UNDERSTANDING MULTIPLICATIVE COMPARISON

4.RA.A.1

CONTENTS

The types of documents contained in the unit are listed below. Throughout the unit, the documents are arranged by lesson.

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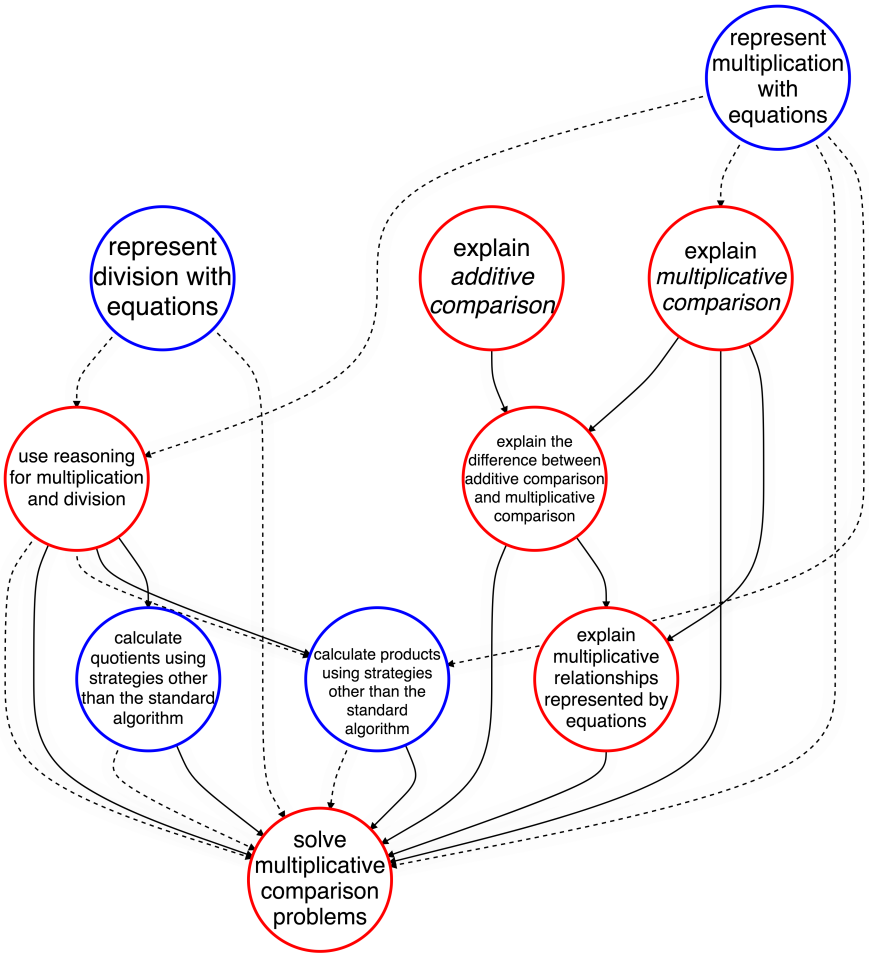
UNDERSTANDING MULTIPLICATIVE COMPARISON

LEARNING MAP INFORMATION

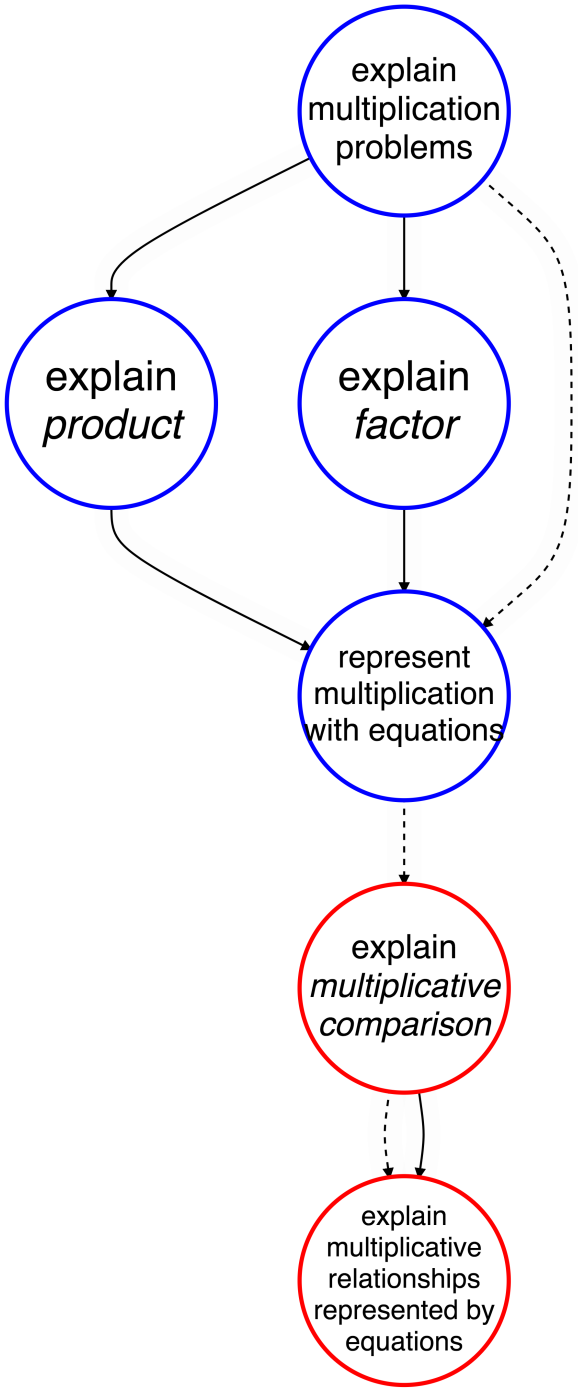
STANDARDS

4.RA.A.1 Multiply or divide to solve problems involving a multiplicative comparison.



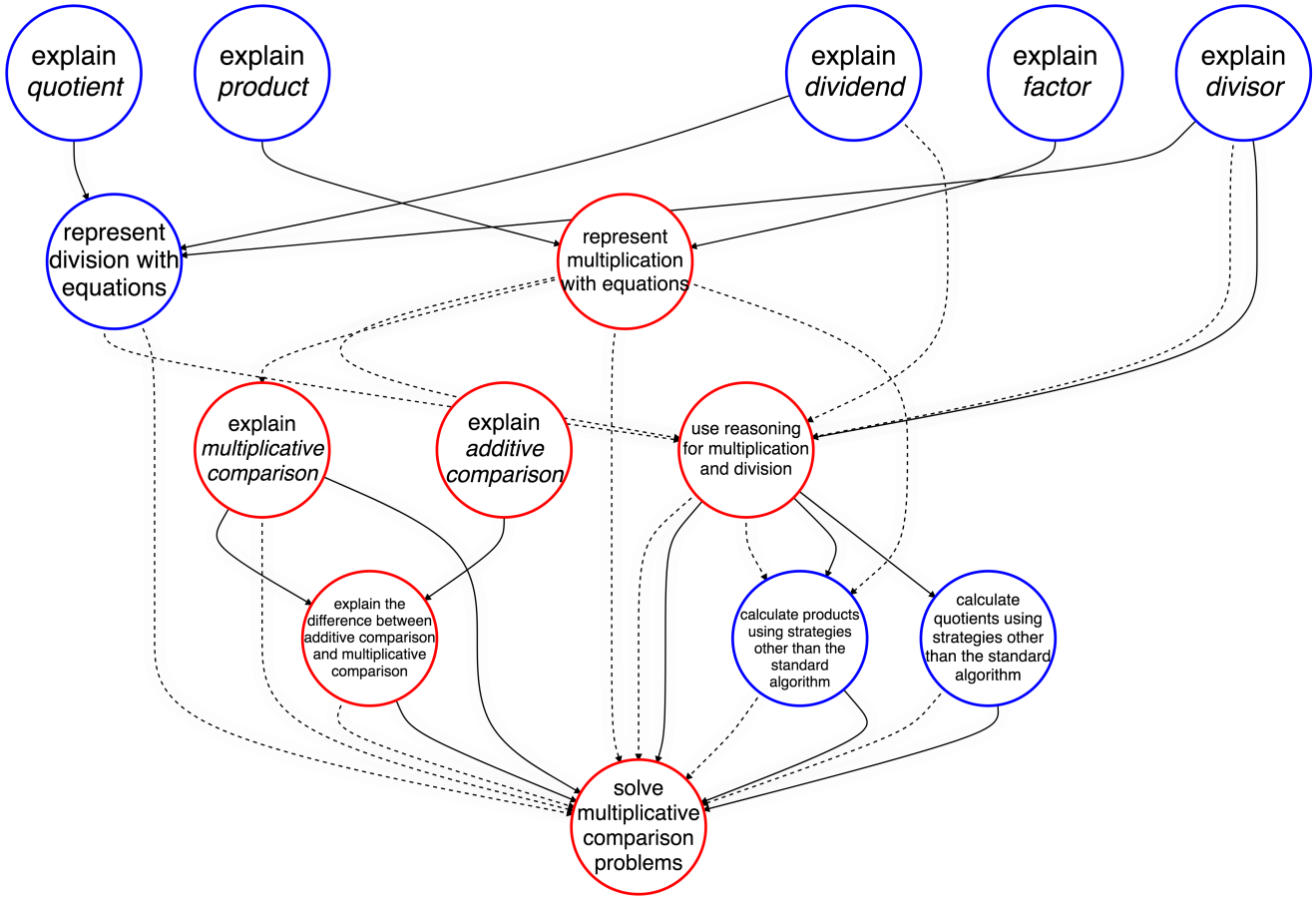
**Learning map model of* 4.OA.1,2

4.RA.A.1 Multiply or divide to solve problems involving a multiplicative comparison.



**Learning map model of* 4.OA.1

4.RA.A.1 Multiply or divide to solve problems involving a multiplicative comparison.



**Learning map model of* 4.OA.2

Node Name	Node Description
CALCULATE PRODUCTS USING STRATEGIES OTHER THAN THE STANDARD ALGORITHM	Demonstrate multiplication of two factors using words, drawings, manipulatives, or other strategies such as partial products.
CALCULATE QUOTIENTS USING STRATEGIES OTHER THAN THE STANDARD ALGORITHM	Demonstrate division or other strategies such as partitioning or fair-sharing.
EXPLAIN *ADDITIVE COMPARISON*	Make known your understanding that, in an additive comparison, the difference between the two elements of the comparison defines the relationship and remains a constant for creating equivalent comparisons.
EXPLAIN *DIVIDEND*	Make known your understanding that the dividend is the number being divided.
EXPLAIN *DIVISOR*	Make known your understanding that the divisor is the number that divides another number. It is also known as a factor.
EXPLAIN *FACTOR*	Make known your understanding that a factor is a number you multiply by another number to get a product.
EXPLAIN *MULTIPLICATION PROBLEMS*	Make known your understanding that, in a multiplication problem, the first factor describes the number of groups, and the second factor describes the number of elements in each group.
EXPLAIN *MULTIPLICATIVE COMPARISON*	Make known your understanding that multiplicative comparison indicates a ratio between the two elements (for example, "Jake has five times as many stickers as Carolyn").
EXPLAIN MULTIPLICATIVE RELATIONSHIPS REPRESENTED BY EQUATIONS	Make known your understanding of multiplicative relationships represented by equations (for example, interpret the equation "5 × 7 = 35" as "35 is five times as many as seven").
EXPLAIN *PRODUCT*	Make known your understanding that a product is the answer when two or more numbers are multiplied together.
EXPLAIN *QUOTIENT*	Make known your understanding that a quotient is the number obtained when dividing one number by another.
EXPLAIN THE DIFFERENCE BETWEEN ADDITIVE COMPARISON AND MULTIPLICATION COMPARISON	Make known your understanding that the constant (ratio or difference) that characterizes the comparison is the difference between additive comparison and multiplicative comparison.
REPRESENT DIVISION WITH EQUATIONS	Use an equation to represent a division problem (for example, 10 ÷ 2 = 5).
REPRESENT MULTIPLICATION WITH EQUATIONS	Use an equation to represent a multiplication sentence (for example, 2 × 3 = 6).
SOLVE MULTIPLICATIVE-COMPARISON PROBLEMS	Use multiplication and division to solve word problems involving multiplicative comparison.
USE REASONING FOR MULTIPLICATION AND DIVISION	Solve multiplication and division problems strategically.

UNDERSTANDING MULTIPLICATIVE COMPARISON

TEACHER NOTES

This unit includes the following documents:

- Learning Map Information
- Instructional Activity (three lessons)
- Instructional Activity Student Handout (for Lessons 2 and 3)
- Instructional Activity Supplement (for Lessons 1 – 3)
- Student Activity
- Student Activity Solution Guide

Throughout this unit, students will learn to interpret and solve multiplicative comparison problem situations by applying their understanding of the inverse relationship between multiplication and division. In addition, students will identify and explain the differences between multiplicative comparison and other types of multiplication (e.g., equal-group multiplication, measurement multiplication, etc.) as well as additive comparison.

RESEARCH

Beginning instruction of multiplicative comparison with contextual situations is central to student success. Lannin, Chval, & Jones (2013) state, "Contextual problems are a good place to begin because students are most familiar with multiplicative situations that they encounter outside classroom mathematics" (p. 36). Student understanding of problem situations is key to determining an answer for the given situation and students are more familiar with situations they encounter daily outside of school. Therefore, framing multiplicative situations in the context of problem scenarios students may encounter outside of school will help them comprehend and ultimately solve the situation. Understanding multiplicative situations is vital to understanding multiplicative comparison; if students are unaware of what problem situations are saying or asking, they will be unable to determine what type of situation they are faced with and how to appropriately solve the situation. For example, if a student is unaware of the differences between a multiplicative comparison scenario and an additive comparison scenario, then the student will not know how to solve the given scenario or could incorrectly add when multiplication is necessary or vice versa. Students build meaning for concepts when they learn mathematics through real contexts, problems, situations, and models (Hiebert, Carpenter, Fennema, Fuson, Wearne, Murray, Olivier, & Human, 1997). When students connect previous understanding and knowledge from life outside of the mathematics classroom with new concepts, they build lasting understanding based on comprehension, not memorization.

When solving multiplicative comparison situations, it is imperative that students understand the scenario as opposed to looking for "key words". For two of the three types of multiplicative comparison, there is an inconsistency between the cue word, *times*, and the operation necessary to solve the computation.

AN EXAMPLE

When students only search for "key words" or cue words, they are likely to answer a problem situation incorrectly. In the case of multiplicative comparison, "when encountering problems with inconsistent language [students] were likely to commit a reversal error" (Xin, 2007, p. 348).

> Veronica has 24 pairs of earrings. She has three times more pairs of earrings than her little sister, Maggie. How many pairs of earrings does Maggie have?

If searching or a "key" or cue word, a student would identify *times* and multiply the given values. However, the problem situation should be represented as an equation such as the following.

$$24 = 3 \times \square$$

To solve the equation, a student must use the inverse relationship between multiplication and division and divide 24 by 3 to determine that Maggie has 8 pairs of earrings.

Xin (2007) goes on to state, "Providing problem-solving opportunities that emphasize mathematical thinking and reasoning is critical for teaching conceptual understanding of fundamental mathematics" (p. 348). In addition, Lambdin (2003) identifies the relationship between problem solving and deep conceptual understanding of mathematics as *symbiotic*. That is, when students are engaged in problem-solving situations during instruction, it increases their conceptual understanding; likewise, a student's conceptual understanding enhances their ability to problem-solve. Therefore, Lambdin (2003) continues, "learning through problem-solving develops understanding" (p. 7). The more students understand, the fewer misconceptions they will hold and the fewer errors they will make. For example, if students can reason about and make sense of a problem situation, as opposed to seeking out "key" or cue words, then they will be less likely to make errors related to comprehension of the problem situation, such as reversal errors. Furthermore, when students are aware of and understand the relationship between multiplication and division, they can represent a problem situation as one operation and solve as another. For example, if students represent multiplicative comparison situations as multiplication equations that accurately represent the scenario, then they will identify what operation is necessary to solve the situation based on the representation and the location of the unknown value.

Due to the fact that students struggle more with situations that involve "indirect/inconsistent" terms than they do with problem situations that involve unnecessary information, it is significant to present students with numerous and varied opportunities to make sense of and solve multiplicative comparison situations (Xin, 2007). Van de Walle, Karp, Lovin, & Bay-Williams (2014) explain that there are three possibilities for the unknown in a multiplicative comparison situation: product unknown, group size unknown, and number of groups unknown. To help students make sense of the multiplicative comparison problem situations, it is important that they do "not simply identify some problems as 'multiplication problems and others as 'division problems'" (Lannin, et al., 2015, p. 55). This way students can utilize a multitude of strategies to solve the situation (e.g., repeated addition, division, using the relationship between multiplication and division, etc.).

AN EXAMPLE

There are three types of situations for multiplication comparison (Van de Walle, et al., 2014).

