THE UNIVERSITY OF KANSAS
Achievement & Assessment Institute

RATIONAL NUMBER ADDITION AND SUBTRACTION

7.NS.1

CONTENTS

The types of documents contained in the unit are listed below. Throughout the unit, the documents are arranged by lesson.

Enhanced Learning Maps developed these materials under a grant from the Department of Education, PR/Award # S368A150013. Contents do not necessarily represent the policy of the Department of Education, and you should not assume endorsement by the Federal Government.

RATIONAL NUMBER ADDITION AND SUBTRACTION

LEARNING MAP INFORMATION

STANDARDS

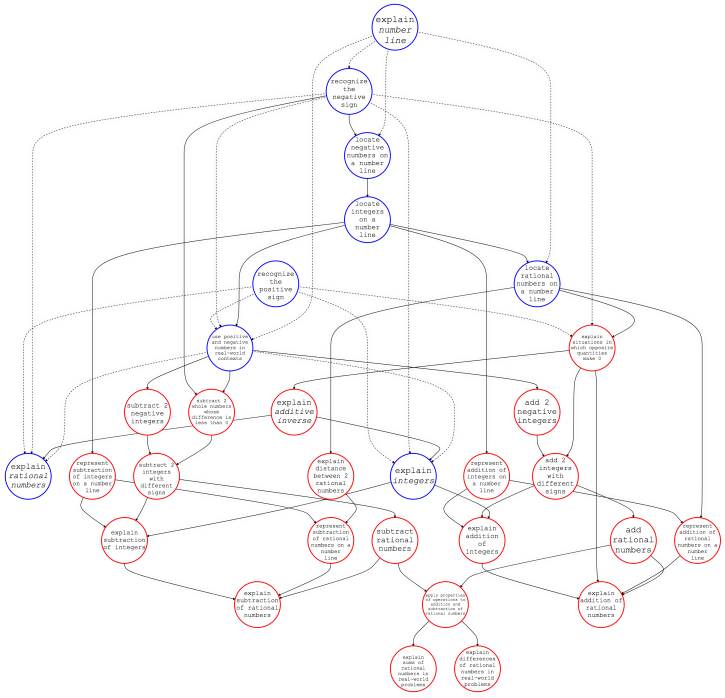
7.NS.1 Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

7.NS.1.a Show that a number and its opposite have a sum of 0 (additive inverses). Describe situations in which opposite quantities combine to make 0. *For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.*

7.NS.1.b Understand addition of rational numbers ($p + q$ as the number located a distance $|q|$ from p, in the positive or negative direction depending on whether q is positive or negative). Interpret sums of rational numbers by describing real-world contexts.

7.NS.1.c Understand subtraction of rational numbers as adding the additive inverse, $p – q = p + (–q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principal in real-world contexts.

7.NS.1.d Apply properties of operations as strategies to add and subtract rational numbers.



**Learning map model of* 7.NS.1

Node Name	Node Description
ADD 2 INTEGERS WITH DIFFERENT SIGNS	Determine the sum of two integers with different signs.
ADD 2 NEGATIVE INTEGERS	Determine the sum of two negative integers.
ADD RATIONAL NUMBERS	Determine the sum of two or more rational numbers.
APPLY PROPERTIES OF OPERATIONS TO ADDITION AND SUBTRACTION OF RATIONAL NUMBERS	Apply properties of operations such as the commutative, associative, and distributive properties to addition and subtraction of rational numbers.
EXPLAIN ADDITION OF INTEGERS	Make known your understanding that the sum of two integers takes into account the magnitude (i.e., size) and direction (i.e., sign) of each of the addends. The result of adding two integers with the same sign is a number farther from zero than either of the addends, whereas the result of adding two integers with different signs is a number closer to zero than one of the addends.
EXPLAIN ADDITION OF RATIONAL NUMBERS	Make known your understanding of how the sizes and signs of the addends justify the result of adding rational numbers. Apply and explain strategies including composition, decomposition, and additive inverses.
EXPLAIN *ADDITIVE INVERSE*	Make known your understanding that a number and its opposite have a sum of zero and are known as additive inverses.
EXPLAIN DIFFERENCES OF RATIONAL NUMBERS IN REAL-WORLD PROBLEMS	Make known your understanding that the difference of two rational numbers may result in a number that is farther or closer to zero depending on the magnitude and direction of the rational numbers being subtracted. For example, when subtracting two debts, the result is a smaller debt because part of the the original debt is removed.
EXPLAIN DISTANCE BETWEEN 2 RATIONAL NUMBERS	Make known your understanding that the distance between two rational numbers is the absolute value of their difference. This can be done in mathematical or real-world problems.
EXPLAIN *INTEGERS*	Make known your understanding that integers include whole numbers and their opposites. Each integer possesses both magnitude and direction, and the set of integers is symmetric around zero.
EXPLAIN *NUMBER LINE*	Make known your understanding that a number line is a graphic representation of numbers; a straight line is partitioned into equal segments representing iteration of quantity. Lesser values are positioned to the left of greater values on the number line.
EXPLAIN *RATIONAL NUMBERS*	Make known your understanding that a rational number is a number that can be written as the quotient of two integers a/b where b is not zero.
EXPLAIN SITUATIONS IN WHICH OPPOSITE QUANTITIES MAKE 0	Make known your understanding of mathematical and real-world situations in which opposite quantities make zero.
EXPLAIN SUBTRACTION OF INTEGERS	Make known your understanding that subtracting an integer is equivalent to adding the additive inverse of the integer being subtracted, and that the distance between two integers on the number line is the absolute value of their difference.
EXPLAIN SUBTRACTION OF RATIONAL NUMBERS	Make known your understanding that subtraction of rational numbers is the equivalent to adding the additive inverse of the number being subtracted.
EXPLAIN SUMS OF RATIONAL NUMBERS IN REAL-WORLD PROBLEMS	Make known your understanding that the sum of two rational numbers may result in a number that is farther or closer to zero depending on the magnitude and direction of the rational numbers being added. For example,

	when adding two debts, the result is a larger debt; or when adding a debt to a credit, the result could be a smaller credit or a smaller debt.
LOCATE INTEGERS ON A NUMBER LINE	When given an integer, locate the number on a number line.
LOCATE NEGATIVE NUMBERS ON A NUMBER LINE	When given a negative number, locate the number on a number line.
LOCATE RATIONAL NUMBERS ON A NUMBER LINE	When given a rational number, locate the number on a number line.
RECOGNIZE THE NEGATIVE SIGN	Identify or name the negative sign and interpret it to indicate a value that is less than zero. Distinguish this use of the '–' symbol from the use of the same symbol to indicate subtraction.
RECOGNIZE THE POSITIVE SIGN	Identify or name the positive sign and interpret it to indicate a value that is greater than zero.
REPRESENT ADDITION OF INTEGERS ON A NUMBER LINE	Through writing or an appropriate assistive technology, represent addition of integers on a vertical or horizontal number line.
REPRESENT ADDITION OF RATIONAL NUMBERS ON A NUMBER LINE	Through writing or an appropriate assistive technology, represent addition of rational numbers on a vertical or horizontal number line.
REPRESENT SUBTRACTION OF INTEGERS ON A NUMBER LINE	Through writing or an appropriate assistive technology, represent subtraction of integers on a vertical or horizontal number line.
REPRESENT SUBTRACTION OF RATIONAL NUMBERS ON A NUMBER LINE	Through writing or appropriate assistive technology, represent subtraction of rational numbers on a vertical or horizontal number line.
SUBTRACT 2 INTEGERS WITH DIFFERENT SIGNS	Determine the difference between two integers with different signs.
SUBTRACT 2 NEGATIVE INTEGERS	Determine the difference between two negative integers. Begin with differences whose results are less than zero (e.g., $-3 - (-1) = -2$). Proceed to differences whose results are greater than zero (e.g., $-2 - (-5) = 3$).
SUBTRACT 2 WHOLE NUMBERS WHOSE DIFFERENCE IS LESS THAN 0	Determine the difference between two whole numbers when the difference is less than zero (e.g., $5 - 7$).
SUBTRACT RATIONAL NUMBERS	Determine the difference between two or more rational numbers.
USE POSITIVE AND NEGATIVE NUMBERS IN REAL-WORLD CONTEXTS	Reason about positive and negative numbers in real-world contexts (e.g., temperature, elevation, credits and debts, etc.).

ADDITIONAL NODES RELATED TO THIS UNIT OF INSTRUCTION

Node Name	Node Description	Related Node
EXPLAIN OPPOSITE NUMBERS ON A NUMBER LINE	Make known your understanding that x and $-x$ are opposite numbers because they are on different sides of zero and are equal distances from zero on the number line.	Prerequisite of EXPLAIN *INTEGERS* (through EXPLAIN THE OPPOSITE OF THE OPPOSITE OF A NUMBER and RECOGNIZE THE OPPOSITE OF THE OPPOSITE OF A NUMBER)

Node Name	Node Description	Related Node
EXPLAIN *SYMMETRY*	Make known your understanding that symmetric figures can be divided into congruent parts, where each of the parts is a mirror image of the other.	Prerequisite of EXPLAIN OPPOSITE NUMBERS ON A NUMBER LINE (through RECOGNIZE OPPOSITE NUMBERS ON A NUMBER LINE)
LOCATE POSITIVE FRACTIONS ON A NUMBER LINE	When given a positive fraction, locate the number on a number line.	Prerequisite of LOCATE RATIONAL NUMBERS ON A NUMBER LINE
RECOGNIZE OPPOSITE NUMBERS ON A NUMBER LINE	Given a number, identify or name the opposite of that number on a number line. For example, given 17, identify and name –17 as the opposite on a number line.	Prerequisite of EXPLAIN OPPOSITE NUMBERS ON A NUMBER LINE

RATIONAL NUMBER ADDITION AND SUBTRACTION

TEACHER NOTES

This unit includes the following documents:

- Learning Map Information
- Instructional Activity (four lessons)
- Instructional Activity Student Handout (for Lessons 3 & 4)
- Instructional Activity Supplement (for Lessons 1, 2, & 4)
- Student Activity (Word Version)
- Student Activity Solution Guide

Students will first explore integer addition and subtraction through a real-world context of credits and debits in an allowance setting. Students will then use the number line to make sense of signed numbers and their properties, including symmetry, additive inverses, and sums and differences of rational numbers.

RESEARCH

Understanding, graphing, and operating with integers comprise important middle school mathematics topics, but these tasks challenge students' prior conceptions about numbers and their representations. The learning map section, therefore, models the use of contextual situations to provide connections to concrete examples of negative quantities (Gregg and Gregg, 2007; Liebeck, 1990). Students benefit from discussing familiar situations involving assets and debts, sea level, or temperature to help them form a conceptual basis for why negative numbers exist and how they relate to things they already know. For example, cases where students must explain that $5 – $7 results in a debt instead of an asset provide productive opportunities for introducing integers. With this preliminary understanding, students can begin to operate with integers and use appropriate notation, but there are several documented difficulties they may face as they gain new knowledge.

Working with integers challenges students who try to apply their whole number schemes to integers (Bishop et al., 2014). Specifically, students who cling to the whole number property that adding always produces larger numbers become confused when they attempt to add a positive number to a negative number. Even when students experience integers initially through familiar real-world contexts, such opportunities may not require students to acknowledge that negative numbers possess both magnitude and direction, because negative values in context can be labeled differently rather than assigned a negative sign (Peled & Carraher, 2008). Thus teachers should incorporate language that describes how far these numbers are from zero (i.e., magnitude) and whether they are positive or negative (i.e., direction) to help students develop their appreciation for these two aspects of integers.

Integer notation also causes confusion because the symbols used for addition and subtraction operations with whole numbers gain new meanings when used with integers. The learning map model depicts how students

must expand their understanding particularly of the "–" sign to incorporate its meaning as negative, opposite, or minus (Lamb et al., 2012). Moreover, the meanings of positive and negative signs can change within a problem, requiring students to incorporate their understanding of signs with their knowledge of operations on integers.

The number line model poses additional challenges to students learning to understand and perform operations with integers. Students initially tend to separate the number line at zero (Peled, Mukhopadhay, & Resnick, 1989). They first view the number line to the left of zero as having similar rules to the number line to the right of zero, and they struggle to coordinate these divided number lines into a continuous number line. These aspects challenge students as they struggle to understand integers as numbers with magnitude and direction and as a set that is symmetric around zero. A productive strategy is to sequence examples involving addition or subtraction of integers such that earlier examples contain operands and solutions all on one side of zero, and later examples contain operands on one side of zero but solutions on the other side of zero. The following is an example of a sequence that would fit this strategy.

- –5 + –3 would allow students to operate as they would in 5 + 3, but realize their result should be negative instead of positive
- –8 – (–7) is similar to 8 – 7 but with a negative result instead of positive
- –2 – (–9) forces students to consider a continuous number line as they operate across zero

This sequence allows a teacher and students to attend specifically to the result of an operation that "crosses over" zero and helps to scaffold students' acknowledgment of the relationships between integers and zero, which supporting their understanding of one continuous number line.

In a comparison of students who learned integer operations through a credit/debit activity versus movement on the number line, Liebeck found that both groups of students enjoyed and operated flexibly in either method during instruction. However, six weeks later, students who learned using the credit/debit model significantly outperformed students who learned using the number line model. Liebeck noted that students whose initial experiences adding and subtracting integers incorporated number line models were successful when the problems remained consistent with adding to or taking away objects or amounts, but these students struggled to extend their understanding to novel problems, such as those with three addends. The instruction provided to these students during the study introduced specific rules for how to use the number line for specific types of problems (e.g., starting the problem by "facing" in a positive direction when adding positive and "facing" in a negative direction when adding negatives) but did not provide students strategies for generalizing those rules for different problem types or connecting the meanings of different numbers to why sums and differences resulted in larger or smaller values. This study's results found that consistently maintaining the physical interpretations of addition as "add" and subtraction as "take away" led to greater success when compared to students who applied particular rules for applying the number line to addition and subtraction of integers (Liebeck, 1990).

Gregg and Gregg also advocate for a credit/debit model and emphasize the importance of students understanding that any integer can be represented in a variety of ways. For example, "four" can be represented as four positives, five positives and one negative, and so on. Similar to Battista, Gregg and Gregg note the ability of the credit/debit model to be relevant to all four basic operations with integers.

LEARNING MAP INFORMATION

The sequences in the learning map and in the lessons in this unit have been influenced by the recommendations of Battista (2014), who advocates for a model that can be extended to all four operations with integers, as well as Liebeck (1990) and Gregg and Gregg (2007), both of whom encourage a credit/debit approach for introducing integer operations to students using a familiar context. The learning map sequence allows for students to add integers and rational numbers using either real-world contexts or a number line model. Independently of addition, the learning map section also models subtraction of integers and rational numbers using either real-world contexts or a number line model. These understandings provide the foundation for students to explain sums and differences of rational numbers in real-world contexts.

INSTRUCTIONAL ACTIVITIES

This unit begins with an intuitive credit and debit activity to establish students' ability to determine whether an action results in an increase or decrease in the initial value, and then students take this understanding to the number line to complete the operation.

Specifically, students will first learn integer operations through a credit and debit activity where addition and subtraction of integers occur in a familiar context. Students will also explore different representations of the same integer value. After reviewing the structure of the number line, students will relate distance on a number line to the difference between two integers and explain opposites, or additive inverses, on the number line. Combining initial understandings of integer operations and knowledge of the number line, students will apply this knowledge to add and subtract integers using a number line as an aid. Because students have built an understanding of whether a particular action will increase or decrease the initial value in their work with credits and debits in the activity using piggy banks to track their allowance, their use of the number line remains consistent with their early use of the number line to "add to" or "take away from" an initial value. Once students are comfortable operating with integers, they are able to extend their knowledge to all rational numbers.

REFERENCES

Battista, M. T.. (1983). A Complete Model for Operations on Integers. *The Arithmetic Teacher, 30*(9), 26–31. Retrieved from http://www.jstor.org/stable/41190724

Gregg, J., & Gregg, D. U.. (2007). A Context for Interger Computation. *Mathematics Teaching in the Middle School, 13*(1), 46–50. Retrieved from http://www.jstor.org/stable/41182493

Jessica Pierson Bishop, Lisa L. Lamb, Randolph A. Philipp, Ian Whitacre, Bonnie P. Schappelle, & Melinda L. Lewis. (2014). Obstacles and Affordances for Integer Reasoning: An Analysis of Children's Thinking and the History of Mathematics. *Journal for Research in Mathematics Education, 45*(1), 19–61.

Liebeck, P.. (1990). Scores and Forfeits: An Intuitive Model for Integer Arithmetic. *Educational Studies in Mathematics, 21*(3), 221–239. Retrieved from http://www.jstor.org/stable/3482594

Lisa L. Lamb, Jessica Pierson Bishop, Randolph A. Philipp, Bonnie P. Schappelle, Ian Whitacre, & Mindy Lewis. (2012). Developing Symbol Sense for the Minus Sign. *Mathematics Teaching in the Middle School, 18*(1), 5–9. http://doi.org/10.5951/mathteacmiddscho.18.1.0005

Peled, I., & Carraher, D. W. (2007). Signed numbers and algebraic thinking. *Algebra in the early grades*, 303-327.

Peled, I., Mukhopadhyay, S., & Resnick, L. B. (1989, July). Formal and informal sources of mental models for negative numbers. In *Proceedings of the 13th international conference for the Psychology of Mathematics Education* (Vol. 3, pp. 106-110).

