

**U.S.-PAKISTAN CENTER  
FOR ADVANCED STUDIES  
IN ENERGY (USPCAS-E)**

# Smart Grid Architecture ESE-909 (Core) Stability Analysis Tools for SG (Ch-4)

By

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# Stability Analysis Tools for SG

- **Analysis tools** provide guidance for operational and planning
  - limited to static and dynamic models of electrical system components.
- **Research Potential:** Little work has been undertaken to upgrade these tools for managing the future power system.
- **Main Objective:** An overview of concepts and techniques used in voltage stability assessment for the design of the new tools needed for smart grid analysis.

# Characteristics of analysis tools for SG

No matter how much the circumstances is

1. **Robustness:** Persistence under perturbations or uncertainty.
2. **Scalability:** Accommodate the scalability Ability to accommodate growing amounts of work.
3. **Stochasticity:** Time based analysis in terms of probability.
4. **Predictivity:** Better take preventive measurements. Rigorous (quantitative) forecast of what will occur under specific conditions.
5. **Adaptability:** System can adapt its behavior according to changes in its environment or in parts of the system itself.
6. **Online real-time data acquisition:** Instantaneous data acquisition.

# Suitable analysis tools for SG

**WAM:** Wide Area Monitoring

Measurements for load state conditions.

Data is usually assumed or computed for static model.

New advances in the design of PMUs, smart meters, state estimation (SE) for high order efficiency, system stability & reliability.

**PMU:** Phasor Measurement Unit  
both in distribution and transmission.

PMU data applies to the following:

1. Asset management
2. Voltage stability
3. Angle stability assessment
4. Designing optimum controls

Voltage stability is the main issue.

**Smart Meters:**

Two - way electronic communication meter.

Advanced Metering Infrastructure (AMI) meters have advance features than simple automated meter reading (AMR).

# Research efforts in defining stability methods

## Voltage stability:

- **Probability distribution function** is used to model resources (wind, PV) that will provide the **average power output**. REG: Renewable Energy Generation
- **Average REG source** is included in the network model.
- **Load models** are measured using stochastic, time series methods;  
Line voltage will never be 1. Sometimes it will 1.05 or sometimes it will be 0.95.
- **Line voltages** are provided with some probabilistic measure to assure their capture. **The steps in voltage stability studies includes:**
  1. **Determine** **operating point** for **stability assessment** by performing a probabilistic **DLF (from base case)** **for prime loading condition**. as called as business as usual.  
Direct Load Flow      Check the reference and make system w.r.t it.
  2. **Define or rank** **contingency** based on model of choice. **The selection of method includes:** **(a)** RT use of ant colony; **(b)** ANN; **(c)** Hybrid methods for optimum condition; **(d)** New index methods; **(e)** V collapse methods.  
RT: Real Time

# Research efforts from SG stability perspective

- To enhance the algorithm for SG, a **RT voltage index** based on PMU data serve as a means of **state estimation**.
- The output or input data **use different scenarios** which are available for **developing visualization tools**.
- The **revised stability tool** for SG will form the basis of new software for **next generation** of **SCADA** or **EMS systems**.  
Energy Management System
- The **stability result margin** concept is a simple way to determine the current or actual system condition **from the critical state**.
  - Since a source perturbation **can cause instability** in SG environment.
  - The indicators generally include:

$$\text{Stability power flow} = 100\% * (P_{\max} - P_{\text{base}}) / P_{\text{base}}$$

$$\text{Stability rescue voltage} = 100\% * (V_{\max} - V_{\text{base}}) / V_{\text{base}}$$

It must not be greater than 5%.

# Old and New Grid Methodology Comparison

## Strengths and weaknesses of existing voltage stability analysis tools

Methodology	Old Grid	New Grid
Model Load	Static	Dynamic
Resources	Deterministic	Stochastic
<small>flexible alternating current transmission system</small> FACTS Devices & Control	Specified	Adaptive
Risk Management	<small>Always constant.</small> Deterministic	Random
Protection Platform	Defined	Adaptive
Planning/Scheduling	Off-line	Real-Time (RT) Online

**Load base Voltage Stability Communication Point (VSCP):** Based on required successive load changes and delivery of voltage desired to determine the MW/MVAr that could lead to voltage collapse.

FACTS Devices: If load is more then capacitive part will come to play.

# Transient stability studies

The **optimization steps** are illustrated as follows:

1. Analyze and define the system configuration with all networks in service.
2. Simulate faults such as **single-line-to-ground**, three phase, or **contingency study**.
3. Evaluate the impact of contingency for **branch, unit outage with voltage stability criteria**.
4. Perform stability analysis for **voltage and angle**.

The **time simulation method** and direct method are often used for transient stability analysis.

- |                                       |   |
|---------------------------------------|---|
| <small>Time simulation method</small> | 1. The <b>first method</b> determines transient stability by <b>solving the system differential equation step by step</b> .<br><small>If two machines then two swing equations. Similarly if three machines then three swing equations.</small> |
| <small>Direct method</small>          | 2. The <b>second method</b> determines the system transient stability <b>without explicitly solving the system differential equations</b> .   |



# Implementation challenges in transient stability studies for SG

The **challenges of implementation** are illustrated as follows:

1. They do not include **time stamp of load change**
2. They do not account for **real-time information** on system topology and **devices contributing to system changes**
3. They do not inform the **decision- maker of real-time status** (location of system parameters)

The **future algorithms** aiming at above challenges, might include:

1. **State estimation (SE) or PMU** data filtering so that **appropriate data** on system parameters
2. **A practical index for voltage stability** based on **input data or estimated system parameters.**
3. **Assessment (preventive/corrective)** scheme to **guide the decision-maker.**
4. **OPF formulation** to overcome the **collapse or instability.**

