READING, WRITING, AND COMPARING MULTI-DIGIT NUMBERS

4.NBT.2

CONTENTS

The types of documents contained in the unit are listed below. Throughout the unit, the documents are arranged by lesson.

LEARNING MAP INFORMATION	An overview of the standards, the learning map section, and the nodes addressed in this unit
TEACHER NOTES	A brief discussion describing the progression depicted in the learning map section with research-based recommendations for focusing instruction to foster student learning and an introduction to the unit's lessons
INSTRUCTIONAL ACTIVITY	A detailed walkthrough of the unit
INSTRUCTIONAL ACTIVITY STUDENT HANDOUT	A handout for the guided activity, intended to be paired with the Instructional Activity
INSTRUCTIONAL ACTIVITY SUPPLEMENT	A collection of materials or activities related to the Instructional Activity
STUDENT ACTIVITY	A work-alone activity for students
STUDENT ACTIVITY SOLUTION GUIDE	A solution guide for the work-alone activity with example errors, misconceptions, and links to the learning map section

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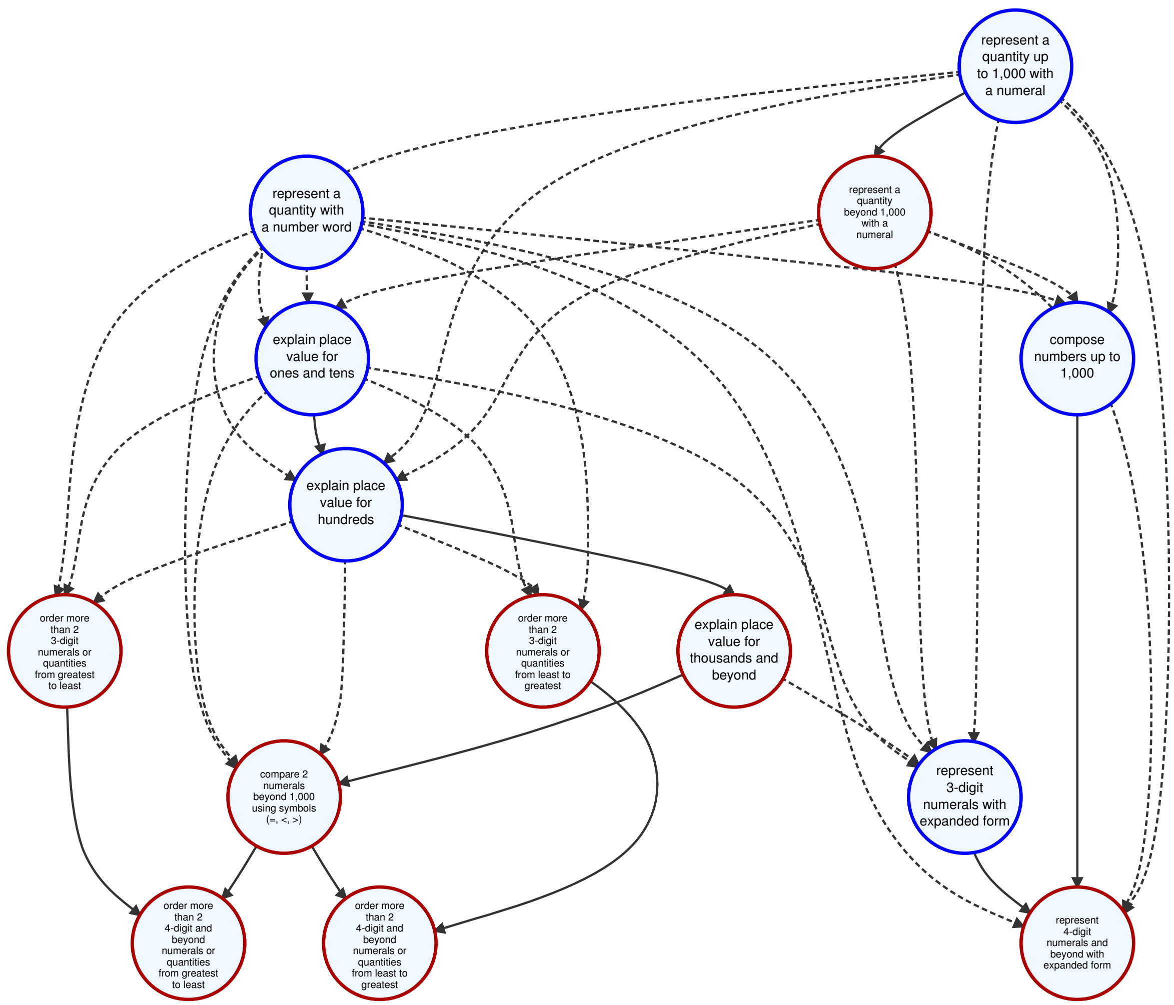
Reading, Writing, and Comparing Multi-Digit Numbers

LEARNING MAP INFORMATION

STANDARDS

4.NBT.2 Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on the value of the digits in each place, using >, =, and < symbols to record the results of comparisons.



**Learning map model of* 4.NBT.2

Node Name	Node Description
COMPARE 2 NUMERALS BEYOND 1,000 USING SYMBOLS (=, <, >)	Use the "more than" symbol, "less than" symbol, and "equal" symbol to create equations and inequalities for numerals with four digits or more.
COMPOSE NUMBERS UP TO 1,000	Combine smaller numbers up to 1,000 to form a larger number.
EXPLAIN PLACE VALUE FOR HUNDREDS	Make known your understanding that the value of a digit is determined by its position in the number. A digit in the hundreds place is worth that many hundreds.
EXPLAIN PLACE VALUE FOR ONES AND TENS	Make known your understanding that the value of a digit is determined by its position in the number. A digit in the ones place is worth that many ones, and a digit in the tens place is worth that many tens.
EXPLAIN PLACE VALUE FOR THOUSANDS AND BEYOND	Make known your understanding that the value of a digit is determined by its position in the number. For example, a digit in the thousands place is worth that many thousands, and a digit in the ten-thousands place is worth that many ten thousands.
ORDER MORE THAN 2 3-DIGIT NUMERALS OR QUANTITIES FROM GREATEST TO LEAST	Using manipulatives or numerals, order more than two three-digit numbers from greatest to least.
ORDER MORE THAN 2 3-DIGIT NUMERALS OR QUANTITIES FROM LEAST TO GREATEST	Using manipulatives or numerals, order more than two three-digit numbers from least to greatest.
ORDER MORE THAN 2 4-DIGIT AND BEYOND NUMERALS OR QUANTITIES FROM GREATEST TO LEAST	Using manipulatives or numerals, order more than two four-digit and beyond numbers from greatest to least.
ORDER MORE THAN 2 4-DIGIT AND BEYOND NUMERALS OR QUANTITIES FROM LEAST TO GREATEST	Using manipulatives or numerals, order more than two four-digit and beyond numbers from least to greatest.
REPRESENT 3-DIGIT NUMERALS WITH EXPANDED FORM	Apply place value understanding to represent three-digit numerals in expanded form through writing or an appropriate assistive technology. For example, 136 in expanded form is 100 + 30 + 6.
REPRESENT 4-DIGIT NUMERALS AND BEYOND WITH EXPANDED FORM	Apply place value understanding to represent four-digit numerals in expanded form through writing or an appropriate assistive technology. For example, 1,368 in expanded form is 1,000 + 300 + 60 + 8.
REPRESENT A QUANTITY BEYOND 1,000 WITH A NUMERAL	Through writing or appropriate assistive technology, use numerals to represent a specific quantity beyond 1,000.
REPRESENT A QUANTITY UP TO 1,000 WITH A NUMERAL	Through writing or appropriate assistive technology, use numerals to represent a specific quantity up to 1,000.
REPRESENT A QUANTITY WITH A NUMBER WORD	Through writing or an appropriate assistive technology, represent the number word for a given quantity.

Reading, Writing, and Comparing Multi-Digit Numbers

TEACHER NOTES

This unit includes the following documents:

- Learning Map Information
- Instructional Activity (three lessons)
- Instructional Activity Student Handout (for Lesson 2)
- Instructional Activity Supplement (for Lessons 1 – 3)
- Student Activity
- Student Activity Solution Guide

In this unit, students will build on their understanding of hundreds, tens, and ones to establish an understanding of multi-digit numbers into the millions. Using a variety of manipulatives, students will develop a flexible understanding of multi-digit numbers and explain place value for thousands and beyond.

RESEARCH

Place value understanding is foundational for computation; understanding the structure of numbers is a necessary component for students to operate with understanding (Richardson, 2012). Developing a deep understanding of numbers in relation to each other takes time, but there is no substitute for good number sense as the foundation of arithmetic and algebraic operations, which are gateways to higher mathematics (Richardson, 2012). Additionally, students who acquire proficiency with whole number concepts of place value, order, and comparisons can build on this understanding to make sense of decimals in later grades (Richardson, 2012), thereby developing a complete and symmetric view of the base-ten system. In contrast, students who learn place value concepts primarily by the position of digits in a multi-digit number generally do not develop a strong sense of number structure; for example, they may be able to state, but not explain, that 10 tens are equal to one hundred (Richardson, 2012).

From kindergarten through third grade, students work to develop an understanding of numbers up to 1,000. Although numbers greater than 1,000 have the same structure as numbers from earlier grades, quantities greater than 1,000 can be difficult for students to conceptualize, yet a good grasp of the size of numbers is one component of number sense (Van de Walle, Lovin, Karp, & Bay-Williams, 2014). To address this need, it is important to discuss large quantities in contexts students can relate to, because their thinking about number and quantity is strongly tied to real objects (Van de Walle et al., 2014; Kastberg & Walker, 2008). It is common for adults to have lost the connections from numbers to real objects and to think about numbers in an abstract way; thus, it is important to purposefully create this connection for students when working with multi-digit numbers (Kastberg & Walker, 2008). Students need adequate opportunities to connect large numbers to objects and contexts so that they form more concrete connections to multi-digit numbers.

As teachers help students make connections within the number structure and from numbers to the real-world, there are two approaches to thinking about multi-digit numbers: a qualitative approach and an abstract formal approach (Kastberg & Walker, 2008). A qualitative approach considers how numbers are related to the real-world and gives the numbers meaning; for example, one might think about one million and wonder if one million pieces of paper would cover a roof or a basketball court (Kastberg & Walker, 2008). An abstract formal approach considers relationships among units; for example, if Jill has 1,000 stickers and Sal has 100 stickers, then Jill has 10 times as many stickers as Sal (Kastberg & Walker, 2008). Both qualitative understanding and abstract formal understanding need to be developed in students in order to support their discussion about and flexibility with numbers (Kastberg & Walker, 2008).

