PARTITIONING SHAPES AND UNIT FRACTIONS

3.GM.A.3

CONTENTS

The types of documents contained in the unit are listed below. Throughout the unit, the documents are arranged by lesson.

LEARNING MAP INFORMATION	An overview of the standards, the learning map section, and the nodes addressed in this unit
TEACHER NOTES	A brief discussion describing the progression depicted in the learning map section with research-based recommendations for focusing instruction to foster student learning and an introduction to the unit's lessons
INSTRUCTIONAL ACTIVITY	A detailed walkthrough of the unit
INSTRUCTIONAL ACTIVITY STUDENT HANDOUT	A handout for the guided activity, intended to be paired with the Instructional Activity
INSTRUCTIONAL ACTIVITY SUPPLEMENT	A collection of materials or activities related to the Instructional Activity
STUDENT ACTIVITY	A work-alone activity for students
STUDENT ACTIVITY SOLUTION GUIDE	A solution guide for the work-alone activity with example errors, misconceptions, and links to the learning map section

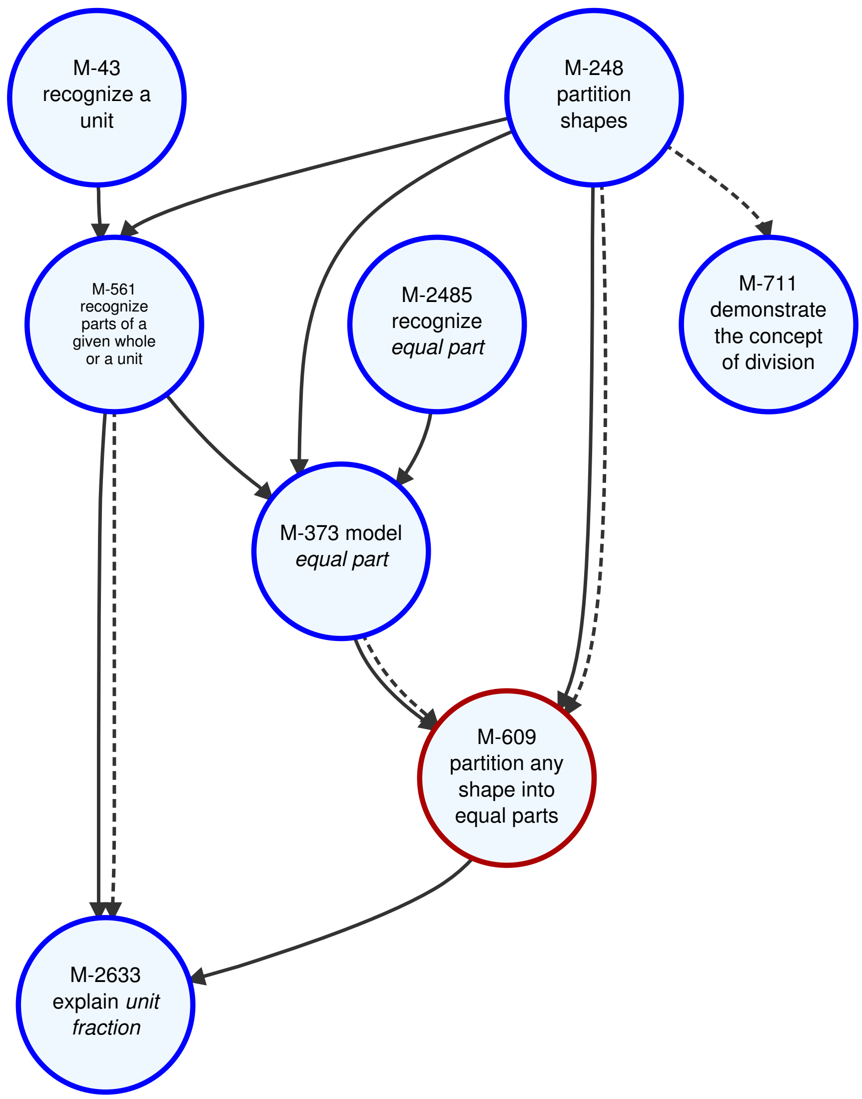
Enhanced Learning Maps developed these materials under a grant from the Department of Education, PR/Award # S368A150013. Contents do not necessarily represent the policy of the Department of Education, and you should not assume endorsement by the Federal Government.

Partitioning Shapes and Unit Fractions

LEARNING MAP INFORMATION

STANDARDS

3.GM.A.3 Partition shapes into parts with equal areas, and express the area of each part as a unit fraction of the whole.



**Learning map model of* 3.G.2

Node Name	Node Description
DEMONSTRATE THE CONCEPT OF DIVISION	Demonstrate division by splitting a set into a number of fair shares or equal sets.
EXPLAIN *UNIT FRACTION*	Make known your understanding that a unit fraction, $\frac{1}{n}$, is the quantity formed by one part when a whole is partitioned into n equal parts.
MODEL *EQUAL PART*	Model equal size parts for continuous amounts.
PARTITION ANY SHAPE INTO EQUAL PARTS	Divide any shape into a specified number of equal parts. For example, when asked to partition a shape into sixths (or six equal parts), the student creates six equal-size parts.
PARTITION SHAPES	Divide or separate a shape into two or more shapes. For example, divide a rectangle into two triangles.
RECOGNIZE A UNIT	Recognize a unit as a group of countable objects that make up a whole.
RECOGNIZE *EQUAL PART*	Identify or name equal parts of a whole. Know that equal parts of a whole do not need to have the same shape, but that they must have the same area.
RECOGNIZE PARTS OF A GIVEN WHOLE OR A UNIT	Make known your understanding that a part-whole relationship indicates that one unit or whole is made up of one or more parts.

Partitioning Shapes and Unit Fractions

TEACHER NOTES

This unit includes the following documents:

- Learning Map Information
- Instructional Activity (three lessons)
- Instructional Activity Student Handout (for Lessons 2 and 3)
- Instructional Activity Supplement (for Lessons 1 – 3)
- Student Activity
- Student Activity Solution Guide

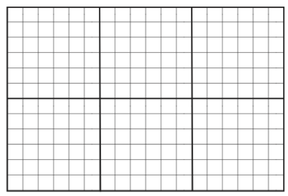
In this unit, students will learn to partition shapes into equal-size parts, as well as identify whether shapes have been partitioned into equal-size parts. In addition, students will connect their understanding of equal-size partitions with unit fractions.

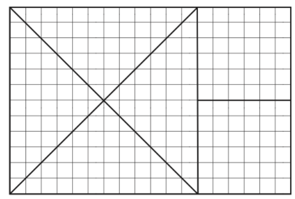
RESEARCH

Many meaningful experiences with equipartitioning and equipartitioned shapes is integral to creating a significant foundation for fraction understanding (Pothier & Swada, 1990). It is important to note the difference between partitioning and equipartitioning: partitioning refers to *any* division of groups or *any*-size parts of a continuous whole, while equipartitioning refers to *equal*-size groups or *equal*-size parts of a continuous whole (Confrey & Maloney, 2010). Students should have prior experience with partitioning shapes, although in their early experiences with partitioning, students will likely identify any subdivision of a shape without regard to the size of the subdivisions. Students should be exposed to shapes that have been partitioned into different-size and equal-size parts. According to Small (2014), students should be asked to distinguish shapes that have been divided into equal-size parts from those that have not. Differentiating between shapes that have been partitioned into parts of equal size and those that have not will support students' visualization of equal or "fair" shares. "Fair" or equal sharing situations help students visualize and comprehend equipartitioning. These situations give students an opportunity to draw on previous personal experiences (Lewis, Gibbons, Kazemi, Lind, 2015). The concept of "fair" sharing helps students recognize the necessity of having equal-size parts (Empson, 2002). In order for students to determine or create parts of equal size, they must be familiar with the concept of area.

AN EXAMPLE

The following images illustrate partitions that are the same size but different shapes. Students' understanding of counting up the squares that make up two or more partitions helps them grasp how areas that do not necessarily look the same, or congruent, can actually be equal in size. This understanding of area supports students' ability to identify and to create equipartitioned shapes.





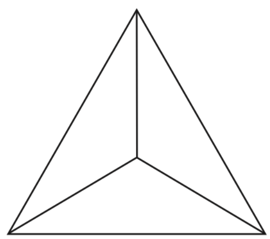
A misconception among students is that parts of a partitioned shape must be the same size and the same shape (Petit, Laird, Marsen, & Ebby, 2015). Students who have a clear understanding of area as the amount of surface within a border will have an easier time accepting that equally sized parts may not always have the same shape, because they can examine two shapes and consider the number of unit squares each contains to evaluate their relative sizes. When students are considering whether a whole is equipartitioned, it is helpful for students to have previous experiences with area. An understanding of the concept of area will support their identification of equal-size parts, especially if those parts are not the same shape. Lewis and colleagues (2015) state that "fractional pieces must be equivalent but do not have to be congruent" (p. 164). Therefore, students should be capable of identifying and eventually creating equal-size partitions regardless of the shape of the parts.

Without sufficient understanding of the equivalence of parts, students become vulnerable to common difficulties in using fraction models. Specifically, Petit et al. (2015) state that some students only consider the number of parts and not the size of the parts when considering fraction models. This could be due to the fact that fraction area models are often only represented as circles or rectangles, and/or that students often only work with pre-partitioned models, which have been partitioned into congruent parts. It is recommended that students' experiences involve different visual models, including differently shaped continuous wholes (e.g., hexagons), number lines, and sets, and that equipartitioning is not implemented in a rote way, which leads students to become dependent on pre-partitioned wholes (Petit et al., 2015). The use of pre-partitioned wholes, and eventually student-partitioned wholes, should consistently be tied to the mathematical concept of equal-area parts, equal-size divisions on a number line, or equal-size sets in a set model (Petit et al., 2015). Yoshida and Swano (2002) found that students gain a stronger understanding of fractions when equal partitioning is taught prior to or in conjunction with traditional fraction content.

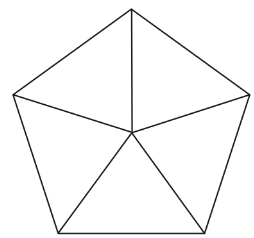
Not only should students be exposed to fractional parts of different shapes, but students should also have experiences with wholes represented by different shapes. Students are most often, or in some cases only, exposed to circles and rectangles, because those shapes are most commonly used in textbooks (Pothier & Sawada, 1990). Pothier and Sawada (1990) write that other shapes are beneficial to "represent particular denominate numbers" (p. 13). For example, an equilateral triangle is useful when representing thirds, a square is beneficial when representing fourths, and so on. Hexagons make nice wholes because they are available as patterns blocks, and they are easily replaced by six equilateral triangle blocks or two trapezoid blocks or three parallelogram blocks. The following example displays images of partitions for specific denominate numbers.

AN EXAMPLE

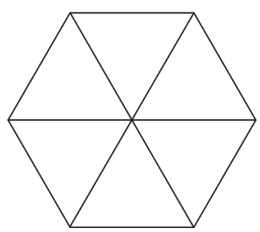
Pothier & Sawada (1990) call attention to the idea that regular polygons are beneficial to model the same number of equal-size partitions as there are the number of sides.



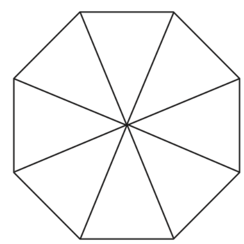
equilateral triangle in thirds



pentagon in fifths



hexagon in sixths



octagon in eighths

When recognizing equal-size partitions or when partitioning shapes into specific numbers of equal-size parts, it is important to consider that students are typically familiar with the notion of "halves" before entering school. This understanding can then be extended to notions of equal sharing and more complex partitions. Allow students to participate in productive struggle as they work to create equal-size parts. As students become more familiar with creating equipartitions such as halves, thirds, and fourths, they will be more proficient at later creating equipartitions such as sixths, eighth, ninths, and tenths. These experiences support students' later work with fractions modeled as continuous wholes and number lines. In particular, experiences partitioning into tenths will benefit students when they begin to think about decimal values on the number line.

Much like partitioning into equal-size parts, students' experiences should also follow a progression to foster conceptual understanding. Students should progress from hands-on experiences with partitioning to identifying visual representations of partitions, and finally, students should draw line segments in order to partition images of shapes (Small, 2014 and Olson, 2000). In addition to identifying equally sized partitions, students should also be able to identify and explain when partitions are not the same size and identify the number of equal-size partitions from a partially partitioned shape or a given part. This variety of partitioning experiences has a later benefit as students use a number line to represent parts and wholes.

Presenting students with partially partitioned shapes or a single part of a whole will help to reveal a students' conceptual understanding of equipartitioning or equivalent shares (Van de Walle, Karp, Lovin, & Bay-Williams, 2014). Students with a strong understanding of equipartitioning will be able to identify a whole from a single partition or identify the number of equivalent shares in a partially divided continuous whole. To demonstrate this understanding, the student must first identify that the given partition or partially divided continuous whole is incomplete. Then, the student must consider what portion of the partially divided continuous whole is represented, or imagine what the whole must look like when given a single partition. Lastly, the student should represent or describe the completely equipartitioned whole.

It is significant to note that manipulatives and images do not "speak" for themselves; a student does not inherently understand partitioned shapes or unit fractions from looking at or manipulating a model (NCTM, 2002). In addition, Lewis et al. (2015) state that fraction vocabulary and notation are not intuitive, meaning it is difficult for students to transition from a whole number mindset to a fractional understanding. Therefore, NCTM (2002) asserts that it is essential to associate clear and concise language with partitioned shapes and fractional models. Students should often be reminded or asked for the type of part (e.g., thirds), how many there are, and what the referent whole is. In addition, Small (2014) states the importance of presenting students with word representations of partitions and fractions before symbols; this emphasizes the concept of a fraction as one number, not a two-part number or two numbers separated by a fraction bar. NCTM (2002) reinforces this recommendation by writing that vocabulary has a powerful impact on students' conceptualization of fractions, and that teachers should avoid using language that encourages students to think of fractions as two-part numbers (e.g., "What is that piece called?" and "Three out of five equal pieces") (NCTM, 2002). Instead, teachers should utilize questioning that emphasizes the part-whole relationship of fractions, such as, "How big is that piece?", "How much brownie does one person get?", and "How many of that piece would fit into the whole?" (NCTM, 2002).

According to van Hiele (1999) students should have ample time exploring and creating with shapes, as well as time to share those creations. Because students' understanding is influenced by past and present circumstances, students should have the opportunity and time to implement and explore their own strategies (Pothier & Sawada, 1990). When students share their strategies with others, they participate in a diversity of ideas and strategies. These strategies also provide the foundation for students' understanding and allows a teacher to determine where to begin and where to go with student interactions (NCTM, 2002). The lessons associated with this unit all allow for using and sharing student-generated strategies.

LEARNING MAP INFORMATION

The learning map section for this sequence of activities begins with the understanding of units and recognizing that multiple parts combine to make up a whole. In addition, students should begin with a general understanding of partitioning shapes. As learning progresses, students should recognize and model equal parts as well as be aware that parts of a whole must be the same size but not necessarily the same shape. Students should also connect their understanding of equipartitioning with the concept of division. From there, the progression leads to the ability to partition any shape into a specified number of equal-size parts, as well as the recognition that one of those parts represents a unit fraction.

INSTRUCTIONAL ACTIVITIES

The activities in this unit are designed to encourage students to employ their own strategies in order to partition shapes into equal-size parts. Several problems are presented in the context of real world situations; even though solving word problems is not a focus of the standard/unit, it is important that students experience partitioning in the context of real-world situations. Therefore, scaffolding for understanding the context of what the question is asking may be necessary to help students work through the task.

In Lesson 1, the focus is on partitioning with the use of hands-on manipulatives such as geoboards, graph paper, pattern shapes, fraction circles, or paper shapes for folding or cutting. The lesson centers around partitioning shapes into equal-size halves, thirds, fourths, sixths, and eighths. Students will experience these partitioning activities in the context of "fair" sharing situations.

Lesson 2 transitions understanding from hands-on experiences to analyzing images of partitioned shapes. When provided an image of a partitioned or partially partitioned shape, students will determine the number of equal-size parts. If the shape is partially partitioned, students will finish partitioning the whole and then color or shade one part to identify the unit fraction for the partitioned shape. Next, in groups of two, students will identify whether images of shapes are partitioned into parts of equal size or different size. With their partners, students will participate in an activity using a set of cards with images of partitioned shapes and a mat that is divided into three columns: "Equal-size Parts", "Not Sure", and "Different-Size Parts." Partners must collaborate in order to determine which cards should be placed in each column.

Lastly, in Lesson 3, students will draw line segments to partition images of shapes. First, students will partition shapes in order to respond to a problem situation. Then, partners will receive a bag with paper strips that have unit fraction names in word form. The partners will select a paper strip from the bag, determine which shape to partition based on the unit fraction, and then model the unit fraction by partitioning and coloring the selected shape on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

REFERENCES

Empson, S. B. (2002). Organizing diversity in early fraction thinking. *Making sense of fractions, ratios, and proportions*, 29-40.

Lewis, R., Gibbons, L., Kazemi, E., Lind, T. (2015). Unwrapping students' idea about fractions. *Teaching Children Mathematics*, 22(3), 158-168.

Olson, M. (2000). No pegs allowed. *Teaching Children Mathematics*, 7(2), 94-100.

Petit, M., Laird, R., Marsden, E., Ebby, C. (2015). *A focus on fractions: Bringing research to the classroom* (2nd ed.). New York, NY: Routledge.

Pothier, Y., & Sawada, D. Partitioning: An approach to fractions. *The Arithmetic Teacher*, 38(4), 12-16.

Small, M. (2014). *Uncomplicating fractions to meet common core standards in math, K – 7*. New York, NY: Teachers College Press.

van Hiele, P. (1999). Developing geometric thinking through activities that begin with play. *Teaching Children Mathematics*, 5(6), 310-316.

Yoshida, H., & Sawano, K. (2002). Overcoming cognitive obstacles in learning fractions: Equal-partitioning and equal-whole. *Japanese Psychological Research*, 44(4), 183-195.

Partitioning Shapes and Unit Fractions

INSTRUCTIONAL ACTIVITY

Lesson 1

LEARNING GOAL

Students will use hands-on manipulatives to partition shapes into halves, fourths, and sixths and identify one part of the whole shape as a unit fraction.

PRIMARY ACTIVITY

Students will use fraction circles, geoboards, pattern blocks, paper shapes with paper strips, and/or paper shapes for folding to partition shapes into halves, fourths, and sixths. Students will also view and explain why some shapes have been partitioned incorrectly.

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

- Rectangle
- Parallelogram
- Circle
- Triangle
- Hexagon
- Partition
- Area
- Half (halves)
- Third(s)
- Fourth(s)
- Sixth(s)
- Whole
- Unit fraction
- Equal shares

MATERIALS

- A variety of manipulatives for students to choose from. The following are some examples:
  - Fraction circles
  - Paper shapes (Refer to the Instructional Activity Supplement.)
  - Geoboards and rubber bands
  - Pattern shapes
  - Thin strips of paper, kebab sticks, or straws

IMPLEMENTATION

Review area by asking students the following questions.

Allow students a minute or so first to discuss their answers with a partner next to them before selecting students to share with the class.

- What is area?
- [Display a rectangle.] Would someone be able to measure the area of this rectangle? Why or why not?
- [Display a circle.] Would someone be able to measure the area of this shape? Why or why not?
- [Display a three-dimensional object such as a ball or a rectangular shoe box.] Would someone be able to measure the area of this? Why or why not?
- [Display a large rectangle and a small rectangle.] Do these two rectangles have the same area? Why or why not?
- [Display a large circle and a small circle.] Do these two circles have the same area? Why or why not?

Display the smaller circle and suggest the students think of the circle as one whole.

Collectively **create** a list of what the circle could represent (e.g., cookie, cake, pie, pizza, etc.).

Note that the text uses the example of pizzas. However, you may opt to use a different example from the list you created as a class.

Explain that if the circle in your hand were a pizza, you would have one whole pizza.

Display the second, larger circle in addition to the smaller circle. **Explain** that both circles represent pizzas.

Ask students the following questions:

- Do these “pizzas” have the same area? How do you know?
- If I were to share one pizza with you, which pizza would you want? Why?

- If you had one pizza and I had the other, would we have equal-size pizzas (or pizzas with the same area)? How do you know?
- [Put down one circle so that you are holding only one circle.] Now, if I were to share with you, would you still get a whole pizza? Why or why not?
- How much of the pizza would you get? Explain.
- [Fold the circle so that there are not equal-size parts. Point to the smaller part.] If I gave you this part of the pizza, would we have equal-size pieces (or pieces with the same area)? How do you know?

Explain that students will work independently to solve a problem situation using a manipulative of their choice. **Display** and **explain** each of the manipulatives that are available for use.

Display and **read** the following problem situation.

> Judith has three extra cookies in her lunch to share with her friends Sam and Petra. How many cookies will Sam and Petra get if Judith gives them all of the cookies?

Display the problem situation somewhere prominent like an interactive whiteboard or a large dry erase board so that students can refer to the problem as they work.

