Here are some exam-style ODE problems to prepare you for the style of question you might see on the final exam.

Note: Some solutions use the integrating factor to solve first-order ODEs, because this was historically taught in this course. This year, showing a given solution is correct by differentiating to check will be sufficient. Students are welcome to use the integrating factor approach if they prefer it.

Q1: Consider the ODE

$$\dot{v} = g \cdot (D - v) + B, \tag{1}$$

where g, D, and B are constants. Show that

$$v(t) = \alpha(t) \cdot v_0 + [1 - \alpha(t)] \cdot v_\infty \tag{2}$$

is a solution to ODE (1) with initial conditions $v(t = 0) = v_0$, where

$$\begin{aligned} \alpha(t) &= e^{-gt} \\ v_\infty &= D + B/g \end{aligned} \tag{3}$$

(Any approach permitted).

Q2: Provide the solution to $\tau\dot{v} = c - v$ and evaluate $v(t)$ at time $t = 10$ for $c = 0$, $\tau = 10$, and $v(0) = -70$.

Q3:

You are asked to write computer code to numerically integrate a linear first-order ODE for a neuron's membrane voltage v, being driven by a known time-varying input $u(t)$

$$C\dot{v} = \tfrac{1}{R}(E - v) + u(t), \tag{4}$$

where C, R, and E are constants.

Q3a: (facts/trivia) Is forward Euler a first or second-order method? **Q3b:** (facts/trivia) What is the order of the error in forward Euler in terms of the time step Δt? (What about over a fixed time interval T?) **Q3c:** You're told that the signal $u(t)$ varies slowly, and is approximately constant $u(t) \approx u_t$ over a short duration between t and $t+\Delta t$, where $\Delta t = 1$ ms. You implement the following (pseudo)code for *forward exponential Euler* to integrate from $t = 0$ to $t = 1000$ ms based on this assumption:

$$\begin{aligned}
&\gamma = ???\\
&v = v_0\\
&\text{for } t \text{ in } 1 \ldots 1000 :\\
&\qquad v_\infty = ???\\
&\qquad \Delta_v = \gamma(v_\infty - v_t)\\
&\qquad v_{t+1} = v_t + \Delta_v
\end{aligned} \tag{5}$$

What expressions belong in the ??? for variables γ and v_∞?