U-3
Maths



Matrices -

A matrix is a rectangular array (arrangement) of No. or function enclosed in a square bracket.

eg:- $A = \begin{bmatrix} 4 & 0 & 2 \\ 3 & 6 & 1 \end{bmatrix}_{2\times 3}$

$B \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}_{2\times 2}$

Trace of matrix (1 mark).

For a square matrix, the sum of main diagonal elements is known as the trace of ~~elements~~ matrix

eg:-

$A \begin{bmatrix} 4 & 5 & 2 \\ -1 & 3 & 6 \\ 2 & 1 & -3 \end{bmatrix}$ For calculating trace we have do the diagonal sum of the matrix.

$trace(A) = 4 + 3 + (-3) = 4$

or $(trA)$

Types- matrices :-

Row matrices :- $A = [1\ 2\ 3]$

Coloumn matrices $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

Zero matrices $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Square matrices :- row and colums are same

Diagonal matrics -

Identity matrices -

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Imp:-
Types.
Symmetric matrix:-
diagonal will same
A square matrix A is called a Symetrical matrix
if $\boxed{A^T = A}$

[2 cross matrix] Eg:- $A = \begin{bmatrix} 1 & 3 \\ 3 & 2 \end{bmatrix}$ $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

$A^T = \begin{bmatrix} 1 & 3 \\ 3 & 2 \end{bmatrix}$

3 cross matrix

$A = \begin{bmatrix} 1 & 5 & 6 \\ 5 & 2 & -2 \\ 6 & -2 & -3 \end{bmatrix}$

$A^T = \begin{bmatrix} 1 & 5 & 6 \\ 5 & 2 & -2 \\ 6 & -2 & -3 \end{bmatrix}$

$\therefore \boxed{A^T = A}$

Property of this matrix is that if you traanverse the matrix you get the same

(2) Skew -Symmetric matrix

A Square matrix A is said to be Skew Symmetric matrix if
$\boxed{A^T = -A}$

$[a_{ij}]^T = -[a_{ij}]$
~~[aij] = -[~~

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Eg - $A = \begin{bmatrix} 0 & 3 \\ -3 & 0 \end{bmatrix}$

$A^T = \begin{bmatrix} 0 & -3 \\ 3 & 0 \end{bmatrix}$

$= -\begin{bmatrix} 0 & 3 \\ -3 & 0 \end{bmatrix}$

$= -A$

Eg $A = \begin{bmatrix} 0 & 5 & 1 \\ -5 & 0 & -2 \\ -1 & 2 & 0 \end{bmatrix}$

$A^T = \begin{bmatrix} 0 & -5 & -1 \\ 5 & 0 & 2 \\ 1 & -2 & 0 \end{bmatrix}$

$= -\begin{bmatrix} 0 & 5 & 1 \\ -5 & 0 & -2 \\ -1 & 2 & 0 \end{bmatrix}$

Note ÷ The main diagonal entry must be zero in Skew matrix.

(iii) Singular matrix -

A square matrix A is said to be Singular matrix.

if $\det(A) = 0$ or $[A] = 0$

Here det(A) is determine of matrix A

eg $A = \begin{bmatrix} 1 & 0 \\ 2 & 0 \end{bmatrix}$

$\det(A) = \begin{bmatrix} 1 & 0 \\ 2 & 0 \end{bmatrix}$

$= 1(0) - 2(0)$

$= 0$

(4) Non-Singular matrix ÷

A square matrix A is said to be non-Singular matrix if $\det(A) \neq 0$ or $|A| \neq 0$

Eg ÷ $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$\det A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = [4 - 6] = -2 \neq 0$

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3 cross 3 matrix - Take ~~two~~ two row and columns same ~~to create~~ 3 ~~cross~~ 3 ~~Non~~-singular ~~matrix~~

(5) ~~Ex~~. $\begin{bmatrix} 1 & 2 & 0 \\ 2 & 3 & 0 \\ 2 & 3 & 0 \end{bmatrix}$

(5) Orthogonal matrix :-

A square matrix is said to be an orthogonal matrix if

$$\boxed{AA^T = A^TA = I}$$

eg. $A = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

$$A^T = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$AA^T = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} (-1)(-1) + 0(0) & (-1)(0) + 0(1) \\ 0(-1) + 1(0) & (0)(0) + 1(1) \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= I$$

A is an orthogonal matrix.