> NOTE: The problem situation can also be typed up and printed out to distribute to the students. However, the goal is for the students to solve the problem situation using the manipulatives—not to draw or complete any computations.

Require students to solve the problem situation using their chosen manipulative.

Allow students to explore and attempt the problem on their own—do not explain the problem situation or correct any student errors yet. This will give you an idea of what the students do and do not understand.

Use the following guiding questions to support student understanding.

GUIDING QUESTIONS

Elicit student thinking:

- What shape could you use to solve this problem? Why?
- Have you ever had to share a whole with someone? Explain.

Determine if the student can RECOGNIZE A UNIT:

- What is the whole that is being divided between Sam and Petra? How do you know?
- Could the set of cookies be a whole? Why or why not?
- Could one single cookie be considered a whole? Why or why not?
- If the set of cookies is the whole, what is one unit that makes up part of the whole? How do you know?
- If a single cookie is the whole, what is one unit that makes up part of the whole? How do you know?
- Do Sam and Petra each get one whole cookie? Why or why not?

Determine if the student can MODEL EQUAL SIZE PARTS:

- If Sam and Petra each get one whole cookie, what happens with the third cookie? How do you know?
- [Point to the manipulatives being used.] Do Sam and Petra both get equal-size parts? How do you know?

Determine if the student can RECOGNIZE *EQUAL PART*:

- [Model a partition that does not result in equal-size parts.] What if you cut the third cookie like this, would Sam and Petra get the same amount? Why or why not?

Determine if the student can PARTITION ANY SHAPE INTO EQUAL PARTS:

- How many cookies does each friend get? How do you know?
- [Point to the manipulative that is partitioned.] Is this the only way to divide the cookie into two equal-size parts? Explain.

Determine if the student can DEMONSTRATE THE CONCEPT OF DIVISION:

- How many friends is Judith sharing her cookies with? How do you know?
- How many people are sharing the cookies altogether? How do you know?
- How many cookies are there? How do you know?

Discuss students' solutions and select students to share who used a variety of manipulatives and strategies.

Ask students the following questions:

- If Sam got one whole cookie and one half of a cookie, what is that fraction called?
- What is one half? What does it mean to have one half of something?

Read aloud and **discuss** the following problem situation.

> Matthew, Thomas, José, and Gabe are sharing a piece of paper shaped like a rectangle. How can they cut the paper into four equal-size parts?

Ask students the following questions. Students should share their responses with a partner before sharing with the class.

- What do the four boys need to do? How do you know?
- What is the whole unit that is being divided? How do you know?
- Who needs a portion of the paper? How do you know?
- How many people are splitting the whole unit? How do you know?

Require students, either independently or in partners, to determine at least one way to partition the paper into four equal-size pieces. Students should select and use a manipulative from those available.

Use the following guiding questions to support student understanding.

GUIDING QUESTIONS

Elicit student thinking:

- What shape would you use to solve this problem? Why?

Determine if the student can RECOGNIZE A UNIT:

- How many pieces of paper are being partitioned? How do you know?
- How do you know the rectangle represents one whole?

Determine if the student can PARTITION SHAPES:

- How many pieces did you divide the rectangle into? Why?
- Show me how you partitioned the rectangle. Why did you divide the rectangle that way?

Determine if the student can RECOGNIZE *EQUAL PART*:

- Are all the pieces from the whole the same size? How do you know?
- Do the pieces have to be the same shape to be the same size? Why or why not?
- [Model two parts that are not of equal sized.] Are these two parts of equal size? How do you know?

Determine if the student can RECOGNIZE PARTS OF A GIVEN WHOLE OR A UNIT:

- What is the whole that is being partitioned in this situation? How do you know?
- What are the parts of the whole? How do you know?
- How many parts make up the whole in this situation? How do you know?

Determine if the student can PARTITION ANY SHAPE INTO EQUAL PARTS:

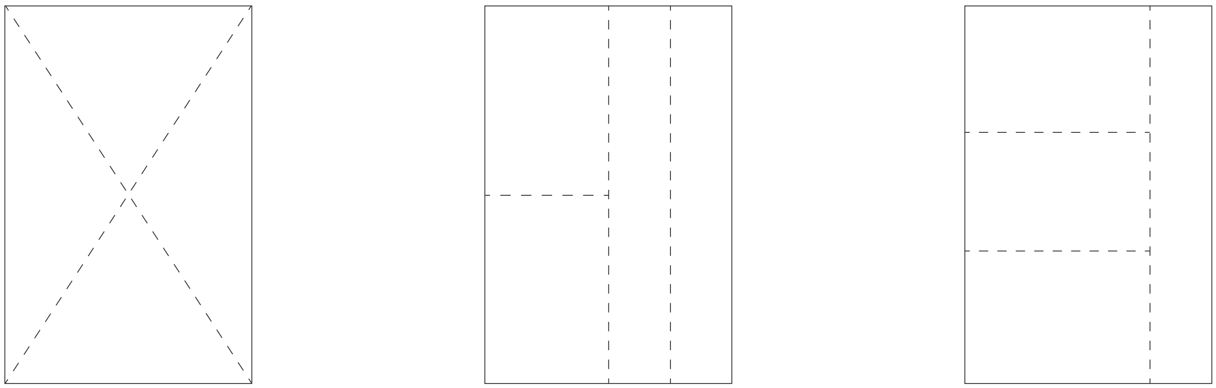
- Are all of the parts the same size? How do you know?
- Show me an example of a rectangle that has not been partitioned equally and explain how you know it is not.
- What is another way you could partition this rectangle into four equal-size parts?

Determine if the student is ready to EXPLAIN *UNIT FRACTION*:

- How many pieces of the whole paper did each boy get? How do you know?
- Did each boy get one whole since they each got one paper? Explain.
- What fraction or part of the whole did Thomas get? How do you know?

Discuss students' solutions, select students who used different partitions and/or manipulatives to share. Select students with more basic partitions—such as halving vertically and then halving horizontally (or vice versa)—to share first, then progress to more complex partitions.

If no students utilized a more complex partition, present one as an example of "previous student work".



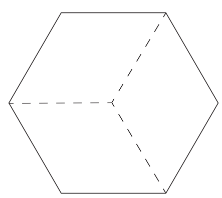
Explain the relationship between a whole and its part.

Display a pattern shape such as a hexagon. Avoid only using circles or rectangles, because students are most often exposed to those shapes during partitioning.

Ask students the following questions:

- What is the name of this shape? How do you know?
- Does this shape represent a whole or a part of the whole? How do you know?

Cover the hexagon with three parallelograms, leaving no gaps and with no overlaps.

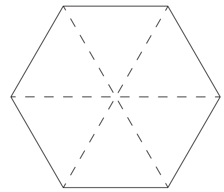


Ask students the following questions:

- Did the shape change?
- What is the whole? How do you know?
- How many parts has the whole been partitioned into? How do you know?
- Are they equal-size parts? How do you know?
- [Hold up one of the parallelograms.] What part of the whole is this parallelogram? How do you know?

Students should identify that one parallelogram is one part of the three parts that make up the whole. **Explain** that this part-whole relationship is also called a *fraction*. **Emphasize** that the fraction name for one part of the three parts that make up the whole parallelogram is *one-third*.

Remove the parallelograms and **cover** the hexagon with triangles, leaving no gaps and with no overlaps.



Ask students the following questions:

- Did the shape change?
- What is the whole? How do you know?
- How many parts has the whole been partitioned into? How do you know?
- Are they equal-size parts? How do you know?
- [Hold up one of the triangles.] What part of the whole is this triangle? How do you know?
- What is the fraction name for this one triangle?

Explain that because one triangle is a single unit from the whole, it is a unit fraction. Further **explain** that any fraction that is a single part of a whole is a unit fraction.

Ask students to name some unit fractions.

NOTE: It is important to refrain from using numerical representations of fractions this early during instruction. Early fraction instruction should emphasize visual models and fractions in word form (spoken and written).

Read aloud and **discuss** the following problem situation.

Madison has a soccer cake. It is the shape of a soccer field and it has six dolls dressed as soccer players. There is a doll for Madison and each of her five friends. How can Madison cut the cake so that each girl will get the same amount of cake?

Ask students the following questions. Students should share their responses with a partner before sharing with the class.

- What does Madison need to do? How do you know?
- What is the whole unit that is being divided? How do you know?
- Who needs a portion of the cake? How do you know?
- How many people are splitting the whole unit? How do you know?

Require students, either independently or in partners, to determine at least one way to partition the cake into six equal-size pieces. Students should select and use a manipulative from those available.

Use the following guiding questions to support student understanding.

GUIDING QUESTIONS

Elicit student thinking:

- What is the shape of a soccer field? What shape could you use to solve this problem?
- When is a time that you had to split something with your friends?

Determine if the student can RECOGNIZE A UNIT:

- What is being partitioned? How do you know?
- How do you know the rectangle represents one whole?

Determine if the student can PARTITION SHAPES:

- What shapes did you partition (divide) the rectangle into? Why?
- How many pieces did you partition (divide) the rectangle into? Why?
- Show me how you partitioned (divided) the rectangle. Why did you partition (divide) the rectangle that way?

Determine if the student can RECOGNIZE *EQUAL PART*:

- Are all the pieces from the whole the same size? How do you know?
- Do the pieces have to be the same shape to be the same size? Why or why not?
- [Model two parts that are not of equal sized.] Are these two parts of equal size? How do you know?

Determine if the student can RECOGNIZE PARTS OF A GIVEN WHOLE OR A UNIT:

- What is the whole that is being partitioned in this situation? How do you know?
- What are the parts of the whole? How do you know?
- How many parts make up the whole in this situation? How do you know?

Determine if the student can PARTITION ANY SHAPE INTO EQUAL PARTS:

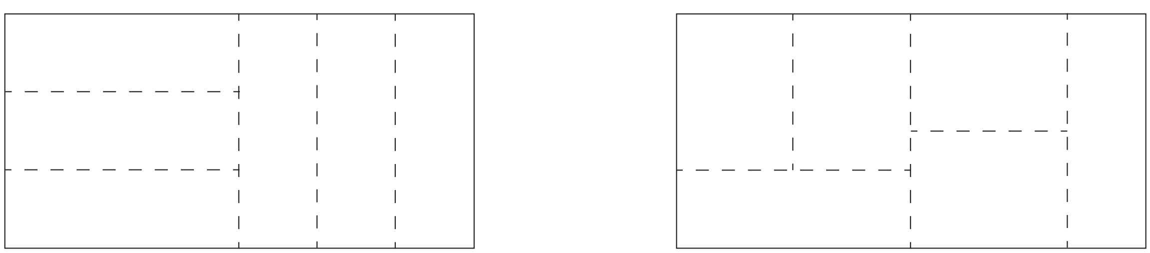
- Are all of the parts the same size? How do you know?
- Show me an example of a rectangle that has not been partitioned equally and explain how you know it is not.
- What is another way you could partition this into six equal-size parts?

Determine if the student is ready to EXPLAIN *UNIT FRACTION*:

- How many pieces of the whole cake did each person get? How do you know?
- Did each person get one whole since they each got one? Explain.
- What fraction or part of the whole did Madison get? How do you know?

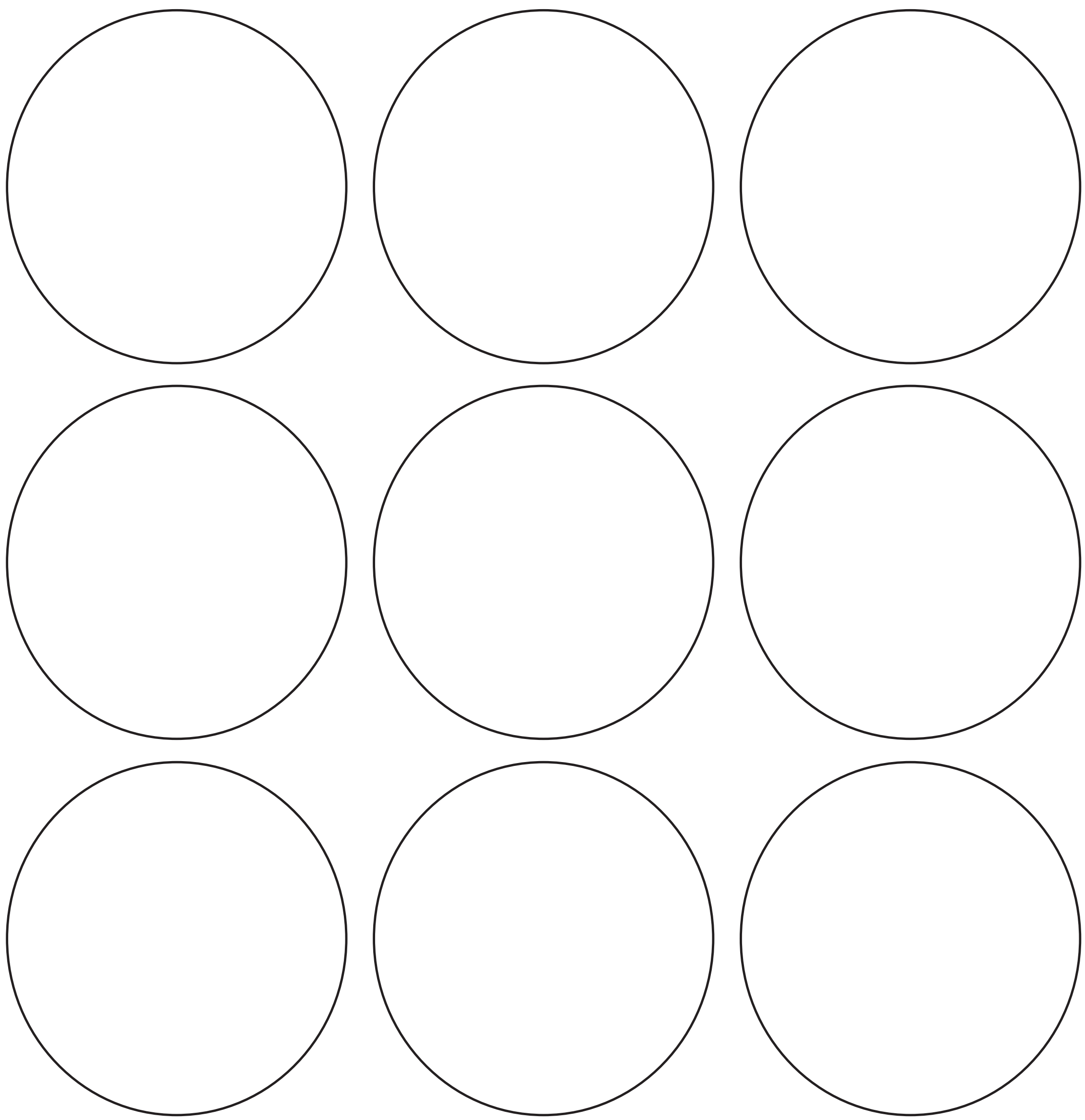
Discuss students' solutions and select students who used different partitions and/or manipulatives to share with the class. Select students with more basic partitions—such as creating six vertical or horizontal parts—to share first, then progress to more complex partitions.

If no students utilized a more complex partition, present one as "previous student work".



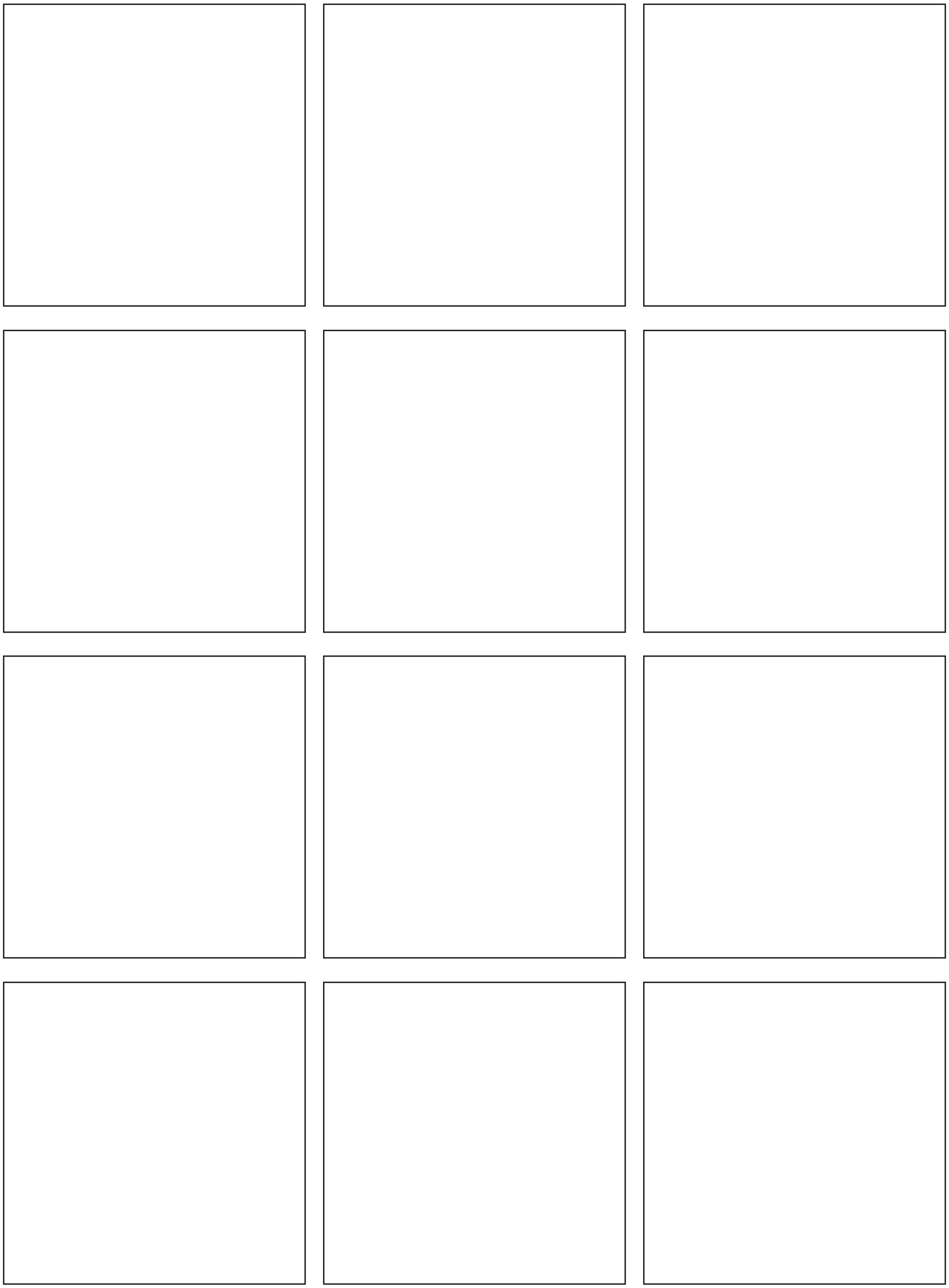
At the end of the activity, display a shape that has been partitioned with parts that are not of equal size. In groups of two, three, or four, students should discuss if the shape is partitioned into equal parts and why or why not. Then, students should determine how to partition the shape into equal parts.

