* Basic Terminologies in Probability:

1. **Experiment :** The term experiment refers to describe an act which can be repeated under some given conditions.

2. **Random experiment :** (also called a trial)
If in an experiment all possible outcomes are known in advance and none of the outcomes can be predicted with certainty, then such an experiment is called random experiment.

Eg. When a fair coin is tossed once, we know that we may get either tail or head but we cannot determine which one of them will come as an outcome.

3. **Sample space :** The set consisting all the possible outcomes of ~~an~~ random experiment is called sample space (S).

Eg. Sample space of tossing two coins simultaneously.
$S = \{HH, HT, TH, TT\}$

Eg. A coin is tossed. If it shows head, we draw a ball from a bag containing 3 blue & 4 white balls if it shows tail we throw a dice.
$S = \{(H,b), (H,w), (T,1), (T,2), (T,3), (T,4), (T,5), (T,6)\}$
[b - blue ball , w - white balls]

4. **Events:** The outcomes of the random experiments are called the events.
→ In simple words,
Any subset of sample space is called event.

Eg. For tossing a fair coin once.
$S = \{H, T\}$
Then $\{H\}, \{T\}, \{H, T\}, \phi$ or $\{\ \}$ are all events.

Eg. Consider an experiment of rolling a dice & the following events:
A: getting an odd no.
B: getting a no. divisible by 2.
C: " " " greater than 2.
D: " " " less than 7
E: " " " greater than 6.
Write the sample space & sample points of all the above events.

⟹ $S = \{1, 2, 3, 4, 5, 6\}$
$A = \{1, 3, 5\}$ $B = \{2, 4, 6\}$ $C = \{3, 4, 5, 6\}$
$D = \{1, 2, 3, 5, 6, 7\}$ $E = \{\ \}$ or ϕ

* **TYPES OF EVENTS:**

1. **Certain or sure:** An event whose occurence is sure or inevitable is called sure or certain event.

→ Eg. The sun will rise tommorrow morning is an sure event.

→ In previous eg. of rolling a dice, the event D is sure event.

NOTE: For a sure event E, E = S.
(S = sample space)

2. Impossible event: An event which is impossible to occur is called impossible event.

Examples:

→ i) The sun will rise in west.

ii) A student will get 120 marks in an exam of total 100 marks.

iii) In previous eg. of rolling a dice, the event E is impossible event.

NOTE: For an impossible event E, E = ∅ = { }

3. Simple event: If an event has only one sample point of a sample space, it is called as simple event.

In short, if E is a simple event then E = singleton set.

→ Examples:

i) Tossing two coins simultaneously
S = {HH, HT, TH, TT}
Then E_1 : getting exactly 2 heads
E_2 : getting exactly 2 tails

E_3 : getting one head on 1st coin
E_4 : " " " " 2nd coin
→ All these events are,
$E_1 = \{HH\}$ $E_2 = \{TT\}$ $E_3 = \{HT\}$ $E_4 = \{TH\}$
simple events.

4. **Mutually exclusive events** : Two events are said to be mutually exclusive or incompatible when both cannot happen simultaneously in a single trial.
In other words, if A & B are two events then they are mutually exclusive iff $A \cap B = \emptyset$.

⇒ **Examples** :
i) Rolling a dice $S = \{1, 2, 3, 4, 5, 6\}$
Consider the events,
E_1 : getting a no. divisible by 2.
E_2 : getting an odd no.
i.e. $E_1 = \{2, 4, 6\}$ $E_2 = \{1, 3, 5\}$
Here $E_1 \cap E_2 = \{\ \} = \emptyset$
∴ Both are mutually exclusive.

ii) Two dice are thrown & the sum of the nos. on the two dice are noted.
Consider the following events :
A : sum is even.
B : sum is multiple of 3
C : " " less than 4.
D : " " greater than 11.
Which pairs of events are mutually exclusive ?

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→ $A = \{(1,1), (1,3), (1,5), (2,2), (2,4), (2,6), (3,1), (3,3), (3,5), (4,2), (4,4), (4,6), (5,1), (5,3), (5,5), (6,2), (6,4), (6,6)\}$

$B = \{(1,2), (2,1), (1,5), (5,1), (3,3), (2,4), (4,2), (3,6), (6,3), (4,5), (5,4)\}, (6,6)\}$

$C = \{(1,1), (2,1), (1,2)\}$ and $D = \{(6,6)\}$

Here,

$A \cap B = \{(1,5), (2,4), (3,3), (4,2), (5,1), (6,6)\}$

$\therefore$ A & B are not mutually exclusive.

$A \cap C = \{(1,1)\}$

$\Rightarrow$ A & C are not mutually exclusive.

$A \cap D = \{6,6\}$ $\Rightarrow$ A & D are also not mutually exclusive.

$B \cap C = \{(1,5), (5,1), (3,3), (2,4), (4,2)\}$

$\Rightarrow$ B & C are not mutually exclusive.

~~$B \cap D$~~ ~~$C \cap A$~~

$B \cap D = \{(6,6)\}$ $\Rightarrow$ B & D not mutually exclusive.

$C \cap D = \{\} = \emptyset$

$\Rightarrow$ C & D are mutually exclusive.

<u>NOTE:</u> A & B are mutually exclusive then the sets A & B are disjoint.

5. Exhaustive events : Events are said to be exhaustive when their totality includes all the possible outcomes.
→ In other words, two events A & B exhaustive if $A \cup B = S$ (S = sample space)

→ Examples :

i) Rolling a dice $S = \{1, 2, 3, 4, 5, 6\}$
Consider the events :
A : a no. less than 4 appears
B : " " greater " 2 but less 5 appears
C : " " " " 4 appears.
Then $A = \{1, 2, 3\}$, $B = \{3, 4\}$ & $C = \{5, 6\}$
Here $A \cup B \cup C = \{1, 2, 3, 4, 5, 6\} = S$
∴ All 3 are together exhaustive events.

ii) ~~Two dice are~~ A coin is tossed 3 times
consider the following events :
A : 'no head appears'
B : exactly one head appears
C : at least two heads appear.
Do they form a set of mutually exclusive & exhaustive events ?

→ $A = \{TTT\}$, $B = \{HTT, THT, TTH\}$
$C = \{HHT, HTH, THH, HHH\}$
Here $A \cap B = \emptyset$, $B \cap C = \emptyset$, $C \cap A = \emptyset$
⇒ A, B & C are mutually exclusive
Also, $A \cup B \cup C = \{TTT, HTT, THT, TTH, HHT, HTH, THH, HHH\} = S$
⇒ They are also exhaustive.

iii) Throwing a dice

Let A: getting an even no.

B: " multiple of 5

Then $A = \{2, 4, 6\}$ $B = \{5\}$

Here $A \cap B = \phi$, $A \cup B \neq S$

$\Rightarrow$ They are mutually exclusive but not exhaustive.

6. Independent events : Two or more events are said to be independent if the outcome of one event does not affect or gets affected by outcome of other events.

$\Rightarrow$ Example : ① If a coin is tossed twice, the second throw of coin is not affected by the 1st toss.

② While rolling a dice the outcomes of none of the events are affected by the other.

