Estimation theory

- Motivation: why estimation theory?

"To measure is to know"
Lord Kelvin

Estimation theory

- Measurement equation

$$y = x + v$$

measurement noise (sensor noise) (we don't know)

true value (we don't know)

measured value (we know)

- Estimation theory
  - we want to estimate the true value x from the measurement y

Estimation theory in everyday life

- Example 1 : body weight measurement : $y = 48\text{Kg}$
  - estimate of x : $\hat{x} =$ _____ Kg
- Example 2 : body weight measurement (three times)
  - measured values : $y_1 = 48, \ y_2 = 47, \ y_3 = 49$

try to derive $\hat{x}$

- Example 3 : body weight measurement : $y = 1000$ Kg
  - estimate of x : $\hat{x} =$ _____ Kg

Estimation theory in everyday life

- Example 1 : body weight measurement : $y = 48$Kg
  - estimate of x : $\hat{x} =$ _____ Kg

- Example 2 : body weight measurement (three times)
  - measured values : $y_1 = 48,\ \ y_2 = 47,\ \ y_3 = 49$
  - three estimators : which one is the best estimator?
    - $\hat{x}_A = \frac{y_1+y_2+y_3}{3}$
    - $\hat{x}_B = y_1$
    - $\hat{x}_C = \min(y_1, y_2, y_3)$

- Example 3 : body weight measurement : $y = 1000$ Kg
  - estimate of x : $\hat{x} =$ _____ Kg

Some questions

- Example 2 : $\hat{x}_A$ is better than $\hat{x}_B$ → better in what sense?
  - an estimator whose estimator error ($e = x - \hat{x}$) is a good estimator?
    - But we cannot compute $e = x - \hat{x}$ because we don't know x
  - How to choose an estimation performance index

- Example 3 : Probably your answer is not $\hat{x} = 1000$
  - Why is that?
    - We know that a human weight cannot be 1000 Kg.
  - Mathematical formula?

- These questions can be answered if you have some knowledge about 'Probability'

Estimator (filter) $y = x + v$

- Estimator

$$y \rightarrow \boxed{\text{estimator}} \rightarrow \hat{x}$$ (x hat)

  - Estimator is also called 'filter'

$$y = x + v$$

$$\Downarrow$$

v

$$\Downarrow$$

$$x$$

$y = x + v$

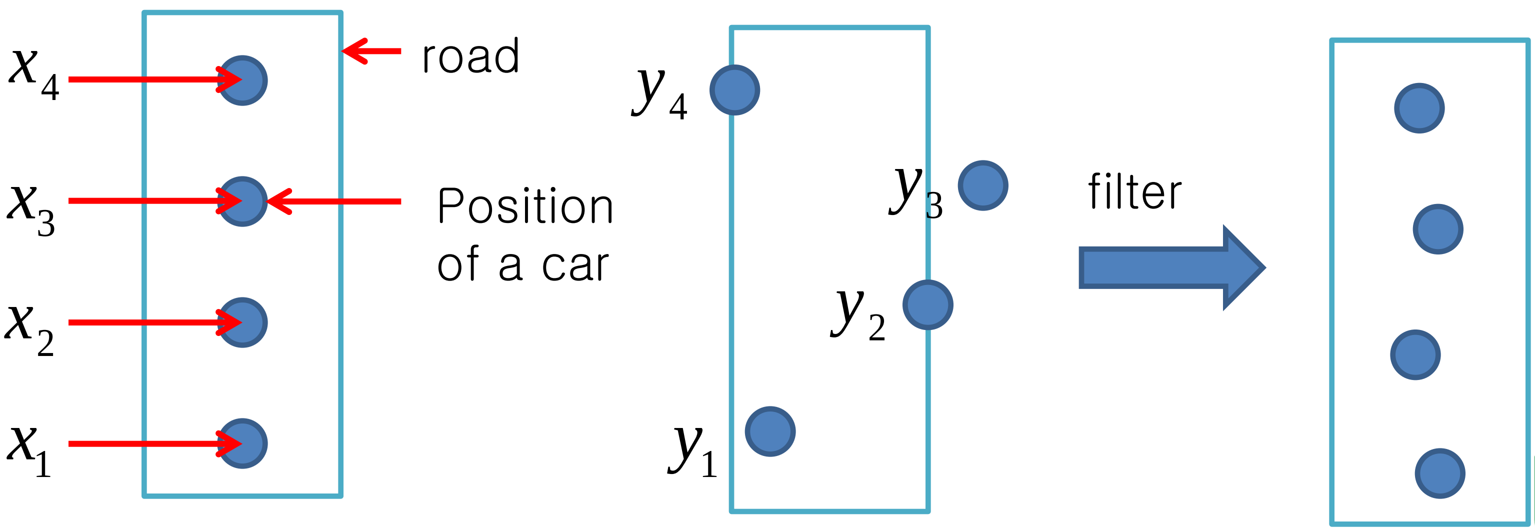
When do we need a filter?

