

# Distribution System Resilience Enhancement through Restoration Paths between DERs and Critical Loads

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**Abstract –** Hardening and operational measures in electrical distribution systems (EDSs) aim at improving the resilience of EDS in case of natural disasters. This paper proposes a novel two-stage framework for establishing optimal restoration paths for supplying critical loads (CLs) in the aftermath of a natural disaster that will improve the resilience of EDSs. To this end, in the first stage, an algorithm is introduced to find all possible candidate paths between available distributed energy resources (DERs) and CLs, the output of which is applied to the second stage, as the inputs. Subsequently, in the second stage, the problem of finding the optimal restoration paths is modeled as a mixed integer linear programming (MILP). In the proposed mathematical formulation, only the status of candidate paths is represented by binary variables, thereby reducing the complexity of the problem. The 33-bus distribution test system is used in order to demonstrate the effectiveness of the proposed framework.

**Index Terms –** Electrical distribution system (EDS), critical loads, distributed energy resource (DER), resilience, natural disasters.

## I. INTRODUCTION

Enhancing the resilience of power grids is a significant issue. The reason for this is that the frequency as well as intensity of extreme weather events have considerably increased in recent years [1]. Hurricane Katrina in 2005, Japan Earthquake in 2011, Hurricane Sandy in 2012, and Hurricane Michael in 2018, to name a few, caused multiple simultaneous outages in power grids. For example, Hurricane Sandy, which hit the Northeast U.S., brought about severe damages to 100000 span wires. In addition, it resulted in power outages for approximately 7.5 million customers [2]. In this regard, power grids are potentially vulnerable to natural disasters. In response, different methods have been proposed to improve the resiliency of power systems against these events [3].

However, the recovery time of electrical distribution systems (EDSs) after extreme weather events or natural disasters is usually longer than that of generation or transmission systems, which is evidenced by some records [4]. Furthermore, at an extreme event onset, multiple permanent faults occur on the components of EDSs, which tends to isolate parts or the entire of an EDS from the upcoming network. Moreover, restoring power to different customers could last for a few days or even weeks [5]. On these circumstances, the loads can be supplied from local distributed energy resources (DERs). However, due to the limits of the

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power supply by DERs, available power generation should serve critical loads, such as hospitals and water stations. In this regard, this paper aims to restore critical loads (CLs) through establishing paths between available DERs and CLs.

In this context, various analytical methods have been presented in the literature for improving the resilience of EDSs, which can be categorized into two groups, namely hardening and operational measures. The first group proposed different strategies for enhancing the design and construction of EDSs. For example, in [6], an optimal hardening strategy is presented that improves the resilience of EDSs against extreme weather events. To this end, this paper formulates the problem as a tri-level optimization framework in which load curtailment as well as hardening investment are taken into account. Likewise, reference [7] outlines different hardening strategies, such as substation elevation and undergrounding. On the other hand, in the second group, the works aim at proposing methods that are associated with actions such as topology switching and generation reschedule during extreme weather events. For instance, authors in [8] present preventive scheduling for microgrids in the case of extreme floods, in which all flood-prone elements are de-energized before flood arrival. Reference [9] develops a framework for constructing multiple microgrids in order to enhance distribution system resilience.

This paper proposes a novel two-stage framework in order to supply CLs after natural disasters. For this purpose, the first stage introduces an algorithm whereby all possible candidate paths between available DERs and CLs are found, which are employed as inputs in stage 2. Subsequently, in the second stage, the problem for constructing restoration paths is modeled as a mixed integer linear programming (MILP). In this stage, the found candidate paths are combined so as to construct final restoration paths. Furthermore, compared with the traditional optimization models, in the proposed formulation, only the status of candidate paths is represented by binary variables, thereby reducing the complexity of the model.

The remainder of this paper is organized as follows. The proposed framework is discussed in Section II. In addition, the algorithm for finding all possible paths between available DERs and CLs is described in this section. Section III presents the problem formulation. Case studies are provided in Section IV. Finally, Section V concludes the paper.

## II. PROPOSED FRAMEWORK FOR ESTABLISHING RESTORATION PATHS IN ELECTRICAL DISTRIBUTION SYSTEMS

The proposed framework consists of two stages. In stage 1, an algorithm is proposed for forming all possible candidate paths between available DERs and all CLs. It should be noted that in this paper, the available DERs refer to generation sources that have not been damaged during an extreme event. Stage 2 evaluates the feasibility of the candidate paths found in stage 1 and then determines the optimal restoration paths.