7. Dependent events : Dependent events are those events in which the occurence or non-occurence of any one trial does not affect the probability of other events in other trials.

$\Rightarrow$ ① For eg., if a card selected from a pack of well-shuffled 52 cards & it is not replaced then the second trial will have 51 cards which will surely affect the probability of any event in 2nd selection of cards.

② For eg., if from a box containing 5 red & 7 black balls, one ball is picked at random ~~is~~ and is kept aside. Then another ball is picked. In this case also, the 2nd selection will ~~get~~ change the probabilities because of the 1st selection of ball.

8. Equally likely events: Events are said to be equally likely if ~~one~~ any one of the event does not occur more often than others.

→ In other words, if events have equal probabilities then they are equally likely events.

⇒ Eg. ① If a roll a dice once, the probability of getting any no. between 1 to 6 will be equal i.e. each face of dice will equal chance of appearing.

⇒ Eg. ② When a coin is tossed once, the probability of getting head & chance of getting tail is same.

9. Complementary events: If two events A & B are mutually exclusive as well as exhaustive events then they are said to be compliment of each other.

* Intersection of Events :

$P(A\cap B) = P($occurence of both A & B$)$

$P(A\cap B)' = 1 - P(A\cap B)$

* Union of events :

$P(A\cup B) = P($occurence of either A or B$)$
$= P($ " " at least one of them$)$

Probability :

- **Definition-1 :** (Mathematical or classical approach)
If an experiment results in 'n' equally likely cases (ways) & out of which, 'm' are favourable to the happening of the event A, then the probability of occurence of A is defined as

$$P(A) = \frac{\text{no. of favourable cases}}{\text{no. of equally likely cases}} = \frac{m}{n}$$

- **Definition-2 :** (Statistical / Empirical / Posterior)
If an experiment is repeated under same conditions for a large no. of times then the limit of the ratio of no. of times the event happens to the total no. of trials is Statistical Probability :

i.e., $P(A) = \lim_{n\to\infty} \left(\frac{m}{n}\right)$

- **Definition-3** (Modern or axiomatic)

If A is any event from the sample space S, then P(A) is called the probability of A if it satisfies the following axioms:

(i) $0 \le P(A) \le 1$

(ii) $P(S) = 1$

(iii) If A & B are mutually exclusive (i.e. $A \cap B = \phi$) the probability of occurence of either A or b is,

$$P(A \cup B) = P(A) + P(B)$$

REMARK:

⟹ For any event A associated with a sample space S of a random experiment the probability of occurence of A is defined as

$$\boxed{P(A) = \frac{n(A)}{n(S)}}$$

where $n(A)$ = no. of sample points in set of A

$n(S)$ = no. of sample points in S.

⟹ $P(A) \nless 0$ & $P(A) \ngtr 1$ for any event A

⟹ $P(A) = 1 \Rightarrow$ A is sure or certain event

$P(A) = 0 \Rightarrow$ A " impossible event.

Q1.) Three unbiased coins are tossed together. Find the probability ~~that~~ getting
(i) exactly 2 heads
(ii) atleast one tail
(iii) ~~at~~ at most 2 heads
(iv) a head on second coin.
(v) exactly 2 heads in succession.

Sol.: Three coins are tossed together.

$S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$

$n(S) = 8$

(i) Let A: exactly 2 heads

Then $A = \{HHT, HTH, THH\}$

$n(A) = 3$

$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{3}{8}$

(ii) Let B: atleast one tail

Then $B = \{HHT, HTH, THH, HTT, THT, TTH, TTT\}$

$n(B) = 7$

$\therefore P(B) = \frac{n(B)}{n(S)} = \frac{7}{8}$

(iii) Let C: atmost 2 heads

Then $C = \{HHT, HTH, THH, HTT, THT, TTH, TTT\}$

$n(C) = 7$

$\therefore P(C) = \frac{7}{8}$

(iv) Let D: a head on 2^{nd} coin

$D = \{HHH, HHT, THH, THT\}$

$\therefore n(D) = 4$

$\therefore P(D) = \frac{n(D)}{n(S)} = \frac{4}{8} = \frac{1}{2}$

(v) Let E : getting 2 heads (exactly) in succession

$E = \{HHT, THH\}$

$n(E) = 2$

$P(E) = \frac{n(E)}{n(S)} = \frac{2}{8} = \frac{1}{4}$

Q2. A card is drawn from a pack of 52 playing cards. Find the probability of following events: That the card drawn is
(i) a spade (III) a face-card
(II) a king (IV) not a club
~~(v)~~ ,

Ans: Total cards = 52

$\therefore n(S) = 52$

i) Let A : getting a spade card

Total spade cards = 13

$\therefore n(A) = 13$

& $P(A) = \frac{n(A)}{n(S)} = \frac{13}{52} = \frac{1}{4}$

ii) Let B : getting a king

Total king = 4

$\therefore n(B) = 4$ & $P(B) = \frac{n(B)}{n(S)} = \frac{4}{52} = \frac{1}{13}$

iii) Let C : getting a face card

$n(C)$ = total face-cards } (4 Jack, 4 king & 4 queens)
$= 12$

$\therefore P(C) = \frac{12}{52} = \frac{3}{13}$

iv) Let D : not a club card

Total cards which are not club $= \cancel{13}\ 52 - 13$
$= 39$

$\therefore n(D) = 39$

& $P(D) = \frac{n(D)}{n(S)} = \frac{39}{52} = \frac{3}{4}$

Q3. Find the probability that an odd no. appears in a single toss of a fair dice.

Ans. Here $S = \{1, 2, 3, 4, 5, 6\} \Rightarrow n(S) = 6$

Let event A be appearance of odd no. on dice.

i.e. $A = \{1, 3, 5\} \Rightarrow n(A) = 3$

$\therefore$ Probability that an odd no. appears

$= P(A) = \frac{n(A)}{n(S)} = \frac{3}{6} = \frac{1}{2}$

Q4. Find the probability of getting at least one head in two throws of an unbiased coin.

Ans.: $S = \{HH, HT, TH, TT\} \Rightarrow n(S) = 4$

Let A : getting at least one head.

Then $A = \{HH, HT, TH\} \Rightarrow n(A) = 3$

$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{4}{3}$

Q5. What is the probability that a leap year will

53 sundays ?

Ans: In leap year, total days = 366

Total complete week = 52

Additional days = 02

∴ Each of the day (out of 7) will be repeated 52 times. Hence there will be 53 sundays iff ~~sunday~~ ~~is~~ one of the 02 additional days one is sunday.

→ Possible outcomes for 2 extra days

S = {(Mon, Tue), (Tue, Wed), (Wed, Thur), (Thur, Fri), (Fri, Sat), (Sat, Sun), (Sun, Mon)}

$n(S) = 7$

→ Favourable outcomes, E = {(Sat, Sun), (Sun, Mon)}

$n(E) = 2$

→ Probability that there will be 53 sundays

$$P(E) = \frac{n(E)}{n(S)} = \frac{2}{7}$$

• Remark: No. of ways to select r objects out of ~~to~~ total n objects is given as,

$$^{n}C_{r} = \frac{n!}{(n-r)!\,r!}$$

Q1. A committee of two persons is selected from two men & two women. What is the probability that the committee will have (i) no man (ii) one man (iii) both man?

