موضوع الدرس ________ التاريخ: / /

6.28

inputs $= x, y, c \implies 2^3 = 8$ columns

outputs $= z, c' \implies 2^2 = 4$ rows

$\therefore z = x \oplus y \oplus c$

$c' = (x \wedge y) \vee (x \wedge c) \vee (y \wedge c)$

Truth Table

x	y	c	z	c'
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

$$\text{matrix} = \begin{array}{c} \\ 00 \\ 01 \\ 10 \\ 11 \end{array} \begin{array}{c} \begin{array}{cccccccc} 000 & 001 & 010 & 011 & 100 & 101 & 110 & 111 \end{array} \\ \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \end{array}$$

S 2-0

classical gates

XOR gates $z = x \oplus y \oplus c$

And, OR gates $c' = xy + xc + yc$

$$M = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

* Code *

```
import numpy as np
matrix = np.zeros((4,8), dtype=int)
rows = {(0,0):0, (0,1):1, (1,0):2, (1,1):3}
col=0
for x in (0,1):
    for y in (0,1):
        for c in (0,1):
            z = (x^y)^c
            c_out = (x&y) | (x&c) | (y&c)

            row = rows[(z, c_out)]
            matrix[row, col] = 1
            col += 1

print("the matrix:")
print(matrix)
```

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i) $X^2 = Y^2 = Z^2 = I$

$\because X^2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}^2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$ ✓

$Y^2 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

$Z^2 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

ii) $H = \frac{1}{\sqrt{2}}(X + Z)$

$\because X + Z = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

$\times \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}(X+Z) = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} = H$ ✓

iii) $X = HZH$

$\because HZ = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$

$\therefore HZH = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1\times1 + (-1\times1) & 1\times1 + (-1\times-1) \\ 1\times1 + 1\times1 & 1\times1 + 1\times-1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = X$ ✓

iv) $Z = HXH$

$\because HX = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$

$HXH = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = Z$ ✓

v) $-1Y = HYH$

$\because HY = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} i & -i \\ -i & -i \end{bmatrix}$

$\therefore HYH = \frac{1}{\sqrt{2}}\begin{bmatrix} i & -i \\ -i & -i \end{bmatrix}\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 0 & +2i \\ -2i & 0 \end{bmatrix} = \begin{bmatrix} 0 & +i \\ -i & 0 \end{bmatrix} \cancel{=} -1Y$ ✓

vi) $S = T^2$

$\because T^2 = \begin{bmatrix} 1 & 0 \\ 0 & \frac{i+1}{\sqrt{2}} \end{bmatrix}^2 = \begin{bmatrix} 1 & 0 \\ 0 & \frac{i+1}{\sqrt{2}} \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & \frac{i+1}{\sqrt{2}} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix} = S$ ✓

vii) $= -1Y = XYX$

$\because XY = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} = \begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix}$

$XYX = \begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & i \\ -i & 0 \end{bmatrix} = -1Y$ ✓