

# Post-disaster Power System Resilience Enhancement Considering Repair Process

Han Zhang, Zhaohong Bie, Chao Yan, Gengfeng Li

**Abstract**—The disturbance and threat of natural disasters and human attacks often lead to the failure of power system equipment, which will cause large-scale power outages and severe economic losses. Consequently, the research on power system resilience has attracted wide attention. This paper proposes a post-disaster power system resilience enhancement strategy, synthetically considering the failure components repair and power system operation dispatching. According to the damage assessment result, the repair tasks, the unit output and transmission switching plan are formed. Combined with the vehicle routing problem (VRP) and the power system DC power flow model, a mixed integer linear program (MILP) is proposed to minimize the power outage loss and rapidly restore the power supply of load with the minimal repair expense. The proposed model is applied on the IEEE RTS 79 system and verifies its effectiveness.

**Index Terms**—damage repair, power system restoration, resilience.

## NOMENCLATURE

### Indices and sets

$g$	Index for generation units.
$i/j$	Index for buses.
$x/y/z$	Index for damaged components.
$c$	Index for repair team.
$t$	Index for time.
$b$	Index for repair center (starting point).
$d$	Index for repair center (return point).
$DC$	Set of damaged components.
$DB$	Set of damaged buses.
$DL$	Set of damaged transmission lines.

### Parameters

$\alpha$	Weight factor
$lp_i$	Active demand at bus $i$ and time $t$ .
$el_i$	Economic loss of lost load at bus $i$ .

$P_g^{max}$	Upper limit of active power generation of unit $g$ .
$P_g^{min}$	Lower limit of active power generation of unit $g$ .
$B_{ij}$	Susceptance of line $i \rightarrow j$ .
$S_{ij}$	Upper limit of apparent power flow on line $i \rightarrow j$ .
$d_{x,y}$	Distance between damaged power component $x$ and $y$ .
$V_c$	Average driving speed of team $c$ .
$Cap_c$	Resource capacity of team $c$ .
$RES_x$	Resources required to fix damaged component $x$ .
$res_d$	Resource amount of repair center.
$RT_{x,c}$	Repair time required to fix damaged component $x$ .
$Num_c$	Number of repair teams.
$M$	Value of big $M$ .
$T$	Time horizon.
$\omega_{crew}$	Wages of a repair team per hour.
$\omega_{road}$	Driving cost of a repair team per km.

### Variables

$PG_{g,t}$	Active power generation of unit $g$ at time $t$ .
$PL_{ij,t}$	Active power flow on line $i \rightarrow j$ .
$LP_{i,t}$	Actual active load at bus $i$ and time $t$ .
$\theta_{i,t}$	Voltage phase at bus $i$ and time $t$ .
$AT_{x,c}$	Arrival time of team $c$ at damaged component $x$ .
$ALS_{ij,t}$	Binary variable equals to 0 if line $i \rightarrow j$ is damaged or under repair and 1 else at time $t$ .
$LS_{ij,t}$	Binary variable equals to 0 if line $i \rightarrow j$ is removed from the system and 1 else at time $t$ .
$BS_{i,t}$	Binary variable equals to 0 if bus $i$ is damaged or under repair and 1 else at time $t$ .
$GS_{g,t}$	Binary variable equals to 1 if unit $g$ is committed and 0 else at time $t$ .
$FT_{x,t}$	Binary variable equals to 1 if damaged component $x$ is repaired at time $t$ and 0 else at time $t$ .
$M_{x,y,c}$	Binary variable equals to 1 if team $c$ moves from damaged component $x$ to $y$ and 0 else.
$N_{x,c}$	Binary variable equals to 1 if damaged component $x$ is repaired by team $c$ and 0 else.
$S_{x,t}$	Binary variable equals to 0 if damaged component $x$ is damaged or under repair and 1 else at time $t$ .