RATIONAL NUMBER ADDITION AND SUBTRACTION

OVERVIEW OF INSTRUCTIONAL ACTIVITIES

Lesson	Learning Goal	Nodes Addressed
Lesson 1	Students will be introduced to integer operations through a credit and debit activity where negative numbers and addition and subtraction of integers surface as a result of familiar operations.	▶ USE POSITIVE AND NEGATIVE NUMBERS IN REAL-WORLD CONTEXTS
Lesson 2	Students will review the number line and its symmetry, plot values on the number line, and answer questions pertaining to distance and opposite values (additive inverses) regarding the values plotted on the number line.	▶ EXPLAIN *NUMBER LINE* ▶ RECOGNIZE THE NEGATIVE SIGN ▶ LOCATE INTEGERS ON A NUMBER LINE ▶ explain situations in which opposite quantities make 0 ▶ explain *additive inverse* ▶ locate negative numbers on a number line ▶ explain distance between 2 rational numbers ▶ represent subtraction of integers on a number line
Lesson 3	Students will extend their understanding of integer addition and subtraction, making connections with the number line and the concepts of credits and debits.	▶ ADD 2 NEGATIVE INTEGERS ▶ ADD 2 INTEGERS WITH DIFFERENT SIGNS ▶ REPRESENT ADDITION OF INTEGERS ON A NUMBER LINE ▶ EXPLAIN ADDITION OF INTEGERS ▶ subtract 2 whole numbers whose difference is less than 0 ▶ subtract 2 negative integers ▶ subtract 2 integers with different signs ▶ represent subtraction of integers on a number line ▶ explain subtraction of integers ▶ apply properties of operations to addition and subtraction of rational numbers
Lesson 4	Students will extend their understanding of integer addition and subtraction to addition and subtraction of rational numbers.	▶ ADD RATIONAL NUMBERS ▶ REPRESENT ADDITION OF RATIONAL NUMBERS ON A NUMBER LINE ▶ EXPLAIN ADDITION OF RATIONAL NUMBERS ▶ SUBTRACT RATIONAL NUMBERS ▶ REPRESENT SUBTRACTION OF RATIONAL NUMBERS ON A NUMBER LINE ▶ EXPLAIN SUBTRACTION OF RATIONAL NUMBERS ▶ APPLY PROPERTIES OF OPERATIONS TO ADDITION AND SUBTRACTION OF RATIONAL NUMBERS ▶ EXPLAIN SUMS OF RATIONAL NUMBERS IN REAL-WORLD PROBLEMS ▶ EXPLAIN DIFFERENCES OF RATIONAL NUMBERS IN REAL-WORLD PROBLEMS

RATIONAL NUMBER ADDITION AND SUBTRACTION

INSTRUCTIONAL ACTIVITY

Lesson 1

LEARNING GOAL

Students will be introduced to integer operations through a credit and debit activity where negative numbers and addition and subtraction of integers surface as a result of familiar operations.

PRIMARY ACTIVITY

Students will use a piggy bank along with credit and debit tickets to simulate addition and subtraction of integers throughout the course of an imaginary week of chores.

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

- Credit
- Payment
- Debit
- Debt

MATERIALS

- INSTRUCTIONAL ACTIVITY STUDENT HANDOUT
- INSTRUCTIONAL ACTIVITY SUPPLEMENT (Recommend one copy of the piggy bank, ten credits, and ten debits for every student.)
- Scissors

IMPLEMENTATION

Students begin seventh grade with previous experience with integers. They should have some understanding that integers are defined by magnitude and direction, though this concept will continue to be emphasized throughout this unit as well.

Students should begin with a picture of a piggy bank and cutouts of 10 credits and 10 debits to participate with. Students can work individually or in pairs during this activity.

The following questions apply throughout the lesson and should be asked regularly to gauge student understanding.

GUIDING QUESTIONS

Elicit student thinking:

- What happens to the balance of the piggy bank when you add a ticket? Can you explain your response? (Students should indicate that it depends whether a credit or debit is added.)
- What happens to the balance of the piggy bank when you remove a ticket? Can you explain your response? (Students should indicate that it depends whether a credit or debit is removed.)
- What is another way to represent the same balance?

Determine if the student can USE POSITIVE AND NEGATIVE NUMBERS IN REAL-WORLD CONTEXTS:

- How does adding a credit to the piggy bank impact the balance?
- How does removing a credit from the piggy bank impact the balance?
- How does adding a debit to the piggy bank impact the balance?
- How does removing a debit from the piggy bank impact the balance?

Explain to students the following scenario to situate the activity. The INSTRUCTIONAL ACTIVITY STUDENT HANDOUT provides a place for students to organize their work for each round.

During the course of a week, students have been assigned chores worth different values that they are to complete. Completing a chore means the students earn a specified amount of money (credit). If they do not complete a chore by the assigned time, they must pay the value of the chore (debit). Instead of exchanging money each time a chore is completed, the students keep track using tickets in a piggy bank. At the end of the week, the students determine how much money they are owed (a positive balance or credit) or must pay (a negative balance or debit).

Begin the week with an empty piggy bank. After each described chore, write an equation to represent the addition or subtraction of credits or debits and the existing balance as a class. The following table provides an example chore series with appropriate equations.

For the first round, **request** that students do exactly as the chore describes rather than trying to determine another method of achieving the same balance. For example, when the student forgets to mow the grass, students should place four –$1 tickets in the piggy bank rather than removing four +$1 tickets.

In subsequent examples, students will need to determine alternate ways to represent the same action or piggy bank balance. However, they need an opportunity to first add or remove tickets exactly as described and determine the current balance before considering alternate methods for achieving the same balance.

Ask the following questions as students work.

- When you add a credit (+$1), does the balance in your piggy bank increase or decrease? By how much?
- When you add a debit (–$1), does the balance in your piggy bank increase or decrease? By how much?
- When you remove –$1, what happens to the balance of the piggy bank? Does it increase or decrease? By how much?

Order	Chore	Equation
1	You took out the trash. Add +$1.	0 + 1 = 1
2	You fed the fish. Add +$1.	1 + 1 = 2
3	You forgot to clean your room. Add –$3.	2 + – 3 = –1
4	You made your bed. Remove –$1.	–1 – (–1) = 0
5	You walked the dog. Add +$2.	0 + 2 = 2
6	You vacuumed the whole house. Add +$5.	2 + 5 = 7
7	You forgot to mow the grass. Add – $4.	7 + –4 = 3

Repeat this activity with several chores so students are able to add or remove the described ticket and report the new value or balance of the piggy bank.

Determine a point to stop describing additional chores and **ask** students whether they would earn or owe money if the week ended then. Once students establish whether they made or owe money, they should calculate the amount.

Next, **prepare** students to establish alternate methods of achieving an equivalent balance in their piggy bank. This can be done in many situations, but it is required if students run out of tickets and need to adjust the balance in another way.

Begin by helping students understand that zero pairs can be removed from the piggy bank and the balance will remain the same. To do this, **pair** up a +$1 and a –$1 ticket and ask students what their balance would be if they get a credit and then a debit. They should be able to communicate what they may have already noticed: that these values make zero.

Indicate these are called "zero pairs" because they add to zero. This provides the basis for a discussion of additive inverses more generally in LESSON 2.

For the next round, **ensure** that students encounter a situation where they eventually run out of +$1 tickets when they complete a chore if they continue to add tickets as they did in the previous rounds. This requires students to problem solve through how to handle this situation in an alternate way.

They can, at this time, remove zero pairs (which will not change the balance) so they have the necessary number of +$1 tickets available to add to the piggy bank. Some students may discover they can also remove –$1 tickets to increase their balance by the desired amount, though this will be the focus of a later lesson. The following table is an example of a list of chores where students will run out of +$1 tickets. Adding or removing tickets is not specified in the following table—only how the balance should change as a result of the completion or failure to complete the chore.

Ask the following questions as students work.

- When you add a credit (+$1), does the balance in your piggy bank increase or decrease? By how much?
- When you add a debit (–$1), does the balance in your piggy bank increase or decrease? By how much?
- What can be removed without changing the value of the piggy bank?
- When you put the dishes away and need to add two +$1 tickets but don't have any left, what could you do to ensure the balance in your piggy bank increases by $2? (Remove zero pairs so it is possible to add the specified amount and increase the balance of your piggy bank.)

Order	Chore	Equation
1	You fed all the animals. +$3	0 + 3 = 3
2	You forgot to take out the trash. –$1	3 + (–1) = 2
3	You cleaned your room. +$3	2 + 3 = 5
4	You forgot to clean the bathroom. –$2	5 + (–2) = 3
5	You walked the dogs. +$4	3 + 4 = 7
At this point, students should have all ten +$1 tickets in their piggy bank along with three –$1 tickets if they have been adding tickets only along the way and reporting the new balance. The next chore will require students to determine an alternate		

way to increase the balance of their piggy bank, either by removing debits or removing zero pairs and then adding back the credits for the chore.		
6	You put the dishes away. +$2	7 + 2 = 9

In subsequent rounds, frequently **ask** students about alternate ways to represent the same action or piggy bank balance.

For the next round, **ensure** that students encounter a situation where they eventually run out of –$1 tickets when they fail to complete a chore if they continue to add tickets as they did in the previous rounds. This requires students to problem solve through how to handle this situation in an alternate way.

They can, at this time, remove zero pairs (which will not change the balance) so that they have the necessary number of –$1 tickets available to add to the piggy bank. Some students may discover they can also remove +$1 tickets in order to decrease their balance by the specified amount, though this will be the focus of a later lesson.

The following table is an example of a list of chores where students will run out of –$1 tickets. Adding or removing tickets is not specified—only how the balance should change as a result of the completion or failure to complete the chore.

Ask the following questions as students work.

- When you add a credit (+$1), does the balance in your piggy bank increase or decrease? By how much?
- When you add a debit (–$1), does the balance in your piggy bank increase or decrease? By how much?
- What can be removed without changing the value of the piggy bank?
- When you forget to mow the grass and need to add four –$1 tickets but don't have enough –$1 tickets left, what could you do to decrease the balance in your piggy bank by $4? (Remove zero pairs so it is possible to add the specified debits and decrease the balance of your piggy bank.)

Order	Chore	Equation
1	You babysat your cousin. +$4	0 + 4 = 4
2	You forgot to feed the fish. –$2	4 + (–2) = 2
3	You forgot to vacuum your room. –$3	2 + – 3 = –1
4	You forgot to make your bed. –$1	–1 + (–1) = –2
5	You walked the dog. +$2	–2 + 2 = 0
6	You did not take out the trash all week. –$3	0 + (–3) = –3
At this point, students should have nine –$1 tickets in their piggy bank along with six +$1 tickets if they have been adding tickets only along the way and reporting the new balance. The next chore will require students to determine an alternate way to decrease the balance of their piggy bank, either by removing credits or removing zero pairs and then adding back the debits for the incomplete chore.		
7	You forgot to mow the grass. – $4	–3 + (–4) = –7

Repeat or continue this activity with several additional chores so students encounter scenarios where they have to remove zero pairs in order to adjust the value according to the chore description.

Determine a point to stop describing additional chores and **ask** students whether they would earn or owe money if the week ended then. Once students establish whether they made or owe money, they should calculate the value.

It is possible that students arrive at the same value with piggy banks that appear different based on decisions they made during the activity. This is an opportunity for students to discuss whether different forms of the same value in their piggy banks could be accurate, and what actions may cause one student's piggy bank to appear different from another student's.

If desired, students could create their own list of chores and values (reasonably small to accommodate the tickets provided) and a coin flip could determine whether or not the chore was completed.

Once students have adjusted their own piggy bank value, provide the class a picture of a piggy bank with five credits ($1 tickets) in the bank.

Using a piggy bank with five +$1 tickets, **ask** students some or all of the following questions.

- Would a piggy bank with seven +$1 tickets and two –$1 tickets have the same value as a piggy bank with five +$1 tickets?
- What are three other ways to represent a piggy bank with a balance of five credits? (six +$1 tickets and one –$1 ticket, eight +$1 tickets and three –$1 tickets, nine +$1 tickets and four –$1 tickets, etc.)
- What would you do if you were asked to remove two debits (–$1)? (Model a balance of five credits using seven credits and two debits so there are two debits available to remove.)
- If you remove two –$1 tickets, will the balance of the piggy bank increase or decrease? By how much?

Students should be required to consistently describe alternate methods of achieving the same balance in the piggy bank and represent these scenarios using equations with integers. Further understanding that addition of a negative value (debit) decreases the balance, and that subtraction or removal of a negative value (debit) increases the balance, sets the foundation for transferring this knowledge to operating on a number line.

At the end of the activity, teachers should provide students with a description of a piggy bank balance and ask students to represent that balance, in pictures or using a table, in two different ways. Then the teacher should request students to add or remove a specified value from each piggy bank and to represent this action in pictures and in an equation underneath each piggy bank.

An extension of this lesson would be to provide the students with an expression or equation containing integer addition or subtraction, and to ask the students to produce a story regarding a realistic context other than credits and debits.

Name_____

Rational Number Addition and Subtraction

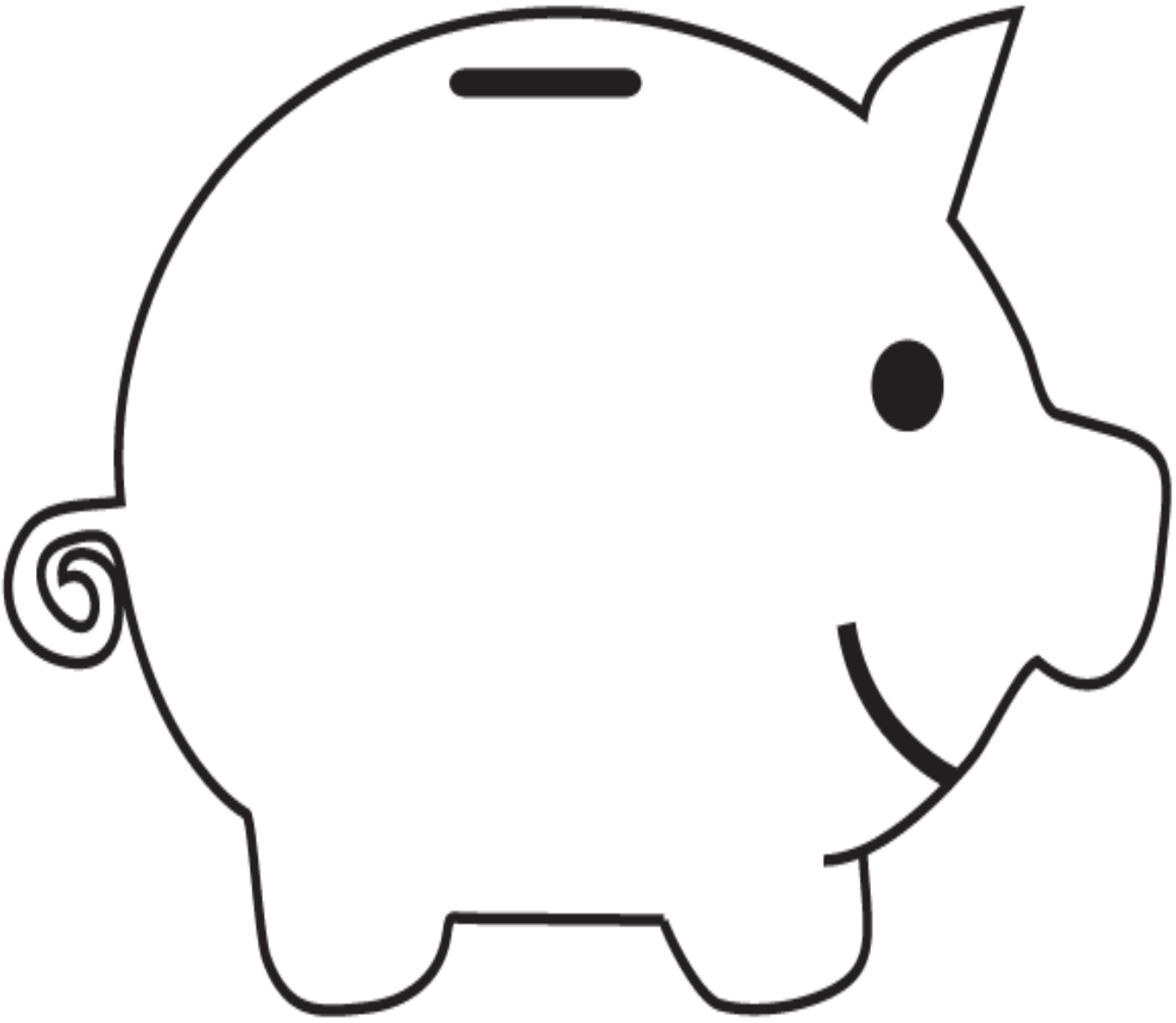
Lesson 1

ORDER	CHORE	+/– SKETCH OF PIGGY BANK CONTENTS	EQUATION

Rational Number Addition and Subtraction

INSTRUCTIONAL ACTIVITY SUPPLEMENT

Lesson 1



+$1	+$1	+$1	+$1	+$1
+$1	+$1	+$1	+$1	+$1

+$1	+$1	+$1	+$1	+$1
+$1	+$1	+$1	+$1	+$1

+$1	+$1	+$1	+$1	+$1
+$1	+$1	+$1	+$1	+$1

–$1	–$1	–$1	–$1	–$1
–$1	–$1	–$1	–$1	–$1

–$1	–$1	–$1	–$1	–$1
–$1	–$1	–$1	–$1	–$1

–$1	–$1	–$1	–$1	–$1
–$1	–$1	–$1	–$1	–$1

RATIONAL NUMBER ADDITION AND SUBTRACTION

INSTRUCTIONAL ACTIVITY

Lesson 2

LEARNING GOAL

Students will review the number line and its symmetry, plot values on the number line, and answer questions pertaining to distance and opposite values (additive inverses) regarding the values plotted on the number line.

