

Coordination of resilience interventions by proactive consumers and the supplying utility

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ABSTRACT

Pursuing resilience enhancement, we propose a hierarchical decision-making framework for proactive consumers and the supplying utility interacting through an electricity distribution system. Such decision framework enables identifying solutions that are optimal for both the leading partner, the electric utility, and the other agents, proactive consumers and non-proactive consumers, that react to the decisions of the leading agent. Resilience improvement interventions include installing batteries and undergrounding distribution lines. The proposed technique can be helpful to distribution system planners, who can use it to optimally coordinate the resilience enhancement investments by the electric utility and proactive consumers. We analyze cases pertaining to realistic distribution systems.

1. Notation

1.1. Set and indexes

I	set of distribution buses, indexed by i
I^{BC}	set of buses with proactive consumers
I^{Sub}	set of buses connected to the substation (typically 1 or 2)
L	set of distribution lines, indexed by l
$r(l)$	receiving-end bus of line l
$s(l)$	sending-end bus of line l
S	set of fault scenarios, indexed by s
Ω_i	set of lines directly connected to bus i

1.2. Variables

$p_{i,s}^{BC}$	power exchange with the proactive consumer at bus i during fault scenario s (kW)
$p_{i,s}^B$	battery discharging power at bus i during fault scenario s (kW)

$p_{i,s}^{B'}$	battery discharging power of proactive consumer at bus i during fault scenario s (kW)
$p_{i,s}^U$	unserved power at bus i during fault scenario s (kW)
$p_{i,s}^{U'}$	unserved power of proactive consumer at bus i during fault scenario s (kW)
\bar{p}_i^B	battery discharging power capacity at bus i (kW)
$\bar{p}_i^{B'}$	battery discharging power capacity of proactive consumer at bus i (kW)
$p_{l,s}^F$	power flow through distribution line l during fault scenario s (kW)
$p_{i,s}^{\text{Sub}}$	power exchange with the substation at bus i during fault scenario s (kW)

1.3. Binary variables

x_i	binary variable indicating whether a battery is installed at bus i (1) or not (0)
z_l	binary variable indicating whether a section of line l is made underground (1) or not (0)

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1.4. Dual variables

$\alpha_{i,s}, \bar{\alpha}_{i,s}$	dual variables associated with the lower/upper bound of the power output of the battery at bus i during fault scenario s
$\gamma_{i,s}, \bar{\gamma}_{i,s}$	dual variables associated with the lower/upper bound of the unserved power at bus i during fault scenario s
μ_i	dual variable associated with the nodal power balance equation at bus i during fault scenario s
χ_i	dual variable associated with the lower bound of the power capacity of the battery at bus i

1.5. Constants

c^B	annualized per unit investment cost of any battery (\$/kW)
C^{Budget}	total investment budget (\$)
c^F	annualized per unit investment of undergrounding a section of equal length of any line (\$)
c^{UE}	cost of unserved energy (\$/kWh)
$d_{l,s}$	binary constant indicating if line l is disabled (1) or not (0) during fault scenario s
f_s	annual frequency (forecast or historical) of fault scenario s (times)
p_i^D	power demand at bus i (kW)
$p_i^{D'}$	power demand of the proactive consumer at bus i (kW)
p_l^L, p_l^U	lower/upper bound of the power flow of line l
t_o	fault duration (h)

2. Introduction

In recent years we have witnessed an increasing number of disastrous power outages. The 2021 Texas Winter Storm caused power outages to over four million houses [1]. In 2020, Hurricane Isaias hit the East Coast of the US and made over two million customers lose access to power [2]. The annual power outages caused by extreme weather has doubled in the US during the past two decades, and hurricanes alone are accountable for over \$900 billion economic losses in the US [3]. The concerns over the increasing number of power outages are further aggravated due to an aging power grid and the rapid replacement of traditional power plants by intermittent renewable units [4].

In this context, both the supplying utility of an electric distribution system and the different consumers connected to that system seek high supply resilience. As stated by the US National Academies [5], the supplying utility has a regulatory responsibility to provide resilient and reliable power supply while investing in the infrastructure to position itself for the future power systems; and consumers invest in household resilience enhancements in order to ensure reliable daily power supply in normal and contingency conditions [6,7].

That is, on one hand, the supplying utility seeks to improve the supply resilience of the distribution system as a whole. In fact, a number of utility resilience enhancement planning methods have been proposed in the literature. Ref. [8] carries out a resilience-oriented distribution expansion planning. Ref. [9] considers distribution resilience under extreme events. Ref. [10] considers distribution resilience in coordination with different energy sources. In [11], a distribution system approach is proposed to harden high-risk distribution power lines. In [12], a heuristic approach is developed to harden the long-term resilience of a distribution system. In [13], a placement approach for storage and PV units is developed to enhance system resilience. In [14], a switch placement technique is developed to facilitate distribution system restoration.