Note : If A is an orthogonal matrix then, $A^{-1} = ?$

Eg $AA^T = I$

$\Rightarrow A^{-1}(AA^T) = A^{-1}(I)$

$\Rightarrow (A^{-1}A)A^T = A^{-1}$

$\Rightarrow IA^T = A^{-1}$

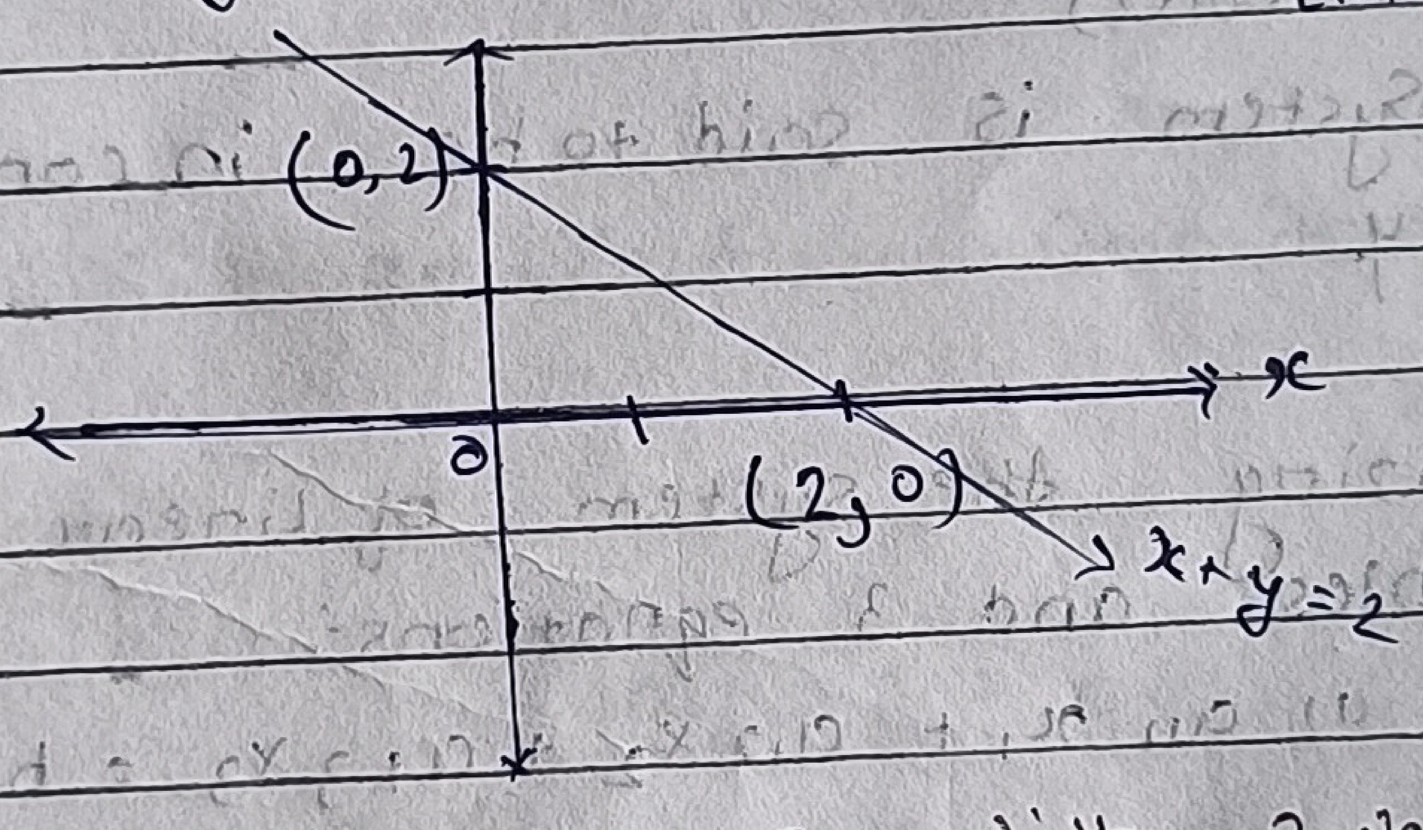
$A^T = A^{-1}$

$\Rightarrow A^{-1} = A^T$

## System of Linear equations:

Linear eqn :- An eqn of the from $ax^1+by^1=c$ is a linear eqn with two variables $x$ & $y$ and here $a, b,$ & $c$ are constant.

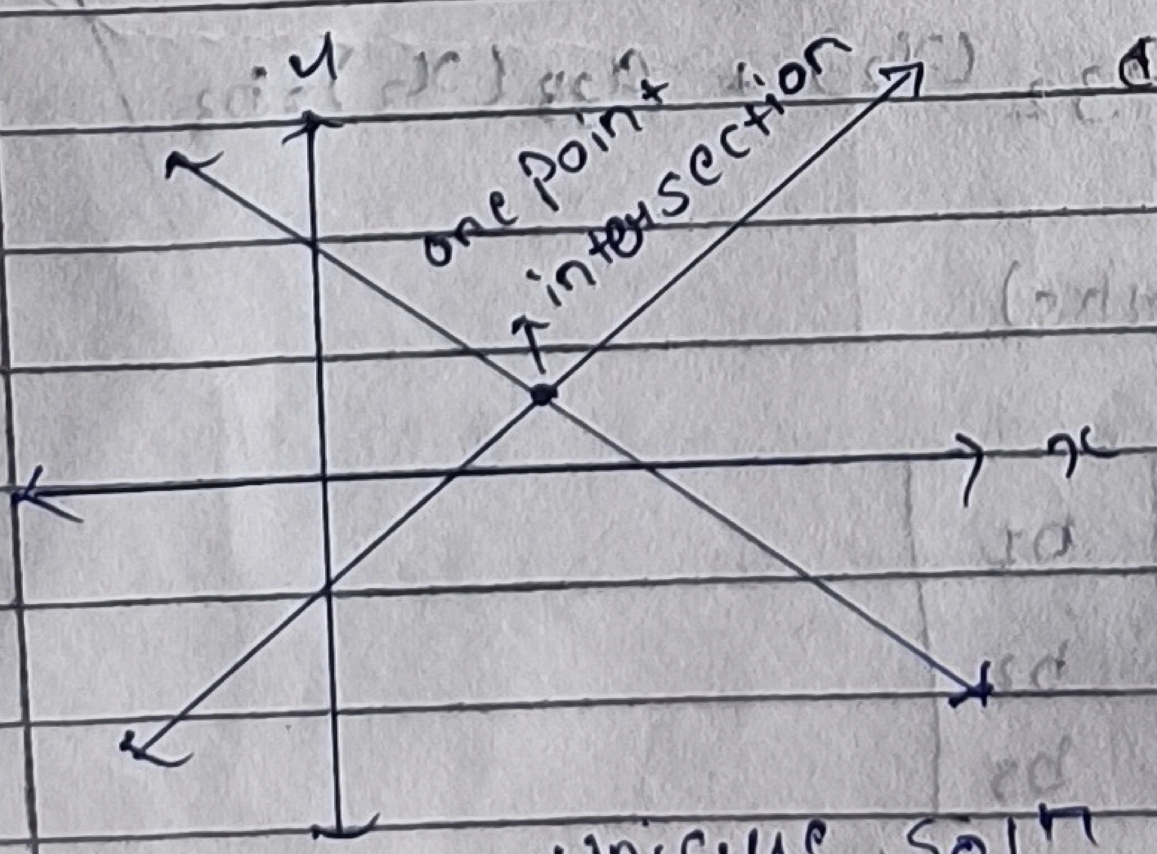
Eg $x^1+y^1=2$ (It shows a straight line because variable are power is one)




