ADDITION AND SUBTRACTION WITHIN 100

2.NBT.B.6

CONTENTS

The types of documents contained in the unit are listed below. Throughout the unit, the documents are arranged by lesson.

Learning Map Information	An overview of the standards, the learning map section, and the nodes addressed in this unit
Teacher Notes	A brief discussion describing the progression depicted in the learning map section with research-based recommendations for focusing instruction to foster student learning and an introduction to the unit's lessons
Overview of Instructional Activities	A table highlighting the lesson goals and nodes addressed in each lesson of this unit
Instructional Activity	A detailed walkthrough of the unit
Instructional Activity Student Handout	A handout for the guided activity, intended to be paired with the Instructional Activity
Instructional Activity Supplement	A collection of materials or activities related to the Instructional Activity
Student Activity	A work-alone activity for students
Student Activity Solution Guide	A solution guide for the work-alone activity with example errors, misconceptions, and links to the learning map section

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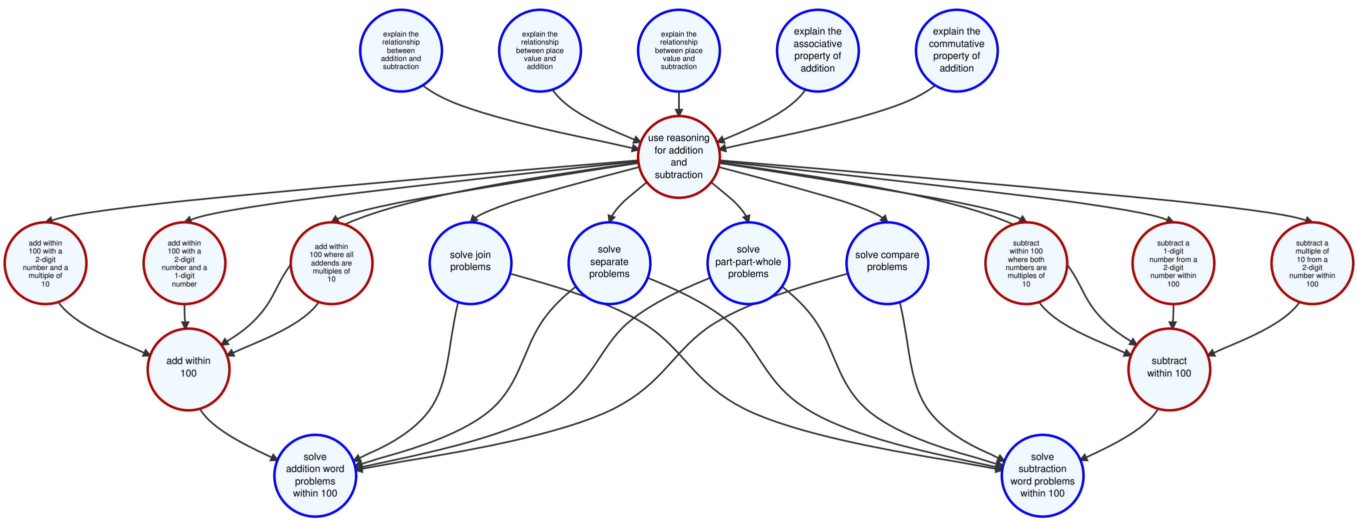
ADDITION AND SUBTRACTION WITHIN 100

LEARNING MAP INFORMATION

STANDARDS

2.NBT.B.6 Demonstrate fluency with addition and subtraction within 100.



**Learning map model of* 2.NBT.5

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Node Name	Node Description
ADD WITHIN 100	Demonstrate combinations of addition within 100 with objects, drawings, equations, etc. All numbers in the equation (including the answer) are from 0-100.
ADD WITHIN 100 WHERE ALL ADDENDS ARE MULTIPLES OF 10	Demonstrate combinations of addition within 100 with objects, drawings, equations, etc., where all of the addends are multiples of 10 (for example, 10 + 90). All numbers in the equation (including the answer) are from 0-100.
ADD WITHIN 100 WITH A 2-DIGIT NUMBER AND A 1-DIGIT NUMBER	Demonstrate combinations of addition within 100 where a one-digit number is added to a two-digit number with objects, drawings, equations, etc. All numbers in the equation (including the answer) are from 0-100.
ADD WITHIN 100 WITH A 2-DIGIT NUMBER AND A MULTIPLE OF 10	Demonstrate combinations of addition within 100 where a two-digit number is added to a multiple of 10 with objects, drawings, equations, etc. All numbers in the equation (including the answer) are from 0-100.
EXPLAIN THE ASSOCIATIVE PROPERTY OF ADDITION	Make known your understanding that in addition problems, you can change the grouping of addends without changing the sum.
EXPLAIN THE COMMUTATIVE PROPERTY OF ADDITION	Make known your understanding that in addition problems, you can change the order of the addends without changing the sum.
EXPLAIN THE RELATIONSHIP BETWEEN ADDITION AND SUBTRACTION	Make known your understanding of subtraction as an unknown addend problem. For example, in order to subtract four from five, find the number that makes five when added to four.
EXPLAIN THE RELATIONSHIP BETWEEN PLACE VALUE AND ADDITION	Make known your understanding that multi-digit numbers represent quantities of ones, tens, hundreds, etc. For example, in a multi-digit whole number, a digit in one place is 10 times the value of the digit to its right. Use this understanding to explain how to calculate sums of multi-digit numbers.
EXPLAIN THE RELATIONSHIP BETWEEN PLACE VALUE AND SUBTRACTION	Make known your understanding that multi-digit numbers represent quantities of ones, tens, hundreds, etc., and that in a multi-digit whole number, a digit in one place represents 10 times what it represents in the place to its right. Use this understanding to explain how to calculate differences between multi-digit numbers.
SOLVE ADDITION WORD PROBLEMS WITHIN 100	Solve word problems with addition within 100 including join, separate, part-part-whole, and compare problems. All numbers in the equation (including the answer) are from 0-100.
SOLVE COMPARE PROBLEMS	Solve one-step problems that involve comparing two amounts. Three quantities are involved (smaller amount, larger amount, and the difference), and one of those amounts is unknown. This may include problems that ask "how many more" and "how many fewer".
SOLVE JOIN PROBLEMS	Solve one-step problems where the action is joining with three quantities involved (initial amount, the change amount, and the resulting amount). One of those amounts is unknown, and the resulting amount is the largest.
SOLVE PART-PART-WHOLE PROBLEMS	Solve one-step problems that involve two parts that are combined to make a whole, where one of those amounts is unknown.
SOLVE SEPARATE PROBLEMS	Solve one-step problems where the action is separating with three quantities involved (initial amount, the change amount, and the resulting amount), and one of those amounts is unknown. The initial amount is the largest.
SOLVE SUBTRACTION WORD PROBLEMS WITHIN 100	Solve word problems with subtraction within 100 including join, separate, part-part-whole, and compare problems.

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SUBTRACT A 1-DIGIT NUMBER FROM A 2-DIGIT NUMBER WITHIN 100	Demonstrate combinations of subtraction within 100 where a one-digit number is subtracted from a two-digit number with objects, drawings, equations, etc. All numbers in the equation are from 0-100.
SUBTRACT A MULTIPLE OF 10 FROM A 2-DIGIT NUMBER WITHIN 100	Demonstrate combinations of subtraction within 100 where a multiple of 10 is subtracted from a two-digit number with objects, drawings, equations, etc. All numbers in the equation are from 0-100.
SUBTRACT WITHIN 100	Demonstrate combinations of subtraction within 100 with objects, drawings, equations, etc. All numbers in the equation (including the answer) are from 0-100.
SUBTRACT WITHIN 100 WHERE BOTH NUMBERS ARE MULTIPLES OF 10	Demonstrate combinations of subtraction within 100 where a multiple of 10 is subtracted from another multiple of 10 with objects, drawings, equations, base-ten blocks, or other assistive technologies. All numbers in the equation are from 0-100.
USE REASONING FOR ADDITION AND SUBTRACTION	Apply strategies to add and subtract and/or explain why addition and subtraction strategies work.

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Addition and Subtraction Within 100

TEACHER NOTES

This unit includes the following documents:

- Learning Map Information
- Instructional Activity (three lessons)
- Instructional Activity Student Handout (for Lessons 1 – 3)
- Instructional Activity Supplement (for Lessons 1 – 3)
- Student Activity (Word version)
- Student Activity Solution Guide

In this unit, students will invent, compare, and discuss strategies for addition and subtraction of two-digit numbers within 100. Throughout the lessons, students will be encouraged to develop more sophisticated strategies utilizing place value understanding and knowledge of the associative and commutative properties of addition.

RESEARCH

To help students construct a strong understanding of the operations, teachers should use contextual problems as the primary teaching tool (Van de Walle, Lovin, Karp, & Bay-Williams, 2014). To support student understanding of these problems, strategies such as talking about what happens in the problem as a group, acting out the problem using models (e.g., counters, diagrams, base-ten blocks, or number lines), and drawing pictures to describe the relationships in the problem are recommended (Van de Walle et al., 2014). It is important that discussions about contextual problems encourage problem analysis and require student explanations, rather than relying on or emphasizing keyword strategies (Van de Walle et al., 2014). Students should think about and explain what is happening with quantities in problems instead of memorizing associations between certain words and operations (e.g., together and addition). In addition, it is important to vary the types of problems (i.e. join, separate, part-part-whole, and comparison) students are exposed to. Students are exposed to join and separate problems more frequently, but the other two problem types (part-part-whole and comparison) should not be neglected (Van de Walle et al., 2014). Exposure to a variety of problem types and structures supports computational fluency and competence (Fuson et al., 1997).

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ADDITION AND SUBTRACTION PROBLEM TYPES (VAN DE WALLE ET AL., 2014)			
Problem Type	**Result Unknown**	**Change Unknown**	**Start Unknown**
Join (Add To)	Diego has five teddy bears. Maria gives him three more. How many teddy bears does Diego have altogether?	Diego has five teddy bears. Maria gives him some more. Now Diego has eight teddy bears. How many teddy bears did Maria give him?	Diego has some teddy bears. Maria gives him three more. Now Diego has eight teddy bears. How many teddy bears did Diego have to begin with?
Separate (Take From)	Diego has eight teddy bears. He gives three teddy bears to Maria. How many teddy bears does Diego have now?	Diego has eight teddy bears. He gives some to Maria. Now he has five teddy bears. How many teddy bears did he give to Maria?	Diego has some teddy bears. He gives three to Maria. Now Diego has five teddy bears. How many teddy bears did Diego have to begin with?
	Whole Unknown	**One Part Unknown**	**Both Parts Unknown**
Part-Part-Whole	Diego has three brown teddy bears and five black teddy bears. How many teddy bears does he have altogether?	Diego has eight teddy bears. Three teddy bears are brown, and the rest are black. How many black teddy bears does Diego have?	Diego has eight teddy bears. Some of the teddy bears are brown, and some of the teddy bears are black. How many of each could he have?
	Difference Unknown	**Bigger Unknown**	**Smaller Unknown**
Comparison	Diego has eight teddy bears, and Maria has five teddy bears. How many more teddy bears does Diego have than Maria? (Or: How many fewer teddy bears does Maria have than Diego?)	Maria has five teddy bears. Diego has three more teddy bears than Maria. How many teddy bears does Diego have? (Or: Maria has three fewer teddy bears than Diego.)	Diego has eight teddy bears. Diego has three more teddy bears than Maria. How many teddy bears does Maria have? (Or: Maria has three fewer teddy bears than Diego.)