Ans: 2 men & 2 women = 4 persons in total
Here two persons are selected from total 4 to ~~form~~ ~~from~~ form committee.

$\therefore n(S) = {}^4C_2 = 6$

1) Let A : committee will have no man.
or both members are women.
(2 women selected from total 2 women)

i.e. $n(A) = {}^2C_2 = 1$.

$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{1}{6}$

ii) Let B : committee will have one man.
(i.e. $\underbrace{\text{one man}}_{\text{1 selected from 2 men}}$ & $\underbrace{\text{one woman}}_{\text{selected from 2 women}}$)

$n(B) = {}^2C_1 \cdot {}^2C_1 = 4$

$\therefore P(B) = \frac{n(B)}{n(S)} = \frac{4}{6} = \frac{2}{3}$

$\boxed{\text{NOTE :}}$ for no. of sample points.
and $\rightarrow \times$
or $\rightarrow +$

iii) Let C : committee will have both members as men.
(i.e. 2 men selected from total 2 men)

$n(C) = {}^2C_2 = 1$

$\therefore P(C) = \frac{n(C)}{n(S)} = \frac{1}{6}$

Q2. From a collection of 10 bulbs of which 4 are defective, 3 bulbs are selected at random & fitted into lamps. Find the probability that

(i) all three bulbs glow, &
(ii) the ~~rom~~ room is lit.

Ans.: Total bulbs = 10
Defective bulbs = 4
$\therefore$ Good bulbs = 6

→ Here ~~2~~ 3 bulbs are selected from Total 10 bulbs.

$\therefore n(S) = {}^{10}C_3 = 120$

i) Let A: all three bulbs glow
(i.e. 3 bulbs selected from ~~3~~ 6 good bulbs)

$n(A) = {}^6C_3 = 20$

$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{20}{120} = \frac{1}{6}$

ii) Let B: the room is lit
(i.e, at least one of the 3 selected bulbs is good)
(i.e. all 3 good or 2 good & 1 defective or 1 good & 2 defective)

$n(\text{all 3 good}) = {}^{6}C_3 = 20$

$n(\text{2 good and 1 defective}) = {}^6C_2 \cdot {}^4C_1 = 60$

$n(\text{1 good and 2 defective}) = {}^6C_1 \cdot {}^4C_2 = 36$

$\therefore n(B) = 20 + 60 + 36 = 116$

& $P(B) = \frac{n(B)}{n(S)} = \frac{116}{120} = \frac{29}{30}$

and → ×
or → +
only for sample points

Q3. If an ~~ub~~ unbiased dice is rolled. Find the probability of getting :
(i) Even no. (ii) a perfect square
(iii) no. divisible by 3.

Ans: $S = \{1, 2, 3, 4, 5, 6\} \Rightarrow n(S) = 6$

i) A : even no. $\Rightarrow A = \{2, 4, 6\}$

$n(A) = 3 \Rightarrow P(A) = \frac{n(A)}{n(S)} = \frac{3}{6} = \frac{1}{2}$

ii) B : a perfect square

$B = \{4, 1\} \Rightarrow n(B) = 2$

$P(B) = \frac{2}{6} = \frac{1}{3}$

iii) C : no. divisible by 3.

$C = \{3, 6\} \Rightarrow n(C) = 2$

$P(C) = \frac{2}{6} = \frac{1}{3}$

Q4. Two cards are drawn from a pack of cards. Find the probability that they will be both red & both pictures.

Ans. Two cards are selected from 52

$n(S) = {}^{52}C_2 = 1326$

Now there are only 6 cards which are both red & both pictures. (Jack, queen & King of heart & diamond)

$\therefore n(E) = {}^{6}C_2 = 15$

Thus the required probability is

$P = \frac{n(E)}{n(S)} = \frac{15}{1326} = \frac{5}{442}$

* Replacements / without replacement examples:
(Dependent / Independent events) :

Q1. There are 5 red & 7 black balls in a box. Two balls are drawn at random one after the other. If they are drawn
1) with replacement (II) without replacement
find the probability that both balls are red ?

Ans: Total balls = 5 red + 7 black = 12

Two balls are drawn from total 12 balls.

$n(S) = \cancel{{}^{12}C_2 = 66}\ {}^{12}C_1 \cdot {}^{12}C_1$ (one after other)

$= 144$

1) with replacement

The event is, E : both are red

(Total red = 5)

$n(E) = {}^{5}C_1 \cdot {}^{5}C_1 = 25$

$\therefore P(E) = \dfrac{n(E)}{n(S)} = \dfrac{25}{144}$

II) without replacement

$n(S) = {}^{12}C_1 \cdot {}^{11}C_1 = 132$

$n(E) = {}^{5}C_1 \cdot {}^{4}C_1 = 20$

$\therefore P(E) = \dfrac{n(E)}{n(S)} = \dfrac{20}{132} = \dfrac{5}{33}$

Q2. Find the probability of a drawing a queen and a king from a pack of cards in 2 consecutive draws, the cards drawn not being replaced.

s: Here 2 consecutive draws from 52 cards

$n(S) = {}^{52}C_1 \, {}^{51}C_1$ (card not replaced)

$= 2652$

Getting a queen & a King $\rightarrow$ A (event)

$n(A) = {}^{4}C_1 \cdot {}^{4}C_1 = 16$

$P(A) = \frac{n(A)}{n(S)} = \frac{16}{2652} = \frac{4}{663}$

Q3. A bag contains 8 red & 5 white balls. Two successive draws of 3 balls each are made such that

i) the balls are replaced before second trial

ii) " " " not replaced " " "

Find the probability that the first draw will give 3 white and the second 3 red balls.

Ans: Total balls = 8 red + 5 white = 13

Here two successive draws of 3 balls are made each time.

i) with replacement before second trial.

$\therefore n(S) = {}^{13}C_3 \; {}^{13}C_3 = 81796$

Let event A : 1st trial gives 3 white balls

& 2nd trial gives 3 red "

$\therefore n(A) = {}^{5}C_3 \cdot {}^{8}C_3 = 560$

Hence the probability of event A

$P(A) = \frac{n(A)}{n(S)} = \frac{560}{817960} = \frac{140}{20449}$

ii) without replacing balls before 2^{nd} trial

Then $n(S) = {}^{13}C_3 \cdot {}^{10}C_3$ (3 less balls in 2^{nd} Trial)

$= 34320$

Again consider event A : 1^{st} Trial gives 3 white & 2^{nd} trial gives 3 red balls.

$\therefore n(A) = {}^5C_3 \cdot {}^8C_3 = 560$

Hence, $P(A) = \frac{n(A)}{n(S)} = \frac{560}{34320} = \frac{7}{429}$

Q4. From a bag containing 4 white & 6 black balls, 2 balls are drawn at random. If the balls are drawn one after the other without replacements, find the probability that the first ball is white and the second ball is black.

Ans.: Total balls = 4 white + 6 black = 10 balls

→ Here two balls are drawn one after the other without replacing 1^{st} picked ball.

