

# Defining and Enabling Resiliency of Electric Distribution Systems with Multiple Microgrids

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May 2016

The INL is a  
U.S. Department of Energy  
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Battelle Energy Alliance



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Prepared for the U.S. Department of Energy  
Office of Nuclear Energy  
Under DOE Idaho Operations Office  
Contract DE-AC07-05ID14517

# Defining and Enabling Resiliency of Electric Distribution Systems with Multiple Microgrids

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**Abstract**—This paper presents a method for quantifying and enabling the resiliency of a power distribution system (PDS) using analytical hierarchical process and percolation theory. Using this metric, quantitative analysis can be done to analyze the impact of possible control decisions to proactively enable the resilient operation of distribution system with multiple microgrids and other resources. Developed resiliency metric can also be used in short term distribution system planning. The benefits of being able to quantify resiliency can help distribution system planning engineers and operators to justify control actions, compare different reconfiguration algorithms, develop proactive control actions to avert power system outage due to impending catastrophic weather situations or other adverse events. Validation of the proposed method is done using modified CERTS microgrids and a modified industrial distribution system. Simulation results show topological and composite metric considering power system characteristics to quantify the resiliency of a distribution system with the proposed methodology, and improvements in resiliency using two-stage reconfiguration algorithm and multiple microgrids.

## I. INTRODUCTION

POWER system resiliency refers to the ability of the network to minimize the negative impacts of infrequently occurring adverse events. Such ‘low-frequency, high impact’ events will be referred to as *unfavorable events* in the remainder of the paper.

There are several working definitions of resiliency as defined by US Department of Homeland Security (DHS) [1], National Infrastructure Advisory Council, National Association of Regulatory Utility Commission (NARUC), and US research facilities like Idaho, Argonne, Los Alamos and Sandia National Laboratories [2], [3]. According to [1], “Resilience includes the ability to withstand and recover from deliberate attacks, accidents, or naturally occurring threats or incidents.” In August 2013, the Executive Office of US President published a white paper on the economic benefits of a more resilient power grid [4], preceded by a policy framework in 2011, which proposed a four-pillared strategy for modernizing the electric grid, a key focus being making the grid more resilient and less vulnerable to weather-related outages.

Resiliency studies of electric grid in existing literature are mostly directed towards their structure and organization [5]–

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The authors would like to thank AVISTA Utilities and Idaho National Lab to inspire this research work. We would also like to thank Farshid Shariatzadeh and Fransiska Martina for providing some of the preliminary work reported here.

[8]. However, the physical constraints (power flow voltage limits, generation limits, thermal limits) of a power distribution system (PDS) provide additional challenges for resiliency analysis.

Resilience of a PDS is also affected by the control decisions taken by the operator such as the reconfiguration method. Resiliency and robustness of the power systems complex network has been studied by several researchers [9]–[13]; however, there exists a few indicators of power grid resilience to compare effectiveness of different control actions.

Numerous optimization-based reconfiguration schemes have been proposed [14], [15], although not much with multiple microgrids. Though most of these schemes are successful in restoring power and meeting their objectives, some schemes make the system more vulnerable to *unfavorable events* than others.

Traditionally performance of PDS over a long period of time, is measured using reliability metrics, like SAIDI, SAIFI, MAIFI [16]. Reliability metrics are designed to measure the performance of system to provide power to all the connected loads. During extreme conditions, it is preferable to design a system to supply power to critical loads first and then to non-critical loads. Ability of the system to supply power to critical loads in extreme unfavorable events is defined as resiliency and can not be captured by reliability metrics [17]. Given existing reliability reporting practice, as of 2013, reliability indices do not take 87% of power outages into consideration. Table I summarizes the important conceptual differences between power system resiliency and reliability.

A large number of publications and scientific articles related to power system resiliency have been published over the last few years. According to IEEEExplore, the number of publications has grown from a single paper by Lachs in 2002 [18] to 163 in mid-2015 for topics related to “power grid resilience”, despite which no ‘gold-standard’ exist for comparing resiliency of two or more distribution systems..

This paper presents a new resiliency metric and approach for enabling resiliency of the distribution system by taking control actions with maximum positive impact on the resiliency. The focus of this paper is not to develop new technique for enabling resiliency though a better reconfiguration algorithm. There is substantial literature that focuses on development of comprehensive operation and self-healing strategy for a distribution system with both dispatchable and nondispatchable distributed generators [19]. The objective of this paper is to enable resiliency by comparing several possible existing reconfiguration strategies and choosing the one, which increases the resiliency most.

TABLE I  
DIFFERENCES BETWEEN RESILIENCY AND RELIABILITY

<b>Resiliency</b>	<b>Reliability</b>
<ul style="list-style-type: none"> <li>- Measured in response to unfavorable event affecting the distribution system.</li> <li>- Will be impacted by network design, operating conditions, control actions and DMS actions, in response to unfavorable events.</li> <li>- Not defined well and metrics to measure still need to be defined.</li> <li>- May be computed before or after an event.</li> <li>- Focus is mostly on critical loads.</li> <li>- All power outages, irrespective of the duration, needs to be considered for resiliency evaluation.</li> </ul>	<ul style="list-style-type: none"> <li>- Measured generally in frequency, time and duration of outage.</li> <li>- Used to indicate ability of distribution systems to meet the connected load demand.</li> <li>- Well defined and measured using SAIFI, SAIDI, ENS, CAIDI, CAIFI</li> <li>- Usually computed over a certain duration of time.</li> <li>- All connected loads are considered</li> <li>- Short duration power outages (generally lasting less than five minutes) are not considered.</li> </ul>

Contribution of this paper includes a) developing a novel method to quantify the resiliency of a distribution system using indices based on network percolation theory and complex network analysis, b) computing the composite resiliency using a multi-criteria decision method analytical hierarchical process (AHP), c) using the developed concept to enable a resilient distribution system using two-stage reconfiguration algorithm, and d) testing and validation using networked modified CERTS microgrid. This information obtained may be used to determine which method of enabling resiliency would be most cost-effective or efficient for the distribution system.