As students are introduced to contextual addition and subtraction problems, researchers recommend that students use their existing knowledge to create a personalized solution strategy, with appropriate support from a teacher or peers, and that students continue to invent solution strategies as they encounter new problems (Heuser, 2005; Bobis, 2007). When students are given opportunities to construct their own

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computation methods and communicate their strategies with others, they develop efficient, flexible, and accurate computation strategies (Scharton, 2004). For example, as students who initially count by ones see their peers use base-ten strategies, they will realize that the use of base-ten concepts is much more efficient as they encounter larger numbers. Using the base-ten strategy modeled by their peers and their previous understandings of base-ten concepts, these students can incorporate this reasoning into their own work. Students who begin using base-ten manipulatives can begin drawing the manipulatives, leaving the concrete manipulatives behind. Drawings that are accompanied by numerals will eventually be replaced by the more efficient use of numerals only, resulting in a strategy that is nearly as efficient as the standard algorithm. Teachers can facilitate this transition through scaffolded questions and discourse with students as well as strategic groupings when students work with peers to solve problems.

It is likely that students' solution methods will vary across different problems and problem settings, and that students will construct alternate algorithms or pathways when solving problems which serve as temporary bridges from simpler to more complex knowledge, a necessary step in constructing understanding (Fuson et al., 1997; Taylor, Breck, & Alijets, 2004). When students draw and write out their solutions, they create records of their thinking. These records serve as memory supports for students and help them engage in reflection and discussion about their solution strategies (Fuson et al., 1997; Van de Walle et al., 2014). The advantage of student-invented algorithms is that they come directly from multi-digit concepts and often specifically label the units that will be combined (Carpenter et al., 1998). Standard algorithms, on the other hand, are efficient and accurate but are sometimes removed from their conceptual basis (Carpenter et al., 1998). In a study conducted by Carpenter, Franke, Jacobs, Fennema, and Empson (1998), students who used invented strategies demonstrated better understanding of base-ten number concepts and performed better on extension problems than students who learned the standard algorithm. For example, by the spring of second grade, 96 percent of the invented strategies group demonstrated knowledge of base-ten number concepts, while only 67 percent of students in the standard algorithm group demonstrated knowledge of base-ten number concepts (Carpenter et al., 1998).

A critical step after students invent strategies is engaging students in classroom discourse. Sharing and relating strategies is crucial as students grow in their understanding and develop more sophisticated strategies. Discussions also provide the opportunity for students to check the accuracy of their work, express their ideas, relate and evaluate the efficiency of different strategies, and make sense of mathematics (Scharton, 2004; Taylor et al., 2004). Teachers should encourage students to listen carefully to other students' logic and to reflect on alternative strategies, so that mathematically sound ideas are acknowledged rather than dismissed because they may be unfamiliar or non-traditional (Fuson et al., 1997; Taylor et al., 2004). Students become mathematical thinkers by asking questions about models and representations, thereby promoting analysis and inference as they encounter mathematical problems in the future (Schielack et al., 2000). Knowledge of a variety of strategies increases students' computational flexibility because they gain an understanding of why arithmetic works (Scharton, 2004). Small-group assignments are effective for promoting classroom discourse; it is recommended that students solve and discuss a problem (where the teacher can provide support as well), share strategies as a whole class to compare representations and solution strategies, then solve a new problem to see how and if their strategies evolve (Scharton, 2004).

Ideally, classroom discussions will include a variety of invented strategies, including some that utilize place value reasoning. Flexibility and efficiency with base-ten materials helps build student understanding of place value concepts; as the numbers become larger, students will likely become more motivated to utilize place

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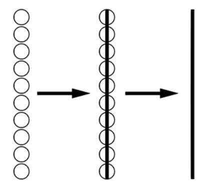
value understanding so they do not have to count every individual object (Fuson et al., 1997). As students gain an understanding of base-ten concepts, they will incorporate a representation of "ten" rather than relying solely on individual counters; eventually, children will construct procedures for adding and subtracting that do not require physical materials of any kind (Carpenter, Franke, Jacobs, Fennema, & Empson, 1998; Fuson et al., 1997). Students understanding of tens sticks can be described through a progression of models, which can be used in real-world problems in place of physical manipulatives (Fuson et al., 1997).

AN EXAMPLE

Student understanding of tens sticks begins with a column of 10 objects, progresses to a column of 10 objects joined by a line, and finally ends with only a vertical stick representing 10 objects (Fuson et al., 1997).



Place value understanding is also critical to student understanding of regrouping strategies that are used in addition and subtraction. Understanding that "ten" can be viewed as 10 ones or one group of ten is an important step in understanding place value (Goodrow & Kidd, 2008) and provides the basis for regrouping strategies. Methods such as opening up a ten to make 10 ones may need support from teachers initially, either in coming up with the strategy to get more ones or to record such a strategy (Fuson et al., 1997). Questions such as "You have several ones here, what could you do with some of them without changing the value of the number?", "If you have a ten and three ones, how can you take away eight ones?", or "You need some more ones; how could you get more ones without changing the value of the number?" support students in conceptualizing regrouping methods (Fuson et al., 1997).

Students who invent strategies to add and subtract may work sequentially (e.g., start with the tens place, then consider the ones place of each number in order), combine units (e.g., tens and ones) separately, or compensate to make the numbers easier to work with (e.g., when solving the problem 39 + 18, a student using the compensation strategy may take one from 18 and "give" it to 39, making the problem 40 + 17) (Carpenter et al., 1998). There are a variety of models that can support students' invented strategies, including base-ten blocks, hundreds charts, and empty number lines. Base-ten materials best model quantities and actual problems, while empty number lines best model distances or measurement (Bobis, 2007). When using manipulatives, keeping all items in view helps supports the connections among addition and subtraction (Van de Walle et al., 2014). An empty number line supports counting by tens and partitioning strategies through jumps on the number line; however, students should only be introduced to an empty number line if they have experience with numbered number lines, it represents their thinking, and they completely understand how it works (Bobis, 2007).

When selecting or creating problems for students, there are advantages to different types of problems. Teachers should go beyond routine problems with two addends to incorporate the addition of more than two

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numbers. Another useful activity is to ask students who have added in different orders with the same results to compare their work to help students build understanding of the associative and commutative properties of addition. It is not necessary for them to name the properties at this time, but it is helpful for them to construct the relationship and recognize the equality of sums (Van de Walle et al., 2014). In addition, open-ended problems provide additional practice, without feeling repetitive, and are more challenging for students because there are a variety of acceptable answers (Britton, 2005).

As students construct understanding of addition and subtraction with multi-digit numbers, the role of the teacher is to pose the problem, let students know there are multiple approaches to solving the problem, coordinate discussion, and ask questions about strategies; teachers should not demonstrate solutions to problems (Fuson et al., 1997). Directly teaching an alternate strategy poses a danger that students could learn the alternate strategy as a rote procedure, similar to how students have learned the standard algorithm; any knowledge that is constructed by linking an algorithm to a task without the necessary conceptual foundation is unstable and can easily be broken (Carpenter et al., 1998). Students typically value the teacher's method as being "the right way" even if it doesn't make sense to them; it is critical that students' strategies truly are invented and do not need to be approved by their teacher (Scharton, 2004; Fuson et al., 1997). Therefore, teachers should avoid giving preference to particular method, because they risk students who don't understand that method trying to imitate and use it in the future (Scharton, 2004). As soon as students are taught an alternative strategy, their tendency is to abandon their invented strategy (Heuser, 2005). Even when students invent strategies, as they eventually learn the often more efficient standard algorithm, it can be linked to conceptual understanding, and at this time it is appropriate to move on from the invented strategy.

MISCONCEPTIONS

There are a few misconceptions students may hold or create as they construct understanding of addition and subtraction of multi-digit numbers. These misconceptions involve the structure of equations, the commutative and associative properties of addition, and keywords in contextual problems. To counter misinterpretations students may hold of the equal sign, it is important to include equations such as $35 = 19 + 16$ or $48 = \square - 13$ during instruction (Van de Walle et al., 2014). Presenting all equations in the form $10 + 65 = \square$ may cause students to believe that the equal sign means to "give an answer" rather than a symbol that states the two sides of the equation are equivalent.

Another misconception students may hold is the belief that the associative and commutative properties apply to subtraction as well as addition (Van de Walle et al., 2014; Fuson et al., 1997). Discussions that clarify when these strategies are appropriate and the use of story problems can help to confront this misconception (Fuson et al., 1997; Van de Walle et al., 2014). In addition, the use of manipulatives for subtraction is compelling because students can literally "take away" some items from a larger number of items, which is very different from reversing the problem with concrete manipulatives. The inappropriate generalization of the commutative and associative properties to subtraction may be one reason multi-digit subtraction tends to be more difficult for students than multi-digit addition (Fuson et al., 1997). Some of the difference in difficulty may also be inherent or the result of limited exposure to take-away problems (Fuson et al., 1997). It is recommended that students are introduced to subtraction soon after addition, as difficulties with subtraction may be greater if there is a long lag between exposure (Fuson et al, 1997).

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Finally, it is possible that students have been trained to look for "keywords" in real-world problems in the past. However, this strategy is dangerous because it is possible that students will connect words to arithmetic operations without considering the context of the meaning of the word (Britton, 2005). For example, a student may see the word "raised" in a problem and believe that addition is appropriate because "raised" means to go higher, without realizing that "raised" does not have anything to do with addition in the context of the problem (Britton, 2005). Because the context of the problem is so critical to determining appropriate operations, it is important to teach students to analyze the relationships or the actions in the problem, rather than to look for keywords (Karp, Bush, & Dougherty, 2014).

KEYWORDS TO SOLVE PROBLEMS (KARP ET AL., 2014)		
Keyword	**Commonly Associated Operation**	**Counterexample (Keyword does not match commonly associated operation)**
Altogether	Add	Todd and Jessie have 18 pencils altogether. Todd has 12 pencils. How many pencils does Jessie have?
Left	Subtract	Laurie has seven pencils in her left hand and five pencils in her right hand. How many pencils does Laurie have?

