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# Optional Logistic Regression: Gradient

**This is an optional reading where I explain gradient descent in more detail. Remember, previously I gave you the gradient update step, but did not explicitly explain what is going on behind the scenes.**

The general form of gradient descent is defined as:

$$\begin{aligned} & \text{Repeat } \{ \\ & \theta_j := \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta) \\ & \} \end{aligned}$$

For all  $j$ . We can work out the derivative part using calculus to get:

$$\begin{aligned} & \text{Repeat } \{ \\ & \theta_j := \theta_j - \frac{\alpha}{m} \sum_{i=1}^m (h(x^{(i)}, \theta) - y^{(i)}) x_j^{(i)} \\ & \} \end{aligned}$$

A vectorized implementation is:

$$\theta := \theta - \frac{\alpha}{m} X^T (H(X, \theta) - Y)$$

## Partial derivative of $J(\theta)$

First calculate derivative of sigmoid function (it will be useful while finding partial derivative of  $J(\theta)$ ):

$$\begin{aligned} h(x)' &= \left( \frac{1}{1 + e^{-x}} \right)' = \frac{-(1 + e^{-x})'}{(1 + e^{-x})^2} = \frac{-1' - (e^{-x})'}{(1 + e^{-x})^2} = \frac{0 - (-x)'(e^{-x})}{(1 + e^{-x})^2} = \frac{-(-1)(e^{-x})}{(1 + e^{-x})^2} \\ &= \left( \frac{1}{1 + e^{-x}} \right) \left( \frac{e^{-x}}{1 + e^{-x}} \right) = h(x) \left( \frac{1 + e^{-x} - 1 + e^{-x}}{1 + e^{-x}} \right) = h(x) \left( \frac{1 + e^{-x}}{1 + e^{-x}} - \frac{1}{1 + e^{-x}} \right) \end{aligned}$$