CHAPTER 1

Signals

Syllabus :

Basic definitions, Classification of signals and systems, Signal operations and properties, Basic continuous time signals, Signal sampling and quantization, Discretization of continuous time signals, Discrete time signals, Representation of digital signals.

1.1 Definition of Signal :

- In a communication system, the word 'signal' is used very commonly. Therefore we must know its exact meaning.
- Mathematically, signal is described as a function of one or more independent variables.
- Basically it is a physical quantity. It varies with some dependent or independent variables.
- So the term signal is defined as "A physical quantity which contains some information and which is function of one or more independent variables."
- The signals can be one-dimensional or multidimensional.

One dimensional signals :

- When a function depends only on a single variable, the signal is said to be one dimensional.
- Example of one dimensional signal is speech signal whose amplitude varies with time.

Multidimensional signals :

- When a function depends on two or more variables, the signal is said to be multidimensional.
- The example of a multidimensional signal is an image because it is a two dimensional signal with horizontal and vertical co-ordinates.

1.2 System :

Definition :

- A system is defined as the entity that operates on one or more signals to accomplish a function, and produces new signals.

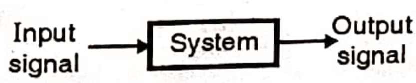


Fig. 1.2.1 : A system

Fig. 1.2.1 demonstrates the interaction between signals and system.

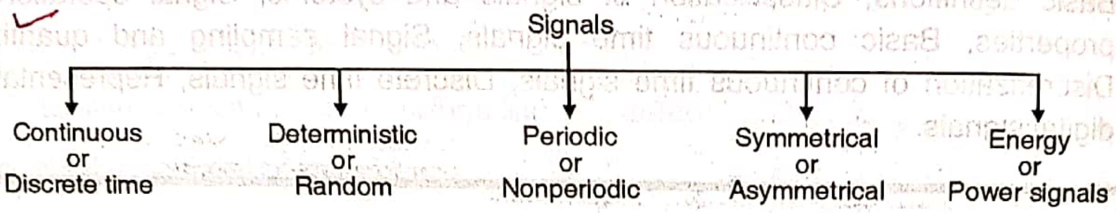
- The types of input and output signals depends on the type of system being used.

1.2.1 Types (Examples) of Systems :

- Signals and systems have several applications. Some of the important types of systems are as follows :
  1. Communication system.
  2. Control system.
  3. Remote sensing system.
  4. Biomedical signal processing.
  5. Auditory system.

1.3 Classification of Signals :

Fig. 1.3.1 shows the classification of signals.



(G-1250) **Fig. 1.3.1 : Classification of signals**

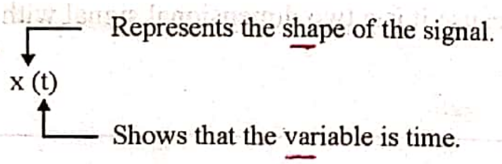
- Out of these we will concentrate only on periodic or non periodic signals.

1.3.1 Continuous and Discrete Time Signals :

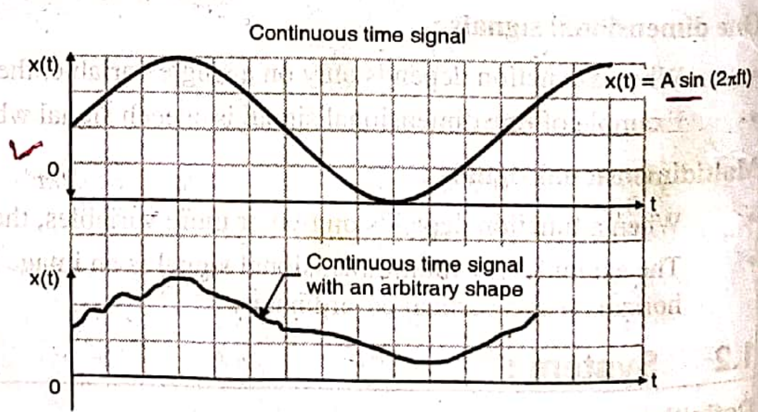
Continuous time (CT) signal :

- A signal of continuous amplitude and time is known as a continuous time signal or an analog signal. This signal will have some finite "value" at every instant of time.
- The electrical signals derived in proportion with the physical quantities such as temperature, pressure, sound etc. are generally continuous time signals.
- The other examples of continuous signals are sine wave, cosine wave, triangular wave etc. Some of the continuous signals are as shown in Fig. 1.3.2(b).

A = Amplit.
Sine wave => A sin
2πft = Angular freqn



(a)



(b) Continuous time signals

Fig. 1.3.2

- The continuous time signals are represented at x(t) where "x" represents the shape of the signal and "t" shows that the variable is time.

Discrete time signals :

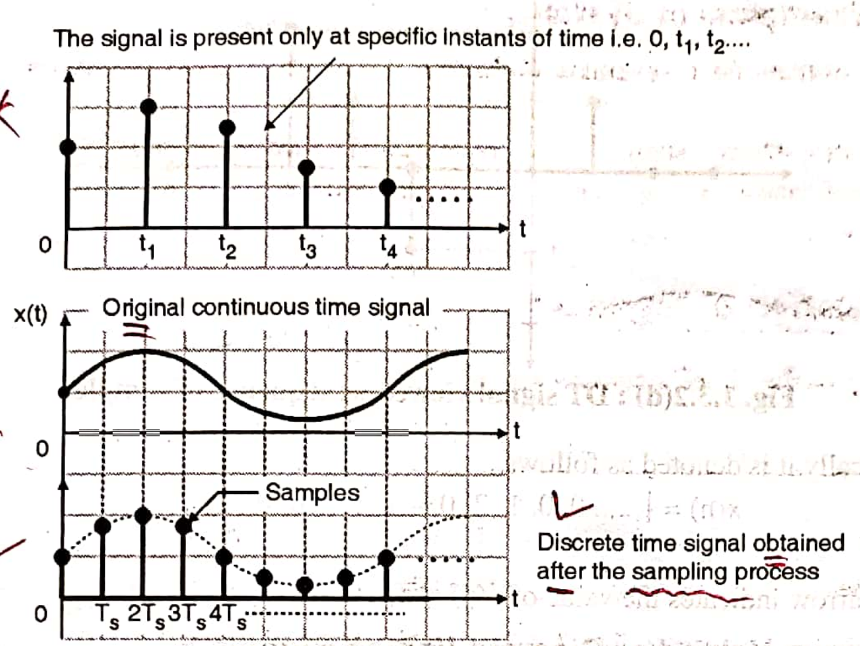


Fig. 1.3.2(c) : Discrete time signals

- If the signal has finite values only at "discrete instants of time," then it is known as a discrete time signal. The discrete time signals have "values" only at certain instants of time.
- If we take the blood pressure readings of a patient after every one hour and plot the graph then the resultant signal will be a discrete signal.
- One more way of obtaining a discrete signals is by "sampling" the analog signals as shown in Fig. 1.3.2(c). The dotted line joining the "sample values" is an imaginary line.
- The sampled version of the continuous time signal is represented by $x_\delta(t)$. The signals which are discrete in time and discrete in amplitude are called as "digital signals".
- The digital signals can be obtained from the continuous time analog signal by a process called "analog to digital conversion".

Sequence of samples :

- The discrete time (DT) signal can also be visualised as a sequence of samples taken at uniform intervals and denoted as x(n).
- Where "x" represents the shape of the signal and "n" is the variable (n is actually an integer). Such a sequence is shown in Fig. 1.3.2(d).

Represents the shape of the signal.

x (n)

Shows that the variable is time.

For example x(1) is the value of x(n) at n = 1, or x(–2) is the value of x(n) at x = – 2 as shown in Fig. 1.3.2(d).

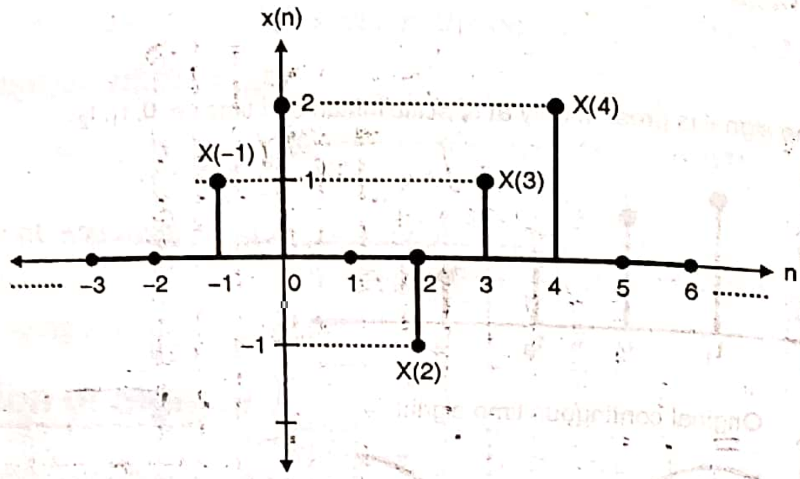


Fig. 1.3.2(d) : DT signal viewed as sequence of samples

- Mathematically it is denoted as follows :

$$x(n) = \{ ...... 0, 0, 1, \underset{\uparrow}{2}, 0, -1, 1, 2, 0, 0 .... \}$$

Where the arrow indicates the value of x(n) at n = 0.

1.3.2 Continuous Valued or Discrete Valued Signals :

Continuous valued signal :

If the variation in the amplitude of signal is continuous then, it is called as continuous valued signal. Such signal may be continuous or discrete in nature. Such signals are as shown in Fig. 1.3.3(a).

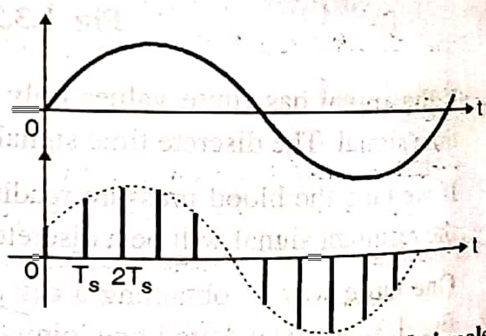
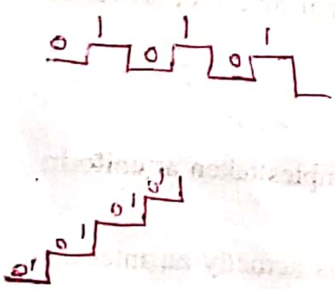


Fig. 1.3.3(a) : Continuous valued signals

Discrete valued signal :



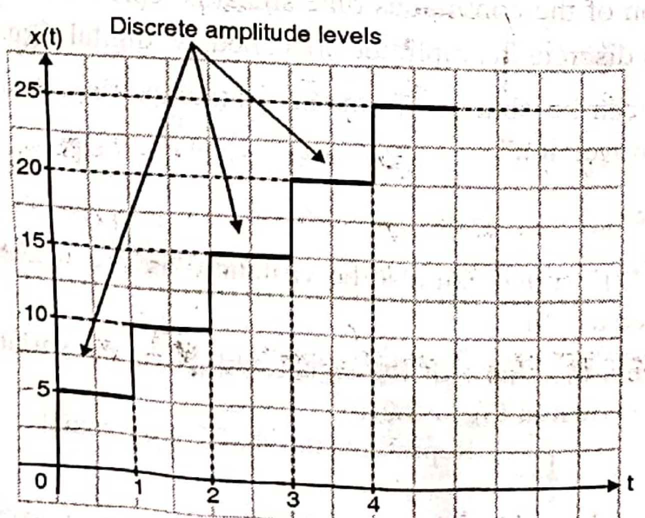


Fig. 1.3.3(b) : Discrete amplitude signal continuous in nature

- If the variation in the amplitude of signal is not continuous; but the signal has certain discrete amplitude levels then such signal is called as discrete valued signal. Such signal may be again continuous or discrete in nature as shown in Figs. 1.3.3(b) and 1.3.3(c).

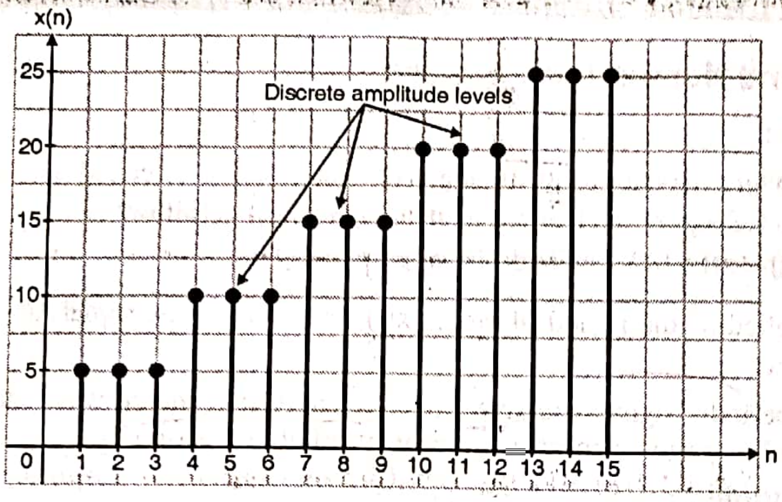


Fig. 1.3.3(c) : Discrete amplitude signal discrete in nature

- As shown in Fig. 1.3.3(b), the signal is defined at all instants of time. So it is continuous signal. But it takes only certain discrete amplitude levels. The amplitude is not continuously changing with time. So it is discrete amplitude signal continuous in nature.
- As shown in Fig. 1.3.3(c), the signal is defined only at discrete intervals of time. So it is discrete signal. And this signal takes only certain discrete amplitude levels. So it is discrete amplitude signal discrete in nature.

Digital signals :

- A **digital signal** is defined as the signal which has only a finite number of distinct values.
- Digital signals are not continuous signal. They are discrete signals as shown in Fig. 1.3.3(d).
- **Binary signal :** If a digital signal has only two distinct values, i.e. 0 and 1 then it is called as a binary signal.

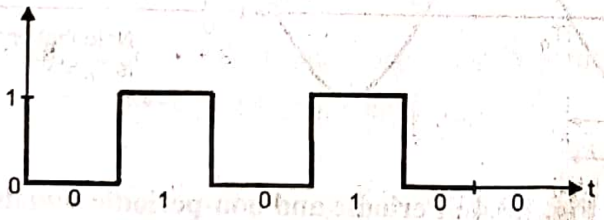


Fig. 1.3.3(d) : Binary signal (Digital signal)

- **Octal signal :** A digital signal having eight distinct values is called as an octal signal.
- **Hexadecimal signal :** A digital signal having sixteen distinct values is called as the hexadecimal number.

Type of digital signal	Number of distinct values
Binary	2
Octal	8
Hexadecimal	16

1.3.3 Periodic and Non-periodic Signals :

Periodic CT signal :

- A CT signal which repeats itself after a fixed time period is called as a periodic signal. The periodicity of a CT signal can be defined mathematically as follows :

$$x(t) = x(t + T_o) \quad : \text{Condition of periodicity} \qquad ...(1.3.1)$$

- where T_o is called as the period of signal x(t), in other words, signal x(t) repeats itself after period of T_o sec.
- Examples of periodic signals are sine wave, cosine wave, square wave etc. Fig. 1.3.4 shows sine wave which is periodic because it repeats itself after a period T_o.

Non-periodic CT signal :

- A CT signal which does not repeat itself after a fixed time period or does not repeat at all is called as a non-periodic or aperiodic signal.
- The non-periodic signals do not satisfy the condition of periodicity stated in Equation (1.3.1).

$$\therefore \quad \text{For a non-periodic signal } x(t) \neq x(t + T_o) \qquad ...(1.3.2)$$

- Sometimes it is said that an aperiodic signal has a period $T_o = \infty$. Fig. 1.3.4 shows a decaying exponential signal.
- This exponential signal is non-periodic but it is deterministic because we can mathematically express it as $x(t) = e^{-\alpha t}$.

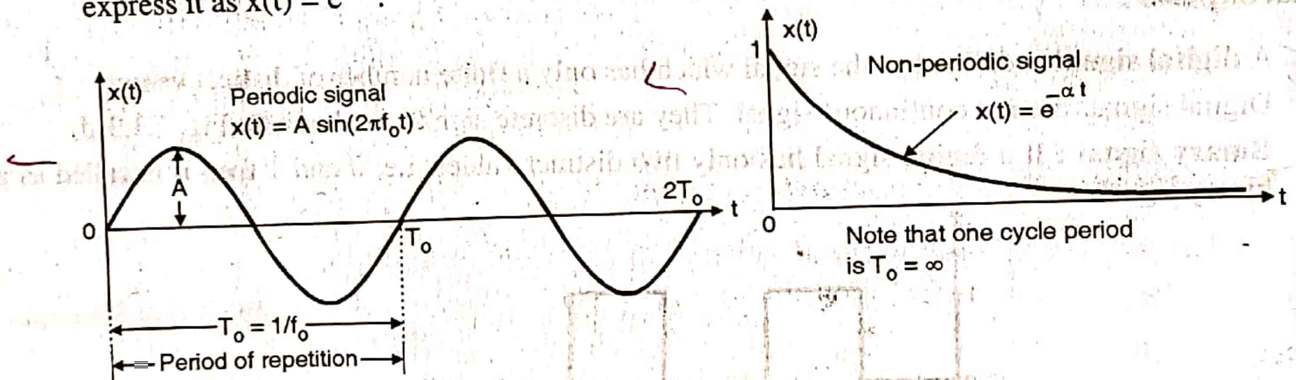


Fig. 1.3.4 : Periodic and non-periodic signals

Periodic discrete time signal :

- For the discrete time signal, the condition of periodicity is,

$$x(n) = x(n + N) \qquad ...(1.3.3)$$

- Here number 'N' is the period of signal. The smallest value of N for which the condition of periodicity exists is called as fundamental period.
- Periodic signals are shown in Figs. 1.3.5(a) and (b).

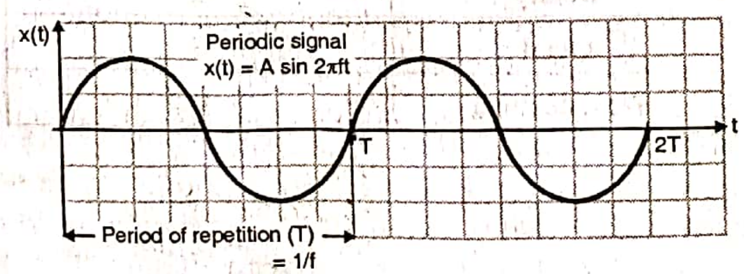


Fig. 1.3.5(a) : Continuous time periodic signal

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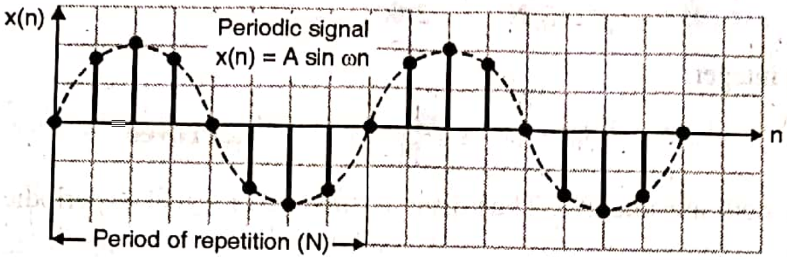


(b) Discrete time periodic signal

Fig. 1.3.5

Non-periodic DT signal :

- A signal which does not repeat itself after a fixed time period or does not repeat at all is called as non-periodic or aperiodic signal. Thus mathematical expression for non-periodic signal is,

$$x(t) \neq x(t+T_0) \quad ...(1.3.4)$$

and $$x(n) \neq x(n+N) \quad ...(1.3.5)$$

Fig. 1.3.5(c) shows a D.T. non-periodic signal, for which $x(n) \neq x(n+N)$.

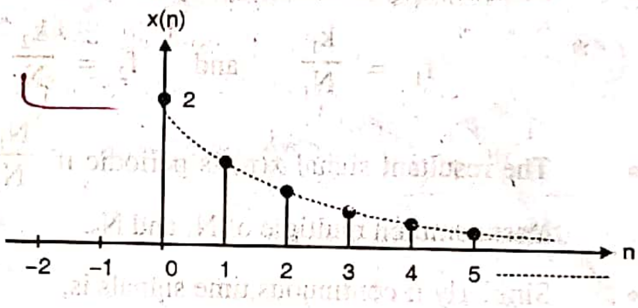


Fig. 1.3.5(c) : Non-periodic D.T. signal

Condition for periodicity of a discrete time signal :

A discrete time sinusoidal signal is periodic only if its frequency (f_0) is rational. That means frequency f_0 should be in the form of ratio of two integers.

Proof :

- For the discrete signal, the condition of periodicity is,

$$x(n+N) = x(n) \quad ...(1.3.6)$$

- Let x(n) be the cosine wave. So it can be expressed as,

$$x(n) = A\cos(2\pi f_0 n + \theta) \quad ...(1.3.7)$$

Here A = Amplitude

and θ = Phase shift

- Now the equation of x(n + N) can be obtained by replacing 'n' by 'n + N' in Equation (1.3.7).

$$\therefore \quad x(n+N) = A\cos[2\pi f_0 (n+N) + \theta] \quad ...(1.3.8)$$

- According to condition of periodicity Equation (1.3.6); we can equate Equations (1.3.7) and (1.3.8).

$$\therefore \quad A\cos[2\pi f_0 (n+N) + \theta] = A\cos(2\pi f_0 n + \theta)$$

$$\therefore \quad A\cos(2\pi f_0 n + 2\pi f_0 N + \theta) = A\cos(2\pi f_0 n + \theta) \quad ...(1.3.9)$$

- To satisfy this equation,

$$2\pi f_0 N = 2\pi k \quad ...(1.3.10)$$

Where k is an integer

$$\therefore \quad f_0 = \frac{k}{N} \quad \text{....Proved.} \quad ...(1.3.11)$$

- Here k and N both are integers. Thus discrete time (DT) signal is periodic if its frequency f_0 i rational.

Periodicity condition for $x(n) = x_1(n) + x_2(n)$:

- Here input sequence x(n) is expressed as summation of two discrete time sequences. We ca calculate the values of f_1 and f_2 corresponding to $x_1(n)$ and $x_2(n)$.
- Let $x_1(n)$ and $x_2(n)$ both be periodic discrete time signals (sequences).
- So according to condition of periodicity,

$$f_1 = \frac{k_1}{N_1} \quad \text{and} \quad f_2 = \frac{k_2}{N_2}$$

- The resultant signal x(n) is periodic if $\frac{N_1}{N_2}$ is ratio of two integers. The period of x(n) will b least common multiple of N_1 and N_2.
- Similarly if continuous time signals is,

$$x(t) = x_1(t) + x_2(t)$$

- We can calculate the values of T_1 and T_2 corresponding to $x_1(t)$ and $x_2(t)$. Then the resultar x(t) is periodic if $\frac{T_1}{T_2}$ is ratio of two integers. The fundamental period of x(t) will be lea common multiple of T_1 and T_2.

Solved examples :

Ex. 1.3.1 : Prove that the sinewave shown in Fig. 1.3.4 is a periodic signal.

Soln. : The sinewave shown in the Fig. 1.3.4 can be mathematically represented as,

$$x(t) = A \sin \omega_o t \quad ...(1$$

Now, let us test if it satisfies the condition for periodicity i.e. if,

$$x(t) = x(t + T_o) \quad ...(2$$

So, let us find the expression for $x(t + T_o)$

$$x(t + T_o) = A \sin \omega_o (t + T_o)$$

$$= A \sin [\omega_o t + \omega_o T_o] \quad ...(3$$

But $\omega_o = 2\pi f_o$ and $T_o = \frac{1}{f_o}$. Therefore $\omega_o T_o = 2\pi f_o \times \frac{1}{f_o} = 2\pi$. Substitute this in Equation (3), w get,

$\sin(A+B) = \sin\cdot\cos + \cos\sin$

$$x(t+T_o) = A \sin[\omega_o t + 2\pi]$$

$$= A[\sin(\omega_o t)\cos 2\pi + \cos(\omega_o t)\sin 2\pi]$$

$$\therefore\ x(t+T_o) = A\sin\omega_o t = x(t) \quad ...(4)$$

Therefore the sinewave shown in Fig. 1.3.4 is a periodic signal.

Ex. 1.3.2 : Prove that the exponential signal shown in Fig. 1.3.4 is non-periodic.

Soln. : The exponential signal shown in Fig. 1.3.4 is expressed mathematically as,

$$x(t) = e^{-\alpha t}$$

Substitute $t = (t+T_o)$ to get,

$$x(t+T_o) = e^{-\alpha(t+T_o)} = e^{-\alpha t}\, e^{-\alpha T_o}$$

But $T_o = \infty$

$$\therefore\ e^{-\alpha T_o} = e^{-\infty} = 0$$

$$\therefore\ x(t+T_o) = e^{-\alpha t}\cdot 0 = 0$$

$$\therefore\ x(t) \neq x(t+T_o)$$

Hence the exponential signal shown in Fig. 1.3.4 is a non-periodic signal.

Ex. 1.3.3 : What is the fundamental frequency of the waveform shown in Fig. P. 1.3.3, in Hz and rad/sec ?

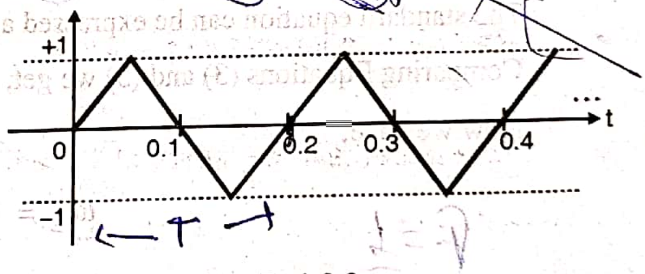


Fig. P. 1.3.3

Soln. :

- One cycle corresponds to 0.2 sec. Hence $T_o = 0.2$ sec.

$$\therefore\ \text{Frequency } f_o = \frac{1}{T_o} = \frac{1}{0.2} = 5 \text{ Hz} \quad ...\textbf{Ans.}$$

- Frequency in rad/sec. $= \omega_o = 2\pi f_o = 2 \times 3.14 \times 5 = 31.4$ rad/s ...**Ans.**

Ex. 1.3.4 : What is the fundamental frequency of the D.T. square wave shown in Fig. P. 1.3.4.

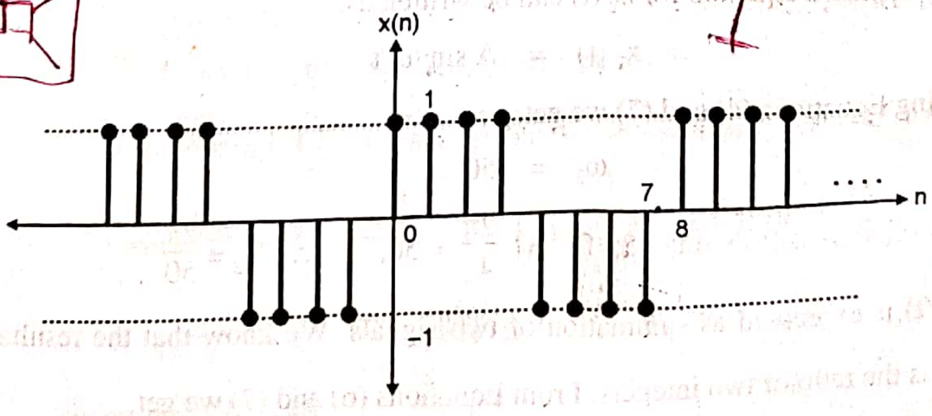


Fig. P. 1.3.4

Soln. :

- The fundamental angular frequency or simply fundamental frequency of x (n) is given by,

$$\Omega = \frac{2\pi}{N}$$

When N = a positive integer indicating number of samples in one cycle.

- For the given signal N = 8.

$$\therefore \quad \Omega = \frac{2\pi}{8} = \frac{\pi}{4} \text{ radians} \quad \text{...A}$$

Ex. 1.3.5 : State whether the following signals x(t) is periodic or not, giving reasons. If it is periodic find the corresponding period.

x (t) = 2 cos 100 π t + 5 sin 50 t

Soln. :

1. The given signal is,

$$x(t) = 2\cos 100\pi t + 5\sin 50t \quad \text{...(1)}$$

$$\text{Let } x(t) = x_1(t) + x_2(t) \quad \text{...(2)}$$

$$\text{Here } x_1(t) = 2\cos 100\pi t \quad \text{...(3)}$$

$$\text{and } x_2(t) = 5\sin 50t \quad \text{...(4)}$$

The standard equation can be expressed as, $x_1(t) = A\cos\omega_1 t$ ...(5)

Comparing Equations (3) and (5) we get, $\omega_1 = 100\pi$

Now we have,

$$\omega_1 = 2\pi f_1 = \frac{2\pi}{T_1}$$

$$\therefore \quad \frac{2\pi}{T_1} = 100\pi$$

$$\therefore \quad T_1 = \frac{2\pi}{100\pi} = \frac{1}{50} \quad \text{...(6)}$$

Similarly standard equation for $x_2(t)$ can be written as,

$$x_2(t) = A\sin\omega_2 t \quad \text{...(7)}$$

Comparing Equations (4) and (7) we get,

$$\omega_2 = 50$$

$$\therefore \quad 2\pi f_2 = \frac{2\pi}{T_2} = 50, \quad \therefore \quad T_2 = \frac{2\pi}{50} \quad \text{...(8)}$$

Here x (t) is expressed as summation of two signals. We know that the resultant signal x (t) is periodic if $\frac{T_1}{T_2}$ is the ratio of two integers. From Equations (6) and (7) we get,

$$\frac{T_1}{T_2} = \frac{1}{50} \cdot \frac{50}{2\pi} = \frac{1}{2\pi}$$

$\sin(a+b) = \sin a \cos b + \cos a \sin b$

It is not the ratio of two integers. Thus x (t) is **non-periodic.**

Ex. 1.3.6 : Determine whether or not each of the following signal is periodic. If periodic, determine its fundamental period :

1. $x(t) = \left[\cos\left(2t - \frac{\pi}{3}\right)\right]^2$ 2. $x(n) = \sin\left(\frac{6\pi}{7}n + 1\right)$

Soln. :

1. $$x(t) = \left[\cos\left(2t - \frac{\pi}{3}\right)\right]^2 = \cos^2\left(2t - \frac{\pi}{3}\right)$$

$$x(t) = \cos\left(2t - \frac{\pi}{3}\right)\cos\left(2t - \frac{\pi}{3}\right) \quad ...(1)$$

The standard equation can be expressed as,

$$x(n) = \cos \omega_1 n \cos \omega_2 n \quad ...(2)$$

Comparing Equations (1) and Equation (2),

$$\omega_1 = \left(2t - \frac{\pi}{3}\right), \omega_2 = \left(2t - \frac{\pi}{3}\right)$$

$$\text{But } \omega = 2\pi f$$

$$\therefore 2\pi f_1 = \left(2t - \frac{\pi}{3}\right)$$

$$6\pi f_1 = (6t - \pi)$$

$$\therefore f_1 = \left(\frac{6t - \pi}{6\pi}\right)$$

$$\therefore f_1 = \left(\frac{6t}{6\pi} - \frac{\pi}{6\pi}\right) = \left(\frac{1}{\pi} - \frac{1}{6}\right) = f_2$$

Hence f_1 and f_2 are ratio of non- integer values, so it is **nonperiodic.**

2. $$x(n) = \sin\left(\frac{6\pi}{7}n + 1\right)$$

$$\sin(a+b) = \sin a \cos b + \sin b \cos a$$

$$\sin\left(\frac{6\pi}{7}n + 1\right) = \sin\frac{6\pi}{7} n \cos 1 + \sin 1 \cos\frac{6\pi}{7} n$$

$$\sin\left(\frac{6\pi}{7}n + 1\right) = \sin\frac{6\pi}{7} n \cdot 1 + 0 \cdot \cos\frac{6\pi}{7} n$$

$$\therefore \left(\frac{6\pi}{7}n + 1\right) = \sin\frac{6\pi}{7} n, \quad ...(3)$$

$$\text{Comparing with, } x(n) = \sin \omega_1 n \quad ...(4)$$

Comparing Equations (3) and (4),

1.3.4 Deterministic and Random Signals :

Deterministic signal :

- A signal which can be described by a mathematical expression, look up table or some defined rule is known as the deterministic signal.
- The examples of deterministic signals are sine wave, cosine wave, square wave etc. Fig. shows a sine wave which is a deterministic signal because it can be represented mathemat as,

$$x(t) = A \sin(2\pi f t)$$

- Where A is the peak amplitude and f is the frequency of the signal.
- The deterministic signals such as sine, cosine etc. are periodic but some deterministic s may not be periodic. Example of such signal is an exponential signal.

Random signals :

- A signal which cannot be described by any mathematical expression is called as a random s Due to this it is not possible to predict about the amplitude of such signals at a given inst time.
- The example of random signal is "Noise" in the communication systems. The determinist random signals are as shown in Fig. 1.3.6.

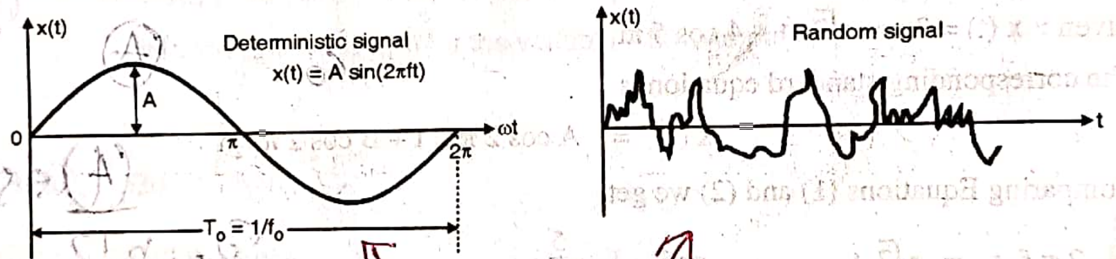


Fig. 1.3.6 : Deterministic and random signals

1.3.5 Symmetrical (Even) or Antisymmetrical (Odd) Signals :

Symmetrical signal (Continuous time) :

- A CT signal x (t) is said to be symmetrical or even if it satisfies the following condition,

Condition for symmetry : $x(t) = x(-t)$ ...(1.

Where, $x(t)$ = Value of the signal for positive "t".

and $x(-t)$ = Value of the signal for negative "t".

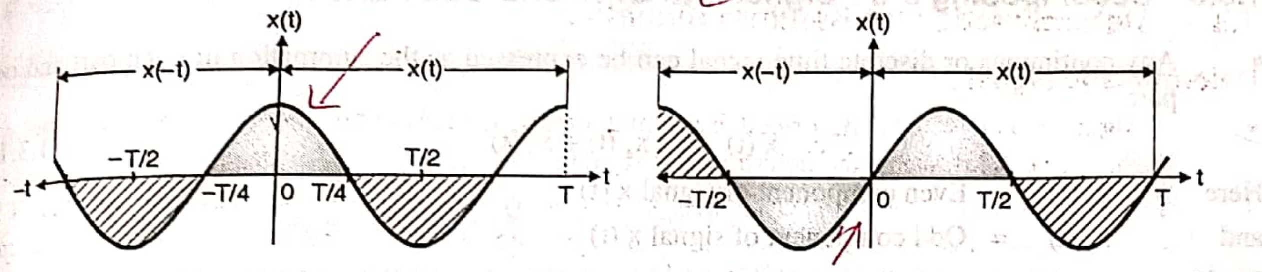
An example of symmetrical signal is a cosine wave shown in Fig. 1.3.7.