# Flow Charts of Voltage Stability Studies

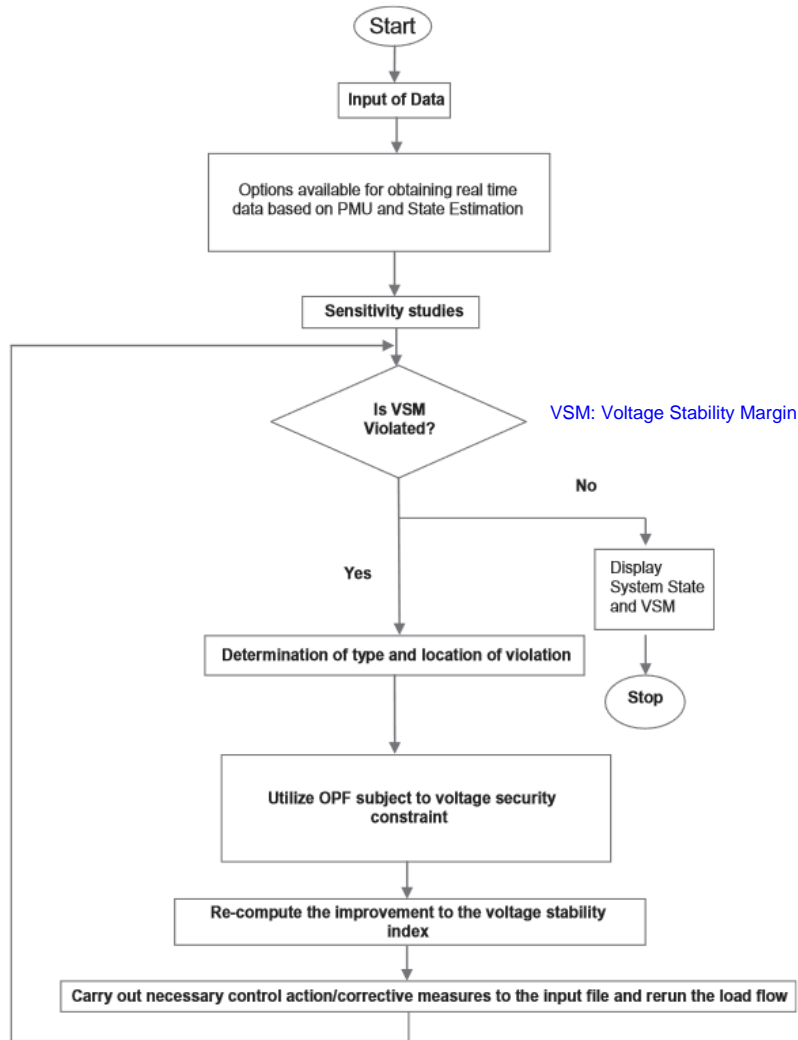


Figure. Flowchart of voltage stability & PMU that enhances the current grid.

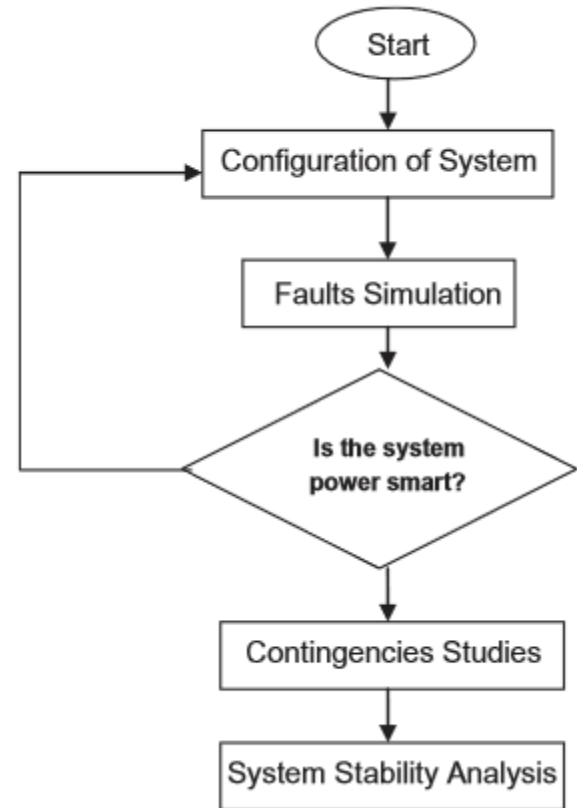


Figure. Voltage stability study algorithm.

# Voltage Stability Assessment Techniques

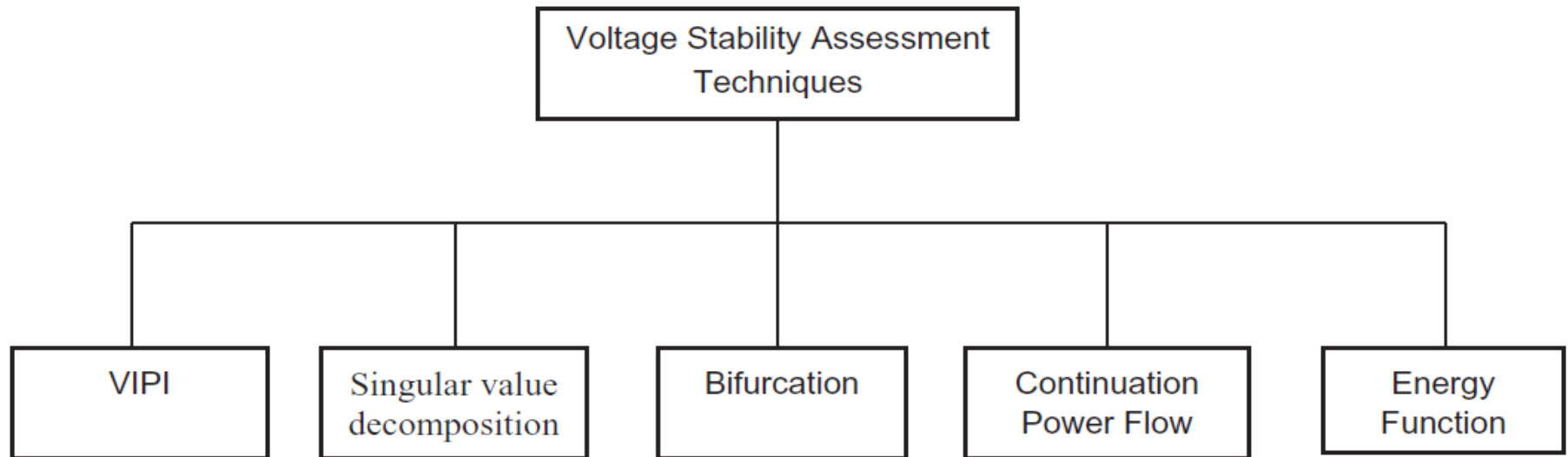


Figure. Voltage stability assessment techniques.

**Home Assignment:** Briefly write/describe about each classical voltage assessment techniques (shown above) with relevant concept. **(2.5 Marks)**

## Voltage Stability Assessment Techniques Contd.

- Voltage instability proximity index (VIPI):
  - It was developed by Tamura et al. based on the concept of voltage stability and multiple load flow solutions.
- Singular value decomposition (SVD):
  - Singular value decomposition method is used to solve the minimum singular value for static voltage stability analysis.
  - When an operating state approaches the collapse point, the J-matrix of the PF equations approaches singularity.

