Assignment - 1

Ques 1)

No. of sales calls (x)	No. of machines sold (y)	$(x-\bar{x})$	$(y-\bar{y})$	$(x-\bar{x})^2$	$(y-\bar{y})^2$	$(x-\bar{x})(y-\bar{y})$
20	30	-2	-15	4	225	30
40	60	18	15	324	225	270
20	40	-2	-5	4	25	10
30	60	8	15	64	225	120
10	30	-12	-15	144	225	180
10	40	-12	-5	144	25	60
20	40	-2	-5	4	25	10
20	50	-2	5	4	25	-10
20	30	-2	-15	4	225	30
30	70	8	25	64	625	200
$\Sigma x = 220$	$\Sigma y = 450$	$\Sigma(x-\bar{x})=0$	$\Sigma(y-\bar{y})=0$	$\Sigma(x-\bar{x})^2 = 760$	$\Sigma(y-\bar{y})^2 = 1850$	$\Sigma(x-\bar{x})(y-\bar{y}) = 900$

$$\bar{x} = \frac{\Sigma x}{n} = \frac{220}{10} = 22$$

$$\bar{y} = \frac{\Sigma y}{n} = \frac{450}{10} = 45$$

$$r = \frac{\sum (x-\bar{x})(y-\bar{y})}{\sqrt{\sum (x-\bar{x})^2}\sqrt{\sum (y-\bar{y})^2}}$$

$$= \frac{900}{\sqrt{760} \times \sqrt{1850}}$$

$$= \frac{900}{1,185.755}$$

$$= 0.759 =$$

$$\therefore \boxed{r = 0.759}$$

Ques 2)

Age	Mean Age (x)	Candidates appeared	Candidates passed	Success % (y)	xy	x^2	y^2
20-22	21	120	102	85	1785	441	7225
22-24	23	100	82	82	1886	529	6724
24-26	25	75	63	84	2100	625	7056
26-28	27	90	72	80	2160	729	6400
28-30	29	50	39	78	2262	841	6084
30-32	31	40	30	75	2325	961	5625
	$\Sigma x = 156$			$\Sigma y = 484$	$\Sigma xy =$ 12518	$\Sigma x^2 =$ 4126	$\Sigma y^2 =$ 39114

$$r = \frac{\Sigma xy - \frac{\Sigma x \Sigma y}{n}}{\sqrt{\Sigma x^2 - \frac{(\Sigma x)^2}{n}} \sqrt{\Sigma y^2 - \frac{(\Sigma y)^2}{n}}}$$

$$= \frac{12518 - \frac{156 \times 484}{6}}{\sqrt{4126 - \frac{156 \times 156}{6}} \sqrt{39114 - \frac{484 \times 484}{6}}}$$

$$= \frac{-66}{\sqrt{4126 - 4056}\ \sqrt{39114 - 39042.66}}$$

$$= \frac{-66}{\sqrt{70}\ \sqrt{71.34}}$$

$$= \frac{-66}{8.366 \times 8.446}$$

$$= \frac{-66}{70.654}$$

$$= -0.934$$

$$\therefore \boxed{r = -0.934}$$

$\Rightarrow$ This indicates a very strong negative correlation between age and success in the examination. As age increases, the success rate tends to decrease significantly. This suggests that younger candidates generally perform better in this examination.

Ques 3)

$n = 8$

$\Sigma x = 40$

$\Sigma y = 88$

$\Sigma xy = 404$

$\Sigma x^2 = 520$

$\Sigma y^2 = 1058$

$r = ?$

$$r = \frac{\Sigma xy - \frac{\Sigma x \Sigma y}{n}}{\sqrt{\Sigma x^2 - \frac{(\Sigma x)^2}{n}} \sqrt{\Sigma y^2 - \frac{(\Sigma y)^2}{n}}}$$

$$= \frac{404 - \frac{40 \times 88}{8}}{\sqrt{520 - \frac{1600}{8}} \sqrt{1058 - \frac{88 \times 88}{8}}}$$

$$= \frac{404 - 440}{\sqrt{320}\sqrt{90}}$$

$$= \frac{-36}{169.695} = -0.212$$

$\therefore$ $\boxed{r = -0.212}$

Ques 4)

$\Sigma x = 100$, $\Sigma y = 80$, $\Sigma xy = 406$, $\Sigma x^2 = 220$, $\Sigma y^2 = 1058$,
$n = 20$

