# Assignment

① Find Eigen Value and Eigen Vector

(i) $\begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -4 & -3 \end{bmatrix}$ (ii) $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$

② Find A.M & G.M

(i) $\begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$

③ Solve

(i) $x_1 + x_2 + 2x_3 = 8$
$-x_1 - 2x_2 + 3x_3 = 1$
$3x_1 - 7x_2 + 4x_3 = 10$

(ii) $2x_1 + x_2 + 5x_3 = 4$
$3x_1 - 2x_2 + 2x_3 = 2$
$5x_1 - 8x_2 - 4x_3 = 1$

④ verify Cayley Hamilton thm. and find $A^{-1}$ & $A^4$

$A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$

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4+4+4+4+2
=18

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MATH-1 ASSIGNMENT

(1) Find Eigen Value and Eigen vector

(i) $\begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -4 & -3 \end{bmatrix}$

$S_1 = 4+3-3 = 4$

$S_2 = \begin{vmatrix} 3 & 2 \\ -4 & -3 \end{vmatrix} + \begin{vmatrix} 4 & 6 \\ -1 & -3 \end{vmatrix} + \begin{vmatrix} 4 & 6 \\ 1 & 3 \end{vmatrix}$

$= (-9+8) + (-12+6) + (12-6)$

$= -1 - 6 + 6$

$= -1$

$S_3 = 4\begin{vmatrix} 3 & 2 \\ -4 & -3 \end{vmatrix} - 6\begin{vmatrix} 1 & 2 \\ -1 & -3 \end{vmatrix} + 6\begin{vmatrix} 1 & 3 \\ -1 & -4 \end{vmatrix}$

$= 4(-9+8) - 6(-3+2) + 6(-4+3)$

$= 4(-1) - 6(-1) + 6(-1)$

$= -4 + 6 - 6$

$= -4$

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$\Rightarrow \quad \lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0$

$\lambda^3 - 4\lambda^2 - 1\lambda + 4 = 0$

$\lambda = 1 \qquad (\lambda^2 - 3\lambda - 4)$

$\cancel{(\lambda+4)(\lambda-1)}$

$(\lambda-4)(\lambda+1)$

| | | | | |
|---|---|---|---|---|
| 1 | 1 | -4 | -1 | 4 |
| | 0 | 1 | -3 | -4 |
| | 1 | -3 | -4 | 0 |

Eigen values $\left\{ \lambda = 1, 4, -1 \right.$

$\Rightarrow \quad |A - \lambda I| = 0$

$$\begin{bmatrix} 4-\lambda & 6 & 6 \\ 1 & 3-\lambda & 2 \\ -1 & -4 & -3-\lambda \end{bmatrix}$$

$\Rightarrow \quad \lambda = 1$

$$\Rightarrow \left[\begin{array}{ccc|c} 3 & 6 & 6 & 0 \\ 1 & 2 & 2 & 0 \\ -1 & -4 & -4 & 0 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 2 & 0 \\ 3 & 6 & 6 & 0 \\ -1 & -4 & -4 & 0 \end{array}\right] \quad R_1 \leftrightarrow R_2$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 2 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -2 & -2 & 0 \end{array}\right] \quad \begin{array}{l} R_2 \rightarrow R_2 - 3R_1 \\ R_3 \rightarrow R_3 + R_1 \end{array}$$

$y = t$

$-2y - 2z = 0$

$-2z = 2t$

$z = -t$

$x + 2y + 2z = 0$

$x = -2y - 2z$

$= -2t + 2t$

$= 0$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = t \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$$

for, $\lambda = 1$

Eigen vector $= \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$

$\Rightarrow$ for $\lambda = 4$

$$\Rightarrow \left[\begin{array}{ccc|c} 4-\lambda & 6 & 6 & 0 \\ 1 & 3-\lambda & 2 & 0 \\ -1 & -4 & -3-\lambda & 0 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{ccc|c} 0 & 6 & 6 & 0 \\ 1 & -1 & 2 & 0 \\ -1 & -4 & -7 & 0 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & -1 & 2 & 0 \\ 0 & 6 & 6 & 0 \\ -1 & -4 & -7 & 0 \end{array}\right] \quad R_1 \leftrightarrow R_2$$

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$$\sim \left[\begin{array}{ccc|c} 1 & -1 & 2 & 0 \\ 0 & 6 & 6 & 0 \\ 0 & -5 & -5 & 0 \end{array}\right] \quad R_3 \to R_3 + R_1$$

$$\sim \left[\begin{array}{ccc|c} 1 & -1 & 2 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & -5 & -5 & 0 \end{array}\right] \quad R_2 \to \frac{1}{6} R_2$$

$$\sim \left[\begin{array}{ccc|c} 1 & -1 & 2 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \quad R_3 \to R_3 + 5R_2$$

