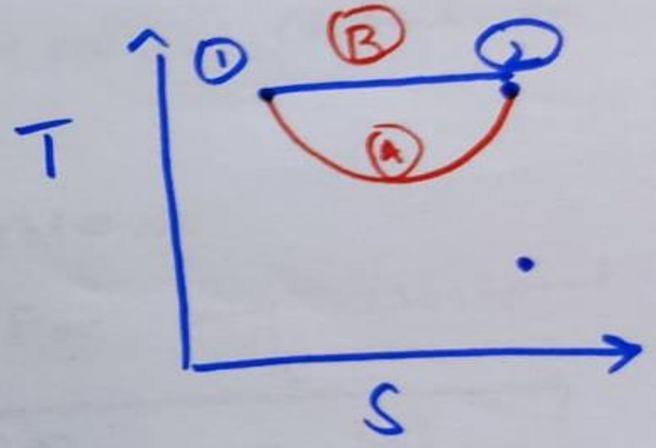
Heat transfer formulas :-

Isobaric Process $\boxed{Q = m c_p \Delta T}$

Isochoric Process $\boxed{Q = m c_v \Delta T}$

Isothermal Process :-

$\boxed{T = C}$



For an Ideal gas

$\boxed{PV = C}$ -> Hyperbolic equation/relationship

B is isothermal
A is not isothermal path

$PV = C$

→ For ideal gas this relation only represents an Isothermal process.

→ But for pure substances or real gases this $PV = C$ is a hyperbolic relation.

$\boxed{Q_{12} = \Delta U + W_{1-2}}$ Closed system

$\boxed{\Delta U = m c_v \Delta T}$

$T = C$

$\Delta T = 0$

$\Delta U = 0$

For any isothermal process

Hence, $\boxed{Q_{1-2} = W_{1-2}}$

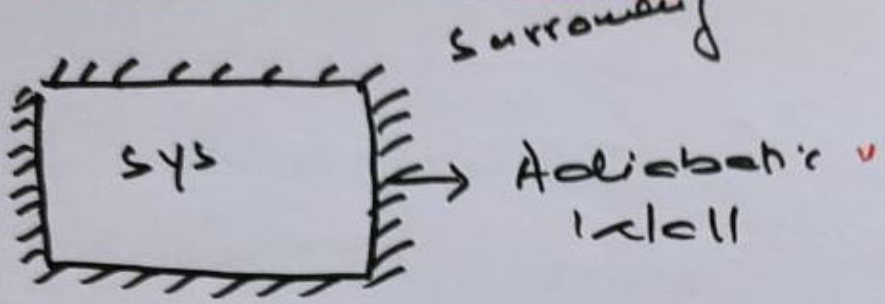
$W_{1-2} = P_1 V_1 \ln V_2/V_1 = Q_{1-2}$

or $P_1 V_1 \ln P_1/P_2$

Same formula

(2

Adiabatic Process



$$Q_{1-2} = \Delta U + W_{1\to 2}$$

$$Q_{1\to 2} = 0 \qquad \text{Adiabatic}$$

$$0 = \Delta U + W_{1\to 2}$$

$$\boxed{\Delta U = -W_{1-2}}$$

In an adiabatic process if work transfer is taking place it will change the internal energy of the system only!

$$W_{1\cdot 2} = \frac{P_1 V_1 - P_2 V_2}{\gamma - 1}$$

$$\gamma = C_p / C_v$$

Energy of an Isolated System

$$dQ = dE + dW$$

Perfectly insulated

$dQ = 0$ or $dW = 0$

$$0 = dE + 0$$

$$dE = 0$$

No change in total energy of the system

For an isolated system their is no change in total energy of system.