LEARNING MAP INFORMATION

The learning map section for this sequence of activities begins with understanding of place value, the commutative and associative properties of addition, and the relationship between addition and subtraction. These concepts all lead to students' ability to use reasoning for addition and subtraction. From there, the learning map model continues to three different sections: addition involving two-digit numbers, subtraction involving two-digit numbers, and different types of contextual problems which result in addition or subtraction, depending on the information provided in the problem. Two-digit addition and subtraction as well as the four types of contextual problems lead to students' ability to solve addition and subtraction word problems within 100. Note that solving addition and subtraction word problems within 100 is not aligned to this standard; it is expected that at this point in students' learning, understanding of contextual problems will need to be scaffolded by the teacher. Students' ability to solve these problems independently should develop as they work with the standard 2.OA.1.

INSTRUCTIONAL ACTIVITIES

The activities in this unit are designed to allow students to invent mathematically accurate and increasingly efficient strategies to add and subtract two-digit numbers. Several problems are presented in a context based on the recommendations from research, though understanding of the context and the goal of the problem

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should be scaffolded through class discussion. Greater consideration of independent problem solving in real-world contexts with one- and two-step problems is explored in 2.OA.1.

Lesson 1 is modeled after Schielack, Chancellor, and Childs' (2000) description of the game "100 or Bust". In this game, students roll a standard, six-sided number cube, determine whether the roll represents tens or ones, and try to get as close to 100 as possible without going over through addition and/or subtraction. Variations on the game are provided in the lesson. This activity emphasizes place value and provides students with practice adding and subtracting multiples of 10 and one-digit numbers.

During Lesson 2, students invent strategies to solve contextual problems where one of the values is unknown. After students create a strategy in a small group, groups who approached the problem differently are asked to share their strategy with the class. This discussion allows groups to check the accuracy of their approach and make connections among strategies. Ideally, students will create more efficient and sophisticated strategies using place value reasoning and the relationship between addition and subtraction after they have a chance to reflect on strategies their classmates used.

Lesson 3 presents students with more open-ended contextual problems, asking students to provide more than one correct answer. When given a problem with three addends, students will discuss informally how the commutative and associative properties can be used in their work. Students should continue using invented strategies, discussing different approaches as a class, and considering how their strategy could be more efficient.

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REFERENCES

Bobis, J. (2007). The empty number line: A useful tool or just another procedure?. *Teaching Children Mathematics, 13*(8), 410-413.

Carpenter, T. P., Franke, M. L., Jacobs, V. R., Fennema, E., & Empson, S. B. (1998). A longitudinal study of invention and understanding in children's multidigit addition and subtraction. *Journal for Research in Mathematics Education*, 3-20.

Fuson, K. C., Wearne, D., Hiebert, J. C., Murray, H. G., Human, P. G., Olivier, A. I., Carpenter, T. P., & Fennema, E. (1997). Children's conceptual structures for multidigit numbers and methods of multidigit addition and subtraction. *Journal for Research in Mathematics Education*, 130-162.

Goodrow, A. M., & Kidd, K. (2008). “We all have something that has to do with tens”: Counting school days, decomposing number, and determining place value. *Teaching Children Mathematics, 15*(2), 74-79.

Heuser, D. (2005). Teaching without telling: Computational fluency and understanding through invention. *Teaching Children Mathematics, 11*(8), 404-412.

Karp, K. S., Bush, S. B., & Dougherty, B. J. (2014). 13 rules that expire. *Teaching Children Mathematics, 21*(1), 18-25.

Scharton, S. (2004). I did it my way: Providing opportunities for students to create, explain, and analyze computation procedures. *Teaching Children Mathematics, 10*(5), 278-282.

Schielack, J. F., Chancellor, D., & Childs, K. M. (2000). Designing questions to encourage children's mathematical thinking. *Teaching Children Mathematics, 6*(6), 398-402.

Taylor, A. R., Breck, S. E., & Aljets, C. M. (2004). What Nathan teaches us about transitional thinking. *Teaching Children Mathematics, 11*(3), 138-143.

Van de Walle, J. A., Karp, K. S., Lovin, L. A. H., & Bay-Williams, J. M. (2014). *Teaching Student-centered Mathematics: Developmentally Appropriate Instruction for Grades Pre-K-2* (Vol. 1). Pearson Higher Ed.

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ADDITION AND SUBTRACTION WITHIN 100

OVERVIEW OF INSTRUCTIONAL ACTIVITIES

Lesson	Learning Goal	Nodes Addressed
Lesson 1	Students will use place value reasoning to add and subtract multiple values during a game in which the goal is to get a result as close to 100 as possible.	▶ EXPLAIN THE COMMUTATIVE PROPERTY OF ADDITION ▶ EXPLAIN THE ASSOCIATIVE PROPERTY OF ADDITION ▶ USE REASONING FOR ADDITION AND SUBTRACTION ▶ ADD WITHIN 100 WHERE ALL ADDENDS ARE MULTIPLES OF 10 ▶ ADD WITHIN 100 WITH A 2-DIGIT NUMBER AND A 1-DIGIT NUMBER ▶ ADD WITHIN 100 WITH A 2-DIGIT NUMBER AND A MULTIPLE OF 10 ▶ SUBTRACT WITHIN 100 WHERE BOTH NUMBERS ARE MULTIPLES OF 10 ▶ SUBTRACT A 1-DIGIT NUMBER FROM A 2-DIGIT NUMBER WITHIN 100 ▶ SUBTRACT A MULTIPLE OF 10 FROM A 2-DIGIT NUMBER WITHIN 100
Lesson 2	Students will invent strategies to solve real-world addition and subtraction problems with two, two-digit numbers.	▶ USE REASONING FOR ADDITION AND SUBTRACTION ▶ ADD WITHIN 100 ▶ ADD WITHIN 100 WHERE ALL ADDENDS ARE MULTIPLES OF 10 ▶ ADD WITHIN 100 WITH A 2-DIGIT NUMBER AND A 1-DIGIT NUMBER ▶ ADD WITHIN 100 WITH A 2-DIGIT NUMBER AND A MULTIPLE OF 10 ▶ SOLVE ADDITION WORD PROBLEMS WITHIN 100 ▶ SUBTRACT WITHIN 100 ▶ SUBTRACT WITHIN 100 WHERE BOTH NUMBERS ARE MULTIPLES OF 10 ▶ SUBTRACT A 1-DIGIT NUMBER FROM A 2-DIGIT NUMBER WITHIN 100 ▶ SUBTRACT A MULTIPLE OF 10 FROM A 2-DIGIT NUMBER WITHIN 100 ▶ SOLVE SUBTRACTION WORD PROBLEMS WITHIN 100
Lesson 3	Students will invent strategies to solve real-world addition and subtraction problems with two-digit numbers.	▶ USE REASONING FOR ADDITION AND SUBTRACTION ▶ ADD WITHIN 100 ▶ SUBTRACT WITHIN 100 ▶ ADD WITHIN 100 WITH A 2-DIGIT NUMBER AND A 1-DIGIT NUMBER ▶ SUBTRACT A 1-DIGIT NUMBER FROM A 2-DIGIT NUMBER WITHIN 100 ▶ ADD WITHIN 100 WITH A 2-DIGIT NUMBER AND A MULTIPLE OF 10 ▶ SUBTRACT A MULTIPLE OF 10 FROM A 2-DIGIT NUMBER WITHIN 100 ▶ SOLVE ADDITION WORD PROBLEMS WITHIN 100 ▶ SOLVE SUBTRACTION WORD PROBLEMS WITHIN 100

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Addition and Subtraction Within 100

INSTRUCTIONAL ACTIVITY

Lesson 1

LEARNING GOAL

Students will use place value reasoning to add and subtract multiple values during a game in which the goal is to get a result as close to 100 as possible.

PRIMARY ACTIVITY

Students will play a game where they roll a standard, six-sided number cube six times, decide whether the values rolled represent ones or tens, and use addition and/or subtraction to obtain a result as close to 100 as possible. Four different rounds of this activity are provided, so that students have an opportunity to practice addition first, then addition and subtraction.

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

- Add
- Subtract
- One
- Ten
- Regroup
- Number
- Value
- Sum
- Difference
- Result

MATERIALS

- Standard, six-sided number cubes (Recommend one per student.)
- Familiar manipulatives (base-ten blocks, counters, etc.)
- Blank number line (optional)

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- Instructional Activity Student Handout
- Word version Instructional Activity Student Handout
- Instructional Activity Supplement (optional)
- Word version Instructional Activity Supplement

IMPLEMENTATION

Throughout this lesson, students should have access to familiar materials they have used in the past to model addition and subtraction. Examples of materials are base-ten blocks, counters, blank number lines, and an empty hundreds chart (provided in the Instructional Activity Supplement).

Review the difference between ones and tens using models that are familiar to students (e.g., base-ten blocks or drawings representing base-ten concepts).

Require students to model four tens and seven ones, then **ask** what number is represented by four tens and seven ones.

Ask students to model additional two-digit numbers using familiar models, if necessary, before beginning the following activity. For example, “Can you create a model to represent the number 56?”

The Instructional Activity Student Handout includes four variations on a game involving place value, addition, and subtraction. In each round, students will roll a standard, six-sided number cube six times and decide whether the number rolled will represent tens or ones. For each round, determine whether students should complete all six rolls and then determine the value of each number, or whether they should determine the value of each number as it is rolled. To differentiate this activity, teachers could predetermine that certain rows will represent tens and other rows will represent ones.

The parameters for each round of the activity are as follows. Rounds 1 and 2 can be completed in either order, as can Rounds 3 and 4.

Round 1: Addition only, sum as close to 100 as possible, without going over.

Round 2: Addition only, sum as close to 100 as possible.

Round 3: Addition and subtraction, result as close to 100 as possible, without going over.

Round 4: Addition and subtraction, result as close to 100 as possible.

Space is provided to the right of the tables in the Instructional Activity Student Handout for students to draw representations of the numbers and to show their work.

Provide students with the first page of the Instructional Activity Student Handout.

Discuss the activity as a group, **explaining** that in the first round, students will roll the number cube six times, decide whether each roll represents ones or tens, then add the values together. In this round, the goal is to get as close to 100 as possible, without going over.

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Inform students whether they will determine the value of each roll before rolling again, or whether they will roll all six times then determine the value of each roll.

Model the process of rolling the number cube and completing the table provided to clarify the directions for students.

> NOTE: It is important that, during the modeling process, instruction is not provided regarding how to add or subtract the values in the table. Students should be allowed to invent meaningful strategies to add and subtract throughout this activity, using manipulatives or models as needed. Teachers can question students about their strategies to deepen their understanding.

Allow students to either work together in pairs or to work individually and compete with a classmate.

Ensure that students show enough work on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT so that someone else could understand the reasoning they used to determine the sum.

Ask the following guiding questions as students work through the activity.

GUIDING QUESTIONS

Elicit student thinking:

- Can you describe the goal of this activity in your own words?
- What is your strategy?

