Lab 2: Leaky Integrate and Fire (LIF)

This coursework is about numerical simulation of an integrate-and-fire model neuron. The intended learning outcome is to check that you can translate the ODE model described in Equation (1) into computer code to simulate and explore its behaviour. You will, most likely, end up writing write a notebook/script/file/function that contains a `for` loop to implement various numerical integration schemes for the LIF model.

Consider leaky integrate-and-fire model

$$\begin{aligned} \tau_m \dot{v} &= (E_L - v) + R_m I_e \\ \text{if } v(t) > V_\theta \text{ then } &\begin{cases} v \leftarrow V_r \text{ reset the membrane voltage} \\ \text{record a spike at time } t \end{cases} \end{aligned} \tag{1}$$

with the following parameters:

$$\begin{aligned} R_m &= 10\ \text{M}\Omega && \text{membrane resistance} \\ C_m &= 1\ \text{nF} && \text{membrane capacitance} \\ \tau_m &= R_m C_m = 10\ \text{ms} && \text{membrane time constant} \\ E_L &= -70\ \text{mV} && \text{leak reversal (resting) voltage} \\ V_r &= -70\ \text{mV} && \text{reset voltage} \\ V_\theta &= -40\ \text{mV} && \text{threshold voltage} \\ I_e &= 3.1\ \text{nA} && \text{externally applied current} \\ \tau_{\text{ref}} &= 5\ \text{ms} && \text{absolute refractory period} \end{aligned} \tag{2}$$

1. **Forward Euler:** Write computer code to simulate this LIF neuron using Euler's (forward) method with timestep $\Delta t = 1$ ms for $T = 1$ second. Plot the voltage as a function of time. You do not need to plot spikes: once membrane potential exceeds threshold, simply set the membrane potential to V_r.
2. **Forward Exponential Euler:** Calculate v_∞, the steady-state membrane voltage assuming the input I_e remains constant and ignoring the spiking dynamics. Implement the forward exponential Euler method by solving analytically for the decay of $v(t)$ toward v_∞ on each Δt width time-step. Is the resulting solution the same as forward Euler? How small do you need to make Δt to get the same number of spikes from both approaches?
3. **Euler-Maruyama:** Starting from your forward Euler solution in part 1, and using $\Delta t = 1$ ms, simulate your model with additive Gaussian white noise with variance $\sigma^2 = 1$ mV^2/ms on each time step. Assuming you've expressed your simulation in units of mV for voltage and ms for time, this should be as simple as adding a zero-mean, unit-variance, Gaussian random number to the voltage on each time step. Play around with I_e and σ^2. Does noise change the the neuron's response to various steady currents I_e?

Optional Bonus:

Consider the Quadratic Integrate and Fire (QIF) neuron with dynamics $\dot{\tau}_m v = -v(1-v) + R_m I$. This model "spikes" by blowing up to $+\infty$ (and "resetting" through $-\infty$) in finite time. In practice, numerical simulations set some finite positive voltage for the threshold, and resets to rest (Choose these thresholds at your discretion). [1]

1. Simulate the QIF model using forward Euler.
2. What features can this model capture that are missing in the LIF neuron?

[1] You can also change coordinates map this "voltage" to a circular variable θ, with θ passing through π corresponding to the 'spike' up to and through $+\infty$ and back around from $-\infty$, allowing you to simulate the equations without choosing arbitrary finite thresholds, but this is beyond the scope of this course.