Antisymmetrical signal (Continuous time) :

A CT signal x (t) is said to be antisymmetric or odd if it satisfies the following condition,

Condition for antisymmetry : $x(t) = -x(-t)$ ...(1.

An example of odd signal is a sine wave shown in Fig. 1.3.7.



Cosine wave x(t) = x(–t) Symmetrical or even signal Sine wave x(t) = –x(–t) Asymmetrical or odd signal

Fig. 1.3.7 : Symmetrical and antisymmetrical signals

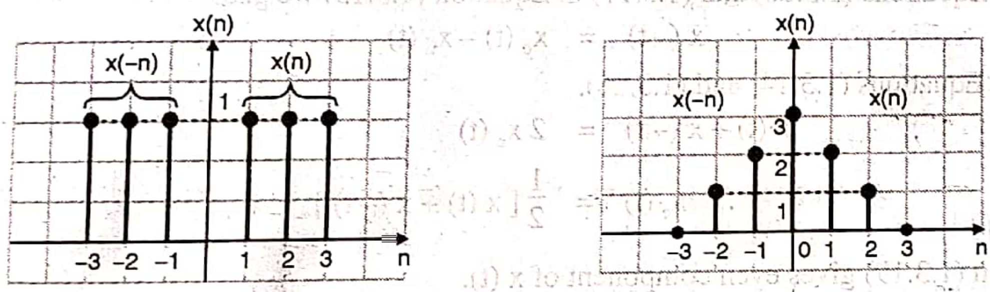
Even and odd discrete time signals :

D.T. symmetric signals :

A discrete time real valued signal is said to be symmetric(even) if they satisfy the following condition,

Condition for symmetry : $x(n) = x(-n)$ ...For D.T. signal

Examples : Fig. 1.3.8(a) and (b) are examples of D.T. even signals.



(a) (b)

Fig. 1.3.8 : D.T. symmetric (even) signals

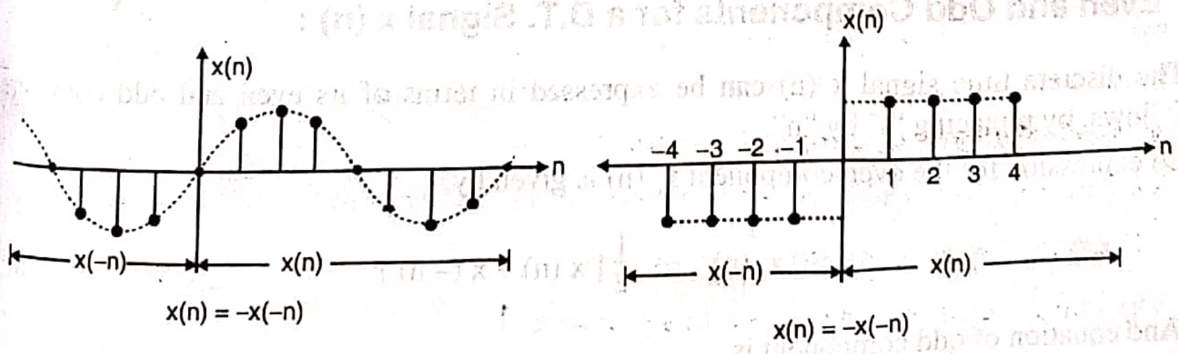
x(n) = –x(–n)

Odd (Antisymmetric) D.T. signal :

- Similarly discrete time (D.T.) signal x (n) is said to be antisymmetric or odd if it satisfies the following condition,

Condition of antisymmetric : $x(n) = -x(-n)$ ... For D.T. signal

Fig. 1.3.9 shows the antisymmetric discrete time signals.



(a) (b)

Fig. 1.3.9 : Antisymmetric (odd) D.T. signals

1.3.6 Decomposing a CT Signal into Even and Odd Parts :

- Any continuous or discrete time signal can be expressed as the summation of even part and odd part.

$$\therefore \quad x(t) = x_e(t) + x_o(t) \qquad \text{...(1.3}$$

Here $x_e(t)$ = Even component of signal x (t)

and $x_o(t)$ = Odd component of signal x (t)

Putting 't' = – t in Equation (1.3.14) we get,

$$x(-t) = x_e(-t) + x_o(-t) \qquad \text{...(1.3}$$

- Let us now obtain the expressions for the even and odd part $x_e(t)$ and $x_o(t)$.

Expression for the even part $x_e(t)$:

- For the even signal we have,

$$x_e(t) = x_e(-t) \qquad \text{...(1.3}$$

- And for odd signal we have,

$$x_o(-t) = -x_o(t) \qquad \text{...(1.3}$$

- Putting Equations (1.3.16) and (1.3.17) in Equation (1.3.15) we get,

$$x(-t) = x_e(t) - x_o(t) \qquad \text{...(1.3}$$

- Adding Equations (1.3.14) and (1.3.18),

$$x(t) + x(-t) = 2\,x_e(t)$$

$$\therefore \quad x_e(t) = \frac{1}{2}[\,x(t) + x(-t)\,] \qquad \text{...(1.3}$$

Equation (1.3.19) gives even component of x (t).

Expression for the odd part $x_o(t)$:

Now subtracting Equation (1.3.18) from Equation (1.3.14) we get,

$$x(t) - x(-t) = 2\,x_o(t)$$

$$\therefore \quad x_o(t) = \frac{1}{2}[\,x(t) - x(-t)\,] \qquad \text{...(1.3}$$

Equation (1.3.20) gives odd component of x(t).

1.3.7 Even and Odd Components for a D.T. Signal x (n) :

- The discrete time signal x (n) can be expressed in terms of its even and odd component follows, by replacing "t" by "n".
- So expression for the even component $x_e(n)$ is given by,

$$x_e(n) = \frac{1}{2}[\,x(n) + x(-n)\,] \qquad \text{...(1.3}$$

- And equation of odd component is,

$$x_o(n) = \frac{1}{2}[\,x(n) - x(-n)\,] \qquad \text{...(1.3}$$

$$x_e(t) = \frac{1}{2}[x(t) + x(-t)]$$

$$\therefore \quad x_e(t) = \frac{1}{2}[1 + t\cos t + t^2 \sin t + t^3 \sin t \cos t + 1 - t \cos t - t^2 \sin t + t^3 \sin t \cos t]$$

$$\therefore \quad x_e(t) = \frac{1}{2}[2 + 2t^3 \sin t \cos t]$$

$$\therefore \quad x_e(t) = 1 + t^3 \sin t \cos t \qquad \text{...Ans.}$$

$$x_o(t) = \frac{1}{2}[x(t) - x(-t)]$$

$$= \frac{1}{2}[1 + t\cos t + t^2 \sin t + t^3 \sin t \cos t - 1 + t \cos t + t^2 \sin t - t^3 \sin t \cos t]$$

$$= \frac{1}{2}[2t \cos t + 2t^2 \sin t]$$

$$\therefore \quad x_o(t) = t \cos t + t^2 \sin t \qquad \text{...Ans.}$$

1.3.8 Energy and Power Signals :

Power signal :

- A signal is called as a power signal if its "average normalized power" is non-zero and finite.
- It has been observed that almost all the periodic signals are power signals.

Energy signals :

- A signal having a finite non-zero total normalized energy is called as an energy signal.
- It has been observed that almost all the non-periodic signals defined over a finite period, are energy signals.
- As these signals are defined over a finite period, they are called as time limited signals.

1.4 Energy and Power of the Signals :

1.4.1 Time Average (dc Value) of a Signal :

- The time average or dc value of a signal x (t) over all time is defined as,

Time average or dc value : $< x(t) > = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t)\, dt$? ...(1.4.1)

- The integration of x (t) from – T / 2 to T / 2 gives us the area under x (t) between the time limits – T / 2 to T / 2.
- This area is then divided by T to obtain the average value of x (t). And as the limit of $T \to \infty$ is taken, the average is calculated over the entire time range from $-\infty$ to $+\infty$.
- If the signal x (t) is periodic signal with period "T_o" then its average value is given by,

Time average of periodic signal : $< x(t) > = \frac{1}{T_o} \int_{-T_o/2}^{T_o/2} x(t)\, dt$ ...(1.4.2)

1.4.2 Power :

Instantaneous power :

- If x (t) is a voltage across a resistor R, the instantaneous power is defined as,

$$\text{Instantaneous power} = \frac{x^2(t)}{R} \quad ...($$

- However if x (t) is a current signal then the expression for the instantaneous power is given

$$\text{Instantaneous power} = x^2(t) \times R \quad ...($$

Normalized power :

- Everytime we may not know whether x (t) is a voltage signal or a current signal. Hence in to make the expression for power independent of the nature of x (t), we normalize substituting R = 1Ω in Equations (1.4.3) and (1.4.4).
- Therefore the normalized instantaneous power is given by,

$$\text{Normalized instantaneous power} = x^2(t) \quad ...($$

Average normalized power :

- Next step is to obtain the average value of this normalized power. From the basic expressi time average defined by Equation (1.4.1) we can write that,

$$\text{Average normalized power} = P = \langle x^2(t) \rangle = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} x^2(t)\, dt \quad ...($$

- The above definition can be generalized for a complex signal x (t) as,

$$P = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2\, dt \quad ...($$

- From Equations (1.4.6) and (1.4.7) it is observed that the signal power P is the time av (mean) of the signal amplitude squared that is the "mean squared" value of x (t).
- Therefore the square root of P is the root mean square (rms) value of x (t).

$$\therefore \; P = \text{Mean square value of } x(t) \qquad \therefore \; \sqrt{P} = \text{rms value of } x(t) \quad ...($$

- For a periodic signal with a period T_o, the Equations (1.4.6) and (1.4.7) get modified to,

$$P = \frac{1}{T_o} \int_{-T_o/2}^{T_o/2} x^2(t)\, dt \quad ...($$

- For a complex periodic signal x (t) the average normalized power is given by,

$$P = \frac{1}{T_o} \int_{-T_o/2}^{T_o/2} |x^2(t)|\, dt \quad ...(1$$

Based on these definitions of average normalized power, we have defined the power signal preceding section.

1.4.3 Energy :

- The total normalized energy for a "real" signal x (t) is given by,

$$E = \int_{-\infty}^{\infty} x^2(t)\, dt \quad ...(1.4.11)$$

- However if the signal is complex then the expression for total normalized energy is given by,

$$E = \int_{-\infty}^{\infty} |x(t)|^2\, dt \quad ...(1.4.12)$$

- These equations indicate that the energy is equal to the area under $x^2(t)$ the curve over $(-\infty \leq t \leq \infty)$ hence it is always positive.

1.4.4 Comparison of Energy and Power Signals :

Table 1.4.1

Sr. No.	Power signals	Energy signals
1	The signal having finite non-zero power are called as power signals.	The signals having a finite non-zero energy are called as energy signals.
2	Almost all the periodic signals in practice are power signals.	Almost all the non-periodic signals are energy signals.
3	Power signals can exist over an infinite time. They are not time limited.	Energy signals exist over a short period of time. They are time limited.
4	Energy of a power signal is infinite.	Power of an energy signal is zero.

1.4.5 Average Power of a DT Signal :

For a discrete time signals, average power P is given by,

$$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} x^2(n) \quad ...(1.4.13)$$

1.4.6 Energy of a DT Signal :

The energy of a D.T. signal is denoted by E and it is given by,

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2 \quad ...(1.4.14)$$

Note : The nonperiodic signals are generally energy signals whereas the periodic signals are generally the power signals.

Signal	Condition
Energy signal	$0 < E < \infty$
Power signal	$0 < P < \infty$

1.4.7 Power of the Energy Signals :

Let x (t) be an energy signal i.e. x (t) has a finite non-zero energy. Let us calculate the power of x (t). By definition, stated in Equation (1.4.10) the power of x (t) is given by,

$$P = \lim_{T_o \to \infty} \frac{1}{T_o} \int_{-T_o/2}^{T_o/2} x^2(t)\, dt = \lim_{T_o \to \infty} \frac{1}{T_o} \cdot \lim_{T_o \to \infty} \int_{-T_o/2}^{T_o/2} x^2(t)\, dt$$

$$= \lim_{T_o \to \infty} \frac{1}{T_o} \int_{-\infty}^{\infty} x^2(t)\, dt$$

$$= \lim_{T_o \to \infty} \frac{1}{T_o} E \quad \text{Using Equation (1.4.12)}$$

$$\therefore \quad P = 0 \times E = 0, \qquad \text{As, } \lim_{T_o \to \infty} \frac{1}{T_o} = 0$$

Thus the power of an energy signal is zero over a finite time.

1.4.8 Energy of a Power Signal :

Let x (t) be a power signal. The normalized energy of this signal is given by,

$$E = \int_{-\infty}^{\infty} x^2(t)\, dt$$

lim To→∞ To/2 −To/2

This expression can be written in a different way as follows :

$$E = \lim_{T_o \to \infty} \int_{-T_o/2}^{T_o/2} x^2(t)\, dt = \lim_{T_o \to \infty} \left[T_o \cdot \frac{1}{T_o} \int_{-T_o/2}^{T_o/2} x^2(t)\, dt \right]$$

P = 1/To ∫ To/2 −To/2 x²(t)

$$= \lim_{T_o \to \infty} T_o \cdot \lim_{T_o \to \infty} \frac{1}{T_o} \int_{-T_o/2}^{T_o/2} x^2(t)\, dt = \lim_{T_o \to \infty} T_o\, [P]$$

$$\therefore \quad E = \infty \qquad \text{As } \lim_{T_o \to \infty} T_o = \infty$$

Thus the energy of a power signal is infinite over a finite time.

Ex. 1.4.1 : What is the average power of the square wave shown in Fig. P. 1.4.1 ?

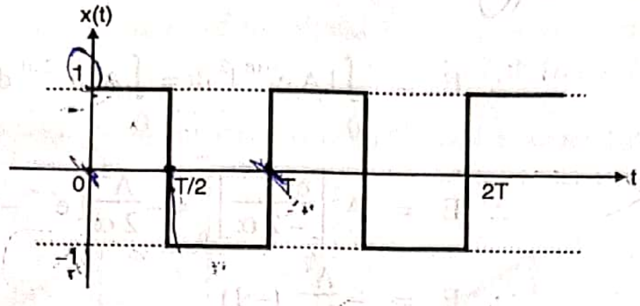


Fig. P. 1.4.1

Soln. : Given signal is periodic. So consider one cycle from 0 to T.

$$P = \frac{1}{T}\int_{0}^{T} x^2(t)\,dt$$

$$\text{But, } x(t) = 1 \qquad \text{For } 0 \le x(t) \le T/2$$

$$T = -1 \qquad \text{For } T/2 \le x(t) \le T$$

$$\therefore P = \frac{1}{T}\left[\int_{0}^{T/2}(1)^2\,dt + \int_{T/2}^{T}(-1)^2\,dt\right] = \frac{1}{T}\left[1\,(t)_{0}^{T/2} + 1\,(t)_{T/2}^{T}\right]$$

$$= \frac{1}{T}\left[\frac{T}{2}+\frac{T}{2}\right] = 1$$

...Ans.

Ex. 1.4.2 : Consider the following signal :

$x(t) = A\,e^{-\alpha t}\,u(t),\ \alpha > 0.$

Is x (t) an energy signal or power signal ? As $\alpha \to 0$, what is the nature of signal x (t) ?

Soln. : The given signal is as shown in Fig. P. 1.4.2. It is positive sided exponential signal; which is non-periodic, so it is an energy signal.

Now energy of continuous signal is given by,

$$E = \int_{-\infty}^{\infty} |x(t)|^2\,dt$$

$$\therefore E = \int_{-\infty}^{\infty} |A\,e^{-\alpha t}\,u(t)|^2\,dt$$

Since the signal $A\,e^{-\alpha t}$ is multiplied by u (t); we will change the limits of integration. The new limits will be from t = 0 to t = ∞; sin t * u (t) exists only in this range and its magnitude is 1.

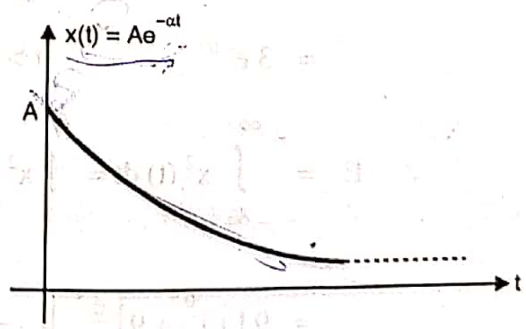


Fig. P. 1.4.2

$$\therefore \quad E = \int_{0}^{\infty} |A e^{-\alpha t}|^2 dt = \int_{0}^{\infty} A^2 e^{-2\alpha t} dt$$

$$\therefore \quad E = A^2 \left[\frac{e^{-2\alpha t}}{-2\alpha}\right]_0^{\infty} = -\frac{A^2}{2\alpha}[e^{-\infty} - e^0] = -\frac{A^2}{2\alpha}\left[\frac{1}{e^{\infty}} - 1\right]$$

$$\therefore \quad E = -\frac{A^2}{2\alpha}(-1)$$

$$\therefore \quad E = \frac{A^2}{2\alpha}$$

Hence 'A' and 'α' and constant terms. Thus energy is finite and non-zero.

Now the given signal is,

$$x(t) = A e^{-\alpha t} u(t), \alpha > 0.$$

As $\alpha \to 0$; the equation of x (t) becomes,

$$x(t) = A\, u(t)$$

Thus as $\alpha \to 0$; x (t) tends to become unit step of magnitude 'A'.

Ex. 1.4.3 : Determine whether the signal shown in Fig. P. 1.4.3 is an energy signal or a power sign

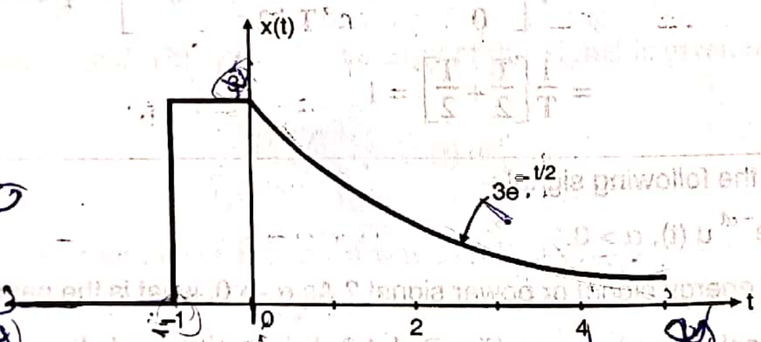


Fig. P. 1.4.3

Soln. : The signal shown in Fig. P. 1.4.3 tends to zero as $|t| \to \infty$. Therefore the average power content of this signal will be zero. Let us calculate the total normalized energy of this signal.

$$x(t) = 3 \qquad -1 \le t \le 0$$

$$= 3 e^{-t/2} \qquad t > 0$$

$$\therefore \quad E = \int_{-\infty}^{\infty} x^2(t)\, dt = \int_{-1}^{0} x^2(t)\, dt + \int_{0}^{\infty} x^2(t)\, dt = \int_{-1}^{0} (3)^2 dt + \int_{0}^{\infty} (3 e^{-t/2})^2 dt$$

$$= 9[t]_{-1}^{0} + 9\left[\frac{e^{-t}}{-1}\right]_0^{\infty} = 9[0+1] - 9[e^{-\infty} - e^0]$$

$$\therefore \quad E = 9 + 9 = 18$$

Thus the total normalized energy of the signal is non-zero and finite. Hence this signal is an energy signal.

Ex. 1.4.4 : Determine whether the signal shown in Fig. P. 1.4.4 is an energy signal or a power signal.

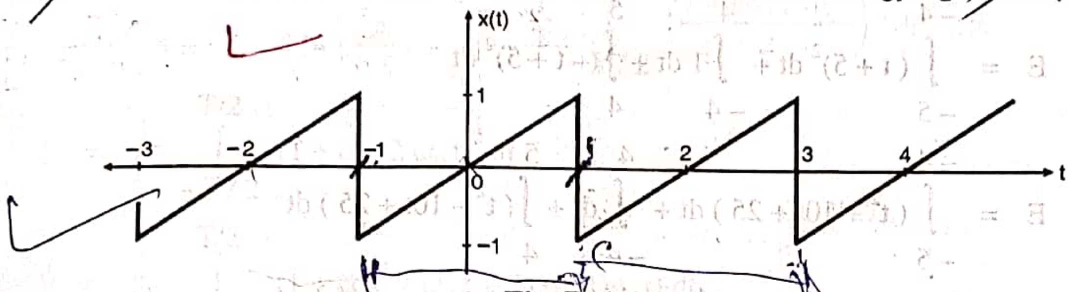


Fig. P. 1.4.4

Soln. : The signal shown in Fig. P. 1.4.4 does not tend to zero as $|t| \to \infty$. Therefore the energy of this signal will be ∞. Let us calculate the average normalized power of this signal. Referring to Equation (1.4.10) we can write that,

$$P = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} x^2(t)\, dt$$

From Fig. P. 1.4.4, it is clear that $T_0 = 2$ sec. and $x(t) = t$

$$\therefore \quad P = \frac{1}{2} \int_{-1}^{1} t^2\, dt = \frac{1}{2 \times 3} [t^3]_{-1}^{1}$$

$$\therefore \quad P = \frac{1}{6} [1 + 1] = \frac{1}{3}$$ ...Ans.

As the power is non-zero and finite the given signal is a power signal.

Ex. 1.4.5 : Find energy of power for the signals shown in Fig. P. 1.4.5 :

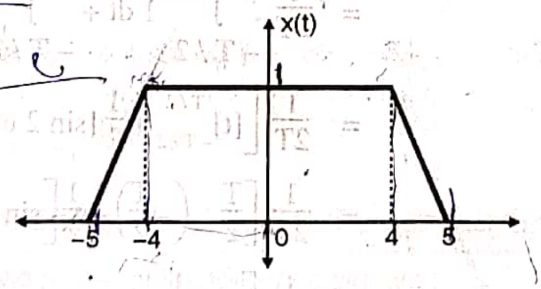


Fig. P. 1.4.5

Soln. :

1. The given signal can be mathematically written as,

$$x(t) = \begin{cases} t + 5 & \text{for } -5 \le t \le -4 \\ 1 & \text{for } -4 \le t \le 4 \\ -t + 5 & \text{for } 4 \le t \le 5 \end{cases}$$

Now energy of a signal is given by,

Thus Equation (4) becomes, $P = \lim_{N \to \infty} \frac{1}{2N+1} [N_2 - N_1 + 1]$

$$= \lim_{N \to \infty} \frac{1}{2N+1} [N - 0 + 1]$$

$$\therefore \quad P = \lim_{N \to \infty} \frac{N+1}{2N+1}$$

Dividing numerator and denominator by N we get,

$$P = \lim_{N \to \infty} \frac{1 + \frac{1}{N}}{2 + \frac{1}{N}} \qquad \therefore \; P = \frac{1}{2}$$

Since the power is finite; unit step is power signal.

1.4.9 Multichannel and Multidimensional Signals :

Multichannel signals :

- As the name indicates, multichannel signals are generated by multiple sources or multiple sensors.
- The resultant signal is the vector sum of signals from all channels.

Example :

- A common example of multichannel signal is ECG waveform. To generate ECG waveform; different leads are connected to the body of a patient.
- Each lead is acting as individual channel. Since there are 'n' number of leads; the final ECG waveform is a result of multichannel signal. Mathematically final wave is expressed as,

$$X(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} \text{; If three leads are used.}$$

Multidimensional signals :

- If a signal is a function of single independent variable, then such a signal is called as one-dimensional signal. On the other hand, if a signal is a function of many independent variables then it is called as multidimensional signal.
- A good example of multidimensional signal is the picture displayed on the TV screen. To locate a pixel (a point) on the TV screen two co-ordinates namely X and Y are required.
- Similarly this point is a function of time also. So to display a pixel, minimum three dimensions are required; namely x, y and t.
- Thus this is multidimensional signal. Mathematically it can be written as, P (x, y, t).

Comparison of Multichannel and Multidimensional signal :

Sr. No.	Multichannel signal	Multidimensional signal
1.	Such signals are generated by multiple sources or multiple sensors.	Such signals are function of many independent variables.

Sr. No.	Multichannel signal	Multidimensional signal
2.	Example : ECG signal. Mathematically it is represented by, $x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix}$	Example : Picture displayed on TV screen. Mathematically it is represented by , P (x, y, t).

Causal and Anticausal signals :

A continuous time signal, x (t) is causal, if it is existing only for positive values of time. That means, x (t) is causal if x (t) = 0 for t < 0. It is anticausal, if x (t) = 0 for t > 0.

A discrete time signal, x (n) is causal if x (n) = 0 for n < 0. It is anticausal if x (n) = 0 for n > 0.

Thus, if a signal is multiplied with unit step then it becomes causal signal.

1.5 Elementary or Basic Signals :

- In signals and systems we need to use some standard or elementary signals.
- In this section we will show some important standard signal graphically and express them mathematically.
- Some of the standard continuous time and discrete time signals are :

1. A dc signal
2. Unit step signal
3. Sinusoidal signal
4. Signum function
5. Triangular signals
6. A rectangular pulse
7. Delta or unit impulse function
8. Exponential signals
9. Sinc function.

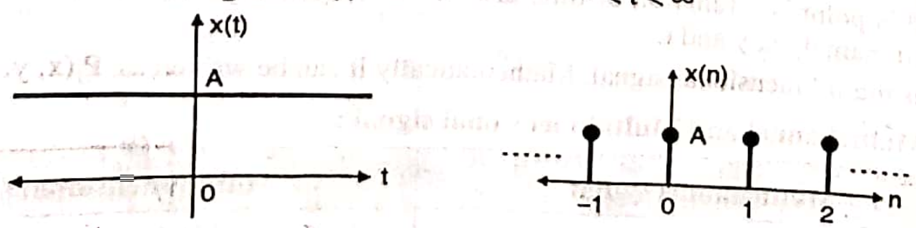
Let us understand them one-by-one.

1.5.1 DC Signal :

Continuous time DC signal :

- A CT dc signal is as shown in Fig. 1.5.1(a). As seen from the waveform the amplitude "A" of a direct current (dc) signal remains constant independent of time.
- The CT dc signal can be represented mathematically as follows :

$$\text{A CT dc signal : } x(t) = A \qquad -\infty < t < \infty \qquad \text{...(1.5.1)}$$



(a) A CT dc signal (b) A discrete time dc signal

Fig. 1.5.1

Discrete time DC signal :

- Fig. 1.5.1(b) shows the discrete time dc signal. It is a sequence of samples each of amplitude A and extending from $-\infty < n < \infty$.
- This signal can be mathematically represented as follows :

$$x(n) = A \qquad \ldots -\infty < n < \infty$$

- Or it can be represented in the infinite sequence form as follows :

$$x(n) = \{\ldots\ldots A, A, \underset{\uparrow}{A}, A, A, \ldots.\}$$

1.5.2 Sinusoidal Signals :

C.T. sinusoidal signals :

- The CT sinusoidal signals include sine and cosine signals. They are as shown in the Fig. 1.5.2.

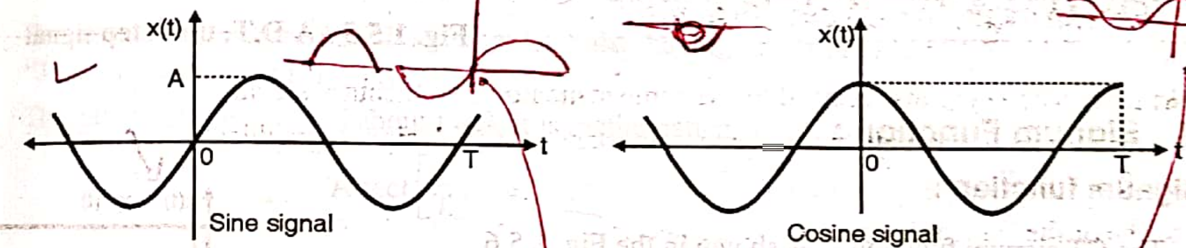


Fig. 1.5.2 : Sinusoidal signals

- Mathematically they can be represented as follows :

A sine signal : $x(t) = A \sin \omega t = A \sin (2\pi f t)$ ...(1.5.2)

A cosine signal : $x(t) = A \cos \omega t = A \cos (2\pi f t)$ ...(1.5.3)

Discrete time sinusoidal wave :

- A discrete time sinusoidal waveform is denoted by,

$$x(n) = A \sin \omega n = A \sin (2\pi f n)$$

Here A = Amplitude

ω = Angular Frequency = $2\pi f$

- This waveform is as shown in Fig. 1.5.3.
- Similarly a discrete time cosine wave is expressed as,

$$x(n) = A \cos \omega n = A \cos (2\pi f n)$$

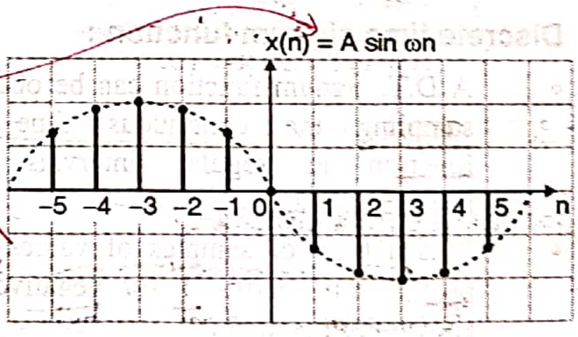


Fig. 1.5.3 : DT sinusoidal waveform

1.5.3 Unit Step Signal :

C.T. unit step signal :

- The CT unit step signal is as shown in Fig. 1.5.4. It has a constant amplitude of unity(1) for the zero or positive values of time "t". Whereas its value is equal to zero for negative values of t.

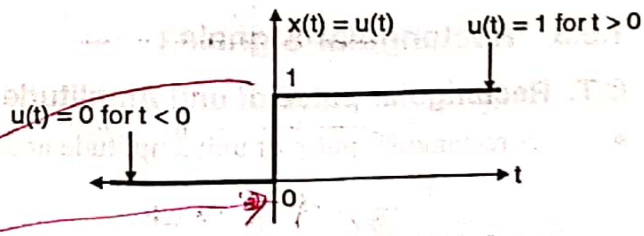


Fig. 1.5.4 : Unit step signal

- The CT unit step signal is mathematically represented as,

Unit step signal : u (t) $= 1$ For $t \geq 0$

$= 0$ For $t < 0$ ...(1.5.4)

A D.T. unit step signal :

- A discrete time unit step signal is denoted by u(n). Its value is unity(1) for all non negative values of "n". That means its value is one for $n \geq 0$. While for other values of n, its value is zero.

$$\therefore \quad u(n) = \begin{cases} 1 & \text{For } n \geq 0 \\ 0 & \text{For } n < 0 \end{cases}$$

- In the form of sequence it can be written as,

$$u(n) = \{\underset{\uparrow}{1}, 1, 1, 1, \ldots.\}$$

$= 1, 2, 3, 4, 5,$

- This signal is graphically represented in Fig. 1.5.5.

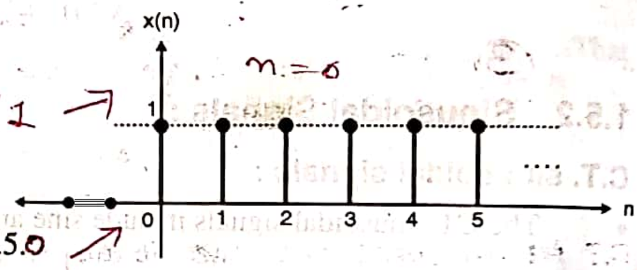


Fig. 1.5.5 : A D.T. unit step signal

1.5.4 Signum Function :

CT Signum function :

- The CT signum function is as shown in the Fig. 1.5.6.
- It is represented mathematically as follows :

Signum function : sgn(t) $= 1$ For $t > 0$

$= -1$ For $t < 0$ ...(1.5.5)

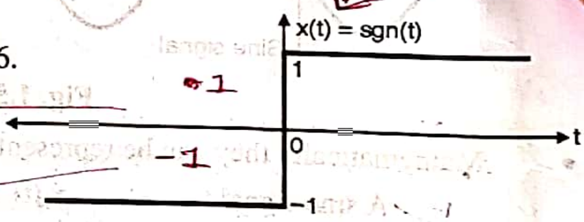


Fig. 1.5.6 : Signum function

- The signum function is an "odd" or antisymmetric function.

Discrete time signum function :

- A D.T. signum function can be obtained by sampling the continuous time signum function at regular intervals i.e. at $n = 0, \pm 1, \pm 2 \ldots.$
- It is a train of samples of values + 1 for positive "n" and – 1 for negative "n" as shown in Fig. 1.5.7.

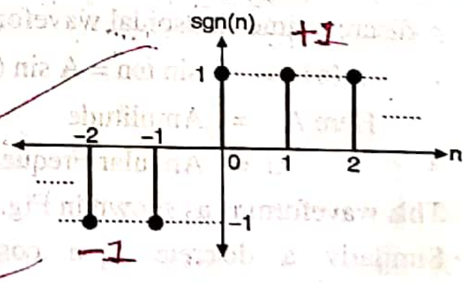


Fig. 1.5.7 : D.T. signum function

- The DT signum function is defined mathematically as follows :

$x(n) = \text{sgn}(n) = 1 \qquad n \geq 0$

$\qquad = -1 \qquad n < 0$ ...(1.5.5(a))

1.5.5 Rectangular Signals :

C.T. Rectangular pulse of unit Amplitude and Unit duration :

- A rectangular pulse of unit amplitude and duration is as shown in Fig. 1.5.8(a).

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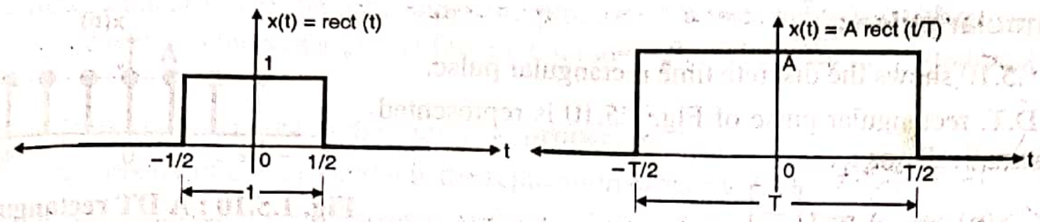


(a) Rectangular pulse of unit duration and unit amplitude

(b) Rectangular pulse of duration T and amplitude A

Fig. 1.5.8

- The unit CT rectangular pulse is centered about the y-axis i.e. about t = 0. It is represented mathematically as follows,

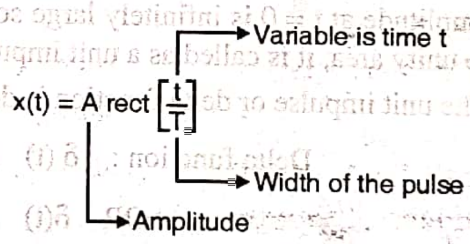
$$\text{rect}(t) = 1 \qquad -1/2 \leq t \leq 1/2$$
$$= 0 \qquad \text{Elsewhere} \qquad \text{...(1.5.6)}$$

C.T. Rectangular pulse of amplitude "A" and duration "T" :

- The other general type of CT rectangular pulse having an amplitude of "A" over a duration of "T" is as shown in Fig. 1.5.8(b).
- This pulse also is centered about t = 0. It is mathematically represented as,

$$A\,\text{rect}\left[\frac{t}{T}\right] = A \qquad -T/2 \leq t \leq T/2$$
$$= 0 \qquad \text{Elsewhere} \qquad \text{...(1.5.7)}$$

- In this expression, "t" shows that it is a function of time and "T" represents the width of the rectangular pulse, and A is the amplitude. Rectangular pulse is an even or symmetrical function.
- Hence it is possible to represent the rectangular signal in frequency domain as follows :



$$x(f) = A\,\text{rect}\left[\frac{f}{2W}\right]$$

and it is graphically represented as shown in Fig. 1.5.9.