PRODUCT UNKNOWN

Scenarios in which the product is the unknown are "multiplication problems".

Rachel has six times as many game tickets as Monica. Monica has 24 game tickets. How many game tickets does Rachel have?

$\square = 6 \times 24$

GROUP SIZE UNKNOWN

Scenarios in which the group size is unknown are "partitive division problems".

Joey won 72 game tickets from the ball game. He won three times as many game tickets from the ball game as he did from the space game. How many tickets did Joey win from the space game?

$3 \times \square = 72$

$72 \div 3 = \square$

NUMBER OF GROUPS UNKNOWN

Scenarios in which the number of groups is unknown are "measurement division problems".

Brisen won 95 tickets from the ball game, and Sierra won 19 tickets from the ball game. How many times as many tickets did Brisen win as Sierra did?

$95 = \square \times 19$

$\square = 95 \div 19$

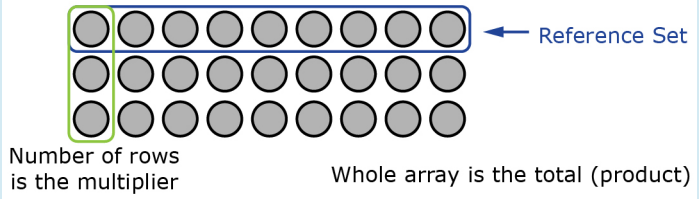
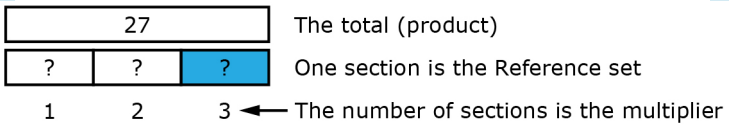
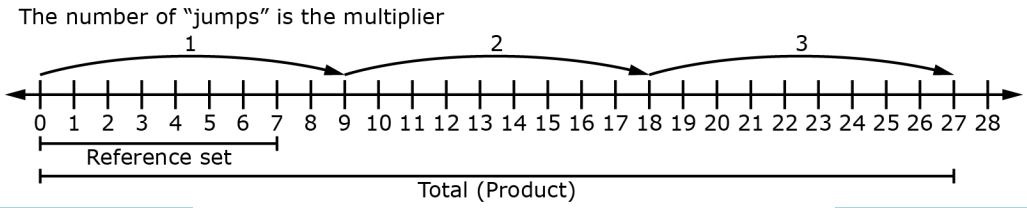
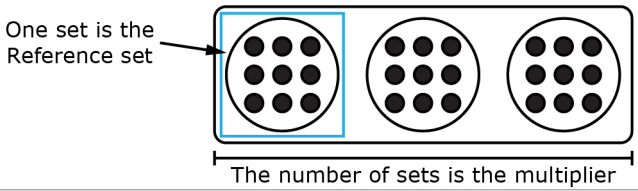
Students should have numerous opportunities to wrestle with contexts and problems, as well as to build solutions using a variety of strategies (Fuson, 2003). Focusing on the strategies that students utilize will support their ability increase their problem-solving abilities. Fuson (2003) also states, "the twenty-first century requires a greater focus on a wider range of problem-solving experiences and a reduced focus on learning and practicing by rote a large body of standard calculation methods" (p. 301). Students should not be explicitly taught methods or strategies by teachers but rather exposed to a variety of strategies and models from their peers while engaging in discussion. Presenting students with strategies and models in the context of something a fictional student has done in the past is a way to increase student experiences with multiple

methods. The strategy or model should be presented as an example, not as explicit instruction. It is important for students to develop fluency between all representations and models of multiplication and division. It is possible for students to develop the ability to solve problems in one context or with one model, but also to lack the capability to solve problems outside of that context. Therefore, students should be asked to explain the similarities and differences between all representations of multiplication and division, and they should be able to identify how the components (e.g., the factors, product, quotient, dividend, etc.) are represented amongst the representations. Students should also begin to identify which strategies are more efficient and appropriate for different types of problems. Each time a model or strategy is presented explicitly, students should identify the multiplier (or number of multiplicative units), the reference set (or the multiplicative unit), and the product.

AN EXAMPLE

Models can help students visualize and interpret problem situations. The models provided are some that can be employed to interpret the following multiplicative comparison problem situation.

> Javier goes to play games with his friends every other Saturday. He has game tickets left over from the last Saturday he went. During his current trip, he won three times as many tickets as he had left over. Javier won 27 game tickets on this trip. How many game tickets did Javier have left over from his trip last Saturday?

Model	Example
Array	
Bar model (linear model)	
Number line (linear model)	
Set model (discrete model)	

LEARNING MAP INFORMATION

The learning map section for this sequence of activities begins with students' understanding of how to represent both multiplication and division as equations. This understanding leads to knowledge of the relationship between multiplication and division. Students will be representing multiplicative comparison scenarios as both multiplication and division equations, therefore an understanding of the relationship between multiplication and division is necessary to interpret the scenario and to determine how to solve a multiplication equation using division or vice versa. In order to accurately represent multiplicative scenarios, students will also need to be able to distinguish between multiplicative and additive comparison situations and to explain the differences between the two. Once students understand and can explain a multiplicative comparison situation, then they should proceed to explaining how the situation relates to an equation representation. Students should be expected and able to solve multiplicative situations utilizing a variety of strategies, including models and algorithms.

INSTRUCTIONAL ACTIVITIES

The activities in this unit are designed to increase students' understanding of multiplicative comparison in relation to additive comparison and other multiplication situations, such as equal-group multiplication.

In Lesson 1, students will review the concept of additive comparison and review previous understandings of multiplication situations, equal-groups multiplication, measurement multiplication, area multiplication, and array multiplication. Students with then learn about multiplicative comparison in relation to their previous understandings. Students will play a game called *Making Matches* in which they will match provided problem scenarios with the type of problem (e.g., additive comparison).

In Lesson 2, students will practice creating models to represent given scenarios. Students will use a table to organize their understanding by identifying what the scenario is asking, representing the scenario with a model, and providing the solution. In some situations, students will need to create a scenario that represents a given model and provide the solution.

In Lesson 3, students will extend their understanding to include equations that represent given scenarios and models. Before organize their understanding in a table, students will use "equation cards" to represent a given scenario or model and then solve the equation. On the table, students will need to identify what the scenario is asking, represent the scenario with a model and an equation, and provide the solution. In some situations, students will need to create a scenario that represents a given model.

REFERENCES

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Hiebert, J., Carpenter, T., Fennema, E., Fuson, K., Wearne, D., Murray, H., Olivier, A., & Human, P. (1997). *Making sense: Teaching and learning mathematics with understanding*. Portsmouth, NH: Heinemann.

Lambdin, D. (2003). Benefits of teaching through problem solving. In F. Lester Jr. & R. Charles (Eds.), *Teaching mathematics through problem solving: Prekindergarten-grade 6* (pp. 3-13). Reston, VA: The National Council of Teachers of Mathematics.

Lanin, J., Chval, K., & Jones, D. (2013). *Putting* ***essential understanding*** *of multiplication and division in practice: 3-5*. K. Chval & B. Dougherty (Eds.). Reston, VA: The National Council of Teachers of Mathematics.

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Xin, Y. (2007). Word problem solving tasks in textbooks and their relation to student performance. *The Journal of Educational Research*, 100(6), 347-359.

UNDERSTANDING MULTIPLICATIVE COMPARISON

OVERVIEW OF INSTRUCTIONAL ACTIVITIES

Lesson	Learning Goal	Nodes Addressed
Lesson 1	Students will extend their knowledge of multiplication and division with equal-groups problems to include multiplicative comparison situations.	▶ EXPLAIN MULTIPLICATION PROBLEMS ▶ EXPLAIN *PRODUCT* ▶ MATHEMATIZE CONTEXTUAL SITUATIONS INVOLVING MULTIPLICATION ▶ EXPLAIN *MULTIPLICATIVE COMPARISON*
Lesson 2	Students will create semi-concrete models to represent multiplicative-comparison problem situations.	▶ MATHEMATIZE CONTEXTUAL SITUATIONS INVOLVING MULTIPLICATION ▶ EXPLAIN *ADDITIVE COMPARISON* ▶ EXPLAIN *MULTIPLICATIVE COMPARISON* ▶ CALCULATE PRODUCTS USING STRATEGIES OTHER THAN THE STANDARD ALGORITHM ▶ EXPLAIN THE DIFFERENCE BETWEEN ADDITIVE COMPARISON AND MULTIPLICATIVE COMPARISON
Lesson 3	Students will use their critical thinking skills to determine what part of the problem situation is unknown and then write an equation representing the problem situation.	▶ MATHEMATIZE CONTEXTUAL SITUATIONS INVOLVING ADDITION ▶ REPRESENT ADDITION WITH EQUATIONS ▶ EXPLAIN ADDITIVE COMPARISON ▶ EXPLAIN MULTIPLICATIVE RELATIONSHIPS REPRESENTED BY EQUATIONS ▶ EXPLAIN THE DIFFERENCE BETWEEN ADDITIVE COMPARISON AND MULTIPLICATIVE COMPARISON ▶ SOLVE MULTIPLICATIVE-COMPARISON PROBLEMS

UNDERSTANDING MULTIPLICATIVE COMPARISON

INSTRUCTIONAL ACTIVITY

Lesson 1

LEARNING GOAL

Students will extend their knowledge of multiplication and division with equal-groups problems to include multiplicative comparison situations.

PRIMARY ACTIVITY

Students use reasoning skills to differentiate between previous concepts of multiplication and division (i.e. equal groups) and newer concepts of multiplicative comparison. Students will also distinguish between additive comparison and multiplicative comparison. The lesson will finish with students applying their new understanding by playing a game of *Making Matches*.

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

- Total/whole
- Multiplier
- Reference set
- Compare
- Multiplicative comparison
- Equal-groups multiplication
- Measurement multiplication
- Additive comparison

MATERIALS

- Manipulatives:
    - Counting manipulatives – i.e. beans, tiles, cubes, etc. (Recommend 40 – 50 counters for each student.)

    - Number lines (Page 5 of the INSTRUCTIONAL ACTIVITY SUPPLEMENT A can be printed and placed in plastic sleeves to be reusable with dry erase markers.)
- INSTRUCTIONAL ACTIVITY SUPPLEMENT A (Recommend one copy of pages 1 – 4 for every two to three students.)
- INSTRUCTIONAL ACTIVITY SUPPLEMENT B – *Making Matches* game (Recommend one copy for every two to four students.)

IMPLEMENTATION

Arrange students into groups of two, three, or four (depending on preference and needs).

Distribute the scenario strips and activity mats from pages 1 – 4 of the INSTRUCTIONAL ACTIVITY SUPPLEMENT A.

> NOTE: The scenario strips should be cut into individual strips. The activity mats can be made by affixing the activity mat labels (from page 4 of the INSTRUCTIONAL ACTIVITY SUPPLEMENT A) to the top of a piece of construction paper.

Require students to read the scenarios in their groups and then place the strip on the map that matches the type of problem they think it is. If they do not know what a type of problem is (e.g., "What is a measurement problem?") make note of the question and asking guiding questions (e.g., "What does measurement make you think of? How might that relate to a multiplication problem?"), and avoid giving them the answer at this point.

Once all groups are finished, **review** the types of problems by **asking** students to share definitions and/or examples (for example, "What is a measurement problem?" or "How do you know if a problem is a measurement problem?").

- Equal-groups multiplication problems
- Measurement multiplication problems
- Area multiplication problems
- Array multiplication problems
- Addition problems

As students share their responses, **write** the definitions of the problem types on chart paper, large whiteboard, or an interactive white board, for later reference. **Allow** students to share their thoughts, making sure to **redirect** any misconceptions.

Provide groups with a few minutes (if needed) to move any of their scenario strips based on the problem definitions. **Note** any groups that *did* move their strips, **ask** them to share what strip(s) they moved and why.

Read aloud each scenario, pausing between each scenario and **requiring** groups to share what type of problem they thought it was and why. Groups that already shared because they moved a strip do not need to explain a second time for the same strip.

Acknowledge each group as they explain *where* they placed the strips, but do not provide correct answers or redirect students with incorrect answers.

Students should move their mats to the side, keeping the strips on them (they can be stacked for more room).

For the next activity, students may stay in the small groups, be arranged into groups of two, or work individually based on class needs.

Distribute or make available the manipulatives (both counting manipulatives and number lines).

Display and **read aloud** the following scenario.

> Ben eats eight carrots in his lunch every day at school. How many carrots does Ben eat in one school week?

Some students may need support with recognizing that there are only five days in a school week as opposed to seven days.

Ask students to discuss in small groups or with a partner what type of problem this is and how they know. Then, **discuss** student/group responses as a whole class.

Tell students to use the manipulatives to represent the problem. Note that you are *not* asking for the solution, but rather a representation of the problem situation.

As students work in groups, **pay attention** to *how* groups/students represent the problem situation, making note of which group(s) use set models, the number line, and an array. Also, **pay attention** to how students are representing the number of groups and the size of one group (i.e., there should be five groups of eight, not eight groups of five).

Use the following guiding questions as students are modeling the situation and during the following discussions.

GUIDING QUESTIONS

Elicit student thinking:

- Why did you select this manipulative to represent the situation?
- Have you solved an equal-groups problem situation before? Explain.

Determine if the student can EXPLAIN MULTIPLICATION PROBLEMS:

- Which number represents the number of equal-size groups? How do you know?
- Which number represents the size of each group? How do you know?

Determine if the student can EXPLAIN *PRODUCT*:

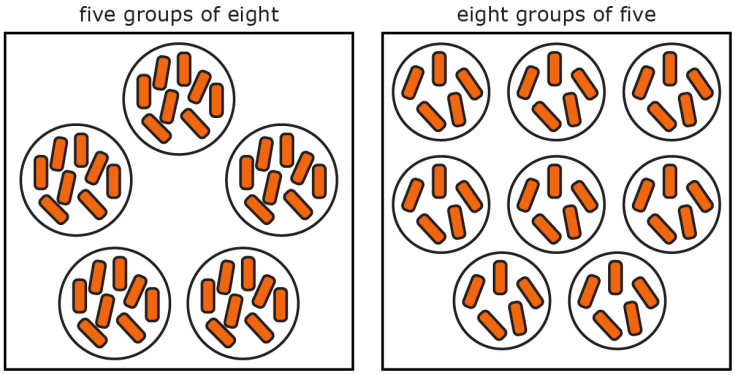
- What is the answer to a multiplication situation called?
- What is the product in a multiplication situation?
- What does the product in this multiplication situation represent? How do you know?

Determine if the student can MATHEMATIZE CONTEXTUAL SITUATIONS INVOLVING MULTIPLICATION:

- What do the carrots represent, the number of groups or the number in each group? How do you know?
- What do the days of the school week represent, the number of groups or the number in each group? How do you know?
- What does the number of carrots Ben eats in one school week represent, the number of groups, the size of each group, or the total value? How do you know?

Select groups to share their representations based on their selected model. Students representing the situation with set models should go first, then students using the number line. Lastly, select a group/student that used an array. If no groups/students used a specific representation, present it to students as a strategy previous students have used.

Direct student attention to the number of groups and the size of each group. **Emphasize** that eight groups of five is a different situation by modeling eight groups of five and comparing it with five groups of eight.



Ask students the following questions. Students should discuss in small groups/partners before a whole class discussion.

- What type of problem is this? How do you know? (*Note that you already asked this question; this is to reaffirm that understanding and keep the idea fresh in students minds as they discuss the next questions.*)
- Which number represents the number of groups? How do you know?
- Which number represents the number in each group? How do you know?
- How many carrots does Ben eat in one school week? How do you know?

Share the following problem situation with students.

> Monique eats 12 carrots in her lunch each school week. Ben eats three times as many carrots as Monique each school week. How many carrots does Ben eat each school week?

Ask students to discuss in small groups or with a partner what type of problem this is and how they know. **Lead** a discussion, using the following questions to help facilitate discourse.

- What do we know about this problem situation? Explain.
- What is the problem situation asking? How do you know?
- Does this problem situation require us to make equal-size groups? How do you know?
- Is this an equal-groups problem? How do you know?
- How is this problem situation the same as the equal-groups problem we just solved?
- How is this problem situation different than the equal-groups problem we just solved?
- Does this problem situation require us to determine area? How do you know?
- Does this problem situation require us to measure a distance or weight? How do you know?
- What do you think the problem situation is asking us to do? Explain.

If no students respond that you are comparing the number of carrots that Ben eats to the number of carrots Monique eats to determine how many carrots Ben eats, **use** questioning techniques to lead students to that conclusion; do not just tell them that is what the question is requiring.

Explain that the problem situation with Monique and Ben is called a comparison situation or a multiplicative comparison situation because the product is the result of comparing one value with another value, i.e. the value for one group is a specified number of copies of the other group. For example, the number of carrots Ben eats is three copies of one of Monique's groups (the number of carrots Monique eats in one day at school).