A system of linear eqn with 2 variables is the form

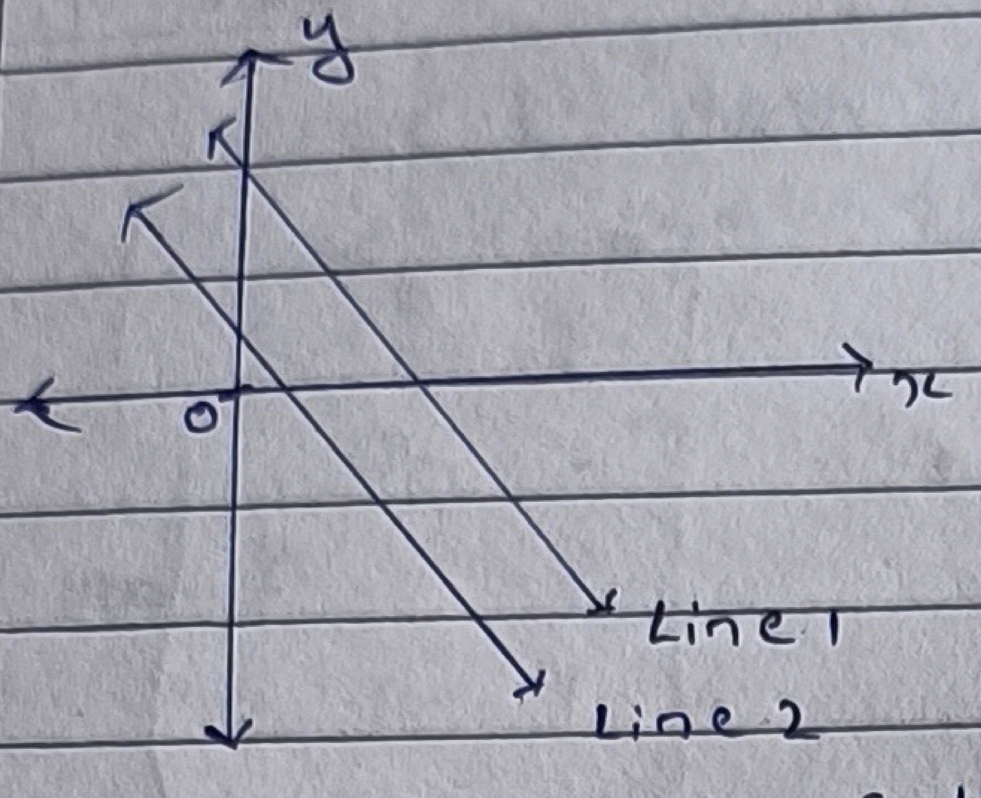
$ax+by=c$ → Line 1

$dx+ey=f$ → Line 2



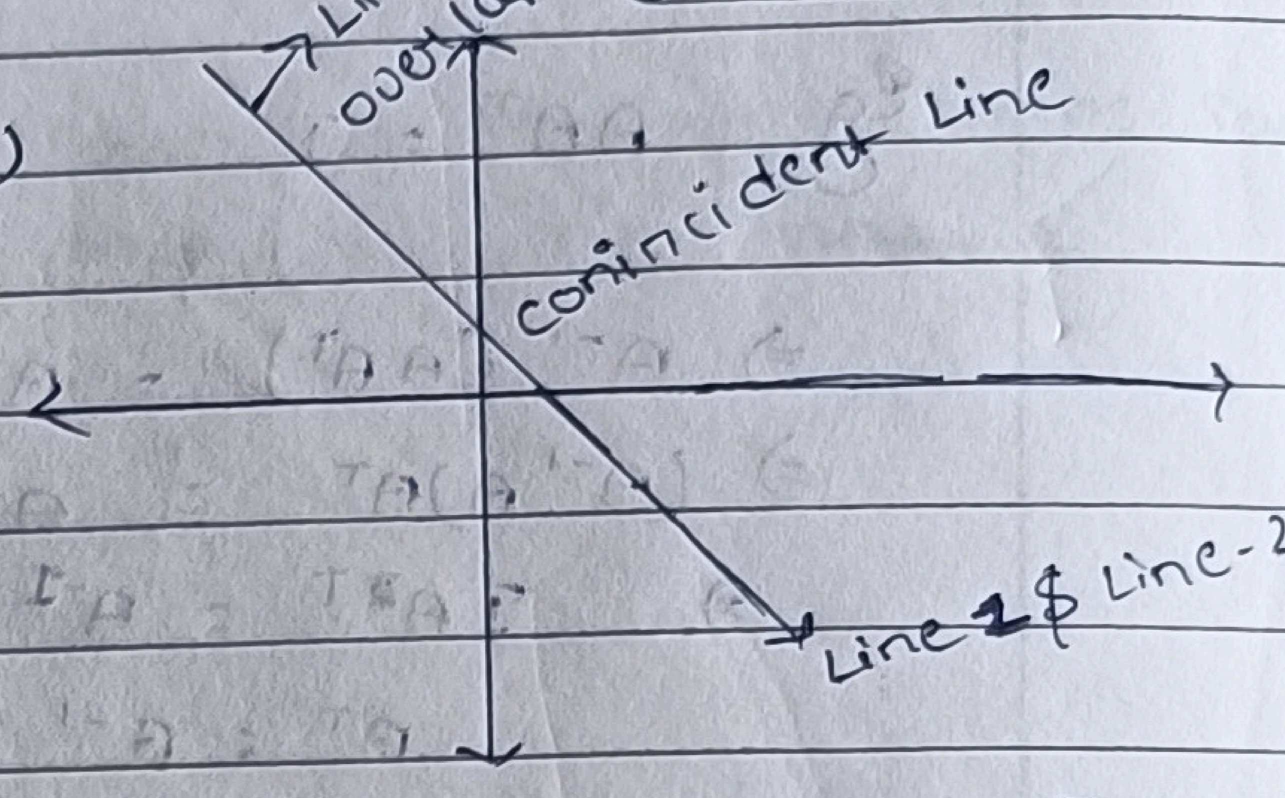

unique sol$^n$

(ii)




no - solution

(iii)




∴ infinite solⁿ

∴ There are 3 possibilities for a ~~solⁿ~~ system of Linear eqⁿ

(i) Unique Solⁿ (ii) No Solⁿ (iii) Infinite Solⁿ

According to this :-

System is said to be conistent if it has possibility (1) or (2)

and System is said to be in consitent if it has possibility.

∴ Generalizing the system of Linear eqⁿ 3 variables and 3 equations :-

(i) $a_{11} x_1 + a_{12} x_2 + a_{13} x_3 = b_1$

(ii) $a_{21} (x_1) + a_{22} (x_2) + a_{23} (x_3) = b_2$

(iii) $a_{31} (x_1) + a_{32} (x_2) + a_{33} (x_3) = b_3$

(Imp) Agimented matrix (1marks)

$$\left[\begin{array}{ccc|c} a_{11} & a_{12} & a_{13} & b_1 \\ a_{21} & a_{22} & a_{23} & b_2 \\ a_{31} & a_{32} & a_{33} & b_3 \end{array}\right]$$

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Eg:-

Que
$$x_1 \quad - x_3 = 3$$
$$4x_1 + 2x_2 = 6$$
$$x_3 = 1$$

Sol^n
$$\left[\begin{array}{ccc|c} 1 & 0 & -1 & 3 \\ 4 & 2 & 0 & 6 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

$$2x_1 - 3x_2 + 4x_3 = 1$$
$$3x_1 + 5x_2 - 6x_3 = 2$$
$$4x_1 + -3x_2 + x_3 = 3$$

Sol^n  Augmented
$$\left[\begin{array}{ccc|c} 2 & -3 & 4 & 1 \\ 3 & 5 & -6 & 2 \\ 4 & -3 & 1 & 3 \end{array}\right]$$

