



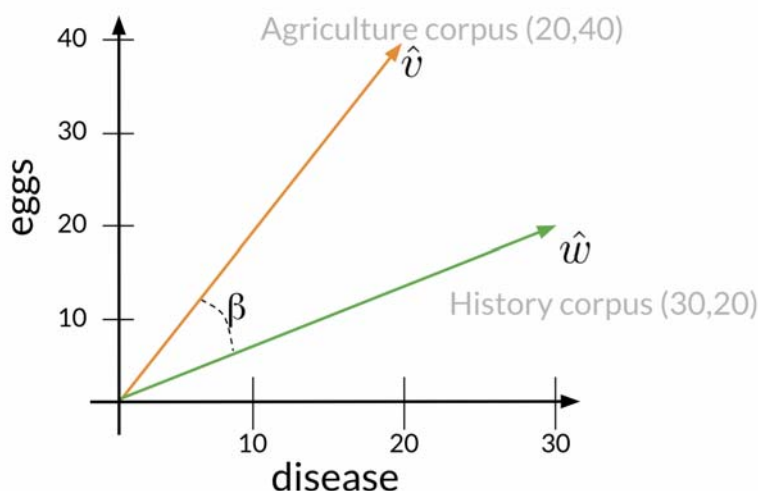

Cosine Similarity

Before getting into the cosine similarity function remember that the **norm** of a vector is defined as:

$$\|\vec{v}\| = \sqrt{\sum_{i=1}^n |v_i|^2}$$

The **dot product** is then defined as:

$$\vec{v} \cdot \vec{w} = \sum_{i=1}^n v_i \cdot w_i$$



$$\hat{v} \cdot \hat{w} = \|\hat{v}\| \|\hat{w}\| \cos(\beta)$$

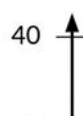
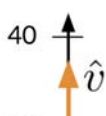
$$\cos(\beta) = \frac{\hat{v} \cdot \hat{w}}{\|\hat{v}\| \|\hat{w}\|}$$

$$= \frac{(20 \times 30) + (40 \times 20)}{\sqrt{20^2 + 40^2} \times \sqrt{30^2 + 20^2}} = 0.87$$

The following cosine similarity equation makes sense:

$$\cos(\beta) = \frac{\hat{v} \cdot \hat{w}}{\|\hat{v}\| \|\hat{w}\|}$$

If \hat{v} and \hat{w} are the same then you get the numerator to be equal to the denominator. Hence $\beta = 0$. On the other hand, the dot product of two orthogonal (perpendicular) vectors is 0. That takes place when $\beta = 90$.



$\beta = 0^\circ$

