Unit:- 5 Fourier Series

Tatorial = 4

1. Find the fourier series of $f(x) = \left(\frac{\pi - x}{2}\right)^2, 0 \le x \le 2\pi$

SoM

$$F(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \cos\frac{n\pi x}{L} + b_n \sin\frac{n\pi x}{L}\right)$$

$$L = \frac{b-a}{2} = \frac{2\pi - 0}{2} = \pi$$

$$F(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx)$$

$$a_0 = \frac{1}{L}\int_a^b f(x)\,dx$$

$$= \frac{1}{\pi}\int_0^{2\pi}\left(\frac{\pi - x}{2}\right)^2 dx$$

$$= \frac{1}{4\pi}\left[\frac{(\pi - x)^3}{-3}\right]_0^{2\pi}$$

$$= \frac{1}{-12\pi}\left[(\pi - 2\pi)^3 - (\pi - 0)^3\right]$$

$$= \frac{1}{-12\pi}\left[-\pi^3 - \pi^3\right]$$

$$= \frac{-2\pi^3}{-12\pi}$$

$$= \frac{\pi^2}{6}$$

$$a_n = \frac{1}{L}\int_0^{2\pi} f(x)\cos nx\,dx$$

$$= \frac{1}{\pi}\int_0^{2\pi}\left(\frac{\pi - x}{2}\right)^2 \cos nx\,dx$$

$$= \frac{1}{4\pi}\int_0^{2\pi}(\pi - x)^2 \cos nx\,dx$$

$$= \frac{1}{4\pi}\left[(\pi - x)^2\left(\frac{\sin nx}{n}\right) - 2(\pi - x)\left(\frac{\cos nx}{n^2}\right) - 2\left(\frac{\sin nx}{n^3}\right)\right]_0^{2\pi}$$

$$= \frac{1}{4\pi}\left[(\pi - 2)^2\frac{\sin nx}{n} - 2(\pi - 2)\frac{(\cos nx)}{n^2} - 2\frac{(\sin nx)}{n^3}\right]_0^{2\pi}$$

$$= \frac{1}{4\pi}\left[\left(-2(\pi - 2\pi)\cdot\frac{1}{n^2}\right) + \frac{2\pi}{n^2}\right]$$

$$= \frac{1}{4\pi}\left[\frac{-2\pi + 2\pi}{n^2} + \frac{2\pi}{n^2}\right]$$

$$= \frac{1}{4\pi}\left(\frac{-2\pi - 2\pi + 2\pi}{n^2}\right)$$

$$= \frac{1}{4\pi}\left(\frac{-4\pi}{n^2}\right)$$

$$= \frac{1}{n^2}$$

$$b_n = \frac{1}{L}\int_0^{2\pi} f(x)\sin nx\,dx$$

$$= \frac{1}{\pi}\int_0^{2\pi}\left(\frac{\pi - x}{2}\right)^2 \sin nx\,dx$$

$$= \frac{1}{4\pi}\int_0^{2\pi}(\pi - x)^2 \sin nx\,dx$$

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$$= \frac{1}{4\pi}\ \cancel{\frac{(\pi-x)^3}{3}}\left[(\pi-x)^2\left(\frac{-\cos nx}{n}\right) - 2(\pi-x)(-1)\left(\frac{-\sin nx}{n^2}\right) + 2\left(\frac{\cos nx}{n^3}\right)\right]_0^{2\pi}$$

$$= \frac{1}{4\pi}\left[-(\pi-x^2)\frac{\cos nx}{n} - 2(\pi-x)\frac{\sin nx}{n^2} + \frac{2\cos nx}{n^3}\right]_0^{2\pi}$$

$$= \frac{1}{4\pi}\left[\left(-\frac{\pi^2}{n} + \pi(0) + \frac{2}{n^3}\right) - \left(-\frac{\pi^2}{n} + \frac{2}{n^3}\right)\right]$$

$$= \frac{1}{4\pi}\left(-\frac{\pi^2}{n} + \frac{2}{n^3} + \frac{\pi^2}{n} - \frac{2}{n^3}\right)$$

$$= \frac{1}{4\pi}(0)$$

$b_n = 0$

$$F(x) = \frac{\pi^2}{12} + \sum_{n=1}^{\infty}\frac{1}{n^2}\cos nx \quad \text{Ans}$$

