UNIVERSITY OF BRISTOL

Fall 2024 Week 6 Exam

FACULTY OF ENGINEERING

SEMT30003
Computational Neuroscience

TIME ALLOWED:
1 hour

This paper contains TWO parts.

[A] The first contains 5 short questions. Each question is worth 2 MARKS. All short questions should be attempted. (10 MARKS)

[B] The second contains 3 in-depth questions. Each of these is worth 10 MARKS. The BEST 2 in-depth questions will be used for assessment. (20 MARKS)

The maximum for this paper is 30 MARKS.

Word counts are not mandatory, they are provided as hints to help you avoid spending more time than needed. You may exceed the advised word count if you wish. You may also use diagrams and mathematics as appropriate in short answer questions.

Other Instructions:
Calculators must have the Faculty of Engineering Seal of Approval.

TURN OVER ONLY WHEN TOLD TO START WRITING

Section A: Five short questions—answer all questions

QA.1 [2 marks] Which of these statements best matches the simplifying heuristic known as Dale's principle (pick one):

1. Most dendrites receive either exclusively excitatory or exclusively inhibitory inputs.
2. Single neurons typically use the same set of chemical neurotransmitters at all their axonal terminals.
3. A single neuron releases either excitatory or inhibitory neurotransmitters depending on the identity of the post-synaptic target at each synapse.
4. When neuron A excites neuron B, and persistently contributes to making B spike, the strength of the synapse from A $\rightarrow$ B is decreased.

QA.2 [2 marks] Which of the following are types of glutamate receptors in the brain (select ALL that apply):

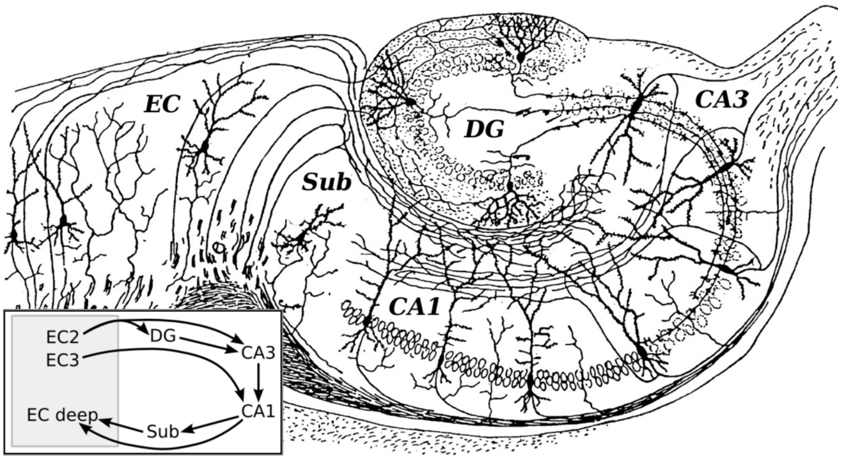
1. GABA
2. AMPA
3. 5HT-2A
4. NMDA
5. nAChR
6. Muscarinic

QA.3 [2 marks] Neurons fire an action potential when the membrane voltage rises above approximately -55 mV, which opens voltage-gated ion channels that allow sodium to enter the cell and raise the voltage further.

- **i** [1 mark] What eventually stops the membrane voltage from continuing to rise? (≈5–25 words)
- **ii** [1 mark] What causes the refractory period following the action potential? (≈5–25 words)

QA.4 [3 marks] The neurotransmitter GABA is inhibitory: It lowers the membrane voltage of the postsynaptic cells. What makes the voltage response inhibitory rather than excitatory? Explain in terms of the role of ionotropic channels and the relative concentration of chloride ions inside versus outside the cell in this process. (≈25-100 words)

QA.5 [3 marks] This an annotated sketch of the circuity of the hippocampus and surrounding structures by Santiago Ramón y Cajal. Circle the region that is thought to play an important role in pattern completion thanks to recurrent self-excitation in the local circuitry.



Section B: In-depth questions—answer TWO questions

QB.1 [10 marks] This question is about attractor networks

(a) [4 marks] What is an attractor network? (≈25–50 words)

Consider a Hopfield network model:

- Network states are a column vector $\boldsymbol{x} = \{x_1, x_2, \ldots, x_K\}^\top$ for K units ($K = 2$ mostly here) States are either positive or negative one $x_i \in \{-1, 1\}$.
- Network weights are a symmetric matrix $\mathbf{W} \in \mathbb{R}^{K \times K}$. Assume that the weight matrix is set to zero along its diagonal $w_{ii} = 0$.
- Threshold is defined such that non-negative activation triggers a 1 output

$$\Theta(r) = \begin{cases} 1 & \text{if } r \geq 0 \\ -1 & \text{otherwise.} \end{cases}$$

(b) [3 marks] Write the mathematical expression of the energy of a pattern in the Hopfield network (or the equivalent in words). State whether (remembered) patterns stored in the weight matrix should have higher or lower energy compared to random patterns.