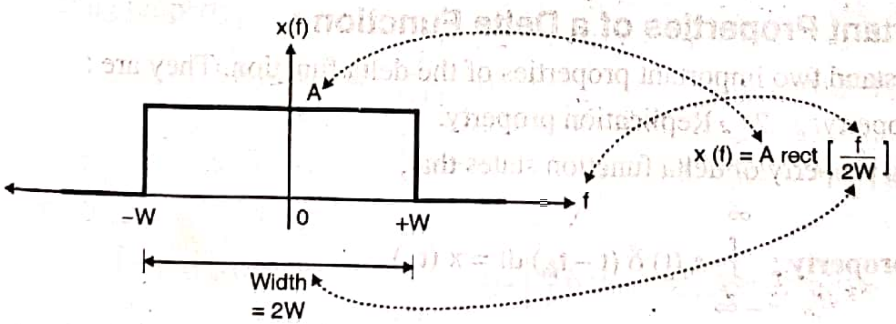


Fig. 1.5.9

D.T. rectangular pulse :

- Fig. 1.5.10 shows the discrete time rectangular pulse.
- The D.T. rectangular pulse of Fig. 1.5.10 is represented mathematically as :

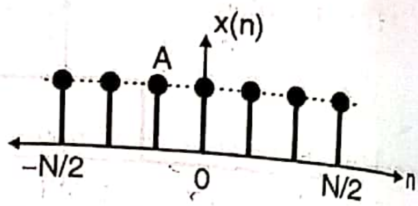


Fig. 1.5.10 : A DT rectangular pulse

$$x(n) = A \operatorname{rect}\left[\frac{n}{N}\right]$$

where A is its amplitude and N is its width.

$$A \operatorname{rect}\left[\frac{n}{N}\right] = A \quad \ldots\ldots -N/2 \le n \le N/2$$

$$= 0 \quad \ldots\ldots \text{elsewhere}$$

1.5.6 Delta or Unit Impulse Function [δ (t)] :

(7) - The delta function δ (t) is an extremely important function which is used for the analysis of communication systems. The impulse response of a system is its response (output) to a delta function applied at the input.

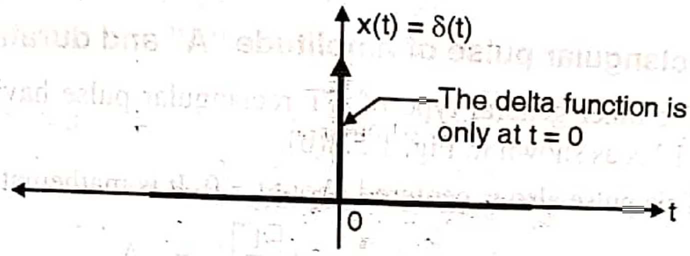


Fig. 1.5.11 : Delta function

- The delta function is as shown in Fig. 1.5.11. It is present only at t = 0, its width tends to 0 and its amplitude at t = 0 is infinitely large so that the area under the delta function is unity (i.e. 1). Due to unity area, it is called as a unit impulse function.
- The unit impulse or delta function is denoted by δ (t) and represented mathematically as follows :

$$\text{Delta function : } \delta(t) = 0 \quad \text{For } t \neq 0 \qquad \ldots(1.5.8)$$

$$\text{OR } \delta(t) \neq 0 \quad \text{For } t = 0$$

The area under the unit impulse is given as,

$$\text{Area under unit impulse : } \int_{-\infty}^{\infty} \delta(t)\, dt = 1 \qquad \ldots(1.5.9)$$

1.5.6.1 Important Properties of a Delta Function :

Here let us understand two important properties of the delta function. They are :

1. Sifting property 2. Replication property.

- The sifting property of delta function states that,

$$\textbf{Sifting property : } \int_{-\infty}^{\infty} x(t)\, \delta(t - t_m)\, dt = x(t_m) \qquad \ldots(1.5.10)$$

- where $\delta(t - t_m)$ represents the time shifted delta function. This delta function is present only at $t = t_m$. The RHS of Equation (1.5.10) represents the value of x (t) at $t = t_m$. The LHS of Equation (1.5.10) is the area under the product term $x(t) \cdot \delta(t - t_m)$.

- This result indicates that the area under the product of a function x (t) with a shifted impulse $\delta(t - t_m)$ is equal to the value of that function at the instant where the unit impulse is located i.e. $x(t_m)$.
- This property is also known as the **sampling property**.
- The other property of delta function is the replication property. It states that,

Replication property : $x(t) * \delta(t) = x(t)$ ...(1.5.11)

- This property can be stated in words as : The convolution of any function x (t) with delta function yields the same function. The sign * in Equation (1.5.11) represents "convolution".

Ex. 1.5.1 : Write the properties of impulse signal δ(t) and evaluate the following using δ(t) properties for given x(t) shown in Fig. P. 1.5.1(a) ahead :

1. $\int_{-\infty}^{+\infty} x(t)\,\delta(t)\,dt$
2. $\int_{-\infty}^{+\infty} x(t-1)\,\delta(t-1)\,dt$
3. $\int_{-\infty}^{+\infty} x(t)\,\delta(4t)\,dt$
4. $x(t) \cdot \sin t \cdot \delta(t)$
5. $\sin[\,x(t)\,\delta(t)\,]$.

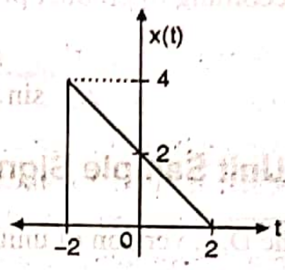


Fig. P. 1.5.1(a)

Soln. :

1. According to sifting property,

$$\int_{-\infty}^{\infty} x(t)\,\delta(t - t_o)\,dt = x(t_o)$$

Let $t_o = 0$

$$\therefore \int_{-\infty}^{\infty} x(t)\,\delta(t)\,dt = x(0) = 2.$$

2. Let $x'(t) = x(t-1)$. It is shown in Fig. P. 1.5.1(b).

Now $$\int_{-\infty}^{\infty} x'(t)\,\delta(t-1)\,dt = x'(1) = 2$$

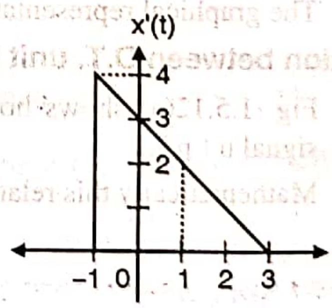


Fig. P. 1.5.1(b)

3. According to scaling property,

$$\delta(\alpha t) = \frac{1}{|\alpha|}\,\delta(t)$$

$$\therefore \delta(4t) = \frac{1}{4}\,\delta(t)$$

$$\therefore \int_{-\infty}^{\infty} x(t)\,\delta(4t)\,dt = \int_{-\infty}^{\infty} x(t)\cdot\left[\frac{1}{4}\,\delta(t)\right] dt$$

$$= \frac{1}{4}\int_{-\infty}^{\infty} x(t)\,\delta(t)\cdot dt \quad \text{Let } t_o = 0$$

Using shifting property,

$$\int_{-\infty}^{\infty} x(t)\,\delta(4t)\,dt = \frac{1}{4}\,x(t_o) = \frac{1}{4} \times 2 = \frac{1}{2}$$

4. Consider the term $\sin t \cdot \delta(t)$

According to product property,

$$y(t) \cdot \delta(t) = y(0)\,\delta(t)$$

$$\therefore \quad \sin t \cdot \delta(t) = \sin(0) \cdot \delta(t) = 0 \times \delta(t) = 0$$

$$\therefore \quad x(t) \cdot \sin t \cdot \delta(t) = 0$$

5. According to product property,

$$x(t) \cdot \delta(t) = x(0) \cdot \delta(t) = 2$$

$$\therefore \quad \sin[x(t) \cdot \delta(t)] = \sin(2) = 0.0348°$$

1.5.7 Unit Sample Signal δ (n) :

- The D.T. version of unit impulse signal is the unit sample signal.
- A discrete time unit impulse function is denoted by δ (n). Its amplitude is 1 at n = 0 and for all other values of n; its amplitude is zero.

$$\therefore \quad \delta(n) = \begin{cases} 1 & \text{For } n = 0 \\ 0 & \text{For } n \neq 0 \end{cases}$$

- In the sequence form it can be represented as,

$$\delta(n) = \{\ldots, 0, 0, 0, \underset{\uparrow}{1}, 0, 0, 0\}$$

$$\text{or} \quad \delta(n) = \{1\}$$

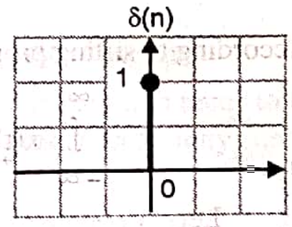


Fig. 1.5.12 : Unit sample signal δ(n)

- The graphical representation of unit sample signal is as shown in Fig. 1.5.12.

Relation between D.T. unit impulse and D.T. unit step signals :

- Fig. 1.5.12(a) shows how to obtain a D.T. unit impulse signal δ [n] from a D.T. unit step signal u [n].
- Mathematically this relation can be expressed as,

$$\delta[n] = u[n] - u[n-1] \quad \ldots(1.5.11(a))$$

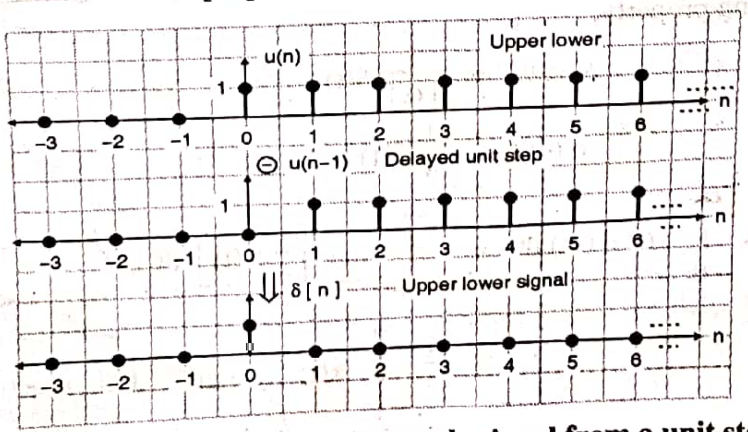


Fig. 1.5.12(a) : How to obtain a unit sample signal from a unit step signal

- We can obtain a D.T. unit step signal u[n] by taking the sum of unit samples. This is mathematically expressed as,

$$u[n] = \ldots + \delta[n] + \delta[n-1] + \delta[n-2] + \ldots$$

$$= \sum_{k=0}^{\infty} \delta[n-k] \qquad \ldots(1.5.11(b))$$

- The summation of unit impulses to obtain a unit step function is shown in Fig. 1.5.12(b).

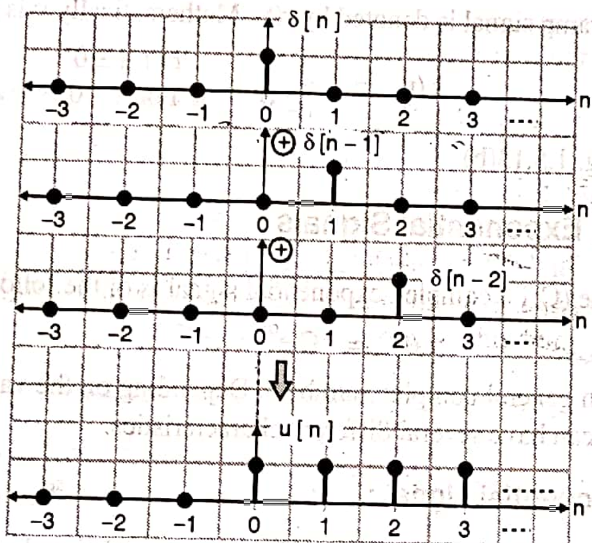


Fig. 1.5.12(b) : How to obtain a D.T. unit step from unit impulses

Sampling using a unit sample signal :

- The unit sample sequence can be used to sample the value of a signal.
- If signal x (n) is multiplied by a unit sample δ (n), then we get the value of x (n) at n = 0 as the product. That means,

$$x(n)\,\delta(n) = x(0) \qquad \ldots(1.5.11(c))$$

- This happens because δ (n) = 1 only at n = 0. Similarly we can write that,

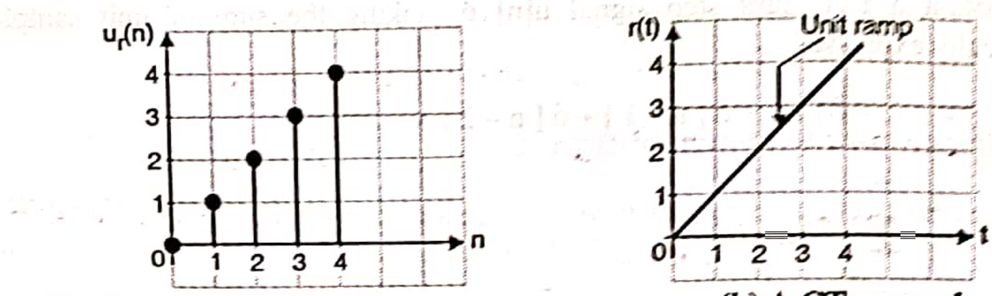
$$x(n)\,\delta(n-n_0) = x(n_0) \qquad \ldots(1.5.11(d))$$

1.5.8 Unit Ramp Signal :

- A discrete time unit ramp signal is denoted by $u_r(n)$. Its value increases linearly with sample number n. Mathematically it is defined as,

$$u_r(n) = \begin{cases} n & \text{For } n \geq 0 \\ 0 & \text{For } n < 0 \end{cases}$$

- Graphically it is represented as shown in Fig. 1.5.13(a).



(a) Unit ramp signal u_r (n) (b) A CT ramp signal r(t)

Fig. 1.5.13

- A continuous time ramp signal is denoted by r(t). Mathematically it is expressed as,

$$r(t) = \begin{cases} t & \text{For } t \geq 0 \\ 0 & \text{For } t < 0 \end{cases}$$

It is as shown in Fig. 1.5.13(b).

1.5.9 C.T. Complex Exponential Signals :

- The continuous time (C.T.) complex exponential signal is of the following form,

$$x(t) = C e^{\alpha t} \quad ...(1.5.12)$$

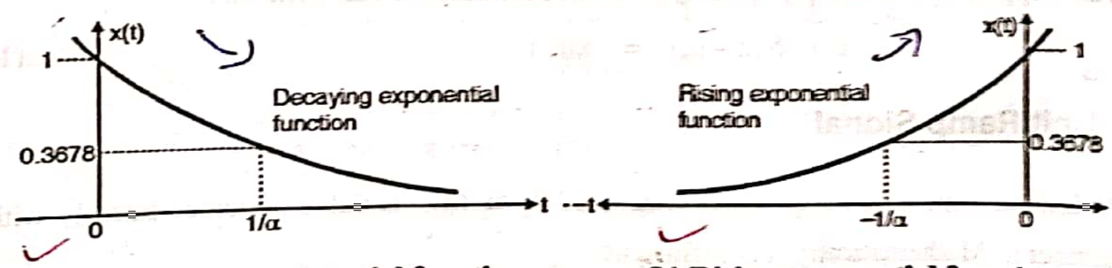
- Here C and α are in general complex numbers. Depending on the values of these parameters, the complex exponent can have several different characteristics.

Types of complex exponential signal :

- Depending on the type of C and α, the complex exponential signal can be classified as follows :
  1. Real exponential signals
  2. Periodic complex exponential signals

1. CT real exponential signals :

- If C and α both are real, then the corresponding exponential signal is called as the real exponential signal.
- The exponential functions also are used extensively in the signal analysis. There are two types of exponential functions viz., rising and decaying exponential functions as shown in the Fig. 1.5.14.



(a) Decaying exponential function (b) Rising exponential function

Fig. 1.5.14

- They are mathematically represented as follows :

1. Decaying exponential function : $x(t) = e^{-\alpha t}$ ...(1.5.13)

2. Rising exponential function : $x(t) = e^{\alpha t}$ ...(1.5.14)

- Note that we have assumed C = 1 for both the equations stated above.

2. CT periodic complex exponential signals :

- This is the second important type of complex exponential signals. For this type, α is assumed to be purely imaginary.
- Such an exponential is mathematically expressed as,

$$x(t) = e^{j\omega_0 t} \quad ...(1.5.15)$$

- The most important property of this signal is that it is a periodic signal. Applying the condition of periodicity we can write that,

$$x(t) = e^{j\omega_0 t} = e^{j\omega_0 (t+T)} \quad ...(1.5.16)$$

$$e^{j\omega_0 t} = e^{j\omega_0 t} \cdot e^{j\omega_0 T}$$

- The above expression will be true if and only if,

$$e^{j\omega_0 T} = 1$$

- So the conclusion is that for $e^{j\omega_0 t}$ to be periodic, $e^{j\omega_0 T}$ has to be equal to 1.
- If $\omega_o = 0$, then x (t) = 1 for any value of T. If $\omega_o \neq 0$, then the fundamental period T_o of x (t) that is the smallest positive value of T for which $e^{j\omega_0 T} = 1$ is

$$T_o = \frac{2\pi}{|\omega_o|} \quad ...(1.5.17)$$

- Thus the signals $e^{j\omega_0 t}$ and $e^{-j\omega_0 t}$ will have the same fundamental period of T_o.

1.5.10 Discrete Time exponential signals :

- A discrete time exponential signal is expressed as,

$$x(n) = a^n \quad ...(1.5.18)$$

Here 'a' is some real constant.

- If 'a' is the complex number then x (n) is written as,

$$x(n) = r e^{j\theta} \quad ...(1.5.19)$$

- Depending on the values of "a" we have four different cases,
  1. Case 1 : $a > 1$ 2. Case 2 : $0 < a < 1$
  3. Case 3 : $a < -1$ 4. Case 4 : $-1 < a < 0$

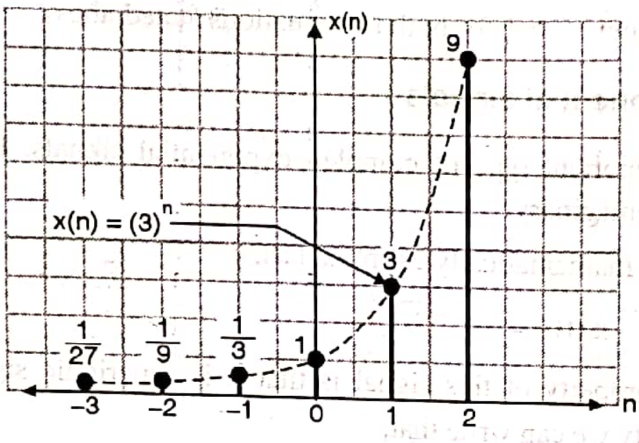
Case 1 : For a > 1 : Rising D.T. exponential

- Let a = 3. Thus we have,

$$x(n) = a^n = 3^n$$

- Graphically such signal is represented as shown in Fig. 1.5.15(a).

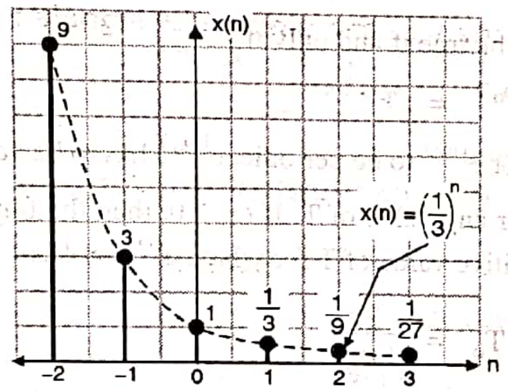
- Since the signal is exponentially growing; it is called as rising exponential signal.



n	3^n
–3	1/27
–2	1/9
–1	1/3
0	1
1	3
2	9

Fig. 1.5.15(a) : Rising exponential signal (a > 1)

Case 2 : For 0 < a < 1 : Decaying exponential signal



n	$(1/3)^n$
–3	27
–2	9
–1	3
0	1
1	1/3
2	1/9
3	1/27

Fig. 1.5.15(b) : Decaying exponential signal

- In this case we will get decaying exponential sequence. Let $a = \frac{1}{3}$

$$\therefore \quad x(n) = a^n = \left(\frac{1}{3}\right)^n.$$

- Graphically such signal is represented as shown in Fig. 1.5.15(b).

Case 3 : For a < – 1 :

- In this case we will get double sided rising exponential signal.

$$\text{Let } a = -3$$

$$\therefore \quad x(n) = (-3)^n$$

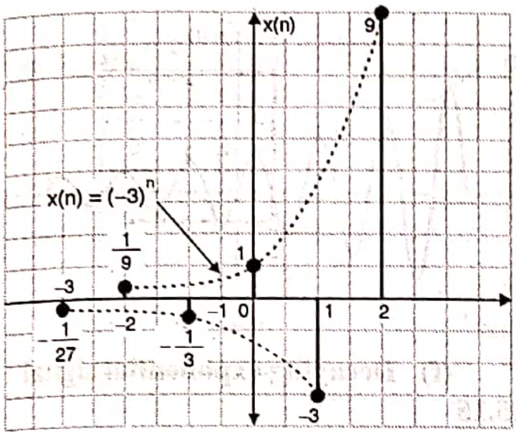
- Graphically such signal is represented as shown in Fig. 1.5.15(c).

Case 4 : For – 1 < a < 0 :

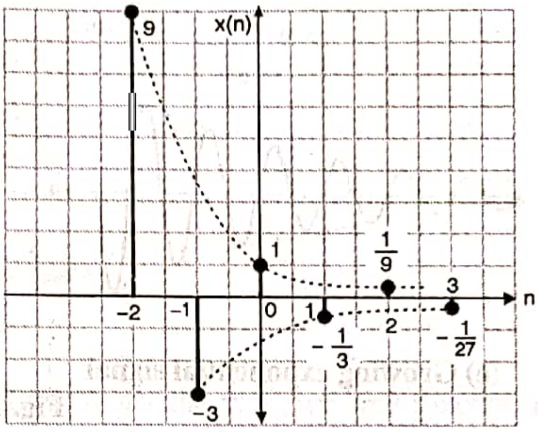
- In this case we will get double sided decaying exponential signal.

$$\text{Let } a = -\frac{1}{3} \qquad \therefore \quad x(n) = a^n = \left(-\frac{1}{3}\right)^n$$

- Graphically such signal is as shown in Fig. 1.5.15(d).



(c) Double sided growing exponential signal



(d) Double sided decaying exponential signal

Fig. 1.5.15

Relation between the complex exponential and sinusoidal signals :

- The C.T. complex exponential signal is given by,

$$x(t) = e^{j\omega_0 t} \quad ...(1.5.20)$$

- By Euler's relation, the complex exponential signal can be written in terms of sinusoidal signals as,

$$e^{j\omega_0 t} = \cos \omega_o t + j \sin \omega_o t \quad ...(1.5.21)$$

General complex exponential signal :

- The general complex exponential signal is given by,

$$x(t) = C e^{\alpha t}$$

- If we express C in the polar form and α in the rectangular form, then

$$C = |C| e^{j\theta}$$

$$\text{and} \quad \alpha = r + j\omega_o$$

- Substituting we get

$$C e^{\alpha t} = |C| e^{j\theta} e^{(r + j\omega_0) t}$$

$$= |C| e^{rt} e^{j(\omega_0 t + \theta)} \quad ...(1.5.22)$$

- Using the Euler's expression, we can expand this equation as follows :

$$C e^{\alpha t} = |C| e^{rt} \cos(\omega_o t + \theta) + j |C| e^{rt} \sin(\omega_o t + \theta) \quad ...(1.5.23)$$

- If $r = 0$ then $e^{rt} = 1$

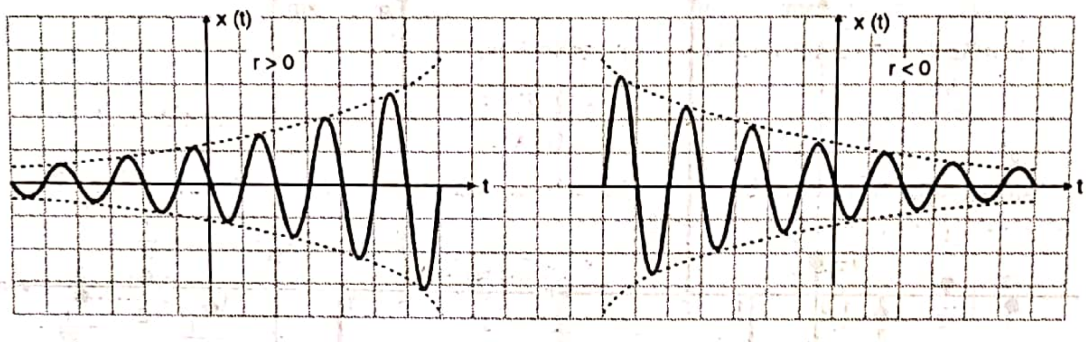
$$\therefore \; C e^{\alpha t} = |C| \cos(\omega_o t + \theta) + j |C| \sin(\omega_o t + \theta) \quad ...(1.5.24)$$

- Thus for $r = 0$, the real and imaginary parts of the complex exponential are sinusoidal.

For r > 0 : Growing exponential

- If $r > 0$, then e^{rt} in Equation (1.5.23) will be a growing exponential signal.
- So in Equation (1.5.24), the cosine and sine terms are multiplied by a growing exponential signal.

- Hence we get a growing sinusoidal signal as shown in Fig. 1.5.15(e).



(e) Growing exponential signal (f) Decaying exponential signal

Fig. 1.5.15

For r < 0 : Decaying exponential

- If r is negative (r < 0), then e^{rt} in Equation (1.5.23) will be a decaying exponential.
- Hence each sinusoidal signal in Equation (1.5.24) is multiplied by a decaying exponential. So we get a decaying sinusoidal signal as shown in Fig. 1.5.15(f).

Damped sinusoids : Sinusoidal signals, multiplied by decaying exponentials are called as damped sinusoids.

9

1.5.11 Sinc Function :

- The sinc function or sinc pulse is mathematically expressed as,

$$\text{Sinc function : } \operatorname{sinc}(x) = \frac{\sin(\pi x)}{(\pi x)} \quad \text{... For } x \neq 0 \qquad \text{...(1.5.25)}$$

where x is the independent variable. The procedure of plotting the sinc function is explained in Ex. 1.5.1.

- It is proved in Ex. 1.5.1, that → $x(t) = A \quad -\infty < t < \infty$ ---- I.5.1.

$$\operatorname{sinc}(x) = 1 \qquad \text{... At } x = 0$$

$$\text{and} \quad \operatorname{sinc}(x) = 0 \qquad \text{... At } x = \pm 1, \pm 2, \pm 3 \text{ ....}$$

- Hence the graphical representation of a sinc function is as shown in Fig. 1.5.16.

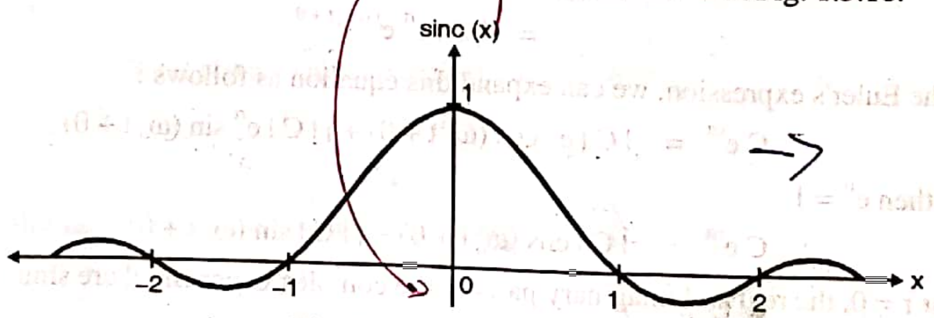


Fig. 1.5.16 : sinc function

- Fig. 1.5.16 shows that sinc function has the shape of a sinewave.
- Its magnitude goes on decreasing as the value of |x| increases.
- sinc x → 0 when |x| → ∞.

Ex. 1.5.2 : Plot graphically the sinc function.

Soln. :

- The sinc function is mathematically represented as,

$$\text{sinc}(x) = \frac{\sin(\pi x)}{(\pi x)} \quad \text{... For } x \neq 0 \quad \text{...(1)}$$

- Here x is an independent variable. We have to find the value of sinc function for different values of x including zero and negative values of x.

Substitute $x = 0$ in Equation (1)

$$\text{sinc}(x) = \frac{\sin 0}{0}.$$

- This is an indefinite form. Therefore we must find out the value of sinc function at $x = 0$ separately.

To find sinc(x) at x = 0 :

- Express $\sin(\pi x)$ in the exponential form. Using Euler's theorem we can express $\sin(\pi x)$ as,

$$\sin(\pi x) = \frac{e^{j\pi x} - e^{-j\pi x}}{2j} \quad \text{...(2)}$$

- To solve Equation (2), let us take help of the standard exponential series. It is given as,

$$e^t = 1 + t + \frac{t^2}{2!} + \frac{t^3}{3!} + \frac{t^4}{4!} + \ldots$$

Substitute $t = j\pi x$ to get, $\therefore\ e^{(j\pi x)} = 1 + j\pi x + \frac{(j\pi x)^2}{2!} + \frac{(j\pi x)^3}{3!} + \frac{(j\pi x)^4}{4!} + \ldots$ ...(3)

$$\text{Similarly, } e^{(-j\pi x)} = 1 - j\pi x + \frac{(-j\pi x)^2}{2!} + \frac{(-j\pi x)^3}{3!} + \frac{(-j\pi x)^4}{4!} + \ldots \quad \text{...(4)}$$

- Subtracting Equation (4) from Equation (3), we get,

$$e^{(j\pi x)} - e^{(-j\pi x)} = 2\,j\pi x + \frac{2}{3\,!}(j\pi x)^3 + \frac{2}{5\,!}(j\pi x)^5 + \ldots \quad \text{...(5)}$$

- Divide both the sides of Equation (5) by "2j" to get,

$$\sin(\pi x) = \frac{e^{(j\pi x)} - e^{(-j\pi x)}}{2j} = \pi xt + \frac{(j\pi x)^3}{j3!} + \frac{(j\pi x)^5}{j5!} + \ldots \quad \text{...(6)}$$

- Now divide both the sides of Equation (6) by (πx) to get,

$$\text{sinc}(x) = \frac{\sin(\pi x)}{\pi x} = 1 + \frac{1}{3!}\frac{(j\pi x)^3}{j\pi x} + \frac{1}{5!}\frac{(j\pi x)^5}{j\pi x} + \ldots$$

$$= 1 + \frac{1}{3!}(j\pi x)^2 + \frac{1}{5!}(j\pi x)^4 + \ldots = 1 + \frac{1}{3!}j^2\pi^2x^2 + \frac{1}{5!}j^4\pi^4x^4 + \ldots$$

$$\therefore\ \text{sinc}(x) = 1 - \frac{\pi^2x^2}{3!} + \frac{\pi^4x^4}{5!} + \ldots \quad \text{...(7)}$$

- Now substitute $x = 0$ in the Equation (7), to get,

$$\text{sinc}(0) = 1 \quad \text{...(8)}$$

Values of sinc(x) at other values of x :

At other values of x, find out sinc(x) using the Equation (1).

Table P. 1.5.2

x	sinc (x)	x	sinc (x)
0.25	0.9	– 0.25	0.9
0.50	0.6366	– 0.5	0.6366
0.75	0.3	– 0.75	0.3
1.5	– 0.2122	– 1.5	– 0.2122
2.5	0.1273	– 2.5	– 0.1273

By plotting these values, we get the waveform for sinc(x) as shown in Fig. P. 1.5.2.

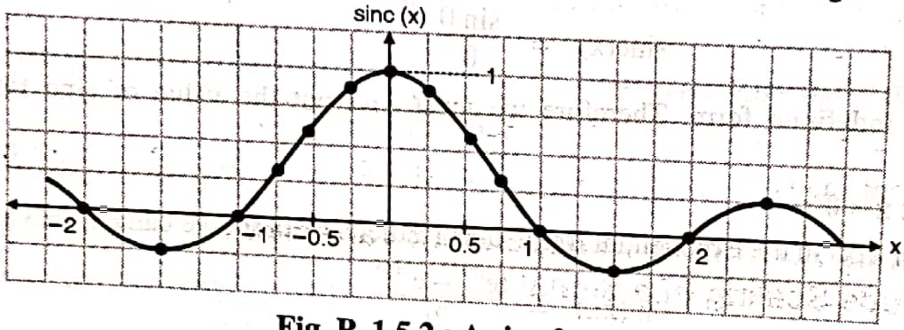


Fig. P. 1.5.2 : A sinc function

Conclusion from the Fig. P. 1.5.2 :

The sinc function passes through zero at the multiple values of x i.e. at $x = \pm 1, \pm 2, \pm 3$..... This is because $\sin(\pi x) = 0$ for multiple values of x and the amplitude of the "sinc" function decreases with increase in the magnitude of x.

Unit triangle function :

A continuous and discrete time unit triangular functions are shown in Fig. 1.5.17 and 1.5.18 respectively.

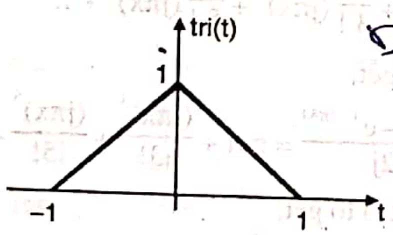


Fig. 1.5.17

Fig. 1.5.18

It is called as unit tringle function because its height and area both are one.

$$\text{tri}(t) = \begin{cases} 1 - |t| , & |t| < 1 \\ 0 , & |t| \geq 1 \end{cases}$$

Similarly D.T. triangular function is defined as,

$$\text{tri}(n) = \begin{cases} 1 - |n| , & |n| < 1 \\ 0 , & |n| \geq 1 \end{cases}$$

1.6 Signal Operations and Properties :

- The independent variable for a C.T. signal is time "t" and for a D.T. signal is "n".
- The transformation of the signal is an important concept in signal and system analysis.
- Transformation of independent variable needs simple modifications of "t" and "n".
- Some of the important transformations of the independent variable are :
  1. Time shifting 2. Time scaling 3. Folding or time reversal.
- In this section, we are going to discuss these three useful signal operations :
- However the discussion is valid for functions which have independent variables other than time (e.g. frequency) as well.
- Some useful signal operations are :
  1. Amplitude scaling 2. Addition and subtraction
  3. Multiplication 4. Time scaling 5. Time shifting.