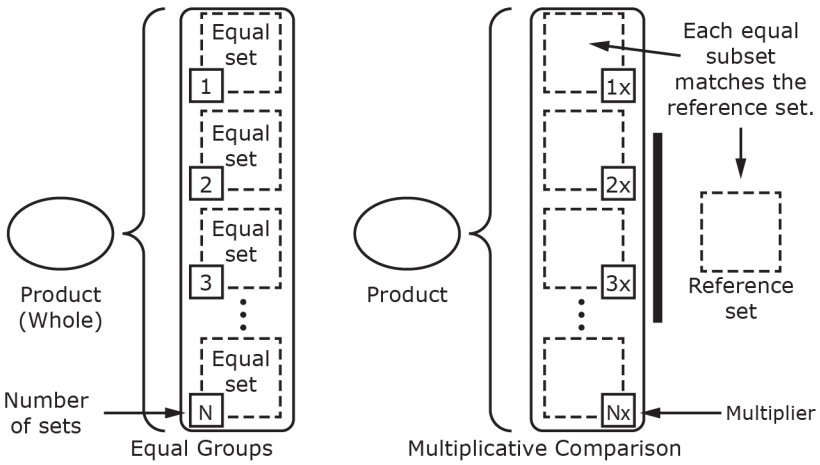
Ask students to represent the problem situation using manipulatives of their choice. Note that you are *not* asking for the solution, but rather a representation of the problem situation. Students may struggle with this; allow them grapple with the problem, and do not rush in to show them.

After students have had enough time to explore attempting to represent the problem situation with manipulatives, **discuss** the process/results.

Ask students to share their representations or describe their attempt at representing the situation (i.e. why their representation does not fit the situation). **Scaffold** this discussion by selecting students who discovered an appropriate representation to share last.

Display or **share** with students the left side (Equal Groups) of the following image, and **ask** students what the image means. Be sure to **clarify** the structure of equal-groups multiplication after students discuss. The larger square represents each equal-size set, and the smaller squares represent the number of equal-size groups. All of the sets combined make up the whole.

Then, **display** or **share** with students the right side (Multiplicative Comparison) of the following image, and **ask** students what the image means. Be sure to **clarify** the structure of multiplicative comparison multiplication after students discuss. The larger square represents the reference set, and the smaller squares represent the multiplier, or the number of times the reference set is repeated. The repeated number of reference sets combined make up the product.



Adapted from (Van de Walle, Karp, Lovin, Bay-Williams, 2014, p. 108).

Highlight the differences between the two types of multiplication: that equal-groups multiplication emphasizes the size of the group and the number of groups, whereas multiplicative comparison multiplication emphasizes the size of one group, or reference set, and how large or small (the multiplier) the other group or product is compared to the reference set.

Refer back to the problem situation with Monique and Ben. **Ask** students *what* is happening in the situation. Are equal groups being combined to determine a total? Is measurement taking place?

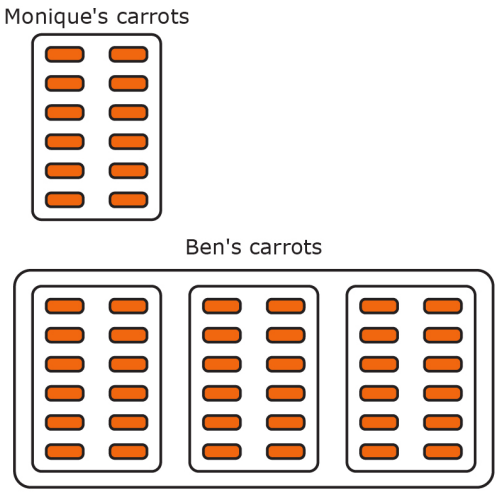
Emphasize that the total number of carrots Ben eats is only given in comparison to how many Monique eats. The number of carrots they each have are compared to each other.

Model (or have a student that modeled correctly previously) the scenario with Monique and Ben using manipulatives.

Start by showing Monique's group of carrots. **Ask** students what the multiplier for the scenario is and how they know (*three times*).

> NOTE: It is significant to avoid teaching "key words" when identifying multiplication, division, addition, and subtraction problems. This removes the emphasis from understanding what the question is saying/asking. And, often times the "key words" are misleading or not included in situations. Instead, focus on analyzing the problem situation and determining what the question is asking. Refrain from telling students to look for the word "times" as an indication of a multiplicative-comparison problem; this can be misleading, and students may always think they need to multiply in order to solve the multiplicative comparison scenario.

Next to Monique's group of carrots, **create** three groups equal to Monique's to represent Ben's carrots. **Ask** students what the reference group is, or what the group being compared to is (*Monique's 12 carrots*). Then, **ask** students what the multiplier represents. Finally, **ask** students how many carrots Ben ate in one school week.



It is important to line the manipulatives up (i.e. Monique's group next to, above, or below Ben's group) so that students can see and internalize the act of comparing the two sets of manipulatives. This will help students transition to the bar model in LESSON 2.

Share the following problem situation with students.

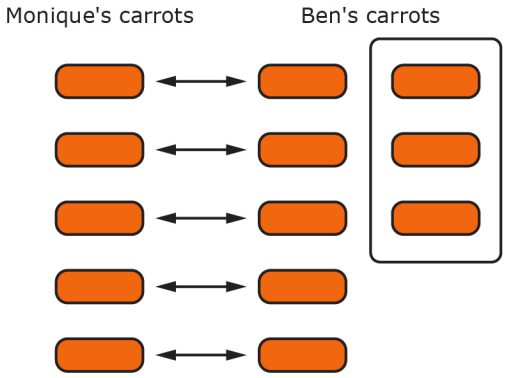
> Monique has five carrots in her lunch today, and Ben has eight carrots in his lunch today. How many more carrots does Ben have than Monique?

Ask students to represent the problem situation using manipulatives for their choice. Note that you are *not* asking for the solution, but rather a representation of the problem situation. Students may struggle with this; allow them grapple with the problem, and do not rush in to show them.

After students have modeled or attempted to model the situation, **lead** a discussion by **asking** the following questions.

- What do you know about the problem situation?
- What is the problem asking? How do you know?
- Share and explain how you modeled the situation.
- Are you comparing the number of carrots Monique and Ben have? How do you know?
- How is this scenario different from the other scenario with Monique and Ben?

Model this situation with manipulatives and compare it to the model for the other scenario with Monique and Ben.



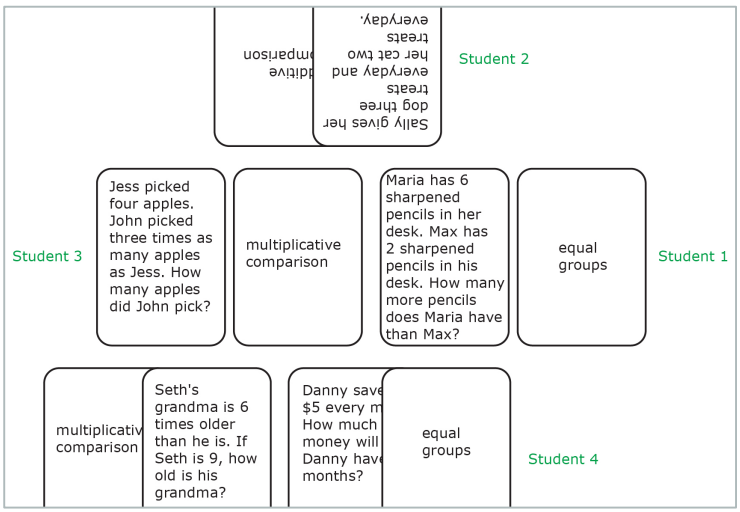
Lead students to the understanding that this scenario with Monique and Ben is *not* multiplicative comparison, but rather additive comparison. **Identify** the fact that there is no multiplier in this scenario. The comparison is between the number of carrots they both have, and the question asks how many more carrots, identifying individual units—not a set, reference set, or multiplier.

Emphasize that because this situation does not contain a multiplier, it is an additive comparison situation, not a multiplicative comparison situation.

Arrange students into groups of two to four, and distribute one set of *Making Matches* cards from the INSTRUCTIONAL ACTIVITY SUPPLEMENT B to each group.

Explain the directions for the *Making Matches* game (based on the card game "Stealing Bundles" or "Casino"). The goal of the game is to collect as many pairs of example cards and problem type cards as possible.

1. One student mixes the cards and passes out four cards to each student in the group. Then that student places four cards face up in the center of the group.
2. On each turn, a student has two choices:
    a. Take a center card and make a pair with a card in their hand.
    b. Leave a card in the center face up.



All pairs must be placed face up so the group can verify that the match is correct.

3. After each student has had one turn, the student who dealt the cards gives everyone in the group one more card.
4. Repeat steps 2 and 3 until there are no more cards.
5. The student with the most pairs/matches wins.

Use the following guiding questions as students are playing *Making Matches* to scaffold student thinking.

GUIDING QUESTIONS

Elicit student thinking:

- Have you played a game like this before? Explain.
- How do you know if you have a match? Explain.

Determine if the student can EXPLAIN MULTIPLICATION PROBLEMS:

- [Point to a student's pair of cards.] Does this problem situation have equal groups? How do you know?
- [Point to a student's pair of cards.] Why did you pair this card with the equal-groups multiplication card?
- [Point to a student's pair of cards.] What represents the number of groups in this problem situation? How do you know?
- [Point to a student's pair of cards.] What represents the size of each group in this problem situation? How do you know?

Determine if the student can MATHEMATIZE CONTEXTUAL SITUATIONS INVOLVING MULTIPLICATION:

- [Point to a student's card.] Is this problem situation additive (adding) or multiplicative (multiplying)? How do you know?
- [Point to a student's card.] What information in the problem situation helped you identify this as a multiplication problem? Explain.
- [Point to a student's card.] What is this problem situation asking? How do you know?

Determine if the student can EXPLAIN *MULTIPLICATIVE COMPARISON*:

- [Point to a student's pair of cards.] Does this problem situation require multiplicative comparison? How do you know?
- [Point to a student's pair of cards.] How do you know this is multiplicative comparison and not additive comparison?
- [Point to a student's pair of cards.] Why did you pair this card with the multiplicative comparison card?
- [Point to a student's pair of cards.] What represents the reference set in this problem situation? How do you know?
- [Point to a student's pair of cards.] What represents the multiplier in this problem situation? How do you know?

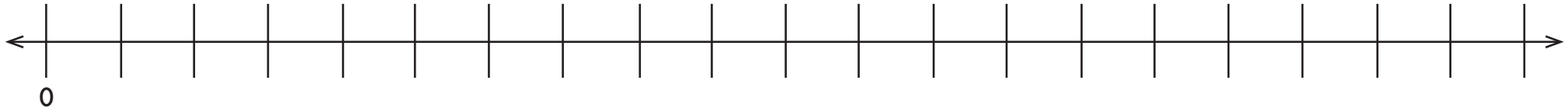
At the end of the activity, teachers should have students share with the class one of their pairs and explain why they paired those two cards together. Encourage students to identify the different parts of the problem situation, such as the number of groups and size of each group for equal-groups problems, the reference set and multiplier for multiplicative comparison, the two sets for additive comparison, and so on.

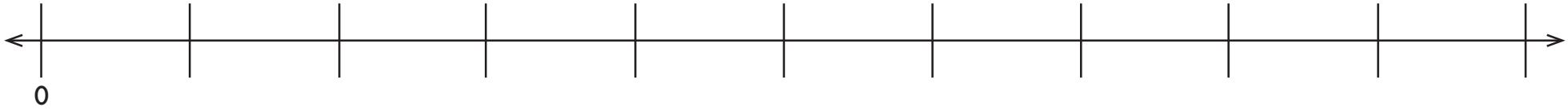
Ms. Smith has three students in each of her small groups for reading. If she has six small groups, how many students does Ms. Smith meet with for reading?
Mitchel, Brenna, and Ana need six notecards for a group project. Mitchel has three notecards, Brenna has two notecards, and Ana has zero notecards. Do they have enough notecards for their group project?
Mr. Gonzales needs 100 two-liter bottles for a science project. If each of his 23 students brings in three two-liter bottles, will Mr. Gonzales have enough two-liter bottles for the science project?
Jackie is getting a new blanket for her bed. The area of her bed is 28 feet. Jackie wants her blanket to be larger than her bed. The blanket she picks is six feet long and four feet wide. Will the blanket Jackie picked be larger or smaller than her bed?
Marco is saving money for a new pair of headphones. He adds his allowance of $10 to his savings every week. How much money will Marco have saved after four weeks if he starts with $0 in his savings?

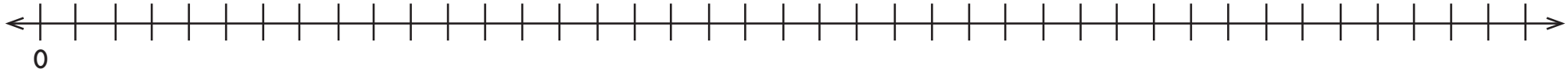
Mr. Jacobson is putting a fence up around Mrs. Jacobson's garden. The garden is 10 feet long and four feet wide. If the fencing is sold in lengths of 20 feet, how many lengths will Mr. Jacobson need to buy?
Liam has to read four books before the school break. The first book has 176 pages, the second book has 103 pages, and the third and fourth books each have 98 pages. How many pages does Liam have to read before the school break?
Emma and Terrance are competing for the tallest cup stack. Emma has 55 cups stacked so far, and Terrance has 62 cups stacked so far. How many cups does Emma need to add to her stack to beat Terrance, if Terrance does not add any more cups?
Becca is picking berries with her family. Each pail holds four pints of berries. Becca fills four pails, and her sister fills three pails. How many pints of berries do Becca and her sister collect altogether?
Mia and Ruby are making a poster for their social studies project. They want their poster to be blue on the top half and green on the bottom half to look like the sky and grass. They buy a blue poster board that is 18 inches long and 24 inches wide. What is the area of the poster board they need to paint?
Each week, Elias brings his lunch from home for three days and buys school lunch for two days. Each school lunch costs $2. How much will Elias spend on school lunches after four weeks?

Each window on a house has four pieces of glass. If there are seven windows, how many total pieces of glass make up all the windows?
Zoey ordered six books online, and Brayden ordered eight books online. If their books arrive together in the same box, how many books will there be?
Aaliyah is hanging curtains in her bedroom. If the area of each of her windows is 6 square feet, will a curtain that is five feet long and 3 feet wide be large enough to cover a whole window?
The area of the playground at Stuart elementary school is 792 square feet. If the school adds a second playground of the same size, what will be the total square feet of the two playgrounds?
Devin and Leah are using their base-ten blocks in math class. Devin has 6 groups with 3 ones in each group. Leah has 9 groups with 2 ones in each group. Are Devin and Leah using the same number of base-ten blocks for the problem they are modeling?

Equal-Groups Multiplication Problems
Measurement Multiplication Problems
Addition Problems







Making Matches Game

The goal of the game is to collect as many pairs of example cards and problem type cards as possible.

Directions:

1. One student mixes the cards and passes out four cards to each student in the group. Then that student places four cards face up in the center of the group.
2. On each turn, a student has two choices:
    a. Take a center card and make a pair with a card in their hand.
    b. Leave a card in the center face up.

All pairs must be placed face up so the group can verify that the match is correct.

3. After each student has had one turn, the student who dealt the cards gives everyone in the group one more card.
4. Repeat steps 2 and 3 until there are not more cards.
5. The student with the most pairs/matches wins.

