The Rotation Matrix (Optional Reading)

Counterclockwise Rotation

If you want to rotate a vector r with coordinates (x, y) and angle α counterclockwise over an angle β to get vector r' with coordinates $(x$', y') then the following holds:

$$x = r * cos(\alpha)$$

$$y = r * sin(\alpha)$$

$$x' = r' * cos(\alpha + \beta)$$

$$y' = r' * sin(\alpha + \beta)$$

Trigonometric addition gives us:

$$cos(\alpha + \beta) = cos(\alpha)cos(\beta) - sin(\alpha)sin(\beta)$$

$$sin(\alpha + \beta) = cos(\alpha)sin(\beta) + sin(\alpha)cos(\beta)$$

For proof, see this Wikipedia page section.

As the length of the vector stays the same,

$$x' = r * cos(\alpha)cos(\beta) - r * sin(\alpha)sin(\beta)$$

$$y' = r * cos(\alpha)sin(\beta) + r * sin(\alpha)cos(\beta)$$

This equates to:

$$x' = x * cos(\beta) - y * sin(\beta)$$

$$y' = x * sin(\beta) + y * cos(\beta)$$