On the other hand, customers seek to improve their individual supply resilience. An increasing number of studies pertaining to behind-the-meter distributed generators and batteries are available [15]. In [16], a distributed generator and back-up storage co-siting solution is proposed to reduce power outages impact on a household. In [17], a battery sizing approach is developed to place batteries in residential houses to ensure power supply in medical emergencies. In [18], the authors propose a planning methodology to integrate batteries and hydrogen vehicles in residential households.

Considering the above and seeking resilience improvement regarding both extreme events and non-extreme events, a natural trade-off arises: if the supplying utility achieves a high level of resilience for the whole distribution system by carrying out appropriate investments, individual consumers have no incentive to invest to improve their individual resilience levels. Conversely, if the supplying utility does not provide an appropriate resilience level for the distribution system as a whole, individual consumers generally have a strong incentive to invest to improve their individual resilience levels.

To model this trade-off, we assume that the supplying utility acts as the leader [19] as it has played such role historically and, as a consequence, proactive consumers react to the actions (or lack of actions) of the supplying utility. We also assume that the resilience level required by each consumer is different, ranging from consumers that require a high resilience level to others that require a low one.

Within this hierarchical framework [20,21], the supplying utility, constrained by a budget, *acts first* to identify the optimal investments to maximize the resilience of the distribution system as a whole [22,23], and then, each individual proactive consumer, constrained by its own budget, *reacts* to the investments of the utility with its own optimal investment [24] to maximize its own individual resilience. This is illustrated in Fig. 1. Since the action of the supplying utility conditions the actions of the individual consumers and, vice versa, the actions of the consumers depend on the actions of the supplying utility, the resulting hierarchical model is a bi-level optimization problem. Such problem can generally be converted into a single-level optimization problem and solved using state-of-the-art solvers [25,26].

Solving this resilience problem allows identifying the simultaneous optima regarding resilience investments by the supplying utility and by each of the consumers in the electricity distribution system. This means that the utility and all individual consumers, within their respective budgets, do their best in terms of resilience, leading to a social optimum [27,28].

Specifically, the options to improve the resilience level for the supplying utility include (i) enabling supply from an alternative substation (supply point), (ii) installing grid-scale batteries, (iii) building distributed generation units at suitable locations throughout the distribution system, and (iv) retrofitting key distribution lines from overhead to underground. On the other hand, resilience improvement options for proactive consumers include (i) installing behind-the-meter batteries and (ii) installing behind-the-meter distributed generation units.

A key benefit of the considered hierarchical approach is that it allows not only determining whether or not an investment is beneficial for either the utility or some (or all) consumers, but also to globally optimize resilience interventions. The proposed technique enables all interested parties to comprehend how to improve the resilience of a distribution system with potential investments from both the supplying utility and the proactive consumers.

In summary, we develop a decision framework to comprehend the interactions between consumers and the supplying utility to enhance resilience. Consumers pursue individual interventions to enhance their own resilience, while the supplying utility has a regulatory mandate to improve the resilience of the distribution system as a whole. Rigorously modeling this utility-consumer interactions allows understanding and identifying the best interventions by both consumers and the supplying utility.

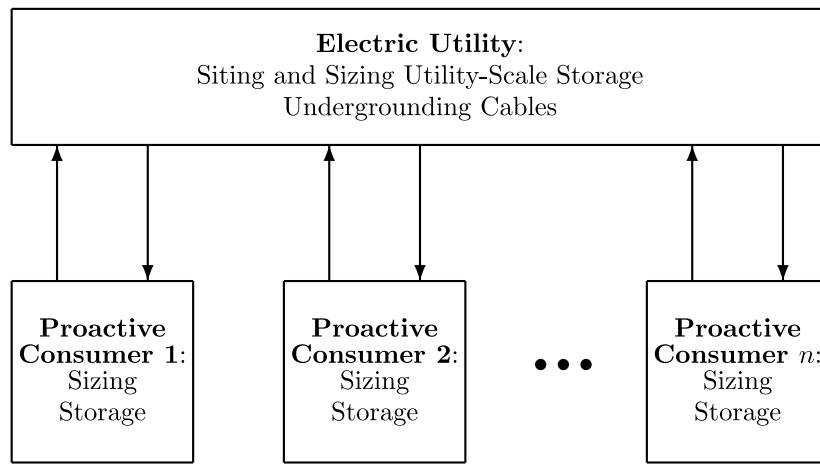


Fig. 1. Structure of the resilience enhancement problem in distribution systems.

