DIVISION OF FRACTIONS

6.NS.1

CONTENTS

The types of documents contained in the unit are listed below. Throughout the unit, the documents are arranged by lesson.

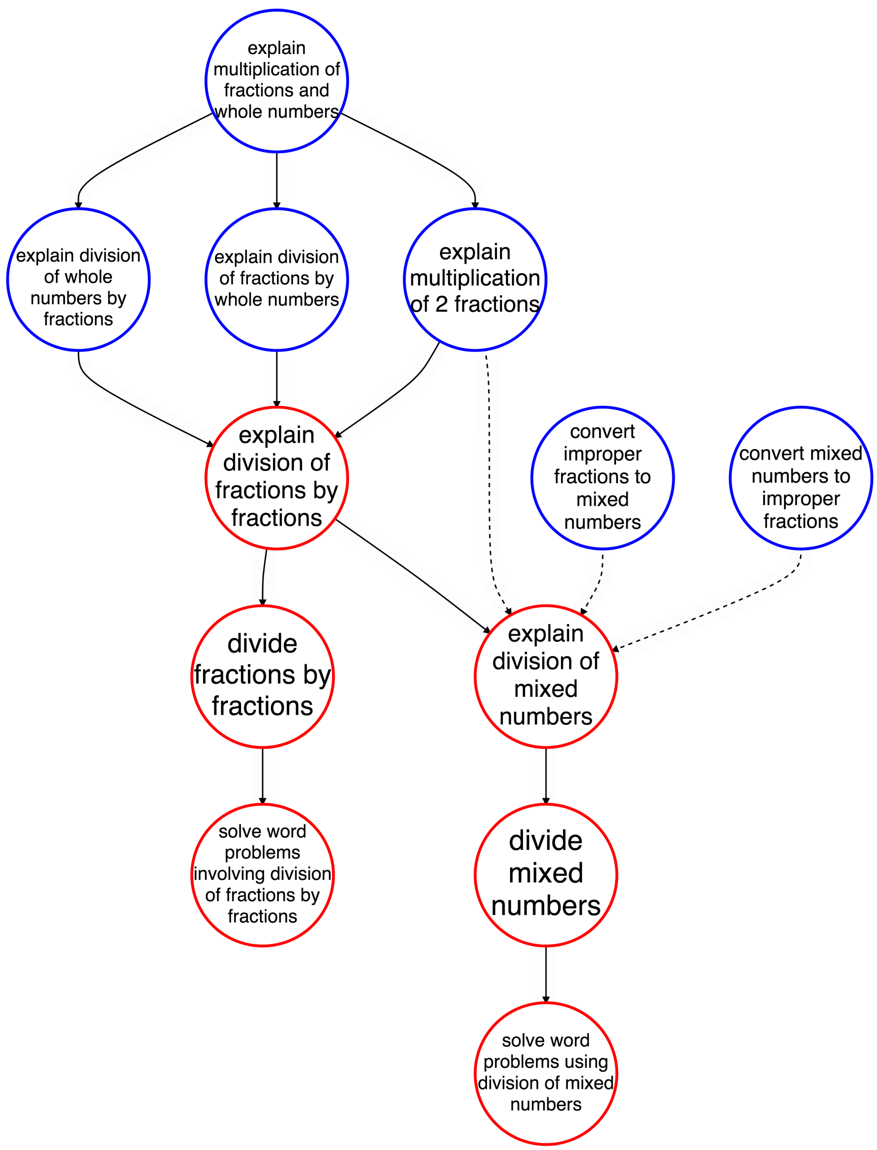
LEARNING MAP INFORMATION	An overview of the standards, the learning map section, and the nodes addressed in this unit
TEACHER NOTES	A brief discussion describing the progression depicted in the learning map section with research-based recommendations for focusing instruction to foster student learning and an introduction to the unit's lessons
OVERVIEW OF INSTRUCTIONAL ACTIVITIES	A table highlighting the lesson goals and nodes addressed in each lesson of this unit
INSTRUCTIONAL ACTIVITY	A detailed walkthrough of the unit
INSTRUCTIONAL ACTIVITY STUDENT HANDOUT	A handout for the guided activity, intended to be paired with the Instructional Activity
INSTRUCTIONAL ACTIVITY SUPPLEMENT	A collection of materials or activities related to the Instructional Activity
STUDENT ACTIVITY	A work-alone activity for students
STUDENT ACTIVITY SOLUTION GUIDE	A solution guide for the work-alone activity with example errors, misconceptions, and links to the learning map section

Division of Fractions

LEARNING MAP INFORMATION

STANDARDS

6.NS.1 Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, requiring multiple exposures connecting various concrete and abstract models.



**Learning map model of* 6.NS.1

Node Name	Node Description
CONVERT IMPROPER FRACTIONS TO MIXED NUMBERS	Change the given improper fraction to a mixed number.
CONVERT MIXED NUMBERS TO IMPROPER FRACTIONS	Change the given mixed number to an improper fraction.
DIVIDE FRACTIONS BY FRACTIONS	Divide a fraction by a fraction using models or algorithms. For example, determine a quotient using the standard algorithm, common-denominator algorithm, or models such as fraction strips.
DIVIDE MIXED NUMBERS	Divide mixed numbers using models or algorithms. For example, convert the mixed number(s) to improper fractions and determine the quotient by using the standard algorithm, common-denominator algorithm, or models such as fraction strips.
EXPLAIN DIVISION OF FRACTIONS BY FRACTIONS	Through words, drawings, manipulatives, or equations make known your understanding of the quotient that results from dividing a fraction by a fraction. Additionally, tell when a fraction divided by a fraction will result in a quotient larger than the dividend, and when a fraction divided by a fraction will result in a quotient smaller than the dividend. For example, use visual or concrete representations to model how, $\frac{2}{3}$ divided by $\frac{1}{6}$ results in small equipartitions and how that correlates with inverting and multiplying when using the standard algorithm (i.e., the same as $\frac{2}{3}$ times $\frac{6}{1}$).
EXPLAIN DIVISION OF FRACTIONS BY WHOLE NUMBERS	Make known your understanding through words, drawings, manipulatives, etc. that when you divide a fraction by a whole number, it will result in a smaller quotient.
EXPLAIN DIVISION OF MIXED NUMBERS	Through words, drawings, manipulatives or equations make known your understanding of the quotient that results from dividing mixed numbers (e.g., using visual or concrete models to represent the problem and determine the quotient).
EXPLAIN DIVISION OF WHOLE NUMBERS BY FRACTIONS	Make known your understanding through words, drawings, manipulatives, etc., that when you divide a whole number by a proper fraction, it will result in a greater quotient.
EXPLAIN MULTIPLICATION OF 2 FRACTIONS	Make known your understanding through words, drawings, manipulatives, etc. that when you multiply two fractions, you multiply the numerators and the denominators.
EXPLAIN MULTIPLICATION OF FRACTIONS AND WHOLE NUMBERS	Make known your understanding that when you multiply a fraction and a whole number, you must write the whole number as a fraction with a denominator of 1, and then multiply the numerators and the denominators.
SOLVE WORD PROBLEMS INVOLVING DIVISION OF FRACTIONS BY FRACTIONS	Use division to solve word problems involving fractions divided by fractions. This can be done with manipulatives, drawings, etc.
SOLVE WORD PROBLEMS USING DIVISION OF MIXED NUMBERS	Solve word problems using division of mixed numbers.

DIVISION OF FRACTIONS

TEACHER NOTES

This unit includes the following documents:

- Learning Map Information
- Instructional Activity (four lessons)
- Instructional Activity Student Handout (for Lesson 2 and 3)
- Instructional Activity Supplement (for Lessons 1, 3, 4)
- Student Activity
- Student Activity Solution Guide

In this unit, students will model division of a fraction by a fraction and learn the two algorithms (invert-and-multiply and common denominator) for dividing fractions.

RESEARCH

"Division of fractions is one of the least understood topics" for students of mathematics (Lappan, Smith, Jones, 2012; Van de Walle, Karp, Lovin, Bay-Williams, 2014). This is partially due to the fact that students often do not understand the conceptual reasoning behind the traditional "invert-and-multiply" algorithm, causing confusion about when to use the algorithm and why it works. In order to provide students with a deeper understanding of the processes to solve division of fractions, students must learn to divide fractions from a conceptual standpoint and be given time to notice patterns and develop the algorithms on their own.

To help students develop a conceptual understanding of division with fractions, students should be presented with contextual problems to solve (Petit, Laird, Marsden & Ebby, 2016). Students find contextual problems more relatable and approachable than non-contextual problems, and contextual problems provide meaning for problem-solving strategies as well as solutions. Real-world problems also provide a natural entry point for students to begin exploring division of fractions. For example, asking a student to solve $\frac{1}{2} \div \frac{3}{4}$ does not naturally lead a student to a solution strategy. However, giving a student a contextual problem to solve invites the student to create a visual model to begin solving. Consider the following example.

> If mowing $\frac{1}{2}$ the yard took $\frac{3}{4}$ of an hour, how much of the yard will be mowed in 1 hour?

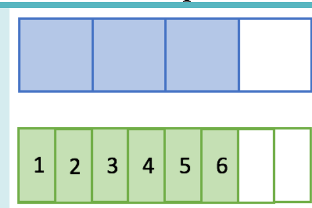
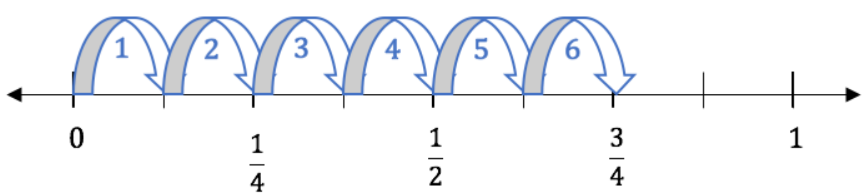
Students can then imagine a lawn and use the visual model strategy to solve. Eventually students can apply this strategy to other more abstract problems.

Visual models and manipulatives should be used abundantly during students' initial exploration of division of fractions (Petit et al., 2016). Because students struggle with estimating and conceptualizing fractions, number lines should be used to help students place fractions and estimate the magnitude of the dividend, divisor, and quotient (Van de Walle et al., 2014).

AN EXAMPLE

Students need to use concrete and semi-concrete models when exploring division of fractions in order to give meaning to algorithms. The following table shows how to represent the given problem situation with two different models.

A serving size is $\frac{1}{8}$ of a pizza. Catherine has $\frac{3}{4}$ of a pizza. How many servings does Catherine have?

Model	Example
Fraction Bars/Area Model	
Number Line	

Initially, students' visual models may contain mistakes or inaccuracies, and teachers should guide students toward correct representations through conversation. Identify aspects of the model that are correct as a jumping-off point for the conversation, such as a correct representation of the whole or correct partitioning. Student mistakes may be related to a misunderstanding of what the question is asking, or to an inability to correctly represent the scenario with a model. A more precise understanding of students' misconceptions can be discovered through student-teacher dialogue.

Another way to strengthen students' abstract understanding of fractions and division with fractions is to require students to estimate the quotient before performing and calculations (Petit et al, 2016; Small, 2014). Quotients can first be approximated as either greater than or less than 1. The inverse relationship between multiplication and division should be emphasized to help students estimate quotients. Multiplying by a number between 0 and 1 will reduce the value, but dividing by a number between 0 and 1 will have the inverse effect (i.e. the value will become greater). Quotients can also be approximated more closely by comparing the factors to benchmark numbers such as 1 and $\frac{1}{2}$. For example, $3 \div \frac{1}{2}$ is 6, so $3 \div \frac{3}{5}$ (3 divided by a number slightly larger than $\frac{1}{2}$) would result in a quotient slightly less than 6. Estimation builds students' number sense and intuition, which will make mistakes in computation more apparent.

When introducing division of fractions, it is important to ground new information in students' prior knowledge of division with whole numbers (Van de Walle et al., 2014). Students may need to see examples with whole numbers in order to remind them of the interpretations and patterns of division. Because there are several interpretations of division, students may be confused about which situations call for division and which call for multiplication. Providing contextual problems allows students to interpret their solution and evaluate their validity, therefore providing students with scenarios containing whole numbers can help a student sort out when division is necessary for solving a problem.

