DETERMINING AREA OF RECTANGLES

3.MD.6,7

CONTENTS

The types of documents contained in the unit are listed below. Throughout the unit, the documents are arranged by lesson.

Learning Map Information	An overview of the standards, the learning map section, and the nodes addressed in this unit
Teacher Notes	A brief discussion describing the progression depicted in the learning map section with research-based recommendations for focusing instruction to foster student learning and an introduction to the unit's lessons
Instructional Activity	A detailed walkthrough of the unit
Instructional Activity Student Handout	A handout for the guided activity, intended to be paired with the Instructional Activity
Instructional Activity Supplement	A collection of materials or activities related to the Instructional Activity
Student Activity	A work-alone activity for students
Student Activity Solution Guide	A solution guide for the work-alone activity with example errors, misconceptions, and links to the learning map section

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Determining Area of Rectangles

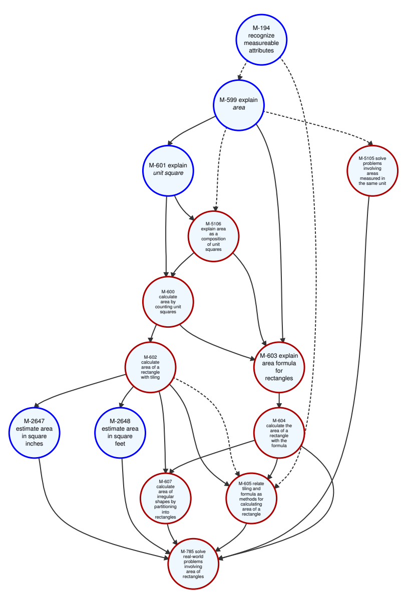
LEARNING MAP INFORMATION

STANDARDS

3.MD.6 Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).

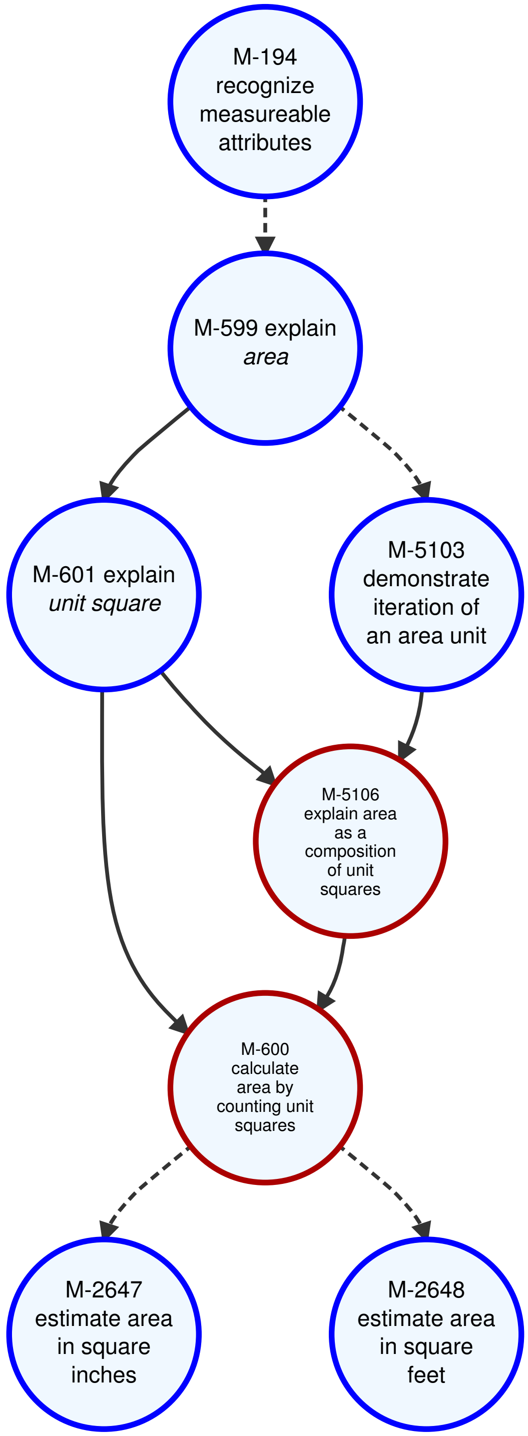
3.MD.7 Relate area to the operations of multiplication and addition.

> **3.MD.7a** Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.
>
> **3.MD.7b** Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.
>
> **3.MD.7c** Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning.
>
> **3.MD.7d** Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.



**Learning map model of* 3.MD.6,7

3.MD.6 Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).



**Learning map model of* 3.MD.6

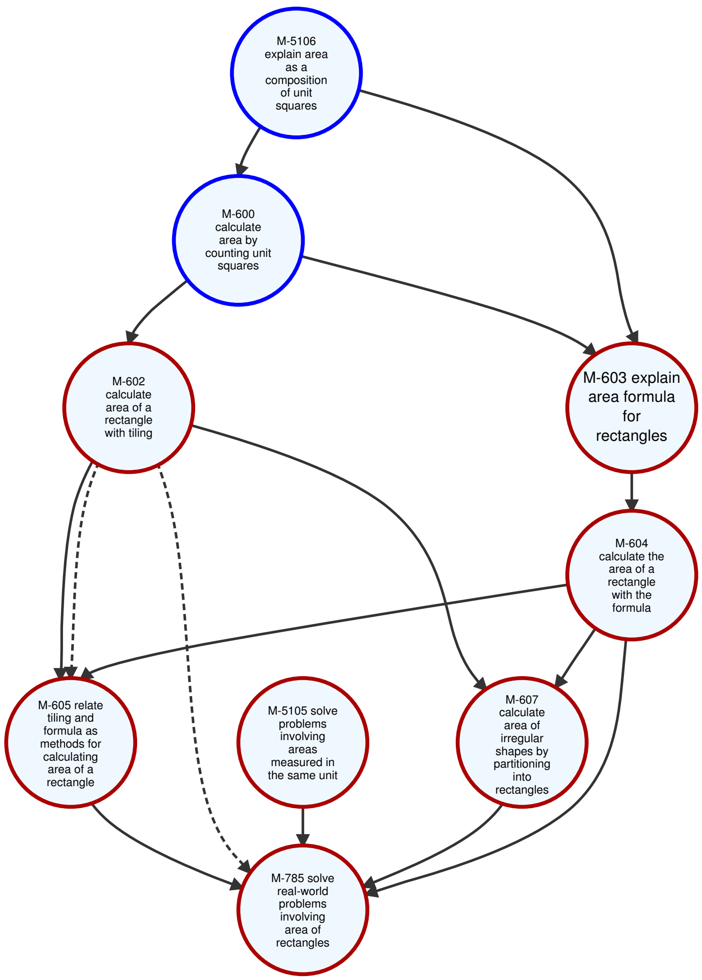
3.MD.7 Relate area to the operations of multiplication and addition.

3.MD.7a Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.

3.MD.7b Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.

3.MD.7c Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning.

3.MD.7d Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.



**Learning map model of* 3.MD.7

Node Name	Node Description
CALCULATE AREA BY COUNTING UNIT SQUARES	Calculate the area of a rectangle by counting the number of square units drawn to cover the area.
CALCULATE AREA OF A RECTANGLE WITH TILING	Calculate the area of a rectangle by using square tiles and then counting or calculating the number of tiles that would be needed to cover the area without any gaps or overlapping tiles.
CALCULATE AREA OF IRREGULAR SHAPES BY PARTITIONING INTO RECTANGLES	Partition irregular shapes into rectangles and find total area as the sum of the area of rectangles.
CALCULATE THE AREA OF A RECTANGLE WITH THE FORMULA	Calculate the area of the rectangle using the formula $A = ab$, where a is the long side length and b is the short side length.
ESTIMATE AREA IN SQUARE FEET	Estimate the area of an object using square feet as the unit of measurement.
ESTIMATE AREA IN SQUARE INCHES	Estimate the area of an object using square inches as the unit of measurement.
EXPLAIN *AREA*	Make known your understanding that area is a two-dimensional quantity representing the amount of space in a surface.
EXPLAIN AREA AS A COMPOSITION OF UNIT SQUARES	Make known your understanding that area of a figure can be determined by covering the figure with a single layer of non-overlapping square units without gaps or spaces and counting the number of square units.
EXPLAIN AREA FORMULA FOR RECTANGLES	Make known your understanding that area of a figure can be determined by covering the figure with a single layer of non-overlapping square units without gaps or spaces and counting the number of square units.
EXPLAIN *UNIT SQUARE*	Make known your understanding that a unit square has an area of one square unit.
RECOGNIZE MEASURABLE ATTRIBUTES	When shown a certain shape, correctly communicate the name of an attribute that can be measured.
RELATE TILING AND FORMULA AS METHODS FOR CALCULATING AREA OF A RECTANGLE	Make known your understanding that both tiling and applying the formula ($A = length \times width$) for calculating area result in the same answer.
SOLVE PROBLEMS INVOLVING AREAS MEASURED IN THE SAME UNIT	Use addition and/or subtraction to solve mathematical or real-world problems involving areas measured in the same unit.
SOLVE REAL-WORLD PROBLEMS INVOLVING AREA OF RECTANGLES	Solve real-world and contextualized mathematical problems involving area of rectangles.

ADDITIONAL NODES RELATED TO THIS UNIT OF INSTRUCTION

Node Name	Node Description	Related Node
APPLY THE DISTRIBUTIVE PROPERTY	Apply distributive property as a strategy for multiplication. For example, 7×18 can be calculated as $(7 \times 10) + (7 \times 8)$.	Prerequisite of CALCULATE AREA OF IRREGULAR SHAPES BY PARTITIONING INTO RECTANGLES through EXPLAIN THE DISTRIBUTIVE PROPERTY
EXPLAIN *LENGTH*	Make known your understanding that length is the distance along a path between two points on that path.	Prerequisite of EXPLAIN *AREA*
EXPLAIN MULTIPLICATION PROBLEMS	Make known your understanding that, in a multiplication problem, the first	Postrequisite of REPRESENT REPEATED ADDITION through DEMONSTRATE THE

	factor describes the number of groups and the second factor describes the number of elements in each group.	CONCEPT OF MULTIPLICATION and SOLVE REPEATED ADDITION PROBLEMS
EXPLAIN REPEATED ADDITION	Make known your understanding, using words or concrete models, that the concept of repeated addition concerns adding the same addend three or more times.	Prerequisite of REPRESENT REPEATED ADDITION WITH A MODEL
EXPLAIN THE DISTRIBUTIVE PROPERTY	Make known your understanding that when multiplying a number over a sum, you distribute the factor to each addend (i.e., multiply the factor by each addend and then add).	Prerequisite of CALCULATE AREA OF IRREGULAR SHAPES BY PARTITIONING INTO RECTANGLES
EXPLAIN THE FUNCTION OF THE EQUAL SIGN	Make known your understanding, using words or manipulatives, that the equal sign is used in an equation to represent an equivalent relationship between expressions.	Prerequisite of EXPLAIN REPEATED ADDITION through REPRESENT ADDITION WITH EQUATIONS
MODEL EQUAL PART	Model equal-size parts for continuous amounts.	Postrequisite of PARTITION LENGTH
PARTITION A RECTANGLE INTO ROWS AND COLUMNS	Divide a rectangle into rows and columns, creating same-size rectangular sections.	Postrequisite of MODEL EQUAL PART
PARTITION LENGTH	Split a length into two or more distinct lengths.	Postrequisite of EXPLAIN *LENGTH*
PARTITION LENGTH INTO EQUAL PARTS	Split a length into two or more equal lengths.	Postrequisite of PARTITION LENGTH and MODEL EQUAL PART
RECOGNIZE *RECTANGLE*	When presented a set of different shapes, select the rectangle.	Prerequisite of EXPLAIN AREA FORMULA FOR RECTANGLES
REPRESENT *RECTANGLE*	Through writing or an appropriate assistive technology, represent a rectangle. Both pairs of opposite sides are parallel, and all adjacent sides are perpendicular (i.e., all interior angles measure 90 degrees).	Postrequisite of RECOGNIZE *RECTANGLE*
REPRESENT REPEATED ADDITION WITH A MODEL	Show repeated addition by using a model such as an array, set, number line, etc.	Postrequisite of EXPLAIN REPEATED ADDITION
SOLVE ARRAY PROBLEMS	Use multiplication and division to solve word problems in situations involving arrays. Know that, in a multiplication expression, the first factor represents the number of rows modeled in the corresponding array, and the second factor represents the number of columns modeled in the corresponding array.	Postrequisite of EXPLAIN THE DISTRIBUTIVE PROPERTY through USE REASONING FOR MULTIPLICATION AND DIVISION

DETERMINING AREA OF RECTANGLES

TEACHER NOTES

This unit includes the following documents:

- Learning Map Information
- Instructional Activity (five lessons)
- Instructional Activity Student Handout (for Lesson 1 – 5)
- Instructional Activity Supplement (for Lesson 3 and 5)
- Student Activity
- Student Activity Solution Guide

In this unit, students will explore the concept of area by creating and covering rectangles with unit squares. Then, students will relate rectangles composed of rows and columns to an array in order to make sense of the area formula for rectangles. In addition to determining the area of a single rectangle, students will also divide rectangles into two smaller rectangles and apply the distributive property of multiplication in order to determine the area. Finally, students will decompose irregular shapes into two rectangles to determine the area of the total figure.

RESEARCH

Before students calculate the area of rectangles with the formula, they should first develop an understanding of the concept of area. Van de Walle, Karp, Lovin, & Bay-Williams (2014) assert, "students must first understand the attribute of area before measuring" (p. 324). Therefore, prior determining the area of a rectangle, students must first understand what area is, not *length* times *width* or *base* times *height*, but the understanding that area is the amount of two-dimensional surface inside a boundary.

Such an understanding can reduce the likelihood of students mixing up the formulas for perimeter and area, which is noted by teachers and researchers as a common difficulty. Moyer (2001) states that, "The ability to connect conceptual understanding to the processes for determining perimeter and area is essential for children if they are to make sense of these two mathematical concepts" (p. 52-53). Furthermore, students often confuse the ideas of area and perimeter because they are learned primarily as a set of procedures; whereas, if meaning is attached to the ideas prior to the procedures being introduced, then students can begin to distinguish between the two concepts because the two measures are perceived as qualitatively different (Moyer, 2001). Likewise, Malloy (1999) agrees that, students "…may not have fully conceptualized the meaning of the words (area and perimeter). They become confused by the formulas and find area when they are asked for perimeter and perimeter when they are asked for area" (p. 88). In addition, students often confuse the idea of area and perimeter because both concepts are presented using similarly shaped figures, often rectangles (Malloy, 1999). To expand students' experiences and avoid some of the confusion, students should be exposed to a variety of shapes during their initial experiences with the concepts of area so that they develop accurate ideas about area as an attribute, as well as how to iterate area units to cover a surface and determine area measures.

Once students understand the idea of area, Van de Walle et al. (2014), recommends that the "objective in the beginning is to develop the idea that area is measured by covering or tiling. Do not introduce formulas yet" (p. 326). Covering or tiling an area and then counting the number of units builds students' understanding through direct measurement. It is important that students connect the idea of counting the units as a means of measurement, not just counting a number of squares. When students are first introduced to the concept of area, and prior to identifying unit squares, students should experience determining area by iterating a variety of non-square units (i.e., triangle, rectangle, pentagon, pattern blocks, etc.) and informal units (e.g., candies, erasers) (McDuffie & Eve, 2009). When selecting units for measuring area, it is important for students to understand that the units should have regular areas, or that they should all take up the same amount of space. Once students have grasped the concept of area and that a unit square measures one unit of area, they are ready to tile or cover rectangles with unit squares of different dimensions (i.e., square inches, square feet, square centimeters, etc.). Teachers should consider the benefits and drawbacks of different manipulatives as tools for measuring area through tiling or covering. The following chart lists the benefits and drawbacks of popular manipulatives.

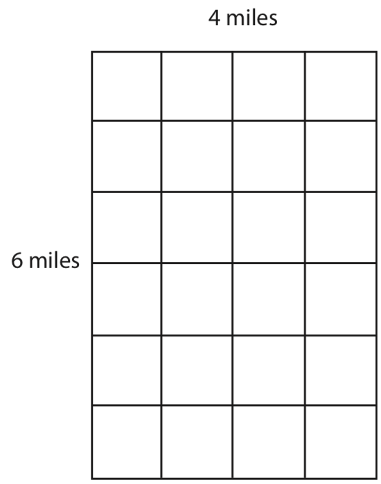
MANIPULATIVE	BENEFITS AND DRAWBACKS
Square tiles (i.e., squares found with pattern shapes)	Benefit: ▸ less three-dimensional appearing than cubes ▸ represent unit squares, all squares are uniform in shape and size Drawback: ▸ only comes in one size, does not allow for flexibility to explore with other unit sizes
Base-ten "ones" blocks Unifix (interlocking) cubes White, "ones" Cuisenaire rods	Benefit: ▸ easily manipulated and moved to cover areas Drawback: ▸ the three-dimensionality of the cube may cause confusion with volume (Civil & Khan, 2001)
Paper squares	Benefit: ▸ flexibility to choose unit size, shape, and/or color ▸ do not appear to be three-dimensional Drawback: ▸ due to self-creation, cutting and or side lengths may not be exact and/or uniform ▸ time and effort of self-creation

McDuffie & Eve (2009) offer four ordered activities students should explore as they gain an understanding of area. First, students should partition a region with a two-dimensional unit of measure. For example, students should divide a rectangle into rows and columns creating unit squares that can be counted. Second, students should iterate an area unit to cover the region without space or overlap. For example, using square-shaped pattern blocks, base-ten unit cubes, or same-size paper squares, students will cover the area of a rectangle without overlapping or leaving gaps between squares, then count the number of unit squares. Third, students should explore conservation of area (i.e., the area stays the same even if a region is cut and rearranged into different shapes). For example, students will decompose rectangles into two smaller rectangles and calculate the areas of the decomposed rectangles to demonstrate that the combined areas of the decomposed rectangles are equal to the area of the original rectangle. Fourth, students should structure an array by partitioning the rectangle into rows and identifying the number of unit squares in a single row, and then repeatedly adding the number of unit squares in one row, or multiplying the number of unit squares in one row by the number of rows. The following images highlight examples of each of these experiences.

AN EXAMPLE

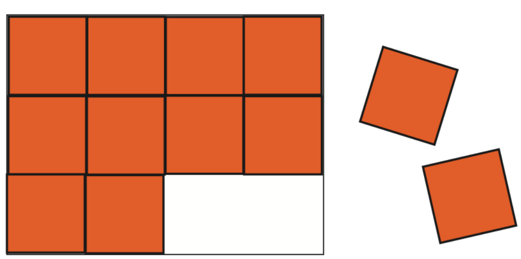
Students should have experiences with the following practices to develop a conceptual understanding of area (McDuffie & Eve, 2009).

PARTITIONING



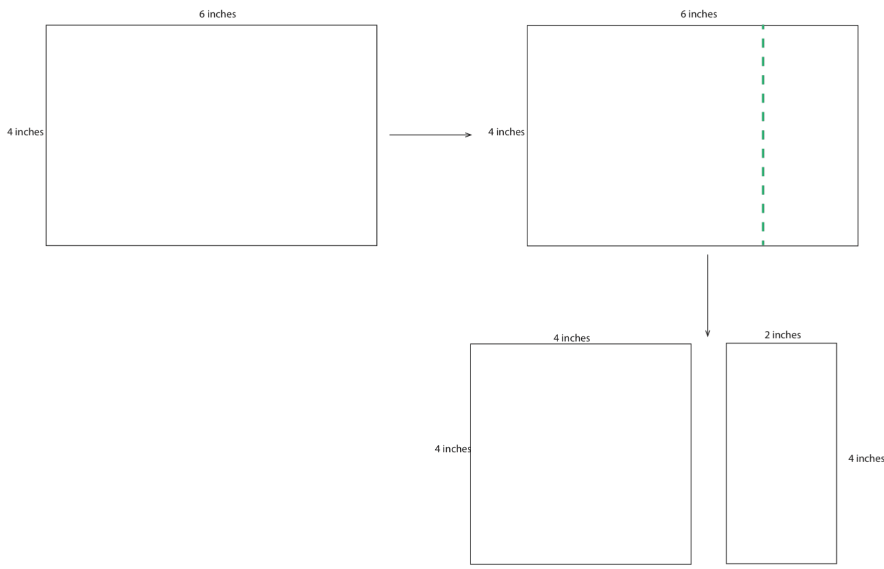
Dividing a shape into same-size unit squares based on one unit of measure. It is significant to note that students may struggle with the idea that while the side length is identified as "6 miles" or "4 miles", the side length does not represent the number of lines that are drawn, but rather the number of rows or columns that should be created.

ITERATING



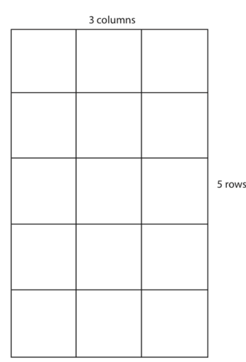
When students tile or cover a shape using like units, such as unit squares, they are iterating, or repeatedly measuring an area using same-size units. It is important that students attend to the detail that there must not be any gaps or overlaps between units when iterating.

CONSERVATION OF AREA



The conservation of area provides students with an understanding that even though a shape has been divided and/or rearranged, the total area remains the same. One way in which students will explore this concept is by applying the distributive property of multiplication when a rectangle has been decomposed into two rectangles.

ARRAY

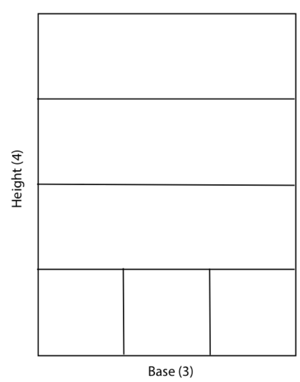


A rectangular array can be thought of as *five groups* (or *five rows*) of *three units* each. In this manner, students can consider the area of a rectangle first as repeated addition (3 + 3 + 3 + 3 +3) and then later as multiplication (5 × 3).

Before students learn or apply any formula for calculating area, they should understand how the formula arises from viewing a rectangular surface as an array of unit tiles. By linking area models and arrays to the area formula for rectangles, students can develop a more conceptual understanding of the formula and better distinguish it from other formulas such as perimeter. The bridge between direct measurement (counting) and indirect measurement (calculating with a formula) is scaffolded by thinking of a rectangle as an array and using repeated addition to find the sum of the units. Students may go from counting each individual unit, to determining how many units are in one row and counting the number of rows in order to repeatedly add the number of units in each row, to multiplying the number of units in one row by the number of rows, and then from that point connecting that conceptual understanding to the formula. An added benefit of visualizing the area formula for rectangles in connection with an area model or an array is that this practice gives students an opportunity to regard variables (i.e., *l*, *w*, *b*, or *h* for length, width, base, or height respectively) as something other than simply placeholders for numeric values (Hines & Bridges, 2003). For example, when considering an area model or an array, students can visualize the relationship between the base or height of a rectangle and the area of the rectangle. With experience, the variables take on more meaning; that is, the base is not just a number but represents how many units are in each row, or the width of the rectangle, and the height is not just a number but represents the number of rows, or the height of the rectangle.

AN EXAMPLE

The following image illustrates a representation of the connection between an array and the area formula for rectangles, base times height (Joram & Oleson, 2004, p. 452).



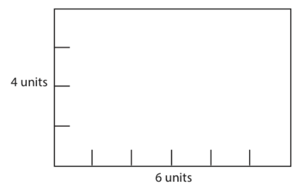
Van de Wall et al. (2014) explains that, "the length of one side indicates how many squares will fit on that side. If that set of squares is taken as a unit, then the length of the adjacent side (not a number of squares) will determine how many rows of squares can fit in the rectangle" (p. 313).

Similar to Hines & Bridges (2003), Van de Walle et al. (2014) recommends that students should "…begin to use spatial reasoning to apply the concept of multiplication using arrays to the area of rectangles" (p. 326). Students should be able to visualize the rectangle as a composition of rows and columns, which leads to the understanding of the area formula for rectangles, *height* × *base* (length × width), as *the number of rows* × *the number of units in one row*. Students should understand that the order of the side lengths in the formula is not fixed—it can be interpreted as both *height* × *base* and *base* × *height*. This understanding should be connected with the commutative property of multiplication, that when a rectangle is rotated the size or area of the rectangle does not change. In addition, it is significant to note that the terms *base* and *height* do not necessarily refer to the sides (line segments) of the rectangle, but rather the *base* refers to the measurement of one side length and the *height* refers to the length of a perpendicular segment from the *base* to the highest point of the figure. This is an important concept to consider, because as students' progress through geometry, the *height* will not always be a side length. Therefore, use caution as you explore the terms *base* and *height* with students, attempt to avoid referring to specific sides as the base and height (i.e., the base is always the bottom side length and the height is always the vertical side length), and focus instead on the lengths of the sides and present those lengths in the context of rows and number of units in one row or rows and columns.

An array also supports students' understanding of the area formula for rectangles as a means for both additive and multiplicative approaches. Students can visualize the rows of rectangular arrays as repeated addition. Because the idea of repeated addition is the foundation of multiplication, this is a natural bridge between addition and multiplication. This understanding can be achieved without dividing the rectangle into rows and columns, but rather, by measuring the number of units along one edge to represent the number of rows and then measuring the number of same-size units along an adjacent, perpendicular edge to represent the number of units in one row (Van de Walle et al., 2014). Students who use the multiplicative approach will visualize the area as the number of rows *times* the number of units in one row. As with the additive approach, students do not have to divide the rectangle into rows and columns to achieve this understanding but can measure out each edge using same-size units. Van de Walle et al. (2014) states that the goal of this understanding is "…not to develop an area formula, but to apply students' developing concepts of multiplication to the area of rectangles. Not all students will use a multiplicative approach" (p. 327).

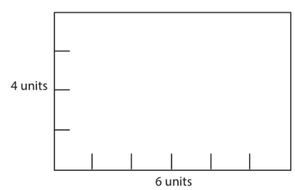
AN EXAMPLE

ADDITATIVE APPROACH



$A = 6 + 6 + 6 + 6$

MULTIPLICATIVE APPROACH



$A = 4 \times 6$

When students are given ample time to explore and practice the concept of area through tiling or covering (iterating), partitioning, comparing the rectangular area model with an array, and decomposing rectangles into smaller rectangles, they will develop a deeper understanding of area that will persist through their geometry experiences and lead to less confusion between the measurements of area and perimeter.

LEARNING MAP INFORMATION

The learning map section for this sequence of activities begins by focusing on recognizing measureable attributes and understanding the concepts of area and unit squares. The map then proceeds to understanding the measurement of area through a composition of unit squares. Students should be able to both explain and calculate area by covering a rectangle with unit squares and then counting the number of unit squares. This path extends to estimating the area of a given rectangle.

The learning map progression then expands to calculating area by tiling, explaining, and calculating area of rectangles using the formula. Students should be able to make known their understanding that both methods—the area formula for rectangles and tiling rectangles to determine area—result in the same answer. In addition, students should be able to decompose rectangles and irregular shapes into smaller rectangles to determine the area of the total figure. Finally, the learning map sequence progresses to the application of solving real-world situations by determining the area of rectangles within a given context.

INSTRUCTIONAL ACTIVITIES

The activities in this unit are designed to scaffold students' conceptual understanding of area and the area formula for rectangles.

The unit begins with an iterating activity designed to reinforce student's understanding of area and unit squares (these are concepts presented in standard 3.MD.5, prior to those which are the focus of this unit). Students will then apply their understanding of unit squares and iterating to creating and modeling rectangles on a geoboard and then visualizing the figure as individual squares.

In the next lesson, students increase their understanding of area by dividing rectangles into rows and columns using rulers to measure side lengths and draw unit squares by creating the given number of rows and columns. Students will also explore the concept of estimation by comparing two rectangles which have differing dimensions but comparable areas. Not only will students be required to determine which rectangle they think has the larger area, but they will need to estimate the area of both rectangles in square inches before measuring, using a ruler or unit squares to determine the actual areas of both rectangles.

Once students have had a sufficient amount of time exploring area by iterating square tiles and dividing rectangles into rows and columns, students will extend their understanding to apply additive and multiplicative understanding to calculate the area of rectangles. Students will use rulers to divide rectangles into rows and then proceed to divide one row into square units. In addition, students will use the distributive property to consider the area of a rectangle as the sum of two smaller rectangles. Finally, students will apply their understanding of area to determining the area of a rectilinear figure by decomposing the shape into smaller rectangles in order to determine the total area of the original figure. Students will encounter real-world situations throughout the lessons in this unit.