1.6.1 Operations Performed on Dependent Variables :

1.6.1.1 Amplitude Scaling of a Signal :

Amplitude scaling of a CT signal :

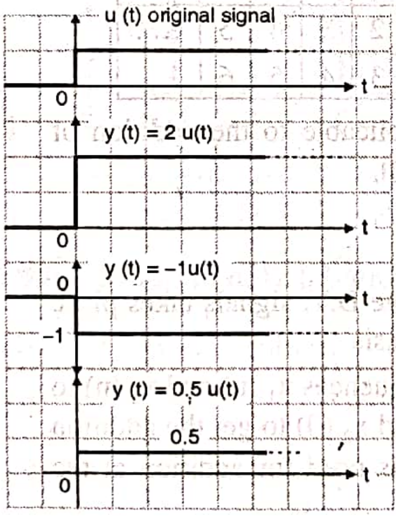
- Amplitude scaling means changing the amplitude of given continuous time signal.
- We will denote a continuous time signal by x (t). If it is multiplied by some constant 'A' then the resulting signal is,

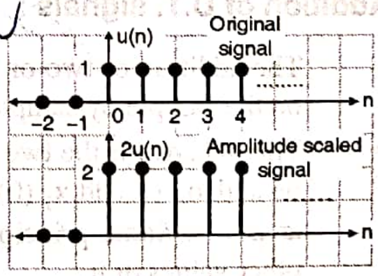
$$y(t) = A\,x(t)$$

- y (t) is the amplitude scaled version of x (t) and A is a constant.
- Depending on the value of A we can get different versions of x (t). This is illustrated in Fig. 1.6.1(a).
- Note that the scaling takes place on the y-axis and not on the x-axis (i.e. time axis).

Example :

- Let x (t) = u (t) be the original signal. Then the time scaled signal y (t) = A u (t) for different values of A is shown in Fig. 1.6.1(a).





(a) Amplitude scaling (b) Amplitude scaling of DT signal

Fig. 1.6.1

- The same concept is applicable to the discrete time signals as well.

Amplitude scaling of a DT signal :

- Fig. 1.6.1(b) illustrates the amplitude scaling of a DT signal.
- The original signal is given by,

$$y(n) = u(n) = \{..., 0, 0, ..., \underset{\uparrow}{1}, 1, 1, 1, ...\}$$

- The amplitude scaled signal is given by,

$$y(n) = 2u(n) = \{..., 0, 0, ..., \underset{\uparrow}{2}, 2, 2, 2, ...\}$$

1.6.1.2 Addition of Signals :

Addition of CT signals :

- Let the two signals to be added by $x_1(t)$ and $x_2(t)$. Then their addition is given by,

$$y(t) = x_1(t) + x_2(t)$$

- Note that the addition is to be made on the basis of instant to instant addition of the two signals.

Example :

Let $x_1(t) = u(t)$ and $x_2(t) = r(t)$

- Suppose we want to obtain $y(t) = x_1(t) + x_2(t)$ that means, $y(t) = u(t) + r(t)$. This operation is performed as follows :

Here $u(t)$ = Unit step = 1 for $t = 0$ to ∞.

and $r(t)$ = Unit ramp = t for $t = 0$ to ∞.

- The addition is shown in the tabulated form in Table 1.6.1 and graphically in Fig. 1.6.2(a).

Table 1.6.1

t	–2	–1	0	1	2	3	4	5	.....
$x_1(t) = u(t)$	0	0	1	1	1	1	1	1	.....
$x_2(t) = r(t)$	–2	–1	0	1	2	3	4	5	.....
y (t)	–2	–1	1	2	3	4	5	6	.....

- The same principle is applicable to the addition of discrete time signals as well.

Addition of D.T. signals :

- The addition of two or more D.T. signals takes place on the sample by sample basis.
- That means for the two sequences $x_1(n)$ and $x_2(n)$ to be added, we add $x_1(0)$ and $x_2(0)$ to get the addition at $n = 0$. Same principle is used for addition at the other values of n.

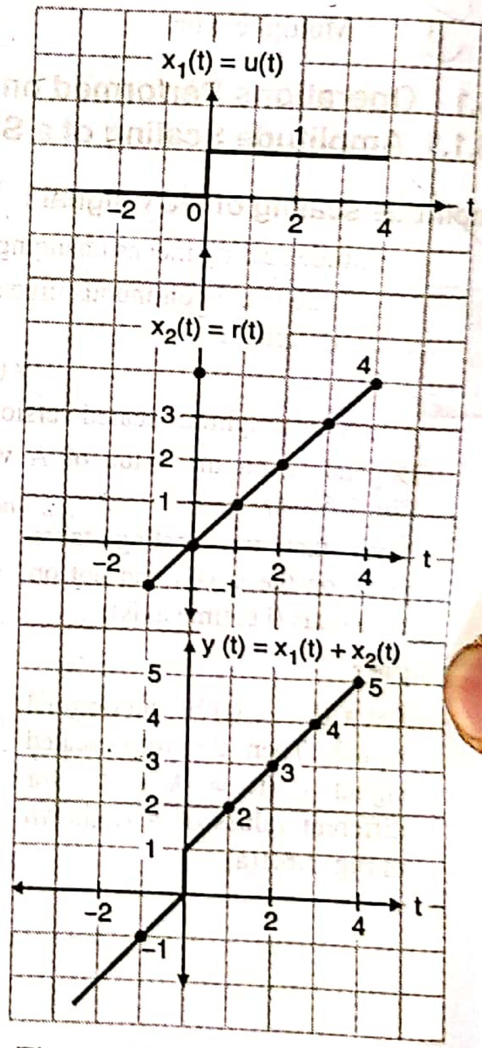


Fig. 1.6.2(a) : Addition of signals

- Let $x_1(n) = u(n)$ and $x_2(n) = u(n)$. Table 1.6.2 shows their addition in the tabular form while Fig. 1.6.2(b) shows the addition graphically.

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Table 1.6.2

n	– 2	– 1	0	1	2	3	4	5	.....
$x_1(n)$	0	0	1	1	1	1	1	1	.....
$x_2(n)$	– 2	– 1	0	1	2	3	4	5	.....
$y(n) = x_1(n) + x_2(n)$	– 2	– 1	1	2	3	4	5	6	.....

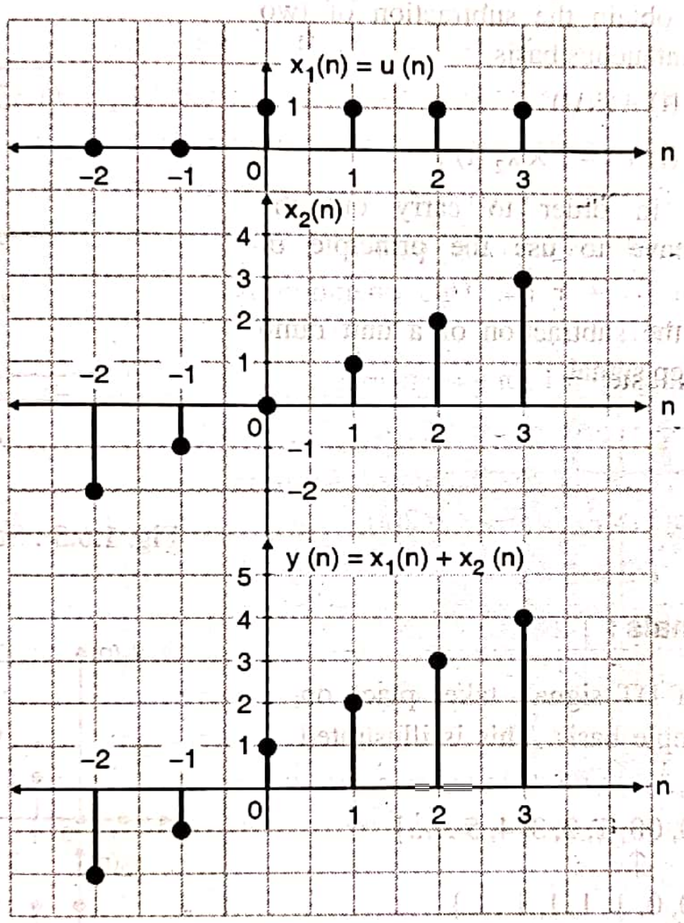


Fig. 1.6.2(b) : Addition of D.T. signals

- This addition can be mathematically represented as follows :

$$x_1(n) = \{0, 0, \underset{\uparrow}{1}, 1, 1, 1, ...\}$$

$$x_2(n) = \{..., -2, -1, \underset{\uparrow}{0}, 1, 2, 3, 4, ...\}$$

- And their addition is

$$y(n) = x_1(n) + x_2(n) = \{..., -2, -1, \underset{\uparrow}{1}, 2, 3, 4, ...\}$$

Example :

A physical example of a device that adds two signals is an audio mixer which adds music and sound signal.

1.6.1.3 Subtraction of Signals :

Subtraction of C.T. signals :

- Similarly we can obtain the subtraction of two signals on the instantaneous basis.

$$y(t) = x_1(t) - x_2(t)$$
$$= x_1(t) + [-1 \times x_2(t)]$$

- This shows that in order to carry out the subtraction, we have to use the principle of addition only.
- Fig. 1.6.3 shows the subtraction of a unit ramp signal and a unit step signal.

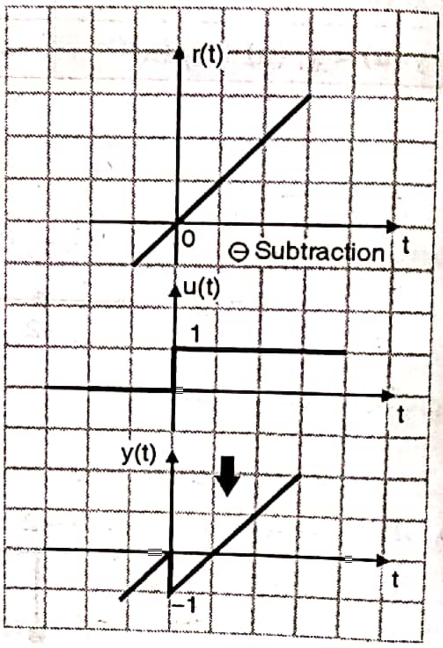


Fig. 1.6.3 : Subtraction of CT signals

Subtraction of DT signals :

- The subtraction of DT signals takes place on the sample by sample basis. This is illustrated in Fig. 1.6.4.

$$x_1(n) = \{0, 0\underset{\uparrow}{0}, 1, 2, 3, 4, 5 \ldots\}$$

$$\text{and } x_2(n) = \{0, 0, \underset{\uparrow}{1}, 1, 1, 1 \ldots\}$$

$$\therefore\ x_1(n) - x_2(n) = \{0, -\underset{\uparrow}{1}, 0, 1, 2, 3, 4, \ldots\}$$

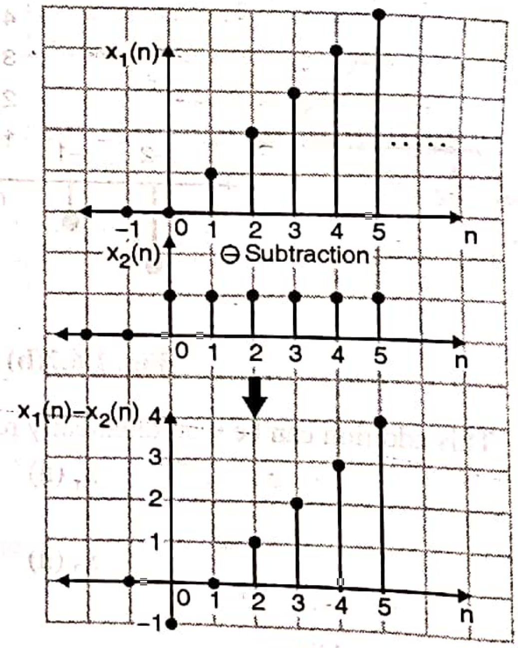


Fig. 1.6.4 : Subtraction of D.T. signals

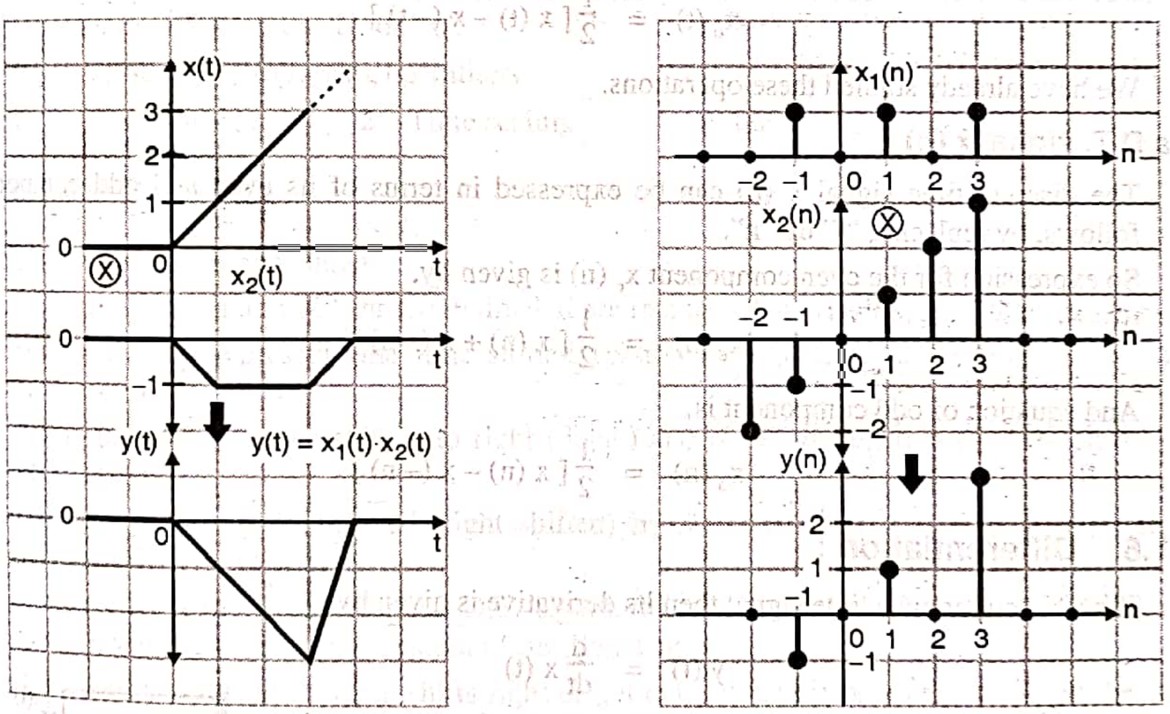
1.6.1.4 Multiplication of Two Signals :

Multiplication of CT Signals :

- If $x_1(t)$ and $x_2(t)$ are two continuous signals then the product of $x_1(t)$ and $x_2(t)$ is,

$$y(t) = x_1(t) \cdot x_2(t)$$

- In this case, the multiplication of amplitudes of two signals take place. For example, $y(t) = u(t) \cdot r(t)$.
- Since the amplitude of u (t) is 1; this multiplication will not change the ramp signal r (t). But if we perform, $y(t) = 2u(t) \cdot r(t)$ then it will make the slope of r (t) equal to 2.
- For each prescribed instant of time (t) the value of y (t) is given by the product of corresponding values of $x_1(t)$ and $x_2(t)$ at the same instant.
- Fig. 1.6.5(a) shows the example of multiplication of two CT signals.



(a) Multiplication of CT signals (b) Multiplication of DT signals

Fig. 1.6.5 : Multiplication of signals

Multiplication of D.T. Signal :

- Similarly for D.T. signals

$$y(n) = x_1(n) \cdot x_2(n)$$

- Fig. 1.6.5(b) shows the multiplication of two sequences. Note that the multiplication of DT signals is performed on the sample by sample basis.
- In Fig. 1.6.5(b) $x_1(n) = \{0, 1, \underset{\uparrow}{0}, 1, 0, 1\}$ and $x_2(n) = \{-2, -1, \underset{\uparrow}{0}, 1, 2, 3\}$

Then their product sequence is given by,

$$y(n) = \{0, -1, \underset{\uparrow}{0}, 1, 0, 3\}$$

Physical example :

A physical example is AM radio signal, in which x_1 (t) consists of an audio signal plus dc component and x_2 (t) is a carrier wave (sine wave).

1.6.1.5 Even and Odd Parts :

For CT signal :

- Even part of signal x (t) is given by,

$$x_e(t) = \frac{1}{2}[x(t) + x(-t)]$$

- and odd part of x (t) is given by,

$$x_o(t) = \frac{1}{2}[x(t) - x(-t)]$$

- We have already studied these operations.

For a D.T. signal x (n) :

- The discrete time signal x (n) can be expressed in terms of its even and odd components as follows, by replacing "t" by "n".
- So expression for the even component x_e (n) is given by,

$$x_e(n) = \frac{1}{2}[x(n) + x(-n)]$$

- And equation of odd component is,

$$x_o(n) = \frac{1}{2}[x(n) - x(-n)]$$

1.6.1.6 Differentiation :

- If x (t) is continuous time signal then its derivative is given by,

$$y(t) = \frac{d}{dt}x(t)$$

- For example, an inductor performs differentiation operation.
- Let the current passing through inductor be denoted by i (t) as shown in Fig. 1.6.6.
- Then the voltage across an inductor is,

$$v(t) = L\frac{d}{dt}i(t)$$

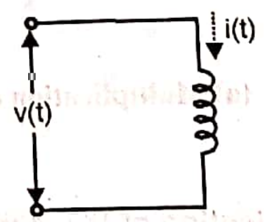


Fig. 1.6.6 : Differentiation

1.6.1.7 Integration :

- Similarly integration of x (t) with respect to time is given by,

$$y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$$

- Here τ is an integration variable.
- For example, a capacitor performs integration operation. Let i (t) be the current passing through capacitor as shown in Fig. 1.6.7.
- Then the voltage developed across the capacitor is,

$$v(t) = \frac{1}{C}\int_{-\infty}^{t} i(\tau)\, d\tau$$

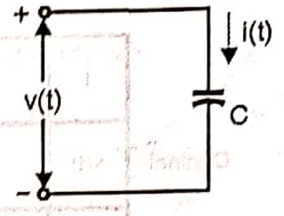


Fig. 1.6.7 : Integration

1.6.1.8 Accumulation for D.T. Signal :

- The integration is basically a process of summation. This process is equivalent to accumulation or summation of sample values over the specified interval of time.

1.6.2 Operations Performed on Independent Variable :

- The independent variable in our discussion is time "t" or "n". So the operations discussed in this section are performed on "t", or "n".
- We will discuss the following operations :

  1. Time shifting 2. Time scaling 3. Folding or time inversion.

1.6.2.1 Time Shifting :

- Let x (t) be the original signal.
- A signal x (t) is said to be "shifted in time" if we replace "t" by (t – T).
- Thus x (t – T) represents the time shifted version of x (t) and the amount of time shift is "T" second.
- If T is positive, then the shift is to right (delay) and if "T" is negative then the shift is to the left (advance).
- Thus x (t – 3) is x (t) delayed (right shifted) by 3 seconds and x (t + 2) is x (t) advanced (left shifted) by 2 seconds.
- Similarly let x (n) be the original D.T. signal. Then x (n – N) represents the shifted version of x (n), and N represents the amount of positional shift.
- It N is positive, then the signal shifts right or get delayed and if N is negative, then the signal is shifted left or it gets advanced.
- Thus x (n – 3) is x (n) delayed (right shifted) by 3 positions whereas x (n + 2) is x (n) advanced (left shifted) by 2 positions.

Time advance of CT Signals :

- Let x (t) = r (t) is the original signal which is present only for $t \geq 0$.

$$\therefore\ x(t) = r(t) = t \qquad \ldots\ t \geq 0$$
$$= 0 \qquad \ldots\ t < 0$$

- Then x (t + 2) represents the time advanced (left shifted) version of x (t).
- Table 1.6.3 shows the values of x (t) and x (t + 2) for different values of t and Fig. 1.6.8(a) shows the time advanced version.

Table 1.6.3 : Time advance

	t	–3	–2	–1	0	1	2	3	....
Original	x(t)	–3	–2	–1	0	1	2	3	....
Time advanced	x(t+2)	x(–3+2) = x(–1) = –1	x(–2+2) = x(0) = 0	1	2	3	4	5	....

Time advance by 2

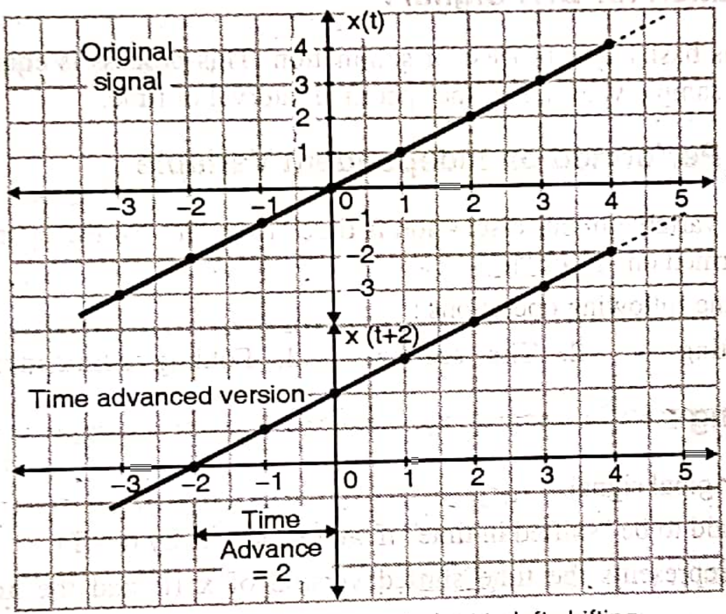


Fig. 1.6.8(a) : Time advancing

Note : When time advanced the signal is shifted left on the time axis (x-axis). So left shifting and time advance are one and the same.

Time advance for a D.T. signal :

Let x (n) = u (n) be the original signal. Then x (n + 3) = u (n + 3) is the advanced version of u (n) and the left shift is of three positions as shown in Fig. 1.6.8(b).

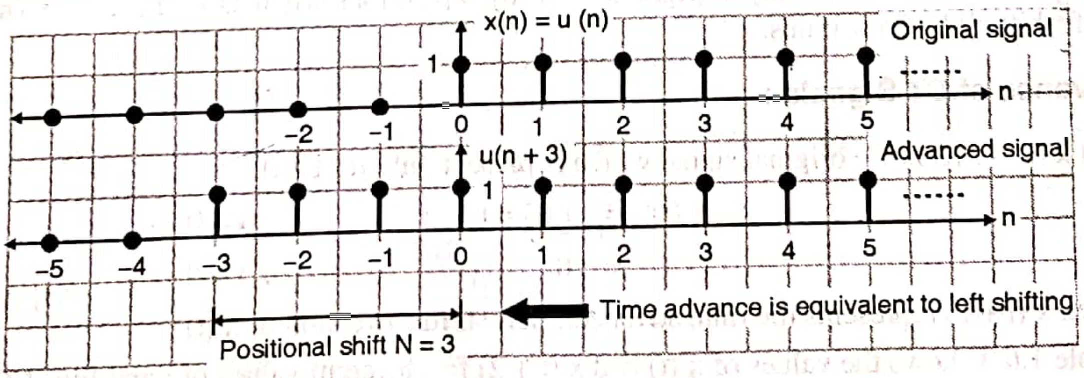


Fig. 1.6.8(b) : Positional advance of D.T. signal

Mathematical representation of shifting operation :

From Fig. 1.6.8(b) we can write the two sequences mathematically as follows :

Original signal : $x(n) = u(n) = \{ \ldots 0, 0, 0, 0, \underset{\uparrow}{1}, 1, 1, 1, 1 \ldots\ldots \}$

Shifted signal : $u(n+3) = \{ \ldots 0, \underbrace{1, 1, 1,}_{N=3} \underset{\uparrow}{1}, 1, 1, 1, 1 \ldots\ldots \}$

Note that the arrow is shifted right by N = 3 positions

Advance - left
Delay + Right

Note : The arrow indicating n = 0, has shifted three positions (N = 3) to the right as compared to the original sequence.

Time delay of CT signals :

- Let x (t) = u (t) be the original signal. Then x (t – 1) i.e. u (t – 1) represents the delayed version of x (t) with a delay equal to 1.
- Table 1.6.4 explains the concept of time delaying and Fig. 1.6.9(a) shows the waveforms.

Table 1.6.4 : Time delay

t	–1	0	1	2	3	4	......
x(t) = u(t)	0	1	1	1	1	1	......
x(t–1) = u(t–1)	u (–1–1) = u(–2) = 0	u (0–1) = u(–1) = 0	u (1–1) = u(0) = 1	u (2–1) = u(1) = 1	1	1	......

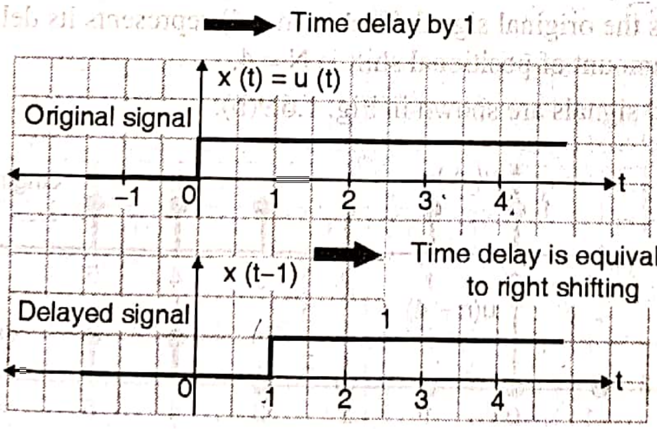


Fig. 1.6.9(a) : Time delay

Note : When time delayed, the signal is shifted right on the x-axis.

Ex. 1.6.1 : If the given signal $x(t) = e^{-at} u(t)$, draw the signals x (t + 2) and x (t – 3).

Soln. :

- x (t + 2) is a time advanced signal with a left shift of 2 places.
- x (t – 3) is a time delayed signal with a right shift of 3 places.
- These waveforms are shown in Fig. P. 1.6.1.

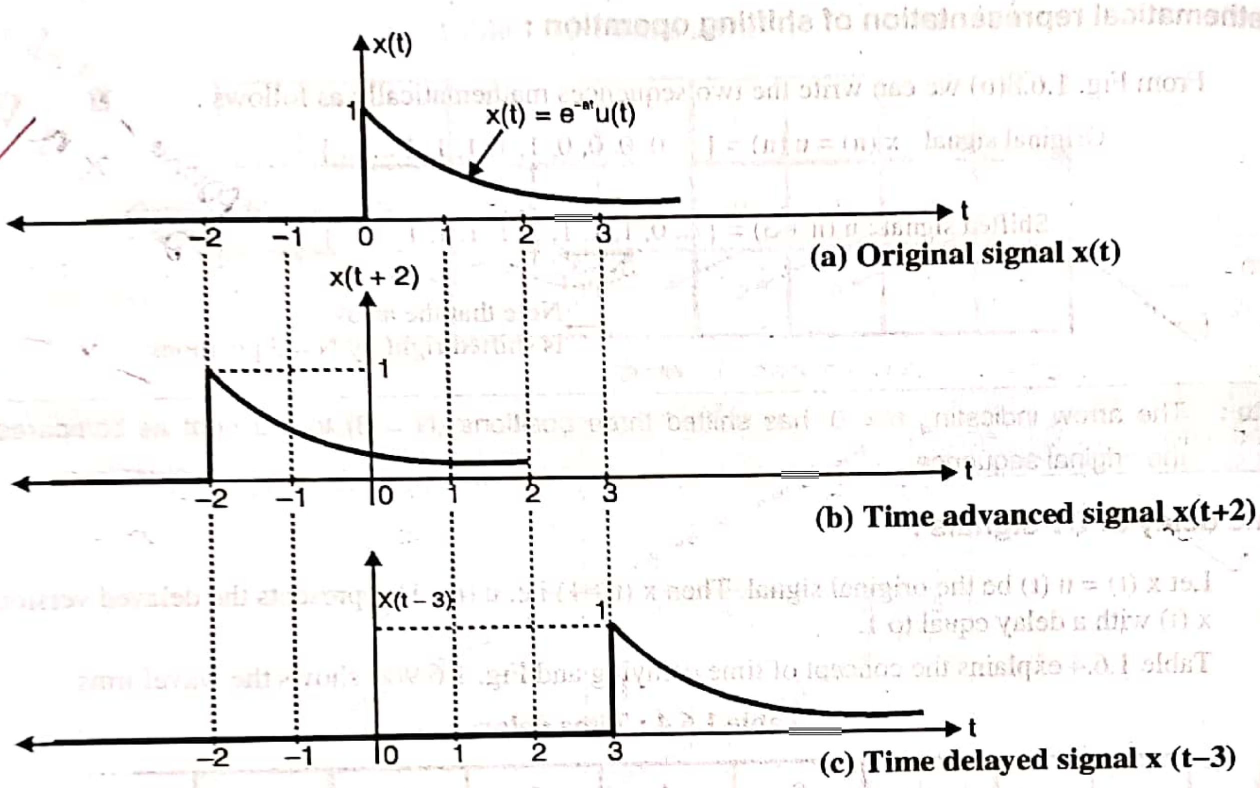


Fig. P. 1.6.1 : Time shifting

Time delay for a D.T. signal :

- For the original D.T. signal x (n), the delayed signal is denoted by x (n – N). The delayed signal is always the right shifted.
- Consider x (n) = u (n) as the original signal. Then x (n – 4) represents its delayed version or right shifted version and the amount of positional shift is N = 4.
- The original and delayed signals are shown in Fig. 1.6.9(b).

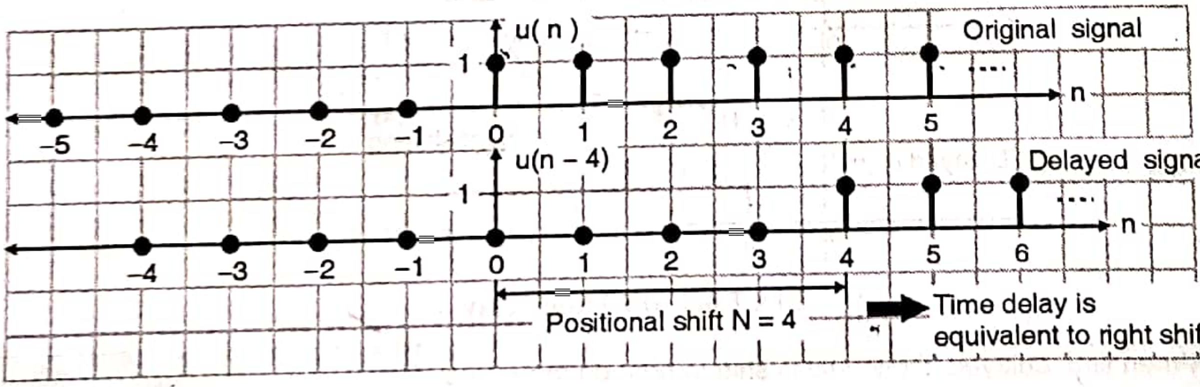


Fig. 1.6.9(b) : Positional delay of a D.T. signal

Mathematical representation :

From Fig. 1.6.9(b) we can express the two sequences mathematically as follows :

Original signal $x(n) = u(n) = \{ \dots 0, 0, 0, 0, \underset{\uparrow}{1}, 1, 1, 1, 1, \dots \}$

Delayed signal $u(n-4) = \{ \dots \underset{\uparrow}{0}, 0, 0, 0, 1, 1, 1, 1, 1 \dots \}$

Note that the arrow is shifted left by N = 4 positions

Note : The arrow indicating n = 0 has shifted N = 4 positions to the left as compared to the original signal.

1.6.2.2 Time Scaling :

Time scaling of CT signals :

- It x (t) is the original signal, then x (at) is its time scaled version, where a is a constant.
- Depending on the value of "a" we can either have signal compression or signal expansion.

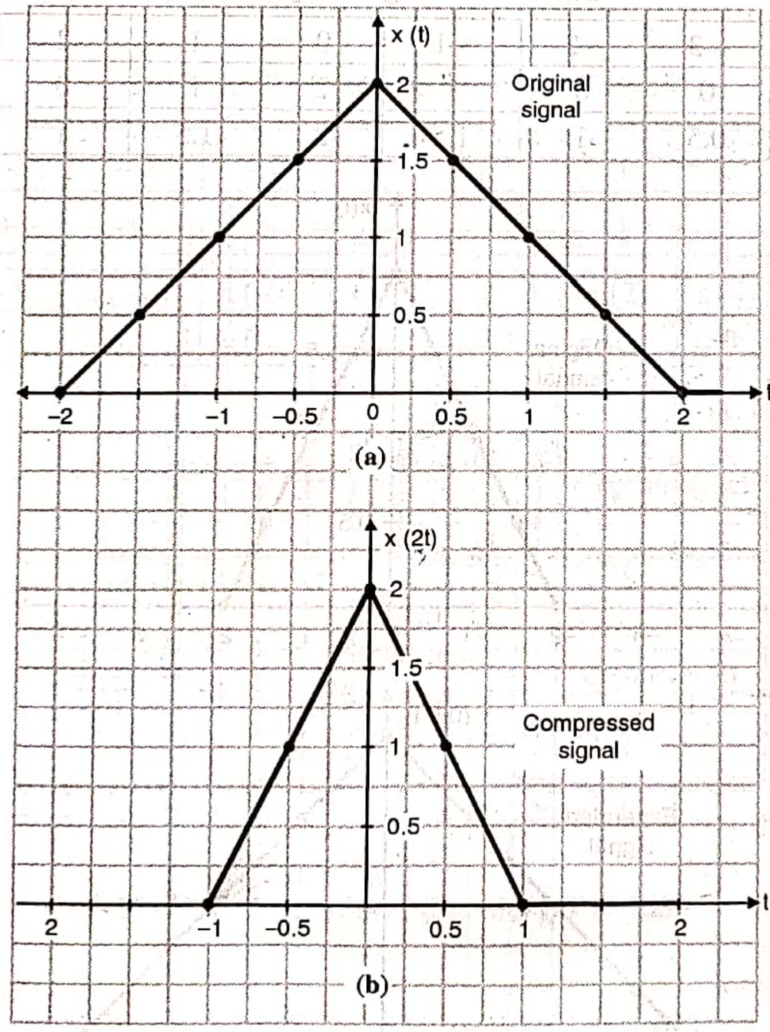


Fig. 1.6.10 : Signal compression

Signal compression a > 1 :

- For the values of "a" greater than 1, compression of the original signal will take place.

- Let the signal x (t) be a triangular wave as shown in Fig. 1.6.10(a). Then for a = 2 the compressed version of x (t) i.e. x (2t) is shown in Fig. 1.6.10(b).
- The time scaling (signal compression) process is also illustrated in Table 1.6.5.

Table 1.6.5 : Signal compression

t	− 2	− 1.5	− 1	0.5	0	0.5	1	1.5
x (t)	0	0.5	1	1.5	2	1.5	1	0.5
x(2t)	x (2x − 2) = x (− 4) = 0	x(− 3) = 0	x(− 2) = 0	x(− 1) = 1	x(0) = 2	x(1) = 1	x(2) = 0	x(3) = 0

Signal expansion :

- In x (at) if a is positive and less than 1 (0 < a < 1) then the time scaled signal is the expanded version of x (t).
- This process is illustrated in Table 1.6.6 and in Fig. 1.6.11.
- Note that the value of a is assumed to be equal to 0.5.