* Recommend printing the cards on tag board or card stock to last longer.

multiplicative comparison	Seth's grandma is 6 times older than he is. If Seth is 9 how old is his grandma?	equal-groups multiplication	Danny saves $5 every month. How much will Danny have in 4 months if he starts with $0?
additive comparison	Every day, Sally gives her dog three treats and her cat two treats. How many more treats does her dog get than her cat every day?	measurement multiplication	Jackie is getting a new blanket for her bed. The area of her bed is 28 square feet. Jackie wants her blanket to be larger than her bed. The blanket she picks is 6 feet long and 4 feet wide. Will the blanket Jackie picked be larger or smaller than her bed?

multiplicative comparison	Jess picked four apples. John picked three times as many apples as Jess did. How many apples did John pick?	equal-groups multiplication	Ms. Smith has three students in each of her small groups for reading. If she has six small groups, how many students does Ms. Smith meet with for reading?
additive comparison	Maria has 6 sharpened pencils in her desk. Max has 2 sharpened pencils in his desk. How many more pencils does Maria have than Max?	measurement multiplication	Juan is making a board game for a math project. He is designing the board like a table, divided into rows and columns. If he has 7 rows and 4 columns, how many squares will he have on his game board?

multiplicative comparison	Makayla did not do all of her reading homework for the week. Braxton did all of his reading homework for the week on time. Braxton read 33 pages, and he read 3 times more than Makayla. How many pages will Makayla have to read to catch up?	equal-groups multiplication	Each window on a house has four pieces of glass. If there are seven windows, how many total pieces of glass make up all the windows?
additive comparison	Evan and Josiah were collecting empty cans to recycle. Evan collected five more cans than Josiah. If Evan collected 12 cans, how many cans did Josiah collect?	measurement multiplication	The grass on the soccer field is being replaced. The soccer field is 20 feet long and 57 feet wide. What is the total area of new grass needed to cover the soccer field?

multiplicative comparison	Trenton's older brother Jaxson has 120 minutes of homework a week. Trenton has 40 minutes of homework a week. How many times more does Jaxson spend on homework than Trenton each week?	equal-groups multiplication	Jayla has 20 minutes of homework each night Monday through Thursday. How many minutes of homework does Jayla have each school week?
additive comparison	Chase has 15 minutes of homework on Tuesday and 27 minutes of homework on Wednesday. How much more time did Chase spend on homework on Tuesday than Wednesday?	measurement multiplication	For homework, Antonio has to make a paper quilt with each square representing an event in the American Revolution. His quilt is made of 42 paper squares, and the length of the paper quilt is 7 paper squares. How many paper squares wide is Antonio's quilt?

multiplicative comparison	During the science experiment, it took the water 2 times longer to have a reaction than the oil. The oil took 13 minutes. How long did the water take?	equal-groups multiplication	During the science experiment, the teacher gave each group of students five tablespoons of oil. There are eight groups in the science class. How many tablespoons of oil will the science teacher need?
additive comparison	For the science experiment, the students used baking soda and oil. Each group was given five tablespoons of oil and two tablespoons of baking soda. How much more oil did the students get than baking soda?	measurement multiplication	The students recorded their data from the science experiment in a table. There were 3 columns and 12 rows. How many total cells are in the table?

MULTIPLICATIVE COMPARISON

INSTRUCTIONAL ACTIVITY

Lesson 2

LEARNING GOAL

Students will create semi-concrete models to represent multiplicative-comparison problem situations.

PRIMARY ACTIVITY

Students first explore the differences between multiplicative-comparison problem situations and other multiplication or additive-comparison problem situations by analyzing models representing the problem situation. Students will then create their own models to represent multiplicative-comparison problem situations.

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

- Multiplicative comparison
- Equal-groups multiplication
- Measurement multiplication
- Additive comparison
- Product
- Group size
- Multiplier (number of groups)
- Set model
- Area model
- Bar diagram

MATERIALS

- INSTRUCTIONAL ACTIVITY STUDENT HANDOUT
- INSTRUCTIONAL ACTIVITY SUPPLEMENT (Recommend one teacher copy.)
- Manipulatives from LESSON 1 (optional)

- Chart paper, butcher paper, or large construction paper (optional)
- Small/individual dry erase boards and dry erase markers (optional) (Recommend one board for every one to three students.)

IMPLEMENTATION

Prior to teaching the lesson, write each of the three problem situations on a piece of chart paper, butcher paper, or large construction paper, or type each problem situation to display on an interactive white board.

Review the different types of problem situations from LESSON 1.

Discuss the different problem types and how to identify the type of problem based on what the situation is asking. As you discuss, **create** a list of the problem types for students to reference during the following activity.

Share the following problem situation with students.

> To celebrate fall, Ms. Thompson's class is going to decorate their lockers. Each student gets 7 pieces of construction paper to make decorations. There are 19 students in Ms. Thompson's class. How many pieces of construction paper will Ms. Thompson need to get from the supply room?

Ask students what type of problem situation this is and how they know.

Write the problem type (*equal-groups multiplication*) under the problem situation on the paper or interactive white board.

Require students to draw a model that represents the problem situation. **Circulate** and **focus** on the models students create.

Use the following guiding questions to support student understanding.

GUIDING QUESTIONS

Elicit student thinking:

- What type of problem situation do you think this is? Why?
- Have you created a model for a word problem before? How did you do it?

Determine if the student can MATHEMATIZE CONTEXTUAL SITUATIONS INVOLVING MULTIPLICATION:

- Does this problem have equal groups? How do you know?
- Is the product of this problem the result of comparing two values? How do you know?
- What is the problem asking? (What do you need to solve?) How do you know?
- Do the pieces of paper represent the size of each group or the number of groups? How do you know?
- Do the number of students represent the size of each group or the number of groups? How do you know?

Select students to share their models. Students with less efficient strategies/models should share before those with more abstract/advanced models.

Record different types of models under the problem situation on the paper or interactive white board.

Discuss which models are most efficient and which models more accurately reflect the problem situation. For example, a set model would support understanding of the problem situation more accurately than an area model. However, an area model is still a valid model; it is not incorrect—it is just not the most accurate representation of the problem situation.

If no students present a model representation, **share** the model as work a previous student had completed. The models to emphasize are set models, arrays, area models, number lines, and bar diagrams.

Repeat the process by **sharing** the following problem situation.

> Mr. Jackson's class is decorating their lockers to celebrate fall. Mr. Jackson's class needs 161 pieces of paper to make decorations. Ms. Thompson's class needs 133 pieces of paper to make decorations. How many more pieces of paper will Mr. Jackson's class need than Ms. Thompson's class?

Ask students what type of problem situation this is and how they know.

Write the problem type (*additive comparison*) under the problem situation on the paper or interactive white board.

Require students to draw a model that represents the problem situation. **Circulate** and **focus** on the models students create.

Use the following guiding questions to support student understanding.

GUIDING QUESTIONS

Elicit student thinking:

- What type of problem situation do you think this is? Why?
- Have you created a model for a word problem before? How did you do it?

Determine if the student can EXPLAIN *ADDITIVE COMPARISON*:

- Does this problem require addition or multiplication? How do you know?
- What is being compared? How do you know?
- What is the problem asking? (What do you need to solve?) How do you know?
- Explain your model for the problem situation.
- How would you determine the difference between the number of pieces of paper Mr. Jackson needs compared to Ms. Thompson?

Select students to share their models. Students with less efficient strategies/models should share before those with more abstract/advanced models.

Record different types of models under the problem situation on the paper or interactive white board.

Discuss which models are most efficient and which models more accurately reflect the problem situation.

If no students present a model representation, **share** the model as work a previous student had completed. The models to emphasize are set models, arrays, area models, number lines, and bar diagrams.

Repeat the process by **sharing** the following problem situation.

> Ryker and Jaidyn want to use their 7 pieces of paper to make leaves. Jaidyn makes 14 large leaves with her paper. Ryker makes small leaves and has 4 times as many leaves as Jaidyn. How many leaves does Ryker have?

Ask students what type of problem situation this is and how they know.

Write the problem type (*multiplicative comparison*) under the problem situation on the paper or interactive white board.

Require students to draw a model that represents the problem situation. **Circulate** and **focus** on the models students create.

Use the following guiding questions to support student understanding.

GUIDING QUESTIONS

Elicit student thinking:

- What type of problem situation do you think this is? Why?
- Have you created a model for a word problem before? How did you do it?

Determine if the student can EXPLAIN *MULTIPLICATIVE COMPARISON*:

- Does this problem require addition or multiplication? How do you know?
- What is being compared? How do you know?
- What is the problem asking? (What do you need to solve?) How do you know?
- Explain your model for the problem situation.
- Who makes more leaves with their pieces of paper, Ryker or Jaidyn? How do you know?
- How can you figure out how many paper leaves Ryker and Jaidyn each make? Explain.

Select students to share their models. Students with less efficient strategies/models should share before those with more abstract/advanced models.

Record different types of models under the problem situation on the paper or interactive white board.

Discuss which models are most efficient and which models more accurately reflect the problem situation.

If no students present a model representation, **share** the model as work a previous student had completed. The models to emphasize are set models, arrays, area models, number lines, and bar diagrams.

Arrange students into groups of two or three if they are not in small groups already.

Display model A on the INSTRUCTIONAL ACTIVITY SUPPLEMENT. **Ask** the students to think about the model individually for about one minute. Then, **tell** students to discuss with their group mates what type of problem they think the model *best* represents and why.

After a few minutes, **discuss** what type of problem the students think the model *best* represents and why. **Require** students to support their answer with their thinking, and **encourage** students to create problem scenarios to support their explanations.

> NOTE: While there are no "right" answers, some models are better suited to represent some problem types than others. The emphasis of this activity should be placed on the students' reasoning abilities and not getting a "correct" answer or response. It is appropriate to encourage students to "see" that some models are more appropriate than others for certain problem situations, but do not force or require a student to use one specific type of model for a problem type.

Repeat the process with each of the models on the INSTRUCTIONAL ACTIVITY SUPPLEMENT.

Distribute the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT. Students may work in their small groups from the previous activity, with new partners, or independently.

Read the directions, and **complete** the first question together as a whole class. **Require** students to complete the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

Use the following guiding questions to support student understanding.

GUIDING QUESTIONS

Elicit student thinking:

- Have you created a model for a word problem before? How did you do it?
- [Point to a table on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.] What do you have to write in each section? How do you know?
- What type of problem situation do you think this is? Why?
- Would you rather solve a written problem situation or write a problem situation when given a model? Explain.

Determine if the student can CALCULATE PRODUCTS USING STRATEGIES OTHER THAN THE STANDARD ALGORITHM:

- [Point to a table on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.] Explain your model to me. How are the factors represented? How is the product represented?
- [Point to a table on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.] What is the product for this question? How do you know?
- [Point to a table on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.] Show me this problem using a different model.
- [Point to a table on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.] Why did you use this model to represent this problem situation?

Determine if the student can EXPLAIN *ADDITIVE COMPARISON*:

- [Point to Question 5.] What is this question asking? How do you know?
- [Point to Question 5.] Will Bella every get her new shoes more or less than three months before Emma? Why or why not?
- [Point to a question that a student has represented as additive comparison.] How do you know this is an additive-comparison problem?

Determine if the student can EXPLAIN *MULTIPLICATIVE COMPARISON*:

- [Point to Question 1 or 4.] What is this question asking? How do you know?
- [Point to Question 1.] Will Rylee always have the same number of cards more than Justice? Why or why not?
- [Point to Question 1.] If Justice found two trading cards under his bed, and Rylee had the same number of trading cards as before, would Rylee still have six times more trading cards than Justice? Why or why not?
- [Point to Question 4.] Will Bella's sister and mom always run the same distance less than Bella and her dad run? Why or why not?
- [Point to Question 4.] If Bella was sore from a soccer game on Friday night, so she and her dad only ran four miles, but her little sister and mom walked the same distance they usually do, would Bella and her dad still run three times farther than her little sister and mom walk? Why or why not?
- [Point to a question that a student has represented as multiplicative comparison.] How do you know this is a multiplicative-comparison problem?

Determine if the student can EXPLAIN THE DIFFERENCE BETWEEN ADDITIVE COMPARISON AND MULTIPLICATIVE COMPARISON:

- Explain how you know if a problem is additive comparison or multiplicative comparison.
- [Point to a table on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.] Does this problem situation (or model) represent additive comparison or multiplicative comparison? How do you know?
- [If the student wrote a multiplicative-comparison problem for Question 3.] Could this model also represent an additive-comparison problem? Why or why not?
- [If a student wrote an additive-comparison problem for Question 3.] Could this model also represent a multiplicative-comparison problem? Why or why not?
- [If a student wrote an additive-comparison problem for Question 3.] Why did you choose to represent this model as additive comparison and not multiplicative comparison?

Discuss student work by strategically **selecting** students with a variety of strategies/problem situations to share with the class.

At the end of the activity, teachers should collect the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT and analyze for student understanding.

Name_____

Understanding Multiplicative Comparison

Lesson 2

Use the information you are given to fill in the blank sections.

1.

Problem situation	Rylee has six times more trading cards than Justice. If Justice has twelve trading cards, how many trading cards does Rylee have?
Restate the question in your own words	
Model	
Solution	

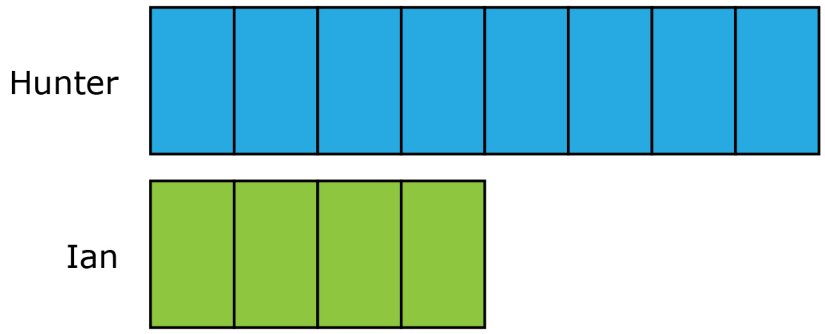
Name_____

2.

Problem situation	Justice, Rylee, and Theo are going to play a game with their trading cards. To prepare, Theo arranges his cards into stacks. He makes five stacks with six cards in each stack. How many trading cards does Theo have?
Restate the question in your own words	
Model	
Solution	

Name_____

3.

Problem situation	
Restate the question in your own words	
Model	
Solution	

Name_____

4.

Problem situation	Bella runs with her dad every Saturday morning. Bella's mom and little sister walk every Saturday morning. Bella and her dad run three times as far as her mom and little sister walk. If Bella and her dad run six miles, how far do her mom and little sister walk?
Restate the question in your own words	
Model	
Solution	

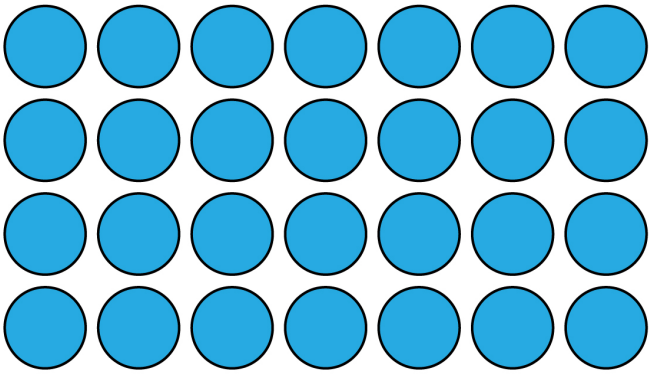
Name_____

5.

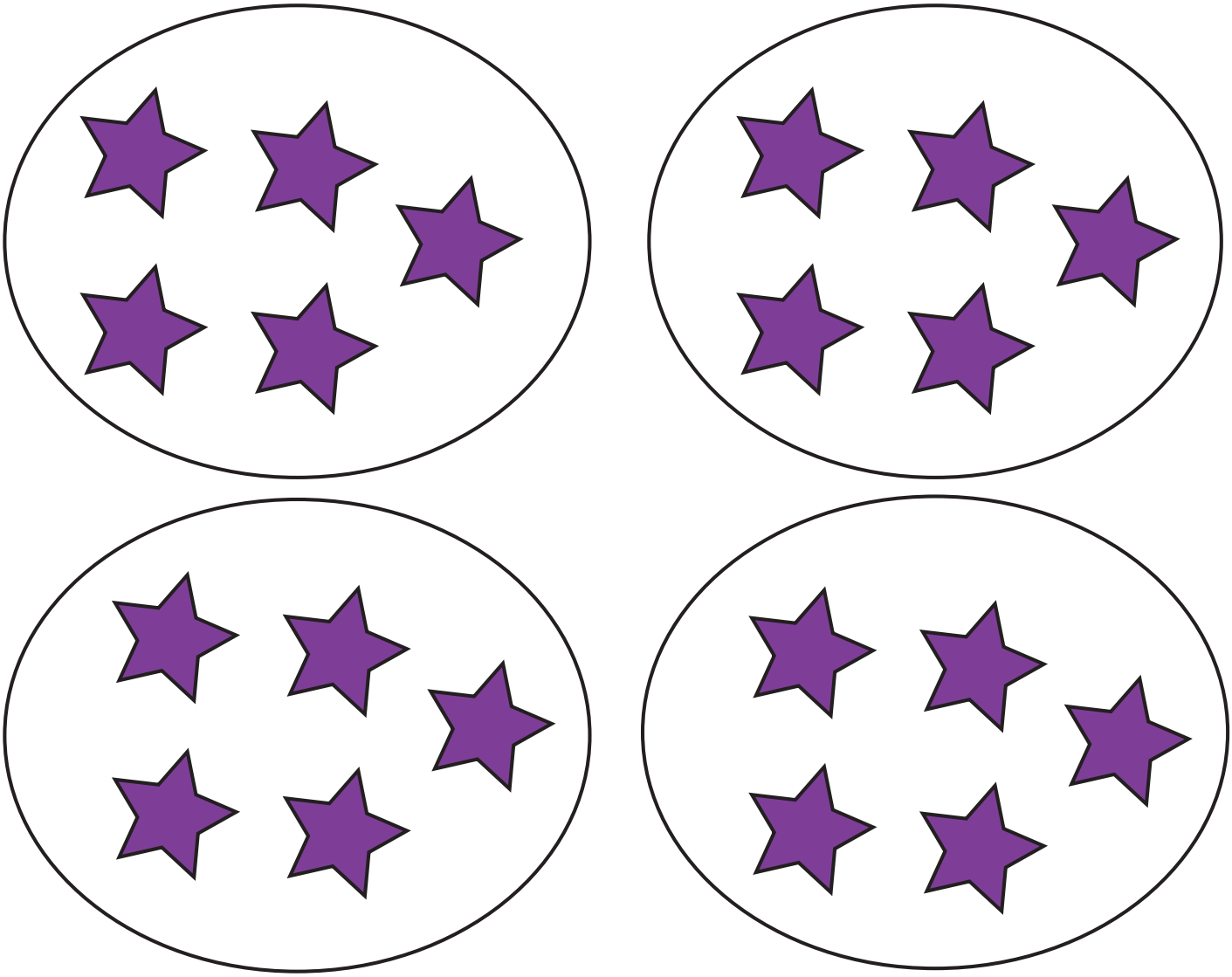
Problem situation	Bella has to get new running shoes every 4 months. Bella's little sister Emma gets new walking shoes every 7 months. How many months after Bella gets new running shoes will Emma get her new walking shoes?
Restate the question in your own words	
Model	
Solution	

Name_____

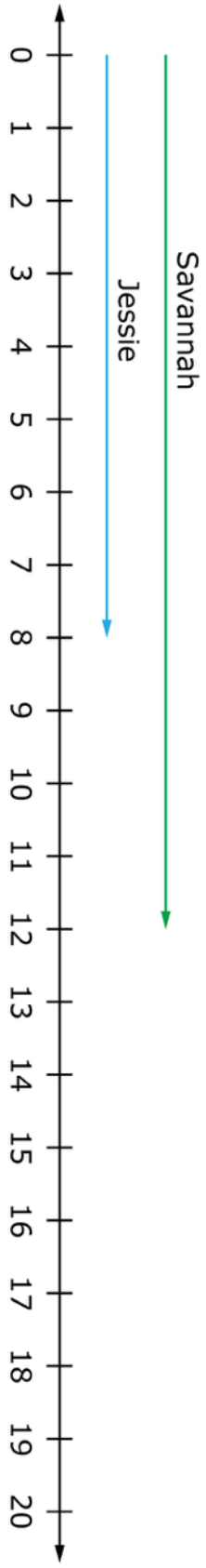
6.