## I. INTRODUCTION

THE save and stable operation of modern power systems has become the one of the most important guarantee for the

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development and prosperity of human race. With the development of electric technology, the strength of the power system equipment is constantly enhanced, the scale of power system interconnection is increasing, and consequently the power system is becoming stronger. However, natural disasters, human attacks and terrorism are increasingly frequent, resulting in a large number of blackouts [1-3]. In order to reduce the losses caused by high-impact low-probability (HILP) disasters, the power system resilience [4] has been the main focus of researchers and scholars in recent years.

According to the evolving nature of general disasters, reference [5] firstly divided the research of power system resilience into three parts: pre-disaster system toughness, during-disaster system resistance and post-disaster system restoration ability. As the core part of resilience research, the enhancement strategies of resilience can also be implemented from the three parts. For post-disaster resilience research, how to rapidly repair the damaged components and restore curtailed load with limited resources [6-8] is the most urgent and significant task for post-disaster restoration.

It has been proved that the coordination of damage repair and power system restoration is a challenging problem [9]. Reference [10] modeled the transportation of repair crews as a vehicle routing problem (VRP) in detail, and a two-stage method for the repair and restoration of distribution networks is proposed to minimize the sizes and durations of outages. However, for generation and transmission systems, existing models are relatively rough. This paper proposes a novel co-optimization model to coordinate the damage repair work with power system operation. Once the damage information is available for utilities, the maintenance department can make the best damage repair route. Simultaneously, according to the anticipated available states of fault components, the dispatching department will change the system operational state. Thus, the economic loss of outage will be minimized, with reasonable repair cost. The proposed model is tested on the IEEE RTS 79 test system [11].

The remainder of this paper is organized as follows. Section 2 describes the mathematical formulation of the proposed model. Numerical studies are provided in Section 3 to exhibit the effectiveness of the proposed method. Finally, the conclusions are drawn in Section 4.

## II. MATHEMATICAL MODEL

The post-disaster restoration problem is modeled as a MILP problem to coordinate the damage repair work and power system operation optimization.

### A. Objective Function

The objective is to minimize the post-disaster power outage loss due to load shedding with smaller repair cost, as shown in (1). The objective function can be formulated as follows:

$$\begin{aligned} \min \quad & \alpha_1 C_{rep} + \alpha_2 C_{loss} \\ \left\{ \begin{aligned} C_{rep} &= \omega_{crew} \sum_{\forall c} AT_{d,c} + \omega_{road} \sum_{\forall x,y,c} M_{x,y,c} d_{x,y} \\ C_{loss} &= \sum_{t=1}^T \sum_{\forall i} el_i LP_{i,t} \end{aligned} \right. \end{aligned} \quad (1)$$

where  $C_{rep}$  represents the damage repair cost, including labor and transportation cost, and  $C_{loss}$  is the total economic loss due to outage.  $\alpha_1$  and  $\alpha_2$  are weight factors. The value of  $\alpha_2$  is correspondingly larger.

Besides, we need to point out that because the repair cost and unit restoration cost are fixed once the damaged components are identified, so they are not involved in the objective function.

### B. Constraints

#### 1) Constraints of damage repair

$$\sum_{\forall x \in DC \cup b} M_{x,y,c} - \sum_{\forall z \in DC \cup d} M_{y,z,c} = 0, \forall c, y \in DC \quad (2)$$

$$\sum_{x \in DC} M_{b,x,c} = 1, \forall c \quad (3)$$

$$\sum_{x \in DC} M_{x,b,c} = 0, \forall c \quad (4)$$

$$\sum_{x \in DC} M_{x,d,c} = 1, \forall c \quad (5)$$

$$\sum_{x \in DC} M_{d,x,c} = 0, \forall c \quad (6)$$

$$\sum_{\forall c} N_{d,c} = Num_c \quad (7)$$

$$\sum_{\forall c} N_{x,c} = 1, \forall x \in DC \quad (8)$$

$$N_{x,c} = \sum_{\forall y \in DC \cup d} M_{x,y,c}, \forall c, x \in DC \cup b \quad (9)$$

$$RES_x N_{x,c} \leq Cap_c, \forall x \in DC, c \quad (10)$$

$$\sum_{\forall x \in DC, c} RES_x N_{x,c} \leq res_d \quad (11)$$

$$AT_{x,c} + RT_{x,c} + d_{x,y} / V_c - AT_{y,c} \leq (1 - M_{x,y,c}) M, \quad (12)$$