$\therefore n(S) = {}^{10}C_1 \cdot {}^9C_1 = 90$

Let event A : 1^{st} picked ball is white and 2^{nd} " " " black

$\therefore n(A) = {}^4C_1 \cdot {}^6C_1 = 24$

Hence $P(A) = \frac{n(A)}{n(S)} = \frac{24}{90} = \frac{4}{15}$

ADDITION THEOREM :

→ If A and B are two events, the probability of occurence of either A or B is,

$$P(A\cup B) = P(A) + P(B) - P(A\cap B)$$

• REMARK :

i) If A and B are mutually exclusive events then $A\cap B = \phi \Rightarrow P(A\cap B) = 0$

$\therefore P(A\cup B) = P(A) + P(B)$

ii) For ~~ev~~ 3 events A, B and C

$$P(A\cup B\cup C) = P(A) + P(B) + P(C) - P(A\cap B) - P(B\cap C) - P(A\cap C) + P(A\cap B\cap C)$$

iii) $P(A\cap B) = P(A) + P(B) - P(A\cup B)$

iv) $P(A\cup B)' = 1 - P(A\cup B)$

EXAMPLES

Q1) If $P(A) = \frac{1}{3}$, $P(B') = \frac{1}{4}$, $P(A\cap B) = \frac{1}{6}$, find

(i) $P(A\cup B)$ (ii) $P(A'\cap B')$ (iii) $P(A'\cup B')$

(iv) $P(\text{only } A)$ (v) $P(\text{only } B)$

Ans.: $P(B) = 1 - P(B') = 1 - \frac{1}{4} = \frac{3}{4}$

i) $P(A\cup B) = P(A) + P(B) - P(A\cap B)$

$$= \frac{1}{3} + \frac{3}{4} - \frac{1}{6} = \frac{11}{12}$$

NOTE:

I) $(A \cup B)' = A' \cap B'$

II) $(A \cap B)' = A' \cup B'$

II) $P(A' \cap B')$

$= P(A \cup B)'$

$= 1 - P(A \cup B)$

$= 1 - \frac{11}{12} = \frac{1}{12}$

III) $P(A' \cup B') = P(A \cap B)'$

$= 1 - P(A \cap B)$

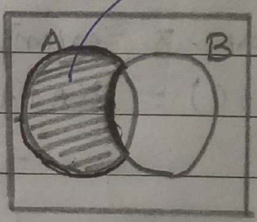
$= 1 - \frac{1}{6} = \frac{5}{6}$

IV) $P(\text{only } A)$

$= P(A) - P(A \cap B)$

$= \frac{1}{3} - \frac{1}{6} = \frac{1}{6}$

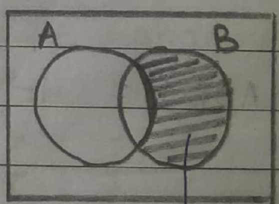
only A



V) $P(\text{only } B)$

$= P(B) - P(A \cap B)$

$= \frac{3}{4} - \frac{1}{6} = \frac{7}{12}$



only B

- **REMARK:**

I) At least one, either ... or ..., minimum one of them
→ union

II) both A and B, A and B together,
→ intersection

III) not ... at least one → $(A \cap B)'$

IV) not both A and B → $(A \cup B)'$

V) only A
→ $P(A) - P(A \cap B)$

VI) only B
→ $P(B) - P(A \cap B)$

Q2. The probability that a contractor will get a contractor will get a contract is 2/3 and the probability that he will get on other contract is 5/9. If the probability of getting at least one contract is 4/5, what is the probability that he will get both the contracts?

Ans.: Let event A be : getting 1^{st} contract
" B : " another contract

Then $P(A) = \frac{2}{3}$, $P(B) = \frac{5}{9}$

and $P(\text{at least one contract}) = P(A \cup B) = \frac{4}{5}$

$\therefore P(\text{both } A \text{ and } B) = P(A \cap B)$
$= P(A) + P(B) - P(A \cup B)$
$= \frac{2}{3} + \frac{5}{9} - \frac{4}{5} = \frac{19}{45}$

Q3. A card is drawn from a well-shufled pack of cards. What is the probability that it is either a spade or an ace?

Ans: Total cards = 52 $\Rightarrow$ $n(S) = 52$
Let A : getting a spade card
B : " an ace card

Then,

$P(A) = \frac{n(A)}{n(S)} = \frac{13}{52} = \frac{1}{4}$

$P(B) = \frac{n(B)}{n(S)} = \frac{4}{52} = \frac{1}{13}$

$P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{1}{52}$

$P(A \cap B)$
$= P(A \text{ and } B \text{ both})$
$n(\text{both spade and ace})$
= only 1 card
(ace of spade)

$\therefore$ P(either A or B)

$= P(A \cup B)$

$= P(A) + P(B) - P(A \cap B)$

$= \frac{1}{4} + \frac{1}{13} - \frac{1}{52} = \frac{4}{13}$

Q4. The probability that a contractor will get a contract is 2/3 and the probability that he will <u>not</u> get another contract is 5/9. If the probability of getting any one contract is 4/5, what is the probability he will get both the contract?

Ans: Let A : getting a contract

B : getting another contract

Then $P(A) = \frac{2}{3}$, $P(B') = \frac{5}{9}$ (B' : not getting)

$\Rightarrow P(B) = 1 - P(B') = \frac{4}{9}$

P(getting any one of A and B)

$= P(A \cup B) = 4/5$

$\rightarrow$ P(getting both A and B)

$= P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$= \frac{2}{3} + \frac{4}{9} - \frac{4}{5} = \frac{14}{45}$

Q6. Two cards are drawn from a pack of cards. Find the probability that they will be both red or both pictures.

Ans.: Let event A : getting both red cards
B : " " picture cards
(Jack, King, queen)

Then $P(A) = \frac{n(A)}{n(S)}$ and $P(B) = \frac{n(B)}{n(S)}$

→ Here two cards selected from 52 cards
$n(S) = {}^{52}C_2 = 1326$

→ Total red cards = 26
For A, 2 cards selected from 26 red cards
$\therefore n(A) = {}^{26}C_2 = 325$

$\therefore P(A) = \frac{325}{1326}$

→ Total picture cards = 12 out of which 2 selected for event B.
$\therefore n(B) = {}^{12}C_2 = 66$

$\therefore P(B) = \frac{66}{1326}$

(same example done earlier)

→ Now, $P(A \cap B) = P(\text{both red and pictures})$
$= \frac{{}^{6}C_2}{{}^{52}C_2} = \frac{15}{1326}$

→ P(both red or both pictures)
$= P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{376}{1326} = \frac{188}{663}$

Q6. A box contains 4 white, 6 red, 5 black and 5 balls of other colours. Two balls are drawn from the box at random. Find the probability that (i) both are white or both are red, and (ii) both are red or both are black.

