ADDING AND SUBTRACTING TO SOLVE ONE-STEP AND TWO-STEP PROBLEMS

2.OA.1

CONTENTS

The types of documents contained in the unit are listed below. Throughout the unit, the documents are arranged by lesson.

LEARNING MAP INFORMATION	An overview of the standards, the learning map section, and the nodes addressed in this unit
TEACHER NOTES	A brief discussion describing the progression depicted in the learning map section with research-based recommendations for focusing instruction to foster student learning and an introduction to the unit's lessons
OVERVIEW OF INSTRUCTIONAL ACTIVITIES	A table highlighting the lesson goals and nodes addressed in each lesson of this unit
INSTRUCTIONAL ACTIVITY	A detailed walkthrough of the unit
INSTRUCTIONAL ACTIVITY STUDENT HANDOUT	A handout for the guided activity, intended to be paired with the Instructional Activity
INSTRUCTIONAL ACTIVITY SUPPLEMENT	A collection of materials or activities related to the Instructional Activity
STUDENT ACTIVITY	A work-alone activity for students
STUDENT ACTIVITY SOLUTION GUIDE	A solution guide for the work-alone activity with example errors, misconceptions, and links to the learning map section

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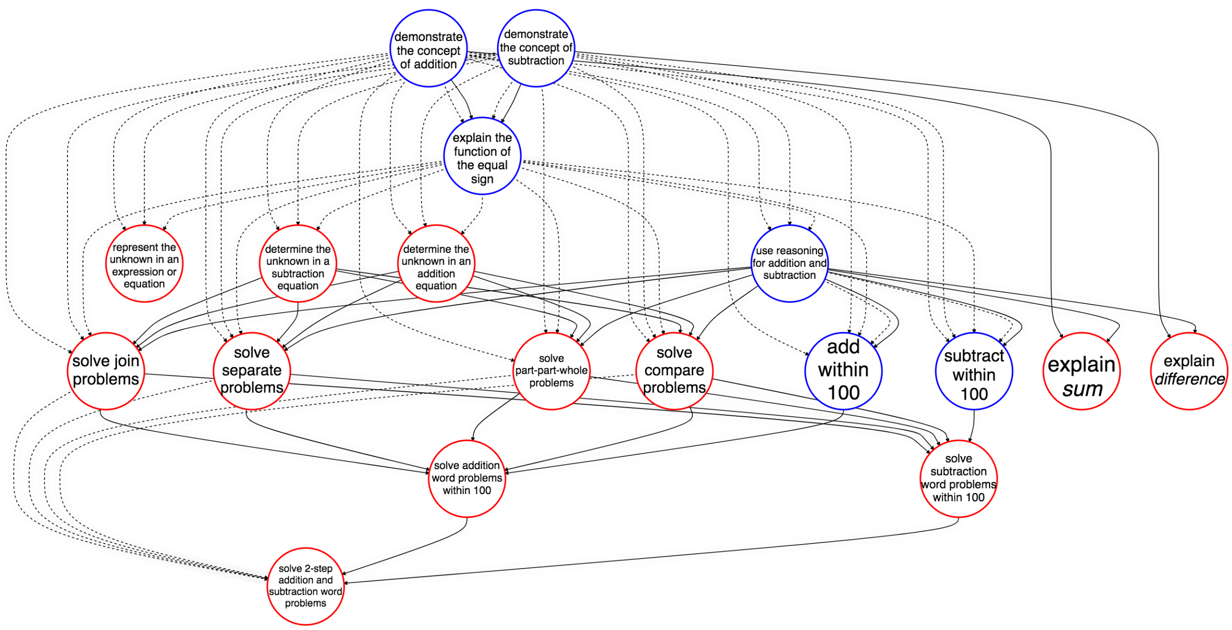
Adding and Subtracting to Solve One-Step and Two-Step Problems

LEARNING MAP INFORMATION

STANDARDS

2.OA.1 Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.



**Learning map model of* 2.OA.1

Node Name	Node Description
ADD WITHIN 100	Demonstrate combinations of addition within 100 with objects, drawings, equations, etc. All numbers in the equation (including the answer) are from 0-100.
DEMONSTRATE THE CONCEPT OF ADDITION	Demonstrate through actions, fingers, mental images, drawings, sounds, acting out situations, verbal explanations, expressions, or use of concrete manipulatives that addition means to put objects together to form a set.
DEMONSTRATE THE CONCEPT OF SUBTRACTION	Demonstrate through actions, fingers, mental images, drawings, sounds, acting out situations, verbal explanations, expressions, or use of concrete manipulatives that subtraction means to take objects away from a set.
DETERMINE THE UNKNOWN IN A SUBTRACTION EQUATION	Determine the unknown/missing minuend, subtrahend, or the difference when given an equation with subtraction operation.
DETERMINE THE UNKNOWN IN AN ADDITION EQUATION	Determine the unknown/missing addend or sum when given an equation with addition operation.
EXPLAIN *DIFFERENCE*	Make known your understanding that a difference is the result of a subtraction problem.
EXPLAIN *SUM*	Make know your understanding that a sum is the result of an addition problem.
EXPLAIN THE FUNCTION OF THE EQUAL SIGN	Make known your understanding that the equal sign is used in an equation to represent an equivalent relationship between expressions.
REPRESENT THE UNKNOWN IN AN EXPRESSION OR EQUATION	Through writing or an appropriate assistive technology, represent the unknown in an expression or equation using a symbol or letter.
SOLVE ADDITION WORD PROBLEMS WITHIN 100	Solve word problems with addition within 100 including join, separate, part-part-whole, and compare problems. All numbers in the equation (including the answer) are from 0-100.
SOLVE COMPARE PROBLEMS	Solve one-step problems that involve comparing two amounts. Three quantities are involved (smaller amount, larger amount, and the difference), and one of those amounts is unknown. This may include problems that ask “how many more” and “how many fewer”.
SOLVE JOIN PROBLEMS	Solve one-step problems where the action is joining with three quantities involved (initial amount, the change amount, and the resulting amount). One of those amounts is unknown, and the resulting amount is the largest.
SOLVE PART-PART-WHOLE PROBLEMS	Solve one-step problems that involve two parts that are combined to make a whole, where one of those amounts is unknown.
SOLVE SEPARATE PROBLEMS	Solve one-step problems where the action is separating with three quantities involved (initial amount, the change amount, and the resulting amount), and one of those amounts is unknown. The initial amount is the largest.
SOLVE SUBTRACTION WORD PROBLEMS WITHIN 100	Solve word problems with subtraction within 100 including join, separate, part-part-whole, and compare problems.
SOLVE 2-STEP ADDITION AND SUBTRACTION WORD PROBLEMS	Use addition and subtraction to solve two-step word problems including join, separate, part-part-whole, and compare problems.
SUBTRACT WITHIN 100	Demonstrate combinations of subtraction within 100 with objects, drawings, equations, etc. All numbers in the equation (including the answer) are from 0-100.
USE REASONING FOR ADDITION AND SUBTRACTION	Apply strategies to add and subtract and/or explain why addition and subtraction strategies work.

Adding and Subtracting to Solve One-Step and Two-Step Problems

TEACHER NOTES

This unit includes the following documents:

- Learning Map Information
- Instructional Activity (two lessons)
- Instructional Activity Student Handout (for Lessons 1 and 2)
- Instructional Activity Supplement (for Lessons 1 and 2)
- Student Activity (Word version)
- Student Activity Solution Guide

In this unit, students will use addition and subtraction within 100 to solve a variety of one- and two-step problems. Throughout the lessons, students will be encouraged to use invented strategies based on place value to add and subtract.