- Body weight measurement case

  - $y = 48 \rightarrow \hat{x} = 48$
  - Usually we don't need a filter in this case because the measured value is accurate enough (or we don't need 99.9999% accuracy for the body weight)

- Car navigation (car position estimation problem)

  - Let's assume the accuracy of GPS is about 10m
  - Filter (map matching algorithm) is used for accurate positioning.



When do we need a filter? (part 1)

- the measurement noise is too large for the required estimation accuracy
  - GPS error 10m → required position accuracy 2m
  - Note: there are two ways to increase the accuracy
    - use 10,000$ GPS sensor instead of 2$ GPS sensor in the car navigation system
    - use a filter to increase accuracy (just few more codes in the software)

- Multiple sensors are used to measure the same quantity
  - how to combine two sensor values (how to choose α)

y_1

estimator

$$\hat{x} = \alpha y_1 + (1-\alpha) y_2$$

y_2

When do we need a filter? (part 2)

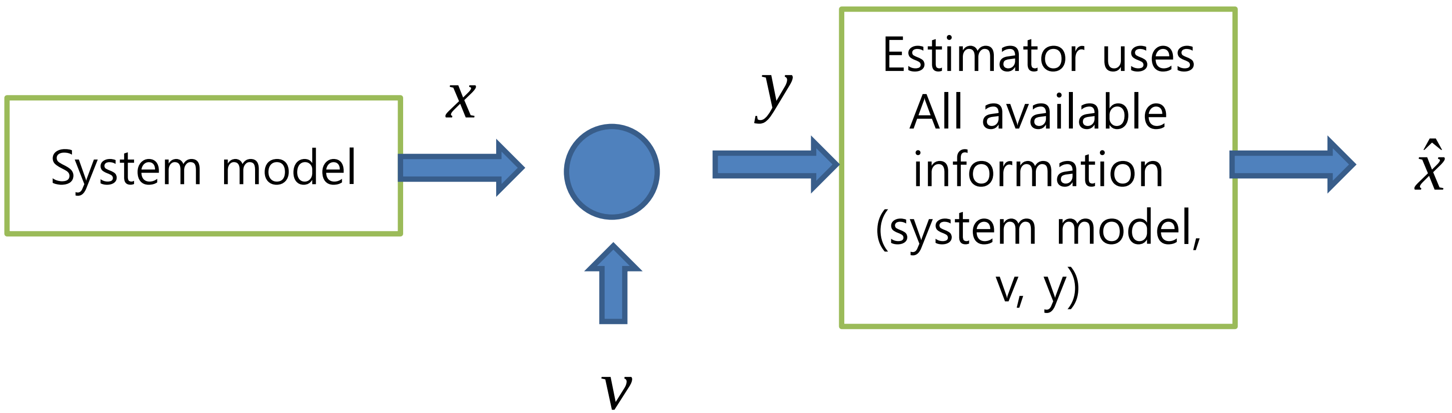
- One sensor is used to estimate multiple quantities
  - Example : temperature measurement with one sensor
    - only y_1 is available
    - We know the following relationship: $0.8\, x_1 \leq x_2 \leq 0.9\, x_1$

x_1 : temperature

x_2 : temperature

The more we know, the more accurately we can estimate

- Filter uses information about x and v



- (implicit) model example
  - Body weight : $y_1 = 48, \quad y_2 = 49, \quad y_3 = 47$ → $\hat{x} = \dfrac{y_1 + y_2 + y_3}{3}$
    - (implicit) model : $x_{k+1} = x_k$
  - Stock price of a company: $y_1 = 191500, \quad y_2 = 157000, \quad y_3 = 182000$
    - stock price of 2011, 2012, 2013 (Jan)
    - Is $x_{k+1} = x_k$ a valid model?

The more we know, the more accurately we can estimate

- Example: position and velocity estimation using accelerometer

position r

$$x = \begin{bmatrix} r \\ \dot{r} \\ \ddot{r} \end{bmatrix}$$

  - model : position (r) → velocity ($\dot{r}$) → acceleration ($\ddot{r}$)
  - model equation

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} w(t)$$

$$y(t) = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} x(t) + v(t)$$

Simple 1D movement exercise

- a vehicle is moving approximately 1m/sec
  - a vehicle position is measured with some sensor (every 1 second)
  - $x_k = x(kT)$ (T = 1 second is a sampling period) : position
  - $y_k = y(kT)$: measured position

- Suppose we have $y_1 = 1.2,\ y_2 = 2.3, y_3 = 3.1$
  - we know that $x_0 = 0$

- Try to derive some equations for $\hat{x}_k$ (k=1,2,3)