Ans.: Total balls = 4 white + 6 red + 5 black + 5 others = 20 balls

→ Here two balls selected at random from box of 20 balls

$\therefore n(S) = {}^{20}C_2 = 190$

i) A : both are white or both are red

Let A_1 : both white, A_2 : both are red

$P(A_1) = \frac{n(A_1)}{n(S)} = \frac{{}^4C_2}{190} = \frac{6}{190}$

$P(A_2) = \frac{n(A_2)}{n(S)} = \frac{{}^6C_2}{190} = \frac{15}{190}$

$\therefore P(A) = P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2)$

$= \frac{6}{190} + \frac{15}{190} - 0$

$= \frac{21}{190}$

$A_1 \cap A_2 = \phi$

$\Rightarrow P(A_1 \cap A_2) = 0$

ii) B : both red or both black

Let B_1 : both red, B_2 : both black

Then $P(B_1) = \frac{{}^6C_2}{190} = \frac{15}{190}$

$P(B_2) = \frac{n(B_2)}{n(S)} = \frac{{}^5C_2}{190} = \frac{10}{190}$

$\therefore P(AB) = P(B_1 \text{ or } B_2) = P(B_1 \cup B_2)$

$= P(B_1) + P(B_2) - P(B_1 \cap B_2)$

$= \frac{15}{190} + \frac{10}{190} - 0 = \frac{25}{190} = \frac{5}{38}$

[$B_1 \cap B_2$ = both red and both black
at a time both balls can't be red and black
$\therefore B_1 \cap B_2 = \emptyset \Rightarrow P(B_1 \cap B_2) = 0$]

Q7. If probability of getting a contract A is 0.25, of getting a contract B is 0.15 and the probability of getting at least one contract is 0.30, calculate the probability of

i) getting both the contracts
ii) " getting contract A only
iii) " " B "
iv) not " both the contracts
v) not getting at least one contract.

Ans: Let A: getting contract A
B: " " B

Then, $P(A) = 0.25$, $P(B) = 0.15$ and
$P(\text{getting both A and B}) = P(A \cap B)$
while $P(\text{getting at least one contract}) = P(A \cup B) = 0.30$

i) $P(\text{both A \& B}) = P(A \cap B)$
$= P(A) + P(B) - P(A \cup B)$
$= 0.25 + 0.15 - 30 = 0.1$

ii) $P(\text{only A}) = P(A) - P(A \cap B) = 0.25 - 0.10 = 0.15$

iii) $P(\text{only } B) = P(B) - P(A \cap B)$
$= 0.15 - 0.10 = 0.05$

iv) $P(\text{not both } A \& B)$
$= P(A \cup B)' = 1 - P(A \cup B) = 1 - 0.3 = 0.7$

v) $P(\text{not at least one})$
$= P(A \cap B)' = 1 - P(A \cap B) = 1 - 0.1 = 0.9$

Q8. Two students Anil and Ashima appeared in an exam. The probability that Anil will qualify exam is 0.05 and that Ashima will qualify is 0.10. The probability that both will qualify is 0.02. Find the probability that
(a) Both Anil and Ashima will not qualify the exam.
(b) at least one of them will not qualify
(c) only one of them will qualify.

Ans.: Let A : Anil will qualify
B : Ashima will qualify
Then $P(A) = 0.05$, $P(B) = 0.10$
$P(\text{both } A \text{ and } B) = P(A \cap B) = 0.02$

1) $P(\text{both will not})$
$= P(A \cup B)' = 1 - P(A \cup B)$
$= 1 - (P(A) + P(B) - P(A \cap B))$
$= 1 - [0.05 + 0.10 - 0.02) = 0.87$

ii) P(at least one of them will not qualify)
$= P(A\cap B)' = 1 - P(A\cap B) = 1 - 0.02 = 0.98$

iii) P(only one will qualify)
$= P(\text{only } A) + P(\text{only } B)$
$= P(A) - P(A\cap B) + P(B) - P(A\cap B)$
$= P(A) + P(B) - 2P(A\cap B)$
$= 0.05 + 0.10 - 2(0.02) = 0.11$

Q9. A card is ~~Ka~~ drawn at random from pack of well shuffled 52 cards. Find the probability that the card drawn is (i) a king (ii) not a diamond (iii) an ace of hearts or diamonds.

Ans: $n(S) = 52$

i) $P(\text{a king}) = \dfrac{n(\text{Kings})}{n(S)} = \dfrac{4}{52} = \dfrac{1}{13}$

ii) P(not a diamond)
$= 1 - P(\text{a doiamond})$
$= 1 - \dfrac{n(\text{diamonds})}{n(S)}$
$= 1 - \dfrac{13}{52} = 1 - \dfrac{1}{4} = \dfrac{3}{4}$

iii) P(ace of hearts or diamonds)
Let A_H : ace of hearts
A_D : " " diamonds
$P(A_H \text{ or } A_D) = P(A_H \cup A_D) = P(A_H) + P(A_D) - P(A_H \cap A_D)$

$= \frac{1}{52} + \frac{1}{52} - \frac{0}{52}$ (no. of cards which are both ace of hearts diamonds = 0)

$= \frac{2}{52} = \frac{1}{26}$

MULTIPLICATION THEOREM :

→ If two events A and B are independent, the probability of occurence of both the events is equal to the product of the individual probabilities of both the events.
i.e., $P(A \cap B) = P(A) \cdot P(B)$.

Q1.) Two students X and Y work independently on a problem. The probability that X will solve it is 3/4 and that Y will solve it is 2/3. What is the probability that the problem will be solved?

Ans: Let event X : student X will solve
Y : " Y " "

Then $P(X) = \frac{3}{4}$, $P(Y) = \frac{2}{3}$

Now, X and Y work independently

$\therefore P(X \cap Y) = P(X) \cdot P(Y) = \frac{3}{4} \cdot \frac{2}{3} = \frac{1}{2}$

Finally, P(problem will be solved)
= P(atleast one of X & Y will solve it)
$= P(X \cup Y) = P(X) + P(Y) - P(X \cap Y)$

$$= \frac{3}{4} + \frac{2}{3} - \frac{1}{2} = \frac{11}{12}$$

Q2. A dice is thrown. If E is the event 'the no. appearing is a multiple of 3' and F be the event 'the no. appearing is even' then find whether E and F are independent?

Ans.: $S = \{1, 2, 3, 4, 5, 6\}$, $E = \{3, 6\}$, $F = \{2, 4, 6\}$ and $E \cap F = \{6\}$

Now, $P(E) = \frac{n(E)}{n(S)} = \frac{2}{6} = \frac{1}{3}$

$P(F) = \frac{n(F)}{n(S)} = \frac{3}{6} = \frac{1}{2}$

and $P(E \cap F) = \frac{n(E \cap F)}{n(S)} = \frac{1}{6}$

Now, if E & F are independent then by the Multiplication theorem,

$$P(E \cap F) = P(E) \cdot P(F)$$

$\therefore$ we check for the same with given values.

Here $P(E) \cdot P(F) = \frac{1}{3} \cdot \frac{1}{2} = \frac{1}{6} = P(E \cap F)$

$\therefore$ E & F are independent events.

Q3. Three coins are tossed simultaneously. Consider the event E: '3 heads or 3 tails', F: 'at least 2 heads' and G: 'at most 2 heads'. Of the pairs (E,F), (E,G) and (F,G), which are independent and which are dependent?