⇒ More generalized form

m eq^n and n variables

$$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1$$
$$a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2$$
$$\vdots$$
$$a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n = b_m$$

Compact form
$$\sum_{i=1}^{m} \sum_{j=1}^{n} a_{ij} x_j = b_i$$

- Note

(i) If $b_1, b_2 \dots b_m$ are all non-zero then, the system is called as (non-homogenous system of linear eq^n)

(ii) If $b_1, b_2 \dots b_m$ are all zeros, then the system is called as homogenous system of linear eq^n

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Eg :- (i)
$$\left.\begin{aligned} 2x_1 + 3x_2 + 4x_3 &= 2 \\ 3x_1 - 2x_2 - 3x_3 &= 9 \\ x_2 + 3x_3 &= 12 \end{aligned}\right] \text{non-}\cancel{\text{zero}}\text{ homo}$$

(ii)
$$\left.\begin{aligned} 2x + 2y + 4z &= 0 \\ 3y + 2z &= 0 \\ 3x - 9y &= 0 \end{aligned}\right] \text{homogenous}$$

Topic

**Row echelon form of matrix :-**
(R.E

A matrix is said to be in R.E from if following 4 condition are satisfied.

(i) If a row is not having all zero entries then the first non-zero entry should be one '1' which is called as leading one.

ii) If a row is having all zeros entries then, it most be in the bottom of matrix.

(iii) The leading one in the below row ~~same~~ should appear farther in the right side as compared to the leading one in the above row

iv Elements below leading '1' must be ~~called~~ all zero in the column.

(v) Note - If in addition $5^{th}$ condition
Elements above leading 1 must be all zero in that column then the matrix is said to be in Reduced row echelon (RRE).

$$RRE \Rightarrow RE$$
$$\not\Leftarrow$$

Topic Classify the following matrices in R.E / R.R.E / Nither

Eg $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ Ans R.R.E

(ii) $\begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ Ans R.E

(iii) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 0 \end{bmatrix}$ Ans Nither

(iv) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ Ans ~~R.R.E~~ Nither

(v) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ Ans R.R.E

(vi) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ Ans ~~Nither~~ R.R.E

(7) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ Ans Nither

(8) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ = Impt

Ans ~~Nither~~ R.R.E

(9) $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ Ans Nither

(10) $\begin{bmatrix} 1 & 1 & 0 & 1 & 1 \\ 0 & 2 & 0 & 2 & 2 \\ 0 & 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 0 & 4 \end{bmatrix}$ Nither

(11) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ ~~An~~ RRE

Note zero matrix is ~~also~~ always R.R.E

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Topic : Gauss Elimination method :

:- To solve system of linear eqⁿ we apply the method

1) Write the aligmented matrix
2) Convert the aligmented matrix into R.E. form by row opperations
3) Write the corresponding linear system.
4) Using back substitution obtain solⁿ:

Examples :-

Solve the system of linear eqⁿ using gauss elimination method.

Solve

$$x + y + z = 6$$
$$x + 2y + 3 = 14$$
$$2x + 4y + 7z = 30$$

Solⁿ

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 14 \\ 2 & 4 & 7 & 30 \end{array}\right] \Rightarrow \begin{array}{l} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - 2R_1 \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 8 \\ 0 & 2 & 5 & 18 \end{array}\right] R_3 \to R_3 - 2R_2$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 8 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

$$x + y + z = 6$$
$$y + 2z = 8$$
$$z = 2$$

$\therefore \quad y = 8 - 2z$
$= 8 - 2(2)$
$= 4$

$x = 6 - y - 2$
$= 6 - 4 - 2$
$= 0$

$x = 0 \quad , y = 4 \quad z = 2$ is the Onique solⁿ.
Hence the given system is consistence.

2) $-2b + 3c = 1$
$3a + 6b - 3c = -2$
$6a + 6b + 3c = 5$

Solⁿ

$$\left[\begin{array}{ccc|c} 0 & -2 & 3 & 1 \\ 3 & 6 & -3 & -2 \\ 6 & 6 & 3 & 5 \end{array}\right] \quad R_{12} \quad R_1 \leftrightarrow R_2$$

$$\sim \left[\begin{array}{ccc|c} 3 & 6 & -3 & -2 \\ 0 & -2 & 3 & 1 \\ 6 & 6 & 3 & 5 \end{array}\right] \quad R_1 = \frac{R_1}{3}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 2 & -1 & -2/3 \\ 0 & -2 & 3 & 1 \\ 6 & 6 & 3 & 5 \end{array}\right] \quad R_3 \to R_3 - 6(R_1)$$

$\cancel{5-6(-1)}$

$$\sim \left[\begin{array}{ccc|c} 1 & 2 & -1 & -2/3 \\ 0 & -2 & 3 & 1 \\ 0 & -6 & 9 & 9 \end{array}\right] \quad R_2/2$$

$5 - 6(2/3)$ | $3 - 6(-1)$
$5 + \frac{12}{3}$ | $3 - (-6)$
$5 + 4 = 9$ | $9$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & -2/3 \\ 0 & 1 & -3/2 & -1/2 \\ 0 & -6 & 9 & 9 \end{array}\right] \quad R_3 \to R_3 + 6R_2$$

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$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & -2/3 \\ 0 & 1 & -3/2 & -1/2 \\ 0 & 0 & 0 & 6 \end{array}\right]$$

$9 + 6\ (-1/2)$

$9 - 6/2 \Rightarrow 9 - 3 = 6$

$\cancel{a + 2b + c}$

$a + 2b - c = -2/3$

$b - 3/2\, c = -1/2$

$0a + 0b + 0c - 6 \Rightarrow 0 = 6$

which is not possible

It has no solⁿ

∴ Hence the system is inconsistent.

Que

$$\left[\begin{array}{ccc|c} 2x & + 2y & + 2z & 0 \\ -2x & + 5 & + 2z & 1 \\ 8x & + 1y & + 4z & -1 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{ccc|c} 2 & 2 & 2 & 0 \\ -2 & 5 & 2 & 1 \\ 8 & 1 & 4 & -1 \end{array}\right] \quad R_1/2$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ -2 & 5 & 2 & 1 \\ 8 & 1 & 4 & -1 \end{array}\right] \quad \begin{array}{l} R_2 \to R_2 + 2R_1 \\ R_3 \to R_3 - 8R_1 \end{array}$$

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$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 7 & 4 & 1 \\ 0 & -7 & -4 & -1 \end{array}\right] \quad R_3 \to R_3 + R_2$$

$$\Rightarrow \sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 7 & 4 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right] \quad R_2/7$$

$$\Rightarrow \sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & 4/7 & 1/7 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$x + y + z = 0$$
$$y + 4/7\, z = 1/7$$

Let $z = t$ (free variable)

$$y = \frac{1}{7} - \frac{4}{7}t$$

$$x = -y - z$$
$$= -(1/7 - 4/7\, t) - t$$
$$\Rightarrow -1/7 + 4/7\, t - t)$$
$$\Rightarrow -1/7 + \frac{4t - 7t}{7}$$
$$= -1/7 - \frac{3t}{7}$$

$x = -1/7 - 3/7\, t$, $y = 1/7 - 4/7\, t$, $z = t$

is the $Sol^n$

As we the change the value of t then there are infinetly many $Sol^n$.

