A New Setting for Some Inverse Potential Problem and The Bubbling Method

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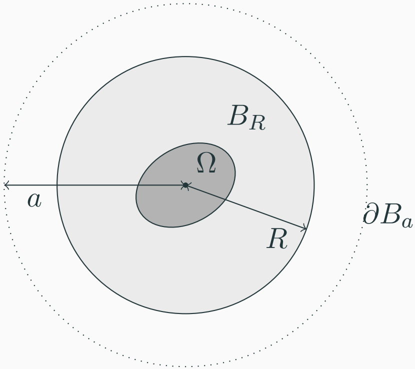
Can the potential take the place of the gravity?

We investigate the influence for reconstructing the shape of Ω when we observe the potential or the gravity on ∂B_a.



Core-Shell Model

Core-Shell Body

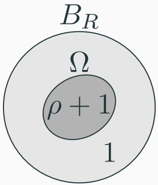
The potential of Core-shell body U is written as

$$U(x) = \underbrace{\int_{B_R} E(x-y)dy}_{U^{B_R}} + \rho \underbrace{\int_{\Omega} E(x-y)dy}_{U^{\Omega}},$$

where E is the fundamental solution of the Laplace equation.

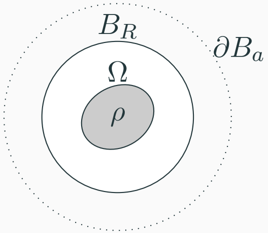
The potential ρU^{Ω} can be calculated on the ∂B_a.

$$\rho U^{\Omega} = U(\text{observed}) - U^{B_R}(\text{known}) \quad \text{on} \quad \partial B_a.$$



Core-shell body

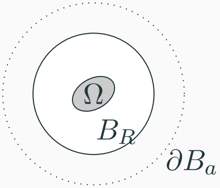
$\rightarrow$



Extract buried body

A New Setting for The Inverse Potential Problem

Observe the gravity or the potential on ∂B_a, and recover the shape of Ω.



We will compare reconstruction by observation of the gravity and the potential.

- Observation of Gravity (traditional)

$$\rho \nabla U^{\Omega} = \overrightarrow{g} \quad \text{on} \quad \partial B_a$$

- Observation of Potential (new)

$$\rho U^{\Omega} = p \quad \text{on} \quad \partial B_a$$

Sketch of Reconstruction Algorithm

We can observe only at the finite points $\{A_n\}_{n=1}^N \subset \partial B_a$.

- Observation of Gravity

$$\rho \nabla U^{\Omega} = \overrightarrow{g} \quad \text{on} \quad \{A_n\}_{n=1}^N \subset \partial B_a$$

- Observation of Potential

$$\rho U^{\Omega} = p \quad \text{on} \quad \{A_n\}_{n=1}^N \subset \partial B_a$$

Reconstruction algorithm consists of two parts.

1. Approximate the body Ω by a set of point masses $\rightarrow$ Optimization method
2. Homogenize the set of point masses $\rightarrow$ Bubbling method

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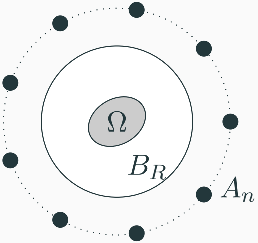
Apprx. Body by a Set of Point Masses (Gravity observation)

Observation points $\{A_n\}_{n=1}^{N} \subset \partial B_a$.



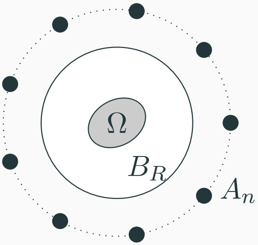
We difine a cost function J_G as following. set of point mass has representation $(X, M) = (X_1, \ldots X_K, M_1, \ldots M_K)$.

$$J_G(X, M) = \frac{1}{N} \sum_{n=1}^{N} \left| \rho \nabla U^{\Omega}(A_n) - G_K(A_n; X, M) \right|^2,$$

$$G_K(A_n; X, M) = \frac{1}{4\pi} \sum_{k=1}^{K} \frac{M_k(A_n - X_k)}{|A_n - X_k|^3}$$

Apprx. Body by a Set of Point Mass (Potential observation)

Observation points $\{A_n\}_{n=1}^{N} \subset \partial B_a$.



We difine a cost function J_P as following. set of point mass has representation $(X, M) = (X_1, \dots X_K, M_1, \dots M_K)$.

$$J_P(X, M) = \frac{1}{N} \sum_{n=1}^{N} \left| \rho U^{\Omega}(A_n) - P_K(A_n; X, M) \right|^2 ,$$

$$P_K(A_n; X, M) = \frac{1}{4\pi} \sum_{k=1}^{K} \frac{M_k}{|A_n - X_k|}$$

Sketch of Reconstruction Algorithm

We can observe only at the finite points $\{A_n\}_{n=1}^N \subset \partial B_a$.

- Observation of Gravity

$$\rho \nabla U^\Omega = \overrightarrow{g} \quad \text{on} \quad \{A_n\}_{n=1}^N \subset \partial B_a$$

- Observation of Potential

$$\rho U^\Omega = p \quad \text{on} \quad \{A_n\}_{n=1}^N \subset \partial B_a$$

Reconstruction algorithm consists of two parts.

