State-feedback control systems

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$\mathbf{5} \quad \mathcal{H}_{\infty} \text{ state-feedback control}$

- The state-feedback controller is developed with the objective of attenuating the impact of disturbances on the closed-loop system, based on full state measurements.
- To begin with, let us consider the following linear system subject to external disturbances belonging to the \mathcal{L}_2 space:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + Nw(t) \\ z(t) = Cx(t) + Du(t). \end{cases}$$
(31)

• For (31), the following state-feedback control law is adopted:

$$u(t) = Kx(t)$$

where K is the control gain to be designed later.

• As a result, the closed-loop control system is described as follows:

$$\begin{cases} \dot{x}(t) = \bar{A}x(t) + Nw(t) \\ z(t) = \bar{C}x(t) \end{cases}$$
 (32)

where

$$\bar{A} = A + BK, \ \bar{C} = C + DK.$$

Theorem 4. System (31) is asymptotically stable and has an \mathcal{H}_{∞} performance level, if there exist matrices $\bar{P} = \bar{P}^T > 0$ and \bar{K} , and a scalar $\gamma > 0$ such that the following condition is satisfied:

Furthermore, the control gain can be recovered as follows:

$$K = \bar{K}\bar{P}^{-1}.$$

Proof:

• Let us recall the following the \mathcal{H}_{∞} stability condition:

$$\dot{V}(t) + z^{T}(t)z(t) - \gamma^{2}w^{T}(t)w(t) < 0.$$
 (34)

• Then, condition (34) is described as

$$x^{T}(t)P(\bar{A}x(t) + Nw(t)) + (\bar{A}x(t) + Nw(t))^{T}Px(t) + x^{T}\bar{C}^{T}\bar{C}x(t) - \gamma^{2}w^{T}(t)w(t) < 0.$$

$$(35)$$

• Furthermore, (35) can be rewritten as

$$\begin{bmatrix} x(t) \\ w(t) \end{bmatrix}^T \begin{bmatrix} x(t) \\ w(t) \end{bmatrix} < 0.$$

• Thus, condition (34) holds if

$$\begin{bmatrix} \mathbf{He}\{P\bar{A}\} + \bar{C}^T\bar{C} & PN \\ N^TP & -\gamma^2I \end{bmatrix} < 0. \tag{36}$$

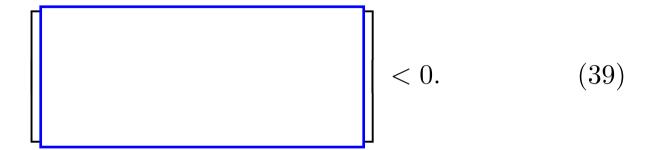
• According to Lemma 3, (36) is equivalent to

$$\begin{bmatrix} \mathbf{He}\{P\bar{A}\} + \bar{C}^T\bar{C} & PN \\ N^TP & -\gamma^2I \end{bmatrix} < 0$$
(37)

which leads to

$$\begin{bmatrix} \mathbf{He}\{\bar{A}P^{-1}\} + P^{-1}\bar{C}^T\bar{C}P^{-1} & N \\ N^T & -\gamma^2 I \end{bmatrix} < 0.$$
 (38)

• By Lemma 2 (called the Schur complement), condition (38) can be transformed into



• By substituting $\bar{A} = A + BK$ and $\bar{C} = C + DK$, and letting $\bar{P} = P^{-1}$ and $\bar{K} = KP^{-1}$, it is derived that