



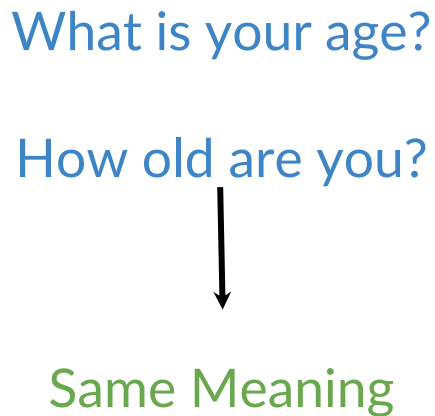
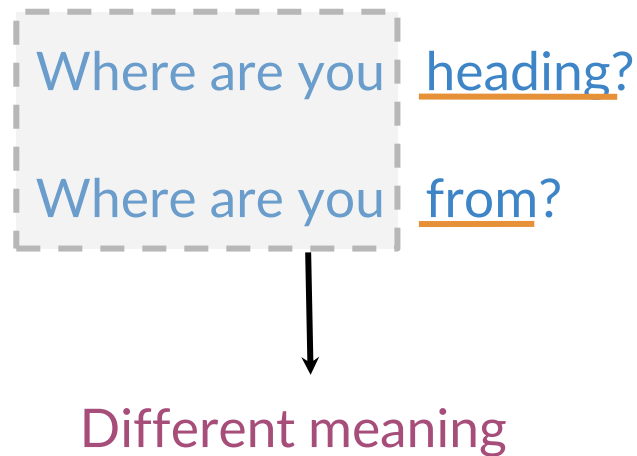
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Vector Space Models

Outline

- Vector space models
- Advantages
- Applications

Why learn vector space models?



Vector space models applications

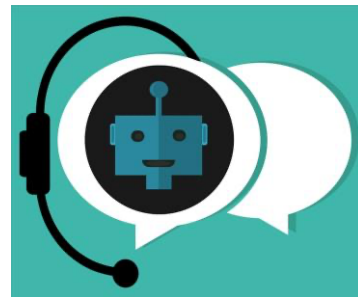
- You eat cereal from a bowl
- You buy something and someone else sells it



Information Extraction



Machine Translation



Chatbots

Fundamental concept

“You shall know a word by the company it keeps”

Firth, 1957



(Firth, J. R. 1957:11)

Summary

- Represent words and documents as **vectors**
- Representation that **captures** relative **meaning**



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Word by Word and Word by Doc.

Outline

- Co-occurrence \longrightarrow Vector representation
- Relationships between words/documents

Word by Word Design

Number of times they *occur together within a certain distance* k

I like simple data

I prefer simple raw data

$k=2$

	simple	raw	like	l
data	2	1	1	0

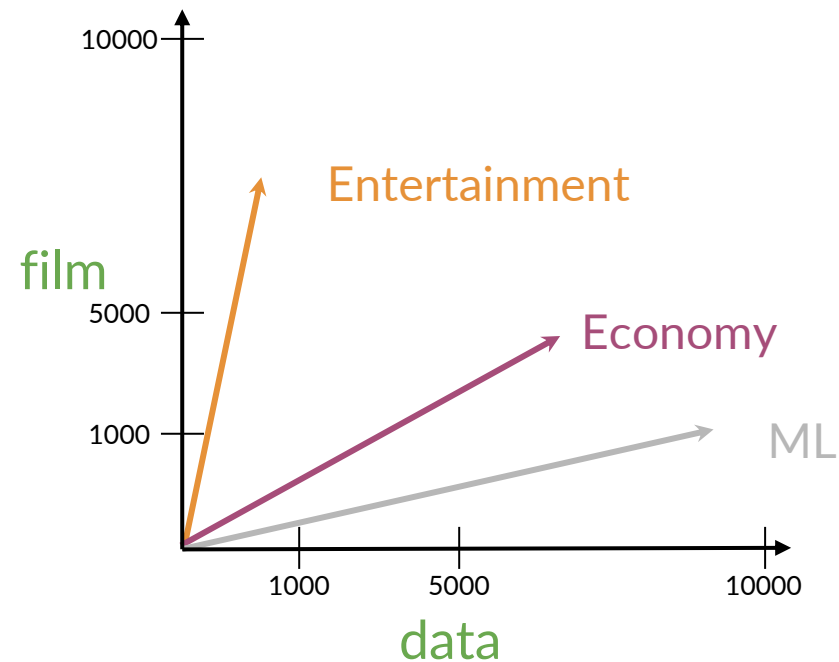
n

Word by Document Design

Number of times a word *occurs within a certain category*

	Corpus		
	Entertainment	Economy	Machine Learning
data	500	6620	9320
film	7000	4000	1000