### A. Establishing Candidate Paths between DERs and CLs

This part presents an algorithm by which all paths with no loops between DERs and CLs are identified. To achieve this aim, at first, an EDS is transferred to a graph in which each node and branch are depicted by a vertex and edge, respectively. Subsequently, the proposed algorithm is implemented in the following steps:

- Step 1: The locations of faulty branches are identified.
- Step 2: The edges corresponding to the faulty branches are removed.
- Step 3: Using the algorithm described in Fig.1, all possible paths between available DERs and CLs are constructed.
- Step 4: Constructed paths are categorized into some groups based on their starting node.

The flowchart illustrated in Fig. 1 describes the algorithm for finding the paths without loops between two nodes. It should be notified that this method is based on depth first search (DFS) algorithm. DFS is a systematic methodology for visiting vertices of graphs [10]. To this end, DFS starts from the initial node (source) of a graph and then goes to the farthest node that it can go down the particular path. Subsequently, DFS backtracks until all unexplored paths are found, and the path is explored. This task is done until all the vertices of the graph have been explored. On these bases, the flowchart for finding all paths between two nodes, namely starting (DERs) and destination (CLs), is given as follows:

- Step 1: A stack is defined. Each DER node, which is called the starting node, is assumed to be the current node.
- Step 2: The current node is pushed into the stack. In addition, the current node is regarded as an explored node.
- Step 3: If there is an unexplored node in the adjacent node set of the current node, the node is selected and the algorithm goes to step 2. Otherwise, the algorithm goes to Step 4.
- Step 4: If the current node is not the target node and there is not any adjacent node for the current node, the current node is popped from the stack.
- Step 5: When the current node is the destination node, two possible cases can occur. If there is a repetitive node in the stack, the current node is popped due to the presence of a loop. Otherwise, the path is stored as a candidate path between the DER and CL node. Subsequently, the current node is popped from the stack.

- Step 6: While the stack is not empty, Steps 3-5 are repeated.

It should be noted that each restoration path can include multiple DERs as well as CLs. In addition, the algorithm finds all possible paths between an available DER and CLs. Indeed, in the aftermath of a natural disaster, multiple permanent faults occur at different sections of an EDS. As a result, the number of candidate restoration paths will be limited. In this paper, a candidate path refers to the buses and lines that are energized through the path. Moreover, a line is in operation when both ending nodes of the line have been energized.

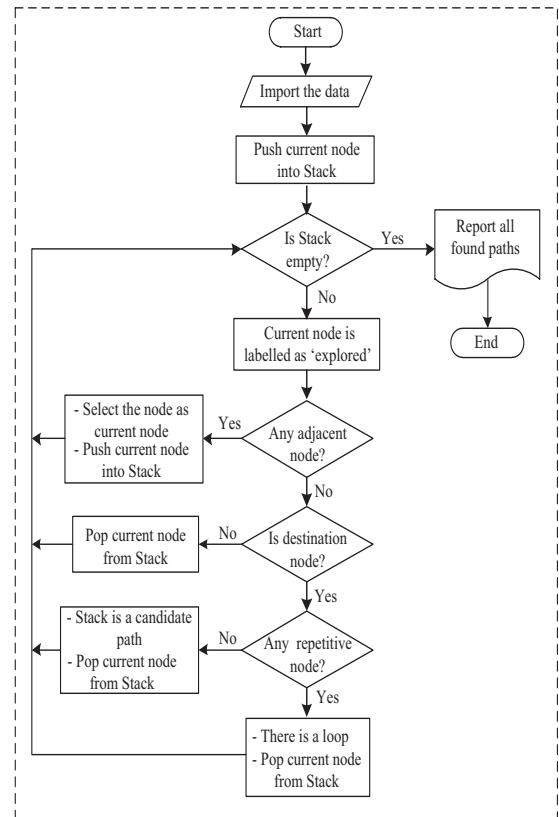


Fig. 1. Flowchart for finding all paths between two nodes of a distribution network

### B Evaluation of Candidate Restoration Paths

When a set of candidate paths are found, it is required to assess the constructed candidate paths so as to find optimal restoration paths that satisfy all of the considered constraints. To this end, the proposed method is modeled as a MILP optimization problem. The objective is to maximize the amount of supplied CLs. In doing so, available DERs could supply high-priority loads. Indeed, priority factors are used to categorize the loads based on their importance. It should be notified that determining priority factors is out of the scope of this paper.