NOTE: Throughout each lesson, there are notes on ways to modify activities to meet students' needs.

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Determining Area of Rectangles

INSTRUCTIONAL ACTIVITY

Lesson 1

LEARNING GOAL

Students will identify and compare the area of rectangles by counting unit squares.

PRIMARY ACTIVITY

Students will identify unit squares, compose different rectangles with the same area, and compare different-size rectangles with the same number of different-size units. Students will also use paper squares or square tiles to practice tiling, demonstrating an understanding that square units cannot overlap and there cannot be gaps between squares. Finally, students will create and trace figures using paper squares or square tiles, then trade with a partner to determine the area.

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

- Unit square
- Square inches
- Square feet
- Rectangle
- Area

MATERIALS

- Instructional Activity Student Handout
- Paper strips: one inch wide × 12 inches long (Recommend one for each student.)
- Rulers
- Paper strip: one foot wide × 12 feet long (Recommend one, already divided into 12 squares.)
- Scissors
- Squares: Pattern blocks or paper squares (Recommend 15 to 20 squares for each student.)
- Graph paper (optional)

IMPLEMENTATION

Begin a discussion to review measureable attributes, area, and unit squares.

Display or draw a line on the board.

Ask students the following questions. If necessary, **clarify** the question by using your finger to outline what you are asking. For example, when asking about perimeter, it may help to trace your finger around the outside of the figure to demonstrate perimeter.

- How can you measure this line?
- Can you measure how much it weighs?
- Can you measure how long it is?
- Can you measure the amount of space inside the line?
- Can you measure the height of the line?
- Can we measure the distance around the outside of the line?

Repeat the questioning with several shapes, such as a square, a triangle, a circle, and a rectangle.

Review the concept of area by asking students what area means. **Require** students to share with a partner.

After one minute or so, **identify** three to four students to share what area means. Be sure to clarify or correct any misconceptions.

Repeat the process by asking about unit squares. **Ensure** students identify that each side length of a unit square measures one unit (e.g., one inch, once centimeter, one foot).

Distribute rulers and the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT, so that every student has one of each.

Review how to use a ruler using the following tasks and questions.

- Please point to the zero on the ruler.
- What is the largest number on the ruler?
- What do the lines and numbers on a ruler represent?
- What unit is a ruler divided into?
- From zero to one is one what?
- How do you read a ruler?
- Show on the ruler how long three inches is. How do you know this distance is three inches?

NOTE: When students demonstrate how long three inches is, ensure that they are pointing out a *length* on the ruler. For example, if the student begins at one inch on the ruler, they should indicate that three inches is the distance from one inch to four inches.

Require students to measure the line segments on Question 1 of the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

Review the correct answers by asking students for the measurements.

- 1.a. seven inches
- 1.b. four inches

Distribute the paper strips that measure one inch by 12 inches.

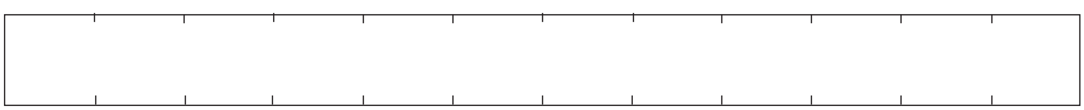
NOTE: For students who struggle, one-inch graph paper may be used along with the ruler, or you may provide the student with a strip that already has the marks, done in thick marker, so the student only has to draw the lines connecting the marks.

Display the strip so that horizontally it is 12 inches and vertically it is one inch.

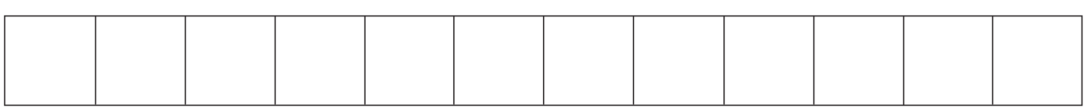
Ask one or two students to explain how they would measure how tall the strip of paper is. **Require** the student(s) that explained to demonstrate with the displayed strip. Then, **require** all students to measure the height of their own strips. Students should identify that the height of the strip is one inch.

Ask students, "If you want to create unit squares, and the height is one inch, how wide does the unit square need to be?" Students should identify that the width of the unit square should be one inch as well.

Require students to mark their strip every inch along the bottom and the top of the strip. This will make it easier for students to create straight lines when dividing the strip into unit squares.



Require students to connect the marks on the top with the marks on the bottoms of the strip to divide the strip into squares.



Use the guiding questions to engage students in discussion.

GUIDING QUESTIONS

Elicit student thinking:

- What is the shape of the paper strip?
- What shapes were created when you drew lines on the paper strip?
- When or why would you use a ruler?

Determine if the student can RECOGNIZE MEASUREABLE ATTRIBUTES:

- What can be measured on the paper strip?
- Can you measure how long the paper strip is? How do you know?
- Can you measure how wide the strip is? How do you know?
- Can you measure the area of the paper strip? How do you know?

Determine if the student can EXPLAIN *AREA*:

- What is area?
- What would you measure if you were to measure the area of the paper strip? Explain.
- Could you measure the area of a line segment? Why or why not?
- How is a piece of paper shaped like a rectangle different from a cereal box or a shoe box that is shaped like a rectangle?
- Can you use unit squares to count the area of a piece of paper shaped like a rectangle? Why or why not?
- Can you use unit squares to count the area of a cereal box or a shoe box that is shaped like a rectangle? Why or why not?

Determine if the student can EXPLAIN *UNIT SQUARE*:

- What is a unit square?
- Point to a unit square on your paper strip. How do you know this is a unit square?
- [Point to a square on the paper strip.] Now that you have measured the side lengths of each of the squares, what would you name this unit square? Why?
- If the units on the paper strip were one foot long and one foot wide instead of one inch long and one inch wide, would it still be a unit square? Why or why not?
- If the units on the paper strip were one inch wide and two inches long, would it still be a unit square? Why or why not?

Ask students to identify how many units make up the paper strip. **Require** students who respond to defend their answer by **asking** how they determined their answer.

Ask students what the area of the paper strip would be and how they know this to be true.

Ensure that each student responds with the correct square unit (square inches). If they do not provide the correct square unit, use questioning to lead them back to the measurement of each square.

Display the paper strip that is one foot wide and 12 feet long and has already been divided into 12 squares.

Require one or two students to measure the height and base of one or more squares on the large paper strip. As a class, identify that each square is one foot wide and one foot long, then **ask** students what the area will be measured in (square feet).

Ask students how many square feet are in the paper strip. Either require one student to count aloud the number of square feet and one student to lead the class in counting the number of square feet, or count the square feet together as a class.

Ask students what the area of the paper strip would be and how they know this to be true.

Display both paper strips side by side so that students can see the difference in the sizes.

Ask students what the area of the smaller paper strip is (12 square inches). **Repeat** with the larger strip (12 square feet).

Ask students how the two strips are the same and how the two strips are different.

Ask students the following questions:

- How are the two strips the same number but different sizes?
- Is 12 feet the same as 12 inches? Why or why not?
- Is 12 square feet the same as 12 square inches? Why or why not?

Emphasize that because area is the amount of space contained in a two-dimensional surface, the two paper strips do not have the same area because they do not have the same about of space—even though the two paper strips have the number 12 in common. One square inch is much smaller than one square foot, and therefore 12 square inches covers a smaller area than 12 square feet.

NOTE: To emphasize this idea even more, demonstrate or display what the larger strip would look like divided into square inches. Time and group permitting, you could require a student to count or lead the class in counting the number of square inches in the larger paper strip.

Direct student attention to the chart on Question 2 of the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

Explain the directions for the chart. Use the example in row 1 to support student understanding.

- Students will create three rectangles
- Draw each rectangle in the *Drawing* column on the chart
- Determine the area of each rectangle and write it in the *Area in square inches* column

Require students to cut apart their paper strips along the lines they drew to create 12 separate squares.

Remind students when they are creating rectangles that the squares should not overlap or leave any gaps, and that each rectangle must contain *all* of the squares cut from the strip.

Use guiding questions to support student understanding.

GUIDING QUESTIONS

Elicit student thinking:

- Why did you cut the paper strip into pieces?
- [Point to one of the squares.] What shape pieces do you have after cutting the strip apart?
- What are you going to do with the square pieces now?
- How many square pieces can you use for each rectangle?

Determine if the student can EXPLAIN AREA AS A COMPOSITION OF UNIT SQUARES:

- [Point to one of the drawn rectangles.] Show me with your squares how you created this rectangle.
- Create a rectangle that you have not yet created using your squares.
- [Move a square so that it is overlapping another square.] Can these two squares overlap when you're determining area? Why or why not?
- [Move a square so that there is a gap between two or more squares.] Can there be a gap between these squares when you're determining area? Why or why not?
- [Point to one of the drawn rectangles.] How many square units are in this rectangle? How do you know?

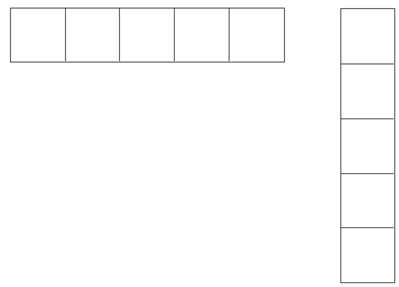
Determine if the student can CALCULATE AREA BY COUNTING UNIT SQUARES:

- [Point to one of the drawn rectangles.] What is the area of this rectangle? How do you know?
- [Point to one of the written areas.] Show me with your squares how you know this is 12 square inches.

Select two or three students with different rectangles to share their work.

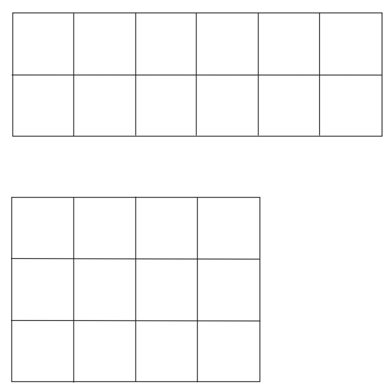
Discuss area by posing the following questions.

[Show or draw the following rectangles.]



- Are these two rectangles the same? Why or why not?
- Do these two rectangles have the same area? Why or why not?

[Show or draw the following rectangles.]



- Are these two rectangles the same? Why or why not?
- Do these two rectangles have the same area? Why or why not?
- Did the number of squares or units change from one rectangle to the next? How do you know?
- How do you know the two rectangles have the same number of unit squares?
- Are the unit squares for both rectangles the same size? How do you know? How could you find out?
- Do the two rectangles cover the same amount of area? How do you know?

Direct student attention to Question 3 on the Instructional Activity Student Handout.

Explain the directions for Question 3. **Model** the activity on an extra Instructional Activity Student Handout if necessary.

- Each student will create two irregular (nonrectangular) figures using the squares provided.
- Figures can be any design, using any number of squares from what is provided.
- Squares in each figure cannot overlap or leave gaps.
- Students will trade shapes with other students in their groups. Each student will determine the area of two different shapes from two different students.
- Students will need to know area of their own figures in order to check their partners' work.

Arrange students into groups of three. Distribute 15 to 20 squares (pattern blocks or paper squares) to each student.

> NOTE: For students who struggle, graph paper may be used in place of tracing squares on a blank page.
>
> To make the activity more challenging, have students leave the inside of the figure blank so that their partners have to use the provided squares (or make available squares of a different size, i.e. centimeters) to determine the area. Using squares of a different size, i.e. centimeters, will provide insight into how the student will encounter partial units.

As groups of three complete Question 3 in the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT, use the following guiding questions to support student understanding.

GUIDING QUESTIONS

Elicit student thinking:

- Does your figure have to be a rectangle?
- How did you decide to what to create?
- Do you have to use all the squares you were given?
- Can you use more squares than the ones you were given?

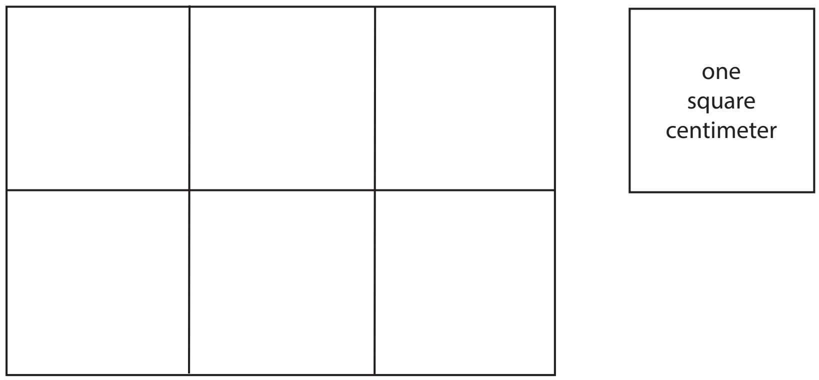
Determine if the student can EXPLAIN AREA AS A COMPOSITION OF UNIT SQUARES:

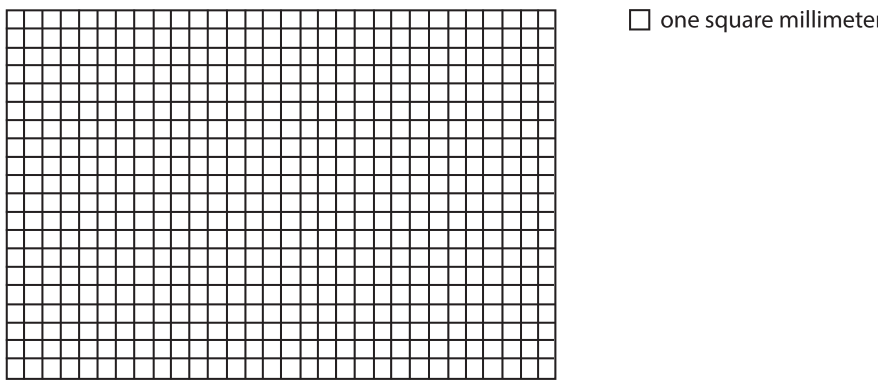
- Can any of the squares in your figure overlap? Why or why not?
- Can there be any gaps between any of the squares in your figure? Why or why not?
- Do all the unit squares need to be the same size?
- Once you have drawn all the unit squares in the figure, how can you find the area?

Determine if the student can CALCULATE AREA BY COUNTING UNIT SQUARES:

- [Point to a figure.] How did you find the area of this figure?
- [Point to a written area.] How do you know this is the area of this figure?

Display or draw the following rectangles. Ensure that the rectangles are the same size, and that the size of the unit square for each rectangle is different.





Ask students the following questions.

- Are these the same shapes? How do you know?
- Do they have the same area? How do you know?
- How are the unit squares the same? How are they different?
- Can two figures have the same area but be measured in different units? Explain.

Emphasize that the area of the first rectangle is six square centimeters and the area of the second rectangle is 60 square millimeters, but that both rectangles are exactly the same size and shape and therefore cover the same amount of area. The units that are used to measure the areas are different, which causes the number of square units to be different. Square millimeters are smaller than square centimeters, therefore it takes more square millimeters to cover the same amount of space. Likewise, square centimeters are larger than square millimeters, therefore it takes fewer square centimeters to cover the same amount of space.

At the end of the activity, students should answer Question 4 on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

Name________________________________

Determining Area of Rectangles

Lesson 1

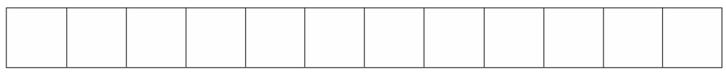
1. Measure the line segments in inches.

a. ________________________________ inches

b. ________________________________ inches

Name________________________________

2. Use all of the squares from your paper strip to create three additional rectangles and complete the chart.

	Drawing	**Area in square inches**
1		12 square inches
2		
3		
4		

Name____________________________

3. Use your squares to create two figures. Trace each figure and all of the squares inside each figure.

Have a partner find and write the area for each figure.

Figure A

Partner Name:

Area:

Name______________________________

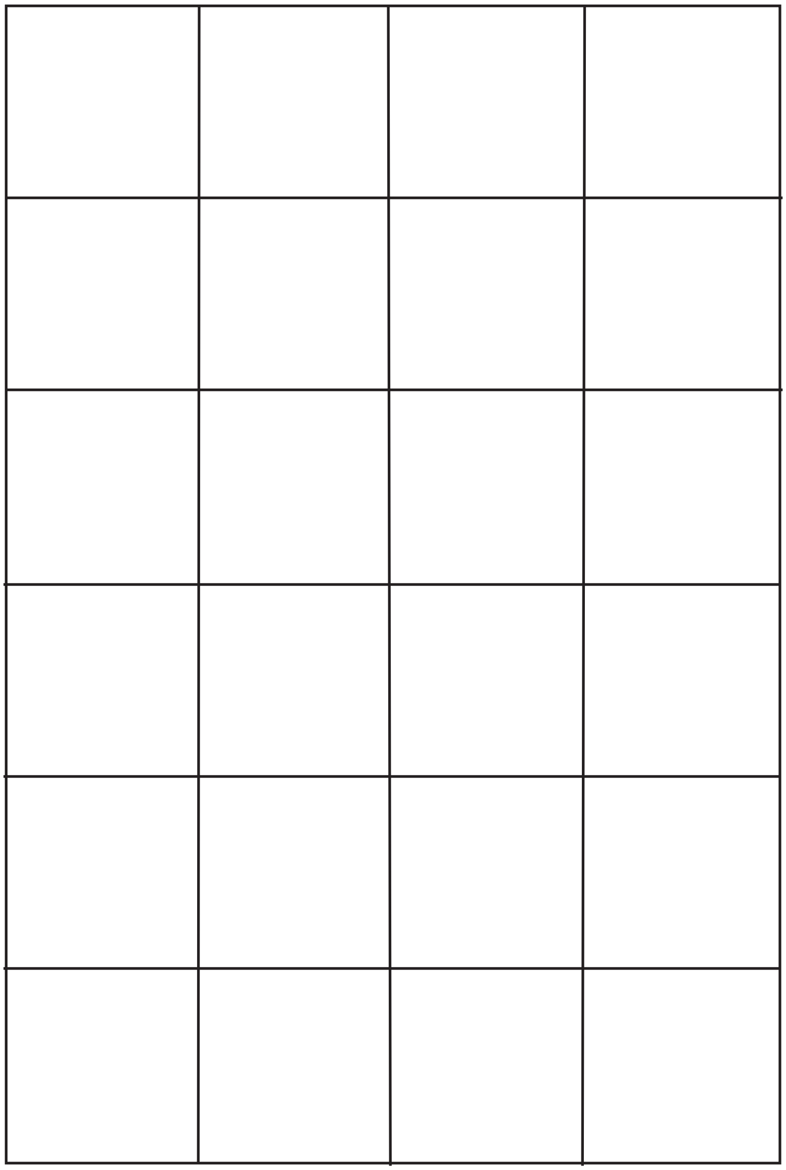
Figure B

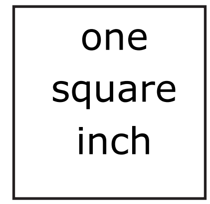
Partner Name:

Area:

Name______________________________

4. Find the area of the rectangle in square inches.





Area = ________________________________

Determining Area of Rectangles

INSTRUCTIONAL ACTIVITY

Lesson 2

LEARNING GOAL

Students will determine the area of rectangles by tiling and then by counting the number of square units.

PRIMARY ACTIVITY

Students will use square tiles to cover a rectangle, then count the number of tiles to determine the area. Next, students will use a geoboard and rubber bands or a digital geoboard to create rectangles and "tile" the shape to determine the area. Students will determine the area of given rectangles in addition to creating corresponding rectangles when given an area, while using the geoboards.

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

- Square unit
- Square inches
- Rectangle
- Area

MATERIALS

- One-square-inch squares: tiles or paper (Recommend 30 to 35 squares for every one to two students.)
- Geoboards with rubber bands (Recommend one board with four to six rubber bands for each student.)

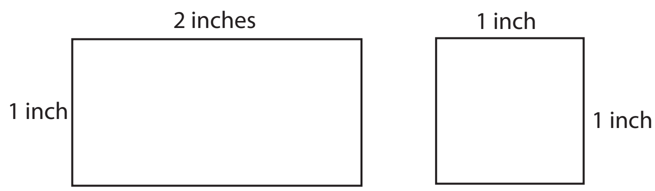
> NOTE: An alternative to handheld geoboards and rubber bands is an interactive website:
>
> http://www.mathlearningcenter.org/web-apps/geoboard/
>
> Also, available as an app for Apple, Windows, and Chrome devices:

https://www.mathlearningcenter.org/resources/apps/geoboard/

- Chart paper and markers (optional)
- INSTRUCTIONAL ACTIVITY STUDENT HANDOUT

IMPLEMENTATION

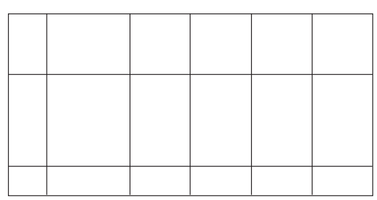
Begin a discussion to review area and square units by showing or drawing the following image with measurements labeled.



Ask students the following questions to start and support discussion. **Allow** students to discuss, agree, and disagree with each other. Make sure that each student who responds explains their reasoning or provides evidence to support their answer.

- How are these shapes the same? How are they different?
- Could you use both of these shapes as a unit square? Why or why not?
- Which shape could be used as a unit square? Why?
- Which shape could not be used as a unit square? Why not?
- What is the name of the unit that is being used to measure the unit square? How do you know?

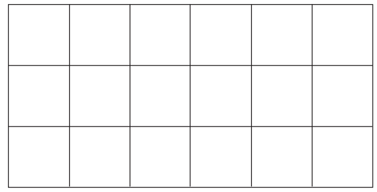
Repeat the same process with the following image.



Ask students the following questions:

- What is the name of this shape? How do you know?
- Thinking of any rectangle, what parts or attributes could you measure? Explain.
- Can you measure the area of this [Point to the rectangle.] rectangle with these units? Why or why not?
- Could you change anything so that you can measure the area of this rectangle? Explain.

Repeat the process with the following image.



Ask students the following questions:

- Is this shape the same as or different than the other shape? [Point to the rectangle with incorrect units.] Explain.
- Can you measure the area of this rectangle? [Point to the rectangle.] Why or why not?
- What is the area of this rectangle? How do you know?

Distribute square tiles and the Instructional Activity Student Handout.

Explain that for Questions 1 – 3 on the Instructional Activity Student Handout, students are going to use the square tiles to measure the areas of different rectangles.

Relate the concept of covering the rectangle in unit squares to the activity in which students created rectangles using the unit squares from the paper strips in Lesson 1.

Require students to complete Questions 1 – 3 on the Instructional Activity Student Handout either independently or in partners.

> NOTE: For students that require extra support, use one-inch-square graph paper and draw the outline of the rectangle on the graph paper. Require the student to fill the shape with square tiles, then count the number of squares. The student can then outline each square on the graph paper instead of trying to trace each square tile.

Use the the following guiding questions to support student understanding.

GUIDING QUESTIONS

Elicit student thinking:

- [Point to one of the rectangles on Questions 1 – 3.] What shape is this? How do you know?
- [Point to one of the square tiles.] What shape is this? How do you know?
- How are squares and rectangles the same? How are they different?

Determine if the student can EXPLAIN *UNIT SQUARE*:

- [Point to a square tile.] Is this a unit square? How do you know?
- [Put two square tiles together to make a rectangle.] Is this a unit square? Why or why not?
- [Make a square that is two tiles long and two tiles wide.] Could this be a unit square? Why or why not?

Determine if the student can EXPLAIN AREA AS A COMPOSITION OF UNIT SQUARES:

- [Move a square so that it is overlapping another square.] Can these two squares overlap when you're determining area? Why or why not?
- [Move a square so that there is a gap between two or more squares.] Can there be a gap between these squares when you're determining area? Why or why not?
- [Point to one of the rectangles on Questions 1 – 3.] Can you measure the area of this rectangle without the tiles? If so, how? If not, why not?

Determine if the student can CALCULATE AREA OF RECTANGLE WITH TILING:

- [Point to one of the rectangles on Questions 1 – 3.] Show me with your squares how you measured the area of this rectangle.
- [Point to one of the rectangles on Questions 1 – 3.] How many unit squares are in this rectangle? How do you know?
- [Point to one of the rectangles on Questions 1 – 3.] Can you identify each of the unit squares in this rectangle? What is the area of the rectangle?
- [Move some of the tiles so that there is space between them and some of the squares go outside the rectangle.] Would you get the same area if there was space between tiles like this? Why or why not?
- [Overlap some tiles so that you can fit more either partially or all the way onto the rectangle.] Would you get the same area if the tiles were overlapping like this? Why or why not?

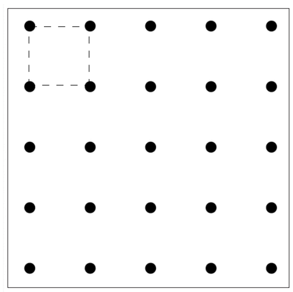
Discuss the answers to Questions 1 – 3 on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

Require several students to share their process for tiling and their answers. **Select** students that used different methods of tiling to share (e.g., some students might fill the rectangle with tiles, count, then trace, while some students might trace as they go, then count the squares they drew).

Collect the square tiles.

Explain and **model** the directions for Questions 4 – 6, using the example on Page 3 of the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT. Consider modeling the example using the digital geoboard on an interactive whiteboard prior to students engaging with handheld geoboards or technology devices.

Establish a unit square on the geoboard.



NOTE: For students that require extra support, draw lines on the geoboard between the pegs for better visualization of the squares or enable lines on the digital geoboard.

Establish the rules for using geoboards or digital devices. Distribute the geoboards or technology devices.

Note that if you do not have access to geoboards or technology, the students can use the INSTRUCTIONAL ACTIVITY STUDENT HANDOUTS without the manipulatives.

Provide students two or three minutes to explore the geoboards (digital or handheld).

Require students to complete Questions 4 – 6 on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT either independently or in partners.

Use the following guiding questions to support student understanding.

GUIDING QUESTIONS

Elicit student thinking:

- [Point to one of the rectangles on Questions 4 – 6.] What shape is this? How do you know?
- Can you create a rectangle on your geoboard? How do you know this is a rectangle?
- Can you create a square on your geoboard? How do you know this is a square?

Determine if the student can EXPLAIN *UNIT SQUARE*:

- Show me a unit square on the geoboard. How do you know this is a unit square?
- [Make a rectangle on the geoboard that is two unit squares.] Is this a unit square? Why or why not?
- [Make a square that is two units long and two units wide.] Could this be a unit square? Why or why not?

Determine if the student can CALCULATE AREA BY COUNTING UNIT SQUARES:

- [Point to one of the rectangles on Questions 4 – 6.] How many unit squares are in this rectangle? How do you know?
- [Point to one of the rectangles on Questions 4 – 6.] Can you identify each of the unit squares in this rectangle? What is the area of the rectangle?

Discuss the answers to Questions 4 – 6 on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

Require several students, who used different strategies for finding the area, to share their answer with the class and explain how they arrived at the answer. Students with less-advanced strategies should present first, proceeding to students with more advanced strategies.

Display a blank geoboard on the interactive whiteboard, a handheld geoboard on a document camera, or a picture of a blank geoboard on a large whiteboard or chart paper.

Share the following problem situation.

I bought a new rectangular rug for my house. The area of my new rug is 12 square feet. What could my new rug look like?

Discuss the attributes of a rectangular rug, and lead students to the understanding that measuring the area of the rug is determining the area of a rectangular space.

Clarify that students will need to create a rectangle that has an area of 12 square feet. **Ensure** that students understand that each unit in this scenario is one foot, and therefore the distance between the pegs on the geoboard will represent one foot.

Provide students with one or two minutes to manipulate their geoboards, to think about the problem, and to discuss with a partner.

Require several students to share and model their answers on the displayed geoboard.

Ask students for other situations that would require determining the area of a rectangle.

Discuss possible scenarios to elicit the fact that area is the measurement of the two-dimensional space in a surface.

To support the discussion, possible scenarios include:

- Covering a garden with soil (without considering depth)
- Installing carpet
- Covering a bed with a blanket
- Painting a wall
- The size of a large poster

Explain that for Questions 7 – 9 on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT, students will need to read the contextualized problem and create a rectangle with the given area.

Require students to complete Questions 7 – 9 on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT either individually or in partners.

Use the following guiding questions to support student understanding.

GUIDING QUESTIONS

Elicit student thinking:

- How is counting the area when you have a rectangle different than creating a rectangle when you have the area? How is it the same?
- What things do you see or use every day in school that are rectangles? Outside of school? (e.g., at home, at the store, in sports, etc.)