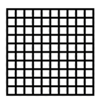
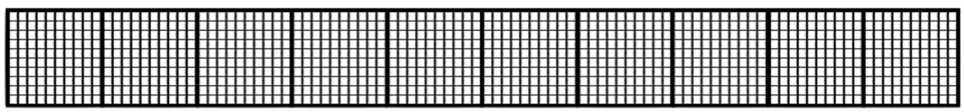
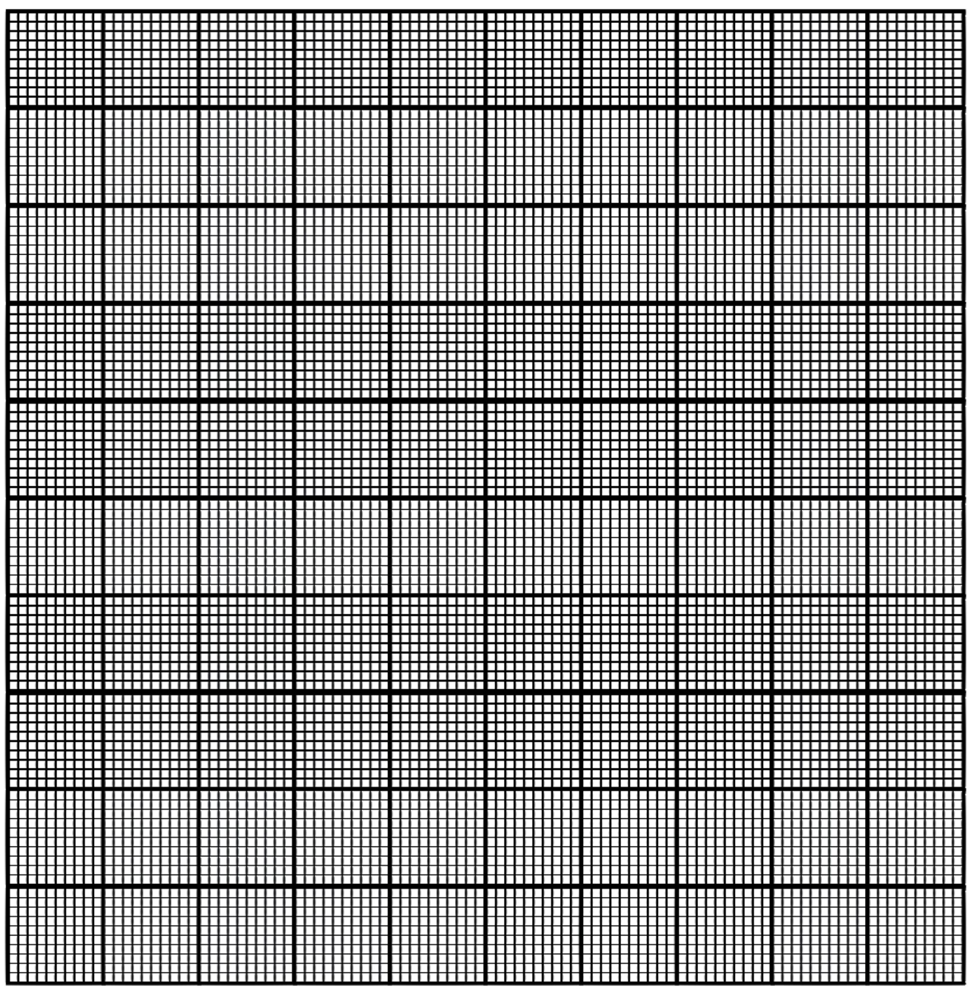
Activities to help students develop both qualitative and abstract formal approaches to numbers involve using concrete base-ten materials to build multi-digit numbers. Three-dimensional base-ten materials demonstrate the repetitive pattern in the number structure (Van de Walle et al., 2014). Using base-ten materials, a small cube can represent "one", 10 small cubes in a row can represent "ten", and 10 "tens" arranged side by side can represent a "hundred". From there, 10 "hundreds" stacked on top of each other to form a larger cube represents a "thousand", the beginning of the second cycle. 10 "thousands" in a row can represent "ten thousand", 10 "ten thousands" arranged side by side represent a "hundred thousand" (Van de Walle et al., 2014). Developing the idea of grouping, or that 10 in any position makes a single group in the next larger position, is an important component of place value instruction (Van de Walle et al., 2014). Likewise, it is important to develop the idea that one in any position makes a group of 10 in the next smaller position. When using three-dimensional base-ten materials, it is important to use base-ten language (rather than terms such as "cube", "stick" or "long", and "flat" without any associated base-ten language) in order to reinforce conceptual understanding of the quantities represented (Van de Walle et al., 2014). This pattern continues for millions, billions, trillions, and so forth, although it likely becomes impossible to model prior to millions due to a limited amount of materials.

With two-dimensional base-ten materials, students can start with 10 hundreds squares and tape them together in a row to make 1,000 (Van de Walle et al., 2014). 10 students can then tape their thousands together to make 10,000 (Van de Walle et al., 2014). This can either continue with physical materials or it can be drawn with sidewalk chalk (or tape) to help students visualize the size difference among quantities and to emphasize the multiplicative relationship and the base-ten number structure (Van de Walle et al., 2014).

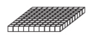
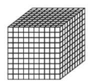
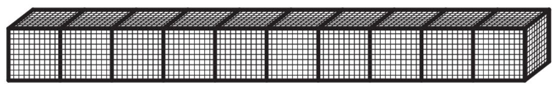
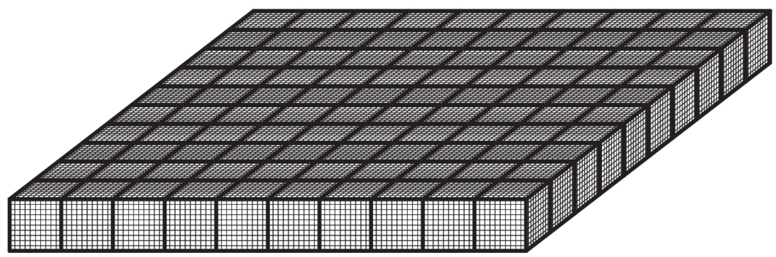
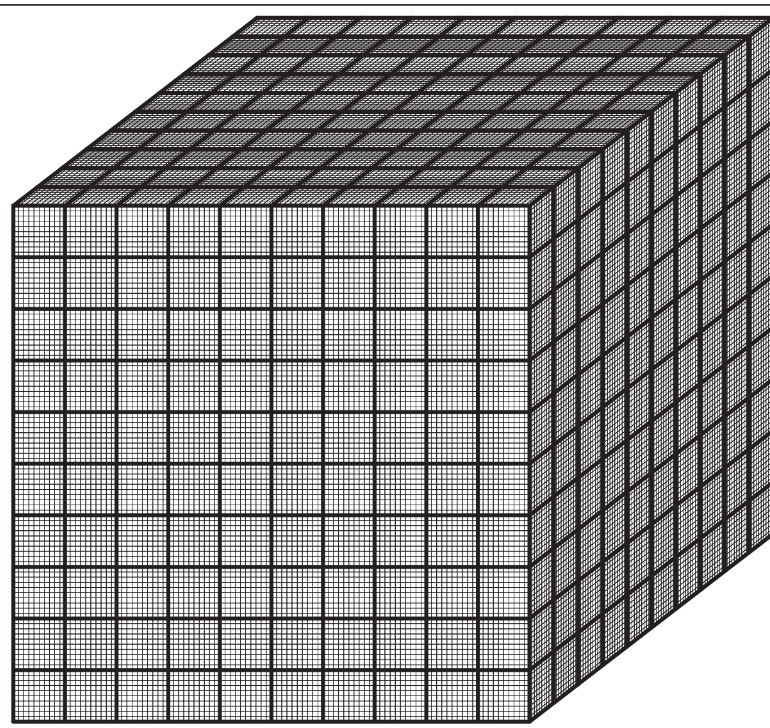
Note that virtual versions of base-ten manipulatives are also available online. These, however, are often limited in their ability to represent multi-digit numbers or make explicit the pattern in number structure. Examples of sites with virtual base-ten manipulatives include http://www.symbaloo.com/mix/onlinemathmanipulatives, where students can work with thousands, hundreds, tens, and ones and the value of the manipulatives is provided along with the number name and expanded form. Alternatively, https://www.eduplace.com/kids/mw/manip/mn_3.html allows students to trade within hundreds, tens, and ones for larger and smaller units. While the benefits of these sites include clean, unlimited manipulatives, there are drawbacks in their limited ability to model multi-digit numbers in the thousands and beyond, and that students are not able to physically trade 10 of one unit for one of the next larger unit or vice versa.

AN EXAMPLE

The following graphics are representations of 1, 10, 100, 1,000, and 10,000 using two-dimensional base-ten models.

TWO–DIMENSIONAL MODELS	
1	
10	
100	
1,000	
10,000	

The following graphics are representations of 1, 10, 100, 1,000, 10,000, 100,000, and 1,000,000 using three-dimensional base-ten models.

THREE–DIMENSIONAL MODELS	
1	
10	
100	
1,000	
10,000	
100,000	
1,000,000	

Generally speaking, students benefit from models that make the quantities visible and help show the relationships among quantities (Richardson, 2012). When working with concrete manipulatives, it is critical that students regularly reflect on their actions in order to build mathematical meaning for the manipulatives they use (Clements, 2000). For example, students should practice arranging concrete blocks in multiple ways to represent the same number. Such practice reinforces the base-ten relationships in the blocks used to represent the number. In addition, manipulatives can help encourage students to think flexibly about numbers in the thousands and beyond (Richardson, 2012). It is important to help students see that the same number can be represented or thought of in many different ways, which is an understanding that is attained as students physically reorganize models and participate in conversations with their peers and their teacher (Richardson, 2012). For example, students should understand that 8,000 is composed of 8,000 ones or 800 tens or 80 hundreds.

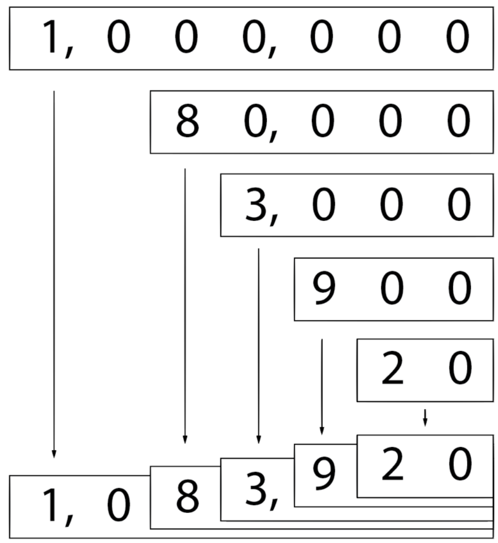
Unlike the relationships among quantities and the structure and pattern within numbers, the way we say and write numbers are conventions that must be explicitly taught to students (Van de Walle et al., 2014). To read multi-digit numbers, students should mark off triples starting from the right; the digits in each triple correspond to the ones, tens, hundreds pattern described with three-dimensional base-ten manipulatives (Van de Walle et al., 2014). The first triple on the right is the units, then thousands, then millions, and so forth. To read each number, students should begin at the left and focus on reading the three-digit number contained in each triple, then stating the units before moving to the next triple (Van de Walle et al., 2014). The following is an example of a structure to help organize multi-digit numbers and scaffold students' experiences reading multi-digit numbers.

MILLIONS			THOUSANDS			UNITS		
1	4	5 ,	0	7	2 ,	9	0	3
One hundred forty-five *million*,			seventy-two *thousand*,			nine hundred three		

Conventions for writing multi-digit numerals as well as the importance of "0" in place value can be a challenging for some students (Van de Walle et al., 2014). The use of number cards can help students properly build numbers in standard form and easily translates to writing numbers in expanded form (Van de Walle et al., 2014).

AN EXAMPLE

The following graphic is a representation of building 1,083,920 using number cards.