$$\forall x \in DC \cup b, y \in DC \cup d, c \quad (13)$$

$$0 \leq AT_{x,c} \leq N_{x,c} M, \forall x \in DC \cup b \cup d, c \quad (13)$$

$$\sum_{t=1}^T FT_{x,t} = 1, \forall x \in DC, t \quad (14)$$

$$\sum_{t=1}^T t FT_{x,t} \geq \sum_{\forall c} (AT_{x,c} + RT_{x,c} N_{x,c}), \forall x \in DC, t \quad (15)$$

$$\sum_{t=1}^T tFT_{x,t} \leq \sum_{\forall c} (AT_{x,c} + RT_{x,c}N_{x,c}) + 1 - \varepsilon, \forall x \in DC, t, c \quad (16)$$

$$S_{x,t} = \sum_{\tau=1}^{t-1} FT_{x,\tau}, \forall x \in DC, t \geq 2 \quad (17)$$

In the repair route of team  $c$ , if  $c$  leaves  $x$  for  $y$ ,  $M_{x,y,c}$  equals to 1. Constraint (2) ensures that repair team  $c$  will leave damaged component  $y$  once  $y$  is repaired. Constraints (3)-(7) ensure that each team starts from the repair center and finally returns to it after all the missions are completed. Constraint (8) ensures that each damage point will be repaired by one and only one team, avoiding the overlap of tasks. Constraint (9) builds the relationship between  $N_{x,c}$  and  $M_{x,y,c}$ .

Constraint (10) indicates that the repair resources needed by team  $c$  to finish its assignment should not exceed its capacity limit, while constraint (11) ensures that the repair center has enough resources to serve all repair teams.

The time it takes for  $x$  to return to the available state is composed of the waiting time, the route time of repair teams and the repair time. Constraint (12) helps to find the arrival time of team  $c$  at fault component  $y$ . Team  $c$  arrives at  $x$  at  $AT_{x,c}$ , and time  $RT_{x,c}$  is spent to repair the damage. Then it takes  $d_{x,y}$  time to drive to  $y$ . The constraint doesn't work if  $M_{x,y,c}$  is 0 by the big  $M$  method. If damaged component  $x$  is not repaired by team  $c$ ,  $N_{x,c}$  equals to 0, and consequently  $AT_{x,c}$  equals to 0, or  $AT_{x,c}$  is not affected by constraint (13). When  $FT_{x,t}$  equals to 1, it means that damaged component  $x$  is repaired at  $t$ . Constraint (14) ensures that each damage is repaired once, which is essential for the model. Constraints (15) and (16) help to calculate the restoration moment of damaged component  $x$ . As mentioned above,  $N_{x,c}$  and  $AT_{x,c}$  are 0 if  $x$  is not repaired by team  $c$ , making no influence on the two constraints.  $\varepsilon$  is a very small positive number. Constraint (17) helps to obtain the available states of damaged components, which is important information for system dispatchers.

## 2) Constraints of power system operation and component operation states

$$B_{ij}(\theta_{i,t} - \theta_{j,t}) - PL_{ij,t} + (1 - LS_{ij,t})M \geq 0, \forall i, j, t \quad (18)$$

$$B_{ij}(\theta_{i,t} - \theta_{j,t}) - PL_{ij,t} + (1 - LS_{ij,t})M \leq 0, \forall i, j, t \quad (19)$$

$$-LS_{ij,t}S_{ij} \leq PL_{ij,t} \leq LS_{ij,t}S_{ij}, \forall i, j, t \quad (20)$$

$$\sum_{g \in i} PG_{g,t} - LP_{i,t} - \sum_{j: i \rightarrow j} PL_{ij,t} + \sum_{e: e \rightarrow i} PL_{ei,t} = 0, \forall i, t \quad (21)$$

$$BS_{i,t}\theta_i^{\min} \leq \theta_{i,t} \leq BS_{i,t}\theta_i^{\max}, \forall i, t \quad (22)$$