The contributions of this work are the following ones:

1. To mathematically describe resilience interventions by the supplying utility and proactive consumers.
2. To develop a decision-making model based on bi-level optimization and mixed-integer linear optimization.
3. To comprehensively analyze the hierarchical interaction (through a hierarchical game model [19]) between the supplying utility and the proactive consumers.
4. To carry out three case studies including an illustrative one, a realistic one based on a suburban distribution network, and a large one to prove the scalability of the proposed technique.

3. Problem formulation

3.1. Problem structure

Considering that x and y represent the investment and operation variables of the utility, and x^i and y^i represent the investment and operation variables of each proactive consumer i , the bi-level problem to be solved has the structure below:

$$\begin{aligned} \min_{x,y,x^i,y^i,\forall i} \quad & f(x,y) \\ \text{s.t.} \quad & h(x,y) = \mathbf{0} : \lambda \\ & g(x,y) \leq \mathbf{0} : \mu \end{aligned} \quad (1a)$$

$$\begin{aligned} \min_{x^i,y^i} \quad & f^i(x,y,x^i,y^i) \\ \text{s.t.} \quad & h^i(x,y,x^i,y^i) = \mathbf{0} : \lambda^i \\ & g^i(x,y,x^i,y^i) \leq \mathbf{0} : \mu^i, \end{aligned} \quad \left. \right\} \forall i, \quad (1b)$$

where (1a) is the investment and operation (under contingencies) upper-level problem for the utility and (1b) are the investment and operation (under contingencies) lower-level problems of the proactive consumers. Functions $f(\cdot)$, $h(\cdot)$ and $g(\cdot)$ are the objective function, equalities and bounds of the utility upper-level problem, respectively. On the other hand, functions $f^i(\cdot)$, $h^i(\cdot)$ and $g^i(\cdot)$ are the objective function, equalities and bounds of the lower-level problem of proactive consumer i , respectively.

Problem (1) is transformed into a single-level one as:

$$\begin{aligned} \min_{x,y,x^i,y^i,\forall i} \quad & f(x,y) \\ \text{s.t.} \quad & h(x,y) = \mathbf{0} : \lambda \\ & g(x,y) \leq \mathbf{0} : \mu \end{aligned} \quad (2a)$$

$$\text{KKT}_i \} \forall i, \quad (2b)$$

where “ KKT_i ” represents the Karush–Kuhn–Tucker conditions of problem (1b) of each proactive consumer i .

The Karush–Kuhn–Tucker conditions are optimality conditions that replace the optimization problem (1b) of each proactive consumer i by a set of equality and inequality constraints. This way, instead of considering the bi-level optimization problem (1), the single-level optimization problem (2) can be solved using a standard optimization solver.

If the problem of each proactive consumer i , (1b), is convex, the single-level problem (2) is fully equivalent to the bi-level one, (1). If, on the other hand, it is non-convex, care should be exercised since multiples solutions may exist of both problem (1b) and its Karush–Kuhn–Tucker optimality conditions.

3.2. Assumptions

We make the following common assumptions:

1. Under contingency conditions (extreme or otherwise), supply from the utility is either non available or just partially available. Under such conditions, local generation facilities are the only means to supply totally or partially the demand. Such local generation facilities are distributed generators (renewable or otherwise) and storage facilities. For the sake of clarity and simplicity, we consider below only storage facilities, specifically electric batteries. Additionally, and for the sake of clarity as well, we consider no alternative substation connections.
2. The supplying utility may perform two resilience enhancement interventions: (1) installing a battery, and (2) converting an overhead line section into an underground cable section.
3. A proactive consumer has available a resilience enhancement solution: Installing a battery at the proactive consumer's location.
4. We consider an unbalanced three-phase distribution system and assume that the three phases have low coupling between each other and that they can be treated independently.
5. The distribution system is equipped with multiple pole-mounted capacitors or regulators to ensure acceptable voltage magnitudes. Therefore, we only consider power in our formulation.
6. The utility historical fault data are available for this planning problem. Based on such information, we can develop a contingency scenario set S whose elements are contingencies of concern. Regarding a contingency scenario, we assume that its occurrence frequency is known a priori, as well as its fault location (e.g., “Line 1 is in fault condition”) and its duration (e.g., the fault lasts from 13:00–15:00).