PRIMARY ACTIVITY

Students will construct a number line, plot points on the number line, and answer questions related to symmetry, distance, and opposites (additive inverses) about values on the number line.

> This lesson will likely require more than one class period to complete.

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

- Integers
- Negative
- Number line
- Opposite
- Plot
- Sum
- Symmetry

MATERIALS

- INSTRUCTIONAL ACTIVITY SUPPLEMENT (Recommend one copy for every student.)
- Counters to plot points (small candies, plastic/paper circles, circles leftover from a 3-hole punch, etc.)

IMPLEMENTATION

GENERATING THE NUMBER LINE

Students should begin with the number line in the INSTRUCTIONAL ACTIVITY SUPPLEMENT. Note that tick marks are present, but only zero is labeled.

Ask students to generate and label the positive side of the number line by counting (by ones).

Have students fold the number line at zero to generate the negative side of the number line (counting by ones). **Note** that negative one and one are symmetric with respect to zero, negative two and two are symmetric with respect to zero, and so forth. Holding the number line up to the light allows students to see the positive side of the number line and they can number the negative side accordingly.

Discuss that opposite numbers have the same magnitude, but different direction, with respect to zero. For example, four and negative four are both four units from zero, but in opposite directions.

Ask students what must be true about the space between marks for evenly spaced numbers (e.g. the distance between three and four on the number line compared to the distance between negative six and negative five). Students should note that intervals of the same size must be represented by equal distances on the number line.

GUIDING QUESTIONS

Elicit student thinking:

- How would you describe the number line?

Determine if the student can EXPLAIN *NUMBER LINE*:

- What is true about the positive and negative sides of the number line?
- What must be true about the intervals on a number line?

Determine if the student can RECOGNIZE THE NEGATIVE SIGN:

- What is the symbol in front of the numbers on the left side of zero?
- What does the negative sign indicate regarding the value of the numbers to the left of zero on the number line?
- Where else have you seen the negative sign?
- What are some different ways you can use the "–" symbol?
- Can you provide examples of different ways you can use the "–" symbol?

ADDITIVE INVERSES ON THE NUMBER LINE

Once the number line is labeled and the fact that opposites have the same magnitude but different direction is established, shift students' focus to the concept that opposites (additive inverses) add to zero. Start with the concepts from the activity in LESSON 1 with credits and debits in a piggy bank. The familiar context of credits and debits along with the idea of symmetry will allow students to develop a more holistic understanding of the number line.

Ask students to recall their work with zero pairs from LESSON 1. They should recall that –$1 and +$1 add to zero and are therefore called zero pairs. Use this as the foundation to generalize the sum of any pair of opposites.

Ask students if they had four credits (+$4) and four debits (–$4), which are opposite pairs, in their piggy bank, what their balance would be. They should respond that the balance is zero.

Note that adding a value and its opposite always results in a balance, or sum, of zero.

Repeat this questioning with a few different values. To extend the concept to other rational numbers, **include** values such as one half of a credit (+$0.50) and one half of a debit (–$0.50) or two and one half credits (+$2.50) and two and one half debits (–$2.50) to solidify students' understanding that all opposite pairs add to zero.

Extend the questioning to inform students the balance of the piggy bank is zero and there are a given number of credits. **Ask** students for the value of debits that confirms the balance of the piggy bank is zero.

Repeat this questioning for a variety of given credits and debits, **asking** the students for the other value that would create a balance of zero in the piggy bank.

Finally, **tell** students the balance in the piggy bank is zero and **ask** them what the possible credits and debits inside could be.

Relate these concepts to the number line after discussing them in terms of credits and debits from LESSON 1. Have students place a counter at a given value on the number line. Then **ask** students to place a second counter at the value that, when added to the first, gives a result of zero.

Ask students what they notice about the two values on the number line. They should note that they are equidistant from zero in opposite directions, or symmetric across zero. In other words, their magnitudes are the same, but in different directions from zero.

Repeat this questioning for a variety of values on the number line. **Include** fractions and decimals in order to more thoroughly address the entire set of rational numbers.

GUIDING QUESTIONS

Elicit student thinking:

- What do you know about opposites?
- How can you model opposites in a piggy bank with credits and debits?

Determine if the student can LOCATE INTEGERS ON A NUMBER LINE:

- [Provide students with two opposite integers.] Where would these two values be located on the number line?

Determine if the student can EXPLAIN SITUATIONS IN WHICH OPPOSITE QUANTITIES MAKE 0:

- In terms of credits and debits, what would be two values that result in a balance of zero?
- On the number line, can you point to a pair of opposite values?
- Can you show me two numbers on the number line that make zero when they are added together?

Determine if the student can EXPLAIN *ADDITIVE INVERSE*:

- Can you point to an example of additive inverses on the number line?
- In terms of credits and debits, can you give an example of additive inverses?
- What is the sum of additive inverses?

SUBTRACTION ON THE NUMBER LINE

Next, students will practice finding the distance between two numbers on the number line. The goal is to help students relate the distance on the number line to subtraction of integers.

Provide students with the expression 5 – 2 and ask them to model this subtraction on the number line with counters or by marking the movement with a pencil. An alternative is to use laminated number lines and for students to use dry erase markers to plot points.

Focus students' attention on how far apart the values five and two are. Start by plotting the values five and two on the number line, then count how far apart the values are. Students should count three spaces between the values, meaning they are three units apart.

Ask students to relate how far apart the values five and two were on the number line to the result of subtracting 5 – 2. They should note that the distance is the result of the subtraction and vice versa.

Next, **present** students with the expression 5 – (–2) and **ask** students how they could determine the difference between the values. Students should suggest plotting the values five and negative two and counting the space between the values. Students should count seven spaces between five and negative two and determine the difference is seven.

Guide students to relate this value to the result of subtracting negative two from five. (**Note** that *order is significant* in subtraction, therefore the result of subtracting five from negative two would actually be negative seven, whose absolute value is seven.) It is helpful to **refer** to credits and debits from LESSON 1 and phrase the question as "If you have a balance of five credits in the piggy bank and then, as a result of completing a chore, you are able to remove two debits, what would the new balance in your piggy bank be?"

When using distances on the number line (which are always positive) to determine the difference, students must consider whether the initial amount is greater than or less than the amount that is subtracted to determine whether the difference is positive or negative. For example, in the expression 5 – (–2), 5 is seven greater than –2, so the difference is positive seven. In the expression –2 – 5, –2 is seven less than 5, so the difference is negative seven.

Rather than resorting to memory "tricks", students need a concrete way to understand why the expression 5 – (–2) is different than –2 – 5.

Repeat this process and questioning with a variety of numbers. **Include** examples such as –2 – (–5) and –8 – 10 to ensure students can determine whether the result of the subtraction is a positive or negative value.

GUIDING QUESTIONS

Elicit student thinking:

- What are benefits to using the number line?

Determine if the student can LOCATE NEGATIVE NUMBERS ON A NUMBER LINE:

- Where, in general, are negative numbers located on the number line?
- Where is negative three located on the number line? How far is it from zero? In what direction?

Determine if the student can LOCATE INTEGERS ON A NUMBER LINE:

- How is the location of negative four different from the location of positive four on the number line?
- How is the location of negative four similar to the location of positive four on the number line?
- [Provide the student with any integer.] Where is this value located on the number line?

Determine if the student can EXPLAIN DISTANCE BETWEEN 2 RATIONAL NUMBERS:

- How far apart are these two values on the number line?
- How does the distance between two values on the number line relate to subtraction?
- Is the distance between two numbers on the number line always the result of subtracting those numbers?
- How does the order of the values in a subtraction expression impact the result?

Determine if the student can REPRESENT SUBTRACTION OF INTEGERS ON A NUMBER LINE:

[Provide students with two integers: a and b.] What is the value of a – b? What is the value of b – a? How do these values relate to the distance between a and b on the number line?

Students should be required to explain their reasoning and relate their work on the number line to the credit and debit activity in LESSON 1. Relating the number line to a more intuitive experience and to symmetry will help students retain their understanding of rational numbers, and it will provide the foundation for students to extend their knowledge to future concepts.

At the end of the activity, require students to draw a point at five on the number line. Then ask students to address or respond to the following questions. Example responses showing student understanding are provided in parentheses.

- Place the opposite of five on the number line. (–5)
- Find the value of five minus its opposite. (5 – (–5) = 10) Describe how this is represented on the number line (5 and –5 are 10 units apart on the number line), then describe this operation in terms of credits and debits in a piggy bank. (If the balance in the piggy bank is five credits and you remove five debits, which is equivalent to adding five credits, the new balance will be ten credits.)
- Would your answer be different if you were asked to find the opposite of five minus five? Justify your response using credits and debits in a piggy bank. (Yes, –5 – 5 = –10 instead of 10. In terms of the piggy bank, if you have a balance of five debits and take away five credits, your balance would be ten debits or the opposite of ten credits.)
- Find the sum of five and its opposite (0). Describe how this is represented on the number line (opposites add to zero), then describe this operation in terms of credits and debits in a piggy bank. (If the piggy bank has five credits and five debits, the balance of the piggy bank is zero.)
- Would your answer be different if you were asked to find the sum of the opposite of five and five? Justify your response using credits and debits in a piggy bank. (No, –5 + 5 = 0. In terms of the piggy bank, if the piggy bank has five debits and five credits, the balance of the piggy bank is still zero.)

Rational Number Addition and Subtraction

INSTRUCTIONAL ACTIVITY SUPPLEMENT

Lesson 2

RATIONAL NUMBER ADDITION AND SUBTRACTION

INSTRUCTIONAL ACTIVITY

Lesson 3

LEARNING GOAL

Students will extend their understanding of integer addition and subtraction, making connections with the number line and the concepts of credits and debits.

PRIMARY ACTIVITY

Students will use their existing understanding of adding and subtracting integers formed in the credit and debit activity to perform operations using a number line, rather than physical counters or objects, as a way to keep track of values. Students may utilize strategies from the credit and debit activity to reason through integer addition and subtraction.

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

- Number line
- Integers
- Sum
- Difference

MATERIALS

- INSTRUCTIONAL ACTIVITY STUDENT HANDOUT
- Counters to plot points (small candies, plastic/paper circles, etc.)

IMPLEMENTATION

Students should begin adding and subtracting integers using a combination of the concepts they learned in LESSON 1 and their number line knowledge in LESSON 2.

Review the equations students wrote in LESSON 1 as they were participating in the activity with credits and debits. Following are a few rows from the example provided in LESSON 1 including an alternate representation of adding a debit to the piggy bank in the third row and removing a debit from the piggy bank in the fourth row.

Discuss the equation 2 + –3 = –1 in terms of credits and debits. (Starting with an initial value of two credits and adding three debits will decrease the value of the piggy bank by three, resulting in a balance of negative one or one debit.)

Then, **discuss** the equation 2 – 3 = –1 on the number line. (Start at two on the number line and decrease the value by three units, resulting in a value of negative one.)

After discussing each scenario individually, **note** that both actions result in a decrease of three and the same simplest form, and are therefore equivalent.

Discuss the equation –1 – (–1) = 0 in terms of credits and debits. (Starting with an initial value of one debit and removing one debit will increase the value of the piggy bank by one, resulting in a balance of zero.)

Then, **discuss** the equation –1 + 1 = 0 on the number line. (Start at negative one on the number line and increase the value by one unit, resulting in a value of zero.)

After discussing each scenario individually, **note** that both actions result in an increase of one and the same simplest form, and are therefore equivalent.

Order	Chore	Equation
1	You took out the trash. +$1	0 + 1 = 1
2	You fed the fish. +$1	1 + 1 = 2
3	You forgot to clean your room. –$3	2 + – 3 = –1 or 2 – 3 = –1
4	You made your bed. Remove –$1	–1 – (–1) = 0 or –1 + 1 = 0

Provide students with an example to think through the following process to add or subtract integers (e.g., 3 – (–4)).

Encourage students to think of the first value as the starting amount or balance (in the piggy bank).

Discuss the following options for the operation following the first number.

If the operation is addition, the students should think of this as *adding* tickets to the piggy bank (they could be adding a credit, which would increase the balance, or a debit, which would decrease the balance).

If the operation is subtraction, the students should think of this as *subtracting*, *removing,* or *taking away* tickets from the piggy bank (they could be removing a credit, which would decrease the balance, or a debit, which would increase the balance).

Encourage students to think of the second value as the amount that is either added to or subtracted from the balance (or the piggy bank).

Discuss whether the balance will increase or decrease as a result of the action described in the expression. Continue to relate the symbols back to the credit and debit activity with chores from LESSON 1.

If possible, **write an alternate expression** to reflect the discussion regarding whether the balance will increase or decrease.

For example, for the expression 3 – (–4), a debit of four is removed from the balance (or the piggy bank). Therefore, the balance will increase by four and this expression could be written as 3 + 4.

Similarly, if the expression was 3 + (–4), a debit of four is being added to the balance (or the piggy bank). Therefore, the balance will decrease by four and this expression could be written as 3 – 4.

Encourage students to use the number line to perform the simplification once these discussions have occurred.

Note that if students have determined the value will increase, then students should always move to the right on the number line (larger values). If students have determined the value will decrease, then students should always move to the left on the number line (smaller values).

In the provided example, students should start at three and count to the left five spaces. Therefore, the result, or the simplified expression, is negative two.

Discuss the first completed example on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT as a class. Students should then be ready to complete the rest of the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT in pairs or small groups.

GUIDING QUESTIONS

Elicit student thinking:

- Does adding always increase the value/balance?
- Does subtracting always decrease the value/balance?

Determine if the student can ADD 2 NEGATIVE INTEGERS:

- What is the current balance of the piggy bank?
- Are you adding or removing a value?
- What value is being added to the piggy bank?
- Will the balance of the piggy bank increase or decrease as a result?
- What is the new balance of the piggy bank?

Determine if the student can ADD 2 INTEGERS WITH DIFFERENT SIGNS:

- What is the current balance of the piggy bank?
- Are you adding or removing a value?
- What value is being added to the piggy bank?
- Will the balance of the piggy bank increase or decrease as a result?
- What is the new balance of the piggy bank?

Determine if the student can REPRESENT ADDITION OF INTEGERS ON A NUMBER LINE:

- How is the original/starting value significant when modeling addition on the number line?
- How is the original value being changed?
- Will the original value increase or decrease as a result?
- What circumstances are modeled by moving to the right on a number line?
- What circumstances are modeled by moving to the left on a number line?
- What is the result of the addition?

Determine if the student can EXPLAIN ADDITION OF INTEGERS:

- What is the original value/starting value/balance of the piggy bank? How does this value help you determine the result of the addition?
- Are you adding or removing a value?
- What value is being added to the original value/balance?
- Will the original value/balance of the piggy bank increase or decrease as a result?
- How can you describe this situation in terms of credits and debits?
- How would addition be represented on a number line?

Determine if the student can SUBTRACT 2 WHOLE NUMBERS WHOSE DIFFERENCE IS LESS THAN 0:

- What is the current balance of the piggy bank?
- Are you adding or removing a value?
- What value is being removed from the piggy bank?
- Will the balance of the piggy bank increase or decrease as a result?
- What is the new balance of the piggy bank?

Determine if the student can SUBTRACT 2 NEGATIVE INTEGERS:

- What is the current balance of the piggy bank?
- Are you adding or removing a value?
- What value is being removed from the piggy bank?
- Will the balance of the piggy bank increase or decrease as a result?
- What is the new balance of the piggy bank?

Determine if the student can SUBTRACT 2 INTEGERS WITH DIFFERENT SIGNS:

- What is the current balance of the piggy bank?
- Are you adding or removing a value?
- What value is being removed from the piggy bank?
- Will the balance of the piggy bank increase or decrease as a result?
- What is the new balance of the piggy bank?

Determine if the student can REPRESENT SUBTRACTION OF INTEGERS ON A NUMBER LINE:

- How is the original/starting value significant when modeling subtraction on the number line?
- How is the original value being changed?
- Will the original value increase or decrease as a result?
- What circumstances are modeled by moving to the right on a number line?
- What circumstances are modeled by moving to the left on a number line?
- What is the result of the subtraction?

Determine if the student can EXPLAIN SUBTRACTION OF INTEGERS:

- What is the original value/starting value/balance of the piggy bank? How does this value help you determine the result of the addition?
- Are you adding or removing a value?
- What value is being removed from the original value/balance?
- Will the original value/balance of the piggy bank increase or decrease as a result?
- How can you describe this situation in terms of credits and debits?
- How would subtraction be represented on a number line?

Determine if the student can APPLY PROPERTIES OF OPERATIONS TO ADDITION AND SUBTRACTION OF RATIONAL NUMBERS:

- If your piggy bank has a balance of 5 credits and you add 3 debits, what is the new balance? If the piggy bank instead starts with a balance of 3 debits and you add 5 credits, what would the balance be? Did the order that the credits and debits were added matter? What mathematical property is demonstrated in this example?

Students should be required to thoroughly complete each question in the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT. If students are struggling to do so, they should utilize scaffolds from the credit and debit activity in LESSON 1. Sketching a piggy bank and individual credits and debits may be beneficial for some students.

At the end of the activity, provide students with a variety of additional integer addition and subtraction questions without the written scaffolds to guide their thinking. Include examples of subtraction that require students to regroup. Require students to use whatever means necessary to simplify each expression.

Name_____

Rational Number Addition and Subtraction

Lesson 3

Use the following structure to simplify each expression. Thinking in terms of the balance and adjusting credits and debits may be beneficial. The first question has been completed for you as an example. Note that there may not be an equivalent expression for all questions (e.g., 2 + 1 is the most simplified form of the expression).