Algorithms should not be introduced until students have had ample time to explore many division problems with manipulatives and visual models and to develop their own strategies for solving (Lappan et al., 2012). Students' strategies should become more sophisticated over time as students gain understanding and confidence. For example, students may begin with a repeated subtraction strategy, but they should find more efficient strategies as they work with more examples. It is helpful if initial examples contain whole numbers, unit fractions, and simpler numbers so that students begin to notice patterns and structure of division.

AN EXAMPLE

Just as there are two types of division for whole numbers, there are two types of division when dividing fractions. Quotitive division, also known as measurement division, is a type of division in which the number of groups is unknown. Van de Walle, et al. (2014) recommend beginning instruction with quotitive division because it is more easily represented with visual models.

Partitive division, when the size of each group is unknown, does not always make sense with fraction division. For example, dividing $\frac{3}{4}$ pound of candy amongst $\frac{2}{3}$ of a person does not make sense. However, partitive division with fractions can be used to find the "whole" in a situation (e.g., when it takes $\frac{1}{2}$ hour to paint $\frac{1}{3}$ of a wall, $\frac{1}{2} \div \frac{1}{3}$ will calculate how long it takes to paint the whole wall).

Division Type	Whole Number Example	Fraction Example
Quotitive (Measurement) Division	Ms. Kramer has 60 flashcards and is making packs of 12 flashcards. How many students get a pack of flashcards? $60 \div 12 = 5$	Ms. Kramer has $\frac{6}{8}$ pound of jelly beans, and each student in her class gets $\frac{1}{4}$ pound of jelly beans. How many students will get jelly beans? $\frac{6}{8} \div \frac{1}{4} = 3$
Partitive Division	Ms. Kramer has 60 flashcards and 12 students. How many flashcards will each student get? $60 \div 12 = 5$	Ms. Kramer eats $\frac{1}{8}$ pound of jelly beans in $\frac{3}{4}$ of an hour. How long will it take her to eat a pound of jelly beans? $\frac{3}{4} \div \frac{1}{8} = 6$

Students' experiences with division of fractions should encompass both types of division, and practice should eventually mix the two types together. Problems should also include all types of remainders, because if students only encounter problems with no remainders, they will assume that this is always the case. Problems should have remainders of zero, whole-number remainders, and fraction remainders. Remainders should also be interpreted in the context of the problem and rounded or adjusted as necessary.

Quotitive division easily associates with the common denominator solving strategy, which students often notice or begin to create on their own through exploration of word problems (Lappan et al., 2012). Begin with whole number examples and examples with unit fractions so students can connect to their prior knowledge and develop their own strategies for solving that can then be applied to more difficult problems. Students should notice that dividing fractions with the same denominator is the same as dividing the numerators. Once students make this observation, teachers can provide formal explanations and appropriate vocabulary.

Partitive division of fractions should follow students' exploration of quotitive division of fractions. Students should realize that while they can use the common denominator strategy to solve partitive division problems, the interpretation of partitive division will lend itself to the discovery of the "invert-and-multiply" strategy (Lappan et al., 2012). As with quotitive division, instruction should begin with whole numbers and then include toward unit fractions for students to begin noticing patterns and develop algorithms on their own.

CAUTIONS AND MISCONCEPTIONS

Many misconceptions about division of fractions derive from misconceptions that students hold about division of whole numbers. One such misconception is that it isn't possible to divide a number by a greater value. For example, students may believe that you cannot divide 3 by 8 because they have a rigid definition of division that does not allow for quotients that are less than 1.

Additionally, students may have developed a misconception that division will always result in a lesser value. This can be especially difficult with division of fractions, because dividing by a fraction between 0 and 1 will cause the dividend to increase. This misconception is also linked to students' prior experiences with whole number division, where dividends are often greater than the quotient.

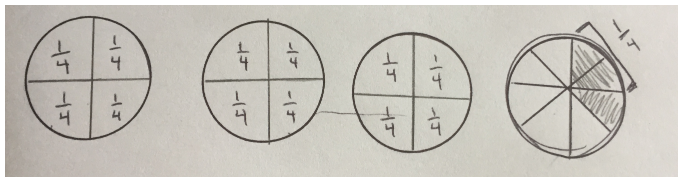
Students may also lack a conceptual understanding of division and will typically confuse situations that call for multiplication with situations that call for division (Small, 2014). Because there are several interpretations of division and they are closely related to multiplication, students find it difficult to identify what operation is required to solve a word problem.

When students expand their knowledge of division to include fractions, they may believe that all quotients of division of fractions must also be written as fractions. For example, if solving the problem $\frac{4}{7} \div \frac{2}{7}$, students may apply the common denominator algorithm incorrectly and believe that the answer should be $\frac{2}{7}$ rather than 2.

AN EXAMPLE

Students also struggle with problems that contain remainders, both in calculating the amount of the remainder and in describing the units of the remainder (Van de Walle et al., 2014).

Consider the example $3\frac{3}{8} \div \frac{1}{4}$.



There are 12 fourths in 3, and another fourth in $\frac{2}{8}$. Therefore, the answer is 13 with a remainder. Students often incorrectly believe that the remainder should be $\frac{1}{8}$ because that is the fraction of the whole left over. However, it must be determined how much of $\frac{1}{4}$ is left over, not how much of the whole.

To help students avoid this misconception, teachers should use deliberate phrasing, such as "How many fourths are in $3\frac{3}{8}$?" Students should realize that the quotient is expressed as fourths, therefore they are determining how many fourths remain. It is also helpful to ask students, "How many of the next piece do you have?", referring to one-fourth sized pieces. Since $\frac{1}{8}$ is half of $\frac{1}{4}$, the answer is $13\frac{1}{2}$.

The unit of the quotient is the divisor, but caution should be exercised so that students do not write their answer with a denominator of 4, as described in the previous paragraph. Just as when answering "How many fives are in 10?", we say "two", not "two fives". Students do not need to answer as "$13\frac{1}{2}$ fourths".

LEARNING MAP INFORMATION

The learning map section for this sequence of activities begins with an understanding of multiplication of fractions both by whole numbers and other fractions. Students then progress to dividing whole numbers by fractions, and eventually to dividing fractions by fractions. Students will also explain division of fractions and use the knowledge to solve word problems. Finally, students extend their knowledge of division of fractions to incorporate division of mixed numbers.

INSTRUCTIONAL ACTIVITIES

The activities in this unit are designed to allow students to develop the algorithms to solve division of fractions. Initially, students explore division scenarios without setting up or solving equations. LESSON 1 requires students to find the number of servings in varying amounts of food in order to lay the groundwork for future division scenarios and algorithms. LESSON 2 requires students to determine quotients using models and to find patterns and connections between the division problems and the quotient. From this experience, students should discover the common-denominator strategy. Similarly, in LESSON 3, students are given strategic problems that lead them to discover the invert-and-multiply strategy. Lastly, LESSON 4 is designed to allow students to practice both algorithms and determine when one method is more efficient than the other.

REFERENCES

Lappan, G., Smith, M. S., & Jones, E. (2012). *Rich & engaging mathematical tasks: Grades 5-9*. Reston, VA: National Council of Teachers of Mathematics.

Petit, M. M., Laird, R. E., Marsden, E. L., & Ebby, C. B. (2016). *A focus on fractions: Bringing research to the classroom* (2nd ed.). New York: Routledge.

Small, M. (2014). *Uncomplicating fractions to meet the common core standards in math, K-7*. New York: Teachers College Press.

Van de Walle, J. A., Karp, K. S., Lovin, L. H., & Bay-Williams, J. M. (2014). *Teaching student-centered mathematics: Developmentally appropriate instruction for grades 3-5* (Vol. 2). Upper Saddle River, NJ: Pearson.

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Division of Fractions

OVERVIEW OF INSTRUCTIONAL ACTIVITIES

Lesson	Learning Goal	Nodes Addressed
Lesson 1	Students will explore division of positive quotients through a real-world exploration, relate division of fractions to division of whole numbers, and informally discover strategies for solving division of fractions by fractions.	▶ CONVERT MIXED NUMBERS TO IMPROPER FRACTIONS ▶ CONVERT IMPROPER FRACTIONS TO MIXED NUMBERS ▶ EXPLAIN DIVISION OF FRACTIONS BY WHOLE NUMBERS ▶ EXPLAIN DIVISION OF FRACTIONS BY FRACTIONS
Lesson 2	Students will formalize the common denominator strategy for diving fractions by fractions.	▶ EXPLAIN DIVISION OF WHOLE NUMBERS BY FRACTIONS ▶ EXPLAIN DIVISION OF FRACTIONS BY FRACTIONS ▶ INSERT NODE NAME
Lesson 3	Students will explore division of fractions in order to formalize the invert-and-multiply strategy.	▶ EXPLAIN DIVISION OF WHOLE NUMBERS BY FRACTIONS ▶ EXPLAIN DIVISION OF FRACTIONS BY FRACTIONS ▶ EXPLAIN *MIXED NUMBERS* ▶ CONVERT MIXED NUMBERS TO IMPROPER FRACTIONS ▶ DIVIDE FRACTIONS BY FRACTIONS ▶ SOLVE WORD PROBLEMS INVOLVING DIVISION OF FRACTIONS BY FRACTIONS ▶ DIVIDE MIXED NUMBERS ▶ SOLVE WORD PROBLEMS INVOLVING DIVISION OF MIXED NUMBERS
Lesson 4	Students will practice the two algorithms for solving division of fractions problems, and justify their method of solving.	▶ EXPLAIN DIVISION OF FRACTIONS BY FRACTIONS ▶ DIVIDE FRACTIONS BY FRACTIONS ▶ SOLVE WORD PROBLEMS INVOLVING DIVISION OF FRACTIONS BY FRACTIONS ▶ DIVIDE MIXED NUMBERS ▶ SOLVE WORD PROBLEMS INVOLVING DIVISION OF MIXED NUMBERS

Division of Fractions

INSTRUCTIONAL ACTIVITY

Lesson 1

LEARNING GOAL

Students will explore division of positive quotients through a real-world exploration, relate division of fractions to division of whole numbers, and informally discover strategies for solving division of fractions by fractions.

PRIMARY ACTIVITY

Students answer questions about serving size using models, which increase in difficulty from whole-number serving sizes to fractional serving sizes. Students will develop their own strategies and algorithms for solving, then share their strategies with the class.

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

- Common denominator
- Dividend
- Divisor
- Quotient
- Serving size

MATERIALS

- Instructional Activity Student Handout (Recommend one for each student.)
- Instructional Activity Supplement – fraction bars (Recommend one set for each student.)

IMPLEMENTATION

Distribute the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT to each student.

Direct students' attention to the nutrition label on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT and review how to read it. Specifically, ensure that students understand where to locate and how to interpret the serving size and the number of servings per container.

Ask students to determine the total number of cookies in the container. **Call on** one or two students to explain their strategy for finding the total number of cookies. **Discuss** how the total number of *cookies* in the container is different than the total number of *servings* in the container.

Tell students that last night you ate 6 cookies. **Show or draw** a picture of 6 cookies (circles), which also appears on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT. **Ask** students to find the number of servings you ate using the picture to model the situation, or students may create another model or an equation.

Call on two or three students to share with the class how they found the number of servings in six cookies. Focus the discussion on the process instead of specific values. Strategies that students may share are:

- Shading the serving size in two cookies and counting the groups
- Repeated subtraction on a number line
- Repeated subtraction of the serving size from the servings eaten
- Division of the servings eaten by the serving size
- Finding the missing factor in the multiplication $2 \times \textit{servings eaten} = 6$

If one of the strategies is not offered, **share** the strategy with students as work a previous student had completed.

GUIDING QUESTIONS

Elicit student thinking:

- Will the number of servings be more or less than 6?
- Can you rephrase the question [more simply]?
- How should you label your answer?
- Does the serving size divide evenly into the number of cookies eaten?

Tell students that the night before last you ate 5 cookies. **Show or draw** a picture of 5 cookies, which also appears on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT. **Ask** students to find the number of servings you ate using the picture to model the situation, or students may create another model or equation.

Call on one or two students to explain their strategy for finding the number of servings in five cookies. Specifically, discuss how the remainder was found.

GUIDING QUESTIONS

Elicit student thinking:

- Will the number of servings be more or less than 5?
- Can you rephrase the question [more simply]?
- How should you label your answer?
- Does the serving size divide evenly into the number of cookies eaten?
- How many cookies are left over?
- What portion of a serving is the leftover amount?
- Can you show this relationship on a number line?