Problem situation	
Restate the question in your own words	
Model	
Solution	

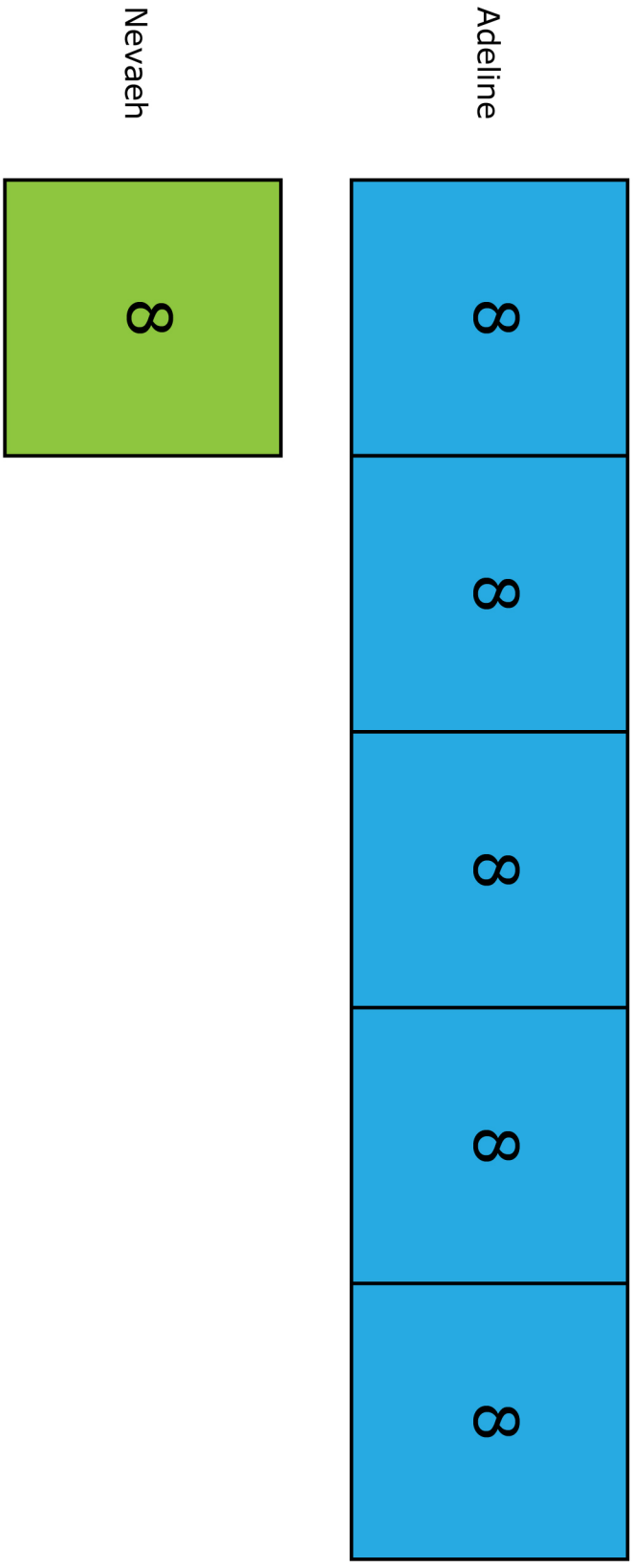
Model A



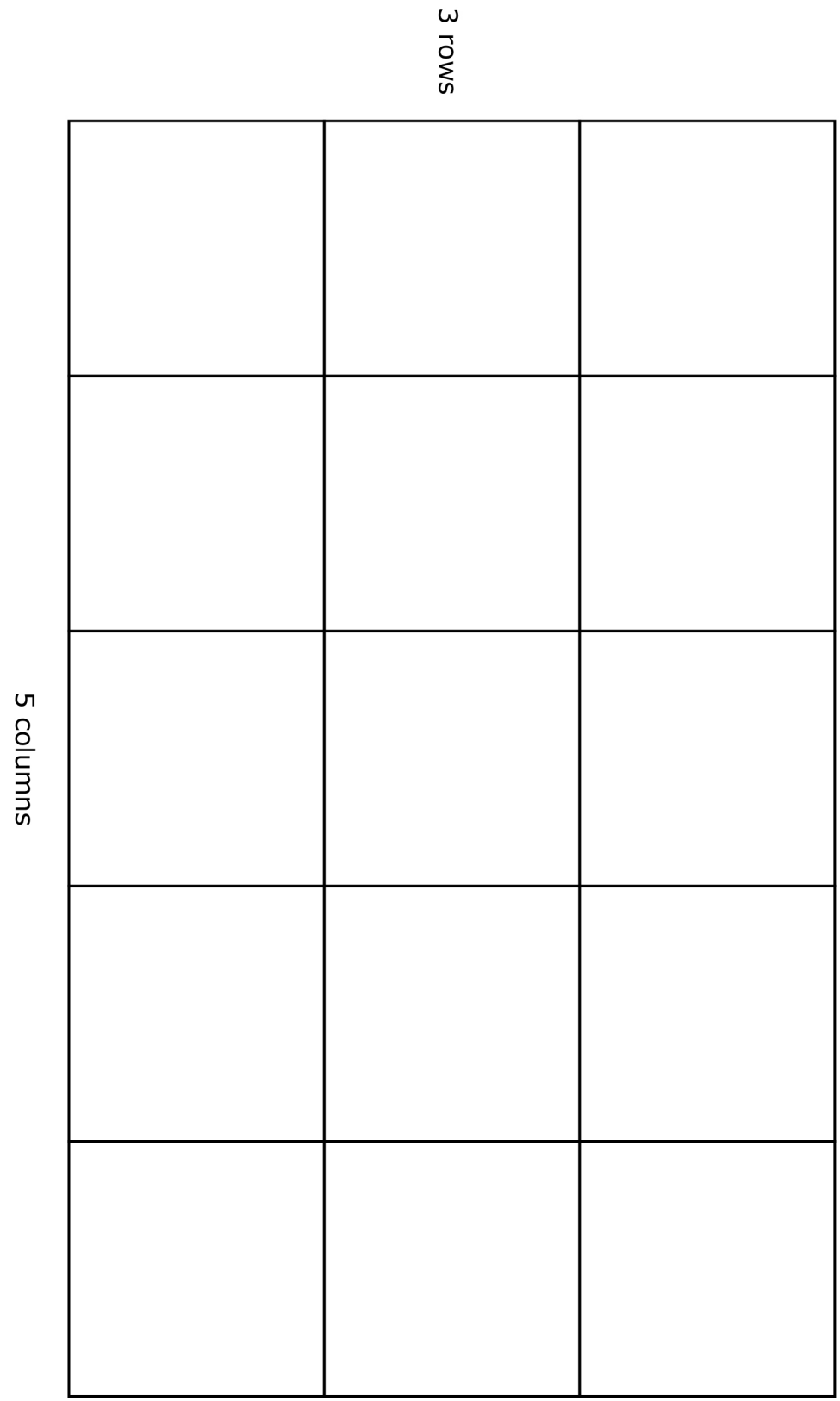
Model B



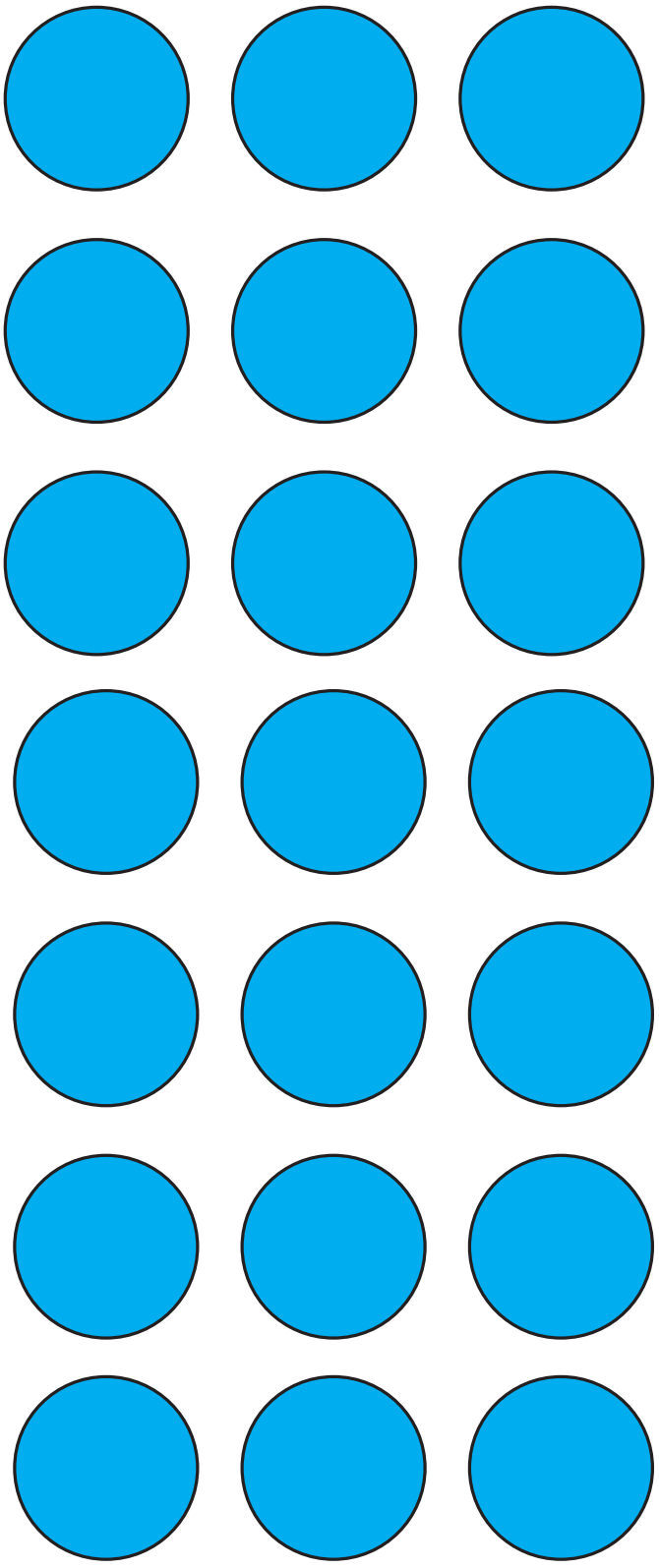
Model C



Model D



Model E



UNDERSTANDING MULTIPLICATIVE COMPARISON

INSTRUCTIONAL ACTIVITY

Lesson 3

LEARNING GOAL

Students will use their critical thinking skills to determine what part of the problem situation is unknown and then write an equation representing the problem situation.

PRIMARY ACTIVITY

Students will analyze problem situations to determine the unknown part of the problem. In small groups, students will use "equation cards" to model the equation that represents the problem situation. Students will finish the lesson by writing equations that represent the problem situation.

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

- Equation
- Unknown value
- Multiplicative comparison
- Product
- Group size
- Multiplier
- Reference set

MATERIALS

- INSTRUCTIONAL ACTIVITY STUDENT HANDOUT
- INSTRUCTIONAL ACTIVITY SUPPLEMENT (Recommend one for every two to three students.)

IMPLEMENTATION

Review the different problem situation types from LESSONS 1 and 2.

Discuss the different problem types and how to identify the type of problem based on what the situation is asking.

Tell students they are going to help you write an equation for a problem situation. **Explain** that an equation contains an equal sign, and that the information on either side of the equal sign must have the same value. For example, in the equation 2 + 4 = 3 + 3, the value on either side of the equal sign is 6. This can be modeled by completing these operations: 2 + 4 = 6, 3 + 3 = 6, and 6 = 6.

Explain that in problem situations, not all of the information is given and you have to solve to determine the missing information. **Tell** students that the missing information will be represented in the equations with a question mark ("?") or an empty box ("☐").

Share the following problem situation with students. **Cover up** or **remove** the numbers from the problem situation to start, so the students focus on the information given in the text and do not just identify the numbers in the problem.

> To celebrate fall, Ms. Thompson's class is going to decorate their lockers. Each student gets ■ pieces of construction paper to make decorations. There are ■ students in Ms. Thompson's class. How many pieces of construction paper will Ms. Thompson need to get from the supply room?

Remind students they analyzed this situation in LESSON 2. **Ask** students what type of problem situation this is and how they know.

Ask students what information is given in the problem situation, requiring students to explain their responses. Students should respond with phrases such as, "pieces of paper to make decorations" and "the students in Ms. Thompson's class".

If they do not share what the problem situation is asking, **ask** students what the problem situation is asking. Students should respond what phrases such as, "how many pieces of paper Ms. Thompson will need for her class".

Write the given information in a visible place (e.g., a large dry erase board, interactive white board, chart paper, etc.).

Once students have identified the information given in the problem situation, **share** the problem situation with the numerals visible.

> To celebrate fall, Ms. Thompson's class is going to decorate their lockers. Each student gets 7 pieces of construction paper to make decorations. There

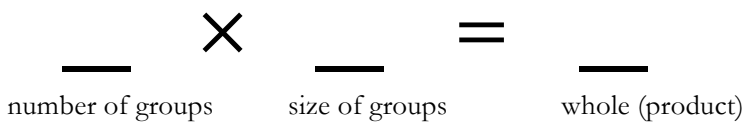
are 19 students in Ms. Thompson's class. How many pieces of construction paper will Ms. Thompson need to get from the supply room?

Refer back to where you wrote the information from the problem situation, and ask students to identify the values that go with each piece of information.

Ask students what the missing piece of information is, and relate this to the question that is being asked in the problem (*the number of pieces of paper Ms. Thompson will need for her class.*).

Remind students that they identified the situation as an equal-groups multiplication problem.

On the same paper or board where you wrote the problem information, set up the equation with three blanks, a multiplication sign, and an equal sign. Be sure to **write** "number of groups," "size of groups," and "total" under the appropriate blanks. **Reference** the following image for an example.

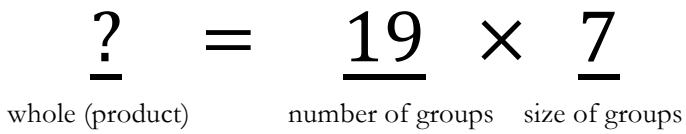


Ask directed questions to help students identify that they are given the number of groups (*19 students*) and the size of the groups (*seven pieces of paper*), but they are missing the total (*number of pieces of paper Ms. Thompson needs to get from the supply room*).

Select a student to fill in the blanks on the equation with the information given in the problem situation and a "?" in the total box since that is the missing information.

After the student adds the numerals and question mark, **emphasize** what they represent. Then, **read aloud** the equation, including the labels as you do; for example, "19 students *times* seven pieces of paper each *equals* the total number of pieces of paper needed."

