Leaky Integrate-and-Fire (LIF)

Part I: Overview

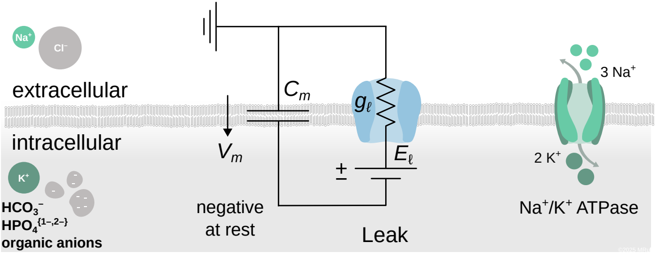
Computational Neuroscience

University of Bristol

M Rule

Learning outcomes:

- Mathematically analyze spiking dynamics of the LIF model
- The f–I curve (rate model) for LIF



$$C_m \dot{v} = g_\ell (E_\ell - v)$$

$$\tau_m \dot{v} = E_\ell - v$$

$C_m =$	100×10^{-12}	Farads	(100 picofarads)	**cell membrane capacitance**
$E_\ell =$	-70×10^{-3}	Volts	(-70 millivolts)	**resting potential**
$g_\ell =$	10×10^{-9}	Siemens	(10 nanosiemens)	**leak conductance**
$R_m = 1/g_\ell =$	100×10^{6}	Ohms	(100 megaohms)	**membrane resistance**
$\tau_m = r_m C_m =$	10×10^{-3}	seconds	(10 milliseconds)	**membrane time constant**

Plausible (albeit invented) numbers for a cortical pyramidal cell neuron

ΔV: arrow tip − arrow base

Modelling Scales

... Quantum Chemistry, Molecular dynamics

Molecules
Gillespie, Master Equation

Concentrations
Mass-Action Kinetics

Conductance Models
Hodgkin–Huxley

Spiking Models
Leaky Integrate and Fire

Rate Neurons
Neural Mass/Field Models

Poisson Neurons
Generalized Linear Models

Binary Neurons
McCulloch–Pitts, Hopfield, Perceptron

Cognitive Neuroscience, Psychology ...

Physiological, Quantitative

Biological Realism, Data needed to identify parameters

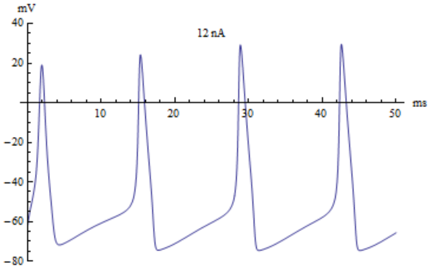
Computational Efficiency, Mathematical Tractability

Phenomenological, Qualitative

Conductance Model Point Neuron e.g. Hodgkin–Huxley

Nonlinear, action potential costly to simulate

Study role of ion channels, conductances, dendritic morphology (shape), etc.



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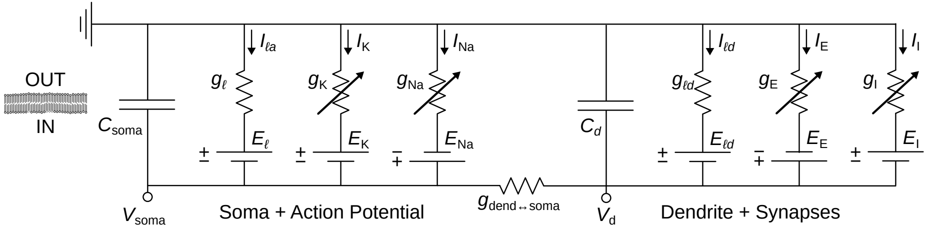

do you need to fix any mistakes in the diagram?

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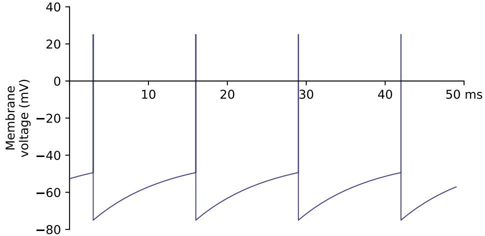
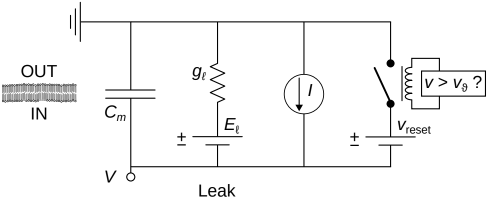
Departs from physiology, but sufficient to build intuition

Easy to integrate





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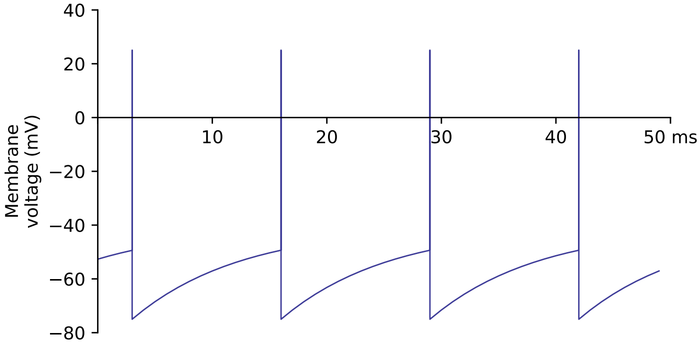

LIF model:

$C\dot{v} = \frac{1}{R}(v_{\mathrm{r}} - v) + I$

if $v(t^-) > v_\vartheta$ then

Emit a spike

$v(t^+) \leftarrow v_{\mathrm{r}}$

(if refracotry)$v(t) = v_{\mathrm{r}}$ for $t \in [t^+, t + T_{\mathrm{r}}]$

$v(t)$: membrane voltage

v_ϑ: Threshold (spike when $v(t) > v_\vartheta$)

v_{r}: Reset voltage ($v(t) \leftarrow v_{\mathrm{r}}$ after spike)

R: Membrane resistance

C: Membrane capacitance

I: Current through the membrane

Solution of LIF up until spike?