Given that (5,4) is used instead of (4,5)

∴ Correct values will be

$\Sigma x = 100 - 5 + 4$
$= 99$

$\Sigma y = 80 - 4 + 5$
$= 81$

$\Sigma xy = 406$ (It will be same)

$\Sigma x^2 = 220 - 25 + 16$
$= 211$

$\Sigma y^2 = 1058 - 16 + 25$
$= 1067$

We know that,

$$r = \frac{\Sigma xy - \Sigma x \Sigma y}{\sqrt{\Sigma x^2 - \frac{(\Sigma x)^2}{n}} \sqrt{\Sigma y^2 - \frac{(\Sigma y)^2}{n}}}$$

$$= \frac{406 - \cancel{99 \times 81}}{\sqrt{211 - \frac{99 \times 99}{20}}\sqrt{1067 - \frac{81 \times 81}{20}}}$$

$$= \frac{406 - \frac{99 \times 81}{20}}{\sqrt{211 - \frac{99 \times 99}{20}}\sqrt{1067 - \frac{81 \times 81}{20}}}$$

$$= \frac{406 - 400.95}{\sqrt{211 - 490.05}\sqrt{1067 - 328.05}}$$

This value will be negative in the underroot which is not possible so there might be a mistake in the question.

Ques 5)

$r = 0.5$

$\text{cov}(x, y) = 4$

S.d [Standard deviation (σx)] $= 2.5$

$\Sigma y^2 = 192.4$

$n = 10$

$\bar{y} = ?$

$$\Rightarrow \quad r = \frac{\text{cov}(x, y)}{\sigma x \, \sigma y}$$

$$0.5 = \frac{4}{2.5 \times \sigma y}$$

$$\boxed{\sigma y = 3.2}$$

We know that

$$\sigma y = \sqrt{\frac{\Sigma (y - \bar{y})^2}{n}}$$

Squaring both sides

$$(\sigma y)^2 = \frac{\Sigma (y - \bar{y})^2}{n}$$

$$(3.2)^2 = \frac{\Sigma (y - \bar{y})^2}{10}$$

$$\Sigma(y-\bar{y})^2 = 102.4$$

$$\Rightarrow \Sigma(y^2 - 2y\bar{y} + \bar{y}^2) = 102.4$$

$$\Rightarrow \Sigma y^2 - 2\Sigma y\bar{y} + \Sigma \bar{y}^2 = 102.4$$

$$\Rightarrow \Sigma y^2 - 2\bar{y}\Sigma y + \Sigma \bar{y}^2 = 102.4 \quad [\bar{y} \text{ is constant}]$$

$$\Rightarrow \Sigma y^2 - 2\bar{y}(n\bar{y}) + \Sigma \bar{y}^2 = 102.4 \quad \left[\because \bar{y} = \frac{\Sigma y}{n}\right]$$

$$\Rightarrow \Sigma y^2 - 2n\bar{y}^2 + \Sigma \bar{y}^2 = 102.4$$

$$\Rightarrow \Sigma y^2 - 2n\bar{y}^2 + n\bar{y}^2 = 102.4 \quad [\bar{y}^2 \text{ is constant so its summation will be } n\bar{y}^2]$$

$$\Rightarrow \Sigma y^2 - n\bar{y}^2 = 102.4$$

$$\Rightarrow 192.4 - n\bar{y}^2 = 102.4$$

$$\Rightarrow n\bar{y}^2 = 90$$

$$\Rightarrow \bar{y}^2 = \frac{90}{10}$$

$$\Rightarrow \bar{y} = \sqrt{9} = 3 \checkmark$$

$$\therefore \boxed{\bar{y} = 3}$$

Ques 6)

Judge 1 (x)	Judge 2 (y)	Rank of x (R_x)	Rank of y (R_y)	$d = R_x - R_y$	d^2
1	3	10	8	2	4
9	10	2	1	1	1
4	6	7	5	2	4
2	1	9	10	-1	1
8	8	3	3	0	0
5	2	6	9	-3	9
7	4	4	7	-3	9
3	5	8	6	2	4
10	9	1	2	-1	1
6	7	5	4	1	1
					$\Sigma d^2 = 34$

$n = 10$, $\Sigma d^2 = 34$

$$r = 1 - \frac{6 \Sigma d^2}{n(n^2-1)}$$

$$= 1 - \frac{6 \times 34}{10(100-1)}$$

$$= 1 - \frac{204}{990} \Rightarrow 1 - 0.206 \Rightarrow 0.794$$

$\therefore$ $r = 0.794$

Ques 7)