Hence the system is consistent.

4) $\frac{3}{x} + \frac{2}{y} - \frac{1}{z} = 9$

$\frac{2}{x} - \frac{1}{y} + \frac{2}{z} = 10$

$-\frac{1}{x} + \frac{3}{y} + \frac{4}{z} = 30$

Sol$^n$ Let $\frac{1}{x} = a$, $\frac{1}{y} = b$, $\frac{1}{z} = c$

$3a + 2b - c = 9$
$2a - b + 2c = 10$
$-a + 3b + 4c = 30$

alinged matrix is

$$\left[\begin{array}{ccc|c} 3 & 2 & -1 & 9 \\ 2 & -1 & 2 & 10 \\ -1 & 3 & 4 & 30 \end{array}\right] \quad R_1 \to R_1 - R_2$$

$$\sim \left[\begin{array}{ccc|c} 1 & 3 & -3 & -1 \\ 2 & -1 & 2 & 10 \\ -1 & 3 & 4 & 30 \end{array}\right] \quad \begin{array}{l} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 + R_1 \end{array}$$

$-1 - 2 \cdot 3$
$2x$
$4 + -(3)$
$30 + \cdot(-1)$

$$\sim \left[\begin{array}{ccc|c} 1 & 3 & -3 & -1 \\ 0 & -7 & 8 & 12 \\ 0 & 6 & 1 & 29 \end{array}\right] \quad \begin{array}{l} R_2 \to R_2 + R_3 \\ R_2 \to (-1)R_2 \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 3 & -3 & -1 \\ 0 & 1 & -9 & -41 \\ 0 & 6 & 1 & \end{array}\right] \quad R_3 \to R_3 - 6R_2$$

$$\sim \left[\begin{array}{ccc|c} 1 & 3 & -3 & -1 \\ 0 & 1 & -9 & -41 \\ 0 & 0 & 55 & 275 \end{array}\right]$$

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$$\sim \left[\begin{array}{ccc|c} 1 & 3 & -3 & -1 \\ 0 & 1 & -9 & -41 \\ 0 & 0 & 1 & 5 \end{array}\right]$$

$a + 3b - 3c = -1$

$b - 9c = -41$

$c = 5$

$b - 9(5) = -41$ ~~$a + 3b - 3c = -1$~~

$b - 45 = -41$ ~~$b - 9c =$~~

$b = -41 + 45$

$b = 4$

$a = -1 - 3b + 3c$

$= -1 - 3(4) + 3(5)$

$= -1 - 12 + 15$

$= 2$

$x = 1/a, \quad y = 1/b \quad z = 1/c$

$\Rightarrow x = 1/2, \quad y = 1/4 \quad z = 1/5$

Hence the system has unique solⁿ which concludes that system is consistent.

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Ex-2 consider the following system.

$x + y + z = 6$
$x + 2y + 3z = 10$
$x + 2y + \lambda = \mu$

for what values of $\lambda$ and $\mu$ do the system has.

i) unique solⁿ
ii) no solⁿ
iii) infinitely many solⁿ

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 10 \\ 1 & 2 & \lambda & \mu \end{array}\right] \quad \begin{array}{l} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 1 & \lambda-1 & \mu-6 \end{array}\right] \quad R_3 \to R_3 - R_2$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & \lambda-3 & \mu-10 \end{array}\right] \quad \cancel{R_3 \to R_3 - R_2}$$

(i) System has unique solⁿ if $\lambda - 3 \neq 0$ and $\mu - 10$ may be or may not be zero
or
$\lambda \neq 3$ and $\mu$ can take any value.

(ii) System has no solⁿ if $\lambda = 3$ & $\mu \neq 10$

(iii) System has infinitely many solⁿ if $\lambda = 3$ & $\mu = 10$

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⇒ Gauss Jordan method :-

Steps

I) Write the augmented matrix

II) Using row operation make the ~~gag~~ augmented matrix into R.R.E form

3) obtain the soln by writing the system.

Que
$2y + 3z = 7$
$3x + 6y - 12z = -3$
$5x - 2y + 2z = -7$

Soln A.M is

$$\left[\begin{array}{ccc|c} 0 & 2 & 3 & 7 \\ 3 & 6 & -12 & -3 \\ 5 & -2 & 2 & -7 \end{array}\right] R_{12}$$

$$\sim \left[\begin{array}{ccc|c} 3 & 6 & -12 & -3 \\ 0 & 2 & 3 & 7 \\ 5 & -2 & 2 & -7 \end{array}\right] R_1/3$$

$$\sim \left[\begin{array}{ccc|c} 1 & 2 & -4 & -1 \\ 0 & 2 & 3 & 7 \\ 5 & -2 & 2 & -7 \end{array}\right] R_3 = R_3 + (-5)R_1$$

$$\sim \left[\begin{array}{ccc|c} 1 & 2 & -4 & -1 \\ 0 & 2 & 3 & 7 \\ 0 & -12 & 22 & -2 \end{array}\right] R_2/2$$

$$\sim \left[\begin{array}{ccc|c} 1 & 2 & -4 & -1 \\ 0 & 1 & 3/2 & 7/2 \\ 0 & -12 & 22 & -2 \end{array}\right] R_3 \to R_3 + 2R_2$$

$$\sim \left[\begin{array}{ccc|c} 1 & 2 & -4 & -1 \\ 0 & 1 & 3/2 & 7/2 \\ 0 & 0 & 40 & 40 \end{array}\right] \quad R_3/40$$

$$\sim \left[\begin{array}{ccc|c} 1 & 2 & -4 & -1 \\ 0 & 1 & 3/2 & 7/2 \\ 0 & 0 & 1 & 1 \end{array}\right] \quad \begin{array}{l} R_1 \to R_1 + 4R_3 \\ R_2 \to R_2 - \frac{3}{2}R_3 \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 2 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{array}\right] \quad R_1 \to [R_1 - R_2]$$

$0 - \frac{3}{2} 0$
$1 - \frac{3}{2} \times 0$
$\frac{7}{2} - \frac{3}{2} \times -1$
$\frac{7-3}{2}$ $\frac{4}{2} = 2$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

$x = -1$
$y = 2$
$z = 1$

There is a Single Sol$^n$ for this System and System is consistent & unique sol$^n$.