Conventional EDSs are operated in radial configuration in which, there is only one path between each node and substations. This leads to lower short-circuit current. Moreover, a radial structure makes coordination of protective

devices easier [11]. In this regard, the proposed framework should enforce the radial topology of final restoration paths.

### III. PROBLEM FORMULATION

This section presents the mathematical formulation of the proposed framework for establishing optimal restoration paths between available DERs and all CLs. It should be noted that the constructed candidate paths found in the previous stage are considered as the inputs of the optimization problem. As noted earlier, in contrast to the traditional approaches in which the status of lines and buses is represented by binary variables in optimization problems, in the proposed framework, only the status of found candidate paths is represented by binary variables.

#### A. Objective Function

In this paper, the proposed framework aimed at increasing supplied CLs is given as:

$$\max \sum_{k \in \Omega_{CL}} \omega_k PL_k \quad (1)$$

where  $\omega_k$  is the priority factor for each CL. In addition,  $PL_k$  denotes the amount of supplied active power at each load point  $k$ .  $\Omega_{CL}$  stands for the set of CLs.

#### B. Constraints

This part expresses different operational as well as structural constraints that are taken into consideration in this study.

**Connectivity:** The connectivity of each candidate path is enforced by the algorithm introduced in Section II.

**Radial Structure:** Each candidate path is radial. However, in the case of candidate path combinations, the candidate paths that form any loops cannot be combined. In other words, prohibited node groups that form loops, cannot be energized simultaneously. This means that the radial structure of each final restoration path is satisfied in two steps, namely when the candidate paths are being formed and when the final restorations paths are being constructed. By doing so, the radial structure of final candidate paths is guaranteed.

**Power Flow Equations:** Due to the nonlinearity of AC power flow equations, the linear version of DistFlow equations [12] is used in this paper. The great merit of this formulation is to guarantee the global optimal solution. Equations (2) and (3) respectively express the active and reactive power balance constraints at each node. In addition, the relationship between the voltage magnitudes of two adjacent nodes is given by Equation (4).

$$P_{Gi} - P_{Di} + P_{Li}^{cur} = \sum_j P_{ij}^b \quad \forall i \in \Omega_N, (ij) \in \Omega_b \quad (2)$$

$$Q_{Gi} - Q_{Di} + Q_{Li}^{cur} = \sum_j Q_{ij}^b \quad \forall i \in \Omega_N, (ij) \in \Omega_b \quad (3)$$

$$V_j = V_i - \left( \frac{r_{ij} P_{ij}^b + x_{ij} Q_{ij}^b}{V_0} \right) \quad \forall i \in \Omega_N, (ij) \in \Omega_b \quad (4)$$

where  $P_{Gi}$  and  $Q_{Gi}$  respectively stand for the active and reactive power generated at node  $i$ . In addition, the active and reactive flows from node  $i$  to node  $j$  on line  $(ij)$  are denoted by  $P_{ij}^b$  and  $Q_{ij}^b$ , respectively. In these equations,  $P_{Di}$  and  $Q_{Di}$  refer to the active and reactive demand at bus  $i$ . The active and reactive amount of load curtailment at bus  $i$  are shown by  $P_{Li}^{cur}$  and  $Q_{Li}^{cur}$ , respectively. In equation (4),  $V_i$  represents the voltage magnitude of bus  $i$ . The resistance and reactance of line  $(ij)$  are respectively shown by  $r_{ij}$  and  $x_{ij}$ . In addition,  $\Omega_N$  as well as  $\Omega_b$  account for the set of energized nodes and lines, respectively. It should be notified that the superscripts “min” and “max” refer to the lower and upper bounds of each variable, respectively.

**Voltage Limits:** Equation (5) is related to the lower and upper bound of voltage magnitude at each bus.

$$V_i^{\min} \leq V_i \leq V_i^{\max} \quad \forall i \in \Omega_N \quad (5)$$

**Generation Limits:** The active and reactive power generated by each DER must be limited by their maximum available capacities, which are specified by (6) and (7). In these equations,  $\Omega_g$  refers to the set of in-service DERs.

$$P_{Gi}^{\min} \leq P_{Gi} \leq P_{Gi}^{\max} \quad \forall i \in \Omega_g \quad (6)$$

$$Q_{Gi}^{\min} \leq Q_{Gi} \leq Q_{Gi}^{\max} \quad \forall i \in \Omega_g \quad (7)$$

