COMPARING FRACTIONS

4.NF.2

CONTENTS

The types of documents contained in the unit are listed below. Throughout the unit, the documents are arranged by lesson.

LEARNING MAP INFORMATION	An overview of the standards, the learning map section, and the nodes addressed in this unit
TEACHER NOTES	A brief discussion describing the progression depicted in the learning map section with research-based recommendations for focusing instruction to foster student learning and an introduction to the unit's lessons
OVERVIEW OF INSTRUCTIONAL ACTIVITIES	A table highlighting the lesson goals and nodes addressed in each lesson of this unit
INSTRUCTIONAL ACTIVITY	A detailed walkthrough of the unit
INSTRUCTIONAL ACTIVITY STUDENT HANDOUT	A handout for the guided activity, intended to be paired with the Instructional Activity
INSTRUCTIONAL ACTIVITY SUPPLEMENT	A collection of materials or activities related to the Instructional Activity
STUDENT ACTIVITY	A work-alone activity for students
STUDENT ACTIVITY SOLUTION GUIDE	A solution guide for the work-alone activity with example errors, misconceptions, and links to the learning map section

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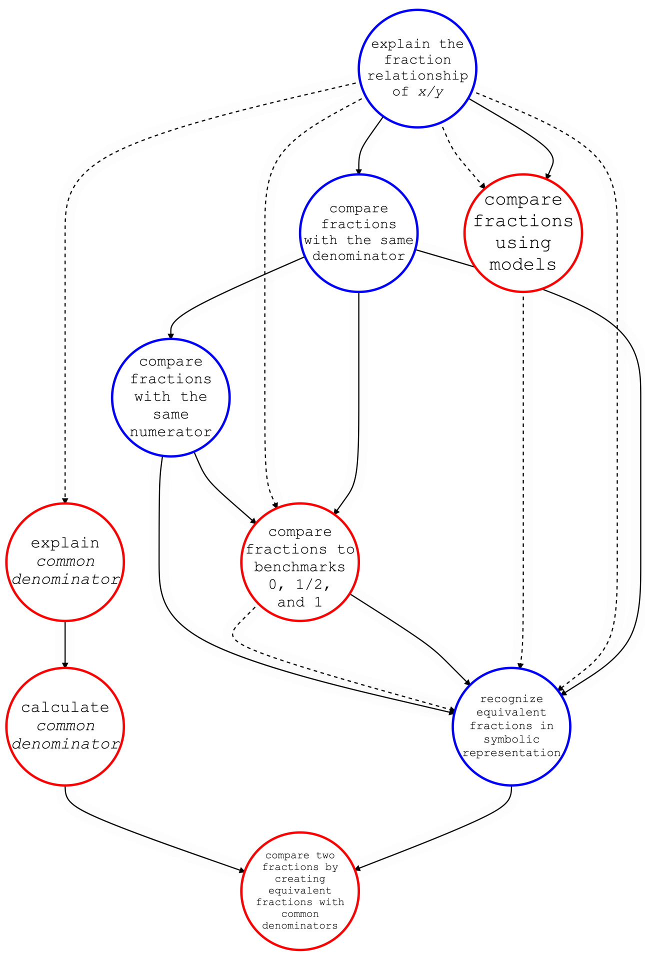
Comparing Fractions

LEARNING MAP INFORMATION

STANDARDS

4.NF.2 Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are only valid when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.



**Learning map model of* 4.NF.2

Node Name	Node Description
CALCULATE *COMMON DENOMINATOR*	Calculate a common multiple of the denominator of the fraction. Multiply each fraction (numerator and denominator) by the appropriate factor in order to have the same denominator.
COMPARE FRACTIONS BY CREATING EQUIVALENT FRACTIONS WITH COMMON DENOMINATORS	Compare two or more fractions by creating equivalent fractions with common denominators. Use the appropriate notation to show if one fraction is greater than, less than, or equal to the other fraction (i.e. >, <, =).
COMPARE FRACTIONS USING BENCHMARKS $0, \frac{1}{2}$, AND 1	Use number sense understanding of zero, one, and one-half to compare the relative size of two or more fractions by reasoning about the relationship each fraction has to either zero, one, or one-half. Use the appropriate notation to show if one fraction is greater than, less than, or equal to another fraction (i.e. >, <, =).
COMPARE FRACTIONS USING MODELS	Use concrete manipulative models or semi-concrete visual representations of fractions with equivalent wholes (i.e. same area for each model, same length on a number line from 0 to 1, same ratio between set models) to compare the relative size of two or more fractions by reasoning the relationship between or among the fractions.
COMPARE FRACTIONS WITH THE SAME DENOMINATOR	When two fractions have a common denominator, use the appropriate notation to show if one fraction is greater than, less than, or equal to the other fraction (i.e. >, <, =).
COMPARE FRACTIONS WITH THE SAME NUMERATOR	When two fractions have a common numerator, use the appropriate notation to show if one fraction is greater than, less than, or equal to the other fraction (i.e. >, <, =).
EXPLAIN *COMMON DENOMINATOR*	Make known your understanding that a common denominator among two or more fractions indicates that these fractions represent some number of same-size parts.
EXPLAIN THE FRACTION RELATIONSHIP OF *X/Y*	Make known your understanding that the symbolic fraction $\frac{x}{y}$ represents x parts of a whole that is partitioned into y equal-size parts. The whole could represent a length, a shape, or a collection of objects (i.e., a set).
RECOGNIZE EQUIVALENT FRACTIONS IN SYMBOLIC REPRESENTATION	When presented with fractions represented symbolically, select two fractions that are equivalent. For example, $\frac{1}{2} = \frac{2}{4}$.

Comparing Fractions

TEACHER NOTES

This unit includes the following documents:

- Learning Map Information
- Instructional Activity (four lessons)
- Instructional Activity Student Handout (for Lessons 1, 2, 3)
- Instructional Activity Supplement (for Lessons 1, 2, 4)
- Student Activity (Word Version)
- Student Activity Solution Guide

In this unit, students will learn to compare fractions with different numerators and denominators using several different strategies. These strategies incorporate concrete models (manipulatives), semi-concrete models or pictorial models (drawings and images), and symbolic representations (numeric). Additionally, students will explore fractions with area models, linear models, and set models. The goal of incorporating many strategies and models is to diversify student understanding and allow students to internalize these representations to create their own mental model of fractions.

RESEARCH

An abundance of research states that fractions are one of the most difficult concepts for students to fully grasp, and a failure to do so can lead to many deficiencies in further mathematic learning (Pitsolantis & Osana, 2013; Van de Walle, Karp, Lovin, Bay-Williams, 2014). One explanation for students' struggle to understand fractions is an overreliance on their existing whole number schema (Behr, Wachsmuth, Post, & Lesh, 1984; Gould, 2005; Post, Wachsmuth, Lesh, & Behr, 1985; Pitsolantis & Osana, 2013; Van de Walle, et al., 2014). For example, students often apply whole number counting practices to the denominator of a fraction, and therefore they reach the conclusion that one-third is less than one-fifth because three is less than five. Therefore, according to Petit, Laird, Mardsen, and Ebby (2016) as well as Bray and Abreu-Sanchez (2010), fraction understanding must develop within the context of overall number sense and should simultaneously incorporate and reinforce students' understanding of numbers. Additionally, algorithms and "tricks" can be introduced too soon and limit students' ability to conceptualize the size of fractions, thus making comparisons between fractions difficult (Gould, 2005; Karp, Bush, & Dougherty, 2015; Mok, Cai, & Fung, 2008).

Because students will rely heavily on their whole number reasoning, McCoy, Barnett, and Stine (2016) recommend that the lessons make direct connections to students' counting intuition. Emphasis should be placed on iterations of fractions, and counting by halves, thirds, fourths, etc. is helpful. Writing out the fractions for all learners to see as students count will help them make a connection between the iterations and their symbolic notation. This informal way to discuss the numerator (iterations) and denominator (partitions) can help students conceptualize the connection between the fraction as one number and its two distinct number components. This will help students distinguish between what is being represented by the numerator and the denominator, and it relates fraction counting to the familiar strategy of counting by whole numbers.

Students should be continually reminded that when comparing fractions, the partitions must be equal in size and taken from the same unit or "universal whole" (Gould, 2005; Van de Walle, et al., 2014). Issues arise when students try to draw their own representations to compare fractions and neglect to partition the same-size or even same-shape whole (although it is possible to create two different shapes with the same size, it is very unlikely that a student will intentionally or successfully do this). Further issues arise when a student partitions their whole into unequal sizes. Deliberate questioning can cause students to confront this matter, such as asking, "How can $\frac{1}{2}$ be less than $\frac{1}{3}$?" Students should be confronted with what assumptions are being made when we compare fractions (i.e. that we are drawing the fractions from equal-size wholes), and care should also be put into the phrasing of questions by expressly stating that the same-size whole is being partitioned.

The use of manipulatives and models is essential in helping students develop their own mental images for solving problems with fractions (Bray, & Abreu-Sanchez, 2010; Petit, Laird, & Marsden, 2010; Post, et al., 1985). However, physical representations should be used to aid students in understanding the mathematics, not presented as the sole way of solving a mathematical problem, as students can develop a dependence on concrete tools and use them even for simple comparisons that they could easily do in their heads (Petit, et al. 2010). Additionally, students who rely on a drawn picture to help them can end up confusing themselves with a lack of precision and uniformity, as well as a misunderstanding of the "universal whole" concept explained in the preceding paragraph. Students' use of physical models should be considered a transitional strategy on the path to manipulative-independent reasoning.

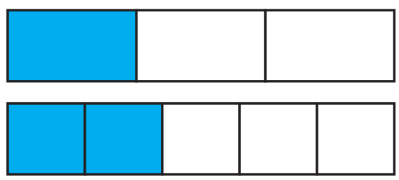
One way to avoid this reliance on manipulatives and models is to expose students to a variety of representations with differing perceptual features, thereby compelling students to create one generalized concept in their minds (Petit, et al., 2010). Another strategy for encouraging the use of mental models is to set the problems in a context. This prompts students to mentally picture the real-world objects, such as pizza slices or distance traveled. Providing a variety of real-world representations further broadens students' mental models of fractions. Class discussion in which students share their chosen methods of solving also benefits all learners by exposing them to several mental strategies and encouraging them to create patterns and generalize the processes that they see their peers using. Discussion should also mention the efficiency and practicality of the different strategies.

For the same reasons that students should not rely on manipulatives, teachers should avoid introducing algorithms without firmly associating them with the students' existing number sense (Gould, 2005; Karp, 2015; Mok, Cai, & Fung, 2008). Shortcuts and "tricks" to help students compare fractions will be counterproductive and inhibit the development of students' conceptual understanding of fractions. According to Karp (2015), the "butterfly method" of solving (as illustrated below) removes all conceptual understanding from the question. Additionally, students over-apply the method to other situations involving fractions, such as adding or subtracting fractions.

There are several strategies students can use to compare fractions that rely on number sense instead of computation (Van de Walle, et al., 2014). Students can compare fractions using manipulatives, which provides them with the opportunity to work with concrete models. Once students have begun to internalize the visual representation and physical manipulation provided by the manipulatives, they can use their knowledge of equivalent fractions to compare fractions to familiar benchmark numbers, such as zero, one-half, and one. This will require students to reason about the relative size of several fractions at one time, and it is a rich exercise steeped in fractional number sense. After spending extensive time employing these strategies and developing a deep conceptual understanding of the size of fractions, students can begin to use strategies that are more reliant on computations, such as finding a common denominator. Students should recognize that this is not always the most efficient method for comparing fractions, and that they may still use prior methods to reason about a fraction's relative size.