The model developed under the assumptions above is illustrated in Fig. 1.

3.3. Decision models

We describe below the decision frameworks proposed for the supplying utility and the proactive consumers.

3.3.1. Supplying utility

A battery at the utility level support a number of individual consumers if being islanded due to a fault. On the other hand, underground conversion of feeders highly reduce/virtually eliminate faults along these feeders, drastically improving resilience, but at a high cost.

The decision model for the utility (corresponding to model (1a)) has the form:

$$\min_{\Xi_1} c^{\text{UE}} \sum_{s \in S} \left(f_s t_o \sum_{i \in I} p_{i,s}^U \right) + \sum_{i \in I} c^B \bar{p}_i^B x_i + \sum_{l \in L} c^F z_l \quad (3a)$$

s.t.

$$p_{i,s}^{\text{Sub}} - \sum_{s(l)|l \in \Omega_i} p_{i,s}^F + p_{i,s}^U = p_{i,s}^D \quad \forall i \in I^{\text{Sub}} \quad \forall s \in S \quad (3b)$$

$$p_{i,s}^{\text{BC}} + p_{i,s}^B - \sum_{s(l)|l \in \Omega_i} p_{i,s}^F + \sum_{r(l)|l \in \Omega_i} p_{i,s}^F + p_{i,s}^U = p_{i,s}^D \quad \forall i \in I^{\text{BC}} \quad \forall s \in S \quad (3c)$$

$$p_{i,s}^B - \sum_{s(l)|l \in \Omega_i} p_{i,s}^F + \sum_{r(l)|l \in \Omega_i} p_{i,s}^F + p_{i,s}^U = p_{i,s}^D \quad \forall i \in I \notin [I^{\text{Sub}}, I^{\text{BC}}] \quad \forall s \in S \quad (3d)$$

$$p_{i,s}^U \leq p_{i,s}^D \quad \forall i \in I \quad \forall s \in S \quad (3e)$$

$$0 \leq p_{i,s}^B \leq \bar{p}_i^B x_i \quad \forall i \in I \quad \forall s \in S \quad (3f)$$

$$p_l^F [1 - d_{l,s}(1 - z_l)] \leq p_{i,s}^F \leq \bar{p}_l^F [1 - d_{l,s}(1 - z_l)] \quad \forall l \in L \quad \forall s \in S \quad (3g)$$

$$\sum_{i \in I} c^B \bar{p}_i^B x_i + \sum_{l \in L} c^F z_l \leq C^{\text{Budget}} \quad (3h)$$

$$x_i, z_l \in \{0, 1\} \quad \forall i \in I \quad \forall l \in L, \quad (3i)$$

where

$$\Xi_1 = [p^{\text{Sub}}, p^{\text{BC}}, p^F, p^B, p^U, \bar{p}^B, x, z].$$

The objective function (3a) is the expected total unserved energy cost, based on scenarios, plus investment costs. Constraints (3b), (3c), and (3d) are nodal balance equations for the substation, proactive consumer, and non-proactive consumer buses, respectively. Constraint (3e) enforces that the unserved power should be lower than the corresponding demand. Constraint (3f) bounds the active-power outputs of utility batteries. Note that binary variables x_i enforces battery i output to be zero if it is equal to 0. The effect of installing an underground power cable is reflected in constraint (3g). If an underground power cable is used for line section l ($z_l = 1$), its power flow is bounded by the thermal limit (p_l^F, \bar{p}_l^F) regardless of the fault status of the line ($d_{l,s}$). However, if line section l is not made underground ($z_l = 0$), under a fault condition ($d_{l,s} = 1$), its power flow is enforced to be zero. Constraint (3h) limits the total investment cost of various enhancement solutions. Constraints (3i) declare variables.

We note that only the discharging mode of batteries is represented in the formulation above. This is so because discharging is the only working mode for batteries under contingencies. Needless to say, we assume that batteries are charged and ready to respond to contingencies.

3.3.2. Proactive consumers

The decision framework of proactive consumer i to improve its resilience relies on investing in a behind-the-meter battery. A battery allows displacing energy across time, which improves resilience throughout the day.