Vector Space



	Entertainment	Economy	ML
data	500	6620	9320
film	7000	4000	1000

Measures of “similarity:”
Angle
Distance

Summary

- W/W and W/D, *counts* of occurrence
- Vector Spaces \longrightarrow Similarity between words/documents



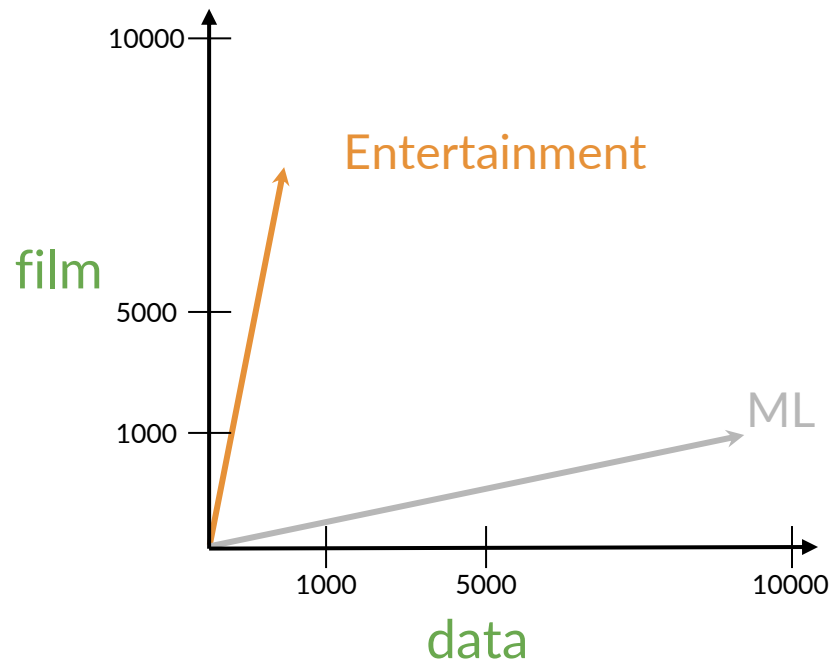
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Euclidean Distance

Outline

- Euclidean distance
- N-dimension vector representations comparison

Euclidean distance

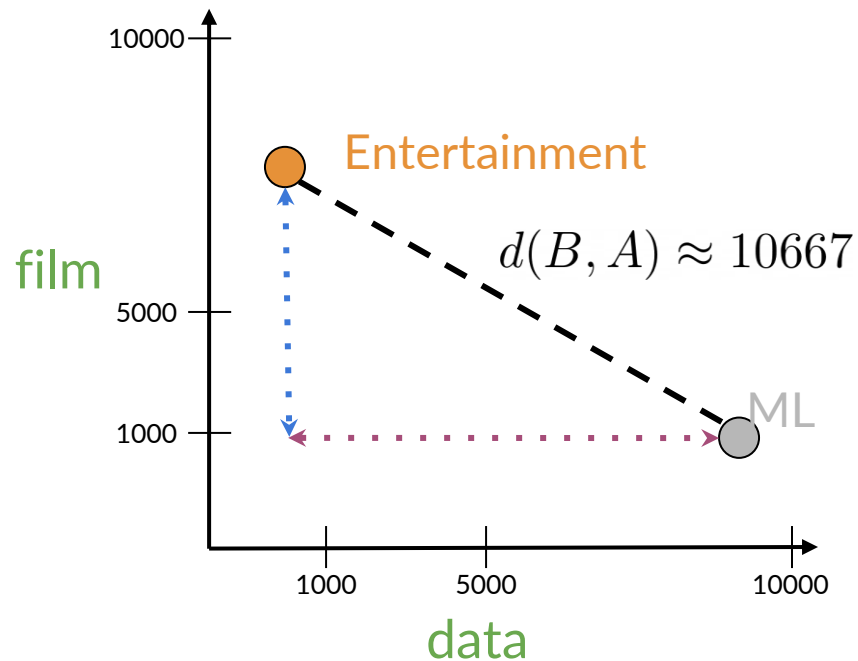


Corpus **A**: (500,7000)



Corpus **B**: (9320,1000)

Euclidean distance



Corpus **A**: (500,7000)