AN EXAMPLE

Students can compare fractions using many strategies. They should first compare fractions using concrete representations leading into abstract reasoning before comparing fractions with a computational method, such as calculating common denominators (Gould, 2005; Karp, Bush, & Dougherty, 2015; Mok, Cai, & Fung, 2008).

Manipulatives	$\frac{1}{3}$ **compared with** $\frac{2}{5}$ 
More than/less than one-half or one whole	$\frac{2}{3}$ compared with $\frac{5}{12}$ Because $\frac{2}{3}$ is greater than one-half and $\frac{5}{12}$ is less than one-half, we can conclude that $\frac{2}{3}$ is greater than $\frac{5}{12}$.
Closeness to one-half or one whole	$\frac{2}{3}$ compared with $\frac{5}{6}$ Because $\frac{2}{3}$ is one-third less than a whole and $\frac{5}{6}$ is one-sixth less than a whole—and sixths are smaller than thirds—we can conclude that $\frac{5}{6}$ is greater than $\frac{2}{3}$.
Common Denominator	$\frac{5}{8}$ compared with $\frac{7}{12}$ The least common denominator of the two fractions is 24. $\frac{5\times3}{8\times3}=\frac{15}{24}$ and $\frac{7\times2}{12\times2}=\frac{14}{24}$, therefore $\frac{5}{8}$ is greater than $\frac{7}{12}$.

When rewriting fractions in equivalent form, students tend to incorrectly apply an additive approach, which comes from an over-application of their whole-number concepts (Van de Walle, et al., 2014). For example, students harboring this misconception would conclude that $\frac{3}{6}=\frac{6}{9}$ because 3 was added to the numerator and denominator. This mistake is made less often with common fractions, such as $\frac{1}{2}$ and $\frac{3}{4}$. 2 could be added to the numerator and denominator of $\frac{1}{2}$ to create $\frac{3}{4}$, but because these fractions are more familiar to students, the mistake is more obvious. To address this misconception, it is helpful to reference these familiar examples, as well as to test the incorrect conclusion with a model, such as the circular fraction manipulatives used in this lesson. These mistakes are often made when a student is uncomfortable with the fractions and is grasping for a method to make sense of them.

LEARNING MAP INFORMATION

The learning map section for this sequence of activities begins with the prerequisite knowledge of an understanding of the relationship between numerator and denominator of a fraction; that is, students must know what a fraction represents before learning to compare them. Students should have already learned how to compare fractions with the same denominator, as well as fractions with the same numerator, and should be ready to compare fractions to the benchmarks 0, $\frac{1}{2}$, and 1. These skills rely on students' ability to reason with numbers. For example, if learners can reason about and compare fractions that are the same number of different-size parts, then they are ready to reason about different-size parts' proximity to more familiar numbers such as 0, $\frac{1}{2}$, and 1. This skill develops through students' number sense and is deeply tied to the development of students' mental models of fractions. Simultaneously, students are able to use models to represent and compare fractions. Concrete manipulatives are important to this skill, and visual representations offer yet another model for students to internalize and incorporate as part of their fraction schema. The sequence of these lessons emphasizes students' development of innate number sense by comparing with benchmarks on a number line prior to working with manipulatives. These two skills are represented in the map model as two alternate pathways to identifying and using equivalent fractions. Using models and comparing to benchmarks can sometimes be inefficient; therefore, it is helpful for students to be able to compare fractions by rewriting them with common denominators. Students should first be introduced to the vocabulary and process for calculating common denominators, and then should be able to compare fractions through use of this process.

INSTRUCTIONAL ACTIVITIES

The activities in this unit are designed to introduce students to several strategies to compare fractions. In LESSON 1, students are guided through an exploration of fractions on a number line in order to encourage students to compare a fraction to a benchmark—such as 0, 1, and $\frac{1}{2}$—in order to assess the size of the fraction. Emphasis is placed on equivalent fractions in order to develop students' sense of approximating the overall size of a fraction. Additionally, students focus on the relative size of unit fractions, or the number of equal-size parts the wholes are divided into. For example, they compare fifths with fourths in order to determine which is greater, and which would be farther away from 0 on the number line, then extend this reasoning to tell how far $\frac{5}{4}$ is from 1. This lesson also includes an opportunity for students to practice counting by fractional parts in order to see structure in fractions (i.e. that the numerator increases when counting to show the number of parts, and the denominator stays the same and indicates the size of the fractional parts being counted). Students should gain a mental model from this activity and gradually move away from the need for a physical number line to aid their thinking.

In LESSON 2, students have the opportunity to work with fraction manipulatives and concrete models to develop the students' ability to visualize an area model of a fraction. This activity is meant to supplement and connect with students' number line understanding from LESSON 1. Students should already be able to determine that $\frac{7}{8}$ is greater than $\frac{2}{3}$, by reasoning that one-eighth is smaller than one-third and therefore $\frac{7}{8}$ is closer to 1 than $\frac{2}{3}$. LESSON 2 aims to connect students' linear model of fractions on a number line with an area model of fractions using drawings and manipulatives. Students can explore equivalent fractions using the manipulatives and comparing the size of equivalent fractions. A connection should be made between area models of fractions and linear models (i.e. that equivalent fractions have equal area and are equidistant from 0). In this lesson, students also practice drawing their own fraction models to enforce the idea of a universal whole and equipartitioning of the wholes.

LESSON 3 introduces students to the strategy of finding a common denominator to compare fractions. This strategy should be used in conjunction with students' fractional number sense—not in place of it. Therefore, students estimate their answers before performing the calculations to improve their mental models of fractions. The common denominator approach is presented to students through a scenario in which the two previously learned processes are cumbersome or impractical to use, thus signaling to students that a chosen comparison method won't necessarily work in all situations.

In LESSON 4, students are given an opportunity to incorporate all the strategies they've learned to compare fractions by participating in a dice rolling activity. Students create two fractions by rolling a die, choose the most convenient or advantageous strategy to compare the two fractions, and decide which strategy is greater. Students also practice writing correct inequality symbols to symbolically represent their comparison. Students are required to estimate which fraction is larger before applying an appropriate strategy, in order to continually foster the development of mental models as a means for comparing two fractions.

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Comparing Fractions

OVERVIEW OF INSTRUCTIONAL ACTIVITIES

Lesson	Learning Goal	Nodes Addressed
Lesson 1	Students will compare fractions using a number line.	▶ COMPARE FRACTIONS WITH THE SAME NUMERATOR ▶ COMPARE FRACTIONS WITH THE SAME DENOMINATOR ▶ COMPARE FRACTIONS USING BENCHMARKS 0, $\frac{1}{2}$, AND 1 ▶ COMPARE FRACTIONS USING MODELS
Lesson 2	Students will compare fractions using models and will make a connection between fractions represented on a number line and fractions represented with manipulatives or pictorially.	▶ COMPARE FRACTIONS USING MODELS ▶ COMPARE FRACTIONS BY CREATING EQUIVALENT FRACTIONS WITH COMMON DENOMINATORS ▶ EXPLAIN *COMMON DENOMINATOR*
Lesson 3	Students will compare fractions by finding a common denominator or a common numerator.	▶ EXPLAIN *COMMON DENOMINATOR* ▶ FIND THE LEAST COMMON MULTIPLE FOR 2 NUMBERS UP TO 12 ▶ CALCULATE *COMMON DENOMINATOR* ▶ COMPARE FRACTIONS BY CREATING EQUIVALENT FRACTIONS WITH COMMON DENOMINATORS
Lesson 4	Students will compare fractions with different numerators and different denominators.	▶ COMPARE FRACTIONS BY CREATING EQUIVALENT FRACTIONS WITH COMMON DENOMINATORS ▶ COMPARE FRACTIONS USING BENCHMARKS 0, $\frac{1}{2}$, AND 1 ▶ COMPARE FRACTIONS USING MODELS

Comparing Fractions

INSTRUCTIONAL ACTIVITY

Lesson 1

LEARNING GOAL

Students will compare fractions using a number line.

PRIMARY ACTIVITY

Students are introduced to several representations of fractions and begin to order and compare unit fractions in preparation for ordering and comparing fractions with different numerators and denominators.

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

- Numerator
- Denominator
- Equivalent
- Greater than
- Less than
- Equal to

MATERIALS

- Large number line, either on a white board or on long poster paper
- Instructional Activity Student Handout
- Instructional Activity Supplement (Recommend one copy with the fractions cut out for individual students or pairs of students.)

IMPLEMENTATION

Introduce the lesson by displaying a large number line that ranges from 0 to 1. No tick marks should be on the number line, as this activity should rely on students' estimation skills and not on any precise measuring or counting.

Distribute a fraction card from the INSTRUCTIONAL ACTIVITY SUPPLEMENT to each student or to pairs of students as needed.

Differentiate the level of difficulty for each student by giving them a fraction card that is appropriate for their level of understanding. Fractions equivalent to one (i.e. $\frac{3}{3}$, $\frac{4}{4}$, etc.) and one-half (i.e. $\frac{2}{4}$, etc.) will be easiest for students to place, and non-unit fractions that do not simplify will likely be more difficult.

Call on the student(s) who have the $\frac{1}{2}$ card and ask them to place the card on the number line where they think it falls in relation to the two marked numbers, 0 and 1. Next, **call on** the student(s) who have the $\frac{1}{4}$ card, followed by the student(s) who have $\frac{3}{4}$.

The next students selected should be the rest of the unit fractions.

NOTE: It is recommended that these unit fractions are placed in the following order once the benchmark fractions have been identified. First, $\frac{1}{8}$, because student can find it by identifying one-half of $\frac{1}{4}$. Then, $\frac{1}{3}$ and $\frac{1}{6}$, because student can find a pattern in the denominators (that they are increasing). Finally, $\frac{1}{5}$, because third grade standards do not explicitly use a denominator of five, and they may be the first time some students are working with fifths.

Once the unit fractions are placed, students can volunteer to place their fraction cards as they start to see patterns and make use of structures.

After placing all the fractions on the number line and discussing their placements as necessary, attention should be drawn to the iterations of fraction parts and how they are shown on the number line. The fractions are color coded for assistance.

As a class, **practice** counting by thirds, fourths, sixths, and eighths while the instructor or a student volunteer points to each fraction as students count.

Ask students, "How did the fractions change as you were counting? Explain."

Conclude as a class that the numerator changed as they counted. This reinforces the idea that the numerator follows whole number counting patterns that they are used to, but the denominator does not.

The following questions should be used to help students struggling to place their fraction card, to guide the activity, and to summarize student thinking after all fraction cards have been placed.

GUIDING QUESTIONS

Elicit student thinking:

- What strategies did you use to place your fraction on the number line?
- [Point to a fraction on the number line.] Do you agree with the placement of this fraction? Why or why not?
- Could you have placed your fraction without any other fractions on the number line? Why or why not?
- What fractions were most helpful to see on the number line when placing your fraction on the number line?

Determine if the student can COMPARE FRACTIONS WITH THE SAME NUMERATOR:

- If you know where $\frac{1}{6}$ is, how does that help you place $\frac{1}{5}$?
- If you know where $\frac{1}{6}$ is, how does that help you place $\frac{1}{8}$?

Determine if the student can COMPARE FRACTIONS WITH THE SAME DENOMINATOR:

- If you know where $\frac{4}{8}$ is, how does that help you place $\frac{5}{8}$?

Determine if the student is ready to COMPARE FRACTIONS USING BENCHMARKS 0, $\frac{1}{2}$, AND 1:

- What do you notice about the relationship between the numerator and denominator for fractions less than one-half? For fractions greater than one-half?