# Voltage Stability Indexing.

IEEE: Power Society for Electrical and Communication

- Voltage stability, defined by the IEEE PSEC as follows:
  - "Voltage stability is the ability of a system to maintain voltage so that, when load admittance is increased, load power will increase, and so that both power and voltage are controllable"

CIGRE: Council on Large Electric Systems

- The IEEE/CIGRE Joint Task Force defines:
  - "Voltage stability refers to the ability of a power system to maintain steady voltages at all buses in the system after being subjected to a disturbance from a given initial operating condition"

# Voltage Stability Indexing. Contd.

The main **incidents** causing voltage instability are:

1. **Load** increase.
2. **Tripping** of power system equipment (such as transmission lines, power transformers & generators).
3. **Exceeding** generators reactive power **limits**.
4. **Malfunction** of on-load tap changer (OLTC) and respective transformers.

When generator is saturate then reactive power will be limited so tripping of generator will cause.

# Voltage Stability Indexing. Contd.

There are countermeasures to avoid voltage instability. The most important ones include:

1. Improvement of weak buses and lines at the planning step of power system (by distributed generation units or other voltage supporting equipment).
2. Shunt capacitor switching.
3. FACTS devices to extend the voltage stability margin (VSM).
4. Tap changer blocking.
5. Load shedding as the last line defense.

# Voltage Stability Indexing. Contd.

- Voltage stability studies utilize **voltage stability index (VSI)** as an **indicator** to a most sensitive node close to voltage collapse.
- Conceptually, if a node or line in a test system is sensitive to voltage collapse, there is a possibility that the whole system approaches the collapse point.
- One of the important applications of the **voltage stability indices (VSIs)** is identifying the weak lines and buses in the power systems.
- The **VSIs are used in offline/online mode**, and the required data are obtained from the **static analysis or phasor measurement units (PMUs)**.
  - The **line/bus with a VSI closest** to the **critical value** is selected as the weakest line/bus.



# Voltage Stability Indexing. Contd.

- This application of VSIs can be used in different cases such as, the placement and sizing of distributed generation (DG) units, capacitor allocation and the planning of power systems.
- In the DG placement and sizing problems, the VSIs are used in two steps.
  - In the first step, the weak buses and lines are selected to determine the candidate location for DG units.
  - In the second one, the optimal location of DG units are determined to maximize the VSM, where the VSM is calculated by the VSIs .

# Voltage Stability Indexing. Contd.

- The problems concerning the **DG placement and sizing** can be **divided into two groups** in terms of their objective functions and each group consists of two steps.
  - In the **first step**, the candidate location for DG units are determined
  - In the **second step**, the optimal locations of DG units are obtained by solving an optimization problem.
- The **first group** only minimizes the power losses and the **candidate locations** for DG units are **usually selected randomly**.
- In the **second group** the objective is to maximize the VSM and simultaneously minimize the power losses where the **candidate locations** for DG units **are chosen by the VSIs**.

# Voltage Stability Indexing. Contd.

- In the **first step of the second group**, the candidate location of DG units is chosen according to VSIs.
  - In the **first step**, nearly all the VSIs that are able to **determine the weak buses and lines** can be used.
  - In the **second step** (second group), optimal location of DG units are determined by **solving an optimization problem that minimizes the power losses and maximizes VSM**.
- With more accurate VSIs, better places can be obtained for the DG units.
  - Moreover, **complex VSIs increase the running time** of the optimization problem **especially in large power systems**.
  - So, **ideally**, an accurate and **simple VSI must be used** in DG placement and sizing problems.
- In order to provide an **overview for the selection of VSIs**, the VSIs are **classified according to their types** (i.e. **line**, **bus** and **overall VSIs**), assumptions, concept, equations and critical values (CV).

# VSI at a glance

	VSI	CV	Equation	Assumptions
1	FVSI	1	$FVSI = (4Z^2 Q_r) / (V_s^2 X)$	$\sin \delta \approx 0, \cos \delta \approx 1$
2	$L_{ij}$	1	$L_{ij} = (4Z^2 Q_r X) / (V_s^2 (R \sin(\delta) - X \cos(\delta))^2)$	$Y \approx 0$
3	$L_{mn}$	1	$L_{mn} = (4X Q_r) / (V_s \sin(\theta - \delta))^2$	Effect of active power neglected, $Y \approx 0$
4	LQP	1	$LQP = 4(X/V_s^2)(Q_r + (P_s^2 X)/V_s^2)$	$R \approx 0, Y \approx 0$
5	$L_p$	1	$L_p = \frac{4RP_s}{(V_s \cos(\theta - \delta))^2}$	Effect of reactive power neglected, $Y \approx 0$
6	NLSI	1	$NLSI = (P_r R + Q_r X) / (0.25 V_s^2)$	$\delta \approx 0, Y \approx 0$
7	VCPI	1	$VCPI(1) = P_r / P_{r(max)}, VCPI(2) = Q_r / Q_{r(max)}$ $VCPI(3) = P_l / P_{l(max)}, VCPI(4) = Q_l / Q_{l(max)}$	Constant power factor, $Y \approx 0$
8	$L_{sr}$	1	$L_{sr} = S_r / S_{r(max)}$	Constant power factor, $Y \approx 0$
9	NVSI	1	$NVSI = (2X \sqrt{P_r^2 + Q_r^2}) / (2Q_r X - V_s^2)$	$R \approx 0, Y \approx 0$
10	$VQI_{line}$	1	$VQI_{line} = (4Q_r) / ( B  V_s^2)$	$\delta \approx 0, Y \approx 0$
11	PTSI	1	$PTSI = (2S_r Z(1 + \cos(\theta - \phi))) / V_s^2$	$Y \approx 0$
12	VSI_1	0	$VSI_1 = \min\left(\frac{P_{max}}{P_{min}}, \frac{Q_{max}}{Q_{min}}, \frac{S_{max}}{S_{min}}\right)$	$R \approx 0, Y \approx 0$
13	VSLI	1	$VSLI = 4[V_s V_r \cos(\delta) - V_r^2 \cos^2(\delta)] / V_s^2$	$Y \approx 0$
14	$L$	1	$L = 4(V_s V_r - V_r^2) / V_s^2$	$Y \approx 0, \delta \approx 0$
15	$VSM_s$	0	$VSM_s = (S_{cr} - S_L) / S_{cr}$	Constant power factor, $Y \approx 0$
16	VCPI_1	0	$VCPI_1 = V_r \cos(\delta) - 0.5 V_s$	$Z_{th(s)} \approx 0$
17	VSI_2	1	$VSI_2 = (4Q_r(R + X^2)) / (X(V_s^2 + 8RQ_r))$	$Y \approx 0, \delta \approx 0$
18	VSMI	0	$VSMI = (\delta_{max} - \delta) / \delta_{max}$	$Y \approx 0$ and $R \approx 0$ in lossless case
19	VSLBI	1	$VSLBI = V_r / \Delta V$	$Z_{th(s)} \approx 0$
20	SI	0	$SI = 2V_s^2 V_r^2 - V_r^4 - 2V_r^2(P_r R + Q_r X)$ $- Z^2(P_r^2 + Q_r^2)$	$Y \approx 0$
21	LCPI	1	$LCPI = \frac{4 A  \cos(\alpha)}{[V_s \cos(\delta)]^2} \left[ \frac{P_r  B  \cos(\beta)}{+ Q_r  B  \sin(\beta)} \right]$	Lines are modeled as pie model
22	$VCPI_{bus}$	1	$VCPI_{bus} = \min\{VCPI_i\}$	Voltage of a bus is not depend on the other bus voltages
23	L-index	1	$L = \max_{j \in \Omega_L} \{L_j\} = \max_{j \in \Omega_L} \left  1 - \sum_{i \in \Omega_G} F_{ji} V_i / V_j \right $	All generator voltages remain unchanged.
24	SDC	0	$SDC =  1 + (\Delta V_r \Delta I_r^*) / (V_r \Delta I_r^*) $	
25	$VSI_{bus}$	0	$VSI_{bus} = [1 + (I_i \Delta V_i) / (V_i \Delta I_i)]^\alpha$	$\Delta V_r \Delta I_r \approx 0$
26	ISI	0	$ISI = (Z_L - Z_{th}) / Z_L = 1 -  I_r \Delta V_r  /  V_r \Delta I_r $	System topology remains unchanged after a disturbance
27	$Z_L / Z_s$ ratio	1	$\frac{Z_L}{Z_s} = \frac{M+1}{-M \cos \beta + [(M \cos \beta)^2 - M^2 + 1]^{0.5}}$	$73^\circ \leq \phi_s \leq 87^\circ$
28	SVSI	1	$SVSI_r = \frac{\Delta V_r}{V_r}$	Voltage of the nearest generator to a load bus is equal to the Thevenin voltage of the network at that bus
29	SG	sharp rise	$SG_p = \frac{P_{pe}}{P_{de}}, SG_q = \frac{P_{qe}}{Q_{de}}$	Power system efficiency is constant