Determine if the student can EXPLAIN THE COMMUTATIVE PROPERTY OF ADDITION:

- [Point to two numbers the student has added.] When you're adding these two numbers, does changing their order affect the sum? Why or why not?
- What is the same about 30 + 20 and 20 + 30? What is different?

Determine if the student can EXPLAIN THE ASSOCIATIVE PROPERTY OF ADDITION:

- [Point to three numbers the student has added.] When you're adding these three numbers, does it matter which two you add first? Why or why not?

Determine if the student can USE REASONING FOR ADDITION AND SUBTRACTION:

- How did you model this problem? Why did you model it this way?
- How did you add these two numbers? Can you explain your strategy?
- Did you use any regrouping in your work? Can you explain this strategy to me?

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Determine if the student can ADD WITHIN 100 WHERE ALL ADDENDS ARE MULTIPLES OF 10:

- [Point to multiples of 10 in the Value Assigned column.] What is the sum of these numbers? How do you know?
- Can you model adding 20 and 40? What is the sum? How do you know?

Determine if the student can ADD WITHIN 100 WITH A 2-DIGIT NUMBER AND A 1-DIGIT NUMBER:

- [Identify a one-digit number and a two-digit number in the student's work.] What is the sum of these numbers? How do you know?
- Can you model adding 52 and 7? What is the sum? How do you know?
- Can you model adding 89 and 5? What is the sum? How do you know?

Determine if the student can ADD WITHIN 100 WITH A 2-DIGIT NUMBER AND A MULTIPLE OF 10:

- [Point to a multiple of 10 in the Value Assigned column.] What is the sum if you add this number to your total? How do you know?
- Can you model adding 36 and 50? What is the sum? How do you know?

Once students have completed Round 1 of the activity, **select** students who used different addition strategies to share how they decided whether to make a number represent ones or tens and how they determined the sum. If possible, select students who counted by ones using drawings or manipulatives to share their work first, then students who used drawings (e.g., base-ten blocks, number lines, or an empty hundred chart) or manipulatives of tens and ones, then students who used symbols to reason about groups of tens and ones.

Ask students to identify similarities and differences among strategies and addition methods.

Make or **clarify** connections among the addition methods as needed, ideally through questions posed to students in the class.

Be sure to incorporate a **discussion** about how 10 ones can be regrouped as one ten (because their values are equivalent) during the course of the class discussion.

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NOTE: Exposure to alternate solution strategies provides students with options as they solve problems in the future. The goal is for students to invent more efficient methods of solving utilizing base-ten understanding as they are exposed to additional problems. This transition can be scaffolded by teachers, though it is recommended that teachers refrain from presenting strategies to students. Instead, allow students to arrive at methods and strategies that they create and are meaningful to them, as long as their work is mathematically accurate.

Next, students will participate in Rounds 2 through 4 of the activity in the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

Guide students through Rounds 2 through 4 in a similar fashion as Round 1.

Explain the parameters for the round, which are provided at the beginning of this lesson and on each page of the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT, as students are ready to begin the next round.

Note that the first two rounds are intended to allow students to use addition only, while the third and fourth rounds are intended to allow students to add and subtract strategically to arrive at a result as close to 100 as possible. **Require** students to use subtraction in Rounds 3 and 4.

Inform students whether they will determine the value of each roll before rolling again, or whether they will roll all six times and then determine the value of each roll.

Allow students either to work individually and compete with a classmate or to work together in pairs to try to get as close to 100 as possible.

Ensure that students show enough work on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT so that someone else could understand the reasoning they used to determine the sum.

Ask the following guiding questions as students work through Rounds 2 through 4 of the activity. **Note** that the guiding questions addressing subtraction apply to Rounds 3 and 4 only.

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GUIDING QUESTIONS

Elicit student thinking:

- Can you describe the goal of this activity in your own words?
- What is your strategy?
- [During Round 3 or Round 4.] How did you decide whether you were going to add or subtract these numbers?

Determine if the student can EXPLAIN THE COMMUTATIVE PROPERTY OF ADDITION:

- [Point to two numbers the student has added.] When you're adding these two numbers, does changing their order affect the sum? Why or why not?
- What is the same about 10 + 50 and 50 + 10? What is different?

Determine if the student can EXPLAIN THE ASSOCIATIVE PROPERTY OF ADDITION:

- [Point to three numbers the student has added.] When you're adding these three numbers, does it matter which two you add first? Why or why not?

Determine if the student can USE REASONING FOR ADDITION AND SUBTRACTION:

- How did you model this problem? Why did you model it this way?
- How did you add (or subtract) these two numbers? Can you explain your strategy?
- Did you use any regrouping in your work? Can you explain this strategy to me?

Determine if the student can ADD WITHIN 100 WHERE ALL ADDENDS ARE MULTIPLES OF 10:

- [Point to multiples of 10 in the Value Assigned column.] What is the sum of these numbers? How do you know?
- Can you model adding 30 and 70? What is the sum? How do you know?

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Determine if the student can ADD WITHIN 100 WITH A 2-DIGIT NUMBER AND A 1-DIGIT NUMBER:

- [Identify a one-digit number and a two-digit number in the student's work.] What is the sum of these numbers? How do you know?
- Can you model adding 41 and 6? What is the sum? How do you know?
- Can you model adding 59 and 9? What is the sum? How do you know?

Determine if the student can ADD WITHIN 100 WITH A 2-DIGIT NUMBER AND A MULTIPLE OF 10:

- [Point to a multiple of 10 in the Value Assigned column.] What is the sum if you add this number to your total? How do you know?
- Can you model adding 48 and 20? What is the sum? How do you know?

[Rounds 3 and 4] Determine if the student can SUBTRACT WITHIN 100 WHERE BOTH NUMBERS ARE MULTIPLES OF 10:

- [Point to multiples of 10 in the Value Assigned column.] What is the difference between these numbers? How do you know?
- Can you model subtracting 30 from 50? What is the difference? How do you know?

[Rounds 3 and 4] Determine if the student can SUBTRACT A 1-DIGIT NUMBER FROM A 2-DIGIT NUMBER WITHIN 100:

- [Identify a one-digit number and a two-digit number in the student's work.] What is the difference between these numbers? How do you know?
- Can you model subtracting 4 from 67? What is the difference? How do you know?
- Can you model subtracting 8 from 43? What is the difference? How do you know?

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[Rounds 3 and 4] Determine if the student can SUBTRACT A MULTIPLE OF 10 FROM A 2-DIGIT NUMBER WITHIN 100:

- [Point to a multiple of 10 in the Value Assigned column.] What is the result if you subtract this number from your total? How do you know?
- Can you model subtracting 30 from 74? What is the difference? How do you know?

Select students who used different addition and subtraction strategies to share how they decided whether to make a number represent ones or tens and how they determined the result.

Make or **clarify** connections among the addition and subtraction methods as needed, ideally through questions posed to students in the class.

Discuss that 10 ones can be regrouped as one ten (or one ten can be regrouped as 10 ones) during the course of the class discussion.

Students should be required to show their reasoning for each problem using a combination of words, pictures, and numbers.

At the end of the activity, collect the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT and observe whether students are using base-ten reasoning, either with symbols or pictures, and whether their invented strategies are mathematically accurate (even if they are not conventional). Use this information to move students toward more efficient strategies in future lessons, based on their current approach and reasoning.

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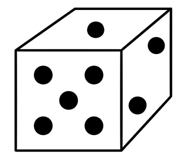
Name________________________________

Example: Addition

Roll the number cube six times. Write each number in the Number Rolled column. Decide whether the number will represent tens or ones, and write the value in the Value Assigned column.



(For example, if you roll a 5 and decide it represents five tens, write "5" in Number Rolled and "50" in Value Assigned.)

Try to get the sum as close as possible to 100, without going over!

Number Rolled	Value Assigned
6	60
5	5
1	10
1	10
3	3
4	4
Sum	92

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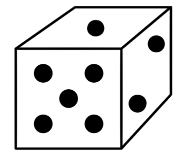
Round 1: Addition

Roll the number cube six times. Write each number in the Number Rolled column. Decide whether the number will represent tens or ones and write the value in the Value Assigned column.



Try to get the sum as close as possible to 100, without going over!

Number Rolled	Value Assigned
Sum	

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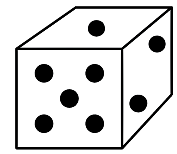
Round 2: Addition

Roll the number cube six times. Write each number in the Number Rolled column. Decide whether the number will represent tens or ones and write the value in the Value Assigned column.



Try to get the sum as close to 100 as possible!

Number Rolled	Value Assigned
Sum	

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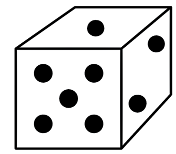
Round 3: Addition & Subtraction

Roll the number cube six times. Write each number in the Number Rolled column. Decide whether the number will represent tens or ones and write the value in the Value Assigned column.



Try to get the result as close as possible to 100, without going over!

Number Rolled	**Value Assigned**
Result	

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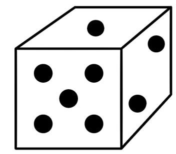
Name______________________________

Round 4: Addition & Subtraction

Roll the number cube six times. Write each number in the Number Rolled column. Decide whether the number will represent tens or ones and write the value in the Value Assigned column.



Try to get the result as close to 100 as possible!

Number Rolled	Value Assigned
Result	

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Addition and Subtraction Within 100

INSTRUCTIONAL ACTIVITY

Lesson 2

LEARNING GOAL

Students will invent strategies to solve real-world addition and subtraction problems with two, two-digit numbers.

PRIMARY ACTIVITY

Students will solve a variety of real-world addition and subtraction problems within 100 using a combination of words, drawings, manipulatives, and numbers. Students should use invented strategies that are meaningful to them, discuss strategies with their peers, and compare the methods used to solve the problem. Students should incorporate components of other strategies as they solve new problems.

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

- Add
- Subtract
- Sum
- Difference
- Ones
- Tens
- Regroup

MATERIALS

- Familiar manipulatives (base-ten blocks, counters, etc.)
- Blank number line (optional)
- Instructional Activity Student Handout
- Word version Instructional Activity Student Handout
- Instructional Activity Supplement (optional)
- Word version Instructional Activity Supplement

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IMPLEMENTATION

The focus of this standard, as well as LESSON 2 and LESSON 3, is on addition and subtraction within 100. Based on research recommendations, each problem is presented in a context, and students are asked to invent strategies to solve the problem. Discussing the information provided in the problem and clarifying the goal of the problem as a class allows students to focus on meaningful computation and prepares students to solve one- and two-step word problems independently, as part of the standard 2.OA.1.

Throughout this lesson, students should have access to familiar materials they have used in the past to model addition and subtraction. Examples of materials are base-ten blocks, counters, blank number lines, and an empty hundreds chart (provided in the INSTRUCTIONAL ACTIVITY SUPPLEMENT).

Present students with the first problem in the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

> Sam has 27 erasers. Jill gives Sam 35 erasers. How many erasers does Sam have now?