## II. QUANTIFYING RESILIENCY OF DISTRIBUTION SYSTEMS

Several researchers have used graph-based metrics to analyze complexity and robustness of power systems networks, from a topological perspective [20]. However, none of these researchers considered distributed energy resources (DERs), microgrids, and resiliency aspects while formulating their approach. Since percolation theory offers a way to find connected paths even in networks with a large number of damaged nodes, it seems to be suited for PDS resiliency analysis. Percolation theory-based analysis of power systems cascading outages has been carried out previously in [21]. In [22], a generalized framework for resiliency analysis has been proposed however constraints The approach proposed in [23], [24] is more suited for long-term resiliency planning of transmission systems, and does not consider short-term proactive actions that can be taken to avoid power outage in distribution systems. Also, priority of loads has not been considered in these works. The proposed metrics for resiliency of distribution systems in this work is based on evaluations of factors affecting the topology of the network. The problem of quantifying resiliency is formulated as a decision making problem. Factors, other than network topology that affect resiliency are: probability of damage of each node in the network given a certain unfavorable event is about to occur, intensity of unfavorable event and type, age, and current level of maintenance of the power system infrastructure.

### A. Percolation Theory to Quantify Resiliency

Quantification of resiliency is done against a threat, which can be expressed as a probability of a node being damaged from an unfavorable event. Using percolation theory, it is possible to easily determine a certain threshold value of

the probability, called percolation threshold. A probability of a node being functional after a contingent event, can be determined from historical data, or typically available mean failure rate information. Values of probability of functionality of each node are statistically independent of each other, as interconnected power system equipment are different and located in different parts of the network. If a path can be formed from a substation node to a load node by connecting nodes that have probability of being functional, higher than the percolation threshold, then it can be guaranteed that at least one critical load would remain connected to the main grid, irrespective of impact of the event. During a powerful unfavorable event, all the nodes of the distribution system are affected such that each node is functional with a probability  $p$  and damaged with a probability  $1 - p$ , where  $p < 1$ . The value of  $p$  can be determined by formulating as a Bayesian conditional probability problem, where probability of a certain event A (in our case, node failure) taking place is computed on the basis of probability of occurrence of another event B (in our case might be, wind speed, isobar data, storm prediction, or other weather modeling data). Based on these studies, the average size of islands can be determined, and secondary sources of energy like distributed generators (DG), solar panels, fuel cells, battery storage, plug-in electric vehicles can be planned, so as to enhance the overall resiliency of the distribution system. This threshold value of probability is referred to as *percolation threshold*  $p_c$ . It is used to determine the critical fraction of the nodes that one can afford to have damaged from any unfavorable event. If the percolation threshold is determined for a given network, then by direct comparison of the probability value of node failure against a threat assigned by other modeling tools to the percolation threshold, it is possible to determine whether at least one path will remain connected to the main power grid during the attack. This information provides useful insights in the amount of power required to be generated from secondary sources during an attack or post-contingency, or plan ahead if any restoration scheme is feasible using the available connected path.

### B. Topological Metrics for Resiliency

To analyze the electric power using complex network theory percolation approach, a graph from the system model is obtained in the first step [25]. A PDS is represented by a graph  $G = (N, E, W)$  comprising of a set of nodes  $N$ , a set of sections or edges  $E$  with each element directed from

a ‘From’ node  $x$  to a ‘To’ node  $y$ , and a set of Weights  $W$  representing the power flowing through the sections. There is a one-to-one correspondence between sets  $E$  and  $W$ .

Topological metrics such as betweenness centrality, graph diameter, average path length, clustering coefficient - help us understand the structural properties of graph [26]. Graph diameter ( $D = \frac{2|E|}{|N|(|N|-1)}$ ) and average path length of the graph ( $l_G = \frac{\sum_{i \neq j} d(n_i, n_j)}{|N|(|N|-1)}$ , where  $d$  is the distance between two nodes  $n_i$  and  $n_j$ ) shows the number of nodes must be traveled through to connect a source node to a load node when considering the whole network, and thus is important for computing topological resiliency. Clustering coefficient ( $C_n = \frac{2e_n}{k_n(k_n-1)}$ , where  $e_n$  is the number of connected pairs between all neighbors of a node  $n$ , and  $k_n$  is the number of neighbors of a node  $n$ ) is useful in informing which nodes in a graph tend to be connected with other adjacent nodes. However, topological metrics alone cannot be sufficient to describe ‘how well connected each node is to each other’, which are adequately answered from the eigenvalue analysis of the adjacency metric of the graph and represented using Spectral Metrics.

The number of nodes in the graph that remain connected after an unfavorable event determine the percolation transition in the network. The degree of each node depends on the number of nodes that is connected to each node. The degree distribution of all nodes in a network is computed as follows,

$$\langle k \rangle = \frac{2|E|}{|N|} \quad (1)$$

The  $n^{th}$  moment of the degree distribution is defined as:

$$k^n = \int_{k_{min}}^{\infty} k^n p(k) dk \quad (2)$$

The first moment is the average node degree distribution  $\langle k \rangle$  defined above, and the second moment  $\langle k^2 \rangle$  is the square of the standard deviation of degree distribution in the network. These lower moments are very significant in the study of percolation in networks. In context of PDS, the degree distribution represents the number of laterals arising out of each feeder node. In terms of topological resiliency, it helps to have a more heterogeneous network for higher topological resiliency.