Corpus **B**: (9320,1000)

$$d(B, A) = \sqrt{\underbrace{(B_1 - A_1)^2}_{\text{purple}}} + \underbrace{(B_2 - A_2)^2}_{\text{blue}}$$
$$c^2 = a^2 + b^2$$

$$d(B, A) = \sqrt{(-8820)^2 + (6000)^2}$$

Euclidean distance for n-dimensional vectors

		\vec{w}	\vec{v}
	data	boba	ice-cream
AI	6	0	1
drinks	0	4	6
food	0	6	8

$$= \sqrt{(1 - 0)^2 + (6 - 4)^2 + (8 - 6)^2}$$

$$= \sqrt{1 + 4 + 4} = \sqrt{9} = 3$$

$$d(\vec{v}, \vec{w}) = \sqrt{\sum_{i=1}^n (v_i - w_i)^2} \longrightarrow \text{Norm of } (\vec{v} - \vec{w})$$

Euclidean distance in Python

```
# Create numpy vectors v and w
v = np.array([1, 6, 8])
w = np.array([0, 4, 6])

# Calculate the Euclidean distance d
d = np.linalg.norm(v-w)
# Print the result
print("The Euclidean distance between v and w is: ", d)
```

The Euclidean distance between v and w is: 3

Summary

- Straight line between points
- Norm of the difference between vectors



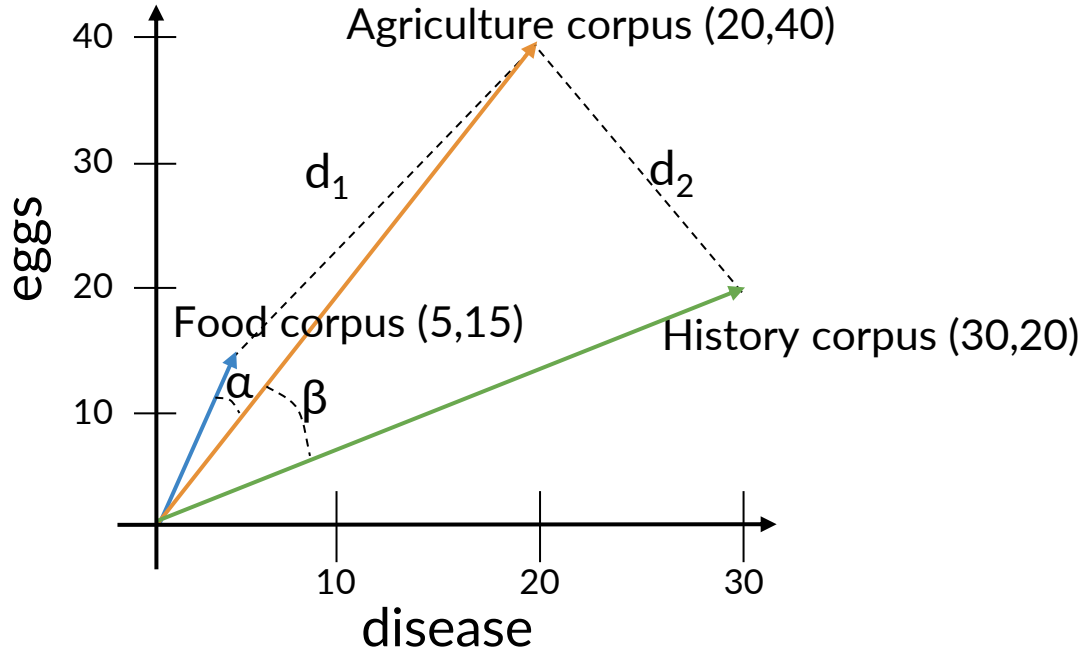
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Cosine Similarity: Intuition

Outline

- Problems with Euclidean Distance
- Cosine similarity

Euclidean distance vs Cosine similarity



Euclidean distance: $d_2 < d_1$

Angles comparison: $\beta > \alpha$

The cosine of the angle between the vectors

Summary

- Cosine similarity when corpora are different sizes



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Cosine Similarity

Outline

- How to get the cosine of the angle between two vectors
- Relation of this metric to similarity

