Tutorial-6

1 Check whether the following sequences whose n^{th} terms are given below, converges or not.

a. $a_n = \dfrac{(-1)^{n+1} n}{n+\sqrt{n}}$ b. $a_n = \left(\dfrac{n+1}{n-2}\right)^n$

2 Show that the sequence $a_n = \frac{n}{n^2+1}$ is a decreasing sequence.

3 Determine whether the following series converges or diverges. Find the sum of the series if it converges: $\sum_{n=1}^{\infty} \left[\tan^{-1}\left(\frac{1}{n^2+n+1}\right)\right]$

4 For which values of p, does the series $\sum_{n=1}^{\infty} \frac{n+1}{n^p}$ is convergent?

5 Test the converges and diverges of following series

a. $\sum \sqrt{\mathrm{n}} - 1/(\mathrm{n}^2+1)$ b. $\sum_{n=1}^{\infty}\left(1+\frac{1}{n}\right)^{n^2}$ c. $\sum_{\mathrm{n}=1}^{\infty} \frac{1}{\mathrm{n}} \sin\left(\frac{1}{n}\right)$ d. $\sum_{n=0}^{\infty} n!x^n$

e. $\sqrt{\frac{1}{2^3}} + \sqrt{\frac{2}{3^3}} + \sqrt{\frac{3}{4^3}} + \cdots$ f. $\sum \frac{1}{n!}$ g, $\sum_{n=1}^{\infty} \frac{n^3+2}{2^n+2}$ h. $\frac{n2^n(n+1)!}{3^n.n!}$

i. $\frac{1.2}{3^2.4^2} + \frac{3.4}{5^2.6^2} + \frac{5.6}{7^2.8^2} + \cdots$.... j. $\frac{3}{1^2-3} + \frac{3}{2^2-3} + \frac{3}{3^2-3} + \frac{3}{4^2-3} + \cdots$

k. $\sum_{n=1}^{\infty} \frac{4^n+5^n}{6^n}$ l. $\left(\frac{1}{2}\right)^{\frac{1}{2}} x + \left(\frac{2}{5}\right)^{\frac{1}{2}} x^2 + \left(\frac{3}{10}\right)^{\frac{1}{2}} x^3 + \cdots \infty, x > 0.$

m. $\sum_{n=1}^{\infty} \frac{n2^n(n+1)!}{3^n n!}$ n. $\sum_{n=1}^{\infty} \frac{n!}{n^n}$ o. $\sum_{n=1}^{\infty} \frac{(n!)^2}{2^{n^2}}$ p. $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{\sqrt{n^2+1}} x^n$

q $1 - \frac{1}{2\sqrt{2}} + \frac{1}{3\sqrt{3}} - \frac{1}{4\sqrt{4}} + \cdots$ r. $\sum_{n=1}^{\infty} \frac{n^n x^n}{(n+1)^n}$ s. $\sum_{n=1}^{\infty} \frac{1}{n^2+1} x^n$

6 If $a_n = \begin{cases} \frac{n}{2^n} & \text{when n is odd} \\ \frac{1}{2^n} & \text{when n is even} \end{cases}$, does $\sum_{n=1}^{\infty} a_n$ converge?

7 Find interval of Convergence for which the series $x - \frac{x^2}{2^2} + \frac{x^3}{3^2} - \frac{x^4}{4^2} + \ldots\ldots$ is convergent.

8 For $\sum_{n=0}^{\infty} \frac{(2x+3)^{2n+1}}{n!}$ Find the radius of convergence. For what values of x does the series converge a. absolutely b. conditionally?

9 Determine absolute or conditional convergence of the series

a $1-\frac{2}{3}+\frac{3}{3^2}-\frac{4}{3^3}+\cdots \ldots$ b $\sum_{n=1}^{\infty}(-1)^n.\frac{2^{3n}}{3^{2n}}$ c $\sum_{n=1}^{\infty}(-1)^n.\frac{5^n}{n!}$

10. Expand log (cos $\left(x+\frac{\pi}{4}\right)$) in powers of x . Hence find the value of log ($cos 48°$).

11. State Taylor's Series for one variable and hence find $\sqrt{25.15}$.

12. Express $2x^3+7x^2+x-6$ in ascending powers of (x-2).

13. Expand $log x$ in powers of $(x-1)$.

14. Obtain $\tan^{-1} x$ *in power of* $(x-1)$.

15. Find Maclaurin's series of a. e^{-x} b. $y=\tan x$.

16. Find the series expansion of $f(x)=e^{e^x}$.