Enrollment No: 23UG051722

PARUL UNIVERSITY
FACULTY OF ENGINEERING & TECHNOLOGY
B.Tech. Mid Semester Exam

Semester: 1
Subject Code: 303191101
Subject Name: Mathematics-1

Date: 25/11/2023
Time: 10:30 a.m. to 12:00 p.m.
Total Marks: 40

Sr. No.		Marks
Q.1	**(A) Answer the following questions:**	05
	(1) Check whether the given matrix $A=\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}$ is orthogonal or not?	
	(2) Find the eigen values of the matrix $A=\begin{bmatrix}-1 & 2 & 6\\ 0 & 3 & 4\\ 0 & 0 & 7\end{bmatrix}$.	
	(3) Find the limit of the function $\lim_{(x,y)\to\left(\frac{\pi}{2},0\right)} \dfrac{\cos(y)+1}{y-\sin x}$.	
	(4) Find $\frac{dy}{dx}$ if $x^3+y^3=6xy$.	
	(5) Express the given quadratic form in matrix notation: $2x^2+3y^2-5z^2-2xy+6xz-10yz$	
	(B) Fill in the blanks for the following:	05
	(1) The eigen values of matrix $A=\begin{bmatrix}9 & -2\\ -2 & 6\end{bmatrix}$ are 5,10. Then the eigen values of $5A^2+3A^{-1}$ are ____ and ____.	
	(2) If $z(x,y)=x^3y^2+y^3\sin x+y^2$ then, $\frac{\partial z}{\partial x}=$____ and $\frac{\partial z}{\partial y}=$____.	
	(3) The degree of the homogeneous function $u=\left(\dfrac{x^{\frac{1}{4}}+y^{\frac{1}{4}}}{x^{\frac{1}{5}}+y^{\frac{1}{5}}}\right)$ is —	
	(4) The matrix $A=\begin{bmatrix}1 & 0 & 0 & 3\\ 0 & 0 & 1 & 6\\ 0 & 1 & 0 & 4\end{bmatrix}$ is in row echelon form____. [State True or False].	
	(5) The characteristic equation of the matrix $A=\begin{bmatrix}5 & -2\\ -2 & 8\end{bmatrix}$.	
Q.2	**Attempt ANY FOUR of the following:**	12
	(1) Verify Caley-Hamilton theorem for the matrix $A=\begin{bmatrix}1 & 4\\ 2 & 3\end{bmatrix}$	

(2) Check whether the function

$$f(x,y)=\begin{cases}\dfrac{2x^2-y^2}{x^2+2y^2} & ,\text{when}(x,y)\neq(0,0)\\ 0 & ,\text{when}(x,y)=(0,0)\end{cases}$$ is continuous at origin or not?

(3) Prove that the function $f(x,y)=\log\left(\sqrt{x^2+y^2}\right)$ satisfies the equation

$$\frac{\partial^2 f}{\partial x^2}+\frac{\partial^2 f}{\partial y^2}=0.$$

(4) If $u=f\left(x^2+2yz, y^2+2zx\right)$, prove that:

$$\left(y^2-zx\right)\frac{\partial u}{\partial x}+\left(x^2-yz\right)\frac{\partial u}{\partial y}+\left(z^2-xy\right)\frac{\partial u}{\partial z}=0.$$

(5) Find the rank of the matrix $A=\begin{bmatrix}1 & 2 & -1 & -4\\ 2 & 4 & 3 & 5\\ -1 & -2 & 6 & -7\end{bmatrix}$ by row-echelon form.

Q.3 Attempt ANY TWO of the following: 08

(1) Investigate for what values of λ and μ, the system of simultaneous equations $x+y+z=6;\ x+2y+3z=10;\ x+2y+\lambda z=\mu$ have (i) no solution, (ii) a unique solution, and (iii) infinite number of solutions.

(2) For $u=\log\left(x^3-x^2y+xy^2-y^3\right)$. Prove the following:

a. $x\dfrac{\partial u}{\partial x}+y\dfrac{\partial u}{\partial y}=3$ b. $x^2\dfrac{\partial^2 u}{\partial x^2}+2xy\dfrac{\partial^2 u}{\partial x\partial y}+y^2\dfrac{\partial^2 u}{\partial y^2}=-3$

(3) Determine the nature of the quadratic form:
$-3x^2-3y^2-3z^2-2xy-2xz+2yz$

Q.4 Answer the following questions:

(A) Find a matrix P that diagonalizes $A=\begin{bmatrix}1 & 0\\ -1 & 2\end{bmatrix}$. Hence find A^5. 05

(B) (i) Find the equation of tangent plane and normal line at a point $(3,4,5)$ to the surface $x^2+y^2-4z=5$. 02

(ii) If $u=\dfrac{y^2}{x}$, $v=\dfrac{x^2}{y}$ Find the Jacobian $J=\dfrac{\partial(x,y)}{\partial(u,v)}$. 03

OR

(B) Find the eigen values and eigen vectors for the matrix – 05

$$A=\begin{bmatrix}4 & 6 & 6\\ 1 & 3 & 2\\ -1 & -4 & -3\end{bmatrix}.$$