$z = t$

$y + z = 0$

$y = -z$

$y = -t$

$x - y + 2z = 0$

$x = y - 2z$

$= -t - 2t$

$= -3t$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = t \begin{bmatrix} -3 \\ -1 \\ 1 \end{bmatrix}$$

for $\lambda = 4$

Eiger vector is $\begin{bmatrix} -3 \\ -1 \\ 1 \end{bmatrix}$

$\Rightarrow$ for $\lambda = -1$

$$\sim \left[\begin{array}{ccc|c} 4-\lambda & 6 & 6 & 0 \\ 1 & 3-\lambda & 2 & 0 \\ -1 & -4 & -3-\lambda & 0 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 5 & 6 & 6 & 0 \\ 1 & 4 & 2 & 0 \\ -1 & -4 & -2 & 0 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 4 & 2 & 0 \\ 5 & 6 & 6 & 0 \\ -1 & -4 & -2 & 0 \end{array}\right] \quad R_1 \leftrightarrow R_2$$

$$\sim \left[\begin{array}{ccc|c} 1 & 4 & 2 & 0 \\ 0 & -14 & -4 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \quad \begin{array}{l} R_2 \to R_2 - 5R_1 \\ R_3 \to R_3 + R_1 \end{array}$$

$z = t$

$-14y - 4z = 0$

$-14y = 4z$

$-14y = 4t$

$y = -\frac{4}{14} t = -\frac{2}{7} t$

$x + 4y + 2z = 0$

$x = -4y - 2z$

$= -4\left(\frac{-2}{7}\right)t - 2(t)$

$= \frac{8}{7} t - 2t = t\left(\frac{8}{7} - 2\right) = t\left(\frac{8-14}{7}\right) = \frac{-6}{7} t$

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$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = t \begin{bmatrix} -6/7 \\ -2/7 \\ 1 \end{bmatrix} = t \begin{bmatrix} -6 \\ -2 \\ 7 \end{bmatrix}$$

for $\lambda = -1$

Eigen vector is $\begin{bmatrix} -6 \\ -2 \\ 7 \end{bmatrix}$

$\Rightarrow$ Eigen value $= 1, 4, -1$

Eigen vector $= \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} -3 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} -6 \\ -2 \\ 7 \end{bmatrix}$

(ii) $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$

$S_1 = -2 + 1 = -1$

$$S_2 = \begin{vmatrix} 1 & -6 \\ -2 & 0 \end{vmatrix} + \begin{vmatrix} -2 & -3 \\ -1 & 0 \end{vmatrix} + \begin{vmatrix} -2 & 2 \\ 2 & 1 \end{vmatrix}$$

$= (0 - 12) + (0 - 3) + (-2 - 4)$

$= -12 - 3 - 6$

$= -21$

$$S_3 = -2\begin{vmatrix} 1 & -6 \\ -2 & 0 \end{vmatrix} - 2\begin{vmatrix} 2 & -6 \\ -1 & 0 \end{vmatrix} - 3\begin{vmatrix} 2 & 1 \\ -1 & -2 \end{vmatrix}$$

$= -2(0-12) - 2(0-6) - 3(-4+1)$

$= -2(-12) - 2(-6) - 3(-3)$

$= 24 + 12 + 9$

$= 45$

$\Rightarrow \lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0$

$\lambda^3 + 1\lambda^2 - 21\lambda - 45 = 0$

$$\begin{array}{c|cccc} 5 & 1 & 1 & -21 & -45 \\ & 0 & 5 & 30 & 45 \\ \hline & 1 & 6 & 9 & 0 \end{array}$$

$\lambda = 5 \quad (\lambda^2 + 6\lambda + 9)$

$(\lambda + 3)^2$

$\lambda = 5, -3, -3 \leftarrow$ Eigen value

$\Longrightarrow |A - \lambda I| = 0$

$$\sim \left[\begin{array}{ccc|c} -2-\lambda & 2 & -3 & 0 \\ 2 & 1-\lambda & -6 & 0 \\ -1 & -2 & 0-\lambda & 0 \end{array}\right]$$