RESEARCH

To help students construct a strong understanding of addition and subtraction within 100, teachers should use contextual problems as the primary teaching tool (Van de Walle, Lovin, Karp, & Bay-Williams, 2014). Students should consider familiar contexts such as giving, receiving, and combining toys or other objects to formulate their understanding of the meaning of addition and subtraction. Applied problems should not just exist at the end of a set of computation problems; this structure does not foster students' reasoning about why certain operations are useful for different contextual situations and has the additional unintended consequence of leading students to simply perform the same operation in the applied problem as they used in the computation problems (Fuson, 2003). Alternatively, it is recommended that students begin with problem solving situations, reason about how to create mathematical models for the situations, and then carry out appropriate operations; this series of activities has been shown to increase students' problem-solving competence and result in the same or better computational competence than students who learn addition and subtraction without a context before being exposed to applied problems (Fuson, 2003).

To support student understanding of applied problems, students and teachers should talk about what happens in the problem, acting out the problem using models (e.g., counters, diagrams, base-ten blocks, or number lines) and drawing pictures to describe the relationships in the problem are recommended (Van de Walle et al., 2014; Fuson, 2003). It is important that discussions about contextual problems encourage problem analysis and require student explanations, rather than relying on or emphasizing keyword strategies (Van de Walle et al., 2014). Tying student models directly to the activity inherent in the problem scenario helps students observe the various combinations of numbers and operations that can be used. For example, rather than leaping to subtraction to solve a missing addend problem, students should model the problem with addition of an unknown quantity. Students should think about and explain what is happening with quantities in problems instead of memorizing associations between certain words and operations (e.g., together and addition); emphasizing keywords does not help students learn to read, interpret, and model the problem (Fuson, 2003).

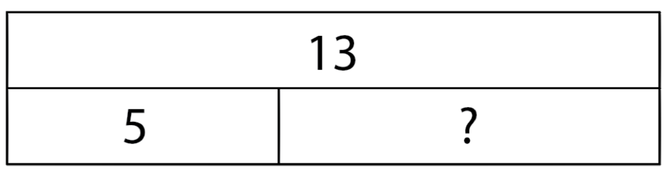
In addition, it is important to vary the types of problems (i.e. join, separate, part-part-whole, and comparison) students are exposed to. Students are exposed to join and separate problems more frequently, but the other two problem types (part-part-whole and comparison) should not be neglected (Van de Walle et al., 2014). Exposure to a variety of problem types and structures supports computational fluency and competence (Fuson et al., 1997). It is not necessary that students know the different problem structures' names; rather, it is important that they consider and solve a variety of problem types.

ADDITION AND SUBTRACTION PROBLEM TYPES (VAN DE WALLE ET AL., 2014)			
Problem Type	**Result Unknown**	**Change Unknown**	**Start Unknown**
Join (Add To)	Diego has five teddy bears. Maria gives him three more. How many teddy bears does Diego have altogether?	Diego has five teddy bears. Maria gives him some more. Now Diego has eight teddy bears. How many teddy bears did Maria give him?	Diego has some teddy bears. Maria gives him three more. Now Diego has eight teddy bears. How many teddy bears did Diego have to begin with?
Separate (Take From)	Diego has eight teddy bears. He gives three teddy bears to Maria. How many teddy bears does Diego have now?	Diego has eight teddy bears. He gives some to Maria. Now he has five teddy bears. How many teddy bears did he give to Maria?	Diego has some teddy bears. He gives three to Maria. Now Diego has five teddy bears. How many teddy bears did Diego have to begin with?
	Whole Unknown	**One Part Unknown**	**Both Parts Unknown**
Part-Part-Whole	Diego has three brown teddy bears and five black teddy bears. How many teddy bears does he have altogether?	Diego has eight teddy bears. Three teddy bears are brown, and the rest are black. How many black teddy bears does Diego have?	Diego has eight teddy bears. Some of the teddy bears are brown, and some of the teddy bears are black. How many of each could he have?
	Difference Unknown	**Bigger Unknown**	**Smaller Unknown**
Comparison	Diego has eight teddy bears, and Maria has five teddy bears. How many more teddy bears does Diego have than Maria? (Or: How many fewer teddy bears does Maria have than Diego?)	Maria has five teddy bears. Diego has three more teddy bears than Maria. How many teddy bears does Diego have? (Or: Maria has three fewer teddy bears than Diego.)	Diego has eight teddy bears. Diego has three more teddy bears than Maria. How many teddy bears does Maria have? (Or: Maria has three fewer teddy bears than Diego.)

As students represent different problem situations with equations and drawings, their representation will be consistent with their view of the problem situation (Fuson, 2003). A student's view of the problem may or may not be consistent with how prepared materials choose to represent the problem, but it is important to allow students to represent a problem in a way that is consistent with their understanding of the problem situation, as long as it is mathematically accurate (Fuson, 2003). Furthermore, it is critical that teachers fairly consider a variety of representations so as not to discount a correct, albeit unfamiliar, model offered by a creative student. For example, when representing the problem situation "Julian has five pencils. Julian gets more pencils from a friend. Now Julian has 13 pencils. How many pencils did Julian's friend give him?", a student may choose to represent the problem with the equation 5 + ? = 13 rather than 13 – 5 = ? (Fuson, 2003). The following is an example of a strip diagram representation that models this problem and could be used to generate an equation to represent the problem.

AN EXAMPLE

The following strip diagram representation models the problem "Julian has five pencils. Julian gets more pencils from a friend. Now Julian has 13 pencils. How many pencils did Julian's friend give him?"



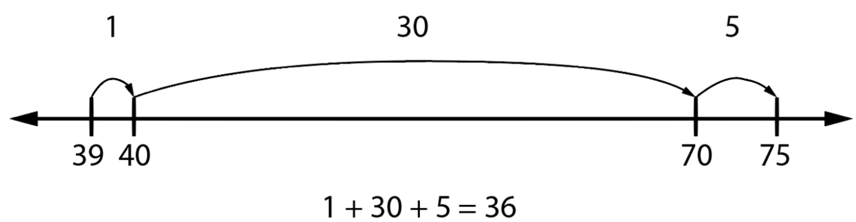
Flexibility with how students represent mathematical problems, as long as the representation is accurate, communicates to students that a major goal of problem solving is to understand and be able to represent the problem (Fuson, 2003).

As students solve contextual addition and subtraction problems, researchers recommend that students use their existing knowledge to create a personalized solution strategy, with appropriate support from a teacher or peers, and that students continue to invent solution strategies as they encounter new problems (Heuser, 2005; Bobis, 2007). When students are given opportunities to construct their own computation methods and communicate their strategies with others, they develop efficient, flexible, and accurate computation strategies (Scharton, 2004). Additionally, they strengthen their understanding by having to explain their representations to their peers and teachers.

It is likely that students will have a variety of levels of proficiency and may require different supports as they add and subtract numbers within 100. Students who initially use more concrete methods (e.g., base-ten manipulatives) can begin drawing the manipulatives, leaving the concrete manipulatives behind. Drawings that are accompanied by numerals can eventually be replaced by the more efficient use of numerals only, resulting in a strategy that is nearly as efficient as the standard algorithms. Encouraging base-ten representations is advantageous because they allow students to translate their representations to algorithmic approaches more efficiently. An empty number line supports counting by tens and partitioning strategies through jumps on the number line; however, students should only be introduced to an empty number line if they have previous experience with numbered number lines, the number line effectively represents their thinking, and they understand how the number line model connects to the original problem (Bobis, 2007). Teachers can facilitate the transition to more sophisticated methods through purposeful sequences of questions that serve as scaffolds and discourse with students as well as strategic groupings in which students work with peers to solve problems.

AN EXAMPLE

The following graphic is a representation of using an empty number line to determine the missing addend in the equation 39 + ? = 75.



It is likely that students' solution methods will vary across different problems and problem settings (Fuson et al., 1997). When students draw and write their solutions, they create records of their thinking. These records serve as memory supports for students and help them engage in reflection and discussion about their solution strategies (Fuson et al., 1997; Van de Walle et al., 2014). The advantage of student-invented algorithms is that they come directly from multi-digit concepts and often specifically label the units that will be combined (Carpenter et al., 1998). Standard algorithms, on the other hand, are efficient and accurate but are not useful for developing reasoning when they are removed from their conceptual basis (Carpenter et al., 1998). Algorithms should not be the strategy students use until they identify them organically and see them as useful for particular situations.