Table 1.6.6 : Signal expansion

t	− 4	− 3	− 2	− 1	0	1	2	3	4
x (t)	0	0	0	1	2	1	0	0	0
x(0.5 t)	0	0.5	1	1.5	2	1.5	1	0.5	0

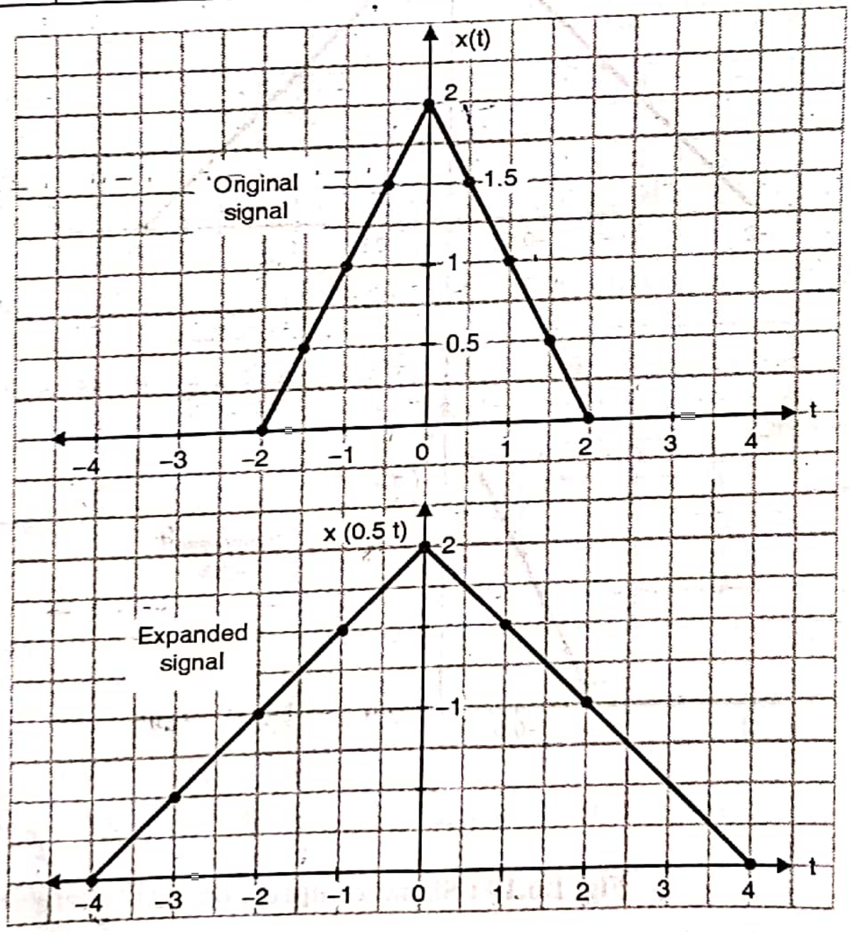


Fig. 1.6.11 : Signal expansion

Time scaling of DT signals :

- If x(n) is the original DT signal then x (an) is called as the time scaled version of x(n), where a is a constant.
- Depending on the value of "a", we can either have signal compression or signal expansion.

Signal compression (a > 1) :

- For a > 1, the signal compression will take place. Let the original signal be represented by,

$$x(n) = \{0, 0, 1, 1, 1, \underset{\uparrow}{1}, 1, 1, 1, 0, 0, \ldots\}$$

This signal is as shown in Fig. 1.6.12(a).

- For a = 2 the compressed version of x(n) i.e. x (2n) is as shown in Fig. 1.6.12(b). The time scaling process is illustrated in Table 1.6.7. The time scaled signal is given by,

$$x(2n) = \{0, 0, 0, 0, 1, \underset{\uparrow}{1}, 1, 0, 0, 0, 0\}$$

Table 1.6.7 : Compression of a DT signal

n	– 4	– 3	– 2	– 1	0	1	2	3	4
x (n)	0	1	1	1	1	1	1	1	0
x (2n)	x (– 8) = 0	x (– 6) = 0	x (– 4) = 0	x (– 2) = 1	x (0) = 1	x (2) = 1	x (4) = 0	x (6) = 0	x (8) = 0

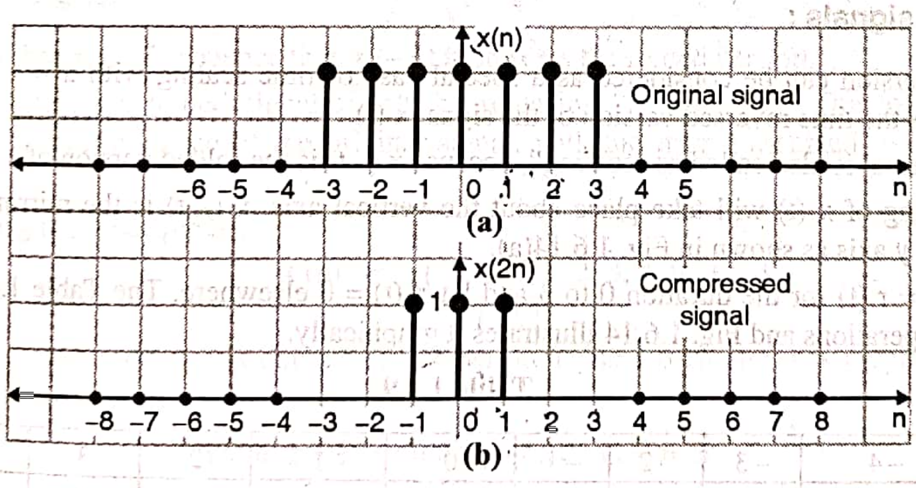


Fig. 1.6.12 : Compression of a DT signal

Signal expansion (a < 1) :

- In x (an) if a is positive and less than 1 (0 < a < 1) then the time scaled signal x (an) is the expanded version of the original signal x(n).
- This process is illustrated in Fig. 1.6.13 and Table 1.6.8 with a = 0.5.

The original signal $x(n) = \{0, 0, 0, 1, 1, 1, 1, \underset{\uparrow}{1}, 1, 1, 0, 0, \ldots\}$

Table 1.6.8 : Expansion of a DT signal

n	– 6	– 5	– 4	– 3	– 2	– 1	0	1	2	3	4	5	6
x (n)	0	0	0	1	1	1	1	1	1	1	0	0	0
x (0.5 n)	x (– 3) = 1	x (– 2.5) = 0	x (– 2) = 1	x (– 1.5) = 0	x (– 1) = 1	x (0.5) = 0	x (0) = 1	x (0.5) = 0	x(1) = 1	0	1	0	1

and the expanded signal is

$$x\,(0.5\,(n)) \;=\; \{0, 1, 0, 1, 0, 1, 0, \underset{\uparrow}{1}, 0, 1, 0, 1, 0, 1, 0....\}$$

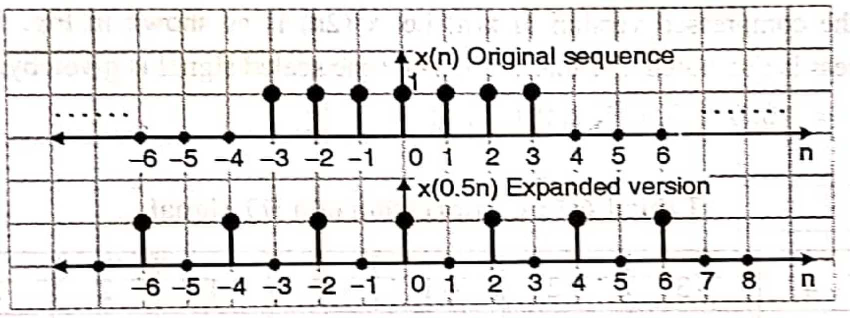


Fig. 1.6.13 : Expansion of a DT signal

1.6.2.3 Time Reversal (Time Inversion) or Folding :

Folding of CT signals :

- Time inversion can be considered as a special case of time scaling with a = – 1. Thus x (– t) represents the time reversed version of the signal x (t).
- Time reversal is also called as "folding", because x (– t) is the folded version of x (t).
- The folding of x (t) will take place about the vertical axis. x (– t) is the mirror image of x (t) about the y axis as shown in Fig. 1.6.14(a).
- Let x (t) = r (t) for the duration 0 to 3 and let x (t) = 0 elsewhere. The Table 1.6.9 explains the folding operations and Fig. 1.6.14 illustrates it graphically.

Table 1.6.9

t	– 4	– 3	– 2	– 1	0	1	2	3	4	.....
x (t)	0	0	0	0	0	1	2	3	0	.....
x(– t)	x[– (– 4)] = x(4) = 0	x(3) = 3	x(2) = 2	x(1) = 1	x(0) = 0	x(– 1) = 0	x(– 2) = 0	x(– 3) = 0	x(– 4) = 0	.....

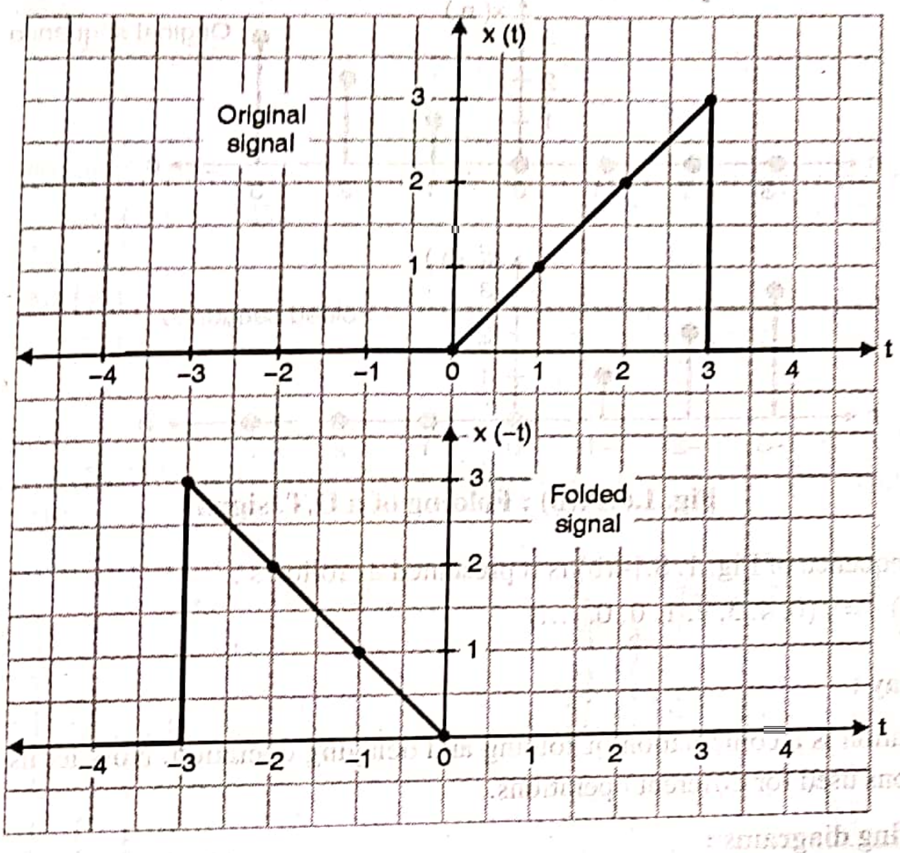


Fig. 1.6.14(a) : Process of folding

Folding of D.T. signal :

- If x (n) is the original sequence then x (– n) represents the folded version.
- We can obtain the folded signal simply by replacing "n" by "– n". The folded sequence is actually the mirror image of the original signal, with the mirror assumed to be placed at the y-axis.
- Let the original sequence be,

$$x(n) = \{..., 0, \underset{\uparrow}{0}, 1, 2, 3, 4, 0, 0, ...\}$$

- Then the folded sequence is obtained as shown in Table 1.6.10 and it is graphically plotted as shown in Fig. 1.6.14(b).

Table 1.6.10 : Folding of a D.T. signal

n	– 5	– 4	– 3	– 2	– 1	0	1	2	3	4	5	6	...
x(n)	0	0	0	0	0	0	1	2	3	4	0	0	...
x(– n)	x(5) = 0	x(4) = 4	x(3) = 3	x(2) = 2	x(1) = 1	x(0) = 0	x(– 1) = 0	x(– 2) = 0	x(– 3) = 0	x(– 4) = 0	x(– 5) = 0	x(– 6) = 0	...

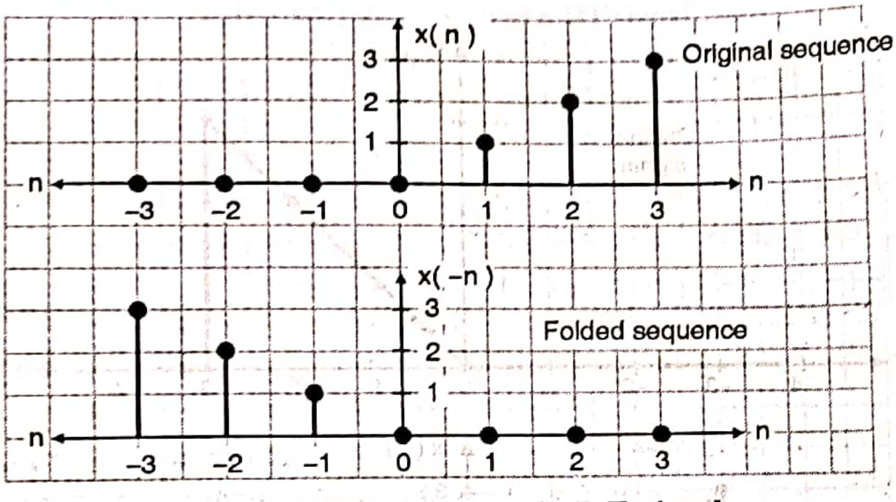


Fig. 1.6.14(b) : Folding of a D.T. signal

- The folded sequence of Fig. 1. 8.14(b) is represented as follows :

$$x(-n) = \{0, 4, 3, 2, 1, \underset{\uparrow}{0}, 0, \ldots\}$$

Folding and Delay :

This operation is a combination of folding and delaying operation. Now let us take a quick view at the modifications used for different operations.

1. For drawing diagrams :

- To obtain delayed sequence, shift the original diagram towards right by 'k' samples.
- To obtain advanced sequence, shift the original diagram towards left by 'k' samples.
- To obtain the folded version, take the mirror image of the diagram at n = 0.

2. For writing the sequence :

- The delayed version of x (n) is denoted by x (n – k)

  Negative sign always indicates delay operation of x (n).
- The advanced version of x (n) is denoted by x (n + k)

  Positive sign always indicates advanced operation of x (n).
- To obtain the folded version of x (n); replace n by – n

Now folding and delay operation means :

- First fold the sequence x (n); that means obtain x (– n)
- Then delay the folded sequence by k samples.

One major difference between x (n) and x (– n) is that x (– n) denotes the mirror image of x (n). We know that in case of mirror image; left and right sides are reversed. Now if x (n) is original sequence then its delayed version is denoted by x (n – k). The folded sequence is denoted by x (– n). Since it is a mirror image the delayed version of folded signal is denoted by x (– n + k).

$$\therefore\ x(n) \xrightarrow{\text{Delay}} x(n-k)$$

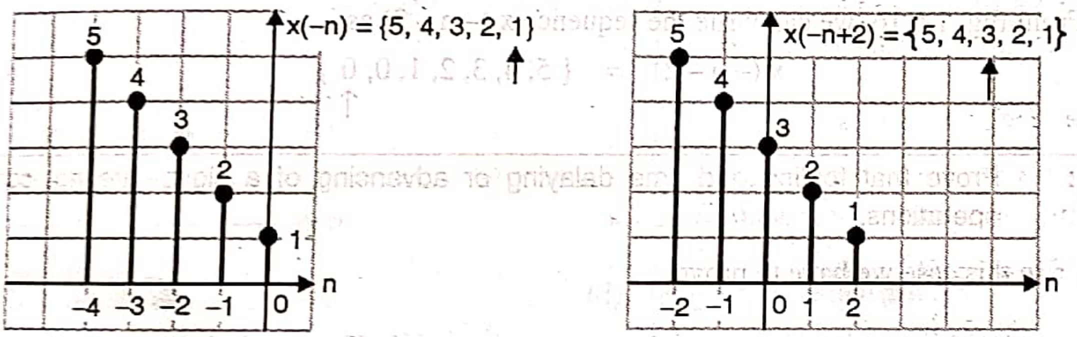
Folding ↓

$$x(-n) \xrightarrow{\text{Delay}} x[-(n-k)] = x(-n+k)$$

But stick to the basic concepts. Delay means shift the diagram towards right by k samples.

Consider the same original sequence $x(n) = \{\underset{\uparrow}{1}, 2, 3, 4, 5\}$ as shown in Fig. 1.6.15(a). The

Folded sequence x (– n) is shown in Fig. 1.6.15(a). Suppose we want to delay this folded sequence x (– n) by '2' samples then it will be denoted by x (– n + 2). This sequence is as shown in Fig. 1.6.15(b).



(a) Folded version x (– n) (b) Delay of folded sequence x (– n + 2)

Fig. 1.6.15

From Fig. 1.6.15(b), the sequence x (– n + 2) can be written as,

$$x(-n+2) = \{5, 4, \underset{\uparrow}{3}, 2, 1\}$$

Folding and Advance :

The advanced version of original sequence x (n) is denoted by x (n + k). The folded version of x (n) is denoted by x (– n). Since folding means mirror image of sequence; the advanced version of folded sequence is denoted by x (– n – k).

$$\therefore\ x(n) \xrightarrow{\text{Advance}} x(n+k)$$

Folding ↓

$$x(-n) \xrightarrow{\text{Advance}} x[-(n+k)] = x(-n-k)$$

Consider the same folded sequence x (– n) shown in Fig. 1.6.15(a). Suppose we want to advance this sequence by '2' samples, then the advanced version is denoted by x (– n – 2). Such a sequence is as shown in Fig. 1.6.16.

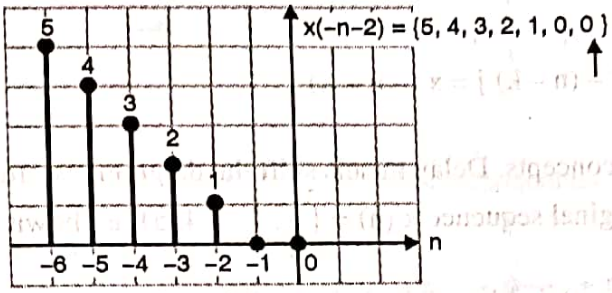


Fig. 1.6.16 : Advance of folded sequence, x (– n – 2)

Remember the basic rule. Advancing the sequence means shifting the diagram towards left by 'k' samples. From Fig. 1.6.16, we can write the sequence x (– n – 2) as,

$$x(-n-2) = \{5, 4, 3, 2, 1, 0, \underset{\uparrow}{0}\}$$

Ex. 1.6.2 : Prove that folding and time delaying or advancing of a signal are not commutative operations.

Soln. : In this case we have to prove,

$$\text{Delaying (Folding)} \neq \text{Folding (Delaying)} \quad \ldots(1)$$

Let the input sequence be,

$$x(n) = \{1, \underset{\uparrow}{1}, 1, 1\}$$

This sequence is represented in Fig. P. 1.6.2(a).

Now consider L.H.S.

Step 1 : First fold the sequence x (n). The folded sequence x (– n) is shown in Fig. P. 1.6.2(b).

Step 2 : Delay this sequence by 1 sample. This gives delaying of folding operation. This sequence is shown in Fig. P. 1.6.2(c).

Now consider the R.H.S.

Step 3 : First delay the sequence x (n) by one sample. This sequence is as shown in Fig. P. 1.6.2(d).

Step 4 : Now take the folded version of signal x (n – 1). This gives folding of delaying operation. Such sequence is as shown in Fig. P. 1.6.2(e).

Compare Fig. P. 1.6.2(c) and Fig. P. 1.6.2(e). Since these two diagrams are not same so;

Delaying (Folding) ≠ Folding (Delaying)

Hence proved.

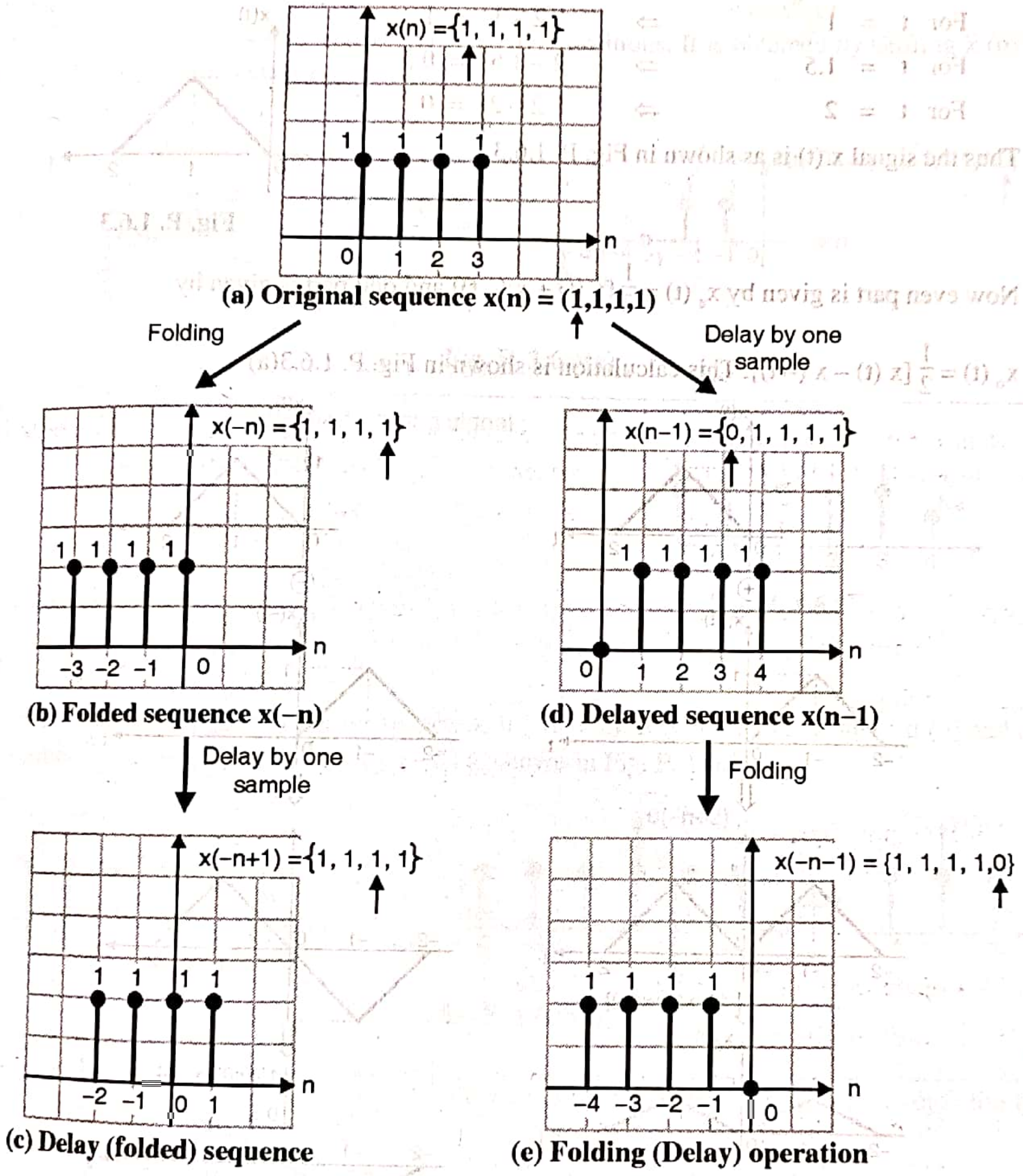


Fig. P. 1.6.2

Ex. 1.6.3 : Find and sketch the even and odd components of the following :

$$x(t) = \begin{cases} t & , 0 \le t \le 1 \\ 2-t & , 1 \le t \le 2 \end{cases}$$

Soln. :

Given : $x(t) = \begin{cases} t & , 0 \le t \le 1 \\ 2-t & , 1 \le t \le 2 \end{cases}$

In the range $0 \le t \le 1$ it is a ramp wave. Now for the range $1 \le t \le 2$, we will put some values of t

For $t = 1 \Rightarrow 2 - 1 = 1$

For $t = 1.5 \Rightarrow 2 - 1.5 = 0.5$

For $t = 2 \Rightarrow 2 - 2 = 0$

Thus the signal x (t) is as shown in Fig. P. 1.6.3.

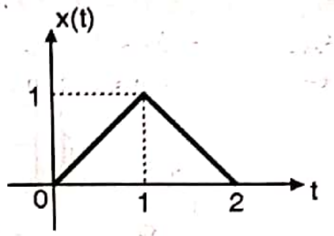


Fig. P. 1.6.3

Now even part is given by $x_e(t) = \frac{1}{2}[x(t) + x(-t)]$ and odd part is given by $x_o(t) = \frac{1}{2}[x(t) - x(-t)]$. This calculation is shown in Fig. P. 1.6.3(a)

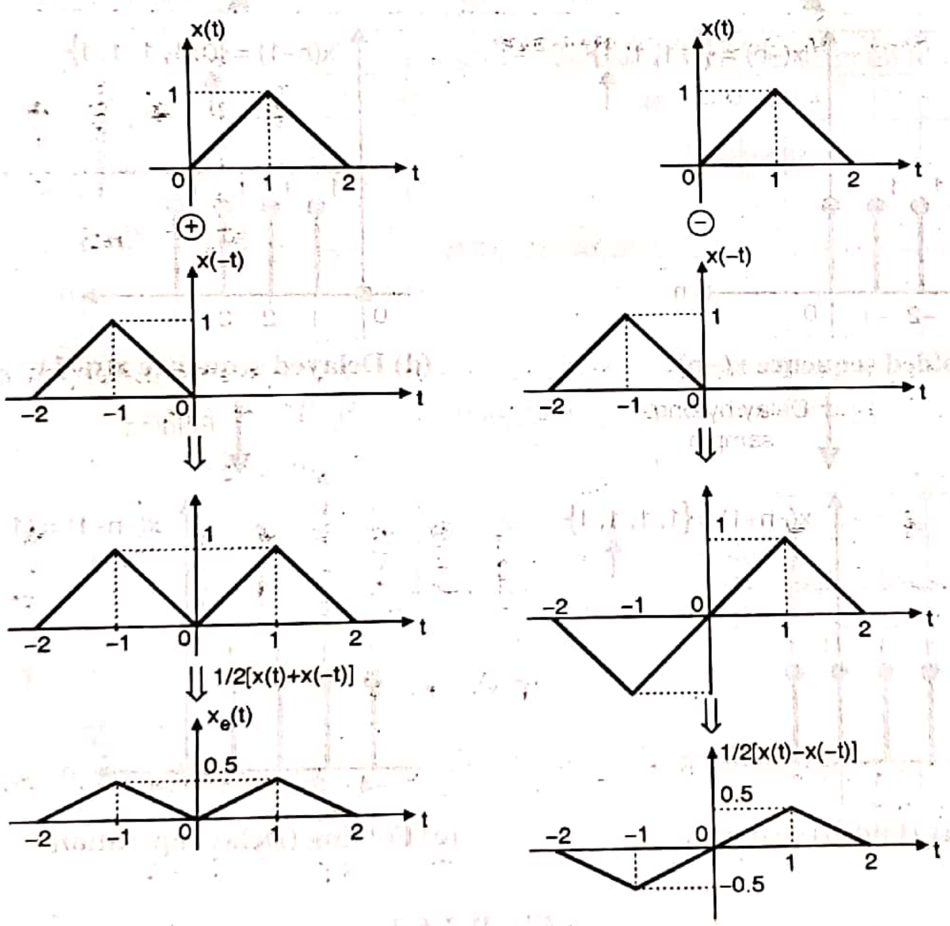


Fig. P. 1.6.3(a)

Ex. 1.6.4 : The discrete time signal :

$$x(n) = \begin{cases} 1, & n = 1, 2 \\ -1, & n = -1, -2 \\ 0, & n = 0 \text{ and } |n| > 0. \end{cases}$$

Find and sketch the signal $y(n) = x(n + 3)$

Soln. : The given sequence can be written as,

$$x(n) = \{-1, -1, \underset{\uparrow}{0}, 1, 1\}$$

Here x (n + 3) indicates advance of x (n) by 3 positions. It is obtained by shifting x (n) towards left by 3 positions, as shown in Fig. P. 1.6.4(a).

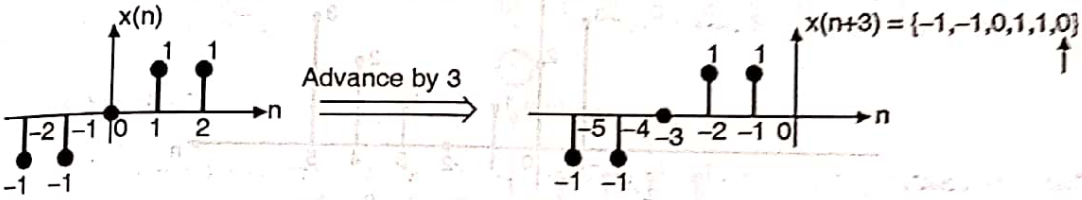


Fig. P. 1.6.4(a)

Ex. 1.6.5 : Sketch and label the following signal :

$$y(n) = x(n) \cdot u(2-n)$$

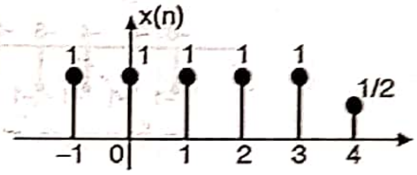


Fig. P. 1.6.5

Soln. :

Here the signal u (2 – n) can be written as u (– n + 2). It represents folding of u (n) and delaying by 2 positions. The operation x (n) · u (– n + 2) is shown in Fig. P. 1.6.5(a).

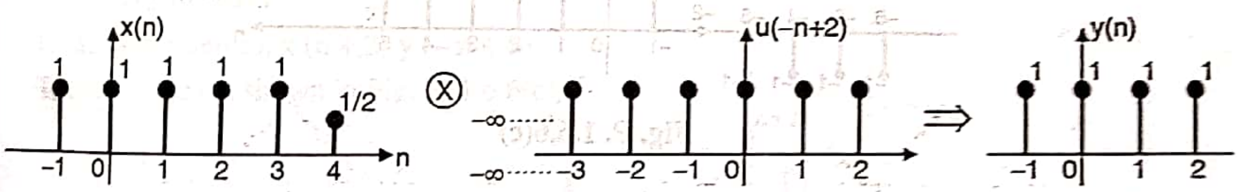


Fig. P. 1.6.5(a)

Ex. 1.6.6 : Let x [n] and y [n] be in Fig. P. 1.6.9(a) and P. 1.6.6(b) respectively. Sketch the following signals :

1. x [n – 2] + y [n + 2] 2. x [3 – n] y [n] 3. x [n + 2] y [6 – n]

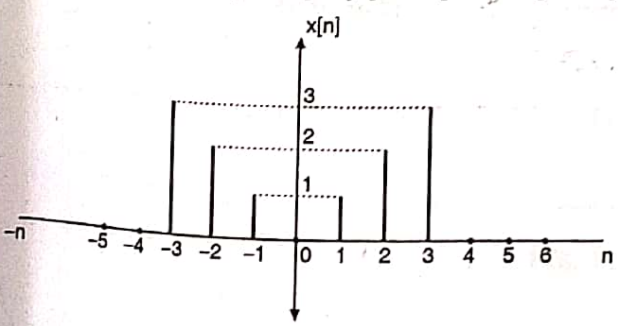


Fig. P. 1.6.6(a)

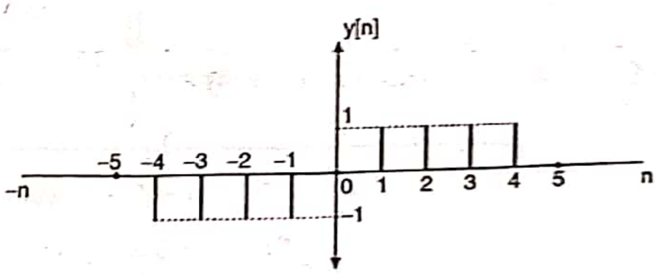


Fig. P. 1.6.6(b)

Soln. :

1. **x (n – 2) + y (n + 2) :**

This operation is shown in Fig. P. 1.6.6(c).

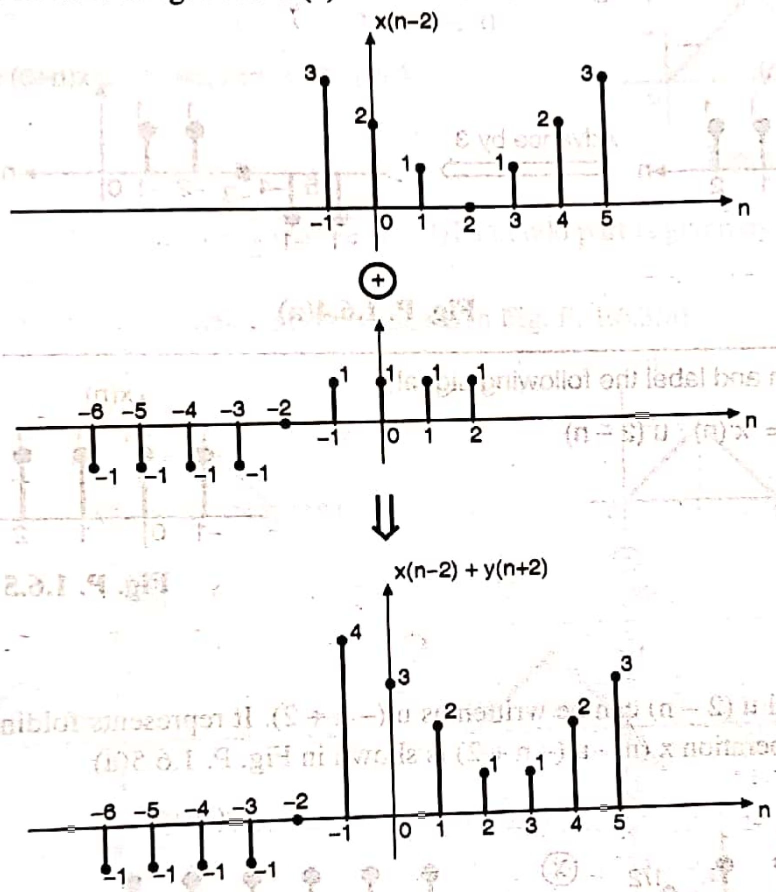


Fig. P. 1.6.6(c)

2. **x (3 – n) y (n) :**

It can be written as, x (– n + 3) y (n).

It is shown in Fig. P. 1.6.6(d).