1. Expression: 3 + (−5)

   Starting value/balance: 3

   Operation meaning (circle one):

   (add to the balance) subtract from the balance

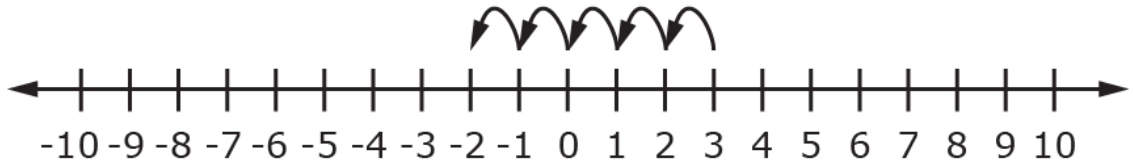
   Value of the change: −5

   In the end, the balance will (circle one):

   increase (decrease)

   Alternate expression (if applicable): 3 − 5

   Number line representation:



   Result: −2

2. Expression: −1 + 7

   Starting value/balance: _____

   Operation meaning (circle one):

   add to the balance subtract from the balance

Name_____

Value of the change: _____

In the end, the balance will (circle one):

increase decrease

Alternate expression (if applicable): _____

Number line representation:

Result:

3. Expression: –2 – 8

Starting value/balance: _____

Operation meaning (circle one):

add to the balance subtract from the balance

Name_____

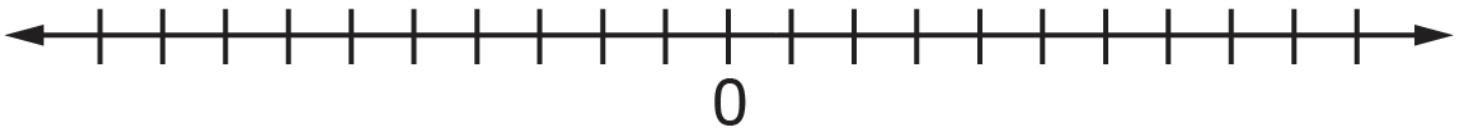
Value of the change: _____

In the end, the balance will (circle one):

increase decrease

Alternate expression (if applicable): _____

Number line representation:



Result:

4. Expression: 7 + (–3)

Starting value/balance: _____

Operation meaning (circle one):

add to the balance subtract from the balance

Name_____

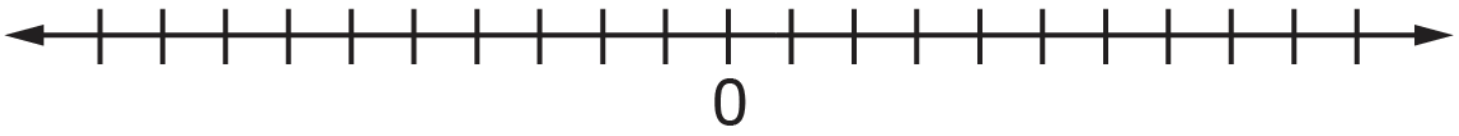
Value of the change: _____

In the end, the balance will (circle one):

increase decrease

Alternate expression (if applicable): _____

Number line representation:



Result:

5. Expression: 5 – (–1)

Starting value/balance: _____

Operation meaning (circle one):

add to the balance subtract from the balance

Name_____

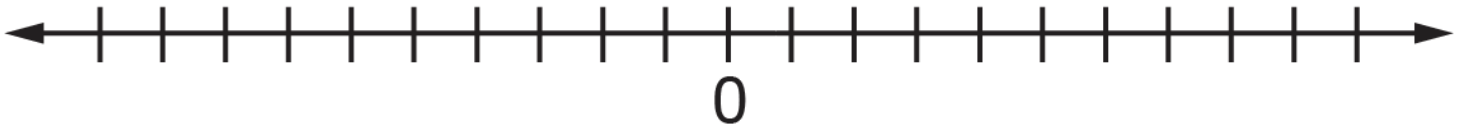
Value of the change: _____

In the end, the balance will (circle one):

increase decrease

Alternate expression (if applicable): _____

Number line representation:



Result:

6. Expression: –3 – (–4)

Starting value/balance: _____

Operation meaning (circle one):

add to the balance subtract from the balance

Value of the change: _____

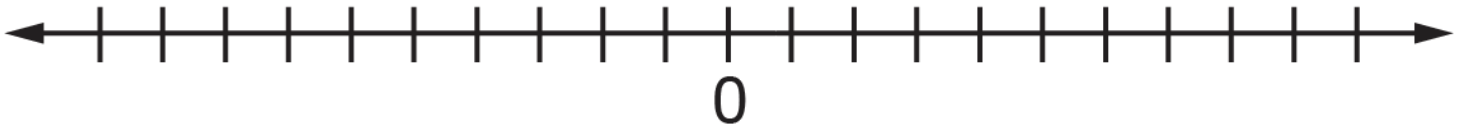
Name_____

In the end, the balance will (circle one):

increase decrease

Alternate expression (if applicable): _____

Number line representation:



Result:

7. Expression: 7 – 9

Starting value/balance: _____

Operation meaning (circle one):

add to the balance subtract from the balance

Value of the change: _____

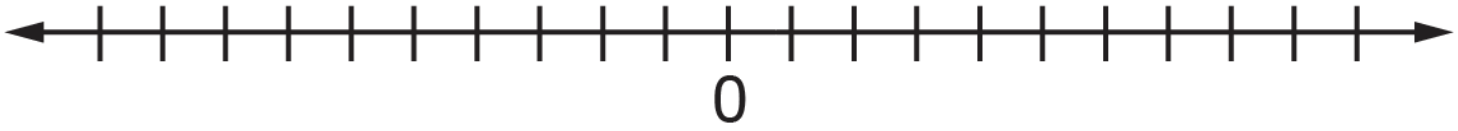
Name_____

In the end, the balance will (circle one):

increase decrease

Alternate expression (if applicable): _____

Number line representation:



Result:

8. Expression: 2 + 6

Starting value/balance: _____

Operation meaning (circle one):

add to the balance subtract from the balance

Value of the change: _____

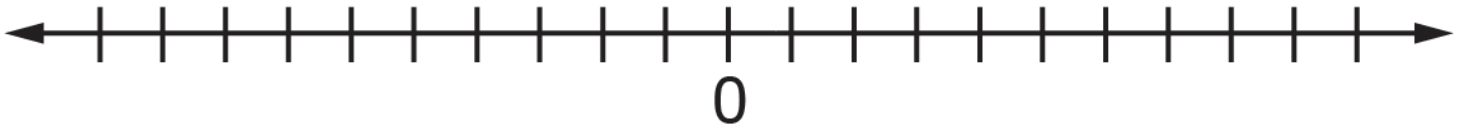
In the end, the balance will (circle one):

Name_____

increase decrease

Alternate expression (if applicable): _____

Number line representation:



Result:

9. Expression: –1 + (–5)

Starting value/balance: _____

Operation meaning (circle one):

add to the balance subtract from the balance

Value of the change: _____

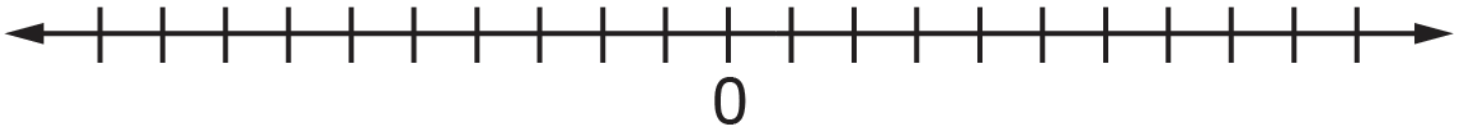
In the end, the balance will (circle one):

Name_____

increase decrease

Alternate expression (if applicable): _____

Number line representation:



Result:

RATIONAL NUMBER ADDITION AND SUBTRACTION

INSTRUCTIONAL ACTIVITY

Lesson 4

LEARNING GOAL

Students will extend their understanding of integer addition and subtraction to addition and subtraction of rational numbers.

PRIMARY ACTIVITY

Students will use their existing understanding of adding and subtracting integers formed in the credit and debit activity to add and subtract rational numbers. Students may utilize strategies from the credit and debt activity to reason through rational number addition and subtraction.

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

- Number line
- Integers
- Rational numbers
- Sum

MATERIALS

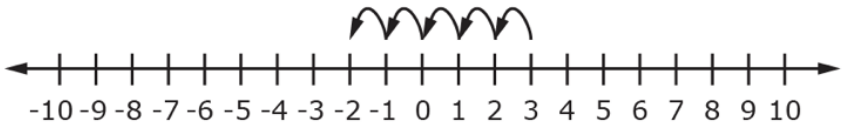
- INSTRUCTIONAL ACTIVITY STUDENT HANDOUT
- INSTRUCTIONAL ACTIVITY SUPPLEMENT (Recommend one copy for every student.)

IMPLEMENTATION

Students should reference their knowledge of adding and subtracting integers from LESSON 3 as they extend their understandings of integer addition and subtraction to all rational numbers.

Review the scaffold students used to add and subtract integers in LESSON 3. Following is an example provided in the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT for LESSON 3.

Expression:	3 + (–5)	
Starting value/balance:	3	
Operation meaning (circle one):	add to the balance	subtract from the balance
Value of the change:	–5	
In the end, the balance will (circle one):	increase	decrease
Alternate expression (if applicable):	3 – 5	
Number line representation:		



Result:	–2

Ask students if they can think of any other types of numbers that can be positive or negative. **Guide** students to consider fractions and decimals, which they should have experience with from earlier grades.

Discuss how students can extend their understandings of addition and subtraction of integers to fractions and decimals (rational numbers).

Review, as needed, addition and subtraction of positive fractions and decimals.

Extend students' knowledge of adding and subtracting positive fractions to include negative fractions, beginning with the worked out example provided in the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

Students should consider and operate with fractions on a number line in a way that is consistent with their work with integers.

NOTE: When adding and subtracting fractions on a number line, once students determine the least common denominator for the fractions they are adding or subtracting, they should eventually use this value to partition the number line. For example, if the least common denominator is 12, students may start by partitioning the number line into fourths because one denominator is 4, but should then continue to partition the number line into twelfths. An awareness of the value of the fraction and whether the initial value will increase or decrease as a result of the operation will help students determine how much and which portion of the number line must be labeled to model the addition or subtraction. A scaffold for students who struggle to set up the number line would be for the teacher to guide the process of setting

up the number line for them, or provide a pre-labeled number line, to focus their attention on concepts of addition and subtraction of rational numbers.

Provide a few additional examples of addition and subtraction expressions containing negative fractions. **Start** with examples that can be modeled on a number line labeled from negative two to positive two, then move to examples that require students to draw their own number line and consider the values that must be present to model the expression.

Review the idea that the unit length must remain constant on the number line, even when it is subdivided to accommodate fractions or decimals. Students may have a tendency to make more space for subdivided units than units that are not subdivided.

Require students to model the addition or subtraction on the number line provided in the INSTRUCTIONAL ACTIVITY SUPPLEMENT.

Relate the symbols back to the credit and debit activity with chores from LESSON 1 and operations with fractions from previous instruction.

Once students show an understanding of addition and subtraction when the expression contains negative fractions, **shift** students' focus to operations with decimals.

NOTE: Adding and subtracting decimals can be modeled with the concept of credits and debits alone or in conjunction with the number line. Solid number sense, additive inverses, the commutative property, and the use of the number line, even if it is an approximation, will allow students to reason through operations with decimals without using a calculator. For example, in the expression 1.2 + (–2.35), in order to operate by hand, it is beneficial to decompose –2.35 into –1.2 and –1.15 so students are able to see that the zero pair, 1.2 and –1.2, simplifies to zero and, therefore, –1.15 is the simplest form of the expression. This should be approximately the resulting location on the number line if a student plots a point at 1.2 and models a decrease of 2.35. When possible, appealing to students' natural tendency to combine negative numbers in the same way as positive numbers is beneficial as well.

Provide a few additional examples of addition and subtraction containing negative decimals to discuss as a class. **Start** with examples that can be modeled on a number line labeled from negative two to positive two, then move to examples that require students to draw their own number line and consider the values that must be present to accommodate the expression.

Require students to model the addition or subtraction on the number line provided in the INSTRUCTIONAL ACTIVITY SUPPLEMENT. Students should be precise as they work with decimals to the tenths place, but may need to plot the initial value and then approximate with decimals to the hundredths place or beyond.

Relate the symbols back to the credit and debit activity with chores from LESSON 1 and operations with decimals from previous instruction.

Students should now be ready to complete the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT in pairs or small groups. Blank number lines are provided for all questions, though it is reasonable to only require students to operate precisely on number lines when operating with fractions and decimals to the tenths place. When expressions include decimals to the hundredths place or beyond, students should accurately place the initial value, then approximate the increase or decrease on the number line in order to estimate the simplified value, which must be calculated by hand for precision. Modeling precisely on the number line serves to enhance students' representational fluency and strengthen the student's conceptual framework for working with signed numbers. Ultimately, students should think abstractly about relative positions on a number line without needing a concrete model to simplify every addition and subtraction expression. An estimation of the number line along with computation is the goal for most students.

GUIDING QUESTIONS

Elicit student thinking:

- Does adding always increase the value/balance?
- Does subtracting always decrease the value/balance?
- How is adding and subtracting fractions on a number line similar to adding and subtracting integers? How is it different?

Determine if the student can ADD RATIONAL NUMBERS:

- What is the original value?
- Are you adding or removing a value?
- What value is being added to the original value/balance?
- Will the original value increase or decrease as a result?
- What is the result of the addition?
- How can you describe this situation in terms of credits and debits?

Determine if the student can REPRESENT ADDITION OF RATIONAL NUMBERS ON A NUMBER LINE:

- What is the initial value? Where is this value located on the number line?
- Are you adding or removing a value?
- What value is being added to the initial value/balance?
- Will the initial value increase or decrease as a result?
- How can you model this on a number line? What intervals would help you model the addition?
- What is the result of the addition?

Determine if the student can EXPLAIN ADDITION OF RATIONAL NUMBERS:

- What is the initial value?
- Are you adding or removing a value?
- What value is being added to the initial value/balance?
- Will the initial value increase or decrease as a result?
- How can you describe this situation in terms of credits and debits?
- How would addition be represented on a number line?

Determine if the student can SUBTRACT RATIONAL NUMBERS:

- What is the initial value?
- Are you adding or removing a value?
- What value is being removed from the initial value/balance?
- Will the original value increase or decrease as a result?
- What is the result of the subtraction?
- How can you describe this situation in terms of credits and debits?

Determine if the student can REPRESENT SUBTRACTION OF RATIONAL NUMBERS ON A NUMBER LINE:

- What is the initial value? Where is this value located on the number line?
- Are you adding or removing a value?
- What value is being removed from the initial value/balance?
- Will the initial value increase or decrease as a result?
- How can you model this on a number line? What intervals would help you model the subtraction?
- What is the result of the subtraction?

Determine if the student can EXPLAIN SUBTRACTION OF RATIONAL NUMBERS:

- What is the initial value?
- Are you adding or removing a value?
- What value is being removed from the initial value/balance?
- Will the initial value increase or decrease as a result?
- How can you describe this situation in terms of credits and debits?
- How would subtraction be represented on a number line?

Determine if the student can APPLY PROPERTIES OF OPERATIONS TO ADDITION AND SUBTRACTION OF RATIONAL NUMBERS:

- [Provide students with an expression containing addition of two rational numbers.] Can you demonstrate this addition either with credits and debits or on the number line? What happens to the sum if the numbers are switched? Does the sum change?

Determine if the student can EXPLAIN SUMS OF RATIONAL NUMBERS IN REAL-WORLD PROBLEMS:

- Can you explain this addition in terms of credits and debits?
- Can you think of a context, other than credits and debits, that this addition could describe?

Determine if the student can EXPLAIN DIFFERENCES OF RATIONAL NUMBERS IN REAL-WORLD PROBLEMS:

- Can you explain this subtraction in terms of credits and debits?
- Can you think of a context, other than credits and debits, that this subtraction could describe?

Students should be required to thoroughly complete each question in the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT. If students are struggling to do so, they should utilize scaffolds from the credit and debit activity in LESSON 1. Sketching a piggy bank and individual credits and debits may be beneficial for some students.

At the end of the activity, provide students with a variety of additional rational number addition and subtraction questions without the written scaffolds to guide their thinking. Include examples of subtraction that require students to regroup. Require students to use whatever means necessary to simplify each expression.

Name_____

Rational Number Addition and Subtraction

Lesson 4

Use the following structure to simplify each expression. Thinking in terms of the balance and adjusting credits and debits may be beneficial. The first question has been completed for you as an example. Note that there may not be an equivalent expression for all questions (e.g., 2 + 1 is the most simplified form of the expression).