Students will repeat the activity on their own number line in INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

Distribute the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

Referencing the number line that students just completed on the INSTRUCTIONAL ACTIVITY SUPPLEMENT, **ask** the class where they think $\frac{11}{12}$ should be placed.

Discuss, as a class, the strategies students used to determine where to place $\frac{11}{12}$ on the number line, such as its proximity to one $\left(\frac{12}{12}\right)$.

Direct students to place $\frac{11}{12}$ on their number line from the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

Instruct students to place a circle around the fractions on their INSTRUCTIONAL ACTIVITY STUDENT HANDOUT that could also be placed on the number line using this strategy.

Verify that students have circled $\frac{9}{10}, \frac{7}{8}, \frac{5}{6}$ and no other fractions.

Ask students to discuss with a partner or group of three to four which of the circled fractions would be closest to 1 on the number line.

Circulate and listen for conversations that discuss the size of the "missing" part needed in order to make a whole and whether tenths, eighths, or sixths is smallest.

Select student volunteers to share their answers and explanations. **Lead** a class discussion which develops an understanding that because one-tenth is smaller than both one-eighth and one-sixth, nine-tenths is closest to one (of the three fractions).

Instruct students to mark and label the three fractions on their number line on their INSTRUCTIONAL ACTIVITY STUDENT HANDOUT, ensuring each fraction is appropriately placed in reference to the already marked $\frac{11}{12}$. They may adjust the placement of $\frac{11}{12}$ if necessary.

Require students to write under each of the fractions an explanation for how they decided to place the three fractions on the lines. Explanations should be specific and reference the size of the fraction and its proximity to 1.

After given time to mark $\frac{9}{10}, \frac{7}{8}$, and $\frac{5}{6}$ on the number line, **ask** students what strategy they would use to place the fraction $\frac{51}{100}$.

Discuss as a class what strategies students used to place $\frac{51}{100}$ on the number line, such as a fraction's proximity to one-half $\left(\frac{50}{100}\right)$.

Direct students to place $\frac{51}{100}$ on the number line on their INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

Instruct students to place a star next to the fractions on their INSTRUCTIONAL ACTIVITY STUDENT HANDOUT that could also be placed on the number line using this strategy.

Verify that students have starred $\frac{6}{10}$, $\frac{5}{8}$, and $\frac{4}{6}$. Some students may already be extending this strategy to fractions that are one part less than one-half. For now, only those fractions that are one part greater than one-half will be starred and placed on the number line.

Ask students to discuss with a partner or group of three or four which of their starred fractions would be closest to $\frac{1}{2}$ on the number line.

Circulate and listen for conversations that discuss the size of the parts. Because each of the fractions have one part more than one-half, it is necessary for students to decide which of the parts is smallest (i.e. whether tenths, eighths, or sixths is smallest).

Select student volunteers to share their answers and explanations**. Lead** a class discussion that develops an understanding that because one-tenth is smaller than one-eighth and one-sixth, $\frac{6}{10}$ is the closest to $\frac{1}{2}$ of the three fractions, followed by $\frac{5}{8}$, and finally $\frac{4}{6}$.

Instruct students to mark and label the three fractions on their number line on their INSTRUCTIONAL ACTIVITY STUDENT HANDOUT, making sure to appropriately place them in reference to the already marked $\frac{51}{100}$. They may adjust the placement of $\frac{51}{100}$ if necessary.

Require students to write an explanation for how they decided to place the three fractions on the lines provided under each of the fractions. Explanations should be specific and reference the size of the fraction and its proximity to $\frac{1}{2}$.

Direct students to draw a rectangle around those fractions on their INSTRUCTIONAL ACTIVITY STUDENT HANDOUT that are one part less than one-half.

Verify that students have identified $\frac{4}{10}$, $\frac{3}{8}$, and $\frac{2}{6}$.

Instruct students to place these fractions on the number line on their INSTRUCTIONAL ACTIVITY STUDENT HANDOUT using the same strategy that was used for the previous two groups of fractions. **Require** students to write their explanations on the lines provided under the fractions.

Lastly**, ask** students which fractions on their INSTRUCTIONAL ACTIVITY STUDENT HANDOUT still need to be placed on the number line. Students should see that $\frac{13}{12}$ has not yet been placed on the number line.

Direct students to discuss in their groups or partners where $\frac{13}{12}$ should be placed and how they know.

Circulate and **listen** for explanations that can be discussed as a class. **Ensure** that students who are having difficulty verbalizing their reasoning get a chance to see or hear their peers give strong mathematical explanations that reference the numerator, denominator, and the relationship between the numerator and denominator.

GUIDING QUESTIONS

Elicit student thinking:

- What kinds of fractions does this strategy work for?
- Give an example of a fraction that you wouldn't be able to place on a number line using this strategy.

Determine if the student can COMPARE FRACTIONS USING BENCHMARKS 0, $\frac{1}{2}$, AND 1:

- How do you know that $\frac{3}{8}$ is less than $\frac{1}{2}$?
- How do equivalent fractions help you in determining where to place fractions on a number line?
- What fractions are equivalent to one-half?
- What fractions are equivalent to one?

Determine if the student is ready to COMPARE FRACTIONS USING MODELS:

- Which would have a greater size: eleven-twelfths of a pie, or nine-tenths of a same-size pie?
- Which would have a greater size: one-half of a cookie, or three-sixths of a same-size cookie?

Require students to answer the following questions on an exit ticket.

- Give a number that is closer to 1 than $\frac{3}{4}$. Explain.
- Give a number that is closer to 0 than $\frac{1}{2}$. Explain.
- Give a number that is closer to $\frac{1}{2}$ than $\frac{2}{3}$. Explain.

At the end of the activity, teachers should check the exit ticket for accuracy and make note of which questions had unique answers, or indicate that more learning is necessary.

Name______________________________

Comparing Fractions

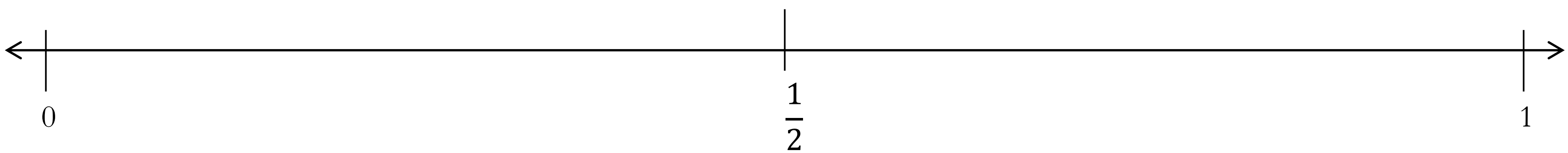
Lesson 1

$\frac{9}{10}$ ______________________ ______________________	$\frac{6}{10}$ ______________________ ______________________
$\frac{7}{8}$ ______________________ ______________________	$\frac{5}{6}$ ______________________ ______________________
$\frac{4}{6}$ ______________________ ______________________	$\frac{13}{12}$ ______________________ ______________________

Name________________________________

$\frac{5}{8}$ ______________________ ______________________	$\frac{3}{8}$ ______________________ ______________________
$\frac{4}{10}$ ______________________ ______________________	$\frac{2}{6}$ ______________________ ______________________
$\frac{7}{12}$ ______________________ ______________________	$\frac{5}{12}$ ______________________ ______________________

Name________________________________



$\frac{1}{8}$	$\frac{1}{6}$	$\frac{1}{5}$
$\frac{1}{4}$	$\frac{2}{8}$	$\frac{1}{3}$
$\frac{2}{6}$	$\frac{3}{8}$	$\frac{1}{2}$
$\frac{2}{4}$	$\frac{4}{8}$	$\frac{3}{6}$

$\frac{5}{8}$	$\frac{2}{3}$	$\frac{4}{6}$
$\frac{3}{4}$	$\frac{6}{8}$	$\frac{5}{6}$
$\frac{7}{8}$	$\frac{3}{3}$	$\frac{2}{2}$
$\frac{4}{4}$	$\frac{6}{6}$	$\frac{8}{8}$

Comparing Fractions

INSTRUCTIONAL ACTIVITY

Lesson 2

LEARNING GOAL

Students will compare fractions using models and will make a connection between fractions represented on a number line and fractions represented with manipulatives or pictorially.

PRIMARY ACTIVITY

Students will use circular manipulatives to compare and order fractions.

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

- Numerator
- Denominator
- Whole/unit
- Part
- Partition
- Equipartition
- Equivalent

MATERIALS

- Circular fraction manipulatives (Recommend five to seven sets, one per group.). The fraction parts should be labeled with their corresponding size (e.g. $\frac{1}{2}$, $\frac{1}{3}$, etc.). If these materials are not available, they are replicated in the Instructional Activity Supplement, which can be printed and cut out.
- Whiteboards (optional)
- Instructional Activity Student Handout

- INSTRUCTIONAL ACTIVITY SUPPLEMENT – Use these fraction circles if other manipulatives are not available. (Recommend one copy for every three to four students.)

IMPLEMENTATION

Begin the lesson by reviewing the exit ticket questions from LESSON 1. If any questions were commonly missed or if students provided unique and/or noteworthy responses, then time should be given to discuss them as a lead-in to LESSON 2.

Group students in groups of three or four.

Distribute one set of fraction manipulatives (or the fraction circles from the INSTRUCTIONAL ACTIVITY SUPPLEMENT) to each group of students.

Allow time for students to become familiar with the fraction manipulatives by setting them out, sorting them, and discussing their observations.

Instruct students to show $\frac{5}{6}$ and $\frac{7}{8}$ using the fraction circles.

Tell students they will be comparing the two fractions and that this comparison is only allowed because the two fractions are parts of the same-size whole—that is, the circle representing "one whole" is equal for both fractions.

Ask students, "Which is greater, $\frac{5}{6}$ or $\frac{7}{8}$?" **Instruct** students to talk with their groups to decide which is greater and explain how the fraction circles support their answer.

Discuss as a class and **allow** several explanations from students. Focus on the greater fraction having a greater area.

Ask students, "Which is greater, $\frac{2}{6}$ or $\frac{3}{8}$?" **Instruct** students to close their eyes and picture the two fractions either with the circular fraction manipulatives or on a number line as in LESSON 1.

Ask them to predict which fraction is greater based on this mental image before they use their circular fraction manipulatives. After they have made a prediction, they should share with their groups to decide which fraction is greater and explain how the fraction circles support their answer.

Repeat this process for the following sets of questions. Students should make an individual prediction before turning to their fraction circles and comparing their answer with the rest of their group.

> NOTE: The following series of questions could be executed as an informal introduction or as a formative assessment. Students can work individually,

in pairs, or in groups. Additionally, students can write their answers on whiteboards, and the score could be kept to make the activity more of a game.

- Which is greater, $\frac{2}{6}$ or $\frac{2}{8}$?
- Which is less, $\frac{2}{6}$ or $\frac{3}{6}$?

Discuss students' strategies as a class so that students can practice justifying their thinking with mathematical vocabulary. Strategies mentioned should address the same number of different-size parts (same numerator, different denominator) and a different number of same-size parts (different numerator, same denominator). These strategies should be familiar to students, as they should have been covered in third grade standard 3.NF.3.D. Prompt students to use appropriate vocabulary and encourage all learners to attempt to explain their reasoning.

- Which is less, $\frac{1}{2}$ or $\frac{2}{3}$?
- Which is less, $\frac{5}{6}$ or $\frac{4}{8}$?

Discuss students' strategies as a class so that students can practice justifying their thinking with mathematical vocabulary. Strategies should include comparisons to the benchmark $\frac{1}{2}$ (or a number equivalent to $\frac{1}{2}$). Prompt students to use appropriate vocabulary and encourage all learners to attempt to explain their reasoning.