2. Find the Fourier series of $f(x) = 2x - x^2$ in $(0,3)$

Sol^n

$$F(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n\cos\frac{n\pi x}{L} + b_n\sin\frac{n\pi x}{L}\right)$$

$$L = \frac{3-0}{2} = \frac{3}{2}$$

$$F(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left[a_n\cos\left(\frac{2n\pi x}{3}\right) + b_n\sin\left(\frac{2n\pi x}{3}\right)\right]$$

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$$a_0 = \frac{1}{L}\int_a^b f(x)\,dx$$

$$= \frac{2}{3}\int_0^3 (2x - x^2)\,dx$$

$$= \frac{2}{3}\left[\frac{2x^2}{2} - \frac{x^3}{3}\right]_0^3$$

$$= \frac{2}{3}\left(\frac{2(3)^2}{2} - \frac{(3)^3}{3}\right)$$

$$= \frac{2}{3}(9 - 9)$$

$$= 0$$

$$a_n = \frac{1}{L}\int_a^b f(x)\cos\frac{2n\pi x}{3}\,dx$$

$$= \frac{2}{3}\int_0^3 (2x - x^2)\cos\left(\frac{2n\pi x}{3}\right)dx$$

$$= \frac{2}{3}\left[(2x-x^2)\frac{\sin\left(\frac{2n\pi x}{3}\right)}{\frac{2n\pi}{3}} - (2-2x)\left(-\frac{\cos\frac{2n\pi x}{3}}{\frac{4n^2\pi^2}{9}}\right) + (-2)\left(-\frac{\sin\frac{2n\pi x}{3}}{\frac{8n^3\pi^3}{27}}\right)\right]_0^3$$

$$= \frac{2}{3}\left[-2\left(\frac{1}{\frac{4n^2\pi^2}{9}}\right) - 2\left(\frac{1}{\frac{4n^2\pi^2}{9}}\right)\right]$$

$$= \frac{2}{3}\left(\frac{-9\times 2}{4n^2\pi^2} - \frac{9\times 2}{4n^2\pi^2}\right) = \frac{2}{3}\left(\frac{-36}{4n^2\pi^2}\right) = \frac{-6}{n^2\pi^2}$$

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$$b_n = \frac{1}{L}\int_0^3 f(x)\sin\left(\frac{n\pi x}{L}\right)dx$$

$$= \frac{2}{3}\int_0^3 (2x-x^2)\sin\left(\frac{2n\pi x}{3}\right)dx$$

$$= \frac{2}{3}\left[(2x-x^2)\left(\frac{-\cos\frac{2n\pi x}{3}}{\frac{2n\pi}{3}}\right) - (2-2x)\left(\frac{-\sin\frac{2n\pi x}{3}}{\frac{4n^2\pi^2}{9}}\right) + (-2)\frac{\cos\frac{2n\pi x}{3}}{\frac{8n^3\pi^3}{27}}\right]_0^3$$

$$= \frac{2}{3}\left(3\left(\frac{1}{\frac{2n\pi}{3}}\right) - 2\left(\frac{1}{\frac{8n^3\pi^3}{27}}\right) + 2\,\frac{1}{\frac{8n^3\pi^3}{27}}\right)$$

$$= \frac{2}{3}\left(\frac{9}{2n\pi}\right) = \frac{3}{n\pi}$$

$$b_n = \frac{3}{n\pi}$$

$$F(x) = \sum_{n=1}^{\infty}\left[\frac{-6}{n^2\pi^2}\cos\left(\frac{2n\pi x}{3}\right) + \frac{3}{n\pi}\sin\left(\frac{2n\pi x}{3}\right)\right] \text{ Ans}$$

3. Find fourier series for the function $f(x) = \{$

$$f(x) = \begin{cases} 1 + \frac{2x}{\pi} & -\pi \le x \le 0 \\ 1 - \frac{2x}{\pi} & 0 \le x \le \pi \end{cases}$$

deduce that : $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} - - - = \frac{\pi^2}{8}$

