈30–90 words) **[5]**

C: Membrane capacitance, relationship between charge flow across membrane and change in membrane voltage, measures stored electrostatic field;

R: The membrane resistance determines how quickly the neuron is pulled toward its resting potential. Smaller values mean the membrane is more leaky.

E_ℓ: Resting potential, the effective reversal potential of all channels contributing to the leak conductance; Determine the baseline membrane voltage without input

I: Injected or applied current, or current arising from other ion channels.

$v_{\text{threshold}}$: The voltage above which positive feedback will trigger an action potential. Often set to -55 mv.

Full marks for answers that (1) attempt a not-overtly-wrong response for each variable and (2) cover at least 60% of the biological detail (e.g. can give further correct details or elaborate on at least 3 out of 5).

Example minimal response for full marks: C and R are the membrane capacitance and resistance, respectively; E is the resting potential; I is applied current. LIF approximates the spiking nonlinearity by resetting the membrane voltage if it goes above v_threshold.

Addendum: "The voltage of the threshold" is not an adequate response for $v_{\text{threshold}}$, some further depth as above is required.

ii. If we define

$$\begin{aligned} \tau &= RC \\ v_\infty &= E_\ell + RI \end{aligned} \tag{2}$$

we can write the solution for (1) *ignoring the spiking reset* as

$$v(t) = e^{-t/\tau}(v(0) - v_\infty) + v_\infty \tag{3}$$

Assume that a constant current is applied to the LIF model, leading to rhythmic spiking. Derive an expression for the time "T" between consecutive spikes as a function of I, v_∞, and τ[1]. **[7]**

[1] Hint: After a spike the membrane voltage starts at $v(0) = v_{\text{reset}}$, and then rises to $v(t) = v_{\text{threshold}}$ at some unknown future time T; Solve for this T in the solution for the voltage provided in Equation(3).

We start with the ODE solution

$$v(t) = e^{-t/\tau}(v(0) - v_\infty) + v_\infty \tag{4}$$

And set $v(0) = v_{\text{reset}}$ and $v(t) = v_{\text{threshold}}$

$$v_{\text{threshold}} = e^{-t/\tau}(v_{\text{reset}} - v_\infty) + v_\infty \tag{5}$$

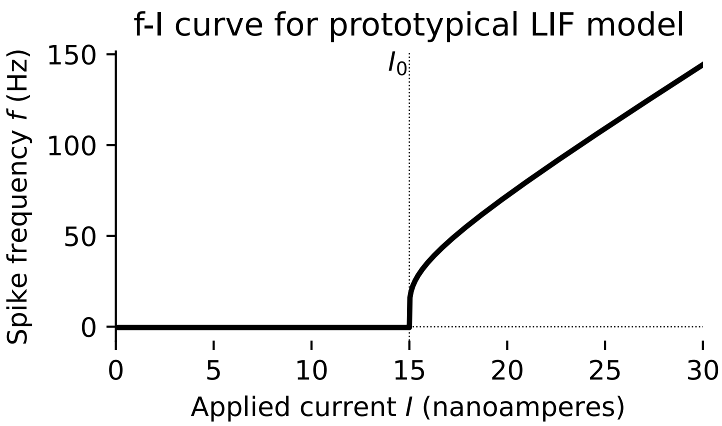
Then solve for t:

$$T = t = -\tau \ln \frac{v_{\text{threshold}} - v_\infty}{v_{\text{reset}} - v_\infty} \tag{6}$$