As students construct understanding of addition and subtraction with multi-digit numbers, the role of the teacher is to pose the problem, let students know there are multiple approaches to solving the problem, coordinate discussion, and ask questions about strategies; teachers should not simply demonstrate solutions to problems (Fuson et al., 1997). Directly teaching an alternate strategy poses a danger that students could learn the alternate strategy as a rote procedure, similar to how students often learn the standard algorithm; any knowledge that is constructed by linking an algorithm to a task without the necessary conceptual foundation is unstable and lacks utility in new or unfamiliar situations (Carpenter et al., 1998). Students typically perceive the teacher's method as being "the right way" even if it doesn't make sense to them; it is critical that students' strategies truly are invented and do not need to be approved by their teacher, though teachers should consistently check student strategies for accuracy and guide students to an accurate representation if needed (Scharton, 2004; Fuson et al., 1997). Therefore, teachers should avoid giving preference to any particular method because students with limited understanding may apply that method by imitation rather than applying their own mathematical reasoning (Scharton, 2004).

The Role of Place Value Understanding

Place value understanding is critical to student understanding of regrouping strategies that are used in addition and subtraction. Understanding that "ten" can be viewed as 10 ones or one group of 10 is an important step in understanding place value (Goodrow & Kidd, 2008) and provides the conceptual basis for regrouping strategies. Students should become comfortable with the idea that 10 ones can be regrouped as a single ten when they are adding, and that a ten can be regrouped as 10 ones when they are subtracting. These methods may need support from teachers initially, either in coming up with the strategy or recording such a strategy (Fuson et al., 1997). Questions such as "You have several ones here; what could you do with some of them without changing the value of the number?", "If you have one ten and three ones, how can you take away eight ones?", or "You need some more ones; how could you get more ones without changing the value of the number?" support students in conceptualizing regrouping methods (Fuson et al., 1997). Regrouping should be a fluid and flexible strategy for students and should be based in working with concrete objects (base-ten blocks, straws, bundled objects, etc.).

MISCONCEPTIONS

As students represent one- and two-step problems using equations, they may demonstrate misconceptions about the structure of equations and the equal sign. To counter misinterpretations of the equal sign that students may hold, it is important to include equations such as $35 = 19 + 16$ or $48 = \square - 13$ during instruction (Van de Walle et al., 2014). Presenting all equations in the form $10 + 65 = \square$ may cause students to believe that the equal sign is a symbol meaning to "give an answer" rather than a symbol that states the two sides of the equation are equivalent.

In addition, although the spacing in representations such as strip diagrams and empty number lines used to represent problems does not need to be perfectly precise, it is important that students show an understanding of the size of the quantity they are representing. For example, when students represent one- and two-step problems using strip diagrams, it is important that students only use equal-size parts to represent equal numbers of objects. Similarly, when students represent problems on an empty number line, a "jump" of one unit should be noticeably shorter than a "jump" of 30 units.

Lastly, it is possible that students have been trained to look for "keywords" in real-world problems in the past. However, this strategy is dangerous because it is possible that students will connect words to arithmetic operations without considering the context of the meaning of the word (Britton, 2005). For example, a student may see the word "raised" in a problem and believe that addition is appropriate because "raised" means to go higher, without realizing that "raised" does not have anything to do with addition in the context of the problem (Britton, 2005). Because the context of the problem is so critical to determining appropriate operations, it is important to teach students to analyze the relationships or the actions in the problem, rather than to look for keywords (Karp, Bush, & Dougherty, 2014). As is shown in the previous table of addition and subtraction problem types, the decision to add or subtract depends on the context of the problem and the quantity that is unknown, rather than isolated words that may appear in the description of the problem.

KEYWORDS TO SOLVE PROBLEMS (KARP ET AL., 2014)		
Keyword	**Commonly Associated Operation**	**Counterexample (Keyword does not match commonly associated operation)**
Altogether	Add	Todd and Jessie have 18 pencils altogether. Todd has 12 pencils. How many pencils does Jessie have?
Left	Subtract	Laurie has seven pencils in her left hand and five pencils in her right hand. How many pencils does Laurie have?

LEARNING MAP INFORMATION

The learning map section for this sequence of activities begins with students' ability to demonstrate the concepts of addition and subtraction. This foundational understanding helps students to explain sums and differences, use reasoning for addition and subtraction, represent unknowns in expressions and equations, and determine the unknowns in addition and subtraction equations. Experiences modeling and solving addition and subtraction problems allow students to approach and solve join, separate, part-part-whole, and compare problems. Exposure to a variety of problem situations prepares students to approach one- and two-step addition and subtraction word problems with numbers up to 100.

INSTRUCTIONAL ACTIVITIES

The activities in this unit are designed to allow students to build on and utilize invented and increasingly efficient strategies to solve one- and two-step problems involving addition and subtraction within 100. In 2.NBT.5, several problems were presented in a context based on the recommendations from research, though understanding of the context and the goal of the problem was scaffolded through class discussion. Throughout this unit, students will learn to model and solve one- and two-step real-world problems through focused classroom discussion of real-world problem scenarios.

Lesson 1 will focus on a variety of one-step problems that require addition and subtraction within 100. Students will discuss the context of the problem, model the problem (with concrete manipulatives, with drawings, and with equations where symbols represent the unknown value), then solve the problems using invented strategies.

Lesson 2 will focus on a variety of two-step problems which require addition and subtraction within 100. Students will again discuss the context of the problem, model the problem (with concrete manipulatives, with drawings, and with equations where symbols represent the unknown value), then solve the problems using invented strategies.

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Adding and Subtracting to Solve One-Step and Two-Step Problems

OVERVIEW OF INSTRUCTIONAL ACTIVITIES

Lesson	Learning Goal	Nodes Addressed
Lesson 1	Students will model and solve a variety of one-step, real-world problems using invented strategies to add and subtract within 100.	▶ REPRESENT THE UNKNOWN IN AN EXPRESSION OR EQUATION ▶ DETERMINE THE UNKNOWN IN AN ADDITION EQUATION ▶ DETERMINE THE UNKNOWN IN A SUBTRACTION EQUATION ▶ SOLVE PART-PART-WHOLE PROBLEMS ▶ SOLVE COMPARE PROBLEMS ▶ SOLVE JOIN PROBLEMS ▶ SOLVE SEPARATE PROBLEMS
Lesson 2	Students will model and solve a variety of two-step, real-world problems using invented strategies to add and subtract within 100.	▶ REPRESENT THE UNKNOWN IN AN EXPRESSION OR EQUATION ▶ SOLVE ADDITION WORD PROBLEMS WITHIN 100 ▶ SOLVE SUBTRACTION WORD PROBLEMS WITHIN 100 ▶ SOLVE 2-STEP ADDITION AND SUBTRACTION WORD PROBLEMS

Adding and Subtracting to Solve One-Step and Two-Step Problems

INSTRUCTIONAL ACTIVITY

Lesson 1

LEARNING GOAL

Students will model and solve a variety of one-step, real-world problems using invented strategies to add and subtract within 100.

PRIMARY ACTIVITY

Students will focus on a variety of one-step, real-world problems that require addition and subtraction within 100. Students will discuss the context of the problem and first model the problem with concrete manipulatives, then with drawings and symbols, and finally with equations where symbols represent the unknown value. After experiencing multiple representations of the problem, students will solve the problems using invented addition or subtraction strategies.

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

- Add
- Subtract
- One
- Ten
- Regroup
- Number
- Value
- Sum
- Difference
- Equation
- Expression

MATERIALS

- Two boxes or containers for every two to three students
- Familiar manipulatives (base-ten blocks, counters, tens frames, etc.)
- Blank number line (optional)
- Instructional Activity Student Handout
- Word version Instructional Activity Student Handout
- Instructional Activity Supplement (optional)
- Word version Instructional Activity Supplement

IMPLEMENTATION

Throughout this lesson, students should have access to familiar materials/manipulatives they have used in the past to model addition and subtraction. Examples of materials/manipulatives are base-ten blocks, drawings of base-ten manipulatives, counters, blank number lines, and an empty hundreds chart (provided in the Instructional Activity Supplement). These materials should be used as tools to model and solve real-world problems. Utilize modeling and discussion, as a whole class and in small groups, helps students conceptualize the context of the problem in order to model the problem mathematically.