$$0 \leq LP_{i,t} \leq BS_{i,t}lp_{i,t}, \forall i, t \quad (23)$$

$$PG_{\min} \leq PG_{g,t} \leq PG_{\max}, \forall g \in i, i \notin DB, t \quad (24)$$

$$BS_{i,t} = 1, \forall i \notin DB, t \quad (25)$$

$$ALS_{ij,t} = 1, \forall i \rightarrow j \notin DL, i \notin DB, j \notin DB, t \quad (26)$$

$$\begin{cases} ALS_{ij,t} \leq BS_{i,t}, \\ ALS_{ij,t} \leq BS_{j,t}, \\ ALS_{ij,t} \geq BS_{i,t} + BS_{j,t} - 1, \\ \forall i \rightarrow j \notin DL, (i \notin DB \vee j \notin DB), t \end{cases} \quad (27)$$

$$LS_{ij,t} \leq ALS_{ij,t}, \forall i \rightarrow j, t \quad (28)$$

$$GS_{g,t} = 1, \forall g \in i, i \notin DB, t \quad (29)$$

$$GS_{g,t} = BS_{i,t}, \forall g \in i, i \notin DB, t \quad (30)$$

The relationship between bus voltage phase and line flow are constrained by constraints (18) and (19). The big  $M$  method is used here to ensure that the voltage magnitudes of bus  $i$  and  $j$  will be irrelevant if the line  $i \rightarrow j$  is out of service. The capacity limit of active power flow on transmission lines is ensured by constraint (20). Constraint (21) indicates the active power balance at each bus and time  $t$ . Constraints (22)-(23) are the limits of bus voltage phase and actual active power supply, which should not exceed their upper and lower bounds. The operation state  $BS_{i,t}$  of bus  $i$  at time  $t$  helps to ensure these variables are limited to 0 if bus  $i$  is damaged or under repair. Constraint (24) defines the active power output limits for unit  $g$ , if unit  $g$  doesn't malfunction or locate on damaged buses.

The operation state variable  $BS_{i,t}$  of bus  $i$  at time  $t$  equals to 1 if the bus is neither damaged nor under repair, as shown in constraint (25). Constraint (26) indicates that the available state variable  $ALS_{ij,t}$  of line  $i \rightarrow j$  at time  $t$  equals to 1 if the line and its linked buses are normal. Constraint (27) presents that the available state  $ALS_{ij,t}$  of the undamaged line  $i \rightarrow j$  is determining by  $BS_{i,t}$  and  $BS_{j,t}$ . As for the operation state of line  $i \rightarrow j$ ,  $LS_{ij,t}$  is 0 when the line is damaged or under repair, and 1 or 0 according to the decision of by system dispatchers, when  $ALS_{ij,t}$  equals to 1, as expressed by constraint (28).

Besides, the operation state variable  $GS_{g,t}$  of unit  $g$  at time  $t$  equals to 1 if the unit is neither damaged nor under repair, as shown in constraint (29). If the unit is on a damaged bus, then the state of the unit follows that of the bus, which is expressed by (30).

## 3) Coupling constraints

$$ALS_{x,t} = S_{x,t}, \forall x \in DL, t \quad (31)$$

$$BS_{x,t} = S_{x,t}, \forall x \in DB, t \quad (32)$$

The fault repair route problem and post-disaster power system operation problem can be extracted from the above constraints. When components are damaged or under repair, they cannot be put into operation and will influence the topology of power systems. Therefore, the two problems are coupled by the two coupling constraints, as shown in (31) and (32).

### III. CASE STUDY

The co-optimization model is tested on the IEEE RTS 79 test system, to demonstrate the effectiveness of the model. To reduce the computational complexity, the clustering method proposed in [10] is used to acquire repair missions of each repair center. The objective of the clustering model is to allocate the repair center of each cluster and reduce the sum of the distances between repair centers and fault points. It should be ensured that each fault point is repaired and each repair center has enough resources to finish its mission. The clustering result is listed in Table I, while 1 indicates the damaged component will be repaired by the repair center, and 0 else.