Any equivalent expression is valid. Partial marks for solutions that are only wrong due to dropped minus signs, etc. Please note the following *non-exhaustive* list of equivalent forms, considering the $E_\ell = v_{\text{reset}}$ assumption:

$$\begin{aligned} T &= \tau \ln \frac{v_{\text{reset}} - v_\infty}{v_{\text{threshold}} - v_\infty} \\ &= \tau \ln \frac{E_\ell - v_\infty}{v_{\text{threshold}} - v_\infty} \\ &= \tau \ln \frac{v_{\text{reset}} - E_\ell - RI}{v_{\text{threshold}} - E_\ell - RI} \\ &= \tau \ln \frac{-RI}{v_{\text{threshold}} - E_\ell - RI} \\ &= \tau \ln \frac{RI}{E_\ell + RI - v_{\text{threshold}}} \\ &= -RC \ln[1 - \frac{v_{\text{threshold}} - E_\ell}{RI}] \end{aligned} \tag{7}$$

iii.α Below is a plot of firing frequency as a function of injected current. The LIF model has a hard threshold: the model abruptly starts spiking above a certain minimum current (labelled I_0 below). Derive an expression for I_0 in terms of v_∞, C, R. **[5]**



Note: Although this plot has units, you do not need to know the values of the model parameters here. No calculation is required and you may leave your answer in terms of v_∞, C, R.

Students should not need the answer from **(ii)** to complete this question, it can be intuited from the provided time-domain solution. The neuron can only spike when $v_\infty > v_{\text{threshold}}$, and we find the point just before this by solving for $v_\infty = v_{\text{threshold}}$. Depending on specific reading of the question, the following answers are valid

$$\begin{aligned} I_0 &> \frac{v_{\text{threshold}} - E_\ell}{R} \\ I_0 &\geq \frac{v_{\text{threshold}} - E_\ell}{R} \\ I_0 &= \frac{v_{\text{threshold}} - E_\ell}{R} \end{aligned} \tag{8}$$

Or any of the above with E_ℓ replaced with v_{reset}, but ***only*** because the question defined these to be equal; If you are using this as a past exam to sudy *expect these voltages to be different in future exams!*

Full marks for *any correct* nontrivial expression for I_0. Partial marks for valiant attempts with hasty errors.

-1 mark for any of the following errors:

- A sign error or writing $<$ in equation (8) (although doing both would cancel

iii.β State whether you'd expect I_0 to increase or decrease if R is *decreased*. Would this make it easier or harder for incoming excitatory synapses to cause this neuron to fire? Briefly speculate about what such a change might correspond to in a binary perceptron, whose output is given by $y = \Theta(\boldsymbol{w}^\top \boldsymbol{x} - b)$. (≈10–20 words) **[5]**

Full marks for saying "increase", followed by any sincere attempt that does not belie obvious misunderstanding. For example, any text or diagram that conveys part of the following intuition, or any display of reasonably related neuroscience understanding.

Example minimalist answer: Increase; Harder; In the binary perceptron this can be modelled as scaling down the weights (or decreasing b, either weights or bias sufficient on its own for marks).

Addendum: 1 mark for a *consistently* wrong attempt (decrease, easier, larger weights and/or less-positive b), suggesting mis-reading of increase/decrease wording in question.

+2 mark for correctly stating that I_0 should *increase*, but other responses incorrect

+1 mark for indicating that this increase makes it **harder** for excitatory input to recruit spiking.

+2 marks for all details as stated above.

-1 marks for a mostly correct attempt but with some technically incorrect misstatements that preclude full marks.

(c) [22 marks] The Hodgkin–Huxley Model: (10–20 minutes)
The voltage in the Hodgkin Huxley model of the action potential evolves according to the following differential equations:

$$\begin{aligned}
C\dot{v} &= g_\ell(E_\ell - v) + \bar{g}_{\mathrm{K}} n^4 (E_{\mathrm{K}} - v) + \bar{g}_{\mathrm{Na}} m^3 h (E_{\mathrm{Na}} - v) + I \\
\dot{n} &= \alpha_n(v) \cdot (1 - n) - \beta_n(v) \cdot n \\
\dot{m} &= \alpha_m(v) \cdot (1 - m) - \beta_m(v) \cdot m \\
\dot{h} &= \alpha_h(v) \cdot (1 - h) - \beta_h(v) \cdot h.
\end{aligned} \tag{9}$$