Que
$x + y + z$
$3x - y - z = 4$
$x + 5y + 5z = -1$

Sol$^n$ A.M is

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$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 3 & -1 & -1 & 4 \\ 1 & 5 & 5 & -1 \end{array}\right] \quad \begin{array}{l} R_2 \to R_2 - 3R_1 \\ R_3 - R_3 - R_1 \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & -4 & -4 & 1 \\ 0 & 4 & 4 & -2 \end{array}\right] \quad R_3 \to R_3 + R_2$$

$4 - 3x \quad -1 - 3\times 1)$
$-1 - 3$
$-1 - 1 \quad -4$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 5 \\ 0 & 0 & 0 & \cancel{2/4} \; -1 \end{array}\right] \quad \begin{array}{l} R_2 \to R_2 + 5(R_1) \\ R_3 \to R_3/4 \end{array}$$

$1 + 5\times 1 = 5$

$2/4$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & -4 \\ 0 & 1 & 1 & 4 \\ 0 & 0 & 0 & -1 \end{array}\right] \quad R_1 \to R_1 - R_2$$

$1 - 5 = -4$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & -4 & -4 & 1 \\ 0 & 0 & 0 & -1 \end{array}\right] \quad R_3/-4$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & -1/4 \\ 0 & 0 & 0 & -1 \end{array}\right] \quad R_1 \to R_1 - R_2$$

$\sim [$

Ques -
$$x + 2y - 3z = -2$$
$$3x - y - 2z = 1$$
$$2x + 3y - 5z = -3$$

(1) $\left[\begin{array}{ccc|c} 1 & 2 & -3 & -2 \\ 3 & -1 & -2 & 1 \\ 2 & 3 & -5 & -3 \end{array}\right]$ $R_2 \to R_2 - 3R_1$, $R_3 \to R_3 - 2R_1$

(ii) $\left[\begin{array}{ccc|c} 1 & 2 & -3 & -2 \\ 0 & -7 & 7 & 7 \\ 0 & -1 & 1 & 1 \end{array}\right]$ $R_2 \to R_2/7$

$\sim$ (iii) $\left[\begin{array}{ccc|c} 1 & 2 & -3 & -2 \\ 0 & 1 & -1 & -1 \\ 0 & -1 & 1 & 1 \end{array}\right]$ $R_3 \to R_3 + R_2$

$\sim$ (iv) $\left[\begin{array}{ccc|c} 1 & 2 & -3 & -2 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right]$ $R_1 \to R_1 - 2R_2$

(v) $\left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$x - z = 0$

$y - z = -1$

Let assume $z = t$ (free variable)

$x = t$, $y = -1 + t$, $z = t$ is the soln

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Hence the system is consistent and its has infinetly many Sol$^n$.

## Topic Homogenous System of Linear eq$^n$

The Linear eq$^n$ system of the form

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = 0$$
$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = 0$$
$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = 0$$

is called Homogenous System of Linear eq$^n$.

There can be two possible Sol$^n$

(i) Trivial Sol$^n$ / obvious Sol$^n$
$x_1 = 0$, $x_2 = 0$, $x_3 = 0$

(ii) Non trivial Sol$^n$
$x_1$, $x_2$ and $x_3$ Have infinety Sol$^n$

Note - homogenous Linear system is always consistent.

Ex-1 Solve the folling Linear System.

(i) $4x + 3y - z = 0$
$3x + 4y + z = 0$
$5x + y - 4z = 0$

A-m, $\left[\begin{array}{ccc|c} 4 & 3 & -1 & 0 \\ 3 & 4 & 1 & 0 \\ 5 & 1 & -4 & 0 \end{array}\right]$ $R_1 \to R_1 - R_2$

$\sim \left[\begin{array}{ccc|c} 1 & -1 & -2 & 0 \\ 3 & 4 & 1 & 0 \\ 5 & -1 & -4 & 0 \end{array}\right]$

$1 - (5 \times -1)$
$1 - -5$
$6$

$\cancel{\left[\begin{array}{ccc|c} 1 & -1 & -2 & 0 \\ 0 & 7 & 7 & 0 \\ 0 & 6 & 6 & 0 \end{array}\right]}$ $R_2 \to R_2 - 3(R_1)$ $\quad 4 - 3(-1)$

$3 - 3 \times 1 \quad \Rightarrow 4 - 3$

$R_3 \to R_3 - 5R_1 \quad 1 \quad 3(-2)$

$1 - (-6)$

$\left[\begin{array}{ccc|c} 1 & -1 & -2 & 0 \\ 0 & 7 & 7 & 0 \\ 0 & 6 & 6 & 0 \end{array}\right]$ $\frac{R_2}{7}, \frac{R_3}{6}$

$-1 - 5 \times -1$
$-1 - 6$
$= 7$

$-4 - 5 \times 2$
$-4 - 10$
$= 14$

$\left[\begin{array}{ccc|c} 1 & -1 & -2 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right]$ $R_3 \to R_3 - R_2$

$-1 - 5 \times -1$
$-1 - (5)$
$= +$

$-4 \times 5(-1)$
$= 4 - -5$

$\left[\begin{array}{ccc|c} 1 & -1 & -2 & 0 \\ 0 & 1 & 1 & \\ 0 & 0 & 0 & \end{array}\right]$

$x - y - 2z = 0$
$y + z = 0$

Let assume $z = t$ (free variable)

$y + z = 0$

$x - t - 2t = 0 \qquad y = -z + 0$

$x - t = +2t + 0 \qquad y = -t$

$x = 3t$

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$x = t,\ y = t,\ z = t$

infintely many soln

The system is consistent and it has non-trivial soln

2) $2x + y + 3z = 0$

$x + 2y = 0$

$y + z = 0$

Ans

$$\left[\begin{array}{ccc|c} 2 & 1 & 3 & 0 \\ 1 & 2 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right] \quad R_1 - R_1 - R_2$$

$2-1 = 1$ $\quad 1-2 = -1$ $\quad 3-0 = 3$

R.F

$$\left[\begin{array}{ccc|c} 1 & -1 & 3 & 0 \\ 1 & 2 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right] \quad R_2 \to R_2 - R_1$$

$1-1 = 0$ $\quad 2-1$ $\quad R_2/3 =$

$$\left[\begin{array}{ccc|c} 1 & -1 & 3 & 0 \\ 0 & 3 & -3 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right] \quad \cancel{R_3 \to R_2 - R_3}$$

$-3\ -3$

$$\left[\begin{array}{ccc|c} 1 & -1 & 3 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] \quad R_3 \to R_3 - R_2$$

$x - y + 3z = 0$

$y = 0$

$z = 0$ $\qquad x = 0,\ y = 0\ \&\ z = 0$

The System is Trivial soln.