Determine if the student can EXPLAIN *UNIT SQUARE*:

- Show me a unit square on the geoboard. How do you know this is a unit square?
- [Make a rectangle on the geoboard that is two unit squares.] Is this a unit square? Why or why not?
- [Make a square that is two units long and two units wide.] Could this be a unit square? Why or why not?

Determine if the student can CALCULATE AREA BY COUNTING UNIT SQUARES:

- [Point to one of the rectangles on Questions 7 – 9.] How do you know the area of your rectangle matches the area given in the problem?
- [Point to one of the rectangles on Questions 7 – 9.] Can you identify each of the unit squares in this rectangle? What is the area of the rectangle? Does it match the area given in the problem?
- If the problem said the area was seven square feet, could you create a rectangle with an area of seven square units? Show me to explain why or why not.

Discuss the answers to Questions 7 – 9 on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

Require two or three students, with different rectangles as solutions, to share their answer with the class and explain how they arrived at the answer.

Collect the geoboards.

Students should be required to share with a partner what a square unit is, what area is, and how to determine the area of a rectangle.

At the end of the activity, collect the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT and review the responses for students' understanding.

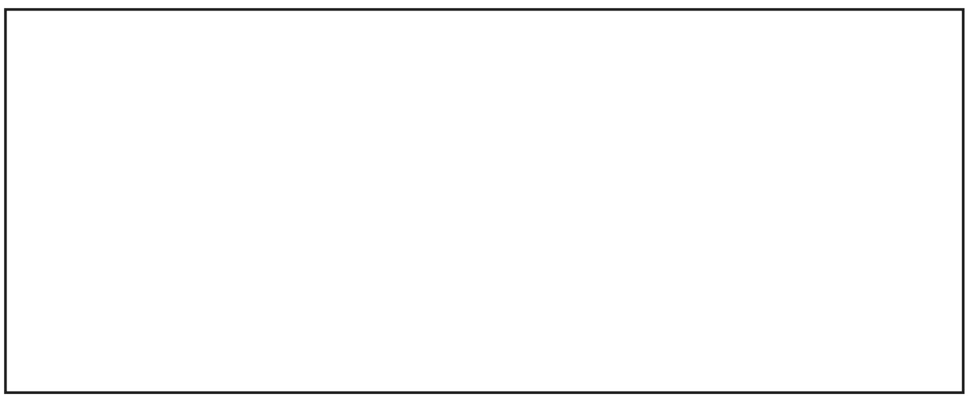
Name________________________________

Determining Area of Rectangles

Lesson 2

Use the square tiles to find the area of each rectangle in square inches. Trace each square tile to show the area of the rectangle. Write the area, in square inches, above the rectangle.

1. Area:



2. Area:

Name______________________________

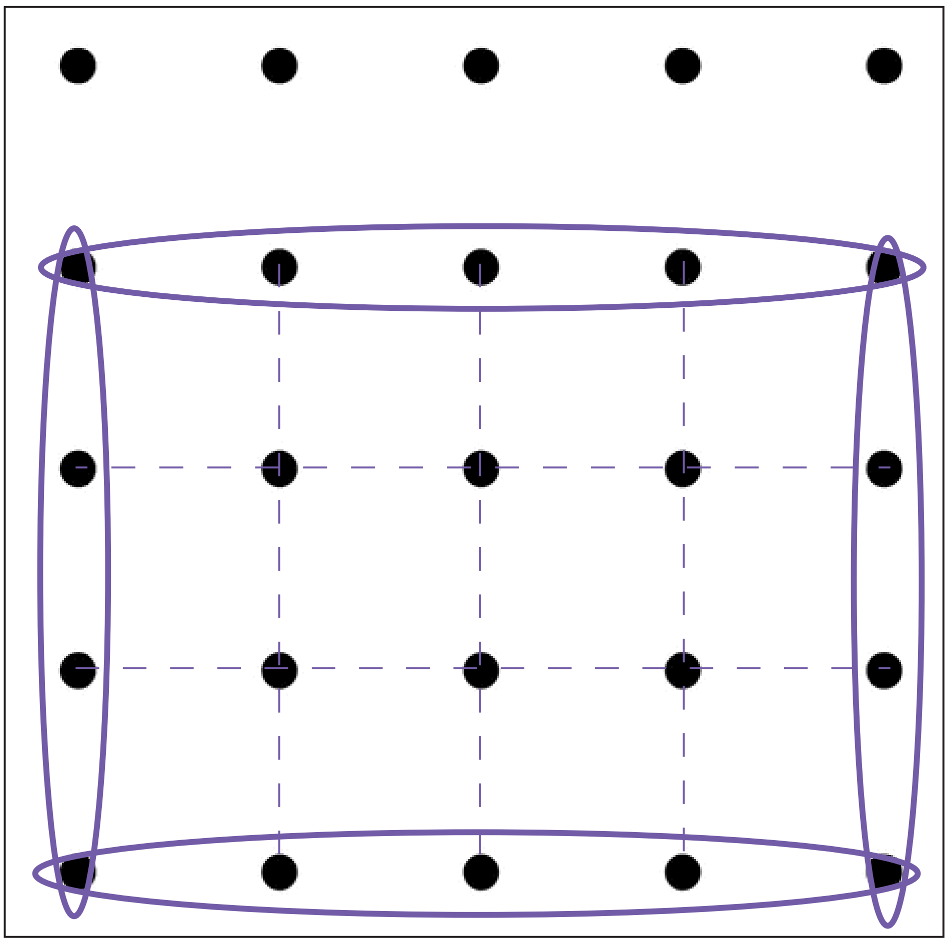
3. Area:

Name________________________________

Use your geoboard to find the area in square units of the rectangles in Questions 4 – 6.

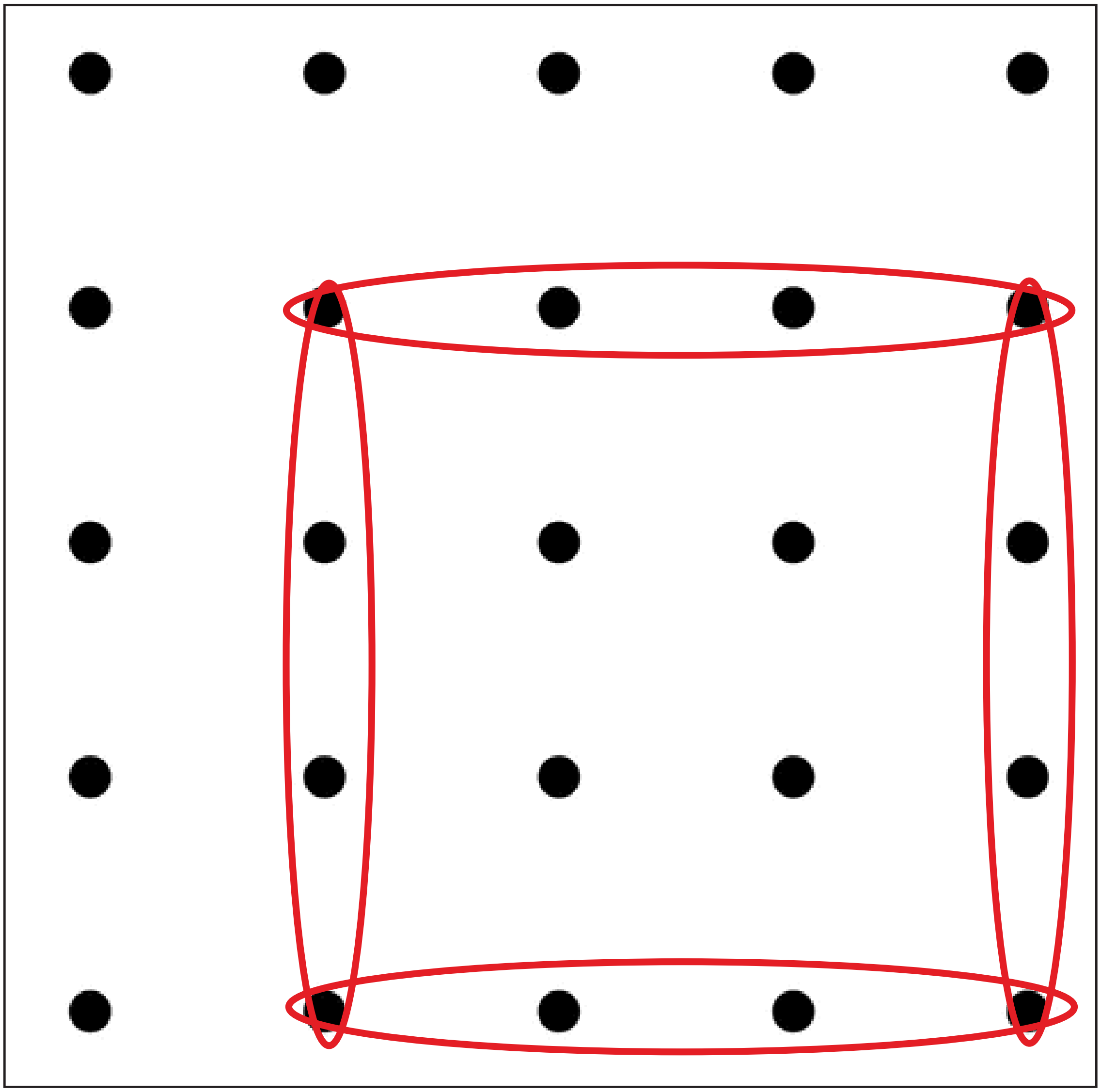
Example:

Area: 12 square units



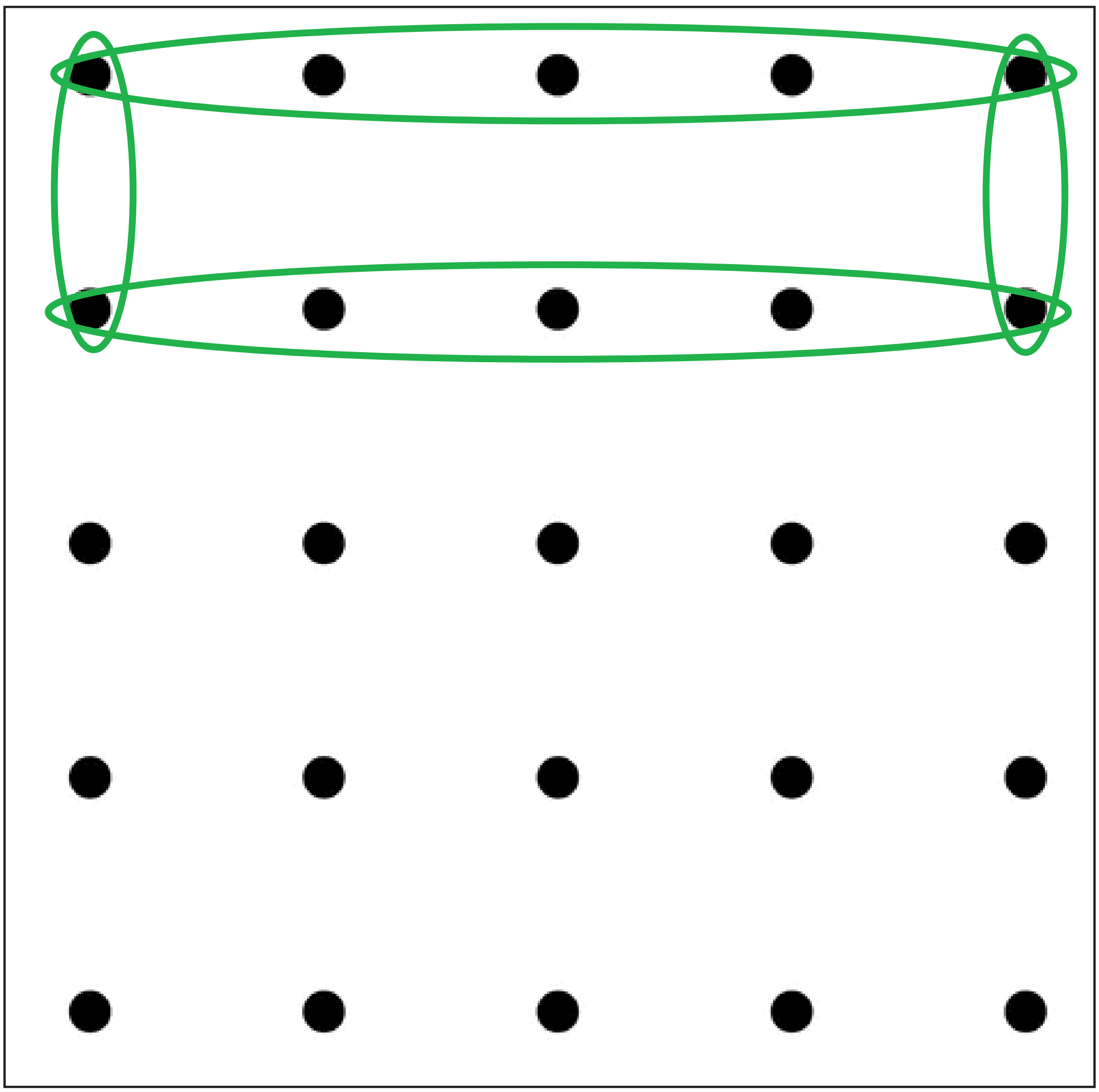
Name________________________________

4. Area:



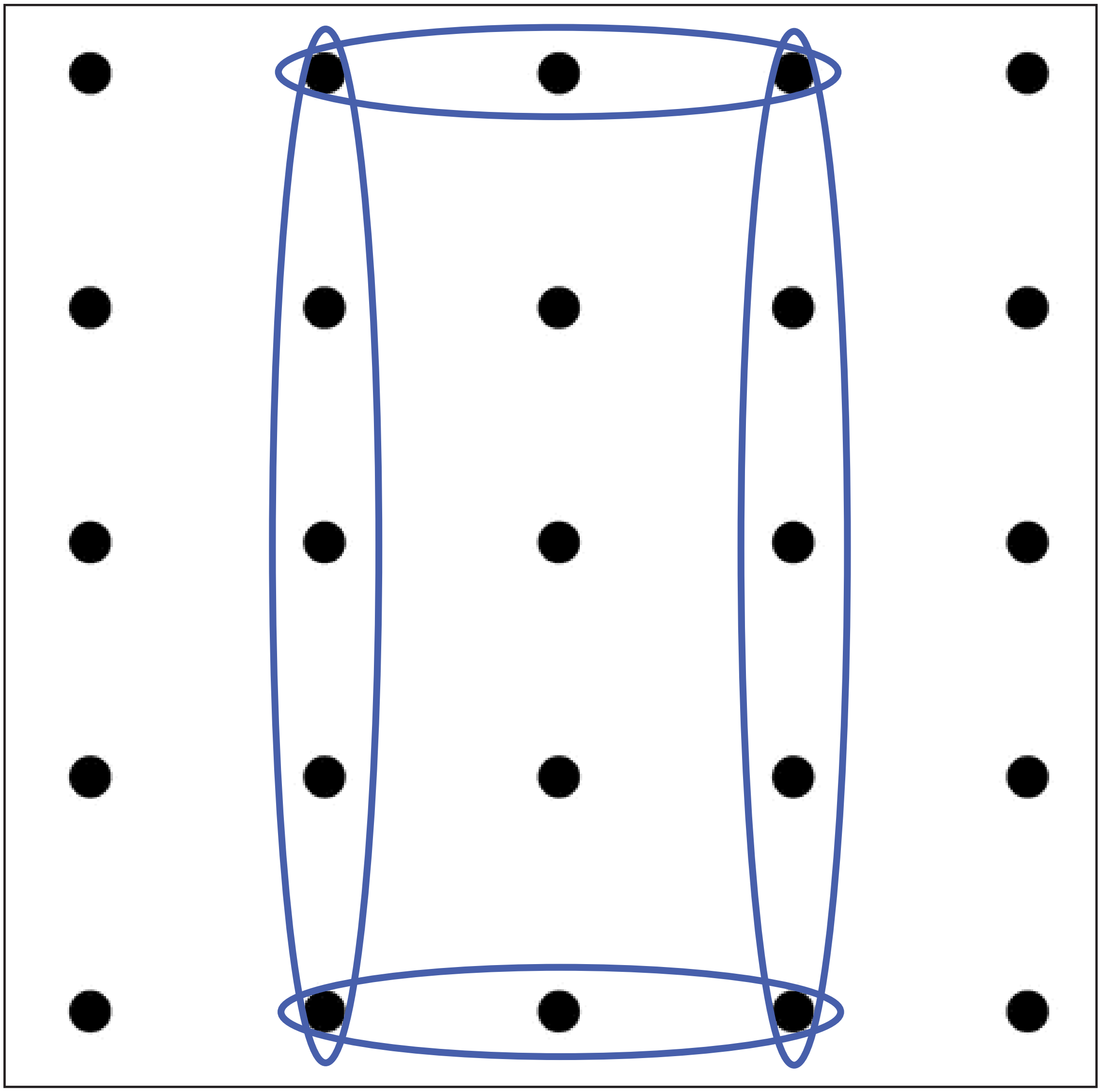
Name________________________________

5. Area:



Name________________________________

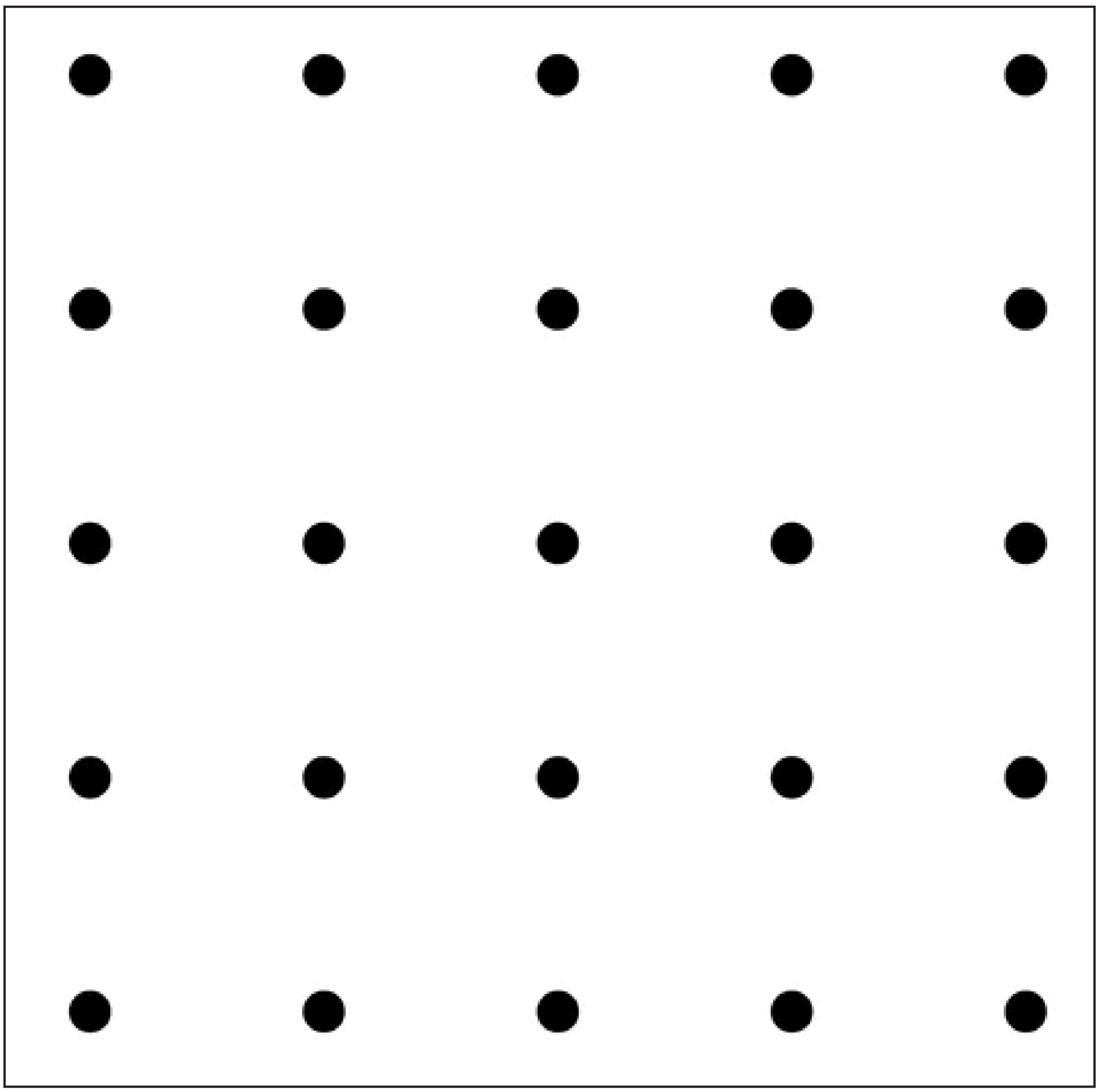
6. Area:



Name______________________________

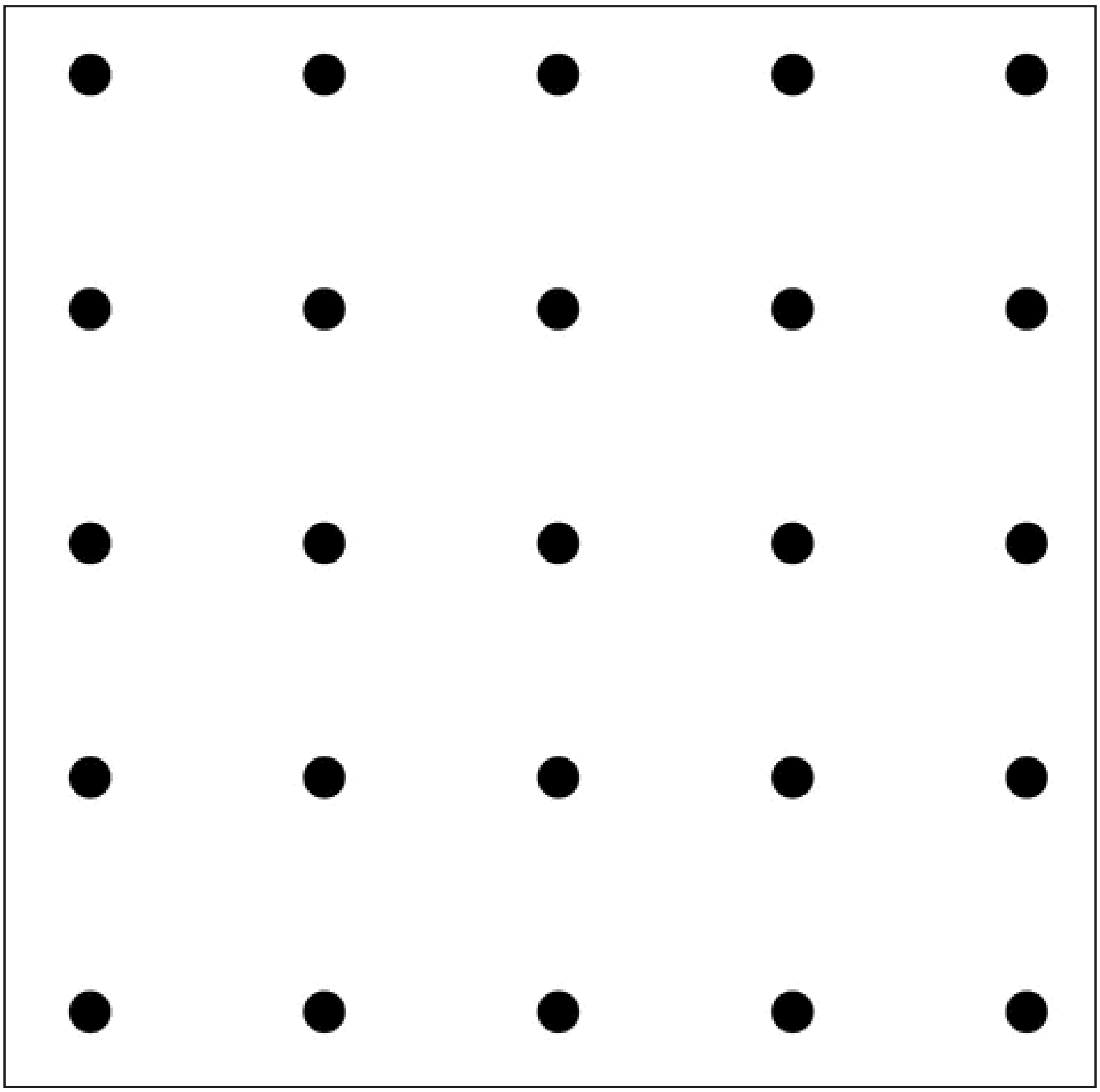
Read each question. Use the geoboard to create a rectangle with the given area. Label the side lengths of each rectangle.

7. Brad was given a new poster for his birthday. The area of Brad’s poster is six square feet. Draw Brad’s new poster.



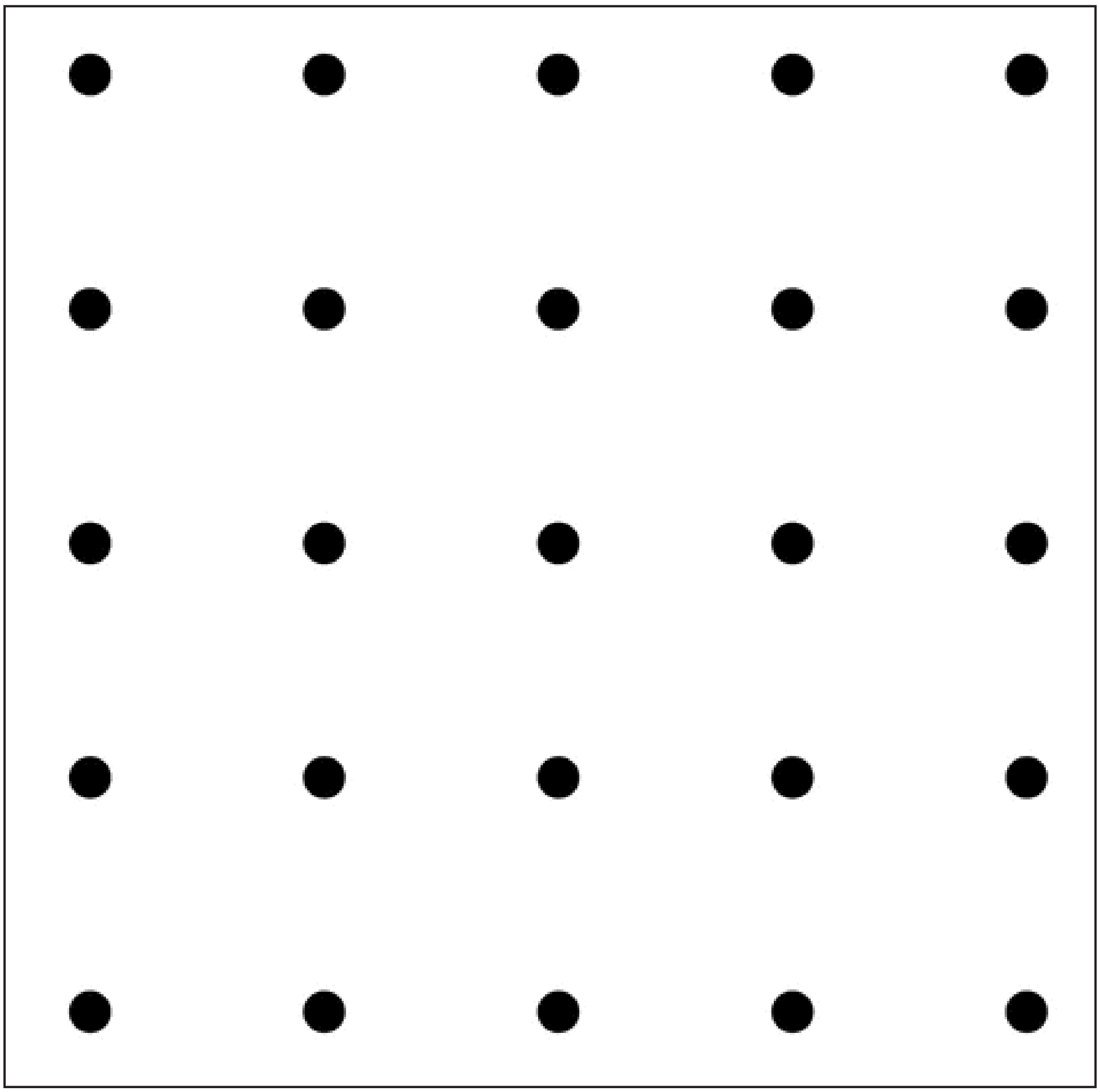
Name______________________________

8. Amy has a rug in her hallway. The area of the rug is 8 square feet. Draw Amy’s rug.



Name______________________________

9. Sierra's grandmother made her a blanket. The area of the blanket is nine square feet. Draw Sierra's blanket.



Determining Area of Rectangles

INSTRUCTIONAL ACTIVITY

Lesson 3

LEARNING GOAL

Students will determine the area of rectangles by tiling, drawing, and counting square units. Then, students will estimate the areas of rectangles in square inches and check their estimation by creating and counting square inches.