1. Expression: $-\frac{3}{4} - \left(-\frac{5}{2}\right)$

   Starting value/balance: $-\frac{3}{4}$

   Operation meaning (circle one):

   add to the balance subtract from the balance

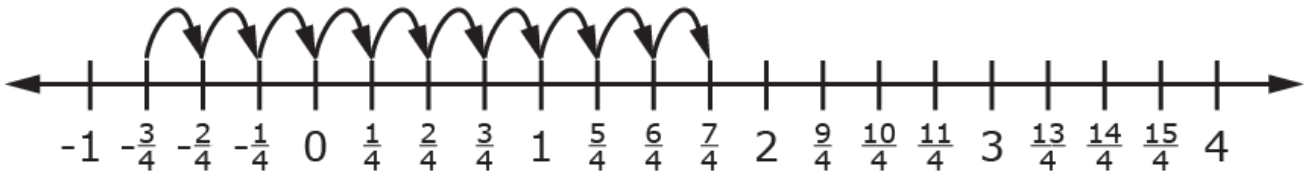
   Value of the change: $-\frac{5}{2}$

   In the end, the balance will (circle one):

   increase decrease

   Alternate expression (if applicable): $-\frac{3}{4} + \frac{10}{4}$

   Number line representation:



   Result: $\frac{7}{4}$

2. Expression: 1.4 + (−2.6)

   Starting value/balance: _____

Name_____

Operation meaning (circle one):

add to the balance subtract from the balance

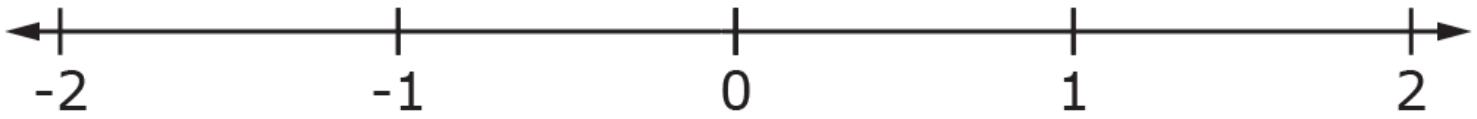
Value of the change: _____

In the end, the balance will (circle one):

increase decrease

Alternate expression (if applicable): _____

Number line representation:



Result:

3. Expression: $\frac{2}{3} - \frac{7}{6}$

Starting value/balance: _____

Operation meaning (circle one):

add to the balance subtract from the balance

Name_____

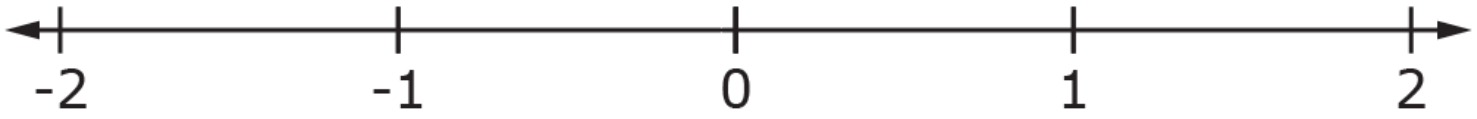
Value of the change: _____

In the end, the balance will (circle one):

increase decrease

Alternate expression (if applicable): _____

Number line representation:



Result:

4. Expression: $-\frac{7}{5}+\frac{1}{2}$

Starting value/balance: _____

Operation meaning (circle one):

add to the balance subtract from the balance

Value of the change: _____

In the end, the balance will (circle one):

increase decrease

Alternate expression (if applicable): _____

Number line representation:

Result:

5. Expression: –5.13 – (–1.47)

Starting value/balance: _____

Operation meaning (circle one):

add to the balance subtract from the balance

Value of the change: _____

In the end, the balance will (circle one):

increase decrease

Alternate expression (if applicable): _____

Name_____

Number line representation (approximate):

Result:

6. Expression: $-\frac{3}{2} - \frac{5}{6}$

Starting value/balance: _____

Operation meaning (circle one):

add to the balance subtract from the balance

Value of the change: _____

In the end, the balance will (circle one):

increase decrease

Alternate expression (if applicable): _____

Number line representation:

Result:

7. Expression: 13.1 – (–4.55)

   Starting value/balance: _____

   Operation meaning (circle one):

   add to the balance subtract from the balance

   Value of the change: _____

   In the end, the balance will (circle one):

   increase decrease

   Alternate expression (if applicable): _____

   Number line representation (approximate):

   Result:

Name_____

8. Expression: $\frac{3}{8} + \frac{3}{2}$

   Starting value/balance: _____

   Operation meaning (circle one):

   add to the balance subtract from the balance

   Value of the change: _____

   In the end, the balance will (circle one):

   increase decrease

   Alternate expression (if applicable): _____

   Number line representation:

   Result:

9. Expression: –2.39 + (–1.47)

   Starting value/balance: _____

Name_____

Operation meaning (circle one):

add to the balance subtract from the balance

Value of the change: _____

In the end, the balance will (circle one):

increase decrease

Alternate expression (if applicable): _____

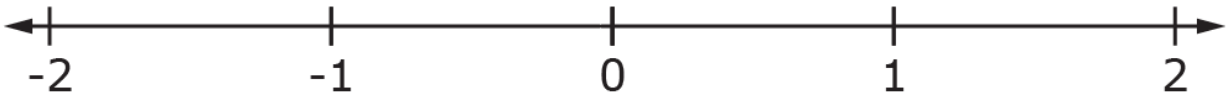
Number line representation (approximate):

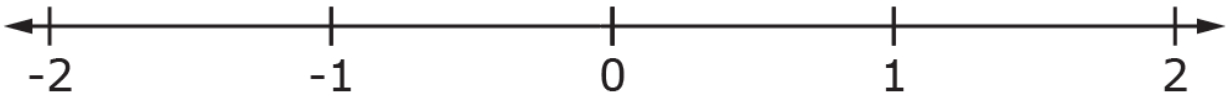
Result:

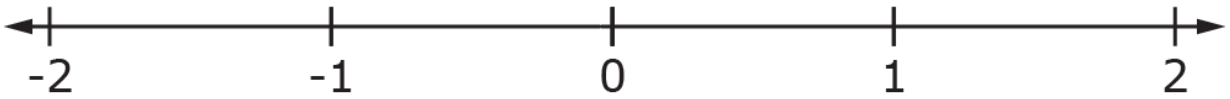
Rational Number Addition and Subtraction

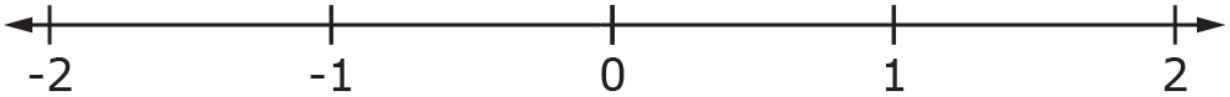
INSTRUCTIONAL ACTIVITY SUPPLEMENT

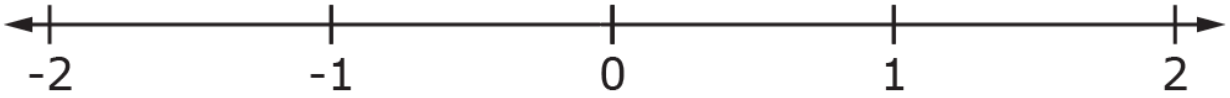
Lesson 4

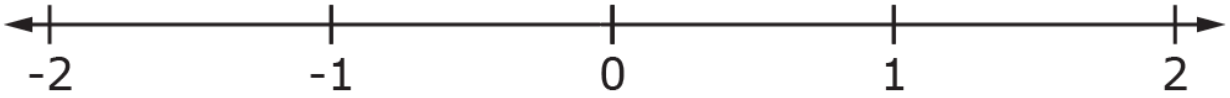












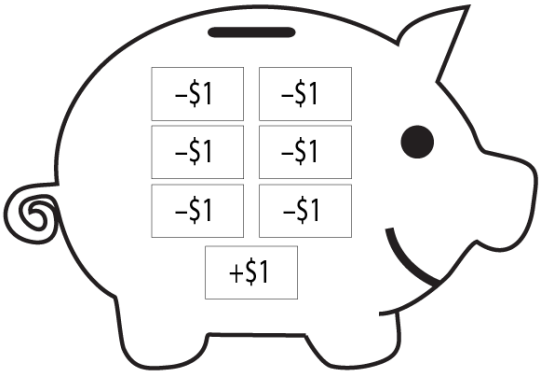
Name_____

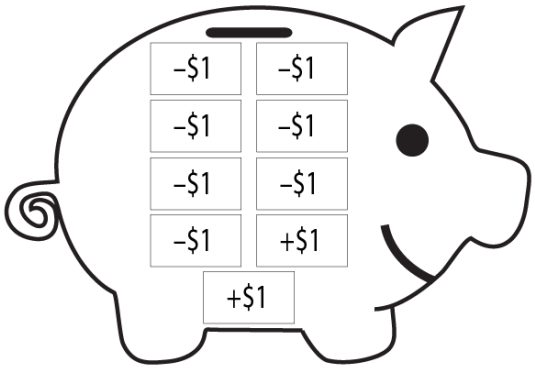
Rational Number Addition and Subtraction

Lessons 1 – 4

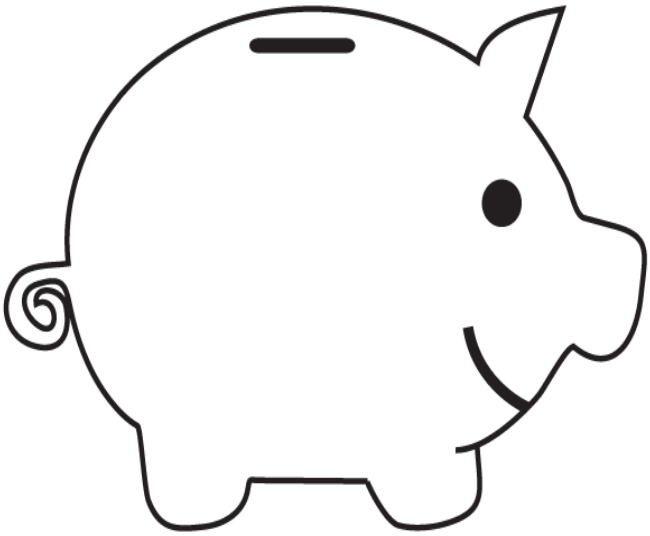
1. Use the piggy banks provided to draw two different representations of the given balance using single credits (+$1) and debits (–$1).

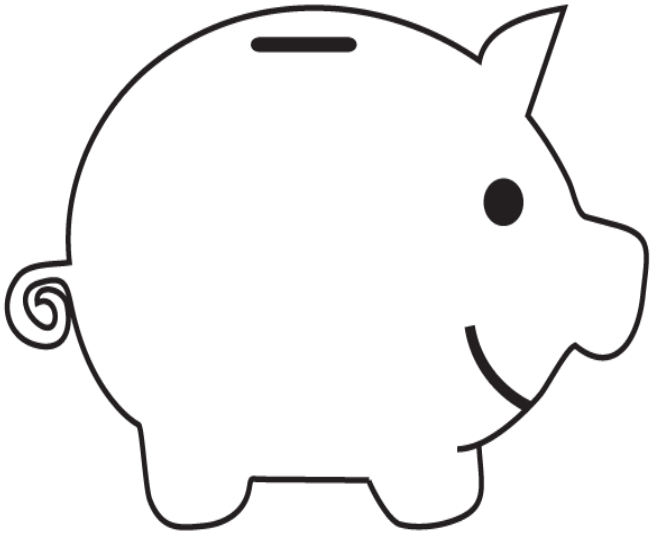
   Example: Two possible representations of a balance of 5 debits (–$5) are shown below.





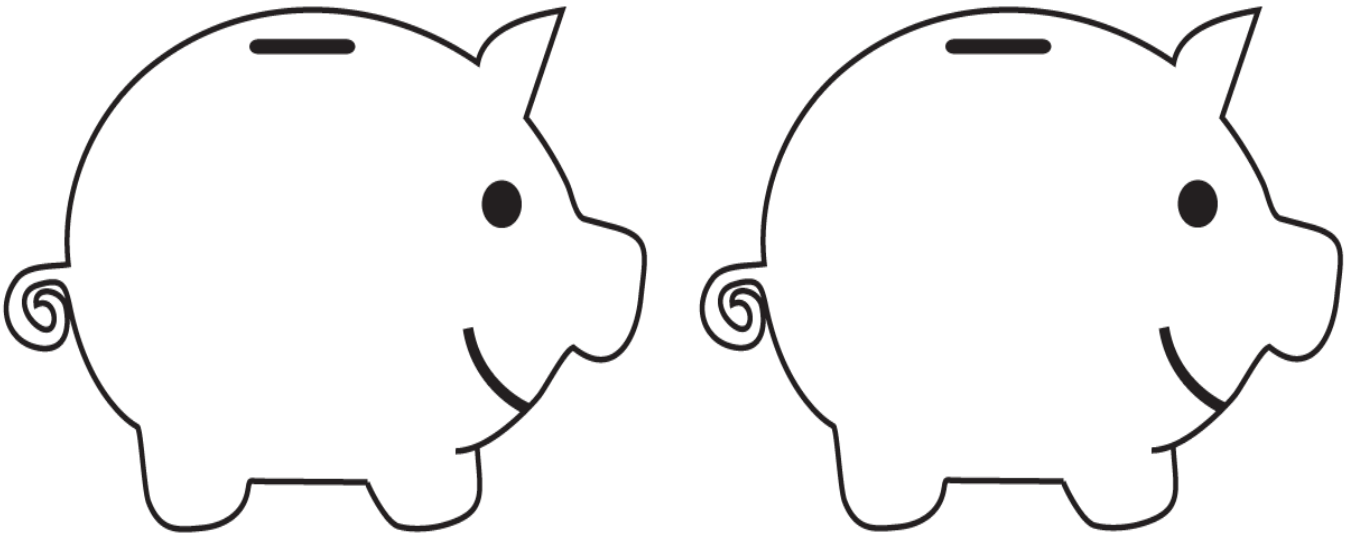
1.a. A balance of 4 credits (+$4)



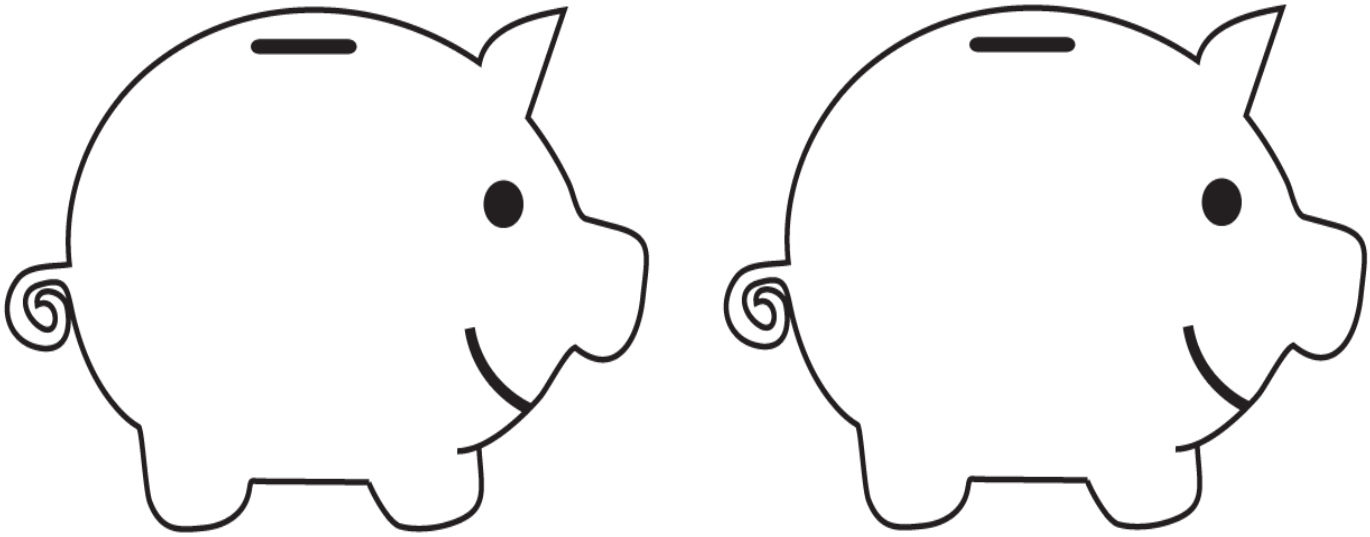


Name_____

1.b. A balance of 3 debits (–$3)

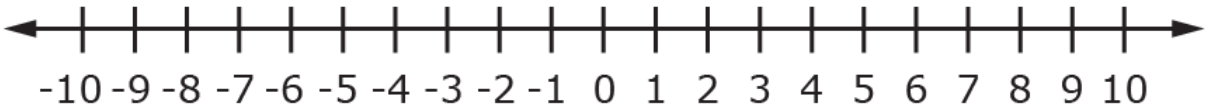


1.c. A balance of 0 ($0)



Name_____

2. A number line is provided to reference as you respond to the following questions.



2.a. Describe how the number line is symmetric with respect to zero.

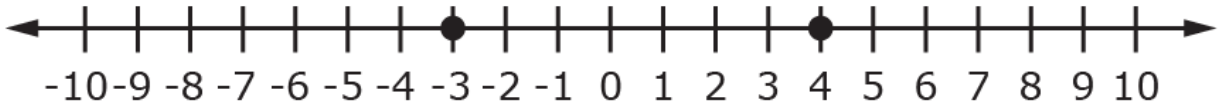
2.b. Compare the position of –8 to the position of 8 by describing both magnitude and direction relative to zero.

Name_____

2.c. Compare the position of −7 to the position of 5 by describing both magnitude and direction relative to zero.

2.d. Compare the position of −1 to the position of −4 by describing both magnitude and direction relative to zero.

3. Two values have been placed on the number line provided. Use the number line to answer the following questions.



3.a. How many units apart are −3 and 4 on the number line?

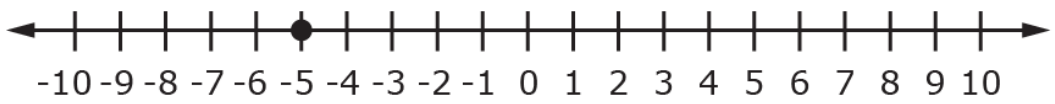
3.b. Describe the expression 4 – (–3) in terms of credits and debits in a piggy bank. What would the balance of the piggy bank be as a result of this action? How does this result relate to your response in part (a)?