Discuss the problem with students to ensure that they understand the information given and understand the question they will answer.

Allow students to work in pairs or small groups to solve the problem, using any manipulatives or materials they need to answer the question.

Ensure that students show enough work on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT so that someone else could understand the reasoning they used to solve the problem.

Ask the following guiding questions as students work through the problem.

GUIDING QUESTIONS

Elicit student thinking:

- Can you describe this problem in your own words?
- What question are you trying to answer?

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Determine if the student can USE REASONING FOR ADDITION AND SUBTRACTION:

- How did you model this problem? Why did you model it this way?
- How did you add these two numbers? Can you explain your thinking and why you chose this strategy?
- Did you use any regrouping in your work? Can you explain this strategy to me?

Determine if the student can ADD WITHIN 100:

- How many erasers does Sam have altogether?
- Can you explain your work to me?
- What part of your work shows how many erasers Sam started with?
- What part of your work shows how many erasers Jill gave Sam?
- What part of your work shows the number of erasers Sam has after Jill gives him erasers?
- Did you use regrouping in your addition? If so, can you show me where?

Determine if the student can ADD WITHIN 100 WHERE ALL ADDENDS ARE MULTIPLES OF 10:

- [If a student is struggling to solve the problem] What if Sam started with 30 erasers and Jill gave him 30 more? Then how many erasers would Sam have?
- [If a student is struggling to solve the problem] What if Sam started with 30 erasers and Jill gave him 40 more? Then how many erasers would Sam have?
- [Referring to one of the previous two questions] How is this problem similar to the problem on your paper? How is it different? Can this question help you provide an answer to the problem on your paper?

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Determine if the student can ADD WITHIN 100 WITH A 2-DIGIT NUMBER AND A 1-DIGIT NUMBER:

- [If a student is struggling to solve the problem] What if Sam started with 27 erasers and Jill gave him two more? Then how many erasers would Sam have?
- [If a student is struggling to solve the problem] What if Sam started with 27 erasers and Jill gave him five more? Then how many erasers would Sam have?
- [Referring to one of the previous two questions] How is this problem similar to the problem on your paper? How is it different? Can this question help you provide an answer to the problem on your paper?

Determine if the student can ADD WITHIN 100 WITH A 2-DIGIT NUMBER AND A MULTIPLE OF 10:

- [If a student is struggling to solve the problem] What if Sam started with 27 erasers and Jill gave him 30 more? Then how many erasers would Sam have?
- [If a student is struggling to solve the problem] What if Sam started with 30 erasers and Jill gave him 35 more? Then how many erasers would Sam have?
- [Referring to one of the previous two questions] How is this problem similar to the problem on your paper? How is it different? Can this question help you provide an answer to the problem on your paper?

Determine if the student can SOLVE ADDITION WORD PROBLEMS WITHIN 100:

- How did you decide whether to add or subtract these numbers?

Once students have had a chance to answer the question, **select** pairs or groups of students that used different strategies to share their work. If possible, select students who counted by ones using drawings or manipulatives to share their work first, then students who used drawings (e.g., base-ten blocks, number lines, or an empty hundred chart) or manipulatives of tens and ones, then students who used symbols to reason about groups of tens and ones.

Ask students to identify similarities and differences among solution methods.

Make or **clarify** connections among solution methods as needed, ideally through questions posed to students in the class.

Discuss how 10 ones can be regrouped as one ten (because their values are equivalent) during the course of the class discussion.

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NOTE: Exposure to alternate solution strategies provides students with options as they solve problems in the future. The goal is for students to invent more efficient methods of utilizing base-ten understanding as they are exposed to additional problems. This transition can be scaffolded by teachers, though it is recommended that teachers refrain from presenting strategies to students. Instead, allow students to arrive at methods and strategies that they create and are meaningful to them, as long as their work is mathematically accurate.

Next, students will consider a comparison problem.

Present students with the second problem in the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

There are 18 dogs and 33 cats at the vet's office. How many more cats are at the vet's office than dogs?

Discuss the problem with students to ensure that they understand the information given and understand the question they will answer.

Allow students to work in pairs or small groups to solve the problem, using any manipulatives or materials they need to answer the question. **Note** that students may approach this as an addition problem, utilizing the relationship between addition and subtraction.

Ensure that students show enough work on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT so that someone else could understand the reasoning they used to solve the problem.

Ask the following guiding questions as students work through the problem.

GUIDING QUESTIONS

Elicit student thinking:

- Can you describe this problem in your own words?
- What question are you trying to answer?

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Determine if the student can USE REASONING FOR ADDITION AND SUBTRACTION:

- How did you model this problem? Why did you model it this way?
- How did you subtract these two numbers? Can you explain your thinking and why you chose this strategy?
- Did you use any regrouping in your work? Can you explain this strategy to me?

Determine if the student can SUBTRACT WITHIN 100 (or ADD WITHIN 100):

- How many more cats are there than dogs?
- Can you explain your work to me?
- What part of your work shows how many dogs there are?
- What part of your work shows how many cats there are?
- What part of your work shows how many more cats there are than dogs?
- Did you use regrouping in your subtraction? If so, can you show me where?

Determine if the student can SUBTRACT WITHIN 100 WHERE BOTH NUMBERS ARE MULTIPLES OF 10 (or ADD WITHIN 100 WHERE ALL ADDENDS ARE MULTIPLES OF 10):

- [If a student is struggling to solve the problem] What if there were 20 dogs and 30 cats? Then how many more cats are there than dogs?
- [If a student is struggling to solve the problem] What if there were 10 dogs and 30 cats? Then how many more cats are there than dogs?
- [Referring to one of the previous two questions] How is this problem similar to the problem on your paper? How is it different? Can this question help you provide an answer to the problem on your paper?

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Determine if the student can SUBTRACT A 1-DIGIT NUMBER FROM A 2-DIGIT NUMBER WITHIN 100 (or ADD WITHIN 100 WITH A 2-DIGIT NUMBER AND A 1-DIGIT NUMBER):

- [If a student is struggling to solve the problem] What if there were three dogs and 33 cats? Then how many more cats are there than dogs?
- [If a student is struggling to solve the problem] What if there were seven dogs and 33 cats? Then how many more cats are there than dogs?
- [Referring to one of the previous two questions] How is this problem similar to the problem on your paper? How is it different? Can this question help you provide an answer to the problem on your paper?

Determine if the student can SUBTRACT A MULTIPLE OF 10 FROM A 2-DIGIT NUMBER WITHIN 100 (or ADD WITHIN 100 WITH A 2-DIGIT NUMBER AND A MULTIPLE OF 10):

- [If a student is struggling to solve the problem] What if there were 10 dogs and 33 cats? Then how many more cats are there than dogs?
- [If a student is struggling to solve the problem] What if there were 20 dogs and 33 cats? Then how many more cats are there than dogs?
- [Referring to one of the previous two questions] How is this problem similar to the problem on your paper? How is it different? Can this question help you provide an answer to the problem on your paper?

Determine if the student can SOLVE SUBTRACTION WORD PROBLEMS WITHIN 100 (or SOLVE ADDITION WORD PROBLEMS WITHIN 100):

- How did you decide whether to add or subtract these numbers?

Once students have had a chance to answer the question, **select** pairs or groups of students that used different strategies to share their work. If possible, select students who counted by ones using drawings or manipulatives to share their work first, then students who used drawings (e.g., base-ten blocks, number lines, or an empty hundred chart) or manipulatives of tens and ones, then students who used symbols to reason about groups of tens and ones.

Ask students to identify similarities and differences among solution methods.

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Make or **clarify** connections among solution methods as needed, ideally through questions posed to students in the class.

Discuss how one ten can be regrouped as 10 ones (because their values are equivalent) during the course of the class discussion.

NOTE: Continuing to expose students to alternate solution strategies provides students with options to consider for future problems. Look for students to use more sophisticated reasoning as they have more experience with and understanding of addition and subtraction of multi-digit numbers. Continue to allow students to solve problems in a way that is meaningful to them, as long as their work is mathematically accurate.

Next, students will consider a problem with an unknown change.

Present students with the third problem in the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

Jose had 72 pennies. Jose bought a piece of candy. Now he has 27 pennies. How many pennies did Jose spend on the piece of candy?

Discuss the problem with students to ensure that they understand the information given and understand the question they will answer.

Allow students to work in pairs or small groups to solve the problem, using any manipulatives or materials they need to answer the question. **Note** that students may approach this as an addition problem, utilizing the relationship between addition and subtraction.

Ensure that students show enough work on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT so that someone else could understand the reasoning they used to solve the problem.

Ask the following guiding questions as students work through the problem.

GUIDING QUESTIONS

Elicit student thinking:

- Can you describe this problem in your own words?
- What question are you trying to answer?

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Determine if the student can USE REASONING FOR ADDITION AND SUBTRACTION:

- How did you model this problem? Why did you model it this way?
- How did you subtract these two numbers? Can you explain your thinking and why you chose this strategy?
- Did you use any regrouping in your work? Can you explain this strategy to me?

Determine if the student can SUBTRACT WITHIN 100 (or ADD WITHIN 100):

- How much money did Jose spend on the piece of candy?
- Can you explain your work to me?
- What part of your work shows how many pennies Jose started with?
- What part of your work shows how many pennies Jose has now?
- What part of your work shows how many pennies Jose spent on the piece of candy?
- Did you use regrouping in your subtraction? If so, can you show me where?

Determine if the student can SUBTRACT WITHIN 100 WHERE BOTH NUMBERS ARE MULTIPLES OF 10 (or ADD WITHIN 100 WHERE ALL ADDENDS ARE MULTIPLES OF 10):

- [If a student is struggling to solve the problem] What if Jose started with 70 pennies and ended up with 20 pennies after he bought the piece of candy? Then how many pennies did Jose spend on the candy?
- [If a student is struggling to solve the problem] What if Jose started with 70 pennies and ended up with 30 pennies after he bought the piece of candy? Then how many pennies did Jose spend on the candy?
- [Referring to one of the previous two questions] How is this problem similar to the problem on your paper? How is it different? Can this question help you provide an answer to the problem on your paper?

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Determine if the student can SUBTRACT A 1-DIGIT NUMBER FROM A 2-DIGIT NUMBER WITHIN 100 (or ADD WITHIN 100 WITH A 2-DIGIT NUMBER AND A 1-DIGIT NUMBER):

- [If a student is struggling to solve the problem] What if Jose started with 72 pennies and ended up with two pennies after he bought the piece of candy? Then how many pennies did Jose spend on the candy?
- [If a student is struggling to solve the problem] What if Jose started with 72 pennies and ended up with six pennies after he bought the piece of candy? Then how many pennies did Jose spend on the candy?
- [Referring to one of the previous two questions] How is this problem similar to the problem on your paper? How is it different? Can this question help you provide an answer to the problem on your paper?