The relative importance of each node in the graph is represented using betweenness centrality of a node ( $n_i \in N$ ), and is given by

$$C_B(i) = \sum_{n_i \neq n_k \neq n_l} \frac{n_k \rightarrow n_l, n_i}{n_k \rightarrow n_l} \quad (3)$$

where  $(n_k \rightarrow n_l, n_i)$  is 1 if the shortest path between the node  $n_k$  to  $n_l$  goes through  $n_i$ , and 0 if  $n_k$  to  $n_l$  does not pass through  $n_i$ . The notation  $n_k \rightarrow n_l$  is to indicate the shortest path between the two nodes mentioned in the term. The smaller the value of  $\max(C_B(i))$  for nodes of a higher degree in the distribution network, the network is more resilient. Spectral Metric, algebraic connectivity ( $\Lambda_2 = \text{eig } L_{(I,j)}^2$ ) is an indicator that indicates: higher the value of second smallest eigenvalue of Laplacian matrix of the network, the network is

more resilient [27], [28]. The Laplacian Matrix is obtained as follows

$$L_{(i,j)} = \begin{cases} \deg(n_i) & \text{if } i = j \\ -1 & \text{if } i \neq j \text{ and } n_i \text{ is adjacent to } n_j \\ 0 & \text{otherwise} \end{cases}$$

In other mathematical papers on percolation theory, in fact, the process of a node being dysfunctional is called site percolation and the process of a section being dysfunctional is called bond percolation. In case of a PDS, the network changes in response to unfavorable events, from distribution poles being uprooted or line-ground faults (sectional failures, i.e. bond percolation) or through malfunctioning of distribution system transformers (node failures, i.e. site percolation).

According to [29], percolation threshold  $p_c$  of a network can be reached and a percolation transition is possible only if the ratio of  $\langle k^2 \rangle$  to  $\langle k \rangle$  equals 2. If this condition is met, we can be assured that irrespective of how many number of nodes in a distribution system is damaged, there will be at least one node within the network, that would continue to be fed from the distribution substation. However for most practical distribution systems,  $\frac{\langle k^2 \rangle}{\langle k \rangle} \neq 2$  and alternative percolation models need to be proposed, considering installed DERs.

The easiest approach to model percolation is by removing an arbitrary fraction of nodes from service, and continue to search for at least one undamaged path to serve the critical loads. This involves assigning a probability of damage of all nodes, and is effective for studying resiliency of a distribution system to unfavorable event, parameterized by the probability ( $p$ ) of a node being damaged. Targeted attacks are designed to damage nodes based on a descending order of importance of a node, measured using betweenness centrality.

While studying an unfavorable event, fraction of damaged nodes is considered to identify the largest connected component. The problem of percolation on networks with degree correlations have been reported with a generating function formalism, or using a more general statistical mechanics approach [26], [30]. The fraction of nodes in the largest cluster is given by

$$f = 1 - \sum_k p_k (\phi_k)^k \text{ where,} \quad (4)$$

$$\phi_k = \sum_{k'} p(k'|k) (\phi'_{k'})^{k'-1} \quad (5)$$

where  $p(k'|k)$  is a conditional probability that a vertex of degree  $k'$  is reached following any edge coming from a vertex of degree  $k$ , and  $\phi_k$  is the average probability that an edge connected to a vertex of degree  $k$  is connected to another vertex that does not belong to the large fraction of connected nodes in a damaged network [26], [30]. According to the results in these papers, the largest fraction of nodes in a connected network while considering node and bond

percolation after  $f'$  nodes and  $f''$  edges are damaged, are given by

$$f_{node} = 1 - f' - (1 - f') \sum_k p_k(u_k)^k \quad (6)$$

$$f_{bond} = 1 - \sum_k p_k(u_k)^k \quad (7)$$

$$\phi_k = f' + (1 - f') \sum_{k'} p(k'|k) (u_{k'})^{k'-1} \quad (8)$$

The second part of the expression in Equation (8) can be arranged in the form of a matrix, the largest value of which will be used to determine the stability of the solution of Equation (8). The solution to Equation (8) is stable if and only if the largest eigenvalue of  $(1 - f')(k' - 1)p(k'|k)$  is greater than 1 [26]. This criteria may be used to determine if the network is resilient even after a certain node is subjected to damage. If critical loads are connected to nodes which yield  $\text{Eig}(1 - f')(k' - 1)p(k'|k) < 1$  and are not supplied with feasible distributed generation, then they are not considered to be resilient nodes. Using this algorithm iteratively, a fraction of nodes,  $f_c$ , is determined which are deemed to be resilient enough to supply critical loads after nodes have been randomly damaged.

### III. ANALYTICAL HIERARCHICAL PROCESSES FOR COMPOSITE RESILIENCY METRIC

Resiliency of power distribution systems is dependent on several factors related to distribution network, resources and controls. To be able to reach a numerical interpretation of all the factors affecting resiliency of the network simultaneously, a decision making problem approach is considered in this paper. Decision making provides the mathematical foundation in choosing most influential and relevant factors among several possible factors to affect the resiliency. In this paper Analytical Hierarchical Process (AHP) was applied to choose most influential factors affecting resiliency.

Topological resiliency of a distribution network depends on the spectral and statistical metrics discussed thus far. All the metrics are dependent on each other, and changes in the network configuration affect other parameters. Thus in essence, we have a vector of metrics, providing various insights into the topological resiliency of a given network. It can be represented as

$$\vec{\mathfrak{R}}_\tau = [f_c, D, l_G, C_B, C_n, \Lambda_2] \quad (9)$$

where  $f_c$  represents the critical fraction of the complex network representing the distribution system,  $D$  represents the diameter of the complex network, i.e. the metric to represent the length of the shortest path between the farthest nodes,  $l_G$  shows the length of the graph,  $C_B$  represents the betweenness centrality of the graph,  $C_n$  represents the clustering coefficient of the network, and  $\Lambda_2$  represents the algebraic connectivity of the network. Equation (9) only represents one example of integrating multiple possible resiliency metrics and how to find trade-off to converge on integrated resiliency metrics. How the network parameters must change for higher topological resiliency of the PDS is shown below.

$$f_c \uparrow D \downarrow C_B \downarrow l_G \uparrow C_n \uparrow \Lambda_2 \uparrow$$

From Equation (9), a traditional weighted summation of the indicator metrics may be used to compute the topological resiliency of the network.