Previous definitions

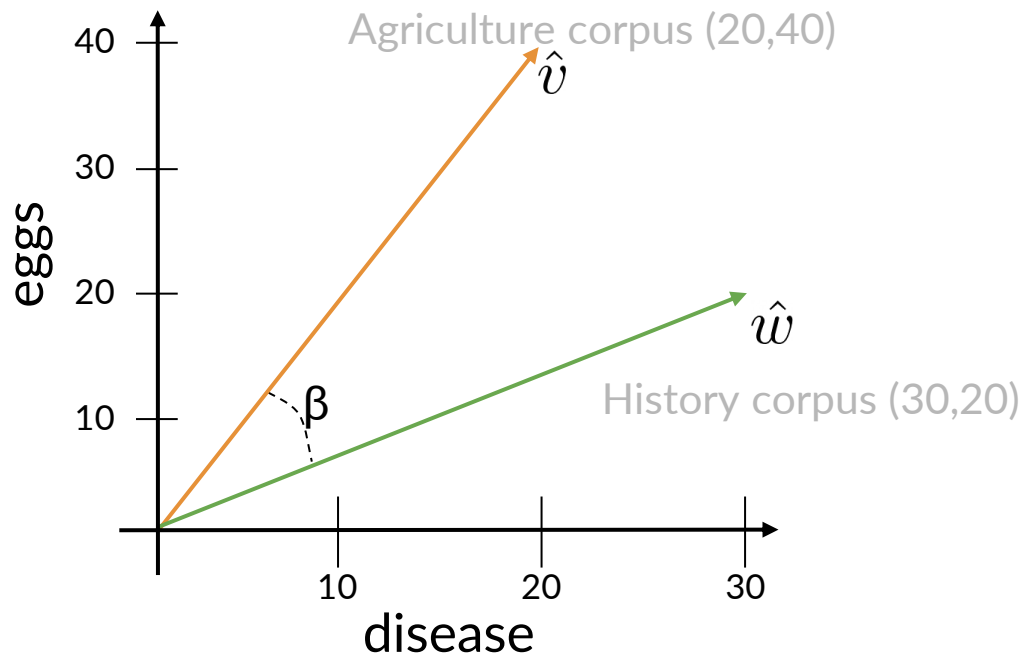
Vector norm

$$\|\vec{v}\| = \sqrt{\sum_{i=1}^n v_i^2}$$

Dot product

$$\vec{v} \cdot \vec{w} = \sum_{i=1}^n v_i \cdot w_i$$

Cosine Similarity

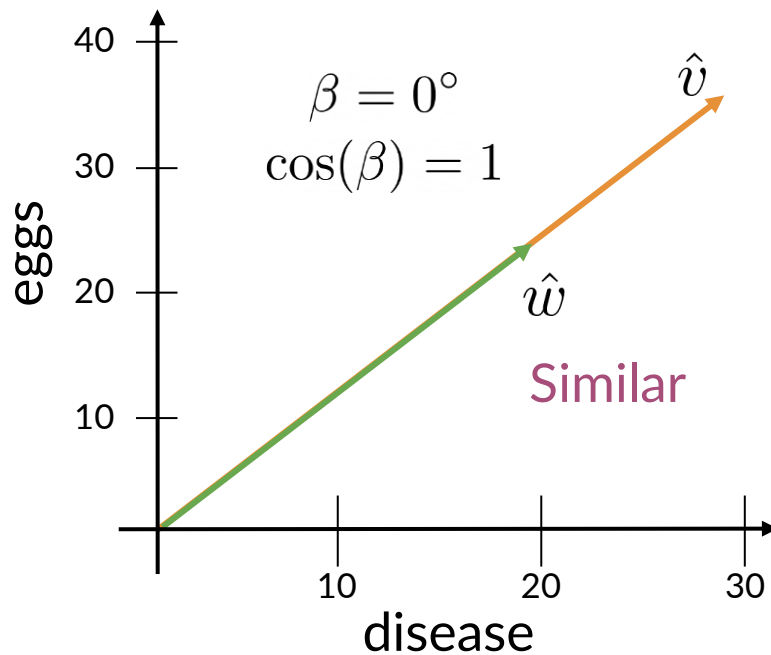
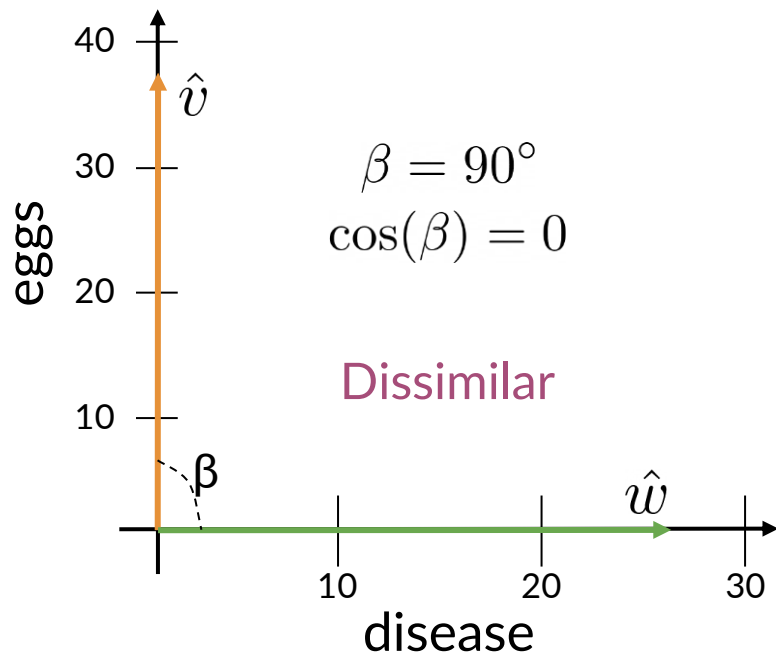