**Line Flow Limit:** The constraints of line flow limits are imposed through (8) and (9).

$$-P_{ij}^{b,\max} \leq P_{ij}^b \leq P_{ij}^{b,\max} \quad \forall (ij) \in \Omega_b \quad (8)$$

$$-Q_{ij}^{b,\max} \leq Q_{ij}^b \leq Q_{ij}^{b,\max} \quad \forall (ij) \in \Omega_b \quad (9)$$

**Load Shedding:** The amount of load shedding for each load point must not exceed the associated demand.

$$0 \leq P_{Li}^{cur} \leq P_{Di} \quad \forall i \in \Omega_{CL} \quad (10)$$

$$0 \leq Q_{Li}^{cur} \leq Q_{Di} \quad \forall i \in \Omega_{CL} \quad (11)$$

**Candidate Paths:** The constraint on candidate path combinations is expressed by (12).

$$\sum_z \delta_z^p \leq 1 \quad (12)$$

in which,  $\delta_z^p$  is a binary variable that represents the presence of path  $p$  in the final restoration paths, which is equal to 1 if the associated elements of the path have been energized. Otherwise, it is equal to 0. In addition,  $z$  refers to the set of meeting paths. It should be noted that some paths may be a part of larger paths. The set of these paths is called meeting paths in this study. With this in mind, in the final restoration paths, at most one candidate path from each set of meeting paths must be chosen. To this end, Equation (12) is added to the mathematical formulation. It should be notified that this is

the only binary variable used in the model formulation. This is because the status of lines, buses, and DERs are specified by candidate paths.

#### IV. NUMERICAL RESULTS

In this section, the proposed framework for constructing restoration paths between available DERs and CLs is implemented on the 33-bus test system, the single line diagram of which is depicted in Fig. 2 [13]. It is assumed that all DERs are available. In addition, the proposed formulation is implemented and solved using MATLAB software.

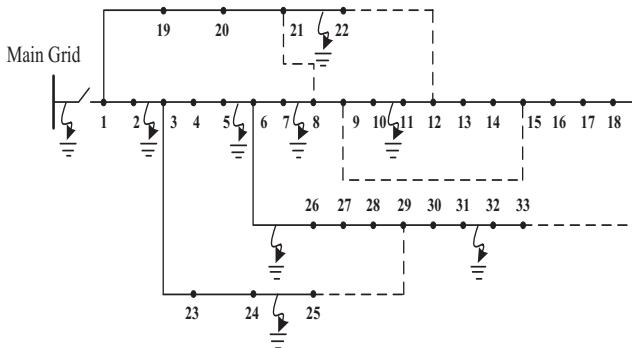


Fig. 2. Single line diagram of 33-bus distribution test system.

As it can be seen in Fig. 2, the faulty lines are 2-3, 5-6, 7-8, 10-11, 21-22, 6-26, 24-25 and 31-32. Moreover, it is assumed that there is a permanent fault in the upstream network, which results in the isolation of the distribution system from the main grid. In this situation, the loads could only be supplied by the available DERs. The corresponding data associated with available DERs are summarized in Table I. Without loss of generality, these DERs are assumed to be dispatchable DERs.

TABLE I  
LOCATIONS AND SIZES FOR DERs

DER No.	Bus No.	Size	PF
DER 1	5	0.45	0.888
DER 2	7	0.5	0.9
DER 3	13	0.55	0.909
DER 4	21	0.9	0.888
DER 5	27	0.95	0.895

It should be noted that in this paper all quantities are expressed in per unit based on 1 MVA and 20 kV. In addition, it is assumed that the distribution system is fully automated. Table II shows the information about the CLs in the test system.

TABLE II  
INFORMATION ABOUT CRITICAL LOADS IN DISTRIBUTION SYSTEM

CL No.	Bus No.	P <sub>D</sub>	Q <sub>D</sub>
CL 1	1	0.3	0.15
CL 2	9	0.4	0.19
CL 3	11	0.6	0.29
CL 4	17	0.8	0.35
CL 5	19	0.3	0.15
CL 6	23	0.7	0.33
CL 7	26	0.55	0.27

CL 9	31	0.29	0.15

#### A. Finding Possible Candidate Paths

In this part, the introduced algorithm in Section II is employed in order to find all possible paths between available DERs and CLs. In the distribution grid under study, there are five DERs and nine CLs. By multiplying the associated number of available DERs and CLs, 45 node pairs are obtained. Moreover, each pair of nodes can be connected by more than one path. However, as discussed earlier, in the case of natural disasters, multiple prolonged faults occur over a wide area in an EDS, restricting the number of candidate paths. For example, in the case under study, 12 candidate paths exist under the disaster event corresponding to the faults shown in Fig. 2. These paths are listed in Table III.