Tell students that tomorrow night you plan on eating $4\frac{1}{2}$ cookies. **Show or draw** a picture of $4\frac{1}{2}$ cookies, which also appears on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT. **Ask** students to find the number of servings you ate using the picture to model the situation, or students may create another model or an equation.

GUIDING QUESTIONS

Elicit student thinking:

- Will the number of servings be more or less than $4\frac{1}{2}$?
- Can you rephrase the question [more simply]?
- How should you label your answer?
- Does the serving size divide evenly into the number of cookies eaten?
- Can you show this relationship on a number line?

Determine if the student can CONVERT MIXED NUMBERS TO IMPROPER FRACTIONS:

- [Point to an answer in mixed-number form.] How do you rewrite your answer as an improper fraction?

Determine if the student can CONVERT IMPROPER FRACTIONS TO MIXED NUMBERS:

- [Point to an answer in improper-fraction form.] How do you rewrite your answer as a mixed number?

Determine if the student can EXPLAIN DIVISION OF FRACTIONS BY WHOLE NUMBERS:

- How many cookies are left over?
- What portion of a serving is the leftover amount?
- How did you find the fraction of a serving left?
- Can you write a division problem to represent the leftover serving amount?
- What is the dividend of the division problem?
- What is the divisor of the division problem?

Tell students that last week you went to a party where cookies were served. Because these cookies were much larger, the serving size is only $\frac{1}{2}$ of a cookie. **Distribute** one set of fraction bars (INSTRUCTIONAL ACTIVITY SUPPLEMENT) to each student, and **tell** students that the fraction bars may be used to help answer the questions on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

Ask students to continue answering the questions about the $\frac{1}{2}$ serving size in the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT. **Circulate** and ask the following guiding questions and help students as needed.

GUIDING QUESTIONS

Elicit student thinking:

- Will the number of servings be more or less than $\frac{1}{2}$?
- Can you rephrase the question [more simply]?
- How should you label your answer?
- Can you show this relationship with fraction bars?
- Can you show this relationship on a number line?
- Can you show this relationship with a division equation?

Determine if the student can EXPLAIN DIVISION OF WHOLE NUMBERS BY FRACTIONS:

- Is the quotient greater or less than the dividend? [i.e. is the number of servings greater or less than the serving size?]
- Why does dividing by a fraction make the dividend larger?
- How are multiplying by a fraction and dividing by a fraction related?

Determine if the student is ready to EXPLAIN DIVISION OF FRACTIONS BY FRACTIONS:

- Compare and contrast the problems with fractional amounts of cookies eaten with problems with whole number amounts of cookies eaten.
- Why does dividing by a fraction make the dividend larger?
- How are multiplying by a fraction and dividing by a fraction related?

Review the answers to the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT with students. LESSON 2 will go over the questions more formally, so this should be an informal discussion and student-sharing opportunity.

Direct students' attention to Question #8. **Encourage** students to use their fraction bars to help answer the following questions.

- How many one-half's fit into $\frac{3}{4}$?
- How many halves are left, after $\frac{1}{2}$ is taken from $\frac{3}{4}$?
- How much of $\frac{1}{2}$ is $\frac{1}{4}$?

Direct students' attention to Question #9. **Encourage** students to use their fraction bars to help answer the following questions.

- How many one-half's fit into $\frac{3}{8}$?
- Can you divide the figure into eighths to help answer the question?
- How many eighths are in $\frac{1}{2}$?
- Is there an equivalent way to write $\frac{1}{2}$ that would help us answer this question?

Direct students' attention to Question #10. **Encourage** students to use their fraction bars to help answer the following questions.

- How many one-half's fit into $\frac{5}{8}$?
- Can you divide the figure into eighths to help answer the question?
- How many eighths are in $\frac{1}{2}$?
- Is there an equivalent way to write $\frac{1}{2}$ that would help us answer this question?
- What fraction of $\frac{1}{2}$ is left, after $\frac{1}{2}$ is taken from $\frac{5}{8}$?

Direct students' attention to Question #11. **Encourage** students to use their fraction bars to help answer the following questions.

- How many one-half's fit into $\frac{11}{8}$?
- Can you divide the figure into eighths to help answer the question?
- How many eighths are in $\frac{1}{2}$?
- Is there an equivalent way to write $\frac{1}{2}$ that would help us answer this question?
- What fraction of $\frac{1}{2}$ is left, after $\frac{2}{2}$ is taken from $\frac{11}{8}$?

Students should be required to answer questions about the following nutrition label.

Nutrition Facts

12 servings per container

Serving size	**2/3 cookies**
Amount Per Serving	
Calories	**180**
	% Daily Value*
Total Fat 7g	**9%**
Saturated Fat 0g	**0%**
Trans Fat 0g	
Sodium 0mg	**0%**
Total Carbohydrate 27g	**10%**
Dietary Fiber 0g	**0%**
Total Sugars 19g	
Includes 0g Added Sugars	**0%**
Protein 1g	**2%**

Not a significant source of cholesterol, vitamin D, calcium, iron, and potassium

*The % Daily Value (DV) tells you how much a nutrient in a serving of food contributes to a daily diet. 2,000 calories a day is used for general nutrition advice.

- How many total cookies are in the container?
- How many servings are in 2 cookies?
- How many servings are in $\frac{4}{3}$ of a cookie?

At the end of the activity, teachers should arrange students into groups of three to four, and students should share observations that they had about the serving-size questions (as answered on Question #12 and #13 of the Instructional Activity Student Handout). Each group should share one observation with the class.

Name________________________________

Division of Fractions

Lesson 1

Nutrition Facts

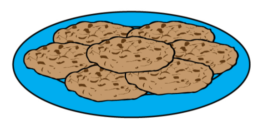
8 servings per container

Serving size	**2 cookies**
Amount Per Serving	
Calories	**210**
	% Daily Value*
Total Fat 10g	**13%**
Saturated Fat 0g	**0%**
Trans Fat 0g	
Sodium 0mg	**0%**
Total Carbohydrate 33g	**12%**
Dietary Fiber 0g	**0%**
Total Sugars 21g	
Includes 0g Added Sugars	**0%**
Protein 1g	**2%**

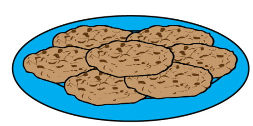
Not a significant source of cholesterol, vitamin D, calcium, iron, and potassium

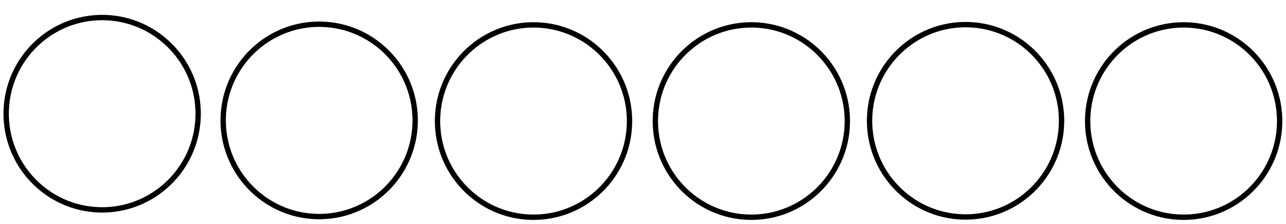
*The % Daily Value (DV) tells you how much a nutrient in a serving of food contributes to a daily diet. 2,000 calories a day is used for general nutrition advice.

1. How many cookies are in the container? Show your work, or explain how you know.



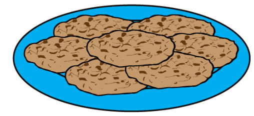
2. Your teacher ate 6 cookies last night. How many servings did your teacher eat?

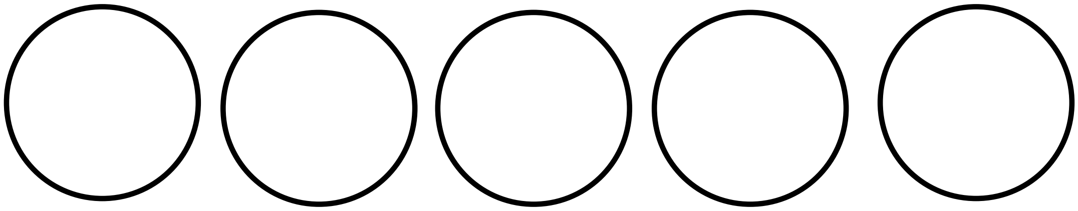




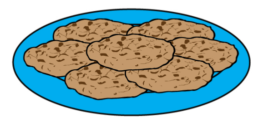
Name______________________________

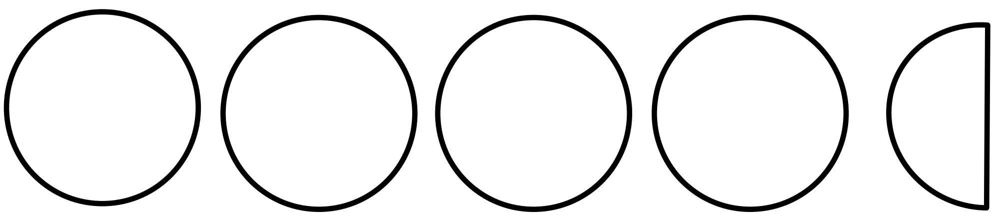
3. Your teacher ate 5 cookies the night before last.
   How many servings did your teacher eat?





4. Your teacher will eat 4 $\frac{1}{2}$ cookies tomorrow night.
   How many servings will your teacher eat?





Name______________________________

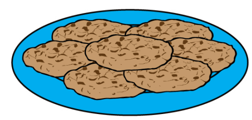
Nutrition Facts

Nutrition Facts	
10 servings per container	
Serving size	**1/2 cookie**
Amount Per Serving	
Calories	**210**
	% Daily Value*
Total Fat 10g	**13%**
Saturated Fat 0g	**0%**
Trans Fat 0g	
Sodium 0mg	**0%**
Total Carbohydrate 33g	**12%**
Dietary Fiber 0g	**0%**
Total Sugars 21g	
Includes 0g Added Sugars	**0%**
Protein 1g	**2%**

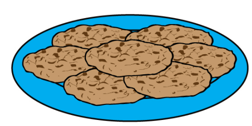
Not a significant source of cholesterol, vitamin D, calcium, iron, and potassium

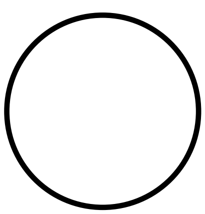
*The % Daily Value (DV) tells you how much a nutrient in a serving of food contributes to a daily diet. 2,000 calories a day is used for general nutrition advice.

5. How many cookies are in the container? Show your work, and explain how you know.



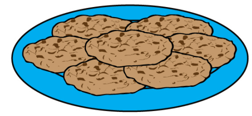
6. If your teacher ate 1 cookie, how many servings did your teacher eat?

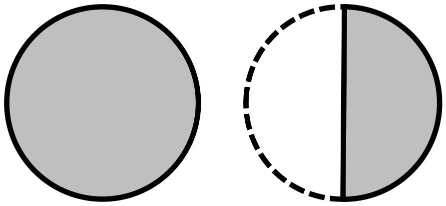




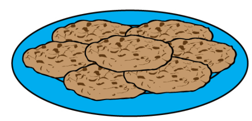
Name______________________________

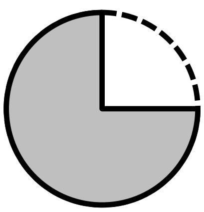
7. If your teacher ate $\frac{3}{2}$ of a cookie, how many servings did your teacher eat?



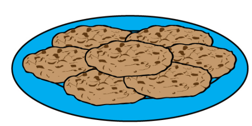


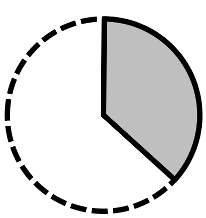
8. If your teacher ate $\frac{3}{4}$ of a cookie, how many servings did your teacher eat?





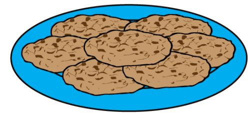
9. If your teacher ate $\frac{3}{8}$ of a cookie, how many servings did your teacher eat?