# Line voltage stability indices.

- Voltage stability analysis can be evaluated by the voltage stability index (VSI) referred to a line.
  - Most of the line VSIs are formulated based on the **two bus representation** of a system.
  - Where the **shunt admittances are neglected**

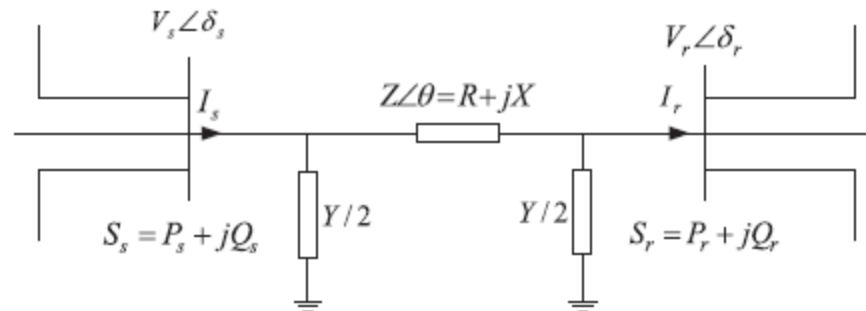


Fig. The two bus representation of a power system.

- The **theoretical base of most of the line VSIs are the same** and the **difference is in the assumptions used in each index**.
- In proving most of the line VSIs, the **discriminant of the voltage quadratic equation is set to be greater than or equal to zero to achieve the stability**.

# Line voltage stability indices. Contd.

## L-VSI-1: Fast voltage stability index (FVSI)

- Musirin et al. proposed the FVSI based on the concept in which the discriminant of the voltage quadratic equation is set to be greater than or equal to zero.

$$V_r^4 + (2P_r R + 2Q_r X - V_s^2)V_r^2 + S_r^2 Z^2 = 0$$

- At voltage collapse point (VCP), above equation have pairs of real identical roots. If the load is increased further, the roots become complex with real and imaginary parts.
  - On the other hand, for the voltage to be stable, the discriminant of must be greater than or equal to zero.
- For a typical line/branch, the FVSI is calculated by:

# Line voltage stability indices. Contd.

## L-VSI-1: Fast voltage stability index (FVSI)

- For a typical line/branch, the FVSI is calculated by:

$$FVSI = \frac{4Z^2 Q_r}{V_s^2 X}$$

- The FVSI must be **below 1 for a stable line/branch**.
  - If FVSI goes **beyond 1**, one of the buses that is connected to the **line will experience a sudden voltage drop leading to system collapse**.
- In this index, the shunt admittance is neglected and the following **assumptions** are also considered:
  - $\sin\delta \approx 0$ ;  $\cos\delta \approx 1$ ;  $R\sin\delta \approx 0$ ;  $X\cos\delta \approx X$**
- If the assumptions are not considered and the index  $L_{ij}$  has been obtained as following:

$$L_{ij} = \frac{4Z^2 Q_r X}{V_s^2 (R \sin(\delta) - X \cos(\delta))^2}$$

# Line voltage stability indices. Contd.

## L-VSI-2: Line Stability Index ( $L_{mn}$ )

It has been proposed by Moghavemmi et al., and is obtained using the **same concept as FVSI** in which the discriminant of the voltage quadratic equation is set to be greater than or equal to zero.

$$L_{mn} = \frac{4XQ_r}{(V_s \sin(\theta - \delta))^2}$$

**In the  $L_{mn}$** , the effect of the active power on the voltage stability as well as the line shunt admittance are neglected.

- As long as the  $L_{mn}$  remains **less than 1**, the **system is stable**
- If index **exceeds the value 1**, the system **loses its stability**.



# Line voltage stability indices. Contd.

## L-VSI-3: Line Stability Factor (LQP)

Mohamed et al. developed the line stability factor, LQP, based on the same concept as two previous line VSIs.

$$LQP = 4 \left( \frac{X}{V_s^2} \right) \left( Q_r + \frac{P_s^2 X}{V_s^2} \right)$$

For the transmission line to be stable, it should be  $LQP < 1$ .

In this index, the lines are assumed to be lossless ( $R/X \ll 1$ ) and the shunt admittance of lines is neglected.