- Which is greater, $\frac{5}{12}$ or $\frac{4}{6}$?
- Which is greater, $\frac{5}{12}$ or $\frac{2}{6}$?
- Which is greater, $\frac{5}{8}$ or $\frac{6}{10}$?
- Which is less, $\frac{9}{10}$ or $\frac{7}{8}$?
- Which is greater, $\frac{11}{10}$ or $\frac{13}{12}$?

Discuss students' strategies as a class after each comparison so that students have practice justifying their thinking with mathematical vocabulary.

NOTE: Teachers should encourage student answers to be specific. For example, when comparing $\frac{11}{10}$ and $\frac{13}{12}$, a student should say that both fractions are one part more than a whole (or one). Specifically, $\frac{11}{10}$ is one-tenth more than one and $\frac{13}{12}$ is one-twelfth more than one. Because one-tenth is larger than one-twelfth, $\frac{11}{10}$ is larger than $\frac{13}{12}$. This language should be encouraged both in students' verbal explanations as well as their written explanations.

The following questions will be helpful to ask as students discuss in pairs or groups of three to four, as well as during a whole-class discussion between each set of questions.

GUIDING QUESTIONS

Elicit student thinking:

- How did the circular fraction manipulatives help you compare fractions?
- Do you find it easier to think of fractions on a number line or picture them with the circular fraction manipulatives?

Determine if the student can COMPARE FRACTIONS USING MODELS:

- Which has a larger area, a greater fraction or a lesser fraction? How do you know?
- How do you know if the fraction is larger than half the circle?
- How do you know if the fraction is larger than one whole circle?
- Can we compare fractions from two different-size circles (wholes)? Why or why not?

Determine if the student is ready to COMPARE FRACTIONS BY CREATING EQUIVALENT FRACTIONS WITH COMMON DENOMINATORS:

- What strategy did you use to compare $\frac{2}{6}$ and $\frac{3}{6}$?
- How do you know that $\frac{1}{2}$ is equivalent to $\frac{2}{4}$?
- Can you compare any fractions with the fraction circles you used today? Why or why not?
- Give an example of a fraction comparison that you cannot answer with the fraction circles you used today.

Direct students to individually complete the questions on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

Encourage students to use the strategies and processes that they used or heard discussed during the whole-class discussion.

The following set of questions should be used to guide students as they work on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

GUIDING QUESTIONS

Elicit student thinking:

- [Point to a fraction.] Show me where this fraction is located on a number line.
- [Point to a fraction] Show me this fraction using the circle manipulatives.
- [Point to a fraction.] Draw a model of this fraction.
- What similarities and differences to do you see between a fraction represented on a number line and the same fraction represented pictorially?

Determine if the student can COMPARE FRACTIONS USING MODELS:

- [Point to a question.] How does drawing a model help you answer this question?
- [Point to Question 2.] How many equal-size parts does the whole need to be divided into? How do you know?
- [Point to a question that requires equipartitioning a whole.] Are all the parts that you drew of equal size (or the same size)? Do they need to be? Why or why not?
- [Point to Question 7.] Why is it important to know that the three pizzas are the same size?

Determine if the student is ready to EXPLAIN *COMMON DENOMINATOR*:

- What questions from our class discussion did you find easiest to answer?
- Can you rewrite any of these fractions in an equivalent form? Which fractions?

Students should be required to complete and turn in the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

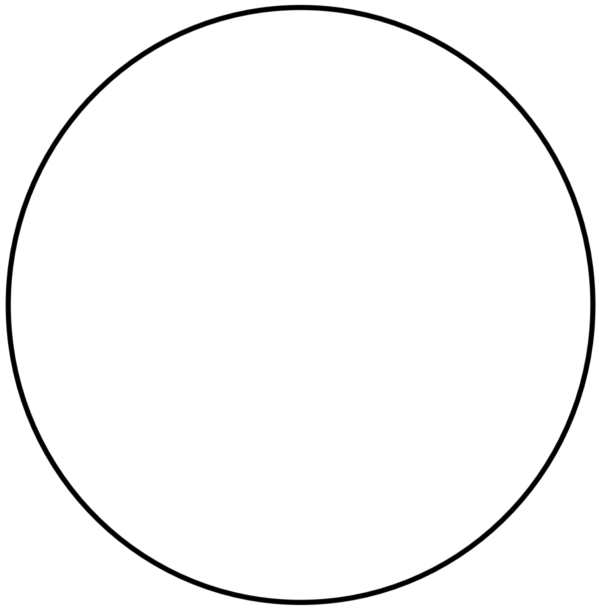
At the end of the activity, teachers should evaluate the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT for accuracy.

Name________________________________

Comparing Fractions

Lesson 2

1. Show $\frac{3}{4}$ using the given circle as a whole.



2. Show $\frac{5}{6}$ using the given rectangle as a whole.

Name______________________________

3. Show $\frac{3}{4}$ on the number line provided.

4. Show $\frac{5}{6}$ on the number line provided.

Name______________________________

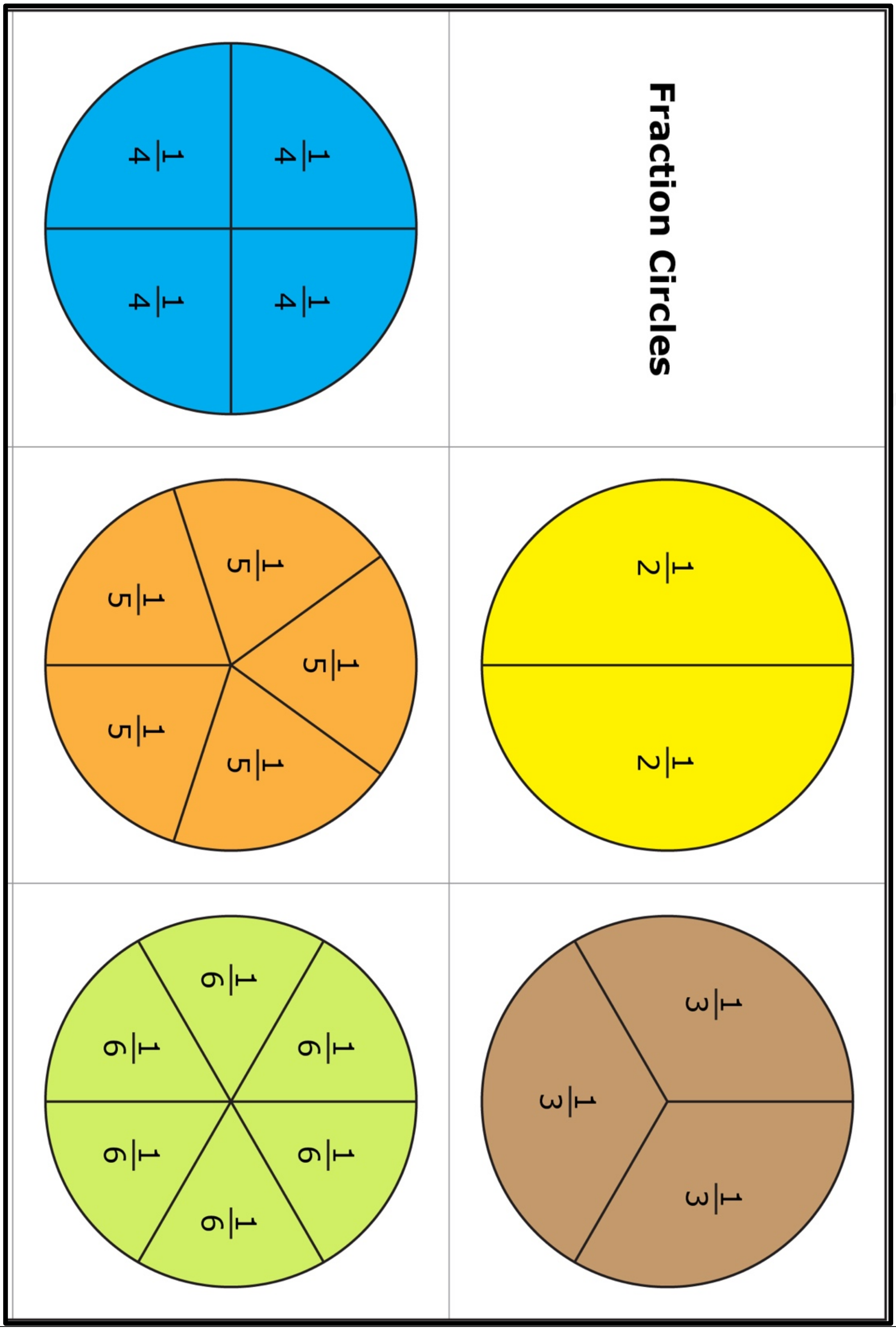
5. Which is greater, $\frac{5}{8}$ or $\frac{2}{4}$?

a. Support your answer using a number line or picture.

b. Support your answer using words.

c. Write the appropriate inequality symbol in the box to compare the two fractions.

$\frac{5}{8} \; \square \; \frac{2}{4}$



Fraction Circles
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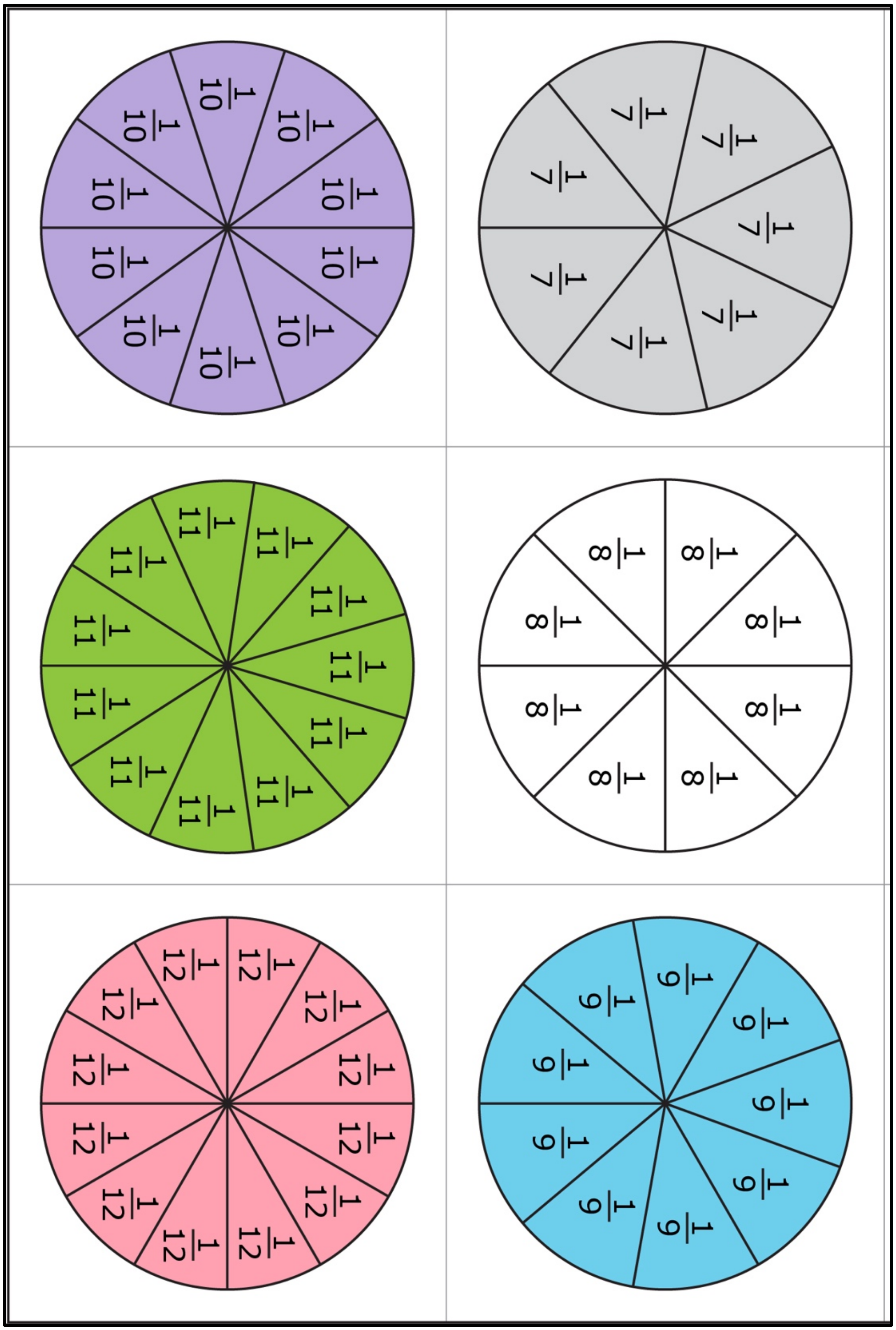

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Comparing Fractions

INSTRUCTIONAL ACTIVITY

Lesson 3

LEARNING GOAL

Students will compare fractions by finding a common denominator or a common numerator.