Determine if the student can SUBTRACT A MULTIPLE OF 10 FROM A 2-DIGIT NUMBER WITHIN 100 (or ADD WITHIN 100 WITH A 2-DIGIT NUMBER AND A MULTIPLE OF 10):

- [If a student is struggling to solve the problem] What if Jose started with 72 pennies and ended up with 20 pennies after he bought the piece of candy? Then how many pennies did Jose spend on the candy?
- [If a student is struggling to solve the problem] What if Jose started with 72 pennies and ended up with 30 pennies after he bought the piece of candy? Then how many pennies did Jose spend on the candy?
- [Referring to one of the previous two questions] How is this problem similar to the problem on your paper? How is it different? Can this question help you provide an answer to the problem on your paper?

Determine if the student can SOLVE SUBTRACTION WORD PROBLEMS WITHIN 100 (or SOLVE ADDITION WORD PROBLEMS WITHIN 100):

- How did you decide whether to add or subtract these numbers?

Once students have had a chance to answer the question, **select** pairs or groups of students that used different strategies to share their work. If possible, select students who counted by ones using drawings or manipulatives to share their work first, then students who used drawings (e.g., base-ten

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blocks, number lines, or an empty hundred chart) or manipulatives of tens and ones, then students who used symbols to reason about groups of tens and ones.

Ask students to identify similarities and differences among solution methods.

Make or **clarify** connections among solution methods as needed, ideally through questions posed to students in the class.

Discuss how one ten can be regrouped as 10 ones (because their values are equivalent) during the course of the class discussion.

NOTE: Continue to encourage base-ten reasoning, but allow students to solve problems in a way that is meaningful to them, as long as their work is mathematically accurate.

Students should be required to show their reasoning for each problem using a combination of words, pictures, and numbers.

At the end of the activity, provide students with the fourth problem in the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT. Require students to solve the problem individually, showing enough work to explain their strategy. Observe whether students are using base-ten reasoning, either with symbols or pictures, and whether their invented strategies are mathematically accurate (even if they are not conventional). Use this information to move students toward more efficient strategies in future lessons, based on their current approach and reasoning.

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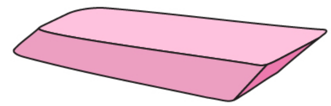
Name________________________________

1. Sam has 27 erasers. Jill gives Sam 35 erasers. How many erasers does Sam have now?



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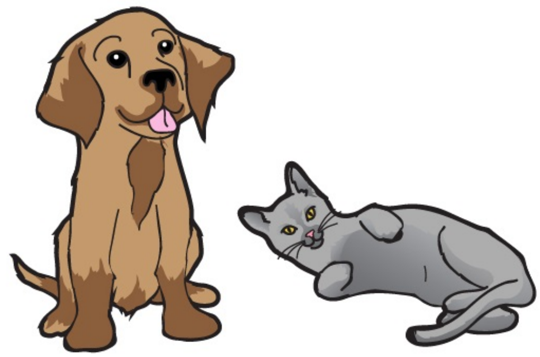
2. There are 18 dogs and 33 cats at the vet's office. How many more cats are at the vet's office than dogs?



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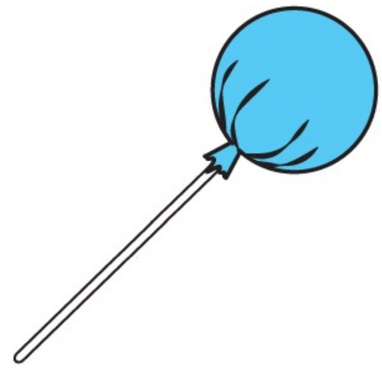
Name______________________________

3. Jose had 72 pennies. Jose bought a piece of candy. Now he has 27 pennies. How many pennies did Jose spend on the piece of candy?



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Name________________________________

4. Hazel has 64 purple and orange flowers. She has 17 orange flowers. How many purple flowers does Hazel have?

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Addition and Subtraction Within 100

INSTRUCTIONAL ACTIVITY

Lesson 3

LEARNING GOAL

Students will invent strategies to solve real-world addition and subtraction problems with two-digit numbers.

PRIMARY ACTIVITY

Students will solve a variety of real-world addition and subtraction problems within 100 using a combination of words, drawings, manipulatives, and numbers. Problems include open-ended questions and questions involving more than two values. Students should use invented strategies that are meaningful to them, discuss strategies with their peers, and compare the methods used to solve the problem. Students should incorporate components of other strategies as they solve new problems.

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

- Add
- Subtract
- Sum
- Difference
- One
- Ten
- Possible
- Regroup
- Solution

MATERIALS

- Familiar manipulatives (base-ten blocks, counters, etc.)
- Blank number line (optional)
- Instructional Activity Student Handout
- Word version Instructional Activity Student Handout
- Instructional Activity Supplement (optional)
- Word version Instructional Activity Supplement

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IMPLEMENTATION

The focus of this standard, as well as LESSON 3, is on addition and subtraction within 100. Based on research recommendations, each problem is presented in a context, and students are asked to invent strategies to solve the problem. Discussing the information provided in the problem and clarifying the goal of the problem as a class allows students to focus on meaningful computation and prepares students to solve one- and two-step word problems independently, as part of the standard 2.OA.1.

Throughout this lesson, students should continue to have access to familiar materials they have used in the past to model addition and subtraction. Examples of materials are base-ten blocks, counters, blank number lines, and an empty hundred chart (provided in the INSTRUCTIONAL ACTIVITY SUPPLEMENT).

Present students with the first problem in the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

> Together, Shana and Kari have 88 colored pencils. How many colored pencils could Shana and Kari each have? Give more than one answer.

Discuss the problem with students to ensure that they understand the information given and understand the question they will answer

Be sure students understand that there is more than one correct answer and that they are to provide more than one solution to the problem.

Allow students to work in pairs or small groups to solve the problem, using any manipulatives or materials they need to answer the question. **Note** that some students may approach this as an addition problem, while other students may approach this as a subtraction problem, utilizing the relationship between addition and subtraction.

Ensure that students show enough work on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT so that someone else could understand the reasoning they used to solve the problem.

Ask the following guiding questions as students work through the problem.

GUIDING QUESTIONS

Elicit student thinking:

- Can you describe this problem in your own words?
- What question are you trying to answer?

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Determine if the student can USE REASONING FOR ADDITION AND SUBTRACTION:

- How did you model this problem? Why did you model it this way?
- How did you add (or subtract) these two numbers? Can you explain your thinking and why you chose this strategy?
- Did you use any regrouping in your work? Can you explain this strategy to me?

Determine if the student can ADD WITHIN 100 or SUBTRACT WITHIN 100:

- How many colored pencils could each person have?
- Can you explain your work to me?
- What part of your work shows how many colored pencils Shana has?
- What part of your work shows how many colored pencils Kari has?
- What part of your work shows how many colored pencils Shana and Kari have together?
- Did you use regrouping in your addition or subtraction? If so, can you show me where?

Determine if the student can ADD WITHIN 100 WITH A 2-DIGIT NUMBER AND A 1-DIGIT NUMBER or SUBTRACT A 1-DIGIT NUMBER FROM A 2-DIGIT NUMBER WITHIN 100:

- [If a student is struggling to solve the problem] What if Shana has 80 colored pencils? Then how many colored pencils would Kari have?
- [If a student is struggling to solve the problem] What if Kari has 87 colored pencils? Then how many colored pencils would Shana have?
- [Referring to one of the previous two questions] How is this problem similar to the problem on your paper? How is it different? Can this question help you provide an answer to the problem on your paper?

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Determine if the student can ADD WITHIN 100 WITH A 2-DIGIT NUMBER AND A MULTIPLE OF 10 or SUBTRACT A MULTIPLE OF 10 FROM A 2-DIGIT NUMBER WITHIN 100:

- [If a student is struggling to solve the problem] What if Shana has 68 colored pencils? Then how many colored pencils would Kari have?
- [If a student is struggling to solve the problem] What if Kari has 40 colored pencils? Then how many colored pencils would Shana have?
- [Referring to one of the previous two questions] How is this problem similar to the problem on your paper? How is it different? Can this question help you provide an answer to the problem on your paper?

Determine if the student can SOLVE ADDITION WORD PROBLEMS WITHIN 100 or SOLVE SUBTRACTION WORD PROBLEMS WITHIN 100:

- How did you decide whether to add or subtract these numbers?

Once students have had a chance to provide multiple responses to the question, **select** pairs or groups of students that used different strategies to share their work. If possible, select students who counted by ones using drawings or manipulatives to share their work first, then students who used drawings (e.g., base-ten blocks, number lines, or an empty hundred chart) or manipulatives of tens and ones, then students who used symbols to reason about groups of tens and ones.

Ask students to identify similarities and differences among solution methods.

Make or **clarify** connections among solution methods as needed, ideally through questions posed to students in the class.

Discuss how 10 ones can be regrouped as one ten (because their values are equivalent) during the course of the class discussion.

NOTE: Exposure to alternate solution strategies provides students with options as they solve problems in the future. Students should begin to invent more efficient and sophisticated methods of utilizing base-ten understanding as they are exposed to additional problems.

Next, students will consider a modified version the previous problem that provides an opportunity for students to work with more than two addends.

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Present students with the second problem in the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

> Three friends—Hans, Isabella, and Cole—have a total of 79 markers. How many markers could each friend have? Give more than one answer.

Discuss the problem with students to ensure that they understand the information given and understand the question they will answer.

Be sure students understand that there is more than one correct answer and that they are to provide more than one solution to the problem. **Note** that some students may approach this as an addition problem, while other students may approach this as a subtraction problem, utilizing the relationship between addition and subtraction.

Allow students to work in pairs or small groups to solve the problem, using any manipulatives or materials they need to answer the question.

Ensure that students show enough work on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT so that someone else could understand the reasoning they used to solve the problem.

Ask the following guiding questions as students work through the problem.

GUIDING QUESTIONS

Elicit student thinking:

- Can you describe this problem in your own words?
- What question are you trying to answer?

Determine if the student can USE REASONING FOR ADDITION AND SUBTRACTION:

- How did you model this problem? Why did you model it this way?
- How did you add (or subtract) these two numbers? Can you explain your thinking and why you chose this strategy?
- Did you use any regrouping in your work? Can you explain this strategy to me?

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Determine if the student can ADD WITHIN 100 or SUBTRACT WITHIN 100:

- How many markers could each person have?
- Can you explain your work to me?
- What part of your work shows how many markers Hans has?
- What part of your work shows how many markers Isabella has?
- What part of your work shows how many markers Cole has?
- What part of your work shows how many markers Hans, Isabella, and Cole have together?
- Did you use regrouping in your addition or subtraction? If so, can you show me where?