# Line voltage stability indices. Contd.

## L-VSI-4: Line Stability Index (Lp)

The Lp has been designated by Moghavvemi et al. based on the same concept as previous line VSIs. For any value of Lp greater than one, the system is considered as unstable. It is defined as:

$$L_p = \frac{4RP_r}{(V_s \cos(\theta - \delta))^2}$$

In this index, the effect of reactive power on voltage stability as well as line shunt admittance are neglected.

It is assumed that only the active power affects the line voltage stability.

# Line voltage stability indices. Contd.

## L-VSI-5: Novel line stability index (NLSI)

Yazdanpanah-Goharrizi et al. derived it, based on the same concept as Lp.

Any line in the system whose NLSI is close to unity indicates that the line is approaching its stability limit It is defined as:

$$NLSI = \frac{P_r R + Q_r X}{0.25 V_s^2}$$

In this index, the angular difference between the sending and the receiving voltage is assumed to be very small ( $\delta \approx 0$ )

Also the line shunt admittance is neglected.

# Line voltage stability indices. Contd.

## L-VSI-6: Voltage collapse proximity index (VCPI)

- Moghavvemi et al. had proposed four VCPIs for the assessment of the line voltage stability based on the concept of maximum power transferable through a line.
  - VCPIs consider the maximum power transferred through a line and maximum power in a line. It is defined as:

$$VCPI(1) = \frac{P_r}{P_{r(max)}}$$

$$VCPI(2) = \frac{Q_r}{Q_{r(max)}}$$

$$VCPI(3) = \frac{P_l}{P_{l(max)}}$$

$$VCPI(4) = \frac{Q_l}{Q_{l(max)}}$$

- Where maximum powers and losses are calculated by:

$$P_{r(max)} = \frac{V_s^2}{Z} \frac{\cos \varphi}{4 \cos^2 \left( \frac{\theta - \varphi}{2} \right)}$$

$$Q_{r(max)} = \frac{V_s^2}{Z} \frac{\sin \varphi}{4 \cos^2 \left( \frac{\theta - \varphi}{2} \right)}$$

$$P_{l(max)} = \frac{V_s^2}{Z} \frac{\cos \theta}{4 \cos^2 \left( \frac{\theta - \varphi}{2} \right)}$$

$$Q_{l(max)} = \frac{V_s^2}{Z} \frac{\sin \theta}{4 \cos^2 \left( \frac{\theta - \varphi}{2} \right)}$$

- Where  $\varphi$  is phase angle of the load impedance or load power factor (PF). For any values of VCPIs greater than one, the system is exposed to a V-collapse. The PF is assumed constant.

# Line voltage stability indices. Contd.

## **L-VSI-7: New Voltage Stability Index (NVSI)**

The NVSI derived by Kanimozhi et al. is obtained using the **same concept as  $L_p$** . The index is defined for a TL as follows:

$$NVSI = \frac{2X\sqrt{P_r^2 + Q_r^2}}{2Q_rX - V_s^2}$$

For transmission line to be **stable  $NVSI < 1$** . In this index, the line resistance as well as the line shunt admittance are neglected.

# Line voltage stability indices. Contd.

## **L-VSI-8:** Voltage reactive power index ( $VQI_{Line}$ )

This index has been derived by Althowibi et al. based on the **same concept as  $L_p$** . This index is given by:

$$VQI_{Line} = \frac{4Q_r}{|B|V_s^2}$$

Where  $B = \text{Im}(Y) = \text{Im}(1/(R+jX))$ . The **critical value (CV) of  $VQI_{Line}$  is 1** and beyond this value, the voltage will collapse.

In this index,  **$\delta$  is assumed to be zero** and the line shunt admittance is neglected.

# Line voltage stability indices. Contd.

## L-VSI-9: Power transfer stability index (PTSI)

The PTSI has been proposed by Nizam et al. as follows :

$$PTSI = \frac{2S_r Z (1 + \cos(\theta - \varphi))}{V_s^2}$$

- This index is based on the same concept as VCPI in which the maximum transferable power through a line is limited.
  - For the safe operation of the network, the PTSI must be  $< 1$ .
- When PTSI reaches 1, it indicates that voltage collapse has occurred.
- In this index, the line shunt admittance is neglected the same as all the previous line VSIs.

# Line voltage stability indices. Contd.

## L-VSI-10: Voltage stability index 1 (VSI\_1)

The VSI\_1 is formulated as follows based on the **same concept as VCPI and the PTSI**. The VSI\_1 proposed by Gong et al. shows:

$$VSI\_1 = \min \left( \frac{P_{margin}}{P_{max}}, \frac{Q_{margin}}{Q_{max}}, \frac{S_{margin}}{S_{max}} \right)$$

- Where:

where

$$P_{margin} = P_{max} - P_r$$

$$Q_{margin} = Q_{max} - Q_r$$

$$S_{margin} = S_{max} - S_r$$

&

$$P_{max} = \sqrt{\frac{V_s^2}{4X^2} - Q_r \frac{V_s^2}{X}}$$

$$Q_{max} = \frac{V_s^2}{4X} - \frac{P_r^2 X}{V_s^2}$$

$$S_{max} = \frac{(1 - \sin \varphi) V_s^2}{2X \cos^2 \varphi}$$

- To maintain a **secure condition**, the value of VSI\_1 index should be maintained **greater than 0**.
- The line resistance and shunt admittance is neglected.



# Line voltage stability indices. Contd.

## L-VSI-11: Voltage Stability Load Index (VSLI)

Proposed by Abdul Rahman et al. for line voltage stability assessment based on the same concept as  $L_p$ , & is given by:

$$VSLI = \frac{4[V_s V_r \cos(\delta) - V_r^2 \cos(\delta)]}{V_s^2}$$

- VSLI must be kept below 1.0 to maintain voltage stability.
  - If VSLI exceeds 1.0, it indicates a voltage collapse in system.
- The line shunt admittances are neglected and if assumption  $\cos(\delta)=1$ , the stability index L is formulated as.

$$L = \frac{4(V_s V_r - V_r^2)}{V_s^2}$$

# Line voltage stability indices. Contd.

## L-VSI-12: Voltage Stability Margin (VSMs)

Guiping et al. proposed VSMs based on the same concept as VCPI and PTSI as follows:

$$VSM_s = \frac{S_{cr} - S_L}{S_{cr}}$$

where

$$S_{cr} = \frac{V_1^2}{2Z[1 + \cos(\theta - \varphi)]}$$

- The CV of VSM index is zero.
- The system will experience the voltage collapse in values less than zero.
- In the VSMs, the power factor is assumed to be constant and the line shunt admittance is neglected.

