Assignment - 3

1) A class consists of 6 girls and 10 boys. If a committee of three is chosen at random from the class, find the probability that (i) three boys are selected and (ii) exactly two girls are selected.

Sol) 6 girls + 10 boys = 16 students

Total ways to choose 3 students:

$^{16}C_3 = 560$.

(i) Three boys are selected:

$P(3 \text{ boys}) = \frac{^{10}C_3}{^{16}C_3} = \frac{120}{560} = \frac{3}{14}$

(ii) exactly two girls are selected:

$P(2 \text{ girls}, 1 \text{ boy}) = \frac{^{6}C_2 \cdot {}^{10}C_1}{^{16}C_3} = \frac{150}{560} = \frac{15}{56}$

2) A person applies for a job in two firms A and B, the probability of his being selected in the firm A is 0.7 and being rejected in the firm B is 0.5. The probability of at least one of the applications being rejected is 0.6. What is the probability that he will be selected in one of the two firms.

Sol) Given

$P(A) = 0.7$

$P(B') = 0.5$ $\quad P(B) = 1 - P(B') = 1 - 0.5 = 0.5$

$P(A' \cup B') = 0.6$

$\therefore P(A' \cup B') = 1 - P(A \cap B)$

$P(A \cap B) = 1 - 0.6 = 0.4$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.7 + 0.5 - 0.4$

$= 0.8$

3) A bag contain 3 red and 4 white balls. Two draws are made without replacement. What is the probability that both the balls are red?

Sol, Total balls 3 red + 4 white = 7 balls

Probability of 1st ball red = $3/7$

Probability of 2nd ball red = $2/6$

combined probability = $\frac{3}{7} \times \frac{2}{6}$

$= \frac{1}{7}$ //

4) Data on readership of a certain magazine show that the probability of male reader under 35 is 0.40 and the over 35 is 0.20. if the probability of readers under 35 is 0.70. find the probability of subscribers that are females over 30 years. Also calculate the probability that a randomly selected male subcriber is under 35 years of age

Sol, $P(M \cap U) = 0.40$

$P(M \cap O) = 0.20$

$P(U) = 0.70$

$\therefore P(O) = 1 - P(U) = 1 - 0.7 = 0.30$

$P(F \cap O) = P(O) - P(M \cap O) = 0.30 - 0.20 = 0.10$

The randomly selected male is under 35 years of age $P(U/M) = \frac{P(M \cap U)}{P(M)}$

$P(M) = P(M \cap U) + P(M \cap O) = 0.60$

$P(U/M) = \frac{0.40}{0.60} = \frac{2}{3}$ //

5) A business man goes to hotels X, Y, Z for 20%, 50%, 30% of the time respectively. It is known that 5%, 4%, 8% of the rooms in X, Y, Z hotels X, Y, Z hotels have faulty plumbing. What is the probability that the businessman's room having faulty plumbing is assigned to hotel Z?

Sol,

$P(X) = 0.2$

$P(Y) = 0.5$

$P(Z) = 0.3$

$P(F/X) = 0.05$

$P(F/Y) = 0.04$

$P(F/Z) = 0.08$

$P(F) = P(F/X) \cdot P(X) + P(F/Y) \cdot P(Y) + P(F/Z) \cdot P(Z)$

$= 0.054$

$P(F/Z) = \dfrac{P(F/Z) \cdot P(F)}{P(F)}$

$= \dfrac{0.08 \times 0.3}{0.054}$

$= 0.444$

6) A factory has two machines, A and B. Past records show that the Machine A produces 30% of the total output and the machine b, the remaining 70%. machine A produces 5% defective articles and Machine B produces 1% defective items. An item is drawn at random and found to be defective. what is the probability that it was produced (i) by the Machine A, and (ii) by the Machine B?

Sol, $P(A) = 0.3$

$P(B) = 0.7$

$P(D/A) = 0.05$

$P(D/B) = 0.01$

$P(D) = P(D/A) \cdot P(A) + P(D/B) \cdot P(B)$

$= 0.022$

(i) $P(A/D) = \frac{P(D/A) \cdot P(A)}{P(D)}$

$= \frac{0.05 \times 0.3}{0.022} = 0.682$

(ii) $P(B/D) = \frac{P(D/B) \cdot P(B)}{P(D)}$

$= \frac{0.01 \times 0.7}{0.022} = 0.318$

7) The probability distribution of a random variable X is given as follows.