circles

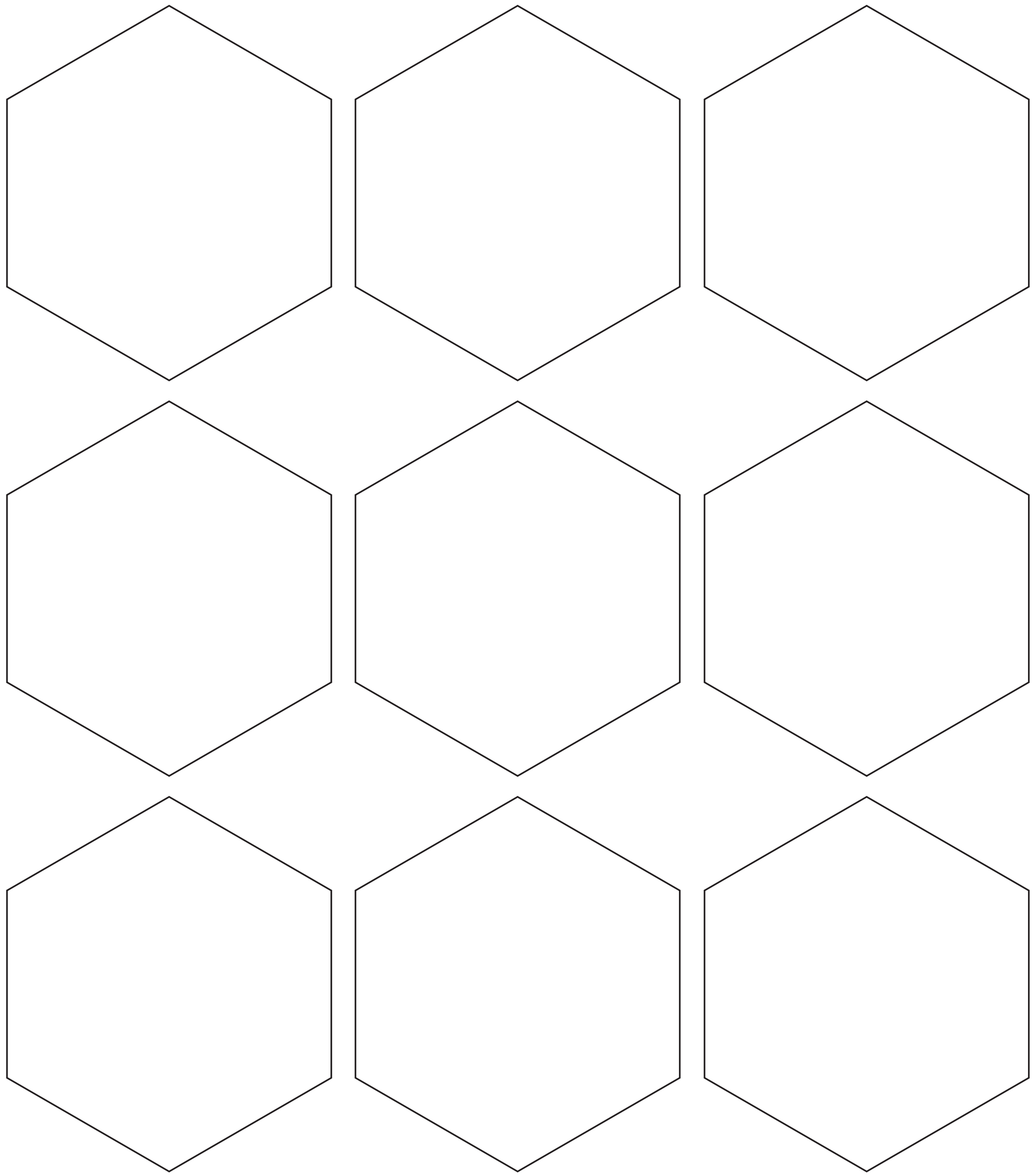


rectangles

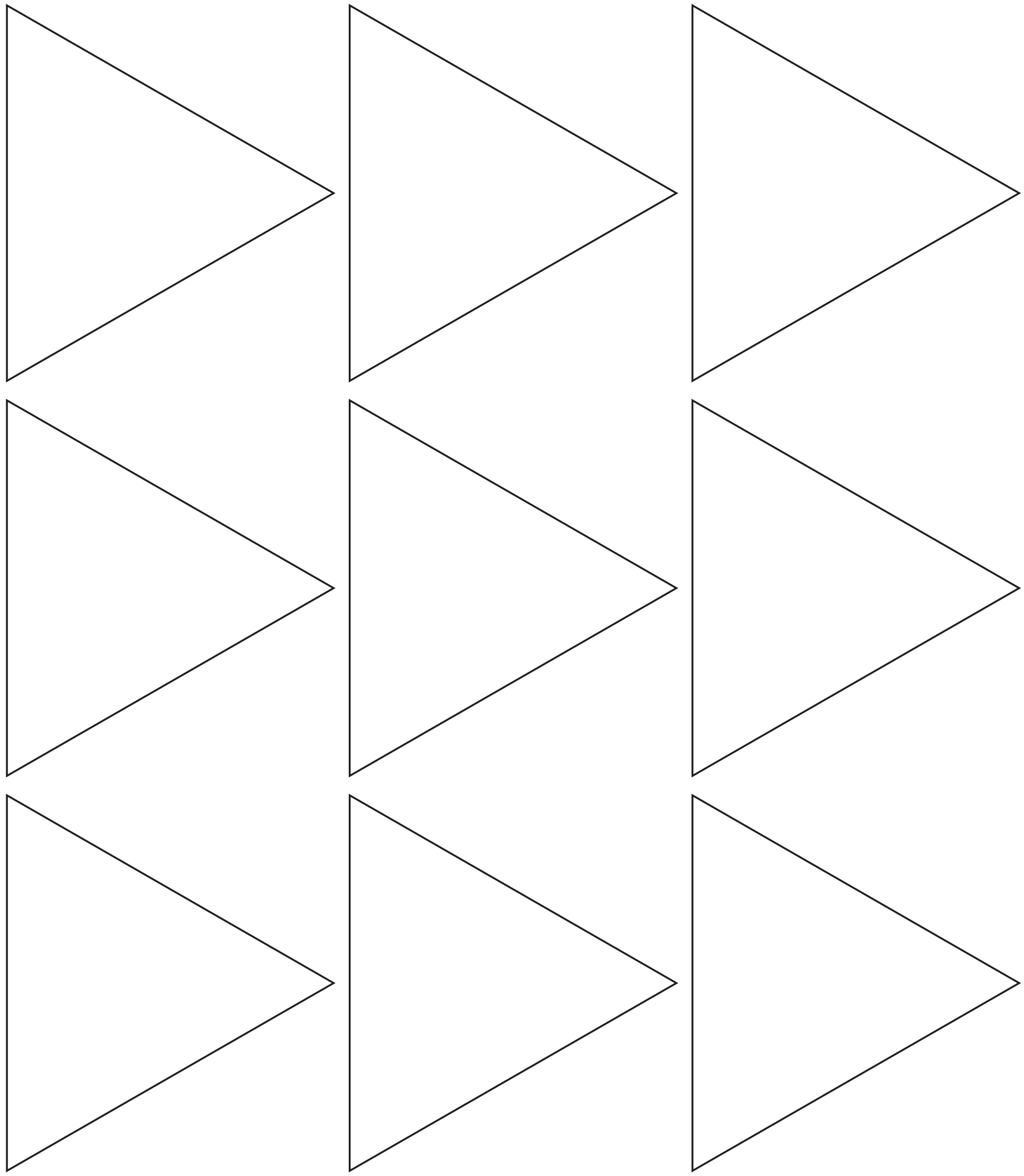
squares



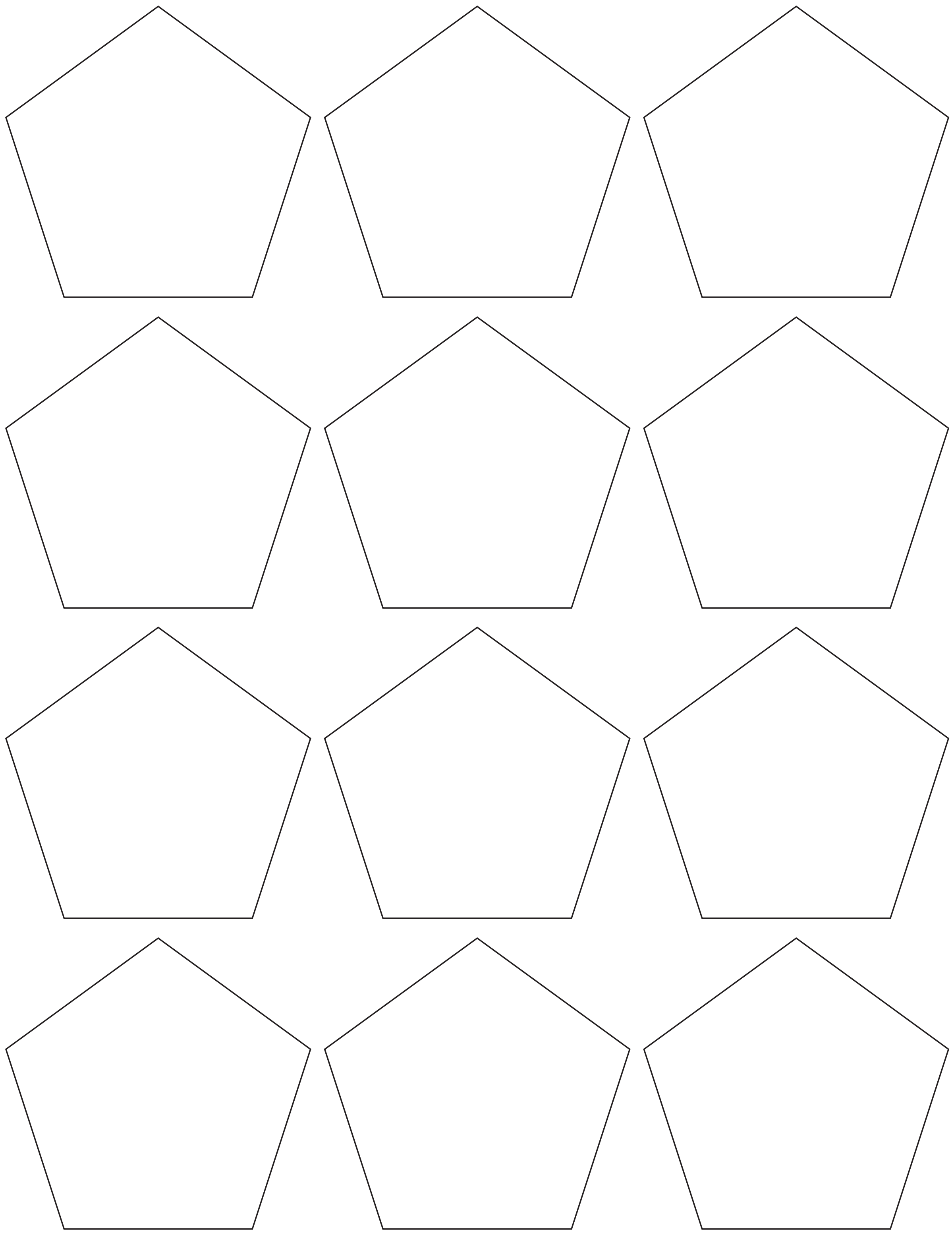
hexagons



triangles



pentagons



Partitioning Shapes and Unit Fractions

INSTRUCTIONAL ACTIVITY

Lesson 2

LEARNING GOAL

Students will identify the unit fraction of partitioned shapes as well as identify equivalent and non-equivalent partitions when provided an image of a partitioned shape.

PRIMARY ACTIVITY

Students will analyze images of shapes that have been completely or partially partitioned to determine the number of equal-size parts and to identify the modeled unit fraction. Students will also play a sorting game in which they will analyze shapes that have been partitioned to determine if the partitions are equal in size.

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

- Unit fraction
- Whole
- Equal share
- Partition
- Half (halves)
- Thirds
- Fourths
- Fifths
- Sixths
- Eighths
- Ninths
- Tenths

MATERIALS

- A variety of manipulatives for students to choose from. The following are some examples:
    - Fraction circles

- Paper shapes (Refer to the INSTRUCTIONAL ACTIVITY SUPPLEMENT from Lesson 1.)
- Geoboards and rubber bands
- Pattern shapes
- Thin strips of paper, kebab sticks, or straws

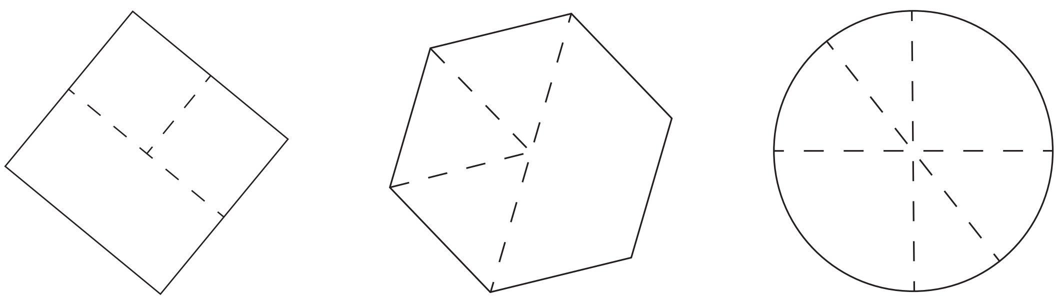
- INSTRUCTIONAL ACTIVITY STUDENT HANDOUT
- INSTRUCTIONAL ACTIVITY SUPPLEMENT

IMPLEMENTATION

Begin the lesson by reviewing unit fractions.

Ask students what a unit fraction is. Students should share with a partner before discussing as a whole group.

Present examples of partitioned and partially partitioned shapes. Use simpler partitions such has halves, fourths, eighths, thirds, and sixths for this activity. Be sure to use a variety of shapes—not just rectangles and circles. The following images are examples of partially partitioned shapes that can be displayed.



Ask students to share with a partner the number of equal-size partitions. Then select students to share their response with the class.

Display one of the partitioned shapes that was used for the previous discussion. **Color** in one of the partitioned parts.

Explain and **model** how to write the name for the unit fraction you just created. Refrain from introducing students to numerical representations of fractions at this point. The focus should be on the visual representation and the fraction name. Therefore, **require** students to write the word form for each unit fraction (e.g. *one-fourth*, *one-sixth*, and *one-eighth*).

Display the names for fractional parts so that students can easily and correctly spell the unit fraction names. **Emphasize** the "ths" ending for the number of parts. For example, a whole partitioned into four equal-size parts is partitioned into *fourths* (the exception to this is *halves* and *thirds*). The following is a list of fractional part names that will need to be available: *halves*, *thirds*, *fourths*, *fifths*, *sixths*, *eighths*, *ninths*, and *tenths*.

Distribute the Instructional Activity Student Handout. **Read** and **review** the directions. Make available the same manipulatives from Lesson 1 for student use.

Require students to complete the Instructional Activity Student Handout either independently or in partners.

Use the following guiding questions to support student understanding.

GUIDING QUESTIONS

Elicit student thinking:

- When is a time you had to divide something into equal-size parts?

Determine if the student can RECOGNIZE A UNIT:

- [Point to a shape on the Instructional Activity Student Handout.] What is the whole? How do you know?
- [Point to one part of a shape.] Is this the whole? How do you know?

Determine if the student can PARTITION SHAPES:

- [Point to a shape on the Instructional Activity Student Handout.] What shape(s) are the parts of this shape? How do you know?

Determine if the student can RECOGNIZE PARTS OF A GIVEN WHOLE OR A UNIT:

- Do parts of a whole have to be the same size? Why or why not?
- [Point to a shape on the Instructional Activity Student Handout.] How many parts make up this whole? How do you know?
- [Point to a partially partitioned shape.] How can you determine the number of equal-size parts in this shape? How many parts are there?
- If you take away some of the parts, will you still have one whole? Why or why not?
- If you add more parts, will you still have one whole? Why or why not?

Determine if the student can RECOGNIZE *EQUAL PART*:

- [Point to a shape on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.] Is this shape completely partitioned (or divided)? How do you know?
- [Point to a shape on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.] Is this shape partially partitioned (or divided)? How do you know?
- Do equal-size parts of a whole have to be the same shape? Why or why not?
- [Point to a shape on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.] Is this shape partitioned (or divided) into equal-size parts? How do you know?
- [Point to a shape on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.] Are all of these parts the same size? How do you know?
- [Point to a shape on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.] Do all of these parts have to be the same size? Why or why not?
- [Point to a shape on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.] Do all of these parts have to be the same shape? Why or why not?

Determine if the student can EXPLAIN *UNIT FRACTION*:

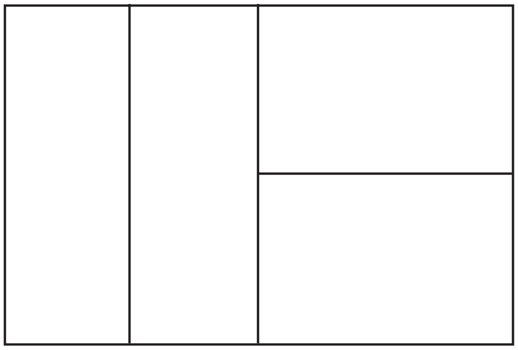
- If a shape is partitioned (or divided) into eight equal-size parts, what unit fraction represents one of the parts? How do you know?
- [Point to a partitioned shape.] What is the unit fraction for one of the parts of this shape? How do you know?
- [Point to a student's solution.] How do you know this is the unit fraction?
- Is two-fourths a unit fraction? Why or why not?
- Is one-twentieth a unit fraction? Why or why not?

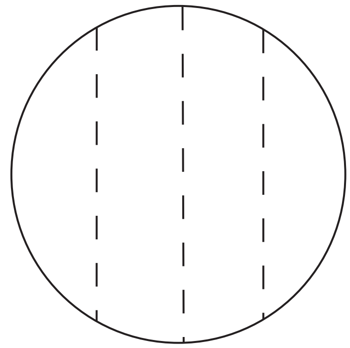
Select students to share and discuss their solutions.

Collect the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT to review student responses.

Review the concept that when shapes are partitioned into equal-size parts, the parts do not have to be the same shape.

Model the following images. **Discuss** whether the partitions result in equal-size parts or parts of different sizes.



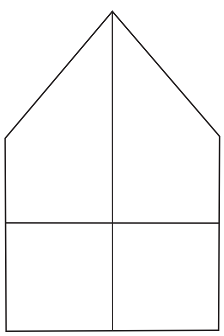


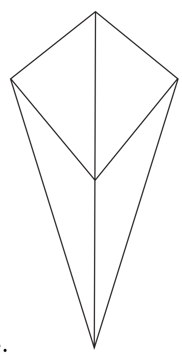
Ask students how they can determine if each part is the same size. **Discuss** students' responses and **require** students to model their thinking with the displayed images.

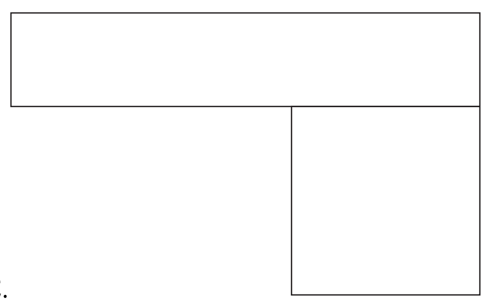
Display the following partitioned shapes one at a time. These shapes can also be found on Pages 1 – 3 of the INSTRUCTIONAL ACTIVITY SUPPLEMENT.

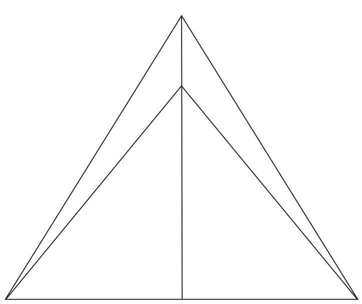
Each time a shape is displayed, **ask** students to display a hand signal, such as thumbs up for equal-size partitions and thumbs down for partitions of different sizes. Then, **select** a student or two that provide the correct response to explain why they responded the way they did.

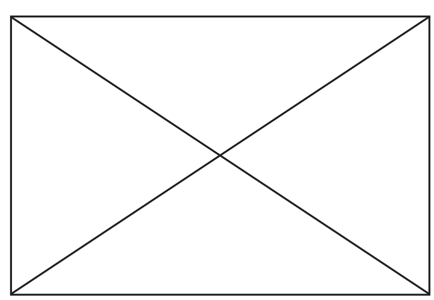
Note that for figure C, the two parts can be measured to demonstrate that both parts have the same area.

A.


B.


C.


D.


E.


Note that shapes A and D are not divided into equal-size parts, but shapes B, C, and E are partitioned into equal-size parts.

Discuss different strategies for determining if parts of a partitioned shape are equal in size.

Arrange students into partners and distribute to each pair one set of cards and one mat from Pages 4 – 10 of the INSTRUCTIONAL ACTIVITY SUPPLEMENT.

Explain that the set of cards should be mixed and placed face down, and that the mat should be placed between the partners so both students can access all the columns. One student will select a card and, along with the second student, determine if the shape on the card is partitioned into equal-size parts or parts of different sizes. If the students cannot agree or they do not know, the card should be placed in the middle column ("Not Sure"). The student that drew the card will place it in the correct column, and then the second partner will draw a card and repeat the process. Students will continue until all cards have been drawn.

Make available the same manipulatives from LESSON 1 for student use.

As students participate in the activity, use the following guiding questions to support student understanding.

GUIDING QUESTIONS

Elicit student thinking:

- What does it mean if a shape has been partitioned (or divided) into parts of equal size?
- What does it mean if a shape has not been partitioned (or divided) into parts of equal size?
- If you were to share a pizza with your friend, would you want the pieces to be the same size? Why or why not?

Determine if the student can PARTITION SHAPES:

- [Point to a partitioned shape on a card.] What is another way you can partition (or divide) this shape?
- [Point to a partitioned shape on a card.] How can you partition (or divide) this shape into the same number of parts, but by using different shapes for the parts?
- [Point to a shape that has been partitioned into four parts.] How can you partition (or divide) this shape into three parts instead of four parts?

Determine if the student can RECOGNIZE *EQUAL PART*:

- [Point to a card in the *equal-size parts* column.] How do you know this shape is partitioned (or divided) into equal-size parts?
- [Point to a card in the *different-size parts* column.] How do you know this shape is partitioned (or divided) into parts of different sizes?
- [Point to a card that has equal-size parts, but the parts are different shapes.] Do the parts have to be the same shape to be the size? Why or why not?
- Are all parts of any whole that are the same size always the same shape? Why or why not?
- Are all parts of any whole that are the same shape always the same size? Why or why not?
- [Point to a card placed on the mat.] Why did you put this card in this column?