PRIMARY ACTIVITY

Students are led through a series of questions that require them to relate the process of finding a common denominator with recognizing equivalent fractions, and recognizing when it is advantageous to find a common denominator to compare fractions.

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

- Common multiple
- Common denominator
- Least common denominator
- Equivalent fractions
- Multiple
- Factor

MATERIALS

- Instructional Activity Student Handout

IMPLEMENTATION

Begin by reminding students of the two methods they have used to represent and compare fractions (on a number line and by equipartitioning a shape and shading the appropriate number of parts).

Ask the class to think about the following question: "Which is greater, $\frac{7}{10}$ or $\frac{3}{5}$?"

Allow students to think about their answer for a moment, then walk them through their previously learned strategies. Do not let students share their own solutions or strategies out loud yet, in order to let all students attempt to answer the question with their own thought process.

Ask students if they can compare the two fractions to the benchmark number $\frac{1}{2}$. Students may struggle with making an equivalent fraction to $\frac{1}{2}$ that has a denominator of 5. If they do think of $\frac{1}{2} = \frac{2.5}{5}$ then they will think of $\frac{3}{5}$ as $\frac{0.5}{5}$ greater than $\frac{1}{2}$, which is difficult to compare to $\frac{7}{10}$ being $\frac{2}{10}$ greater than $\frac{1}{2}$.

State that this method does not work as easily as it did in previous problems.

Ask if they can compare the two fractions to one whole. This is a little easier since it won't require decimals. However, comparing $\frac{3}{10}$ less than a whole to $\frac{2}{5}$ less than a whole may still be difficult for some students.

Direct students to partition a shape into 10 equal-size pieces and then into five equal-size pieces in order to represent the fractions with an area model. Give students a few minutes to attempt this. Suggest that they try partitioning a circle or a rectangle. **Select** two or three students to draw their equipartitioned shapes for the class to see.

Emphasize that the shape must be equally divided (i.e. all of their parts must be the exact same size). Students might suggest free-hand drawing a whole with five (or 10) equal-size partitions, or measuring out each partition to make them all equal-size parts.

Discuss as a class whether or not they found it easy to partition their shape into five equal-size parts. Into 10 equal-size parts? What if the fraction had a denominator of 75? Would it be easy to partition a shape into 75 equal-size parts?

Summarize these discussions and come to a consensus that comparing the fractions to a benchmark was not very efficient, and some fractions are not easy to represent pictorially, or at the very least it can be cumbersome and time consuming to do so.

Emphasize the fact that even when you have drawn representations of fractions, it can sometimes be difficult to tell which is greater based on an inexact sketch.

Introduce the idea of transforming one (or both) of the fractions in order to compare them. At this point, some students may have already noticed that $\frac{3}{5}$ is equivalent to $\frac{6}{10}$. This is an appropriate time to have students share this observation.

If no students volunteer the information, **ask** if there is an equivalent way to write $\frac{3}{5}$ with a denominator of 10. For those students who had already seen the equivalence, have them explain how they know the two fractions are equivalent.

Prompt their use of vocabulary by encouraging their answer to include the words "numerator", "denominator", and "multiple/multiply".

Show the process for rewriting $\frac{3}{5} = \frac{3}{5} \times \frac{2}{2} = \frac{6}{10}$ on the board so all students can see, and stress that both the numerator and denominator must be *multiplied* by the same number to create an equivalent fraction. Additionally, **state** that because the fraction is being multiplied by $\frac{2}{2}$, or 1, the fraction value does not change. **Emphasize** that $\frac{3}{5} = \frac{6}{10}$. The fraction has been rewritten in an equivalent form.

NOTE: Students tend to incorrectly apply an additive approach to equivalent fractions. For example, students harboring this misconception would conclude that $\frac{3}{6} = \frac{6}{9}$ because 3 was added to the numerator and denominator. This mistake is made less often with common fractions, such as $\frac{1}{2}$ and $\frac{3}{4}$. 2 could be added to the numerator and denominator of $\frac{1}{2}$ to create $\frac{3}{4}$, but because these fractions are more familiar to students, the mistake is more obvious. To address this misconception, it is helpful to reference these familiar examples, as well as to test the incorrect conclusion with a model, such as the circular fraction manipulatives used in this lesson. These mistakes are often made when a student is uncomfortable with the fractions and is grasping for a method to make sense of them. Therefore, referring back to the methods and strategies for comparing fractions in LESSON 1, LESSON 2, and LESSON 3 will help guide students away from incorrect strategies.

Define "common denominator" for students by explaining that the two fractions now have a *common denominator* of 10.

Answer the original question as a class by comparing seven copies of $\frac{1}{10}$ to six copies of $\frac{1}{10}$ and concluding that $\frac{7}{10}$ is greater, therefore $\frac{7}{10}$ is greater than $\frac{3}{5}$.

Write $\frac{7}{10} > \frac{3}{5}$ on the board.

Instruct students to complete questions 1 – 5 on THE INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

Circulate while students work and assist by asking the following questions when appropriate.

GUIDING QUESTIONS

Elicit student thinking:

- Why would it be advantageous to rewrite a fraction in an equivalent form?
- Why is it important to multiply the fraction by a fraction equivalent to 1?
- How do you compare two fractions with the same denominator?
- How do you compare two fractions with the same numerator?

Determine if the student can EXPLAIN *COMMON DENOMINATOR*:

- How do you know if two fractions have a common denominator?
- What is a common denominator?
- What does it mean if two fractions have a common denominator?

Determine if the student is ready to FIND THE LEAST COMMON MULTIPLE FOR 2 NUMBERS UP TO 12:

- What is a common denominator of $\frac{3}{5}$ and $\frac{5}{6}$?
- What is a common denominator of $\frac{3}{2}$ and $\frac{5}{12}$?
- What is a common denominator of $\frac{1}{4}$ and $\frac{2}{10}$?

Practice a few problems as a class in which students have to rewrite one fraction to get a common denominator.

Show students the fractions $\frac{1}{2}$ and $\frac{5}{12}$.

Ask students to discuss with a partner what they think the common denominator of the two fractions could be.

Circulate while students are discussing and make note of any students who discuss the common denominator of 24, whether it is their final answer or not.

Select two to three students to volunteer their chosen common denominator. **Specifically call on** those who discussed 24 (or even six) as an option. If no students discuss common denominators other than 12, **ask** students if they think that 12 is the only common denominator that the two fractions can share. Through small group and/or whole class discussion, come to the conclusion that both 12 and 24 (and 36, 48, etc.) are common denominators of $\frac{1}{2}$ and $\frac{5}{12}$.

Define *least common denominator* as the smallest number that is a multiple of both of the denominators.

Now **ask** students, "What is the least common denominator of $\frac{1}{2}$ and $\frac{5}{12}$?" Students should easily answer "12".

Direct students to complete questions 6 – 8 on their INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

Circulate while students work to help and ask the following guiding questions.

GUIDING QUESTIONS

Elicit student thinking:

- Why is it helpful to find the common denominator of two fractions?

Determine if the student can CALCULATE *COMMON DENOMINATOR*:

- [Point to a denominator.] Can you list the multiples of this number?
- [Point to a problem set.] Do you need to multiply one or both of the fractions in order for them to have a common denominator?

Determine if the student is ready to COMPARE FRACTIONS BY CREATING EQUIVALENT FRACTIONS WITH COMMON DENOMINATORS:

- [Point to Question 8.a, 8.b or 8.c.] Now that you've rewritten the fractions, can you tell which fraction is greater? Explain.
- Write a statement comparing the two fractions symbolically.

Practice a few problems in which students have to rewrite both fractions in order to get a least common denominator.

Ask students to find the least common denominator for $\frac{1}{3}$ and $\frac{3}{4}$. Students cannot find the answer by multiplying only one fraction as before, however some students will likely see that both denominators are a factor of 12.

Select a student volunteer to show how they found the common denominator.

Model the process for all students on the board by writing out multiples of 3 in one color and multiples of 4 in another color. Include a few multiples past 12 for each.

Ask students to identify the smallest number that appears in each list. When students answer "12", reiterate that this is called the *least common multiple* of 3 and 4, and it will become the *least common denominator* of the rewritten fractions.

Circle "12" in both lists of multiples.

Direct students to rewrite $\frac{1}{3}$ to have a denominator of 12 by writing their calculations in the margins of their INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

Demonstrate and **verify** students' work by writing the process on the board as $\frac{1}{3} = \frac{1}{3} \times \frac{4}{4} = \frac{4}{12}$. **Emphasize** that multiplying by $\frac{4}{4}$ is the same as multiplying by 1, therefore we have not changed the value of the fraction—we have only rewritten it in an equivalent form. **Ask** students to show $\frac{1}{3}$ and $\frac{4}{12}$ with their circular fraction manipulatives. **Verify** that they are equivalent fractions by laying them on top of each other.

Direct students to rewrite $\frac{3}{4}$ to have a denominator of 12 by writing their calculations in the margins of their INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

Demonstrate and **verify** students' work by writing the process on the board as $\frac{3}{4} = \frac{3}{4} \times \frac{3}{3} = \frac{9}{12}$. **Emphasize** that multiplying by $\frac{3}{3}$ is the same as multiplying by 1, therefore we have not changed the value of the fraction—we have only rewritten it in an equivalent form.

Ask students to show $\frac{3}{4}$ and $\frac{9}{12}$ with the circular fraction manipulatives. **Verify** that they are equivalent by laying them on top of each other.

Ask students, "Which fraction is smaller, $\frac{1}{3}$ or $\frac{3}{4}$?"

State that students may answer using any strategy that they are comfortable with, but they should observe how the equivalent fractions with a common denominator relate to their answer.

Discuss the solution and **write** $\frac{1}{3} < \frac{3}{4}$ on the board.

Repeat this series of steps if necessary with another pair of fractions: $\frac{1}{2}$ and $\frac{2}{5}$.

Direct students to complete questions 9 – 11 on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

Use the following questions to help students while they work on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

GUIDING QUESTIONS

Elicit student thinking:

- Why is it helpful to find the common denominator of two fractions?

Determine if the student can CALCULATE *COMMON DENOMINATOR*:

- [Point to a denominator.] Can you list the multiples of this number?
- [Point to a problem set.] Do you need to multiply one or both of the fractions in order for them to have a common denominator?