Rewrite the equation so that the total is to the left of the equal sign. This will support student understanding of the meaning of the equal sign: that values on both sides are equal as opposed to the left side being a procedure and the right side an answer.



Ask students the following questions. **Require** students to share with a partner before you **discuss** the answers as a whole group.

- Has the equation changed in any way? How do you know?
- Does the equation still represent the problem situation? Why or why not?
- Will the question mark still be the same value? How do you know?

After the discussion, **allow** students sufficient time to solve the equation.

Recognize that solving multiplication problems is not the focus of this unit, and therefore **allow** students to use whatever strategy they are comfortable with when solving. The focus of the unit and lessons is on making meaning of multiplication-comparison problems and distinguishing them from other types of multiplication- and additive-comparison problems.

Select one or two students to share and explain their strategy with the class, **ensuring** that each student who shares includes the labels for each numeral.

Share the following problem situation with students.

Cover up or **remove** the numbers from the problem situation to start, so the students focus on the information given in the text and do not just identify the numbers in the problem.

> Mr. Jackson's class is decorating their lockers to celebrate fall. Mr. Jackson's class needs ■ pieces of paper to make decorations. Ms. Thompson's class needs ■ pieces of paper to make decorations. How many more pieces of paper will Mr. Jackson's class need than Ms. Thompson's class?

Remind students they analyzed this situation in LESSON 2. **Ask** students what type of problem situation this is and how they know.

Ask students what information is given in the problem situation, requiring students to explain their responses. Students should respond with phrases such as, "pieces of paper to make decorations" and "pieces of paper to make decorations".

If they do not share what the problem situation is asking, **ask** students what the problem situation is asking. Students should respond what phrases such as, "how many *more* pieces of paper Mr. Jackson will need than Ms. Thompson".

Write the given information in a visible place (e.g., a large dry erase board, interactive white board, chart paper).

Ask students how this problem is different from and the same as the equal-groups multiplication problem situation. **Require** students to share with a partner before discussing whole group.

Once students have identified the information given in the problem situation, **share** the problem situation with the numerals visible.

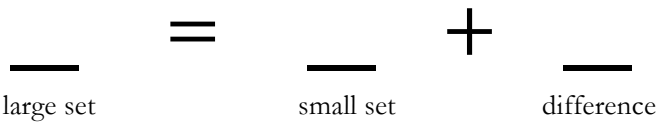
> Mr. Jackson's class is decorating their lockers to celebrate fall. Mr. Jackson's class needs 161 pieces of paper to make decorations. Ms. Thompson's class needs 133 pieces of paper to make decorations. How many more pieces of paper will Mr. Jackson's class need than Ms. Thompson's class?

Refer back to where you wrote the information from the problem situation, and ask students to identify the values that go with each piece of information.

Ask students what the missing piece of information is, and relate this to the question that is being asked in the problem (*how many more pieces of paper Mr. Jackson will need than Ms. Thompson*).

Remind students that they earlier identified the problem situation as an additive-comparison problem. **Explain** that the parts of the equation will be different for this problem situation because it is not an equal-groups multiplication situation.

On the same paper or board where you wrote the problem information, set up the equation with three blanks, an addition sign, and an equal sign. Be sure to **write** "large set," "small set," and "difference" under the appropriate blanks. **Reference** the following image for an example.



Arrange students into groups of two or three. **Distribute** one set of the *Equation Cards* from the INSTRUCTIONAL ACTIVITY SUPPLEMENT to each group.

Require students to use the *Equation Cards* to model the equation with the information given in the problem situation.

Use the following guiding questions to support student understanding.

GUIDING QUESTIONS

Elicit student thinking:

- How is this equation different from the first equation?
- How is this equation the same as the first equation?

Determine if the student can MATHEMATIZE CONTEXTUAL SITUATIONS INVOLVING ADDITION:

- How do you know this is an addition problem?
- What are you adding together in this problem situation? How do you know?

Determine if the student can REPRESENT ADDITION WITH EQUATIONS:

- [Point to the equation modeled with the *Equation Cards*.] Why did you place these numbers here?
- [Point to the equation modeled with the *Equation Cards*.] Would you get the same sum if you put the number for the large set and the number for the small set on the same side of the equal sign? Why or why not?

Determine if the student can EXPLAIN ADDITIVE COMPARISON:

- What is being compared in this problem situation? How do you know?
- [Point to the equation modeled with the *Equation Cards*.] Which value represents the amount of paper Mr. Jackson needs? Why did you place it there?
- [Point to the equation modeled with the *Equation Cards*.] Which value represents the amount of paper Ms. Thompson needs? Why did you place it there?
- [Point to the equation modeled with the *Equation Cards*.] Should the number of pieces of paper Mr. Jackson needs be on the same side of the equal sign or the opposite side of the equal sign from Ms. Thompson's number? Why?

Determine if the student is ready to EXPLAIN MULTIPLICATIVE RELATIONSHIPS REPRESENTED BY EQUATIONS:

- If the number of pieces of paper Mr. Jackson needed was three times more than Ms. Thompson, would the values for the number of pieces of paper for Mr. Jackson go on the same side of the equal sign or the opposite side of the equal sign from Ms. Thompson's number? How do you know?

Select groups to share and explain their equations. **Ensure** that the groups who share have the correct values in the correct places.

$$\underset{\text{large set}}{161} = \underset{\text{small set}}{133} + \underset{\text{difference}}{?}$$

Emphasize that the unknown in the problem situation is the difference between the larger set and the smaller set. The problem situation is comparing the two sets to each other; one set is defined by

the size of the other set. For example, the larger set is defined by *how much larger* than the smaller set it is, and the smaller set is defined by *how much smaller* it is than the larger set.

Allow students sufficient time to solve the equation. **Make note** of strategies students/groups use to solve the equation (e.g., counting up/adding up, subtraction, etc.).

Select students to share and explain their strategies for solving the equation. Choose students with more basic strategies and explanations to share first and students with more efficient strategies and advanced explanations to share after.

Share the following problem situation with students.

Cover up or **remove** the numbers from the problem situation to start, so the students focus on the information given in the text and do not just identify the numbers in the problem.

> Ryker and Jaidyn want to use their ■ pieces of paper to make leaves. Jaidyn makes ■ large leaves with her paper. Ryker makes small leaves and has ■ times as many leaves as Jaidyn. How many leaves does Ryker have?

Remind students that they analyzed this situation in LESSON 2. **Ask** students what type of problem situation this is and how they know.

Ask students what information is given in the problem situation, requiring students to explain their responses. Students should respond with phrases such as, "pieces of paper," "large leaves," and "times more leaves".

If they do not share what the problem situation is asking, **ask** students what the problem situation is asking. Students should respond what phrases such as, "how many leaves Ryker has".

Write the given information in a visible place (e.g., a large dry erase board, interactive white board, chart paper).

Ask students how this problem is different from and the same as the additive-comparison problem situation. **Require** students to share with a partner before discussing whole group.

Once students have identified the information given in the problem situation, **share** the problem situation with the numerals visible.

> Ryker and Jaidyn want to use their 7 pieces of paper to make leaves. Jaidyn makes 14 large leaves with her paper. Ryker makes small leaves and has 4 times as many leaves as Jaidyn. How many leaves does Ryker have?

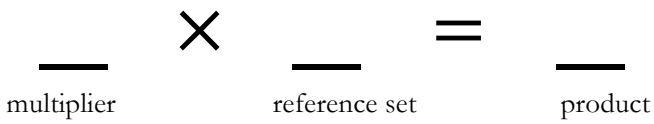
Refer back to where you wrote the information from the problem situation, and ask students to identify the values that go with each piece of information. **Discuss** the "extra" or "unnecessary" information in the problem situation.

If no students identify the values, **point out** that the number could be removed from the first sentence and it would not affect the problem situation; for example, "Ryker and Jaidyn want to use their pieces of paper to make leaves." **Avoid** teaching this as a "rule" since it will not always be the case. **Ensure** that students understand that, in the context of *this* particular problem situation, the third given "extra" value is unnecessary to understand and solve the context of the problem.

Ask students what the missing piece of information is, and relate this to the question that is being asked in the problem (*how many leaves Ryker has*).

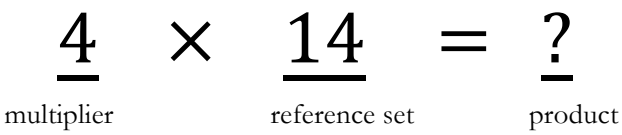
Remind students that they earlier identified the problem situation as a multiplicative-comparison problem. **Explain** that the parts of the equation will be different for this problem situation because it is not an equal-groups multiplication situation and it is not an additive-comparison problem situation.

On the same paper or board where you wrote the problem information, set up the equation with three blanks, a multiplication sign, and an equal sign. Be sure to **write** "reference set," "multiplier," and "product" under the appropriate blanks. **Reference** the following image for an example.



Require students to use the *Equation Cards* to model the equation with the information given in the problem situation.

Select groups to share and explain their equations. **Ensure** that the groups who share have the correct values in the correct places.



Ensure that students recognize that although 14 × 4 would yield the same product; it does not have the same meaning. The equation 14 × 4 = ? would represent 14 *times* instead of four *times* and would not accurately reflect the problem situation.

Read the equation as, "The product is four times as many as 14," or "The product is four times as much as 14." **Require** the students to read the equation as you did aloud as a whole class.

Rewrite the equation so that the total is to the left of the equal sign.

Ask students the following questions. **Require** students to share with a partner before you **discuss** the answers as a whole group.

- Has the equation changed in any way? How do you know?
- Does the equation still represent the problem situation? Why or why not?
- Will the question mark still be the same value? How do you know?
- Would the equation still represent the problem situation if it was "Question mark (missing value) equals 14 times four?" Why or why not?

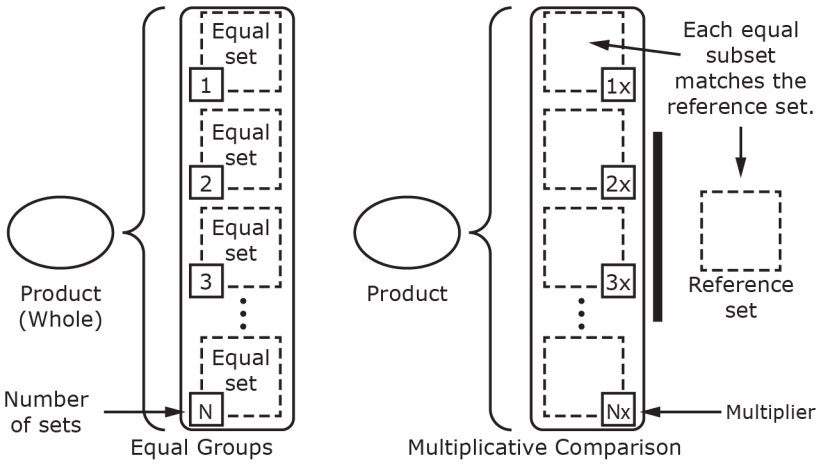
After the discussion, **allow** students sufficient time to solve the equation.

Make note of strategies students/groups use to solve the equation (e.g., different models, standard algorithm, partial products, etc.).

Select students to share and explain their strategies for solving the equation. Choose students with more basic strategies and explanations to share first and students with more efficient strategies and advanced explanations to share after.

Read the equation again, this time including the value of the product in the statement. **Require** the students to read the equation as you did aloud as a whole class.

Display or **share** with students the image from Lesson 1 that compares equal-groups multiplication with multiplicative comparison using models.



Adapted from (Van de Walle, Karp, Lovin, Bay-Williams, 2014, p. 108).

Review how the model relates to an equal-groups multiplication equation. The larger box is the size of each equal set, the smaller box is the number of sets, and the whole or product is the oval off to the side that represents all of the sets combined. The equation set up for equal-groups multiplication is "number of sets times the size of each set equals the whole".

Compare that model and equation set up to the multiplicative-comparison model and equation. The larger box represents the reference set, the smaller box represents the multiplier, and the oval is the product or the combination of the reference set repeated the number of times indicated by the

multiplier. The equation set up for multiplicative comparison is "multiplier times reference set equals the product".

Ask students how the models and equations for both types of multiplication are the same and how they are different. **Use** directed questioning to **ensure** that students draw a connection between the meanings of the first factor in both types of multiplication. **Make known** that the multiplier (multiplicative comparison) and the number of equal-size groups (equal-groups multiplication) essentially represent the same concept: a repetition of an identified set or group.

Share the following problem situation with students.

Cover up or **remove** the numbers from the problem situation to start, so the students focus on the information given in the text and do not just identify the numbers in the problem.

> Bella runs with her dad every Saturday morning. Bella's mom and little sister walk every Saturday morning. Bella and her dad run ■ times as far as her mom and little sister walk. If Bella and her dad run ■miles, how far do her mom and little sister walk?

Ask students what type of problem situation this is and how they know.

Ask students what information is given in the problem situation, requiring students to explain their responses. Students should respond with phrases such as, "miles they run" and "times more miles".

If they do not share what the problem situation is asking, **ask** students what the problem situation is asking. Students should respond what phrases such as, "how many miles Bella's mom and little sister walk".

Write the given information in a visible place (e.g., a large dry erase board, interactive white board, chart paper).

Ask students how this problem is different from and the same as the multiplicative-comparison problem situation they just analyzed. **Require** students to share with a partner before discussing whole group.

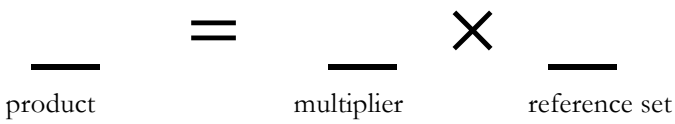
Once students have identified the information given in the problem situation, **share** the problem situation with the numerals visible.

> Bella runs with her dad every Saturday morning. Bella's mom and little sister walk every Saturday morning. Bella and her dad run three times as far as her mom and little sister walk. If Bella and her dad run six miles, how far do her mom and little sister walk**?**

Refer back to where you wrote the information from the problem situation, and ask students to identify the values that go with each piece of information.

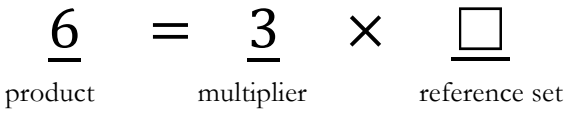
Ask students what the missing piece of information is, and relate this to the question that is being asked in the problem (*how many miles Bella's mom and little sister walk*).

On the same paper or board where you wrote the problem information, set up the equation with three blanks, a multiplication sign, and an equal sign. Be sure to **write** "reference set," "multiplier," and "product" under the appropriate blanks. **Reference** the following image for an example.



Require students to use the *Equation Cards* to model the equation with the information given in the problem situation.

Select groups to share and explain their equations. **Ensure** that the groups who share have the correct values in the correct places.



Ask students how this equation is different from the equation for the other multiplicative-comparison problem situation. Students should identify that the missing information is one of the factors, not the product.

After the discussion, **allow** students sufficient time to solve the equation.

Make note of strategies students/groups use to solve the equation (e.g., different models, division, guess and check, repeated grouping, etc.).

Select students to share and explain their strategies for solving the equation. Choose students with more basic strategies and explanations to share first and students with more efficient strategies and advanced explanations to share after.

Distribute one INSTRUCTIONAL ACTIVITY STUDENT HANDOUT to each student and one set of task cards from pages 11 – 13 the INSTRUCTIONAL ACTIVITY SUPPLEMENT to every two to three students.

Require students to use the *Equation Cards* to model the problem situation on each task card as an equation. Students should select four equations to record along with the task card numbers on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

Tell students that, at the end of the activity, each group will be responsible for explaining and solving one task card for the whole class.

Use the following guiding questions to support students understanding.

GUIDING QUESTIONS

Elicit student thinking:

- Which symbol are you going to use for the unknown value? Why?
- [Point to a task card.] Tell me about this problem situation.
- In an equation, does the unknown value always have to be on the opposite side of the equal sign from the other values in the equation? Why or why not?
- Would you rather use the *Equation Cards* to show an equation or write the equation on paper or a white board? Explain.

Determine if the student can EXPLAIN THE DIFFERENCE BETWEEN ADDITIVE COMPARISON AND MULTIPLICATIVE COMPARISON:

- [Point to a task card.] How do you know this is an addition problem and not a multiplication problem (or vice versa)?
- [Point to task card #2.] How would this problem be different if it was multiplicative comparison instead of additive comparison?
- Does an additive-comparison problem have a multiplier? Explain.
- How do you know if a problem situation is an additive-comparison situation?
- How do you know if a problem situation is a multiplicative-comparison situation?
- [Point to task card #5.] Is this problem situation an additive-comparison situation or a multiplicative-comparison situation? How do you know?
- How is a multiplicative-comparison problem different from an additive-comparison problem? How are they the same? Show me an example.