Present the context of two people, Sally and Julia, who work in a hat factory. At the hat factory, Sally and Julia put hats in boxes.

Pose the following problem.

> Sally has a box with 46 hats. Julia has a box with 29 hats. How many hats are in both Sally and Julia's boxes?

Provide groups of two or three students with two boxes or containers and manipulatives (e.g., base-ten blocks or counters) to model the problem.

Ask pairs or groups of students to model the hats in Sally's box and the hats in Julia's box using the manipulatives and boxes or containers.

Observe how students model Sally's hats and Julia's hats. **Encourage** students to think in groups of 10 and to use place value reasoning as they are able, though the priority should be that students have modeled the problem in a way that makes sense to them.

Discuss as a class the information they already know and what they are still trying to determine in the problem situation. Students should identify that they are trying to determine the total number of hats in Sally and Julia's boxes.

Ask students how they could model the total number of hats using the manipulatives they are working with, but do not have students determine the total yet.

Next, students will connect the concrete manipulatives to drawings and symbols.

Ask pairs or groups of students to draw a picture to represent the number of hats is Sally's box, the number of hats in Julia's box, and the total number of hats in both boxes.

Require students to include in their drawing the numeral for the number of hats in Sally's box and the numeral for the number of hats in Julia's box.

Ask the class for suggestions regarding how to represent the total number of hats, a number they don't yet know. Suggestions could include a question mark, an empty box, or a letter.

Require pairs or groups of students to include an agreed upon symbol for the total number of hats in both boxes to the portion of their drawing that represents the total number of hats.

Finally, students will connect the concrete manipulatives, drawings, and symbols to an equation that represents the problem.

Ask the class for ideas regarding how the problem can be modeled with an equation.

NOTE: Refrain from discussing keywords; all discussions should promote a deep understanding of the context of the problem and how the problem can be modeled using mathematics.

Through class discussion about the problem, **arrive** at the common understanding that they will need to add together the number of hats in Sally's box and the number of hats in Julia's box and that this will equal the total number of hats in both boxes together.

Guide students to symbolically represent combining or adding Sally and Julia's hats together with an expression (46 + 29 or 29 + 46).

NOTE: It is important to use consistent, precise vocabulary when referring to *expressions* (mathematical phrases that include numbers, variables, or both and may include operations but no equal sign) and *equations* (mathematical sentences relating two equivalent numerical or algebraic expressions with an equal sign).

Remind students that because they have not yet added the numbers together, they do not know the sum.

Explain to students that the equation that represents this problem can be written as ? = 46 + 29 or as 46 + 29 = ? (29 + 46 is also accurate because of the commutative property). Modeling the equation in both ways and allowing students to use either will encourage understanding that the equal sign is used to relate two equivalent expressions and does not simply mean to give an answer.

Ask students to use a strategy that they have invented and makes sense to them to add the two numbers together.

As students add, **observe** their work to ensure they are properly utilizing place value understanding in the strategy they choose.

> NOTE: It is important that, during the modeling process, instruction is <u>not</u> provided regarding how to add or subtract the values. Students should be allowed to invent and use meaningful strategies to add and subtract throughout this activity, using manipulatives or models as needed. Teachers can question students about their strategies to deepen their understanding.

Ask students what the result of their addition was and what that number means in the context of the problem. Students should be able to explain that there are a total of 75 hats in Sally and Julia's boxes.

Now that the value of the unknown amount has been determined, **require** students to write the equation with 75 in place of the symbol (i.e., 75 = 46 + 29, 75 = 29 + 46, 46 + 29 = 75, or 29 + 46 = 75).

Ask students to write a sentence about the total number of hats in Sally and Julia's boxes (e.g., "There are 75 hats in Sally and Julia's boxes." or "Sally and Julia have 75 hats altogether [in their boxes].").

Use the following guiding questions to check for student understanding.

GUIDING QUESTIONS

Elicit student thinking:

- Can you describe this problem in your own words?
- How can you model this problem?

Determine if the student can REPRESENT THE UNKNOWN IN AN EXPRESSION OR EQUATION:

- What can you use to represent a number you don't know?
- What number do you not know in this problem?
- Can you write an equation for the number of hats in Sally's box, the number of hats in Julia's box, and the total number of hats?

Determine if the student can DETERMINE THE UNKNOWN IN AN ADDITION EQUATION:

- In your equation for the number of hats in Sally's box, the number of hats in Julia's box, and the total number of hats, how would you figure out the unknown number (the total number of hats)?
- What is the unknown number in this equation?

Determine if the student can SOLVE PART-PART-WHOLE PROBLEMS:

- Can you tell me about the different parts of this problem?
- What are you trying to figure out in this problem?
- What would you do to solve the problem? How do you know?
- What is the answer to the question this problem is asking?
- How many total hats are in both Sally and Julia's boxes?

Pose a new problem to students. Note that the initial information is the same as the previous problem, but the nature of the question is different (comparing two quantities).

> Sally has a box with 46 hats. Julia has a box with 29 hats. How many more hats are in Sally's box than in Julia's box?

Repeat the process of pairs or small groups of students discussing the problem, modeling the problem first with concrete manipulatives, then with a drawing and symbols, and finally with an equation.

As students write equations, it is possible that some students will choose to model this problem with addition (i.e., 29 + ? = 46, 46 = 29 + ?, ? + 29 = 46, or 46 = ? + 29) while other students will choose to model this problem with subtraction (i.e., 46 – 29 = ?).

Allow students to use an equation that makes sense to them (as long as it is mathematically accurate).

Once students have modeled the problem, they should solve the problem using invented strategies and describe their answer in terms of the context of the problem.

Select students or pairs of students who represented and solved the problem in different ways and **ask** them to share their work and their reasoning with the class.

Scaffold student understanding of the modeling and solving process during the discussion using guiding questions.

GUIDING QUESTIONS

Elicit student thinking:

- Can you describe this problem in your own words?
- How can you model this problem?

Determine if the student can REPRESENT THE UNKNOWN IN AN EXPRESSION OR EQUATION:

- What can you use to represent a number you don't know?
- What number do you not know in this problem?
- Can you write an equation for how many more hats are in Sally's box than in are in Julia's box?

Determine if the student can DETERMINE THE UNKNOWN IN A SUBTRACTION EQUATION (or DETERMINE THE UNKNOWN IN AN ADDITION EQUATION):

- In your equation for how many more hats are in Sally's box than in Julia's box, how would you figure out the unknown number?
- What is the unknown number in this equation?

Determine if the student can SOLVE COMPARE PROBLEMS:

- Can you tell me about the different parts of this problem?
- What are you trying to figure out in this problem?
- What would you do to solve the problem? How do you know?
- What is the answer to the question this problem is asking?
- How many more hats are in Sally's box than Julia's box?

The INSTRUCTIONAL ACTIVITY STUDENT HANDOUT includes four real-world problems related to hats at a hat factory. Space is provided for students to record how they modeled each problem with concrete manipulatives or to model the problem strictly through a drawing.

Provide partners with the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT, either one page at a time or all pages at once.

Allow students to work together in pairs to model each problem with concrete manipulatives and/or a drawing, model each problem with an equation, solve the problem, and describe what their answer means in terms of the context of the problem.

Ensure that students show enough work on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT so that someone else could understand the reasoning they used to solve the problem.

> NOTE: A differentiation strategy for students who are struggling would be to provide the student with the same problem situation but with numbers less than 10. Once the student feels comfortable with the context and how they would solve using lesser values, ask the student to consider the problem as written, prompting them to use place-value reasoning to operate with two-digit numbers.

Ask the following guiding questions as students work through the problems in the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

GUIDING QUESTIONS

Elicit student thinking:

- Can you describe this problem in your own words?
- How can you model this problem?