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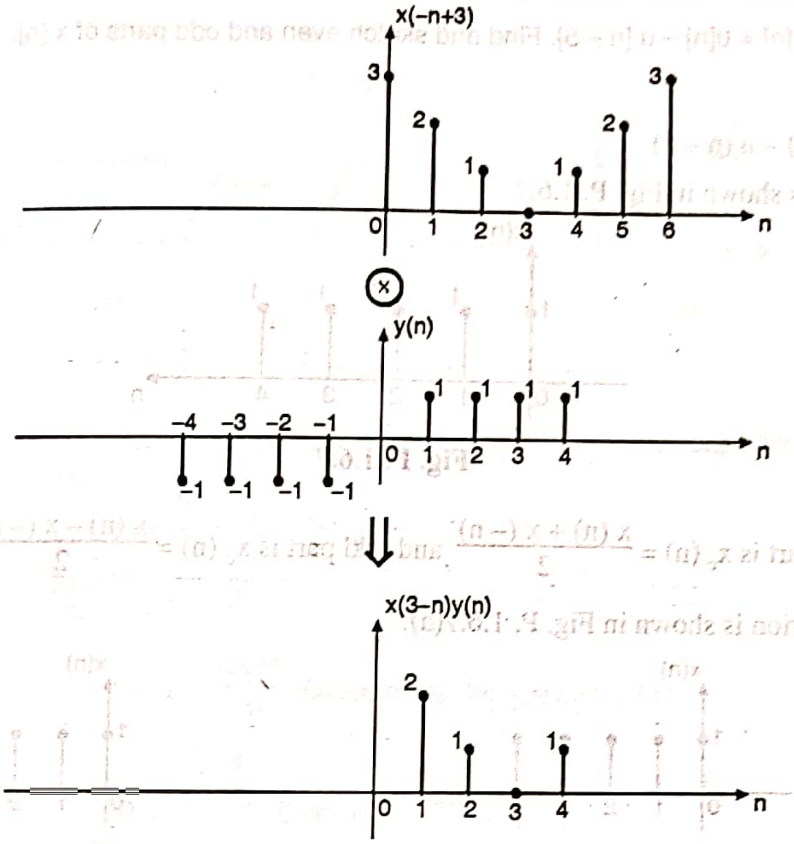

Fig. P. 1.6.6(d)

3. **x (n + 2) y (6 – n) :**

It can be written as, x (n + 2) y (– n + 6).

This operation is shown in Fig. P. 1.6.6(e).

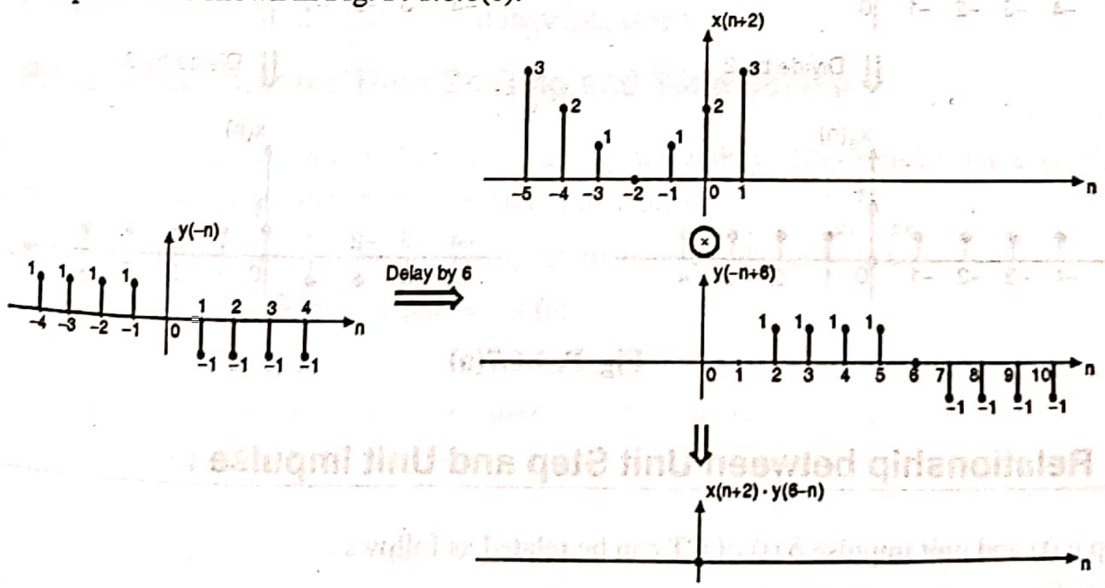

Fig. P. 1.6.6(e)

Ex. 1.6.7 : Let x[n] = u[n] – u [n – 5]. Find and sketch even and odd parts of x [n].

Soln. :

Given : $x(n) = u(n) - u(n-5)$

This signal is shown in Fig. P. 1.6.7

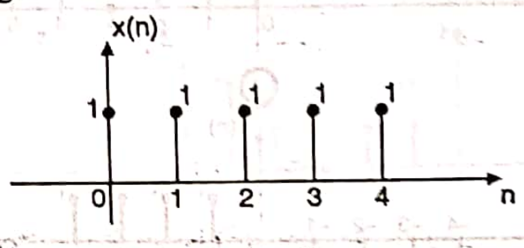

Fig. P. 1.6.7

Now even part is $x_e(n) = \frac{x(n) + x(-n)}{2}$ and odd part is $x_o(n) = \frac{x(n) - x(-n)}{2}$

This calculation is shown in Fig. P. 1.6.7(a).

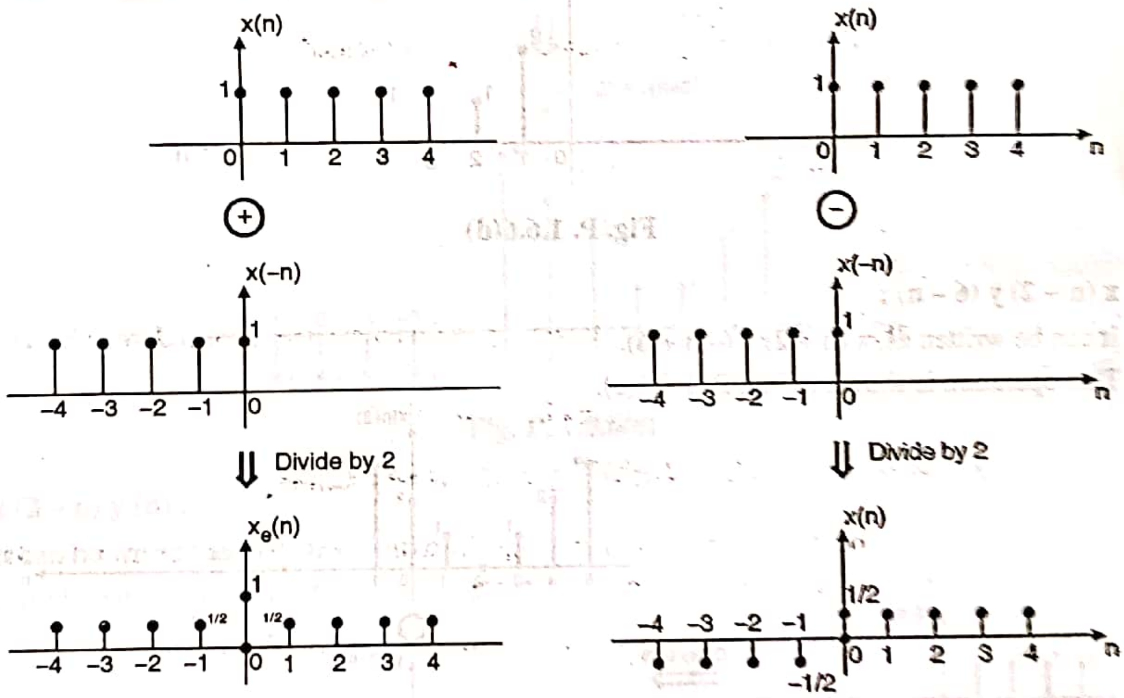

Fig. P. 1.6.7(a)

1.7 Relationship between Unit Step and Unit Impulse :

Unit step u (t) and unit impulse δ (t) of CT can be related as follows :

- Mathematically δ (t) is derivative of u (t) or
  u (t) is integral of δ (t)
- Unit step is not differentiable because there is discontinuity at t = 0 as shown in Fig. 1.7.1(a).
- Hence u (t) has limiting case of $u_\Delta(t)$ as shown in Fig. 1.7.1(b).

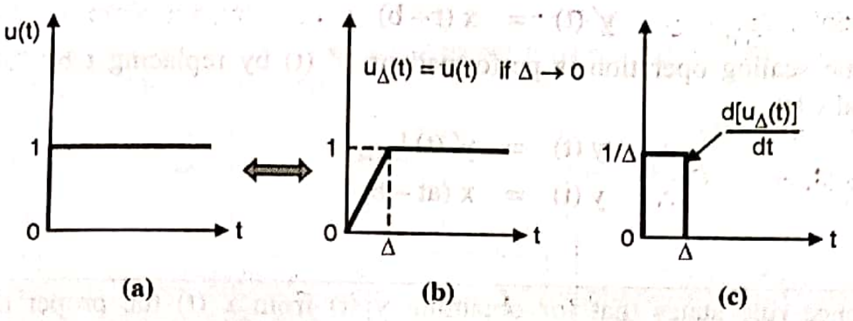

Fig. 1.7.1

- We know that u_Δ (t) is continuous function and hence it is differentiable. Derivative of u_Δ (t) as shown in Fig. 1.7.1(c).
- As $\Delta \to 0$ Fig. 1.7.1(c) becomes unit impulse.

$$\therefore \lim_{\Delta \to 0} \frac{d\,[u_\Delta (t)\,]}{dt} = \delta t$$

$$\therefore \delta (t) = \frac{du(t)}{dt}$$ Or same can be represented as

$$u(t) = \int_{-\infty}^{t} \delta(\tau)\, d\tau$$

Importance of impulse :

- To convert CT-signal into DT-signal samples of CT signals are required to take at regular interval of time.
- For sampling of CT signal a train of unit impulses is required.

1.7.1 Precedence Rule for Time Shifting and Time Scaling :

- Let x (t) and y (t) be two continuous time signals with y (t) derived from x (t) through a combination of time shifting and time scaling operations.
- Let the relation between x (t) and y (t) be given mathematically as follows :

$$y(t) = x(at - b) \qquad \text{...(1.7.1)}$$

where a and b are constants.
- Let the relation between x (t) and y (t) satisfy the following two conditions :

Condition - 1 $\quad y(0) = x(-b)$

and

Condition - 2 $\quad y(b/a) = x(0)$
- In order to obtain y (t) correctly from x (t), it is necessary to perform the time shifting and time scaling operations in the correct order.
- The proper order is decided on the basis of the fact that in the scaling operation "t" is replaced by "at" and in the time shifting operation "t" is replaced by (t – b).
- So the time shifting operation is carried out on x (t) to get an intermediate signal y′ (t) as,

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$$y'(t) = x(t-b) \quad ...(1.7.2)$$

- Then the time scaling operation is performed on y′ (t) by replacing t by "at". The result is, desired signal y (t).

$$\therefore \quad y(t) = y'(t)\big|_{t=at}$$

$$\therefore \quad y(t) = x(at-b) \quad ...(1.7.3)$$

Conclusion :

The precedence rule states that for obtaining y (t) from x (t) the proper order needs to be followed. First time shifting and then the time scaling needs to be performed.

Diagrammatic representation of precedence rule :

The precedence rule can be represented diagrammatically as shown in Fig. 1.7.2.

$$x(t) \longrightarrow \boxed{\text{Time shifting}} \xrightarrow{x(t-b)} \boxed{\text{Time scaling}} \longrightarrow y(t) = x(at-b)$$

Fig. 1.7.2 : Precedence rule

What if the precedence rule is not followed ?

If the rule is not followed, then we do not obtain the desired signal y (t). This is demonstrated as follows :

- First perform the time scaling operation on x (t) to get y′ (t)

$$y'(t) = x(at)$$

- Then perform the time shifting operation on y′ (t) by replacing t by (t – b).

$$\therefore \quad y(t) = x[a(t-b)] = x[at-ab]$$

- We have not obtained the desired signal y (t) = x (at – ab).
- To understand the precedence rule, solve the following example :

Ex. 1.7.1 : If x (t) = rect(t/3) then obtain y (t) = x (2t – 3) first by following the precedence rule and then by violating the rule.

Soln. :

Step 1 : Draw x (t) :

x (t) is given by,

$$x(t) = \text{rect}\left[\frac{t}{3}\right]$$

So it is a rectangular signal of width 3 time units and an amplitude equal to 1. It is plotted as shown in Fig. P. 1.7.1(a).

Part I : Solution by following the precedence rule :

Step 2 : Delay x (t) by 3 units :

As shown in Fig. P. 1.7.1(b), x (t) is delayed by 3 time units to obtain the intermediate signal y′ (t) = x (t – 3).

- The number of quantization levels Q = Number of combinations of bits/word.

$$\therefore \; Q = 2N \quad ...(1.9.25)$$

- Thus if N = 4 i.e. 4 bits per word then the number of quantization levels will be 2^4 i.e.16.

Signal to quantization noise ratio (SNR_q) :

- This ratio is the figure of merit for the PCM systems. The signal to quantization noise ratio with a sinusoidal input signal to the PCM system is expressed as,

$$\frac{S_i}{N_q} = [1.8 + 6N] \text{ dB} \quad \textbf{For a sinusoidal signal} \quad ...(1.9.26)$$

- This equation shows that the signal to quantization noise ratio is solely dependent on the number of bits per word i.e. N.
- This ratio should be as high as possible, which can be achieved by increasing N. But this increases the bit rate and hence bandwidth of the PCM system.
- Therefore the number of bits per word is a compromise between high SNR_q and bandwidth requirements.

Review Questions

Q. 1 Define signal.

Q. 2 Give the classification of signals.

Q. 3 Define continuous time (CT) and discrete time (DT) signals.

Q. 4 Differentiate between CT and DT signals.

Q. 5 How is a CT signal represented mathematically ?

Q. 6 How do you obtain a DT signal from a CT signal ?

Q. 7 Define a digital signal.

Q. 8 Define periodic and nonperiodic signals.

Q. 9 State the condition of periodicity.

Q. 10 Compare periodic and nonperiodic signals.

Q. 11 Prove that $x(t) = A \cos \omega_0 t$ is a periodic signal.

Q. 12 Define deterministic and random signals.

Q. 13 Compare deterministic and random signals.

Q. 14 Define even and odd signals.

Q. 15 Compare energy and power signals.

Q. 16 Define signal energy.

Q. 17 Define signal power.

Q. 18 Define multichannel and multidimensional signals.

Q. 19 Define the following signals and write their mathematical expressions :

1. DC signal
2. Exponential signal
3. Rectangular pulse
4. Signum function.

Q. 20 State mathematical expression for unit step and rectangular signals.

Q. 21 State the important properties of delta function.

Q. 22 State the relation between DT unit impulse and DT unit step signals.

Q. 23 Define a unit ramp signal and draw it graphically.

Q. 24 State different types of complex exponential signal.

Q. 25 What is the relation between the complex exponential and sinusoidal signals.

Q. 26 Draw the x(t) = sinc (t) signal graphically.

Q. 27 Define the following :

1. Time shifting.
2. Time scaling

Q. 28 Define folding or time reversal.

Q. 29 What is the difference between amplitude scaling and time scaling ?

Q. 30 State the relationship between unit step and unit impulse signals.

Q. 31 State the precedence rule for time shifting and time scaling.

❑❑❑

CHAPTER 2

Systems

Syllabus :

Basic system properties, Classification of systems, Case study of different signals from communication and biomedical field.

2.1 System :

Definition :

- A system is defined as an entity that operates on one or more signals to implement a function, thereby producing new signals.
- The block diagram representation of a system is shown in Fig. 2.1.1.

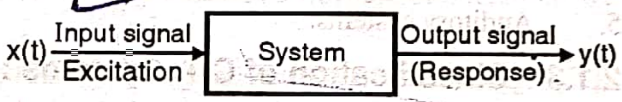


Fig. 2.1.1 : Block diagram representation of a signal

- A system may be defined as a set of elements and functional blocks interconnected to produce an output y (t) in response to an input x (t).
- The input signal x (t) is also called as excitation of a system and y (t) is also called as response of the system.
- Some of the examples of continuous time (CT) systems around us are : amplifier, filters, and other electronic systems.
- The response y (t) of a system to the excitation x (t), in the time domain depends on the impulse response h (t) of the system. The relation between them is as follows :

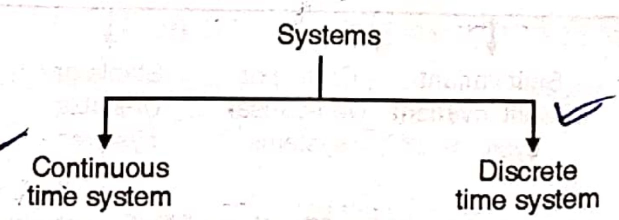
$$y(t) = x(t) * h(t) \quad \text{...(2.1.1)}$$

where * represents the convolution of x (t) and h (t).

- In the frequency domain, the system response depends on the transfer function of the system.

2.1.1 Types of Systems :

Depending on the type of input signal (continuous time or discrete time) the systems are classified into two types as follows :



Continuous time (CT) system :

- A continuous time system is defined as the system which processes a continuous time (CT) signal and produces another continuous time (CT) signal at its output.
- The CT signal at the input of CT system is denoted by x(t) and the CT response of the system is denoted by y(t) as shown in Fig. 2.1.1.

Discrete time (DT) system :

- A discrete time (DT) system is defined as the system which processes a discrete time (DT) signal and produces another discrete time (DT) signal at its output.
- The DT signal at the input of DT system is denoted by x (n) and the DT response of the system is denoted by y (n).

  Fig. 2.1.2 shows the block diagram of a D.T. system.

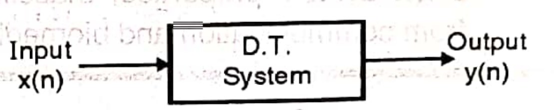


Fig. 2.1.2 : Block schematic of a D.T. system

2.1.2 Examples of Practical Systems :

There are several types of practical systems. We can classify them into following categories :

1. Communication system
2. Control system
3. Remote sensing
4. Biomedical signal processing
5. Auditory systems.

2.1.3 Classification of C.T. Systems :

The systems are classified as follows :

1. Linear and non-linear systems
2. Time variant and time invariant systems
3. Causal or non-causal systems
4. Stable and unstable systems.
5. Static or dynamic systems.

This classification is made based on various properties of systems.

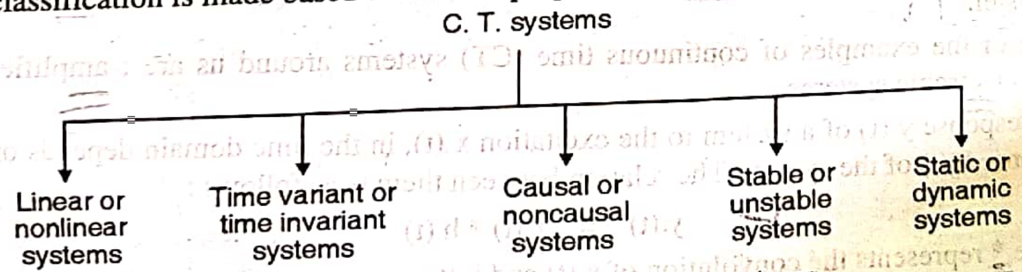


Fig. 2.1.2(a) : Classification of C.T. systems

2.1.4 Classification of D.T. Systems :

Similarly the D.T. systems can be classified as shown in Fig. 2.1.2(b).

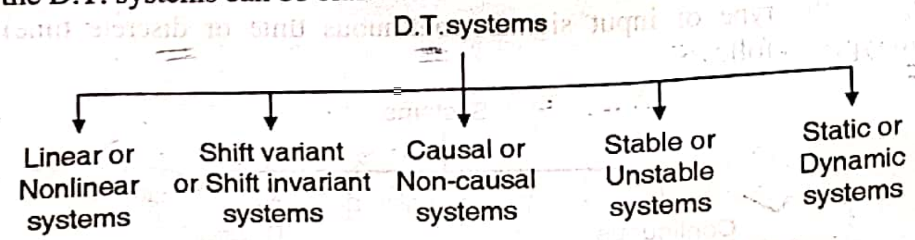


Fig. 2.1.2(b) : Classification of D.T. systems

2.2 Linear and Non linear Systems :

2.2.1 Linear and Nonlinear C.T. Systems :

- A C.T. system is said to be "linear" if it satisfies the superposition theorem. Let the two C.T. systems be defined as follows :

$$y_1(t) = f[x_1(t)]$$

and

$$y_2(t) = f[x_2(t)]$$

- Here $y_1(t)$ and $y_2(t)$ are responses and $x_1(t)$ and $x_2(t)$ are the excitations. Then a C.T. system will be called as a linear system if it satisfies the following expression :

$$f[a_1 x_1(t) + a_2 x_2(t)] = a_1 y_1(t) + a_2 y_2(t) \quad ...(2.2.1)$$

where a_1 and a_2 are constants.

- If you see the LHS of Equation (2.2.1), it shows that the inputs $x_1(t)$ and $x_2(t)$ are multiplied with a_1 and a_2, then the product terms are added and finally the addition is passed through the system. This is as shown in Fig. 2.2.1.
- Whereas the RHS of Equation (2.2.1) shows that $x_1(t)$ and $x_2(t)$ are passed through the individual systems to produce $y_1(t)$ and $y_2(t)$. The individual system responses are then multiplied by a_1 and a_2 and finally the product terms are added. This is as shown in Fig. 2.2.1.
- Equation (2.2.1) shows that even after following completely different sequences of operations we get the same output.
- This is called as linearity or superposition. This property is diagrammatically shown in Fig. 2.2.1.

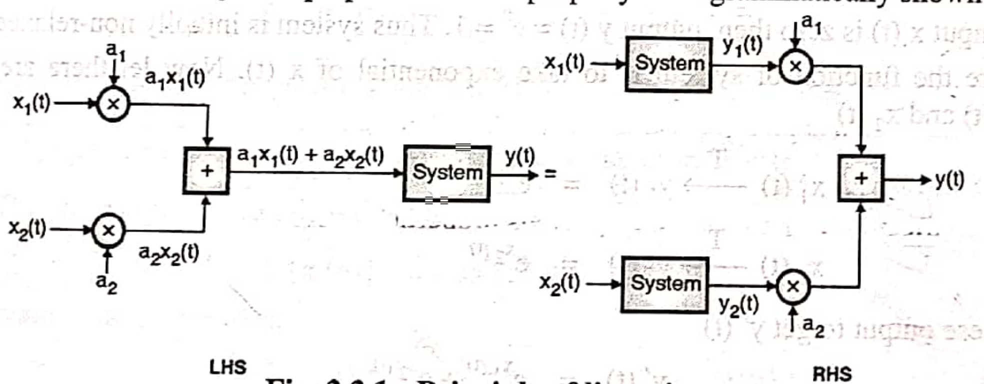


Fig. 2.2.1 : Principle of linearity

- A C.T. system is said to be nonlinear if it does not satisfy the superposition theorem stated in Equation (2.2.1). Communication channels and filters are examples of linear systems.

2.2.2 Linearity of D.T. System :

- We can define the linearity of D.T. system in the similar way as that of a C.T. system.
- A D.T. system is said to be linear if it satisfies the superposition theorem. Let the two D.T. systems be defined as follows :

$$y_1(n) = f[x_1(n)] \text{ and } y_2(n) = f[x_2(n)]$$

- Then a D.T. system is called as a linear system if it satisfies the following expression :

$$f[a_1 x_1(n) + a_2 x_2(n)] = a_1 y_1(n) + a_2 y_2(n) \quad ...(2.2.2)$$

where a_1 and a_2 are constants.

- The procedure to be followed to check the linearity of a D.T. system is same as that followed for the C.T. system.
- This condition can be represented diagrammatically as shown in Fig. 2.2.2.

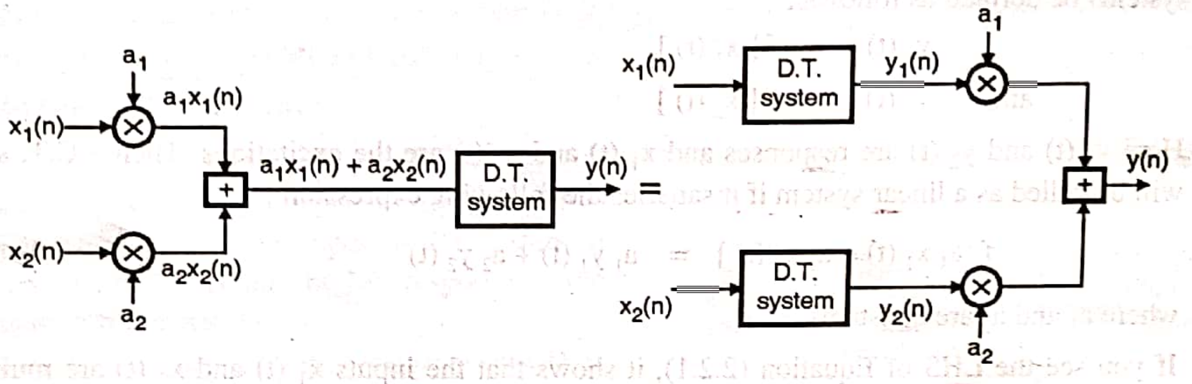


Fig. 2.2.2 : Principle of linearity

- A D.T system is said to be **non-linear** if it does not satisfy the superposition theorem stated earlier.

Ex. 2.2.1 : Check whether the following system is linear or not $y(t) = e^{x(t)}$

Soln. : We can write the given equation as,

$$y(t) = e^{x(t)}$$

Step 1 : If input x (t) is zero then, output $y(t) = e^0 = 1$. Thus system is initially non-relaxed.

Step 2 : Here the function of system is to take exponential of x (t). Now let there are two inputs, $x_1(t)$ and $x_2(t)$

$$\therefore\ x_1(t) \xrightarrow{T} y_1(t) = e^{x_1(t)}$$

$$x_2(t) \xrightarrow{T} y_2(t) = e^{x_2(t)}$$

Add these output to get $y'(t)$

$$\therefore\ y'(t) = e^{x_1(t)} + e^{x_2(t)} \qquad ...(1)$$

Step 3 : First add input $x_1(t)$ and $x_2(t)$ and then apply it to the system. We know that the system produces exponential of input.

$$\therefore\ [x_1(t) + x_2(t)] \xrightarrow{T} y''(t) = e^{[x_1(t) + x_2(t)]}$$

$$\therefore\ y''(t) = e^{[x_1(t) + x_2(t)]}$$

$$\therefore\ y''(t) = e^{x_1(t)} \cdot e^{x_2(t)} \qquad ...(2)$$

Step 4 : Compare Equations (1) and (2)

Since $y'(t) \neq y''(t)$; the system is non-linear

Note : If applied input is zero and the system produces zero output then the system is said to be initially relaxed; otherwise it is initially non-relaxed. Step 1 is used just to check, whether the system is initially relaxed or not. But for checking linearity other steps must be performed.

Ex. 2.2.2 : Determine whether the following system is linear or not.

$$y(t) = \frac{1}{12}x(t) - \frac{5}{6}$$

Soln. :

Step 1 : Apply zero input to the system. Then,

$$y(t) = -\frac{5}{6}$$

Thus system is initially non-relaxed

Step 2 : Apply individual input to the system.

$$\therefore \quad x_1(t) \xrightarrow{T} \frac{1}{12}x_1(t) - \frac{5}{6}$$

$$\text{and} \quad x_2(t) \xrightarrow{T} \frac{1}{12}x_2(t) - \frac{5}{6}$$

$$\therefore \quad y'(t) = \frac{1}{12}x_1(t) - \frac{5}{6} + \frac{1}{12}x_2(t) - \frac{5}{6}$$

Step 3 : Combine both input and apply to the system.

$$\therefore \quad [x_1(t) + x_2(t)] \xrightarrow{T} \frac{1}{12}[x_1(t) + x_2(t)] - \frac{5}{6}$$

$$\therefore \quad y''(t) = \frac{1}{12}x_1(t) + \frac{1}{12}x_2(t) - \frac{5}{6}$$

Step 4 : Since $y'(t) \neq y''(t)$; the system is non-linear.

Ex. 2.2.3 : $T\{x(n)\} = ax(n) + 6$. Check if the system is linear or nonlinear.

Soln. : The given equation can be written as,

$$T\{x(n)\} = ax(n) + 6$$

Step 1 : If input x (n) is zero then,

$$y(n) = 0 + 6 \qquad \therefore \quad y(n) = 6$$

Thus system is initially non-relaxed.

Step 2 : Consider two inputs $x_1(n)$ and $x_2(n)$. We will apply these two inputs separately to the system.

$$\therefore \quad x_1(n) \xrightarrow{T} y_1(n) = ax_1(n) + 6$$

$$\text{and} \quad x_2(n) \xrightarrow{T} y_2(n) = ax_2(n) + 6$$

Add these outputs to get $y'(n)$

Note that the given equation contains the coefficients 'a' and 6. So to check the linearity we will add the outputs as follows :

$$y'(n) = a_1 y_1(n) + a_2 y_2(n)$$

$$\therefore \quad y'(n) = a_1[a x_1(n) + 6] + a_2[a x_2(n) + 6] \qquad \text{...(1)}$$

Here 'a_1' and 'a_2' are arbitrary constants used to check the linearity.

Step 3 : We will add two inputs and then we will apply this signal to the system. Again we will use arbitrary constants a_1 and a_2.

$$\therefore \quad [a_1 x_1(n) + a_2 x_2(n)] \xrightarrow{T} a[a_1 x_1(n) + a_2 x_2(n)] + 6$$

$$\therefore \quad y''(n) = a[a_1 x_1(n) + a_2 x_2(n)] + 6 \qquad ...(2)$$

Note that in this case the function of system is to multiply input by constant 'a' and then add 6. The term $a_1 x_1(n) + a_2 x_2(n)$ acts as combined input signal. Thus the output $y''(n)$ is obtained as given by Equation (2).

Step 4 : Compare Equations (1) and (2),

Since $y'(n) \neq y''(n)$; the system is non-linear. **...Ans.**

Note : Whenever the given equation contains some constants and the equation contains addition of two or more terms; then to check the linearity, use arbitrary constants a_1 and a_2. In other cases we are assuming $a_1 = a_2 = 1$.

Ex. 2.2.4 : $y(n) = \cos x(n)$. Check the linearity of the system.

Soln. :

Step 1 : If input $x(n)$ is zero then,

$$y(n) = \cos(0) \quad \therefore \quad y(n) = 1$$

Thus system is initially non-relaxed

Step 2 : Consider two inputs $x_1(n)$ and $x_2(n)$. Apply these two inputs separately to the system. Observe that the function of system is to take 'cos' of input signal.

$$\therefore \quad x_1(n) \xrightarrow{T} y_1(n) = \cos x_1(n)$$

$$\text{and} \quad x_2(n) \xrightarrow{T} y_2(n) = \cos x_2(n)$$

Now add two outputs to get $y'(n)$

$$\therefore \quad y'(n) = y_1(n) + y_2(n)$$

$$\therefore \quad y'(n) = \cos x_1(n) + \cos x_2(n) \qquad ...(1)$$

Step 3 : Add two inputs and pass it through the system.

$$\therefore \quad [x_1(n) + x_2(n)] \xrightarrow{T} \cos[x_1(n) + x_2(n)]$$

$$\therefore \quad y''(n) = \cos[x_1(n) + x_2(n)] \qquad ...(2)$$

Use trignometric identity,

$$\cos[A + B] = \cos A \cos B + \sin A \sin B$$

Thus Equation (2) becomes,

$$y''(n) = \cos x_1(n) \cos x_2(n) + \sin x_1(n) \sin x_2(n) \qquad ...(3)$$

Step 4 : Compare Equations (1) and (2)

Since $y'(n) \neq y''(n)$; the system is non-linear. ...Ans.

Ex. 2.2.5 : $y(n) = |x(n)|$. Is the system linear or non-linear ?

Soln. :

Step 1 : Apply zero input to the system that means $x(n) = 0$.

$$\therefore \quad y(n) = |x(n)| = 0$$

Thus system is initially relaxed.

Step 2 : Consider two inputs $x_1(n)$ and $x_2(n)$ and apply it to the system.

$$\therefore \quad x_1(n) \xrightarrow{T} y_1(n) = |x_1(n)|$$

$$\text{and} \quad x_2(n) \xrightarrow{T} y_2(n) = |x_2(n)|$$

Add these two outputs to get $y'(n)$,

$$\therefore \quad y'(n) = y_1(n) + y_2(n)$$

$$\therefore \quad y'(n) = |x_1(n)| + |x_2(n)| \qquad ...(1)$$

Step 3 : Add two inputs and apply it to the system. Observe that the function of system is to take magnitude of input signal.

$$\therefore \quad [x_1(n) + x_2(n)] \xrightarrow{T} |x_1(n) + x_2(n)|$$

$$\therefore \quad y''(n) = |x_1(n) + x_2(n)| \qquad ...(2)$$

Step 4 : Compare Equations (1) and (2),

Since $y'(n) \neq y''(n)$; the system is non-linear. ...Ans.

Ex. 2.2.6 : Explain the following system with respect to linearity property :

$y(n) = x(n) + n\,x(n+1)$

Soln. :

Step 1 : Apply input $x(n) = 0$ to the system. Thus system is initially relaxed.

Step 2 : Consider two inputs $x_1(n)$ and $x_2(n)$. Apply two inputs separately to the system.

$$\therefore \quad x_1(n) \xrightarrow{T} y_1(n) = x_1(n) + n\,x_1(n+1)$$

$$\text{and} \quad x_2(n) \xrightarrow{T} y_2(n) = x_2(n) + n\,x_2(n+1)$$

Add these two outputs to get $y'(n)$,

$$\therefore \quad y'(n) = y_1(n) + y_2(n)$$

$$\therefore \quad y'(n) = x_1(n) + n\,x_1(n+1) + x_2(n) + n\,x_2(n+1)$$

$$\therefore \quad y'(n) = x_1(n) + x_2(n) + n[x_1(n+1) + x_2(n+1)] \qquad ...(1)$$

Step 3 : Combine two inputs and apply it to the system.

$$\therefore \quad [x_1(n) + x_2(n)] \xrightarrow{T} [x_1(n) + x_2(n)] + n[x_1(n+1) + x_2(n+1)]$$

$$\therefore \quad y''(n) = [x_1(n) + x_2(n)] + n[x_1(n+1) + x_2(n+1)] \quad ...(2)$$

Step 4 : Compare Equations (1) and (2),

Since $y'(n) = y''(n)$; the system is linear. **...Ans.**

Ex. 2.2.7 : Explain the following system with respect to linearity property :

$y(n) = x(2n)$.

Soln. :

Step 1 : When input is zero, output y (n) is also zero. Thus system is initially relaxed.

Step 2 : Consider two separate inputs $x_1(n)$ and $x_2(n)$. Apply it to the system.

$$\therefore \quad x_1(n) \xrightarrow{T} y_1(n) = x_1(2n)$$

$$\text{and} \quad x_2(n) \xrightarrow{T} y_2(n) = x_2(2n)$$

Add $y_1(n)$ and $y_2(n)$ to get $y'(n)$,

$$\therefore \quad y'(n) = x_1(2n) + x_2(2n) \quad ...(1)$$

Step 3 : Combine two inputs and apply it to the system.

$$\therefore \quad [x_1(n) + x_2(n)] \xrightarrow{T} x_1(2n) + x_2(2n)$$

$$\therefore \quad y''(n) = x_1(2n) + x_2(2n) \quad ...(2)$$

Step 4 : Compare Equations (1) and (2),

Since $y'(n) = y''(n)$; the system is linear. **...Ans.**

Ex. 2.2.8 : Find whether the following system is linear :

$y(n) = x(-n+2)$

Soln. :

Step 1 : Apply zero input to the system. It produces zero output. Thus system is initially relaxed.