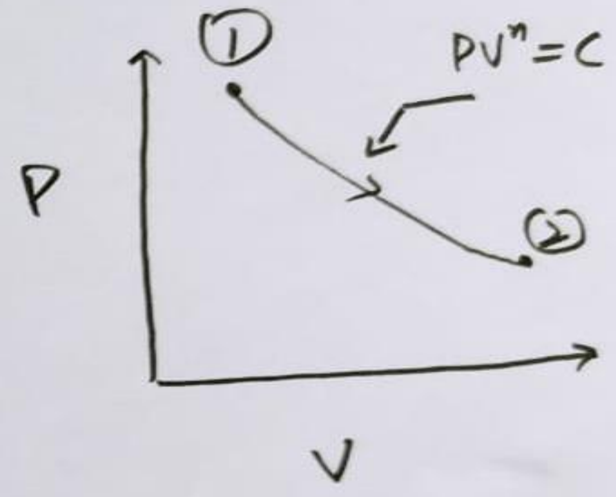
$$\boxed{E = \text{constant}}$$

(2

Heat Transfer in a Polytropic Process

Closed system $\rightarrow$

$$\boxed{Q_{12} = \Delta U + W_{12}}$$



$$Q_{1-2} = mC_v(T_2 - T_1) + \frac{P_1V_1 - P_2V_2}{n-1}$$

$$C_v = \frac{R}{\gamma - 1}$$

$$Q_{1-2} = \frac{mR(T_2 - T_1)}{\gamma - 1} + \frac{P_1V_1 - P_2V_2}{\gamma - 1}$$

for an Ideal gas

$PV = mRT$

$$Q_{12} = \frac{P_2V_2 - P_1V_1}{\gamma - 1} + \frac{P_1V_1 - P_2V_2}{n-1}$$

$mRT = PV$

$mR(T_2 - T_1) = P_2V_2 - P_1V_1$

$$= \frac{P_1V_1 - P_2V_2}{n-1} - \frac{P_1V_1 - P_2V_2}{\gamma - 1}$$

$$= (P_1V_1 - P_2V_2)\left(\frac{1}{n-1} - \frac{1}{\gamma-1}\right)$$

$$= P_1V_1 - P_2V_2\left(\frac{\gamma \cancel{-1} - n \cancel{+1}}{(n-1)(\gamma-1)}\right)$$

$$= (P_1V_1 - P_2V_2)\left(\frac{\gamma - n}{(n-1)(\gamma-1)}\right)$$

$$Q_{1-2} = \left(\frac{P_1V_1 - P_2V_2}{n-1}\right)\left(\frac{\gamma - n}{\gamma - 1}\right)$$

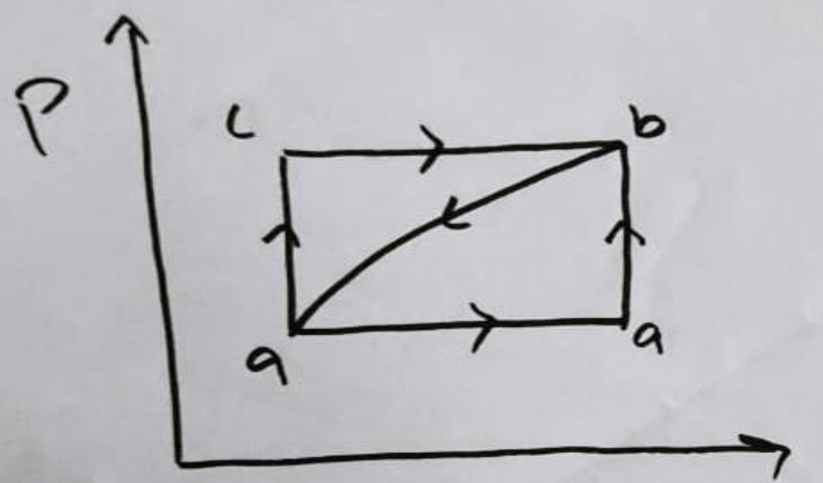
(3)