Determine if the student can REPRESENT THE UNKNOWN IN AN EXPRESSION OR EQUATION:

- What can you use to represent a number you don't know?
- What number do you not know in this problem?
- Can you write an equation for this situation using a symbol for the number you do not know yet?

Determine if the student can DETERMINE THE UNKNOWN IN AN ADDITION EQUATION:

- [Point to the unknown in an addition equation.] How can you figure out what this number is?
- [Point to the unknown in an addition equation.] What is the unknown number in this equation?

Determine if the student can DETERMINE THE UNKNOWN IN A SUBTRACTION EQUATION:

- [Point to the unknown in a subtraction equation.] How can you figure out what this number is?
- [Point to the unknown in a subtraction equation.] What is the unknown number in this equation?

[Question 1] Determine if the student can SOLVE JOIN PROBLEMS:

- Can you tell me about the different parts of this problem?
- What are you trying to figure out in this problem?
- What would you do to solve the problem? How do you know?
- What is the answer to the question this problem is asking?
- How many hats are in Sally's box in the end?

[Question 2] Determine if the student can SOLVE PART-PART-WHOLE PROBLEMS:

- Can you tell me about the different parts of this problem?
- What are you trying to figure out in this problem?
- What would you do to solve the problem? How do you know?
- What is the answer to the question this problem is asking?
- How many hats did Sally put in the box?

[Question 3] Determine if the student can SOLVE SEPARATE PROBLEMS:

- Can you tell me about the different parts of this problem?
- What are you trying to figure out in this problem?
- What would you do to solve the problem? How do you know?
- What is the answer to the question this problem is asking?
- How many hats were in the box in the beginning?

[Question 4] Determine if the student can SOLVE COMPARE PROBLEMS:

- Can you tell me about the different parts of this problem?
- What are you trying to figure out in this problem?
- What would you do to solve the problem? How do you know?
- What is the answer to the question this problem is asking?
- How many fewer hats are in Julia's box than Sally's box?

Select students who used different strategies to share their work for each problem as time allows. All students should be required to show their reasoning for each problem using a combination of pictures, symbols, numbers, and equations in the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

At the end of the activity, collect the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT and observe whether students understand the context of each problem and have properly represented each problem using an equation. In addition, check to see whether students are using base-ten reasoning and mathematically accurate strategies (even if they are not conventional) as they solve for the unknown value in each problem.

Use this information to evaluate whether students need to continue modeling problems with concrete manipulatives so they can act out and physically manipulate the problem as well as determine whether it is necessary to provide additional practice writing equations or to revisit addition and subtraction strategies.

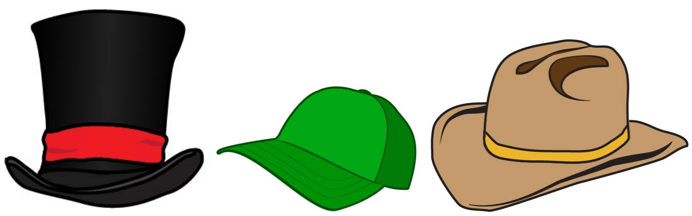
Name________________________________

Adding and Subtracting to Solve One-Step and Two-Step Problems

Lesson 1

Model the problem with a drawing and an equation. Then, solve the problem to answer the question.

Sally and Julia work at a hat factory.

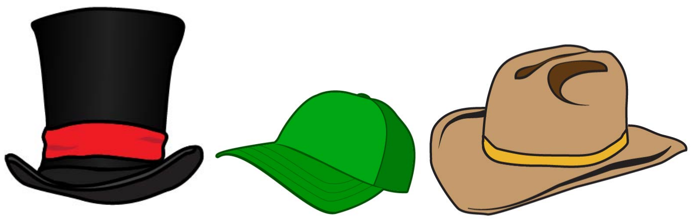


1. There are 38 hats in Sally’s box. Sally puts six more hats in the box. How many hats are in the box now?

Name______________________________

Model the problem with a drawing and an equation. Then, solve the problem to answer the question.

Sally and Julia work at a hat factory.

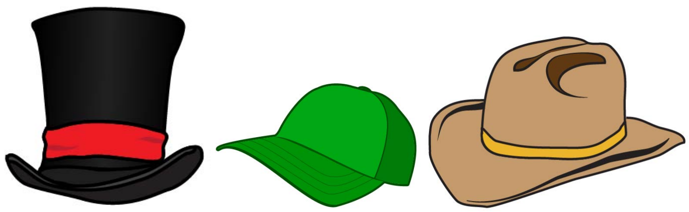


2. Sally and Julia both put hats in a box. Julia puts 13 hats in the box. There are 47 hats in the box altogether. How many hats did Sally put in the box?

Name_______________________________

Model the problem with a drawing and an equation. Then, solve the problem to answer the question.

Sally and Julia work at a hat factory.

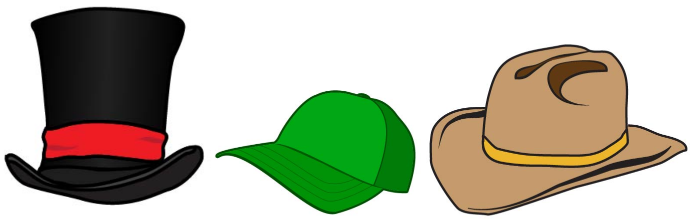


3. There are hats in a box. Julia removes 19 hats because they are torn. Now there are 72 hats in the box. How many hats were in the box to begin with?

Name______________________________

Model the problem with a drawing and an equation. Then, solve the problem to answer the question.

Sally and Julia work at a hat factory.