Mathematically, the decision framework of each proactive consumer i can be recast as a linear programming problem (problem i in (1b)):

$$\min_{\Xi_2} c^{\text{UE}} \sum_{s \in S} \left(f_s t_o p_{i,s}^U \right) + c^B \bar{p}_{i,s}^B \quad (4a)$$

s.t.

$$-p_{i,s}^{\text{BC}} + p_{i,s}^B + p_{i,s}^U = p_{i,s}^{D'} \quad i \in I^{\text{BC}} \quad \forall s \in S : (\mu_{i,s}) \quad (4b)$$

$$0 \leq p_{i,s}^B \leq \bar{p}_i^B \quad i \in I^{\text{BC}} \quad \forall s \in S : (\underline{\alpha}_{i,s}, \bar{\alpha}_{i,s}) \quad (4c)$$

$$0 \leq p_{i,s}^U \leq p_{i,s}^{D'} \quad i \in I^{\text{BC}} \quad \forall s \in S : (\underline{\gamma}_{i,s}, \bar{\gamma}_{i,s}) \quad (4d)$$

$$-\bar{p}_i^B \leq 0 \quad i \in I^{\text{BC}} : (\chi_i), \quad (4e)$$

where

$$\Xi_2 = [p^B, p_{i,s}^{\text{BC}}, \bar{p}_i^B].$$

The objective function (4a) is the proactive consumer's expected unserved energy, based on scenarios, and the investment cost in a battery. Constraint (4b) is the nodal balance equation at the proactive consumer bus, including the power exchange with the utility network and the outputs from a battery if installed. Constraints (4c) and (4d) bound the outputs from the battery and the unserved power, respectively. Note that the upper bound of the battery output (\bar{p}_i^B) is a variable in this problem, which must be greater than or equal to 0 as defined by (4e).

We note that the coupling variable between proactive consumer i lower-level problem and the utility upper-level problem is $p_{i,s}^{\text{BC}}$, the power exchange with the proactive consumer at bus i during fault scenario s .

Since we seek to convert (1) into (2), the KKT conditions of problem (4) are [20] (that correspond to KKT_i in (2b)):

$$-p_{i,s}^{\text{BC}} + p_{i,s}^B + p_{i,s}^U = p_{i,s}^{D'} \quad i \in I^{\text{BC}} \quad \forall s \in S \quad (5a)$$

$$\mu_{i,s} + \bar{\alpha}_{i,s} - \underline{\alpha}_{i,s} = 0 \quad i \in I^{\text{BC}} \quad \forall s \in S \quad (5b)$$

$$f_s t_o + \mu_{i,s} + \bar{\gamma}_{i,s} - \underline{\gamma}_{i,s} = 0 \quad i \in I^{\text{BC}} \quad \forall s \in S \quad (5c)$$

$$c^B - \bar{\alpha}_{i,s} - \chi_i = 0 \quad i \in I^{\text{BC}} \quad \forall s \in S \quad (5d)$$

$$0 \leq \underline{\alpha}_{i,s} \perp p_{i,s}^B \geq 0 \quad i \in I^{\text{BC}} \quad \forall s \in S \quad (5e)$$

$$0 \leq \bar{\alpha}_{i,s} \perp (-p_{i,s}^B + \bar{p}_i^B) \geq 0 \quad i \in I^{\text{BC}} \quad \forall s \in S \quad (5f)$$

$$0 \leq \underline{\gamma}_{i,s} \perp p_{i,s}^U \geq 0 \quad i \in I^{\text{BC}} \quad \forall s \in S \quad (5g)$$

$$0 \leq \bar{\gamma}_{i,s} \perp (p_{i,s}^{D'} - p_{i,s}^U) \geq 0 \quad i \in I^{\text{BC}} \quad \forall s \in S \quad (5h)$$

$$0 \leq \chi_i \perp \bar{p}_i^B \geq 0 \quad i \in I^{\text{BC}} \quad (5i)$$

Constraint (5a) is (4b). Constraints (5b), (5c), and (5d) are dual constraints associated with primal variables $p_{i,s}^B$, $p_{i,s}^U$, and \bar{p}_i^B , respectively. Constraints (5e) through (5i) represent complementarity conditions for inequality constraints (4c) through (4e). Those constraints can be linearized using Fortuny-Amat McCarl linearization method [29], which involves auxiliary binary variables and sufficiently large constants that need to be carefully tuned up.

3.4. Optimization model

We assume that the supplying utility acts first investing (within a limited budget) with the purpose of achieving maximum resilience improvement. Such action by the utility triggers the individual reactions of the proactive consumers that individually invest (within limited individual budgets) seeking their own maximum resilience improvements. It is apparent that the actions of the utility and that of each proactive consumer are linked and need to be determined jointly. That is, if the utility invests heavily to improve the resilience of the system as a whole, most consumers will not invest as they are satisfied with the resilience level provided by the utility. On the other hand, if the utility invests lightly, barely improving the resilience of the system as a whole, most consumers will be compelled to carry out their own investments for resilience improvement. Within this multi-agent decision framework, the key question is: which is the best balance regarding utility investment and consumers' individual investments.