It is assumed that the damage situations of the post-disaster power system are known, as shown in Fig. 1. The distance between different fault points, the distance between fault points and repair centers, the needed damage repair resources and time of fault points, the resource amount of repair centers and the economic loss of lost load are displayed from Table II to Table V.

The time length of restoration horizon  $T$  is 50h. An hourly simulation step is adopted. The economic loss of lost load is divided into four categories: \$10/kWh for the most important load, \$6.979/kWh for important load, \$3.706/kWh for ordinary load and \$0.110/kWh for least important load. Each repair team consists of 5 members, and the average wage of each member is assumed to be \$70/h. The gas cost for each team is \$33/100km. The average driving speed of each team is 50km/h. Rating of each transmission line is 100MVA. The weights in the objective function are set to be  $\alpha_1=1$ ,  $\alpha_2=10$ , to highlight the necessity of reducing the economic loss due to power outage.

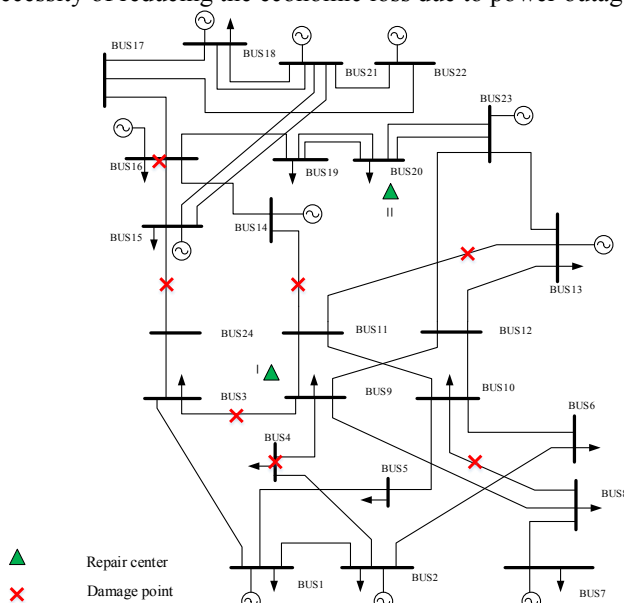


Fig.1. The damage locations of the post-disaster IEEE RTS 79 system

TABLE I  
DAMAGE REPAIR ASSIGNMENT

Repair center	Bus number		Line number				
	4	16	18	19	27	13	6
1	1	0	1	0	1	1	1
2	0	1	0	1	0	0	0

TABLE II  
DISTANCE BETWEEN FAULT POINTS

Numbers of damaged components		Bus number		Line number				
		4	16	18	19	27	13	6
Bus	4	0	180	80	60	60	30	40
	16	180	0	45	25	65	30	40
Line	6	80	45	0	20	50	30	40
	13	60	25	20	0	40	30	40
	18	60	65	50	40	0	30	40
	19	30	30	30	30	30	0	40
	27	40	40	40	40	40	40	0

TABLE III  
DISTANCE BETWEEN FAULT POINTS AND REPAIR CENTERS

Repair center	Bus number		Line number				
	4	16	18	19	27	13	6
1	60	100	60	30	50	30	40
2	80	70	90	40	50	100	110

TABLE IV  
DAMAGE REPAIR RESOURCE AND TIME OF FAULT POINTS

Repair requirements	Bus number		Line number				
	4	16	18	19	27	13	6
resources	30	35	10	6	14	8	6
repair time	15	18	8	10	7	10	8

TABLE V  
RESOURCE CAPABILITY OF REPAIR CENTERS

Repair center	Resource (capability of each team)
1	70 (30; 40)
2	45

Two cases are designed in this part to prove the advantages of the proposed co-optimization model.

Case 1: the damage repair scheme and the power system operation optimization are co-optimized using the proposed method.

Case 2: the damage repair scheme is formulated independently to realize the minimization of the repair expenses. According to the damage repair scheme and the available states of system components, the power system operation optimization is conducted to minimize the power generation cost and power outage loss.