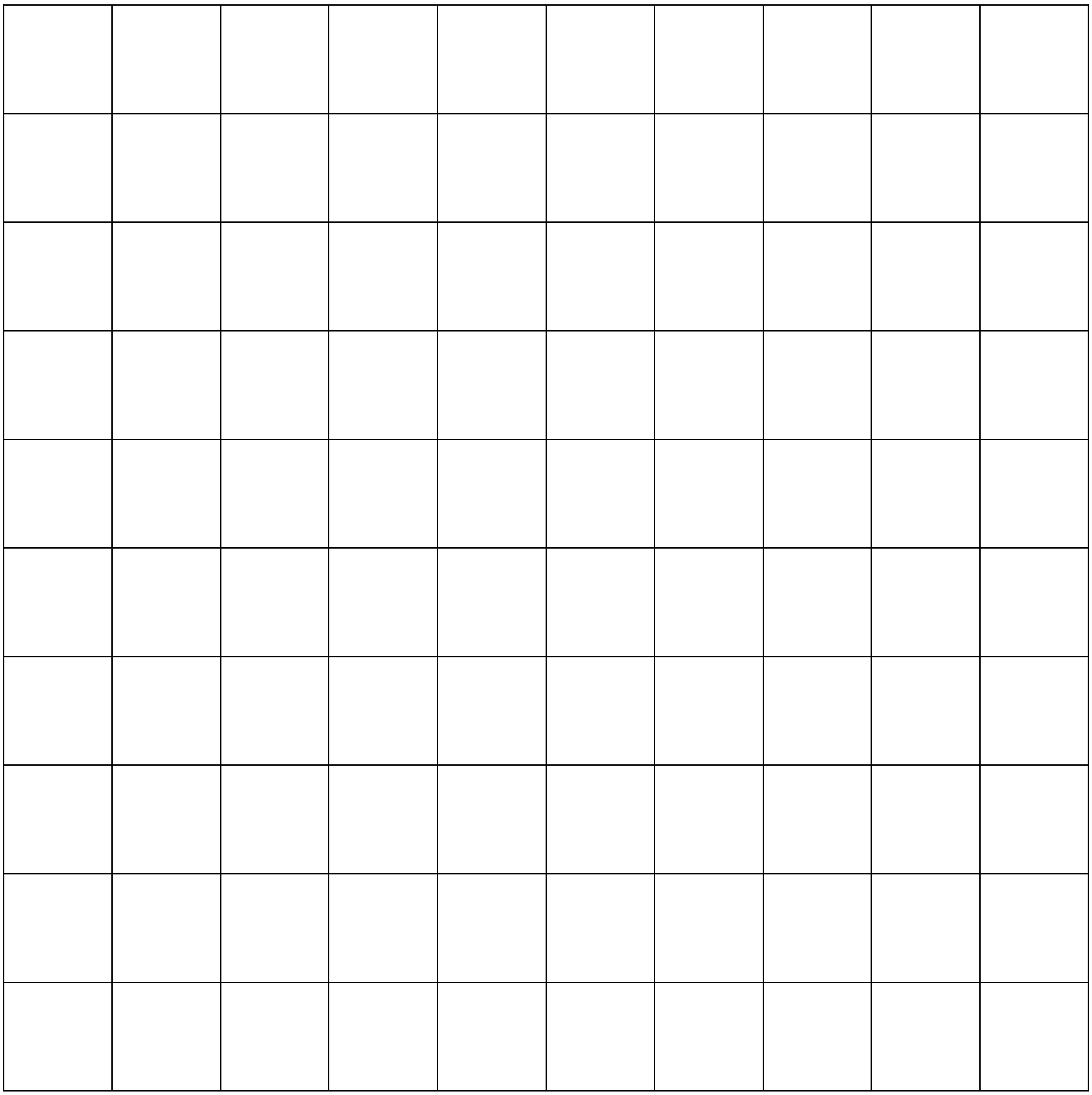


4. Sally’s box has 43 hats. Julia’s box has 28 hats. How many fewer hats are in Julia’s box than in Sally’s box?

Adding and Subtracting to Solve One-Step and Two-Step Problems

INSTRUCTIONAL ACTIVITY SUPPLEMENT

Lesson 1



Adding and Subtracting to Solve One-Step and Two-Step Problems

INSTRUCTIONAL ACTIVITY

Lesson 2

LEARNING GOAL

Students will model and solve a variety of two-step, real-world problems using invented strategies to add and subtract within 100.

PRIMARY ACTIVITY

Students will focus on a variety of two-step, real-world problems that require addition and subtraction within 100. Students will discuss the context of the problem and model the problem—first with concrete manipulatives, then with drawings and symbols, and finally with equations where symbols represent the unknown value. After experiencing multiple representations of the problem, students will solve the problems using invented addition or subtraction strategies.

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

- Add
- Subtract
- One
- Ten
- Regroup
- Number
- Value
- Sum
- Difference
- Equation

MATERIALS

- Familiar manipulatives (base-ten blocks, counters, tens frames, etc.)
- Blank number line (optional)
- Instructional Activity Student Handout
- Word version Instructional Activity Student Handout
- Instructional Activity Supplement (optional)
- Word version Instructional Activity Supplement

IMPLEMENTATION

Throughout this lesson, students should have access to familiar materials/manipulatives they have used in the past to model addition and subtraction. Examples of materials/manipulatives are base-ten blocks, drawings of base-ten manipulatives, counters, blank number lines, and an empty hundreds chart (provided in the Instructional Activity Supplement). These materials should be used as tools to model and solve real-world problems. Utilize modeling and discussion, as a whole class and in small groups, helps students conceptualize the context of the problem in order to model the problem mathematically.

Pose the following problem to students.

> Nico has 68 cents. Nico finds 25 cents on the ground. Then, Nico buys a piece of candy for 75 cents. How many cents does Nico have now?

Provide groups of two or three students with manipulatives (e.g., base-ten blocks or counters) to model the problem.

Ask pairs or groups of students to act out the first part of the problem (the amount Nico starts with and the amount Nico finds) using manipulatives, and **record** their actions with a drawing on paper.

Require students to include in their drawing the numeral for the number of cents Nico starts with as well as the numeral for the number of cents Nico finds.

Observe how students model the problem. **Encourage** students to think in groups of 10 and to use place value reasoning as they are able, though the priority should be that students have modeled the problem in a way that makes sense to them.

Discuss as a class what they do not yet know but are trying to determine. Students should identify that they need to figure out how much money Nico has after he finds 25 cents.

Ask pairs or groups of students to write an equation, using a symbol to represent the unknown, to determine the amount of money Nico has after he finds 25 cents.

Observe and **guide** students' work as they write equations to represent the problem.

Discuss the equations students came up with and how they represent the problem.

Allow students to write equations in a way that makes sense to them and is mathematically accurate. Examples include: ? = 68 + 25, 68 + 25 = ?, ? = 25 + 68, and 25 + 68 = ?.

Ask students to use a strategy that they have invented and that makes sense to them to add the two numbers together.

As students add, **observe** their work to ensure they are properly utilizing place value understanding in the strategy they choose.

> NOTE: It is important that, during the modeling process, instruction is <u>not</u> provided regarding how to add or subtract the values. Students should be allowed to invent and use meaningful strategies to add and subtract throughout this activity, using manipulatives or models as needed. Teachers can question students about their strategies to deepen their understanding.

Use the following guiding questions to check for student understanding as they work.

GUIDING QUESTIONS

Elicit student thinking:

- Can you describe this problem in your own words?
- How can you model this problem?

Determine if the student can REPRESENT THE UNKNOWN IN AN EXPRESSION OR EQUATION:

- What can you use to represent a number you don't know?
- What number do you not know in this problem?
- Can you write an equation for the amount of money Nico has after he finds 25 cents?

Determine if the student can SOLVE ADDITION WORD PROBLEMS WITHIN 100:

- Can you tell me about the different parts of this problem?
- What are you trying to figure out in this problem?
- What would you do to solve the problem? How do you know?
- What is the answer to the question this problem is asking?
- How much money does Nico have before he buys a piece of candy?

Ask students what the result of their addition was and what that number means in the context of the problem. Students should be able to explain that Nico has 93 cents before he buys the piece of candy.

Ask students whether they have solved the whole problem that was posed at the beginning of the lesson. Students should identify that they have not yet figured out how much money Nico has after he buys a piece of candy for 75 cents.

Ask pairs or groups of students to begin with what they learned from the first part of the problem (Nico now has 93 cents) to act out the second part of the problem (buying the piece of candy) using manipulatives, and **record** their actions with a drawing on paper.

Require students to include in their drawing the numeral for how much money Nico has before he buys the piece of candy (93 cents) and how much money he spends on the piece of candy (75 cents).

Observe how students model the problem. **Encourage** students to think in groups of 10 and to use place value reasoning as they are able, though the priority should be that students have modeled the problem in a way that makes sense to them.

Discuss as a class what they do not yet know but are trying to determine now. Students should identify that they need to figure out how much money Nico has after he buys the piece of candy.

Ask pairs or groups of students to write an equation, using a symbol to represent the unknown, to determine the amount of money Nico has after he finds 25 cents.

Observe and **guide** students' work as they write equations to represent the problem.

Discuss the equations students came up with and how they represent the problem.

Allow students to write equations in a way that makes sense to them and is mathematically accurate. Examples include: 93 – 75 = ?, 75 + ? = 93, and ? + 75 = 93.

Ask students to use a strategy that they have invented and that makes sense to them to solve the problem.

Observe students' work to ensure they are properly utilizing place value understanding in the strategy they choose.

Use the following guiding questions to check for student understanding as they work.

GUIDING QUESTIONS

Elicit student thinking:

- Can you describe the whole problem in your own words?
- How can you model this problem?

Determine if the student can REPRESENT THE UNKNOWN IN AN EXPRESSION OR EQUATION:

- What can you use to represent a number you don't know?
- What number do you not know in this problem?
- Can you write an equation for the amount of money Nico has after he buys the piece of candy?

Determine if the student can SOLVE SUBTRACTION WORD PROBLEMS WITHIN 100 (or SOLVE ADDITION WORD PROBLEMS WITHIN 100):

- Can you tell me about the different parts of this problem?
- What are you trying to figure out in this problem?
- What would you do to solve the problem? How do you know?
- What is the answer to the question this problem is asking?
- How much money does Nico have after he buys the piece of candy?

