Parul University
Faculty of Engineering & Technology
Department of Applied Sciences and Humanities
1st Year B. Tech. Programme (All Branches)
Mathematics – 1 (303191101)
First Order Differential Equations
Tutorial- 5

Q1. Find the order and degree of the following differential equations:

a. $\left(\frac{dy}{dx}\right)^2 + 2y = x$

b. $\left(\frac{d^2y}{dx^2}\right) + \left(\frac{dy}{dx}\right)^2 + y = 0$

c. $\left(\frac{d^2y}{dx^2}\right)^2 + \frac{dy}{dx} + 3y = 0$

d. $\left[1 + \left(\frac{d^2y}{dx^2}\right)^2\right]^{\frac{3}{4}} = \frac{d^2y}{dx^2}$

e. $\left(\frac{d^4y}{dx^4}\right)^2 + \frac{d^3y}{dx^3}\frac{dy}{dx} + x^3\left(\frac{dy}{dx}\right)^4 = 0$

f. $\sqrt{\left(\frac{dy}{dx}\right)^4 + 4} = \left(\frac{d^2y}{dx^2}\right)^6$

Q2. Form the differential equation from the following:

a. $y = Ae^{2x} + Be^{5x}$; where A and B are arbitrary constants.
b. $y = e^x(A\cos x + b\sin x)$; A and B are arbitrary constants.
c. Family of circles of radius r whose centers lies on the x-axis.

Q3. Verify that the given function is a solution of the corresponding given differential equation, where, a, b, c are arbitrary constants.

a. $y' + y = x^2 - 2$, $\quad\quad y = ce^{-x} + x^2 - 2x$
b. $x + yy' = 0$, $\quad\quad\quad x^2 + y^2 = 1$

Q4. Check whether the differential equation $\left(x^2 + y^2 + 3\right)dx - 2xydy = 0$ is exact or not?

Q5. Solve the following Differential equations:

1. $2xy\,dx + x^2dy = 0$	10. $\left(1+y^2\right)dx = \left(\tan^{-1}y - x\right)dy$
2. $xy' + y = 0,\ y(2) = -2$	11. $xy - \frac{dy}{dx} = y^3e^{-x^2}$
3. $\left(x^2 + y^2 + 3\right)dx - 2xydy = 0$	12. $xy\left(1 + xy^2\right)\frac{dy}{dx} = 1$
4. $\left(y^4 + 2y\right)dx + \left(xy^3 + 2y^4 - 4x\right)dy = 0$	13. $\frac{dy}{dx} = e^{x-y}\left(e^x - e^y\right)$
5. $\left(x^2y - 2xy^2\right)dx - \left(x^3 - 3x^2y\right)dy = 0$	14. $2xydx + x^2dy = 0$
6. $\left(xy + 2x^2y^2\right)ydx + \left(xy - x^2y^2\right)xdy = 0$	15. $\frac{dy}{dx} - y = e^{2x}$
7. $x\frac{dy}{dx} + (1+x)y = x^3$	16. $\frac{dy}{dx} + y = -\frac{x}{y}$
8. $x(1-2y)dx - \left(x^2 + 1\right)dy = 0$ with $y(2) = 1$	17. $2xy\frac{dy}{dx} = y^2 - x^2$
9. $(2x - 4y + 5)\frac{dy}{dx} + x - 2y + 3 = 0$	18. $\left(x^3 + 3xy^2\right)dx + \left(3x^2y + y^3\right)dy = 0$

Q6. The tank contains 1000 gal of water in which 200 lb of salt are dissolved. Fifty gallons of brine, each containing $(1 + \cos t)$ lb of dissolved salt, run into the tank per minute. The mixture, kept uniform by stirring, runs out at the same rate. Find the amount of salt $y(t)$ in the tank at any time t.