The use of precise language, by both teacher and student, is critical to student understanding of place value (Van de Walle et al., 2014). Thus, whether or not a student uses precise language when discussing place value concepts can help teachers gauge their students' conceptual understanding. When referring to a digit that is not in the ones place, it is important to use base-ten language or the value of the number to describe the value of that digit (Van de Walle et al., 2014). For example, in the number 8,234, refrain from saying "three" to describe the "3" in the tens place; instead, say "three tens" or "thirty" (Van de Walle et all, 2014). In addition, the word "and" should be used with caution; it is acceptable to use "and" when listing base-ten materials that compose a number, but not when reading a whole number in standard form. For example, reading 342 as "three hundred *and* forty-two" (rather than "three hundred forty-two") is inaccurate and must be unlearned in later grades as students work with decimals and the word "and" is specifically used to read the decimal point.

Once students have a strong understanding of place value for a multi-digit number, they can continue to utilize place value reasoning to compare and order two or more multi-digit numbers. As students compare and order two or more numbers, they should first determine the largest place value (unit) in the numbers being compared and what this means in terms of the relative size of the numbers. In instances where the largest units are different (e.g., one number's largest unit is millions and the other number's largest unit is hundred thousands), students should understand how to immediately compare or order the values. In instances where the largest units are the same (e.g., each numbers' largest unit is ten thousands), students should consider how those quantities compare. If the largest units' values are equal, then students should progress to the next largest unit to make the comparison. Students should understand the importance of place value as they make these comparisons; for example, a nine in the ones place does not have a greater value than a one in the tens place (even though nine is greater than one).

MISCONCEPTIONS

Student misunderstandings often can be traced to a lack of place value understanding or incomplete knowledge of number structure. Children may see multi-digit numbers as concatenated single-digits (e.g., viewing 42 as the single-digits four and two) (Fuson, 1998). This concatenated, single-digit perspective is common among students and carries into multi-digit addition and subtraction, causing many types of well-documented errors (Fuson, 1998). To prevent such confusion, teachers should consistently model proper vocabulary and encourage the same in students to avoid this misconception (e.g., refer to the "4" in 42 as "forty" or "four tens" rather than "four") (Fuson, 1998). In addition, making connections among numerals and the corresponding concrete base-ten representation can help students better understand the difference between the value of a digit in the tens place compared to the value of a digit in the ones place.

In addition, look for and point out common mistakes, such as writing 1,042 as "1000402" or "1,000402" (Van de Walle et al., 2014). This misunderstanding is almost the reverse misconception of the one previously stated (where students view numbers as concatenated single digits) and is likely the result of the way numbers are pronounced. For example, 20,003 is read "twenty thousand three" and therefore may be written as it sounds: "200003" or "20,0003" (Fuson, 1998). If students hold this misconception, continue using mathematically accurate number names while emphasizing place value understanding and the structure of base-ten numerals. The use of place value cards in conjunction with base-ten manipulatives can help scaffold understanding of the standard form of multi-digit numbers (Van de Walle et al., 2014). Research indicates that students make more errors with four-digit number names than with three-digit number names; students do not easily generalize their place value understanding and need time to explore examples and tasks (Cayton & Brizuela, 2007). It is possible that this increase in student errors relates back to the likelihood that students have fewer concrete opportunities to make sense of numbers greater than 1,000.

LEARNING MAP INFORMATION

The learning map section for this sequence of activities begins with students' ability to represent a quantity with a number word and to represent quantities up to 1,000 with numerals. These understandings lead to representing quantities beyond 1,000 with numerals, composing numbers up to 1,000, and explaining place value for ones, tens, and hundreds.

Students' understanding of place value for hundreds is the foundation for ordering three-digit numbers and for understanding place value for thousands and beyond. Once students are able to explain place value for thousands and beyond and order three-digit numbers, they should be ready to compare and order numerals beyond 1,000.

Students' understanding of place value for thousands and beyond, along with students' ability to compose numbers up to 1,000, provides the foundation for representing multi-digit numerals in expanded form.

INSTRUCTIONAL ACTIVITIES

The activities in this unit are designed to foster student understanding of place value and multi-digit numbers, such that students are able to compare and order multi-digit numbers and to write multi-digit numbers in expanded form.

In Lesson 1, students will be guided through the structure of numbers in the thousands and millions using familiar base-ten manipulatives. Students will first observe the patterns that exist in the number structure through three-dimensional base-ten manipulatives, then through two-dimensional base-ten manipulatives. Once students have been exposed to one million, they will consider the magnitude of multi-digit numbers in familiar, real-world contexts.

In Lesson 2, students will review ones, thousands, and millions and discuss the structure for writing and reading numerals and number names. Students will then use number strips to write standard and expanded form for a variety of multi-digit numbers.

Finally, in Lesson 3, students will first compare two multi-digit numbers using the symbols >, <, and =. Students will then order more than two multi-digit numbers from least to greatest or from greatest to least by physically rearranging numbers printed on strips of paper. Finally, students will play a mystery number game on the number line where they will try to guess a multi-digit number based on its position on the number line and the placement of previous guesses.

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Reading, Writing, and Comparing Multi-Digit Numbers

INSTRUCTIONAL ACTIVITY

Lesson 1

LEARNING GOAL

Students will be introduced to thousands and millions using two- and three-dimensional base-ten manipulatives, making connections to the structure of three-digit numbers. Students will then discuss large multi-digit numbers in real-world contexts.

PRIMARY ACTIVITY

Students will be guided through the structure of numbers in the thousands and millions using familiar base-ten manipulatives. Students will first observe the patterns that exist in the number structure through three-dimensional base-ten manipulatives, then through two-dimensional base-ten manipulatives. Once students have been exposed to one million, they will consider the magnitude of multi-digit numbers in familiar, real-world contexts.

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

- Ones
- Tens
- Hundreds
- Thousands
- Millions
- Units
- Base-ten

MATERIALS

- Base-ten manipulatives (three-dimensional; 10 ones, 10 tens, 10 hundreds, and 10 thousands cubes, if possible)

- Tape
- Scissors
- Sidewalk chalk
- INSTRUCTIONAL ACTIVITY SUPPLEMENT (Recommend one copy—10 hundreds squares—for every student.)

IMPLEMENTATION

Begin the lesson by modeling multi-digit numbers with familiar three-dimensional base-ten manipulatives.

Review place value concepts for three-digit numbers using three-dimensional base-ten manipulatives.

- 10 ones make a "ten". (**Model** lining up 10 ones to create the same shape as one ten.)
- 10 tens make a "hundred". (**Model** lining up 10 tens to create the same shape as a hundred.)

Stack 10 hundreds to create the same shape as a thousand cube.

Note that you began with a unit cube, and that this new, larger cube is referred to as a thousands cube.

> NOTE: It is important to model stacking 10 hundreds to create a thousand to help students understand that the thousands cube is filled with unit squares and is not a hollow shape where only the visible exterior units are considered.

Next, **line up** 10 thousands to make ten thousand. **Point out** that 10 thousands creates a shape that is similar to but larger than 10 ones (or 10 units).

It is likely that at this point, the necessary number of physical models will no longer be available to model larger numbers. In addition to describing the pattern to help students visualize larger and larger quantities with base-ten manipulatives, there is an image of the pattern that can be found in the TEACHER NOTES or on Wikipedia (https://en.wikipedia.org/wiki/Million).

Describe setting 10 ten thousands next to each other to make a hundred thousand.

Note that the shapes cycle: a cube for the unit, a stick for the tens, and a flat for the hundreds.

Ask students to imagine stacking 10 hundred thousands: "What shape would it make?"

Establish that 10 hundred thousands creates a cube, a new unit which stands for millions. This pattern, based on tens, continues to create larger and larger numbers.

GUIDING QUESTIONS

Elicit student thinking:

- What patterns do you notice in these numbers?
- What makes the number 10 important when talking about large numbers?

Determine if the student can COMPOSE NUMBERS UP TO 1,000:

- If you put 10 ones together, what is another name for this amount?
- If you put 10 tens together, what is another name for this amount?
- If you put 10 hundreds together, what is another name for this amount?

Determine if the student can EXPLAIN PLACE VALUE FOR HUNDREDS:

- In the numeral 100, what does the 1 represent? What do the zeros represent?
- What is different about the numbers 100 and 500?

Determine if the student is ready to REPRESENT A QUANTITY BEYOND 1,000 WITH A NUMERAL:

- How would you write "one thousand" using numerals?
- How would you write "ten thousand" using numerals?
- How would you write "one hundred thousand" using numerals?
- How would you write "one million" using numerals?

Continue the lesson by modeling multi-digit numbers with two-dimensional base-ten manipulatives.

Each student will need 10 hundreds squares (cut out) from the INSTRUCTIONAL ACTIVITY SUPPLEMENT.

Ask students to count the hundreds ("100, 200, 300, 400, …" or "one hundred, two hundreds, three hundreds, four hundreds, …"). When students get to "10 hundred", **remind** them that 10 hundreds make one thousand.

Note the pattern: 10 ones make a ten, 10 tens make a hundred, 10 hundreds make a thousand.

Write “1,000” and “one thousand” on the board for students to see.

Ask each student to tape together the 10 hundreds in a single row to make a thousand. Note that each hundreds square is 10 centimeters by 10 centimeters, so the thousands rectangle will be 10 centimeters by 100 centimeters (one meter).

100	100	100	100	100	100	100	100	100	100

Pose the question, “How many total unit squares would there be if you combine 10 thousands?” without requesting an immediate response.

Ask 10 students to set their thousands next to each other, and allow students time to respond to the question. Note that each hundreds square is 10 centimeters by 10 centimeters, so the ten-thousands square will be 100 centimeters by 100 centimeters (one meter by one meter).

100	100	100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100	100	100

As a class, **discuss** and **establish** that 10 thousands make a ten thousand. Write "10,000" and "ten thousand" on the board for students to see.

Tape the 10 thousands together to create a ten-thousand square.

To continue the pattern, it is likely necessary to relocate the class due to the size of the models for 100,000 and 1,000,000. It is recommended that students use sidewalk chalk in a large, open area outside due to the size of the subsequent models. Alternatively, masking tape and a large indoor area could be used as well. A 10 meter by 10 meter area will be needed to model one million.

Take the class and a ten-thousand square and sidewalk chalk (or tape) to a large, open area.

Set the ten-thousand square on the ground and **trace** the perimeter.

Ask students what they think 10 ten thousands would look like. Students should suggest lining up 10 ten thousands in a row, as they did with 10 hundreds in order to create a thousand.

Use the ten-thousand square to **trace** 10 ten thousands in a row.

Ask students what number they think is equivalent to 10 ten thousands.

Through discussion and, if necessary, reminders of the previous activity with three-dimensional base-ten manipulatives, **establish** that 10 ten thousands is one hundred thousand. Note that the hundred-thousands rectangle will be 10 meters by one meter.

Write "100,000" and "one hundred thousand" in chalk on the ground.

Continue using the ten-thousands square to model 10 hundred thousands on the ground.

Place and **draw** 10 hundred thousands rectangles (using the ten-thousands square as a guide) in a row to form a square composed of 10 rectangles (each representing 100,000). This square will be 10 meters by 10 meters.

Inform or **remind** students that 10 hundred thousands is referred to as a million.

Write "1,000,000" and "one million" on the ground in chalk.

Compare the size of one million to the ten-thousands square, the hundreds square, and the unit square to impress upon students the size difference of the quantities.