$$Q_{1-2} = W_{1-2}\left(\frac{\gamma - n}{\gamma - 1}\right)$$

$$\boxed{Q_{poly} = W_{poly}\left(\frac{\gamma - n}{\gamma - 1}\right)}$$

Relationship b/w heat transfer & work transfer in a polytropic process

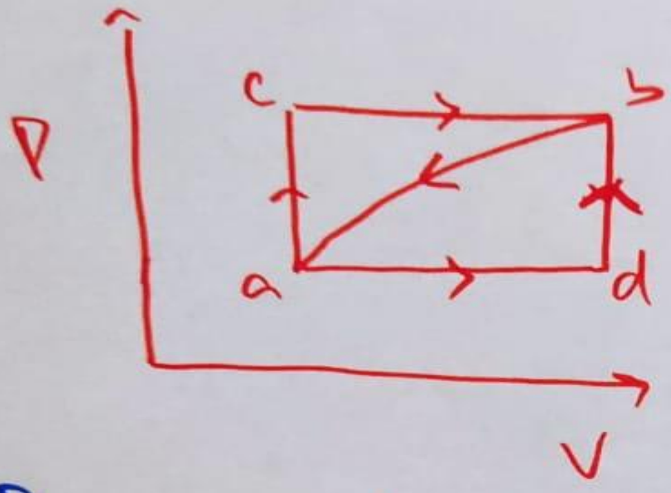
Problems:



When a system is taken from state "a" to state "b" along path "acb", 84 KJ of heat flows into the system and system does 32 KJ of work.

1) How much will the heat flow into the system along path "adb" if the work done is 10.5 KJ.

Solution::



For Process acb

$Q = 84\ KJ$ $\quad W = 32\ KJ$

$Q_{acb} = \Delta U + W_{acb}$

$84 = \Delta U + 32$

$\Delta U = 84 - 32$

$\boxed{\Delta U = 52\ KJ}$

$\therefore$ U = Point function

Property of the system

Irrespective of the path followed
Internal energy would be same.

1) Path adb

$$Q_{adb} = \Delta U + W_{adb}$$

$Q_{adb} = ?$ $\quad \Delta U = 52 \text{ KJ}$

$W_{adb} = 10.5 \text{ KJ}$

$$= 52 + 10.5$$

$$\boxed{Q_{adb} = 62.5 \text{ KJ}}$$

(35

(2) When system returns from b-a following the curved path work done on the system is 21 KJ. Does the system absorbs or liberates heat?

Solution:

$W = -21 KJ$ ∴ ON The system from b-a

For b-a

$\Delta U = -52 KJ$

$Q_{ba} = \Delta U + W_{ba}$

a-b $= U_b - U_a$

$\boxed{\Delta U = 52 KJ}$

$Q_{ba} = -52 - 21$

$\boxed{Q_{ba} = -73 KJ}$ liberates heat.