3.c. Describe the expression –3 – 4 in terms of credits and debits in a piggy bank. What would the balance of the piggy bank be as a result of this action? How does this result relate to your response in part (a)?

Name_____

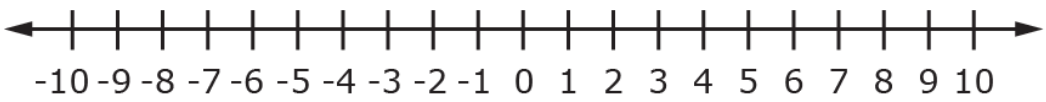
4. The balance of your piggy bank is +$9 (9 credits). What value, when added to the piggy bank, would result in a balance of 0? Explain your reasoning.

5. A value is represented on the number line provided. Plot the opposite of this value and describe the criteria for two numbers to be opposites.



Name_____

6. Using the number line provided, plot two values that are additive inverses. How are additive inverses and opposite numbers related?



7. Use the expression to answer the following questions.

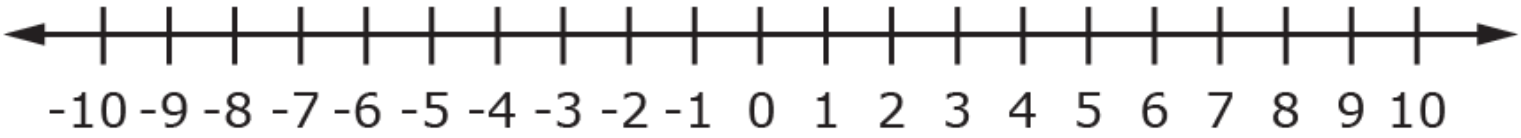
$$-2 + 5$$

7.a. Create a story about credits and debits to match this expression.

Name_____

7.b. Will the initial value increase or decrease as a result of this operation? Explain your reasoning.

7.c. Represent the scenario on the number line provided.



7.d. What is the simplest form of this expression?

Name_____

8. Use the expression to answer the following questions.

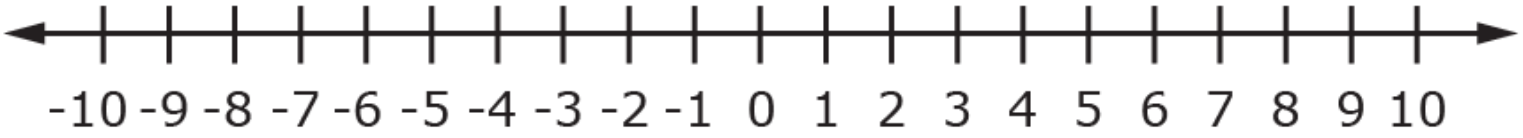
$$-3 - 6$$

8.a. Create a story about credits and debits to match this expression.

8.b. Will the initial value increase or decrease as a result of this operation? Explain your reasoning.

Name_____

8.c. Represent the scenario on the number line provided.



8.d. What is the simplest form of this expression?

9. Use the expression to answer the following questions.

$$10 + (-15)$$

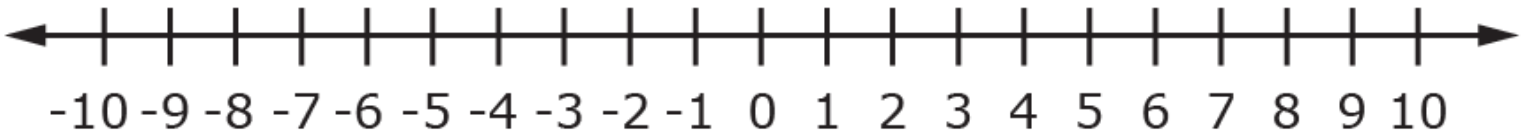
9.a. Create a story about credits and debits to match this expression.

Name_____

9.b. Will the initial value increase or decrease as a result of this operation? Explain your reasoning.

9.c. What other expression could be used to represent the same operation?

9.d. Represent the scenario on the number line provided.



9.e. What is the simplest form of this expression?

Name_____

10. Use the expression to answer the following questions.

$$-1 - (-8)$$

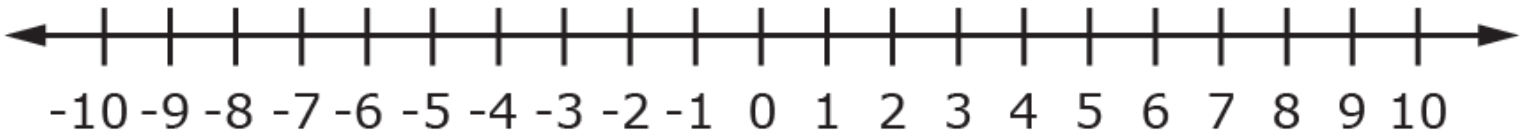
10.a. Create a story about credits and debits to match this expression.

10.b. Will the initial value increase or decrease as a result of this operation? Explain your reasoning.

Name_____

10.c. What other expression could be used to represent the same operation?

10.d. Represent the scenario on the number line provided.



10.e. What is the simplest form of this expression?

Name_____

11. Label the following number line such that it will be helpful as a precise model of the expression and its simplified form. Be sure to subdivide properly and show the entire range needed to simplify the expression. You do not need to perform the subtraction on the number line.

$$\frac{3}{2} - \left(-\frac{4}{3}\right)$$

12. Use the expression to answer the following questions.

$$\frac{7}{6} - \left(-\frac{8}{3}\right)$$

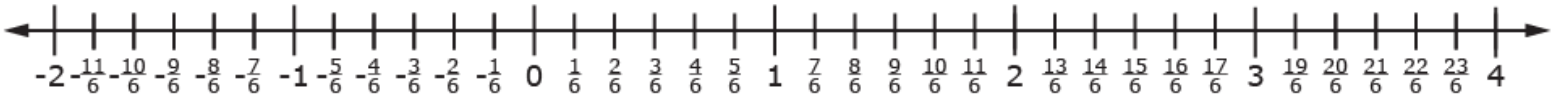
12.a. Create a story about credits and debits to match this expression.

Name_____

12.b. Will the initial value increase or decrease as a result of this operation? Explain your reasoning.

12.c. What other expression could be used to represent the same operation?

12.d. Represent the scenario on the number line provided.



12.e. What is the simplest form of this expression?

Name_____

13. Use the expression to answer the following questions.

$$-0.6 + (-1.2)$$

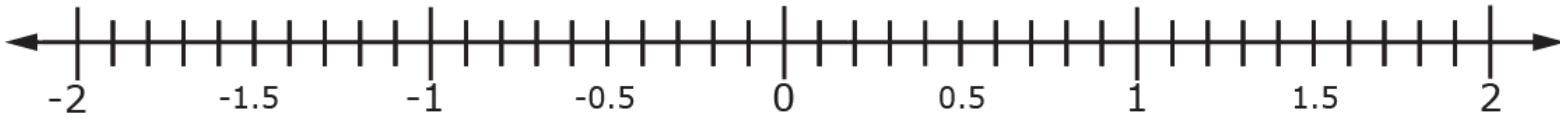
13.a. Create a story about credits and debits to match this expression.

13.b. Will the initial value increase or decrease as a result of this operation? Explain your reasoning.

Name_____

13.c. What other expression could be used to represent the same operation?

13.d. Represent the scenario on the number line provided.



13.e. What is the simplest form of this expression?

14. Use the expression to answer the following questions.

$$\frac{1}{4} + \left(-\frac{2}{5}\right)$$

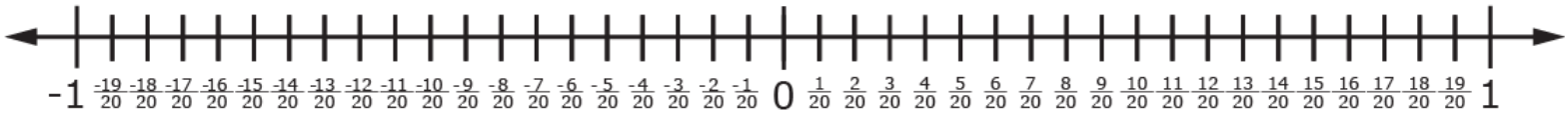
14.a. Create a story about credits and debits to match this expression.

Name_____

14.b. Will the initial value increase or decrease as a result of this operation? Explain your reasoning.

14.c. What other expression could be used to represent the same operation?

14.d. Represent the scenario on the number line provided.



14.e. What is the simplest form of this expression?

Name_____

15. Use the expression to answer the following questions.

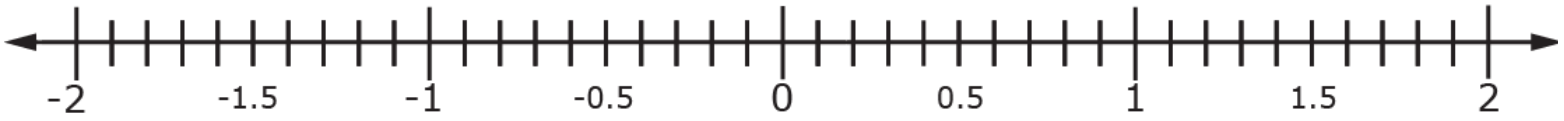
$$-0.13 + 1.39$$

15.a. Create a story about credits and debits to match this expression.

15.b. Will the initial value increase or decrease as a result of this operation? Explain your reasoning.

Name_____

15.c. Represent the scenario on the number line provided. Place the initial value as precisely as possible, then approximate the amount of increase or decrease on the number line.



15.d. What is the simplest form of this expression?

RATIONAL NUMBER ADDITION AND SUBTRACTION

STUDENT ACTIVITY SOLUTION GUIDE

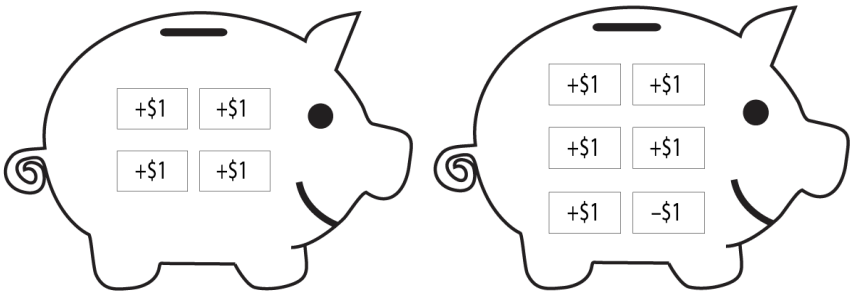
Lessons 1 – 4

1. Use the piggy banks provided to draw two different representations of the given balance using single credits (+$1) and debits (–$1).

   1.a. A balance of 4 credits (+$4)

CORRECT ANSWER

Answers will vary, but correct responses must have 4 more credits than debits. Two example responses are provided.



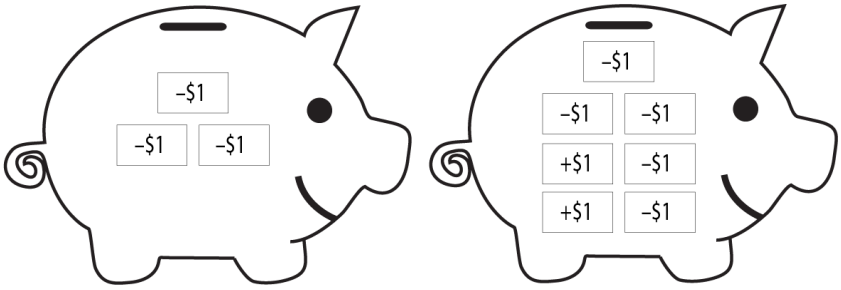
ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Student represents a balance of 4 credits using 4 "+$1" tickets but does not provide a second representation of 4 credits.	does not understand that opposite quantities add to zero and, therefore, cannot add an equal number of "+$1" and "–$1" tickets and know that the balance remains the same	EXPLAIN SITUATIONS IN WHICH OPPOSITE QUANTITIES MAKE 0
Student places 4 tickets total in both piggy banks without attention to the value on each ticket.	does not distinguish between the "+" and the "–" signs on the tickets and the impact they have on the balance of the piggy bank	RECOGNIZE THE NEGATIVE SIGN and RECOGNIZE THE POSITIVE SIGN

1.b. A balance of 3 debits (–$3)

CORRECT ANSWER

Answers will vary, but correct responses must have 3 more debits than credits. Two example responses are provided.



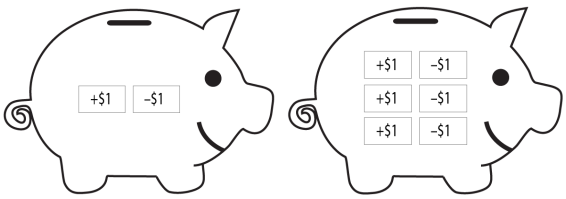
ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Student represents a balance of 3 debits using 3 "–$1" tickets but does not provide a second representation of 3 debits.	does not understand that opposite quantities add to zero and, therefore, cannot add an equal number of "+$1" and "–$1" tickets and know that the balance remains the same	EXPLAIN SITUATIONS IN WHICH OPPOSITE QUANTITIES MAKE 0
Student places 3 tickets total in both piggy banks without attention to the value on each ticket.	does not distinguish between the "+" and the "–" signs on the tickets and the impact they have on the balance of the piggy bank	RECOGNIZE THE NEGATIVE SIGN and RECOGNIZE THE POSITIVE SIGN

1.c. A balance of 0 ($0)

CORRECT ANSWER

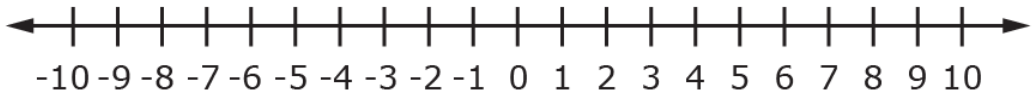
Answers will vary, but correct responses must have 3 more debits than credits. Two example responses are provided.



ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Student represents a balance of 0 using an empty piggy bank but does not provide an additional representation of 0.	does not understand that opposite quantities add to zero and, therefore, cannot add an equal number of "+$1" and "–$1" tickets and know that the balance remains the same	EXPLAIN SITUATIONS IN WHICH OPPOSITE QUANTITIES MAKE 0
Student places an even number of tickets in both piggy banks without attention to the value on each ticket.	does not distinguish between the "+" and the "–" signs on the tickets and the impact they have on the balance of the piggy bank	RECOGNIZE THE NEGATIVE SIGN and RECOGNIZE THE POSITIVE SIGN

2. A number line is provided to reference as you respond to the following questions.



2.a. Describe how the number line is symmetric with respect to zero.

CORRECT ANSWER

> The positive and negative numbers are symmetric with respect to zero on a number line. If you were to fold the number line on itself with the fold at zero, negative one would be a mirror image of one, negative two would be a mirror image of two, and so forth.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Student describes basic items they see on the number line without reference to symmetry, folding the number line at zero, or mirror image across zero.	does not understand what symmetry is	EXPLAIN *SYMMETRY*
Student states the number line has a line of symmetry without specifying where the line of symmetry is or which numbers are symmetric.	knows that if a number line is symmetric it must have a line of symmetry, but cannot identify the location on the number line or which values are symmetric	EXPLAIN *INTEGERS*

2.b. Compare the position of –8 to the position of 8 by describing both magnitude and direction relative to zero.

CORRECT ANSWER

> –8 and 8 have the same magnitude relative to zero, but in different (opposite) directions.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
States –8 and 8 have the same direction and the same magnitude.	does not notice or understand the impact of the "–" symbol on the direction	RECOGNIZE THE NEGATIVE SIGN
States –8 has a smaller magnitude than 8.	thinks the magnitude of –8 is less than 8 because –8 is less than 8	EXPLAIN *INTEGERS*

2.c. Compare the position of –7 to the position of 5 by describing both magnitude and direction relative to zero.

CORRECT ANSWER

> –7 is farther from zero than 5, so they have the different magnitudes relative to zero and are in different (opposite) directions.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
States –7 and 5 have the same direction.	does not notice or understand the impact of the "–" symbol on the direction	RECOGNIZE THE NEGATIVE SIGN
States –7 has a smaller magnitude than 5.	thinks the magnitude of –7 is less than 5 because –7 is less than 5	EXPLAIN *INTEGERS*

2.d. Compare the position of –1 to the position of –4 by describing both magnitude and direction relative to zero.

CORRECT ANSWER

–4 is farther from zero than –1, so they have the different magnitudes relative to zero, but are in the same direction from zero.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
States –4 has a smaller magnitude than –1.	thinks the magnitude of –4 is less than –1 because –4 is less than –1	EXPLAIN *INTEGERS*
States –4 and –1 have different directions because 4 and 1 are different numbers.	believes direction depends on the digit rather than the sign of the number	EXPLAIN *INTEGERS*

3. Two values have been placed on the number line provided. Use the number line to answer the following questions.

3.a. How many units apart are –3 and 4 on the number line?

CORRECT ANSWER

–3 and 4 are 7 units apart on the number line.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Student states –3 and 4 are 8 units apart.	counts the tick marks instead of the spaces between the tick marks	EXPLAIN *NUMBER LINE*
Student states –3 and 4 are 1 unit apart.	counts the interval between 3 and 4 instead of –3 and 4	RECOGNIZE THE NEGATIVE SIGN

3.b. Describe the expression 4 – (–3) in terms of credits and debits in a piggy bank. What would the balance of the piggy bank be as a result of this action? How does this result relate to your response in part (a)?

