A New Setting for Some Inverse Potential Problem and The Bubbling Method

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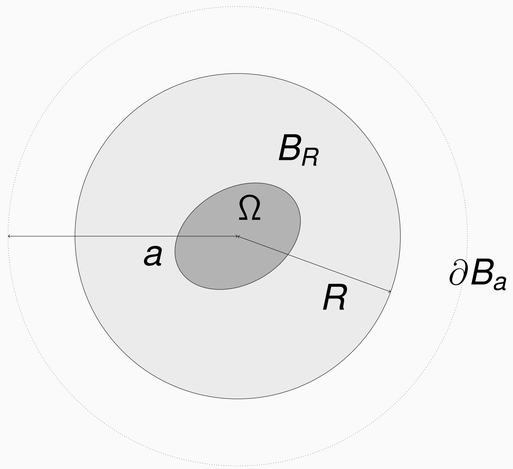
Abstract

Can the potential take the place of the gravity?

We investigate the influence for reconstructing the shape of Ω when we observe the potential or the gravity on ∂B_a.



Core-Shell Body

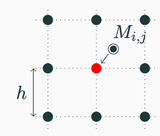
Sketch of Reconstruction Algorithm

Reconstruction algorithm consists of two parts.

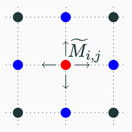
1. Approximate the body Ω by a set of point masses
   $\rightarrow$ Optimization method
2. Homogenize the set of point masses
   $\rightarrow$ Bubbling method

The Bubbling Method

We homogenize the set of point masses to body with density ρ.



Move to lattice point



Mass Scattering

Core-Shell Body

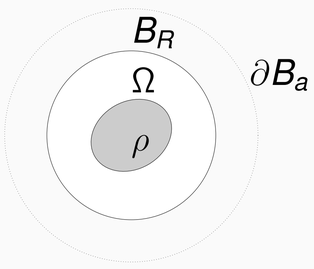
The potential of Core-shell body U is written as

$$U(x) = \underbrace{\int_{B_R} E(x-y)dy}_{U^{B_R}} + \rho \underbrace{\int_{\Omega} E(x-y)dy}_{U^{\Omega}},$$

where E is the Newton potential.

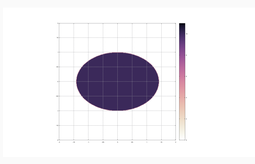
The potential ρU^{Ω} can be calculated on the ∂B_a.

$$\rho U^{\Omega} = U(\text{observed}) - U^{B_R}(\text{known}) \quad \text{on} \quad \partial B_a.$$



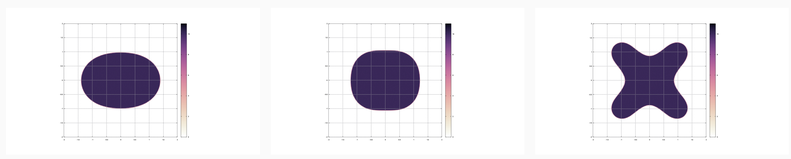
Extract buried body

Reconstruction of Ellipzoid

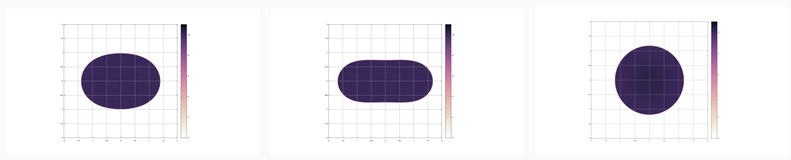


Source

- Observation of Gravity ($a = 10, 30, 200$)



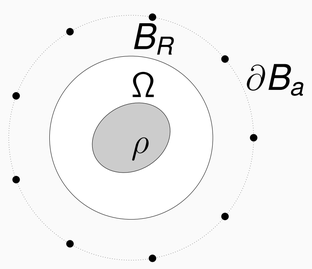
- Observation of Potential ($a = 10, 30, 200$)



Inverse Potential Problem

The potential observation has became possible by atomic clock!

Observe the gravity or the potential on ∂B_a, and recover the shape of Ω.



- Observation of Gravity (Traditional)

$$\rho \nabla U^{\Omega} = \overrightarrow{g} \quad \text{on} \quad \{A_n\}_{n=1}^{N} \subset \partial B_a$$

- Observation of Potential (New)

$$\rho U^{\Omega} = p \quad \text{on} \quad \{A_n\}_{n=1}^{N} \subset \partial B_a$$

Conclusion

We observe the potential and reconstruct the shape of the body, compared to observation of the gravity.

- Reconstruction of Ellipzoid
  In this case, reconstruction by observation of the potential can reconstruct source body more correctly.