X	-2	-1	0	1	2
$P(X=x)$	0.2	0.1	0.3	0.3	0.1

find (i) $E(X)$ (ii) $V(X)$ (iii) $E(2X-3)$ iv) $(2X-3)$

Sol, (i) $E(X) = \mu = \Sigma X P(X)$

$= -0.4 + (-0.1) + (0.3) + (0.2)$

$= 0$

(ii) $V(X) = \sigma^2 = \Sigma X^2 P(X) - [\Sigma X \cdot P(X)]^2$

$= 0.8 + 0.1 + 0.3 + 0.4$

$= 1.6$

(iii) $E(2X-3) = 2E(X) - 3$

$= 2 \times 0 - 3$

$= -3$

(iv) $V(ax+b) = a^2 V(x)$

$= 4(1.6)$

$= 6.4$ //

8) There are two defective pencils in a pack of dozen pencils. If three pencils are taken at random. find the probabilities that (i) at the most one pencil is defective (ii) two pencils are defective.

Sol// Total pencils = 12

Defective pencils = 2

The total ways to choose 3 pencils

$12C_3 = 220$.

(i) At most one pencil is defective.

$2C_0 \times 10C_3 = 1 \times 120 = 120$

$2C_1 \times 10C_2 = 2 \times 45 = 90$

Total ways = $120 + 90 = 210$

P(C one defective) = $\frac{210}{220} = \frac{21}{22}$

(ii) Two pencils are defective

$2C_2 \times 10C_1 = 1 \times 10 = 10$

P(C 2 defective) = $\frac{10}{220} = \frac{1}{22}$ //

9) A sample of 3 items is selected at random from a box containing 10 items of which 4 are defective. find the expected no. of defective item.

Sol, P(C 0 defective) = $\frac{6C_3}{10C_3} = 0.167$

$P(X=1) = \frac{{}^4C_1 \cdot {}^6C_2}{{}^{10}C_3} = 0.500$

$P(X=2) = \frac{{}^4C_2 \cdot {}^6C_1}{{}^{10}C_3} = 0.300$

$P(X=3) = \frac{{}^4C_3}{{}^{10}C_3} = 0.033$

$E(X) = 0 \cdot P(X=0) + 1 \cdot P(X=1) + 2 \cdot P(X=2) + 3 \cdot P(X=3)$

$= 0.167(0) + 0.500(1) + 0.300(2) + 0.033(3)$

$= 1.199$ //

10) Let X be a continuous random variable with pdf $f(x) = Kx(x-1)$, $0 \le x \le 1$ then find then find the value of K.

Sol // Given $f(x) = Kx(x-1)$, $0 \le x \le 1$

$$\int_0^1 Kx(x-1)\,dx = 1$$

$$K\int_0^1 x(x-1)\,dx = 1$$

$$K\int_0^1 (x^2 - x)\,dx = 1$$

$$K\left[\frac{x^3}{3} - \frac{x^2}{2}\right]_0^1 = 1$$

$$K\left[\frac{1}{3} - \frac{1}{2}\right] = 1$$

$$K\left(-\frac{1}{6}\right) = 1$$

$K = -6$ //

11) let the continuous random variable X have the probability density function.

$$f(x) = \begin{cases} 2/x^3 & 1 < x < \infty \\ 0 & \text{otherwise} \end{cases}$$

find $f(x)$.

Sol, $f(x) = P(X \le x) = \int_{-\infty}^{x} f(t)\,dt$.

Since $f(x) = 0$ for $x \le 1$, we focus on $x > 1$

$f(x) = \int_{1}^{x} 2/t^3\,dt$.

$\int 2/t^3\,dt = 2\int t^{-3}\,dt = 2(t^{-2}/-2) = -1/t^2$

Apply the limits from 1 to x

$F(x) = [-1/t^2]_1^x = -1/x^2 - (-1/1^2) = 1 - 1/x^2$

$\therefore F(x) = \begin{cases} 0, & \text{if } x \le 1, \\ 1 - 1/x^2 & \text{if } x > 1. \end{cases}$

12) The Incidence of an occupational disease in an industry is such that the workers have a 20% chance of suffering from it. what is the probability that out of 6 workers chosen at random four or more will suffer from the disease?

Sol, $n = 6, \quad P = 0.2 \quad q = 0.8$

$P(X \ge 4) = P(X=4) + P(X=5) + P(X=6)$

$P(X=4) = {}^6C_4 \cdot (0.2)^4 \cdot (0.8)^2 = 0.015$

$P(X=5) = {}^6C_5 \cdot (0.2)^5 \cdot (0.8)^1 = 0.002$

$P(X=6) = {}^6C_6 \cdot (0.2)^6 \cdot (0.8)^0 = 0.000$

$P(X \ge 4) = 0.015 + 0.002 + 0.000$

$= 0.017$,

13) The probability that a patient will get reaction of a particular injection is 0.001. If 2000 patients are given that injection. find the probabilities that (i) 3 patient will get reaction (ii) at least 2 reaction (iii) at most 3 accidents.