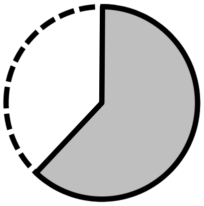




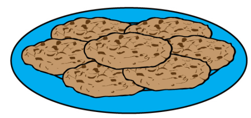
Name_______________________________

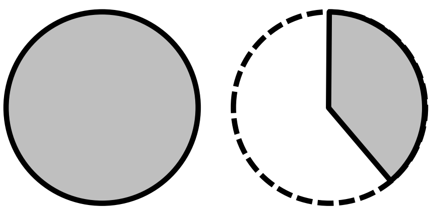
10. If your teacher ate $\frac{5}{8}$ of a cookie, how many servings did your teacher eat?





11. If your teacher ate $\frac{11}{8}$ of a cookie, how many servings did your teacher eat?





12. What strategies did you use to answer the questions?

13. What patterns do you see in your answers?

1																			
$\frac{1}{2}$										$\frac{1}{2}$									
$\frac{1}{3}$							$\frac{1}{3}$							$\frac{1}{3}$					
$\frac{1}{4}$					$\frac{1}{4}$					$\frac{1}{4}$					$\frac{1}{4}$				
$\frac{1}{5}$				$\frac{1}{5}$				$\frac{1}{5}$				$\frac{1}{5}$				$\frac{1}{5}$			

$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$									
$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$								
$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$							
$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$						
$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$					
$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$			
$\frac{1}{15}$	$\frac{1}{15}$	$\frac{1}{15}$	$\frac{1}{15}$	$\frac{1}{15}$	$\frac{1}{15}$	$\frac{1}{15}$	$\frac{1}{15}$	$\frac{1}{15}$	$\frac{1}{15}$	$\frac{1}{15}$	$\frac{1}{15}$	$\frac{1}{15}$	$\frac{1}{15}$	$\frac{1}{15}$

Division of Fractions

INSTRUCTIONAL ACTIVITY

Lesson 2

LEARNING GOAL

Students will formalize the common-denominator strategy for diving fractions by fractions.

PRIMARY ACTIVITY

Students will review strategies for division and practice finding common denominators in order to divide fractions by fractions.

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

- Common denominator
- Dividend
- Divisor
- Quotient

MATERIALS

- Instructional Activity Student Handout
- Fraction Bars

IMPLEMENTATION

Distribute a set of fraction bars to each student to use as a tool during this lesson.

Remind students of the first problem in the Instructional Activity Student Handout Lesson 1.

A serving size is 8 cookies, and there are 8 servings per container. How many cookies are there per container?

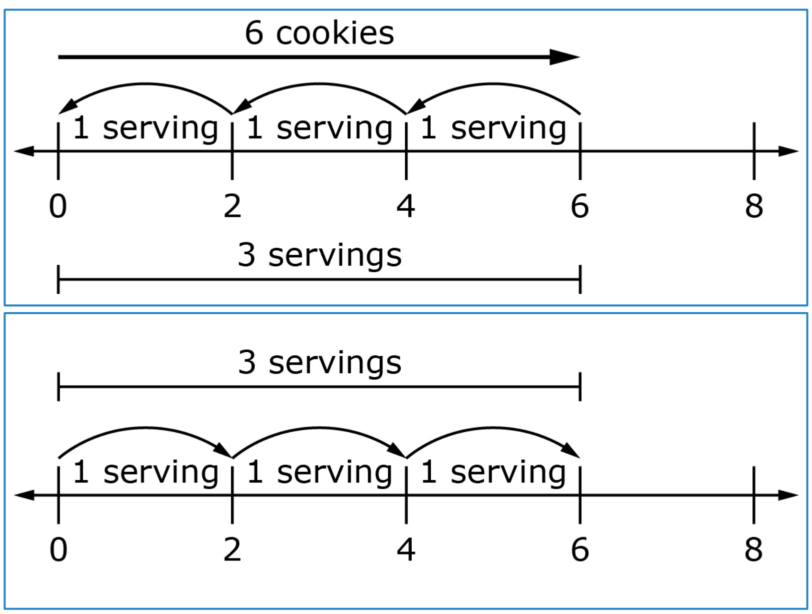
Call on two to three student volunteers to tell how they solved this question. If possible, call on students with less sophisticated approaches first, such as using a visual model or repeated addition, and have subsequent students show more sophisticated approaches, such as multiplication of 8×2. The focus of the lesson is not on multiplication, but it will be helpful to remind students of the models used, as well as the relationship between multiplication and division, as they begin to explore division of fractions.

Present the second question from the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT LESSON 1 to students.

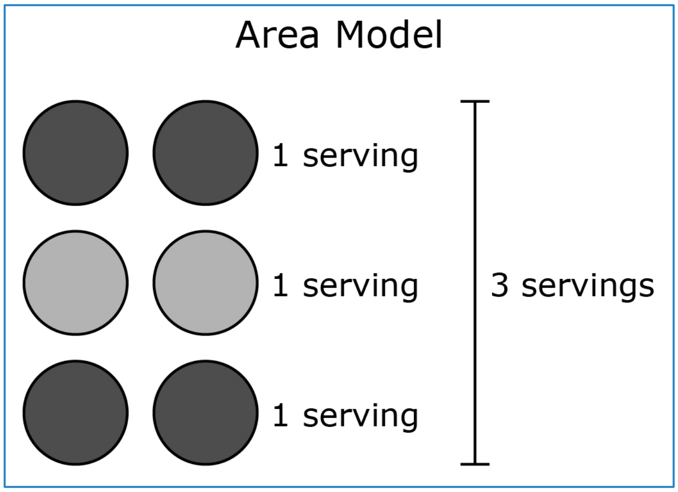
A serving size is 2 cookies. If your teacher eats 6 cookies, how many servings did your teacher eat?

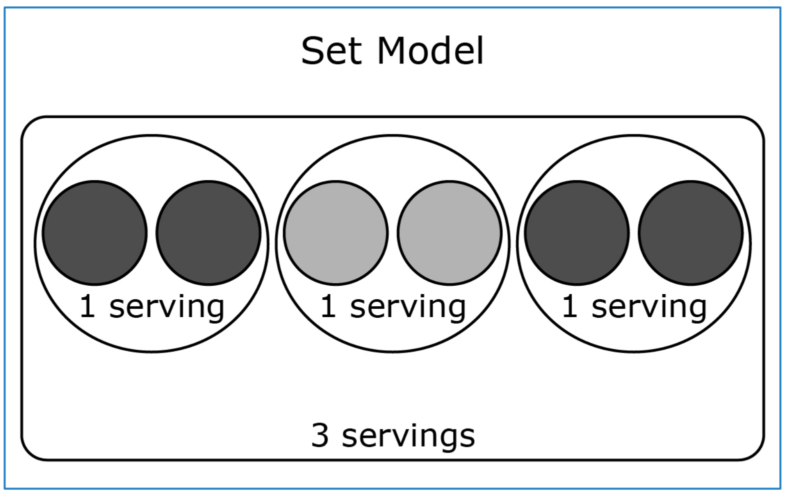
Ask, "How do you model this with a number line?" **Call on** a student volunteer to show and explain their model to the class, or **present** it as work done by a previous student. The number line should look similar to the following image.

NOTE: Students may begin from 0 counting by 2s to reach 6, or begin at 0, go to 6 and count back making groups of 2.



Ask, "How do you model this with an area model or a set model?" **Call on** a student volunteer to show and explain their model to the class, or **present** it as work done by a previous student. The model should look similar to the following image.





Ask, "How do you model this with a division equation?" **Call on** a student volunteer to show and explain their model to the class, or **present** it as work done by a previous student. The equation should be $6 \div 2 = 3$.

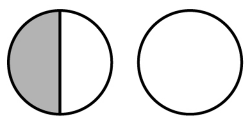
Ask, "How do you model this with a missing factor multiplication equation?" **Call on** a student volunteer to show the class, or **present** it as work done by a previous student. The equation should be $\square \times 2 = 6$.

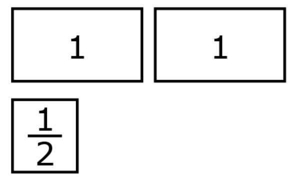
> NOTE: The missing factor multiplication strategy will be used in developing the invert-and-multiply strategy in LESSON 3.

Repeat this process with the remaining questions for the nutrition label from the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT LESSON 1 (finding the number of servings in 5 and $4\frac{1}{2}$ cookies). This will require interpretation of the remainder, and students should be guided to see that when finding the remainder, you are finding the portion of the divisor (i.e. the serving size) that remains.

For example, when 5 cookies are divided by a serving size of 2, there is 1 cookie left over that doesn't make a full serving. This is 1 cookie out of a serving size of 2 cookies, therefore the remainder is $\frac{1}{2}$ servings.

Similarly, when $\frac{1}{2}$ of a cookie is left out of a serving size of 2 cookies, this is $\frac{\frac{1}{2}}{2}$. **Help** students decide that this is $\frac{1}{4}$ by having a student volunteer draw and explain a visual model. Students may find the answer more easily through a visual model, so it is important to **provide** a connection to the algebraic strategies for solving.





Tell students that these strategies will be useful when solving other problems with more difficult values (e.g., values that are greater in magnitude, or fractions, which make the answer less obvious).

Focus on the final questions in INSTRUCTIONAL ACTIVITY LESSON 1, which have a fractional serving size instead of a whole-number serving size. The questions will be reviewed formally as a class so all students see the possible strategies for solving.

Show students the final question in INSTRUCTIONAL ACTIVITY LESSON 1.

> The serving size is $\frac{2}{3}$ of a cookie, and there are 12 servings in the container. How many total cookies are in the container?

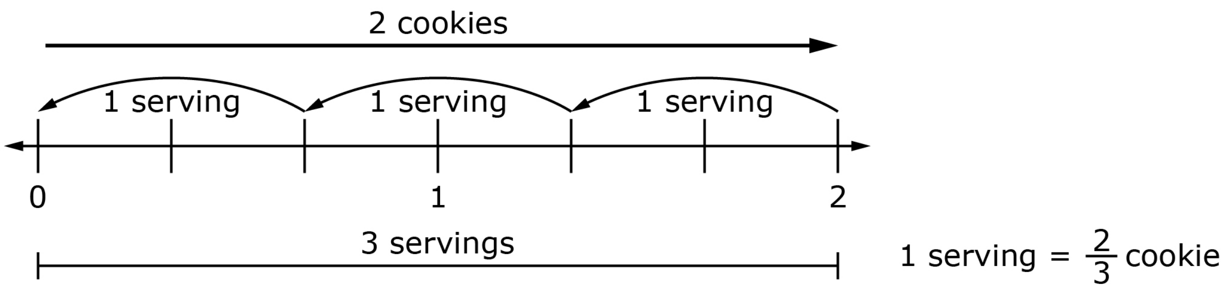
Call on two or three student volunteers to share how to find the total number of cookies in the container. They might use repeated addition, multiplication, or a visual model to determine the total. If possible, call on students with less sophisticated approaches first, such as using a visual model or repeated addition, and have subsequent students use more sophisticated approaches, such as multiplying $12 \times \frac{2}{3}$.

Direct students to consider, but not yet answer, the following question.

> The serving size is $\frac{2}{3}$ of a cookie, and there are 12 servings in the container. How many servings are in 2 cookies?

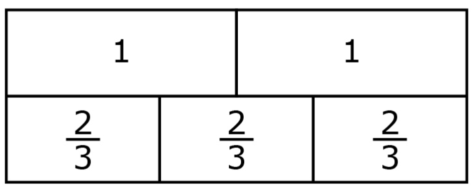
Ask, "What is another way to phrase this question?" **Call on** a student volunteer to rephrase the question. If no student volunteers, **rephrase** the question as "How many times does $\frac{2}{3}$ go into 2?"

Ask, "How do you model this question with a number line?" **Call on** a student volunteer to show how.



Say, "There are three $\frac{2}{3}$'s in 2."

Ask, "How do you model this question with fraction bars?" **Call on** a student volunteer to show and explain how.



Say, "There are three $\frac{2}{3}$ in 2."

Ask, "How do you model this question with a division equation?" **Call on** a student volunteer to show and explain how.

$$2 \div \frac{2}{3} = 3$$

Ask, "How can we rewrite 2 to have a denominator of 3?" **Tell** students that, to answer this question, they should determine how many thirds are in 2. **Call** on a student to rewrite the equation to have a common denominator.

$$\frac{6}{3} \div \frac{2}{3} = 3$$

Ask students what observations they have about the equation. Students should notice that the quotient can be determined by dividing only the numerators.