$\Rightarrow$ for $\lambda = 5$

$$\sim \left[\begin{array}{ccc|c} -7 & 2 & -3 & 0 \\ 2 & -4 & -6 & 0 \\ -1 & -2 & -5 & 0 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} +1 & +2 & +5 & 0 \\ 2 & -4 & -6 & 0 \\ -7 & 2 & -3 & 0 \end{array}\right] \quad R_1 \longleftrightarrow R_3$$

$$\sim \left[\begin{array}{ccc|c} 1 & 2 & 5 & 0 \\ 0 & -8 & -16 & 0 \\ 0 & 16 & 32 & 0 \end{array}\right] \quad \begin{array}{l} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 + 7R_1 \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 2 & 5 & 0 \\ 0 & +1 & +2 & 0 \\ 0 & 16 & 32 & 0 \end{array}\right] \quad R_2 \to \frac{-1}{8} R_2$$

$$\sim \left[\begin{array}{ccc|c} 1 & 2 & 5 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \quad R_3 \to R_3 - 16R_2$$

$z = t$

$\cancel{x+2}\, y + 2z = 0$

$y = -2z$

$y = -t$

$x + 2y + 5z = 0$

$x = -2y - 5z$

$= -2(-t) - 5(t) = 2t - 5t = -3t$

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$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = t \begin{bmatrix} -3 \\ -1 \\ 1 \end{bmatrix}$$

for $\lambda = 5$

Eigen vector $= \begin{bmatrix} -3 \\ -1 \\ 1 \end{bmatrix}$

$\Rightarrow$ for $\lambda = -3$

$$\sim \left[\begin{array}{ccc|c} -2+3 & 2 & -3 & 0 \\ 2 & 1+3 & -6 & 0 \\ -1 & -2 & 3 & 0 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 2 & -3 & 0 \\ 2 & 4 & -6 & 0 \\ -1 & -2 & 3 & 0 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 2 & -3 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \quad \begin{array}{l} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 + R_1 \end{array}$$

$y = t_1, \ z = t_2$

$x + 2y - 3z = 0$

$x = -2y + 3z$

$= -2t_1 + 3t_2$

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$$\sim \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2t_1 + 3t_2 \\ t_1 \\ t_2 \end{bmatrix}$$

$$\sim \begin{bmatrix} x \\ y \\ z \end{bmatrix} = t_1 \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} + t_2 \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}$$

for $\lambda = -3$

Eigen vector is $= \{ t_1(-2,1,0), t_2(3,0,1) \; t_1, t_2 \in R \}$

$\Rightarrow$ $\boxed{\text{Eigen Values} = 5, -3, -3 \quad \text{Eigen Vectors} = \begin{bmatrix} -3 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}}$

2. find A.M & G.M.

(i) $\begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$

$S_1 = 2+2+2 = 6$

$S_2 = \begin{vmatrix} 2 & 1 \\ 0 & 2 \end{vmatrix} + \begin{vmatrix} 2 & 0 \\ 0 & 2 \end{vmatrix} + \begin{vmatrix} 2 & 1 \\ 0 & 2 \end{vmatrix}$

$= (4-0) + (4-0) + (4-0) = 12$

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$$S_3 = 2\begin{vmatrix} 2 & 1 \\ 0 & 2 \end{vmatrix} - 1\begin{vmatrix} 0 & 1 \\ 0 & 2 \end{vmatrix} + 0$$

$$= 2(4-0) - 1(0-0)$$

$$= 8$$

$$\Rightarrow \lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0$$

$$\lambda^3 - 6\lambda^2 + 12\lambda - 8 = 0$$

$\lambda = 2$ $(\lambda^2 - 4\lambda + 4)$

$(\lambda - 2)^2$

$$\begin{array}{c|cccc} 2 & 1 & -6 & 12 & -8 \\ & 0 & 2 & -8 & 8 \\ \hline & 1 & -4 & 4 & 0 \end{array}$$

$\lambda = 2, 2, 2$

for $\lambda = 2$

$$|A - \lambda I| = 0$$

$$\sim \left[\begin{array}{ccc|c} 2-\lambda & 1 & 0 & 0 \\ 0 & 2-\lambda & 1 & 0 \\ 0 & 0 & 2-\lambda & 0 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Rank = 2

A.M for $\lambda = 2$ is 3

G.M for $\lambda = 2$ is $3 - 2 = \boxed{1}$

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(3) Solve.