In pairs, students should select one card with a shape they believe is partitioned into equal-size parts, and one card with a shape they believe is partitioned into different-size parts. Each pair should share their cards and explanations with the class.

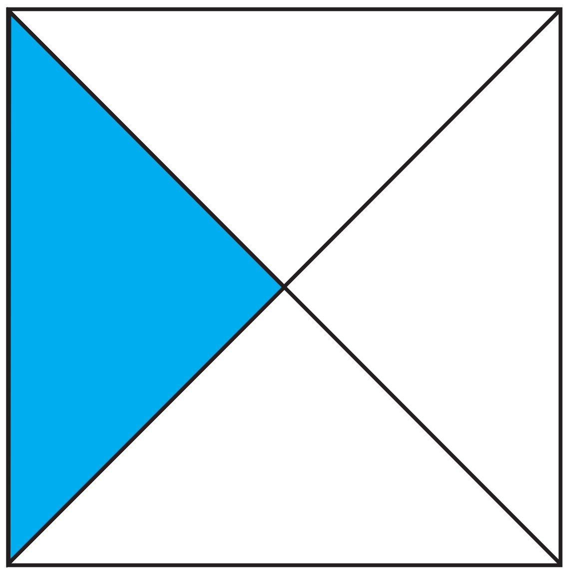
Name________________________________

Partitioning Shapes and Unit Fractions

Lesson 2

Use each shape to answer the questions.

1.



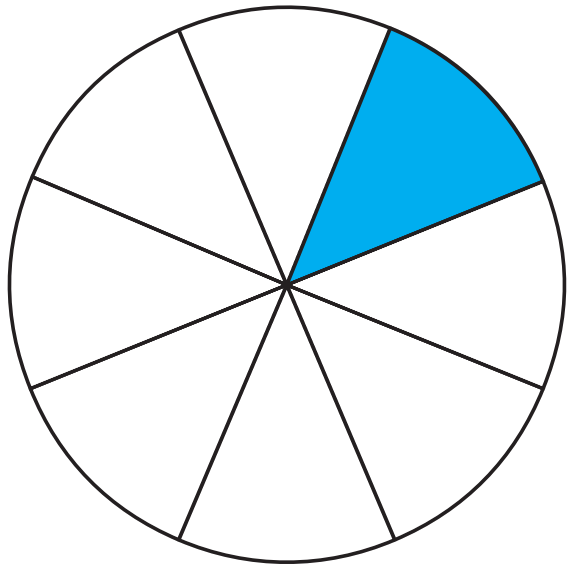
1.a. Number of equal-size parts you see:

1.b. Number of equal-size parts in the whole:

1.c. Name of the unit fraction:

Name______________________________

2.



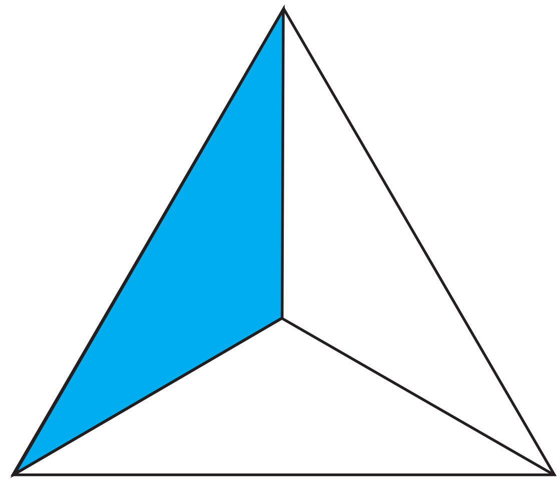
2.a. Number of equal-size parts you see:

2.b. Number of equal-size parts in the whole:

2.c. Name of the unit fraction:

Name______________________________

3.



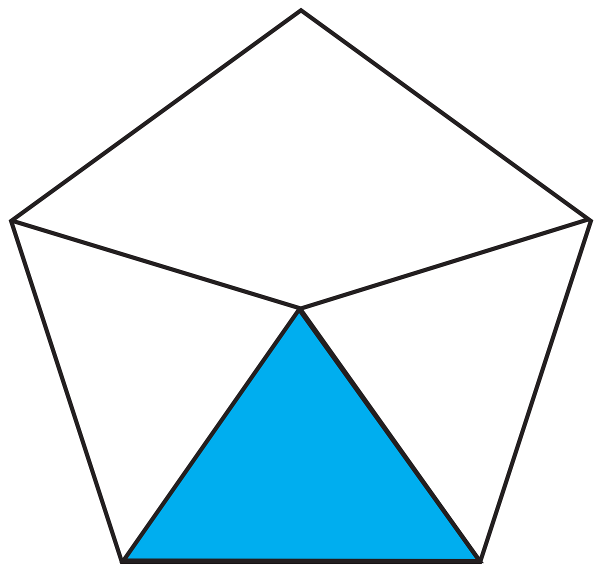
3.a. Number of equal-size parts you see:

3.b. Number of equal-size parts in the whole:

3.c. Name of the unit fraction:

Name______________________________

4.



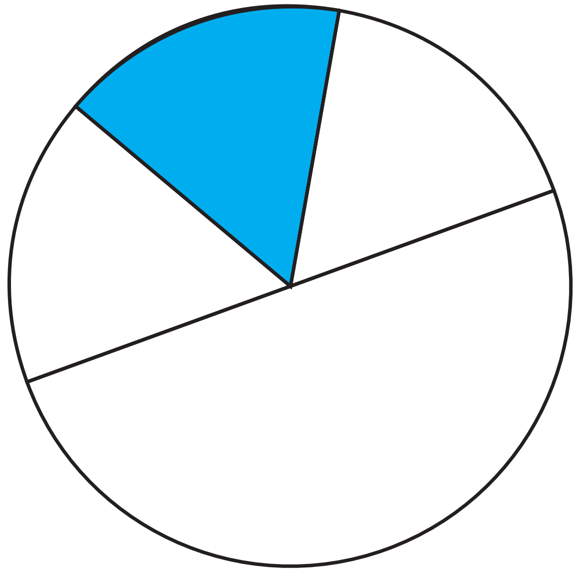
4.a. Number of equal-size parts you see:

4.b. Number of equal-size parts in the whole:

4.c. Name of the unit fraction:

Name______________________________

5.



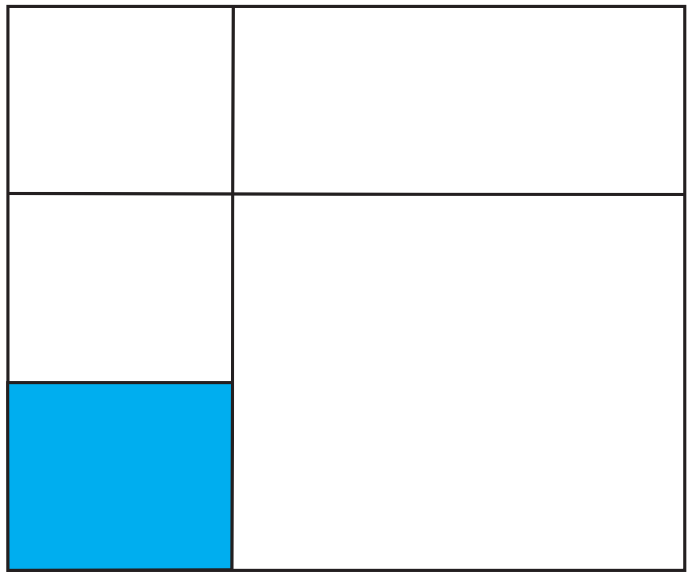
5.a. Number of equal-size parts you see:

5.b. Number of equal-size parts in the whole:

5.c. Name of the unit fraction:

Name______________________________

6.



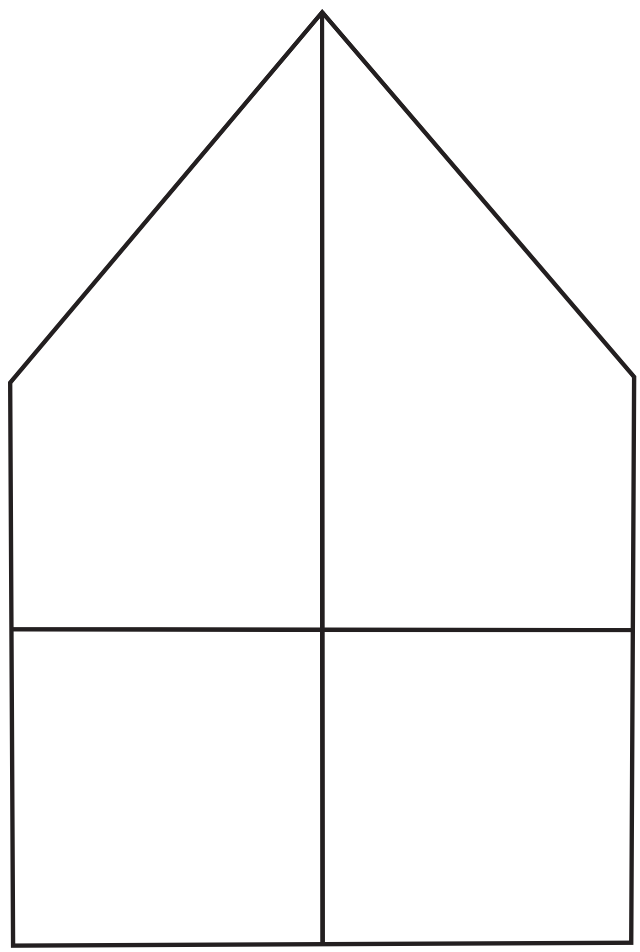
6.a. Number of equal-size parts you see:

6.b. Number of equal-size parts in the whole:

6.c. Name of the unit fraction:

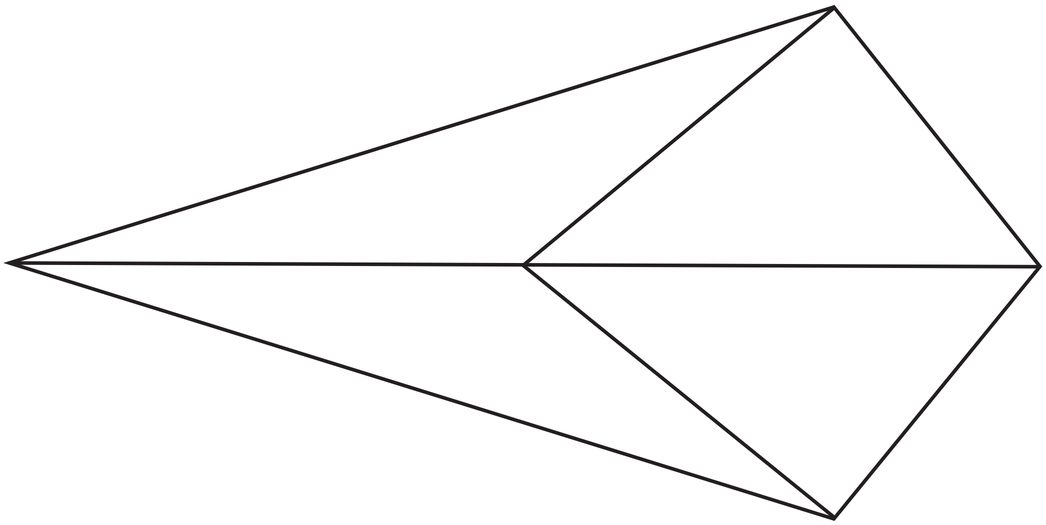
Recommend one copy of this page for teacher use.

A.

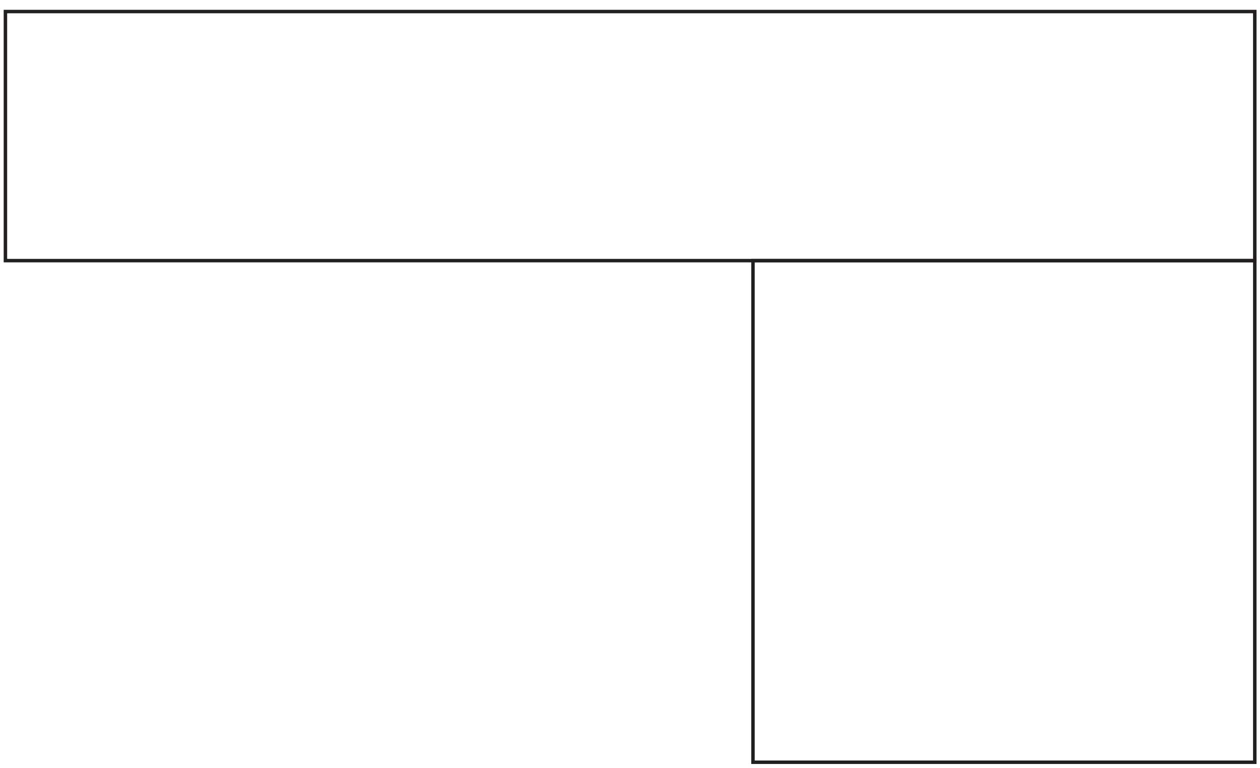


Recommend one copy of this page for teacher use.

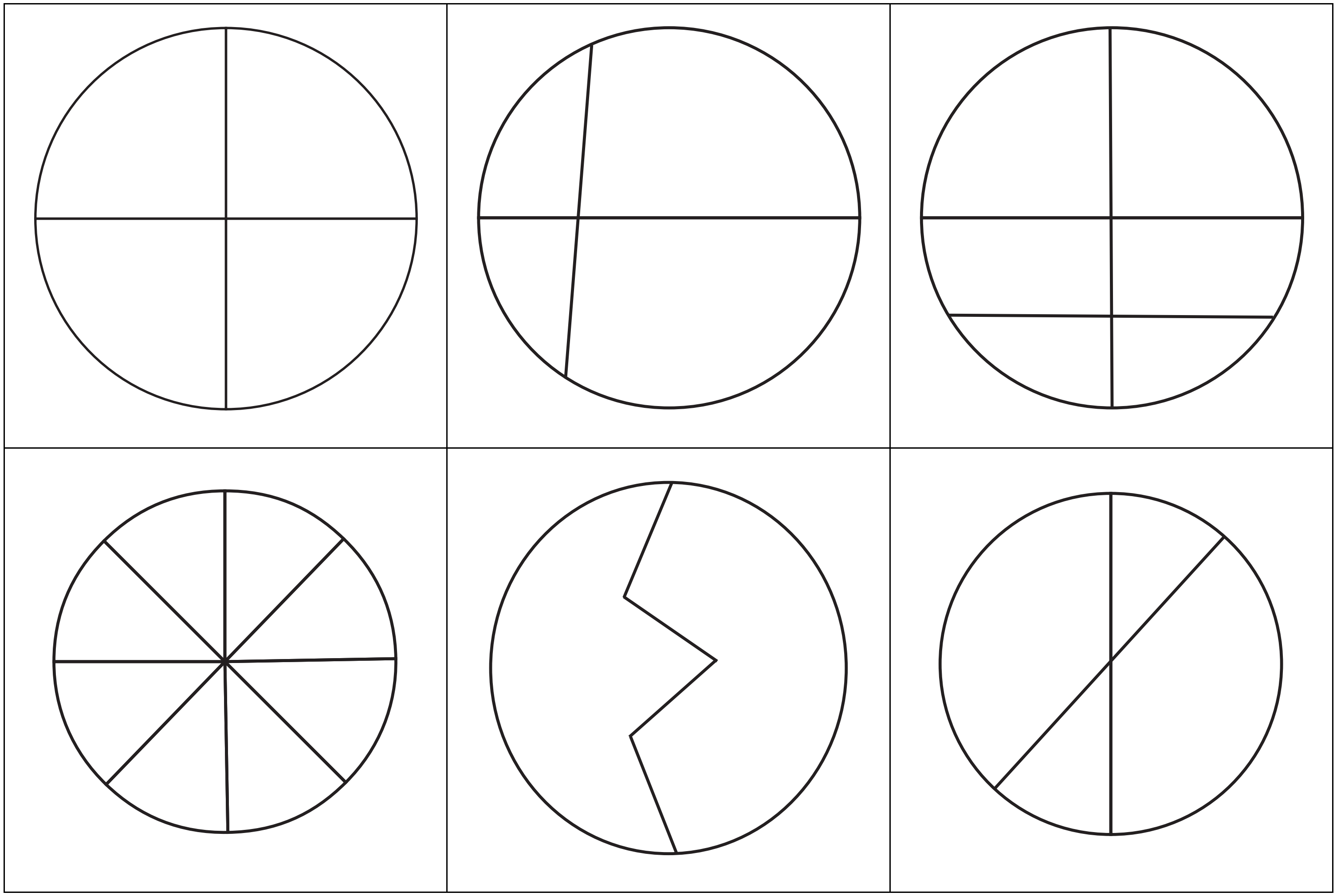
B.