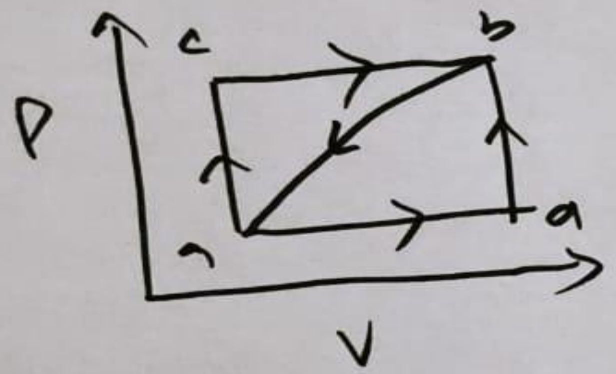
b-a $= U_a - U_b$

$\boxed{\Delta U = -52 KJ}$

(3b

(3) If $U_A = 0$ Find heat absorbed

$U_B = 42\ KJ$ in process db



$Q_{adb} = 62.5\ KJ$

$Q_{db} = ?$

$Q_{adb} = Q_{ad} + Q_{db}$

$Q_{ad} = ?$

$Q_{db} = Q_{adb} - Q_{ad}$

$Q_{ad} = \Delta U_{ad} + W_{ad}$

$\Delta U_{ad} = U_d - U_a = 42 - 0 = 42\ KJ$

$W_{ad} = ?$ $W_{adb} = W_{ad} + W_{db}$

$$W_{adb} = 10.5\ KJ$$

$$W_{db} = 0 \qquad V = C$$

$$W_{adb} = W_{ad} + \cancel{W_{db}}^{\,0}$$

$$\boxed{W_{ad} = 10.5\ KJ}$$

$$Q_{ad} = 42 + 10.5$$

$$\boxed{Q_{ad} = 52.5\ KJ}$$

$$Q_{db} = Q_{ab} - Q_{ad}$$

$$= 62.5 - 52.5$$

$$\boxed{Q_{db} = 10\ KJ}$$

(3θ

Problem A fluid is confined in a cylinder by a spring loaded frictionless piston so that the pressure in the fluid is a linear function of volume

$$P = a + bV$$

The internal energy of fluid is given by

$$U = 34 + 3.15\,PV$$

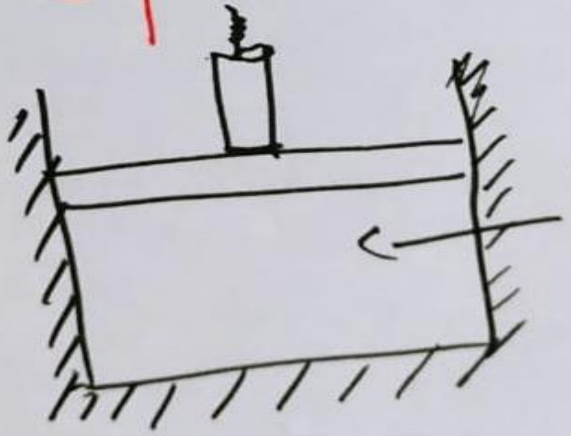
U is in KJ, P in KPa

V is in m^3

BS

if change in initial state of 170 kPa, 0.03 m^3 to a final state of 400 kPa, 0.06 m^3 with no work other than that done by piston.

Find the direction & magnitude of heat transfer.