(i)
$$x_1 + x_2 + 2x_3 = 8$$
$$-x_1 - 2x_2 + 3x_3 = 1$$
$$3x_1 - 7x_2 + 4x_3 = 10$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 2 & 8 \\ -1 & -2 & 3 & 1 \\ 3 & -7 & 4 & 10 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 2 & 8 \\ 0 & -1 & 5 & 9 \\ 0 & -10 & -2 & -14 \end{array}\right] \quad \begin{array}{l} R_2 \to R_2 + R_1 \\ R_3 \to R_3 - 3R_1 \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 2 & 8 \\ 0 & 1 & -5 & -9 \\ 0 & -10 & -2 & -14 \end{array}\right] \quad R_2 \to -R_2$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 2 & 8 \\ 0 & 1 & -5 & -9 \\ 0 & 0 & -52 & -104 \end{array}\right] \quad R_3 \to R_3 + 10R_2$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 2 & 8 \\ 0 & 1 & -5 & -9 \\ 0 & 0 & 1 & 2 \end{array}\right] \quad R_3 \to \frac{-1}{52} R_3$$

$x_3 = 2$

$x_2 - 5x_3 = -9$

$x_2 = 5x_3 - 9$

$= 5(2) - 9$

$x_2 = 1$

$x_1 + x_2 + 2x_3 = 8$

$x_1 + 1 + 4 = 8$

$x = 8 - 5$

$x_1 = 3$

$$\boxed{(x_1, x_2, x_3) = (3, 1, 2)}$$

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ii) $2x_1$
$3x_1$
$5x_1$

ii) $2x_1 + x_2 + 5x_4 = 4$

$3x_1 - 2x_2 + 2x_3 = 2$

$5x_1 - 8x_2 - 4x_3 = 1$

$$\sim \left[\begin{array}{cccc|c} 2 & 1 & 0 & 5 & 4 \\ 3 & -2 & 2 & 0 & 2 \\ 5 & -8 & -4 & 0 & 1 \end{array}\right]$$

$$\sim \left[\begin{array}{cccc|c} 1 & \frac{1}{2} & 0 & \frac{5}{2} & 2 \\ 3 & -2 & 2 & 0 & 2 \\ 5 & -8 & -4 & 0 & 1 \end{array}\right] \quad R_1 \to \frac{1}{2}R_1$$

$$\sim \left[\begin{array}{cccc|c} 1 & \frac{1}{2} & 0 & \frac{5}{2} & 2 \\ 0 & -\frac{7}{2} & 2 & -\frac{15}{2} & -4 \\ 0 & -\frac{21}{2} & +4 & -\frac{25}{2} & -9 \end{array}\right] \quad \begin{array}{l} R_2 \to R_2 - 3R_1 \\ R_3 \to R_3 - 5R_1 \end{array}$$

$$\sim \left[\begin{array}{cccc|c} 1 & \frac{1}{2} & 0 & \frac{5}{2} & 2 \\ 0 & 1 & -\frac{4}{7} & \frac{15}{7} & \frac{8}{7} \\ 0 & -\frac{21}{2} & -4 & -\frac{25}{2} & -9 \end{array}\right] \quad R_2 \to -\frac{2}{7}R_2$$

$$\sim \left[\begin{array}{cccc|c} 1 & \frac{1}{2} & 0 & \frac{5}{2} & 2 \\ 0 & 1 & -\frac{4}{7} & \frac{15}{7} & \frac{8}{7} \\ 0 & 0 & -10 & 10 & 3 \end{array}\right] \quad R_3 \to R_3 + \frac{21}{2}R_2$$

$$\sim \left[\begin{array}{cccc|c} 1 & \frac{1}{2} & 0 & \frac{5}{2} & 2 \\ 0 & 1 & -\frac{4}{7} & \frac{15}{7} & \frac{8}{7} \\ 0 & 0 & 1 & -1 & -\frac{3}{10} \end{array}\right] \quad R_3 \to -\frac{1}{10}R_3$$

$\Rightarrow x_4 = t$

$x_3 - x_4 = -\frac{3}{10}$

$x_3 = -\frac{3}{10} + t \qquad x_3 = t - \frac{3}{10}$

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$$\Rightarrow x_2 - \frac{4}{7}x_3 + \frac{15}{7}x_4 = \frac{8}{7}$$

$$x_2 = \frac{4}{7}x_3 - \frac{15}{7}x_4 + \frac{8}{7}$$

$$= \frac{4}{7}\left(t - \frac{3}{10}\right) - \frac{15}{7}(t) + \frac{8}{7}$$

$$x_2 = \frac{4}{7}t - \frac{12}{70} - \frac{15}{7}t + \frac{8}{7}$$

$$= \left(\frac{4}{7} - \frac{15}{7}\right)t + \left(\frac{8}{7} - \frac{1.2}{7}\right)$$