PRIMARY ACTIVITY

Students will use rulers to mark square inches on rectangles in order to count unit squares to determine the area of the rectangle. Then, students will estimate the area of given rectangles, then compare the estimated areas of different rectangles before determining each area.

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

- Square unit
- Square inches
- Rectangle
- Area
- Base
- Height
- Line segment (segment)

MATERIALS

- Rulers
- Instructional Activity Student Handout Version 1 (Version 2 is optional.)
- Instructional Activity Supplement (Recommend one copy for every two students.)
- Chart paper and markers (optional)

IMPLEMENTATION

Review how to use a ruler. **Select** one or two students to model how to use a ruler to measure inches and/or centimeters.

Display a rectangle that is seven inches long and four inches tall. (This can be created on a piece of paper and displayed on a document camera, represented on an interactive whiteboard, or drawn on a large dry erase board or chart paper.)

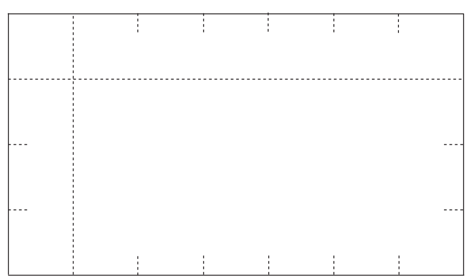
Explain that the *base* of the rectangle measures how wide the rectangle is, and the *height* of the rectangle measures how long or tall the rectangle is. Avoid creating the misconception that the height must be a side length, because that will not always be true as students progress through geometry. Understand that the *height* of a figure will *always* be a perpendicular segment (90° angle) to the *base* of the figure. In addition, the *base* does not always need to be the bottom of the figure.

NOTE: Be sure to use the vocabulary *base* and *height* (as opposed to length and width), so that as students proceed through geometry, the vocabulary is consistent for calculating volume and area of figures such as parallelograms, triangles, and trapezoids.

Select one or two students to help measure the base and height of the rectangle.

Require students to partition one side of the rectangle into inches. Then, **repeat** this process on the opposite, parallel side of the rectangle.

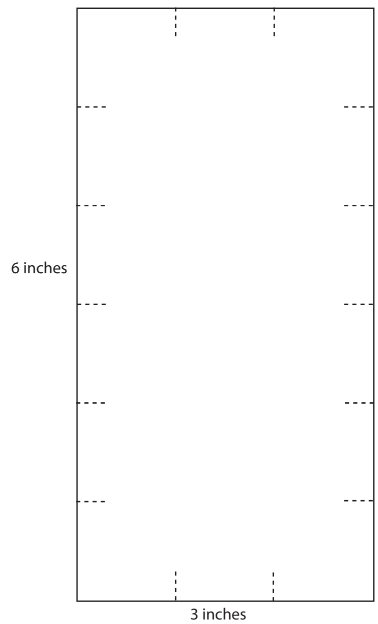
A student should **model** connecting, using the ruler, the marks on opposite sides of the rectangle to divide the shape into a grid of square units.



Once the student(s) have drawn the perpendicular line segments connecting the marks, **ask** students if they can determine the area of the rectangle.

Require students to share with a partner the area and how they determined the area. **Listen** for students using correct units, i.e. square inches.

Repeat the process with a rectangle where the sides are labeled.



Emphasize that the number of units labeled on the base and height are not the number of marks that need to be drawn. If there are three marks on the base, which will result in three perpendicular line segments, then there will be four units across the base (not three). **Make explicit** that the labels are the number of units that result from the marks and the perpendicular line segments.

Distribute the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT VERSION 1 and/or VERSION 2 (see the following note).

> NOTE: For students that benefit from enrichment, use INSTRUCTIONAL ACTIVITY STUDENT HANDOUT VERSION 2 in place of INSTRUCTIONAL ACTIVITY STUDENT HANDOUT VERSION 1. These students will need additional directions.

Require students to complete Questions 1 – 3 on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT, either individually or in partners. If you utilize the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT VERSION 2, ensure that students are paired appropriately.

> NOTE: For students that require extra support, use one-inch-square graph paper to support drawing and visualizing the line segments, or have the rectangle side lengths pre-divided so the student only has to draw the perpendicular line segments connecting marks on opposite sides.
>
> **Provide** enrichment by requiring students to measure the sides using a different unit such as, centimeters. Measuring in centimeters will provide insight into how the student will encounter partial units.

Use the following guiding questions to support student understanding.

GUIDING QUESTIONS

Elicit student thinking:

- What do the numbers on a ruler represent?
- Why would you need to use a ruler?
- When would you need to use a ruler outside of school?
- Can you use a ruler to measure the area of a rectangle?

Determine if the student can EXPLAIN *UNIT SQUARE*:

- [Point to one of the rectangles on Questions 1 – 3.] How are you going to show the unit squares in this rectangle?
- [Point to one of the rectangles on Questions 1 – 3.] What will be the unit for the square units in this rectangle? How do you know?
- [Point to a side length of one of the rectangles on Questions 1 – 3.] How do you know how many marks there are on this side length? Is that the same as the number of units you will have on that side length? Why or why not?
- When you draw the segments connecting your marks, do they have to be straight? Why or why not?
- [In one of the rectangles on Questions 1 – 3, draw or indicate line segments that do not align with opposite marks, so that the segments are not perpendicular, creating shapes that are not squares inside the rectangle.] Are these unit squares? Why or why not?

Determine if the student can CALCULATE AREA BY COUNTING UNIT SQUARES:

- [Point to one of the rectangles on Questions 1 – 3.] How do you know how many unit squares are in this rectangle?
- [Point to one of the rectangles on Questions 1 – 3.] How many unit squares are in this rectangle? What is the area of this rectangle? How do you know?
- [Point to one of the rectangles on Questions 1 – 3.] Do you have to draw all the unit squares in the rectangle to count the area? Why or why not?

Discuss the answers to Questions 1 – 3 on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT. During discussion, be aware that some students may have used centimeters.

Require several students to share their process for measuring, drawing their unit squares, and determining the area. **Select** students that used different methods of completing the task to share their methods with the class (e.g., some students might measure out all the one-inch marks and then connect the marks, while other students might measure one-inch and connect the marks before moving on to the next inch segment).

Discuss estimation. Students do not need to know the word or definition of estimation; however, they should be familiar with the concepts of estimating, as well as guessing and checking.

Display a square, without any markings, that is four square inches. Ideally the square should be drawn on an interactive whiteboard or a large dry-erase board, so that the dimensions are not distorted when projected.

Ask students to estimate or guess the area of the square. Students should share their answers by holding up the number of fingers they think represents the area.

Select several students to share why they chose the area they did.

Require a student to come and measure the area of the square using either the square tiles or a ruler to draw square inches.

Discuss what students discovered. **Consider** the following questions to help guide the discussion.

- Did they guess too many? Too few?
- Was their estimate close to the area or not at all?
- How did they come up with their estimate?
- Did they use any strategies to help them estimate?

Repeat the process two more times with different rectangles. Be sure to represent rectangles that have different dimensions (e.g., one that is taller and narrower, and one that is shorter and wider).

Require students to complete Questions 4 and 5 on either version of the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT by estimating the area of the rectangle and then measuring the side lengths and dividing the rectangle into square units to determine the actual area.

Use the following guiding questions to support student understanding.

GUIDING QUESTIONS

Elicit student thinking:

- Have you ever had to guess how large or small something is before? Explain.
- [Point to one of the rectangles on Questions 4 or 5.] Would you measure the area of this rectangle in square inches or square feet? Why?
- What is an example of something you would measure in square feet? Why?
- What is an example of something you would measure in square inches? Why?

Determine if the student can ESTIMATE AREA IN SQUARE INCHES:

- [Point to one of the rectangles on Questions 4 or 5.] What do you think the area of this rectangle is? Why?
- [Point to one of the rectangles on Questions 4 or 5.] How many square units do you think will fit in this rectangle? Why?
- [Point to one of the rectangles on Questions 4 or 5.] Did you guess the correct area for this rectangle? Explain.
- [Point to one of the rectangles on Questions 4 or 5.] Was your guess close to the actual area? Explain.
- [Point to one of the rectangles on Questions 4 or 5.] Why do you think this will be the area of this rectangle?

Determine if the student can CALCULATE AREA BY COUNTING UNIT SQUARES:

- [Point to one of the rectangles on Questions 4 or 5.] How many unit squares are in this rectangle? What is the area of this rectangle? How do you know?
- [Point to one of the rectangles on Questions 4 or 5.] Do you have to draw all the unit squares in the rectangle to count the area? Why or why not?

Review the answers to Questions 4 and 5 on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT. The answers listed below apply to both versions, unless the student used centimeters.

- 4. 12 square inches
- 5. 8 square inches

Discuss what students discovered regarding their estimates and the actual area. Consider the following questions to help guide the discussion.

- Did they guess too many? Too few?
- Was their estimate close to the area or not at all?
- How did they come up with their estimate?
- Did they use any strategies to help them estimate?

Arrange students into partners. Distribute one set of Page 1 and Page 2 (Rectangles A and B) of the INSTRUCTIONAL ACTIVITY SUPPLEMENT to each group.

Require student partners to estimate which of the rectangles has the larger area. Students should write their answers and explanations on either version of the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT on Question 6.a.

Display a table, like the following, on the board or a piece of chart paper. **Ask** students which rectangle they think has the larger area by a show of hands.

Fill in the first row, "Student Predictions", with tally marks for each student.

Which Rectangle Has the Larger Area?		
	Rectangle A	Rectangle B
Student Predictions		
Actual Area		

Require student partners to determine the area of both rectangles. Students should record the areas for each rectangle on either version of the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT on Questions 6.b and 6.c, then answer Question 6.d.

Refer back to the table and **ask** student partners to share the areas they determined for rectangles A and B. **Require** students to share how they determined the area.

Correct any misconceptions or errors that are shared.

Record the areas on the second row of the table.

Discuss what students discovered as they completed the activity. Consider the following questions to help guide the discussion.

- Which rectangle did more students think had the larger area?
- Why did they think that rectangle (either A or B) was larger?
- Did they use any strategies to help them decide which rectangle had the larger area?

Repeat this process with Rectangles C and D on Pages 3 and 4 of the INSTRUCTIONAL ACTIVITY SUPPLEMENT, and with Question 7 on either version of the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

At the end of the activity, collect each student's INSTRUCTIONAL ACTIVITY STUDENT HANDOUT and review the responses for students' understanding.

Name________________________________

DETERMINING AREA OF RECTANGLES

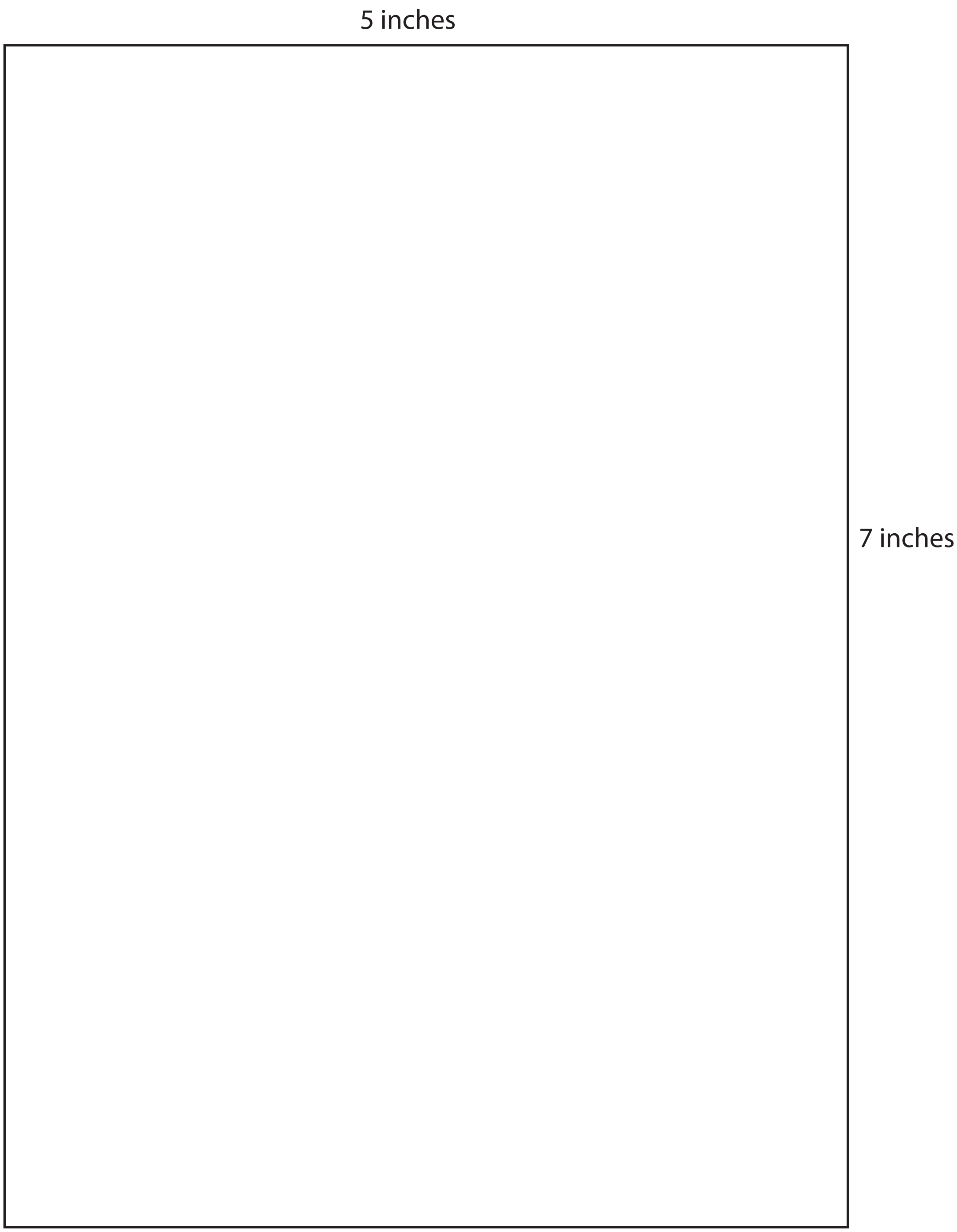
Lesson 3 Version 1

Using a ruler, measure and mark all side lengths in inches. Draw a segment to connect opposite marks. Use a ruler to make sure the segments are straight. Count and identify the area.

1. Area:

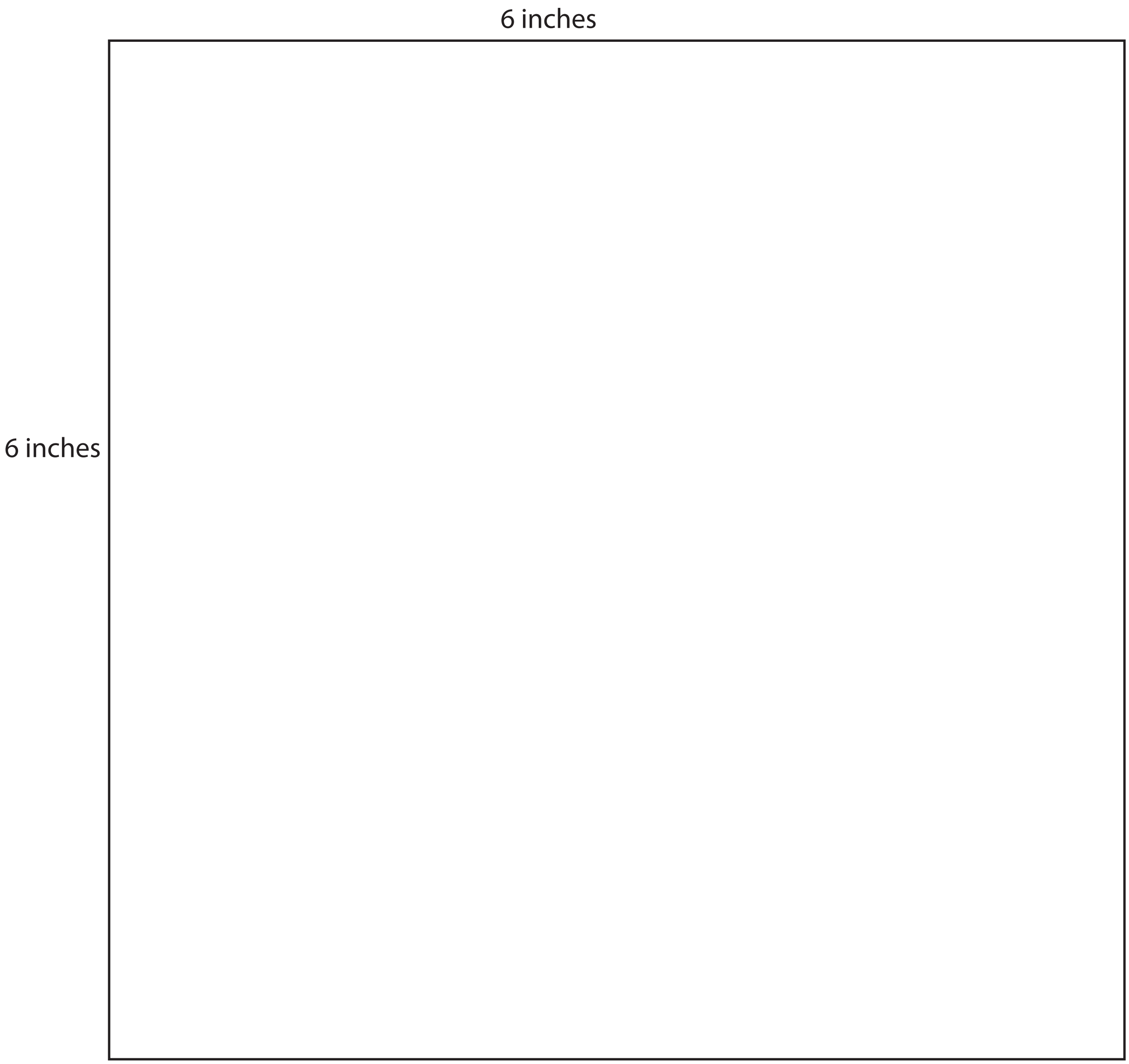
Name________________________________

2. Area:



Name________________________________

3. Area:



Name______________________________

Before you find the area of the rectangle, guess what you think the area will be. Then measure each side length and draw square inches inside the rectangle to find the area.

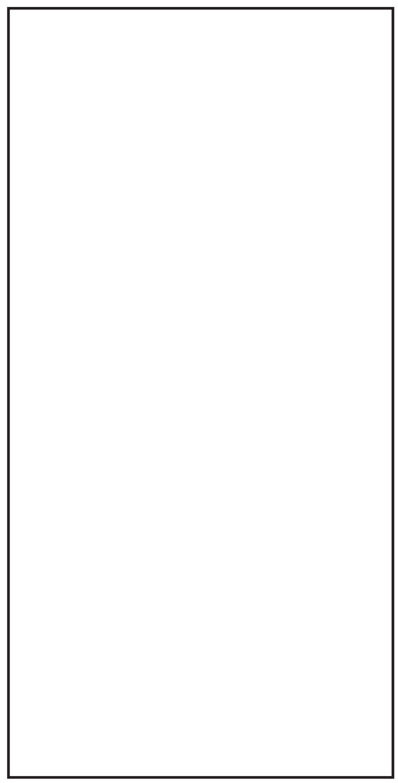
4. I think the area will be ________________________________.

The area is ____.

Name______________________________

Before you find the area of the rectangle, guess what you think the area will be. Then measure each side length and draw square inches inside the rectangle to find the area.

5. I think the area will be ________________________________.



The area is ____.

Name____________________________

6. Use Rectangles A and B to answer the following questions.

a. Which rectangle do you think has the larger area, Rectangle A or Rectangle B? Why?

b. What is the area of Rectangle A?

c. What is the area of Rectangle B?

d. After finding the areas, which rectangle is larger? How do you know?

Name_______________________________

7. Use Rectangles C and D to answer the following questions.

a. Which rectangle do you think has the larger area, Rectangle C or Rectangle D? Why?

b. What is the area of Rectangle C?

c. What is the area of Rectangle D?

d. After finding the areas, which rectangle is larger? How do you know?

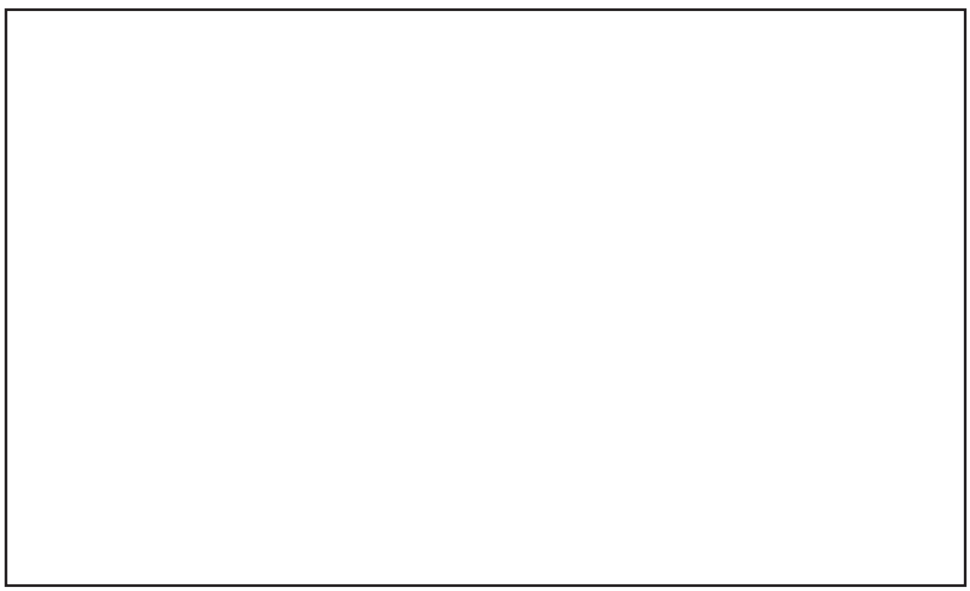
Name______________________________

Determining Area of Rectangles

Lesson 3, Version 2

Using a ruler, measure and mark all side lengths. Draw a segment to connect opposite marks. Use a ruler to make sure the segments are straight. Count and identify the area.

1. Area:



Name_______________________________

2. Area:

Name______________________________

3. Area:

Name______________________________

Before you find the area of the rectangle, guess what you think the area will be. Then measure each side length and draw square inches inside the rectangle to find the area.

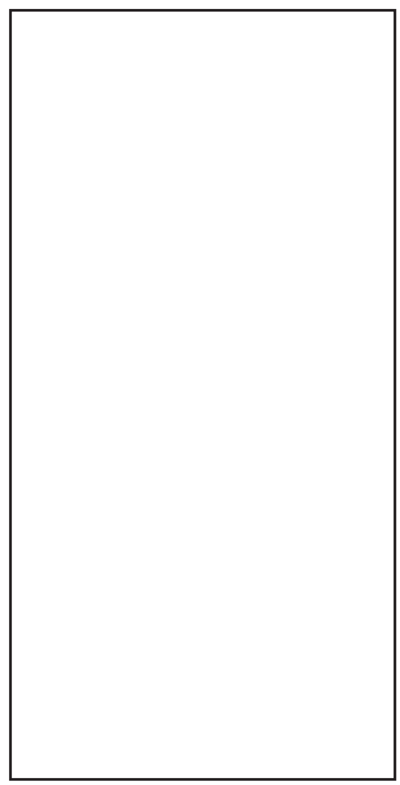
4. I think the area will be ________________________________.

The area is ____.

Name______________________________

Before you find the area of the rectangle, guess what you think the area will be. Then measure each side length and draw square inches inside the rectangle to find the area.

5. I think the area will be ______________________________.



The area is ____________________________________.

Name____________________________

6. Use Rectangles A and B to answer the following questions.

a. Which rectangle do you think has the larger area, rectangle A or rectangle B? Why?

b. What is the area of Rectangle A?

c. What is the area of Rectangle B?

d. After finding the areas, which rectangle is larger? How do you know?

Name____________________________

7. Use Rectangles C and D to answer the following questions.

a. Which rectangle do you think has the larger area, Rectangle C or Rectangle D? Why?

b. What is the area of Rectangle C?

c. What is the area of Rectangle D?

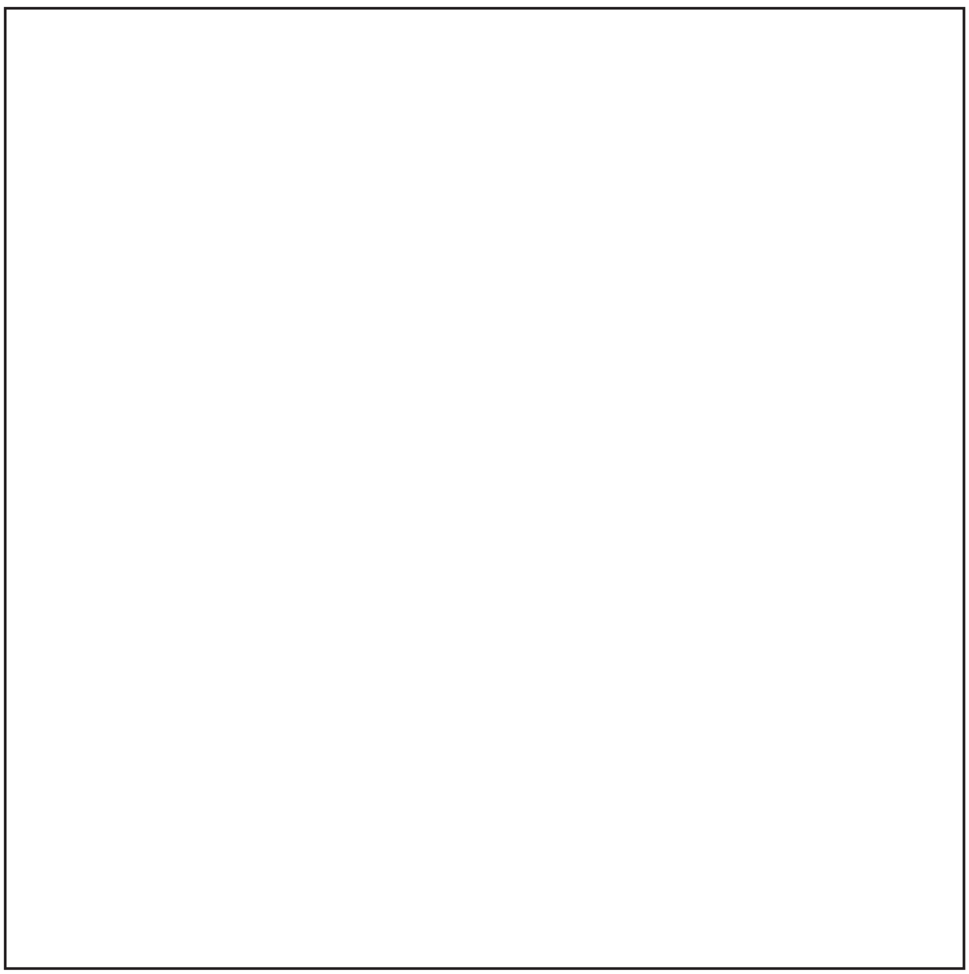
d. After finding the areas, which rectangle is larger? How do you know?