Determine if the student is ready to COMPARE FRACTIONS BY CREATING EQUIVALENT FRACTIONS WITH COMMON DENOMINATORS:

- [Point to Question 8.a, 8.b or 8.c.] Now that you've rewritten the fractions, can you tell which fraction is greater? Explain.
- Write a statement comparing the two fractions symbolically.

Students should be required to compare their answers with a partner or compare in groups of three or four.

At the end of the activity, teachers should use guided writing strategies to lead a discussion and develop written summaries of the strategies:

- Place the fractions on a number line to compare (i.e. linear/length model).
- Compare two fractions to a benchmark (i.e. 0, $\frac{1}{2}$, 1).
- Use the circular fraction manipulatives to represent the fractions and compare their sizes (i.e. area model).
- Draw a representation of the two fractions to compare their sizes (e.g., area model, or length model depending on the shape).
- Rewrite the fractions with a common denominator and compare.
- Rewrite the fractions with a common numerator and compare.

Name_______________________________

Comparing Fractions

Lesson 3

1. What is a common denominator?

2. Give an example of two fractions that have a common denominator.

3. Give an example of two fractions that do not have a common denominator.

Name_______________________________

4. a. Rewrite the fraction $\frac{5}{6}$ to have a **denominator** of 12.

b. Use your answer from part (a) to tell whether $\frac{5}{6}$ or $\frac{9}{12}$ is greater, or whether the two fractions are equivalent.

Name______________________________

5. a. Rewrite the fraction $\frac{3}{5}$ to have a **numerator** of 6.

b. Use your answer from part (a) to tell whether $\frac{3}{5}$ or $\frac{6}{12}$ is greater, or whether the two fractions are equivalent.

Name______________________________

6. Write the least common denominator for $\frac{1}{3}$ and $\frac{5}{12}$.

7. Write the least common denominator for $\frac{1}{5}$ and $\frac{6}{10}$. Then list another common denominator for the two fractions.

Name______________________________

8. Find the least common denominator for the following pairs of fractions. **Then**, rewrite one or both fractions with the least common denominator. **Lastly**, circle the greater fraction.

a. $\frac{1}{4}$ and $\frac{7}{12}$

b. $\frac{2}{3}$ and $\frac{5}{6}$

c. $\frac{3}{8}$ and $\frac{5}{2}$

Name______________________________

9. Write the least common multiple of the following pairs of numbers.

    a. 3 and 6

    b. 3 and 7

    c. 2 and 5

d. 9 and 4

10. State the least common denominator for the following pairs of fractions.

a. $\frac{1}{4}$ and $\frac{7}{5}$

b. $\frac{2}{3}$ and $\frac{4}{5}$

c. $\frac{3}{2}$ and $\frac{5}{3}$

Name______________________________

11. Find the least common denominator for the following pairs of fractions. **Then**, rewrite one or both fractions with the least common denominator. **Lastly**, circle the greater fraction.

a. $\frac{2}{5}$ and $\frac{7}{4}$

b. $\frac{2}{3}$ and $\frac{5}{8}$

c. $\frac{2}{4}$ and $\frac{7}{6}$

Comparing Fractions

INSTRUCTIONAL ACTIVITY

Lesson 4

LEARNING GOAL

Students will compare fractions with different numerators and different denominators.

PRIMARY ACTIVITY

Students play a dice game with a partner to compare fractions using a variety of strategies.

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

- Numerator
- Denominator
- Greater than
- Less than
- Equal to

MATERIALS

- One die for each student
- Instructional Activity Supplement (Recommend one copy for each student.)

IMPLEMENTATION

Students will roll a die to create a fraction and record it on their Instructional Activity Supplement. Their partner will also roll a die to create a fraction, then the two students will work together to write an inequality statement with symbols.

Arrange students into pairs and **distribute** one INSTRUCTIONAL ACTIVITY SUPPLEMENT and two dice to each pair of students.

Demonstrate how to play the game with a student volunteer using a document camera or other appropriate technology so that all students can see the process.

Roll the die and record the roll as the numerator on the INSTRUCTIONAL ACTIVITY SUPPLEMENT. **Roll** again and record your roll as the denominator.

Direct the student volunteer to roll once and record the number as the numerator, then roll again and record the number as the denominator.

Before performing any calculations, **put a star** next to the fraction you estimate to be greater. Because each student has their own INSTRUCTIONAL ACTIVITY SUPPLEMENT, the partners may disagree at first. Estimation is an important step, because it allows students to develop fraction number sense and to develop a sense of the "bigness" of fractions.

Discuss with the student volunteer to determine which of the two created fractions is greater. Allow the student to lead the discussion and **model** appropriate and specific vocabulary for the class. The discussion should center around one of the strategies used in LESSON 1, LESSON 2, and LESSON 3. Actual calculations may be performed, and manipulatives can be used.

Once a conclusion has been reached, **write** the appropriate inequality symbol in the middle box.

Direct students to begin the activity with their partner.

Circulate and assist students as needed. Some students who are struggling might benefit from using the circular fraction manipulatives from LESSON 2, however these should be used as a supplement and not as a student's sole means of comparing the fractions.

Use the following guiding questions to support student understanding as the engage in the activity.

GUIDING QUESTIONS

Elicit student thinking:

- Is it possible to use the same comparison strategy for all problems?
- How did you decide which strategy to use?

Determine if the student can COMPARE FRACTIONS BY CREATING EQUIVALENT FRACTIONS WITH COMMON DENOMINATORS:

- [Point to a fraction comparison.] Did you have to find common denominators to compare these two fractions? Why or why not?
- How did you know which inequality symbol to use?

Determine if the student can COMPARE FRACTIONS USING BENCHMARKS 0, $\frac{1}{2}$, AND 1:

- [Point to a fraction comparison.] What benchmark did you compare these fractions to? Why?
- [Point to a fraction comparison.] Were these fractions both greater than your chosen benchmark? Both less than your chosen benchmark?
- How do you know which benchmark to compare the given fractions to?

Determine if the student can COMPARE FRACTIONS USING MODELS:

- [Point to a fraction comparison that a student has drawn an area model for.] Why did you decide to represent these fractions with an area model?
- When is it advantageous to represent the fractions pictorially?
- How does this visual representation help you picture the fraction?

Students should be required to complete an exit ticket explaining which of the strategies they like best and why.

At the end of the activity, teachers should review students' exit tickets.

Example:

Student 1 Student 2 $\frac{\boxed{4 \text{ 1st roll}}}{\boxed{3 \text{ 2nd roll}}} \ \boxed{<} \ \frac{\boxed{6 \text{ 1st roll}}}{\boxed{4 \text{ 2nd roll}}}$ Explanation: $\frac{4}{3}$ is $\frac{1}{3}$ greater than 1, and $\frac{6}{4}$ is $\frac{2}{4}$, (or $\frac{1}{2}$) greater than 1. Since $\frac{1}{2}$ is greater than $\frac{1}{3}$, $\frac{6}{4}$ is greater than $\frac{4}{3}$.	Student 1 Student 2 Explanation:	Student 1 Student 2 Explanation:
Student 1 Student 2 Explanation:	Student 1 Student 2 Explanation:	Student 1 Student 2 Explanation:

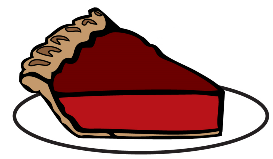
Student 1 Student 2 Explanation:	Student 1 Student 2 Explanation:	Student 1 Student 2 Explanation:
Student 1 Student 2 Explanation:	Student 1 Student 2 Explanation:	Student 1 Student 2 Explanation:

Name_______________________________________

Comparing Fractions

Lesson 1-4

1. James ate three slices of a cherry pie that was cut into eight equal-size pieces. Luke ate three slices of an equally sized apple pie that was cut into six equal-size pieces.



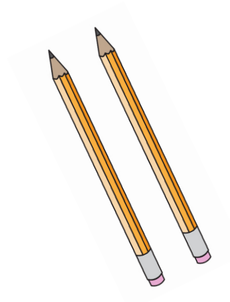
1.a. What fraction of a whole pie did each boy eat?

1.b. Which boy ate more pie? Tell how you know with words.

Name____

1.c. Represent the two fractions on the number line.

2. Ms. Harvey has a set of 20 pencils. Yesterday $\frac{3}{4}$ of the pencils were used. Today $\frac{4}{5}$ of the pencils are being used. On which day were more pencils used? Explain in words how you know.

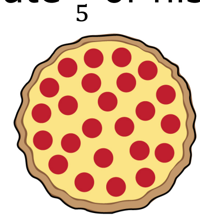


2.a. Support your answer by creating and using a number line.

Name____

2.b. Support your answer by creating and using an area model.

3. Olivia, Peyton, and Justin each ordered a large pizza. Olivia ate $\frac{1}{2}$ of her pizza, Peyton ate $\frac{5}{8}$ of her pizza, and Justin ate $\frac{2}{5}$ of his pizza. Who ate the most pizza? Explain how you know.

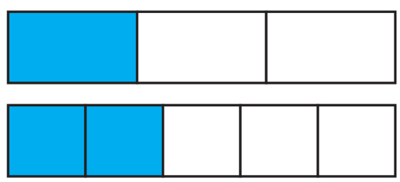


Name____

4. Order the following fractions from least to greatest.

$$\frac{9}{10}, \frac{1}{12}, \frac{5}{8}, \frac{2}{10}, \frac{5}{12}, \frac{5}{6}$$

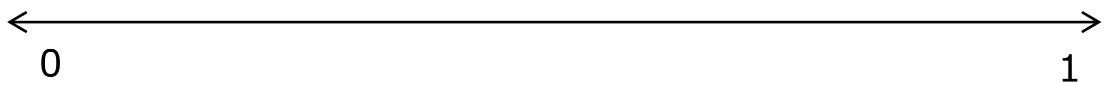
5. Use the fraction model to answer the following questions. One whole is one long rectangle.



5.a. What two fractions are shown by the shaded regions?

Name____

5.b. Place both fractions on the same number line.



5.c. Write a statement comparing the two fractions symbolically.

6. Tell whether the following fractions are less than or greater than one-half. Explain how you know.

6.a. $\frac{3}{5}$

Name____

6.b. $\frac{4}{10}$

6.c. $\frac{9}{8}$

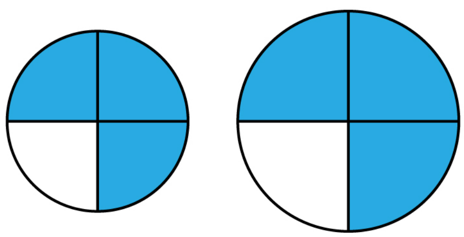
7. Rewrite the following fraction(s) to have a common denominator.

$\frac{5}{8}$ and $\frac{4}{5}$

Name____

7.a. What do you know about the rewritten fractions?

8. Use the two fraction models to answer the following questions.

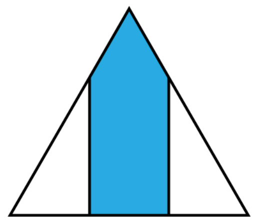


8.a. What fraction is represented by the two drawings?

8.b. Are the two fractions equal? Why or why not?

Name________________________________

9. Jessica partitioned this triangle in an attempt to represent $\frac{1}{3}$. Explain her mistake.



10. Look at $\frac{4}{2}$ and $\frac{10}{4}$. Do the 4s represent the same thing in these two fractions? Explain.

Name____

11. Are $\frac{3}{5}$ and $\frac{5}{3}$ equal because they have the same numbers? Explain.

12. How can $\frac{5}{3}$ be greater than $\frac{5}{8}$ when both have a numerator of 5?

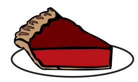
13. How does the numerator relate to the denominator when the amount is less than one? Equal to one? More than one?