$$= \frac{-11}{7}t + \frac{6.8}{7}$$

$$= \frac{1}{7}\left(-11t + 6.8\right)$$

$$\Rightarrow x_1 + \frac{1}{2}x_2 + 0 + \frac{5}{2}x_4 = 2$$

$$x_1 = 2 - \frac{1}{2}x_2 - \frac{5}{2}x_4$$

$$= 2 - \frac{1}{2}\left(\frac{-11}{7}t + \frac{6.8}{7}\right) - \frac{5}{2}(t)$$

$$= 2 + \frac{11}{14}t - \frac{6.8}{14} - \frac{5}{2}t$$

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$= 2 - \frac{6.8}{14} + \frac{11}{14}t - \frac{5}{2}t$

$= \frac{28-6.8}{14} + \frac{11}{14}t - \frac{35}{14}t$

$= \frac{21.2}{14} - \frac{24}{14}t$

$x_1 = \frac{1}{14}\left(-24t + 21.2\right)$

$\boxed{(x_1, x_2, x_3, x_4) = \left\{\frac{21.2}{14} - \frac{24t}{14}, \frac{-11}{7}t + \frac{6.8}{7}, t - \frac{3}{10}, t\right\}}$

(4) Verify cayley Hamilton thm and find $A^{-1}$ & $A^4$.

$$\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$$

$S_1 = 3+1 = 4$

$S_2 = 3-8 = -5$

$\lambda^2 - S_1\lambda + S_2 = 0$

$\Rightarrow \cancel{\lambda^2 - 4\lambda - 5 = 0}$ $\Rightarrow \lambda^2 - 4\lambda - 5 = 0$

$\Rightarrow \cancel{(\lambda-5)(\lambda+1)}$.

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$\Rightarrow A^2 - 4A - 5I = 0$

$A^2 = A \cdot A$

$= \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$

$= \begin{bmatrix} 1+8 & 4+12 \\ 2+6 & 8+9 \end{bmatrix} \Rightarrow \begin{bmatrix} 9 & 16 \\ 8 & 17 \end{bmatrix}$

$\Rightarrow \text{LHS} = A^2 - 4A - 5I$

$= \begin{bmatrix} 9 & 16 \\ 8 & 17 \end{bmatrix} - 4 \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} - 5 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 9-4-5 & 16-16-0 \\ 8-8-0 & 17-12-5 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$\therefore$ Hence Cayley Hamilton thm satisfied.

$\Rightarrow A^2 - 4A - 5I = 0$

$\cancel{A^{-1}(A - 4I - 5A^{-1}) = 0}$

$A^{-1}(A^2 - 4A - 5AI) = 0$

$A - 4I - 5A^{-1} = 0$

$\Rightarrow 5A$

$\Rightarrow A$

$$\Rightarrow 5A^{-1} = A - 4I$$

$$= \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} - 4\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1-4 & 4 \\ 2 & 3-4 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & 4 \\ 2 & -1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{5}\begin{bmatrix} -3 & 4 \\ 2 & -1 \end{bmatrix}$$

$\Rightarrow$ for $A^4$:

$$A^2 - 4A - 5I = 0$$

$$A^2(A^2 - 4A - 5I) = 0$$

$$A^4 - 4A^3 - 5A^2 = 0$$

$$A^4 = 4A^3 + 5A^2$$

$$\cancel{\therefore A^2 = 4A + 5I}$$

$$A^3 = A^2 \cdot A = \begin{bmatrix} 9 & 16 \\ 8 & 17 \end{bmatrix}\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 9+32 & 36+48 \\ 8+34 & 32+51 \end{bmatrix}$$

$$= \begin{bmatrix} 41 & 84 \\ 42 & 83 \end{bmatrix}$$

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$$\Rightarrow A^4 = 4A^3 + 5A^2$$

$$= 4\begin{bmatrix} 41 & 84 \\ 42 & 83 \end{bmatrix} + 5\begin{bmatrix} 9 & 16 \\ 8 & 17 \end{bmatrix}$$

$$= \begin{bmatrix} 164 & 336 \\ 168 & 332 \end{bmatrix} + \begin{bmatrix} 45 & 80 \\ 40 & 85 \end{bmatrix}$$

$$= \begin{bmatrix} 209 & 416 \\ 208 & 417 \end{bmatrix}$$

So, $\boxed{A^{-1} = \frac{1}{5}\begin{bmatrix} -3 & 4 \\ 2 & -1 \end{bmatrix} \qquad A^4 = \begin{bmatrix} 209 & 416 \\ 208 & 417 \end{bmatrix}}$

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