Determining Area of Rectangles

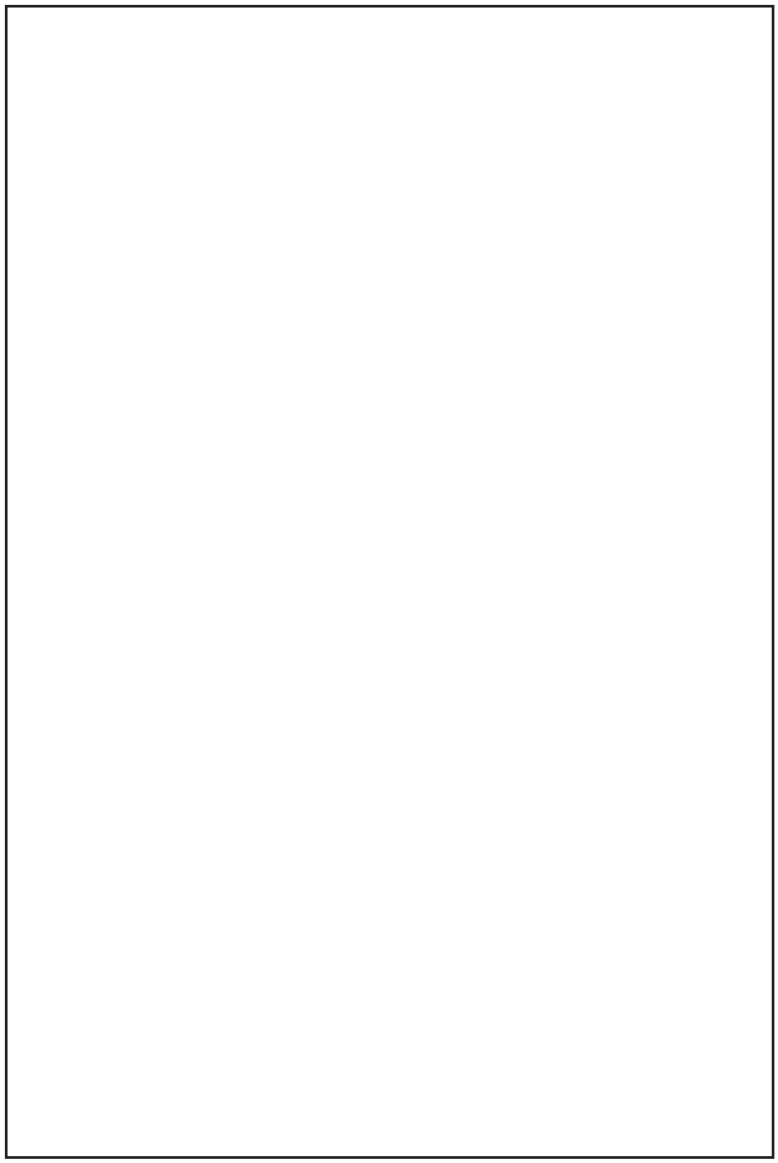
INSTRUCTIONAL ACTIVITY SUPPLEMENT

Lesson 3

Rectangle A



Rectangle B



Rectangle C

Rectangle D

Determining Area of Rectangles

INSTRUCTIONAL ACTIVITY

Lesson 4

LEARNING GOAL

Students will connect their understanding of multiplication using arrays with the rectangle area formula.

PRIMARY ACTIVITY

Students will collaborate in small groups to determine the area of a rectangle using a number of small square tiles that is less than the area. Then, as a class, students will apply their strategies to determine the area of a large rectangular space using a number of square tiles that is less than the area. Students will link their understanding of multiplication with arrays (i.e. rows and the number of items in each row) to discover the formula for area of rectangles.

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

- Area
- Base
- Height
- Square units (inches, feet, centimeters, etc.)
- Array
- Row
- Rectangle
- Square
- Distributive property

MATERIALS

- One-square-inch squares: tiles or paper (Recommend 23 squares for every one to two students.)
- One-square-foot squares: tiles or paper (Recommend one square for every three to four students.)
- 30-square-foot rectangle: outlined on the floor with tape or created using large butcher paper
- Instructional Activity Student Handout

IMPLEMENTATION

Review how to determine the area of a rectangle using tiling with one-square-inch squares. **Display** a rectangle and **select** a student to model tiling it.

Discuss the example utilizing the following guiding questions.

GUIDING QUESTIONS

Elicit student thinking:

- Does this rectangle with unit squares look like anything else you have used in math? Explain.
- Do you think you have to fill the entire rectangle to find the area? Why or why not?

Determine if the student can EXPLAIN AREA AS A COMPOSITION OF UNIT SQUARES:

- What do each one of these squares represents? Explain.
- Do the unit squares need to be touching? Why or why not?
- Can there be spaces between the unit squares? Why or why not?
- Can the unit squares be on top of (or covering) each other? Why or why not?

Determine if the student can CALCULATE AREA OF A RECTANGLE WITH TILING:

- How would you find the area of this rectangle? Explain.
- What is the area of this rectangle?

Determine if the student is ready to EXPLAIN AREA FORMULA FOR RECTANGLES:

- Show me a row on your rectangle using unit squares. How many units are in that one row? How do you know?
- Show me how many rows are in the rectangle. How many rows are there? How do you know?

Arrange students into groups of three or four. Each group should receive one one-square-foot square and 23 one-square-inch squares.

> **Require** students to work collaboratively to use the one-square-inch squares to determine the area of the larger one-square-foot square (in square inches).
>
> **Explain** that students do not have enough smaller squares to completely cover the larger square.
>
> **Use** the previous set of guiding questions (Page 2) to support student understanding.
>
> Once students have had a sufficient amount of time to determine the area, **ask** students to share their strategies with the class.
>
> **Collect** the one-square-inch squares.

Bring students' attention to the 30-square-foot rectangle. Students will take their understanding from the previous small group activity and apply it to the following whole class activity.

> **Explain** that students will collaborate as a class, using their one-square-foot squares, to determine the area of the large rectangle.
>
> **Explain** that, once again, the students do not have enough squares to cover the entire rectangle.
>
> **Watch** and **observe** as students work together.

Use the previous set of guiding questions in this document to support student understanding.

> Once students have determined an area, **discuss** the strategy/strategies that were used. **Consider** the following questions during the discussion:
>
> - Did the students agree and work together?
> - Was there dissention among the students? If so, how was it resolved?
> - Did some students stand back and watch the others? (If so, be sure to engage them in the discussion.)
> - Were all students engaged in the problem solving? Why or why not? (Be sure to engage the unengaged students in the discussion and determine why they were not engaged.)
>
> If necessary, **rearrange** the students' work on the 30-square-feet rectangle so that only the base of the rectangle is a row of one-square-foot squares. [**Add** more squares if necessary.]
>
> **Review** the vocabulary from LESSON 3 by **requiring** one or two students to identify the *base* and the *height* of the 30-square-foot rectangle.
>
> **Select** one student to walk across a row on the rectangle, counting aloud the number of squares in the row.
>
> **Select** another student to identify how many rows there are in the rectangle. [**Make** more squares available if necessary.]

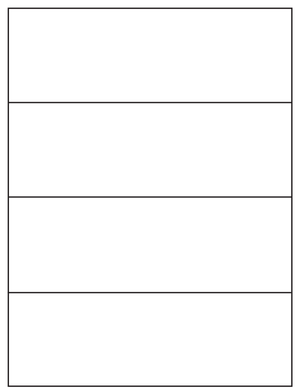
Ask students if they can make any connections between the number of square feet in one row, the number of rows, and the area of the rectangle. **Require** each student that shares to explain their thinking.

Distribute rulers and the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT to each student.

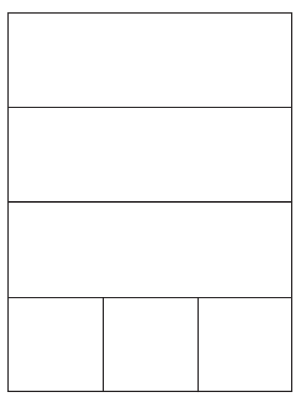
Complete Question 1 together as a class, asking students to provide responses as you work through the question.

Explain the directions on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

- First, using a ruler, students should measure along the *height* and create rows that are one inch tall.



- Second, using a ruler, students should measure along the *base* and create square inches in one row of the rectangle. (**Note** that the row that contains the square units does not have to be the bottom row—it can be any row.)



- Third, students should identify and write the number of squares in one row in addition to the number of rows.

Ask students for strategies to determine the area of the rectangle with this information. (One strategy could be repeatedly adding the number of units in one row for each of the rows. For example, there are three units in the base row, and there are four rows, so you would add: 3 + 3 + 3 + 3.)

- Lastly, student should write the area of the rectangle using the correct units, square inches.

NOTE: Let students discover the formula A = *height* • *base* on their own. When learned through self-discovery, students will understand the formula conceptually, increasing the likelihood that the student maintains the knowledge over time. Guide students towards the formula by comparing the rectangle with an array or reiterating the repeated addition of the rows.

The multiplication for *height* • *base* should be thought of as the number of rows times the number of units in each row. This connects the idea of multiplying to determine area with the concept of an array. With regards to area, know that the formula A = *height* • *base* is equivalent to A = *base* • *height*.

Require students to complete Questions 2 – 4 on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT, either independently or in partners.

Use the following guiding questions to support student understanding.

GUIDING QUESTIONS

Elicit student thinking:

- When do you use/see rectangles outside of school?
- Do you use/see rectangles at home? In sports? In music?
- Why would you need to know the area of a rectangular space?

Determine if the student is ready to EXPLAIN AREA FORMULA FOR RECTANGLES:

- [Point to a rectangle on Questions 2 – 4.] What would you consider the *base* of this rectangle? What would you consider the *height* of this rectangle?
- [Point to a rectangle on Questions 2 – 4.] What can you tell me about the *base*, the *height*, and the area?
- When you divide a rectangle into square units, does it look like anything that you have used in math class before? (an array)
- If you have three groups of six squares, how many squares do you have? Draw what that would look like.

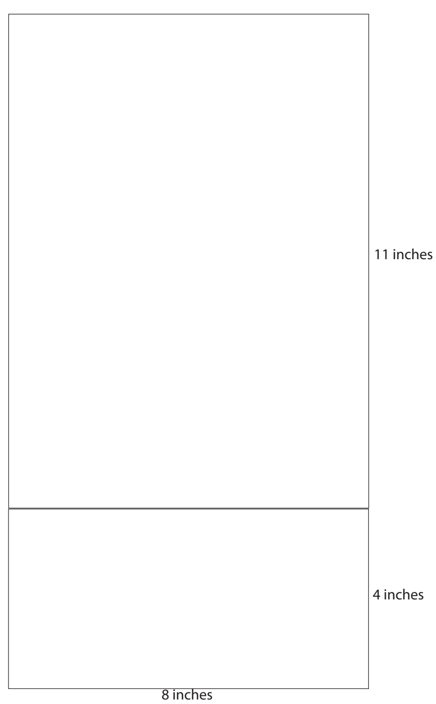
Determine if the student is ready to CALCULATE THE AREA OF A RECTANGLE WITH THE FORMULA:

- [Point to a rectangle on Questions 2 – 4.] How many rows have you divided this rectangle into? How many square units are in each row? What is the area? How do you know?
- [Point to the rectangle on Question 4.] If this rectangle has four rows and there are two square feet in each row, how many square feet are in the rectangle? How do you know?

Discuss the answers to Questions 2 – 4 on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT. **Select** a few students to share their strategies and answers for each question. Try to select students that repeatedly added to share before students that multiplied.

Present and **discuss** the following problem situation and rectangles. Ask students to provide responses as you work through the situation.

Marcus is making a map for his geography project. He taped two pieces of paper together to make a large rectangle. What is the area of Marcus's paper in square inches? [**Display** the following rectangles.]



Ask students the following questions (As students answer, draw lines with a ruler dividing the rectangles into rows and unit squares/columns.):

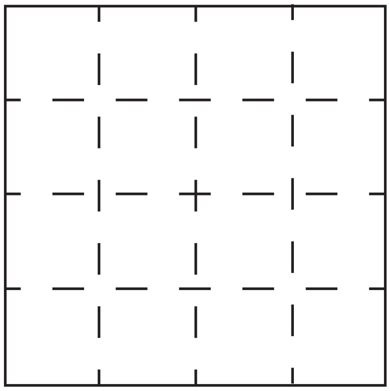
- [Point to the top/larger rectangle.] How many one-inch rows are in the paper Marcus started with?
- [Point to the bottom/smaller rectangle.] How many one-inch rows are in the paper Marcus added?
- How many square inches are in each row?
- What is the area of the piece of paper Marcus started with?
- What is the area of the piece of paper Marcus added?
- What is the area of Marcus's combined paper?
- Would the area be different if Marcus had a piece of paper that had 15 rows (or the height were 15 inches)? Why or why not?
- [Draw or display two new rectangles as you explain the new dimensions.] Suppose the paper that Marcus started with had a height of seven inches (or seven rows) and a base of eight inches, and the paper that Marcus added had a height of eight inches (or eight rows) and a base of eight inches. Would the area be the same or different than the original problem situation? Why?
- [Repeat the previous question with different height dimensions that equal 15, such as five inches and 10 inches, or six inches and nine inches.]

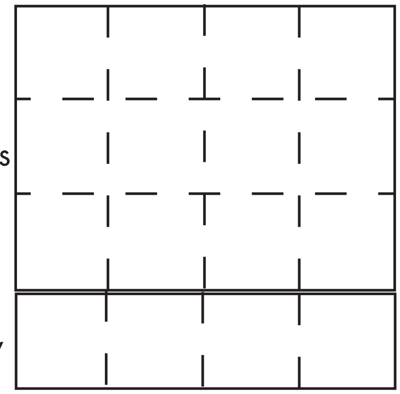
Emphasize that the height of a rectangle can be a single value or a decomposition of that value, and that the area will not change. This concept models the *distributive property*.

First **display** the images on the left. **Explain** that because $3 + 1 = 4$, the image could be divided and drawn as the image on the right. **Ask** students to count unit squares to establish that the area of each image is 16 square units.

Using multiplication to determine the area of the image on the left would result in the equation $A = 4 \times 4$. Note that the result is equal to using the equation $A = (3 + 1) \times 4$.

For the image on the right, using multiplication to determine the area requires first multiplying the *base* (4 units) by three rows (resulting in 12 square units), then multiplying the *base* (4 units) by the remaining one row (resulting in 4 square units). Because both rectangles compose the original image, it is necessary to add the products, 12 square units and 4 square units, together. The sum results in the same area (16 square units) as the product the *height* and the *base*.





$A = 4 \times 4$
$A = (3 + 1) \times 4$

A = area of top rectangle + area of bottom rectanlge
$A = (3 \times 4) + (1 \times 4)$
$A = 12 + 4$
A = 16 square units

NOTE: Make explicit that the rectangle can be divided by rows or columns. Model and/or explain the same process mentioned previously, dividing the rectangle into two smaller rectangles by columns instead of by rows.

Keep in mind that when writing the equation and/or expression, the first value should *always* represent the number of rows and the second value should *always* represent the number of columns (or the number in each row). Therefore, if you model splitting by columns, the equation would be $A = 4 \times (3 + 1)$ or $A = (4 \times 3) + (4 \times 1)$.

Explain the directions for Questions 6 – 8 on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT by modeling how to complete Question 5.

Emphasize that the rows in the rectangle are already divided into two groups. **Ask** students how many rows are in each group, then write that as an addition expression (2 + 2) next to “Number of rows”. Then complete the table by **asking** students for the values. See the following completed table.

Number of rows	**2**	**2**
Number in each row	4	4
Area	**8**	**8**
Total area of rectangle	**8 + 8 = 16 square units**	I checked the area by counting **16 square units**

Write the total area next to the rectangle after, “Area”.

Model counting the unit squares then filling in the “I checked the area by counting” section on the table. **Explain** that students can check the area by tiling and counting the unit squares, or by using a ruler to draw unit squares on the rectangle.

Explain that students do not have to divide the rectangle by rows—they can choose to divide the rectangles by columns instead. When completing the table, instead of having potentially two different values in the "Number of rows" row, there could be two different values in the "Number in each row" row.

Arrange students into groups of two and distribute 20 square-inch squares to each group.

Require students to complete Questions 6 – 8 on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

Use the following guiding questions to support student understanding.

GUIDING QUESTIONS

Elicit student thinking:

- [Point to a rectangle on Questions 6 – 8.] Can you split this rectangle into two smaller rectangles? How?
- [Point to a rectangle on Questions 6 – 8.] When might it help to split a rectangle into two smaller rectangles in order to find the area?

Determine if the student can EXPLAIN AREA FORMULA FOR RECTANGLES:

- [Point to a rectangle on Questions 6 – 8.] How would you determine the area of this rectangle using the *height* and *base* (or rows and columns)?
- [Point to a rectangle on Questions 6 – 8.] Why would you multiply the *height* and the *base* to find the area?
- [Point to a rectangle on Questions 6 – 8.] How are finding the area of this rectangle and multiplying using an array the same? How are they different?

Determine if the student can CALCULATE THE AREA OF A RECTANGLE WITH THE FORMULA:

- [Point to a rectangle on Questions 6 – 8.] What is the area of this rectangle? How did you determine the area?

Determine if the student can SOLVE PROBLEMS INVOLVING AREAS MEASURED IN THE SAME UNIT:

- [Point to a rectangle on Questions 6 – 8.] When you split this rectangle into two rectangles and then add the two areas together, is the area the same or different than the original area? Explain.
- Would the area change if you split this rectangle by rows or columns? Why or why not?
- [Point to a rectangle on Questions 6 – 8.] If you determined the total area of this rectangle and then split it into two rectangles, could you determine the area of one of the smaller rectangles by subtracting? Explain.
- [Point to a split rectangle on Questions 6 – 8.] How would you determine the combined area of these two rectangles?

Determine if the student can RELATE TILING AND FORMULA AS METHODS FOR CALCULATING AREA OF A RECTANGLE:

- [Point to a rectangle on Questions 6 – 8.] Is the area you found when you multiplied the same as the area you found when you counted the number of square units? Why or why not?
- [Point to a rectangle on Questions 6 – 8.] Which strategy do you prefer when determining area, multiplying or counting unit squares? Why?
- How is counting unit squares to determine the area of a rectangle different than multiplying the *height* and the *base* to determine area of a rectangle? How are they the same?

Review the answers for Questions 6 – 8 on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT by selecting students with different strategies to share their strategy and answer.

At the end of the activity, teachers should collect the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT and review the responses for students' understanding.

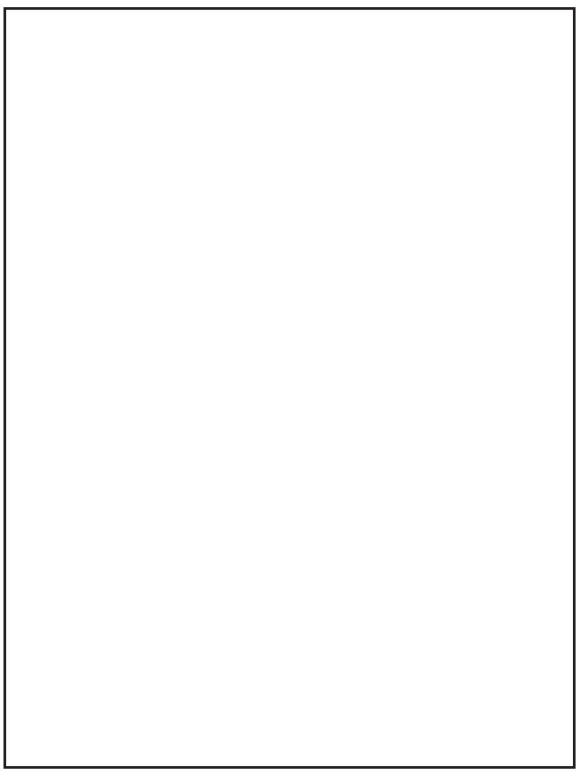
Name________________________________

Determining Area of Rectangles

Lesson 4

Use a ruler to measure the height and draw each row. Then measure and draw each unit in one of the rows. Use this information to find the area of the rectangle.

1.



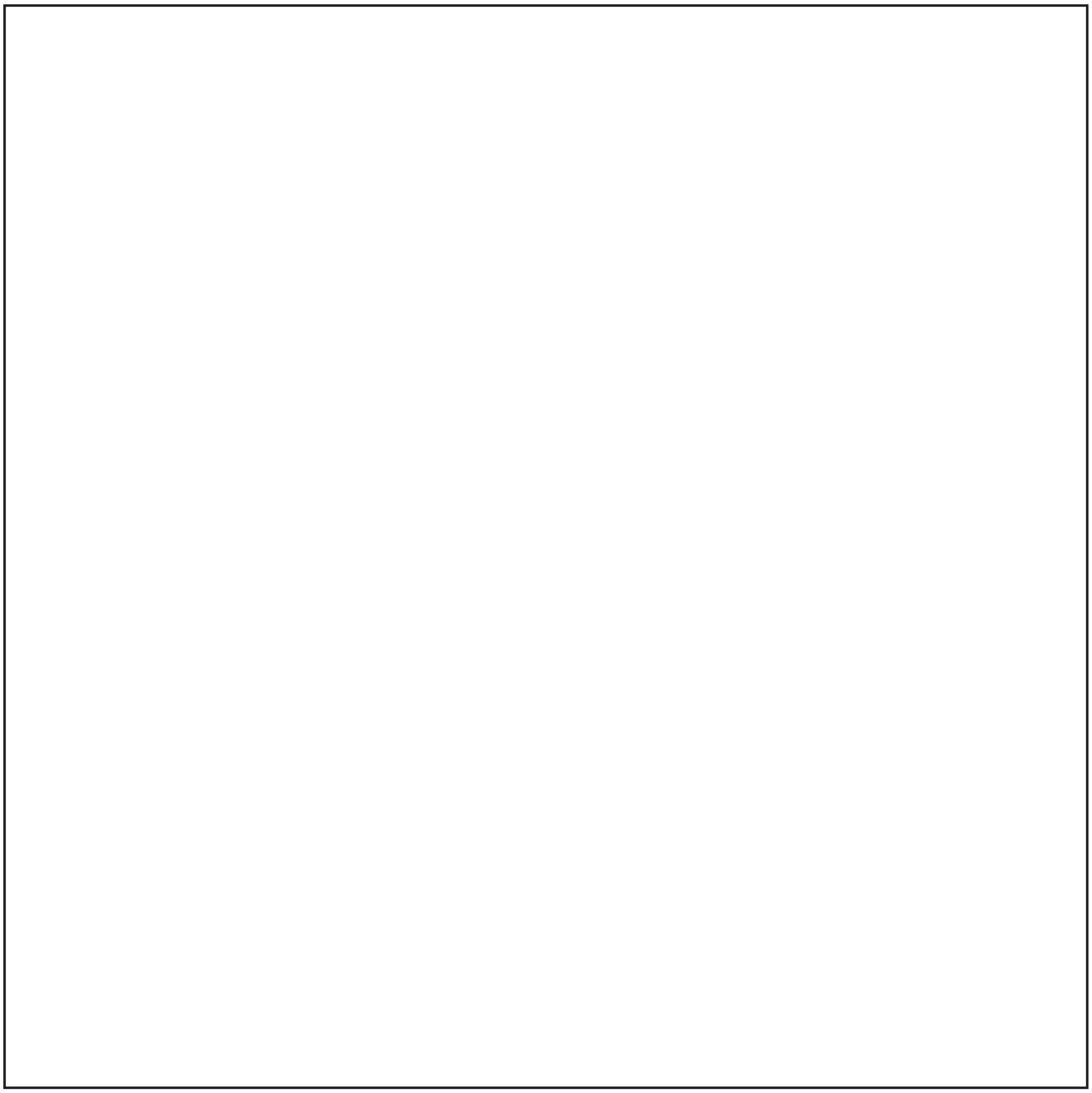
Number of rows:

Number of units in one row:

Area:

Name______________________________

2. James wants to make treats for his friends. James uses the pan below to make treats. If James cuts each piece into one-inch squares, how many treats will James have?



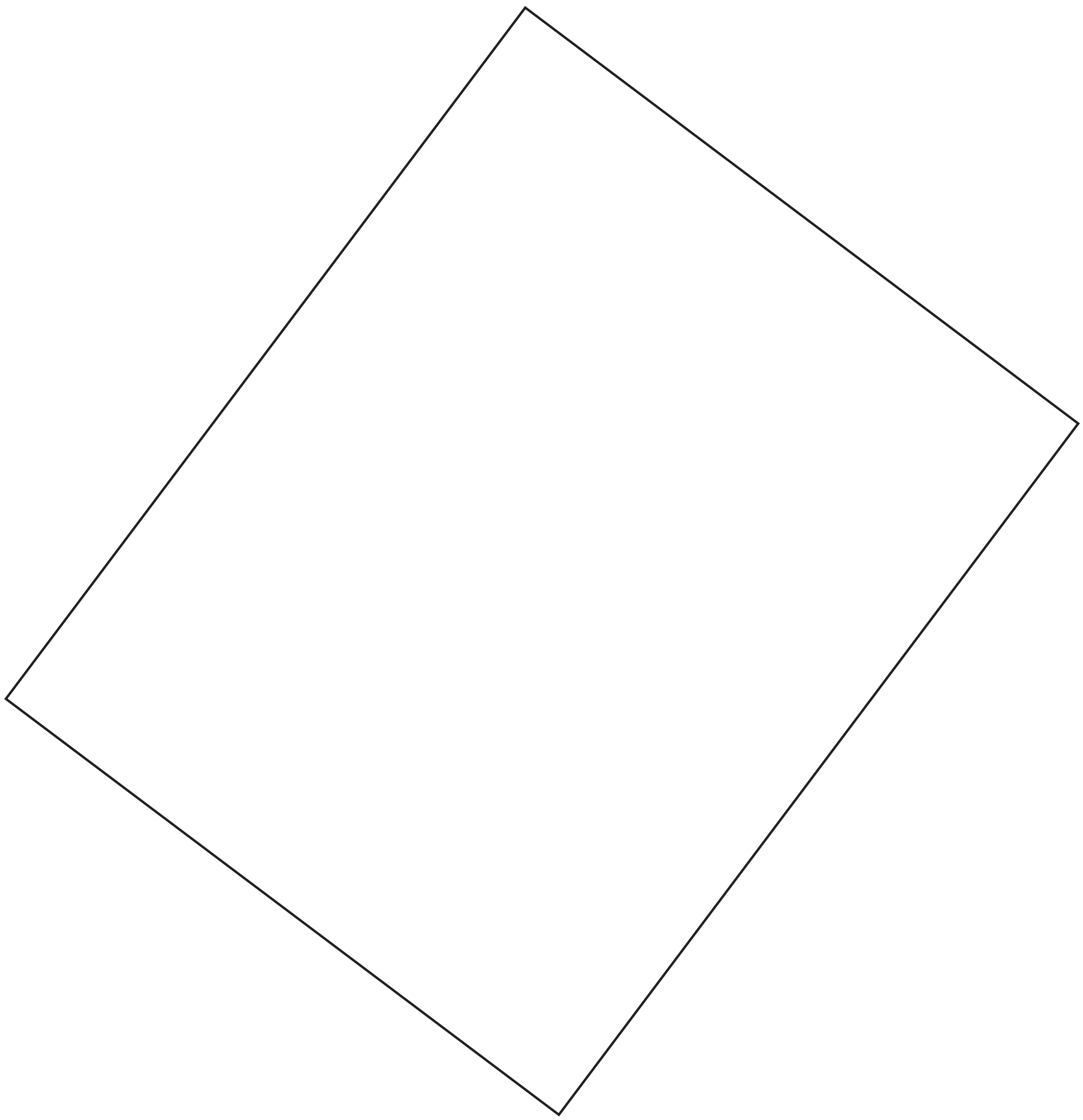
Number of rows:

Number of units in one row:

Area:

Name______________________________

3. The school is adding a sand pit to the play area. How many square feet will the sand pit cover? On the rectangle, one inch represents one foot.



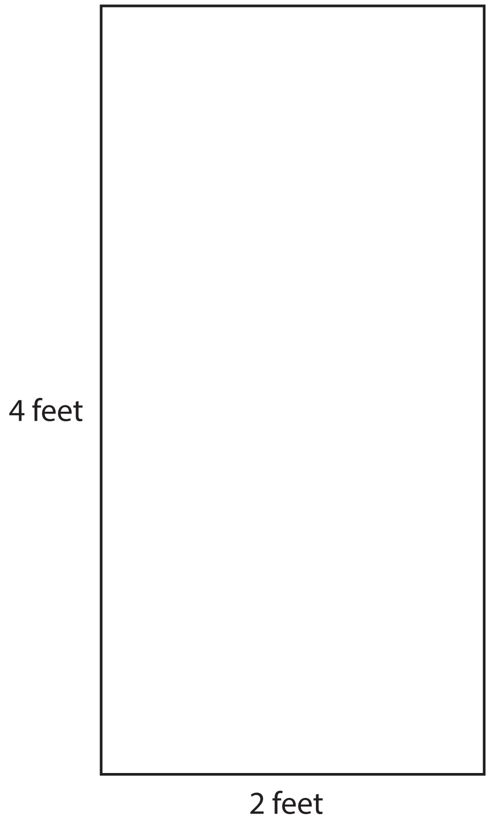
Number of rows:

Number of units in one row:

Area:

Name______________________________

4. Sally and Megan are making blankets. Each blanket will be four feet long and two feet wide. What is the area of one blanket?