Not all the parameters of  $\vec{\mathfrak{R}}_\tau$  are equally good indicators of resiliency. As common in AHP-based based complex decision making [31], weights are assigned based on a relative interaction between all the resiliency indicator metrics using fractions  $a$  through  $u$  in the interval  $(0, 1]$ . For example, algebraic connectivity has  $\frac{1}{e}$  times the influence on the resiliency as the critical fraction of the network, and betweenness centrality has  $m$  times the influence of clustering coefficient. The dominant eigenvector of  $\vec{\mathfrak{R}}_\tau \vec{\mathfrak{R}}_\tau^T$  obtained from Equation (10) can be used as weights accompanying each factor ( $f_c$ ,  $D$ ,  $C_B$ , etc.) that affects topological resiliency of the network under consideration.

$$\vec{\mathfrak{R}}_\tau \vec{\mathfrak{R}}_\tau^T = \begin{pmatrix} f_c & D & C_B & l_G & C_n & \Lambda_2 \\ f_c & 1 & a & b & c & d & e \\ D & 1/a & 1 & f & g & h & i \\ C_B & 1/b & 1/f & 1 & i & j & k \\ l_G & 1/c & 1/g & 1/i & 1 & l & m \\ C_n & 1/d & 1/h & 1/j & 1/m & 1 & n \\ \Lambda_2 & 1/e & 1/i & 1/k & 1/m & 1/n & 1 \end{pmatrix} \quad (10)$$

Let  $\rho(i, j)$  be used to represent an element of  $\vec{\mathfrak{R}}_\tau \vec{\mathfrak{R}}_\tau^T$ .  $\rho(i, j)$  undergoes a linear transformation:

$$\rho(i, j)' = \frac{\rho(i, j) - \min_{i=1}^\eta(\rho(i, j))}{\max_{i=1}^\eta(\rho(i, j)) - \min_{i=1}^\eta(\rho(i, j))} \quad (11)$$

when the number of indicators of resiliency are  $\eta$ . This number may be different from 6 (as considered in our analysis so far) where there is discrimination or strict dominance, i.e. if one indicator outperforms other indicator metrics against all criteria. A higher value of  $\rho(i, j)'$  means higher performance of a resiliency indicator. Let  $V$  be the spectrum of a decision matrix  $\vec{\mathfrak{R}}_\tau \vec{\mathfrak{R}}_\tau^T$ , which would indicate which resiliency metric has the greatest influence upon the overall resiliency of the system for a given configuration. Based on the value of the metric, the overall resiliency of the system can be ascertained. The most dominant eigenvector has the form

$$V = [A_{f_c} B_D C_{C_B} D_{l_G} E_{C_n} F_{\Lambda_2}]^T \quad (12)$$

where  $A, B, \dots, F$  are the derived weights of importance of the metric in its suffix. Thus the overall topological resiliency of the distribution system for a certain configuration is given by

$$\mathfrak{R}_\tau = \sum_{j=1}^{\eta} V_j \rho(i, j)' \quad (13)$$

It is possible to design a distribution network with great topological resilience, but may not be able to sustain physical constraints. In order to maximize our resilience, we need to ensure that none of the power flow limits are violated. The overall resiliency is a complex decision making process

involving: (1) topological resiliency, (2) load flow feasibility, (3) Failure rate of distribution system equipments and (4) intensity of the unfavorable event. To factor these in, we propose the use of Analytical Hierarchical Process (AHP) to determine the composite resiliency of the distribution system [32].

The most important constraint imposed is that of power flow feasibility (PFF). Let the power distribution system be described as a set of state equations  $\Delta t$  interval apart.

$$\bar{\mathbf{x}}_t^i = \phi(P_t^i, Q_t^i, V_t^i, \delta_t^i) \quad (14)$$

$$\bar{\mathbf{x}}_{t+\Delta t}^i = \phi(P_{t+\Delta t}^i, Q_{t+\Delta t}^i, V_{t+\Delta t}^i, \delta_{t+\Delta t}^i) \quad (15)$$

If a fault occurs within  $\Delta t$  and the relays are not able to clear the fault within that time, or in events of excess or under generation,  $V^i$  shall be beyond the acceptable range  $114 \leq V^i \leq 126$  volt [33], and power flow will be infeasible. If there are no operational constraints (like thermal limits, voltage range limits) and the network was lossless, the injection region of a feeder due to DERs would be the set of all generation satisfying the law of conservation of energy. Since the controllers in the system will try to modify power flow in the network, there will be additional optimization problems due to convexity or non-convexity of constraints in the injection region of the network [34], which may be defined as:

$$\begin{aligned} \wp = P \in R^N : P &= Re(\text{diag}(VV^*Y^*)) \\ V_{114} \leq V^i &\leq V_{126} \\ P_{lb} \leq P^i &\leq P_{ub} \\ Q_{lb} \leq Q^i &\leq Q_{ub}, \forall i \end{aligned} \quad (16)$$

where  $\wp$  defines the feasible injection region for any continuous path of the network, and  $Y$  is the admittance matrix of the network depending upon cable type. In our analysis, the geometry of  $\wp$  is of not much importance, as long as our operating point (Equation (14)) lies within the contour. If it does so, the solution is deemed to be convergent. Some of the terms in Equation (14) are independent of other terms, like nodal active and reactive power injections while nodal voltage magnitudes are dependent on changes in real and reactive power. Operating point of a PDS is the vector of all voltage, current, real and reactive power values at all nodes of the power system. Power flow convergence for each operating condition is determined, and a flag PFF is raised for each legitimate operating condition. PFF can be determined for each operating point on a case-by-case basis, or may be determined by determining a boundary of convergence limits [35]. If an operating point is *inside* the boundary, a power flow is deemed to be solvable, and accepted to be a feasible solution for enabling resilience in the PDS.

