Parul University
Faculty of Engineering & Technology
Department of Applied Sciences and Humanities
1st Year B.Tech Programme (All Branches)
Mathematics – 1 (303191101)
Unit – 6 Multivariable Calculus

Tutorial-2

1.	Evaluate the following limits, if exists: a) $\lim\limits_{(x,y)\to(-1,2)} \frac{x^3+y^3}{x^2+y^2}$ b) $\lim\limits_{(x,y)\to(0,0)} \frac{x^2+y^2+1}{3+x^2+3y^2}$ e) $\lim\limits_{(x,y)\to(0,0)} \frac{x^2y}{y^2+x^4}$ c) $\lim\limits_{(x,y)\to(0,0)} \frac{xy}{x^2+y}$ d) $\lim\limits_{(x,y)\to(0,0)} \frac{2x^2y^2}{x^4+y^2}$
2.	Check whether the given function is continuous or not, if yes then find point of continuity. a) $f(x,y)=\begin{cases} \frac{x^2y^2}{2x^2+y^2} & ,if \ (x,y)\neq(0,0) \\ 1 & ,if \ (x,y)=(0,0) \end{cases}$ b) $f(x,y)=\begin{cases} \frac{xy}{x^2+xy+y^2} & ,if \ (x,y)\neq(0,0) \\ 0 & ,\ if \ (x,y)=(0,0) \end{cases}$ c) $f(x,y)=\begin{cases} \frac{x^2y}{x^2+y^2} & ,if \ (x,y)\neq(0,0) \\ 1 & ,if \ (x,y)=(0,0) \end{cases}$ d) $f(x,y)=\begin{cases} \frac{xy}{\sqrt{x^2+y^2}} & ,if \ (x,y)\neq(0,0) \\ 0 & ,if \ (x,y)=(0,0) \end{cases}$
3.	Find the first order partial derivatives at a given point a) $f(x,y)=y\sin(xy)$ at $\left(0,\frac{\pi}{2}\right)$ b) $f(x,y)=x^2+3xy+y-1$ at $(4,-5)$ c) $f(x,y,z)=\frac{x}{y+z}$ at $(3,2,1)$ d) $f(x,y,z)=e^{(x^2+y^2+1)}$ at $(1,1,1)$
4	Find all second order partial derivatives of the following functions a) x^2ysinx b) x^3ysiny c) x^3+y^3 d) $\sin(xy)$
5.	Find $\frac{\partial^3 u}{\partial x\,\partial y\,\partial z}$ for $u=e^{5xyz}$
6.	If $u(x,y)=\log\left(\sqrt{x^2+y^2}\right)$ then show that: $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$
7.	a) If $u=\frac{e^{(x+y+z)}}{e^x+e^y+e^z}$ then show that: $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=2u.$ b) If $u=\log(x^3+y^3+z^3-3xyz)$, then prove that: $\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z}\right)^2 u=-\frac{9}{(x+y+z)^2}$.

8.	If resistors of R_1, R_2, R_3 ohms are connected in parallel to make an R-ohm resistor, the value of R can be found from the equation $\frac{1}{R}=\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}$. Find the value of $\frac{\partial R}{\partial R_2}$ when R_1 =30, R_2 =45 and R_3 =90 ohms.
9.	If $u(x,y)=\log\left(\frac{x^2+y^2}{x+y}\right)$, then prove that: $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=1$
10.	If $u=x^3y^2 sin^{-1}\left(\frac{y}{x}\right)$, show that: a. $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=5u$; $b.$ $x^2\frac{\partial^2 u}{\partial x^2}2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=20u$
11.	For $u=\tan^{-1}\left(\frac{y}{x}\right)$; show that: $a.$ $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$ $b.$ $\frac{\partial^2 u}{\partial x\partial y}=\frac{\partial^2 u}{\partial y\partial x}$
12.	If $f(x,y,z)=\log(x^2+y^2+z^2)$ then prove that: $xf_{yz}=yf_{zx}=zf_{xy}$