Ask students what the result of their work was and what that number means in the context of the problem. Students should be able to explain that Nico has 18 cents after he buys the piece of candy.

Pose a new problem to students.

> Juno has 17 more stickers than Ari. Juno has 29 blue stickers and 34 green stickers. How many stickers does Ari have?

Repeat the process of pairs or small groups of students discussing the problem, modeling the problem first with concrete manipulatives, then with a drawing and symbols, and finally with equations for both parts of the problem.

Allow students to use equations that make sense to them (as long as they are mathematically accurate).

Once students have modeled the problem, they should solve the problem using invented strategies and describe their answer in terms of the context of the problem.

Select students or pairs of students who represented and solved the problem in different ways, and **ask** them to share their work and their reasoning with the class.

Scaffold student understanding of the modeling and solving process during the discussion using guiding questions.

GUIDING QUESTIONS

Elicit student thinking:

- Can you describe this problem in your own words?
- How can you model this problem?

Determine if the student can SOLVE 2-STEP ADDITION AND SUBTRACTION WORD PROBLEMS:

- Can you tell me about the different parts of this problem?
- What did you do first to solve the problem? Why?
- After you figured out how many stickers Juno has, what did you do next? Why?
- How many stickers does Ari have? How do you know?

The INSTRUCTIONAL ACTIVITY STUDENT HANDOUT includes four real-world problems as additional practice. These problems include join, separate, part-part-whole, and compare problem types. Space is provided for students to record how they modeled each problem with concrete manipulatives or to model the problem strictly through a drawing.

Provide partners with the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT, either one page at a time or all pages at once.

Allow students to work together in pairs to model each problem with concrete manipulatives and/or a drawing, model each part of the problem with an equation, solve the problem, and describe what their answer means in terms of the context of the problem.

Ensure that students show enough work on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT so that someone else could understand the reasoning they used to solve the problem.

NOTE: A differentiation strategy for students who are struggling would be to provide the student with the same problem situation but with numbers less than 10. Once the student feels comfortable with the context and how they would solve using lesser values, ask the student to consider the problem as written, prompting them to use place-value reasoning to operate with two-digit numbers.

Ask the following guiding questions as students work through the problems in the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

GUIDING QUESTIONS

Elicit student thinking:

- Can you describe this problem in your own words?
- How can you model this problem?

Determine if the student can SOLVE 2-STEP ADDITION AND SUBTRACTION WORD PROBLEMS:

- Can you tell me about the different parts of this problem?
- What are you trying to figure out in this problem?
- What did you do first to solve the problem? Why?
- After you figured out this number, what did you do next? Why?
- What is the answer to the question the problem is asking? How do you know?

Select students who used different strategies to share their work for each problem as time allows. All students should be required to show their reasoning for each problem using a combination of pictures, symbols, numbers, and equations in the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

At the end of the activity, collect the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT and observe whether students are understanding the context of each problem and properly representing the problem using an equation. In addition, check to see whether students are using base-ten reasoning and mathematically accurate strategies (even if they are not conventional) as they solve for the unknown value in each problem. Provide additional practice with problem solving and concrete manipulatives as needed.

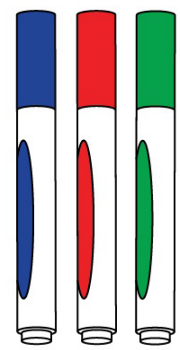
Name________________________________

Adding and Subtracting to Solve One-Step and Two-Step Problems

Lesson 2

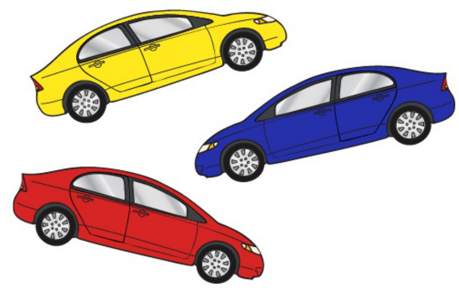
Model and solve each problem using drawings and equations.

1. A box has 25 blue markers, 36 red markers, and 29 green markers. How many markers are in the box?



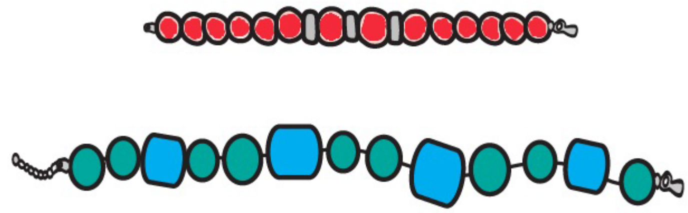
Name________________________________

2. Maya has 41 toy cars. Luz has 13 toy cars. Roan also has some toy cars. Altogether, Maya, Luz, and Roan have 72 toy cars. How many toy cars does Roan have?



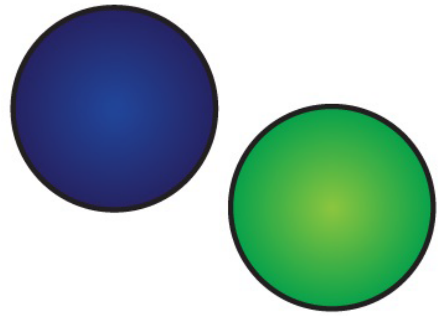
Name_______________________________

3. Raven sells bracelets. Raven sells nine bracelets to Misha and 15 bracelets to Lola. Raven has 36 bracelets left to sell. How many bracelets did Raven have to begin with?



Name______________________________

4. Milo had 43 high-bounce balls. Milo’s friend gave him eight more high-bounce balls. Now Milo has 15 more high bounce balls than Pax. How many high-bounce balls does Pax have?



Adding and Subtracting to Solve One-Step and Two-Step Problems

INSTRUCTIONAL ACTIVITY SUPPLEMENT

Lesson 2

Name________________________________

ADDING AND SUBTRACTING TO SOLVE ONE-STEP AND TWO-STEP PROBLEMS

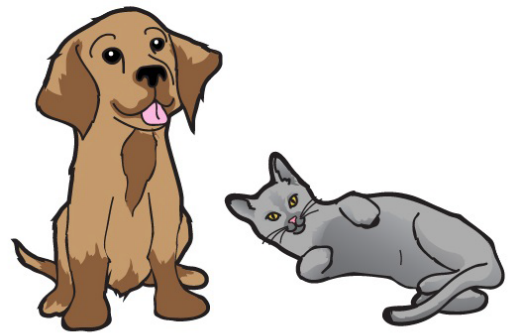
Lessons 1 – 2

For each of the following questions, use words, pictures, equations, and/or numbers to show your thinking.

1. There are 13 more students in 2nd grade than in 3rd grade. There are 81 students in 2nd grade. How many students are in 3rd grade?

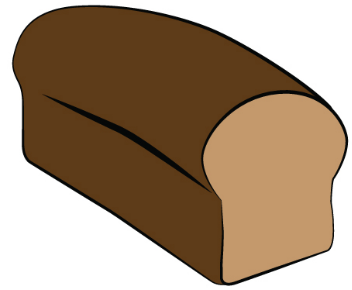
Name____

2. Altogether, there are 54 dogs and cats at the pet shelter. There are 17 dogs. How many cats are there?



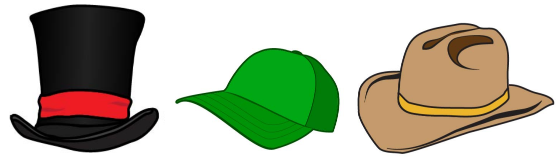
Name____

3. Taylor bakes 19 loaves of bread on Friday, 34 loaves of bread on Saturday, and 27 loaves of bread on Sunday. How many loaves of bread does Taylor bake altogether?



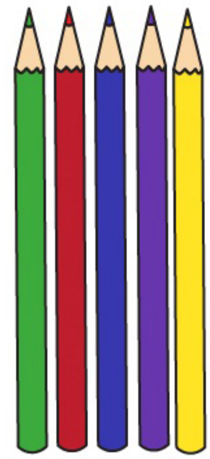
Name____

4. Suzie starts with 47 hats in her box. Suzie puts eight more hats in her box. Joey has 39 hats in his box. How many more hats are in Suzie's box than in Joey's box?