$$\hat{v} \cdot \hat{w} = \|\hat{v}\| \|\hat{w}\| \cos(\beta)$$

$$\cos(\beta) = \frac{\hat{v} \cdot \hat{w}}{\|\hat{v}\| \|\hat{w}\|}$$

$$= \frac{(20 \times 30) + (40 \times 20)}{\sqrt{20^2 + 40^2} \times \sqrt{30^2 + 20^2}} \\ = 0.87$$

Cosine Similarity



Summary

- Cosine \propto Similarity
- Cosine Similarity gives values between 0 and 1



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Manipulating Words in Vector Spaces

Outline

- How to use vector representations

Manipulating word vectors



USA



Washington
DC

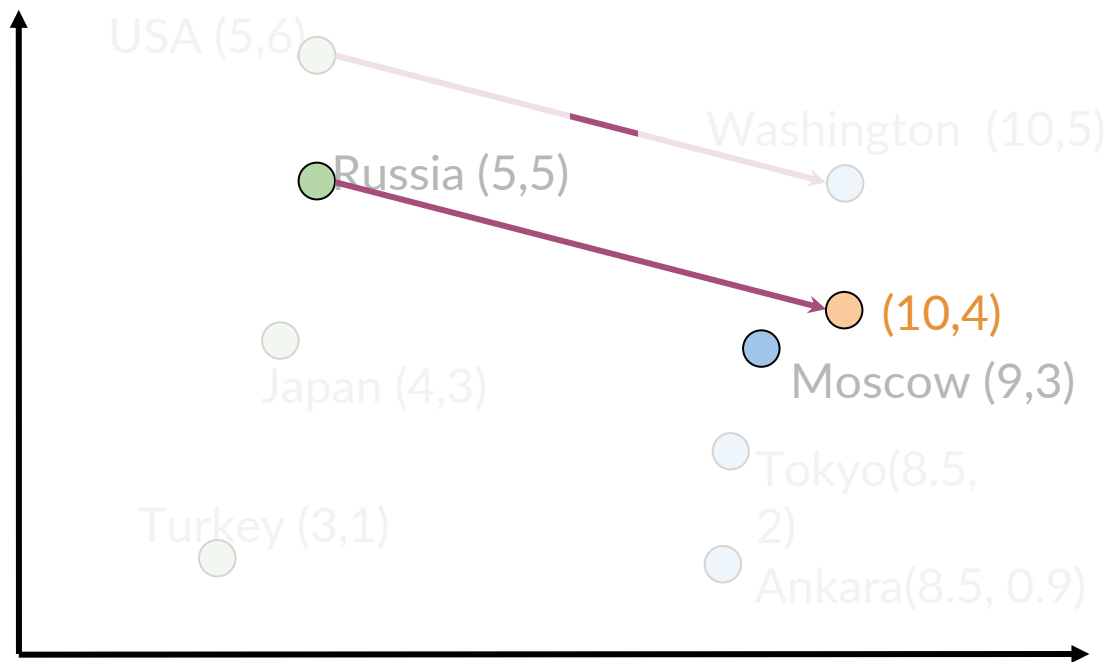


Russia



?