Comparing Fractions

STUDENT ACTIVITY SOLUTION GUIDE

Lessons 1 – 4

1. James ate three slices of a cherry pie that was cut into eight equal-size pieces. Luke ate three slices of an equally-sized apple pie that was cut into six equal-size pieces.



1.a. What fraction of a whole pie did each boy eat?

CORRECT ANSWER

James ate $\frac{3}{8}$ of the pie. Luke ate $\frac{3}{6}$ of the pie.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
James ate three slices and Luke ate three slices.	does not consider the size of the slices or part of the whole that each student ate	EXPLAIN THE FRACTION RELATIONSHIP OF X/Y
James ate 38 slices and Luke ate 36 slices.	does not use a fraction bar in their notation and just writes the two numbers that relate to each boy next to each other	EXPLAIN THE FRACTION RELATIONSHIP OF X/Y
James ate 11 slices and Luke ate nine slices.	adds the number of slices each boy ate with the total number of slices in the pie	EXPLAIN THE FRACTION RELATIONSHIP OF X/Y
James ate $\frac{3}{14}$ of a pie and Luke ate $\frac{3}{14}$ of a pie.	considers both pies as the "whole" instead of two individual wholes.	EXPLAIN *DENOMINATOR*
James ate $\frac{8}{3}$ and Luke ate $\frac{6}{3}$.	knows to write a fraction containing the numbers given in the problem, but does not understand the relationship between numerator and denominator	EXPLAIN THE FRACTION RELATIONSHIP OF X/Y

1.b. Which boy ate more pie? Tell how you know with words.

CORRECT ANSWER

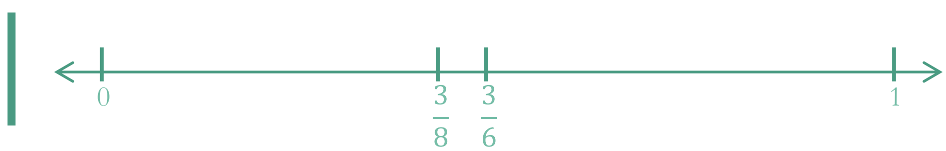
Since $\frac{4}{8}$ is half a pie, James' $\frac{3}{8}$ is less than half of the pie. Luke ate $\frac{3}{6}$ which is equal to half of the pie. Therefore, Luke ate more than James.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
James ate more because 8 is larger than 6.	sees a larger number and assumes that it represents more	COMPARE FRACTIONS WITH THE SAME NUMERATOR; EXPLAIN RELATIONSHIPS BETWEEN UNIT FRACTIONS
James ate more. *or* Luke ate more.	student cannot explain their answer	COMPARE FRACTIONS WITH THE SAME NUMERATOR
James and Luke both ate the same amount because they both ate three slices.	does not consider the denominator of the fractions (i.e. the size of the slices)	COMPARE FRACTIONS WITH THE SAME NUMERATOR

1.c. Represent the two fractions on the number line.

CORRECT ANSWER



ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Places $\frac{3}{6}$ somewhere other than at the halfway point.	does not understand that $\frac{3}{6}$ is equivalent to $\frac{1}{2}$	RECOGNIZE EQUIVALENT FRACTIONS IN SYMBOLIC REPRESENTATION; REPRESENT ONE-HALF ON A LENGTH MODEL
Places either of the fractions past 1.	does not think of the fraction as one number; looks at the individual components which are all past 1	LOCATE POSITIVE FRACTIONS ON A NUMBER LINE
Places $\frac{3}{8}$ is placed between $\frac{3}{6}$ and 1.	thinks that $\frac{3}{8}$ is greater than $\frac{3}{6}$ because 8 is greater than 6.	EXPLAIN *DENOMINATOR*

2. Ms. Harvey has a set of 20 pencils. Yesterday $\frac{3}{4}$ of the pencils were used. Today $\frac{4}{5}$ of the pencils were used. On which day were more pencils used? Explain in words how you know.

CORRECT ANSWER

More pencils were used today because $\frac{4}{5}$ is greater than $\frac{3}{4}$. Each fraction is one fractional part away from one whole, and because fifths are smaller than fourths, $\frac{4}{5}$ is closer to 1 than $\frac{3}{4}$.

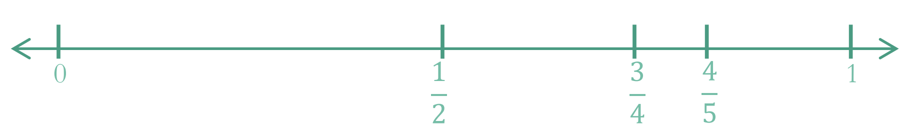
The student may also be able to tell that 20 partitioned into four groups would be groups of five, and therefore $\frac{3}{4}$ of 20 is three groups of five, or 15. 20 partitioned into five groups would be groups of four, so $\frac{4}{5}$ is four groups of four, or 16.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
More pencils were used today because five is greater than four.	only looks at the denominators or sees that the numbers in $\frac{4}{5}$ are in general larger than the numbers in $\frac{3}{4}$ and chooses that fraction	COMPARE FRACTIONS USING BENCHMARKS 0, $\frac{1}{2}$, AND 1
More pencils were used yesterday.	gives no explanation and only guesses at the answer	COMPARE FRACTIONS USING BENCHMARKS 0, $\frac{1}{2}$, AND 1
$\frac{3}{4}$ is greater than $\frac{4}{5}$ because fourths are larger than fifths.	focuses on the size of the parts (i.e. the denominator) and not the number of parts (i.e. the numerator)	EXPLAIN *DENOMINATOR*
The same amount of pencils were used both days because $\frac{3}{4}$ and $\frac{4}{5}$ are both one fractional part away from a whole.	does not consider or compare the size of the missing fractional part	EXPLAIN RELATIONSHIPS BETWEEN UNIT FRACTIONS

2.a. Support your answer by creating and using a number line.

CORRECT ANSWER

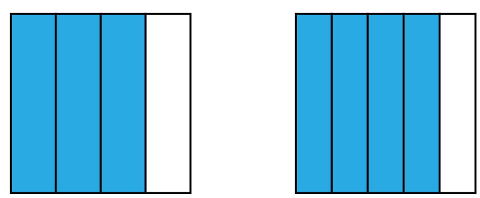


ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Places either of the fractions past 1.	does not think of the fraction as one number; looks at the individual components which are all greater than one	LOCATE POSITIVE FRACTIONS ON A NUMBER LINE
Neither fraction is greater than $\frac{1}{2}$.	does not understand the relationship between the fractions and $\frac{1}{2}$	COMPARE FRACTIONS USING BENCHMARKS 0, $\frac{1}{2}$, AND 1; RECOGNIZE EQUIVALENT FRACTIONS IN SYMBOLIC REPRESENTATION
Places $\frac{3}{4}$ greater than $\frac{4}{5}$.	thinks that $\frac{3}{4}$ is greater than $\frac{4}{5}$	EXPLAIN *DENOMINATOR*

2.b. Support your answer by creating and using an area model.

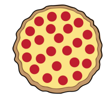
CORRECT ANSWER



ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Student does not equipartition the shape.	does not understand that a fraction represents a number of equal parts of a whole	PARTITION ANY SHAPE INTO EQUAL PARTS
Student does not draw the same sized shape for each fraction.	does not understand the concept of "universal whole"	COMPARE FRACTIONS USING MODELS
Student attempts to determine a common denominator but does not adjust the numerator resulting in $\frac{3}{20}$ and $\frac{4}{20}$.	uses incorrectly rewritten fractions for their area model	CALCULATE *COMMON DENOMINATOR*
Student creates one area model for both fractions by combining into $\frac{7}{9}$.	does not know that there are two different wholes	EXPLAIN THE FRACTION RELATIONSHIP OF X/Y

3. Olivia, Peyton, and Justin each ordered a large pizza. Olivia ate $\frac{1}{2}$ of her pizza, Peyton ate $\frac{5}{8}$ of her pizza, and Justin ate $\frac{2}{5}$ of his pizza. Who ate the most pizza? Explain how you know.



CORRECT ANSWER

Peyton ate the most pizza, because she ate more than one-half of her pizza, while Olivia ate one-half, and Justin ate less than one-half.

The student may also draw area or length models in order to explain their answer, or may also rewrite the fractions to have a common denominator in order to compare them.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	**Misconception**	**Missing Knowledge**
Olivia ate the most because halves are bigger than fifths or eighths.	focuses on the size of the parts (i.e. the denominator) and not the number of parts (i.e. the numerator), and also does not consider the fraction one number	EXPLAIN *DENOMINATOR*
Peyton ate the most because 5 and 8 are larger than the other numbers.	does not consider the size of the fraction, only the whole number components; their thinking is dominated by whole number rationale	EXPLAIN THE FRACTION RELATIONSHIP OF X/Y
Peyton ate the most because she ate five slices, and Olivia only ate one and Justin only ate two.	focuses on the number of parts (i.e. the numerator) and not the size of the parts (i.e. the denominator), and also does not consider the fraction one number	EXPLAIN *NUMERATOR*
Peyton and Justin ate the same amount because they both ate one fractional part less than one whole.	does not consider or compare the size of the missing fractional part	EXPLAIN RELATIONSHIPS BETWEEN UNIT FRACTIONS

4. Order the following fractions from least to greatest.

$$\frac{9}{10}, \frac{1}{12}, \frac{5}{8}, \frac{2}{10}, \frac{5}{12}, \frac{5}{6}$$

CORRECT ANSWER

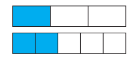
$$\frac{1}{12}, \frac{2}{10}, \frac{5}{12}, \frac{5}{8}, \frac{5}{6}, \frac{9}{10}$$

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Orders the fractions by their denominators from least to greatest, i.e. $\frac{5}{6}, \frac{5}{8}, \frac{2}{10}, \frac{9}{10}, \frac{1}{12}, \frac{5}{12}$	does not interpret the fraction as a single value	EXPLAIN *DENOMINATOR*
Orders the numerators from least to greatest, likely mixing up the three fractions with a numerator of five.	does not consider the fraction as one number	EXPLAIN THE FRACTION RELATIONSHIP OF X/Y
Orders the fractions from greatest to least.	does not read the directions carefully	COMPARE FRACTIONS USING MODELS; COMPARE FRACTIONS USING BENCHMARKS 0, $\frac{1}{2}$, AND 1; COMPARE FRACTIONS BY CREATING EQUIVALENT FRACTIONS WITH COMMON DENOMINATORS
Places $\frac{1}{12}$ first and $\frac{5}{12}$ second.	shows the ability to compare fractions with a common denominator, but cannot correctly include the fractions with unlike denominators in the process	COMPARE FRACTIONS USING MODELS; COMPARE FRACTIONS USING BENCHMARKS 0, $\frac{1}{2}$, AND 1; COMPARE FRACTIONS BY CREATING EQUIVALENT FRACTIONS WITH COMMON DENOMINATORS
Correctly orders the fractions with a numerator of five, but places that sequence in the wrong spot amongst the rest of the fractions; for example, $\frac{1}{12}, \frac{5}{12}, \frac{5}{8}, \frac{5}{6}, \frac{2}{10}, \frac{9}{10}$.	shows the ability to compare fractions with the same numerator, but cannot correctly include the fractions with different numerators in the process	COMPARE FRACTIONS USING MODELS; COMPARE FRACTIONS USING BENCHMARKS 0, $\frac{1}{2}$, AND 1
Places $\frac{5}{6}$ and $\frac{9}{10}$ interchangably or as equals because they are both one fraction part away from a whole.	does not consider or compare the size of the missing fraction part	EXPLAIN RELATIONSHIPS BETWEEN UNIT FRACTIONS

5. Use the fraction model to answer the following questions. One whole is one long rectangle.

5.a. What two fractions are shown by the shaded regions?