Determine if the student can ADD WITHIN 100 WITH A 2-DIGIT NUMBER AND A 1-DIGIT NUMBER or SUBTRACT A 1-DIGIT NUMBER FROM A 2-DIGIT NUMBER WITHIN 100:

- [If a student is struggling to solve the problem] What if Hans and Isabella have 71 markers (combined)? Then how many markers does Cole have?
- [If a student is struggling to solve the problem] What if Hans has five markers? Then how many markers would Isabella and Cole have (combined)?
- [Referring to one of the previous two questions] How is this problem similar to the problem on your paper? How is it different? Can this question help you provide an answer to the problem on your paper?

Determine if the student can ADD WITHIN 100 WITH A 2-DIGIT NUMBER AND A MULTIPLE OF 10 or SUBTRACT A MULTIPLE OF 10 FROM A 2-DIGIT NUMBER WITHIN 100:

- [If a student is struggling to solve the problem] What if Hans and Cole have 50 markers (combined)? Then how many markers does Isabella have?
- [If a student is struggling to solve the problem] What if Cole has 19 markers? Then how many markers would Hans and Isabella have (combined)?
- [Referring to one of the previous two questions.] How is this problem similar to the problem on your paper? How is it different? Can this question help you provide an answer to the problem on your paper?

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Determine if the student can SOLVE ADDITION WORD PROBLEMS WITHIN 100 or SOLVE SUBTRACTION WORD PROBLEMS WITHIN 100:

- How did you decide whether to add or subtract these numbers?

Once students have had a chance to answer the question, **select** pairs or groups of students that used different strategies to share their work. If possible, select students who used a variety of methods, being sure to include students who used more efficient methods to solve the problem.

Ask students to identify similarities and differences among solution methods.

Make or **clarify** connections among solution methods as needed, ideally through questions posed to students in the class.

Using a solution provided by students, **ask** whether it matters in what order the numbers are added to arrive at the total of 79 markers.

Through a discussion, **establish** that the order of the addition does not impact the result (commutative property), and that when adding three or more numbers, it does not matter which numbers are added first (associative property).

Discuss how one ten can be regrouped as 10 ones (because their values are equivalent) during the course of the class discussion.

NOTE: Continuing to expose students to alternate solution strategies provides students with options to consider for future problems. Continue to allow students to solve problems in a way that is meaningful to them, as long as their work is mathematically accurate.

Next, students will consider a problem where the starting amount and the amount of change (decrease) is unknown, but the result is known. Students are again encouraged to determine multiple solutions to the problem.

Present students with the third problem in the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

Jerrod had a lot of books. He donated some of his books to the library. Now Jerrod has 36 books. How many books could Jerrod have started with, and how many books could Jerrod have donated to the library to end up with 36 books? Give more than one answer.

Discuss the problem with students to ensure that they understand the information given and understand the question they will answer.

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Be sure students understand that there is more than one correct answer and that they are to provide more than one solution to the problem. **Note** that some students may approach this as an addition problem, while other students may approach this as a subtraction problem.

Allow students to work in pairs or small groups to solve the problem, using any manipulatives or materials they need to answer the question.

Ensure that students show enough work on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT so that someone else could understand the reasoning they used to solve the problem.

Ask the following guiding questions as students work through the problem.

GUIDING QUESTIONS

Elicit student thinking:

- Can you describe this problem in your own words?
- What question are you trying to answer?

Determine if the student can USE REASONING FOR ADDITION AND SUBTRACTION:

- How did you model this problem? Why did you model it this way?
- How did you add (or subtract) these two numbers? Can you explain your thinking and why you chose this strategy?
- Did you use any regrouping in your work? Can you explain this strategy to me?

Determine if the student can ADD WITHIN 100 or SUBTRACT WITHIN 100:

- How many books could Jerrod have started with? If he started with that many books, how many would he have to give away to end up with 36 books?
- Can you explain your work to me?
- What part of your work shows how many books Jerrod started with?
- What part of your work shows how many books Jerrod gave away?
- What part of your work shows how many books Jerrod ended up with?
- Did you use regrouping in your addition or subtraction? If so, can you show me where?

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Determine if the student can ADD WITHIN 100 WITH A 2-DIGIT NUMBER AND A 1-DIGIT NUMBER or SUBTRACT A 1-DIGIT NUMBER FROM A 2-DIGIT NUMBER WITHIN 100:

- [If a student is struggling to solve the problem] What if Jerrod starts with 39 books? Then how many books did he give to the library?
- [If a student is struggling to solve the problem] What if Jerrod gave seven books to the library? Then how many books did Jerrod start with?
- [Referring to one of the previous two questions] How is this problem similar to the problem on your paper? How is it different? Can this question help you provide an answer to the problem on your paper?

Determine if the student can ADD WITHIN 100 WITH A 2-DIGIT NUMBER AND A MULTIPLE OF 10 or SUBTRACT A MULTIPLE OF 10 FROM A 2-DIGIT NUMBER WITHIN 100:

- [If a student is struggling to solve the problem] What if Jerrod starts with 76 books? Then how many books did he give to the library?
- [If a student is struggling to solve the problem] What if Jerrod gave 20 books to the library? Then how many books did Jerrod start with?
- [Referring to one of the previous two questions] How is this problem similar to the problem on your paper? How is it different? Can this question help you provide an answer to the problem on your paper?

Determine if the student can SOLVE ADDITION WORD PROBLEMS WITHIN 100 or SOLVE SUBTRACTION WORD PROBLEMS WITHIN 100:

- How did you decide whether to add or subtract these numbers?

Once students have had a chance to answer the question, **select** pairs or groups of students that used different strategies to share their work. If possible, **select** students who used a variety of methods. Be sure to **include** students who used more efficient methods to solve the problem.

Ask students to identify similarities and differences among solution methods.

Make or **clarify** connections among solution methods as needed, ideally through questions posed to students in the class.

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Using a solution provided by students, **ask** whether it matters in what order the numbers are subtracted to arrive at 36 books.

Through a discussion, **establish** that the order of the subtraction does impact the result.

Note that subtraction is different than addition; the order of addition does not impact the result.

NOTE: It is important to refrain from telling students that they cannot take a larger number from a smaller number. While elementary students are not expected to solve problems of this type, they can be solved (and will be in the middle grades). Instead, emphasize that the result of subtracting the larger number from the smaller number is different than the result of subtracting the smaller number from the larger number. In the context of this problem, it is appropriate to start with the larger number of books and subtract, or take away, a smaller number of books (you can't give away more books than you have).

Discuss how one ten can be regrouped as 10 ones (because their values are equivalent) during the course of the class discussion.

Continue to **encourage** base-ten reasoning, but allow students to solve problems in a way that is meaningful to them as long as their work is mathematically accurate.

Students should be required to show their reasoning for each problem using a combination of words, pictures, and numbers. In addition, students should be required to continue working with two-digit addition and subtraction within 100 beyond the end of this lesson. It is likely that the ability to reason flexibly and solve problems fluently will require additional time and practice for some students.

At the end of the activity, collect the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT. Observe whether students are using base-ten reasoning, either with symbols or pictures, and whether their invented strategies are mathematically accurate (even if they are not conventional). Use this information to move students toward more efficient strategies in future lessons, based on their current approach and reasoning.

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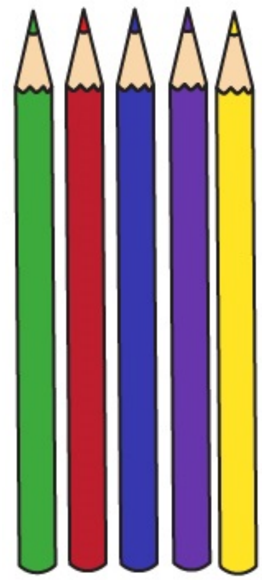
Name________________________________

1. Together, Shana and Kari have 88 colored pencils. How many colored pencils could Shana and Kari each have? Give more than one answer.



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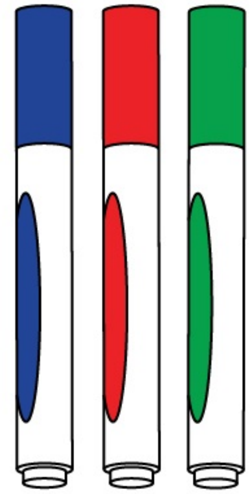
Name______________________________

2. Three friends—Hans, Isabella, and Cole—have a total of 79 markers. How many markers could each friend have? Give more than one answer.



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Name________________________________

3. Jerrod had a lot of books. He donated some of his books to the library. Now Jerrod has 36 books. How many books could Jerrod have started with, and how many books could Jerrod have donated to the library to end up with 36 books? Give more than one answer.

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Addition and Subtraction Within 100

INSTRUCTIONAL ACTIVITY SUPPLEMENT

Lesson 3

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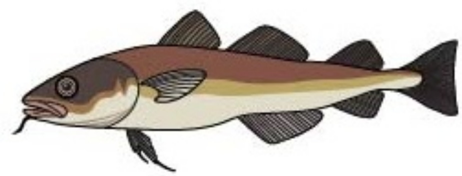
Name____

Addition and Subtraction Within 100

Lessons 1 – 3

For each of the following questions, use words, pictures, and/or numbers to show your thinking.

1. A fish tank has 30 tiger barb fish and 20 cherry barb fish. How many fish are in the fish tank?



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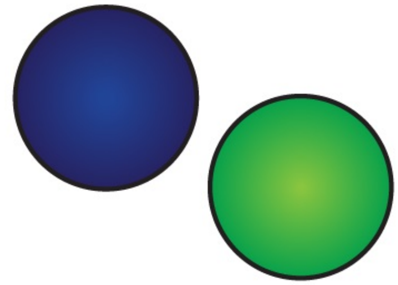
Name____

2. James has 46 high-bounce balls. Kyle has 30 high-bounce balls. How many more high-bounce balls does James have than Kyle?



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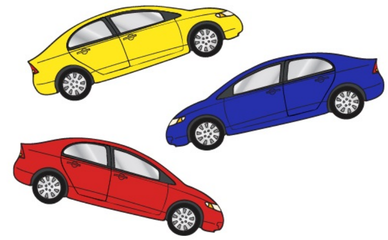
Name____

3. Kate has toy cars. Her brother takes eight toy cars to play with. Kate counts 27 toy cars left. How many toy cars did Kate have to begin with?



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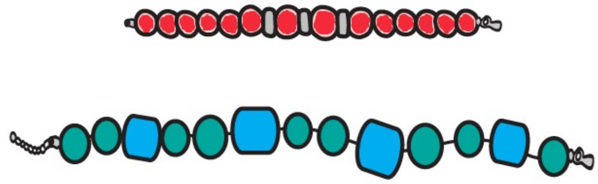
Name____

4. A store sells red and blue bracelets. They have 61 bracelets total. How many red bracelets and how many blue bracelets could they have? Give three different answers.