# Line voltage stability indices. Contd.

## L-VSI-13: Voltage Collapse Proximity Index (VCPI\_1)

- **Main idea of VCPI\_1** is that **at the collapse point, the V-drop across the Thevenin impedance is equal to the load voltage.**
  - Therefore, to assess the risk of voltage collapse, VCPI\_1 proposed by Wang et al. has been defined as

$$VCPI_1 = V_r \cos(\delta) - 0.5V_s$$

- When **VCPI\_1 ≥ 0**, **line voltage is stable** & **otherwise unstable.**
- In an improved variant, the path is transformed into a two-bus equivalent and stability index of the power transmission path (PVSI), also known as equivalent node voltage collapse index (ENVCI), is defined as follows.

$$PVSI = V'_2 \cos(\delta) - 0.5V'_1$$

- where the  $V'_1$  and  $V'_2$  are the voltage magnitudes of the sending and receiving buses in the two-bus equivalent system.

# Line voltage stability indices. Contd.

## L-VSI-14: Voltage Stability Indicator (VSI\_2)

This index is obtained using the same concept as  $L_p$ , in which the discriminant of the voltage quadratic equation is set to be greater than or equal to zero.

$$VSI\_2 = \frac{4Q_r(R+X)^2}{X(V_s^2 + 8RQ_r)}$$

- For the safe operation of the network, the  $VSI\_2$  must be less than unity.
- In this index, the voltage angle is assumed to be very small and the line shunt admittance is neglected.

# Line voltage stability indices. Contd.

## L-VSI-15: Voltage Stability Margin Index (VSMI)

- VSMI has been defined by He et al. based on the relationship between maximum transferable power through a line and the angle difference between sending and receiving buses.
- Operating at secure and stable conditions requires the value of VSMI to be maintained greater than 0.

$$VSMI = (\delta_{max} - \delta) / \delta_{max}$$

- where  $\delta_{max}$  is the maximum angle difference between sending and receiving buses which can be calculated in lossless and lossy transmission lines as follows.

$$\delta_{max} = \frac{\frac{\pi}{2} - \phi}{2}$$

$$\delta_{max} = \frac{\theta - \varphi}{2}$$

# Line voltage stability indices. Contd.

## L-VSI-16: Voltage Stability Load Bus Index (VSLBI)

- Milosevic had proposed the VSLBI for voltage stability assessment using PMU.
  - This index is based on the fact that; **under the maximum power conditions, voltage drop across the transmission line impedance is equal to the load bus voltage.**

$$VSLBI = \frac{V_r}{\Delta V}$$

- where  $\Delta V$  is the dropped voltage across the transmission line.
- As long as the **VSLBI is >1, the system is stable** and if value is <1 then the system loses its stability.
- In this index, the **same assumptions as VCPI\_1** are used.

# Line voltage stability indices. Contd.

## L-VSI-17: Stability Index (SI)

Eminoglu et al. proposed SI based on the voltage quadratic equation which is mostly used for the calculation of the line VSIs.

$$SI = 2V_s^2 V_r^2 - V_r^4 - 2V_r^2 (P_r R + Q_r X) - Z^2 (P_r^2 + Q_r^2)$$

- The line, at which the value of the SI is at minimum, is the most sensitive to the voltage collapse.
- When the SI value reaches zero, the voltage collapses.
- In this index, the line shunt admittance is neglected as many of previous indices.

# Line voltage stability indices. Contd.

## L-VSI-18: Line Collapse Proximity Index (LCPI)

The LCPI is proposed by Tiwari et al. based on same concept as SI.

$$LCPI = \frac{4|A| \cos(\alpha)[P_r|B| \cos(\beta) + Q_r|B| \sin(\beta)]}{[V_s \cos(\delta)]^2}$$

- where  $A$  and  $B$  are the transmission line parameters of the two-port network and  $\alpha$  and  $\beta$  are the phase angles of parameters  $A$  and  $B$ , respectively. The  $A$  and  $B$  can be expressed as:

$$\begin{bmatrix} V_s \\ I_s \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_r \\ I_r \end{bmatrix}$$

- Where  $A$ ,  $B$ ,  $C$  and  $D$  are calculated by the pie model of a transmission line of a two bus system as follows:

$$\begin{aligned} A &= (1 + Z^*Y/2) \\ B &= Z \\ C &= Y^*(1 + Z^*Y/4) \\ D &= A \end{aligned}$$

- Stability: LCPI must be less than unity.



# Bus voltage stability indices.

- Bus VSIs determine the voltage stability of system buses.
  - They **do not provide any information about the weak facilities** with potential voltage problems in the system.
  - So, the bus voltage stability indices **cannot be used for the determination of the weak facilities.**

# Bus voltage stability indices. Contd.

## B-VSI-1: Voltage collapse prediction index ( $VCPI_{bus}$ )

- The  $VCPI_{bus}$  is derived from the basic power flow equation and its value varies between 0 and 1.
- If the value of  $VCPI_{bus}$  reaches 1, the voltage at a bus has collapsed.

$$VCPI_{bus} = \min\{VCPI_i\}$$

$$VCPI_i = 1 - \frac{\sum_{\substack{m=1 \\ m \neq i}}^N V'_m}{V_i}$$

and

$$V'_m = \frac{Y_{im}}{\sum_{\substack{j=1 \\ j \neq i}}^N Y_{ij}} V_m$$

- Where  $V_i$  and  $V'_m$  are the voltage phasors at bus  $m$  and bus  $i$ ,  $N$  is the number of buses, and  $Y_{im}$  is the admittance between the buses  $i$  and  $m$ .
- This index is based on the concept that the voltage equations must have a solution. In the matrix form, the determinant of a matrix must not be zero.

# Bus voltage stability indices. Contd.

## B-VSI-2: L-index

Kessel et al. proposed L-index based on the solution of the power flow equations and formulates as follows.

$$L = \max_{j \in \alpha_L} \{L_j\} = \max_{j \in \alpha_L} \left| 1 - \frac{\sum_{i \in \alpha_G} F_{ji} V_i}{V_j} \right|$$

Where  $\alpha_L$  is the set of load buses,  $\alpha_G$  is the set of generator buses,  $V_j$  and  $V_i$  are the voltage phasors at bus  $j$  and bus  $i$ , and  $F_{ji}$  is the element in  $j$ -th row and  $i$ -th column of matrix  $F$  whose elements are generated from the admittance matrix as:

$$F = -Y_{LL}^{-1} Y_{LG}$$

and

$$\begin{bmatrix} I_L \\ I_G \end{bmatrix} = \begin{bmatrix} Y_{LL} & Y_{LG} \\ Y_{GL} & Y_{GG} \end{bmatrix} \begin{bmatrix} V_L \\ V_G \end{bmatrix}$$

The values of L-index vary between 0 (no load condition) and 1 (voltage collapse) and the concept of this index is the same as that of VCIbus.