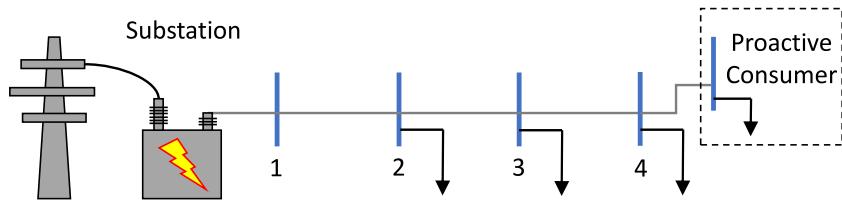


Fig. 2. Four-bus system.

Table 1
Simple case study results.

Case	Input	PC	Utility battery (kW)			Utility cable			Utility cost	UE
			Battery	Bus2	Bus3	Bus4	Line1	Line2		
1	Base	100	0	0	0	0	0	0	0	2.25 MWh
2	UE Cost = \$58.19/kWh	200	0	0	0	0	0	1	\$0.1 M	1.25 MWh
3	$f_2 = 25$	100	0	0	0	0	1	0	\$0.1 M	1.5 MWh
4	No PC, $f_3 = 20$	0	0	0	200	0	0	0	\$0.2 M	3.0 MWh
5	Load 2 = 500 kW	100	0	0	0	1	0	0	\$0.1 M	3.25 MWh
6	Cost (Utility Battery) = \$200/kW	100	0	100	0	0	0	0	\$0.32 M	1.25 MWh

PC: Proactive consumer, UE: Unserved energy.

Table 2
Real-world case study results.

Case	UE cost	PC battery	Utility battery	Utility cost	UE
1	\$18.19/kWh	17 kW	Bus 18: 36 kW	\$0.14 M	3.08 MWh
2	\$8.19/kWh	17 kW	None	\$0	4.12 MWh
3	\$58.19/kWh	17 kW	Bus 16: Bus 18: Bus 19: 14 kW 45 kW 50 kW	\$0.41 M	1.63 MWh

PC: Proactive consumer, UE: Unserved energy.

Mathematically, the above decision framework materializes into the MPEC (mathematical program with equilibrium constraints) below (similar to (2)):

$$\min \quad (3a) \quad (6a)$$

$$\text{s.t.} \quad (3a)-(3i) \quad (6b)$$

$$(5) \quad \forall i \quad (6c)$$

We note that (6) is constrained by the individual decisions of the proactive consumers (6c), and conversely, the decisions of the proactive consumers as expressed through (6c) influence the decision of the utility represented by (6a)–(6b).

4. Numerical results

In this section, three case studies are presented involving an illustrative 4-bus system, a real-world 26-bus system representing a suburban distribution system in Oklahoma, US, and the IEEE 123-bus system. We assume that the unserved energy cost is \$18.19/kWh based on the Interruption Cost Estimate (ICE) Calculator [30].

The models of all case studies are implemented in MATLAB [31] and solved using a laptop with an Intel Core i7 CPU clocking at 2.60 GHz and 8 GB of RAM. The optimization solver Gurobi 6.5 [26] is used within MATLAB to solve the resulting MILP problems.

4.1. Illustrative 4-bus system

We consider an illustrative 4-bus system including four agents: the supplying utility at bus 1 (substation), three non-proactive consumers at buses 2, 3, and 4, and one proactive consumer at bus 4, as shown in Fig. 2. We note that the single bus enclosed in the inner dashed-line quadrangular box in Fig. 2 represents a point-of-interconnection bus

between the proactive consumer and the utility, which is not part of a utility system and therefore not considered for the fault analysis.

We consider the following data:

1. The annual frequency of failure of lines 1, 2, and 3 (f_1 , f_2 and f_3 , respectively) are all equal to 5 (or 5 times per year).
2. The restoration duration of any failure is assumed to be $t_o = 2$ hours.
3. The cost of replacing any of the overhead lines (1, 2, or 3) with an underground cable is \$100 thousand.
4. The cost of installing a 2-hour battery by the utility is assumed to be \$1000 per kW, and \$800 per kW if installed by the proactive consumer.
5. The demands at nodes 2, 3, and 4 are all 100 kW.
6. The peak load of the proactive consumer is 50 kW.

The proactive consumer has the option of installing a battery to improve its resilience. On the other hand, to improve the overall resilience of the system, the utility may install batteries and/or retrofit overhead conductors to underground cables.