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Sol

$$F(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left[a_n\cos\frac{n\pi x}{L} + b_n\sin\frac{n\pi x}{L}\right]$$

$$L = \frac{\pi+\pi}{2} = \frac{2\pi}{2} = \pi$$

$$F(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left[a_n\cos nx + b_n\sin nx\right]$$

$$a_0 = \frac{1}{L}\int_a^b f(x)\,dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{0}\left(1+\frac{2x}{\pi}\right)dx + \frac{1}{\pi}\int_0^{\pi}\left(1-\frac{2x}{\pi}\right)dx$$

$$= \frac{1}{\pi}\left[x + \frac{2x^2}{2\pi}\right]_{-\pi}^{0} + \frac{1}{\pi}\left[x - \frac{2x^2}{2\pi}\right]_0^{\pi}$$

$$= \frac{1}{\pi}\left(-\pi + \frac{\pi^2}{\pi}\right) + \frac{1}{\pi}\left(\pi - \frac{\pi^2}{\pi}\right)$$

$$= 0$$

$$a_n = \frac{1}{L}\int_a^b f(x)\cos nx\,dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{0}\left(1+\frac{2x}{\pi}\right)\cos nx\,dx + \frac{1}{\pi}\int_0^{\pi}\left(1-\frac{2x}{\pi}\right)\cos nx\,dx$$

$$= \frac{1}{\pi}\left[\left(1+\frac{2x}{\pi}\right)\frac{\sin nx}{n} + \frac{2}{\pi}\left(-\frac{\sin nx}{n^2}\right)\right]_{-\pi}^{0} + \frac{1}{\pi}\left[\left(1-\frac{2x}{\pi}\right)\frac{\sin nx}{n} + \frac{2}{\pi}\frac{\sin nx}{n^2}\right]_0^{\pi}$$

$$a_n = \frac{1}{\pi}\left(\frac{2}{\pi n^2} - \frac{2(-1)^n}{\pi n^2}\right) + \frac{1}{\pi}\left(\frac{-2(-1)^n}{\pi n^2} + \frac{2}{\pi n^2}\right)$$

$$a_n = \frac{4}{n^2\pi^2}\left(1-(-1)^n\right)$$

$$b_n = \frac{1}{L}\int_a^b f(x)\sin nx\,dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{0}\left(1+\frac{2x}{\pi}\right)\sin nx\,dx + \frac{1}{\pi}\int_{0}^{\pi}\left(1-\frac{2x}{\pi}\right)\sin nx\,dx$$

$$= \frac{1}{\pi}\left[\left(1+\frac{2x}{\pi}\right)\left(-\frac{\cos nx}{n}\right) - \frac{2}{\pi}\left(-\frac{\sin nx}{n^2}\right)\right]_{-\pi}^{0} + \frac{1}{\pi}\left[\left(1-\frac{2x}{\pi}\right)\left(-\frac{\cos nx}{n}\right) - (-2)\left(-\frac{\sin nx}{n^2}\right)\right]_{0}^{\pi}$$

$$= \frac{1}{\pi}\left(\frac{-1}{n} + \frac{(-1)^n}{n}\right) + \frac{1}{\pi}\left(\frac{(-1)^n}{n} + \frac{1}{n}\right)$$

$$b_n = 0$$

$$a_n = \frac{4}{n^2\pi^2}(1-(-1)^n)$$

For even	For odd
$a_n = \frac{8}{n^2\pi^2}$	$a_n = 0$

Find odd or even function

Put $x = -x$

$$f(-x) = 1 + \frac{2(-x)}{\pi} = 1 - \frac{2x}{\pi} = f(x) \quad (0 \le x \le \pi)$$

$$f(-x) = 1 - \frac{2(-x)}{\pi} = 1 + \frac{2x}{\pi} = f(x) \quad (-\pi \le x \le 0)$$