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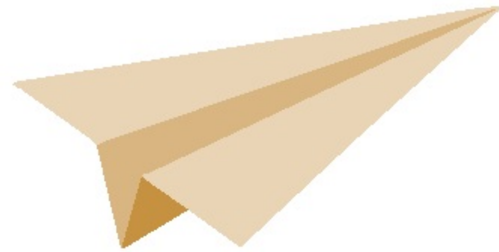
Name____

5. Jackson, Emma, and Mia made a total of 72 paper airplanes. Jackson made 18 paper airplanes, and Emma made 37 paper airplanes. How many more paper airplanes did Emma make than Jackson?



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Name____

6. Chloe is trying to add the following numbers.

_____ = 17 + 49 + 23

6.a. Write the two numbers you think Chloe should add first.

6.b. Why did you choose those numbers?

6.c. What is the answer to Chloe’s addition problem?

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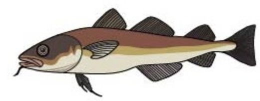
Addition and Subtraction Within 100

STUDENT ACTIVITY SOLUTION GUIDE

Lessons 1 – 3

1. A fish tank has 30 tiger barb fish and 20 cherry barb fish. How many fish are in the fish tank?



CORRECT ANSWER

Check the words, pictures, and/or numbers students provide to explain their thinking.

30 + 20 = ___
30 + 20 = 50

There are 50 fish in the fish tank.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
There are 5 fish in the fish tank.	adds the tens is if they are ones; does not see a difference between 30 and 3 or between 20 and 2	EXPLAIN THE RELATIONSHIP BETWEEN PLACE VALUE AND ADDITION and ADD WITHIN 100 WHERE ALL ADDENDS ARE MULTIPLES OF 10
There are 23 fish in the fish tank.	adds 3 to 20 instead of adding 30 to 20	EXPLAIN THE RELATIONSHIP BETWEEN PLACE VALUE AND ADDITION and ADD WITHIN 100 WHERE ALL ADDENDS ARE MULTIPLES OF 10
There are 32 fish in the fish tank.	adds 2 to 30 instead of adding 20 to 30	EXPLAIN THE RELATIONSHIP BETWEEN PLACE VALUE AND ADDITION and ADD WITHIN 100 WHERE ALL ADDENDS ARE MULTIPLES OF 10
There are 10 fish in the fish tank.	subtracts 20 from 30	SOLVE PART-PART-WHOLE PROBLEMS

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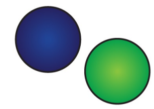
2. James has 46 high-bounce balls. Kyle has 30 high-bounce balls. How many more high-bounce balls does James have than Kyle?



CORRECT ANSWER

Check the words, pictures, and/or numbers students provide to explain their thinking.

46 = 30 + ___
46 = 30 + 16

or

46 – 30 = ___
46 – 30 = 16

James has 16 more high-bounce balls than Kyle.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
There are 76 high-bounce balls.	adds 46 and 30; does not understand that the question asks them to compare the quantities and determine how many more high-bounce balls James has than Kyle	SOLVE COMPARE PROBLEMS and SUBTRACT A MULTIPLE OF 10 FROM A 2-DIGIT NUMBER WITHIN 100
James has 43 more high-bounce balls than Kyle.	subtracts 3 from 46 instead of subtracting 30 from 46	EXPLAIN THE RELATIONSHIP BETWEEN PLACE VALUE AND SUBTRACTION and SUBTRACT A MULTIPLE OF 10 FROM A 2-DIGIT NUMBER WITHIN 100
There are 49 high-bounce balls	adds 3 and 46; does not understand that the question asks them to compare the quantities and determine how many more high-bounce balls James has than Kyle	SOLVE COMPARE PROBLEMS, EXPLAIN THE RELATIONSHIP BETWEEN PLACE VALUE AND SUBTRACTION, and SUBTRACT A MULTIPLE OF 10 FROM A 2-DIGIT NUMBER WITHIN 100

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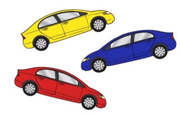
3. Kate has toy cars. Her brother takes eight toy cars to play with. Kate counts 27 toy cars left. How many toy cars did Kate have to begin with?



CORRECT ANSWER

Check the words, pictures, and/or numbers students provide to explain their thinking.

___ – 8 = 27
35 – 8 = 27

or

8 + 27 = ___
8 + 27 = 35

Kate had 35 toy cars to begin with.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Kate had 19 toy cars.	subtracts 8 from 27 instead of adding 8 and 27	SOLVE SEPARATE PROBLEMS or ADD WITHIN 100 WITH A 2-DIGIT NUMBER AND A 1-DIGIT NUMBER
Kate has 17 toy cars.	adds 8, 2, and 7; treats each digit as representing ones	EXPLAIN THE RELATIONSHIP BETWEEN PLACE VALUE AND ADDITION and ADD WITHIN 100 WITH A 2-DIGIT NUMBER AND A 1-DIGIT NUMBER

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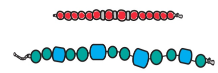
4. A store sells red and blue bracelets. They have 61 bracelets total. How many red bracelets and how many blue bracelets could they have? Give three different answers.



CORRECT ANSWER

Check the words, pictures, and/or numbers students provide to explain their thinking. Students should provide three pairs of addends whose sum is 61. An example correct response is provided.

61 = ___ + ___

61 = 31 + 30
The store could have 31 red bracelets and 30 blue bracelets.

61 = 32 + 29
The store could have 32 red bracelets and 29 blue bracelets.

61 = 60 + 1
The store could have 60 red bracelets and 1 blue bracelet.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Student gives pairs of numbers that do not add up to 61.	does not understand the question or cannot determine the sum of a pair of two-digit numbers	SOLVE PART-PART-WHOLE PROBLEMS, ADD WITHIN 100, or SUBTRACT WITHIN 100
Student gives three pairs of numbers whose difference is 61.	does not understand the question; believes the numbers should have a difference of 61 rather than a sum of 61	SOLVE PART-PART-WHOLE PROBLEMS
Student gives three numbers whose sum is 61.	does not understand the question	SOLVE PART-PART-WHOLE PROBLEMS
Student does not provide an answer.	does not understand the question, cannot add or subtract within 100, or cannot solve a problem that is open-ended and has multiple solutions	SOLVE PART-PART-WHOLE PROBLEMS and ADD WITHIN 100 or SUBTRACT WITHIN 100

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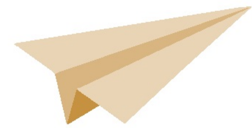
5. Jackson, Emma, and Mia made a total of 72 paper airplanes. Jackson made 18 paper airplanes, and Emma made 37 paper airplanes. How many more paper airplanes did Emma make than Jackson?



CORRECT ANSWER

Check the words, pictures, and/or numbers students provide to explain their thinking.

37 = 18 + ___
37 = 18 + 19

or

37 – 18 = ___
37 – 18 = 19

Emma made 19 more paper airplanes than Jackson.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
55 paper airplanes.	does not understand the problem and adds 37 and 18 instead of subtracting 18 from 37; may not be able to subtract two-digit numbers	SOLVE COMPARISON PROBLEMS or SUBTRACT WITHIN 100
Emma made 54 more paper airplanes.	subtracts 18 from 72 instead of subtracting 18 from 37	SOLVE COMPARISON PROBLEMS
Emma made 35 more paper airplanes.	subtracts 37 from 72 instead of subtracting 18 from 37	SOLVE COMPARISON PROBLEMS
Emma made 17 more paper airplanes.	subtracts 20 from 37 to make the subtraction simpler, but forgets to add 2 back (to compensate for subtracting 2 extra)	SUBTRACT WITHIN 100
Emma made 21 more paper airplanes.	in the ones place, subtracts 7 from 8 (the smaller number from the larger number) rather than regrouping a ten as 10 ones so a ten and 8 ones can be taken away from 37	SUBTRACT WITHIN 100
Emma made 29 more paper airplanes.	adds 10 ones to 37 when subtracting 18 rather than trading a ten for 10 ones, resulting in a difference 10 greater than the actual difference	SUBTRACT WITHIN 100

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6. Chloe is trying to add the following numbers.

$$\_\_\_\_\_\_ = 17 + 49 + 23$$

6.a. Write the two numbers you think Chloe should add first.

CORRECT ANSWER

It is most important to consider the reasoning students provide for their choice in the next part of this problem. A response that displays understanding of the commutative and associative properties of addition is provided.

17 and 23

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
17 and 49 *NOTE: While this answer is not incorrect, a student who provides this response may lack understanding of the associative property.*	may think the first two addends must be added first; if a student answers this way, check their explanation in part b and look for understanding of the associative property of addition	EXPLAIN THE ASSOCIATIVE PROPERTY OF ADDITION
49 and 23 *NOTE: While this answer is not incorrect, a student who provides this response may lack understanding of the commutative property.*	may not know the order of the addends can be changed and chooses the greatest numbers first; if a student answers this way, check their explanation in part b and look for understanding of the commutative property of addition	EXPLAIN THE COMMUTATIVE PROPERTY OF ADDITION
7 and 9 or 7 and 3 or 9 and 3	only considers digits in the ones place when discussing where to begin the addition (NOTE: an alternate interpretation of the question could lead to this response with accurate student understanding; consider the student's explanation for their choice in part b)	USE REASONING FOR ADDITION AND SUBTRACTION
1 and 4 or 1 and 2 or 4 and 2	views the digit in the tens place as representing ones in the addition problem	EXPLAIN THE RELATIONSHIP BETWEEN PLACE VALUE AND ADDITION

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6.b. Why did you choose those numbers?

CORRECT ANSWER

Look for appropriate mathematical reasoning in student responses. An example response is provided based on the response given in part a.

I chose 17 and 23 because the sum is 40. 40 is a multiple of 10 and is easy to add to 49.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
I chose 17 and 49 because they are first.	believes that addition must occur from left to right	EXPLAIN THE ASSOCIATIVE PROPERTY OF ADDITION
I chose 1 and 2 because they are easy to add. or I chose 1 and 7 because they are the first numbers.	views the digit in the tens place as representing ones in the addition problem	EXPLAIN THE RELATIONSHIP BETWEEN PLACE VALUE AND ADDITION
Student does not provide an explanation for their choice in part a.	cannot create or describe a strategy for adding two-digit numbers	USE REASONING FOR ADDITION AND SUBTRACTION

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6.c. What is the answer to Chloe's addition problem?

CORRECT ANSWER

The answer to Chloe's addition problem is 89.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The answer is 26.	adds each digit as though it represents ones (1 + 7 + 4 + 9 + 2 + 3)	EXPLAIN THE RELATIONSHIP BETWEEN PLACE VALUE AND ADDITION and ADD WITHIN 100
The answer is 90.	adds 20, 50, and 20 but forgets to subtract 1 to compensate for the adjustments to the original addends	ADD WITHIN 100
The answer is 40. or The answer is 66. or The answer is 72.	adds only two of the numbers, possibly the numbers identified in part a of this question; may not know how to add three or more two-digit numbers	ADD WITHIN 100

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