Next, it is important to determine how many loads remain online during or after the unfavorable event. A load may be remain online in a number of ways-

- connected to local substation through one feeder.
- connected to local substation through a redundant feeder, after closing normally open switches,

- Have distributed renewable resources (DERs) connected close to load, e.g., battery charged using rooftop solar panels, fuel cells, or, vehicle to grid (V2G) connection, etc.

Thus even if a power distribution network is completely disconnected at multiple nodes, and the critical loads can be picked up following an unfavorable event using local short-term resources, the resiliency of the network is to be considered high. The quantity of power that could have been lost if it was connected to only substation via one feeder, but was not lost due to network redundancy and secondary resources - are crucial to improving the resiliency of the distribution system. A bottom-up modified depth-first-search algorithm is used for reconfiguration of critical load in a damaged network [36]. A steady-state value of current and voltage at each node is determined using forward-backward sweep method [37]. Since the goal during planning stage is to ensure that the end-user has power supply for all anticipated contingency scenarios, we begin the search at a terminal node of a feeder and work our way upstream till a source is identified, and perform a local forward-backward power flow to ensure power system constraints are not violated. If a source can be identified upstream, a flag for PFF is raised, and the number of loads picked up are counted. Load Not Lost Factor (LNLF) is a measure of this quantity, and expressed as a simple mathematical expression as shown in Equation (17).

$$\text{LNLF} = \frac{\sum_{i=1}^m \bar{\mathbf{x}}_t^i}{\sum_{i=1}^m \bar{\mathbf{x}}_0^i} = \frac{\sum_{i=1}^m f(P_t^i)}{\sum_{i=1}^m f(P_0^i)} \quad (17)$$

Failure rate of distribution system parameters is based on age of equipment being used, overhead or underground installation, quality of servicing and maintenance it undergoes. Typically, most distribution system lines and equipment in North America are approximated to have a failure rate  $\lambda = 0.05$  failures/year [38].

The other factor upon which resiliency depends is the intensity of unfavorable events. Such ‘low-frequency, high impact’ events usually follow a log-normal distribution (described using a weather factor  $WF$ ). The weather events may be classified into categories denoting the anticipated intensity of the event [39] High-intensity events are given a higher value of  $WF$ , which is eventually used in the AHP decision making process.

AHP has been used to propose resilience evaluation framework of urban infrastructure, such as water distribution system [40], and the same concept can be used for PDS if power flow constraints are not violated.

In order to compute the weights for the factors deciding the resiliency, AHP needs to develop a pairwise comparison matrix  $M$ . The matrix  $M$  is a  $4 \times 4$  real matrix. Each element  $m_{jk}$  of the matrix  $M$  represents the importance of the  $j^{th}$  criterion relative to the  $k^{th}$  criterion. If  $m_{jk} > 1$ , then the  $j^{th}$  criterion is more important than the  $k^{th}$  criterion, and if  $m_{jk} < 1$ , then the  $j^{th}$  criterion is less important than the  $k^{th}$  criterion. If two criteria have the same importance, then  $m_{jk}$  is 1 and

$\mathbf{m}_{jj}$  is always 1. The elements of the pairwise comparison matrix  $\mathbf{m}_{jk}$  and  $\mathbf{m}_{kj}$  satisfy the following constraint:

$$\mathbf{m}_{jk} \times \mathbf{m}_{kj} = 1 \quad (18)$$

The relative importance between two criteria is measured according to a numerical scale from 1 to 9. This subjective nature of decision making process allows us to identify the most important factor affecting resiliency in a given situation. For example, if we are computing resiliency of a network during an earthquake where there will be lot of infrastructure damage, the weather factor will be key, compared to equipment failure rate and topological resiliency; and for evaluating resiliency of a network with very old equipment under normal weather circumstances will be the equipment failure rate will be a more crucial factor than weather.

There are two possible ways to determine the weight vector to determine the ranking of the most crucial factors for composite resiliency. *First approach:* Once the matrix  $\mathbf{M}$  is built, we can derive the normalized pairwise comparison matrix  $\mathbf{M}_{norm}$  by making the sum of the entries on each column equal to 1, i.e. each entry  $\mathbf{m}_{jk}$  of the matrix  $\mathbf{M}_{norm}$  is computed as

$$\bar{\mathbf{m}}_{jk} = \frac{\mathbf{m}_{jk}}{\sum_{i=1}^4 \mathbf{m}_{ik}} \quad (19)$$

The weight vector  $\mathbf{w}, \mathbf{w} \in R^4$  is written by taking averages of the elements of each row of  $\mathbf{M}_{norm}$  :

$$\mathbf{w}_j = \frac{\sum_{i=1}^4 \bar{\mathbf{m}}_{ik}}{4} \quad (20)$$

such that  $\sum_{j=1}^4 \mathbf{w}_j = 1$ .