$f(x)$ is even

$$F(x) = \frac{8}{\pi^2}\sum_{n=1}^{\infty}\frac{\cos nx}{n^2}$$

$n \to$ odd

$n \to$ even $= 0$

Put $x = 0$

$$1 = \frac{8}{\pi^2}\left[\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \cdots\right]$$

$$\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \cdots$$

Q. Find fourier series for the function $f(x) = \begin{cases} -\pi & -\pi < x < 0 \\ x & 0 < x < \pi \end{cases}$

Sol^n

$$F(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left[a_n \cos\frac{n\pi x}{L} + b_n \sin\frac{n\pi x}{L}\right]$$

$$L = \frac{\pi+\pi}{2} = \pi$$

$$F(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left[a_n \cos nx + b_n \sin nx\right]$$

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$$a_0 = \frac{1}{L}\int_a^b f(x)\,dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{0} -\pi\,dx + \frac{1}{\pi}\int_0^{\pi} x\,dx$$

$$= \frac{1}{\pi}\left[-\pi x\right]_{-\pi}^{0} + \frac{1}{\pi}\left[\frac{x^2}{2}\right]_0^{\pi}$$

$$= \frac{1}{\pi}\left(-\pi^2\right) + \frac{1}{\pi}\left(\frac{\pi^2}{2}\right)$$

$$= -\pi + \frac{\pi}{2}$$

$$= -\pi$$

$$a_n = \frac{1}{L}\int_a^b f(x)\cos nx\,dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{0} (-\pi)\cos nx\,dx + \frac{1}{\pi}\int_0^{\pi} x\cos nx\,dx$$

$$= \frac{1}{\pi}\left[-\pi\frac{\sin nx}{n}\right]_{-\pi}^{0} + \frac{1}{\pi}\left[\left(x\frac{\sin nx}{n}\right) + \frac{\cos nx}{n^2}\right]_0^{\pi}$$

$$= \frac{1}{\pi}(0) + \frac{1}{\pi}\left(\frac{(-1)^n}{n^2} - \frac{1}{n^2}\right)$$

$$= \frac{1}{\pi n^2}\left((-1)^n - 1\right)$$

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$$b_n = \frac{1}{L}\int_a^b f(x)\sin nx\,dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{0}(-\pi)\sin nx\,dx + \frac{1}{\pi}\int_0^{\pi} x\sin nx\,dx$$

$$= \frac{1}{\pi}\left[\pi\frac{\cos nx}{n}\right]_{-\pi}^{0} + \frac{1}{\pi}\left[\frac{x(-\cos nx)}{n} + \frac{x\sin nx}{n^2}\right]_0^{\pi}$$

$$= \frac{1}{\pi}\left[\left(\frac{\pi}{n}\right) - \frac{\pi(-1)^n}{n}\right] + \frac{1}{\pi}\left(-\frac{\pi}{n}(-1)^n\right)$$

$$= \frac{1}{n}\left(1-(-1)^n\right) - \frac{1}{\pi}(-1)^n$$

$$= \frac{1}{n}\left(1-2(-1)^n\right)$$

$$F(x) = -\frac{\pi}{2} + \sum_{n=1}^{\infty}\left[\frac{1}{\pi n^2}\left((-1)^n - 1\right)\cos nx + \frac{1}{n}\left(1-2(-1)^n\right)\sin nx\right]$$

5. Express half range sine series for $f(x) = e^x$, $0 < x < 1$

Sol^n

$$F(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left[a_n \cos\frac{n\pi x}{L} + b_n \sin\frac{n\pi x}{L}\right]$$

$$L = \frac{1-0}{2} = \frac{1}{2}$$

$$F(x) = \frac{a_0}{2} \sum_{n=1}^{\infty}\left[a_n \cos(2n\pi x) + b_n \sin(2n\pi x)\right]$$

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$$a_0 = \frac{1}{L}\int_a^b f(x)\,dx$$

$$= 2\int_0^1 e^x\,dx$$

$$= 2\left[e^x\right]_0^1$$

$$= 2(e^1 - e^0)$$

$$= 2(e-1)$$

$$a_n = \frac{1}{L}\int_a^b f(x)\cos n\pi x\,dx$$

$$a_n = 2\int_0^1 e^x \cos(n\pi x)\,dx$$

$$a_n = 2\left[\frac{e^x}{1+n^2\pi^2}\left[\sin n\pi x - n\pi \sin n\pi x\right]\right]_0^1$$

