Laws of lenses and optical instruments

Related Topics
Law of lenses, magnification, focal length, object distance, telescope, microscope, path of a ray, convex lens, concave lens, real image, virtual image.

Principle
The focal lengths of unknown lenses are determined by measuring the distances of image and object and by Bessel's method. Simple optical instruments are then constructed with these lenses.

Equipment

1	Lens, mounted, f = +20 mm	08018-01
1	Lens, mounted, f = +50 mm	08020-01
1	Lens, mounted, f = +100 mm	08021-01
1	Lens, mounted, f = +300 mm	08023-01
1	Lens, mounted, f = –50 mm	08026-01
1	Lens, mounted, f = –200 mm	08028-01
1	Screen, translucent, 250×250 mm	08064-00
1	Screen, with arrow slit	08133-01
1	Ground glass screen, 50×50×2 mm	08136-01
1	Double condenser, f = 60 mm	08137-00
1	Stage micrometer, 1 mm - 100 div.	62171-19
1	Dog flea, Ctenocephalus, mip	87337-10
1	Slide -Emperor Maximilian-	82140-00
1	Optical profile-bench, l = 1000 mm	08282-00
2	Base f. opt.profile-bench, adjust.	08284-00
5	Slide mount f. opt.pr.-bench, h = 30 mm	08286-01
1	Slide mount f. opt.pr.-bench, h = 80 mm	08286-02
2	Diaphragm holder	08040-00
2	Lens holder	08012-00
1	Condenser holder	08015-00
1	Swinging arm	08256-00
1	Experiment lamp 5, with stem	11601-10
1	Power supply 0-12 V DC/6 V, 12 V AC	13505-93
2	Connecting cord, l = 500 mm, blue	07361-04
1	Rule, plastic, l = 200 mm	09937-01

Tasks
1. To determine the focal length of two unknown convex lenses by measuring the distances of image and object.
2. To determine the focal length of a convex lens and of a combination of a convex and a concave lens using Bessel's method.
3. To construct the following optical instruments:
    1. Slide projector; image scale to be determined
    2. Microscope; magnification to be determined
    3. Kepler-type telescope
    4. Galileo's telescope (opera glasses).

Set-up and procedure
The experiment is set up as shown in Fig. 1. A parallel light beam is produced with the lamp and the double condenser.

1. The object (screen with arrow slit) is directly behind the condenser, and a clear image is projected on to the screen with a lens. The distances of image and object from the lens are measured (assume that the lenses are thin). The measurement of distances of image and object is repeated, using both lenses and with the lens and the screen in different positions.