*Second Approach:* We can directly obtain the eigenvector of  $\mathbf{M}$  from  $\mathbf{MX} = \lambda_{eig}\mathbf{X}$ , where  $\mathbf{X}$  is an eigenvector of  $\mathbf{M}$  associated to  $\lambda_{eig}$  (highest eigenvalues). The terms in the eigenvector is normalized and linear transformation is used to determine the composite resiliency weight vector, say  $\rho'_{ij}$ , so that a commensurate scale of 0 to 1 may be used. The utility score due to each of key factors is then determined by

$$\mathfrak{R} = \rho'_{ij} \mathbf{w}_j \quad (21)$$

For each possible configuration of the distribution system, a composite resiliency score can be obtained using Equation (21). If  $n$  configurations of a distribution system is deemed to be feasible, a vector  $\mathfrak{R}_{total} = [\mathfrak{R}_1, \mathfrak{R}_2, \dots, \mathfrak{R}_n]$  can be written. Thus, the composite resiliency of the whole network can be written as

$$\mathfrak{R}_{total} = \nu + (1 - \nu) \sum_{i=1}^{n-1} w_i \mathfrak{R}_i \quad (22)$$

where  $w_i$  is the normalized weights assigned in descending order to resilient configurations,  $\mathfrak{R}_i$  are all the composite resilient values of the networks that does not have the maximum resiliency and arranged in descending order,  $\nu = \max [\mathfrak{R}_1, \mathfrak{R}_2, \dots, \mathfrak{R}_n]$ . This way, we can ensure that the representation of the fact that overall composite resiliency increases with an increase in number of paths and  $0 \leq \mathfrak{R}_{total} \leq 1$ .

It must be noted that the four criteria used for computing the composite resiliency of a distribution system are those which also enable us to identify the scope for improving the resilience of an infrastructure. In the following section, we demonstrate how distribution system resiliency can be improved by using a restoration algorithm involving two microgrids.

#### IV. ENABLING RESILIENCY OF DISTRIBUTION SYSTEMS

In this section, application of two stage reconfiguration with microgrids have been presented to enable the resiliency of PDS. This reconfiguration scheme developed within our research group can be evaluated in ability to enhance resiliency of distribution system.

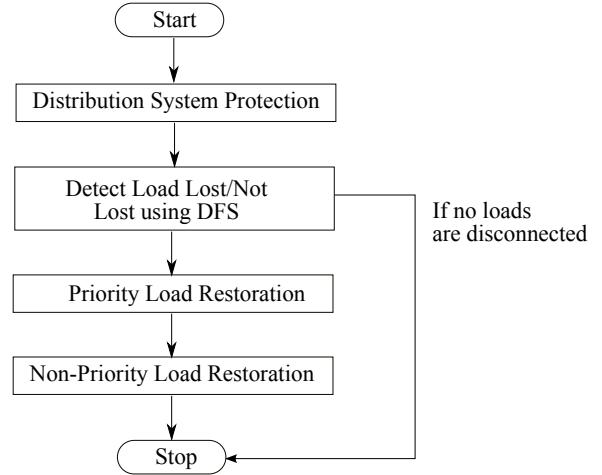


Fig. 1. General Algorithm of Two-Stage Reconfiguration Method

Two proximal microgrids can be operated together to take advantage of shared resources and maintaining power to critical loads of both microgrids. The concept of a multiple microgrids was introduced by the EU MORE Microgrid projects [41] with the objective of enhancing the resiliency of distribution systems [42]. The reconfiguration problem involving multiple microgrids to increase resiliency discussed in this paper has three objectives, which are to maximize the number of the energized loads, to minimize the number of switching operations, and to maximize the utilization of power from renewable generation under contingent situations, while meeting the overall goal of a more resilient PDS.

The weighted-sum method is used to formulate this multi-objective problem. The weight is based on the relative importance of one objective to another objective. Our previous work have proposed a two-stage reconfiguration algorithm (see Figure 1) [36] using multiple microgrids. The first stage is to solve the priority load restoration, while the second stage is the non-priority load restoration.

Using the load-priority based two-stage reconfiguration algorithm, critical loads in both microgrids are restored first, which improves the security and then an attempt is made to recover the non-priority loads. In this work, enabling resiliency means choosing best possible reconfiguration scheme based on resiliency metric.

TABLE II  
CERTS MULTIPLE MICROGRIDS TOPOLOGICAL RESILIENCY ANALYSIS RESULTS

Scenario	Nodes	$l_G$	$\langle k \rangle$	$\langle k^2 \rangle$	$p_c$	$f_c$	$\lambda_2$	$\Re_\tau$
Normal	29	3.248	2.029	5.63	0.44	0.584	2.317	0.79
$\mu G^1$ Island	29	2.444	2.294	5.36	0.62	0.572	3.213	0.81
$\mu G^2$ Island	29	2.667	1.931	7.84	0.48	0.621	3.563	0.84
$\mu G^1, \mu G^2$ Island	28	2.704	7.96	0.48	0.593	0.83	1.11	0.85

TABLE III  
CERTS MULTIPLE MICROGRID COMPOSITE RESILIENCY ANALYSIS RESULTS

Scenario	$p$	$p_c$	$\lambda_{eqp}$	PFF	LNLF	$\Re_\tau$	$\Re$
Without DER	0.333	0.64	0.005	0.62	0.33	0.79	0.43
With DER, $\mu G^1$ Island	0.333	0.62	0.005	0.84	0.93	0.67	0.83
With DER, $\mu G^2$ Island	0.333	0.66	0.005	0.94	0.85	0.71	0.72
With DER, $\mu G^1, \mu G^2$ Island	0.333	0.66	0.005	0.86	0.91	0.65	0.68

## V. SIMULATION RESULTS

### A. A Modified CERTS Microgrid

The proposed theory is tested on two proximally located CERTS microgrids [43]. The differences from the original system is documented in [36].

