Branch & Bound and Knapsack Lab

Objectives

- Implement the branch and bound algorithm
- Apply branch and bound to the knapsack problem
- Implement a greedy algorithm for the knapsack problem

Work Approach: Unlike the lab, you'll be working on a single lab throughout problem sets with the branch and bound algorithm. We will also have time setting a coding plan ideas to reducing the computational time and effort of the algorithm. Lastly, we will explore the dynamics of the knapsack problem.

```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import time
import sys
sys.stdout.encoding = 'UTF-8'
```

Part 1: Branch and Bound Algorithm

Recall that branch and bound algorithm is an addition to the simplex method allows us to solve integer programs. Before applying the algorithm to the knapsack problem, we will first apply it to the simplex method. Recall that the simplex method finds optimal solutions to linear programs. Let's start with a linear program and a branch and bound tree and solve it by hand.

Q1: What are the different ways a node can be fathomed during the branch and bound algorithm? Describe each.

A: Nodes can be pruned in a variety ways: they can be pruned because they are infeasible, because their objective value is worse than the current best solution, or because they have an integer solution.

Q2: Suppose you have a maximization integer program and you solve its linear program relaxation. What does the LP relaxation optimal value tell you about the IP optimal value? What about the LP relaxation optimal solution?

A: The LP optimal value gives an the lowest optimal value the IP can take. If it is a minimization problem it gives us the minimum value the IP can take. It can't be larger than the IP optimal value for a maximization problem.

Q3: Describe one way to fathom/prune a branch-bound algorithm with LP relaxations applied to the knapsack function. Suppose you are running the branch and bound algorithm on a knapsack problem with 10 items and you have found a feasible solution. Suppose the current best solution is worth 100 and the current node you are at has an LP relaxation optimal value of 75. Is it worthwhile to explore this branch?

A: No, because the optimal value is at the most a dot on integer greater than 75, so you fathomed branch remains pruned for the IP.

Q4: Describe one way to find an initial solution of the knapsack program. Suppose you have found an optimal solution to a your integer program, consider adding a new constraint.

A: This is true because all feasible solutions to the original problem are feasible to the new problem. So the IP must be equal to or worse than the old IP solution. Suppose the original IP has a solution of 100. Suppose we add a constraint of these that the new IP must be worthwhile related that the original IP. Then, the optimal solution the original IP must be at most 100.

Q5: Consider the knapsack problem from the current course notes: $\max\ 5x_1 + 8x_2$. How do you know about the optimal value of these models? What if x_1 and x_2 are binary? What about if x_1 and x_2 are integer? x_1 is because notes mentioned are being asked or it off where the optimal solution is greater or it off where the optimal solution is greater or it off where the optimal solution is greater.

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Part 2: The Knapsack Problem

In this lab, you will explore a branch and bound algorithm for the knapsack problem. The integer program for the knapsack problem is:

Knapsack Problem: We are given a collection of n items, where each item i has a value v_i and a weight w_i and a capacity c. In addition, there are n binary variables, $x_1, \ldots, x_n$ where x_i is 1 if we include item i in our knapsack and 0 if we do not. The goal is to find a subset of items that can be packed into the knapsack, such that the total value of the items we include is maximized.

$$\max \sum_{i=1}^{n} v_i x_i$$
$$\text{s.t.} \sum_{i=1}^{n} w_i x_i \leq c$$
$$x_i \in \{0, 1\}$$

Consider the following data which we input into the code.

```
v = [10, 7, 25, 24]
w = [2, 3, 6, 5]
c = 7
```

Q7: Are there any items or sac remove from our input to simplify the problem? What if we replace `[10, 7, 25, 24]` with the item number that our knapsack takes, that, how many ones does the input contain?

A: No, no items. Yes, it would be easier to input the item number that our knapsack takes, that, how many ones.

```
v = [1, 1, 1, 1]
w = [2, 3, 6, 5]
c = 7
```

Q8: Consider the following data. Suppose the knapsack. It does not change the optimal solution to the integer program. Explain why.

A: Yes, it would be easier to input the item number that our knapsack takes, that, how many ones.

Q9: Consider removing items from our input, $v = [1, 1, 1, 1]$ from consequent input. How does the IP relaxation's optimal value change?

A: Yes, it would be easier to input the item number that our knapsack takes, that, how many ones.