Manipulating word vectors



$$\text{Washington} - \text{USA} = \begin{bmatrix} 5 & -1 \end{bmatrix}$$

$$\text{Russia} + \begin{bmatrix} 5 & -1 \end{bmatrix} = \begin{bmatrix} 10 & 4 \end{bmatrix}$$



Moscow

[Mikolov et al, 2013, Distributed Representations of Words and Phrases and their Compositionality]

Summary

- Use known relationships to make predictions



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Visualization and PCA

Outline

- Some motivation for visualization
- Principal Component Analysis

Visualization of word vectors

$d > 2$

oil	0.20	...	0.10
gas	2.10	...	3.40
city	9.30	...	52.1
town	6.20	...	34.3

How can you visualize if your representation captures these relationships?



oil & gas

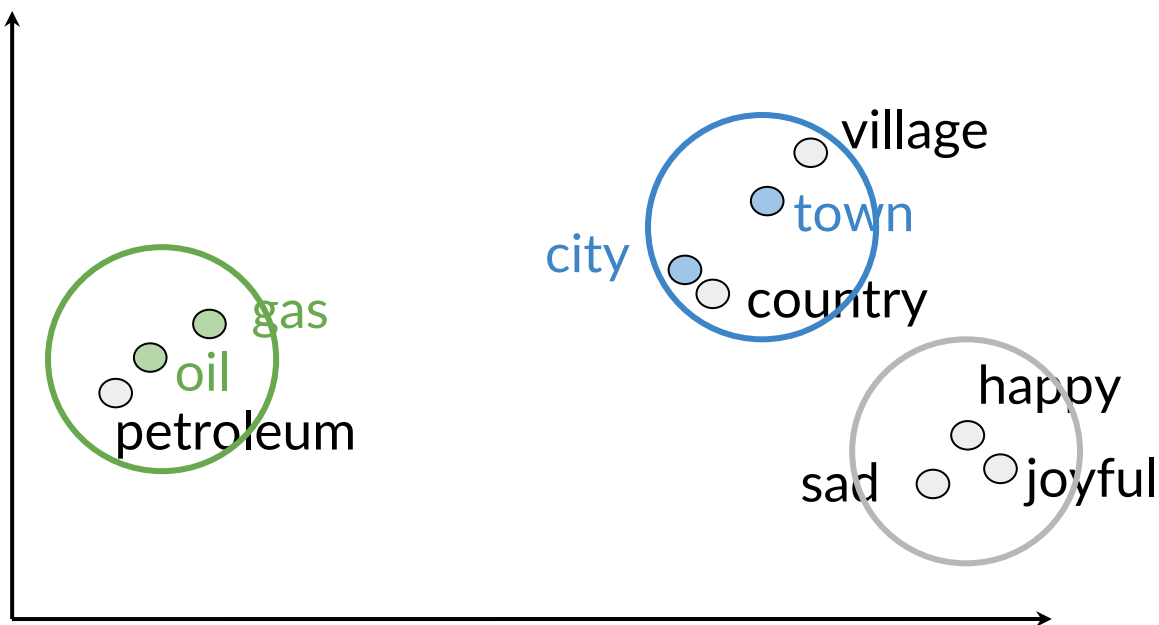


town & city

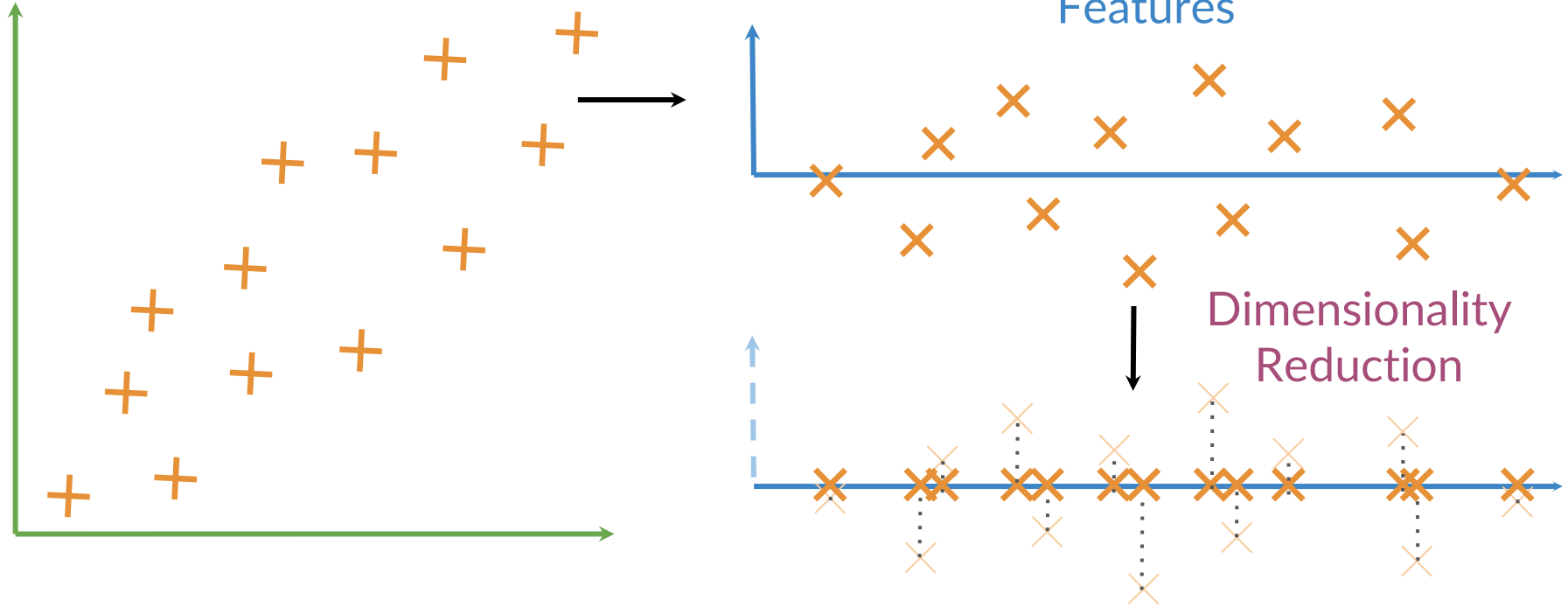
Visualization of word vectors

	$d > 2$				$d = 2$	
oil	0.20	...	0.10	PCA →	oil	2.30 21.2
gas	2.10	...	3.40		gas	1.56 19.3
city	9.30	...	52.1		city	13.4 34.1
town	6.20	...	34.3		town	15.6 29.8



Visualization of word vectors



Principal Component Analysis



Summary

- Original Space  Uncorrelated features  Dimension reduction
- Visualization to see words relationships in the vector space



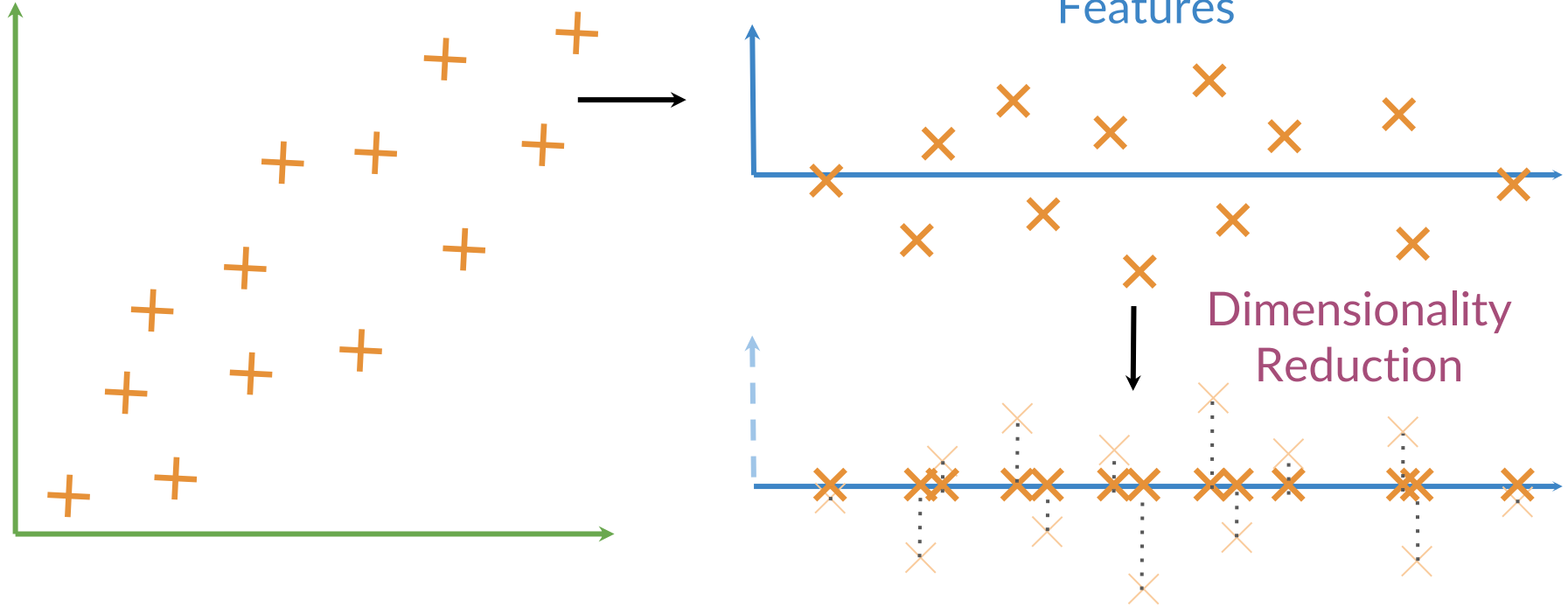
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PCA Algorithm