1. Approximate the body Ω by a set of point masses $\rightarrow$ Optimization method
2. Homogenize the set of point masses $\rightarrow$ Bubbling method

Bubbling Method (Partial Mass Scattering)

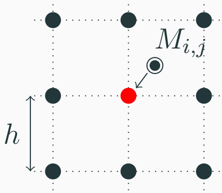
We homogenize the set of point masses to homogeneous body with density ρ.

Point mass (X_k, M_k) move to lattice point $\widetilde{X}_k = (ih, jh)$.

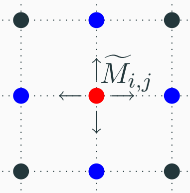
$\widetilde{M}_{i,j} = M_k$. When $\Delta\widetilde{M}_{i,j} = \widetilde{M}_{i,j} - \rho h^2 > \varepsilon$,

$$\widetilde{M}^{(1)}_{i,j} = \rho h^2 - \varepsilon,$$

$$\widetilde{M}^{(1)}_{i\pm1,j} = \widetilde{M}_{i\pm1,j} + \frac{1}{4}(\Delta\widetilde{M}_{i,j} + \varepsilon), \quad \widetilde{M}^{(1)}_{i,j\pm1} = \widetilde{M}_{i,j\pm1} + \frac{1}{4}(\Delta\widetilde{M}_{i,j} + \varepsilon).$$



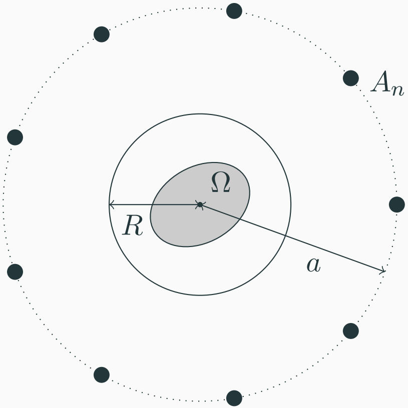
Move to lattice point



Mass Scattering

Reconfirm The Aid of Computation

When we change radius of observation a, we verify influence for the reconstruction which is caused by this change.

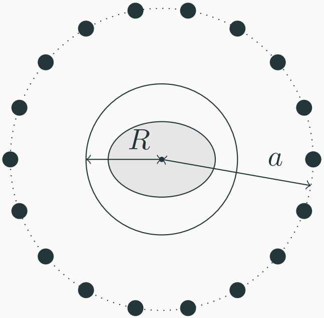


Observation at $\{A_n\}_{n=1}^N$ and Reconstruction $\Omega \subset B_R$

Example: Reconstruction of Ellipzoid

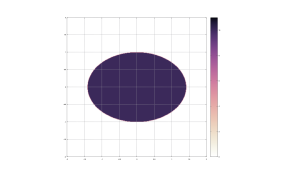
Set $R = 2$. We reconstruct an elliptic shape with long radius $\sqrt{2}$, short radius 1 and density $\rho = 10$. Limit of resolution is 10^{-4}.

We select Levenberg-Marquardt method as a optimization method.



Example: Observation of Gravity

Number of point masses is $K = 100$, number of observation points is $N = 300$. a is observation radius.



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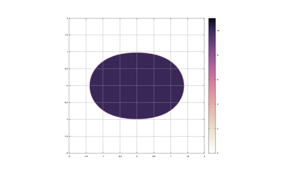
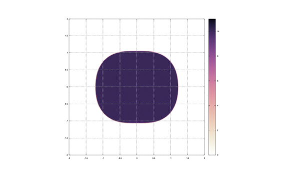
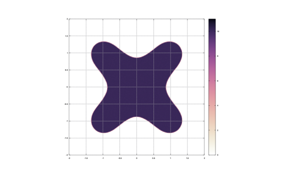


$a = 10$



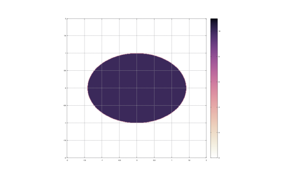
$a = 30$



$a = 200$

Example: Observation of Potential

Number of point masses is $K = 100$, number of observation points is $N = 300$. a is observation radius.



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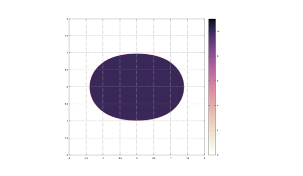

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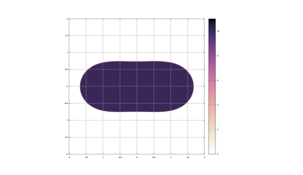

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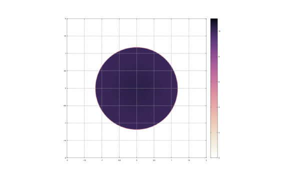

$a = 200$

Conclusion

We observe the potential and reconstruct the shape of the body, compared to observation of the gravity.

- Reconstruction of Ellipzoid
  Reconstruction by observation of the potential can reconstruct source body more correctly.