GUIDING QUESTIONS

Elicit student thinking:

- What patterns do you notice in these numbers?
- What makes the number 10 important when talking about large numbers?

Determine if the student can COMPOSE NUMBERS UP TO 1,000:

- If you put 10 ones together, what is another name for this amount?
- If you put 10 tens together, what is another name for this amount?
- If you put 10 hundreds together, what is another name for this amount?

Determine if the student can EXPLAIN PLACE VALUE FOR HUNDREDS:

- In the numeral 100, what does the 1 represent? What do the zeros represent?
- What is different about the numbers 100 and 500?

Determine if the student is ready to REPRESENT A QUANTITY BEYOND 1,000 WITH A NUMERAL:

- How would you write "one thousand" using numerals?
- How would you write "ten thousand" using numerals?
- How would you write "one hundred thousand" using numerals?
- How would you write "one million" using numerals?

Return to the classroom to discuss patterns and the structure of large numbers, as well as real-world examples.

Review the pattern in the structure of numbers:

- 10 ones = one ten (10)
- 10 tens = one hundred (100)
- 10 hundreds = one thousand (1,000)
- 10 thousands = one ten thousand (10,000)

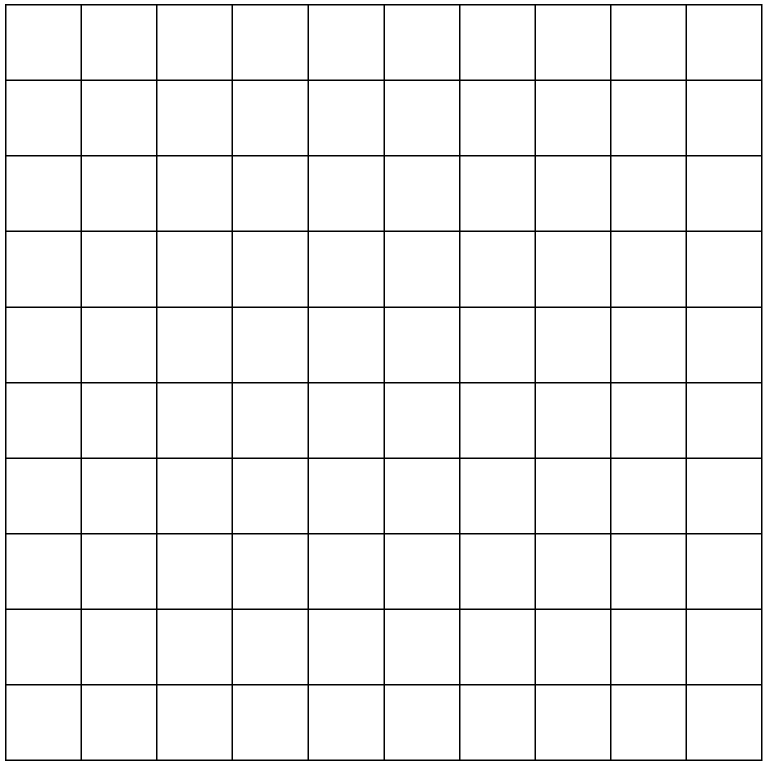
- 10 ten thousands = one hundred thousand (100,000)
- 10 hundred thousands = one million (1,000,000)

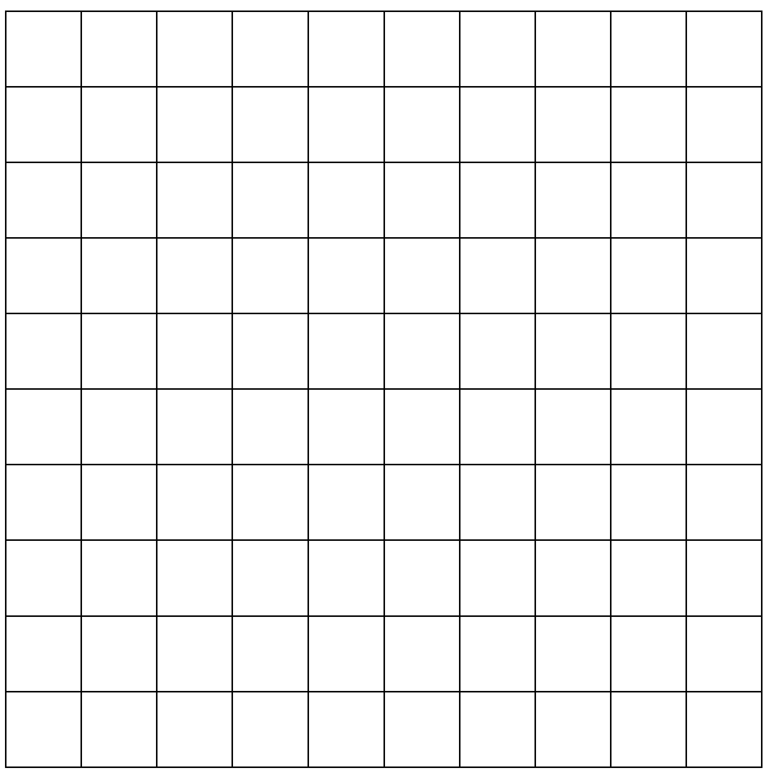
Note that this pattern continues to larger numbers and is based on tens; generally speaking, 10 in any position makes a single group in the next larger position and one in any position makes a group of 10 in the next smaller position.

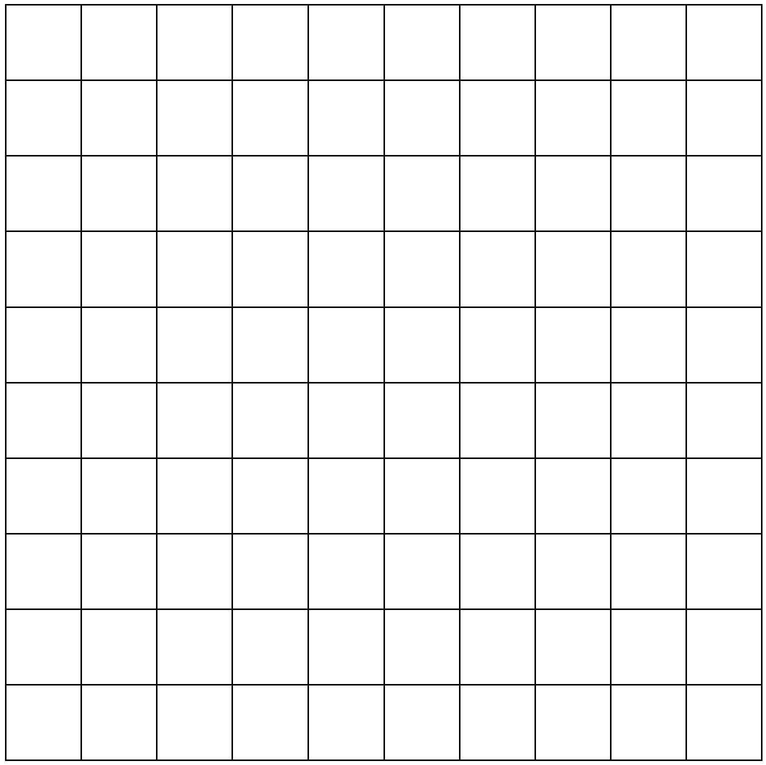
Discuss real-world examples of when students could encounter large quantities, in order to help explain the magnitude of large numbers. Some examples are:

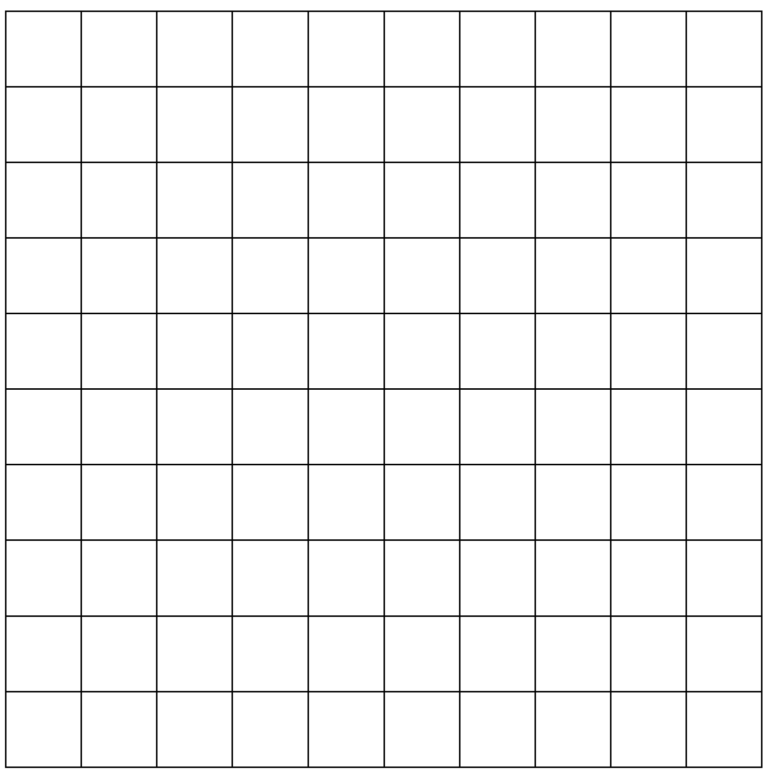
- 1,000 seconds is 16 minutes and 40 seconds; 1,000,000 seconds is just over $11\frac{1}{2}$ days
- Michigan Stadium, where the University of Michigan football team plays, holds just over 100,000 fans (display an image of the stadium). It is the largest stadium in the United States. It would take 10 of these stadiums to hold 1,000,000 people.

At the end of the activity, ask students to write or draw as much as they can to compare the numbers 100,000 and 1,000,000. Collect student responses to evaluate their understanding of large numbers based on the activities in the lesson.









Reading, Writing, and Comparing Multi-Digit Numbers

INSTRUCTIONAL ACTIVITY

Lesson 2

LEARNING GOAL

Students will write and read multi-digit numbers, then write expanded form for numbers into the millions.

PRIMARY ACTIVITY

Students will review ones, thousands, and millions and discuss the structure for writing and reading numerals and number names. Students will then use number strips to write standard and expanded form for a variety of multi-digit numbers.

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

- Ones
- Tens
- Hundreds
- Units
- Thousands
- Millions
- Numeral
- Number name
- Expanded form

MATERIALS

- Scissors
- Individual white boards
- Dry erase markers
- Instructional Activity Student Handout
- Instructional Activity Supplement (Recommend one copy for every two students.)

IMPLEMENTATION

Begin the lesson by reviewing the structure of numbers established and discussed in LESSON 1.

Ask students to recall the patterns they discovered about the structure of numbers in LESSON 1.

Provide time for students to discuss with a partner before asking students to share with the class.

Establish that the ones, tens, hundreds pattern that they are familiar with for three-digit numbers extends to thousands and millions (and beyond).

After reminding students of the patterns that exist in multi-digit numbers, continue the lesson by discussing how to write and read numerals and number names.

Write "Millions", "Thousands", and "Units" horizontally across the board with three rectangles under each word to represent the ones, tens, and hundreds place, using commas to separate the triples.

	Millions				Thousands				Units	
			,				,			

Connect the format for written numerals to the cyclical pattern with three-dimensional base-ten models from LESSON 1.