For the sake of simplicity, we assume that the capacities of the substation and of all distribution lines are all equal to 400 kVA, which prevent congestion.

Table 1 provides outcomes for different cases for which input parameters change one at a time. These results include different resilience enhancement solutions that are optimal for both the proactive consumer and the utility.

The outcomes of Table 1 are briefly discussed below:

1. If the base assumptions are considered (Case 1 in Table 1), a 100-kW 2-hour battery is installed by the proactive consumer at its bus (bus 4). If appropriate contractual arrangements are in place, such battery can supply power to some consumers downstream of the faulted lines, which reduces the annual unserved energy from 3.75 MWh to 2.25 MWh. No utility battery or underground cable retrofitting is selected.
2. If the unserved energy cost increases from \$18.19/kWh to \$58.19/kWh (Case 2 in Table 1), the proactive consumer increases the size of its battery to 200 kW and the utility undergrounds line 3 (between buses 3 and 4). Thus, no faults can occur on line 3 and the proactive consumer battery can supply backup power to both buses 3 and 4. This is the reason why the proactive consumer battery size is increased to support buses 3 and 4 (provided that appropriate contractual arrangements are in place).

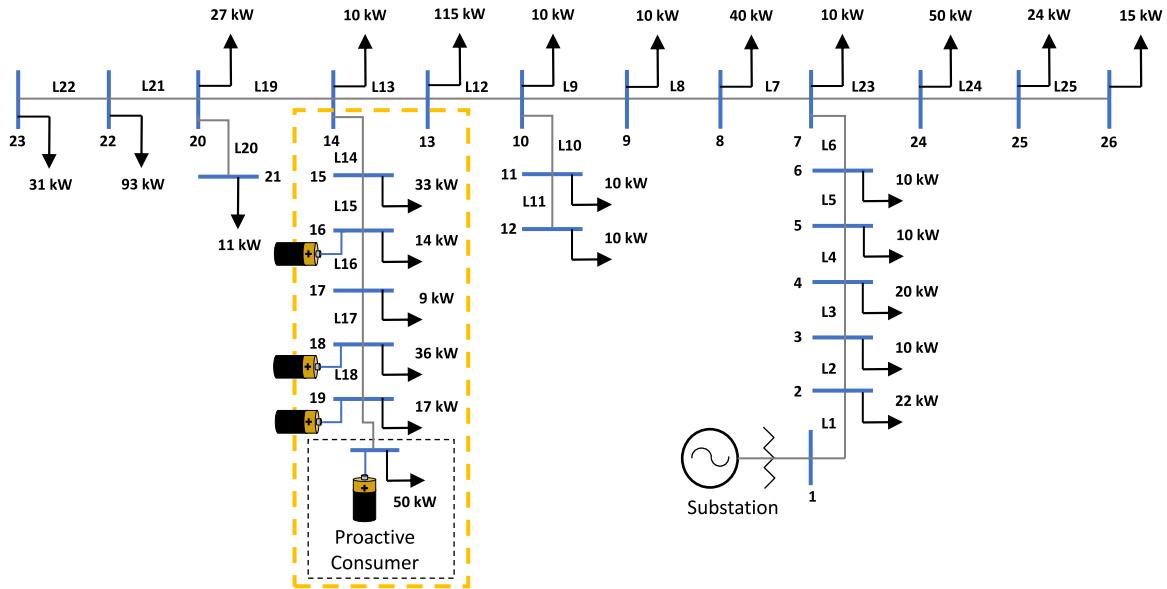


Fig. 3. 26-bus system.

3. If line 2 is expected to fail 25 times, not 5 (Case 3 in [Table 1](#)), the utility makes line 2 underground.
4. If there is no proactive consumer and line 3 is expected to fail 20 times (Case 4 in [Table 1](#)), the optimal decision is to install 200-kW battery at bus 4, which allows supplying power to bus 4 when line 3 fails and supplying power to buses 2 and/or 3 if line 1 or 2 fails (if appropriate contractual arrangements are in place). Another alternative could be undergrounding line 3. However, this option will not help with the failures on lines 1 or 2.
5. If the load at bus 2 increases to 500 kW (Case 5 in [Table 1](#)), making line 2 underground is the most cost-effective solution.
6. A decrease in the cost of the utility battery (Case 6 in [Table 1](#)) leads to install an additional 100-kW battery at bus 3 while the proactive consumer continues installing a battery at bus 4. This shows that the proactive consumer installs a battery regardless of what the utility battery cost is since its goal is to improve its own resilience.

This simple example illustrates the expected outcomes that involve optimal investments by both the supplying utility and the proactive consumers.