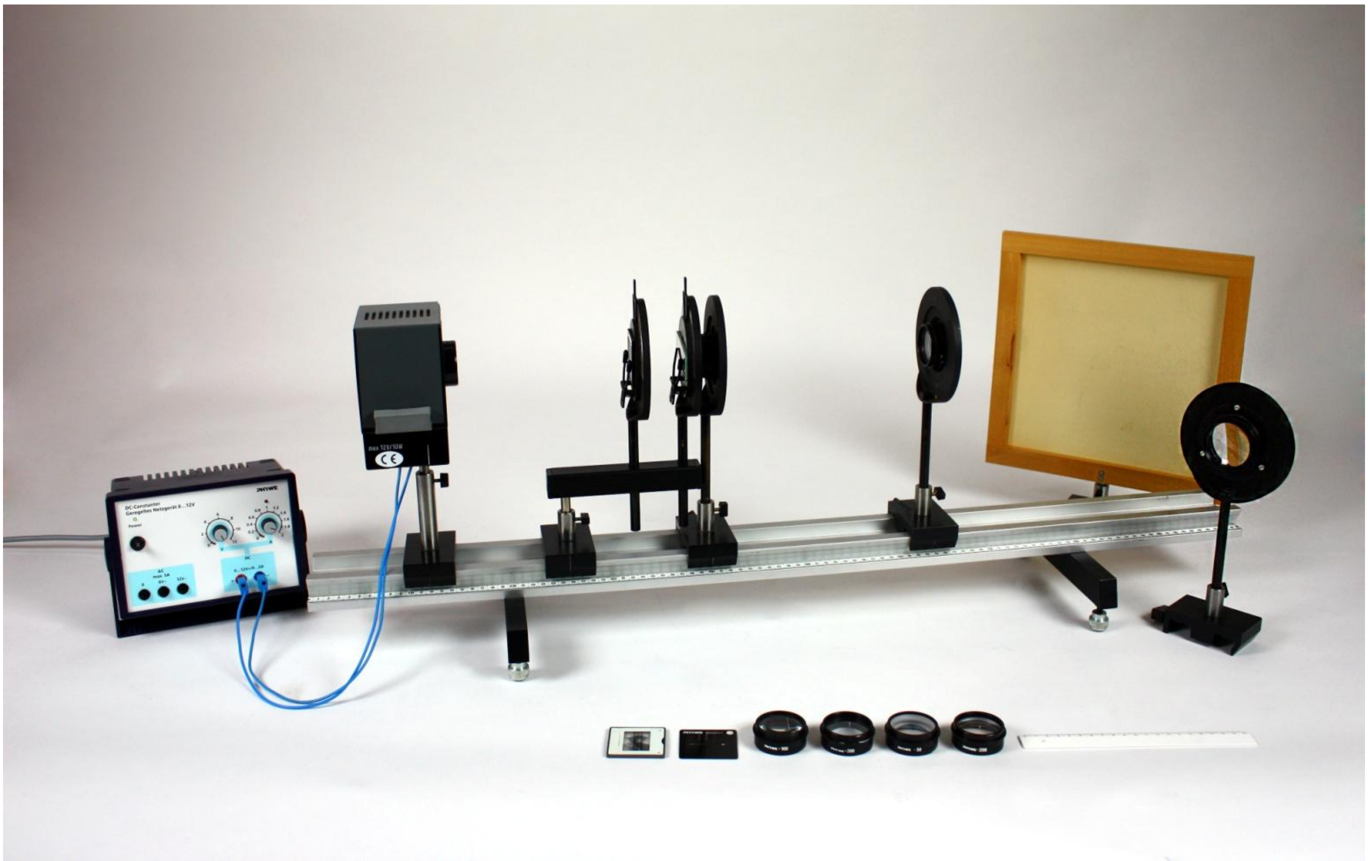

Fig. 1: Experimental set-up (microscope).

2. If, at a fixed distance *d* between object and image (case I), we alter the position of the lens so that the image and object distances are transposed (case II), we still obtain a clear image of the object. In case I the image is magnified, in case II it is reduced (Fig. 2).

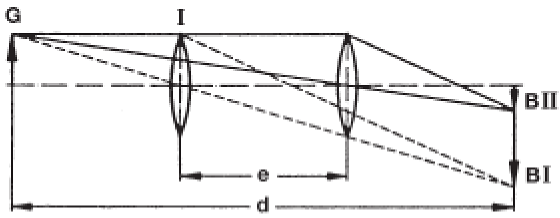

Fig. 2: Determination of focal length after Bessel.

Using a convex lens of focal length +100 mm, for instance, measure the distance *e* at which a sharp image is obtained for both possible lens positions (repeat the measurement and calculate the average value $\bar{e}$. Now take a measurement in the same way but using the convex lens from the first measurement and a concave lens (-200 mm for example). Make the distance *d* as large as possible, and measure at least four times the combined focal length.

3.1. Slide projector

Place the slide – Emperor Maximilian – immediately behind the condenser and project an image on the screen with the lens L_2 (f_2 = +100 mm).

To obtain the best image illumination set the condenser so that the image of the lamp coil is in the plane of objective lens L_2 (Fig. 3).