The repair routes with and without co-optimization are shown in Fig. 2 and Fig. 3. The economic indices with and without co-optimization are depicted in Table VI.

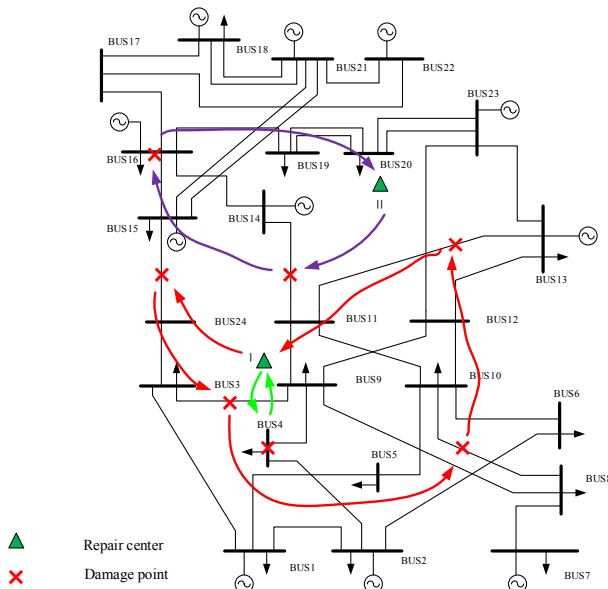


Fig.2. The repair routes with co-optimization

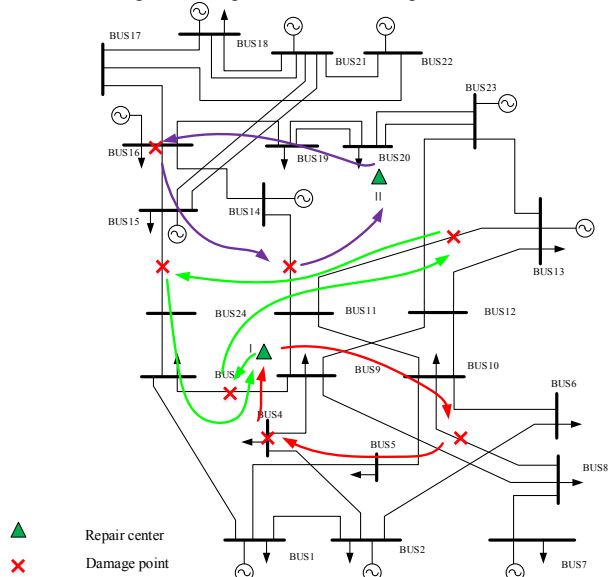


Fig.3. The repair routes without co-optimization

TABLE VI

THE RESTORATION INDICES WITH AND WITHOUT CO-OPTIMIZATION

Restoration index	Restoration with co-optimization	Restoration without co-optimization
Damage repair expense(\$1000)	28.42	27.26
Power outage loss(\$1000)	13036.46	15757.63
Load loss(MWh)	67.22	65.25

When the post-disaster power system doesn't restore with co-optimization, the damage repair will be reduced marginally. However, the power outage loss is greatly increased. Moreover, though the restoration ignoring co-optimization cannot ensure the fast and effective restoration of the important load.

By contrast, if the vehicle routing problem is coordinated with the power system operation and dispatching, the power outage loss will be greatly reduced, especially for the most important load, which will bring more obvious social benefits and accord better with requirements of the post-disaster power system restoration. Hence, the calculation results demonstrate that the effect and necessity of the proposed post-disaster restoration model.

#### IV. CONCLUSION

Resilience of power system has emerged as a novel concept to resist the damage of various disasters. This article proposes a new MILP model for post-disaster power system and resilience enhancement. The model coordinates the problem of VRP and power system operation problem together to restore load curtailed and repair faulty components due to the destruction of disasters, which helps to minimize the power outage loss with possible small expenses of damage repair. The proposed model is validated by the IEEE RTS 79 test system, and the results indicate that the proposed model can realize the enhancement of resilience and effective restoration of post-disaster power systems.

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