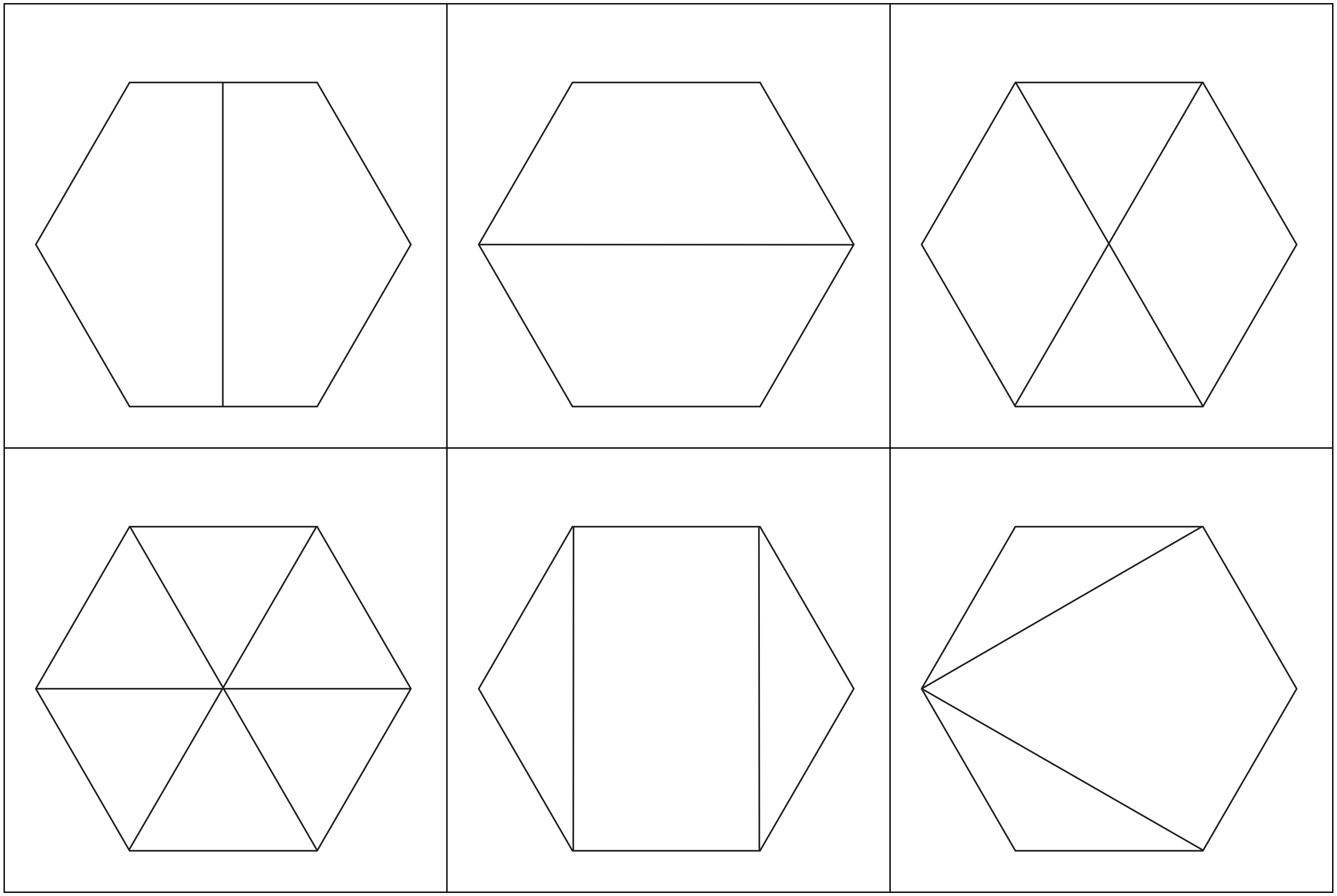


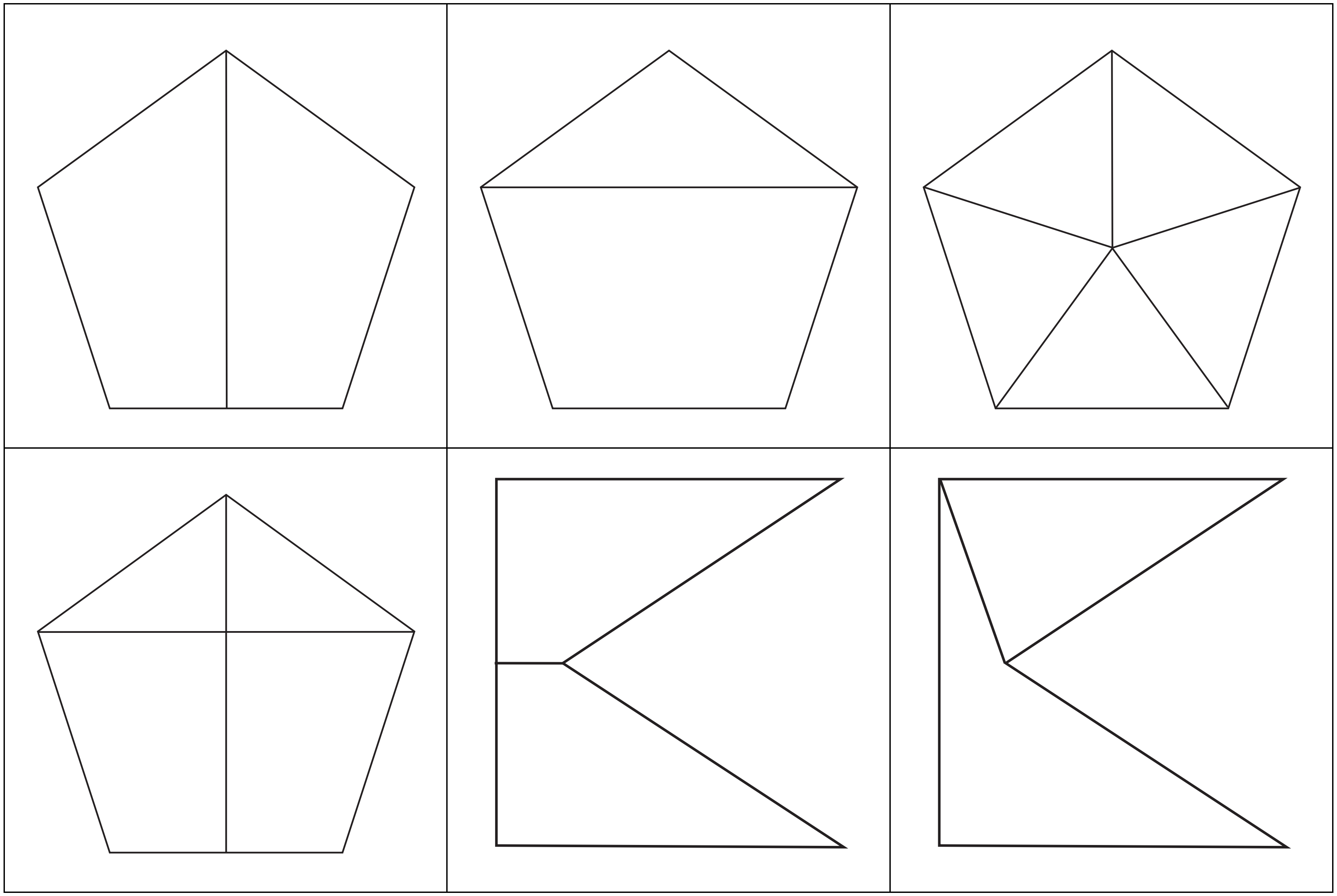
C.

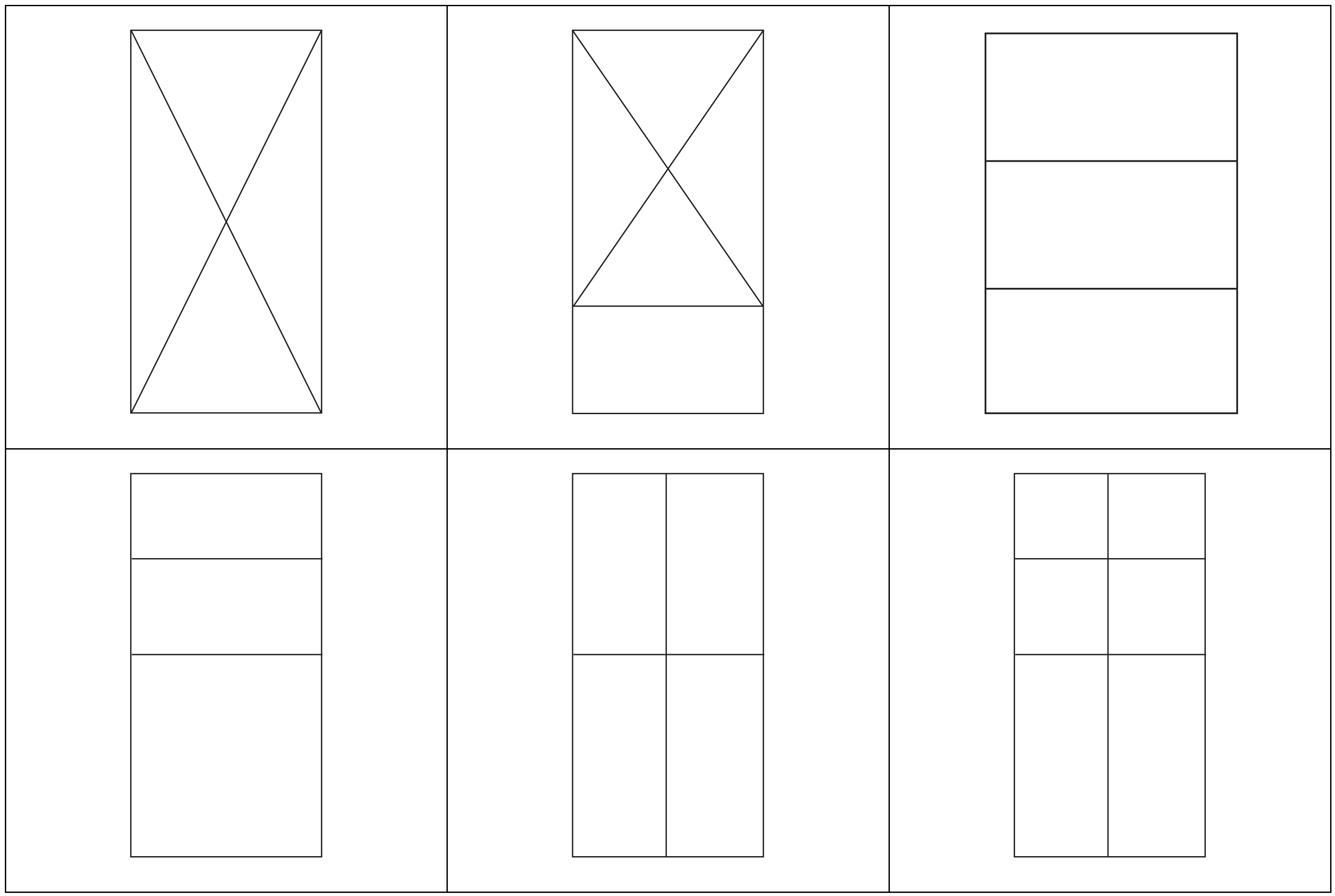


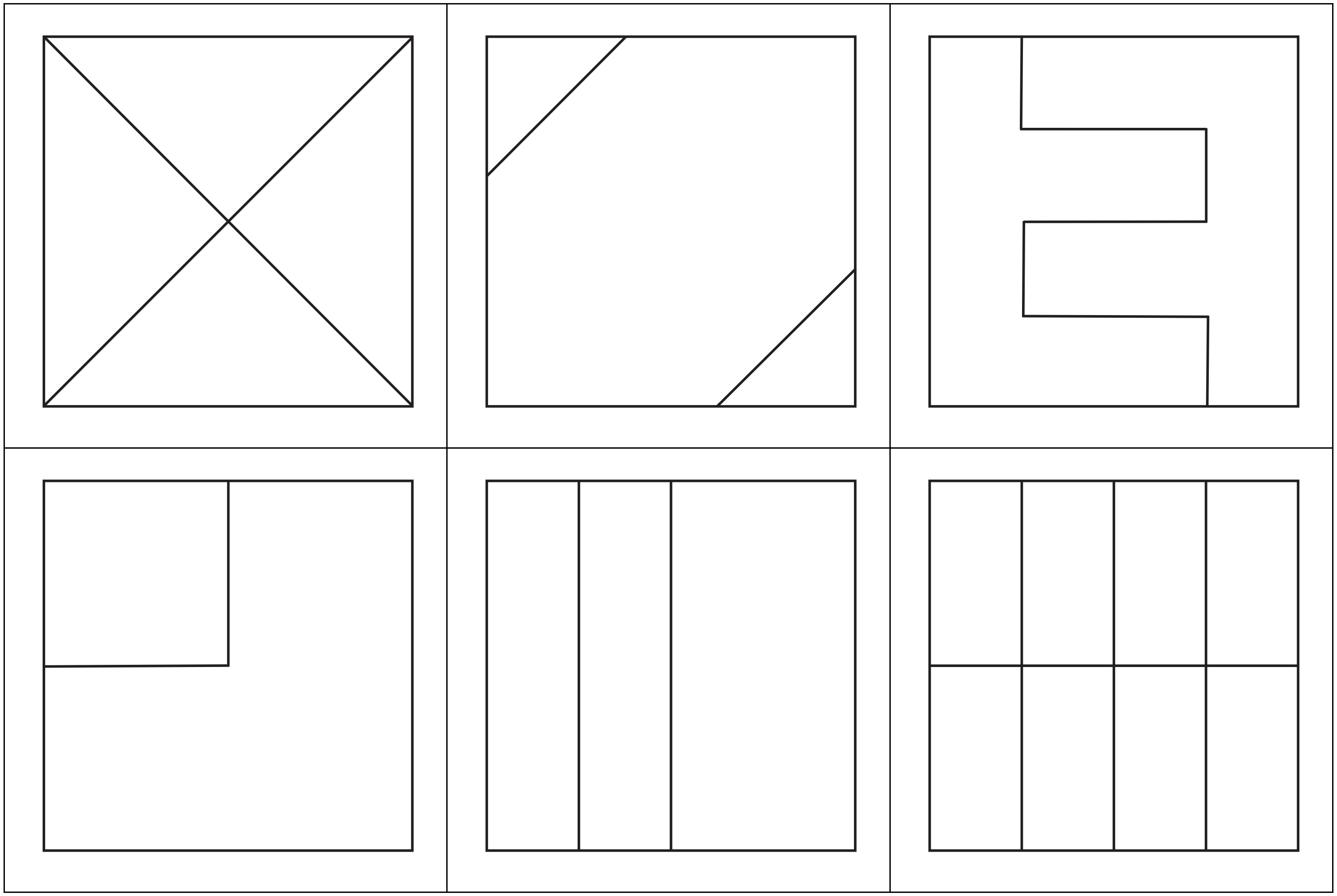
Equal-Size Parts	Don’t Know	Different-Size Parts

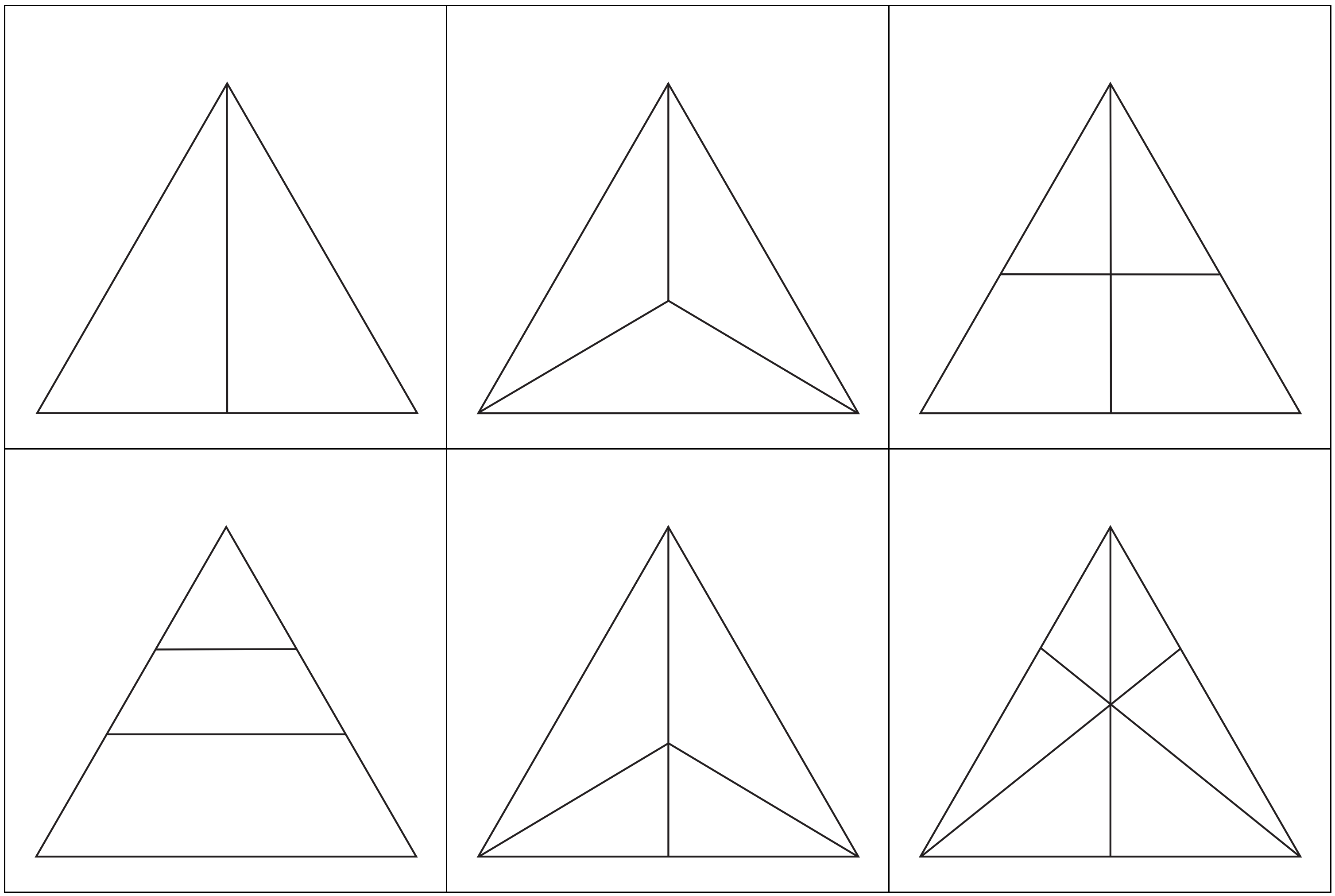












Partitioning Shapes and Unit Fractions

INSTRUCTIONAL ACTIVITY

Lesson 3

LEARNING GOAL

Students will partition images of shapes into equal-size parts, when given the number of equal-size parts or a unit fraction.

PRIMARY ACTIVITY

Students will draw line segments to partition provided shapes into a given number of equal-size parts. Students will start by partitioning rectangles and circles and then progress to other shapes such as triangles, pentagons, and hexagons. Next, students will partition shapes when given a unit fraction. In partners, students will draw a unit fraction chosen from a bag, select a shape to partition, draw the necessary line segment(s), and color in the unit fraction.

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

- Unit fraction
- Whole
- Partition
- One-half
- One-third
- One-fourth
- One-fifth
- One-sixth
- One-eighth

MATERIALS

- A variety of manipulatives for students to choose from. The following are some examples:
    - Fraction circles
    - Paper shapes (Refer to the INSTRUCTIONAL ACTIVITY SUPPLEMENT from Lesson 1.)

  - Geoboards and rubber bands
  - Pattern shapes
  - Thin strips of paper, kebab sticks, or straws
- Paper bag (Recommend one bag for every two students.)
- Markers, crayons, colored pencils
- Rulers (optional)
- INSTRUCTIONAL ACTIVITY STUDENT HANDOUT
- INSTRUCTIONAL ACTIVITY SUPPLEMENT (Recommend one copy of pages 1 – 3 and, for the remaining pages, one copy for every two to three students.)

IMPLEMENTATION

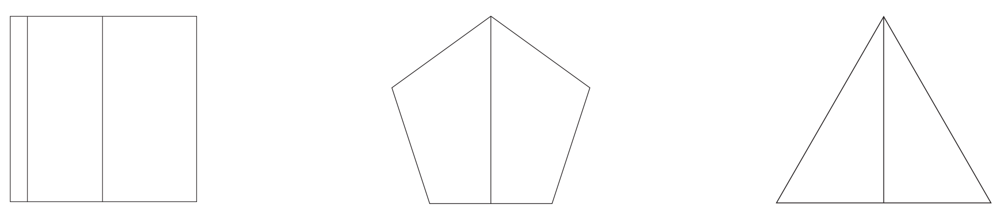
Begin the lesson by reviewing area.

Ask students what area is. Students should share with a partner before discussing as a whole group.

Present examples of partitioned shapes that are not divided into parts of equal size.

Ask students if they think the parts are the same size, and why or why not. Students should share with a partner before discussing as a whole group.

The following partitioned shapes are examples that could be used.



Present the first shape again. Students should watch as you use a ruler partition the shape into eight equal-size parts. **Ask** students whether they think the parts are the same size now, and why or why not. Let students share with a partner before discussing as a whole group.

Repeat the process with the other shapes you displayed. Be sure to partition some shapes that were divided into equal-size groups the first time into parts that are not the same size the second time. Each time, **ask** students to share their thinking after first discussing with a partner.

Display a circle that has not been partitioned.

Ask students how you could divide the circle into two equal-size parts. **Link** using manipulative circle pieces to the drawing, partitioning using a circle manipulative first, then drawing the segments on the shape.

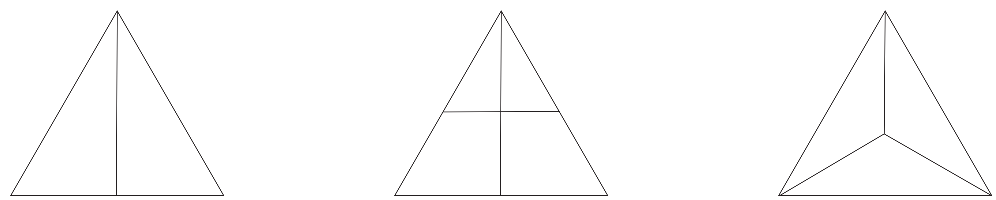
Model, using a ruler to partition the circle into two equal-size parts, by estimating where the center of the circle is. **Think aloud** as you model, pointing out that each piece has the same area or is the same size.

Continue **modeling** by partitioning the halves in half again, creating fourths, and again to create eighths. **Think aloud** as you model.

Emphasize each time you partition that each of the parts are the same size, and state the mathematical term for the number of equal-size parts (i.e. fourths, fifths, sixths, etc.).

Ask a student or two to model the same process for the class with a rectangle. The student(s) should explain their thinking aloud as well.

Repeat the process with a triangle, **modeling** that once you partition a triangle in half, it is difficult to partition it in half again and create two parts of the same area. **Demonstrate** that you can create three parts of equal size instead. For example:



halves *not equal-size parts* *thirds*

Repeat the process with a pentagon, either by modeling aloud yourself or selecting a student or two to share and model.

Emphasize that when partitioning a shape into the number of equal-size parts that is the same as the number of sides, a segment can be drawn from each vertex (corner) to the center of the shape. For example, reference the triangle modeled as thirds previously and the following pentagon and hexagon.

Distribute the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT. Make available rulers and the manipulatives from LESSON 1 and LESSON 2 for student use.

Require students to complete Questions 1 – 4 on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT, either independently or in partners.

Use the following guiding questions to support student understanding.