Formally **introduce** the idea that when dividing fractions with the same denominator, it is only necessary to divide the numerators. **Explain** to students that dividing whole-number fractions could be considered dividing a differing number of parts from the same-size whole. This can be used as a strategy for fractions that already have common denominators, or fractions can be rewritten to have common denominators in order to use this strategy.

Emphasize to students that the denominator of the dividend and divisor (in this case 3) is not the denominator of the quotient.

Introduce a similar way of considering the common-denominator problem. Consider the whole-number problem:

> Adam walked 10 miles, and Hannah walked 5 miles. How many times more miles did Adam walk than Hannah?

The problem can be solved as shown:

$$\frac{10\ \cancel{miles}}{5\ \cancel{miles}} = 2$$

Adam walked 2 times more miles than Hannah. We divided miles by miles, and therefore the quotient is *not* written with the unit "miles".

Bring the students' attention back to the rewritten division problem with a common denominator.

$$\frac{6}{3} \div \frac{2}{3}$$

This expression can be thought of as $6\ thirds \div 2\ thirds$, or $\frac{6\ thirds}{2\ thirds}$. Just as with other units (such as $\frac{10\ miles}{5\ miles}$) when thirds are divided by thirds, the unit of the quotient is not thirds. Therefore, $\frac{6\ \cancel{thirds}}{2\ \cancel{thirds}} = 3$. **Remind** students that the question was asking how many servings (of $\frac{2}{3}$ of a cookie) are in two cookies. Therefore, the units of the quotient are servings.

Emphasize again that "thirds" is not the unit or the denominator of the quotient.

GUIDING QUESTIONS

Elicit student thinking:

- What do you notice about dividing fractions with common denominators?
- How do you know that this scenario calls for division?

Determine if the student can EXPLAIN DIVISION OF WHOLE NUMBERS BY FRACTIONS:

- How do you rewrite the whole number as a fraction?
- How would you use the common-denominator strategy to divide a whole number by a fraction?

Determine if the student can EXPLAIN DIVISION OF FRACTIONS BY FRACTIONS:

- What does the common-denominator strategy help you do?
- When is it efficient to use the common-denominator strategy?
- When is it less efficient to use the common-denominator strategy?

Students should be required to complete the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT to practice the common-denominator strategy.

At the end of the activity, teachers should collect and review the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT for accuracy.

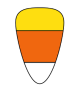
Name______________________________

Division of Fractions

Lesson 2

Use the **common-denominator strategy** to solve the following problems.

1. Shelley has $\frac{4}{5}$ of a pound of candy corn. She is splitting the candy corn into snack bags. Each snack bag has $\frac{1}{10}$ of a pound of candy corn. How many snack bags can she fill?



   a. Write a division problem representing the scenario.

   b. Rewrite the division problem with common denominators.

   c. Solve the division problem from part b to tell how many bags Shelley can fill.

Name______________________________

2. Solve: $\frac{7}{9} \div \frac{1}{3}$

3. Solve: $\frac{1}{3} \div \frac{7}{9}$

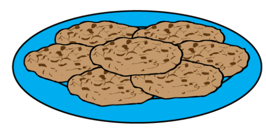
4. Solve: $\frac{3}{4} \div \frac{2}{6}$

5. Solve: $\frac{6}{8} \div \frac{3}{8}$

Name______________________________

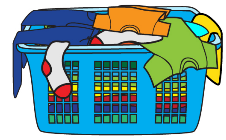
6. Sohail needs to measure out $1\frac{2}{3}$ cups of flour for a recipe. Unfortunately, he only has a $\frac{1}{4}$ measuring cup. How many $\frac{1}{4}$ cups will he need to measure out $1\frac{2}{3}$ cups?

7. Curtis is making cookies for a party. Each dozen calls for $\frac{2}{3}$ of a cup of sugar. If Curtis has $4\frac{2}{3}$ cups of sugar to use, how many dozens of cookies can he make?



Name______________________________

8. Josh is doing several loads of laundry. Each load takes $\frac{1}{5}$ of a cup of laundry detergent. How many loads can Josh do if there are 5 cups of laundry detergent in the container?



Division of Fractions

INSTRUCTIONAL ACTIVITY

Lesson 3

LEARNING GOAL

Students will explore division of fractions in order to formalize the invert-and-multiply strategy.

PRIMARY ACTIVITY

Students participate in a gallery walk to explore division of fractions. After finding the solutions to several division problems in the gallery walk, students make observations about the patterns and structures present in the problems. The invert-and-multiply strategy is then formally introduced and given meaning through missing-factor equations.

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

- Coefficient
- Denominator
- Division
- Inverse
- Missing factor
- Multiplication
- Numerator
- Reciprocal

MATERIALS

- Instructional Activity Supplement A (Recommend one copy per class.)
- Instructional Activity Supplement B (Recommend one copy per student.)
- Instructional Activity Handout
- Fraction bars (optional)

IMPLEMENTATION

Display the 10 division-of-fractions problems from the INSTRUCTIONAL ACTIVITY SUPPLEMENT A around the classroom or in the hallway.

The students will participate in a "gallery walk", so named because they will quietly go to each posted problem to read and answer the question, as if the problems were hanging in an art gallery. The problems do not need to be answered in order, but the problems should be answered on the corresponding problem space on the INSTRUCTIONAL ACTIVITY SUPPLEMENT B.

Tell students how to complete the gallery walk.

Inform them that they must complete an area model for each station even if they can find the answer without it. The area models have been started for them on the INSTRUCTIONAL ACTIVITY SUPPLEMENT B. Their instructions are to copy the division expression on the lines provided, then to write the quotient that they determine from the area model.

Allow students enough time to complete each station. The activity can be differentiated by pairing students homogenously, heterogeneously, pairing ELL students with a bilingual peer, having students work individually, etc.

GUIDING QUESTIONS

Elicit student thinking:

- Is there anything that most or all of the questions have in common?
- Do you notice a pattern in your answers?
- Do you notice a more efficient way to determine the answer?
- Which problems are easiest to notice a pattern?

Determine if the student can EXPLAIN DIVISION OF WHOLE NUMBERS BY FRACTIONS:

- [Point to a problem with a whole number divided by a fraction.] Are the quotients greater than or less than the dividend? How do you know?
- [Point to a problem with a whole number divided by a fraction.] Is the dividend between 0 and 1, or greater than 1? How do you know?

Determine if the student is ready to EXPLAIN DIVISION OF FRACTIONS BY FRACTIONS:

- [Point to a problem with a fraction divided by a fraction.] Are the quotients greater than or less than the dividend? How do you know?
- [Point to a problem with a whole number divided by a fraction.] Is the dividend between 0 and 1, or greater than 1? How do you know?
- Explain how the area model helps you determine the quotient.

When students finish the gallery walk, **divide** them in groups of two or three. **Require** students to share their answers with their group members. If there are any answers that students disagree on, have them explain their thinking and come to a consensus.

Direct all students to look at Question #1. **Ask** if they notice a way to find the answer without drawing an area model.

Call on one or two students to share their ideas and strategies. Do not tell any students that they are correct or incorrect at this point.

Direct students to look at Question #2. **Ask** if they notice a way to find the answer without drawing an area model, and if that strategy is consistent with the answer to Question #1.

Call on one or two students to share their ideas and strategies. At this point, a student has likely shared that if you multiply the whole number by the denominator of the fraction, the result is the quotient.

Direct students to verify if this strategy works on Questions 3, 5, and 7. Students should observe that the strategy works for these questions.

Show students why the strategy works with the following example (Question 1):

$$3 \div \frac{1}{2} = ?$$

Tell students that this division problem can be interpreted as a *missing-factor* problem. For example, the problem $20 \div 5$ can be thought of as, "What do you multiply 5 by to get 20?" Therefore, in Question 1, we are asking, "What can we multiply $\frac{1}{2}$ by to get 3?"

Write the corresponding equation.

$$\frac{1}{2}x = 3$$

Tell students that to solve this equation, the goal is to make the coefficient of the variable 1.

Ask, "What value multiplies with $\frac{1}{2}$ to create a product of 1?"

Students should answer that $\frac{1}{2}$ should be multiplied by 2 to create a product of 1. **Write** the corresponding equation.

$$2 \times \frac{1}{2}x = 3 \times 2$$

Simplify the equation.

$$1x = 6$$

$$x = 6$$

Reiterate for students that the dividend was multiplied by the denominator of the divisor, and that this solved the equation.

Direct students to look at Question #9. **Ask** students to use the strategy of multiplying by the denominator to solve this problem.

Call on one or two students (as necessary) to show or explain why the quotient is 2 (i.e. that $\frac{1}{2} \div \frac{1}{4} = \frac{1}{2} \times 4 = \frac{4}{2} = 2$).

Direct students to look at Question #8. **Display** for students the following portion of the multiplying-by-the-denominator strategy.

$$6 \div \frac{2}{3} \rightarrow 6 \times 3 = 18$$

Call on a student to share the quotient based on the area model. The student should share that the quotient is 9, not 18.

Ask students to think, individually, about how they could have used the multiply-by-the-denominator strategy to determine the correct quotient (9). Once they have had a moment to think individually, ask students to turn to a partner or form a group of three and share their ideas. **Call on** one or two groups to share how they think the problem should be solved.

Students should conclude that the whole number must be multiplied by the denominator and divided by the numerator to find the correct quotient.

Model this conclusion for students.

$$6 \div \frac{2}{3} = 6 \times \frac{3}{2} = \frac{18}{2} = 9$$

Ask students why they were able to multiply by the denominator in the previous examples and get the right answer without addressing the numerator.

Call on one or two students (as necessary) to explain that in reality, they were dividing by the numerator of 1, which caused the value of the quotient to remain the same.

GUIDING QUESTIONS

Determine if the student can EXPLAIN DIVISION OF WHOLE NUMBERS BY FRACTIONS:

- [Point to a problem with a whole number divided by a fraction.] Are the quotients greater than or less than the dividend? How do you know?
- [Point to a problem with a whole number divided by a fraction.] How do you know whether the quotient will be greater than or less than the dividend?

Determine if the student is ready to EXPLAIN DIVISION OF FRACTIONS BY FRACTIONS:

- [Point to a problem with a fraction divided by a fraction.] How do you know whether the quotient will be greater than or less than the dividend?
- [Point to a problem with a fraction divided by a fraction.] What algorithm will you use to find the quotient? Why?
- What are the two algorithms we have for dividing fractions by fractions?

Direct students to look at Question #6. **Ask** students to think about strategies that could be used to solve this mixed-number division problem. Give students time to think about it individually, then have them share with a partner or a group of three (as class size allows).

Circulate and **observe** student strategies.

In order for students to confront their misconceptions, students will explore the strategies they attempted that did not yield the correct quotient.

Ask students to share their correct and incorrect attempts at finding an algorithm that works with mixed numbers.

Ask students to share a strategy that they thought would work but did not yield a quotient of 5. If no students volunteer, **show** an attempt to multiply only the fractional component of the mixed number by the reciprocal of the divisor.

NOTE: As an alternative to students sharing non-working strategies, have students participate in a "My Favorite No" activity. Students write their work anonymously on a notecard, then turn it in to be reviewed. Once all the notecards have been quickly reviewed, the class discusses the teacher's "favorite no", which is an example/are examples of students' incorrect work that illustrate a common misconception or mistake.

$$2\frac{1}{2} \times \frac{2}{1} \rightarrow 2\frac{2}{2} \rightarrow 3$$

Discuss why this strategy did not work by calling on a student volunteer to offer an explanation or telling students that they must consider both the whole number and fractional component of the mixed number when performing the multiplication.

Continue to have a conversation about which strategies did not work and why they did not work.

There are several ways that students could solve Question #6:

- Students may count by $\frac{1}{2}$ until they get to $2\frac{1}{2}$.
- Some students may convert the mixed number into an improper fraction and use the common denominator algorithm to solve.
- Others may convert the mixed number to an improper fraction and use the invert-and-multiply algorithm now that they have had an introduction to it.

Call on student volunteers to share their successful strategies, beginning with less sophisticated strategies and moving toward more efficient and sophisticated strategies.

Conclude that the most efficient way to divide mixed numbers using the invert-and-multiply algorithm is by converting to improper fractions.

Direct students' attention to Question #10.

Ask students if they found it easy to solve Question #10 with an area model. **Call on** two or three students to share how easily they were able to solve the problem with the area model.