Linear relation

$P = a + bV$

$P_1 = 170$ kPa

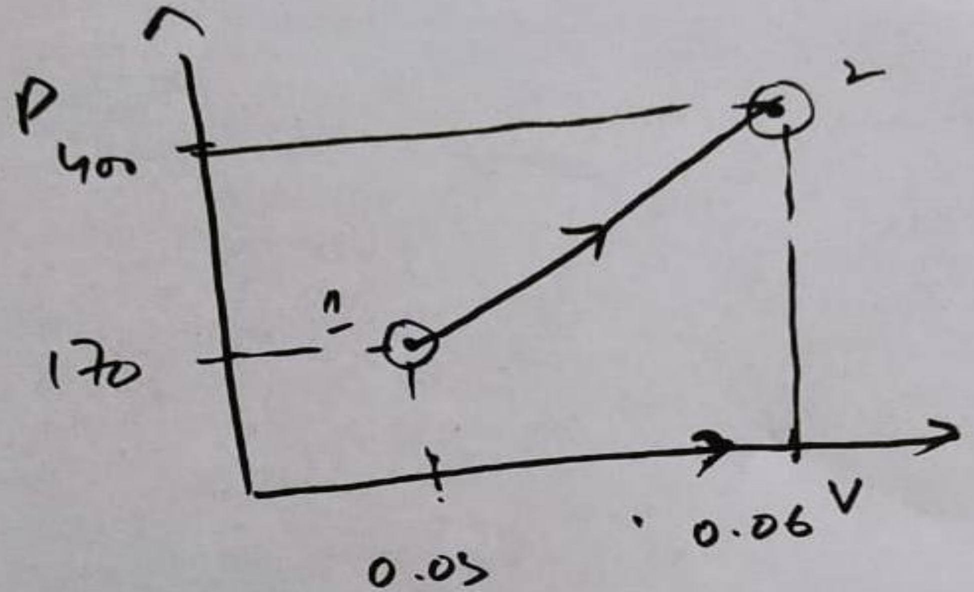
$V_1 = 0.03\ m^3$

$P_2 = 400$ kPa

$V_2 = 0.06\ m^3$

(40

$Q_{1-2} = \Delta U + W_{1-2}$



(41)

$\Delta U = ?$

$W_{1-2} = ?$

$\Delta U = U_2 - U_1$

$U = 34 + 3.15\,PV$

$U_1 = 34 + 3.15\,P_1V_1$

$U_2 = 34 + 3.15\,P_2V_2$

$= 34 + 3.15(P_2V_2) - 34 - 3.15(P_1V_1)$

$= 3.15(P_2V_2 - P_1V_1)$

$= 3.15\,(400(0.06) - 170(0.03))$

(42

$$\boxed{\Delta U = 59.346 \text{ KJ}}$$

$W_{1-2} = ?$

$$W_{1-2} = \int_{V_1}^{V_2} P\,dV$$

$P = a + bV$

$P_1 = a + bV_1$

$P_2 = a + bV_2$

$P_1 = a + 0.03b$

$P_2 = a + 0.06b$

$170 = a + 0.03b$

$400 = a + 0.06b$

$b = 7667 \text{ KPa} \quad a = -60 \text{ KPa}$

$$= \int_{0.03}^{0.06} (a + bV)\,dV$$

$$= \int_{0.03}^{0.06} (-60 + 7667V)\,dV$$

$$W_{1-2} = 8.55 \text{ KJ}$$

$$Q_{1-2} = \Delta U + W_{1-2}$$

$$Q_{1-2} = 59.346 + 8.55$$

$$\boxed{Q_{1-2} = 67.896 \text{ KJ}}$$

+ve value

$\Rightarrow$ Heat is being added into the system.

(43

Problem

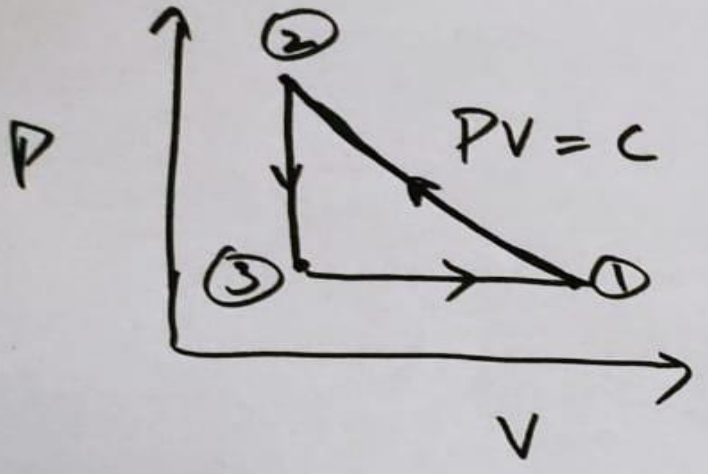
A gas undergoes a thermodynamic cycle consisting of the process beginning at an initial state where $P_1 = 1$ bar, $V_1 = 1.5\ m^3$ and $U_1 = 512$ KJ. The processes are as follows.

1. Process 1-2 compression with PV = constant
   $P_2 = 2$ bar $U_2 = 690$ KJ

2. Process 2-3 $W_{23} = 0$, $Q_{23} = -150$ KJ

3. Process 3-1 $W_{31} = +50$ KJ Neglecting KE & PE changes. Determine heat interaction Q_{12} & Q_{31}?

(44



$$\boxed{PV = C}$$

$$W_{1-2} = P_1 V_1 \ln \frac{V_2}{V_1}$$

or

$$= P_1 V_1 \ln P_1/P_2$$

Is it Isothermal process?