4.2. Real-world 26-bus system

A realistic case study is considered next. It is based on a 26-bus system corresponding to a real-world 13 kV distribution system in Oklahoma (illustrated in [Fig. 3](#)). We consider phase C of this unbalanced three-phase distribution system. The peak load in kW of all buses are provided in [Fig. 3](#). We analyze resilience enhancement solutions for this system with a proactive consumer connected at bus 19, whose peak load is 50 kW. We note that historical outage information indicates that the most problematic portion of the circuit includes the buses enclosed by the dashed-line rectangular box in [Fig. 3](#). We note as well that the single bus enclosed in the inner dashed-line quadrangular box in [Fig. 3](#) represents a point-of-interconnection bus between the proactive consumer and the utility, which is not part of a utility system and therefore not considered for the fault analysis.

The restoration duration of any failure, the cost of converting overhead lines to underground cables, and the cost of installing batteries

by the utility and the proactive consumer are 2 h, \$100,000 per line section, \$1000/kW, and \$800/kW, respectively. Note that all the above values are selected based on historical data from a utility in the Midwest, US. These data have been slightly modified for the sake of confidentiality.

Outcomes are reported in [Table 2](#). The proactive consumer and the utility install a 17-kW and a 36-kW battery at bus 19 (proactive consumer bus) and bus 18, respectively. Note that the proactive consumer does not install a battery that can supply its full demand (50 kW). This is so because only 17-kW battery is worth to invest to improve its resilience. If the unserved energy cost increases to \$58.19/kWh, the utility installs additional batteries at buses 16, 18, and 19 to mitigate the increased cost of unserved energy. If the unserved energy cost decreases to \$8.19/kWh, the incentive of investing in batteries and/or underground distribution lines is further reduced from the utility perspective. Therefore, no utility investment takes place, while the proactive consumer retains its investment. We note that batteries are generally installed towards the end of the branches of the circuit.

The computation time for each case of this real-world system is between 10 and 12 s.

4.3. IEEE 123-bus system

To demonstrate the scalability of the proposed optimization model, we have also analyzed resilience improvements for the IEEE 123-bus system [32]. Only phase A of this system is studied. We consider three cases with one (connected at bus 90), two (connected at buses 33 and 90), and three (connected at buses 33, 68, and 90) proactive consumers. We consider as well 13 line-failure scenarios.

The optimal solutions are summarized in [Table 3](#). Such solutions indicate that as the number of proactive consumers increases, the investment from the utility decreases (utility cost decreases with fewer investments in utility batteries and/or underground lines). Although the system unserved energy increases, the extent of such increase (from 6.66 MWh to 7.74 MWh) translated to cost is much smaller than the utility cost savings (from \$0.92 M to \$0.26 M). In other words, an increasing number of proactive consumers provides a utility with more flexibility in optimizing its investment in resilience improvement.

The computation time for each case of this 123-bus system is between 860 and 900 s.

Table 3
123-bus case study results.

Case	PC battery	Utility battery	Utility cable	Utility cost	UE			
1	Bus 90: 200 kW	Bus 79: 40 kW	Bus 97: 40 kW	Bus 113: 60 kW	Bus 122: 80 kW	Line 88, Line 120	\$0.92 M	6.66 MWh
2	Bus 33: 120 kW	Bus 90: 200 kW	Bus 109: 40 kW	Bus 113: 60 kW		Line 88, Line 120	\$0.5 M	7.34 MWh
3	Bus 33: 120 kW	Bus 68: 200 kW	Bus 90: 40 kW	Bus 113: 60 kW		Line 120	\$0.26 M	7.74 MWh

PC: Proactive consumer, UE: Unserved energy.

5. Conclusion

Since electricity enables virtually everything in modern life, maintaining high supply resilience in an electricity distribution system is a must. We address the problem of improving the resilience of a power distribution system by optimally coordinating investment interventions by the supplying utility and the different proactive consumers of the distribution system. We approach this problem using a hierarchical decision framework that optimally coordinates the conflicting objectives of the supplying utility and the proactive consumers. We analyze three cases pertaining to distribution systems with different sizes, which result in the following findings.

1. The resilience enhancement solution varies as the input parameters change. The load, fault, and cost information can significantly influence solution choices by both the utility and the proactive consumers.
2. Hybrid solutions including investments from both the utility and proactive consumers are common. This demonstrates the advantage of the proposed framework for coordinating resilience interventions by both consumers and the supplying utility.
3. The proposed technique is proven computationally efficient to analyze real-world distribution systems.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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