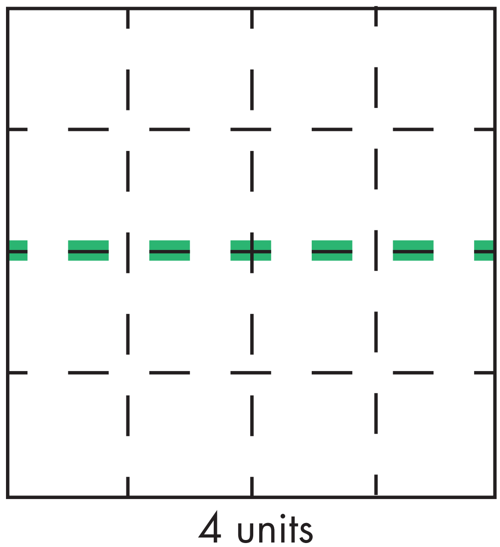
Number of rows:

Number of units in one row:

Area:

Name________________________________

5. Use the rectangles to fill in the blanks and complete the tables. Check your work using square tiles or using a ruler to draw unit squares.



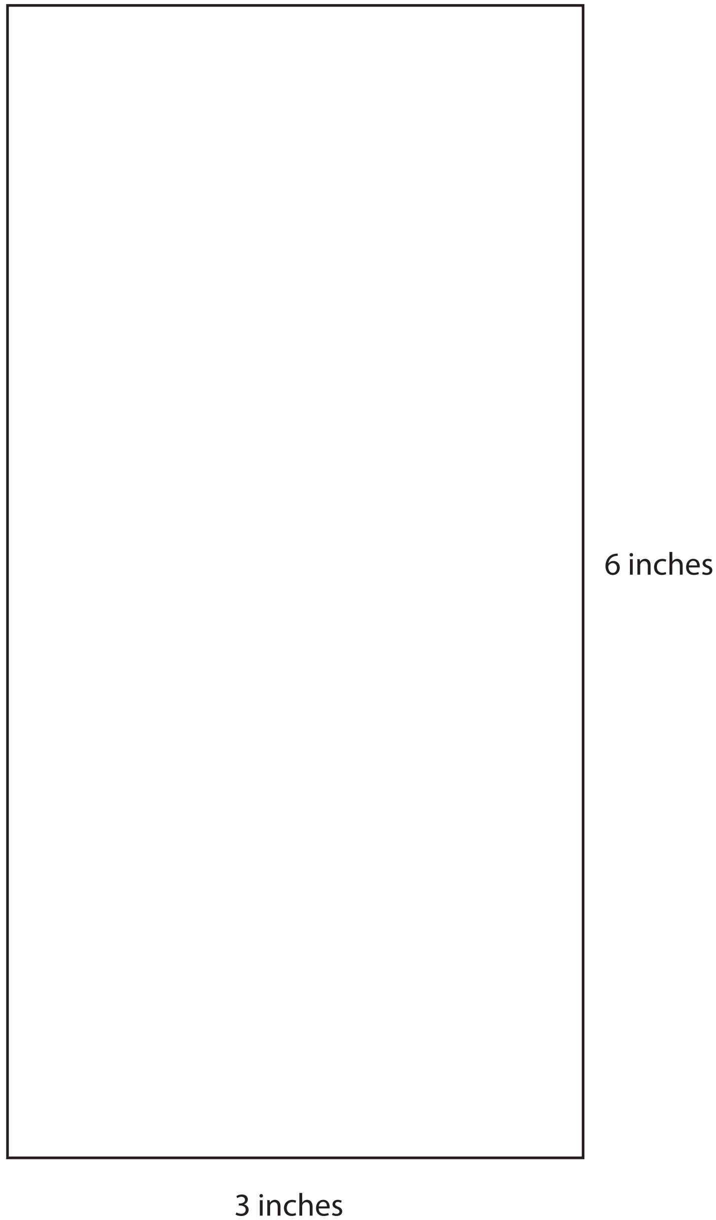
Number of rows:

Number in each row: 4 units

Area: square units

Number of rows		
Number in each row	4 units	4 units
Area		
Total area of rectangle		I checked the area by counting

6.



Number of rows:

Number in each row:

Area:

Name______________________________

Number of rows		
Number in each row		
Area		
Total area of rectangle		I checked the area by counting

Name________________________________

7.

Number of rows:

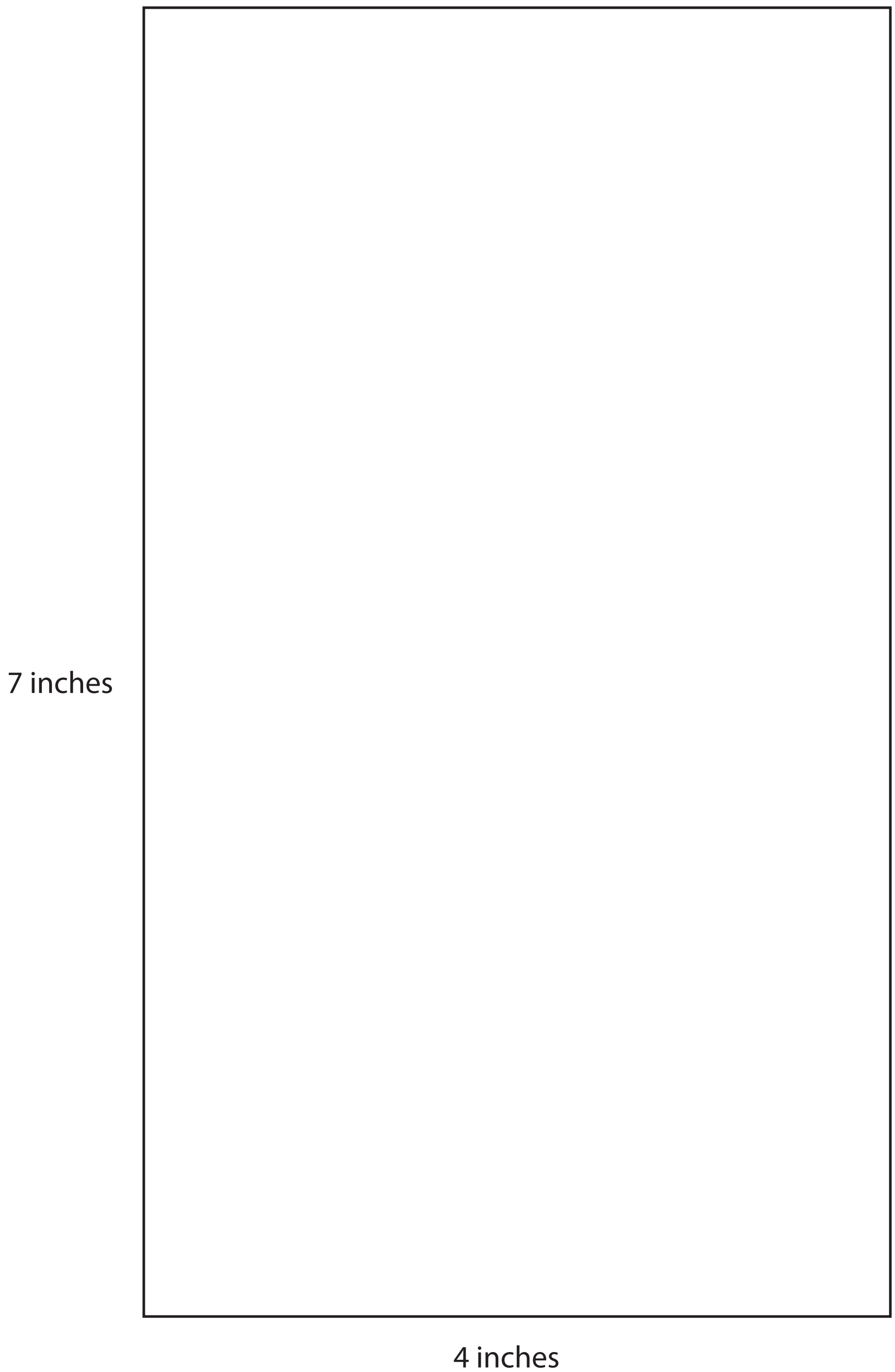
Number in each row:

Area:

Number of rows		
Number in each row		
Area		
Total area of rectangle		I checked the area by counting

Name______________________________

8.



Number of rows:

Number in each row:

Area:

Name________________________________

Number of rows		
Number in each row		
Area		
Total area of rectangle		I checked the area by counting

Determining Area of Rectangles

INSTRUCTIONAL ACTIVITY

Lesson 5

LEARNING GOAL

Students will decompose rectilinear figures into two rectangles to determine the total area of the figure.

PRIMARY ACTIVITY

Students will use scissors to cut rectilinear figures into two rectangles, determine the area of each rectangle, then rejoin the two rectangles to model adding the two areas of the individual rectangles together to determine the total area. Students will finish with two tasks, both of which require determining areas in real-world situations.

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

- Area
- Rectangle
- Base
- Height
- Square units (inches, feet, centimeters, etc.)

MATERIALS

- One-square-inch squares: tiles or paper (Recommend 20 squares for every one to two students.)
- Rulers
- Scissors
- Glue
- Paper rectangles: 17 inches by 12 inches (Recommend one rectangle for every two to three students.)
- Instructional Activity Student Handout
- Instructional Activity Supplement
    - Pages 1 – 4 (Recommend one copy for each student.)
    - Pages 5 – 12 (Recommended quantities are provided on each page.)

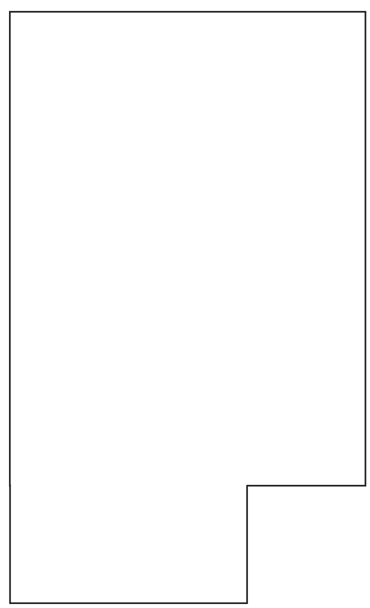
IMPLEMENTATION

Display or **draw** a rectangle.

Ask students to share with a partner if it is possible to cover the rectangle with square tiles to determine the area, and to explain why or why not. **Verify** that the students all agree that the rectangle could be covered with square tiles to determine the area.

Display or **draw** a rectilinear shape by adding a square or another rectangle onto the original rectangle. The following is an example of what the figure could look like.

NOTE: Ensure that the shape is composed of side lengths that measure a whole number of inches (i.e. no fractions of an inch) so that students are able to tile the figure precisely, with square-inch tiles.



Ask students to share with a partner their responses to the following questions.

- Is this shape a rectangle? Why or why not?
- Would it be possible to cover this shape with tiles to determine the area of this shape? Why or why not?

Select one or two students to explain or model how to cover the shape to determine the area.

Arrange students into groups of two and distribute one-square-inch square tiles (or paper squares) and/or rulers and the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

Require students to use the square tiles (or paper squares) or rulers to "tile" the rectilinear shapes on Questions 1 – 3 on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT in order to determine the area.

NOTE: For students that require extra support, use one-inch-square graph paper and draw the outline of the shape on the graph paper. Require the student to fill the shape with square tiles, then count the number of tiles.

For students who have mastered tiling, an option would be to allow those students to measure side lengths and use the formula to calculate the area.

Use the following guiding questions to support student understanding.

GUIDING QUESTIONS

Elicit student thinking:

- [Point to one of the tiles.] What shape is this? How do you know?
- [Point to one of the rectilinear shapes on Questions 1 – 3.] What can you tell me about this shape?
- Why would you need to find the area of a shape (or a space)?

Determine if the student can CALCULATE AREA OF A RECTANGLE WITH TILING:

- [Point to one of the rectilinear shapes on Questions 1 – 3.] Show me with your squares how you measured the area of this shape.
- [Point to one of the rectilinear shapes on Questions 1 – 3.] How many unit squares are in this shape? How do you know?
- [Point to one of the rectilinear shapes on Questions 1 – 3.] Can you identify each of the square units in this shape? What is the area of this shape?
- [Move some of the tiles so that there is space between them and some of the squares go outside the shape.] Would you get the same area if there were space between tiles like this? Why or why not?
- [Overlap some tiles so that you can fit more either partially or all the way onto the shape.] Would you get the same area if the tiles were overlapping like this? Why or why not?
- [Point to one of the rectilinear shapes on Questions 1 – 3.] How is using tiles to determine the area of this shape the same as using tiles to determine the area of a rectangle? How is it different?

Determine if the student can EXPLAIN *UNIT SQUARE*:

- What is a unit square?
- [Point to one of the tiles.] Is this a unit square? Why or why not?
- Do unit squares measuring one shape have to be the same size? Why or why not?

Discuss the answers to Questions 1 – 3 on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT. **Require** several students to share their solutions as well as how they determined their solution.

- 1. 11 square inches
- 2. 20 square inches
- 3. 26 square inches

Ask students if the area can be determined without tiling the shape. **Require** students to share their ideas with someone next to them.

Distribute Page 1 of the INSTRUCTIONAL ACTIVITY SUPPLEMENT along with scissors.

Require students to draw the shape in the first row of the table on Question 4 of the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT, under the column “Draw the shape.”

Require students to cut the rectilinear shape into rectangles. This should be an exploratory activity—do not provide the students with modeling or guidance with the instructions.

For students who struggle, consider guiding the student by asking questions to support their process.

Arrange students into groups of three or four. Try to arrange groups of students who decomposed their shapes differently.

Require students to share with their groups how they cut their shapes and why they cut them that way.

Monitor student discussions for understanding.

Require students to find the area of each rectangle by tiling with the square tiles or by drawing units with a ruler.

Discuss, as a class, the areas students determined.

Ask students how they would find the area of the original shape.

Model putting the two rectangles together, creating the original rectilinear shape.

Emphasize that in order to determine the area of the original shape, the two areas of the rectangles have to be added together.

Model and **require** students to complete the first row in the table on Question 4 of the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

Arrange students into groups of two and distribute Pages 2 – 4 of the INSTRUCTIONAL ACTIVITY SUPPLEMENT.

Require students to use the same process (i.e. draw, cut, determine area, combine) to complete the table on Question 4 of the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT using the rectilinear shapes from the INSTRUCTIONAL ACTIVITY SUPPLEMENT.

Use the following guiding questions to support student understanding.

GUIDING QUESTIONS

Elicit student thinking:

- [Point to a rectilinear shape.] What do you know about the area of this shape?
- [Point to a rectilinear shape.] Do you have to divide this shape to find the area? Why or why not?
- [Point to a rectilinear shape.] When you divide this shape into two shapes, what two shapes will you have? How do you know?
- [Point to a rectilinear shape.] Can you divide this shape into more than two shapes? Explain.
- [Point to a rectilinear shape.] How is this shape different from a rectangle? How is it the same?

Determine if the student can CALCULATE AREA OF A RECTANGLE WITH TILING:

- [Point to a rectangle decomposed from the rectilinear shape.] What is the area of this rectangle? How do you know?
- [Point to a rectangle decomposed from the rectilinear shape.] How did you find the area of this rectangle? What is the area of this rectangle?

Determine if the student can CALCULATE AREA OF IRREGULAR SHAPES BY PARTITIONING INTO RECTANGLES:

- [Point to a rectilinear shape.] How could you divide this shape? Is that the only way to divide the shape? Show me how you know.
- [Point to a rectangle decomposed from the rectilinear shape.] What shape is this? How do you know?
- [Point to a rectangle decomposed from the rectilinear shape.] What are you going to do with this rectangle? Why?
- [Point to a rectangle decomposed from the rectilinear shape.] How did you find the area of this rectangle? What is the area of this rectangle?
- [Point to the drawing of the rectilinear shape on Question 4.] Now that you have the areas of each of the rectangles, how do you find the area of the original shape? What is the area of the original shape?
- What do you do after you find the area of the two rectangles to find the area of the original shape? Why? Show me using your rectangles.

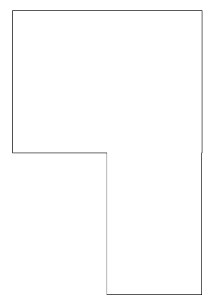
Determine if the student can SOLVE PROBLEMS INVOLVING AREAS MEASURED IN THE SAME UNIT:

- [Point to a decomposed rectilinear shape.] Once you have determined the areas of these two rectangles, how would you determine the total area of the original shape?
- [Point to a decomposed rectilinear shape.] If the area of an irregular shape like this was 36 square units, and it was divided into two rectangles and one rectangle was 12 square units, how would you determine the area of the second rectangle? What is the area of the second rectangle?
- [Point to a decomposed rectilinear shape.] If an irregular shape like this was divided into two rectangles and one rectangle was 14 square units, and the second rectangle was 12 square units, how would you determine the total area of the original shape? What is the area of original shape?

Review student answers to the table on Question 4. **Select** students to explain their answers and process for each rectilinear shape.

Ask students if they must cut the figure to determine the area of the shape. **Select** a few students to share and explain their reasoning.

Explain that the shape does not have to be cut apart to separate the shape into rectangles. **Display** a rectilinear figure, like the following shape.

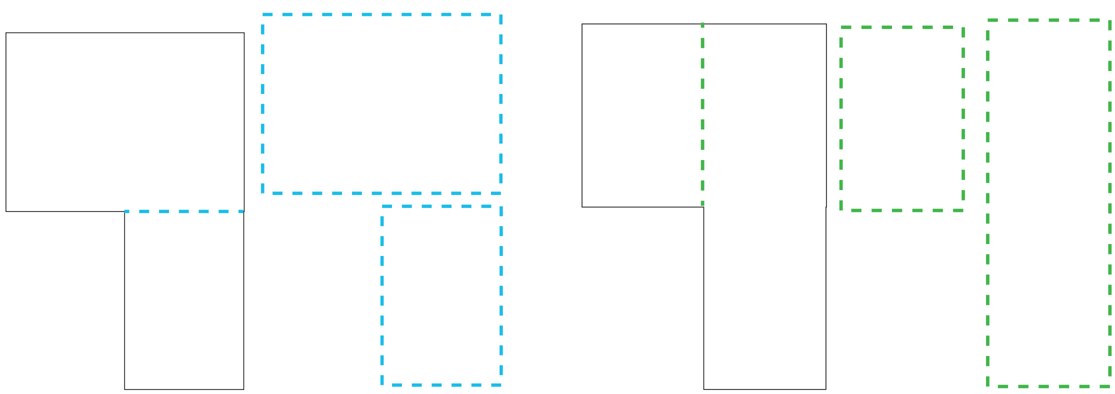


Select a student to show how to decompose the shape into two rectangles using a highlighter or marker. **Require** the student to redraw the two rectangles to show that they are two separated rectangles.

Repeat the process with a second student, a different color, and a different decomposition.

Emphasize that either decomposition is acceptable, as long as the student has separated the shape into rectangles and/or squares.

The following are two possible decompositions.



Ask students how they would determine the area of original shape from this point. Students should share their response with a partner.

Select a few students to share different strategies to determine the area of the original shape (i.e., tiling, repeatedly adding the number or units in one row, and multiplying).

Ensure that each student identifies that they must determine the area of each rectangle and then add the areas to determine the area of the original figure.

Arrange students into groups of two or three.

Read and **discuss** Question 5 on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

Maria's Café is having a large party. The private dining room is 16 feet wide and 20 feet long. There are three rectangular tables that are three feet wide and four feet long, and there are two rectangular tables that are three feet wide and 12 feet long. Draw a table arrangement using all the tables. Each table must have one side touching the side of another table (for example, the table makes an "L" shape).

Emphasize that the table arrangement must fit inside the private dining room. Students may use additional paper to draw their solution. **Provide** graph paper as an additional option.

Use the following guiding questions to support student understanding.

GUIDING QUESTIONS

Elicit student thinking:

- What is the shape of the private dining room? How do you know?
- What is the shape of the tables in Maria's Café? How do you know?
- Can you find a solution to this situation without drawing a picture? Explain.
- Do you think you are going to have to find the area of one shape or multiple shapes in this situation? Explain.

Determine if the student can SOLVE REAL-WORLD PROBLEMS INVOLVING AREA OF RECTANGLES:

- [Point to Question 5.] What is this question asking? How do you know?
- [Point to Question 5.] What is the area of the private dining room? Is that information important? Why or why not?
- [Point to Question 5.] How many tables need to be arranged in the private dining room? How do you know?
- [Point to the student's drawing for Question 5.] What is the area of this group of tables? How do you know?
- [Point to the student's drawing for Question 5.] Do the tables that are 12 feet long have to be arranged a certain way in the room? Explain.
- [Point to the length of the room that is 10 feet wide.] Can the tables that are 12 feet long go along this wall? Why or why not?
- [Point to the student's drawing for Question 5.] When you combine two tables, does it change any of the lengths of the table(s)? Why or why not?

Determine if the student can SOLVE PROBLEMS INVOLVING AREAS MEASURED IN THE SAME UNIT:

- If you combine a table that is three feet wide and eight feet long, with a table that is three feet wide and 12 feet long, what is the total area of the two tables?
- If three of the small tables have a total area of 72 square feet, what is the total area if you take away one of the tables? How do you know?

Review student solutions to Question 5. **Require** each group to share and explain their table arrangement.

Read and **discuss** Question 6 on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

> Ms. Smith is setting up her third-grade classroom. The total area of her room is 816 square feet. She will have 20 students in her class. Use the list below, the available materials, and what you know about area to design a classroom for Ms. Smith. To make sure there is enough space to move around the room, the total area of furniture cannot be more than 500 square feet.

Display and **explain** each of the available materials on the list. Materials should be made available to students but not passed out to each group. Students should be able to select different rectangles, which represent different furniture, for their classroom design.

Students will glue their rectangles on a 17-inch-by-12-inch rectangle, which will model Ms. Smith's classroom.

Students should arrange the classroom before gluing anything on the paper to ensure that they have enough furniture (i.e. desks for students)—but not so much furniture that they exceed 500 square feet.

Explain that to determine the area of furniture, the students should use the dimensions listed on Question 5. **Note** that the square and rectangles students will use to model the classroom are proportional to the dimensions provided. Some students may not follow the directions and try to calculate area using the measurements of the paper rectangles.

Distribute the 17-inch-by-12-inch piece of paper and glue.

Require students to complete the task in their arranged groups of two or three.

Use the following guiding questions to support student understanding.

GUIDING QUESTIONS

Elicit student thinking:

- [Point to the 17-inch-by-12-inch rectangle that represents the classroom.] What is the shape of the classroom? How do you know?
- What is the shape of the top of your desk? Is that shape the same or different from the shape which represents desks in Ms. Smith's classroom? Explain.
- Can you find a solution to this situation without creating a model? Explain.
- Do you think you are going to have to find the area of one shape or multiple shapes in this situation? Explain.

Determine if the student can SOLVE PROBLEMS INVOLVING AREAS MEASURED IN THE SAME UNIT:

- If the area of one student desk is four square feet, what is the total area of three student desks?
- If six student desks have a total area of 24 square feet, what is the remaining area if you remove one of the desks? How do you know?

Determine if the student can SOLVE REAL-WORLD PROBLEMS INVOLVING AREA OF RECTANGLES:

- [Point to Question 6.] What is this question asking? How do you know?
- [Point to Question 6.] What is the area of the classroom? Is that information important? Why or why not?
- [Point to Question 6.] How many student desks will need to be arranged in the classroom? How do you know? How many teacher desks? How do you know?
- [Point to the student's drawing for Question 6.] What is the area of this group of furniture? How do you know?
- [Point to the student's drawing for Question 6.] Do you have to use all the furniture on the list? Explain.
- [Point to the student's drawing for Question 6.] When you combine student desks into groups does it change any of the lengths of the desks)? Why or why not?

Students should be required to share their classroom arrangements with two other small groups.

At the end of the activity, collect all student work including the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT and review the responses for students' understanding.

Name________________________________

Determining Area of Rectangles

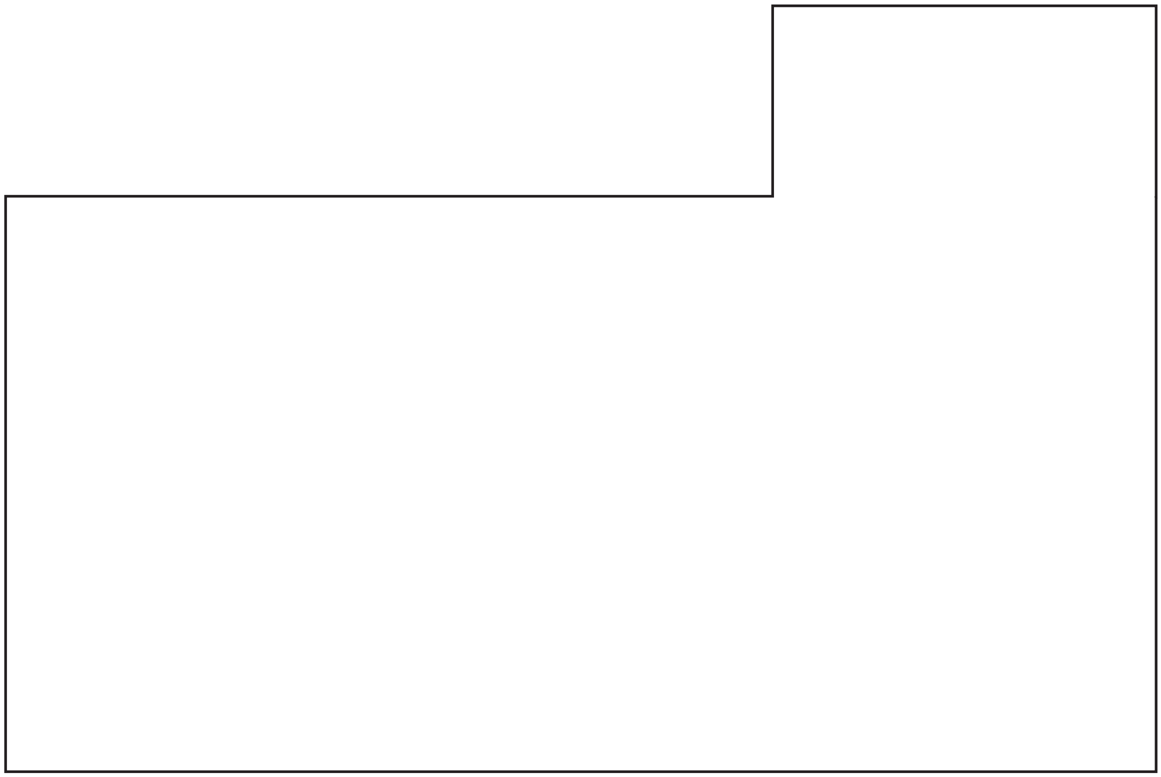
Lesson 5

Use the square tiles to find the area of each rectangle.