Fill in the rectangles with digits to represent a value in the millions (e.g., 1,305,016). **Note** that while the standards only require students to read and write numbers up to 1,000,000, exposing students to numbers in the millions will help emphasize the structure of multi-digit numbers.

Explain to students that to read multi-digit numbers, they should begin on the left-hand side of the numeral, read the first triple in the same way three-digit numbers are read, then include the units before moving to the next triple ("units" does not need to be read at the end of the number). The following are examples that can be practiced with students.

- 1,305,016: "one *million*, three hundred five *thousand*, sixteen"
- 47,090,415: "forty-seven *million*, ninety *thousand*, four hundred fifteen"
- 610,571,008: "six hundred ten *million*, five hundred seventy-one *thousand*, eight"
- 200,004,070: "two hundred *million*, four *thousand*, seventy"
- 000,392,128: "three hundred ninety-two *thousand*, one hundred twenty-eight"

Once students are able to comfortably read multi-digit numbers, **pass out** scissors to each student and one copy of the INSTRUCTIONAL ACTIVITY SUPPLEMENT to pairs of students.

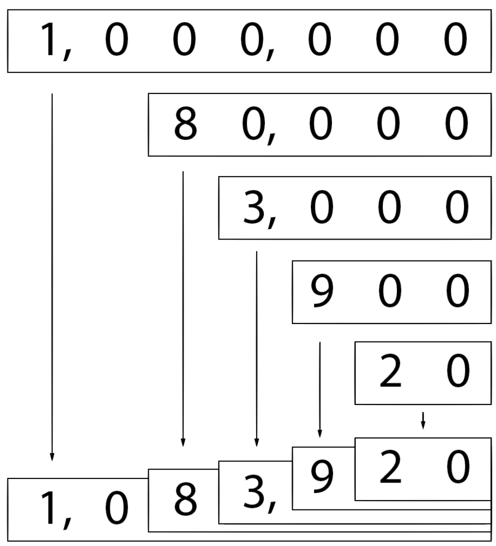
Require students in pairs to cut out the number strips (as well as the addition symbols "+") and read each number as they cut it out, confirming the number name with their partner. The number strips are restricted to numbers less than 10 million so that students have a manageable number of pieces to work with and keep track of.

Circulate among groups to ensure students are correctly reading each numeral.

Once students have all the number strips and addition symbols cut out, **suggest** that they organize the strips based on place value (a stack of millions, hundred thousands, ten thousands, etc.).

Provide students with a variety of written number names and spoken multi-digit numbers (less than 10,000,000 due to the available number strips) to build using their number strips. Note that the strips should be right-aligned as students build numerals, with the largest value on the bottom of the stack and the smallest value on top of the stack.

The following is an example of how pairs of students would build "one million, eighty-three thousand, nine hundred twenty".



R**emind** students once the number is assembled that the digit "1" represents 1,000,000 based on its position in the number, the digit "8" represents 80,000, the digit "3" represents 3,000, the digit "9" represents 900, and the digit "2" represents 20. These values are consistent with the values on the number strips. The first zero indicates that there are zero hundred thousands, and the second zero indicates that there are zero ones.

Note that the structure of the number strips helps reinforce the structure of multi-digit numbers and the presence of zero. In this example, there is not a digit in the hundred thousands place, therefore students do not use any of the hundred thousands number strips and a zero appears in the hundred thousands position.

Ensure that students have adequate practice listening to and reading number words and assembling multi-digit numbers using the number strips before continuing to the next part of the lesson. Ask the following guiding questions to check for student understanding.

GUIDING QUESTIONS

Elicit student thinking:

- [Point to a number.] What can you tell me about this number?
- What patterns do you notice in numbers with multiple digits?

Determine if the student can REPRESENT A QUANTITY WITH A NUMBER WORD:

- [Point to a numeral.] How would you read this numeral?
- [Point to a numeral.] How would you write the number name for this number?

Determine if the student can REPRESENT A QUANTITY BEYOND 1,000 WITH A NUMERAL:

- Can you use number strips to represent the number five thousand, one hundred twenty-three?
- Can you use number strips to represent the number fourteen thousand, sixty-two?
- Can you write the numeral "three million, six hundred forty-one thousand, nine"?

Determine if the student can EXPLAIN PLACE VALUE FOR THOUSANDS AND BEYOND:

- [Point to a digit in the number a student has represented (or is representing) with the number strips.] What is the value of this digit? How do you know?
- How do the number strips help you to know the value of each digit?

Next, students will write the number name and the expanded form for multi-digit numbers. Students should continue using the number strips from the INSTRUCTIONAL ACTIVITY SUPPLEMENT. In addition, each student will need an individual-size white board and a dry erase marker.

> **Write** a multi-digit number (less than 10,000,000 due to the available number strips) using numerals for students to consider (e.g., 4,601,837).

Ask students to independently write the number name (e.g., four million, six hundred one thousand, eight hundred thirty-seven) on their white board and check their work with a partner.

Once students have written the number name and confirmed with a partner, **confirm** the correct number name as a class.

Next, **ask** students to work in pairs to build the number with their number strips.

Circulate to check students work, and **inform** the class that this representation is the *numeral* or *standard form* of the number.

Inform students that numbers can also be written in *expanded form*, which represents the number as the sum of the values of each digit. **Note** that the number strips, along with the addition symbol (+), will help students build the expanded form of the number.

Guide students to pull apart their number strips and position the addition symbol to create the expanded form of the number.

In the example 4,601,837, expanded form using the number strips and addition symbols would appear as follows:

4,000,000	+	600,000	+	1,000	+	800	+	30	+	7

Repeat this process (providing students with a multi-digit numeral and asking them to write the number name on their white board, then create expanded form with the number strips) with a variety of multi-digit numerals to provide students with adequate practice writing number names and expanded form.

As students become more comfortable with expanded form, **challenge** them to write the expanded form of the number on the white board without using the number strips.

NOTE: An extension for students who are able to write and understand expanded form as described previously would be to write expanded form using products to emphasize place value. For example:

4,000,000 + 600,000 + 1,000 + 800 + 30 + 7 could be written as

$4 \cdot 1{,}000{,}000 + 6 \cdot 100{,}000 + 1 \cdot 1{,}000 + 8 \cdot 100 + 3 \cdot 10 + 7 \cdot 1$.

Ask the following guiding questions to check for student understanding.

GUIDING QUESTIONS

Elicit student thinking:

- [Point to a numeral.] How would you read this numeral?
- [Point to a number in any form.] What can you tell me about this number?
- What patterns do you notice in numbers with multiple digits?

Determine if the student can EXPLAIN PLACE VALUE FOR THOUSANDS AND BEYOND:

- [Point to a digit in the numeral a student has represented (or is representing) with the number strips.] What is the value of this digit? How do you know?
- How do the number strips help you to know the value of each digit?
- [Write a number using numerals (e.g., 6,213,578).] What is the value of the digit "2"? [Ask about a variety of digits in the number provided.]

Determine if the student can REPRESENT 4-DIGIT NUMERALS AND BEYOND WITH EXPANDED FORM:

- [Point to a numeral or number name.] Can you show me the expanded form of this number using your number strips?
- [Point to a numeral or number name.] Could you write expanded form for this number without using your number strips?

After students have practiced with expanded form and writing number names using white boards and number strips, provide each student with the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT for additional practice, either independently or in pairs. If necessary, discuss the worked-out example as a class before asking students to work independently or in pairs.

At the end of the activity, collect the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT to check for student understanding of numerals, number names, and expanded form for multi-digit numbers.

Name________________________________

Reading, Writing, and Comparing Multi-Digit Numbers

Lesson 2

For each problem, fill in the missing number representations. An example is provided.

MILLIONS			THOUSANDS			UNITS		
1	4	5 ,	0	7	2 ,	9	0	3
Number Name: One hundred forty-five million, seventy-two thousand, nine hundred three.								
Expanded Form: 100,000,000 + 40,000,000 + 5,000,000 + 70,000 + 2,000 + 900 + 3								

1.

MILLIONS			THOUSANDS			UNITS		
0	0	1 ,	3	0	6 ,	8	2	0
Number Name:								
Expanded Form:								

Name________________________________

2.

MILLIONS			THOUSANDS			UNITS		
		,			,			
Number Name: Two million, four hundred sixty-one thousand, thirty-seven.								
Expanded Form:								

3.

MILLIONS			THOUSANDS			UNITS		
		,			,			
Number Name:								
Expanded Form: 700,000 + 40,000 + 200 + 20 + 9								

Name________________________________

4.

MILLIONS			THOUSANDS			UNITS		
0	3	0 ,	0	8	6 ,	5	1	5
Number Name:								
Expanded Form:								

5.

MILLIONS			THOUSANDS			UNITS		
		,			,			
Number Name:								
Expanded Form: 3,000,000 + 400,000 + 10,000 + 6,000 + 700 + 80 + 9								

Name________________________________

6.

MILLIONS			THOUSANDS			UNITS		
		,			,			
Number Name: Eleven million, sixty-three thousand, four hundred seven.								
Expanded Form:								

7.

MILLIONS			THOUSANDS			UNITS		
2	1	3 ,	5	6	0 ,	7	2	5
Number Name:								
Expanded Form:								

1,	0	0	0,	0	0	0
2,	0	0	0,	0	0	0
3,	0	0	0,	0	0	0
4,	0	0	0,	0	0	0
5,	0	0	0,	0	0	0

6,	0	0	0,	0	0	0
7,	0	0	0,	0	0	0
8,	0	0	0,	0	0	0
9,	0	0	0,	0	0	0
	1	0	0,	0	0	0
	2	0	0,	0	0	0

3	0	0,	0	0	0
4	0	0,	0	0	0
5	0	0,	0	0	0
6	0	0,	0	0	0
7	0	0,	0	0	0
8	0	0,	0	0	0

9	0	0,	0	0	0
	1	0,	0	0	0
	2	0,	0	0	0
	3	0,	0	0	0
	4	0,	0	0	0
	5	0,	0	0	0

6	0,	0	0	0
7	0,	0	0	0
8	0,	0	0	0
9	0,	0	0	0
	1,	0	0	0
	2,	0	0	0

3, 0 0 0	1 0 0
4, 0 0 0	2 0 0
5, 0 0 0	3 0 0
6, 0 0 0	4 0 0
7, 0 0 0	5 0 0
8, 0 0 0	6 0 0

9,	0	0	0	7	0	0
	8	0	0	9	0	0
	1	0	2	0	3	0
	4	0	5	0	6	0
	7	0	8	0	9	0
1	2	3	4	5	6	7

8	9	+	+	+	+	+
		+	+	+	+	+

Reading, Writing, and Comparing Multi-Digit Numbers

INSTRUCTIONAL ACTIVITY

Lesson 3

LEARNING GOAL

Students will utilize place value understanding to compare two multi-digit numbers using symbols (>, <, =) and order more than two multi-digit numbers.

PRIMARY ACTIVITY

Students will first compare two multi-digit numbers using the symbols >, <, and =. Students will then order more than two multi-digit numbers from least to greatest or from greatest to least by physically rearranging numbers printed on strips of paper. Finally, students will play a mystery number game on the number line where they try to guess a multi-digit number based on its position on the number line and the placement of previous guesses.

