Parul University

Faculty of Engineering & Technology

Department of Applied Sciences and Humanities

1st Year B.Tech Programme (All Branches)

Mathematics – 1 (303191101)

Unit – 6 Multivariable Calculus

Tutorial-3

1 Use the Chain Rule to find $\frac{dw}{dt}$ for $w = xe^{y/z}$, $x = t^2$, $y = 1-t$, $z = 1+2t$

2 Use the Chain Rule to find $\partial z/\partial s$ and $\partial z/\partial t$, $z = e^r \cos\theta$, $r = st$, $\theta = \sqrt{s^2+t^2}$

3 Find $\frac{dy}{dx}$ for 1) $x^2 - y^2 + \sin xy = 0$, 2) $x^3 + y^3 = 7xy$.

4 Find $\frac{dw}{dt}$ if $w = xy + z, x = \cos t, y = \sin t, z = t$.

5 Use the chain Rule to find the indicated partial derivatives.

a. $z = x^2 + xy^3$, $x = uv^2 + w^3$, $y = u + ve^w$; $\frac{\partial z}{\partial u}, \frac{\partial z}{\partial v}, \frac{\partial z}{\partial w}$ $when\ u = 2, v = 1, w = 0$

b. $R = \ln(u^2 + v^2 + w^2)$, $u = x + 2y$, $v = 2x - y$, $w = 2xy$; $\frac{\partial R}{\partial x}, \frac{\partial R}{\partial y}$, $when\ x = y = 1$

6 Find $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial s}$ in terms of r and s if $w = (x + 2y + z^2)$, $x = r/s$, $y = r^2 + \ln s$, $z = 2r$.

7 Find $\frac{\partial z}{\partial u}$ when $u = 0, v = 1$ if $z = \sin xy + x\sin y$, $x = u^2 + v^2$, $y = uv$.

8 If $z = f(x,y)$ where $x = e^u + e^{-v}$, $y = e^{-u} - e^v$, prove that $\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v} = x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial v}$

9 a) Find the $Jacobian$ $\frac{\partial(u,v)}{\partial(x,y)}$ for the following functions:

(i) $u = x^2 - y^2, v = 2xy$

(ii) $u = \frac{x^2+y^2}{1-xy}, v = tan^{-1}x + tan^{-1}y$

b) For the transformations $x = e^v secu, y = e^v tanu$,prove that $\frac{\partial(x,y)\partial(u,v)}{\partial(u,v)\partial(x,y)} = 1$

10 a) Find the equation of the tangent plane and normal line to the surface $2xz^2 - 3xy - 4x = 7$ at (1, -1,2).

b) Find the equation of the normal line of the sphere $x^2 + y^2 + z^2 = 6$ at the point (a, b, c).

Show that the normal line passes through the origin.

11 a) Find the stationary value of $x^3 + y^3 - 3axy, a > 0$.

b) Examine the function $x^3y^2(12 - 3x - 4y)$ for extreme values.

12 a) Find the minimum values of xyz, subject to the condition $2x + 2y + 2z = 36$

b) Find the minimum values of $x^2 + y^2$, subject to the condition $ax + by = c$