Step 2 : Consider two inputs $x_1(n)$ and $x_2(n)$. Apply these inputs separately to the system.

$$\therefore \quad x_1(n) \xrightarrow{T} y_1(n) = x_1(-n+2)$$

$$\text{and} \quad x_2(n) \xrightarrow{T} y_2(n) = x_2(-n+2)$$

Add these two outputs to get $y'(n)$,

$$\therefore \quad y'(n) = y_1(n) + y_2(n)$$

$$\therefore \quad y'(n) = x_1(-n+2) + x_2(-n+2) \quad ...(1)$$

Step 3 : Add two inputs and apply it to the system.

$$\therefore \quad x_1(n) + x_2(n) \xrightarrow{T} x_1(-n+2) + x_2(-n+2)$$

$$\therefore \quad y''(n) = x_1(-n+2) + x_2(-n+2) \quad ...(2)$$

Step 4 : Since $y'(n) = y''(n)$; the system is linear. ...Ans.

Ex. 2.2.9 : $y(n) = n\,x^2(n)$ check the linearity :

Soln. :

Step 1 : When input $x(n)$ is zero; output is zero. Thus system is initially relaxed.

Step 2 : Consider two inputs $x_1(n)$ and $x_2(n)$. Apply each input separately to the system.

$$\therefore\ x_1(n) \xrightarrow{T} y_1(n) = n\,x_1^2(n)$$

$$\text{and}\ \ x_2(n) \xrightarrow{T} y_2(n) = n\,x_2^2(n)$$

Add these outputs to get $y'(n)$,

$$\therefore\ y'(n) = n\left[x_1^2(n) + x_2^2(n)\right] \quad ...(1)$$

Step 3 : Combine two inputs and apply it to the system.

$$\therefore\ [x_1(n) + x_2(n)] \xrightarrow{T} n\,[x_1(n) + x_2(n)]^2$$

$$\therefore\ y''(n) = n\,[x_1(n) + x_2(n)]^2 \quad ...(2)$$

Step 4 : Compare Equations (1) and (2),

Since $y'(n) \neq y''(n)$; the system is non-linear. ...Ans.

Ex. 2.2.10 : $y(n) = g(n)\,x(n)$ check the linearity.

Soln. :

Step 1 : When input $x(n) = 0$; output is zero.

Thus system is initially relaxed.

Step 2 : Apply each input separately to the system.

$$\therefore\ x_1(n) \xrightarrow{T} g(n)\,x_1(n)$$

$$\text{and}\ \ x_2(n) \xrightarrow{T} g(n)\,x_2(n)$$

$$\therefore\ y'(n) = g(n)\,x_1(n) + g(n)\,x_2(n) \quad ...(1)$$

Step 3 : Combine two inputs and apply it to the system.

$$\therefore\ [x_1(n) + x_2(n)] \xrightarrow{T} g(n)\,[x_1(n) + x_2(n)]$$

$$\therefore\ y''(n) = g(n)\,x_1(n) + g(n)\,x_2(n) \quad ...(2)$$

Step 4 : Since $y'(n) = y''(n)$; the system is linear. ...Ans.

Ex. 2.2.11 : Check the following system for linearity :

$$y(t) = x(t) + x(t-100)$$

Soln. :

Step 1 : **Apply zero input, $y(t) = 0$.**

Step 2 : **Apply each input separately.**

$$\therefore \quad x_1(t) \xrightarrow{T} x_1(t) + x_1(t-100)$$

$$\therefore \quad x_2(t) \xrightarrow{T} x_2(t) + x_2(t-100)$$

$$\therefore \quad y'(t) = x_1(t) + x_1(t-100) + x_2(t) + x_2(t-100)$$

Step 3 : Apply both inputs combinely.

$$\therefore \quad [x_1(t) + x_2(t)] \xrightarrow{T} y''(t) = x_1(t) + x_1(t-100) + x_2(t) + x_2(t-100)$$

Step 4 : Since $y'(t) = y''(t)$; It is linear system.

Ex. 2.2.12 : Find whether the following systems are linear or non-linear systems :

1. $y(t) = t\,x(t)$ 2. $y(n) = n^2 x(n)$

Soln. :

1. $y(t) = t\,x(t)$:

Step 1 : If input is zero then output is zero. So system is initially relaxed :

Step 2 : Let there be two inputs $x_1(t)$ and $x_2(t)$:

$$\therefore \quad x_1(t) \xrightarrow{T} y_1(t) = t\,x_1(t)$$

$$x_2(t) \xrightarrow{T} y_2(t) = t\,x_2(t)$$

Adding these outputs to get $y'(t)$

$$\therefore \quad y'(t) = t\,[x_1(t) + x_2(t)]$$

Step 3 :

$$[x_1(t) + x_2(t)] \xrightarrow{T} y''(t) = t\,[x_1(t) + x_2(t)] \qquad ...(2)$$

Step 4 : Since $y'(t) = y''(t)$; system is linear :

2. $y(n) = n^2 x(n)$:

Step 1 : If input is zero then output is zero. So, system is initially relaxed :

Step 2 : Let there be two inputs $x_1(n)$ and $x_2(n)$:

$$\therefore \quad x_1(n) \xrightarrow{T} y_1(n) = n^2 x_1(n)$$

$$\text{and} \quad x_2(n) \xrightarrow{T} y_2(n) = n^2 x_2(n)$$

Adding these outputs to get y′ (n)

$$\therefore y'(n) = n^2 [x_1(n) + x_2(n)]$$

Step 3 : Combining two inputs and applying it to the system :

$$\therefore [x_1(n) + x_2(n)] \xrightarrow{T} n^2 x_1(n) + n^2 x_2(n)$$

$$\therefore y''(n) = n^2 [x_1(n) + x_2(n)]$$

Step 4 : Since y′(n) = y″ (n); the system is linear :

2.3 Time Variant or Time Invariant Systems :

2.3.1 Time Invariant and Time Variant C.T. Systems :

- A C.T. system is said to be a **time invariant** system if a time shift in the input signal results in a corresponding time shift in the output. If this condition is not satisfied, then the system is called time variant system.
- Let the relation between input and output of a C.T. system be defined as,

$$y(t) = f[x(t)]$$

- Now for a time invariant system if the input signal x (t) is delayed by "t_d" sec. then the output also will be delayed by the same time i.e.

$$f[x(t-t_d)] = y(t-t_d) \quad ...(2.3.1)$$

- A C.T. system is said to be a **time variant system** if it does not satisfy the Equation (2.3.1). Most of the practical systems which we come across are linear time invariant systems **(LTI).**
- Examples of the C.T. linear time invariant systems are amplifiers, filters, etc.
- This property can be checked by following the procedure given below.

Procedure to test linearity of a C.T. system :

Step 1 : Apply x (t) to a delay unit to obtain x $(t-t_d)$ i.e. the delayed version of x (t).

Step 2 : Pass the delayed signal x $(t-t_d)$ through the C.T. system to produce f [x $(t-t_d)$].

Step 3 : Now pass the signal x (t) through the C.T. system to produce y (t) .

Step 4 : Pass y (t) through the delay unit to obtain delayed version of y (t) i.e. y $(t-t_d)$.

Step 5 : Compare the results of step 2 and 4 to decide about the linearity of the C.T. system.

- This procedure is demonstrated in Fig. 2.3.1.

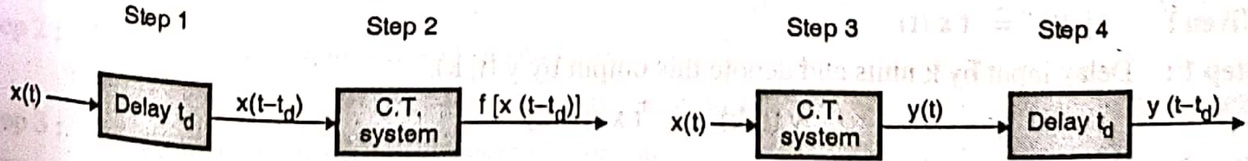


Fig. 2.3.1 : Principle of time invariance

2.3.2 Shift Invariant and Shift Variant D.T. Systems :

- A D.T. system is said to be a **shift invariant** system if a shift in the input signal results in corresponding shift in the output. Otherwise the system is **shift variant.**
- Let the relation between input and output of a D.T. system be given by,

$$y(n) = f[x(n)]$$

- Now if the input signal x (n) is delayed by "k", then the output signal y (n) will also get delayed by "k" if the D.T. system is a **shift invariant** system.
- This is expressed mathematically as follows :

$$f[x(n-k)] = y(n-k) \quad \text{...(2.3.2)}$$

- A D.T. system is said to be a **shift variant** system if it does not satisfy the above expression.
- The principle of shift invariance is demonstrated in Fig. 2.3.2.

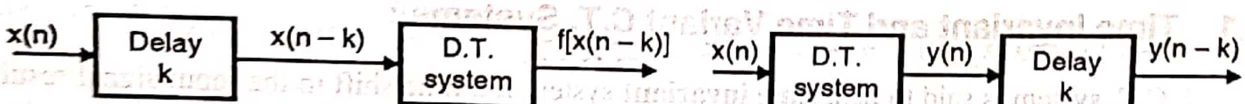


Fig. 2.3.2 : Principle of shift invariance

Theorem : A system is shift invariant if and only if

$$x(n) \xrightarrow{T} y(n) \quad \text{implies that} \quad x(n-k) \xrightarrow{T} y(n-k)$$

How to determine whether D.T. system is shift invariant or not ?

To check whether the D.T. system is shift invariant or shift variant follow the steps given below :

Step 1 : Delay the input x (n) by 'k' units to get x (n – k). Denote the corresponding output by y (n, k).

That means $x(n-k) \xrightarrow{T} y(n-k)$

Step 2 : In the given equation of system y (n) replace 'n' by (n – k) for every "n". This output is y (n – k).

Step 3 : If y (n, k) = y (n – k) then system is shift invariant and if y (n, k) ≠ y (n – k) then system is shift variant.

Ex. 2.3.1 : Show the following system is time variant :

$y(t) = t\,x(t)$

Soln. :

Given : $y(t) = t\,x(t)$

Step 1 : Delay input by k units and denote this output by y (t, k).

$$\therefore \; y(t, k) = t\,x(t-k)$$

Step 2 : Replace t by t – k throughout the equation.

$$\therefore \; y(t-k) = (t-k)\,x(t-k)$$

Step 3 : Since y (t, k) ≠ y (t – k); the system is time variant.

Ex. 2.3.2 : Determine whether the following system is time invariant or not :

$$\frac{d}{dt} y(t) + t y(t) = x(t)$$

Soln. : The given equation contain a term ty (t). That means output changes with time. So this system is time variant.

Ex. 2.3.3 : Classify the following system for linearity and time invariance :

$$y''(t) + 3y'(t) = 2x'(t) + x(t)$$

Soln. :

$$y''(t) + 3y'(t) = 2x'(t) + x(t) \quad \text{... Given}$$

(A) Linearity :

Consider there are two inputs $x_1(t)$ and $x_2(t)$. Then if we apply individual input to the system; it satisfies following equations.

$$y_1''(t) + 3y_1'(t) = 2x_1'(t) + x_1(t) \quad \text{...(1)}$$

$$\text{and } y_2''(t) + 3y_2'(t) = 2x_2'(t) + x_2(t) \quad \text{...(2)}$$

The linear combinations of Equations (1) and (2) is,

$$y_1''(t) + y_2''(t) + 3y_1'(t) + 3y_2'(t) = 2x_1'(t) + 2x_2'(t) + x_1(t) + x_2(t)$$

$$\therefore \frac{d^2}{dt^2} y_1(t) + \frac{d^2}{dt^2} y_2(t) + 3\frac{d}{dt} y_1(t) + 3\frac{d}{dt} y_2(t) = 2\frac{d}{dt} x_1(t) + 2\frac{d}{dt} x_2(t) + x_1(t) + x_2(t)$$

$$\therefore \frac{d^2}{dt^2}[y_1(t) + y_2(t)] + 3\frac{d}{dt}[y_1(t) + y_2(t)] = 2\frac{d}{dt}[x_1(t) + x_2(t)] + x_1(t) + x_2(t) \quad \text{...(3)}$$

This equation can be put in the form of original equation. Thus it is linear system.

(B) Time invariance :

This equation does not contain any term like t y (t) or t x (t). That means this equation does not vary with time. So it is time invariant system.

Ex. 2.3.4 : Determine whether the following system is time variant or not :

$$y(t) = x(t) - 3u(t)$$

Soln. :

$$y(t) = x(t) - 3u(t)$$

Step 1 : Delay input by k samples and denote output by y (t, k)

$$\therefore \quad y(t, k) = x(t-k) - 3u(t) \quad \text{...(1)}$$

Step 2 : Replace 't' by t – k throughout the equation.

$$\therefore \quad y(t-k) = x(t-k) - 3u(t-k) \quad \text{...(2)}$$

Step 3 : Since $y(t, k) \neq y(t-k)$; the system is time variant.

Ex. 2.3.5 : Determine whether the following system is shift invariant or not :

$$T[x(n)] = e^{x(n)}$$

Soln. : We know that T [x(n)] means x(n) is passed through the system to produce output. Thus given equation can be written as,

$$\therefore \quad y(n) = e^{x(n)} \text{ that means } x(n) \xrightarrow{T} y(n) = e^{x(n)}$$

Step 1 : Delay input by k samples and denote this output by y (n, k)

$$\therefore \quad y(n, k) = e^{x(n-k)} \quad ...(1)$$

Step 2 : Replace 'n' by 'n – k' throughout the given equation,

$$\therefore \quad y(n-k) = e^{x(n-k)} \quad ...(2)$$

Step 3 : Compare Equations (1) and (2),

Since y (n, k) = y (n – k); the system is shift invariant ...Ans.

Ex. 2.3.6 : Determine whether the following system is shift invariant or not :

$$y(n) = \cos x(n)$$

Soln. :

Step 1 : Delay the input by 'k' units and denote this output by y (n, k)

$$\therefore \quad y(n, k) = \cos x(n-k) \quad ...(1)$$

Step 2 : Replace 'n' by 'n – k' throughout the equation.

$$\therefore \quad y(n-k) = \cos x(n-k) \quad ...(2)$$

Step 3 : Compare Equations (1) and (2)

Since y (n, k) = y (n – k); the system is shift invariant. ...Ans.

Ex. 2.3.7 : Determine whether the following system is shift invariant or not :

$$y(n) = x(n) + nx(n-1)$$

Soln. :

Step 1 : Delay input by 'k' units and denote this output by y (n, k)

$$\therefore \quad y(n, k) = x(n-k) + nx(n-k-1)$$

Note that only input x (n) is delayed. That means in the given equation wherever x (n) is present, replace it by x (n – k). You should not replace 'n' present in the second term by n – k. That means in this step do not write the equation as,

$$y(n, k) = x(n-k) + (n-k)\, x(n-k-1)$$

This is not valid since we are delaying input, x (n) and not only n.

Step 2 : Replace 'n' by 'n – k' throughout the equation.

$$\therefore \quad y(n-k) = x(n-k) + (n-k)\, x(n-k-1) \quad ...(2)$$

Step 3 : Compare Equations (1) and (2)

Since y (n, k) ≠ y (n – k); the system is shift variant. ...Ans.

Ex. 2.3.8 : Determine whether the following system is shift invariant or not :

$$y(n) = x(2n)$$

Soln. :

Step 1 : Delay input by k units and denote this output by y (n, k)

$$y(n, k) = x[2(n-k)] \quad ...(1)$$

Step 2 : Replace 'n' by 'n – k' throughout the equation

$$\therefore \quad y(n-k) = x[2(n-k)] \quad ...(2)$$

Step 3 : Compare Equations (1) and (2)

Since $y(n, k) = y(n-k)$; the system is shift invariant. **...Ans.**

Ex. 2.3.9 : Determine whether the following systems are time invariant or not

1. $y(t) = t\,x(t)$
2. $y(n) = x(2n)$
3. $3y(n) = 2x(n) - y(n-1)$

Soln. : 1. Refer Ex. 2.3.1.

2. Refer Ex. 2.3.8.

3. $3y(n) = 2x(n) - y(n-1)$

$$y(n) = \frac{1}{3}[2x(n) - y(n-1)]$$

Step 1 : Delay input by 'k' units and denote output by y (n, k)

$$y(n, k) + \frac{1}{3}y(n-1) = \frac{1}{3}2x(n-k)$$

Step 2 : Replace n by (n – k) throughout the equation,

$$y(n-k) + \frac{1}{3}y(n-k-1) = \frac{1}{3}\cdot 2x(n-k)$$

$$\therefore \quad y(n, k) \neq y(n-k)$$

The given system is **time varient.**

Ex. 2.3.10 : Determine whether the following system is shift invariant or not :

$$y(n) = y(n-4) + x(n-4)$$

Soln. : Note that the term y (n – 4) denotes the delayed output. It also indicates that the feedback is taken from the output y (n) and it is delayed by 4 units.

Step 1 : Delay input by k units and denote this output by y (n, k)

$$\therefore \quad y(n, k) = y(n-4) + x(n-k-4) \quad ...(1)$$

Step 2 : Replace 'n' by 'n – k' throughout the equations

$$\therefore \quad y(n-k) = y(n-k-4) + x(n-k-4) \quad ...(2)$$

Step 3 : Compare Equations (1) and (2)

Since $y(n, k) \neq y(n-k)$; the system is shift variant. **...Ans.**

Ex. 2.3.11 : Determine whether the following system is shift invariant or not :

$y(n) = x(-n)$

Soln. :

Step 1 : Delay input by 'k' units and denote this output by y(n, k)

$$\therefore \quad y(n, k) = x(-n-k) \quad \text{...(1)}$$

Note that only 'n' should not be replaced by n – k. This is because we want to delay the whole input x (n) by 'k' units. Thus x (– n) should be delayed by k samples.

Step 2 : Replace 'n' by 'n – k' throughout the equation

$$\therefore \quad y(n-k) = x[-(n-k)] = x(-n+k) \quad \text{...(2)}$$

Step 3 : Compare Equations (1) and (2),

Since $y(n, k) \neq y(n-k)$; the system shift variant system. **...Ans,**

Ex. 2.3.12 : Find whether the following systems are time variant or time invariant

1. $y(t) = e^x(t)$ 2. $y(n) = x(n^2)$

Soln. :

Step 1 : Delay input by t_0 and obtain the output :

1. $$\therefore \quad y(t, t_0) = e^x(t-t_0)$$

Step 2 : Replacing t by $t - t_0$ throughout the given equation :

$$y(t-t_0) = e^x(t-t_0)$$

Step 3 : Since $y(t, t_0) = y(t-t_0)$; the system is shift (time) invariant :

2. Given : $y(n) = x(n^2)$

Step 1 : Delay input by 'k' samples and output is denoted by $y(n, k) = x(n^2 - k)$.

Step 2 : Replacing n by n – k throughout the equation :

$$\therefore \quad y(n-k) = x(n-k)^2$$

Step 3 : Since $y(n, k) \neq y(n-k)$; the system is time variant.

Ex. 2.3.13 : Determine whether the following systems are time invariant or not.

1. $y(t) = x(t) - 3u(t)$
2. $y(n) = x(2n+2) + x(n)$
3. $y(n) = x^2(n)$

Soln. :

1. Refer Ex. 2.3.4
2. $y(n) = x(2n+2) + x(n)$

Step 1 : Delay input by k units and denote this output by y (n, k) :

$$y(n, k) = x(2n + 2 - k) + x(n - k)$$

Step 2 : Replace n by (n – k) throughout the equation :

$$y(n - k) = x(2n + 2 - k) + x(n - k)$$

Step 3 : Since y, (n – k) = y (n – k) the system is time invariant.

3. Refer Ex. 2.3.12

2.4 Causal or Noncausal C.T. Systems :

2.4.1 Causal and Noncausal C.T. Systems :

- A C.T. system is said to be "causal" if it produces a response y (t) only after the application of excitation x (t).
- That means for a causal C.T. system the response does not begin before the application of input x (t).
- The other way of defining a causal system is as follows : A system is "causal" if its output depends only on the present and past values of input and does not depend on its future values.
- If the input is applied at $t = t_m$ then the output at $t = t_m$ i.e. $y(t_m)$ will be dependent only on the values of x (t) for $t \le t_m$.

Condition for causality : $y(t_m) = f[x(t)\,;\, t \le t_m]$ ...(2.4.1)

- Causal C.T. systems are physically realizable systems. The **noncausal systems** do not satisfy Equation (2.4.1). It is not possible to physically realize noncausal systems.

Condition for causality in terms of impulse response h (t) :

- The relation between y (t) and x (t) is given by,

$$y(t) = x(t) * h(t)$$

- where * represents convolution and h (t) is the impulse response of the system. The condition for causality in terms of the impulse response is as follows :

Condition for causality : $h(t) = 0$ for $t < 0$ ...(2.4.2)

- This condition states that a C.T. linear time invariant (LTI) system is "causal" if its impulse response h (t) has a zero value for all the negative values of time.

2.4.2 Causal DT Systems :

Definition :

A DT system is said to be causal system if output at any instant of time depends only on present and past inputs. But the output does not depend on future inputs.

The condition for causality of a D.T. system in terms of its impulse response is as follows :

Condition for causality $h(n) = 0$ for $n < 0$ ...(2.4.2(a))

Examples :

The output of system depends on present and past inputs that means output, y (n) is a function of x (n), x (n – 1), x (n – 2) ... etc. some examples of causal systems are :

1. y (n) = x (n) + x (n – 1) 2. y (n) = 3 x (n) 3. y (n) = x (n) + 4 x (n – 1)

Significance :

Since causal system does not include future input samples; such system is practically realizable. That means such system can be implemented practically. Generally all real time systems are causal systems; because in real time applications only present and past samples are present.

Since future samples are not present; **causal system is memoryless system.**

Anticausal or non-causal DT systems :

Definition :

A DT system is said to be anticausal system if its output depends not only on present and past inputs but also on future inputs.

Examples :

For a noncausal system, output y (n) is function of x (n), x (n – 1), x (n – 2) .... etc. as well x (n + 1), x (n + 2) ... etc. some examples of non-causal systems are :

1. y (n) = x (n) + x (n + 1)
2. y (n) = B x (n + 2)
3. y (n) = x (n) + nx (n + 1)

Significance :

A non-causal system is practically not realizable. That means in practical cases it is not possible to implement a non-causal system.

But if the signals are stored in the memory and at a later time they are used by a system as the time advanced or future signal. Then in such cases it is possible to implement a non-causal system.

Ex. 2.4.1 : Determine if the systems described by following equations are causal or non-causal :

1. y (n) = cos x (n) 2. y (n) = | x (n) |
3. y (n) = x (n) + nx (n – 1) 4. y (n) = x (n) + nx (n + 1)
5. y (n) = x (2n) 6. y (n) = x (– n + 2)

Soln. :

1. y (n) = cos x (n) :

This is a causal system because the function of system is to obtain cosine value of present input.

2. y (n) = | x (n) | :

This is a causal system because output depends on present input.

3. y (n) = x (n) + nx (n – 1) :

This is a causal system because output depends on present and past input but not on the future input.

4. **$y(n) = x(n) + nx(n+1)$:**

Here output depends on future input i.e. $x(n+1)$. So this is non-causal system.

5. **$y(n) = x(2n)$:**

This is non-causal system because output expects future input. This can be verified by putting different values of n.

At $n = 0 \Rightarrow y(0) = x(0)$ Here present output excepts present input only.

At $n = 1 \Rightarrow y(1) = x(2)$ Here present output i.e. at $n = 1$, expects future value of input i.e. $x(2)$.

6. **$y(n) = x(-n+2)$:**

This is non-causal system. This is because at $n = -1$ we get,

$$y(-1) = x[-(-1)+2] = x[1+2] = x(3)$$

Thus present output at $n = -1$, expects future input i.e. $x(3)$.

Ex. 2.4.2 : Determine if the system described by the following equation is causal or non causal.

$y(t) = e^{x(t)}$

Soln. : The given equation is,

$$y(t) = e^{x(t)}$$

This is causal system since output depends on present input, $x(t)$.

Ex. 2.4.3 : Check the following system for causality :

$$y(t) = \frac{1}{c}\int_{-\infty}^{\infty} x(t)\,dt$$

Soln. : Given $$y(t) = \frac{1}{c}\int_{-\infty}^{\infty} x(t)\,dt$$

Here limits of integration are from $-\infty$ to $+\infty$ and output $y(t)$ depends on integration of input. That means output at any instant depends on future inputs also. Thus the system is non-causal.

Ex. 2.4.4 : Determine if the systems described by following equations are causal or non-causal :

1. $y(n) = x(n) + x(n-2)$ 2. $T[x(n)] = ax(n) + 6$

Soln. :

1. **$y(n) = x(n) + x(n-2)$:**

It is a causal system because output depends only on present input i.e. $x(n)$ and past input i.e. $x(n-2)$.

2. **$T[x(n)] = ax(n) + 6$:**

The given equation is,

$$y(n) = ax(n) + 6$$

This is causal system because output depends on present input x (n). There is no future input signal.

2.5 Stable or Unstable Systems :

2.5.1 Stable and Unstable C.T. Systems :

Before defining the stable and unstable systems, let us first define the "bounded" and "unbounded" signals.

Bounded signal :

A signal x(t) is called as a bounded signal if its magnitude is always finite i.e. If

$$|x(t)| \leq M < \infty \qquad ...(2.5.1)$$

Then x (t) is a bounded signal. Here M is a real positive number. A signal which does not satisfy the condition stated in Equation (2.5.1) is called as an unbounded signal.

Definition of stability of a C.T. system :

A C.T. system is said to be BIBO (Bounded Input Bounded Output) stable if it produces a bounded output for every bounded signal applied at its input. The system which does not satisfy this condition is an unstable system. The same definition is applicable to the discrete time (D.T.) system as well.

That means for a BIBO (Bounded Input Bounded Output) stable system, the output signal y (t) satisfies the condition,

$|y(t)| \leq M_y < \infty$ for all t

Whenever the input signal satisfies the condition

$|x(t)| \leq M_x < \infty$ for all t

M_x and M_y represent some finite positive numbers.

Condition for stability in terms of impulse response h (t) :

The condition for stability in terms of the impulse response h (t) of a system is given by,

$$\text{For an LTI system to be stable : } \int_{-\infty}^{\infty} |h(t)|\, dt < \infty \qquad ...(2.5.2)$$

The C.T. system which does not satisfy this equation is an unstable system.

2.5.2 Stable and Unstable D.T. Systems :

An initially relaxed system is BIBO stable if and only if every bounded input produces a bounded output. A relaxed system is that system which produces a zero output when its input is zero.

Mathematical representation :

Let us consider some finite number M_x whose value is less than infinity. That means $M_x < \infty$, so it is a finite value. Then if input is bounded, we can write,

$$|x(n)| \leq M_x < \infty \quad ...(2.5.3)$$

Similarly consider some finite number M_y whose value is less than infinity. That means $M_y < \infty$, so it is a finite value. Then if output is bounded, we can write,

$$|y(n)| \leq M_y < \infty \quad ...(2.5.4)$$

Definition of unstable system :

An initially relaxed system is said to be unstable if bounded input produces unbounded (infinite) output.

Significance :

- Unstable system shows erratic and extreme behaviour.
- When unstable system is practically implemented then it causes overflow.

Ex. 2.5.1 : Determine whether the following system is stable or not.

$$y(t) = e^{x(t)}$$

Soln. : $y(t) = e^{x(t)}$

The given equation is,

$$y(t) = e^{x(t)}$$

We have to check the stability of a system by applying bounded input. That means the value of x (t) should be finite (bounded). The value of 'e' is 2.718. So for bounded input, the output y (t) will be bounded. Thus this is stable system.

Ex. 2.5.2 : Determine whether the following discrete-time systems are stable or not :

1. $T[x(n)] = ax(n) + 6$
2. $y(n) = \cos[x(n)]$
3. $y(n) = x(-n)$
4. $y(n) = x(2n)$

Soln. :

1. $T[x(n)] = ax(n) + 6$:

The given equation is,

$$y(n) = ax(n) + 6$$

Here 'a' is some arbitrary constant. So as long as x (n) is bounded the output y (n) is also bounded. Thus it is stable system.

2. $y(n) = \cos[x(n)]$:

For every bounded value of x (n), its cosine value is always bounded (finite). So given system is stable.

3. **y (n) = x (– n) :**

Here x (– n) means folding of input sequence. That means taking mirror image of x (n). But in case of folding operation; the amplitude of signal is not changed. So if input is bounded the output y (n) = x (– n) will be bounded. Thus it is stable system.

4. **y (n) = x (2n) :**

If input x (n) is bounded then output y (n) is also bounded. So it is a stable system.

2.6 Static or Dynamic D.T. Systems (Dynamicity Property) :

2.6.1 Static DT Systems :

Definition : It is a DT system in which output at any instant of time depends on input sample at the same time.

Example :

1. y (n) = 5x (n)

Here 5 is constant which multiplies input x (n). But output at n^{th} instant that means y (n) dep on input at the same (n^{th}) time instant x (n). So this is static system.

2. y (n) = x^2 (n) + 5x (n) + 10

Here also output at n^{th} instant, y (n) depends on the input at n^{th} instant. So this is static system.

Significance :

Observe the input-output relations of static system. Output does not depend on delayed [x (n – k)] or advanced [x (n + k)] input signals. It depends only on present (n^{th}) input signal. If output depends on delayed input signals then such signals should be stored in memory to calculate the output at n^{th} instant. This is not required in static systems. Thus for static systems, memory is not required. So **static systems are memoryless systems.**

2.6.2 Dynamic DT Systems :

Definition :

It is a system in which output at any instant of time depends on input sample at the same time as well as at other instants of time. Here other times means, other than the present time instant. It may be past time or future time. Note that if x (n) represents input signal at present instant then,

(a) x (n – k); that means delayed input signal is called as **past signal**.

(b) x (n + k); that means advanced input signal is called as **future signal**.

Thus in dynamic systems the output depends on present input as well as past or future inputs.

Examples :

1. y (n) = x (n) + 5x (n – 1)

Here output at n^{th} instant depends on input at n^{th} instant, x (n) as well as $(n – 1)^{th}$ instant x (n – 1) is previous (past) sample. So the system is dynamic.

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2. $y(n) = 3x(n+2) + x(n)$

Here $x(n+2)$ indicates advanced version of input sample that means it is future sample; so this is dynamic system.

Significance :

Observe input-output relations of dynamic system. Since output depends on past or future input sample; we need a memory to store such samples. Thus **dynamic system has a memory.**

2.6.3 A Static or Dynamic C.T. System :

A C.T. system is static or memoryless if its output depends upon the present input only.

Example :

Voltage drop across a resistor.

It is given by, $v(t) = i(t) \cdot R$

Here the voltage drop depends on the value of current at that instant of time only so it is static system.

On the other hand a C.T. system is called as dynamic if output depends on present as well as past values.

Example :

A current flowing through inductor i (t) is related to applied voltage, v (t) as,

$$i(t) = \frac{1}{L} \int_{-\infty}^{t} v(\tau)\, d\tau$$

Here the limits of integration are from $-\infty$ to $+t$. Thus the current at time 't' depends on all past values of voltage extending right from $-\infty$ upto that instant "t".

Ex. 2.6.1 : Check whether the following D. T. systems are static or dynamic :

1. $y(n) = n\,x(n) + b\,x^2(n)$
2. $y(n) = x(n^2)$
3. $y(n) = x(n) \cos \omega_0 n$
4. $y(n) = \sum_{k=0}^{2} x(n-k)$
5. $y(n) = x(-n)$

Soln. :

1. **$y(n) = nx(n) + b\,x^2(n)$:**

Note that $x^2(n)$ means, square the amplitude of input signal. So it is not related to time shifting. (It is not $x(n^2)$). Here 'b' is some constant. Since the equation does not contain any advanced or delayed input term; it is static system.

2. $y(n) = x(n^2)$:

It is dynamic system, because at n^{th} instant, system expects future inputs (n^2). For example,

$$\text{At } n = 1, y(1) = x(1^2) = x(1)$$

$$\text{At } n = 2, y(2) = x(2^2) = x(4)$$

Thus at n = 2, system requires future input, x (4). So it is dynamic system.

3. $y(n) = x(n) \cos \omega_0 n$:

It is static system because output depends on present input only.

4. $y(n) = \sum_{k=0}^{2} x(n-k)$:

First we will expand this summation.

$$\therefore \quad y(n) = x(n) + x(n-1) + x(n-2)$$

Since output contains delayed input terms; it is dynamic system.

5. $y(n) = x(-n)$:

We know that $x(-n)$ means folded input signal. Now we will put different values of n

$$\text{At } n = 0, y(0) = x(-0) = x(0)$$

$$\text{At } n = 1, y(1) = x(-1)$$

$$\text{At } n = 2, y(2) = x(-2)$$

Here $x(-1)$, $x(-2)$ ... etc. indicates past input sample. So the system is dynamic.

Ex. 2.6.2 : Check whether the following D.T. systems are static or dynamic :

1. $y(n) = n\,x(n)$,
2. $y(n) = 3x(n) + 5$,
3. $y(n) = 3|x(n)|$
4. $y(n) = x(n) - 3x(n-2)$
5. $y(n) = \sum_{k=0}^{2} x(n-k)$

Soln. :

1. $y(n) = n\,x(n)$:

It is static system because output depends on present input only.

2. $y(n) = 3x(n) + 5$:

It is static system because output, y (n) depends only on present input x (n).

3. $y(n) = 3|x(n)|$:

It is static system because output depends on present input only.

4. $y(n) = x(n) - 3x(n-2)$:

It is dynamic system because output requires delayed input.

5. $y(n) = \sum_{k=0}^{2} x(n-k)$:

First we will expand this summation.

$$\therefore \quad y(n) = x(n) + x(n-1) + x(n-2)$$

Since output contains delayed input terms; it is dynamic system.

Ex. 2.6.3 : Determine whether the following C.T. systems are static or dynamic :

1. $y(t) = 3x(t) + 10$
2. $y(t) = x(t) \cos 50\pi t$
3. $\frac{dy(t)}{dt} + 10\, y(t) = x(t)$
4. $y(t) = e^{x(t)}$

Soln. :

1. **Given $y(t) = 3x(t) + 10$:**

Output depends on present input only. So it is static system.

2. **$y(t) = x(t) \cos 50\pi t$:**

Output at any instant depends on input at that instant. So it is static system.

3. **$\frac{dy(t)}{dt} + 10\, y(t) = x(t)$:**

This system involves differentiation with time. So it is dynamic system.

4. **$y(t) = e^{x(t)}$:**

Here output depends on present input only. So it is static system.

2.7 Invertibility :

- A system is said to be **invertible** if the input of the system can be recovered from the system output.
- A set of operations will be needed to be carried out in order to recover the input from output. This may be viewed as connecting a second system in cascade with the given system, such that the output signal of the second system is equal to the input signal of the given system. i.e. x(t) as shown in Fig. 2.7.1(a).

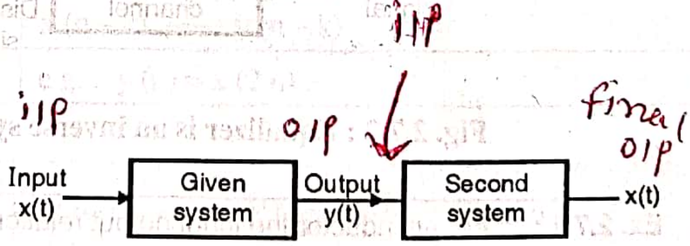


Fig. 2.7.1(a) : Concept of invertibility

- In order to represent the invertibility, mathematically assume that operator H represents a C.T. given C.T. system with input signal x(t) and produces the output signal y(t).
- Let y(t) be applied to a second C.T. system represented by the operator H^{-1} as shown in Fig. 2.7.1(b). Then the output signal of the second system is given by,

$$\text{Output of second system} = H^{-1}\{y(t)\} = H^{-1}\{H\,x(t)\}$$
$$= H^{-1}H\{x(t)\}$$

- If we want the output of second system to equal to the original input signal $x(t)$, then it is necessary that

$$H^{-1}H = I \qquad \text{...(2.7.1)}$$

Where I denotes the identity operator.