CORRECT ANSWER

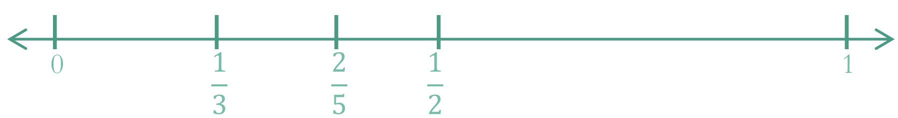
The fractions shown are $\frac{1}{3}$ and $\frac{2}{5}$.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
$\frac{2}{3}$ and $\frac{3}{5}$	does not understand what fraction parts are included in the fraction, or did not read directions clearly enough	COMPARE FRACTIONS USING MODELS
$\frac{1}{8}$ and $\frac{2}{8}$	considers all parts to be the whole instead of two individual wholes	COMPARE FRACTIONS USING MODELS

5.b. Place both fractions on the same number line.

CORRECT ANSWER



ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Places both fractions past $\frac{1}{2}$.	orders the fractions by focusing on the denominator and treating them like whole numbers	EXPLAIN THE POSITION OF FRACTIONS ON THE NUMBER LINE
Places $\frac{2}{5}$ before $\frac{1}{3}$ because fifths are smaller than thirds.	does not consider the whole fraction, only the denominator	EXPLAIN *DENOMINATOR*
Tries to get a common denominator and adds $\frac{1}{3}+\frac{2}{2}=\frac{3}{5}$ and concludes that $\frac{1}{3}$ is greater than $\frac{2}{5}$.	adds an equivalent of 1 to get a common denominator instead of multiplying	CALCULATE *COMMON DENOMINATOR*

5.c. Write a statement comparing the two fractions symbolically.

CORRECT ANSWER

$\frac{1}{3} < \frac{2}{5}$

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
$\frac{1}{3} > \frac{2}{5}$	does not know which inequality symbol to use	COMPARE FRACTIONS BY CREATING EQUIVALENT FRACTIONS WITH COMMON DENOMINATORS
Writes an inequality statement that is inconsistent with the order in which they placed the fractions on the number line.	cannot fluently translate between representations of fractions	COMPARE FRACTIONS USING MODELS; COMPARE FRACTIONS BY CREATING EQUIVALENT FRACTIONS WITH COMMON DENOMINATORS

6. Tell whether the following fractions are less than or greater than one-half. Explain how you know.

6.a. $\frac{3}{5}$

CORRECT ANSWER

$\frac{3}{5}$ is more than $\frac{1}{2}$ because the numerator is larger than $\frac{1}{2}$ of the denominator. Three copies of $\frac{1}{5}$ is greater than two and a half copies of $\frac{1}{5}$.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
$\frac{3}{5}$ is greater than $\frac{1}{2}$ because three is greater than five and five is greater than two.	only compares numerator with numerator and denominator with denominator; does not have a concept of the fraction as one number	COMPARE FRACTIONS USING BENCHMARKS 0, $\frac{1}{2}$, AND 1
$\frac{3}{5}$ is less than $\frac{1}{2}$ because fifths are smaller than halves.	focuses on the size of the parts (i.e. the denominator) and not the number of parts (i.e. the numerator), and also does not consider the fraction one number	EXPLAIN *DENOMINATOR*

6.b. $\frac{4}{10}$

CORRECT ANSWER

$\frac{4}{10}$ is less than $\frac{1}{2}$ because the numerator is less than $\frac{1}{2}$ of the denominator. $\frac{5}{10}$ is equivalent to $\frac{1}{2}$, therefore four copies of $\frac{1}{10}$ is less than five copies of $\frac{1}{10}$.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
$\frac{4}{10}$ is greater than $\frac{1}{2}$ because four is greater than five and 10 is greater than two.	only compares numerator with numerator and denominator with denominator; does not have a concept of the fraction as one number	COMPARE FRACTIONS USING BENCHMARKS 0, $\frac{1}{2}$, AND 1
$\frac{4}{10}$ is less than $\frac{1}{2}$ because fifths are smaller than halves.	focuses on the size of the parts (i.e. the denominator) and not the number of parts (i.e. the numerator), and also does not consider the fraction one number	EXPLAIN *DENOMINATOR*

6.c. $\frac{9}{8}$

CORRECT ANSWER

$\frac{9}{8}$ is greater than $\frac{1}{2}$ because it is also greater than one. The fraction is greater than one because the numerator is larger than the denominator. Nine copies of $\frac{1}{8}$ is greater than four copies of $\frac{1}{8}$ (or $\frac{1}{2}$).

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
$\frac{9}{8}$ is not greater than $\frac{1}{2}$ because it is greater than one.	thinks that a fraction can only be greater than one number	COMPARE FRACTIONS USING BENCHMARKS 0, $\frac{1}{2}$, AND 1
$\frac{9}{8}$ is greater than $\frac{1}{2}$ because nine and eight are greater than one and two.	does not have a mental model for the size of a fraction; still relies on whole number reasoning	COMPARE FRACTIONS USING BENCHMARKS 0, $\frac{1}{2}$, AND 1

7. Rewrite the following fraction(s) to have a common denominator. $\frac{5}{8}$ and $\frac{4}{5}$

CORRECT ANSWER

$\frac{5}{8} \times \frac{5}{5} = \frac{25}{40}$ and $\frac{4}{5} \times \frac{8}{8} = \frac{32}{40}$

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
$\frac{4}{5} + \frac{3}{3} = \frac{7}{8}$	adds a number equivalent to one instead of multiplying by a number equivalent to one	CALCULATE *COMMON DENOMINATOR*
$\frac{5}{8} + \frac{5}{5} = \frac{25}{40}$ and $\frac{4}{5} + \frac{8}{8} = \frac{32}{40}$	recalls that finding a common denominator can sometimes be done by multiplying the numerator and denominator by the other fraction's denominator, but incorrectly adds instead of multiplies	CALCULATE *COMMON DENOMINATOR*
Finds an incorrect common mulitple of 8 and 5, such as 20 or 32.	does not know the multiples of 5 or 8, or does not know that the common denominator needs to be a multiple of both denominators	CALCULATE *COMMON DENOMINATOR*

7.a. What do you know about the rewritten fractions?

CORRECT ANSWER

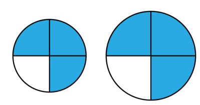
> The two rewritten fractions have the same denominator, therefore they are referring to the same sized parts. They can now be compared using strategies for fractions with the same denominator.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
They have the same denominator.	offers only a simple explanation with no reference to what a denominator is	EXPLAIN THE FRACTION RELATIONSHIP OF X/Y
They are the same.	offers a simple explanation that does not show understanding of how the fractions are similar and how they are different	EXPLAIN THE FRACTION RELATIONSHIP OF X/Y

8. Use the two fraction models to answer the following questions.

8.a. What fraction is represented by the two drawings?



CORRECT ANSWER

> Both models represent $\frac{3}{4}$.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Student makes an attempt to address the different sizes by answering $\frac{3}{4}$ and some fraction greater or less than $\frac{3}{4}$.	does not understand that two different sized models can represent the same fraction	RECOGNIZE PROPER FRACTIONS WITH AN AREA MODEL
The fractions represent $\frac{6}{8}$.	considers both circles as the "whole" instead of two individual wholes	EXPLAIN THE FRACTION RELATIONSHIP OF X/Y

8.b. Are the two fractions equal? Why or why not?

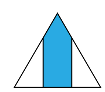
CORRECT ANSWER

> No, the two fractions are not equal, because their wholes are different sizes.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Yes, the two fractions are equal because they are both $\frac{3}{4}$.	student does not understand that in order for two fractions to be compared they must be from the same sized whole	RECOGNIZE EQUIVALENT FRACTIONS WITH AREA MODELS
They are not equal because one is bigger.	does not make reference to the size of the whole; only relies on the size of the images instead of the mathematical idea	COMPARE FRACTIONS USING MODELS

9. Jessica partitioned this triangle in an attempt to represent $\frac{1}{3}$. Explain her mistake.



CORRECT ANSWER

> Jessica did not partition her shape into equal-size parts. The area representing one out of three parts is much larger than the other two parts.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Jessica did not divide the fraction into small enough parts.	makes an attempt at explaining that the shape isn't partitioned into equal-size parts but lacks some vocabulary and mathematical explanation	COMPARE FRACTIONS USING MODELS
Jessica used the wrong shape.	student may be unfamiliar with using a triangle to partition and believes that the shape is not able to be partitioned equally	RECOGNIZE WHOLE ON AN AREA MODEL

10. Look at $\frac{4}{2}$ and $\frac{10}{4}$. Do the 4s represent the same thing in these two fractions? Explain.

CORRECT ANSWER

> No, the 4 in $\frac{4}{2}$ represents the number of halves there are. The 4 in $\frac{10}{4}$ represents the size of the fractional parts, of which there are 10.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
No, one is in the top and one is in the bottom.	only relates the difference to the location of the 4s instead of their mathematical meaning	EXPLAIN THE FRACTION RELATIONSHIP OF X/Y
Yes, both 4s are in the fraction.	does not know what the numerator and denominator represent	EXPLAIN THE FRACTION RELATIONSHIP OF X/Y

11. Are $\frac{3}{5}$ and $\frac{5}{3}$ equal because they have the same numbers? Explain.

CORRECT ANSWER

> No, $\frac{3}{5}$ is 3 copies of $\frac{1}{5}$, and $\frac{5}{3}$ is 5 copies of $\frac{1}{3}$.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Yes, the fractions are the same because $\frac{5}{3} \times \frac{3}{5} = \frac{15}{15}$.	incorrectly applies the idea of finding a common denominator and multiplying the fraction to create an equal numerator and denominator	EXPLAIN THE FRACTION RELATIONSHIP OF X/Y
No, the fractions look different.	student does not have a mathematical justification for how the two fractions differ	EXPLAIN THE FRACTION RELATIONSHIP OF X/Y

12. How can $\frac{5}{3}$ be greater than $\frac{5}{8}$ when both have a numerator of 5?

CORRECT ANSWER

Both have a count of five, but are referencing differently sized fractional parts. $\frac{5}{3}$ is five copies of $\frac{1}{3}$, and $\frac{5}{8}$ is five copies of $\frac{1}{8}$. Because thirds are larger than eighths, $\frac{5}{3}$ is greater than $\frac{5}{8}$.

$\frac{5}{3}$ is greater than $\frac{5}{8}$, because $\frac{5}{3}$ is greater than one whole and $\frac{5}{8}$ is less than one whole.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
$\frac{5}{8}$ is greater than $\frac{5}{3}$ because 8 is greater than 3.	student relies on whole number thinking to compare the two fractions	COMPARE FRACTIONS WITH THE SAME NUMERATOR
Thirds are bigger than eighths.	student gives a brief explanation with little mathematical vocabulary	COMPARE FRACTIONS WITH THE SAME NUMERATOR

13. How does the numerator relate to the denominator when the fraction is less than one? Equal to one? More than one?

CORRECT ANSWER

When a fraction is less than 1, the numerator will be less than the denominator. When a fraction is equal to 1, the numerator and denominator will be equivalent. When a fraction is greater than 1, the numerator is greater than the denominator.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Fractions are only less than 1.	student harbors a misconception that fractions only refer to values less than 1	RECOGNIZE FRACTION
When the fraction is less than 1, both the numerator and denominator will be less than 1.	isn't considering the fraction as one number	EXPLAIN THE FRACTION RELATIONSHIP OF X/Y