1. Area:

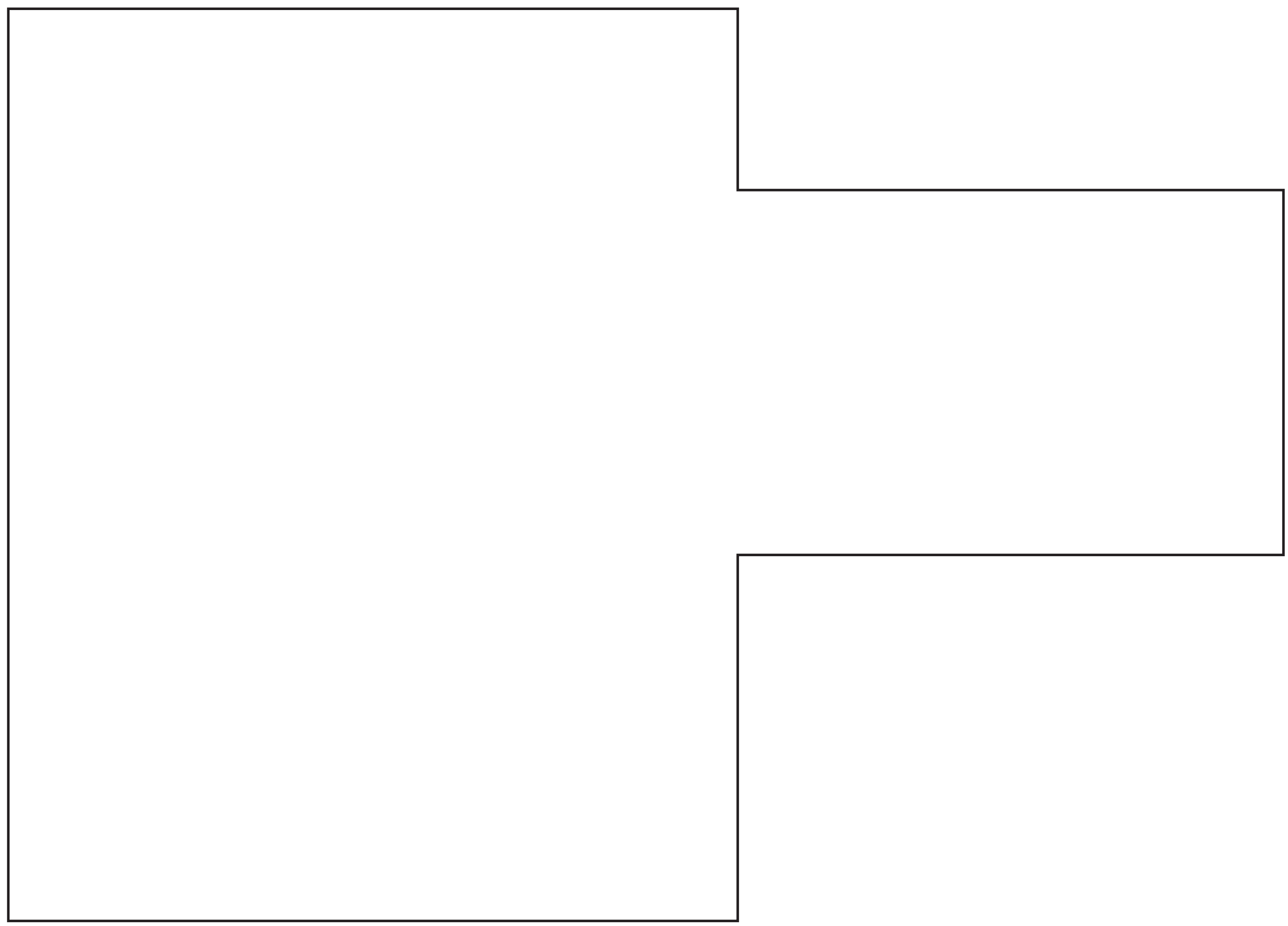
Name_______________________________

2. Area:



Name______________________________

3. Area:



Name________________________________

4. Complete the table.

DRAW THE SHAPE	AREA OF RECTANGLE 1	AREA OF RECTANGLE 2	TOTAL AREA

Name____________________________

5. Maria's Café is having a large party. The private dining room is 10 feet wide and 20 feet long. There are 3 rectangular tables that are 3 feet wide and 8 feet long, and there are 2 rectangular tables that are 3 feet wide and 12 feet long. Draw a table arrangement using all the tables. Each table must have one side touching the side of another table (for example, the tables make an "L" shape).

Name____________________________

6. Ms. Smith is setting up her third-grade classroom. The area of the classroom is 816 square feet. She will have 20 students in her class. Use the list below, the available materials, and what you know about area to design a classroom for Ms. Smith. To make sure there is enough space to move around the room, the total area of furniture cannot be more than 500 square feet.

Supplies:

Teacher desk (blue rectangle): 3 feet by 5 feet

Student desk (orange square): 2 feet by 2 feet

Large bookcase (pink rectangle): 9 feet by 1 foot

Small bookcase (green rectangle): 4 feet by 1 foot

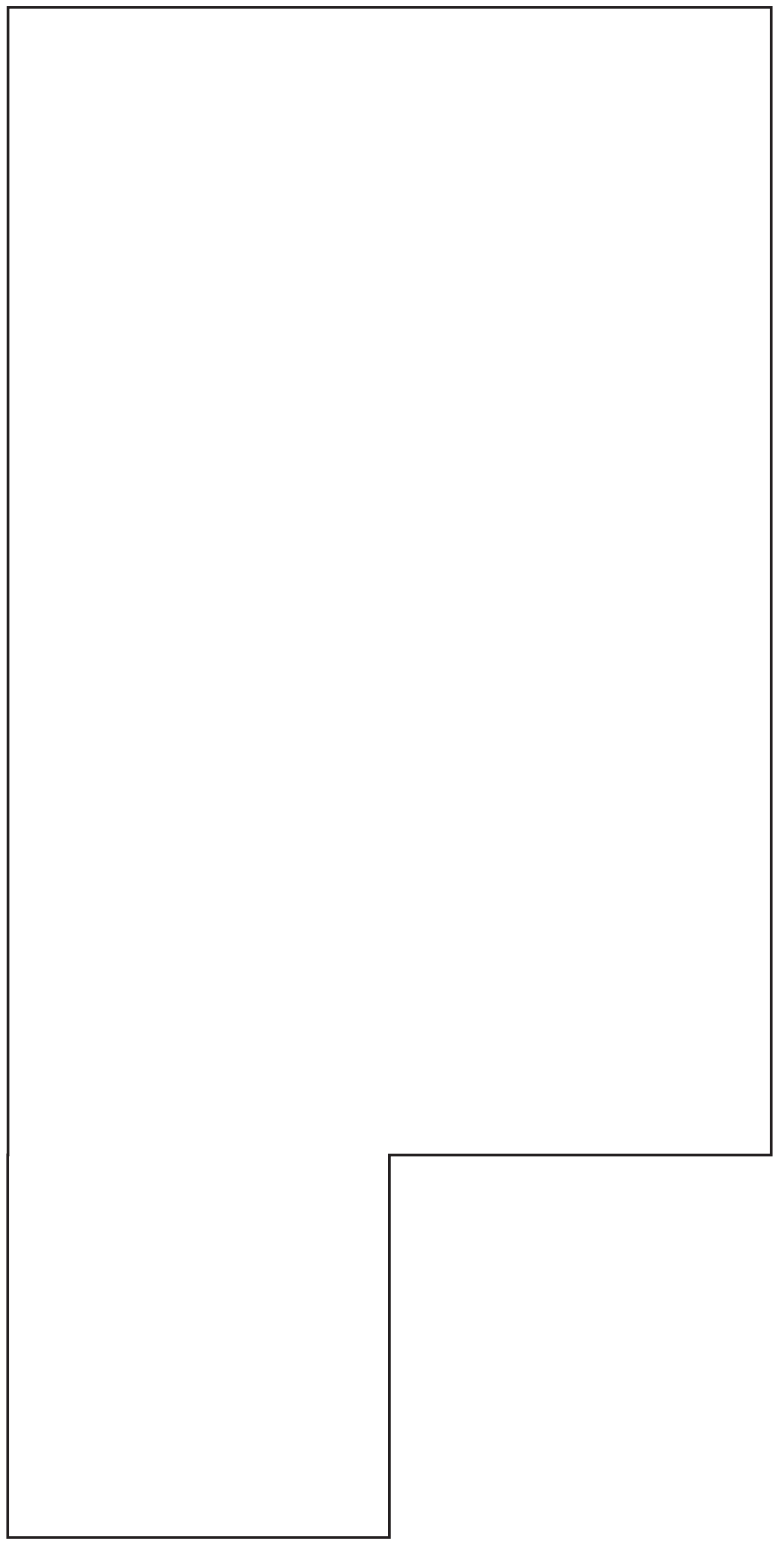
Storage closet (red rectangle): 3 feet by 2 feet

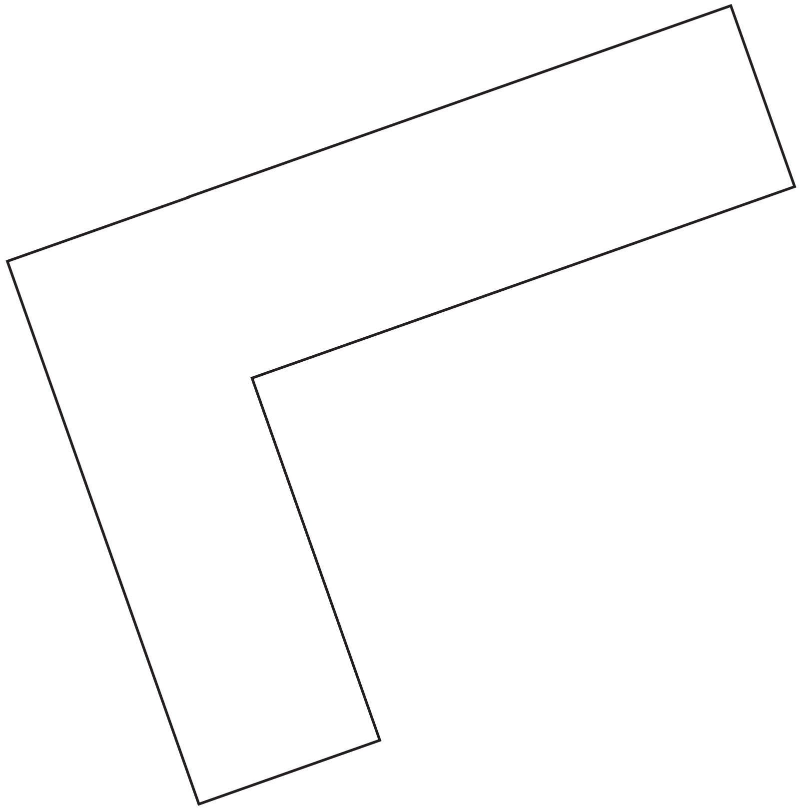
Large rug (yellow rectangle): 8 feet by 13 feet

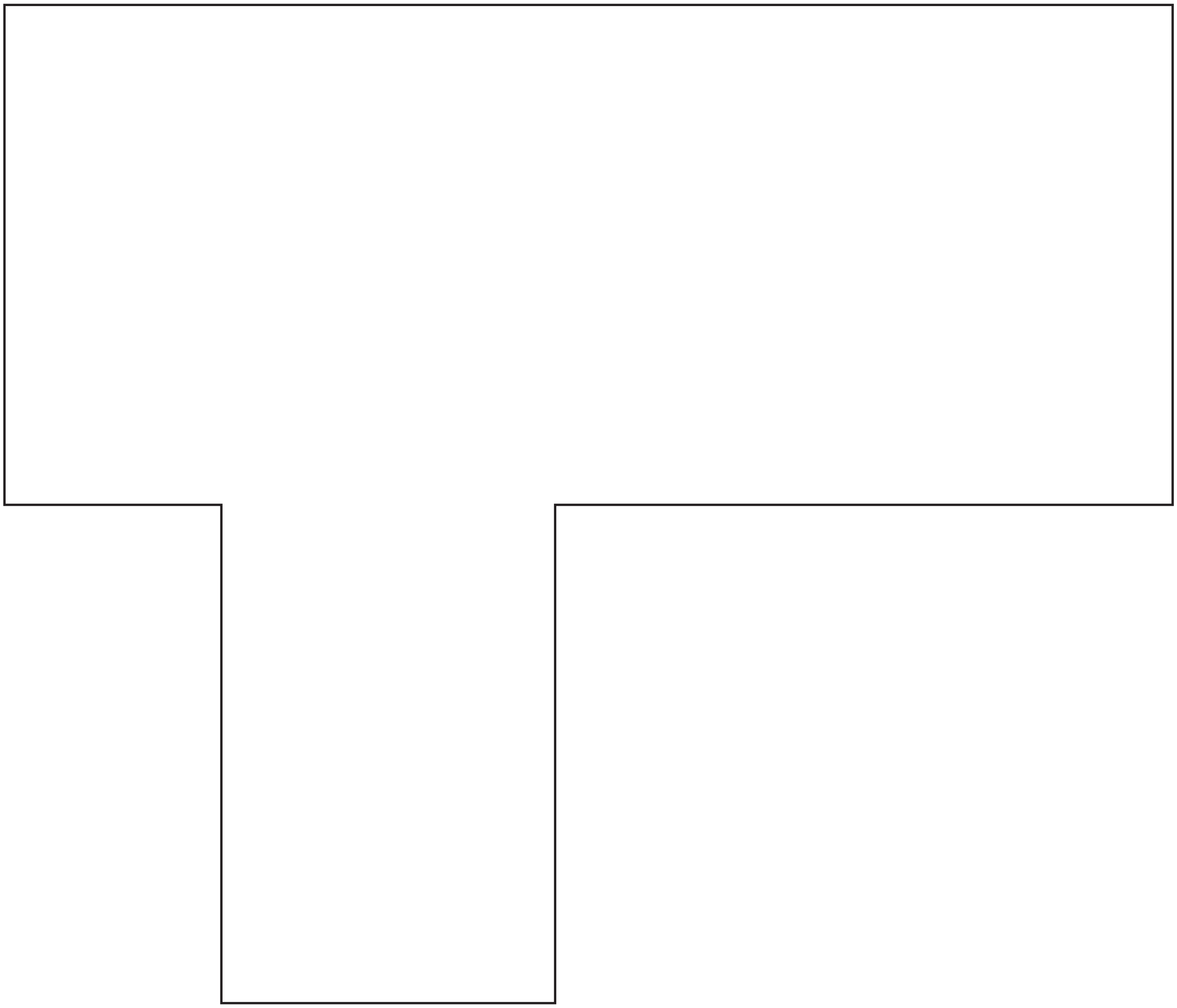
Table for small groups or computers (purple rectangle): 4 feet by 8 feet

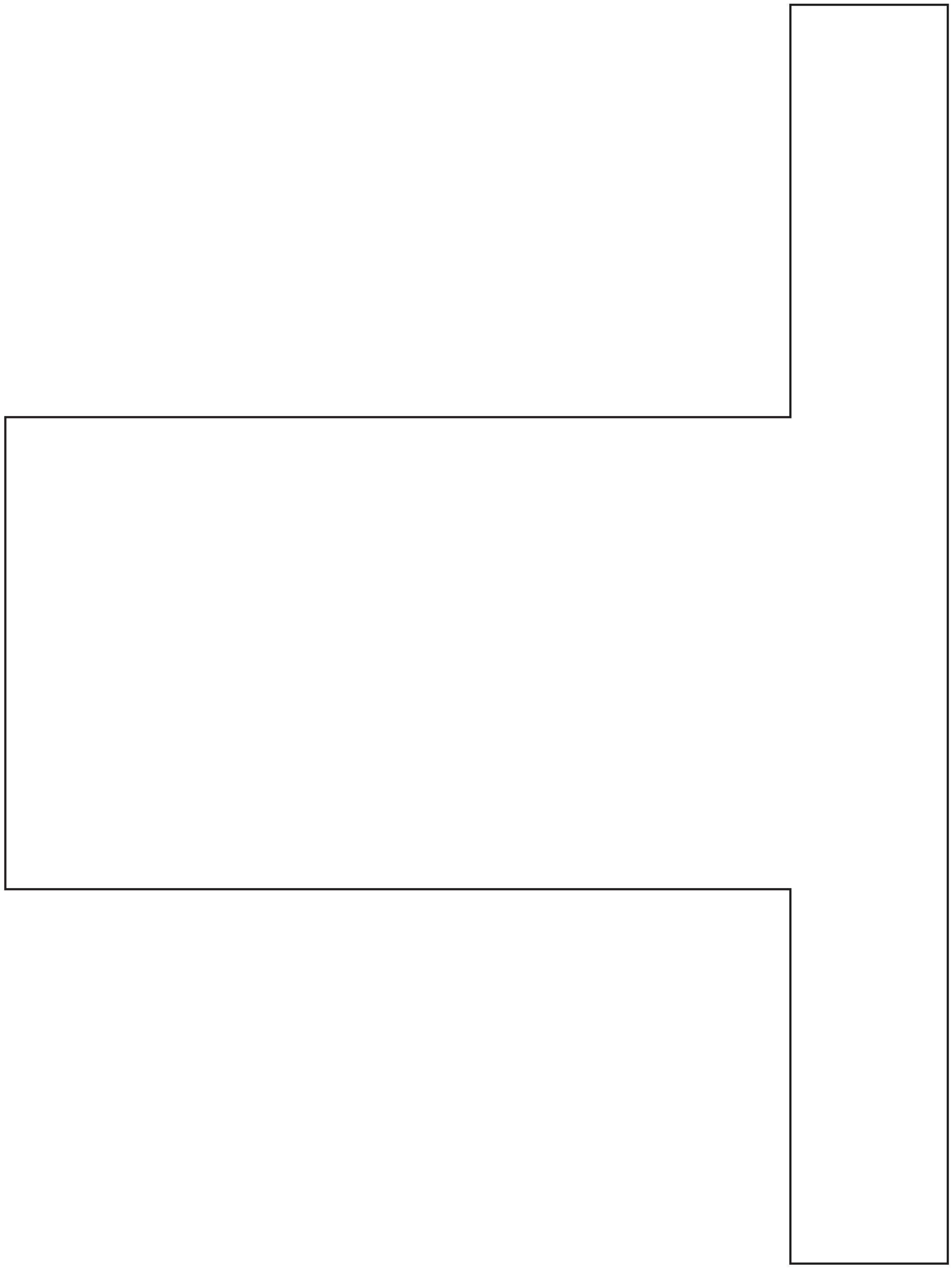
Big Chair (light green): 4 feet by 2 feet

Total area of furniture:





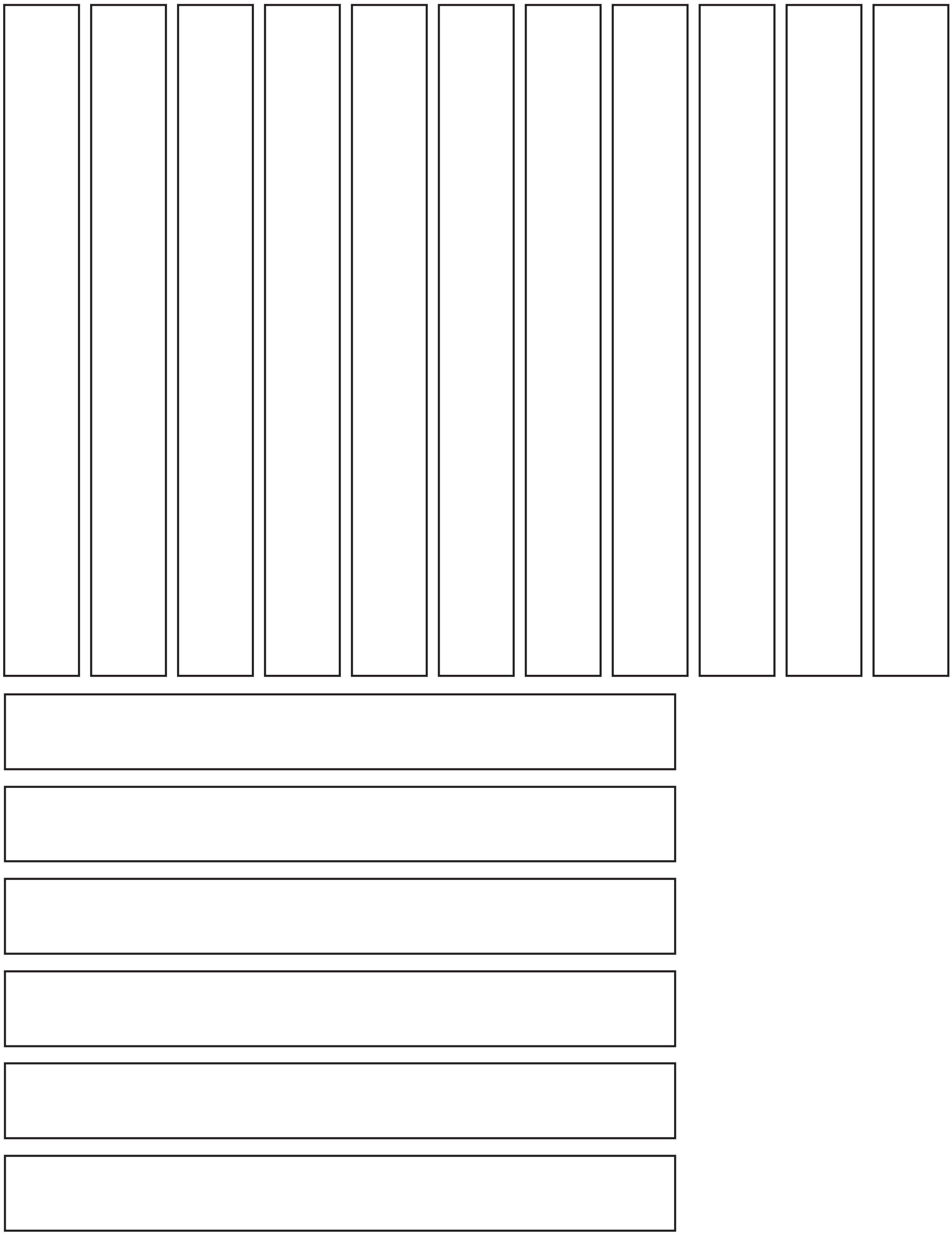




Recommend printing three to four copies on blue paper.

Recommend printing six to seven copies on orange paper.

Recommend printing three to four copies on pink paper.



Recommend printing one to two copies on green paper.

Recommend printing two to three copies on red paper.

Recommend printing six to seven copies on yellow paper.

Recommend printing six to seven copies on purple paper.

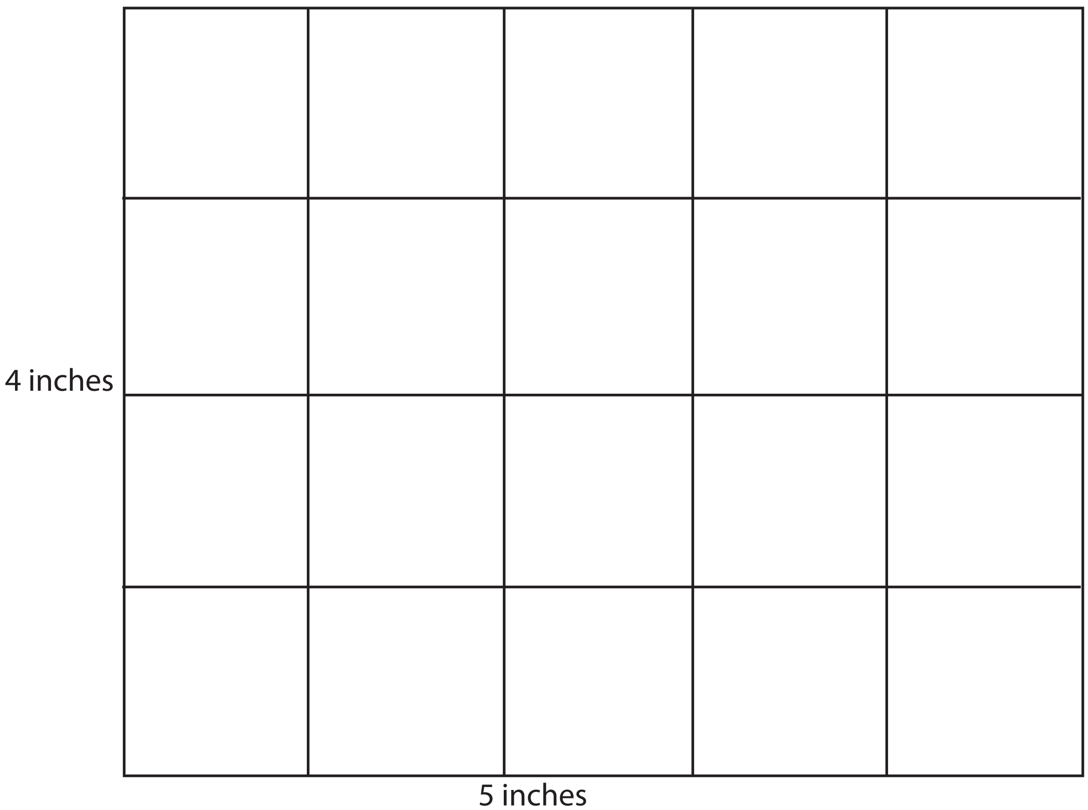
Recommend printing two to three copies on light green paper.

Name____

DETERMINING AREA OF RECTANGLES

Lessons 1 – 5

1. Use the following rectangle to answer Questions 1.a. – 1.c.



1.a. What unit square is shown to measure the area of the rectangle? How do you know?

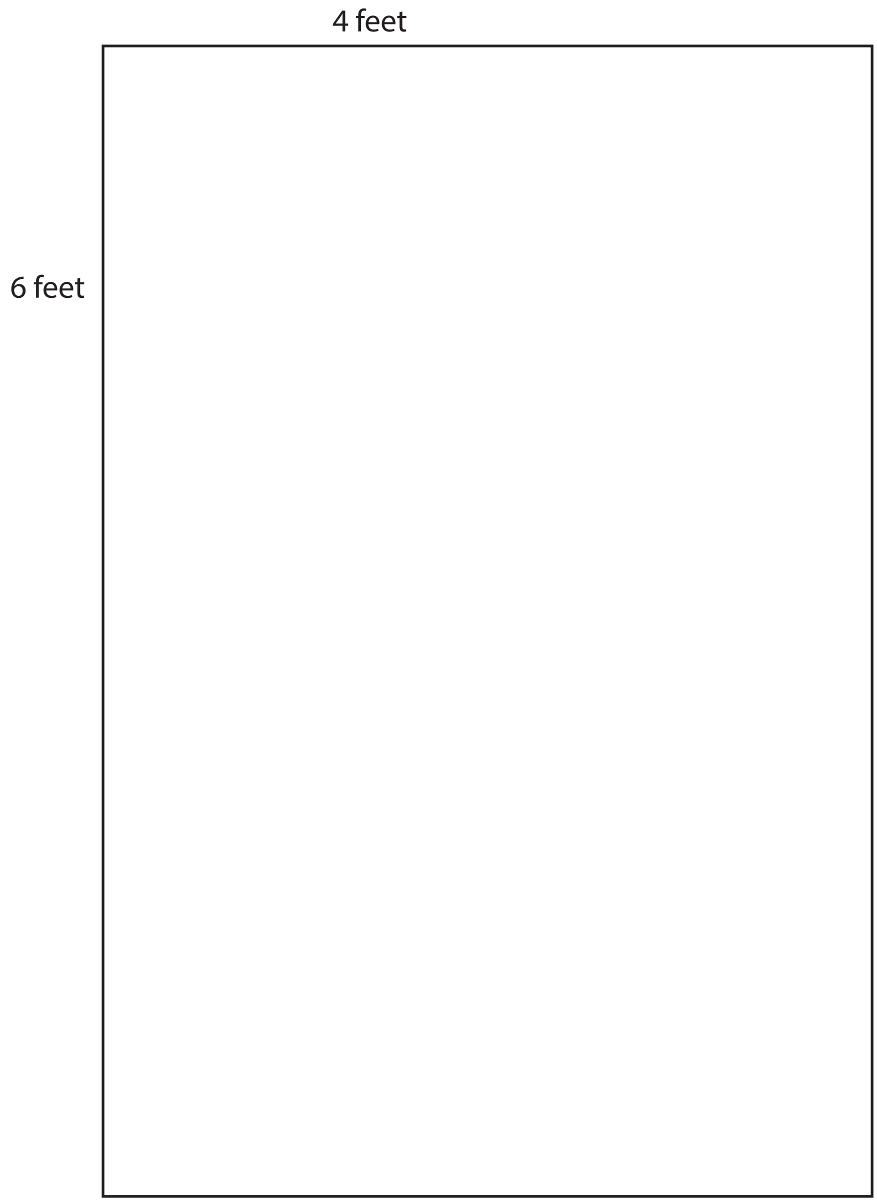
Name_______________________________________

1.b. What is the area of the rectangle?

1.c. Draw a different rectangle that has the same area.

Name____

2. Use the following rectangle to answer Questions 2.a. – 2.d.



2.a. What unit square is shown to measure the area of the rectangle? How do you know?