You do not need to know the specific form of the nonlinear functions $\alpha_{\dots}(v)$ and $\beta_{\dots}(v)$ for this question.

i. What part of the action potential does the gating variable n on the potassium conductance capture? (≈5–10 words) **[2]**

n captures the voltage dependent (or voltage gating) of the potassium conductance. Opening of potassium channels allows K^+ ions to flow out of the cell, lowering and typically hyperpolarizing the membrane following the action potential. This contributes to the refractory period. (Full marks for mentioning either refractory period or post-spike hyperpolarization; 1 mark for mentioning voltage gating but failing to correctly connect to the action potential; 0 marks for incorrect responses, e.g. confusing potassium and sodium)

Addendum: The original low word-count lower-range of 1–10 words implied that something very simple could suffice.

In light of this, any of ”repolarization”, ”hyperpolariaztion”, or ”relative refractory period” get 2 marks.

Scientifically true responses that fail to discuss the action potential get $\frac{1}{2}/2$ points. For example, commenting on voltage gating, or the fact that the exponentiation by 4 reflects an approximate model of four statistically independent voltage gates.

ii. Solving first-order ODEs:[2] Assume that v is constant (so that $\alpha_m(v)$ and $\beta_m(v)$ are also constants). Find an expression for the steady-state value for the gating variable m, denoted as $m_\infty = \lim_{t\to\infty} m(t)$. **[10]**

This is a first-order ODE problem in disguise. Set $\dot{m}$ to zero and solve for the steady state.

$$\begin{aligned} 0 &= \alpha \cdot (1 - m_\infty) - \beta \cdot m_\infty \\ \beta \cdot m_\infty &= \alpha \cdot (1 - m_\infty) \\ \beta \cdot m_\infty &= \alpha - \alpha m_\infty \\ (\beta + \alpha) m_\infty &= \alpha \\ m_\infty &= \frac{\alpha}{\beta + \alpha} \end{aligned} \tag{10}$$

Full marks for any equivalent expression that isolates m_∞. Partial marks for minor errors, or if the answer is stated without derivation (suggesting rote memorization). If the student forgets the simple way to obtain m_∞ and tried a time-consuming method, e.g. integrating factors, nearly full marks will be given to essentially correct answers missing the odd minus sign due to haste.

iii. Assume that α_m and β_m are constants. Re-write the differential equation for the sodium gating variable $\dot{m} = \alpha \cdot (1 - m) - \beta \cdot m$ in the form

$$\tau \dot{m} = m_\infty - m \tag{11}$$

Give an expression for τ and m_∞ in terms of α_m and β_m.[3]

[10]

[2]Hints for (ii): You may assume that $\alpha + \beta > 0$, so that the ODE for m is stable; You do not *need* to integrate the ODE for $\dot{m}$ to find this; A step-by-step algebraic transformation of expressions is fine here.

[3]Hints for (iii): A step-by-step algebraic transformation of expressions is fine here and you may omit the subscripts in α_m and β_m for convenience.

$$
\begin{aligned}
\dot{m} &= \alpha \cdot (1 - m) - \beta m \\
\dot{m} &= \alpha - \alpha m - \beta m \\
\dot{m} &= \alpha - (\alpha + \beta) \cdot m \\
\tfrac{1}{\alpha+\beta}\dot{m} &= \tfrac{\alpha}{\alpha+\beta} - m \\
\text{let } \tau &:= \tfrac{1}{\alpha+\beta} \\
\text{let } m_\infty &:= \tfrac{\alpha}{\alpha+\beta} \\
\text{then } \tau\dot{m} &= m_\infty - m
\end{aligned}
\tag{12}
$$

Since this question involves transforming two given expressions into each other, showing work is required. Only partial marks can be given for answers that try to work from both ends but show errors or vagueness in the middle. Correct expressions for τ and m_∞ without incorrect mathematics hint at rote memorisation, but these answers will still receive most of the marks.