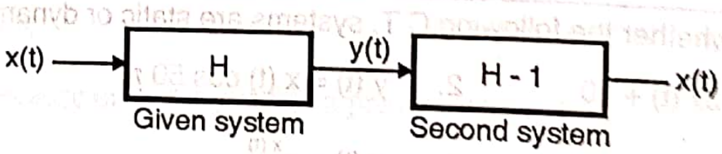


Fig. 2.7.1(b) : Invertibility expressed mathematically

- The operator H^{-1} is called as the **inverse operator** and the second system is called as the inverse system.
- However it is important to understand that H^{-1} is not the reciprocal of H. It is inverse of H.
- There has to be a one-to-one mapping between the input and output signals if a system is be invertible.
- The meaning of one to one mapping is that an invertible system produces distinct outputs for distinct inputs.
- All the discussion done for the C.T. invertible system is applicable to the D.T. systems as well.

Application of invertibility :

- The property of invertibility is very important in designing the **communication systems.** When a signal travels over a communication channel, it gets distorted due to physical characteristics of the channel.
- An **equalizer** can be used to compensate for this distortion and recover back the original signal. The equalizer is added on the receiver side as shown in Fig. 2.7.2.
- An equalizer is an inverse system of the communication channel.

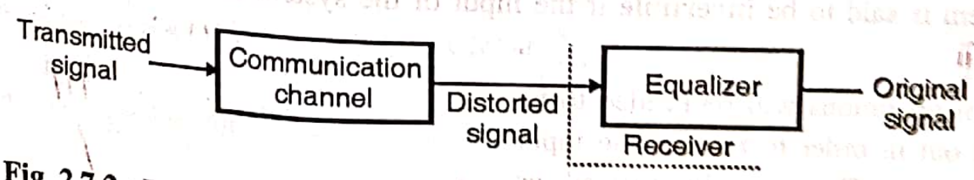


Fig. 2.7.2 : Equalizer is an inverse system of the communication channel

Ex. 2.7.1 : For an inductor the input output relation is as follows :

$$y(t) = \frac{1}{L}\int_{-\infty}^{t} x(t)\,dt$$

Find the operation that represents its inverse system.

Soln. :

$$H = \frac{1}{L} \int_{-\infty}^{t} dt$$

$$\therefore \quad H^{-1} = L \times \frac{d}{dt}$$ ...Ans.

This is the operator of the inverse system so that

$$H^{-1} H = I$$

2.7.1 Comparisons :

Causal system and non-causal system :

Sr. No.	Causal system	Non-causal system
1.	A system is said to be causal system if output at any instant of time depends only on present and past inputs. But the output does not depend on future input.	A system is said to be non-causal system if its output depends not only on present and past inputs but also on future inputs.
2.	Causal system is practically realizable.	Non-causal system is practically not realizable.
3.	Condition for causality, h (t) = 0 for $t < 0$. i.e. h (t) is zero for all negative values of t only.	For non-causal system $h(t) \neq 0$ for $t < 0$. i.e. h (t) exist for negative values of t also.
4.	Example : All real time systems.	Examples : Population growth, Weather broadcasting etc.

Time variant system and time invariant system :

Sr. No.	Time variant system	Time invariant system
1.	A system is time variant if its input-output characteristics changes with time.	A system is time invariant system if its input-output characteristic does not changes with time.
2.	If $x(n) \xrightarrow{T} y(n)$ then for time variant system, $x(n-k) \xrightarrow{T} y(n-k)$	If $x(n) \xrightarrow{T} y(n)$ then for TVI system, $x(n-k) \xrightarrow{T} y(n-k)$
3.	e.g. : $y(n) = x(-n)$.	e.g. : $y(n) = x(2n)$.

Static system and dynamic system :

Sr. No.	Static system	Dynamic system
1.	It is a system in which output at any instant of time depends on input sample at the same time.	It is a system in which output at any instant of time depends on input sample at the same time as well as at other inputs.
2.	Output does not depend on delayed [x (n – k)] or advanced [x (n + k)] inputs.	Output depends on present [x (n)], delayed [x (n – k)] and advanced [x (n + k)] inputs.

Sr. No.	Static system	Dynamic system
3.	These are memoryless systems.	These system has a memory.
4.	e.g. : y (n) = 5x (n)	e.g. : y (n) = x (n) + 5x (n – 1)

2.8 Solved Examples :

Ex. 2.8.1 : Determine whether the system described by :

$y(t) = e^{tx(t)}$ is

1. Memoryless
2. Time invariant
3. Linear
4. Causal
5. invertible
6. Stable

Give proper justification.

Soln. :

1. **Memoryless :** This is memoryless system because past samples are not present.
2. **Time invariant :**

Step 1 : Delay input by 'k' units and denote output by y (t, k).

$$\therefore \quad y(t, k) = e^{tx(t-k)}$$

Step 2 : Replace 't' by (t – k) throughout the equation.

$$\therefore \quad y(t-k) = e^{(t-k)\cdot x(t-k)}$$

Step 3 : Since $y(t, k) \neq y(t-k)$ the system is time variant.

3. **Linear :**

Step 1 : Apply zero input then output is,

$$y(t) = e^0 = 1$$

Step 2 : Consider two inputs $x_1(t)$ and $x_2(t)$,

$$x_1(t) \xrightarrow{T} y_1(t) = e^{tx_1(t)}$$

$$x_2(t) \xrightarrow{T} y_2(t) = e^{tx_2(t)}$$

$$\therefore \quad y'(t) = y_1(t) + y_2(t) = e^{tx_1(t)} + e^{tx_2(t)}$$

Step 3 :

$$[x_1(t) + x_2(t)] \xrightarrow{T} y''(t) = e^{t[x_1(t) + x_2(t)]}$$

$$\therefore \quad y''(t) = e^{tx_1(t)} \cdot e^{tx_2(t)}$$

Step 4 : Since $y'(t) \neq y''(t)$; the system is nonlinear.

4. **Causal :** The system does not contain any future term; so it is causal system.

5. **Invariable :** The function of system is to multiply input by 't' and then take exponential value of input. This is invertible system; because invertible system is the system in which we can recover the input.

6. **Stable :** As long as an input is bounded, the output of system is always bounded. Thus it is stable system.

Ex. 2.8.2 : Determine whether the following systems are :

1. Linear 2. Memoryless
3. Stable 4. Causal 5. Time invariant

(a) $y(t) = x(0.5\,t)$ (b) $y(n) = \log_{10}(|x[n]|)$

Soln. :

(a) $y(t) = x(0.5\,t)$:

1. Let $x(t) = x_1(t) + x_2(t)$

$$x_1(t) \xrightarrow{T} x_1(0.5\,t)$$

$$x_2(t) \xrightarrow{T} x_2(0.5\,t)$$

$$\therefore\ y'(t) = x_1(0.5\,t) + x_2(0.5\,t)$$

Now,

$$[x_1(t) + x_2(t)] \xrightarrow{T} y''(t) = x_1(0.5\,t) + x_2(0.5\,t)$$

$$y'(t) = y''(t)$$

Thus it is linear system.

2. Put $t = 2 \Rightarrow y(2) = x(1)$:

That means at $t = 2$ it needs past input so it requires memory.

3. For any bounded input it produces bounded output; so it is stable system.

4. It is causal system.

5. $y(t, k) = x[0.5(t-k)]$

$$y(t-k) = x[0.5(t-k)]$$

so it is time invariant.

(b) $y(n) = \log_{10}(|x(n)|)$:

1. Let $x(n) = x_1(n) + x_2(t)$

$$x_1(n) \xrightarrow{T} \log_{10}(|x_1(n)|)$$

$$x_2(n) \xrightarrow{T} \log_{10}(|x_2(n)|)$$

$$y'(n) = \log(|x_1(n)|) + \log(|x_2(n)|)$$

Now, $[x_1(n) + x_2(n)] \xrightarrow{T} \log_{10}|[x_1(n) + x_2(n)]| = y''(n)$

Since $y'(n) \neq y''(n)$; the system is non-linear.

2. It is memoryless system; because output at any instant depends on input at that instant.
3. Output depends on present input only; so system is causal.
4. The system is stable.
5. $y(n, k) = \log_{10}|x(n-k)|$

and $y(n-k) = \log|x(n-k)|$

Since $y(n, k) = y(n-k)$; it is time invariant system.

Ex. 2.8.3 : A discrete time system is both linear and time invariant. Suppose the output due to an input $x[n] = \delta[n]$ is given in Fig. P. 2.8.3(a), then find the output due to an input :

1. $x[n] = \delta[n-1]$ 2. $x[n] = 2\delta[n] - \delta[n-2]$
3. $x[n]$ as depicted in Fig. P. 2.8.3(b).

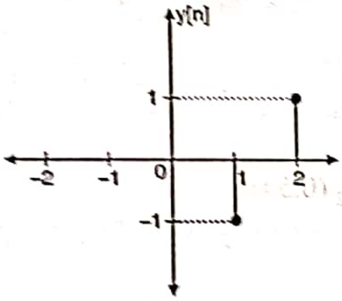


Fig. P. 2.8.3(a)

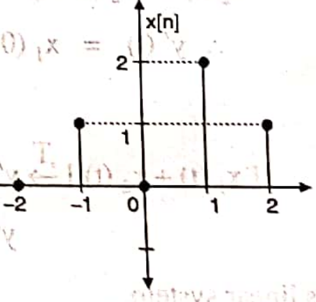


Fig. P. 2.8.3(b)

Soln. :

1. $x(n) = \delta(n-1)$

Since the system is time invariant ; if input is delayed by '1' position then output will be also delayed by '1' position. This output is shown in Fig. P. 2.8.3(c).

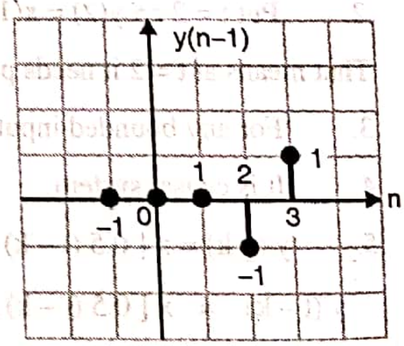


Fig. P. 2.8.3(c)

2. $x(n) = 2\delta(n) - \delta(n-2)$

Consider the term $2\delta(n)$. It will produce output $y_1(n) = 2y(n)$ and the term $\delta(n-2)$ will produce output $y(n-2)$.

Thus final output is,

$$y'(n) = 2y(n) - y(n-2).$$

It is shown in Fig. P. 2.8.3(d).

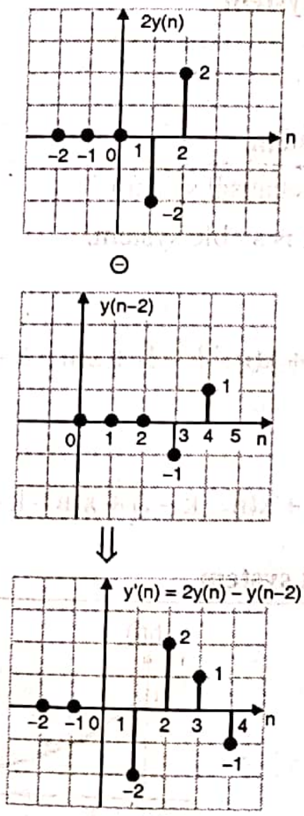
3. The signal $x(n)$ can be represented in terms of $\delta(n)$ as,

$$x(n) = \delta(n+1) + 2\delta(n-1) + \delta(n-2)$$

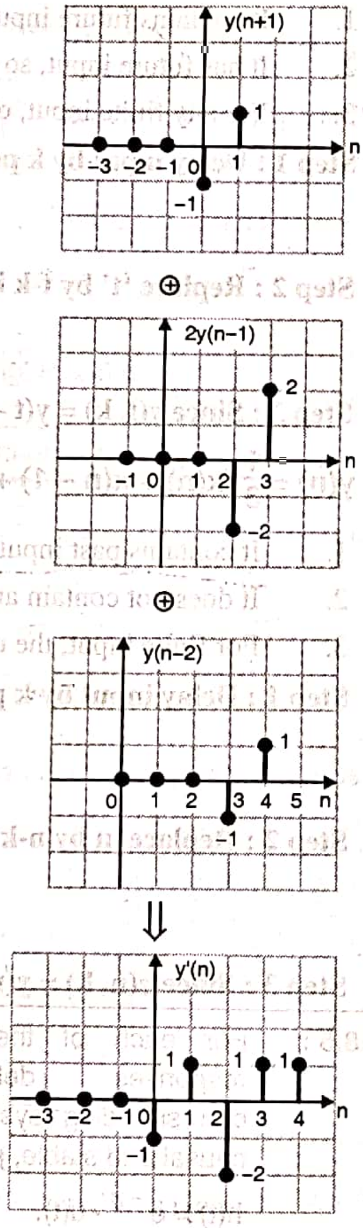
It will produce the corresponding output

$$y'(n) = y(n+1) + 2y(n-1) + y(n-2)$$

This output is calculated as shown in Fig. P. 2.8.3(e).



(d)



(e)

Fig. P. 2.8.3

Ex. 2.8.4 : Check whether the following systems are :

1. Static/dynamic
2. Causal/non-causal
3. Stable/unstable
4. Time invariant/time variant

1. $y(t) = x(t+10) + x^2(t)$
2. $y[n] = \frac{1}{3}\{x[n] + x[n-1] + x[n-2]\}$.

Soln. :

1. $y(t) = x(t + 10) + x^2(t)$:

1. It contains future input term ; so it is dynamic system.
2. It has future input, so it is non-causal.
3. For any finite input, output is always finite ; so it is stable system.

Step 1 : Delay input by k positions :

$$y(t, k) = x(t - k + 10) + x^2(t - k)$$

Step 2 : Replace 't' by t-k in the given equation :

$$\therefore y(t - k) = x(t - k + 10) + x^2(t - k)$$

Step 3 : Since y(t, k) = y(t – k) ; it is time in variant system.

2. $y(n) = \frac{1}{3}\{x(n) + x(n - 1) + x(n - 2)\}$:

1. It contains past input terms ; so it is dynamic system.
2. It does not contain any future input term ; so it is causal system.
3. For finite input, the output is always finite. So it is stable system.

Step 1 : Delay input by k positions :

$$y(n, k) = \frac{1}{3}\{x(n - k) + x(n - k - 1) + x(n - k - 2)\}$$

Step 2 : Replace n by n-k in the given equation :

$$y(n - k) = \frac{1}{3}\{x(n - k) + x(n - k - 1) + x(n - k - 2)\}$$

Step 3 : Since y(n, k) = y(n – k) ; it is time invariant system.

Ex. 2.8.5 : For each of the following impulse responses determine whether corresponding system is memoryless, causal and stable, justify answer.

$h(t) = e^{-2t} \cdot u(t)$.

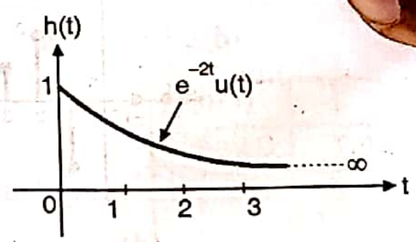


Fig. P. 2.8.5

Soln. :

1. Given $h(t) = e^{-at} u(t)$

This is a positive sided, decaying exponential signal as shown in Fig. P. 2.8.5.

It contains present and past samples, so it is not memoryless system.

2. The condition of causality is,

$$h(t) = 0 \text{ for } t < 0$$

This signal is only at positive side as shown in Fig. P. 2.8.5. So it is causal system.

3. According to the condition of stability,

$$S = \int_{-\infty}^{\infty} |h(t)| \cdot dt < \infty$$

Given $\quad h(t) = e^{-2t} u(t)$

$$S = \int_{0}^{\infty} |e^{-2t}| \, dt = \left[\frac{e^{-2t}}{2}\right]_0^{\infty} = \left(-\frac{1}{2}\right) [e^{-\infty} - e^{0}]$$

$$\therefore \quad S = \frac{1}{2} < \infty$$

So it is a stable system.

Ex. 2.8.6 : Find the input-output relation of the system shown in Fig. P. 2.8.6.

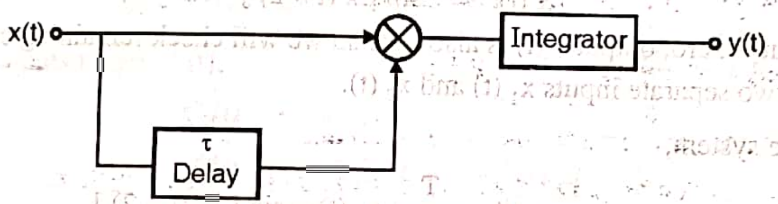


Fig. P. 2.8.6

Soln. :

The signal x (t) is directly applied to multiplier. While x (t) is delayed by τ and then it is applied to the multiplier. So output of multiplier is,

$x(t) \cdot x(t-\tau)$

The result is integrated to produce output y (t)

$$\therefore y(t) = \int_{-\infty}^{\infty} x(t) \cdot x(t-\tau) \, dt$$

This gives equation of autocorrelation.

Ex. 2.8.7 : Determine if the following system described by :

$y(t) = \sin[x(t+2)]$

is memoryless, causal, linear, time invariant, stable.

Soln. :

Given :

$$y(t) = \sin[x(t+2)]$$

(A) Memoryless :

As present output depends on future inputs, the system is not memoryless (Static).

(B) Causal :

As output depends on future input i.e. x (t + 2) the system is non-causal.

(C) Time invariant :

$$\text{Given,} \quad y(t) = \sin[x(t+2)]$$

Step 1 : Delay the input by 'k' samples and denote corresponding output by y (t, k).

$$\therefore \; y(t,k) = \sin[x(t-k+2)] \qquad ...(1)$$

Step 2 : Replace 't' by 't – k' throughout the given equation.

$$\therefore \; y(t-k) = \sin[x(t-k+2)] \qquad ...(2)$$

Step 3 : Comparing Equations (1) and (2),

$$\text{Here,} \quad y(t,k) = y(t-k)$$

Thus the system is time invariant (TIV).

(D) Linear :

$$y(t) = \sin[x(t+2)]$$

Step 1 : When input is zero, output y (t) is also zero so we will check remaining conditions.

Step 2 : Consider two separate inputs $x_1(t)$ and $x_2(t)$.

Apply it to the system,

$$\therefore \; x_1(t) \xrightarrow{T} y_1(t) = \sin[x_1(t+2)]$$

$$\text{and} \; x_2(t) \xrightarrow{T} y_2(t) = \sin[x_2(t+2)]$$

Add $y_1(t)$ and $y_2(t)$ to get $y'(t)$.

$$\therefore \; y'(t) = y_1(t) + y_2(t)$$

$$\therefore \; y'(t) = \sin[x_1(t+2)] + \sin[x_2(t+2)] \qquad ...(3)$$

Step 3 : Combine two inputs and apply it to the system.

$$\therefore \; [x_1(t) + x_2(t)] \xrightarrow{T} \sin[x_1(t+2) + x_2(t+2)]$$

$$\therefore \; y''(t) = \sin[x_1(t+2) + x_2(t+2)] \qquad ...(4)$$

Step 4 : Compare Equations (3) and (4),

Since $y'(t) \neq y''(t)$; the system is non-linear.

(E) Stable : For every bounded value of x (t + 2), its sine value is always bounded (finite). So given system is stable.

Ex. 2.8.8 : Determine whether the following continuous time systems are :

1. Static or dynamic
2. Linear or non-linear
3. Shift varient or shift invarient
4. Causal or non causal
5. Stable or unstable

1. $y(t) = 10\,x(t) + 5$
2. $\frac{d}{dt} y(t) + t\,y(t) = x(t)$
3. $y(t) = x(t) \cos 100\pi t$
4. $\frac{dy(t)}{dt} + 10\,y(t) = x(t)$
5. $y(t) = x(t+10) + x^2(t)$
6. $y(t) = x(t^2)$

Soln. :

1. **$y(t) = 10\,x(t) + 5$**

1. **Static or dynamic :**

Here output, $y(t)$ depends on present input only. So it is **static system.**

2. **Linear or non linear :**

Step 1 : Apply zero input to the system; that means $x(t) = 0$

$$\therefore \quad y(t) = 5$$

Thus system is non-relaxed initially.

Step 2 : Consider two inputs $x_1(t)$ and $x_2(t)$ and apply it to the system.

$$\therefore \quad x_1(t) \xrightarrow{T} y_1(t) = 10\,x_1(t) + 5$$

$$\text{and} \quad x_2(t) \xrightarrow{T} y_2(t) = 10\,x_2(t) + 5$$

Add these two outputs to get $y'(t)$,

$$\therefore \quad y'(t) = y_1(t) + y_2(t)$$

$$\therefore \quad y'(t) = 10\,x_1(t) + 5 + 10\,x_2(t) + 5$$

$$\therefore \quad y'(t) = 10\,x_1(t) + 10\,x_2(t) + 10$$

Step 3 : Combine all inputs and apply it to the system.

$$[\,x_1(t) + x_2(t)\,] \xrightarrow{T} 10\,[\,x_1(t) + x_2(t)\,] + 5$$

$$\therefore \quad y''(t) = 10\,x_1(t) + 10\,x_2(t) + 5$$

Step 4 : Since $y'(t) \neq y''(t)$, the system is non- linear.

3. **Shift varient or shift invarient :**

Step 1 : Delay input by t_0 units and denote output by $y(t, t_0)$

$$y(t, t_0) = 10x(t-k) + 5$$

Step 2 : Replace 't' by $t - t_0$ throughout the equation,

$$\therefore \quad y(t - t_0) = 10x(t-k) + 5$$

Step 3 : Since $y(t, t_0) = y(t - t_0)$, the system is time invarient.

4. **Causal or non causal :**

Given equation is,

$$y(t) = 10x(t) + 5$$

Since output depends on present input only; **the system is causal.**

5. **Stable or unstable :**

If we apply bounded input $x(t)$, then output will be always bounded. Thus **this system is stable.**

Thus given system is static non-linear time invarient, causal and stable.

2. $\frac{d}{dt} y(t) + t\, y(t) = x(t)$

1. **Static or dynamic :**

The given equation contains differentiation operation, so it is **dynamic system.**

2. **Linear or non-linear :**

Step 1 : Apply zero input :

$$\therefore \quad \frac{d}{dt} y(t) + t\, y(t) = 0$$

But we will check the remaining steps.

Step 2 : Apply each input separately to the system :

$$\therefore \quad x_1(t) \overset{T}{\longleftrightarrow} \frac{d}{dt} y_1(t) + t\, y_1(t)$$

$$\text{and} \quad x_2(t) \overset{T}{\longleftrightarrow} \frac{d}{dt} y_2(t) + t\, y_2(t)$$

$$\therefore \quad y'(t) = \frac{d}{dt} y_1(t) + t\, y_1(t) + \frac{d}{dt} y_2(t) + t\, y_2(t)$$

Step 3 : Combine both inputs and apply to the system :

$$\therefore \quad [x_1(t) + x_2(t)] \longleftrightarrow \frac{d}{dt}[y_1(t) + y_2(t)] + t\,[y_1(t) + y_2(t)]$$

$$\therefore \quad y''(t) = \frac{d}{dt} y_1(t) + \frac{d}{dt} y_2(t) + t\, y_1(t) + t\, y_2(t)$$

Step 4 : Since $y'(t) = y''(t)$; this system is linear :

3. **Shift varient or shift invarient :**

The given equation is,

$$\frac{d}{dt} y(t) + t\, y(t) = x(t)$$

It contains the term $t\, y(t)$. So system varies with time. Thus it is **time varient system.**

4. Present output depends on present input so it is **causal system.**

5. If we apply bounded input then output will be always bounded. **Thus it is stable system.**

Thus this system is dynamic, linear, time varient, causal and stable.

3. $y(t) = x(t) \cos 100\pi t$

1. ***Static or dynamic :***

In the given equation output depends to present input only. **So it is static system.**

2. **Linear or non-linear :**

***Step 1 :* Apply zero input then output $y(t) = 0$.**

So system is initially relaxed.

Step 2 : **Apply each input separately to the system.**

$$\therefore \quad x_1(t) \xrightarrow{T} x_1(t) \cos \pi t$$

$$\text{and} \quad x_2(t) \xrightarrow{T} x_2(t) \cos \pi t$$

$$\therefore \quad y'(t) = x_1(t) \cos \pi t + x_2(t) \cos \pi t$$

Step 3 : **Combine both inputs and apply it to the system.**

$$\therefore \quad [x_1(t) + x_2(t)] \xrightarrow{T} x_1(t) \cos \pi t + x_2(t) \cos \pi t = y''(t)$$

Step 4 : **Since $y'(t) = y''(t)$; it is linear system.**

3. **Shift varient or shift invarient :**

Step 1 : **Delay input by 't0' units and denote output by y (t, k)**

$$\therefore \quad y(t, t_0) = x(t - t_0) \cos 100\pi t$$

Step 2 : **Replace t by t – t0 throughout the system.**

$$\therefore \quad y(t - t_0) = x(t - t_0) \cos 100\pi (t - k)$$

Step 3 : **Since $y(t, t_0) \neq y(t - t_0)$; the system is shift varient.**

4. Here output depends on present input; thus **the system is causal system.**

5. Here equation of output contains cosine function. The maximum value of cosine function is 1. So if we apply bounded input then output is always bounded. **Thus it is stable system.**

So this system is static, linear, time varient, causal and stable.

4. $\dfrac{dy(t)}{dt} + 10\, y(t) = x(t)$

1. **Static or dynamic :**

The given equation contains differential term. So it is **dynamic system.**

2. **Linear or non-linear :**

Step 1 : **Applying zero input to the system we get,**

$$\frac{dy(t)}{dt} + 10\, y(t) = 0.$$

Step 2 : **Apply each input separately to the system.**

$$\therefore \quad x_1(t) \xrightarrow{T} \frac{d}{dt} y_1(t) + 10\, y_1(t)$$

$$\text{and} \quad x_2(t) \xrightarrow{T} \frac{d}{dt} y_2(t) + 10\, y_2(t)$$

$$\therefore \quad y'(t) = \frac{d}{dt} y_1(t) + 10\, y_1(t) + \frac{d}{dt} y_2(t) + 10\, y_2(t)$$

Step 3 : **Combine all inputs and apply it to the system.**

$$\therefore \quad [x_1(t) + x_2(t)] \xrightarrow{T} \frac{d}{dt}[y_1(t) + y_2(t)] + 10\,[y_1(t) + y_2(t)]$$

$$\therefore \quad y''(t) = \frac{d}{dt} y_1(t) + \frac{d}{dt} y_2(t) + 10\, y_1(t) + 10\, y_2(t)$$

Step 4 : **Since $y'(t) = y''(t)$; this system is linear.**

3. **Shift varient or shift invarient :**

The given equation does not contain any term which is multiplied by 't'. So output does not vary with time. That means **system is shift invarient.**

4. Output depends on present input only so **it is causal system.**

5. If we apply bounded input then bounded output is obtained. So **it is stable system.**

Thus system is dynamic, linear, shift invarient, causal and stable.

5. $y(t) = x(t + 10) + x^2(t)$

1. **Static or dynamic :**

The given equation contains the term x(t + 10); which is future (advanced) input. So the system **is dynamic.**

2. **Linear or non-linear :**

Step 1 : **Apply zero input that is x (t) = 0 then y (t) = 0.**

Step 2 : **Apply each input separately to the system**

$$\therefore \quad x_1(t) \xrightarrow{T} x_1(t+10) + x_1^2(t)$$

$$\text{and } x_2(t) \xrightarrow{T} x_2(t+10) + x_2^2(t)$$

$$\therefore \quad y'(t) = x_1(t+10) + x_1^2(t) + x_2(t+10) + x_2^2(t)$$

Step 3 : **Combine both inputs and apply to the system**

$$\therefore \quad [x_1(t) + x_2(t)] \xrightarrow{T} x_1(t+10) + x_2(t+10) + [x_1 + x_2]^2 = y''(t)$$

Step 4 : **Since $y'(t) \neq y''(t)$; the system is non- linear.**

3. **Shift varient or invarient :**

Step 1 : **Delay input by 'k' units and denote the output by $y(t, t_0)$.**

$$y(t, t_0) = x(t + 10 - t_0) + x^2(t - t_0)$$

Step 2 : **Replace 't' by $t - t_0$ throughout the given equation**

$$\therefore \quad y(t - t_0) = x(t - t_0 + 10) + x^2(t - t_0)$$

$$\therefore \quad y(t - t_0) = x(t + 10 - t_0) + x^2(t - t_0)$$

Step 3 : **Since $y(t, t_0) = y(t - t_0)$; the system is shift invarient.**

4. Here output depends on future input x (t + 10). Thus system is **non causal.**

5. If we apply bounded input then bounded output is obtained. Hence **the system is stable.**

Thus the system is dynamic, non linear, shift invarient, non causal and stable.

6. $y(t) = x(t^2)$

1. **Static or dynamic :**

Given $y(t) = x(t^2)$

We will put few values of 't'

For $t = 2 \Rightarrow y(2) = x(4)$

For $t = 3 \Rightarrow y(3) = x(9)$

It indicates that output at any instant requires future input. So the **system is dynamic.**

2. **Linear or non-linear :**

Step 1 : Apply zero input then $y(t) = 0$

So the system is initially relaxed.

Step 2 : Apply individual input separately to the system

$$\therefore \; x_1(t) \xrightarrow{T} x_1(t^2)$$

$$\text{and } x_2(t) \xrightarrow{T} x_2(t^2)$$

$$\therefore \quad y'(t) = x_1(t^2) + x_2(t^2)$$

Step 3 : Combine both inputs and apply to the system.

$$\therefore \quad [x_1(t) + x_2(t)] \xrightarrow{T} x_1(t^2) + x_2(t^2) = y''(t)$$

Step 4 : Since $y'(t) = y''(t)$; this system is linear.

3. **Shift varient or shift invarient :**

Step 1 : Delay input by t_0 and denote the corresponding output by $y(t, t_0)$.

$$\therefore \quad y(t, t_0) = x(t^2 - t_0)$$

Step 2 : Replace t by $t - t_0$ throughout the given equation

$$\therefore \quad y(t - t_0) = x(t - t_0)^2$$

Step 3 : Since $y(t, t_0) \neq y(t - t_0)$; the system is shift varient.

4. Since output depends on future input **it is non-causal system.**

5. If we apply bounded input then bounded output is obtained. Hence **it is stable system.**

Thus given system is dynamic, linear, shift varient, non-causal and stable.

Ex. 2.8.9 : Classify the following systems for linearity and time invariance

1. $y(n) = x(n) - x(n-1) + x(n+1)$
2. $y''(t) + 3y'(t) = 2x'(t) + x(t)$

Soln. :

1. $y(n) = x(n) - x(n-1) + x(n+1)$

Step 2 : **Replace n by n – K :**

$$\therefore \quad y(n-K) = x_1(n-K) + (n-K)\, x_2(n-K)$$

Step 3 : **Since y (n, K) ≠ y (n – K) :**

The system is time invariance.

Causality : Since the equation contains advanced term; it is non-causal system.

Review Questions

Q. 1 Define system and state its types.

Q. 2 Define the CT and DT systems.

Q. 3 Classify CT and DT systems.

Q. 4 Define linear and nonlinear systems.

Q. 5 What do you understand by time variant and time invariant systems ? Give examples.

Q. 6 Define causal and noncausal systems.

Q. 7 What is the condition for causality in terms of impulse response of the system ?

Q. 8 Define the stable and unsatable systems.

Q. 9 What is a bounded signal ? What is BIBO stability ?

Q. 10 Define static and dynamic systems and state their examples.

Q. 11 Compare :

1. time variant / invariant systems 2. stable / unstable systems.

Q. 12 Explain the concept of invertibility.

2.9 Examples for Practice :

Ex. 2.9.1 : Check whether the following discrete time system is linear or not :

$y(n) = x(n) \cos \omega_0 n.$

Ans. : **Linear**

Ex. 2.9.2 : Determine whether the following system is linear or not :

$$y(n) = \frac{1}{3}[x(n) + x(n-1) + x(n-2)]$$

Ans. : **Linear**

Ex. 2.9.3 : y (n) = n x (n) check for the linearity.

Ans. : **Linear**

Ex. 2.9.4 : $y(n) = x(n - n_0)$, n_0 is constant. Check for the linearity.

Ans. : **Linear**

Ex. 2.9.5 : $y(n) = x(n) \cdot n(n - n_0)$ check for the linearity.

Ans. : Linear

Ex. 2.9.6 : Examine the following system for linearity,

$y(n) = x(-n)$

Ans. : Linear

Ex. 2.9.7 : Show the following system is shift variant :

$y(n) = n\,x^2(n)$

Ans. : Shift variant

Ex. 2.9.8 : Find out if the following systems are shift invariant or not :

1. $y(n) = x(n - n_0)$, n_0 is constant
2. $y(n) = ax(n) + b$, a and b are constants
3. $y(n) = x(n)\,n(n - n_0)$, n_0 is constant.

Ans. : 1. Shift invariant 2. Shift invariant 3. Shift variant

Ex. 2.9.9 : Determine whether the following system is shift invariant or not :

$y(n) = x(n)\cos(\omega_0 n)$

Ans. : Shift variant

Ex. 2.9.10 : Determine whether the following system is time invariant or not :

$T[x(n)] = ax(n) + 6$

Ans. : Time invariant

Ex. 2.9.11 : Determine whether the following system is time invariant or not :

$y(n) = |x(n)|$

Ans. : Time invariant

Ex. 2.9.12 : Determine whether the following systems are time invariant or not :

1. $y(n) = g(n)\,x(n)$ 2. $y(n) = x(n) + 3u(n+1)$

Ans. : 1. Time variant 2. Time variant

Ex. 2.9.13 : Determine whether the following system is time invariant or not :

$$y(n) = \frac{1}{3}[x(n) + x(n-1) + x(n-2)]$$

Ans. : Time invariant

Ex. 2.9.14 : Determine if the following systems described by :

1. $y(t) = \sin[x(t+2)]$ 2. $y[n] = x[2-n]$

are memoryless, causal, linear, time invariant, stable.

Ans. :

1. **Static, non-causal, TIV, non linear, stable**
2. **Dynamic, non-causal, T.V., linear, stable**

Ex. 2.9.15 : 1. Show that a system with excitation x (t) and response y (t) described by $y(t) = x(t/2)$ is linear, time variant, non-causal.

Ex. 2.9.16 : Show that the moving average system described by input-output relation :

$$y(n) = \frac{1}{3}[x(n+1) + x(n) + x(n-1)]$$ is BIBO stable.

Ex. 2.9.17 : Check whether the following CT systems are time invariant :

1. $y(t) = \sin x(t)$
2. $y[n] = x[-n]$
3. $y(t) = x(t)\cos 200\pi t$
4. $y[n] = x[n] - x[n-1]$

Ans. : **1. T.V. 2. T.V. 3. T.V. 4. TIV**

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