GUIDING QUESTIONS

Elicit student thinking:

- Why would you need to partition (or divide) something outside of school?
- What have you, or what have you seen, partitioned (or divided) outside of school?

Determine if the student can PARTITION SHAPES:

- [Point to a shape on Questions 1 – 4.] How did you partition (or divide) this shape?
- [Point to a shape on Questions 1 – 4.] How can you partition (or divide) this shape differently?
- [Point to a shape on Questions 1 – 4.] Why did you partition (or divide) this shape like this?

Determine if the student can PARTITION ANY SHAPE INTO EQUAL PARTS:

- [Point to a shape on Questions 1 – 4.] Are these parts the same size? How do you know?
- How do you make sure that each part has the same area (or is the same size)?
- [Point to a shape on Questions 1 – 4.] Is there another way to partition (or divide) this shape into equal-size parts? Explain.
- How would you partition (or divide) a triangle into three equal-size parts? Why would you divide it that way?
- Can you partition a rectangle into five equal parts? Why or why not?

Determine if the student can RECOGNIZE PARTS OF A GIVEN WHOLE:

- [Point to a shape on Questions 1 – 4.] How many parts make up the whole? How do you know?

Determine if the student can EXPLAIN *UNIT FRACTION*:

- [Point to a shape on Questions 1 – 4.] What is the unit fraction for one part of this partitioned shape? How do you know?
- What is a unit fraction?
- If a shape is partitioned (or divided) into six equal-size parts, what is the unit fraction for one of the parts? How do you know?
- If a shape is partitioned (or divided) into six different-size parts, can you identify the unit fraction for one of the parts? Why or why not?

Review student solutions by **selecting** students to share and explain their work.

Review unit fractions.

Display shapes that represent a unit fraction, one at a time. Display as many shapes as is necessary to ensure student understanding.

Ask students to show, silently with their fingers, the number of equal-size parts the shape has been partitioned into each time a shape is displayed.

Ask students the name of the unit fraction if you were to select one part from the whole shape.

Read and **explain** the directions for Question 5 on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

In partners, students will select a unit fraction card from a bag. Either together or separately, students will determine which shape on Question 5 to partition for the given unit fraction. Then, students will color in the number of parts that represent the unit fraction. The students will then choose a second unit fraction card from the bag and repeat the process until there are no more cards, or no more shapes, remaining.

NOTE: There are multiple cards for some fractions, such as one-fourth and one-eighth, and fewer cards for other fractions, such as one-third and one-fifth. There are two cards that are optional and can be removed: *one-ninth* and *one-twelfth*.

Arrange students into groups of two or three. Distribute one set of unit fraction cards from the INSTRUCTIONAL ACTIVITY SUPPLEMENT that have been cut apart and mixed up in a paper bag to each group.

Require students to choose unit fraction cards until they run out of cards or shapes on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT, or set a specific time limit.

As students work in small groups, use the following guiding questions to support student understanding.

GUIDING QUESTIONS

Elicit student thinking:

- Why would you need to partition (or divide) something outside of school?
- What have you, or what have you seen, partitioned (or divided) outside of school?

Determine if the student can PARTITION SHAPES:

- [Point to a shape on Questions 5.] How did you partition (or divide) this shape?
- [Point to a shape on Questions 5.] How can you partition (or divide) this shape differently?
- [Point to a shape on Questions 5.] Why did you partition (or divide) this shape like this?

Determine if the student can PARTITION ANY SHAPE INTO EQUAL PARTS:

- [Point to a shape on Questions 5.] Are these parts the same size? How do you know?
- How do you make sure that each part has the same area (or is the same size)?
- [Point to a shape on Questions 5.] Is there another way to partition (or divide) this shape into equal-size parts? Explain.
- How would you partition (or divide) a triangle into three equal-size parts? Why would you divide it that way?
- Can you partition a rectangle into five equal parts? Why or why not?
- [Point to a shape on Questions 5.] Why did you choose this shape for this partition?

Determine if the student can RECOGNIZE PARTS OF A GIVEN WHOLE:

- [Point to a shape on Questions 5.] How many parts make up the whole? How do you know?

Determine if the student can EXPLAIN *UNIT FRACTION*:

- [Point to a shape on Questions 5.] What is the unit fraction for one part of this partitioned shape? How do you know?
- [Point to a shape on Questions 5 that has been partitioned and colored.] How many parts did you color? Why?
- If you colored in three equal-size parts of a shape would that be a unit fraction? Why or why not?
- What is a unit fraction?
- If a shape is partitioned (or divided) into six equal-size parts, what is the unit fraction? How do you know?
- If a shape is partitioned (or divided) into six different-size parts, can you identify the unit fraction? Why or why not?

Require students to share and explain one unit fraction card and the corresponding partitioned shape.

At the end of the activity, teachers should collect the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT and review student work for understanding.

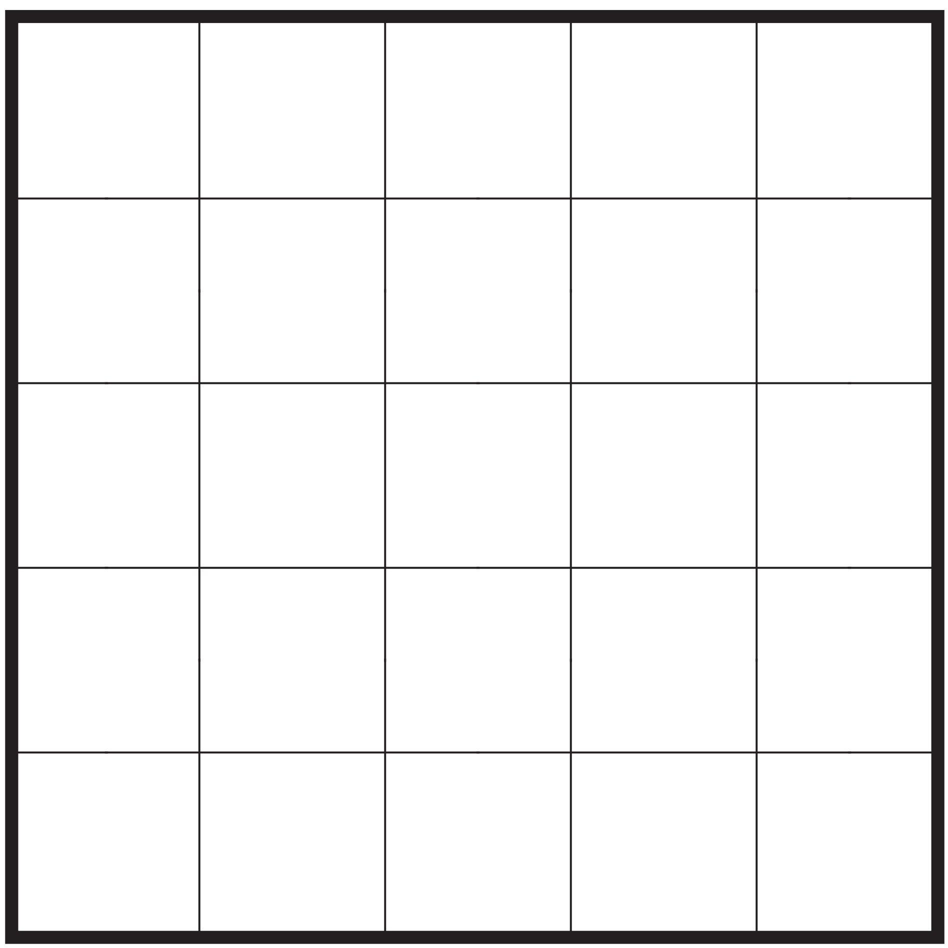
Name________________________________

Partitioning Shapes and Unit Fractions

Lesson 3

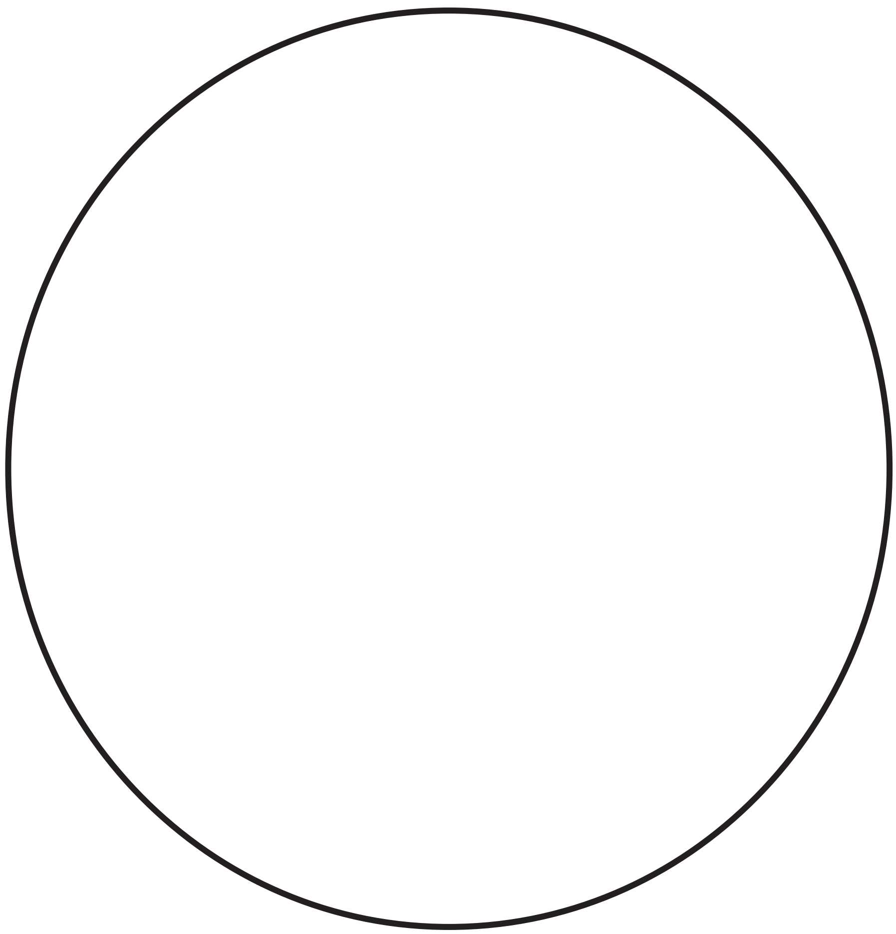
Divide each shape to answer the questions.

1. Mitch, Sam, JaShawn, and Darius are splitting a tray of brownies. How can they divide the brownies so that they each get the same size piece?



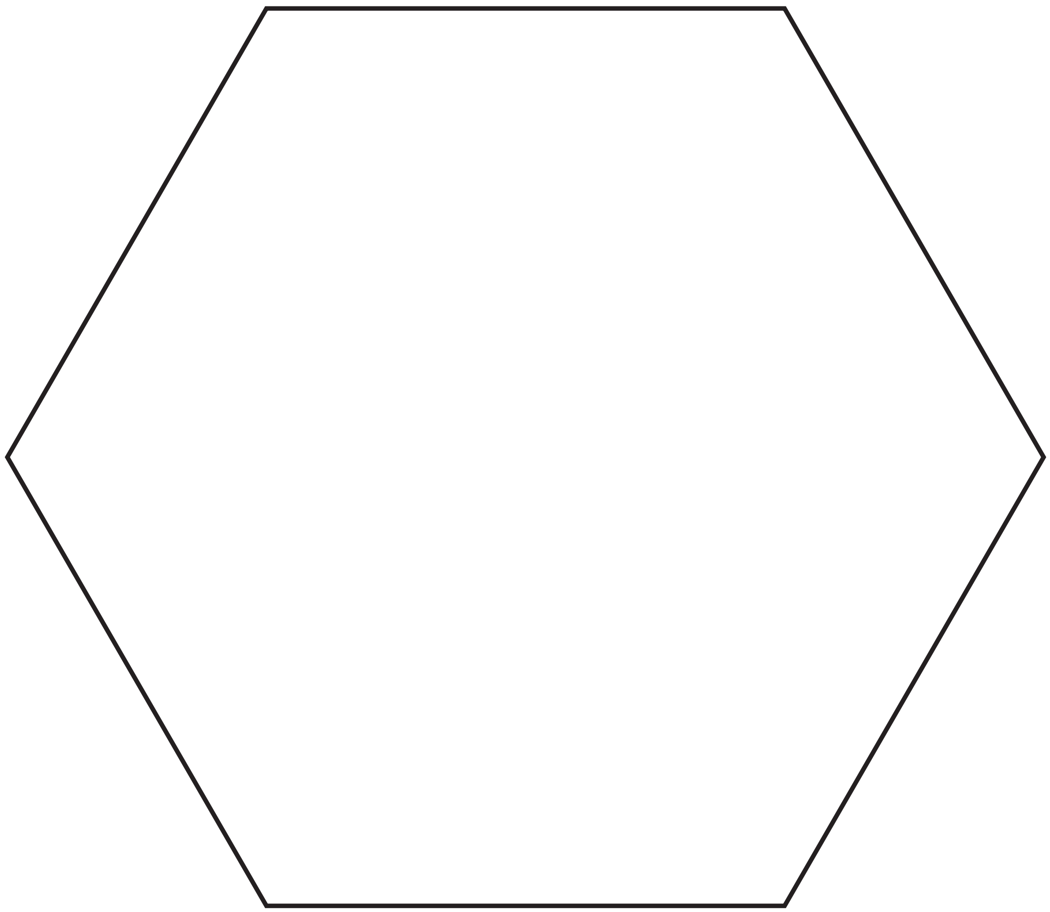
Name____________________________

2. Mr. Burke’s class is taking part in a contest to create a new recess game. Their game is called “Eight Ball Bounce”, which is like four square but it is played in a circle with eight players instead of four. How can Mr. Burke’s class divide the circle into eight equal-size parts?



Name______________________________

3. Julie wants to share her cake with five of her friends. How can Julie cut her cake so there are six equal-size parts?



Name______________________________

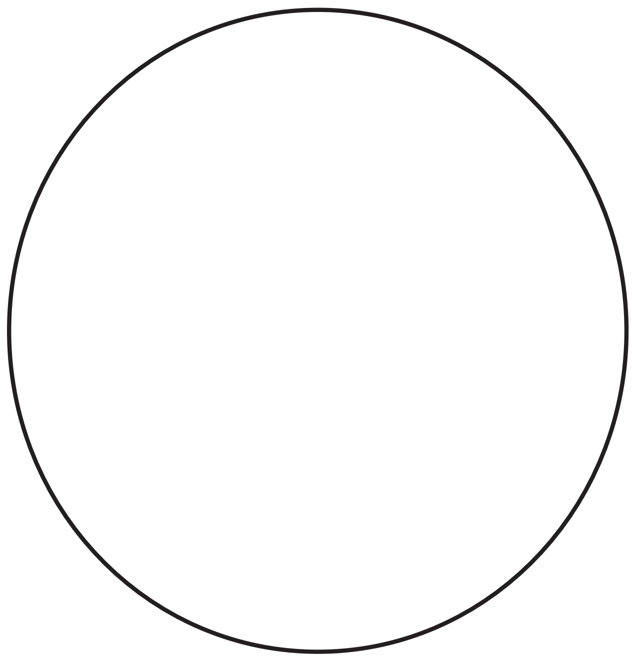
4. Divide the following rectangles into four equal-size parts in two different ways.

Name______________________________

5. Use the following shapes to divide and identify unit fractions that are given on the unit fraction cards.

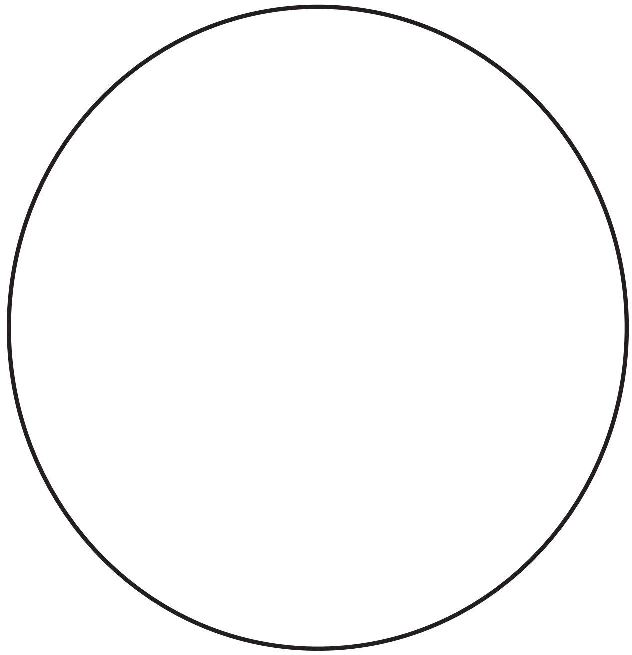
- Write the name of the unit fraction above the shape.
- Color in the unit fraction dividing the shape.

Unit fraction:

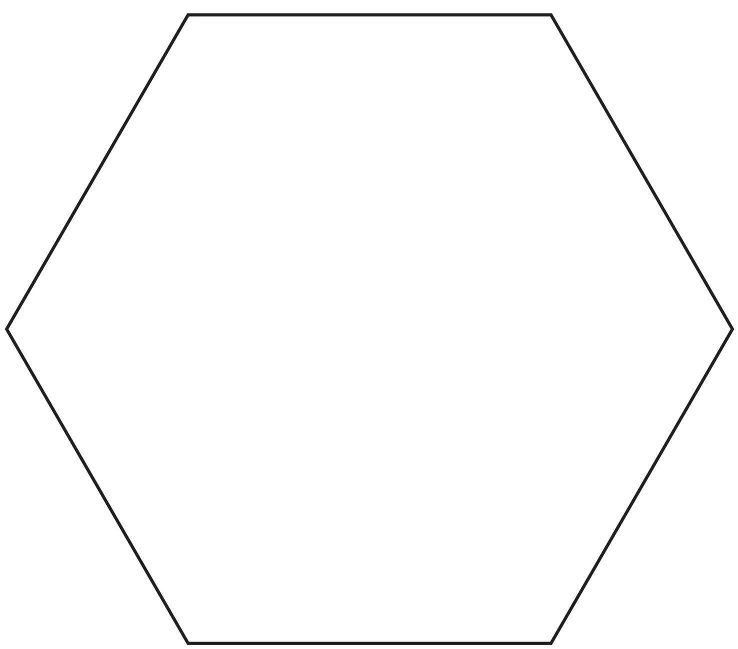


Name_______________________________

Unit fraction:

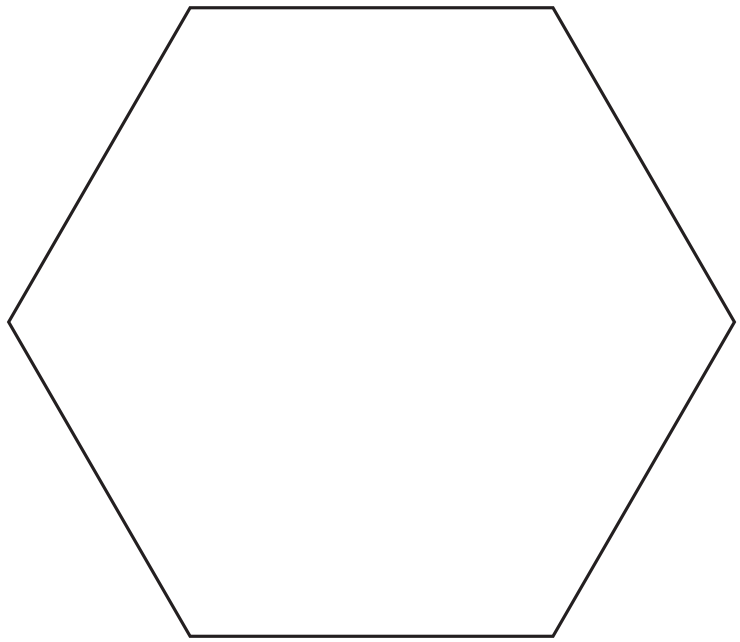


Unit fraction:

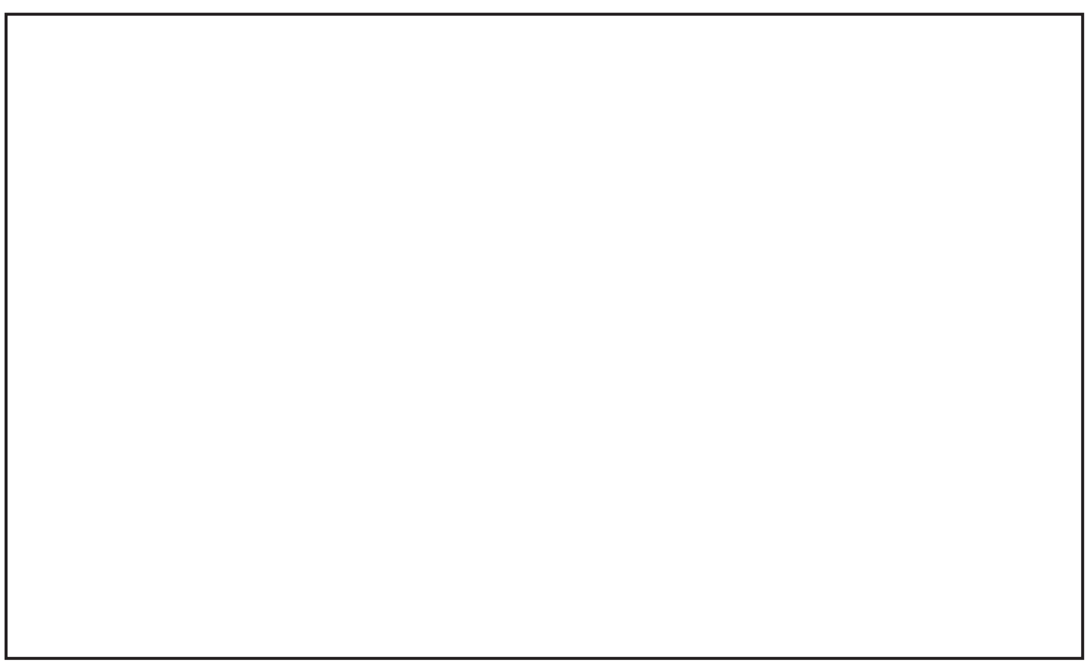


Name________________________________

Unit fraction:

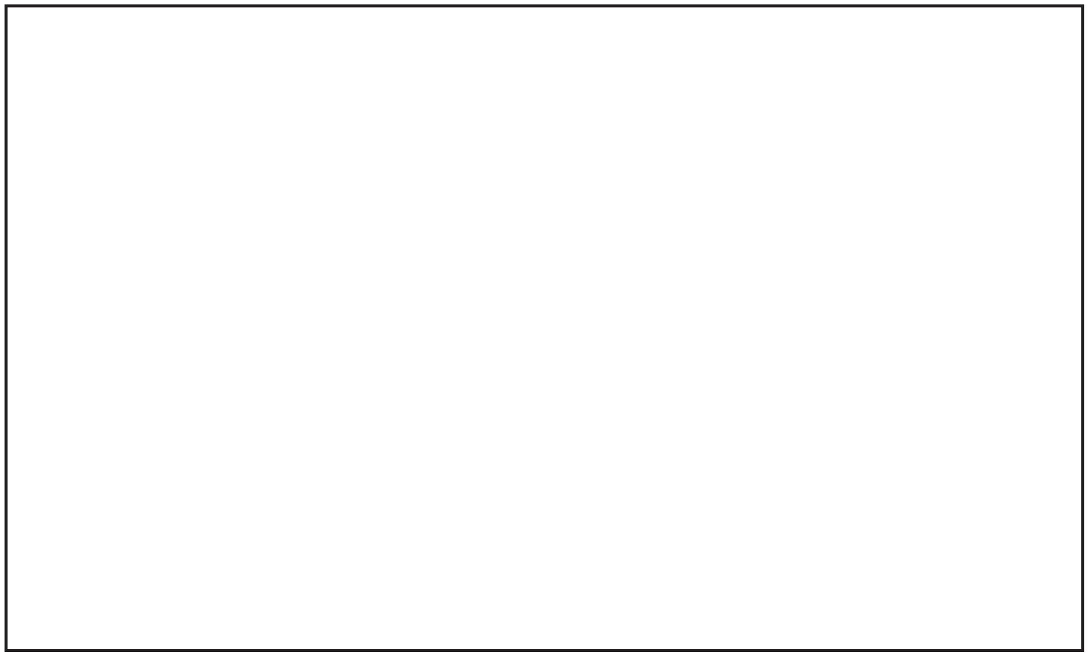


Unit fraction:

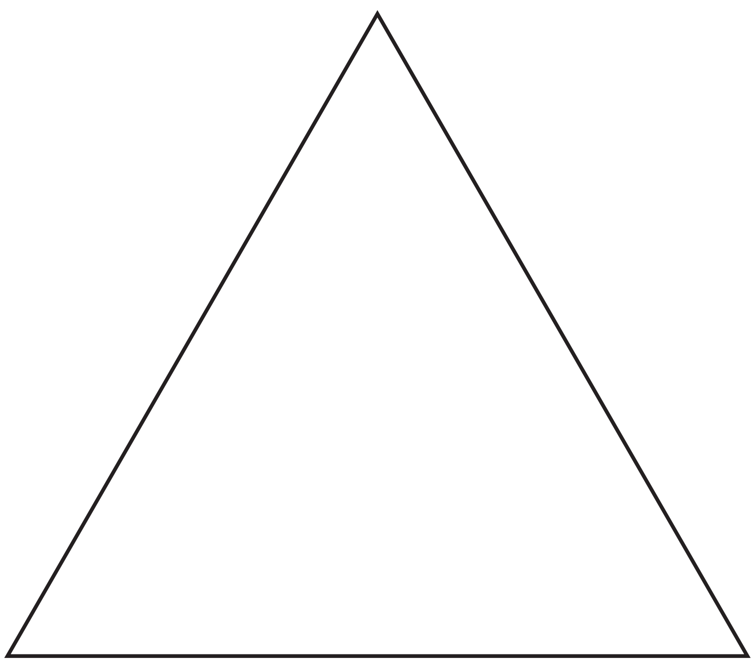


Name______________________________

Unit fraction:

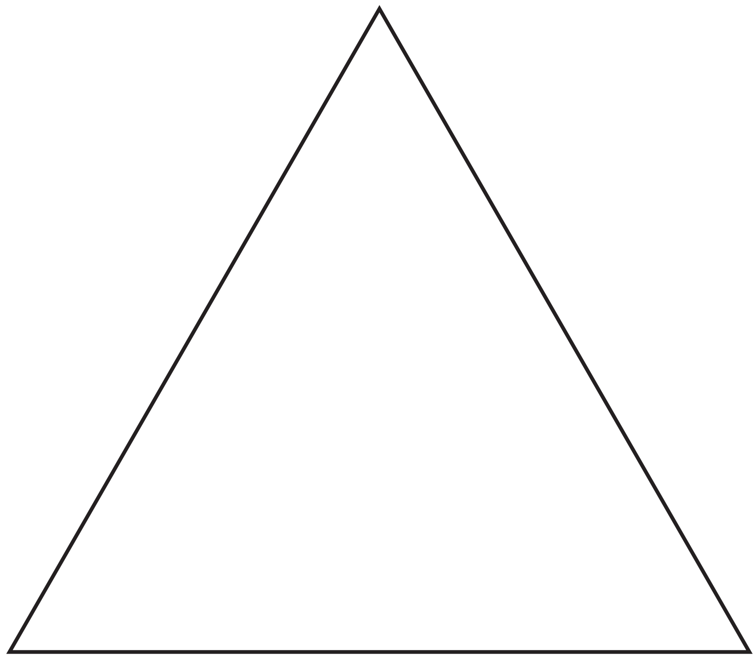


Unit fraction:

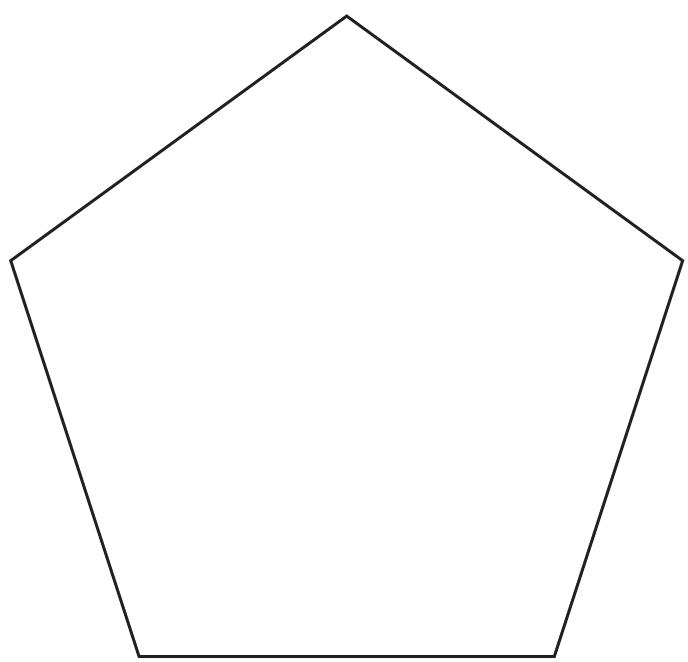


Name______________________________

Unit fraction:

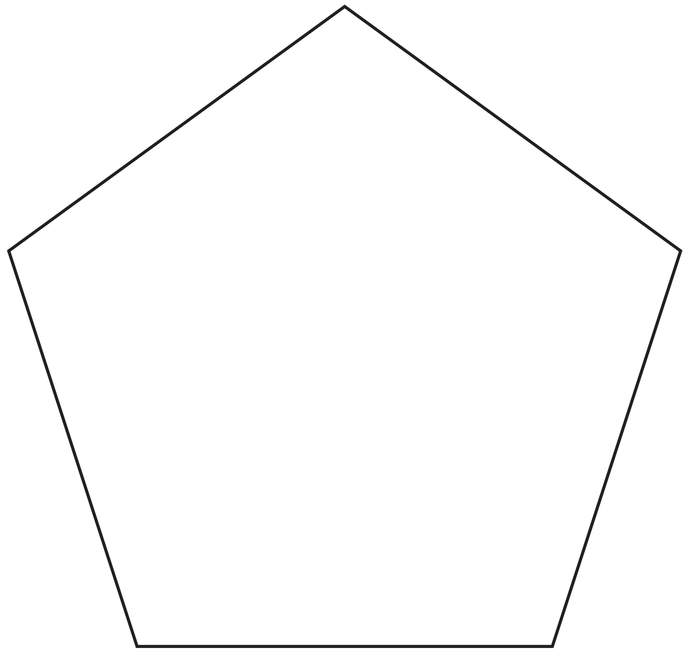


Unit fraction:

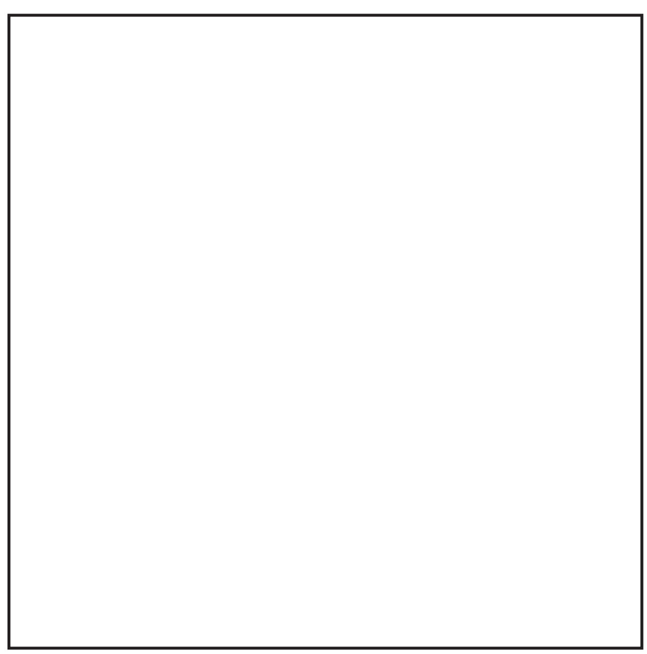


Name______________________________

Unit fraction:



Unit fraction:



One-half	One-half
One-half	One-half
One-fourth	One-fourth
One-fourth	One-fourth
One-eighth	One-eighth
One-eighth	One-eighth
One-twelfth	One-third
One-third	One-third
One-sixth	One-sixth
One-sixth	One-fifth
One-fifth	One-ninth

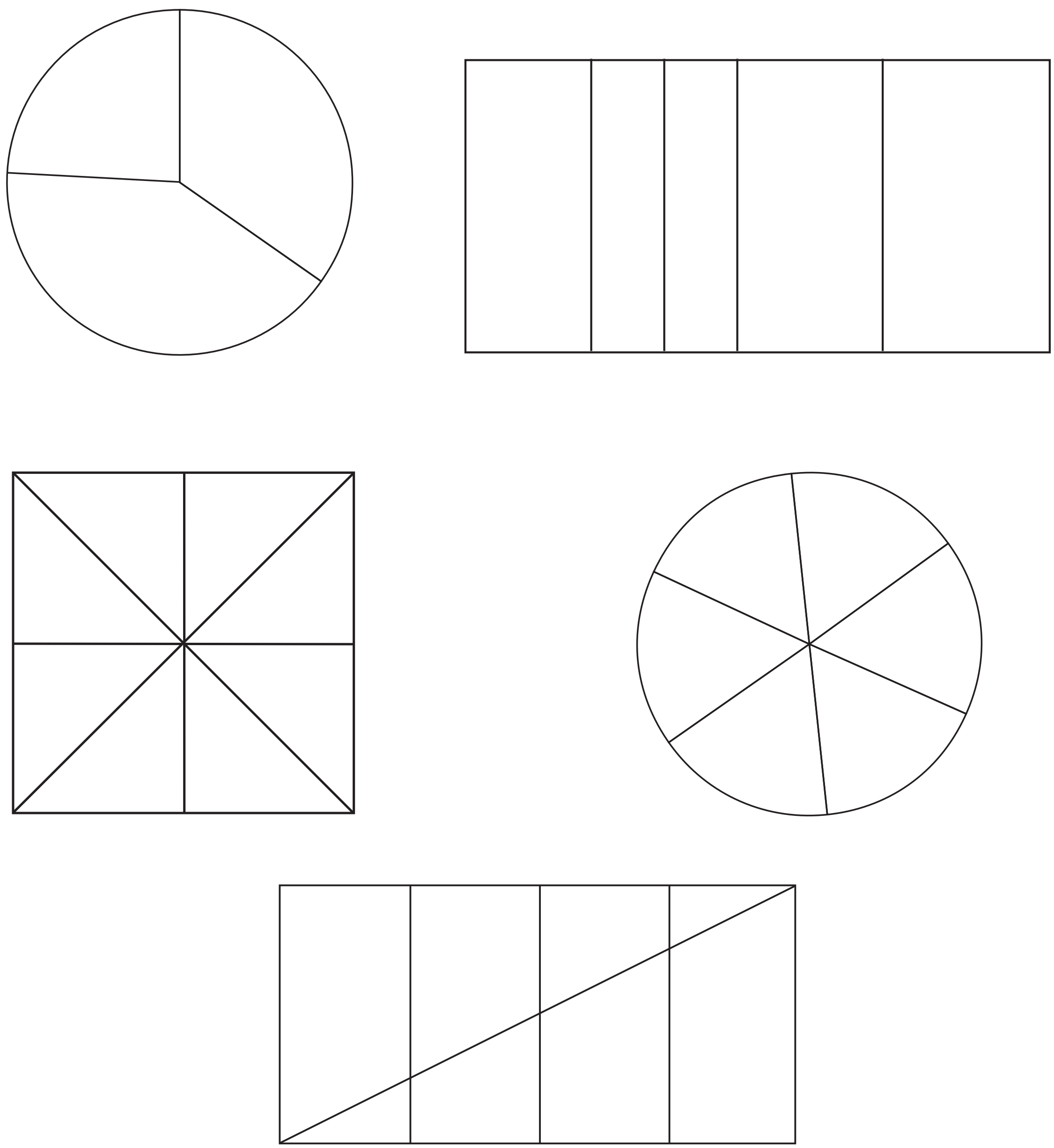
Name____

Partitioning Shapes and Unit Fractions

Lessons 1 – 3

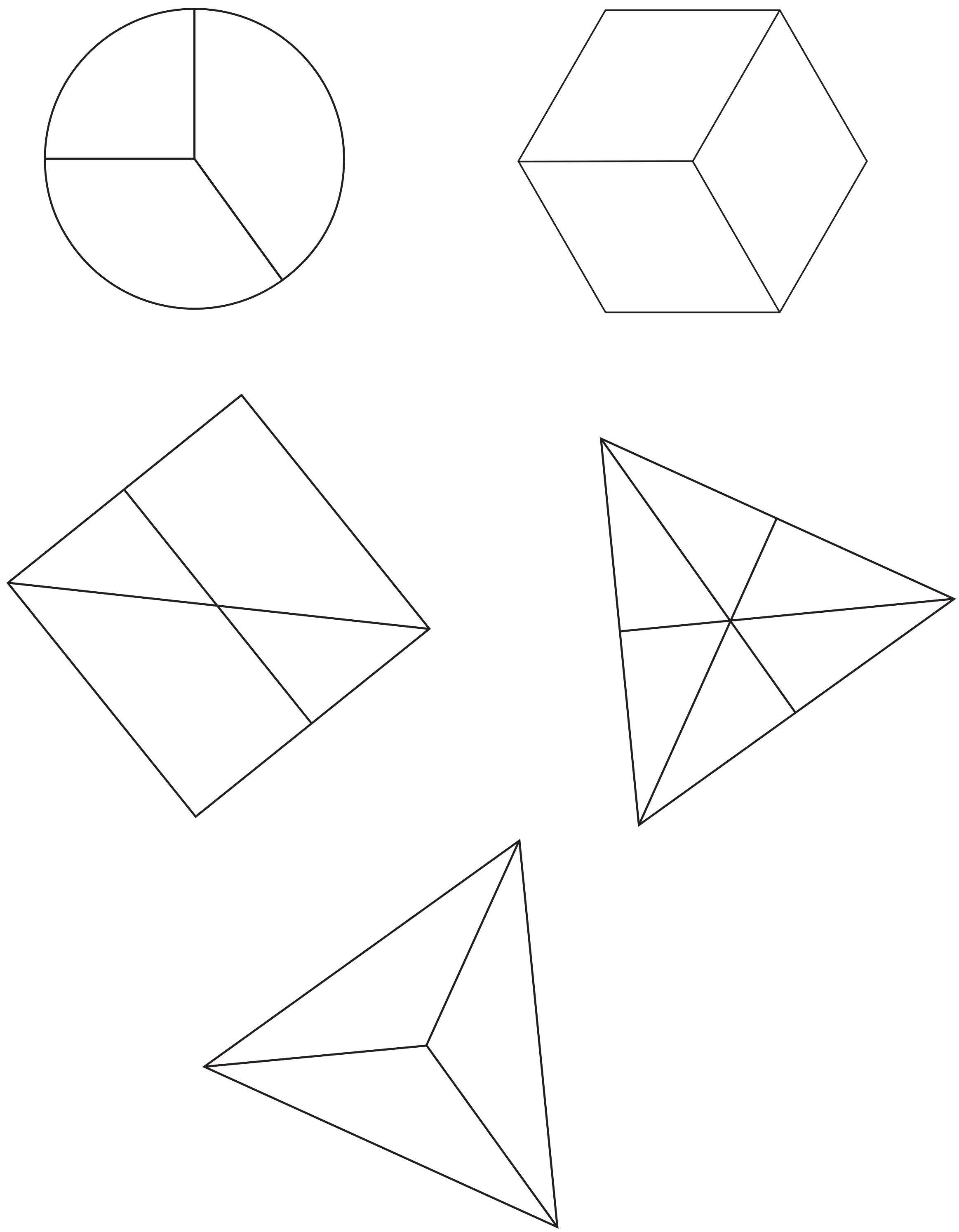
1. Circle the shapes that are divided into equal-size parts.

1.a.