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

- Ones
- Tens
- Hundreds
- Units
- Thousands
- Millions
- Greatest
- Least
- Greater than
- Less than
- Equal to
- Numeral

MATERIALS

- Individual white boards
- Dry erase markers
- Scissors
- Blank paper (lined or unlined)
- Number strips from LESSON 2 INSTRUCTIONAL ACTIVITY SUPPLEMENT (Recommend one copy for every two students.)
- LESSON 3 INSTRUCTIONAL ACTIVITY SUPPLEMENT (Recommend one copy for every two students.)

IMPLEMENTATION

Students will begin the lesson by comparing two multi-digit numbers using place value reasoning.

Display two multi-digit numerals for students to consider (e.g., 4,837 and 5,210).

Ask students to think about which number is greater and compare their thoughts with a partner. Do not yet confirm or correct student thinking.

Provide pairs of students with the number strips from the LESSON 2 INSTRUCTIONAL ACTIVITY SUPPLEMENT.

Ask students to build each number using the number strips, then reconsider the question about which number is greater.

Once students have had a chance to think and discuss the question with and without number strips, **begin** a class discussion about which number is greater.

Ask students to explain their thinking as they state which number they believe is greater. **Encourage** students to discuss the value of each digit using place value understanding.

Note that when comparing two values, it is important to focus on the largest unit—in this case the thousands place—and determine which number has the greater value in that place. **Explain** that if the largest unit is equal in both values, then the next largest unit should be compared to determine which value is greater.

In the example provided, 4,000 is less than 5,000, therefore 4,837 is less than 5,210. The fact that 800 is greater than 200, 30 is greater than 10, and 7 is greater than 0 does not change the fact that 4,837 has fewer thousands, and is therefore less than, 5,210.

NOTE: Precise language is important when referring to digits that are not located in the ones place. Use base-ten language and encourage students to do the same. For example, when referring to the digit “4” in the numeral “41,328”, say “four ten thousands” or “forty thousand”.

Build each number using either two- or three-dimensional base-ten manipulatives from LESSON 1 to show, in a concrete way, that 4,837 is less than (and has fewer units than) 5,210.

Write “4,837 is less than 5,210” and “4,837 < 5,210” for students to see.

Remind students that “<” is a symbol that means “less than”.

Ask students if they know what the “>” symbol means. **Establish** that “>” is a symbol that means “greater than”.

Ask students if they know what the “=” symbol means. **Establish** that “=” is a symbol that means “equal to”.

Provide pairs of students with a whiteboard and a dry erase marker to work through additional guided examples. As students work to compare each pair of numbers, **circulate** to ensure student understanding of the symbols they are using. While the greater than and less than symbols should have been introduced in third grade, it is possible that students are able to correctly compare two values but confuse the symbols to express their understanding.

Sequence examples such that students first compare two values with the same number of digits and a different value in the largest unit, then two values with the same number of digits and the same value in the largest unit, and finally, two values with a different number of digits.

The following is a sample progression of examples for students to consider.

- 33,765 ☐ 40,142
- 783,009 ☐ 821,031
- 47,100 ☐ 43,999
- 129,534 ☐ 129,612
- 5,001,436 ☐ 5,001,436
- 1,000,000 ☐ 999,999
- 54,629 ☐ 110,030

NOTE: If students struggle to compare values when they are written horizontally, suggest writing the numerals vertically and lining up each place value to aid in the comparison of units.

To challenge more advanced students, replace one or both numerals with either the number name or the expanded form of the numeral (e.g., one million, four hundred ☐ 900,000 + 90,000 + 9,000 + 900 + 90 + 9).

Ask the following guiding questions as students compare values.

GUIDING QUESTIONS

Elicit student thinking:

- [Point to each of the numbers the student is comparing, one at a time.] What can you tell me about this number?
- What is a large number you can think of?
- Where might you see a large number outside of school?

Determine if the student can EXPLAIN PLACE VALUE FOR THOUSANDS AND BEYOND:

- [Point to a digit in one of the numbers the student is comparing.] What is the value of this digit? How do you know?
- How could you use the number strips to help you know the value of each digit?

Determine if the student can COMPARE 2 NUMERALS BEYOND 1,000 USING SYMBOLS (=, <, >):

- What do you notice about the number of digits in these numbers?
- Where should you look first to determine which number is greater (or less than)?
- If the largest unit in each number is the same, how would you continue comparing the numbers?
- If one number has one ten thousand as the largest unit and another number has nine thousands as the largest unit, which number is greater? How do you know?
- [Provide two numbers or point to two numbers.] Which symbol (=, <, >) correctly describes the relationship between these two numbers?

Next, students will extend their place value understanding and ability to compare two values to order more than two multi-digit numbers from greatest to least and from least to greatest.

Collect the number strips (from the LESSON 2 INSTRUCTIONAL ACTIVITY SUPPLEMENT), the white boards, and the dry erase markers.

Pass out the LESSON 3 INSTRUCTIONAL ACTIVITY SUPPLEMENT and scissors to pairs of students.

Direct students to cut out the first set of numerals and order them either least to greatest or greatest to least.

Allow time for pairs of students to place the numbers in the order requested.

If students struggle to compare or order the numbers, **suggest** lining the numerals up vertically to focus on each place value, beginning with the largest unit.

Once students have ordered the numbers, **discuss** the correct order as a class. **Emphasize** place value understanding during the class discussion. **Relate** the numbers to their representations using base-ten manipulatives (two- or three-dimensional) during the discussion when possible.

Repeat this process for the remaining numeral sets in the LESSON 3 INSTRUCTIONAL ACTIVITY SUPPLEMENT.

Be sure to **ask** students to order the numerals from least to greatest as well as from greatest to least.

Ask the following guiding questions to check for student understanding.

GUIDING QUESTIONS

Elicit student thinking:

- [Point to a numeral.] What do you know about this number?
- [Point to a numeral.] How would you describe this number?
- [Point to a numeral.] How would you model this number with base-ten manipulatives (the materials we used in the first lesson)?

Determine if the student can EXPLAIN PLACE VALUE FOR THOUSANDS AND BEYOND:

- [Point to a digit in one of the numerals the student is ordering.] What is the value of this digit? How do you know?
- How could you use the number strips to help you know the value of each digit?

Determine if the student can ORDER MORE THAN 2 3-DIGIT NUMERALS OR QUANTITIES FROM GREATEST TO LEAST:

- [For the first and second set of numerals] Which of these numerals is the least? How do you know?
- [For the first and second set of numerals] Which of these numerals is the greatest? How do you know?
- [For the first and second set of numerals] Once you have determined the least (or the greatest) numeral, how would you order the remaining numerals from greatest to least?

Determine if the student can ORDER MORE THAN 2 3-DIGIT NUMERALS OR QUANTITIES FROM LEAST TO GREATEST:

- [For the first and second set of numerals] Which of these numerals is the least? How do you know?
- [For the first and second set of numerals] Which of these numerals is the greatest? How do you know?
- [For the first and second set of numerals] Once you have determined the least (or the greatest) numeral, how would you order the remaining numerals from least to greatest?

Determine if the student can ORDER MORE THAN 2 4-DIGIT AND BEYOND NUMERALS OR QUANTITIES FROM GREATEST TO LEAST:

- [For the third through eighth sets of numerals] Which of these numerals is the least? How do you know?
- [For the third through eighth sets of numerals] Which of these numerals is the greatest? How do you know?
- [For the third through eighth sets of numerals] Once you have determined the least (or the greatest) numeral, how would you order the remaining numerals from greatest to least?

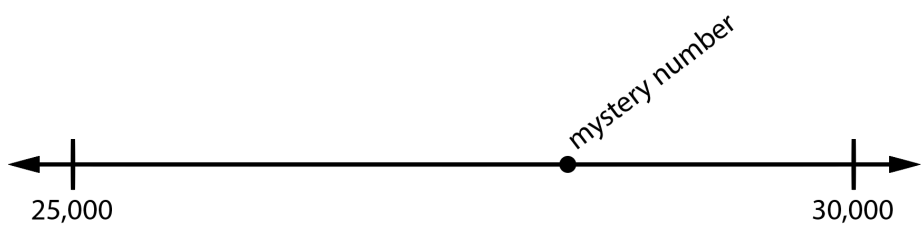
Determine if the student can ORDER MORE THAN 2 4-DIGIT AND BEYOND NUMERALS OR QUANTITIES FROM LEAST TO GREATEST:

- [For the third through eighth sets of numerals] Which of these numerals is the least? How do you know?
- [For the third through eighth sets of numerals] Which of these numerals is the greatest? How do you know?
- [For the third through eighth sets of numerals] Once you have determined the least (or the greatest) numeral, how would you order the remaining numerals from least to greatest?

The last activity in the lesson aims to strengthen student understanding of multi-digit numbers through a mystery number game on the number line.

Draw a number line for students to see, and mark a low and a high value on the number line.

Determine a "mystery number" and place it as accurately as possible on the number line. In this example, the mystery number is 27,904.



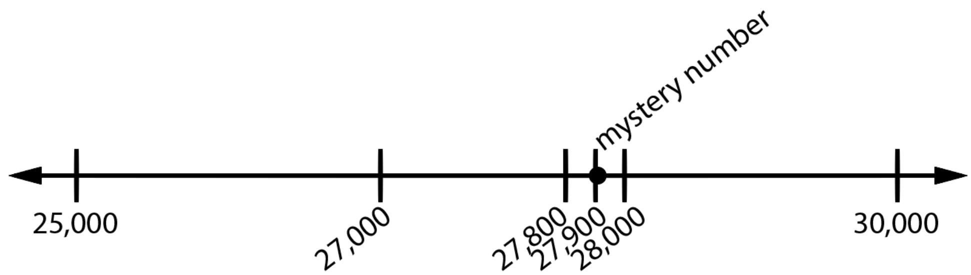
NOTE: It is recommended that the range of the low and high value be somewhat small initially. To increase the difficulty, provide a larger range of numbers (e.g., 10,000 to 1,000,000). Depending on the range provided, it is likely that as students get closer to guessing the mystery number, the number line may need to be redrawn with the two closest guesses as the new low and high values.

Have students take turns guessing the mystery number. **Place** each guess on the number line, as accurately as possible, to inform the next student's guess.

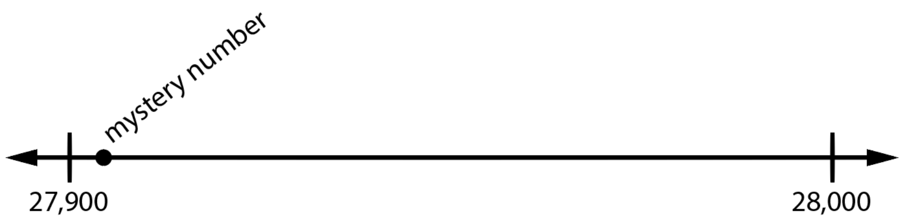
Think aloud as you place each number on the number line to help students understand its placement compared to the other numbers on the number line.

Ensure that students correctly pronounce the numbers they guess, and **correct** their pronunciations as needed.