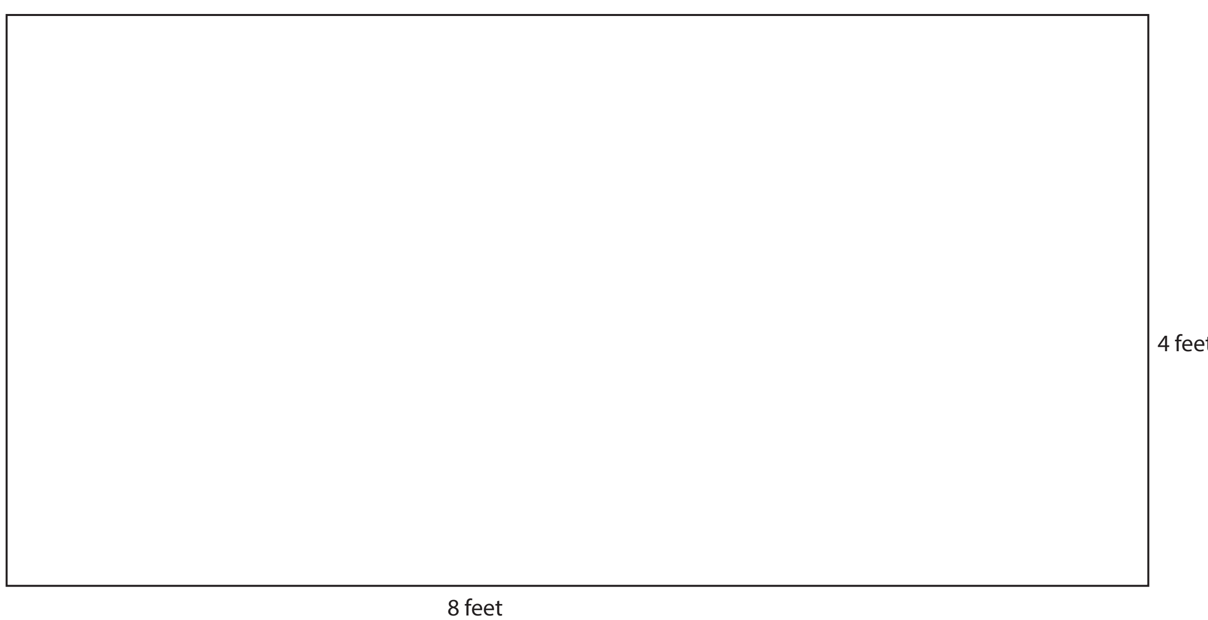
Name____

2.b. Tile the rectangle by drawing lines with a ruler to show the unit squares. One inch represents one foot.

2.c. What is the area of the rectangle in square feet?

2.d. If the height of the rectangle is four feet, what is the base of the rectangle? How do you know?

3. Use the following rectangle to answer Questions 3.a. – 3.e.



3.a. Use the color blue to draw the number of rows inside the rectangle.

3.b. Use the color green to draw the number of square units in <u>one</u> row.

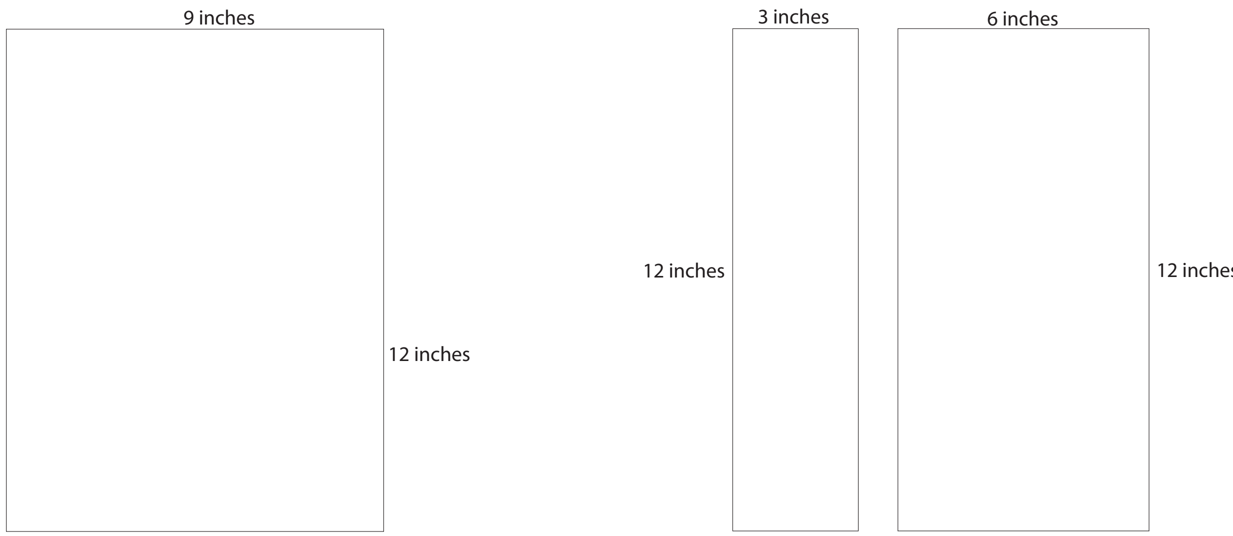
3.c. Use the number of square units in one row and the number of rows to write a repeated addition expression for the area of the rectangle.

3.d. Use the number of square units in one row and the number of rows to write a multiplication expression for the area of the rectangle.

3.e. What is the area of the rectangle in square feet?

Name_______________________________________

4. Parker cut a piece of paper into two rectangles.



4.a. What is the area of the piece of paper before Parker cut it?

4.b. What is the combined area of the two paper rectangles Parker has after cutting the piece of paper?

4.c. Use the distributive property to show that the area of Parker's two rectangles is the same as the area of the paper before it was cut.

Name____

5. Use the following rectangle to answer Questions 5.a. – 5.b.

5.a. Use the color blue to divide the shape into two rectangles.

5.b. How would you find the area of the shape?

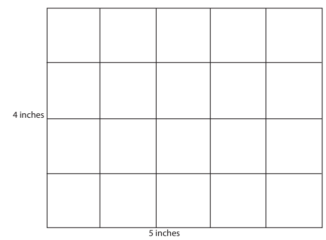
6. Josh shares a room with his twin brother James. They took their bunk beds apart and put them together in an "L" shape, so that their feet share a corner. If each of their beds is three feet by six feet, what is the area of the "L" shape in square feet?

Determining Area of Rectangles

STUDENT ACTIVITY SOLUTION GUIDE

Lessons 1 – 5

1. Use the following rectangle to answer Questions 1.a. – 1.c.



1.a. What unit square is shown to measure the area of the rectangle? How do you know?

CORRECT ANSWER

> *Check student explanation for understanding. The following is an example of a possible response:*
>
> Square inches. I know the unit squares are square inches because the side lengths are measured in inches.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student writes square feet or square units.	does not understand that because the side lengths on the rectangle are shown in inches, and the interior of the rectangle is divided accordingly, the measurement for the area will be given in square inches	EXPLAIN *UNIT SQUARE*
The student writes square inches but does not provide an explanation.	identifies the unit square but does not understand why the unit square is square inches	EXPLAIN *UNIT SQUARE*
The student does not respond.	does not understand what the question is asking or is unsure of what a square unit is	EXPLAIN *UNIT SQUARE*

1.b. What is the area of the rectangle?

CORRECT ANSWER

20 square inches

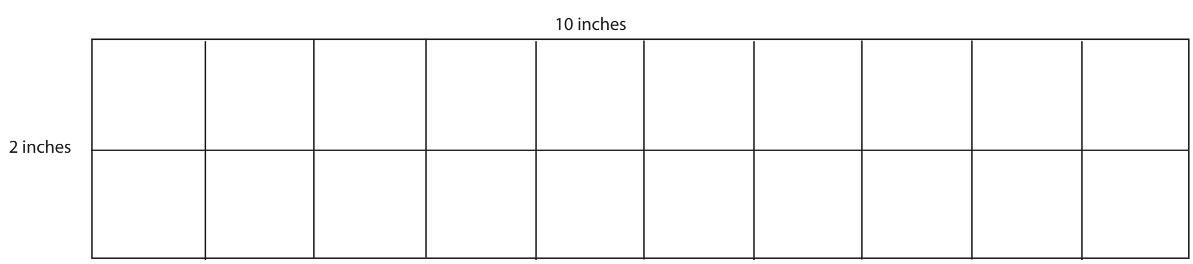
ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
9 square inches	does not understand that the square units inside the rectangle represent the area; adds the two side lengths together	CALCULATE AREA BY COUNTING UNIT SQUARES
18 square inches	misinterprets the area as perimeter and adds all the side lengths together	EXPLAIN *AREA*, CALCULATE AREA BY COUNTING UNIT SQUARES and/or EXPLAIN AREA AS A COMPOSITION OF UNIT SQUARES

1.c. Draw a different rectangle that has the same area.

CORRECT ANSWER

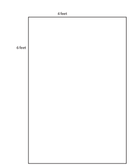
Check student work for understanding. Ensure that the student draws a rectangle whose area is 20 square inches. The following is an example of a possible solution:



ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student creates a shape other than a rectangle.	does not recognize or know how to represent a rectangle	REPRESENT *RECTANGLE* and/or RECOGNIZE *RECTANGLE*
The student creates a rectangle and does not identify the area by creating unit squares or labeling side lenghts, just draws a rectangle.	does not understand that the square units represent the area of the rectangle and are needed to show the area of the rectangle	EXPLAIN AREA AS A COMPOSITION OF UNIT SQUARES and/or CALCULATE AREA BY COUNTING UNIT SQUARES
The student creates a rectangle and does not make the unit squares uniform and/or does not have the same number of units in each column and/or row.	does not understand that unit squares must be uniform and cannot overlap or leave gaps	EXPLAIN *UNIT SQUARE* and/or EXPLAIN AREA AS A COMPOSITION OF UNIT SQUARES

2. Use the following rectangle to answer Questions 2.a. – 2.d.



2.a. What unit square is shown to measure the area of the rectangle? How do you know?

CORRECT ANSWER

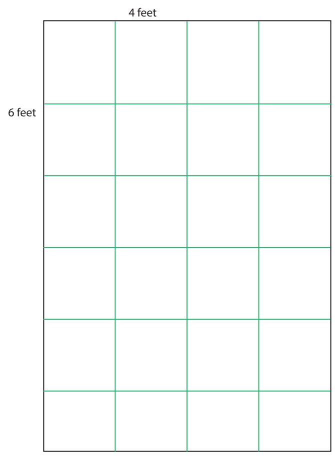
> *Check student explanation for understanding. The following is an example of a possible response:*
>
> Square feet. I know the unit squares are square feet because the side lengths are measured in feet.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student writes square inches or square units.	does not understand that because the side lengths on the rectangle are shown in feet, the measurement for the area will be given in square feet	EXPLAIN *UNIT SQUARE*
The student does not respond.	does not understand what the question is asking or is unsure of what a square unit is	EXPLAIN *UNIT SQUARE*

2.b. Tile the rectangle by drawing lines with a ruler to show the unit squares. One inch represents one foot.

CORRECT ANSWER



ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student draws six lines making seven rows and four lines, making five columns for a total of 35 squares.	does not understand that the side lengths represent the number of rows or columns, not the number of lines drawn to divide the shape	CALCULATE AREA OF A RECTANGLE WITH TILING
The student does not respond.	does not understand what the question is asking or is unsure of what to do; does not know what it means to tile the rectangle	EXPLAIN AREA AS A COMPOSITION OF UNIT SQUARES and/or CALCULATE AREA OF A RECTANGLE WITH TILING

2.c. What is the area of the rectangle in square feet?

CORRECT ANSWER

24 square feet

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
10 square inches	does not understand that the square units inside the rectangle represent the area; adds the two side lengths together	CALCULATE AREA BY COUNTING UNIT SQUARES
20 square inches	misinterprets the area as perimeter and adds all the side lengths together	EXPLAIN *AREA*, *CALCULATE* AREA BY COUNTING UNIT SQUARES and/or EXPLAIN AREA AS A COMPOSITION OF UNIT SQUARES

2.d. If the height of the rectangle is four feet, what is the base of the rectangle? How do you know?

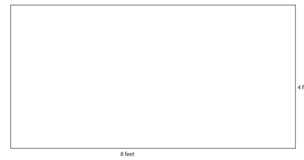
CORRECT ANSWER

> *Check student explanations for accuracy. The following is an example of a possible response:*
>
> The base is 6 feet. The height is the number of rows and the base is the number of units in a single row. Therefore, if there are four rows then the base must be six, because that is how many units would be in one row.
>
> *The student could also include something about the idea that the height and the base meet at a 90° angle.*

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
4 *or* 4 feet.	does not understand that the base and height are perpendicular sides of the rectangle	EXPLAIN AREA FORMULA FOR RECTANGLES
The student does not respond.	does not understand what the question is asking or is unsure of what to do *or* does not know the meaning of the terms *base* and *height*	RECOGNIZE MEASURABLE ATTRIBUTES and/or EXPLAIN AREA FORMULA FOR RECTANGLES
The student writes 6 feet but does not provide an explanation.	identifies the corresponding base; does not understand why that value is the base	EXPLAIN AREA FORMULA FOR RECTANGLES

3. Use the following rectangle to answer Questions 3.a. – 3.e.



3.a. Use the color blue to draw the number of rows inside the rectangle.

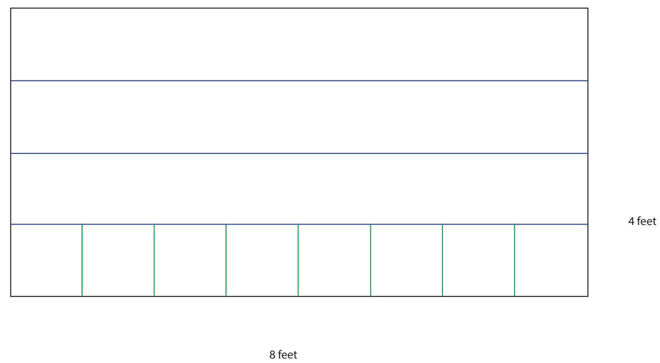
CORRECT ANSWER

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

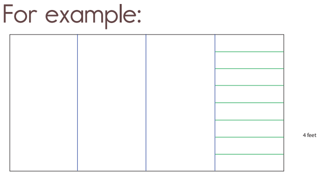
Example Error	Misconception	Missing Knowledge
The student draws three vertical lines to make four columns.	does not understand what rows are and/or does not read or does not understand the side length labels	EXPLAIN *LENGTH* and/or PARTITION A RECTANGLE INTO ROWS AND COLUMNS
The student draws four lines, either horizontally or vertically, making five rows or columns.	does not understand that the value listed as the side length represents the distance or number of rows, not the number of lines drawn	EXPLAIN LENGTH and PARTITION LENGTH
The student does not respond.	does not understand what the question is asking or is unsure of what to do	EXPLAIN *LENGTH* and/or PARTITION A RECTANGLE INTO ROWS AND COLUMNS
The student does not draw lines that are one inch apart or consistently spaced.	does not recognize that the rows should be consistently spaced in order to create square units of the same size	EXPLAIN *UNIT SQUARE*, MODEL EQUAL PART, and PARTITION LENGTH INTO EQUAL PARTS

3.b. Use the color green to draw the number of square units in <u>one</u> row.

CORRECT ANSWER



ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student draws eight lines, either vertically or horizontally, to create nine rows or columns.	does not understand that the value listed as the side length represents the distance or number of rows, not the number of lines drawn	EXPLAIN LENGTH and PARTITION LENGTH and PARTITION A RECTANGLE INTO ROWS AND COLUMNS
The student draws seven horizontal lines, continuing from an error on 3.a. For example: 	does not understand what a unit square is and/or does not read or does not understand the side length labels	EXPLAIN *UNIT SQUARE* and/or EXPLAIN *LENGTH* and/or PARTITION LENGTH
The student draws seven horizontal lines. For example: 	does not understand that, to create square units in one row, vertical lines must be drawn	EXPLAIN *UNIT SQUARE* and/or PARTITION A RECTANGLE INTO ROWS AND COLUMNS
The student does not respond.	does not understand what the question is asking or is unsure of what to do	EXPLAIN *LENGTH* and/or EXPLAIN *UNIT* and/or PARTITION A RECTANGLE INTO ROWS AND COLUMNS
The student does not draw lines that are one inch apart or consistently spaced.	does not recognize that the columns should be consistently spaced in order to create square units of the same size	EXPLAIN *UNIT SQUARE*, PARTITION LENGTH INTO EQUAL PARTS, and MODEL EQUAL PARTS

3.c. Use the number of square units in one row and the number of rows to write a repeated addition expression for the area of the rectangle.

CORRECT ANSWER

8 + 8 + 8 + 8

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
4 + 4 + 4 + 4 + 4 + 4 + 4 + 4	does not understand the difference between rows and columns	PARTITION A RECTANGLE INTO ROWS AND COLUMNS and CALCULATE AREA OF A RECTANGLE WITH TILING
8 + 8	does not understand the concept of repeated addition	CALCULATE AREA OF A RECTANGLE WITH TILING, EXPLAIN REPEATED ADDITION and REPRESENT REPEATED ADDITION WITH A MODEL
4 + 4	does not understand the different between rows and columns and does not understand the concept of repeated addition	CALCULATE AREA OF A RECTANGLE WITH TILING, PARTITION A RECTANGLE INTO ROWS AND COLUMNS, EXPLAIN REPEATED ADDITION, and REPRESENT REPEATED ADDITION WITH A MODEL
8 + 8 + 8	does not account for the one row that has unit squares draw and only adds the three rows that do not have unit squares drawn	CALCULATE AREA OF A RECTANGLE WITH TILING and REPRESENT REPEATED ADDITION WITH A MODEL
8 + 4	does not understand the concept of repeated addition; confuses the area formula for rectangles by adding the two side lengths	CALCULATE AREA OF A RECTANGLE WITH TILING, EXPLAIN REPEATED ADDITION and REPRESENT REPEATED ADDITION WITH A MODEL and/or EXPLAIN AREA FORMULA FOR RECTANGLES
8 + 4 + 8 + 4	adds all the side lengths, confusing the concept of area with perimeter	EXPLAIN *AREA* and CALCULATE AREA OF A RECTANGLE WITH TILING
The student does not respond.	does not understand what the question is asking or is unsure of what to do	CALCULATE THE AREA OF A RECTANGLE WITH TILING, EXPLAIN REPEATED ADDITION and REPRESENT REPEATED ADDITION WITH A MODEL

3.d. Use the number of square units in one row and the number of rows to write a multiplication expression for the area of the rectangle.

CORRECT ANSWER

4 × 8

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
8 × 4	does not understand that with an array or area model that the first factor represents the number of rows and the second factor represents the number in each row or the number of columns	SOLVE ARRAY PROBLEMS and/or EXPLAIN MULTIPLICATION PROBLEMS
8 + 4	does not understand the question is asking to write a multiplication expression; confuses the area formula for rectangles by adding the two side lengths	EXPLAIN MULTIPLICATION PROBLEMS and/or EXPLAIN AREA FORMULA FOR RECTANGLES
8 × 8 × 4 × 4	confuses the formula for area of a rectangle with the repeated addition of perimeter by multiplying all of the side lengths	EXPLAIN AREA FORMULA FOR RECTANGLES and/or EXPLAIN *AREA*
8 × 8 × 8 × 8	does not understand the difference between repeated addition and repeated multiplication	EXPLAIN MULTIPLICATION PROBLEMS and/or EXPLAIN AREA FORMULA FOR RECTANGLES and/or EXPLAIN REPEATED ADDITION
The student does not respond.	does not understand what the question is asking or is unsure of what to do	EXPLAIN MULTIPLICATION PROBLEMS

3.e. What is the area of the rectangle in square feet?

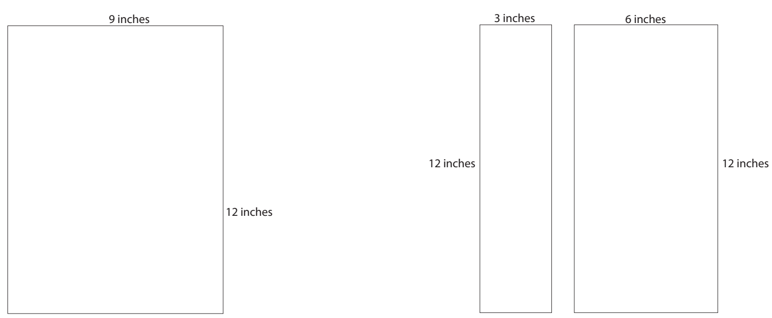
CORRECT ANSWER

32 square feet

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
12 square feet	adds the two side lengths instead of multiplying the two side lengths	CALCULATE THE AREA OF A RECTANGLE WITH THE FORMULA
24 square feet	calculates the perimeter of the rectangle by adding all the side lengths instead of multiplying the base and the height	CALCULATE THE AREA OF A RECTANGLE WITH THE FORMULA and/or EXPLAIN *AREA*
The student does not respond.	does not understand what the question is asking or is unsure of what to do	CALCULATE THE AREA OF A RECTANGLE WITH THE FORMULA and/or EXPLAIN *AREA*

4. Parker cut a piece of paper into two rectangles.



4.a. What is the area of the piece of paper before Parker cut it?

CORRECT ANSWER

108 square inches

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
21 square inches	adds the two given side lengths instead of multiplying the base and height	CALCULATE THE AREA OF A RECTANGLE WITH THE FORMULA
42 square inches	calculates the perimeter of the rectangle by adding all the side lengths instead of multiplying the base and height	CALCULATE THE AREA OF A RECTANGLE WITH THE FORMULA and/or EXPLAIN *AREA*
36 square inches *or* 72 square inches	does not understand what the question is asking and determines the area of one of the smaller rectangles	SOLVE REAL-WORLD PROBLEMS INVOLVING AREA OF RECTANGLES
216 square inches	does not understand what the question is asking and determines the area of all of the rectangles shown	SOLVE REAL-WORLD PROBLEMS INVOLVING AREA OF RECTANGLES
The student does not respond.	does not understand what the question is asking	SOLVE REAL-WORLD PROBLEMS INVOLVING AREA OF RECTANGLES

4.b. What is the combined area of the two paper rectangles Parker has after cutting the piece of paper?

CORRECT ANSWER

36 square inches + 72 square inches = 108 square inches

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
33 square inches	adds the two given side lengths from each rectangle, then adds those sums together instead of multiplying the base and height for each rectangle and adding those products	CALCULATE THE AREA OF A RECTANGLE WITH THE FORMULA
66 square inches	calculates the perimeter of each rectangle, then adds the perimeters instead of multiplying the base and height for each rectangle and adding the products	CALCULATE THE AREA OF A RECTANGLE WITH THE FORMULA and/or EXPLAIN *AREA*
36 square inches *or* 72 square inches	does not understand what the question is asking and determines the area of one of the smaller rectangles	SOLVE REAL-WORLD PROBLEMS INVOLVING AREA OF RECTANGLES
216 square inches	does not understand what the question is asking and determines the area of all of the rectangles shown	SOLVE REAL-WORLD PROBLEMS INVOLVING AREA OF RECTANGLES
144 square inches *or* 180 square inches	does not understand what the question is asking and adds the area of the largest (original) rectangle and the area of one of the smaller rectangles	SOLVE REAL-WORLD PROBLEMS INVOLVING AREA OF RECTANGLES
The student does not respond.	does not understand what the question is asking	SOLVE REAL-WORLD PROBLEMS INVOLVING AREA OF RECTANGLES

4.c. Use the distributive property to show that the area of Parker's two rectangles is the same as the area of the paper before it was cut.

CORRECT ANSWER

Check student response for accuracy. The following is an example of a possible response:

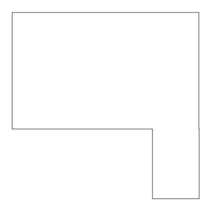
$12 \times 9 = 12 \times (3 + 6)$
$12 \times 9 = (12 \times 3) + (12 \times 6)$
$108 = 36 + 72$
$108 = 108$

The area of the original paper and the area of the combined rectangles after Parker cut the paper are both 108 square inches.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student only writes these equations: $12 \times 9 = 108$ $12 \times 3 = 36$ $12 \times 6 = 72$ $36 + 72 = 108$	does not show an understanding of the distributive property, only that the area of the largest rectangle is the same as the area of the smaller two rectangles combined	SOLVE PROBLEMS INVOLVING AREA MEASURED IN THE SAME UNIT and/or APPLY THE DISTRIBUTIVE PROPERTY and/or EXPLAIN THE DISTRIBUTIVE PROPERTY
$12 + 9 = (12 + 3) + (12 + 6)$	adds the side lengths instead of multiplying them to represent the area and does not recognize that the two values on either side of the equal sign are not equivalent	CALCULATE THE AREA OF A RECTANGLE WITH THE FORMULA and/or EXPLAIN *AREA* and EXPLAIN THE FUNCTION OF THE EQUAL SIGN
The student does not respond.	does not understand what the question is asking	APPLY THE DISTRIBUTIVE PROPERTY and/or EXPLAIN THE DISTRIBUTIVE PROPERTY

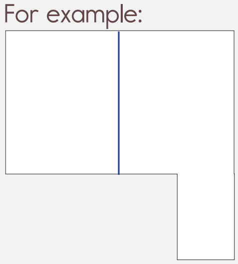
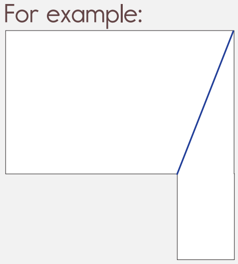
5. Use the following rectangle to answer Questions 5.a – 5.b.



5.a. Use the color blue to divide the shape into two rectangles.

CORRECT ANSWER

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student draws a line that does not create two rectangles but a rectangle and a rectiliniear shape. For example:  *or* The student draws a line that creates two irregular shapes. For example: 	does not understand what the question is asking or does not know how to divide the shape to make two rectangles	CALCULATE AREA OF IRREGULAR SHAPES BY PARTITIONING INTO RECTANGLES, REPRESENT *RECTANGLE*, and/or RECOGNIZE *RECTANGLE*
The student does not respond.	does not understand what the question is asking	CALCULATE AREA OF IRREGULAR SHAPES BY PARTITIONING INTO RECTANGLES, REPRESENT *RECTANGLE*, and/or RECOGNIZE *RECTANGLE*

5.b. How would you find the area of the shape?

CORRECT ANSWER

Check student response for accuracy. The following is an example of a possible response:

To find the area of the total shape, you would first need to find the area of each of the rectangles that you divided the shape into. You have to multiply the base and the height to find the area of each rectangle. After you find the area of the two rectangles, you would then add those two areas together for a total area of the shape.

> *NOTE: The student could write about tiling or covering the shape with unit squares. Understand that this explanation, while correct, does not show an understanding of decomposing the shape into rectangles, using the formula to determine the area of each rectangle, and then combining the two areas for a total area of the original shape.*

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student explains how to determine the area of each rectangle but does not explain that the two areas need to be combined to determine the total area.	does not understand the idea of recomposing the two rectangles into the original shape by adding the two areas together	CALCULATE AREA OF IRREGULAR SHAPES BY PARTITIONING INTO RECTANGLES and/or SOLVE PROBLEMS INVOLVING AREA MEASURED IN THE SAME UNIT
The student does not respond.	does not understand what the question is asking	SOLVE PROBLEMS INVOLVING AREA MEASURED IN THE SAME UNIT and/or CALCULATE AREA OF IRREGULAR SHAPES BY PARTITIONING INTO RECTANGLES

6. Josh shares a room with his twin brother James. They took their bunk beds apart and put them together in an "L" shape, so that their feet share a corner. If each of their beds is three feet by six feet, what is the area of the "L" shape in square feet?

CORRECT ANSWER

> One bed: $3 \times 6 = 18$
>
> Two beds: $18 + 18 = 36$ *or* $18 \times 2 = 36$
>
> The area of the "L" shape is 36 square feet.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
18 square feet	does not understand that the areas of the two beds should be added together, only determines the area of one bed	SOLVE PROBLEMS INVOLVING AREA MEASURED IN THE SAME UNIT and/or SOLVE REAL-WORLD PROBLEMS INVOLVING AREA OF RECTANGLES
9 square feet. (one bed) *or* 18 square feet. (two beds)	does not understand the area formula, adds the two side lengths together instead of multiplying	CALCULATE THE AREA OF A RECTANGLE WITH THE FORMULA and/or SOLVE PROBLEMS INVOLVING AREA MEASURED IN THE SAME UNIT and/or SOLVE REAL-WORLD PROBLEMS INVOLVING AREA OF RECTANGLES
18 square feet (one bed) *or* 36 square feet (two beds)	calculates the perimeter of one bed or calculates the perimeter of each bed and determines the sum instead of determining the area of both beds	EXPLAIN *AREA* and CALCULATE THE AREA OF A RECTANGLE WITH THE FORMULA and/or SOLVE PROBLEMS INVOLVING AREA MEASURED IN THE SAME UNIT and/or SOLVE REAL-WORLD PROBLEMS INVOLVING AREA OF RECTANGLES
The student does not respond.	does not understand what the question is asking	SOLVE PROBLEMS INVOLVING AREA MEASURED IN THE SAME UNIT and/or SOLVE REAL-WORLD PROBLEMS INVOLVING AREA OF RECTANGLES