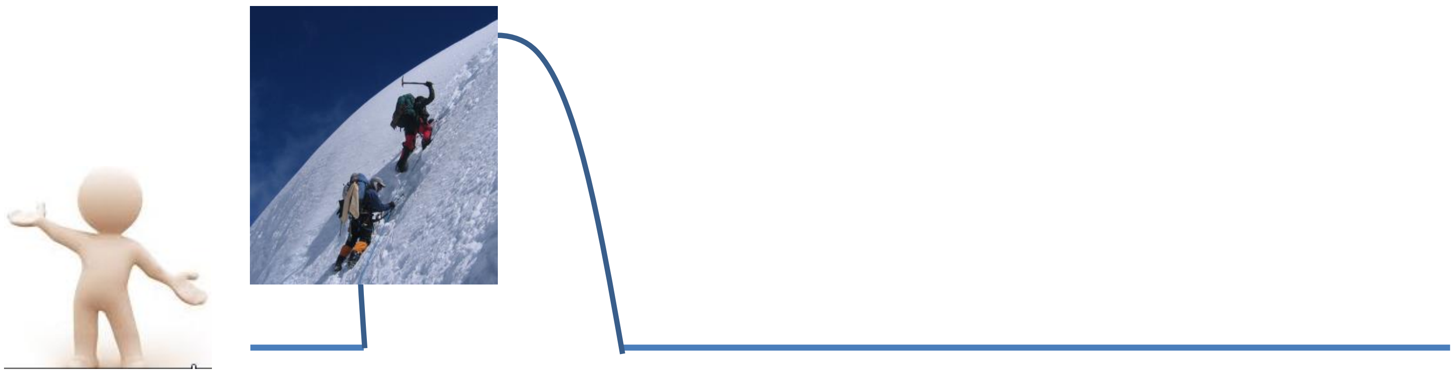
Summary so far

- Estimation
  - Try to estimate some state x from the measurement y
  - In addition to the measurement, we can also use information (model) about x and v to increase estimation accuracy
    - Think of a outfield baseball player trying to catch a ball
  - Possible application for fault detection of sensors and actuators

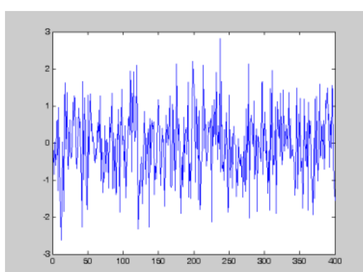
- How to express the information about x and v
  - We need some knowledge about 'Probability'
  - Guassian, Random variable, Random process, white noise…

Lecture outline(1)

first few weeks



Probability
(Random variable and Random Process)
(white Gaussian noise)



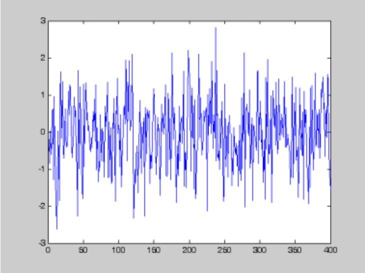
Lecture outline(2)

first few weeks

next few weeks

probability

Kalman filter



R. E. Kalman
(1930~2016)

A new approach to linear filtering and prediction problems [PDF] 160.78.24.2

RE Kalman

1960 · asmedigitalcollection.asme.org

The classical filtering and prediction problem is re-examined using the Bode-Shannon representation of random processes and the "state-transition" method of analysis of dynamic systems. New results are: (1) The formulation and methods of solution of the problem apply without modification to stationary and nonstationary statistics and to growing-memory and infinite-memory filters. (2) A nonlinear difference (or differential) equation is derived for the covariance matrix of the optimal estimation error. From the

자세히 보기

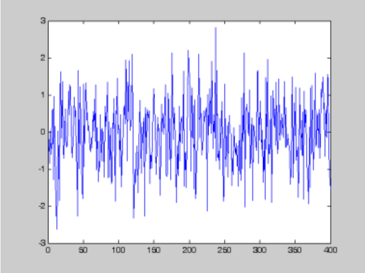
Lecture outline

first few weeks

next few weeks

and then...

probability

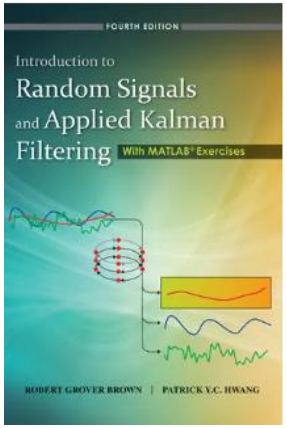


Kalman filter

Application of Kalman filter

Textbook & grading

- Introduction to Random Signals and Applied Kalman Filtering with Matlab Exercises, 4th Edition
  - Robert Grover Brown, Patrick Y. C. Hwang



- Grading
  - Homework 30%, mid exam 30%, final exam 40%
  - Maybe term project?