Ans.: $P(E) = P(3H \text{ or } 3T)$
$= P(3 \text{ heads}) + P(3 \text{ tails}) - P(3 \text{ heads \& tails both})$

$= \frac{1}{8} + \frac{1}{8} - 0 = \frac{2}{8} = \frac{1}{4}$

[S = {HHH, HHT, HTT, TTT, TTH, THH, HTH, THT}
$\Rightarrow n(S) = 8$ (E = {HHH, TTT}, $n(E) = 2$)]

$P(F) = \frac{n(F)}{n(S)} = \frac{4}{8} = \frac{1}{2}$

[F = {HHH, HHT, HTH, THH}
$n(F) = 4$]

$P(G) = \frac{7}{8}$

[G = {~~HHH~~, HHT, THH, HTH, HTT, THT, TTH, TTT}
$n(G) = 7$]

→ For the pair (E, F):

$E \cap F$ = {HHH} $\Rightarrow n(E) = 1$

$\therefore P(E \cap F) = \frac{n(E \cap F)}{n(S)} = \frac{1}{8}$

→ To check independency:

$P(E) \cdot P(F) = \frac{1}{4} \cdot \frac{1}{2} = \frac{1}{8} = P(E \cap F)$

$\therefore$ E and F are independent.

→ For the pair (E, G):

$E \cap G$ = {TTT} $\Rightarrow n(E \cap G) = 1$

$\therefore P(E \cap G) = \dfrac{n(E \cap G)}{n(S)} = \dfrac{1}{8}$

→ To check independency:

$P(E) \cdot P(G) = \dfrac{1}{4} \cdot \dfrac{7}{8} = \dfrac{7}{32} \neq P(E \cap G)$

$\therefore$ E and G are not independent or they are dependent.

→ For the pair (F, G):

$F \cap G = \{HHT, HTH, THH\} \Rightarrow n(F \cap G) = 3$

$\therefore P(F \cap G) = \dfrac{n(F \cap G)}{n(S)} = \dfrac{3}{8}$

→ To check independency:

$P(F) \cdot P(G) = \dfrac{1}{2} \cdot \dfrac{7}{8} = \dfrac{7}{16} \neq P(F \cap G)$

$\therefore$ F and G are dependent.

CONDITIONAL PROBABILITY:

- Definition: If E and F are two events associated with the sample space of a random experiment, the conditional probability of the event E given that F has already occured,

i.e. $P(E|F) = \dfrac{P(E \cap F)}{P(F)}$

provided $P(F) \neq 0$

- **Properties of conditional probability:**

→ Let E and F be events of a sample space S of an experiment, then we have

i) $P(S|F) = P(F|F) = 1$

ii) $P(E'|F) = 1 - P(E|F)$

iii) If ~~any~~ G is another event of same random experiment Then,

$$P(E \cup F | G) = P(E|G) + P(F|G) - P(E \cap F | G)$$

(Addition theorem works similarly for conditional probability)

Q1. Consider the random experiment of tossing three fair coins simultaneously. Let E be the event 'at least two heads appear' and F be the event 'first coin shows tail'. Then find the probability of E if F has occured.

Ans.: Here $S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$

$E = \{HHH, HHT, HTH, THH\}$

$F = \{THH, THT, TTH, TTT\}$

$\therefore P(E) = \dfrac{n(E)}{n(S)} = \dfrac{4}{8} = \dfrac{1}{2}$ and

$P(F) = \dfrac{n(F)}{n(S)} = \dfrac{4}{8} = \dfrac{1}{2}$

Here, $E \cap F = \{THH\} \Rightarrow P(E \cap F) = \dfrac{n(E \cap F)}{n(S)} = \dfrac{1}{8}$

$\therefore$ P(E if F has occurred)

$= P(E|F) = \dfrac{P(E \cap F)}{P(F)} = \dfrac{1/8}{1/2} = \dfrac{2}{8} = \dfrac{1}{4}$

Q1. A family has two children. What is the probability that both the children are boys given that at least one of them is a boy?

Ans: Let B : the child is a boy.
G : " " " " girl.

Then $S = \{(B,G), (B,B), (G,B), (G,G)\}$

Now consider the following events:

A : both childrens are boys

B : at least one of the child is a boy.

Then $A = \{(B,B)\}$ and $B = \{(B,B), (B,G), (G,B)\}$

Also $A \cap B = \{(B,B)\}$

Hence, P(A given that B)

$$= P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{n(A \cap B)/n(S)}{n(B)/n(S)}$$

$$= \frac{n(A \cap B)}{n(B)} = \frac{1}{3}$$

• REMARK: $P(A|B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{\frac{n(A \cap B)}{n(S)}}{\frac{n(B)}{n(S)}} = \dfrac{n(A \cap B)}{n(B)}$

i.e. $P(A|B) = \dfrac{n(A \cap B)}{n(B)}$ [Use this formula when probability not given]

Q3. In a certain college, 25% students failed in Maths, 15% students failed in Statistics and 30% students failed in at least one of the ~~pro~~ subject selected at random. Find the probability that

i) he failed in both the subjects.

ii) " " " Maths if he also failed in Statistics.

iii) he failed in Statistics if he also failed in Maths.

iv) he failed in Maths given that he passed in Statistics.

Ans: Let M : failed in Maths

S : " " Statistics

Then $P(M) = 25\% = 0.25$

$P(S) = 15\% = 0.15$

and $P(\text{at least one}) = P(M \cup S) = 30\% = 0.30$

i) P(failed in both M and S)

$= P(M \cap S)$

$= P(M) + P(S) - P(M \cup S)$

$= 0.25 + 0.15 - 0.30 = 0.10$

ii) P(failed in Maths if also failed in Stats)

$= P(M|S)$

$$= \frac{P(M \cap S)}{P(S)} = \frac{0.10}{0.15} = \frac{2}{3}$$

iii) P(failed in Statistics if also failed in Maths)

$$= P(S|M) = \frac{P(S \cap M)}{P(M)} = \frac{0.10}{0.25} = \frac{2}{5}$$

iv) P(failed in Maths given that passed in Statistics)

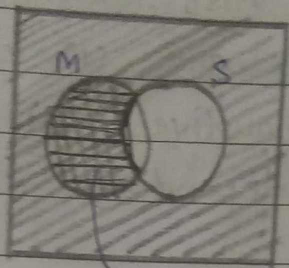
$= P(M|S')$ [S' : passed in Statistics]

$= \dfrac{P(M \cap S')}{P(S')}$

$= \dfrac{P(M) - P(M \cap S)}{1 - P(S)}$

$= \dfrac{0.25 - 0.10}{1 - 0.15}$

$= 0.1765$



$P\ M \cap S'$ = only M

Q4. If A and B are two events with $P(A) = 1/3$, $P(B) = 1/4$, $P(A \cap B) = 1/12$. Find
(i) $P(A|B)$ (ii) $P(B|A)$ (iv) $P(B|\bar{A})$ (iii) $P(A \cap B')$

Ans.: (i) $P(A|B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{1/12}{1/4} = \dfrac{1}{3}$

(ii) $P(B|A) = \dfrac{P(B \cap A)}{P(A)} = \dfrac{1/12}{1/3} = \dfrac{1}{4}$

(iii) $P(A \cap B') = P(A) - P(A \cap B)$
$= \dfrac{1}{3} - \dfrac{1}{12} = \dfrac{3}{12} = \dfrac{1}{4}$

(iv) $P(B|A') = \dfrac{P(B \cap A')}{P(A')}$

$= \dfrac{P(B) - P(B \cap A)}{1 - P(A)}$

$= \dfrac{\frac{1}{4} - \frac{1}{12}}{1 - \frac{1}{3}} = \dfrac{1/6}{2/3} = \dfrac{1}{4}$

Q5. If A and B are 2 events such that $P(A) = 2/3$, $P(A' \cap B) = 1/6$, $P(A \cap B) = 1/3$, find $P(B)$, $P(A \cup B)$, $P(A' \cup B)$, $P(A|B)$, $P(B|A)$ and $P(B')$. Also examine whether the events A and B are (i) equally likely, (ii) exhaustive, (iii) mutually exclusive, and (iv) independent.