$\Delta U = 0$

$T = C$

For Isotherm in an ideal gas

$$\boxed{Q_{1-2} = W_{1-2}}$$

But here the $\Delta U \neq 0$

$U_1 = 512$ KJ $\qquad U_2 = 690$ KJ

So it is not always necessary that $\boxed{PV = C}$ would mean an isotherm process.

Here it represents a hyperbolic relation

(45)

$$W = P_1 V_1 \ln \frac{V_2}{V_1} \quad \text{or} \quad P_1 V_1 \ln \frac{P_1}{P_2}$$

$P_1 = 1\,\text{bar} = 100\,\text{KPa}$

$P_2 = 2\,\text{bar} = 200\,\text{KPa}$

$U_1 = 512\,\text{KJ}$

$U_2 = 690\,\text{KJ}$

$V_1 = 1.5\,m^3$

$$Q_{1-2} = \Delta U + W_{1-2}$$

$$W_{1-2} = P_1 V_1 \ln \frac{P_1}{P_2}$$

$$= 100(1.5) \ln \frac{100}{200}$$

$$\boxed{W_{1-2} = -103.97\ \text{KJ}}$$

(46

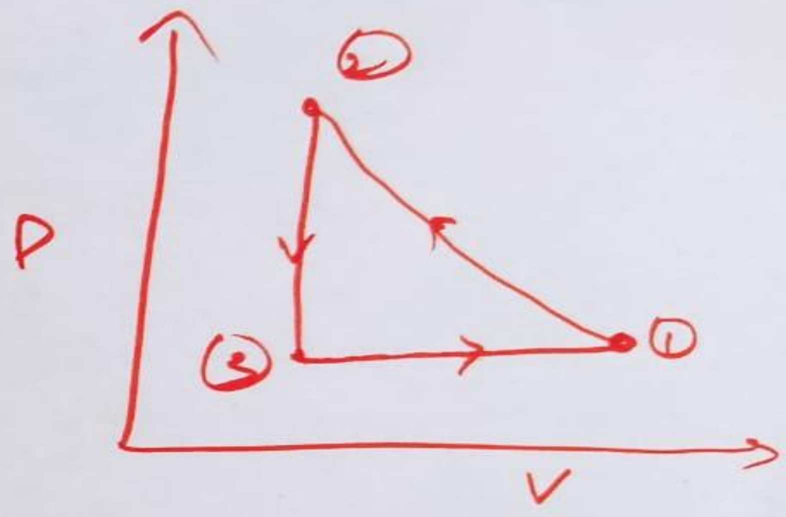
$$Q_{1-2} = \Delta U + W_{1-2}$$

$$= (U_2 - U_1) + 103.97$$

$$= (690 - 512) - 103.97$$

$$\boxed{Q_{1-2} = 74.03 \text{ KJ}}$$

$Q_{31} = ?$



$$Q_{31} = \Delta U + W_{31}$$

$$\Delta U = U_1 - U_3$$

$U_1 = 512$ KJ

$U_3 = ?$

(47

For Process 2-3

$W_{23} = 0$

$Q_{23} = -150\,KJ$

$Q_{23} = \Delta U + \cancel{W_{23}}$

$\Delta U = -150\,KJ$

$U_3 - U_2 = -150\,KJ$

$U_3 = -150 + 690\,KJ$

$\boxed{U_3 = 540\,KJ}$

(c)

$Q_{31} = \Delta U + W_{31}$

$\Delta U = U_1 - U_3$

$= 512 - 540$

$\boxed{\Delta U = -28\,KJ}$

$W_{31} = +50\ KJ$

$Q_{31} = -28 + 50$

$\boxed{Q_{31} = 22\ KJ}$