Sol, Probability of reaction (p) = 0.001

No. of Patients (n) = 2000

Possion distribution $\lambda = n \cdot p$

$\lambda = 2000 \times 0.001 = 2$

(i) Probability of 3 patients will get a reaction.

$P(X = K) = \frac{x^K e^{-\lambda}}{K!}$

$P(X = 3) = \frac{2^3 e^{-2}}{3!} = 0.180$ or 18%.

(ii) Probability that more than 2 Patients will get a reaction.

$P(X > 2) = 1 - P(X \leq 2)$

$P(X \leq 2) = P(X = 0) + P(X = 1) + P(X = 2)$

for $K = 0$

$P(X = 0) = \frac{2^0 e^{-2}}{0!} = 0.135$

$P(X = 1) = \frac{2^1 e^{-2}}{1!} = 0.271$

$P(X = 2) = \frac{2^2 e^{-2}}{2!} = 0.271$

$P(X > 2) = 1 - 0.135 + 0.271 + 0.271 = 0.323$,

The probability that more than 2 patients 32.3%.

14) The distribution of the number of road accidents per day in a city is possion with a mean of 4. find the no. of days of 100 days when there will be (i) no accident (ii) at least 2 accidents, (iii) at most 3 accidents

Sol)

$\lambda = 4$

(i) probability of no accident

$$P(X=0) = \frac{4^0 \cdot e^{-4}}{0!} = 0.018 = 1.8$$

(ii) probability of at least 2 accidents

$$P(X \geq 2) = 1 - P(X < 2)$$

$$= 1 - P(X=0) + P(X=1)$$

$$P(X=1) = \frac{4^1 \cdot e^{-4}}{1!} = 0.073$$

$$P(X \geq 2) = 1 - 0.018 + 0.073$$

$$= 1 - 0.091$$

$$= 0.909$$

$$= 90.9\%$$

(iii) probability of at most 3 accidents.

$$P(X \leq 3) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$$

$$P(X=3) = \frac{4^3 \cdot e^{-4}}{3!} = 0.195$$

$$P(X \leq 3) = 0.018 + 0.073 + 0.147 + 0.195$$

$$= 0.433$$

$$= 43.3$$

15) The expenditures on break fast of customers of a restaurant follow normal distribution with mean Rs. 200 and standard deviation Rs. 50. on a particular day 40 customers spent more than Rs. 275. find the expected number of customers visited the restaurant today.

Sol, $\mu = 200$

$\sigma = 50$

No. of customers who spent more than RS. 275 = 40

$$z = \frac{x - \mu}{\sigma}$$

$$z = \frac{275 - 200}{50} = 1.5$$

$P(X > 275) = 1 - P(z \leq 1.5)$

$= 1 - 0.9332$

$= 0.0668$,,

$\Rightarrow 0.0668 \times N = 40$

$N = 40/0.0668 = 598.802$,,

16) A coin is tossed 900 times. find the probability that the No. of head is b/w 435 & 465.

Sol,, $n = 900$

probability of success (head) $= p = 0.5$

$\mu = n \cdot p = 900 \times 0.5 = 450$

$\sigma = \sqrt{n \cdot p \cdot q} = \sqrt{900 \times 0.5 \times 0.5} = \sqrt{225} = 15$

$z_1 = \frac{x - \mu}{\sigma} = \frac{435 - 450}{15} = -1$

$z_2 = \frac{x - \mu}{\sigma} = \frac{465 - 450}{15} = 1$

$P(435 \leq x \leq 71) = P(-1 \leq z \leq 1)$

$= 2P(0 < z < 1)$

$= 2 \times 0.3413$

$= 0.6826$,,

17) (A coin is) daily profit of a businessmen is RS. 120 and the S.d of the profit is RS. 15. find the no. of days out of 365 days on

which his profit will be less than RS. 100.

Sol,

$\mu = 120$

$\sigma = 15$

$z = \frac{x - \mu}{\sigma} = \frac{100 - 120}{15} = \frac{-20}{15} = -1.33$

$P(z \leq -1.33) = 0.5 - P(-1.33 < z < 0)$

$= 0.5 - P(0 < z < 1.33)$

$= 0.5 - 0.4082$

$= 0.0918$

The no. of Days out of 365 days on which his profit will be less than RS 100 is $= 365 \times 0.0918$

$= 33.580$

$= 34$ days.