Discuss the idea that some methods of solving are more efficient than others, and it doesn't make sense to solve all division problems with an area or length model.

Call on a student to use the invert-and-multiply algorithm to solve.

$$\frac{2}{3} \div \frac{3}{4} = \frac{2}{3} \times \frac{4}{3} = \frac{8}{9}$$

Introduce an alternative way of considering the invert-and-multiply algorithm, which serves to establish the validity of the method.

$$\frac{\frac{2}{3}}{\frac{3}{4}} = \frac{\frac{2}{3} \times \frac{4}{3}}{\frac{3}{4} \times \frac{4}{3}} = \frac{\frac{2}{3} \times \frac{4}{3}}{1} = \frac{2}{3} \times \frac{4}{3}$$

GUIDING QUESTIONS

Elicit student thinking:

- Why was it difficult to divide with the value in the form of a mixed number?
- Why is it beneficial to rewrite the mixed number as an improper fraction?

Determine if the student can EXPLAIN *MIXED NUMBERS*:

- Why must you consider the whole-number component and the fraction component of the mixed number together?

Determine if the student can CONVERT MIXED NUMBERS TO IMPROPER FRACTIONS:

- How can you rewrite this mixed number as an improper fraction?

Students should be required to complete the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT. **Ask** the following guiding questions as students work.

GUIDING QUESTIONS

Determine if the student can EXPLAIN DIVISION OF FRACTIONS BY FRACTIONS:

- Why do you need to rewrite the equation to find the quotient?

Determine if the student can DIVIDE FRACTIONS BY FRACTIONS:

- How will you rewrite the equation to find the quotient?
- Which method do you prefer? Why?
- [Point to a division problem.] Which method seems more efficient for this problem? Why?

Determine if the student can SOLVE WORD PROBLEMS INVOLVING DIVISION OF FRACTIONS BY FRACTIONS:

- [Point to a word problem.] How do you know to use division to solve the problem?
- [Point to a word problem.] How do you know which value is the dividend and which value is the divisor?

Determine if the student can DIVIDE MIXED NUMBERS:

- How will you rewrite the equation to find the quotient?

Determine if the student can SOLVE WORD PROBLEMS INVOLVING DIVISION OF MIXED NUMBERS:

- [Point to a word problem.] How do you know to use division to solve the problem?
- [Point to a word problem.] How do you know which value is the dividend and which value is the divisor?

At the end of the activity, teachers should engage the class in writing a summary of the two algorithms used to divide fractions by fractions. An example would be a table with two columns, one for the common-denominator algorithm and one for the invert-and-multiply algorithm. Students can offer information to include in each column, such as pros and cons of using each algorithm when it makes sense to use one or the other, tips and tricks on using the algorithms, and an overview of the algorithm itself. Following is an example of the two-column-table writing strategy.

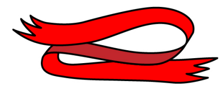
Common Denominator	Invert-and-Multiply
▸ Use if the two fractions have a common denominator or can easily be multiplied to have a common denominator. ▸ How to use: Rewrite the two fractions to have the same denominator, then divide the numerators to find your answer. ▸ Con: More calculations are needed if both fractions need to be multiplied for a common denominator. ▸ Pro: Some problems already have common denominators and it makes the division quite easy. ▸ Works because you are dividing amounts of equally sized parts.	▸ Can be used on any fractions, with no "set up" (i.e. multiplying for a common denominator). ▸ How to use: Rewrite the division as the dividend multiplied by the reciprocal of the divisor. ▸ Works because of the inverse relationship between multiplication and division.

Name______________________________

DIVISION OF FRACTIONS

Lesson 3

1. Ms. Underwood has $3\frac{1}{2}$ feet of ribbon. She wants to use her ribbon to make smaller pieces that are $\frac{3}{5}$ of a foot in length. How many smaller pieces can she make? Write an equation, and solve the problem using the invert-and-multiply strategy.

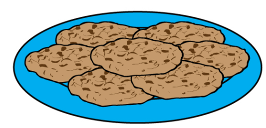


2. It takes Marie $\frac{3}{5}$ of an hour to mow $\frac{1}{2}$ of her yard. How long will it take to mow her whole yard? Write an equation, and solve the problem using the invert-and-multiply strategy. Give your answer in hours as an improper fraction or mixed numbers.

Name______________________________

3. It takes Marie $\frac{3}{5}$ of an hour to mow $\frac{1}{2}$ of her yard. How much of the yard will be mowed in 1 hour? Write an equation, and solve the problem using the invert-and-multiply strategy.

4. Susan eats $5\frac{1}{4}$ cookies. If a serving size is $\frac{3}{8}$ of a cookie, how many servings did Susan eat? Write an equation, and solve the problem using the invert-and-multiply strategy.



Name____________________________

5. Solve the problem using the invert-and-multiply strategy.

$$\frac{2}{3} \div \frac{6}{7}$$

6. Solve the problem using the invert-and-multiply strategy.

$$\frac{10}{2} \div \frac{1}{3}$$

7. Solve the problem using the invert-and-multiply strategy.

$$\frac{4}{5} \div \frac{7}{12}$$

Name____________________________

8. Solve the problem using the invert-and-multiply strategy.

$$\frac{3}{4} \div \frac{3}{4}$$

Division of Fractions

INSTRUCTIONAL ACTIVITY SUPPLEMENT A

Lesson 3

Display the following 10 division-of-fractions problems around the classroom or in the hallway. The students will participate in a “gallery walk”, so named because they will quietly go to each problem and read and answer the question, as if the problems were hanging in an art gallery. They do not need to be answered in order, but the problems should be answered on the corresponding problem space in the Instructional Activity Supplement B.

Question 1

$$3 \div \frac{1}{2}$$

Question 2

$$3 \div \frac{1}{4}$$

Question 3

$$2 \div \frac{1}{4}$$

Question 4

$$\frac{1}{4} \div \frac{1}{4}$$

Question 5

$$2 \div \frac{1}{5}$$

Question 6

$$2\frac{1}{2} \div \frac{1}{2}$$

Question 7

$$5 \div \frac{1}{3}$$

Question 8

$$6 \div \frac{2}{3}$$

Question 9

$$\frac{1}{2} \div \frac{1}{4}$$

Question 10

$$\frac{2}{3} \div \frac{3}{4}$$

Division of Fractions

INSTRUCTIONAL ACTIVITY SUPPLEMENT B

Lesson 3

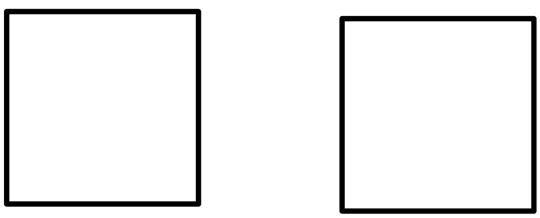
Question 1: _______ ÷ _______ = _______

Question 2: _______ ÷ _______ = _______

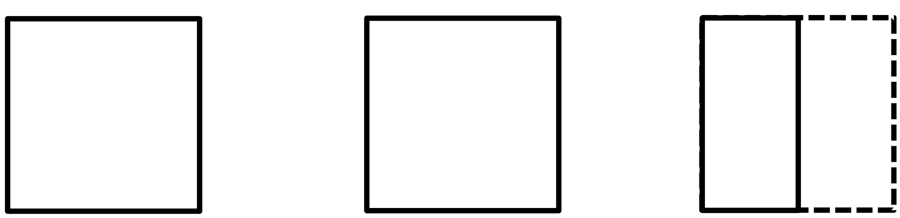
Question 3: _______ ÷ _______ = _______

Question 4: _______ ÷ _______ = _______

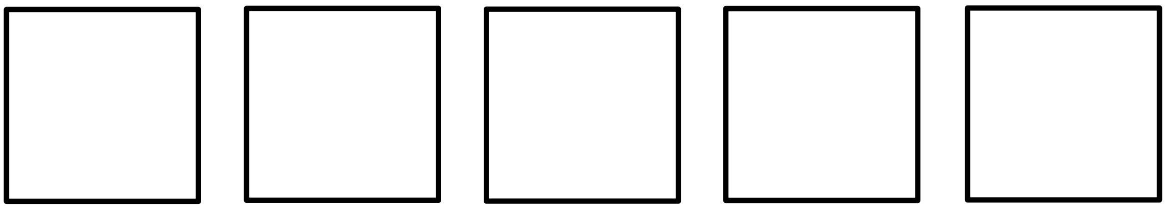
Question 5: _______ ÷ _______ = _______



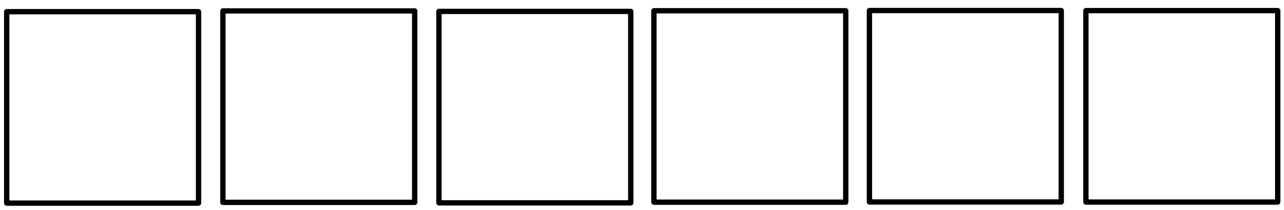
Question 6: _______ ÷ _______ = _______



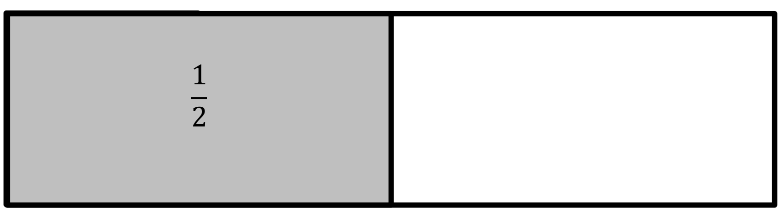
Question 7: _______ ÷ _______ = _______



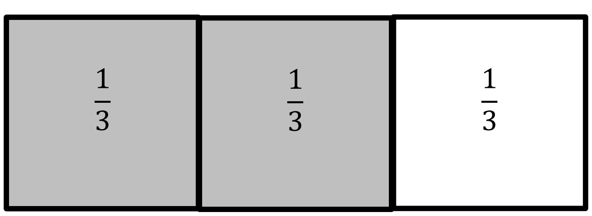
Question 8: _______ ÷ _______ = _______



Question 9: _______ ÷ _______ = _______



Question 10: _______ ÷ _______ = _______



Division of Fractions

INSTRUCTIONAL ACTIVITY

Lesson 4

LEARNING GOAL

Students will practice the two algorithms for solving problems involving division of fractions and justify their method selection.

PRIMARY ACTIVITY

Students participate in a card-matching activity to demonstrate their knowledge of the two algorithms for solving division of fraction problems, as well as the semi-concrete models that represent the problems.

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

- Common denominator
- Dividend
- Division
- Divisor
- Invert
- Multiplication
- Quotient
- Reciprocal
- Invert

MATERIALS

- Instructional Activity Supplement (Recommend one copy for every three to four students.)
- Fraction bars (for students that need additional support)

IMPLEMENTATION

> NOTE: One INSTRUCTIONAL ACTIVITY SUPPLEMENT contains the cards needed for one group of three or four students. It is suggested that the cards be cut out ahead of time in preparation for this lesson.

The first page of the INSTRUCTIONAL ACTIVITY SUPPLEMENT contains a division-of-fractions expression. The second page contains each problem's corresponding invert-and-multiply expression, and the third page contains each problem's corresponding common-denominator expression. The fourth page contains a visual model, word problem, or interpretation of the corresponding division problem. Students will work with their peers to match the equivalent forms and representations of fractions divided by fractions.

Introduce the activity to students, and explain that students will be matching cards with equivalent representations.

Divide students into groups of three or four, and **distribute** one set of cards to each group.

Differentiate the task as needed. For example, the cards can be copied onto different colored sheets of paper so that the students know to make groups consisting of one card of each color. To make the task more difficult, they can be copied on one color and scrambled so that students don't know how many equivalent forms exist. Additionally, struggling students could be given two card types to practice on before moving on to others. For example, if a student is struggling with the invert-and-multiply method, then they could be provided with only the cards showing the original division problem and the cards with the problem rewritten as multiplication by the reciprocal.