CORRECT ANSWER

This expression indicates an initial balance of 4 credits, then 3 debits are removed from the piggy bank. This increases the balance to 7 credits (+$7). The (absolute value of the) result of subtracting 4 – (–3) is equal to the distance between –3 and 4 on the number line.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Student only describes 4 – (–3) in terms of credits and debits without providing a result or relating to the number line.	cannot find the difference or relate the difference between two numbers to their distance apart on the number line	EXPLAIN DISTANCE BETWEEN 2 RATIONAL NUMBERS and SUBTRACT 2 INTEGERS WITH DIFFERENT SIGNS
Student performs the subtraction without relating the expression to credits and debits.	cannot relate an expression with subtraction to a real-world problem	EXPLAIN DIFFERENCES OF RATIONAL NUMBERS IN REAL-WORLD PROBLEMS
Student simplifies 4 – (–3) to 1.	treats subtraction of a negative value the same as subtraction of a positive value	SUBTRACT 2 INTEGERS WITH DIFFERENT SIGNS

3.c. Describe the expression –3 – 4 in terms of credits and debits in a piggy bank. What would the balance of the piggy bank be as a result of this action? How does this result relate to your response in part (a)?

CORRECT ANSWER

This expression indicates an initial balance of 3 debits, then 4 credits are removed from the piggy bank. This decreases the balance to 7 debits (–$7). The absolute value of the result of subtracting –3 – 4 is equal to the distance between –3 and 4 on the number line.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Student only describes –3 – 4 in terms of credits and debits without providing a result or relating to the number line.	cannot find the difference or relate the difference between two numbers to their distance apart on the number line	EXPLAIN DISTANCE BETWEEN 2 RATIONAL NUMBERS and SUBTRACT 2 INTEGERS WITH DIFFERENT SIGNS
Student performs the subtraction without relating the expression to credits and debits.	cannot relate an expression with subtraction to a real-world problem	EXPLAIN DIFFERENCES OF RATIONAL NUMBERS IN REAL-WORLD PROBLEMS
Student simplifies –3 – 4 to –1.	ignores the negative sign in front of the 3	SUBTRACT 2 INTEGERS WITH DIFFERENT SIGNS and RECOGNIZE THE NEGATIVE SIGN

4. The balance of your piggy bank is +$9 (9 credits). What value, when added to the piggy bank, would result in a balance of 0? Explain your reasoning.

CORRECT ANSWER

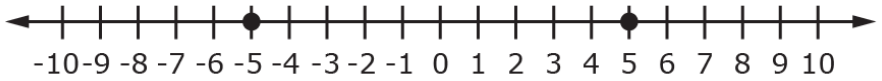
> –$9 (9 debits). I know that adding –$9 (9 debits) would result in a balance of 0 because it is the opposite (additive inverse) of +$9 (9 credits) and +9 and –9 are symmetric across 0 on a number line.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Student responds with –$9 or 9 debits without an explanation.	cannot explain why +9 + –9 = 0	EXPLAIN SITUATIONS IN WHICH OPPOSITE QUANTITIES MAKE 0
Student describes removing +$9 or 9 credits.	knows removing the existing amount results in a value of zero, but does not know that adding the opposite value also results in zero	EXPLAIN SITUATIONS IN WHICH OPPOSITE QUANTITIES MAKE 0

5. A value is represented on the number line provided. Plot the opposite of this value and describe the criteria for two numbers to be opposites.

CORRECT ANSWER



I know 5 is the opposite of –5 because they add to zero and are symmetric across zero on the number line. Opposites must be the same distance (magnitude) from zero but in opposite directions on the number line.

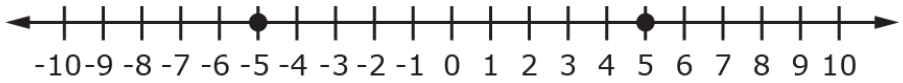
ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Student only states 5 is the opposite of –5 and plots a point at 5 on the number line.	does not understand the relationship between opposites on a number line	EXPLAIN OPPOSITE NUMBERS ON A NUMBER LINE
Places a point at a value other than 5 on the number line.	does not know how to identify the opposite value of a number on a number line	RECOGNIZE OPPOSITE NUMBERS ON A NUMBER LINE

6. Using the number line provided, plot two values that are additive inverses. How are additive inverses and opposite numbers related?

CORRECT ANSWER

Answers will vary, but students should plot a pair of opposite values on the number line. An example response is provided.



Additive inverses are opposite values. I know –5 and 5 are additive inverses because they are opposites and –5 + 5 = 0.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Student only plots two opposite values on the number line.	may have memorized but cannot explain additive inverses as values that add to zero	EXPLAIN *ADDITIVE INVERSE*
Student plots two values that are not additive inverses.	does not know what additive inverses are	EXPLAIN *ADDITIVE INVERSE*

7. Use the expression to answer the following questions.

$$-2 + 5$$

7.a. Create a story about credits and debits to match this expression.

CORRECT ANSWER

I start out owing 2 dollars, then I get 5 dollars.

or

This expression indicates the piggy bank starts with a balance of 2 debits, then 5 credits are added.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Start at –2 and add 5.	reads the expression without relating to credits and debits	EXPLAIN SUMS OF RATIONAL NUMBERS IN REAL-WORLD PROBLEMS

7.b. Will the initial value increase or decrease as a result of this operation? Explain your reasoning.

CORRECT ANSWER

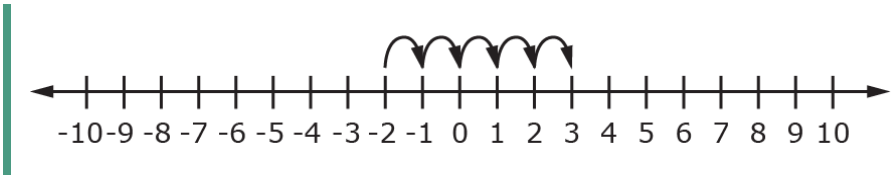
The initial value will increase, because credits are added to the piggy bank.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The initial value will decrease because there is a negative sign.	believes a negative sign in an expression indicates a decrease in value regardless of the location of the negative sign	EXPLAIN ADDITION OF INTEGERS
The student only states the value will increase.	may be working on the assumption that addition always increases the initial value	EXPLAIN ADDITION OF INTEGERS

7.c. Represent the scenario on the number line provided.

CORRECT ANSWER



ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Student circles or marks 3 without indicating the intial value or the amount of increase.	can plot the result of an addition problem but cannot demonstrate the process to arrive at the result on the number line	REPRESENT ADDITION OF INTEGERS ON A NUMBER LINE
Student models 2 + 5 on the number line.	does not understand the meaning of the negative sign	RECOGNIZE THE NEGATIVE SIGN

7.d. What is the simplest form of this expression?

CORRECT ANSWER

3

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
7	ignores the negative sign in front of the 2, therefore adding 2 and 5	RECOGNIZE THE NEGATIVE SIGN
–3	does not understand the importance of the location of the negative sign and simplifies the expression 2 – 5 instead of –2 + 5	ADD 2 INTEGERS WITH DIFFERENT SIGNS and RECOGNIZE THE NEGATIVE SIGN

8. Use the expression to answer the following questions.

$-3 - 6$

8.a. Create a story about credits and debits to match this expression.

CORRECT ANSWER

> I start out owing 3 dollars, then I lose 6 more dollars.
>
> or
>
> This expression indicates the piggy bank starts with a balance of 3 debits, then 6 credits are taken away.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Start at –3 and subtract 6.	reads the expression without relating to credits and debits	EXPLAIN DIFFERENCES OF RATIONAL NUMBERS IN REAL-WORLD PROBLEMS

8.b. Will the initial value increase or decrease as a result of this operation? Explain your reasoning.

CORRECT ANSWER

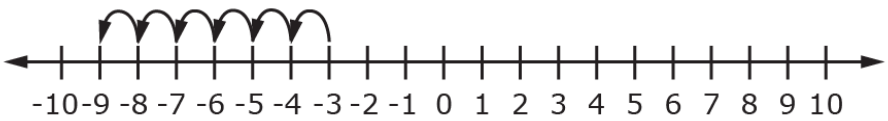
> The initial value will decrease, because credits are taken away from the piggy bank.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The initial value will increase because there are two negative signs.	believes two negative signs in an expression always indicates an increase in value regardless of the location of the negative signs	EXPLAIN SUBTRACTION OF INTEGERS
The student only states the value will decrease.	may be working on the assumption that subtraction always decreases the initial value	EXPLAIN SUBTRACTION OF INTEGERS

8.c. Represent the scenario on the number line provided.

CORRECT ANSWER



ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Student circles or marks –9 without indicating the intial value or the amount of decrease.	can plot the result of a subtraction problem but cannot demonstrate the process to arrive at the result on the number line	REPRESENT SUBTRACTION OF INTEGERS ON A NUMBER LINE
Student models 3 – 6 on the number line.	ignores the negative in front of the 3, may believe it is not needed because there is subtraction in the problem already	RECOGNIZE THE NEGATIVE SIGN
Student models 6 – 3 on the number line.	believes subtraction always has to be the bigger number minus the smaller number, does not pay attention to signs	EXPLAIN SUBTRACTION OF INTEGERS

8.d. What is the simplest form of this expression?

CORRECT ANSWER

–9

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
9	believes two negative signs in an expression can always be written as addition regardless of the location of the negative signs, therefore the student adds 3 and 6	SUBTRACT 2 INTEGERS WITH DIFFERENT SIGNS

Example Error	Misconception	Missing Knowledge
–3	ignores the negative in front of the 3, may believe it is not needed because there is subtraction in the problem already, therefore the student simplifies 3 – 6	RECOGNIZE THE NEGATIVE SIGN
3	believes subtraction always has to be the bigger number minus the smaller number, does not pay attention to signs	EXPLAIN SUBTRACTION OF INTEGERS

9. Use the expression to answer the following questions.

$$10 + (-15)$$

9.a. Create a story about credits and debits to match this expression.

CORRECT ANSWER

I start out with 10 dollars, then I add a debt of 15 dollars.

or

This expression indicates the piggy bank starts with a balance of 10 credits, then 15 debits are added.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Start at 10 and add –15.	reads the expression without relating to credits and debits	EXPLAIN SUMS OF RATIONAL NUMBERS IN REAL-WORLD PROBLEMS

9.b. Will the initial value increase or decrease as a result of this operation? Explain your reasoning.

CORRECT ANSWER

The initial value will decrease, because debits are added to the piggy bank.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The initial value will increase because there is addition.	believes addition always increases the initial value and does not account for the fact that a negative value is added rather than a positive value	EXPLAIN ADDITION OF INTEGERS
The initial value will decrease because there is a negative.	does not consider the placement of the negative sign, only that there is one in the expression	EXPLAIN ADDITION OF INTEGERS

9.c. What other expression could be used to represent the same operation?

CORRECT ANSWER

This expression can also be written as 10 – 15.

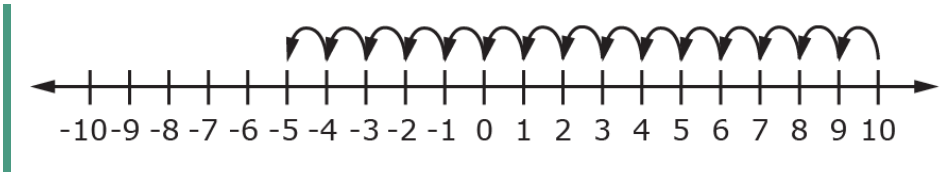
Alternatively, the commutative property allows the expression to be written as –15 + 10 although this does not model the scenario described in part (a). This expression could be modeled on an extended number line and would have the same result.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
There is not an alternate expression.	does not understand that adding a negative value is equivalent to subtracting a positive value	EXPLAIN ADDITION OF INTEGERS
15 + (–10)	believes the position of the 10 and 15 can occur in any order based on the commutative property, not realizing the importance of keeping the negative sign with the 15 if the order is changed	EXPLAIN ADDITION OF INTEGERS and RECOGNIZE THE NEGATIVE SIGN

9.d. Represent the scenario on the number line provided.

CORRECT ANSWER



ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Student circles or marks –5 without indicating the intial value or the amount of decrease.	can plot the result of a problem involving addition of a negative value but cannot demonstrate the process to arrive at the result on the number line using the alternate expression	REPRESENT ADDITION OF INTEGERS ON A NUMBER LINE
Student extends the number line and models 15 – 10 on the number line.	does not realize the importance of keeping the negative sign with the 15 if the order is changed, wants to subtract the smaller number from the larger number	EXPLAIN ADDITION OF INTEGERS

9.e. What is the simplest form of this expression?

CORRECT ANSWER

–5

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
5	sees the negative sign and subtracts the smaller number from the larger number	ADD 2 INTEGERS WITH DIFFERENT SIGNS
25	adds 10 and 15 without considering the negative sign	ADD 2 INTEGERS WITH DIFFERENT SIGNS and RECOGNIZE THE NEGATIVE SIGN

10. Use the expression to answer the following questions.

$$-1 - (-8)$$

10.a. Create a story about credits and debits to match this expression.

CORRECT ANSWER

I start out owing 1 dollar, then 8 of my debts are taken away.

or

This expression indicates the piggy bank starts with a balance of 1 debit, then 8 debits are removed.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Start at –1 and subtract –8.	reads the expression without relating to credits and debits	EXPLAIN DIFFERENCES OF RATIONAL NUMBERS IN REAL-WORLD PROBLEMS
Start at –1 and subtract 8 twice.	believes because there are two minus signs between the –1 and the 8, they must subtract twice	EXPLAIN SUBTRACTION OF INTEGERS

10.b. Will the initial value increase or decrease as a result of this operation? Explain your reasoning.

CORRECT ANSWER

The initial value will increase, because debits are removed from the piggy bank.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The initial value will decrease because there is subtraction.	believes subtraction always decreases the initial value and does not account for the fact that a negative value is subtracted rather than a positive value	EXPLAIN SUBTRACTION OF INTEGERS
The initial value will decrease because there are (an odd number of) negative signs.	does not consider the placement of the negative signs, only that they exist in the expression; may refer to an odd number of negative signs (thinking this indicates a decrease in the initial value) compared to an even number of negative signs (thinking this indicates an increase in the initial value)	EXPLAIN SUBTRACTION OF INTEGERS

10.c. What other expression could be used to represent the same operation?

CORRECT ANSWER

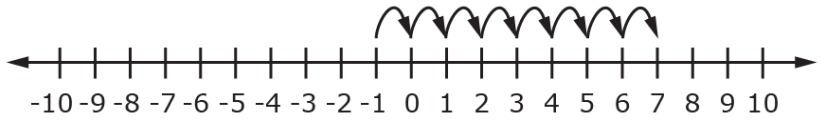
> This expression can also be written as $-1 + 8$.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
$-8 - (-1)$	thinks the commutative property applies to subtraction	APPLY PROPERTIES OF OPERATIONS TO ADDITION AND SUBTRACTION OF RATIONAL NUMBERS
$1 + 8$	thinks that because subtracting a negative value increases the initial value, the negative sign in front of the 1 is unnecessary	EXPLAIN SUBTRACTION OF INTEGERS and RECOGNIZE THE NEGATIVE SIGN
$8 - 1$	believes subtraction always has to be the bigger number minus the smaller number, does not pay attention to signs	EXPLAIN SUBTRACTION OF INTEGERS

10.d. Represent the scenario on the number line provided.

CORRECT ANSWER



ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Student circles or marks 7 without modeling the intial value or the amount of increase.	can plot the result of a problem involving subtraction of a negative value but cannot demonstrate the process to arrive at the result on the number line using the alternate expression	REPRESENT SUBTRACTION OF INTEGERS ON A NUMBER LINE
Student models 1 + 8 on the number line.	thinks that because subtracting a negative value increases the initial value, the negative sign in front of the 1 is unnecessary	EXPLAIN SUBTRACTION OF INTEGERS and RECOGNIZE THE NEGATIVE SIGN
Student models –1 – 8 – 8 on the number line.	believes because there are two minus signs between the –1 and the 8, they must subtract twice	EXPLAIN SUBTRACTION OF INTEGERS
Student models 8 – 1 on the number line.	believes subtraction always has to be the bigger number minus the smaller number, does not pay attention to signs	EXPLAIN SUBTRACTION OF INTEGERS

10.e. What is the simplest form of this expression?

CORRECT ANSWER

7

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

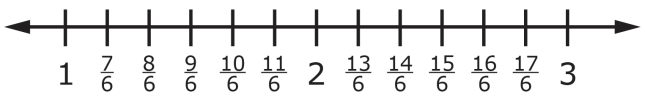
Example Error	Misconception	Missing Knowledge
–7	thinks the commutative property applies to subtraction	APPLY PROPERTIES OF OPERATIONS TO ADDITION AND SUBTRACTION OF RATIONAL NUMBERS
9	thinks that because subtracting a negative value increases the initial value, the negative sign in front of the 1 is unnecessary	SUBTRACT 2 NEGATIVE INTEGERS and RECOGNIZE THE NEGATIVE SIGN
7	believes subtraction always has to be the bigger number minus the smaller number, does not pay attention to signs	EXPLAIN SUBTRACTION OF INTEGERS
–17	believes because there are two minus signs between the –1 and the 8, they must subtract twice	EXPLAIN SUBTRACTION OF INTEGERS

11. Label the following number line such that it will be helpful as a precise model of the expression and its simplified form. Be sure to subdivide properly and show the entire range needed to simplify the expression. You do not need to perform the subtraction on the number line.