The representative one line diagram of two CERTS microgrids are shown in Figure 2. Two CERTS microgrids are redundant with normally open paths shown by tie-lines between N11, N7, N21 and N17. DGs are connected to nodes N6, N8, N10, N16, N18, N19 and N20. The network is analyzed to accommodate the study of the following scenarios:

- 1) Fault occurs in feeder connecting substation to microgrid 1, and the network is fed partially from main grid and DERs
- 2) Fault occurs in feeder connecting substation to microgrid 2, and the network is fed partially from main grid and DERs
- 3) Fault occurs in both feeders connecting substation to microgrids 1 and 2, and the network is fed completely using DERs

This network is converted to a graph and percolation network for resiliency analysis. Two proximal CERTS microgrid can be connected to or islanded from the same Utility substation has 29 nodes when either or both of microgrids are connected to the grid, and 28 nodes when both the microgrids are islanded. Using the definitions in [26], topological and spectral properties of the network is computed. It has been assumed that the probability of each node being functional during an unfavorable event is  $p = 0.333$ . Since this probability is computed using weather factor, there is no need to separately account for intensity of weather phenomena in this simulation. The percolation threshold for each network is subsequently determined for the network to determine the critical fraction of the network. Considering the factors that affect resiliency as inputs for the AHP process, the following values of relative weights are assigned in Equation (10):  $a = 1/3$ ,  $b = 1/3$ ,  $c = 1/7$ ,  $d = 1/5$ ,  $e = 9$ ,  $f = 1/5$ ,  $g = 3$ ,  $h = 1/3$ ,  $i = 7$ ,  $j = 5$ ,  $k = 1/5$ ,  $l = 1/3$ ,  $m = 1/7$  and  $n = 1/9$ . The topological resiliency is computed after taking the product of each linearized element of the decision matrix and the

derived weights of importance of each discerning factor. The results for obtaining the topological resiliency is presented in Table II. Since the topology of the network changes due to switching operations that island the distribution system into microgrids ( $\mu G$ ) 1 and 2, the factors affecting the topological resiliency of the whole network has been computed.  $C_B$  values are concerned with individual nodes within in each network, and thus not reported directly. When a fault occurs in the system, the protection system isolates the faulted area. Then the load not lost identification method [11] searches for the load that is left without power. If there is no disconnected load found, then the program terminates. If there is a disconnected load, the disconnected load identification method will continue to identify which load is classified as the priority load type and which load is classified as the non-priority load type. The priority load type will be restored by the priority load restoration stage, as described in detail in [36]. If there are disconnected priority and non-priority loads, the restoration process is performed in two stages, started by the priority load restoration first, and followed by the non-priority load restoration.

After the topological resiliency of the distribution network has been quantified, other factors affecting resiliency are again subjected to a second iteration of AHP to determine the composite resiliency. The failure rate of each distribution system infrastructure has been assumed to be 0.005 failures/year. Using backward-forward power flow, the power flow feasibility of each network configuration is determined to check for violations. Increasing number of DERs in the grid increased LNLF, and thus increasing the resiliency of the network. LNLF is assigned the highest relative weight of 9 in the second iteration of AHP. Though relative weights of 5 and 3 were assigned to weather factor, and failure rate, their effects was not captured as a part of this test system, as they were assumed to same for all the different scenarios. The composite resiliency for each scenario is presented in Table III.

The betweenness centrality of the nodes in the network reduces, when  $\mu G^2$  is islanded. This in turn increases the topological resiliency of the network. When both  $\mu G^1$  and  $\mu G^2$  are working together after the entire network is isolated from the network the betweenness centrality lies between the

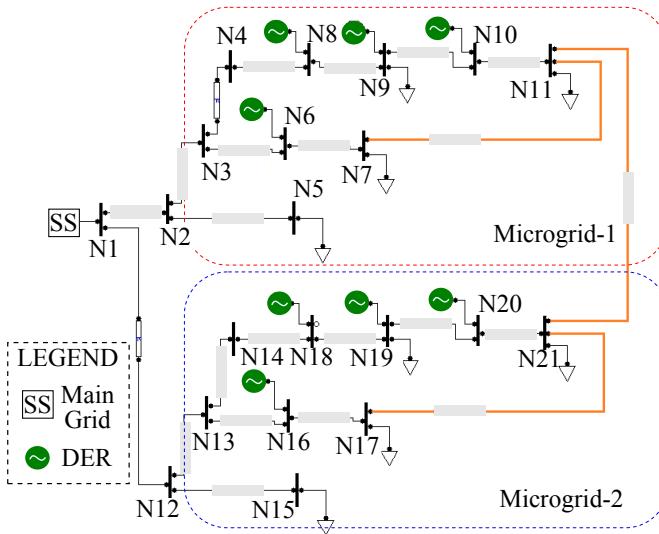


Fig. 2. Multiple Microgrids based on CERTS Concept

previous two cases when either of the microgrids are islanded. Also, since the eccentricity of  $\mu G^2 < \mu G^1$ , the topological resiliency of the network operating partially fed by the main grid and  $\mu G^2$  has the greatest resiliency.

These simulation results demonstrate the ability of developed algorithm to quantify the resiliency of the distribution system with multiple microgrids.

### B. Larger Industrial Distribution System

The method proposed in this paper is scalable, and the approach to deal with a larger system is consistent to the one used for the smaller system. The larger system studied is a modified real power distribution system in Pullman, WA, comprising 4928 nodes and edges in the AVISTA distribution system network. Among four substations in the city, six feeders downstream a chosen substation are studied for resiliency. For confidentiality of feeder information, they have been labeled as F-1 through F-5. To optimize the computation of resiliency of a large network, a combination of software tools have been used as shown in Table IV.