GUIDING QUESTIONS

Elicit student thinking:

- What do you notice when you use the invert-and-multiply method on a problem that already has common denominators?
- What do the common-denominator method and the invert-and-multiply method have in common? How are they different?

Determine if the student can EXPLAIN DIVISION OF FRACTIONS BY FRACTIONS:

- Why do we need to rewrite the equation to find the quotient?

Determine if the student can DIVIDE FRACTIONS BY FRACTIONS:

- How will you rewrite the equation to find the quotient?
- Which method do you prefer? Why?
- [Point to a division problem.] Which method seems more efficient for this problem?

Determine if the student can SOLVE WORD PROBLEMS INVOLVING DIVISION OF FRACTIONS BY FRACTIONS:

- [Point to a word problem.] How do you know to use division to solve the problem?
- [Point to a word problem.] How do you know which value is the dividend and which value is the divisor?

Determine if the student can DIVIDE MIXED NUMBERS:

- How will you rewrite the equation to find the quotient?

Determine if the student can SOLVE WORD PROBLEMS INVOLVING DIVISION OF MIXED NUMBERS:

- [Point to a word problem.] How do you know to use division to solve the problem?
- [Point to a word problem]. How do you know which value is the dividend and which value is the divisor?

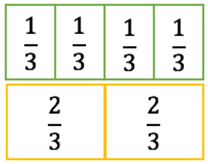
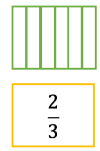
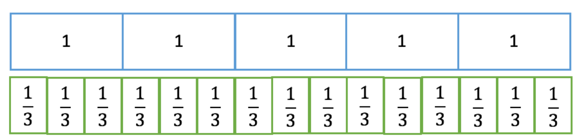
Students should be required to create poster summaries with the matched cards. Assign each group one of the problems, or allow them to choose one. Students should write the original division equation at the top of a piece of paper, then glue all the cards that show equivalent expressions or representations of the division problem to the paper.

At the end of the activity, teachers should choose a selection of student-created posters to display in the classroom or hallway.

$\frac{4}{3} \div \frac{2}{3}$	$\frac{2}{3} \div \frac{3}{4}$
$\frac{2}{3} \div \frac{4}{3}$	$\frac{1}{2} \div \frac{5}{9}$
$\frac{5}{9} \div \frac{1}{2}$	$\frac{6}{9} \div \frac{2}{3}$
$\frac{2}{3} \div \frac{6}{9}$	$5 \div 1\frac{1}{5}$
$\frac{12}{5} \div \frac{1}{3}$	$1\frac{3}{5} \div 1\frac{5}{7}$
$\frac{1}{3} \div \frac{12}{5}$	$1\frac{5}{7} \div 1\frac{3}{5}$
$\frac{3}{4} \div \frac{2}{3}$	$\frac{10}{2} \div \frac{3}{9}$

$\frac{4}{3} \times \frac{3}{2}$	$\frac{2}{3} \times \frac{4}{3}$
$\frac{2}{3} \times \frac{3}{4}$	$\frac{1}{2} \times \frac{9}{5}$
$\frac{5}{9} \times \frac{2}{1}$	$\frac{6}{9} \times \frac{3}{2}$
$\frac{2}{3} \times \frac{9}{6}$	$5 \times \frac{5}{6}$
$\frac{12}{5} \times \frac{3}{1}$	$\frac{8}{5} \times \frac{12}{7}$
$\frac{1}{3} \times \frac{5}{12}$	$\frac{12}{7} \times \frac{8}{5}$
$\frac{3}{4} \times \frac{3}{2}$	$\frac{10}{2} \times \frac{9}{3}$

$\frac{4}{3} \div \frac{2}{3}$	$\frac{8}{12} \div \frac{9}{12}$
$\frac{2}{3} \div \frac{4}{3}$	$\frac{9}{18} \div \frac{10}{18}$
$\frac{10}{18} \div \frac{9}{18}$	$\frac{6}{9} \div \frac{6}{9}$
$\frac{6}{9} \div \frac{6}{9}$	$\frac{25}{5} \div \frac{6}{5}$
$\frac{36}{15} \div \frac{5}{15}$	$\frac{56}{35} \div \frac{60}{35}$
$\frac{5}{15} \div \frac{36}{15}$	$\frac{60}{35} \div \frac{56}{35}$
$\frac{9}{12} \div \frac{8}{12}$	$\frac{90}{18} \div \frac{6}{18}$

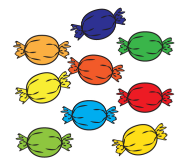
	How many $\frac{3}{4}$ are in $\frac{2}{3}$?
How many $\frac{2}{3}$ are in $\frac{3}{4}$?	It takes Lori half an hour to knit $\frac{5}{9}$ of a scarf. How long does it take her to knit the whole scarf?
How many halves are in $\frac{5}{9}$?	
$\frac{6}{9} \times ? = \frac{2}{3}$	How much chocolate will 5 people get if they fairly divide $1\frac{1}{5}$ pounds?
$\frac{1}{3} \times ? = \frac{12}{5}$	Eric paints $\frac{8}{5}$ walls in $\frac{12}{7}$ hours. How many walls can he paint in 1 hour?
Patrick mows $\frac{1}{3}$ of a lawn in $2\frac{2}{5}$ hours. How many lawns can he mow in 1 hour?	$1\frac{3}{5} \times ? = 1\frac{5}{7}$
How many $\frac{2}{3}$ are in $\frac{3}{4}$?	

Name____

Division of Fractions

Lesson 1-3

1. Annabelle has $4\frac{3}{4}$ pounds of candy to share with her class. She wants to give each classmate $\frac{1}{8}$ of a pound of candy. How many classmates can Annabelle give candy to?



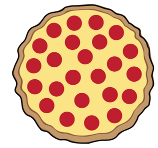
1.a. Find the solution using a length model.

1.b. Find the solution using the common-denominator method. Show all your work.

1.c. Find the solution using the invert-and-multiply method. Show all your work.

Name____

2. Kirbie has $3\frac{2}{3}$ pizzas. If a serving size is $\frac{1}{3}$ of a pizza, how many servings of pizza does he have?



2.a. Find the solution using fraction bars.

2.b. Find the solution using the common-denominator method. Show all your work.

2.c. Find the solution using the invert-and-multiply method. Show all your work.

Name____

3. Use the following graphic to answer the questions.

Nutrition Facts

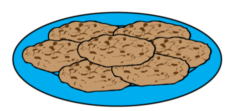
10 servings per container

Serving size	**3/5 cookies**
Amount Per Serving	
Calories	**50**
	% Daily Value*
Total Fat 6g	**8%**
Saturated Fat 0g	**0%**
Trans Fat 0g	
Sodium 0mg	**0%**
Total Carbohydrate 0g	**0%**
Dietary Fiber 0g	**0%**
Total Sugars 15g	
Includes 0g Added Sugars	**0%**
Protein 2g	**4%**

Not a significant source of cholesterol, vitamin D, calcium, iron, and potassium

*The % Daily Value (DV) tells you how much a nutrient in a serving of food contributes to a daily diet. 2,000 calories a day is used for general nutrition advice.

3.a. Write and solve an equation to find the total number of cookies in the container.



3.b. How many servings is 4 cookies?

3.c. How many servings is $\frac{1}{2}$ of a cookie?

Name____

4. Write a story problem that would be solved with the division problem $\frac{4}{5} \div \frac{1}{8}$. Include the answer to the division problem.

5. Solve the following division problems using the common-denominator method. Show all your work.

5.a. $\frac{6}{7} \div \frac{3}{4}$

5.b. $\frac{5}{2} \div \frac{3}{10}$

5.c. $5\frac{2}{3} \div \frac{5}{6}$

Name____

6. Solve the following division problems using the invert-and-multiply method. Show all your work.

6.a. $\frac{3}{5} \div \frac{1}{4}$

6.b. $\frac{3}{10} \div \frac{4}{3}$

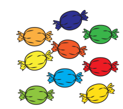
6.c. $3\frac{1}{5} \div 2\frac{1}{20}$

DIVISION OF FRACTIONS

STUDENT ACTIVITY SOLUTION GUIDE

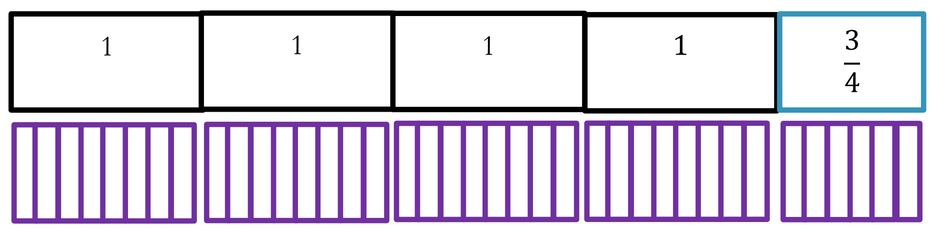
Lesson 1 – 4

1. Annabelle has $4\frac{3}{4}$ pounds of candy to share with her class. She wants to give each classmate $\frac{1}{8}$ of a pound of candy. How many classmates can Annabelle give candy to?



1.a. Find the solution using a length model.

CORRECT ANSWER



Annabelle can give candy to 38 classmates.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Student does not accurately draw eighths.	cannot represent a fraction with a length model	REPRESENT PROPER FRACTIONS WITH A LENGTH MODEL
Student attempts to solve the problem with a method other than with fraction bars.	does not know how to solve using fraction bars, or does not read the directions	EXPLAIN DIVISION OF FRACTIONS BY FRACTIONS
Student only represents the dividend, or only represents the divisor, with fraction bars.	cannot represent the concept of division with fraction bars (or a length model)	EXPLAIN DIVISION OF FRACTIONS BY FRACTIONS, DIVIDE FRACTIONS BY FRACTIONS

1.b. Find the solution using the common-denominator method. Show all your work.

CORRECT ANSWER

$4\frac{3}{4} \div \frac{1}{8}$

$\frac{19}{4} \div \frac{1}{8}$

$\frac{2\times19}{2\times4} \div \frac{1}{8}$

$\frac{38}{8} \div \frac{1}{8}$

$38 \div 1 = 38$

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Student solves using a method other than the common-denominator method.	does not know what the common-denominator method is, or does not read the direction	DIVIDE MIXED NUMBERS
Student does not find the least common denominator. *Note: Not all students may be expected to find the least common denominator, but a conversation can be had with those who calculate a common denominator other than the least common denominator.*	cannot calculate the least common denominator	CALCULATE *LEAST COMMON DENOMINATOR*
$4\frac{6}{8} \div \frac{1}{8} = 4\frac{6}{1}$	only divides the fraction component of the mixed number by $\frac{1}{8}$	DIVIDE MIXED NUMBERS
Student incorrectly converts the mixed number to an improper fraction.	converts the mixed number to an improper fraction in order to divide, but does so incorrectly	CONVERT MIXED NUMBERS TO IMPROPER FRACTIONS

1.c. Find the solution using the invert-and-multiply method. Show all your work.

CORRECT ANSWER

$$4\frac{3}{4} \div \frac{1}{8}$$

$$\frac{19}{4} \div \frac{1}{8}$$

$$\frac{19}{4} \times \frac{8}{1}$$

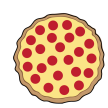
$$\frac{19}{4} \times \frac{8}{1}$$

$$\frac{152}{4} = 38$$

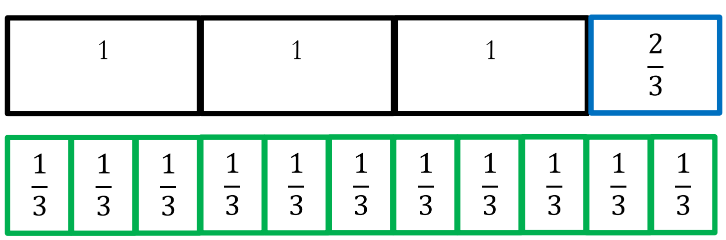
ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Student solves using a method other than the invert-and-multiply method.	does not know what the common-denominator method is, or does not read the direction	DIVIDE MIXED NUMBERS, DIVIDE FRACTIONS BY FRACTIONS
$\frac{4}{19} \times \frac{1}{8} = \frac{4}{152}$	writes the reciprocal of the dividend rather than the divisor	DIVIDE FRACTIONS BY FRACTIONS
$\frac{19}{4} \div \frac{8}{1}$	divides by the reciprocal of the divisor rather than multiplying by the reciprocal of the divisor	DIVIDE MIXED NUMBERS, DIVIDE FRACTIONS BY FRACTIONS

2. Kirbie has $3\frac{2}{3}$ pizzas. If a serving size is $\frac{1}{3}$ of a pizza, how many servings of pizza does he have?