Determine if the student can EXPLAIN MULTIPLICATIVE RELATIONSHIPS REPRESENTED BY EQUATIONS:

- [Point to a multiplication equation, on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT or using the *Equation Cards*.] What does this equation mean if it represents a multiplicative-comparison situation? How do you know?
- [Point to a multiplication equation, on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT or using the *Equation Cards*.] Which value is the reference set? Which value is the multiplier? How do you know?
- [Reverse the factors in an equation shown with *Equation Cards*.] Would you read the multiplicative-comparison equation the same as or different from it was before I moved the factors? Explain.

Determine if the student can SOLVE MULTIPLICATIVE-COMPARISON PROBLEMS:

- [Point to a task card.] What type of problem is this problem situation? How do you know?
- [Point to a task card or equation.] How did you solve this problem?
- [Point to task card #5.] Show me your equation for this problem situation. How did you solve this problem situation?
- [Point to task card #5.] Do you have to use multiplication to solve this problem situation? How else can you determine how many pages Makayla will have to read?
- [Point to task card #7.] Do you have to use multiplication to solve this problem situation? How else can you determine how many times more minutes Jaxson spends on homework than Trenton?
- [Point to task card #5, #7, #9, or #12.] How did you determine the unknown value in the equation? Is there any other way to determine the unknown value?

At the end of the activity, **assign** each group a task card to present to the whole class. **Require** them to explain the problem situation on the task card, model the equation, and describe how they solved for the unknown value.

Collect and **analyze** the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT for student understanding.

Name_____

Understanding Multiplicative Comparison

Lesson 3

Choose four task cards to use to complete the tables.

1.

Task Card Number	
Restate the question in your own words	
Model	
Equation	
Solution	

Name_____

2.

Task Card Number	
Restate the question in your own words	
Model	
Equation	
Solution	

Name_____

3.

Task Card Number	
Restate the question in your own words	
Model	
Equation	
Solution	

Name_____

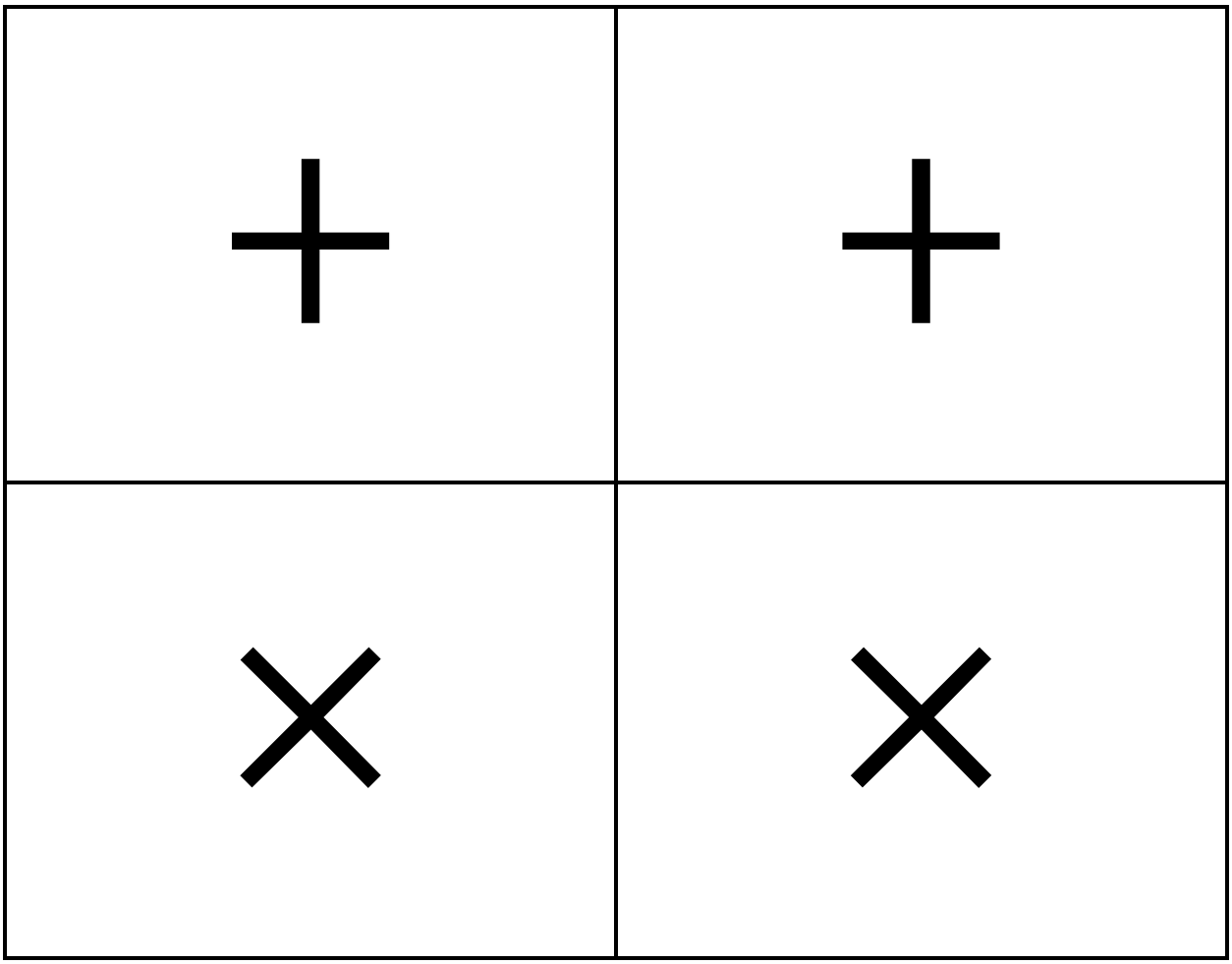
4.

Task Card Number	
Restate the question in your own words	
Model	
Equation	
Solution	

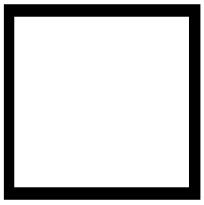
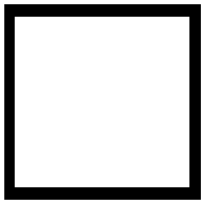
Understanding Multiplicative Comparison

INSTRUCTIONAL ACTIVITY SUPPLEMENT

Lesson 3



÷	÷
=	=
?	?

	
1	1

1	2
2	2

3	3
3	4

4	4
5	5

5	6
6	6

7	7
7	8

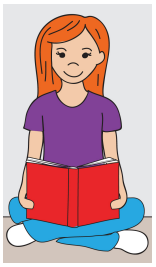
8	8
9	9

9	0
0	0

Task Cards

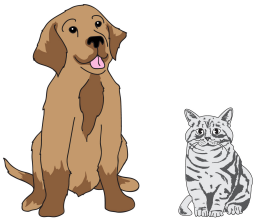
TASK #1

Ms. Smith has 3 students in each of her small groups for reading. If she has 6 small groups, how many students does Ms. Smith meet with for reading?



TASK #2

Every day, Sally gives her dog three treats and her cat two treats. How many more treats does her dog get than her cat every day?

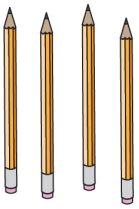


TASK #3

Jackie is getting a new blanket for her bed. The area of her bed is 28 square feet. Jackie wants her blanket to be larger than her bed. The blanket she picks is 6 feet long and 4 feet wide. Will the blanket Jackie picked be larger or smaller than her bed?

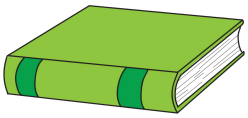
TASK #4

Maria has six sharpened pencils in her desk. Max has two sharpened pencils in his desk. How many more pencils does Maria have than Max?



TASK #5

Makayla did not do all of her reading homework for the week. Braxton did all of his reading homework for the week on time. Braxton read 33 pages, and he read 3 times more than Makayla. How many pages will Makayla have to read to catch up?

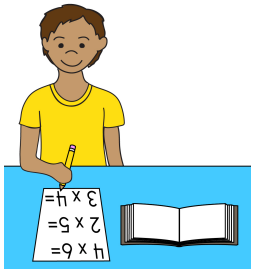


TASK #6

Danny saves $5 every month. How much will Danny have in 4 months if he starts with $0?

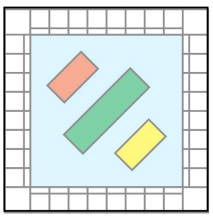
TASK #7

Trenton's older brother Jaxson has 120 minutes of homework a week. Trenton has 40 minutes of homework a week. How many times more does Jaxson spend on homework than Trenton?



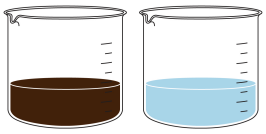
TASK #8

Juan is making a board game for a math project. He is designing the board like a table, divided into rows and columns. If he has seven rows and four columns, how many squares will he have on his game board?



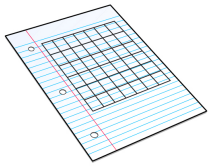
TASK #9

During the science experiment, it took the water 2 times longer to have a reaction than the oil. The oil took 13 minutes. How long did the water take?



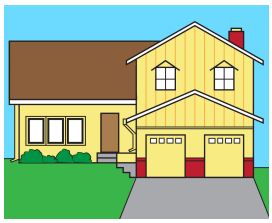
TASK #10

The students recorded their data from the science experiment in a table. There were 3 columns and 12 rows. How many total cells are in the table?



TASK #11

Each window on a house has four pieces of glass. If there are seven windows, how many total pieces of glass make up all the windows?



TASK #12

Seth's grandma is six times older than he is. If Seth is nine, how old is his grandma?

Name_____

Understanding Multiplicative Comparison

Lessons 1 – 3

1. Identify the following problem situations by writing the type under each problem. Explain your answer using words, numbers, and/or models.

Word Bank	
Additive comparison	Equal-groups multiplication
Measurement multiplication	Multiplicative comparison

Use the word bank to help you write the type of problem. Notice that there are more types than there are problems.

1.a. The school student council is having a food drive. So far, the fourth-grade classes combined have brought in three times as many items as the fifth-grade classes. The fourth-grade classes have brought in 132 items. How many items have the fifth-grade classes collected?

Problem type:

I know this because:

Name_____

1.b. So far, Mr. Snow's fourth-grade class has brought in 32 more items for the food drive than Ms. Hart's third-grade class. Ms. Hart's class has brought in 86 items. How many items has Mr. Snow's class brought in for the food drive?

Problem type:

I know this because:

1.c. The student council has to box up all of the items from the food drive so the items can be delivered. They can fit eight items in each box. So far, they have filled twenty-three boxes. How many items have the student council boxed up so far?

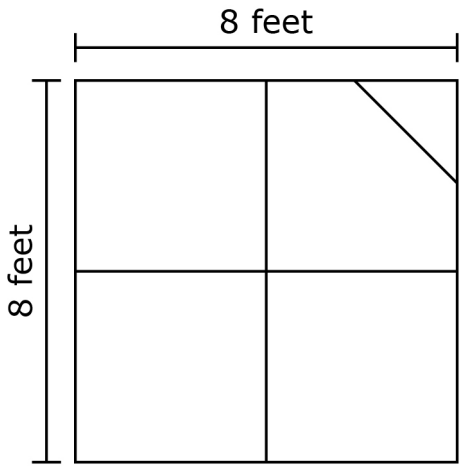
Problem type:

I know this because:

Name_____

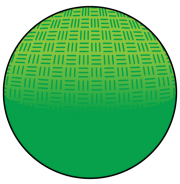
2. Finish each table using the information provided.

2.a.

Problem situation	
Restate the question in your own words	
Model	**Four Square Court** 
Solution	

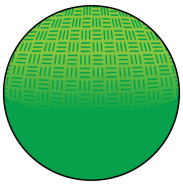
Name_____

2.b.

Problem situation	Ms. Griggs, the P.E. teacher, has 15 playground balls that can be used for kickball or four square. The fourth-grade teachers have 3 playground balls that can be used for kickball or four square. Ms. Griggs has how many times more playground balls than the fourth-grade teachers? 
Restate the question in your own words	
Model	
Solution	

Name_____

2.c.

Problem situation	Ms. Griggs, the P.E. teacher, has 15 playground balls for kickball and four square. There are three different colors: red, blue, and green. There are five green balls and four more blue balls than red balls. If there are three red balls, how many blue balls are there? 
Restate the question in your own words	
Model	
Solution	

Name_____

3. Using the information given, write an equation.

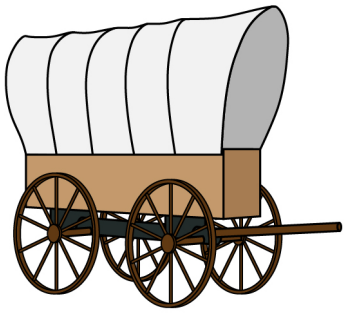
3.a. Forty-two is six times more than seven.

3.b.

3.c. ☐ is 9 times less than 45.

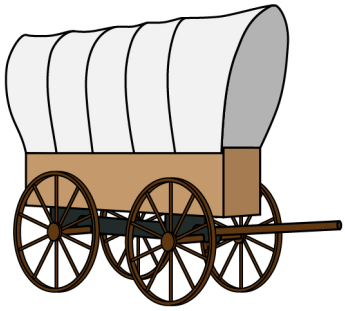
Name_____

4. Pedro and Jayce are creating a digital poster for their social studies assignment on Westward Expansion. They have to have at least 6 facts and 4 pictures. If Pedro finds 2 facts, and Jayce finds 3 facts more than Pedro, will they have enough facts? Show your work using words, models, and/or equations.



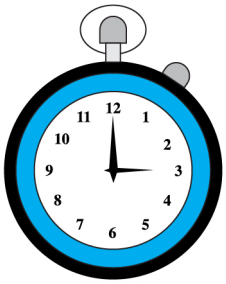
Name_____

5. Harper and Mason are making a video for their social studies assignment on Westward Expansion. They are using four times more pictures than Pedro and Jayce. If Harper and Mason are using twenty pictures, how many pictures are Pedro and Jayce using? Show your work using words, models, and/or equations.



Name_____

6. Mr. Swaim, the social studies teacher, gives the students 10 minutes to present their Westward Expansion projects. Pedro and Jayce take 7 minutes to present. When Harper and Mason timed their video, it was 3 times longer than the presentation by Pedro and Jayce. How much time will Harper and Mason need to present their video in full? Will that amount of time be longer than, shorter than, or the same amount of time Mr. Swaim will allow? Show your work using words, models, and/or equations.



UNDERSTANDING MULTIPLICATIVE COMPARISON

STUDENT ACTIVITY SOLUTION GUIDE

Lessons 1 – 3

1. Identify the following problem situations by writing the type under each problem. Explain your answer using words, numbers, and/or models.

Word Bank	
Additive comparison	Equal-groups multiplication
Measurement multiplication	Multiplicative multiplication

Use the word bank to help you write the type of problem. Notice that there are more types than there are problems.

1.a. The school student council is having a food drive. So far, the fourth-grade classes combined have brought in three times as many items as the fifth-grade classes. The fourth-grade classes have brought in 132 items. How many items have the fifth-grade classes collected?

CORRECT ANSWER

Check the words, numbers, models, and/or equations students provide to explain their thinking. The following is a possible student response.

Problem Type: Multiplicative multiplication

I know this because: the problem says there are three times as many as another group. The other group is the reference group, and the three times is the multiplier. You have to multiply one group by the multiplier to get the number of items for the other group.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Additive comparison	does not understand that the comparison is based on the size of one group that is a multiple of the other group	MATHEMATIZE CONTEXTUAL SITUATIONS INVOLVING MULTIPLICATION and EXPLAIN *ADDITIVE COMPARISON*
Equal-groups multiplication	does not understand that the problem does not involve a specific whole that is divided into group of equal size	MATHEMATIZE CONTEXTUAL SITUATIONS INVOLVING MULTIPLICATION and SOLVE EQUAL-GROUPS PROBLEMS
Measurement multiplication	does not recognize that the problem does not involve measurement such as area or rate	MATHEMATIZE CONTEXTUAL SITUATIONS INVOLVING MULTIPLICATION and SOLVE MEASUREMENT-QUANTITIES PROBLEMS
The student writes *additive comparison*, *equal-groups multiplication*, or *measurement multiplication* but does not provide an explanation.	is unable to explain why the problem type was chosen *or* does not understand the problem situation	MATHEMATIZE CONTEXTUAL SITUATIONS INVOLVING MULTIPLICATION
The student writes *multiplicative comparison* but does not provide an explanation.	is unable to explain why the problem is multiplicative comparison *or* does not understand the problem situation	MATHEMATIZE CONTEXTUAL SITUATIONS INVOLVING MULTIPLICATION and EXPLAIN *MULTIPLICATIVE COMPARISON*

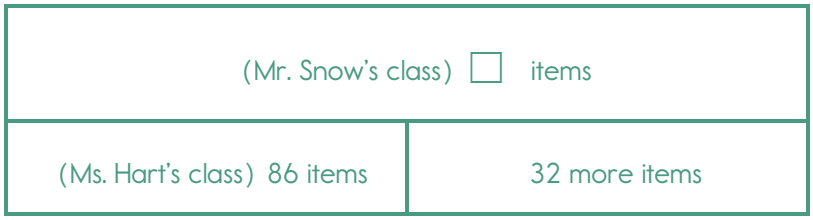
1.b. So far, Mr. Snow's fourth-grade class has brought in 32 more items for the food drive than Ms. Hart's third-grade class. Ms. Hart's class has brought in 86 items. How many items has Mr. Snow's class brought in for the food drive?