Q10: Suppose we are given a collection of n items, where each item i has a value v_i and a weight w_i and a capacity c. Describe a greedy algorithm that finds a feasible solution to the knapsack problem.

Recall that a branch and bound tree can be fathomed if it leads to a better than the value of the best feasible integer solution found so far. Hence, it helps to have a good feasible integer solution as quickly as possible. Let's use the above greedy heuristic. Let's fix the data and input it into the code below. Suppose we are given a collection of n items, where each item i has a value v_i and a weight w_i and a capacity c.

In **knapsack_greedy()**, fill in the knapsack function. Suppose we are given a collection of n items, where each item i has a value v_i and a weight w_i and a capacity c. In addition, there are n binary variables, $x_1, \ldots, x_n$ where x_i is 1 if we include item i in our knapsack and 0 if we do not. The goal is to find a subset of items that can be packed into the knapsack, such that the total value of the items we include is maximized.

```
v = [10, 7, 25, 24]
w = [2, 3, 6, 5]
c = 7
```

```
def knapsack_greedy(v, w, c):
    '''
    Input: v - list of values
           w - list of weights
           c - capacity
    Output: x - list of 0/1 values, 1 if item is taken, 0 otherwise
    '''
    n = len(v)
    x = [0]*n
    ratio = [(v[i]/w[i], i) for i in range(n)]
    ratio.sort(reverse=True)
    for r, i in ratio:
        if w[i] <= c:
            x[i] = 1
            c -= w[i]
    return x, sum(x[i]*v[i] for i in range(n))
```

```
x, v = knapsack_greedy(v, w, c)
print("x =", x)
print("Value =", v)
```

```
x = [1, 0, 0, 1]
Value = 35
```

Now we can create a linear relaxation of our knapsack problem. Step, x_i represents our model and x_i represents our decision variables.

```
x = [1, 0, 0, 1]
```

```
c = 7
Decision Vars = [1
Objective Fn = 35
Constraint 1 = 5
x_0 = 1.0
x_1 = 0.0
x_2 = 0.0
x_3 = 1.0
```

Q11: How does the optimal value compare to the value you found using the heuristic integer solution?

A: The optimal value is higher than the value found using the heuristic, but the optimal solution is not an integer.

Q12: Suppose we are given a collection of n items, where each item i has a value v_i and a weight w_i and a capacity c and decision variables should be imposed for each of the items. Write the LP relaxation.

A: Instead of the fathomed branch it is a better solution that the one we currently have. The additional constraints should be $x_i \leq 1$.

After implementing the linear relaxation model using `Knapsack_Model()`, LP, we can solve and visualize solutions. For example, we can solve the LP relaxation of the knapsack problem.

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x = [1, 0, 0, 1]
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NOTE: The line `x, v = Knapsack_Model(v, [1]*n, w, c, True)` means that x_i is the LP relaxation. All constraints from branching have to be added manually.

Q13: Write the optimal value? Can this node be fathomed? Why? (Hint: In BBS you found a feasible integer solution with value 40.)

A: The optimal value is higher than the value found using the heuristic, so it is more than the current best solution. Yes, this node can be pruned because the current best solution is worth 40 and the LP relaxation optimal value is worth 35. Since the current best solution is worth more than the current node, we can prune the branch.

Q14: Write the optimal value? Can this node be fathomed? Why? (Hint: In BBS you found a feasible integer solution with value 40.)

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Q21: What is the optimal solution at the current (dark) node? Do we have to further explore this branch? Explain.

A: No, we do not have to explore further, since the lower bound, we have gotten a feasible IP solution.

Q22: Recall shown nodes have been explored and the nodes shaded darker pink/feasible region shaded darker) correspond to the current node and its feasible region. Render all shaded floor or ceiling nodes. How many nodes have not yet been explored?

A: 3

Q23: How many nodes have a degree of one in the branch and bound tree? (That is, they are only connected to one edge). These nodes are called leaf nodes. What is the relationship between the leaf nodes and the remaining feasible region?

A: 2. Nodes degree have not yet been explored.

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Geometric Interpretation of LP's

Constraint(s)
x1 + x2 ≤ 4
x1 ≤ 2.5
x2
x1
Objective Value: 0.0
0.0
1.0
2.0
3.0
4.0
0.0
1.0
2.0
3.0
4.0
x1 ≤ 2.5
x2 ≤ 2.5
x1 ≤ 2.5
x2 ≤ 2.5
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