2.a. Find the solution using fraction bars.



ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Student incorrectly draws thirds.	cannot represent a fraction with a length model	REPRESENT THIRDS ON A LENGTH MODEL
Student only represents the dividend, or only represents the divisor, with fraction bars.	cannot represent the concept of division with fraction bars (or an area model)	EXPLAIN DIVISION OF FRACTIONS BY FRACTIONS, DIVIDE FRACTIONS BY FRACTIONS

2.b. Find the solution using the common-denominator method. Show all your work.

CORRECT ANSWER

$3\frac{2}{3} \div \frac{1}{3}$

$\frac{11}{3} \div \frac{1}{3}$

$11 \div 1 = 11$

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Student finds the answer using a method other than the common-denominator method.	does not know what the common-denominator method is, or does not read the direction	DIVIDE MIXED NUMBERS
Student does not find the least common denominator. *Note: Not all students may be expected to find the least common denominator, but a conversation can be had with those who calculate a common denominator other than the least common denominator.*	cannot calculate the least common denominator	CALCULATE *LEAST COMMON DENOMINATOR*
Student only divides the fraction component of the mixed number by $\frac{1}{3}$.	does not consider the mixed number as a single value	DIVIDE MIXED NUMBERS

2.c. Find the solution using the invert-and-multiply method. Show all your work.

CORRECT ANSWER

$3\frac{2}{3} \div \frac{1}{3}$

$\frac{11}{3} \div \frac{1}{3}$

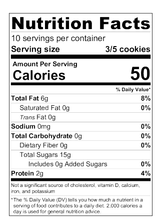
$\frac{11}{3} \times \frac{3}{1}$

$\frac{33}{3} = 11$

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Student solves the problem using a method other than the invert-and-multiply method.	does not know how to apply the invert-and-multiply method, or does not read the directions	DIVIDE MIXED NUMBERS
$\frac{3}{11} \times \frac{1}{3} = \frac{1}{11}$	writes the reciprocal of the dividend rather than the divisor	DIVIDE FRACTIONS BY FRACTIONS
$3\frac{2}{3} \div \frac{1}{3} = 3\frac{2}{1}$	only divides the fraction component of the mixed number by $\frac{1}{3}$	DIVIDE MIXED NUMBERS
$\frac{11}{3} \div \frac{8}{1}$	divides by the reciprocal of the divisor rather than multiplying by the reciprocal of the divisor	DIVIDE FRACTIONS BY FRACTIONS

3. Use the following graphic to answer the questions.


Nutrition Facts
10 servings per container
Serving size 3/5 cookies

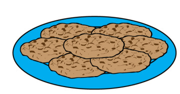
Amount Per Serving
Calories 50

	% Daily Value*
Total Fat 6g	8%
Saturated Fat 0g	0%
Trans Fat 0g	
Sodium 0mg	0%
Total Carbohydrate 0g	0%
Dietary Fiber 0g	0%
Total Sugars 15g	
Includes 0g Added Sugars	0%
Protein 2g	4%

Not a significant source of cholesterol, vitamin D, calcium, iron, and potassium

*The % Daily Value (DV) tells you how much a nutrient in a serving of food contributes to a daily diet. 2,000 calories a day is used for general nutrition advice.

3.a. Write and solve an equation to find the total number of cookies in the container.



CORRECT ANSWER

$10 \times \frac{3}{5}$

$\frac{30}{5} = 6$

There are 6 cookies in the jar.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
There are 10 cookies in the container.	confuses the number of servings with the number of cookies	SOLVE WORD PROBLEMS INVOLVING MULTIPLICATION OF A FRACTION AND A WHOLE NUMBER
Student multiplies 10 by a number other than $\frac{3}{5}$.	recognizes that there are 10 servings but does not multiply by the correct size of each serving	SOLVE WORD PROBLEMS INVOLVING MULTIPLICATION OF A FRACTION AND A WHOLE NUMBER
Student multiplies $\frac{3}{5}$ by a number other than 10.	recognizes that the size of each servings is $\frac{3}{5}$ of a cookie but does not multiply by the correct number of servings	SOLVE WORD PROBLEMS INVOLVING MULTIPLICATION OF A FRACTION AND A WHOLE NUMBER
$10 \div \frac{3}{5}$ or $\frac{3}{5} \div 10$	does not recognize when a problem calls for multiplication instead of division (uses division because they believe they are only practicing division, or the "problem of the day")	SOLVE WORD PROBLEMS INVOLVING MULTIPLICATION OF A FRACTION AND A WHOLE NUMBER

3.b. How many servings is 4 cookies?

CORRECT ANSWER

$4 \div \frac{3}{5}$

$4 \times \frac{5}{3} = \frac{20}{3} = 6\frac{2}{3}$

There are $6\frac{2}{3}$ servings in 4 cookies.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
$4 \times \frac{3}{5}$	does not understand the concept of division	SOLVE WORD PROBLEMS INVOLVING DIVISION OF WHOLE NUMBERS BY FRACTIONS
Student sets up the correct equation but does not solve it correctly.	cannot solve division of fractions by fractions	DIVIDE WHOLE NUMBERS BY FRACTIONS
$\frac{3}{5} \div 4 = \frac{3}{20}$	divides $\frac{3}{5}$ by 4 rather than dividing 4 by $\frac{3}{5}$	DIVIDE WHOLE NUMBERS BY FRACTIONS, EXPLAIN DIVISION OF WHOLE NUMBERS BY FRACTIONS

3.c. How many servings is $\frac{1}{2}$ of a cookie?

CORRECT ANSWER

> $\frac{1}{2} \div \frac{3}{5}$
>
> $\frac{1}{2} \times \frac{5}{3} = \frac{5}{6}$
>
> There are $\frac{5}{6}$ servings in $\frac{1}{2}$ of a cookie.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
$\frac{1}{2} \times \frac{3}{5}$	does not understand the concept of division	EXPLAIN DIVISION OF FRACTIONS BY FRACTIONS
Student sets up the correct equation but does not solve it correctly.	cannot solve division of fractions by fractions	DIVIDE FRACTIONS BY FRACTIONS
$\frac{3}{5} \div \frac{1}{2}$	divides $\frac{3}{5}$ by $\frac{1}{2}$ rather than dividing $\frac{1}{2}$ by $\frac{3}{5}$	DIVIDE FRACTIONS BY FRACTIONS

4. Write a story problem that would be solved with the division problem $\frac{4}{5} \div \frac{1}{8}$. Include the answer to the division problem.

CORRECT ANSWER

Student answers may vary.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Student writes a story problem that would be solved with $\frac{1}{8} \div \frac{4}{5}$.	confuses the dividend and the divisor	EXPLAIN DIVISION OF FRACTIONS BY FRACTIONS
Student writes a story problem that would be solved with $\frac{4}{5} \times \frac{1}{8}$	confuses the concept of division with multiplication	EXPLAIN DIVISION OF FRACTIONS BY FRACTIONS and/or EXPLAIN MULTIPLICATION OF 2 FRACTIONS
Student writes a story problem that would be solved with $\frac{1}{8} - \frac{4}{5}$.	confuses the concept of division with subtraction	EXPLAIN DIVISION OF FRACTIONS BY FRACTIONS and/or SUBTRACT FRACTIONS WITH UNLIKE DENOMINATORS

5. Solve the following division problems using the common-denominator method. Show all your work.

5.a. $\frac{6}{7} \div \frac{3}{4}$

CORRECT ANSWER

$\frac{6}{7} \div \frac{3}{4}$

$\frac{24}{28} \div \frac{21}{28}$

$\frac{24}{21} = \frac{8}{7} = 1\frac{1}{7}$

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Student correctly solves the problem with a method other than the common-denominator method.	does not know how to use the common-denominator method or does not read the directions	DIVIDE FRACTIONS BY FRACTIONS
Student writes the quotient with a denominator of 28.	mistakenly uses the denominator of the dividend and divisor as the denominator of the quotient	DIVIDE FRACTIONS BY FRACTIONS

5.b. $\frac{5}{2} \div \frac{3}{10}$

CORRECT ANSWER

$$\frac{5}{2} \div \frac{3}{10}$$

$$\frac{25}{10} \div \frac{3}{10}$$

$$\frac{25}{3} = 8\frac{1}{3}$$

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Student correctly solves the problem with a method other than the common-denominator method.	does not know how to use the common-denominator method or does not read the directions	DIVIDE FRACTIONS BY FRACTIONS
Student writes the quotient with a denominator of 10.	mistakenly uses the denominator of the dividend and divisor as the denominator of the quotient	DIVIDE FRACTIONS BY FRACTIONS

5.c. $5\frac{2}{3} \div \frac{5}{6}$

CORRECT ANSWER

$5\frac{2}{3} \div \frac{5}{6}$

$\frac{17}{3} \div \frac{5}{6}$

$\frac{34}{6} \div \frac{5}{6} = \frac{34}{5} = 6\frac{4}{5}$

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Student correctly solves the problem with a method other than the common-denominator method.	does not know how to use the common-denominator method or does not read the directions	DIVIDE FRACTIONS BY FRACTIONS
Student writes the quotient with a denominator of 6.	mistakenly uses the denominator of the dividend and divisor as the denominator of the quotient	DIVIDE FRACTIONS BY FRACTIONS

6. Solve the following division problems using the invert-and-multiply method. Show all your work.

6.a. $\frac{3}{5} \div \frac{1}{4}$

CORRECT ANSWER

$\frac{3}{5} \div \frac{1}{4}$

$\frac{3}{5} \times \frac{4}{1}$

$\frac{12}{5} = 2\frac{2}{5}$

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Student correctly solves the problem with a method other than the invert-and-mlutiply method.	does not know how to use the invert-and-multiply method or does not read the directions	DIVIDE FRACTIONS BY FRACTIONS
$\frac{5}{3} \times \frac{1}{4}$	inverts the dividend rather than the divisor	DIVIDE FRACTIONS BY FRACTIONS
$\frac{3}{1.25}$	divides the numerators and the denominators	DIVIDE FRACTIONS BY FRACTIONS
$\frac{3}{5} \div \frac{4}{1}$	inverts the divisor but divides instead of multiplies	DIVIDE FRACTIONS BY FRACTIONS

6.b. $\frac{3}{10} \div \frac{4}{3}$

CORRECT ANSWER

$\frac{3}{10} \div \frac{4}{3}$

$\frac{3}{10} \times \frac{3}{4}$

$\frac{9}{40}$

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Student correctly solves the problem with a method other than the invert-and-mlutiply method.	does not know how to use the invert-and-multiply method or does not read the directions	DIVIDE FRACTIONS BY FRACTIONS
$\frac{3}{10} \times \frac{4}{3}$	inverts the dividend rather than the divisor and multiplies	DIVIDE FRACTIONS BY FRACTIONS
$\frac{10}{3} \div \frac{3}{4}$	inverts the divisor but divides instead of multiplies	DIVIDE FRACTIONS BY FRACTIONS
Student says the problem cannot be solved.	believes that the dividend must be greater than the divisor	DIVIDE FRACTIONS BY FRACTIONS

6.c. $4\frac{1}{5} \div 2\frac{1}{20}$

CORRECT ANSWER

$4\frac{1}{5} \div 2\frac{1}{20}$

$\frac{21}{5} \div \frac{41}{20}$

$\frac{21}{5} \times \frac{20}{41}$

$\frac{420}{205} = \frac{84}{41} = 2\frac{2}{41}$

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Student correctly solves the problem with a method other than the invert-and-mlutiply method.	does not know how to use the invert-and-multiply method or does not read the directions	DIVIDE MIXED NUMBERS
$2\frac{1}{4}$	divides whole numbers, numerators, and denominators separately, and divides 20 by 5 rather than 5 by 20	DIVIDE MIXED NUMBERS
$4\frac{1}{5} \times 2\frac{20}{1}$	multiplies by the reciprocal only of the fraction component of the mixed number	DIVIDE MIXED NUMBERS