Ans.: i) $P(B) = ?$

Given $P(A' \cap B) = P(B \cap A')$

$\Rightarrow 1/6 = P(B) - P(B \cap A)$

$\Rightarrow 1/6 = P(B) - 1/3$

$\Rightarrow P(B) = 1/6 + 1/3 = 3/6 = 1/2$

ii) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{2}{3} + \frac{1}{2} - \frac{1}{3} = \frac{5}{6}$

iii) $P(A' \cup B) = P(A') + P(B) - P(A' \cap B)$ (addition theorem)

$= 1 - P(A) + P(B) - P(A' \cap B)$

$= 1 - \frac{2}{3} + \frac{1}{2} - \frac{1}{6} = \frac{4}{6} = \frac{2}{3}$

iv) $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{1/3}{1/2} = \frac{2}{3}$

v) $P(B|A) = \frac{P(B \cap A)}{P(A)} = \frac{1/3}{2/3} = \frac{1}{2}$

vi) $P(B') = 1 - P(B) = 1 - \frac{1}{2} = \frac{1}{2}$

2nd part of que. :

I) To check for equally likely : $(P(A) = P(B))$

→ Since $P(A) \neq P(B)$, ∴ they (A and B) are not equally likely events.

II) To check for exhaustive ; $[P(A \cup B) = P(S) = 1]$

→ Since $P(A \cup B) = \frac{5}{6} \neq 1$

∴ A and B are not exhaustive events.

III) To check for mutually exclusive :
$[P(A \cap B) = 0]$

→ Since $P(A \cap B) = \frac{1}{3} \neq 0$,

∴ A and B are not mutually exclusive events.

IV) To check for independent :
$[P(A \cap B) = P(A) \cdot P(B)]$

→ Here $P(A) \cdot P(B) = \frac{2}{3} \cdot \frac{1}{2} = \frac{1}{3} = P(A \cap B)$

∴ A and B are independent events.

• REMARK :

→ If the ~~sample~~ no. of sample points of the events are given then

$$P(A|B) = \frac{n(A \cap B)}{n(B)}$$

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Q6. The personnel department of a company has records of its 200 engineers as follows :

Age (year)	Bachelor's degree only	Master's degree	Total
Under 30	90	10	100
30 to 40	20	30	50
Over 40	40	10	50
Total	150	50	200

If one engineer is selected at random from the company, find the probability that

(i) he has only a Bachelor's degree

(ii) he has a Master's degree given that he is over 40.

(iii) he is under 30 given that he has only a Bachelor's degree.

Ans.: i) P(only Bachelor's degree)

$= P(B)$

$= \frac{n(B)}{n(S)} = \frac{150}{200} = \frac{3}{4}$

ii) Let M : he has a Master's degree

O_{40} : he is over 40

Then $P(M|O_{40}) = \frac{n(M \cap O_{40})}{n(O_{40})}$

$= \frac{10}{50} = \frac{1}{5}$

iii) Let U_{30} : he is under 30

and B : he has only Bachelor's degree.

Then $P(U_{30} | B) = \dfrac{n(U_{30} \cap B)}{n(B)}$

$= \dfrac{90}{150} = \dfrac{3}{5}$

Q7. A department store has been the target of many shoplifters during the past month, but owing to increased security precautions, 250 shoplifters have been caught. and their details are noted which are as follows:

Gender	1^{st}-time offender	Repeat offender	Total
Males	60	70	130
Females	44	76	120
Total	104	146	250

Assume that a shoplifter is selected at random, find: the probability that

a) shoplifter is male

b) shoplifter is a 1^{st} time offender, given that the shoplifter is male.

c) shoplifter is female, given that the shoplifter is a repeat offender.

d) shoplifter is female, given that she is a 1^{st}-time offender.

Ans.: Let M : shoplifter is male

F : " " female

O_1 : shoplifter is 1^{st} time offender
O_R : " " repeat "

a) $P(M) = \dfrac{n(M)}{n(S)} = \dfrac{130}{250} = \dfrac{13}{25}$

b) $P(O_1|M) = \dfrac{n(O_1 \cap M)}{n(M)} = \dfrac{60}{130} = \dfrac{6}{13}$

c) $P(F|O_R) = \dfrac{n(F \cap O_R)}{n(O_R)} = \dfrac{76}{146} = 0.5205$

d) $P(F|O_1) = \dfrac{n(F \cap O_1)}{n(O_1)} = \dfrac{44}{104} = 0.4231$

Baye's Theorem :

→ "If an event A corresponds to a no. of mutually exclusive events $B_1, B_2, \ldots, B_n$ and if $P(B_1), P(B_2), \ldots, P(B_n)$ and $P(A|B_1), P(A|B_2), \ldots, P(A|B_n)$ are known and $P(A) > 0$, then

$$P(B_i|A) = \frac{P(B_i)\, P(A|B_i)}{\sum_{i=1}^{n} P(B_i)\, P(A|B_i)}, \quad i = 1, 2, \ldots, n$$

→ In particular case for $n=2$:
If an event A corresponds to two

mutually exclusive events B_1 and B_2 and if $P(B_1)$, $P(B_2)$, $P(A|B_1)$, $P(A|B_2)$ are known and $P(A) > 0$, then

$$P(B_1|A) = \frac{P(A|B_1) \cdot P(B_1)}{P(A|B_1) \cdot P(B_1) + P(A|B_2) \cdot P(B_2)},$$

and $$P(B_2|A) = \frac{P(A|B_2) \cdot P(B_2)}{P(A|B_1) \cdot P(B_1) + P(A|B_2) \cdot P(B_2)}$$

→ Similarly for $n = 3$: $[A, B_1, B_2, B_3]$

$$P(B_1|A) = \frac{P(A|B_1) \cdot P(B_1)}{P(A|B_1)\, P(B_1) + P(A|B_2)\, P(B_2) + P(A|B_3)\, P(B_3)}$$

$$P(B_2|A) = \frac{P(A|B_2)\ P(B_2)}{P(A|B_1)\ P(B_1) + P(A|B_2)\ P(B_2) + P(A|B_3)\ P(B_3)}$$

$$P(B_3|A) = \frac{P(A|B_3)\ P(B_3)}{P(A|B_1)\ P(B_1) + P(A|B_2)\ P(B_2) + P(A|B_3)\ P(B_3)}$$

81.) In a pharmaceutical factory, machines A and B manufacture 40% and 60% of the total output. Of this production, machines A and B produce 5% and 10% defective tablets. A tablet is picked at random and is found to be defective. What is the probability that tablet was produced by the machine A?