$x = y - 3z$

$= 0 - 3 \times 0$

$= 0 - 0$

$= 0$

Q.

$$x + y - z + w = 0$$
$$x - y + 2z - w = 0$$
$$3x + y + w = 0$$

Am -
$$\left[\begin{array}{cccc|c} 1 & 1 & -1 & 1 & 0 \\ 1 & -1 & 2 & -1 & 0 \\ 3 & 1 & 0 & 1 & 0 \end{array}\right]$$

R.E

$3 - 3\times1$

$3 - 3$

$$\sim \left[\begin{array}{cccc|c} 1 & 1 & -1 & 1 & 0 \\ 0 & -2 & 3 & -2 & 0 \\ 0 & -2 & 3 & -2 & 0 \end{array}\right] \quad \begin{array}{l} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - 3(R_1) \end{array}$$

$-1 - 1$

$2 - -1$ $\quad 0 - 3\times1$

$-1 - 1$ $\quad 0 - 3$

$1 - 3\times1$

$$\left[\begin{array}{cccc|c} 1 & 1 & -1 & 1 & 0 \\ 0 & -2 & 3 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \quad \begin{array}{l} R_3 \to R_3 - R_2 \\ R_2/-2 \end{array}$$

$\frac{3}{-2}$ $= 1 - 3$

$\frac{1}{1}$ $= 2$ $\quad 1 \times 3$

$1 - 3$

$= 2$

$$\left[\begin{array}{cccc|c} 1 & 1 & -1 & 1 & 0 \\ 0 & 1 & -3/2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$$x + y - z + w = 0$$
$$y - \tfrac{3}{2}z + w = 0$$

Let $\quad z = t_1, \ w = t_2$

$$y = \tfrac{3}{2}z - w \Rightarrow y = \tfrac{3}{2}t_1 - t_2$$

and $x = -y + z - w)$

$\Rightarrow x = -\left(\frac{3}{2}t_1 - \cancel{t/2}\right) + t_1 - \cancel{t/2}$

$\Rightarrow -\frac{3}{2}t_1 + t_1$

$= \quad x = \frac{-3t_1 + 2t_1}{2} = \frac{-3 \cdot 2t_1}{2} = \frac{-t_1}{2}$

$= \frac{t_1}{-2}$

$\therefore$ The system is consistent & it has non-trivial solⁿ.

<u>Que</u>

$-2x + 2y - 3z = 0$

$2x + y - 6z = 0$

$-x - 2y + 2z = 0$

$3x + y + 4z = 0$

A.m

$$\sim \left[\begin{array}{ccc|c} -2 & 2 & -3 & 0 \\ 2 & 1 & -6 & 0 \\ -1 & -2 & 2 & 0 \\ 3 & 1 & 4 & 0 \end{array}\right]$$

R.E

$\sim$

Rank of MATRIX :-

Method : 2 :-

By determinant method -

$[A]_{r \times r}$

determinant $(A) \neq 0$ $\rightarrow$ $\rho(A) = r$

$\det(A) = 0$ $\rightarrow$ compute minors of A of order $r-1$

(i) If at least one minor of $r-1$ is non zero then, $\rho(A) = r-1$

(ii) If all minors of order $r-1$ are zero $\rho(A) < r-1$

Exp-2 Find the rank of by determinant method.

$$A = \begin{bmatrix} 2 & 3 & 4 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{bmatrix}$$

$$\det(A) \Rightarrow 2\begin{vmatrix} 3 & 1 \\ 2 & 4 \end{vmatrix} - 3\begin{vmatrix} 4 & 1 \\ 1 & 4 \end{vmatrix} + 4\begin{vmatrix} 4 & 3 \\ 1 & 2 \end{vmatrix}$$

$= 2[12-2] - 3(16-1) + 4(8-3)$

$= 2(10) - 3(15) + 4(5)$

$= 20 - 45 + 20$

$= 40 - 45$

$= -5 \neq 0$

$\rho(A) = 3$ (is same as order of matrix A)

2) $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 5 & 7 \end{bmatrix}_{3\times3}$

$\det(A) = 1\begin{vmatrix} 3 & 4 \\ 5 & 7 \end{vmatrix} - 2\begin{vmatrix} 2 & 4 \\ 3 & 7 \end{vmatrix} + 3\begin{vmatrix} 2 & 3 \\ 3 & 5 \end{vmatrix}$

$= 1(21-20) - 2(14-12) + 3(10-9)$

$= 1-4+3$

$= 4-4$

$= 0$

$\rho(A) < 3$

$M_{11} = \begin{vmatrix} 3 & 4 \\ 5 & 7 \end{vmatrix}_{2\times2}$

$= 21-20$

$= 1 \neq 0$

As $M_{11} \neq 0$

$\therefore \rho(A) = 2$ (same as the order non-zero minor)

$A = \begin{bmatrix} 4 & 2 & 3 \\ 8 & 4 & 6 \\ -2 & -1 & -3/2 \end{bmatrix}$

$\det A = 4\begin{vmatrix} 4 & 6 \\ -1 & -3/2 \end{vmatrix} - 2\begin{vmatrix} 8 & 6 \\ -2 & -3/2 \end{vmatrix} + 3\begin{vmatrix} 8 & 4 \\ -2 & -1 \end{vmatrix}$

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$\Rightarrow 4\left(4(-3/2)-(-6)\right) - 2\left(8(-3/2)-(-2)6\right) + 3\left(-8-(-8)\right)$