$$a_n = 2\,\frac{e}{1+n^2\pi^2}(-1)^n - \frac{2}{1+n^2\pi^2}$$

$$a_n = \frac{2}{1+n^2\pi^2}\left(e(-1)^n - 1\right)$$

Half Range Sine Series

$$f(x) = \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{L}$$

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$$b_n = \frac{2}{l}\int_0^l f(x)\sin\frac{n\pi x}{l}\,dx \qquad l = 1$$

$$b_n = \frac{2}{1}\int_0^1 e^x \sin n\pi x\,dx$$

$$b_n = 2\left[\frac{e^x}{1+n^2\pi^2}\left[\sin n\pi x - n\pi\cos n\pi x\right]\right]_0^1$$

$$b_n = \frac{2e^1}{1+n^2\pi^2}\left(-n\pi(-1)^n\right) + \frac{2e^0 n\pi}{1+n^2\pi^2}$$

$$b_n = \frac{2n\pi}{1+n^2\pi^2}\left[-e\cos n\pi + 1\right]$$

$$b_n = \frac{2n\pi}{1+n^2\pi^2}\left[1 - e(-1)^n\right]$$

Half Range Sine Series

$$f(x) = \sum_{n=1}^{\infty} \frac{2n\pi}{1+n^2\pi^2}\sin n\pi x$$

6. Express half range cosine series for $f(x) = x,\ 0 < x < 3$

Sol:
$$F(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{L}$$

$$l = \text{last limit} = 3$$

$$F(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{3}$$

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$$a_0 = \frac{2}{L}\int_0^b f(x)\,dx$$

$$= \frac{2}{3}\int_0^3 x\,dx$$

$$= \frac{2}{3}\left[\frac{x^2}{2}\right]_0^3$$

$$= \frac{2}{3}\left[\frac{3^2}{2}\right]$$

$$= 3$$

$$a_n = \frac{2}{L}\int_0^l f(x)\cos\frac{n\pi x}{3}\,dx$$

$$= \frac{2}{3}\int_0^3 x\cos\frac{n\pi x}{3}\,dx$$

$$= \frac{2}{3}\left[\frac{x\sin\frac{n\pi x}{3}}{\frac{n\pi}{3}} + \frac{\cos\frac{n\pi x}{3}}{\frac{n^2\pi^2}{9}}\right]_0^3$$

$$= \frac{2}{3}\left(\frac{(-1)^n}{\frac{n^2\pi^2}{9}} - \frac{1}{\frac{n^2\pi^2}{9}}\right)$$

$$= \frac{2}{3}\,\frac{9}{n^2\pi^2}\left((-1)^n - 1\right)$$

$$a_n = \frac{6}{n^2\pi^2}\left((-1)^n - 1\right)$$

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Half Range Cosine Series

$$F(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n\cos\frac{n\pi x}{L}$$

$$x = \frac{3}{2} + \sum_{n=1}^{\infty}\frac{6}{n^2\pi^2}\left((-1)^n - 1\right)$$

$$x = \frac{3}{2} + \frac{6}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\left((-1)^n - 1\right) \quad \text{Ans}$$

7. Find the Fourier Series of $f(x) = 1 - x^2$ in the interval $(-1,1)$

Sol^n

$$F(x) = \frac{a_0}{2} + \sum_{n}\left(a_n\cos\frac{n\pi x}{L} + b_n\sin\frac{n\pi x}{L}\right)$$

$$L = \frac{1+1}{2} = 1$$

$$a_0 = \frac{1}{L}\int_a^b f(x)\,dx$$

$$= \int_{-1}^{1}(1 - x^2)\,dx$$

$$= \left[x - \frac{x^3}{3}\right]_{-1}^{1}$$

$$= \left(1 - \frac{1}{3}\right) - \left(-1 + \frac{1}{3}\right)$$

$$= 1 - \frac{1}{3} + 1 - \frac{1}{3}$$

$$= \frac{4}{3}$$

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$$a_n = \frac{1}{L}\int_a^b f(x)\cos n\pi x$$