CORRECT ANSWER

Check the words, numbers, models, and/or equations students provide to explain their thinking. The following is a possible student response.

Problem type: Additive comparison

I know this because:



ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Equal-groups multiplication	does not understand that the problem does not involve a specific whole that is divided into groups of equal size	MATHEMATIZE CONTEXTUAL SITUATIONS INVOLVING MULTIPLICATION and SOLVE EQUAL-GROUPS PROBLEMS
Measurement multiplication	does not recognize that the problem does not involve measurement such as area or rate	MATHEMATIZE CONTEXTUAL SITUATIONS INVOLVING MULTIPLICATION and SOLVE MEASUREMENT-QUANTITIES PROBLEMS
Multiplicative comparison	does not understand that the comparison is based on the difference between the sizes of the two group	MATHEMATIZE CONTEXTUAL SITUATIONS INVOLVING MULTIPLICATION and EXPLAIN *MULTIPLICATIVE COMPARISON*
The student writes *equal-groups multiplication*, *measurement multiplication*, or *multiplicative comparison* but does not provide an explanation.	is unable to explain why the problem type was chosen *or* does not understand the problem situation	MATHEMATIZE CONTEXTUAL SITUATIONS INVOLVING MULTIPLICATION
The student writes *additive comparison* but does not provide an explanation.	is unable to explain why the problem is additive comparison *or* does not understand the problem situation	MATHEMATIZE CONTEXTUAL SITUATIONS INVOLVING MULTIPLICATION and EXPLAIN *ADDITIVE COMPARISON*

1.c. The student council has to box up all of the items from the food drive so the items can be delivered. They can fit eight items in each box. So far, they have filled twenty-three boxes. How many items have the student council boxed up so far?

CORRECT ANSWER

Check the words, numbers, models, and/or equations students provide to explain their thinking. The following is a possible student response.

Problem type: Equal-groups multiplication

I know this because: each box is a group, and there are eight items in each box.

$23 \times 8 = \square$

number of groups × size of each group = $\square$

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

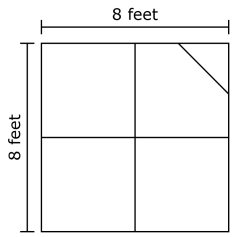
Example Error	Misconception	Missing Knowledge
Additive comparison	does not understand that the problem does not involve a comparison that is based on the size of one group that is a multiple of the other group	MATHEMATIZE CONTEXTUAL SITUATIONS INVOLVING MULTIPLICATION and EXPLAIN *ADDITIVE COMPARISON*
Measurement multiplication	does not recognize that the problem does not involve measurement such as area or rate	MATHEMATIZE CONTEXTUAL SITUATIONS INVOLVING MULTIPLICATION and SOLVE MEASUREMENT-QUANTITIES PROBLEMS
Multiplicative comparison	does not understand that the problem does not involve a comparison that is based on the difference between the sizes of the two groups	MATHEMATIZE CONTEXTUAL SITUATIONS INVOLVING MULTIPLICATION and EXPLAIN *MULTIPLICATIVE COMPARISON*
The student writes *additive comparison*, *measurement multiplication*, or *multiplicative comparison* but does not provide an explanation.	is unable to explain why the problem type was chosen *or* does not understand the problem situation	MATHEMATIZE CONTEXTUAL SITUATIONS INVOLVING MULTIPLICATION
The student writes *equal-groups multiplication* but does not provide an explanation.	is unable to explain why the problem is equal-groups multiplication *or* does not understand the problem situation	MATHEMATIZE CONTEXTUAL SITUATIONS INVOLVING MULTIPLICATION and EXPLAIN *MULTIPLICATION* or SOLVE EQUAL-GROUPS PROBLEMS

2. Finish each table using the information provided.

2.a.

CORRECT ANSWER

Check the words, numbers, and models that students provide to complete the table. The following is a possible student response.

Problem situation	Jimmy, Johnny, Sue, Nancy, and David like to play four square during recess. The four-square court is four feet long and four feet wide. What is the area of the four square court?
Restate the question in your own words	The problem situation is asking for the area of the four-square court (the size of the four square court).
Model	**Four Square Court** 
Solution	The area of the four-square court is 64 square feet.

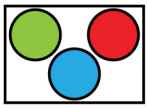
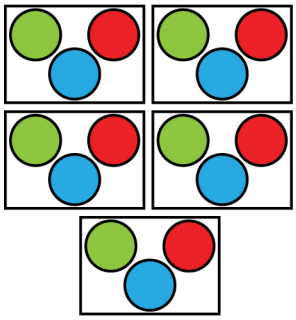
ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student does not provide a problem situation. *or* The problem situation does not represent the area model.	does not understand how to interpret an area model and/or provide a situation that can provide context to the model	EXPLAIN MULTIPLICATION PROBLEMS and/or MATHEMATIZE CONTEXTUAL SITUATIONS INVOLVING MULTIPLICATION
The student leaves the problem expalanation section empty. *or* The explanation does not match the problem situation and/or the model.	is unaware of what the problem situation means	MATHEMATIZE CONTEXTUAL SITUATIONS INVOLVING MULTIPLICATION
The student does not provide a product. *or* The student provides a product other than 64 square feet.	does not understand the area model	SOLVE REAL-WORLD PROBLEMS INVOLVING AREA OF RECTANGLES
The student provides a product other than 64 square feet.	does not correctly multiply 8 × 8	CALCULATE PRODUCTS USING STRATEGIES OTHER THAN THE STANDARD ALGORITHM and MULTIPLY BY 8

2.b.

CORRECT ANSWER

Check the words, numbers, and models that students provide to complete the table. The following is a possible student response.

Problem situation	Ms. Griggs, the P.E. teacher, has 15 playground balls that can be used for kickball or four square. The fourth-grade teachers have three playground balls that can be used for kickball or four square. Ms. Griggs has how many times more playground balls than the fourth-grade teachers? 
Restate the question in your own words	How many times more playground balls does Ms. Griggs have compared to the fourth-grade teachers?
Model	fourth-grade teacher's playground balls  Ms. Griggs' playground balls 
Solution	Ms. Griggs has five times more playground balls than the fourth-grade teachers.

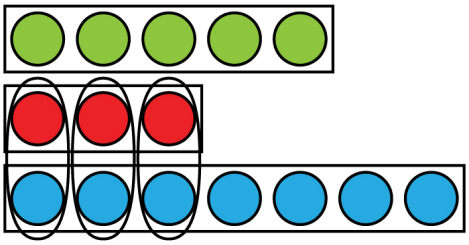
ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student leaves the problem expalanation section empty. *or* The explanation does not match the problem situation.	is unaware of what the problem situation means	MATHEMATIZE CONTEXTUAL SITUATIONS INVOLVING MULTIPLICATION
The student does not provide a model. *or* The model does not accurately represent the problem situation.	does not understand how to represent the multiplicative-comparison problem situation as a model	MATHEMATIZE CONTEXTUAL SITUATIONS INVOLVING MULTIPLICATION and CALCULATE PRODUCTS USING STRATEGIES OTHER THAN THE STANDARD ALGORITHM
The student does not provide a solution. *or* The student provides a solution other than *five times more*.	does not understand the problem situation or multiplicative comparison	SOLVE MULTIPLICATIVE-COMPARISON PROBLEMS
The student provides a solution other than *five times more*.	does not correctly solve $\square \times 3 = 15$ or $15 \div 3 = \square$	MULTIPLY BY 3 or DIVIDE BY 3

2.c.

CORRECT ANSWER

> *Check the words, numbers, and models that students provide to complete the table. The following is a possible student response.*

Problem situation	Ms. Griggs, the P.E. teacher, has 15 playground balls for kickball and four square. There are three different colors: red, blue, and green. There are five green balls and four more blue balls than red balls. If there are three red balls, how many blue balls are there?
Restate the question in your own words	How many blue playground balls does Ms. Griggs have?
Model	
Solution	Ms. Griggs has seven blue playground balls.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student leaves the problem expalanation section empty. *or* The explanation does not match the problem situation.	is unaware of what the problem situation means	MATHEMATIZE CONTEXTUAL SITUATIONS INVOLVING ADDITION
The student does not provide a model. *or* The model does not accurately represent the problem situation.	does not understand how to represent the additive-comparison problem situation as a model	MATHEMATIZE CONTEXTUAL SITUATIONS INVOLVING ADDITION and EXPLAIN *ADDITIVE COMPARISON*
The student does not provide a solution. *or* The student provides the solution *three blue playground balls*; does not add the four more balls to the same number of red and blue playground balls.	does not understand the problem situation or multiplicative comparison	SOLVE COMPARE PROBLEMS and EXPLAIN *ADDITIVE COMPARISON*
The student provides a solution other than *seven blue playground balls*.	does not correctly solve the addition	ADD WITHIN 20

3. Using the information given, write an equation.

3.a. Forty-two is six times more than seven.

CORRECT ANSWER

$42 = 6 \times 7$
or
$6 \times 7 = 42$

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
$42 = 6 + 7$	does not understand the difference between additive comparison and multiplicative comparison	EXPLAIN MULTIPLICATIVE RELATIONSHIPS REPRESENTED BY EQUATIONS and/or EXPLAIN *ADDITIVE COMPARISON*
$42 = 7 \times 6$ *or* $7 \times 6 = 42$	does not understand that, in a multiplication problem, the first factor represents the number of groups (or multiplier), and the second factor represents the number of elements in each group (or the size of the reference set)	EXPLAIN *MULTIPLICATION PROBLEMS*

3.b. 

CORRECT ANSWER

$4 \times 8 = 32$
or
$32 = 4 \times 8$

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
8 + 4	does not understand the difference between additive comparison and multiplicative comparison	EXPLAIN *MULTIPLICATIVE COMPARISON* and/or EXPLAIN *ADDITIVE COMPARISON*
8 + 8 + 8 + 8 = 32 *(Note: this is correct work and could be interperted from the model, however the focus on the unit is on multiplicative comparsion, therefore this answer would be incorrect.)*	does not understand the difference between repeated addition and multiplicative comparison	EXPLAIN *MULTIPLICATIVE COMPARISON* and REPRESENT MULTIPLICATION WITH EQUATIONS
8 × 4 = 32 *or* 32 = 8 × 4	does not understand that, in a multiplication problem, the first factor represents the number of groups (or multiplier), and the second factor represents the number of elements in each group (or the size of the reference set)	EXPLAIN *MULTIPLICATION PROBLEMS*

3.c. ☐ is 9 times less than 45.

CORRECT ANSWER

Any of the following equations would be correct.

9 × ☐ = 45 45 = 9 × ☐

45 ÷ 9 = ☐ 45 ÷ ☐ = 9

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
45 – 9 *or* 45 – ☐ *or* 9 – ☐	does not understand multiplicative comparison and interprets the phrase *less than* to mean subtraction	EXPLAIN *MULTIPLICATIVE COMPARISON* and/or MATHEMATIZE CONTEXTUAL SITUATIONS INVOLVING QUANTITY
☐ × 9 = 45 *or* 45 = ☐ × 9	does not understand that, in a multiplication problem, the first factor represents the number of groups (or multiplier), and the second factor represents the number of elements in each group (or the size of the reference set)	EXPLAIN *MULTIPLICATION PROBLEMS*

4. Pedro and Jayce are creating a digital poster for their social studies assignment on Westward Expansion. They have to have at least 6 facts and 4 pictures. If Pedro finds 2 facts, and Jayce finds 3 more facts than Pedro, will they have enough facts? Show your work using words, models, and/or equations.

CORRECT ANSWER

> Pedro and Jayce will have enough facts, they will have seven facts altogether.
> Pedro's facts: 2
> Jayce's facts: 2 + 3 = 5
> Both boy's facts: 2 + 5 = 7
> 6 < 7

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
2 + 3 = 5	interprets Jayce's number of facts as three and adds the three to Pedro's two facts for a total of five facts	MATHEMATIZE CONTEXTUAL SITUATIONS INVOLVING ADDITION and SOLVE 2-STEP ADDITION AND SUBTRACTION WORD PROBLEMS
3 × 2 = 6 total facts *or* 3 × 2 = 6, 6 + 2= 8 total facts	interprets the problem situation as a multiplicative-comparison problem and identifies the difference as the multiplier	MATHEMATIZE CONTEXTUAL SITUATIONS INVOLVING MULTIPLICATION and EXPLAIN THE DIFFERENCE BETWEEN ADDITIVE COMPARISON AND MULTIPLICATIVE COMPARISON
The student performs any operation including the values 6 and 4.	does not understand the context of the problem situation	MATHEMATIZE CONTEXTUAL SITUATIONS INVOLVING ADDITION

5. Harper and Mason are making a video for their social studies assignment on Westward Expansion. They are using four times more pictures than Pedro and Jayce. If Harper and Mason are using twenty pictures, how many pictures are Pedro and Jayce using? Show your work using words, models, and/or equations.

CORRECT ANSWER

Pedro and Jayce are using five pictures.

$20 = 4 \times \square$

$20 = 4 \times 5$

$\square = 5$

Note: The student can use a variety of strategies to solve for the unknown in the equation, e.g., division, multiplication, repeated addition, etc.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
$20 + 4$ *or* $20 - 4$	does not understand multiplicative comparison and interprets the phrase *more* to mean additive comparison	EXPLAIN THE DIFFERENCE BETWEEN ADDITIVE COMPARISON AND MULTIPLICATIVE COMPARISON and MATHEMATIZE CONTEXTUAL SITUATIONS INVOLVING MULTIPLICATION
Pedro and Jayce are using 80 pictures. $4 \times 20 = 80$	does not understand what the question is asking, assumes the two values need to be multiplied *or* identifies the situation as a multiplicative-comparison problem but does not recognize what the missing value is	MATHEMATIZE CONTEXTUAL SITUATION INVOLVING MULTIPLICATION and SOLVE MULTIPLICATIVE-COMPARISON PROBLEMS
$\square \times 5 = 20$ $\square = 4$ $4 \times 5 = 20$ *(Note: the computation is correct, however the set up of the equation does not relfect the problem situation: multiplier times size of reference set equals product.)*	does not understand that, in a multiplication problem, the first factor represents the number of groups (or multiplier), and the second factor represents the number of elements in each group (or the size of the reference set)	EXPLAIN *MULTIPLICATION PROBLEMS*

6. Mr. Swaim, the social studies teacher, gives the students 10 minutes to present their Westward Expansion projects. Pedro and Jayce take 7 minutes to present. When Harper and Mason timed their video, it was 3 times longer than the presentation by Pedro and Jayce. How much time will Harper and Mason need to present their video in full? Will that amount of time be longer than, shorter than, or the same amount of time Mr. Swaim will allow? Show your work using words, models, and/or equations.

CORRECT ANSWER

Harper and Mason's video is 21 minutes, which is longer than the time Mr. Swaim allows.
$3 \times 7 = \square$
$3 \times 7 = 21$
21 minutes > 10 minutes

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Harper and Mason's video is 10 minutes long, which is the same amount of time Mr. Swaim allows. 3 + 7 = 10 *or* 7 + 3 = 10 10 minutes = 10 minutes	does not understand multiplicative comparison and interprets the phrase *longer* to mean additive comparison	EXPLAIN THE DIFFERENCE BETWEEN ADDITIVE COMPARISON AND MULTIPLICATIVE COMPARISON and MATHEMATIZE CONTEXTUAL SITUATIONS INVOLVING MULTIPLICATION
Harper and Mason's video is four minutes long, which is less than the amount of time Mr. Swaim allows. 7 – 3 = 4 10 minutes > 4 minutes	does not understand multiplicative comparison, interprets the phrase *longer* to mean additive comparison and subtracts the two values	EXPLAIN THE DIFFERENCE BETWEEN ADDITIVE COMPARISON AND MULTIPLICATIVE COMPARISON and MATHEMATIZE CONTEXTUAL SITUATIONS INVOLVING MULTIPLICATION
The student performs any operation including the value 10.	does not understand the context of the problem situation	MATHEMATIZE CONTEXTUAL SITUATIONS INVOLVING MULTIPLICATION
$7 \times 3 = \square$ 7 × 3 = 21 *(Note: the computation is correct, however the set up of the equation does not relfect the problem situation: multiplier times size of reference set equals product.)*	does not understand that, in a multiplication problem, the first factor represents the number of groups (or multiplier), and the second factor represents the number of elements in each group (or the size of the reference set)	EXPLAIN *MULTIPLICATION PROBLEMS*