$= 4(-6+6) - 2(-12+12) + 3(-8+8)$

$= 4(0) - 2(0) + 3(0)$

$= 0$

$\rho(A) < 3$

Minor of 2×2

$M_{11} = \begin{vmatrix} 4 & 6 \\ -1 & -3/2 \end{vmatrix}$

$= -6 + 6$

$= 0$

$M_{12} = \begin{vmatrix} 8 & 6 \\ -2 & -3/2 \end{vmatrix}$

$= -12 + 12$

$= 0$

$M_{21} = \begin{vmatrix} 2 & 3 \\ -1 & -3/2 \end{vmatrix}$

$\Rightarrow 2(-3/2) - (-1)(3)$

$= -3 + 3$

$= 0$

$M_{22} = \begin{vmatrix} 4 & 3 \\ -2 & -3/2 \end{vmatrix}$

$= 4(-3/2) - (-2)(3)$

$= -6 + 6$

$= 0$

$M_{23} = \begin{vmatrix} 4 & 2 \\ -2 & -1 \end{vmatrix}$

$\Rightarrow -4 - (-2)\times(2)$

$\Rightarrow -4 + 4$

$= 0$

$m_{31}: \begin{vmatrix} 2 & 3 \\ 4 & 6 \end{vmatrix}$ $m_{32} = \begin{vmatrix} 4 & 3 \\ 8 & 6 \end{vmatrix}$ $m_{33} \begin{vmatrix} 4 & 2 \\ 8 & 4 \end{vmatrix}$

$= 12-12$ $= 24-24$ $= 16-16$

$= 0$ $= 0$ $= 0$

$\rho = A < 2$ (as all minour are zero)

$\therefore \rho A = 1$

Note :-

1) $\rho(AB) \leq \rho(A)$ and $\rho(AB) \leq \rho(B)$

2) $\rho(A/B) = \rho(A) =$ no. of unknown
$\therefore$ The system $Ax = B$ has unique solⁿ

Eg :- R.E form of a system be

$$[A/B] = \left[\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 6 \end{array}\right]$$

$\rho(A/B) = 3$, $\rho(A) = 3$, no. of unkowns $= 3$

$x + 2y = 3$
$y = 3$
$z = 6$

Unige solⁿ

3) $\rho(A/B) = \rho(A) <$ no. of unknows
The system $Ax = B$ has infinite soln

R.E from of a system be

$$[A/B] = \left[\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\rho(A/B) = 2$, $\rho(A) = 2$, no. of unkowns = 3

$x + 2y + z = 3$

$y = 2$

Infinite Soln

4)

$\rho(A) < \rho(A/B)$

Then, the system has soln

$$A/B = \left[\begin{array}{ccc|c} 1 & 2 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 2 \end{array}\right]$$

then, $\rho(A) = 2$, $\rho(A/B) = 3$

$\rho(A) < (A/B)$

The System has soln

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## Eigens values and Eigen vectors :-

Compulsory square matrix :-

Let $[A]_{n\times n}$ be square matrix

if $Ax = \lambda x$

then $\lambda \to$ eigen value

& $x \to$ eigen vector

$$\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}_{n\times 1} \neq 0$$

$Ax = \lambda x$

$\Rightarrow Ax = \lambda x = 0$

$\Rightarrow (A - \lambda I)x = 0$

$x$ will be non-zero if?

1st method $\det(A - \lambda I) = 0 \Rightarrow$ characteristic eqn

or

For 2x2 matrix :-

$\lambda^2 - S_1 + S_2 = 0$

where

$S_1 =$ trace(A) diagonal sum value of

$S_2 = \det(A)$ 2x2 matrix

For 3x3 matrix :-

$\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0$

where

$S_1 =$ trace(A)

$S_2 =$ Sum of the minors of principal diagonal element of A

$S_3 = \det(A)$

$\lambda = 0$ is not possible

2 $\lambda$ values will be there one can be zero

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Note: On solving characteristic eqⁿ, we get egien values.

⇒ To determine egen vector $x$
on substituting each value of $x$.

in
$[A-\lambda I]x=0$

we generate the homogenous system on solving by R.E
from we get.

Note (i) For a triangular matrix upper or lower triangular
the eigen values are its diagonal elements.

Eg:
$$A=\begin{bmatrix}1 & 2\\ 0 & 3\end{bmatrix} \Rightarrow \lambda=1,3$$

$$A=\begin{bmatrix}1 & 0 & 0\\ 3 & 2 & 0\\ 2 & 1 & 2\end{bmatrix} \Rightarrow \lambda=1,2,2$$

(ii) If eigen values of $A$ are $\lambda_1, \lambda_2, \lambda_3$ then, eigen values of $A^{-1}$ are $\frac{1}{\lambda_1}, \frac{1}{\lambda_2}, \frac{1}{\lambda_3}$

$A^2$ are $\lambda_1^2, \lambda_2^2, \lambda_3^2$

$A^3$ are $\lambda_1^3, \lambda_2^3, \lambda_3^3$

$A^T$ are $\lambda_1^T, \lambda_2^T, \lambda_3^T$

$nA$ are $n\lambda_1, n\lambda_2, n\lambda_3$ ... 3n FIM

Find the eigen values of $A=\begin{bmatrix}1 & 2 & -3\\ 0 & 2 & 3\\ 0 & 0 & 3\end{bmatrix}$

Also determine the egien values of
$A^T$, $A^5$, $5A$, $A^{-1}$

| Matrix | Eigen values |
|---|---|
| $A$ | $1, 2, 3$ |
| $A^T$ | $1^T, 2^T, 3^T$ |
| $A^9$ | $1^9, 2^9, 3^9$ |
| $5A$ | $5, 10, 15$ |
| $A^{-1}+$ | $1, \frac{1}{2}, \frac{1}{3}$ |
| $A^3+3I$ | $1^3+3(1) + 2^3+3(1) + 3^3+(3)$ |

use of Eigen values & Eigen values

To find the

i) increase of the matrix $A$

ii) Power of the matrix $A$

Find the eigen values of

$$A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}_{3\times3}$$

we can get $[A-\lambda I]$ by subtracting $\lambda$ from diagonal

$$\Rightarrow [A-\lambda I] = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -\lambda & 1 & 1 \\ 1 & -\lambda & 1 \\ 1 & 1 & -\lambda \end{bmatrix}$$

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$[A - \lambda I] = 0$

$\Rightarrow \begin{vmatrix} -\lambda & 1 & 1 \\ 1 & -\lambda & 1 \\ 1 & 1 & -\lambda \end{vmatrix} = 0$

$\Rightarrow -\lambda \begin{vmatrix} -\lambda & 1 \\ 1 & -\lambda \end{vmatrix} - (1) \begin{vmatrix} 1 & 1 \\ 1 & -\lambda \end{vmatrix} + 1 \begin{vmatrix} 1 & -\lambda \\ 1 & 1 \end{vmatrix} = 0$

$\Rightarrow -\lambda(\lambda^2 - 1) - 1(-\lambda - 1) + 1(1 + \lambda) = 0$

$= \lambda^3 + \lambda + \lambda + 1 + 1 + \lambda = 0$

$= \lambda^3 + 3\lambda + 2 = 0$

$\boxed{\lambda^3 - 3\lambda - 2 = 0}$ Characterstic Eqn