$$= \int_{-1}^{1}(1-x^2)\cos n\pi x\,dx$$

$$= \left[(1-x^2)\frac{\sin n\pi x}{n\pi} - 2x\frac{\cos n\pi x}{n^2\pi^2} + 2\frac{\sin n\pi x}{n^3\pi^3}\right]_{-1}^{1}$$

$$= \frac{-2(-1)^n}{n^2\pi^2} - \frac{2(-1)^n}{n^2\pi^2}$$

$$a_n = \frac{-4(-1)^n}{n^2\pi^2}$$

$$b_n = \frac{1}{L}\int_a^b f(x)\sin n\pi x\,dx$$

$$= \int_{-1}^{1}(1-x^2)\sin n\pi x\,dx$$

$$= \left[(1-x^2)\left(-\frac{\cos n\pi x}{n\pi}\right) - 2x\frac{\sin n\pi x}{n^2\pi^2} - 2\frac{\cos n\pi x}{n^3\pi^3}\right]_{-1}^{1}$$

$$= \left[\frac{-2(-1)^n}{n^3\pi^3} + \frac{2(-1)^n}{n^3\pi^3}\right]$$

$$b_n = 0$$

$$F(x) = \frac{4}{3} + \sum_{n=1}^{\infty}\left(\frac{-4(-1)^n}{n^2\pi^2}\right)\cos n\pi x$$

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8. Express $f(x) = \sin ax$ in the interval $(-\pi, \pi)$

Sol^n

$$F(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left[a_n\cos\frac{n\pi x}{L} + b_n\sin\frac{n\pi x}{L}\right]$$

$$L = \frac{\pi+\pi}{2} = \pi$$

$$F(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left[a_n\cos nx + b_n\sin nx\right]$$

$$a_0 = \frac{1}{L}\int_a^b f(x)\,dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{\pi}\sin ax\,dx$$

$$= \frac{1}{\pi}\left[\frac{-\cos ax}{a}\right]_{-\pi}^{\pi}$$

$$= \frac{1}{\pi}\left(-\frac{\cos a\pi}{a} + \frac{\cos a\pi}{a}\right)$$

$$a_0 = 0$$

$$a_n = \frac{1}{L}\int_a^b f(x)\cos nx\,dx$$

$$a_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} 2\sin ax\cos nx\,dx$$

$$a_n = \frac{1}{2\pi}\int_{-\pi}^{\pi}\left[\sin(a+n)x + \sin(a-n)x\right]dx$$

$$a_n = \frac{1}{2\pi}\left[-\frac{\cos(a+n)x}{a+n} - \frac{\cos(a-n)x}{a-n}\right]_{-\pi}^{\pi}$$

$a_n = 0$

$b_n = \frac{1}{l}\int_0^b f(x)\sin nx\,dx$

$= \frac{1}{\pi}\int_{-\pi}^{\pi} \sin ax \sin nx\,dx$ $\quad f(x)$ is even.

$= \frac{(2)1}{\pi}\int_0^{\pi} \sin ax \sin nx\,dx$

$= \frac{21}{2\pi}\int_0^{\pi} 2\sin ax \sin nx\,dx$

$= \frac{1}{2\pi}\int_0^{\pi}\left[\cos(a-n)x - \cos(a+n)x\right]dx$

$= \frac{1}{\pi}\left[\frac{\sin(a-n)x}{a-n} - \frac{\sin(a+n)x}{a+n}\right]_0^{\pi}$

$= \frac{1}{\pi}\left[\frac{\sin(a-n)\pi}{a-n} - \frac{\sin(a+n)\pi}{a+n}\right]$

$= \frac{1}{\pi}\left[\frac{\sin a\pi\cos n\pi}{a-n} - \frac{\sin a\pi \cos n\pi}{a+n}\right]$

$= \frac{1}{\pi}\left(\frac{\sin(a\pi)\cos(n\pi)(a+n) - \sin a\pi\cos n\pi(a-n)}{a^2-n^2}\right)$

$= \frac{1}{\pi}\sin a\pi\cos n\pi\left(\frac{a+n-a+n}{a^2-n^2}\right)$

$= \frac{2n\sin a\pi\cos n\pi}{\pi(a^2-n^2)} = \frac{(2\sin a\pi)(n\cos n\pi)}{\pi(a^2-n^2)}$