Outline

- How to get uncorrelated features
- How to reduce dimensions while retaining as much information as possible

Principal Component Analysis

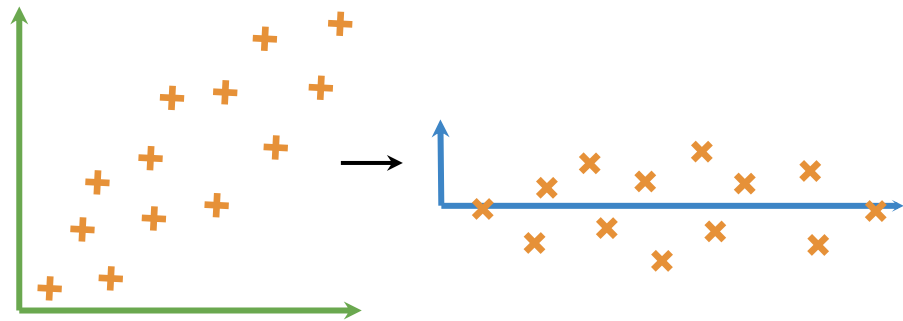


PCA algorithm

Eigenvector: Uncorrelated features for your data

Eigenvalue: the amount of information retained by each feature

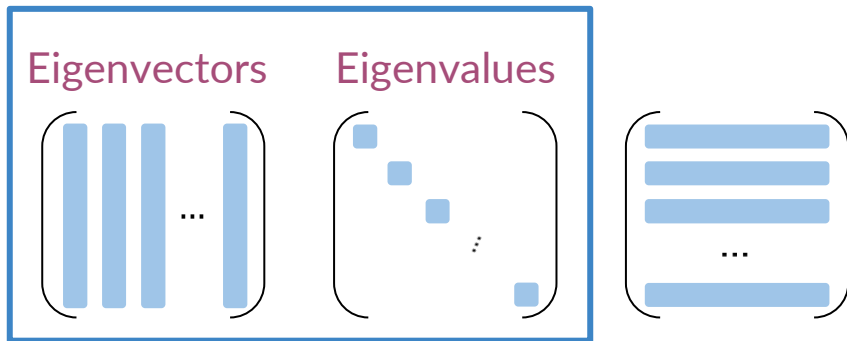
PCA algorithm



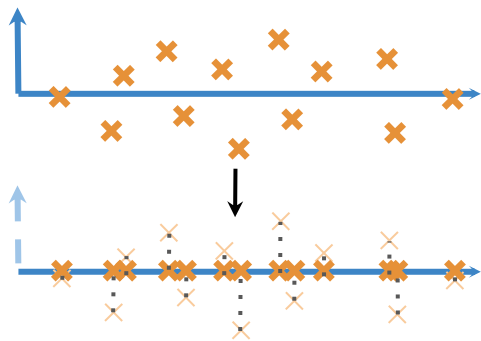
Mean Normalize Data $x_i = \frac{x_i - \mu_{x_i}}{\sigma_{x_i}}$

Get Covariance Matrix Σ

Perform SVD $\text{SVD}(\Sigma)$



PCA algorithm



Eigenvectors

$$\begin{pmatrix} | & | & | & | \\ U & & & \\ | & | & | & | \end{pmatrix}$$

Eigenvalues

$$\begin{pmatrix} \blacksquare & & & \\ & \blacksquare & & \\ & & \blacksquare & \\ & & & \blacksquare \\ & & & & S \\ & & & & & \ddots \\ & & & & & & \blacksquare \end{pmatrix}$$

Dot Product to
Project Data

$$X' = XU[:, 0 : 2]$$

Percentage of
Retained Variance

$$\frac{\sum_{i=0}^1 S_{ii}}{\sum_{j=0}^d S_{jj}}$$

Summary

- Eigenvectors give the direction of uncorrelated features
- Eigenvalues are the variance of the new features
- Dot product gives the projection on uncorrelated features