# Bus voltage stability indices. Contd.

## B-VSI-3: S difference criterion (SDC)

The SDC proposed by Verbic et al. is **based on the fact** that **in the vicinity of voltage collapse point** an increase in the apparent power flow at the sending end of the line no longer yields an increase in the received power and formulates as follows.

$$SDC = \left| 1 + \frac{\Delta V_r I_r^*}{V_r \Delta I_r^*} \right|$$

**At the point of the voltage collapse, when  $\Delta S_r=0$ , the SDC equals 0.** In this index very **small values such as  $\Delta V_r \Delta V_r$**  are neglected in the process of proof. Moreover, this index is calculated when the following conditions are satisfied:

1.  **$|\Delta I_r| < \Delta I_{min}$**
2. **The studied line is a reactive power consumer.**

Otherwise, the SDC is set to 1. **The SDC cannot determine the exact time of collapse because it is always a positive value and never goes below zero.**

# Bus voltage stability indices. Contd.

## B-VSI-4: Voltage stability index (VSIbus)

Haque proposed the VSIbus based on the same concept and assumption of SDC.

$$VSI_i = \left[ 1 + \left( \frac{I_i}{V_i} \right) \left( \frac{\Delta V_i}{\Delta I_i} \right) \right]^\alpha$$

- Where  $I_i$  and  $V_i$  are the current and voltage at bus  $i$ ,  $\Delta I_i$  and  $\Delta V_i$  are current and voltage deviation at bus  $i$ , and  $\alpha$  is a constant number equal or greater than 1 which is used to give a more or less linear characteristic to the index.
- The value of **VSIbus varies** between **unity (at no load)** and **zero (at voltage collapse point)**.

# Bus voltage stability indices. Contd.

## B-VSI-5: Impedance matching Stability Index (ISI)

- According to the circuit theory, when the amplitude of load impedance is equal to the magnitude of the Thevenin impedance, the system reaches the maximum transferable power.
  - The ISI which has been proposed by Smon, uses the above law to estimate the VSM.
  - This index takes values around 1 in normal conditions and at the voltage-collapse point diminishes to 0.

$$ISI = \frac{Z_L - Z_{th}}{Z_L} = 1 - \frac{|I_r \Delta V_r|}{|V_r \Delta I_r|}$$
$$Z_L = \frac{V_r}{I_r}, \quad Z_{th} = \frac{\Delta V_r}{\Delta I_r}$$

- where  $\Delta V_r$  and  $\Delta I_r$  are the voltage and current differences between two consecutive measurement samples from a PMU at the receiving bus. The ISI is more sensitive to smaller changes in the system voltage and current and determining the threshold for  $\Delta V_r$  and  $\Delta I_r$  requires a lot of tests on the system.

# Bus voltage stability indices. Contd.

## B-VSI-6: $Z_L/Z_S$ ratio

- The index of  $Z_L/Z_S$  ratio has been defined by Wiszniewski based on the **same concept as ISI**.
- In this index, **ratio of the load impedance at the load bus ( $Z_L$ ) to the network Thevenin impedance ( $Z_S$ ) at that bus** is calculated as follows.

$$\frac{Z_L}{Z_S} = \frac{M+1}{-M \cos \beta + [(M \cos \beta)^2 - M^2 + 1]^{0.5}}$$

where  $\beta = \phi_S - \phi_L$  and,  $\phi_S$  and  $\phi_L$  are the phase angles of impedances  $Z_S$  and  $Z_L$ , respectively. The factor  $M$  is calculated by measuring the variation of the load apparent power  $S$  and the load admittance  $Y = 1/Z_L$  between the two measurements as .

$$M = \frac{(S_2 - S_1)(Y_2 + Y_1)}{(S_2 + S_1)(Y_2 - Y_1)}$$

The  $Z_L/Z_S$  ratio index is very sensitive to the small change of the load admittance. So, the time difference between the two measurements ought to be about 500 ms . To maintain a secure condition, the value of this index should be kept above 1. In this index, it is assumed that  $73^\circ \leq \phi_S \leq 87^\circ$ .

# Bus voltage stability indices. Contd.

## B-VSI-7: Simplified Voltage Stability Index (SVSI)

Perez-Londono et al. have proposed the SVSI based on the same concept as VSLBI and is shown as follows.

$$SVSI_r = \frac{\Delta V_r}{\beta V_r}$$

Where  $\beta$  is the correction factor which is associated with the highest difference of the voltage magnitude between two buses (m and l).

$$\beta = 1 - [\max(|V_m| - |V_l|)]^2$$

The  $\Delta V_r$  is the voltage drop on the Thevenin impedance and it is estimated by:

$$\Delta V_r \cong |V_g - V_r|$$

Where  $V_g$  and  $V_r$  are the voltage phasors of the generator to the load bus and the analyzed load bus, respectively.

When the SVSI value reaches 1 in a bus, the voltage collapse occurs at that bus.



# Overall voltage stability indices

This type of VSIs are not related to the system buses and lines. So, the **overall VSI cannot determine the weakest bus or line and can only predict the system collapse point.**

## O-VSI-1: Network sensitivity approach (SG)

- SG has been proposed by Althowibi et al. to **calculate voltage stability and senses how far the system is from its collapse point.**

– SG contains two indices.

$$SG_p = \frac{P_{gt}}{P_{dt}} \quad (1)$$

$$SG_q = \frac{P_{gt}}{Q_{dt}} \quad (2)$$

- Where  $P_{gt}$  ,  $P_{dt}$  and  $Q_{dt}$  are total active power generation, total active power demand and total reactive power demand, respectively.
- The system approaches to its **collapse point** when  $SG_p$  and  $SG_q$  increment gradually, **causing a sharp rise to infinite values.**
- The SG is **based on the P-V curve** and it assumed that the power system efficiency is constant.