Determine the magnification M of the image

$$M = \frac{B}{G}$$

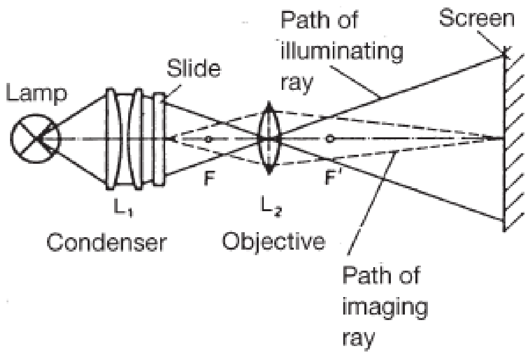

Fig. 3: Path of a ray in a slide projector.

3.2 Microscope

A magnified image of a small object (stage micrometer and micro-slide of a dog flea) is produced with a lens L1 of short focal length f_1 = +20 mm. The real intermediate image is observed through an eyepiece L_2 (f_2 = +50 mm) (Fig. 4). The ground glass and the object holder with the object are fixed in the swinging arm. L_1 is brought as close to the object as possible. The object is illuminated through a ground glass screen.

The size of the image and thence the overall magnification are roughly determined by comparing it with a scale at the least distance of distinct vision (approximately 25 cm). To do this we look through the microscope with the right eye and at the scale with the left. With practice the two images can be superimposed.

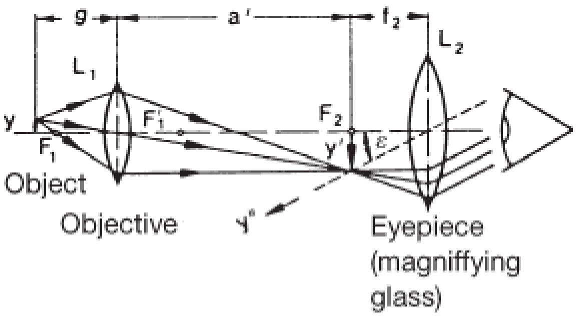

Fig. 4: Path of a ray in the microscope.

3.3 Telescope after Kepler

A convex lens of long focal length f_1 (+300 mm, for example), and one of short focal length f_2 (e.g. +50 mm) are secured to the optical bench at a distance of $f_1 + f_2$ (Fig. 5).

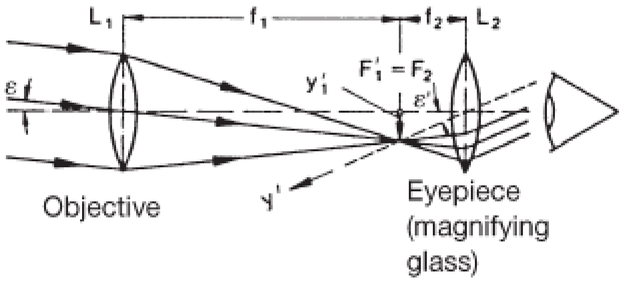

Fig. 5: Path of a ray in a Kepler telescope.

If we look through the lens of short focal length, we can see an inverted, magnified image of a distant object.

3.4 Galileo telescope (opera glasses)

A convex lens of long focal length f_1 (+300 mm, for example) and a concave lens of short focal length f_2 (e. g. -50 mm) are set up at a distance of $f_1 - |f_2|$ (Fig. 6).

Through the concave lens we can see distant object magnified and the right way up.

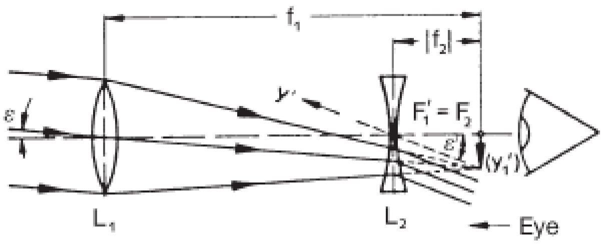

Fig. 6: Path of a ray in Galileo telescope.

Theory and evaluation

The relationship between the focal length *f* of a lens, the object distance *g* and the image distance *b* is obtained from geometrical optics. Three particular rays, the focal ray, the parallel ray and the central ray, are used to construct the image (Fig. 7).

