Parul University
Faculty of Engineering & Technology
Department of Applied Sciences and Humanities
1st Semester B. Tech Programme (All Branches)
Mathematics – 1 (303191101)
[Matrices and Multivariable Calculus] 2023-24

<u>Assignment-1</u>

Q.1 Answer the following:

1. Solve the following system of equations by using Gauss elimination method.

a. $2x - 3y - z = 3$, $x + 2y - z = 4$, $5x - 4y - 3z = -2$

b. $x + 2y - z = 1$, $x + y + 2z = 9$, $2x + y - z = 2$

c. $2x - y - z = 2$, $x + 2y + z = 2$, $4x - 7y - 5z = 2$

2. Solve the following system of equations by using Gauss- Jordan method.

a. $-2y + 3z = 1$, $3x + 6y - 3z = -2$, $6x + 6y + 3z = 5$

b. $x + y + 2z = 8$, $-x - 2y + 3z = 1$, $3x - 7y + 4z = 10$

c. $2x + y + 5w = 4$, $3x - 2y + 2z = 2$, $5x - 8y - 4z = 1$

3. Investigate for what values of λ and μ, the system of simultaneous equations

$$\begin{aligned} x + y + z &= 6 \\ x + 2y + 3z &= 10 \\ x + 2y + \lambda z &= \mu \end{aligned}$$

have [i] No solution, [ii] A unique solution, and [iii] infinite number of solutions.

3. Find the rank the following matrices by determinant method: $A = \begin{bmatrix} 1 & 2 & -1 & -4 \\ 2 & 4 & 3 & 5 \\ -1 & -2 & 6 & -7 \end{bmatrix}$

4. Find the rank the following matrices by reducing to row echelon form: $A = \begin{bmatrix} 3 & -2 & 0 & -1 \\ 0 & 2 & 2 & 1 \\ 1 & -2 & -3 & 2 \\ 0 & 1 & 2 & 1 \end{bmatrix}$

5. Find the eigen values, eigen vectors of the matrix A= $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$.

6. Find the value of μ which satisfy the equation $A^{100}x = \mu x$, where A= $\begin{bmatrix} 2 & 1 & -1 \\ 0 & -2 & -2 \\ 1 & 1 & 0 \end{bmatrix}$.

7. Determine the algebraic multiplicity and geometric multiplicity of the matrix A= $\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

8. If A= $\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$. Find eigen values of the following matrices:

[i] $A^3 + I$ [ii] A^{-1} [iii] $A^2 - 2A + I$ [iv] $A^3 - 3A^2 + A$

9. Verify Caley-Hamilton theorem and hence find the inverse of $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ and A^4. Also, express $A^5 - 4A^4 - 7A^3 + 11A^2 - A - 10I$ as a linear polynomial in A.

10. Find a matrix P that diagonalizes matrix A and determine $P^{-1}AP$ where $A = \begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$.

    Hence, find A^{13}.

11. Describe the nature of quadratic forms $Q = -3x_1^2 - 3x_2^2 - 3x_3^2 - 2x_1x_2 - 2x_1x_3 + 2x_2x_3$

12. Discuss the continuity of $f(x,y) = \begin{cases} \frac{x^3-y^3}{x^3+y^3}, & (x,y) \neq (0,0) \\ 15 & (x,y) = (0,0) \end{cases}$ at point (0,0).

13. If $f(x,y,z) = \log(x^2 + y^2 + z^2)$, prove that: $xf_{yz} = yf_{zx} = zf_{xy}$

14. If $f(x,y) = 3x^3 + 2y^3 - 2x^2y^2$. Find all second order and mixed partial derivatives of $f(x,y)$ at $(2,-1)$.

15. If $u = tan^{-1}(x^2 + 2y^2)$, then show that $(i)\ x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = sin2u$

$$(ii) x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = 2sinu\ cos3u$$

16. If $u = sin^{-1}\left(\frac{x^{\frac{1}{4}}+y^{\frac{1}{4}}}{x^{\frac{1}{5}}+y^{\frac{1}{5}}}\right)$, then prove that $(i) x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{20}tanu$

17. If $u = \sin\left(\frac{x}{y}\right)$, $y = t^2, x = e^t$ find $\frac{du}{dt}$

18. If $u = f(x^2 - y^2, y^2 - z^2, z^2 - x^2)$, show that $\frac{1}{x}\frac{\partial u}{\partial x} + \frac{1}{y}\frac{\partial u}{\partial y} + \frac{1}{z}\frac{\partial u}{\partial z} = 0$.

19. If $y^{x^y} = sinx$ then, find $\frac{dy}{dx}$.

20. If $u = xyz$, $v = xy + yz + zx$ and $w = x + y + z$ then, evaluate $\frac{\partial(u,v,w)}{\partial(x,y,z)}$

21. Find the equations of tangent plane and normal line to the surface $xyz = 4\ at\ (1,2,2)$.

22. Show that tangent plane to the surface $x^2 = y(x+z)$ at any point passes through the origin

23. Find the extreme values of the function $f(x,y) = x^2 + 2y^2$ on the circle $x^2 + y^2 = 1$.

24. Find the shortest and longest distance from the point $(1,2,-1)$ to the sphere $x^2 + y^2 + z^2 = 24$.

25. Find maxima and minima of the function $f(x,y) = x^3 + y^2 - 3x - 12y + 20$.

26. Expand $tan^{-1}\frac{y}{x}$ in powers of $(x-1)$ and $(y-1)$ using Taylor's expansion.