UNIVERSITY OF BRISTOL

Fall 2024 Topics in CS Question

SEMT30003/4
Computational Neuroscience

TIME ALLOWED:
1 hour

This paper contains one in-depth question with three sections that contain multiple parts. **All** components must be attempted and will be used for assessment. The maximum for this section is 50 marks.

Word counts are not mandatory, but rather hints to help you avoid spending more time than needed. You may exceed the advised word count if you wish. You may also use diagrams and mathematics as appropriate in short answer questions.

Please write your student number and/or username on the line below.
(To ensure blinded marking, do not write your name)

____

Other instructions:

- Only approved non-programmable calculators are permitted
- Write your answers in the question paper
- Blank pages are provided at the end if you need extra space. Clearly label which answer you are writing.

(a) [6 marks] General knowledge: (3–9 minutes, 1–3 minutes per question)

i. What do neuroscientists mean when they say "rate coding"? Describe a scenario where rate coding might be a *poor* model of neuronal activity (hint: what information in spiking codes does rate coding ignore?). (≈20–90 words) **[2]**

ii. Where in the human brain can one find neurons that respond to visual input, with receptive fields that detect oriented edges and exhibit phase invariance ("complex cells")? (≈1–5 words) **[2]**

iii. Briefly define *sparsity* in the context of population coding, and discuss its role either in encoding visual information in the retina, or encoding memories in hippocampus. (≈30–90 words) **[2]**

(b) [22 marks] Leaky integrate-and-fire dynamics: (10–20 minutes)

Consider a Leaky Integrate-and-Fire (LIF) neuron obeying the following dynamics:

$$\begin{aligned} &C\dot{v} = \tfrac{1}{R}(E_\ell - v) + I \\ &\text{if } v(t) > v_{\text{threshold}} \text{ then } v(t) \leftarrow v_{\text{reset}} \text{ and emit a spike} \end{aligned} \tag{1}$$

Assume that $v_{\text{reset}} = E_\ell$.

i. Discuss the physiological interpretation of C, R, E_ℓ, I, and $v_{\text{threshold}}$, and how this model approximates the ionic currents across the neuronal membrane during the action potential. (address all 5 variables ≈30–90 words) **[5]**

ii. If we define

$$\begin{aligned} \tau &= RC \\ v_\infty &= E_\ell + RI \end{aligned} \tag{2}$$

we can write the solution for (1) *ignoring the spiking reset* as

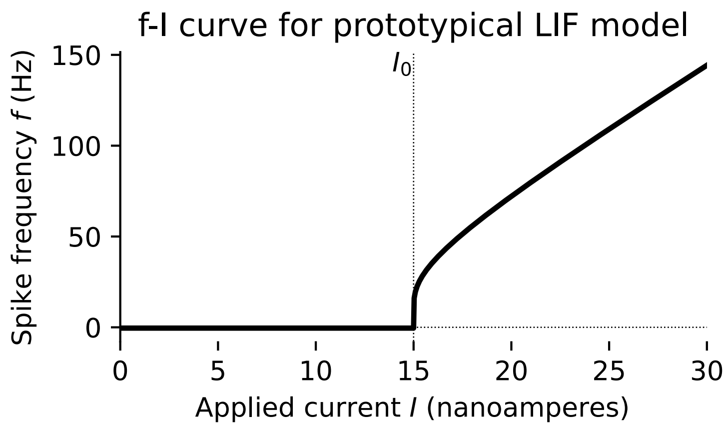
$$v(t) = e^{-t/\tau}(v(0) - v_\infty) + v_\infty \tag{3}$$

Assume that a constant current is applied to the LIF model, leading to rhythmic spiking. Derive an expression for the time "T" between consecutive spikes as a function of I, v_∞, and τ[1]. **[7]**

iii.α Below is a plot of firing frequency as a function of injected current. The LIF model has a hard threshold: the model abruptly starts spiking above a certain minimum current (labelled I_0 below). Derive an expression for I_0 in terms of v_∞, C, R. **[5]**

Note: Although this plot has units, you do not need to know the values of the model parameters here. No calculation is required and you may leave your answer in terms of v_∞, C, R.

[1] Hint: After a spike the membrane voltage starts at $v(0) = v_{\text{reset}}$, and then rises to $v(t) = v_{\text{threshold}}$ at some unknown future time T; Solve for this T in the solution for the voltage provided in Equation(3).



iii.β State whether you'd expect I_0 to increase or decrease if R is *decreased*. Would this make it easier or harder for incoming excitatory synapses to cause this neuron to fire? Briefly speculate about what such a change might correspond to in a binary perceptron, whose output is given by $y = \Theta(\boldsymbol{w}^\top \boldsymbol{x} - b)$. (≈10–20 words) **[5]**

(c) [22 marks] The Hodgkin–Huxley Model: (10–20 minutes)
The voltage in the Hodgkin Huxley model of the action potential evolves according to the following differential equations:

$$\begin{aligned} C\dot{v} &= g_\ell(E_\ell - v) + \bar{g}_{\mathrm{K}} n^4 (E_{\mathrm{K}} - v) + \bar{g}_{\mathrm{Na}} m^3 h (E_{\mathrm{Na}} - v) + I \\ \dot{n} &= \alpha_n(v) \cdot (1 - n) - \beta_n(v) \cdot n \\ \dot{m} &= \alpha_m(v) \cdot (1 - m) - \beta_m(v) \cdot m \\ \dot{h} &= \alpha_h(v) \cdot (1 - h) - \beta_h(v) \cdot h. \end{aligned} \tag{4}$$

You do not need to know the specific form of the nonlinear functions $\alpha_{\dots}(v)$ and $\beta_{\dots}(v)$ for this question.

i. What part of the action potential does the gating variable n on the potassium conductance capture? (≈5–10 words) **[2]**

ii. Solving first-order ODEs:[2] Assume that v is constant (so that $\alpha_m(v)$ and $\beta_m(v)$ are also constants). Find an expression for the steady-state value for the gating variable m, denoted as $m_\infty = \lim_{t\to\infty} m(t)$. **[10]**

iii. Assume that α_m and β_m are constants. Re-write the differential equation for the sodium gating variable $\dot{m} = \alpha \cdot (1 - m) - \beta \cdot m$ in the form

$$\tau \dot{m} = m_\infty - m \tag{5}$$

Give an expression for τ and m_∞ in terms of α_m and β_m.[3]

[10]

[2]Hints for (ii): You may assume that $\alpha + \beta > 0$, so that the ODE for m is stable; You do not *need* to integrate the ODE for $\dot{m}$ to find this; A step-by-step algebraic transformation of expressions is fine here.

[3]Hints for (iii): A step-by-step algebraic transformation of expressions is fine here and you may omit the subscripts in α_m and β_m for convenience.