Ans: Let A : tablet is produced by machine A
B : " " " " machine B

$\therefore P(A) = 40\% = \frac{40}{100} = 0.40$, $P(B) = 60\% = 0.60$

Let D : the tablet is defective.
Then $P(D|A) = 5\% = \frac{5}{100} = 0.05$

$P(D|B) = 10\% = \frac{10}{100} = 0.10$

Now the selected tablet is already found to be defective, then the probability that it is from machine A is given by,

$$P(A|D) = \frac{P(D|A) \cdot P(A)}{P(D|A)\, P(A) + P(D|B)\, P(B)}$$

$$= \frac{(0.05)(0.40)}{(0.05)(0.40) + (0.10)(0.60)}$$

$$= \frac{0.02}{0.08} = 0.25$$

Q2. In a bolt factory machines A, B and C manufacture 25%, 35% and 40% of the total. Of their output 5%, 4% and 2% are defective bolts. A bolt is drawn at random from the product and is found to be defective. What is the probability that it was manufactured by machine A?

Ans.: Let A : Selected bolt is from machine A
B : " " " " " B
C : " " " " " C

$\therefore P(A) = 25\% = 0.25$,
$P(B) = 35\% = 0.35$ and $P(C) = 40\% = 0.40$

Let D : Selected bolt is defective.
Then $P(D/A) = 0.05$, $P(D|B) = 0.04$, $P(D|C) = 0.02$
Now if selected bolt is known to be defective in advance then probability that it is from machine A is given by,

$$P(A|D) = \frac{P(D|A) \cdot P(A)}{P(D|A)\,P(A) + P(D|B)\,P(B) + P(D|C)\,P(C)}$$

$$= \frac{(0.05)(0.25)}{(0.05)(0.25) + (0.35)(0.04) + (0.40)(0.02)}$$

$$= \frac{0.0125}{0.0345} = 0.3623$$

Q3. In a railway reservation office, two clerks are in checking reservation forms. On an average the clerk A checks 55% of the forms, while the clerk B does the remaining work. The clerk A has an error rate of 0.03 and clerk B has an error rate of 0.02. A reservation form is selected at random from the total no. of forms checked and is found to have an error. Find the probability that it was checked by clerk A and B respectively.

Ans: Let A : Selected form is checked by clerk A
B : " " " " " B

$\therefore P(A) = 0.55$ and $P(B) = 1 - 0.55 = 0.45$
(B does remaining work $= 100 - 55 = 45\%$)

Let E : selected form has error.
Then $P(E/A) = 0.03$ and $P(E/B) = 0.02$
Now if a selected form is known to have an error then the probability that it is checked by A is, $P(A/E)$ and that it is " " B is $P(B/E)$ given as follows :

$$P(A/E) = \frac{P(E/A) \cdot P(A)}{P(E/A)\, P(A) + P(E/B)\, P(B)}$$

$$= \frac{(0.03)(0.55)}{(0.03)(0.55) + (0.02)(0.45)}$$

$$= \frac{0.0165}{0.0255} = 0.6471$$

and

$$P(B/E) = \frac{P(E/B) \cdot P(B)}{P(E/A)\, P(A) + P(E/B)\, P(B)}$$

$$= \frac{(0.02)(0.45)}{(0.03)(0.55) + (0.02)(0.45)}$$

$$= \frac{0.0090}{0.0255} = 0.3529$$

Q4. Suppose that 5 men out of 100 and 25 women out of 10000 are colour blind. A colour blind person is chosen at random. What is the probability of his being male? (Assume males and females to be in equal numbers)

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Ans: Let M : Selected person is male
F : Selected " " female

Since it is assumed that males and females are equal in nos.

$P(M) = \frac{1}{2} = 0.5$ and $P(F) = \frac{1}{2} = 0.5$

(50 % males) (50% females)

Let B : Selected person is colour blind

Then $P(B|M) = \frac{5}{100} = 0.05$ (5 men out of 100)

and $P(B|F) = \frac{25}{10000} = 0.0025$ (25 women out of 10000)

Now if the selected person is known to be colour blind then the probability the person is male, is given by

$$P(M|B) = \frac{P(B|M)\ P(M)}{P(M)\ P(B|M) + P(F)\ P(B|F)}$$

$$= \frac{(0.05)(0.5)}{(0.05)(0.5) + (0.0025)(0.5)}$$

$$= \frac{0.0250}{0.0263} = 0.9524$$

Q5. A company has 2 plants to manufacture hydraulic machines. Plant I manufactures 70% of the hydraulic machines, and plant II manufactures 30%. At plant I, hydraulic ~~pd~~ machines are rated standard quality, and at plant II, 90% are rated standard quality. A machine is picked at random and is found to be of standard quality. What is the chance it has come from plant I?

Ans: Let P_1 : machine is from plant I

P_2 : " " " " II

$\therefore P(P_1) = 0.70$ and $P(P_2) = 0.30$

Let S : machine is of standard quality

Then $P(S|P_1) = 0.80$ and $P(S|P_2) = 0.90$

Now if selected machine is known to be of standard quality then probability of its being from plant I is,

$$P(P_1|S) = \frac{P(S|P_1) \cdot P(P_1)}{P(S|P_1) \cdot P(P_1) + P(S|P_2) \cdot P(P_2)}$$

$$= \frac{(0.80)(0.70)}{(0.80)(0.70) + (0.90)(0.30)}$$

$$= \frac{0.56}{0.83} = 0.6747$$

Q6. A bag 'A' contains 2 white and 3 red balls and a bag 'B' contains 4 white and 5 red balls. One ball is drawn at random from one of the bags & it is found to be red. Find the probability that the red ball is from bag B.

Ans. In bag A : 2 white + 3 Red = 5 balls
B : 4 White + 5 Red = 9 balls

Total = 14 balls

Let, R = Selected ball is Red
A = Selected ball is from bag A
B = Selected ball is from bag B

Now, Probability E of a ball to be from B if it is known to be Red is given by,

$$P(B/R) = \frac{P(R/B)\,P(B)}{P(R/A)\,P(A) + P(R/B)\,P(B)}$$

where,

$$P(A) = \frac{n(A)}{n(S)} = \frac{1}{2}$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{1}{2}$$

Also, $P(R/A) = \dfrac{n(R \cap A)}{n(A)} = \dfrac{3}{5}$

$$P(R/B) = \frac{n(R \cap B)}{n(B)} = \frac{5}{9}$$

$$\therefore \quad P(B/R) = \frac{P(R/B)\,P(B)}{P(R/A)\,P(A) + P(R/B)\,P(B)}$$

$$= \frac{\frac{5}{9} \cdot \frac{1}{2}}{\left[\left(\frac{3}{5} \cdot \frac{1}{2}\right) + \left(\frac{5}{9} \cdot \frac{1}{2}\right)\right]}$$

$$= \frac{\frac{5}{18}}{\frac{3}{10} + \frac{5}{18}}$$

$$= 0.4808$$

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