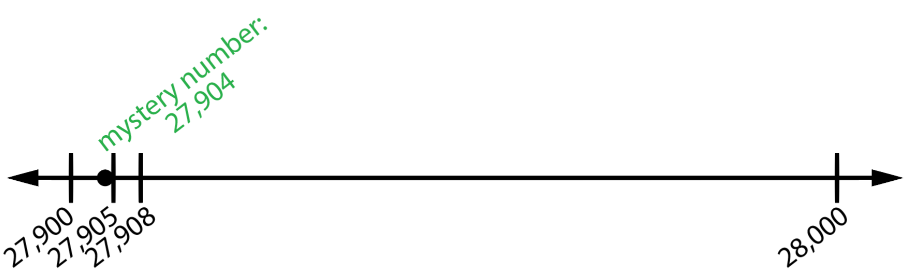
The following is an example of the number line after four guesses.



At this point, the mystery number has not yet been guessed, but there is a very limited amount of space to continue placing student guesses. Therefore, it is recommended that the number line be redrawn with the two closest surrounding values (27,900 and 28,000) as the new low and high values, and that the mystery number is situated accordingly.



Continue taking student guesses until the mystery number has been guessed (or the number line needs to be redrawn again).



Repeat the mystery number game as many times as desired with different multi-digit whole numbers. If students are ready, increase the range of initial high and low values. **Recall** that this might require that the number line be redrawn more than once during the course of the game.

At the end of the activity, have students play the mystery number game in pairs on paper. Each student should take a turn guessing the mystery number while the other student marks the number line.

Require students to turn this paper in at the end of class so their ability to order multi-digit numbers (from least to greatest) can be considered.

1.

613	609	587

2.

307	370	730
703	373	337

3.

4,654	6,017	5,455
3,090		

4.

9,800	9,788	9,194
8,006	8,100	8,090

5.

10,438	10,199	11,000
11,080	9,654	

6.

432,001	610,757	802,963
609,527	756,802	590,900

7.

80,916	400,740	93,608
5,110,675	673,219	

8.

1,000,050	300,700	9,750
55,682	1,100,000	301,500

Name____

Reading, Writing, and Comparing Multi-Digit Numbers

Lessons 1 – 3

1. Draw a model or write a numeral to show how many hundreds you would need to put together to make a thousand.

2. Write the number name for the following numerals.

   2.a. 72,508

   2.b. 460,032

   2.c. 16,002,479

Name____

3. Write the numeral for the following number names.

3.a. Seven hundred thirteen thousand, nine hundred two

3.b. Twenty-one million, six thousand, four hundred eighty

3.c. One hundred fifty million, six hundred thirty-seven thousand, ninety

4. Write the expanded form for the following numerals.

4.a. 45,761

4.b. 234,098

4.c. 1,070,625

Name____

5. Compare the following numerals by writing >, <, or = in the box. Explain your reasoning.

5.a. 51,023 $\square$ 49,675

5.b. 208,453 $\square$ 208,453

5.c. 956,732 $\square$ 957,010

5.d. 882,946 $\square$ 1,300,017

Name____

6. Order the following numerals from least to greatest.

5,104	4,958	4,899	5,110

7. Order the following numerals from greatest to least.

2,000,100	839,004	879,116	910,435	1,567,900

8. Use the following number line to estimate the "mystery number". Explain how you made your estimate.

Reading, Writing, and Comparing Multi-Digit Numbers

STUDENT ACTIVITY SOLUTION GUIDE

Lessons 1 – 3

1. Draw a model or write a numeral to show how many hundreds you would need to put together to make a thousand.

CORRECT ANSWER

You would need to put together 10 hundreds to make a thousand.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
You would need 1,000.	provides the number of ones needed to make 1,000, but does not consider or know how many hundreds are needed to make a thousand	COMPOSE NUMBERS UP TO 1,000
You would need 100.	either provides the number of tens required to make 1,000 or simply reads the word “hundreds” and writes 100	COMPOSE NUMBERS UP TO 1,000

2. Write the number name for the following numerals.

2.a. 72,508

CORRECT ANSWER

Seventy-two thousand, five hundred eight

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Seventy-two, five hundred eight	does not know to include the unit “thousands” after “seventy-two”	EXPLAIN PLACE VALUE FOR THOUSANDS AND BEYOND and REPRESENT A QUANTITY WITH A NUMBER WORD
Seven, two, five, zero, eight	reads the digits from left to right	EXPLAIN PLACE VALUE FOR THOUSANDS AND BEYOND and REPRESENT A QUANTITY WITH A NUMBER WORD
Seventy-two, fifty-eight	does not attend to the zero, then reads the two-digit numbers on either side of the comma separately	EXPLAIN PLACE VALUE FOR THOUSANDS AND BEYOND and REPRESENT A QUANTITY WITH A NUMBER WORD

2.b. 460,032

CORRECT ANSWER

Four hundred sixty thousand, thirty-two

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Four hundred sixty, thirty-two	does not know to include the unit “thousands” after “four hundred sixty”	EXPLAIN PLACE VALUE FOR THOUSANDS AND BEYOND and REPRESENT A QUANTITY WITH A NUMBER WORD
Four, six, zero, zero, three, two	reads the digits from left to right	EXPLAIN PLACE VALUE FOR THOUSANDS AND BEYOND and REPRESENT A QUANTITY WITH A NUMBER WORD
Forty-six, thirty-two	does not attend to the zero, then reads the two-digit numbers on either side of the comma separately	EXPLAIN PLACE VALUE FOR THOUSANDS AND BEYOND and REPRESENT A QUANTITY WITH A NUMBER WORD

2.c. 16,002,479

CORRECT ANSWER

Sixteen million, two thousand, four hundred seventy-nine

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Sixteen, two, four hundred seventy-nine	does not know to include the unit “millions” after “sixteen” or the unit “thousands” after “two”	EXPLAIN PLACE VALUE FOR THOUSANDS AND BEYOND and REPRESENT A QUANTITY WITH A NUMBER WORD
One, six, zero, zero, two, four, seven, nine	reads the digits from left to right	EXPLAIN PLACE VALUE FOR THOUSANDS AND BEYOND and REPRESENT A QUANTITY WITH A NUMBER WORD

3. Write the numeral for the following number names.

3.a. Seven hundred thirteen thousand, nine hundred two

CORRECT ANSWER

713,902

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
700, 13,000, 900, 2	lists separate numerals for “seven hundred”, “thirteen thousand”, “nine hundred”, and “two”	EXPLAIN PLACE VALUE FOR THOUSANDS AND BEYOND and REPRESENT A QUANTITY BEYOND 1,000 WITH A NUMERAL
713,000,902	writes the numeral for “seven hundred thirteen thousand”, then follows with the numeral for “nine hundred two”	EXPLAIN PLACE VALUE FOR THOUSANDS AND BEYOND and REPRESENT A QUANTITY BEYOND 1,000 WITH A NUMERAL
700,13,000,900,2 *or* 700,013,000,900,002	writes the numerals for “seven hundred”, “thirteen thousand”, “nine hundred”, and “two” separated by commas	EXPLAIN PLACE VALUE FOR THOUSANDS AND BEYOND and REPRESENT A QUANTITY BEYOND 1,000 WITH A NUMERAL
713,92	does not include a zero in the tens place to indicate that there are not any tens in the number	EXPLAIN PLACE VALUE FOR HUNDREDS and REPRESENT A QUANTITY UP TO 1,000 WITH A NUMERAL
713,920 *or* 713,092	does not know where the zero should be placed, or does not understand that the position of the zero is significant when writing the number “nine hundred two”	EXPLAIN PLACE VALUE FOR HUNDREDS and REPRESENT A QUANTITY UP TO 1,000 WITH A NUMERAL

3.b. Twenty-one million, six thousand, four hundred eighty

CORRECT ANSWER

21,006,480

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
21,000,000, 6,000, 400, 80	lists separate numerals for "twenty-one million", "six thousand", "four hundred", and "eighty"	EXPLAIN PLACE VALUE FOR THOUSANDS AND BEYOND and REPRESENT A QUANTITY BEYOND 1,000 WITH A NUMERAL
21,000,000,006,000,480 *or* 21,000,000,6,000,480	writes the numeral for "twenty-one million", then follows with the numeral for "six thousand" and the numeral for "four hundred eighty"	EXPLAIN PLACE VALUE FOR THOUSANDS AND BEYOND and REPRESENT A QUANTITY BEYOND 1,000 WITH A NUMERAL
21,000,000,6,000,400,80 *or* 21,000,000,006,000,400,080	writes the numerals for "twenty-one million", "six thousand", "four hundred", and "eighty" separated by commas	EXPLAIN PLACE VALUE FOR THOUSANDS AND BEYOND and REPRESENT A QUANTITY BEYOND 1,000 WITH A NUMERAL
21,648	identifies the appropriate non-zero digits from the number name but does not attend to units and place value	EXPLAIN PLACE VALUE FOR THOUSANDS AND BEYOND and REPRESENT A QUANTITY BEYOND 1,000 WITH A NUMERAL
216,480	does not attend to the units "millions", or does not know how to use zeros as place holders when representing numbers greater than 1,000	EXPLAIN PLACE VALUE FOR THOUSANDS AND BEYOND and REPRESENT A QUANTITY BEYOND 1,000 WITH A NUMERAL
21,600,480 *or* 21,060,480	does not know where the zeros should be placed within the thousands triple in order to indicate "six thousand", or does not understand that that the position of the zero is significant	EXPLAIN PLACE VALUE FOR THOUSANDS AND BEYOND and REPRESENT A QUANTITY BEYOND 1,000 WITH A NUMERAL

3.c. One hundred fifty million, six hundred thirty-seven thousand, ninety

CORRECT ANSWER

150,637,090

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
100, 50,000,000, 600, 37,000, 90	lists separate numerals for "one hundred", "fifty million", "six hundred", "thirty-seven thousand", and "ninety"	EXPLAIN PLACE VALUE FOR THOUSANDS AND BEYOND and REPRESENT A QUANTITY BEYOND 1,000 WITH A NUMERAL
150,000,000,637,000,090	writes the numeral for "one hundred fifty million", then follows with the numeral for "six hundred thirty-seven thousand" and the numeral for "ninety"	EXPLAIN PLACE VALUE FOR THOUSANDS AND BEYOND and REPRESENT A QUANTITY BEYOND 1,000 WITH A NUMERAL
100,50,000,000,600,37,000,90 *or* 100,050,000,000,600,037,000,090	writes the numerals for "one hundred", "fifty million", "six hundred", "thirty-seven thousand", and "ninety" separated by commas	EXPLAIN PLACE VALUE FOR THOUSANDS AND BEYOND and REPRESENT A QUANTITY BEYOND 1,000 WITH A NUMERAL
150,637,900 *or* 150,637,009	does not know where the zero should be placed, or does not understand that the position of the zero is significant when writing the number "ninety"	EXPLAIN PLACE VALUE FOR HUNDREDS and REPRESENT A QUANTITY UP TO 1,000 WITH A NUMERAL
156,379	identifies the appropriate non-zero digits from the number name but does not attend to units and place value	EXPLAIN PLACE VALUE FOR THOUSANDS AND BEYOND and REPRESENT A QUANTITY BEYOND 1,000 WITH A NUMERAL