# Other voltage stability indices

**Another classification** of VSIs can be performed based on the main concepts of the VSIs. The VSIs have main concepts as follows:

1. Maximum transferable power through a line
2. Existence of solutions for voltage equation
3. P–V curve
4. Lyapunov stability theory
5. Jacobian matrix
6. Maximum power transfer theorem

# Voltage stability indices Characteristics

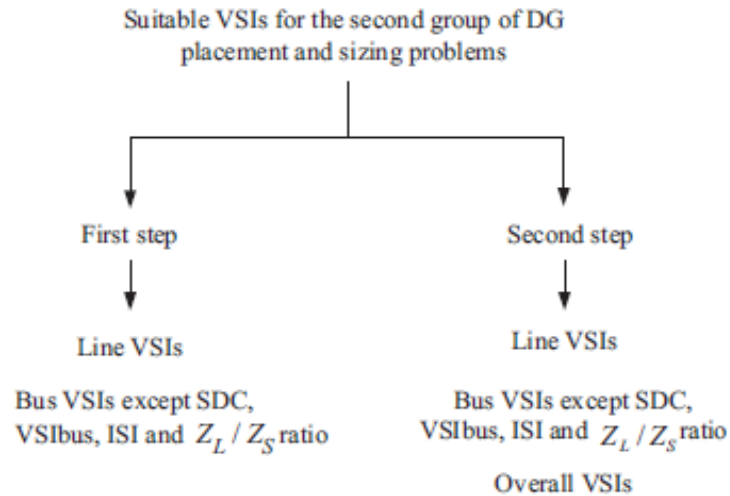


Fig: Suitable VSIs for the second group of DG placement and sizing problems

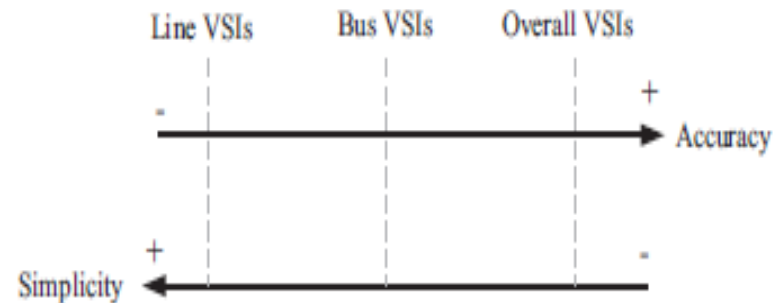


Fig. Comparison between accuracy and simplicity of different types of VSIs .

# Voltage stability indices Characteristics. Contd.

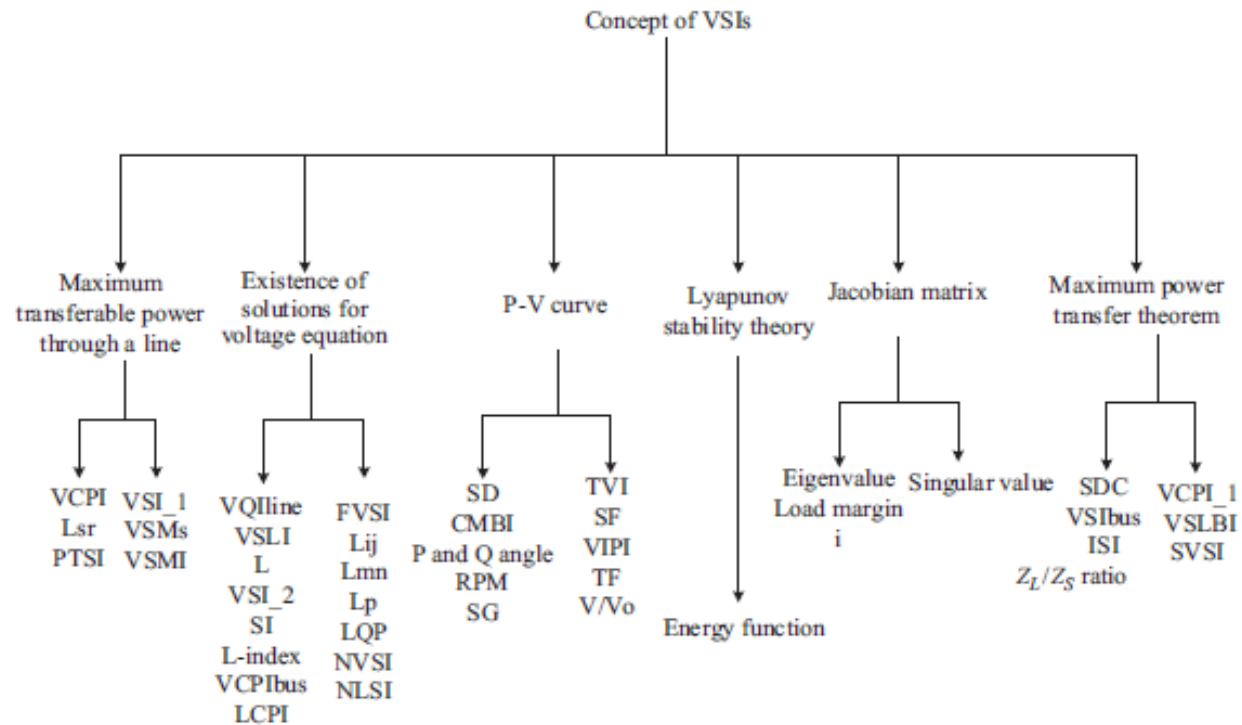


Fig: Concepts of the voltage stability indices.

# Voltage stability indices Characteristics. Contd.

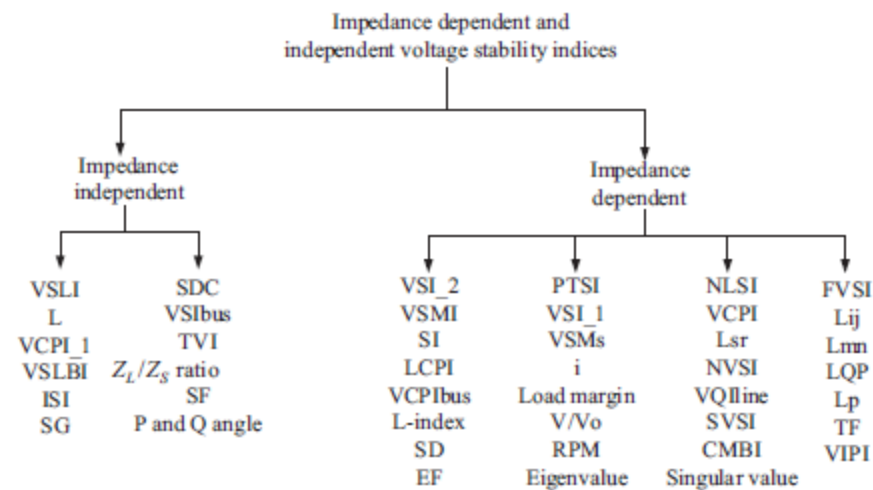


Fig: Impedance dependent and independent voltage stability indices.