Parul University

Faculty of Engineering & Technology

Department of Applied Sciences & Humanities

1st Year B.Tech Programme 2023-2024

Mathematics-1 (303191101) (For all branches)

Question Bank

1. Solve the system of linear equations using the Gauss-elimination method

$$x - 2y + 3z = 9;\ -x + 3y - z = -6;\ 2x - 5y + 5z = 17$$

2. Solve the system of linear equations using the Gauss-Jordan method

$$2x + y + z = 10;\ \ x + 2y + 3z = 1;\ -x - y - z = 2$$

3. Find the rank of the following matrices

(1) $A = \begin{bmatrix} 1 & 2 & 0 & 1 \\ 2 & 4 & 1 & 3 \\ 3 & 6 & 2 & 5 \\ -4 & -8 & 1 & -3 \end{bmatrix}$ (2) $A = \begin{bmatrix} 1 & 1 & 0 & -2 \\ 2 & 0 & 2 & 0 \\ 4 & 1 & 3 & 1 \end{bmatrix}$

4. Find the value of a, so that the rank of the matrix is 2.

(1) $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ a & 7 & 9 \end{bmatrix}$ (2) $A = \begin{bmatrix} 2 & 3 \\ 4 & a \end{bmatrix}$ (Objective question)

5. Determine the eigenvalue and eigenvector for a matrix $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 3 \\ 0 & 0 & 3 \end{bmatrix}$.

6. Find the modal matrix P that diagonalizes $A = \begin{bmatrix} 4 & 2 \\ 3 & -1 \end{bmatrix}$.

7. State Calay-Hamilton theorem. Hence, verify Calay-Hamilton theorem for matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$. Hence compute A^{-1}.

8. Expand as a Fourier series of $f(x) = |x|$ in the interval $(-\pi, \pi)$.

9. Expand as a Fourier sine series $f(x) = \pi - x$ in the interval $(0, \pi)$.

10. Discuss the convergence of the following series

(1) $\sum_{n=1}^{\infty} \frac{n^2+2}{3n^4+1}$ (2) $\sum_{n=1}^{\infty} \left(\frac{5^n+1}{3^n}\right)$ (3) $\sum_{n=0}^{\infty} \left(\frac{n}{n+1}\right)^{n^2}$ (4) $\frac{2!}{3} + \frac{3!}{3^2} + \frac{4!}{3^3} + \cdots$

11. If $z = f(x + ay) + \varphi(x - ay)$ then show that $z_{yy} = a^2 z_{xx}$

12. If $u = e^{xyz}$ find, $\frac{\partial^3 u}{\partial x \partial y \partial z}$

13. If $u = sin^{-1}\left(\frac{x^{\frac{1}{4}} + y^{\frac{1}{4}}}{x^{\frac{1}{6}} + y^{\frac{1}{6}}}\right)$, evaluate $x^2 u_{xx} + 2xyu_{xy} + y^2 u_{yy}$.

14. If $u = f(x - y, y - z, z - x)$, show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$.

15. If $u = x^2 y^3, x = logt, y = e^t$, Find $\frac{dy}{dt}$.

16. If $x^y = y^x$ then find $\frac{dy}{dx}$.

17. Find the equations of the tangent plane and normal line to the surface $x^2yz + 3y^2 = 2xz^2 - 8z$ at the point (1, 2, -1).

18. Find the linearization of $f(x, y) = 3 + \frac{x^2}{16} + \frac{y^2}{9}$ at $(-4, 3)$.

19. Discuss the maxima and minima of the function $3x^2 - y^2 + x^3$.

20. Find the Jacobian $\frac{\partial(u,v)}{\partial(x,y)}, u = x + y, v = x - y$.

21. Solve, $\frac{dy}{dx} - x^3 y^3 + xy = 0$

22. Solve, $\frac{dy}{dx} = ytanx - 2sinx$.

23. Solve, $ysin2xdx - (1 + y^2 + cos^2 x)dy = 0$

24. Solve, $\left(1 + e^{\frac{x}{y}}\right)dx + e^{\frac{x}{y}}\left(1 - \frac{x}{y}\right)dy = 0$

25. Solve, $\int_0^{\infty} e^{-x^2} dx$

26. Evaluate, $\int_0^1 x^2(1 - x^2)^4 \, dx$