4. Write the expanded form for the following numerals.

4.a. 45,761

CORRECT ANSWER

$40{,}000 + 5{,}000 + 700 + 60 + 1$

or

$4 \cdot 10{,}000 + 5 \cdot 1{,}000 + 7 \cdot 100 + 6 \cdot 10 + 1 \cdot 1$

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
$45{,}000 + 761$	does not fully expand the base-ten numeral	REPRESENT 4-DIGIT NUMERALS AND BEYOND WITH EXPANDED FORM
$45 + 761$	does not understand that the units for "45" is thousands and does not fully expand the base-ten numeral	EXPLAIN PLACE VALUE FOR THOUSANDS AND BEYOND and REPRESENT 4-DIGIT NUMERALS AND BEYOND WITH EXPANDED FORM
$40 + 5 + 700 + 60 + 1$	does not understand that the units for "4" is ten thousands rather than tens and the units for "5" is thousands rather than ones	EXPLAIN PLACE VALUE FOR THOUSANDS AND BEYOND
$4 + 5 + 7 + 6 + 1$	does not understand the value of each digit in the number; views 45,761 as a list of five single digits	EXPLAIN PLACE VALUE FOR THOUSANDS AND BEYOND

4.b. 234,098

CORRECT ANSWER

200,000 + 30,000 + 4,000 + 90 + 8

or

$2 \cdot 100{,}000 + 3 \cdot 10{,}000 + 4 \cdot 1{,}000 + 9 \cdot 10 + 8 \cdot 1$

Note that it is also accurate to add "0" in the hundreds place to represent that there are zero hundreds.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
234,000 + 98 *or* 234,000 + 098	does not fully expand the base-ten numeral	REPRESENT 4-DIGIT NUMERALS AND BEYOND WITH EXPANDED FORM
234 + 098 *or* 234 + 98	does not understand that the units for "234" is thousands and does not fully expand the base-ten numeral	EXPLAIN PLACE VALUE FOR THOUSANDS AND BEYOND and REPRESENT 4-DIGIT NUMERALS AND BEYOND WITH EXPANDED FORM
200 + 30 + 4 + 90 + 8 *or* 200 + 30 + 4 + 000 + 90 + 8	does not understand that the units for "2" is hundred thousands rather than hundreds, the units for "3" is ten thousands rather than tens, and the units for "4" is thousands rather than ones	EXPLAIN PLACE VALUE FOR THOUSANDS AND BEYOND
2 + 3 + 4 + 0 + 9 + 8	does not understand the value of each digit in the number; views 234,098 as a list of six single digits	EXPLAIN PLACE VALUE FOR THOUSANDS AND BEYOND

4.c. 1,070,625

CORRECT ANSWER

1,000,000 + 70,000 + 600 + 20 + 5

or

$1 \cdot 1{,}000{,}000 + 7 \cdot 10{,}000 + 6 \cdot 100 + 2 \cdot 10 + 5 \cdot 1$

Note that it is also accurate to add "0" in the hundred thousands place and the thousands place to represent that there are zero hundred thousands and zero thousands.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
1,000,000 + 70,000 + 625 *or* 1,000,000 + 070,000 + 625	does not fully expand the base-ten numeral	REPRESENT 4-DIGIT NUMERALS AND BEYOND WITH EXPANDED FORM
1 + 070 + 625 *or* 1 + 70 + 625	does not understand that the units for "1" is millions or that the units for "70" is thousands and does not fully expand the base-ten numeral	EXPLAIN PLACE VALUE FOR THOUSANDS AND BEYOND and REPRESENT 4-DIGIT NUMERALS AND BEYOND WITH EXPANDED FORM
1 + 70 + 600 + 20 + 5 *or* 1 + 000 + 70 + 0 + 600 + 20 + 5	does not understand that the units for "1" is millions rather than ones or that the units for "7" is ten thousands rather than tens	EXPLAIN PLACE VALUE FOR THOUSANDS AND BEYOND
1 + 0 + 7 + 0 + 6 + 2 + 5	does not understand the value of each digit in the number; views 1,070,615 as a list of seven single digits	EXPLAIN PLACE VALUE FOR THOUSANDS AND BEYOND

5. Compare the following numerals by writing >, <, or = in the box. Explain your reasoning.

5.a. 51,023 ☐ 49,675

CORRECT ANSWER

51,023 > 49,675

I know 51,023 is greater than 49,675 because the largest unit in each number is ten thousands and five ten thousands is greater than four ten thousands.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
51,023 < 49,675	may believe that because many of the digits in 49,675 are greater than the digits in 51,023 that 49,675 is the greater value; does not utilize place value reasoning appropriately	EXPLAIN PLACE VALUE FOR THOUSANDS AND BEYOND and COMPARE 2 NUMERALS BEYOND 1,000 USING SYMBOLS (=, <, >)
51,023 = 49,675	may believe they are equal because there are five digits in each number	COMPARE 2 NUMERALS BEYOND 1,000 USING SYMBOLS (=, <, >)
Student does not explain their reasoning.	may not understand how to compare two numerals but was able to guess the correct symbol	COMPARE 2 NUMERALS BEYOND 1,000 USING SYMBOLS (=, <, >)

5.b. 208,453 ☐ 208,453

CORRECT ANSWER

208,453 = 208,453

I know 208,453 is equal to 208,453 because the largest unit in each number is hundred thousands and both numbers have the same amount of hundred thousands, ten thousands, thousands, hundreds, tens, and ones.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
208,453 > 208,453	does not carefully attend to and compare the values; guesses a comparison symbol	COMPARE 2 NUMERALS BEYOND 1,000 USING SYMBOLS (=, <, >)
208,453 < 208,453	does not carefully compare the values; guesses a comparison symbol	COMPARE 2 NUMERALS BEYOND 1,000 USING SYMBOLS (=, <, >)
Student does not explain their reasoning.	may not understand how to compare two numerals but was able to guess the correct symbol	COMPARE 2 NUMERALS BEYOND 1,000 USING SYMBOLS (=, <, >)

5.c. 956,732 ☐ 957,010

CORRECT ANSWER

956,732 < 957,010

I know 956,732 is less than 957,010 because the largest unit in each number is hundred thousands and both numbers have the same amount of hundred thousands and ten thousands, but six thousands is less than seven thousands.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
956,732 > 957,010	may believe that because many of the digits in 956,732 are greater than the digits in 957,010 that 956,732 is the greater value; does not utilize place value reasoning appropriately	EXPLAIN PLACE VALUE FOR THOUSANDS AND BEYOND and COMPARE 2 NUMERALS BEYOND 1,000 USING SYMBOLS (=, <, >)
956,732 = 957,010	may believe they are equal because there are six digits in each number	COMPARE 2 NUMERALS BEYOND 1,000 USING SYMBOLS (=, <, >)
Student does not explain their reasoning.	may not understand how to compare two numerals but was able to guess the correct symbol	COMPARE 2 NUMERALS BEYOND 1,000 USING SYMBOLS (=, <, >)

5.d. 882,946 ☐ 1,300,017

CORRECT ANSWER

882,946 < 1,300,017

I know 882,946 is less than 1,300,017 because the largest unit in 882,946 is hundred thousands and the largest unit in 1,300,017 is millions. Hundred thousands is a smaller unit than millions, therefore 882,946 is less than 1,300,017.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
882,946 > 1,300,017	may believe that because many of the digits in 882,946 are greater than the digits in 1,300,017 that 882,946 is the greater value; does not utilize place value reasoning appropriately	EXPLAIN PLACE VALUE FOR THOUSANDS AND BEYOND and COMPARE 2 NUMERALS BEYOND 1,000 USING SYMBOLS (=, <, >)
882,946 = 1,300,017	does not carefully compare the values; guesses a comparison symbol	COMPARE 2 NUMERALS BEYOND 1,000 USING SYMBOLS (=, <, >)
Student does not explain their reasoning.	may not understand how to compare two numerals but was able to guess the correct symbol	COMPARE 2 NUMERALS BEYOND 1,000 USING SYMBOLS (=, <, >)

6. Order the following numerals from least to greatest.

5,104	4,958	4,899	5,110

CORRECT ANSWER

4,899
4,958
5,104
5,110

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
5,110 5,104 4,958 4,899	orders the numerals from greatest to least instead of from least to greatest	ORDER MORE THAN 2 4-DIGIT AND BEYOND NUMERALS OR QUANTITIES FROM LEAST TO GREATEST
4,958 4,899 5,110 5,104	correctly compares the largest unit in each numeral, then only considers which numeral has greater remaining digits (in the hundreds, tens, and ones place) without considering place value for each digit	EXPLAIN PLACE VALUE FOR ONES AND TENS, EXPLAIN PLACE VALUE FOR HUNDREDS, and ORDER MORE THAN 2 4-DIGIT AND BEYOND NUMERALS OR QUANTITIES FROM LEAST TO GREATEST
5,110 5,104 4,958 4,899	orders the numerals based on the sum of the digits rather than the value of each number	EXPLAIN PLACE VALUE FOR THOUSANDS AND BEYOND and ORDER MORE THAN 2 4-DIGIT AND BEYOND NUMERALS OR QUANTITIES FROM LEAST TO GREATEST

7. Order the following numerals from greatest to least.

2,000,100	839,004	879,116	910,435	1,567,900

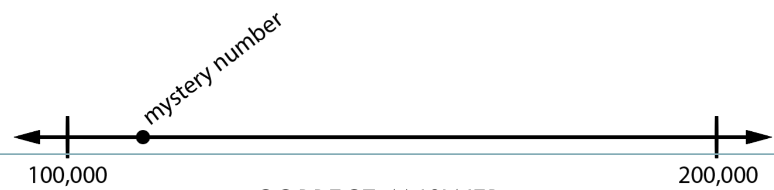
CORRECT ANSWER

2,000,100
1,567,900
910,435
879,116
839,004

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
839,004 879,116 910,435 1,567,900 2,000,100	orders the numerals from least to greatest instead of from greatest to least	ORDER MORE THAN 2 4-DIGIT AND BEYOND NUMERALS OR QUANTITIES FROM GREATEST TO LEAST
879,116 1,567,900 839,004 910,435 2,000,100	orders the numerals based on the sum of the digits rather than the value of each number	EXPLAIN PLACE VALUE FOR THOUSANDS AND BEYOND and ORDER MORE THAN 2 4-DIGIT AND BEYOND NUMERALS OR QUANTITIES FROM GREATEST TO LEAST
910,435 879,116 839,004 2,000,100 1,567,900	orders the numerals based on the first digit without attending to place value	EXPLAIN PLACE VALUE FOR THOUSANDS AND BEYOND and ORDER MORE THAN 2 4-DIGIT AND BEYOND NUMERALS OR QUANTITIES FROM GREATEST TO LEAST

8. Use the following number line to estimate the “mystery number”. Explain how you made your estimate.



CORRECT ANSWER

Answers will vary, but they should range from approximately 110,000 to 115,000. Students should explain that the mystery number is much closer to 100,000 than 200,000, and that the distance from 100,000 to the mystery number appears to be about 10,000 to 15,000 units (given that the range of the number line is 100,000). Students may also describe partitioning the number line into halves or fourths to make their estimate.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
100,010 (or a similar value) because the mystery number is a little more than 100,000.	does not consider the range of the number line when making an estimate	ORDER MORE THAN 2 4-DIGIT AND BEYOND NUMERALS OR QUANTITIES FROM LEAST TO GREATEST
150,000 because the mystery number is between 100,000 and 200,000.	does not carefully consider the position of the mystery number on the number line and the fact that it is not positioned halfway between 100,000 and 200,000	ORDER MORE THAN 2 4-DIGIT AND BEYOND NUMERALS OR QUANTITIES FROM LEAST TO GREATEST
NOTE: This item is an extension of the concept of ordering multi-digit numbers from least to greatest.		