TABLE III  
FOUND CANDIDATE PATHS UNDER A DISASTER EVENT

Path No.	Candidate Paths
P1	5-4-3-23
P2	13-14-15-9-8-21-20-19-1
P3	13-14-15-9
P4	13-12-11
P5	13-14-15-16-17
P6	13-14-9-8-21-20-19
P7	21-20-19-1
P8	21-8-9
P9	21-8-9-15-14-13-12-11
P10	21-8-9-15-16-17
P11	21-20-19
P12	27-26

Each path starts from a DER node and reaches a CL node. In addition, no loops exist in each path. As can be seen in Table III, each path can include multiple DERs and CLs. For example, P2 contains 2 available DERs (DER 3 and 4) and 3 CLs (CL 1, 2, and 5). Furthermore, the combination of P3, P7, P8, and P11 generates P2. Therefore, several meeting paths are constructed from the found candidate paths.

#### B. Establishing Final Restoration Paths

After the candidate paths have been determined, they are used as the input of the optimization problem. In this paper, the value of  $V_i^{\max}$  and  $V_i^{\min}$  are assumed to be 1.05 and 0.95 p.u., respectively. In order to analyze the impacts of priority factors on the constructed final paths, two different cases are considered.

##### Case 1: The priority factors of CLs are the same.

In this case, for the sake of simplicity, the priority factor for each CL is assumed to be 1. The final constructed restoration paths that satisfy operational, connectivity, and radiality constraints are given in Table IV.

TABLE IV  
FINAL RESTORATION PATHS OF CASE 1

Path No.	Final Paths	P <sub>L(p.u.)</sub>
P1	5-4-3-23	0.4
P2, P9	11-12-13-14-15-9-8-21-20-19-1	1.3
P12	27-26	0.55

As can be seen in this table, four candidate paths are chosen for constructing final restoration paths. By doing so,

the distribution system is split into three self-adequate islands. It should be noted that the combination of P2 and P9 leads to the second final restoration path. The value of the objective function in this case is 2.25. It should be noted that DER 2 placed at bus 7 is not in any candidate path. This is because DER 2 is surrounded by 3 permanent faults on lines 5-6, 7-8, and 6-26. Consequently, the DER has been isolated from the rest of the distribution system.

#### Case 2: The priority factors of CLs are different.

This case examines the effect of priority factors of CLs on the final restoration paths. To this end, the priority factors for CL4 and CL2 are assumed to be 3 and 2, respectively. In addition, it is assumed that the priority factor for other CLs is equal to 1. The results of this case are listed in Table V.

TABLE V  
FINAL RESTORATION PATHS OF CASE 2

Path No	Final Paths	P <sub>L</sub> (p.u.)
P1	5-4-3-23	0.4
P4, P5, P10	11-12-13-14-15-16-17-9-8-21	1.3
P12	27-26	0.55

Supplying the high-priority critical loads leads to different results compared to Table IV. Similar to Case 1, the amount of supplied load is 1.3 p.u. in this case. However, the value of the objective function is 3.3. It should be noted that P1 as well as P12 are common in the two cases. The reason is that the locations of permanent faults in the distribution test system result in the isolation of some sections. For example, P1 is an isolated section that includes one DER and one CL. In this situation, if the operational constraints are satisfied, DER 1 can only feed CL6.

#### V. SUMMARY AND CONCLUSION

Natural disasters can significantly disturb the operation of power grids. Hence, it is necessary to improve the resilience of EDSs by application of the hardening and the operational measures. This paper presented a novel framework whereby critical loads are fed through available DERs in the aftermath of extreme events, such as storms. This framework consists of two stages. In the first stage, all possible paths between available DERs and CLs are found. For this purpose, a method based on the DFS algorithm has been introduced in the paper. Subsequently, the second stage finds the optimal restoration paths.

In order to examine the performance of the proposed framework after natural disasters, two case studies have been defined and studied. Furthermore, multiple prolonged faults over a wide area have been considered for simulating the impact of an extreme event on the distribution systems. The results confirmed the effectiveness of the proposed framework for supplying high-priority loads during extreme events. In addition, the results showed how priority factors of different CLs can change the final restoration paths. Furthermore, by considering the binary variables only for the status of candidate paths, the complexity of the optimization problem has been reduced. On these bases, the presented framework can be employed for the study of EDSs in order to continue supplying CLs after occurrence of extreme events.

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