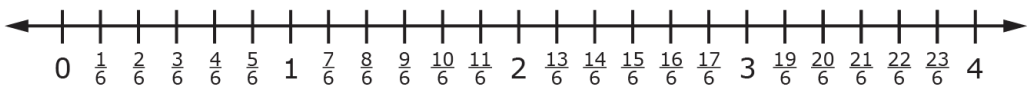
$$\frac{3}{2} - \left(-\frac{4}{3}\right)$$

CORRECT ANSWER

Answers may vary, but the number line should show values from $\frac{3}{2}$ or $\frac{9}{6}$ though $\frac{17}{6}$, divided into sixths. Following are two examples.



or



ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Student divides the number line into halves or thirds.	does not consider the common denominator when dividing the number line	REPRESENT SUBTRACTION OF RATIONAL NUMBERS ON A NUMBER LINE
Student allows space to start at $\frac{3}{2}$ and decrease by $\frac{4}{3}$ instead of increase by $\frac{4}{3}$.	believes subtraction will decrease the initial value without considering the value that is subtracted	EXPLAIN SUBTRACTION OF RATIONAL NUMBERS
Student allows space to start at $\frac{3}{2}$ and decrease by $\frac{4}{3}$ twice.	believes because there are two minus signs between the $\frac{3}{2}$ and the $\frac{4}{3}$, they must subtract twice	EXPLAIN SUBTRACTION OF RATIONAL NUMBERS
Student copies the number line from question 12 onto this number line.	may know the number line should be divided into sixths, but does not pay close attention to the range required to model the subtraction on the number line	REPRESENT SUBTRACTION OF RATIONAL NUMBERS ON A NUMBER LINE

12. Use the expression to answer the following questions.

$$\frac{7}{6} - \left(-\frac{8}{3}\right)$$

12.a. Create a story about credits and debits to match this expression.

CORRECT ANSWER

I start with $\frac{7}{6}$ of a dollar, then take away a debt of $\frac{8}{3}$.
or
This expression indicates the piggy bank starts with a balance of $\frac{7}{6}$ credits, then $\frac{8}{3}$ debits are removed.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Start at $\frac{7}{6}$ and subtract $-\frac{8}{3}$.	reads the expression without relating to credits and debits	EXPLAIN DIFFERENCES OF RATIONAL NUMBERS IN REAL-WORLD PROBLEMS

Example Error	Misconception	Missing Knowledge
Start at $\frac{7}{6}$ and subtract $\frac{8}{3}$ twice.	believes because there are two minus signs between the $\frac{7}{6}$ and the $\frac{8}{3}$, they must subtract twice	EXPLAIN SUBTRACTION OF RATIONAL NUMBERS

12.b. Will the initial value increase or decrease as a result of this operation? Explain your reasoning.

CORRECT ANSWER

> The initial value will increase, because debits are removed from the piggy bank.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The initial value will decrease because there is subtraction.	believes subtraction always decreases the initial value and does not account for the fact that a negative value is subtracted rather than a positive value	EXPLAIN SUBTRACTION OF RATIONAL NUMBERS
The student only states the value will increase.	knows subtracting a negative value increases the initial value but cannot explain why this is the case	EXPLAIN SUBTRACTION OF RATIONAL NUMBERS

12.c. What other expression could be used to represent the same operation?

CORRECT ANSWER

> This expression can also be written as $\frac{7}{6} + \frac{8}{3}$ or $\frac{7}{6} + \frac{16}{6}$.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
There is not an alternate expression.	does not understand that subtracting a negative value is equivalent to adding a positive value	EXPLAIN SUBTRACTION OF RATIONAL NUMBERS

Example Error	Misconception	Missing Knowledge
$\frac{7}{6}-\frac{8}{3}$ or $\frac{7}{6}-\frac{16}{6}$	believes subtracting a negative value and subtracting a positive value are equivalent operations	EXPLAIN SUBTRACTION OF RATIONAL NUMBERS
$\frac{8}{3}-\frac{7}{6}$ or $\frac{16}{6}-\frac{7}{6}$	believes subtraction always has to be the bigger number minus the smaller number, does not pay attention to signs	EXPLAIN SUBTRACTION OF RATIONAL NUMBERS

12.d. Represent the scenario on the number line provided.

CORRECT ANSWER

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Student determines the initial value on the number line but moves only $\frac{8}{6}$ instead of $\frac{16}{6}$ in the positive direction.	moves eight spaces because the value in the numerator is an eight, does not consider the value of the denominator compared to the intervals on the number line	REPRESENT SUBTRACTION OF RATIONAL NUMBERS ON A NUMBER LINE
Student determines the initial value on the number line but moves $\frac{16}{6}$ in the negative direction.	believes subtracting a negative value and subtracting a positive value are equivalent operations	EXPLAIN SUBTRACTION OF RATIONAL NUMBERS
Student models $\frac{8}{3}-\frac{7}{6}$ or $\frac{16}{6}-\frac{7}{6}$ on the number line.	believes subtraction always has to be the bigger number minus the smaller number, does not pay attention to signs	EXPLAIN SUBTRACTION OF RATIONAL NUMBERS
Student models $\frac{7}{6}-\frac{8}{3}-\frac{8}{3}$ or $\frac{7}{6}-\frac{16}{6}-\frac{16}{6}$ on the number line.	believes because there are two minus signs between the $\frac{7}{6}$ and the $\frac{8}{3}$, they must subtract twice	EXPLAIN SUBTRACTION OF RATIONAL NUMBERS
Student starts or places the initial value at a point other than $\frac{7}{6}$ (e.g., $\frac{5}{6}$).	may be unable to plot positive fractions on a number line	LOCATE POSITIVE FRACTIONS ON A NUMBER LINE

12.e. What is the simplest form of this expression?

CORRECT ANSWER

$\frac{23}{6}$ or $3\frac{5}{6}$

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
$\frac{-9}{6}$ or $\frac{-3}{2}$ or $-1\frac{1}{2}$	believes subtracting a negative value and subtracting a positive value are equivalent operations and computes $\frac{7}{6}-\frac{8}{3}$	EXPLAIN SUBTRACTION OF RATIONAL NUMBERS
$\frac{15}{9}$ or $\frac{5}{3}$ or $1\frac{2}{3}$	incorrectly computes $\frac{7}{6}+\frac{8}{3}$ by adding the numerators and adding the denominators independently of each other without finding a common denominator to add	SUBTRACT RATIONAL NUMBERS
$\frac{9}{6}$ or $\frac{3}{2}$ or $1\frac{1}{2}$	believes subtraction always has to be the bigger number minus the smaller number, does not pay attention to signs	EXPLAIN SUBTRACTION OF RATIONAL NUMBERS
$-\frac{25}{6}$ or $-4\frac{1}{6}$	believes because there are two minus signs between the $\frac{7}{6}$ and the $\frac{8}{3}$, they must subtract twice	EXPLAIN SUBTRACTION OF RATIONAL NUMBERS

13. Use the expression to answer the following questions. –0.6 + (–1.2)

13.a. Create a story about credits and debits to match this expression.

CORRECT ANSWER

I owe 60 cents (or 0.6 dollars) and then I add another debt of $1.20 (or 1.2 dollars).

or

This expression indicates the piggy bank starts with a balance of 0.6 debits, then 1.2 debits are added.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Start at –0.6 and add –1.2.	reads the expression without relating to credits and debits	EXPLAIN SUMS OF RATIONAL NUMBERS IN REAL-WORLD PROBLEMS

13.b. Will the initial value increase or decrease as a result of this operation? Explain your reasoning.

CORRECT ANSWER

> The initial value will decrease, because debits are added to the piggy bank.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The initial value will increase because there is addition.	believes addition always increases the initial value and does not account for the fact that a negative value is added rather than a positive value	EXPLAIN ADDITION OF RATIONAL NUMBERS
The initial value will decrease because there are negatives.	does not consider the placement of the negative signs, only that they exist in the expression	EXPLAIN ADDITION OF RATIONAL NUMBERS

13.c. What other expression could be used to represent the same operation?

CORRECT ANSWER

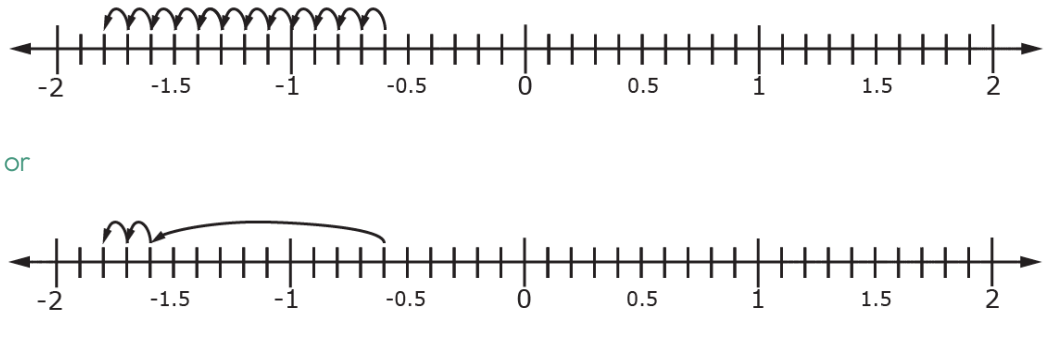
> This expression can also be written as –0.6 – 1.2.
>
> Alternatively, the commutative property allows the expression to be written as –1.2 + –0.6 or –1.2 – 0.6, although this does not model the scenario described in part (a).

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
There is not an alternate expression.	does not understand that adding a negative value is equivalent to subtracting a positive value	EXPLAIN ADDITION OF RATIONAL NUMBERS
0.6 + 1.2	thinks the two negative signs can be replaced with addition	EXPLAIN ADDITION OF RATIONAL NUMBERS and RECOGNIZE THE NEGATIVE SIGN

13.d. Represent the scenario on the number line provided.

CORRECT ANSWER



ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Student determines the initial value on the number line but moves a little more than one interval instead of 12 intervals in the negative direction.	moves a little more than one space because the value added is –1.2, does not consider the intervals on the number line are tenths	REPRESENT SUBTRACTION OF RATIONAL NUMBERS ON A NUMBER LINE
Student determines the initial value on the number line but moves 1.2 (12 intervals) in the positive direction.	believes adding a negative value and adding a positive value are equivalent operations	EXPLAIN ADDITION OF RATIONAL NUMBERS
Student starts or places the initial value at a point other than –0.6.	may be unable to plot rational numbers on a number line	LOCATE RATIONAL NUMBERS ON A NUMBER LINE

13.e. What is the simplest form of this expression?

CORRECT ANSWER

−1.8

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
1.8	replaces the two negative signs with addition and simplifies the expression 0.6 + 1.2	ADD RATIONAL NUMBERS

14. Use the expression to answer the following questions.

$$\frac{1}{4} + \left(-\frac{2}{5}\right)$$

14.a. Create a story about credits and debits to match this expression.

CORRECT ANSWER

I start with $\frac{1}{4}$ of a dollar, then I add a debt of $\frac{2}{5}$ of a dollar.

or

This expression indicates the piggy bank starts with a balance of $\frac{1}{4}$ credits, then $\frac{2}{5}$ debits are added.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Start at $\frac{1}{4}$ and add $-\frac{2}{5}$.	reads the expression without relating to credits and debits	EXPLAIN SUMS OF RATIONAL NUMBERS IN REAL-WORLD PROBLEMS

14.b. Will the initial value increase or decrease as a result of this operation? Explain your reasoning.

CORRECT ANSWER

The initial value will decrease, because debits are added to the piggy bank.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The initial value will increase because there is addition.	believes addition always increases the initial value and does not account for the fact that a negative value is added rather than a positive value	EXPLAIN ADDITION OF RATIONAL NUMBERS
The initial value will decrease because there is a negative.	does not consider the placement of the negative sign, only that it exists in the expression	EXPLAIN ADDITION OF RATIONAL NUMBERS

14.c. What other expression could be used to represent the same operation?

This expression can also be written as $\frac{1}{4} - \frac{2}{5}$ or $\frac{5}{20} - \frac{8}{20}$.

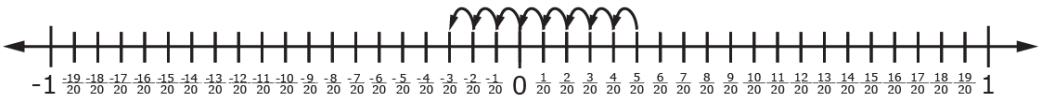
Alternatively, the commutative property allows the expression to be written as $-\frac{2}{5} + \frac{1}{4}$ or $-\frac{8}{20} + \frac{5}{20}$, although this does not model the scenario described in part (a).

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
There is not an alternate expression.	does not understand that adding a negative value is equivalent to subtracting a positive value	EXPLAIN ADDITION OF RATIONAL NUMBERS
$\frac{2}{5} + \left(-\frac{1}{4}\right)$	believes the position of the $\frac{1}{4}$ and $\frac{2}{5}$ can occur in any order based on the commutative property, not realizing the importance of keeping the negative sign with the $\frac{2}{5}$ if the order is changed	EXPLAIN ADDITION OF RATIONAL NUMBERS and RECOGNIZE THE NEGATIVE SIGN

14.d. Represent the scenario on the number line provided.

CORRECT ANSWER



ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Student determines the initial value on the number line but moves only $\frac{2}{20}$ instead of $\frac{8}{20}$ in the negative direction.	moves two spaces because the value in the numerator is a two, does not consider the value of the denominator compared to the intervals on the number line	REPRESENT SUBTRACTION OF RATIONAL NUMBERS ON A NUMBER LINE
Student determines the initial value on the number line but moves $\frac{8}{20}$ in the positive direction.	believes adding a negative value and adding a positive value are equivalent operations	EXPLAIN ADDITION OF RATIONAL NUMBERS
Starts at $-\frac{5}{20}$ and moves $\frac{8}{20}$ in the positive direction.	changes the position of $\frac{1}{4}$ and $\frac{2}{5}$ and does not know to keep the negative sign with $\frac{2}{5}$	ADD RATIONAL NUMBERS and RECOGNIZE THE NEGATIVE SIGN
Student starts or places the initial value at a point other than $\frac{5}{20}$.	may be unable to plot rational numbers on a number line	LOCATE POSITIVE FRACTIONS ON A NUMBER LINE

14.e. What is the simplest form of this expression?

CORRECT ANSWER

$-\frac{3}{20}$

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
$\frac{3}{20}$	changes the position of $\frac{1}{4}$ and $\frac{2}{5}$ and does not know to keep the negative sign with $\frac{2}{5}$ or starts at $\frac{5}{20}$ on the number line and moves two units to the left	ADD RATIONAL NUMBERS and RECOGNIZE THE NEGATIVE SIGN
$\frac{13}{20}$	adds the fractions as if both fractions are positive	ADD RATIONAL NUMBERS
$\frac{-1}{-1}$ or 1	independently subtracts the values in the numerators (1 – 2) and in the denominators (4 – 5) and writes the results in the numerator and denominator respectively	ADD RATIONAL NUMBERS

15. Use the expression to answer the following questions.

$$-0.13 + 1.39$$

15.a. Create a story about credits and debits to match this expression.

CORRECT ANSWER

I start with a debt of 13 cents (or 0.13 dollars) and then I get $1.39 (or 1.39 dollars).

or

This expression indicates the piggy bank starts with a balance of 0.13 debits, then 1.39 credits are added.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Start at –0.13 and add 1.39.	reads the expression without relating to credits and debits	EXPLAIN SUMS OF RATIONAL NUMBERS IN REAL-WORLD PROBLEMS

15.b. Will the initial value increase or decrease as a result of this operation? Explain your reasoning.

CORRECT ANSWER

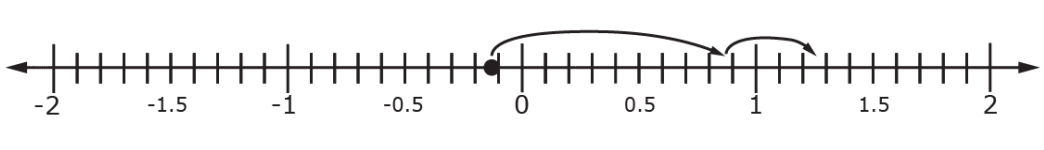
> The initial value will increase, because credits are added to the piggy bank.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The initial value will decrease because there is a negative.	does not consider the placement of the negative sign, only that it exists in the expression	EXPLAIN ADDITION OF RATIONAL NUMBERS
The initial value will increase because there is addition.	believes addition always increases the initial value and does not consider whether the value added is positive or negative	EXPLAIN ADDITION OF RATIONAL NUMBERS

15.c. Represent the scenario on the number line provided. Place the initial value as precisely as possible, then approximate the amount of increase or decrease on the number line.

CORRECT ANSWER



ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Student determines the initial value on the number line but moves a little more than one interval in the positive direction.	moves a little more than one space because the value added is 1.39, does not consider the intervals on the number line are tenths	REPRESENT SUBTRACTION OF RATIONAL NUMBERS ON A NUMBER LINE
Student moves 1.39 in the negative direction from the initial value.	believes the negative sign in the expression indicates a decrease	EXPLAIN ADDITION OF RATIONAL NUMBERS
Student starts or places the initial value on the interval between 0 and –0.1.	cannot locate rational numbers on a number line, in this case the student locates –0.1 then moves a little to the right to account for the 3 hundredths	LOCATE RATIONAL NUMBERS ON A NUMBER LINE

15.d. What is the simplest form of this expression?

CORRECT ANSWER

1.26

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
–1.26	does not understand the importance of the location of the negative sign and simplifies the expression 0.13 – 1.39 instead of –0.13 + 1.39	ADD RATIONAL NUMBERS and RECOGNIZE THE NEGATIVE SIGN
1.52	ignores the negative sign in front of 0.13, therefore adding 0.13 and 1.39	RECOGNIZE THE NEGATIVE SIGN