TABLE IV  
SUMMARY OF THE SOFTWARE USAGE FOR RESILIENCY CALCULATION

Data	Power Flow Engines	Create Complex Network	Network Analysis	Compute Resiliency
MS Excel	SynerGEE	Cytoscape	MATLAB	MATLAB
MATLAB	GridLAB-D	MATLAB		Cytoscape

First, the building level details are added on the existing F-1 feeder models using DNV-GL SynerGEE Electric 3.8, as shown in Figure 3. Based on this model, power flow of the network was solved, and a complex network model of the network was created. Relationships based on the betweenness centrality of the nodes in the network, degree and eccentricity are plotted. Based on the load flow analysis, the two largest loads of this feeder are identified as critical loads. These

critical loads are assigned a weight value equal to three times the weight of a normal load for the analysis.

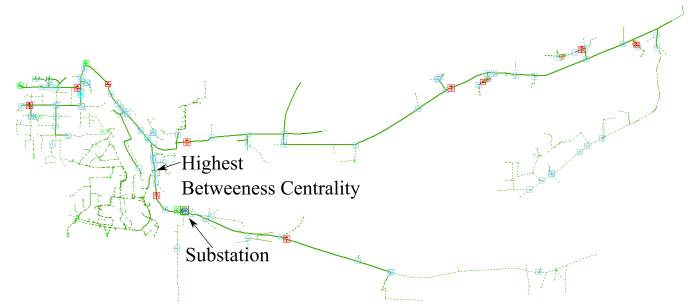


Fig. 3. Modeling of F-1 using DNV-GL SynerGEE Electric 3.8

Additionally, two distributed generators of comparable size are connected near the critical loads. The complex network analysis of F-1 shows that the node with highest betweenness centrality has a degree of 2, and higher degree nodes have a lesser betweenness centrality. However, the eccentricity of the node with highest betweenness centrality have large eccentricity. This means that the probability of loosing a large number of loads increases, and affects the topological resiliency of this network.

Certain scenarios such that the probability of each node being functional after a disruptive event be  $\frac{1}{3}$  and  $\frac{2}{3}$  has been studied. The analysis is presented in Table V and VI.

TABLE V  
F-1 TOPOLOGICAL RESILIENCY ANALYSIS RESULTS

Nodes	$l_G$	$\langle k \rangle$	$\langle k^2 \rangle$	$p_c$	$f_c$	$\lambda_2$	$\mathfrak{R}_\tau$
974	21.237	2.029	5.636	0.437	0.64	1.722	0.419

TABLE VI  
F-1 COMPOSITE RESILIENCY ANALYSIS RESULTS

$p$	$p_c$	$\lambda_{eqp}$	PFF	LNLF	$\mathfrak{R}_\tau$	$\mathfrak{R}$
0.333	0.437	0.005	0.79	0.96	0.42	0.48
0.666	0.437	0.005	0.83	0.98	0.42	0.81

Results indicate that if an disruptive event causes two out of three nodes (which is below the percolation threshold,  $p = 0.333$ ) in the distribution system to be damaged, the resiliency of the system is 0.68. Comparing this event to a stronger disruptive event in which one out of three nodes (which is above the percolation threshold) is damaged, the resiliency of the system improved to 0.81.

As reported in Table VIII, inclusion of additional distributed generators close to the critical loads are observed to boost the overall resiliency of a feeder, compensating for its topological resiliency effects. Individually, F-2 is the most resilient. Also, if the resiliency is computed of the distribution system as a whole and not just individual feeders, the resiliency of the distribution system increases by 25.38% of its least resilient feeder. This is due to additional possibilities of reconfigurations involving multiple feeders and increased LNLF factor.

TABLE VII  
F-2, F-3, F-4 & F-5 TOPOLOGICAL RESILIENCY ANALYSIS RESULTS

Feeder	Nodes	$\langle k \rangle$	$p_c$	$f_c$	$\lambda_2$	$\mathfrak{R}_\tau$
F-2	34	2.029	0.44	0.85	1.17	0.64
F-3	592	2.294	0.82	0.62	1.43	0.49
F-4	52	2.154	0.38	0.78	1.25	0.61
F-5	41	2.049	0.72	0.83	1.11	0.55
All	4928	2.029	0.79	0.64	1.72	0.73

TABLE VIII  
F-2, F-3, F-4 & F-5 COMPOSITE RESILIENCY ANALYSIS RESULTS

Feeder	$p$	$p_c$	$\lambda_{eqp}$	PFF	LNLF	$\mathfrak{R}_\tau$	$\mathfrak{R}$
F-2	0.333	0.44	0.005	0.79	0.27	0.64	0.77
F-3	0.333	0.82	0.005	0.84	0.93	0.49	0.69
F-4	0.333	0.38	0.005	0.94	0.85	0.61	0.73
F-5	0.333	0.72	0.005	0.86	0.91	0.55	0.67
All	0.333	0.79	0.005	0.91	0.95	0.73	0.84

## VI. CONCLUSIONS

A novel formulation using graph theory, percolation theory and analytical hierarchical processes has been presented for measuring resiliency of power distribution systems while meeting physical system constraints. This work also addresses improvement in resiliency using microgrids and DERs. Unlike other papers that have presented network based metrics to quantify resiliency, the metric presented here is not absolute for given a network, but varies according to the forecast of the intensity of an unfavorable event, and level of maintenance and disruption-mitigation already built into the system (example: servicing frequency, underground or overhead lines, etc.). Using a model of two geographically proximal CERTS microgrids and an industrial distribution system, it is demonstrated that the resiliency of the power distribution system may be quantified, and improved. This paper serves to provide a direction for understanding the need for quantifying resiliency of PDS, leveraging existing and upcoming technologies for a stronger and more resilient power grid. This work gives a framework of steps that may be followed to measure the resiliency of distribution systems. In future, time consideration will be integrated in the resiliency metric. The factor of time can be incorporated in the multiple criteria decision making process proposed in this paper, to get a better idea of resiliency of the distribution system. Additionally, DER and switch location to maximize the resiliency will be done in the future. The research in this paper can also be extended in the future to investigate resiliency in time domain and considering system dynamics.

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