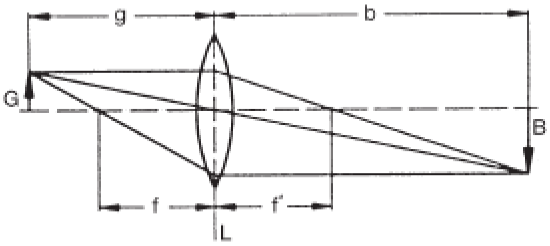

Fig. 7: Image construction with three principal rays.

From the laws of similar triangles,

$$\frac{B}{G} = \frac{b}{g} \text{ and } \frac{G}{B} = \frac{f}{b-f}$$

where *B* is the image size and *G* is the object size.

By transforming we obtain the lens formula

$$\frac{1}{f} = \frac{1}{b} + \frac{1}{g} \text{ or } f = \frac{b \cdot g}{b+g}$$

1. From the values of *b* and *g* measured in Task 1 we calculate *f*, the average value of *f* and its standard deviation. For the first lens (100 mm) f_1 was 100.2 mm with a standard deviation s_{f_1} of 0.6 mm; for the second (50 mm), f_2 was 53.1 mm with a standard deviation s_{f_2} of 0.9 mm. (The focal lengths marked on the lenses have a tolerance of $\pm$ 5%.)

2. Since $g_I = b_{II}$ (the object distance in case I = image distance in case II) and since $g_I = b_{II}$,

$$g_I + b_I = d$$

$$g_I - b_I = e$$

(see Fig. 2).

If we solve the equations for g_I and b_I we obtain

$$g_I = \frac{1}{2}(d+e)$$

$$b_1 = \frac{1}{2}(d-e)$$

Substituting into the lens formula gives

$$f = \frac{d^2 - e^2}{4d}$$

The focal length of the convex lens can therefore be determined
from the measured values of *d* and *e*.

If we now use a lens system of focal length f_{comb}. Consisting of the convex lens already measured (focal length f_s) and a concave lens, and carry out the measurement in the same way, we obtain the following for the focal length of the concave lens f_z:

$$\frac{1}{f_z} = \frac{1}{f_{comb}} - \frac{1}{f_s} \text{ or } f_z = \frac{f_{comb} \cdot f_s}{f_s - f_{comb}}$$

Here we assume that

$$\frac{1}{|f_s|} > \frac{1}{|f_z|}$$

as otherwise no real images would be produced. f_2 was 99.7 mm for the convex lens (+100 mm), f_{comb}. was 180 mm for the combination of two lenses (+100 mm/-200 mm) so that f_z = -223 mm represents the focal length of the concave lens. (The combination of two lenses involves a systematic error as the distance between the principal planes is disregarded).

3.1 The magnification *M* is obtained from the relationship between object size and image size *b*

$$M = \frac{B}{G} = \frac{b-f}{f}$$

When the image distance *b* is 700 mm and the focal length *f* =100 mm, then *M* = 6.

3.2 The overall magnification is obtained by multiplying the magnification due to the objective (Fig. 4),

$$ß_{objective} = \frac{Y'}{Y} = \frac{a'}{g} = \frac{a'}{f_1} - 1$$

by the angular magnification of the eyepiece

$$\Gamma_L = \frac{250mm}{f_2}$$

With the lenses used we obtain an overall magnification *M* = 60.

3.3 The objective L_1 provides a real, inverted image of size Y'_1 of a very distant object, and this image is observed through the eyepiece L_2. The angular magnification (for small angles) is

$$\Gamma_L = \frac{\varepsilon'}{\varepsilon} = \frac{Y'_1 / f_2}{Y'_1 / f_1} = \frac{f_1}{f_2}$$

3.4 A concave lens is placed in the path of the ray in front of the real first image produced by objective L_1 so that the focal points F_1 and F_2 coincide. The eye then sees a virtual, upright image. The magnification is once again

$$\Gamma_L = \frac{f_1}{|f_2|}$$

Note

You may wish to cover the markings on the lenses used to measure focal length by means of opaque adhesive tape while performing the experiment, and reveal these values only when the evaluation is finished.