Marks in Statistics (x)	Rank in Statistics (R_x)	Rank in Economics (R_y)	$d = R_x - R_y$	d^2
39	7	8	-1	1
65	4	3	1	1
76	2	1	1	1
45	6	7	-1	1
90	1	2	-1	1
31	8	6	2	4
72	3	4	-1	1
59	5	5	0	0
				$\Sigma d^2 = 10$

$n = 8, \quad \Sigma d^2 = 10$

$$r = 1 - \frac{6\Sigma d^2}{n(n^2-1)}$$

$$= 1 - \frac{6 \times 10}{8(64-1)} = 1 - \frac{60}{504} = 1 - 0.119 = 0.881$$

$\therefore \boxed{r = 0.881}$

⇒ This indicates a strong positive correlation between the students' marks in Statistics and their ranks in Economics.

Ques 8)

$r = 0.5$

$\Sigma d^2 = 10$

$n = ?$

$$r = 1 - \frac{6\Sigma d^2}{n(n^2-1)}$$

$$0.5 = 1 - \frac{6 \times 10}{n(n^2-1)}$$

$$\frac{60}{n(n^2-1)} = 1 - 0.5$$

$$\frac{60}{0.5} = n(n^2-1)$$

$$120 = n^3 - n$$

$$n^3 - n - 120 = 0$$

By hit & trial method,

At $n = 5$, the equation is satisfied

$\therefore$ No. of observation $(n) = 5$

Ques 9)

X	Y	$x-\bar{x}$	$y-\bar{y}$	$(x-\bar{x})^2$	$(y-\bar{y})^2$	$(x-\bar{x})(y-\bar{y})$
2	10	-7	-9.5	49	90.25	66.5
4	15	-5	-4.5	25	20.25	22.5
8	18	-1	-1.5	1	2.25	1.5
11	20	2	0.5	4	0.25	1
14	24	5	4.5	25	20.25	22.5
15	30	6	10.5	36	110.25	63
$\Sigma X =$ 54	$\Sigma Y =$ 117	$\Sigma(x-\bar{x}) = 0$	$\Sigma(y-\bar{y}) = 0$	$\Sigma(x-\bar{x})^2 = 140$	$\Sigma(y-\bar{y})^2 = 243.5$	$\Sigma(x-\bar{x})(y-\bar{y}) = 177$

$$\bar{x} = \frac{\Sigma x}{n} = \frac{54}{6} = 9$$

$$\bar{y} = \frac{\Sigma y}{n} = \frac{117}{6} = 19.5$$

$$r = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{\sqrt{\Sigma(x-\bar{x})^2}\sqrt{\Sigma(y-\bar{y})^2}}$$

$$= \frac{177}{\sqrt{140}\sqrt{243.5}}$$

$$= \frac{177}{11.832 \times 15.604}$$

$$= \frac{177}{184.627}$$

$$= 0.96$$

$$\therefore \boxed{r = 0.96}$$

Ques 10)

Student No.	Accountancy (X)	Statistics (Y)	Rank of X R_x	Rank of Y R_y	$d = R_x - R_y$	d^2
1	45	35	8	10	-2	4
2	70	90	3	2	1	1
3	65	70	4	5	-1	1
4	30	40	10	8.5	1.5	2.25
5	90	95	1	1	0	0
6	40	40	9	8.5	0.5	0.25
7	50	60	7	6	1	1
8	57	80	6	3.5	2.5	6.25
9	85	80	2	3.5	-1.5	2.25
10	60	50	5	7	-2	4
						$\Sigma d^2 =$
						22

$$r = 1 - 6 \sum \left\{ \frac{d^2 + \frac{m}{12}(m^2-1) + \frac{m}{12}(m^2-1)}{n(n^2-1)} \right\}$$

$$= 1 - 6 \left\{ \frac{22 + \frac{2}{12}(4-1) + \frac{2}{12}(4-1)}{10(100-1)} \right\}$$

$$= 1 - \frac{6 \times (22 + 0.5 + 0.5)}{10 \times 99}$$

$$= 1 - \frac{6 \times 23}{10 \times 99}$$

$$= 1 - \frac{138}{990}$$

$$= 1 - 0.139$$

$$= 0.861$$

$$\therefore \boxed{x = 0.861}$$