(c) [3 marks] In a Hopfield network with three neurons there are two patterns to store

$$\boldsymbol{x}_1 = (1, -1, -1) \text{ and } \boldsymbol{x}_2 = (1, 1, 1);$$

Calculate the weight matrix storing *both* patterns $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$. Use the approach form coursework 1, which calculates the weight matrix for all patterns that are stored in a single step.

QB.2 [10 marks] This question is about linear ordinary differential equations.

(a) [3 marks] You are given a model of a neuron's membrane voltage, whose state v evolves in time according the ordinary differential equation

$$C\dot{v} = \tfrac{1}{R}(E - v) + I. \tag{1}$$

The variables C, R, E, and I are all constants. Re-arrange Equation (1) in the format

$$\tau\dot{v} = v_\infty - v \tag{2}$$

by finding the values of the constants τ and v_∞ in terms of C, R, E, and I.

(b) [3 marks] If v starts at value v_0 at time $t = 0$, show that Equation (2) has a solution of the form $v(t) = e^{-at}(v_0 - v_\infty) + v_\infty$ and state the relationship between a and τ.

Turn Over for QB.2(c)!

(c) [4 marks] Your colleague wrote computer code to integrate Equation (2) numerically. They used the parameter values $\tau = 10$ ms and $v_\infty = -30$ mV, and initial conditions $v(t = 0) = v_0 = -70$. They used the forward Euler integration method with time step $\Delta t = 5$ ms.

i. [2 marks] What is the order of the error in the Forward Euler method in terms of Δt?

ii. [2 marks] What value for v do you get when calculating $v(t = 5)$ using Forward Euler with $\Delta t = 5$? Does Euler integration over- or under-estimate compared to the true solution of $v(t = 5) \approx -54.26$?

QB.3 This question is about the McCulloch–Pitts neuron and the delta learning rule.

(a) [3 marks] Consider a binary perceptron unit with inputs $\boldsymbol{x} = \{x_1, x_2, \ldots, x_n\}^\top$ and weights $\boldsymbol{w} = \{w_1, w_2, \ldots, w_n\}^\top$, whose output is defined as

$$\hat{y} = \Theta(\textstyle\sum_i w_i x_i) = \Theta(\boldsymbol{w}^\top \boldsymbol{x}), \tag{3}$$

where $\Theta(\cdot)$ is the Heaviside step function (that is, binary threshold)

$$\Theta(r) = \begin{cases} 1 & \text{if } r > 0 \\ 0 & \text{otherwise.} \end{cases}$$

Explain the interpretation of the variables $\boldsymbol{x}$, $\boldsymbol{w}$, and $\boldsymbol{y}$ in terms of both *computation* **and** which aspects of *neuronal signalling and/or anatomy* each corresponds to.

(b) [3 marks] The delta learning rule for updating weight w_k in (3) is

$$\Delta w_k = \eta \cdot \mathrm{error} \cdot x_k. \tag{4}$$

Re-write Equation (4), replacing error with its expression in terms of $\boldsymbol{w}$ and a single pair of supervised training data $(\boldsymbol{x}, y^*)$, where y^* is the target output.

 Turn Over for QB.3(c)!

(c) [4 marks] Consider the following function,

$$f(\boldsymbol{w}; \boldsymbol{x}, y^*) = \mathrm{relu}(r) - y^* \cdot r \qquad \text{where } r = \boldsymbol{w}^\top \boldsymbol{x} = \textstyle\sum_i w_i x_i, \tag{5}$$

where $\mathrm{relu}(r) = \mathrm{maximum}(0, r)$ is the “rectified linear unit”, which clips negative values of r to zero.

Suppose that we want to minimise $f(\boldsymbol{w}; \boldsymbol{x}, y^*)$ by adjusting the synaptic weights $\boldsymbol{w}$ (for a given fixed $(\boldsymbol{x}, y^*)$. Show that using gradient descent to change $\boldsymbol{w}$ to minimise Equation (5) leads to the “delta rule” weight update in Equation (4).[1]

[1] You may assume that the derivative of $\mathrm{relu}(r)$ at $r = 0$ is zero, that is: $\frac{\mathrm{d}}{\mathrm{d}r} \mathrm{relu}(r)\big|_0 = 0$.