Name____

5. Joel has 83 pencils at the beginning of school. During the year, Joel gives 18 pencils to his friends and loses 12 pencils. How many pencils does Joel have at the end of the year?



Adding and Subtracting to Solve One-Step and Two-Step Problems

STUDENT ACTIVITY SOLUTION GUIDE

Lessons 1 – 2

1. There are 13 more students in 2nd grade than in 3rd grade. There are 81 students in 2nd grade. How many students are in 3rd grade?

CORRECT ANSWER

Check the words, pictures, numbers, and/or equations students provide to explain their thinking.

? + 13 = 81 *or* 81 – 13 = ?

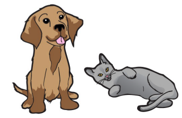
68 + 13 = 81 81 – 13 = 68

There are 68 students in 3rd grade.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
There are 94 students in 3rd grade.	the student adds 81 and 13 instead of subtracting 13 from 81 or determining the number that, when added to 13, results in a sum of 81	SOLVE COMPARE PROBLEMS and SOLVE SUBTRACTION WORD PROBLEMS WITHIN 100 or SOLVE ADDITION WORD PROBLEMS WITHIN 100
There are 72 students in 3rd grade.	the student subtracts the lesser digit from the greater digit in the ones place instead of regrouping a ten as 10 ones	SUBTRACT WITHIN 100
There are 70 students in 3rd grade.	the student subtracts a ten and a one from 81, but does not subtract the remaining two ones	SUBTRACT WITHIN 100

2. Altogether, there are 54 dogs and cats at the pet shelter. There are 17 dogs. How many cats are there?



CORRECT ANSWER

Check the words, pictures, numbers, and/or equations students provide to explain their thinking.

? + 17 = 54 *or* 54 – 17 = ?

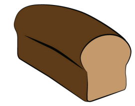
37 + 17 = 54 54 – 17 = 37

There are 37 cats at the pet shelter.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
There are 71 cats at the pet shelter.	adds 54 and 17 (rather than subtracting 17 from 54 or determining the number that, when added to 17, results in a sum of 54)	SOLVE PART-PART-WHOLE PROBLEMS and SOLVE SUBTRACTION WORD PROBLEMS WITHIN 100 or SOLVE ADDITION WORD PROBLEMS WITHIN 100
There are 61 cats at the pet shelter.	adds 54 and 17 without regrouping 10 ones as one ten (rather than subtracting 17 from 54 or determining the number that, when added to 17, results in a sum of 54)	SOLVE PART-PART-WHOLE PROBLEMS, ADD WITHIN 100, and SOLVE SUBTRACTION WORD PROBLEMS WITHIN 100 or SOLVE ADDITION WORD PROBLEMS WITHIN 100
There are 43 cats at the pet shelter.	subtracts the lesser digit from the greater digit in the ones place instead of regrouping a ten as 10 ones	SUBTRACT WITHIN 100
There are 40 cats at the pet shelter.	subtracts one ten and four ones from 54, but does not subtract the remaining three ones	SUBTRACT WITHIN 100

3. Taylor bakes 19 loaves of bread on Friday, 34 loaves of bread on Saturday, and 27 loaves of bread on Sunday. How many loaves of bread does Taylor bake altogether?



CORRECT ANSWER

Check the words, pictures, numbers, and/or equations students provide to explain their thinking. Student may add any two pairs of numbers, then add the third, or represent all three quantities together and count or add them all at once.

19 + 34 + 27 = ?
19 + 34 + 27 = 80

Taylor baked 80 loaves of bread altogether.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Taylor baked 53 loaves of bread. *or* Taylor baked 61 loaves of bread. *or* Taylor baked 46 loaves of bread.	adds two of the quantities but does not complete the problem by adding the third quantity	SOLVE 2-STEP ADDITION AND SUBTRACTION WORD PROBLEMS
Taylor baked 26 loaves of bread.	adds the digits 1, 9, 3, 4, 2, and 7	ADD WITHIN 100

4. Suzie starts with 47 hats in her box. Suzie puts eight more hats in her box. Joey has 39 hats in his box. How many more hats are in Suzie's box than in Joey's box?

CORRECT ANSWER

Check the words, pictures, numbers, and/or equations students provide to explain their thinking. It is likely that students will first determine how many hats are in Suzie's box and then compare that amount to the number of hats in Joey's box. Example work is provided.

47 + 8 = ?
47 + 8 = 55 (Suzie has 55 hats in her box.)

55 – 39 = ?	*or*	39 + ? = 55
55 – 39 = 16		39 + 16 = 55

There are 16 more hats in Suzie's box than in Joey's box.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
There are 94 hats.	adds all quantities given in the problem situation; may believe that the word “more” always indicates addition	SOLVE 2-STEP ADDITION AND SUBTRACTION WORD PROBLEMS
There are 74 hats. *or* There are 84 hats.	adds all quantities given in the problem without proper regrouping	SOLVE 2-STEP ADDITION AND SUBTRACTION WORD PROBLEMS and ADD WITHIN 100
There are 15 more hats in Suzie’s box.	correctly determines the sum of 47 and 8 (55), then subtracts 40 from 55 to make the subtraction simpler in the second part of the problem, but forgets to add 1 back (to compensate for subtracting 1 extra)	SUBTRACT WITHIN 100
There are 26 more hats in Suzie’s box.	correctly determines the sum of 47 and 8 (55), then adds 10 ones to 55 when subtracting 39 rather than trading a ten for 10 ones, resulting in a difference 10 greater than the actual difference	SUBTRACT WITHIN 100
There are 24 more hats in Suzie’s box.	correctly determines the sum of 47 and 8 (55), then subtracts the lesser digit from the greater digit in the ones place instead of regrouping a ten as 10 ones when subtracting 39 from 55	SUBTRACT WITHIN 100
They have the same number of hats.	adds 8 to the number of hats in Joey’s box, resulting in a sum of 47 hats, the same number of hats that are in Suzie’s box	SOLVE 2-STEP ADDITION AND SUBTRACTION WORD PROBLEMS

5. Joel has 83 pencils at the beginning of school. During the year, Joel gives 18 pencils to his friends and loses 12 pencils. How many pencils does Joel have at the end of the year?

CORRECT ANSWER

Check the words, pictures, numbers, and/or equations students provide to explain their thinking. Students may subtract 18 from 83 and then subtract 12, or they may add 18 and 12 and then subtract the sum from 83. Example work is provided.

83 – 18 = ?	*or*	18 + 12 = ?
83 – 18 = 65		18 + 12 = 30
65 – 12 = ?		83 – 30 = ?
65 – 12 = 53		83 – 30 = 53

Joel has 53 pencils at the end of the year.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Joel has 77 pencils at the end of the year.	subtracts 18 from 83, then adds 12 instead of subtracting 12	SOLVE 2-STEP ADDITION AND SUBTRACTION WORD PROBLEMS
Joel has 65 pencils at the end of the year.	subtracts 18 from 83 and gives the difference as the answer; does not consider the 12 pencils lost during the year	SOLVE 2-STEP ADDITION AND SUBTRACTION WORD PROBLEMS
Joel has 71 pencils at the end of the year.	subtracts 12 from 83 and gives the difference as the answer; does not consider the 18 pencils given away during the year	SOLVE 2-STEP ADDITION AND SUBTRACTION WORD PROBLEMS
Joel has 113 pencils at the end of the year.	adds all quantities given in the problem situation	SOLVE 2-STEP ADDITION AND SUBTRACTION WORD PROBLEMS
Joel has 63 pencils at the end of the year.	adds 10 ones to 83 when subtracting 18 rather than trading a ten for 10 ones *or* subtracts the lesser digit from the greater digit in the ones place instead of regrouping a ten as 10 ones when subtracting 18 from 83; either error results in a difference 10 greater than the actual difference in the first part of the problem (75 rather than 65) before the student subtracts 12	SUBTRACT WITHIN 100