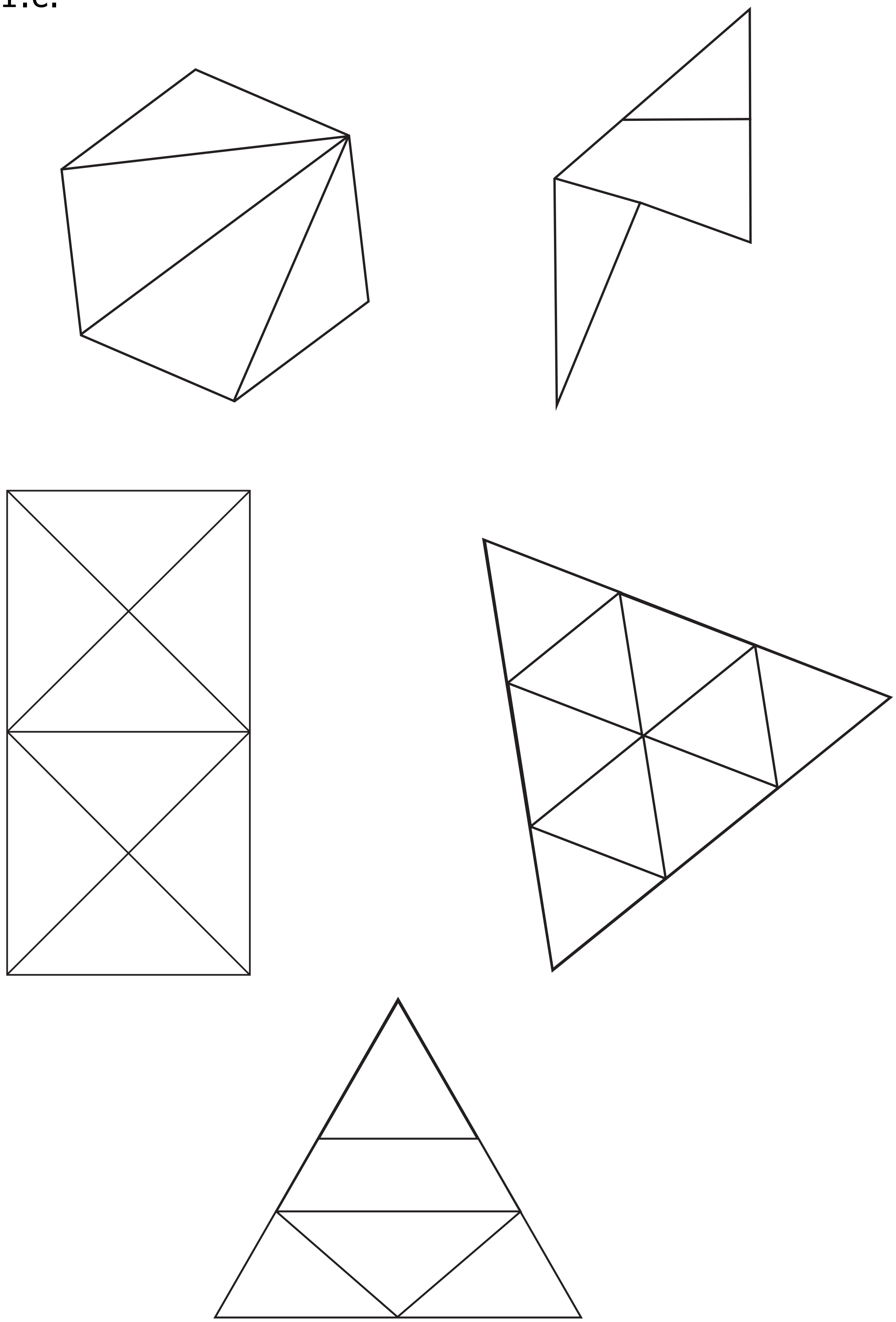
Name____

1.b.



Name____

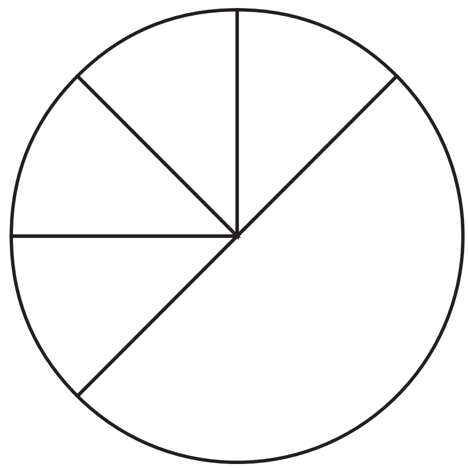
1.c.



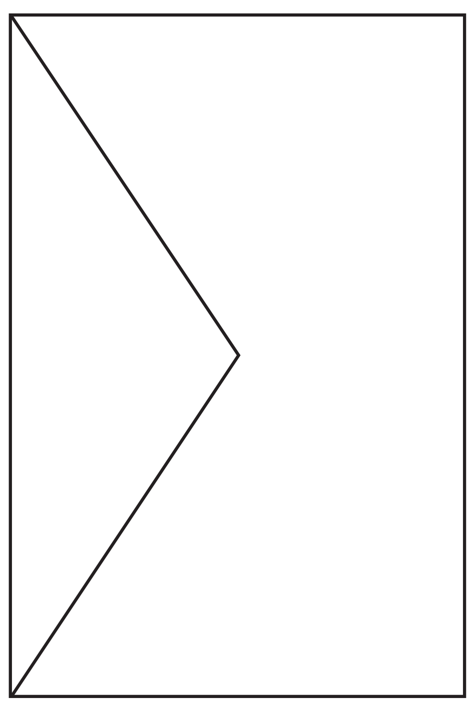
Name_______________________________________

2. Finish dividing the shapes into equal-size parts.

2.a.

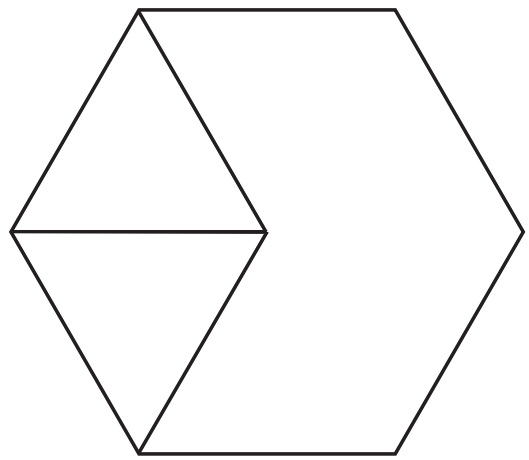


2.b.

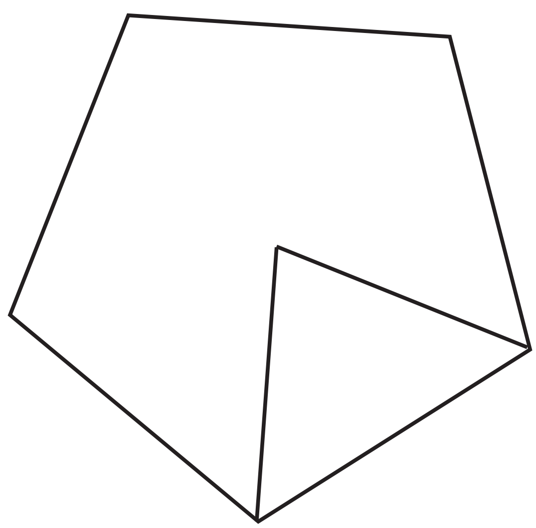


Name____

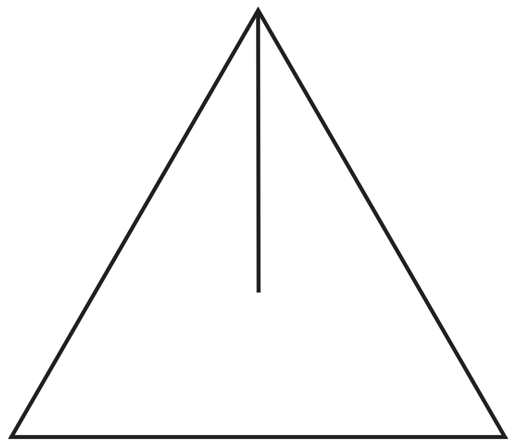
2.c.



2.d.

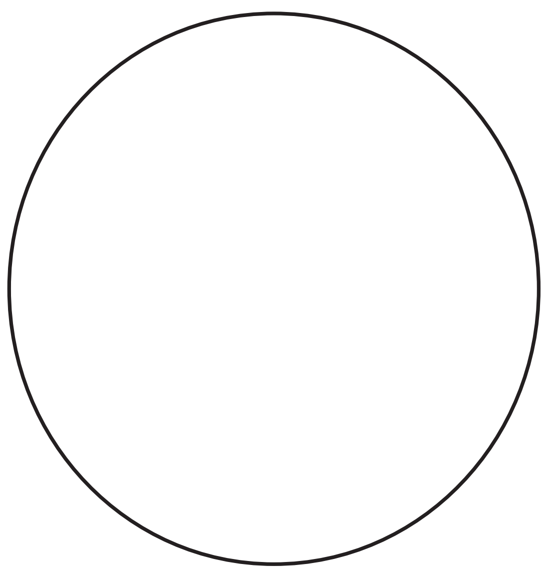


2.e.



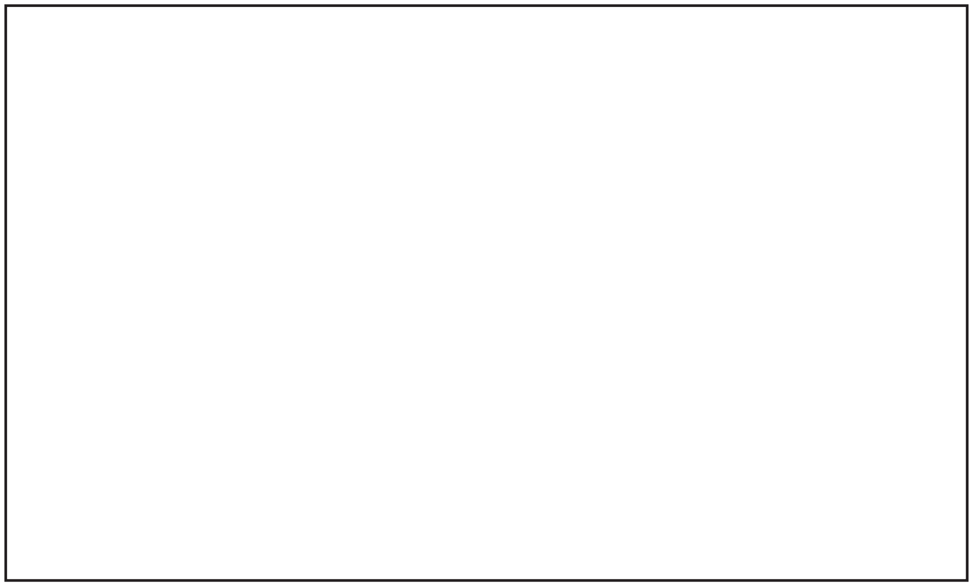
3. Divide the shapes into the given number of equal-size parts.

3.a. Eighths

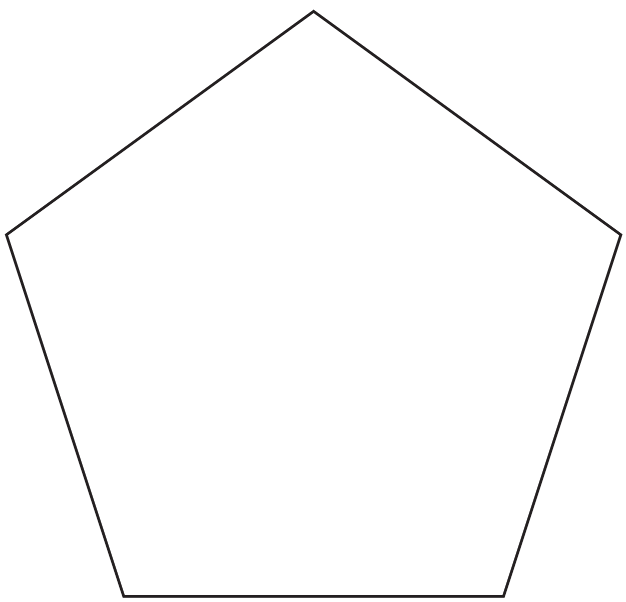


Name____

3.b. Fourths

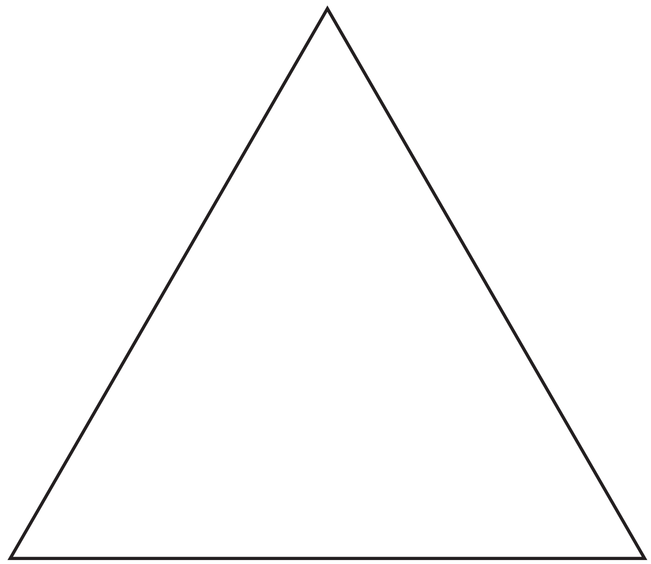


3.c. Halves

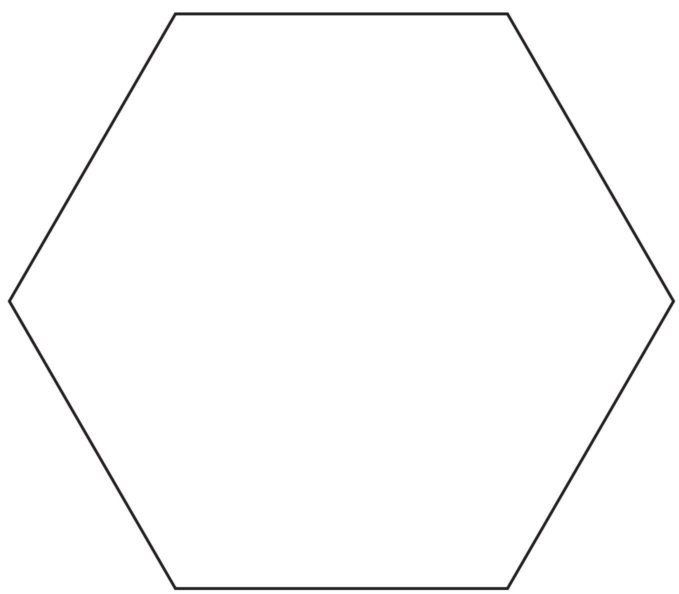


Name____

3.d. Thirds

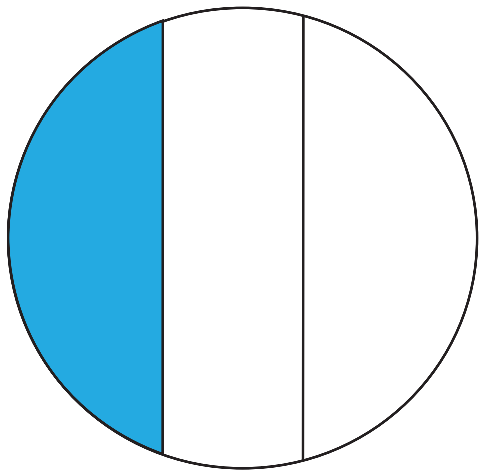


3.e. Sixths



Name____

4. Pete has divided the circle into three parts and colored one part to model one-third. Is Pete's work correct? If yes, explain how you know. If no, explain how you know and draw the correct solution.



Name____

5. Match the divided shape with the correct unit fraction.

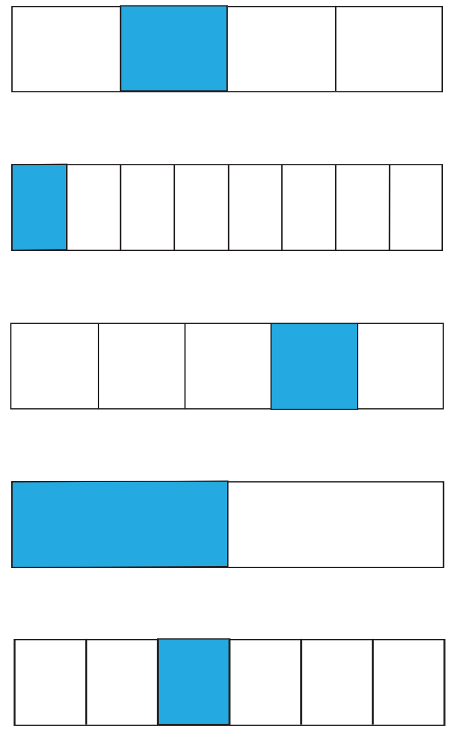
one-half

one-fourth

one-fifth

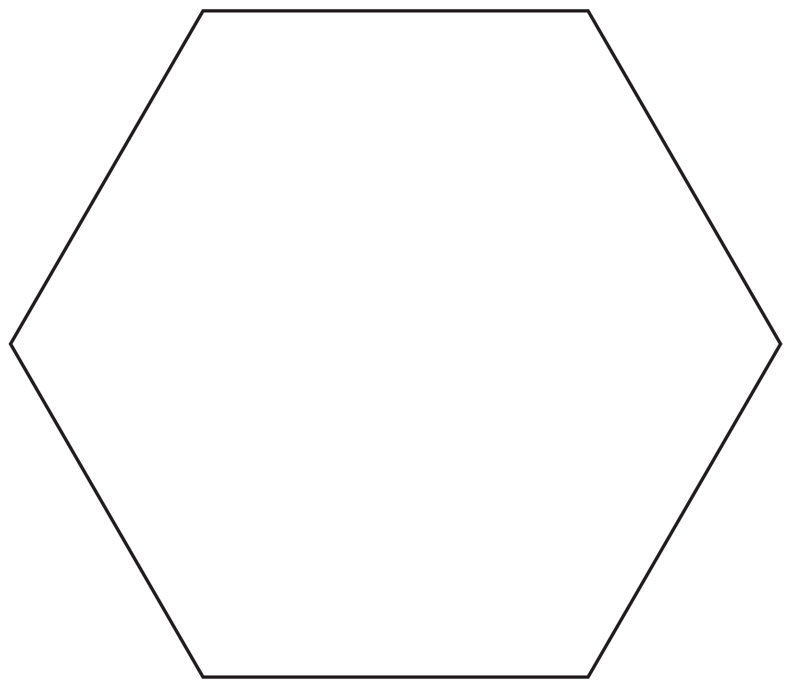
one-sixth

one-eighth



Name____

6. Jenny is making a game board for her math class. The board is shaped like a hexagon. Jenny wants to have four-equal size parts, each a different color. One-fourth of the board will be blue. Use the hexagon to show the blue section of Jenny's game board.

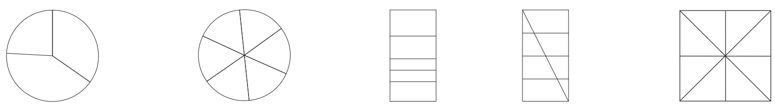


Partitioning Shapes and Unit Fractions

STUDENT ACTIVITY SOLUTION GUIDE

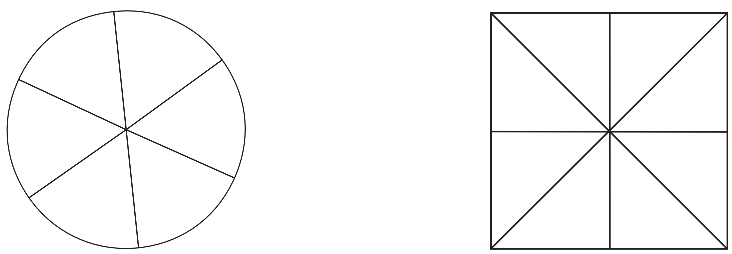
Lessons 1 – 3

1. Circle the shapes that are divided into equal-size parts.

1.a. 

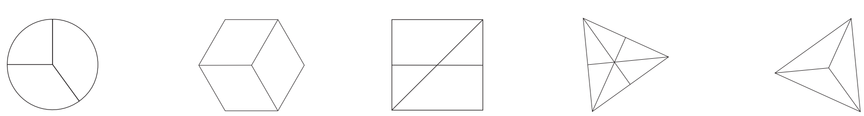
CORRECT ANSWER

The following shapes should be circled:



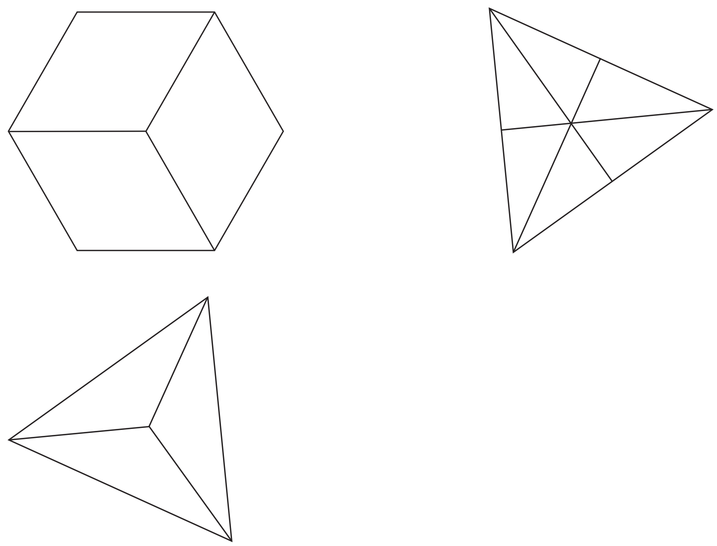
ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student selects shapes that are not partitioned into equal-size pieces. *or* The student does not include both of the shapes that are partitioned correctly.	does not recognize parts that have the same area	RECOGNIZE *EQUAL PART*
The student does not answer the question.	does not understand what the question is asking *or* does not recognize that two of the shapes are partitioned into equal-size parts	RECOGNIZE *EQUAL PART*

1.b. 

CORRECT ANSWER

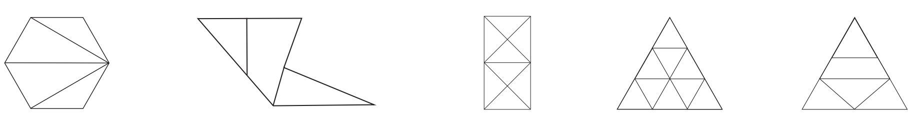
The following shapes should be circled:



ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

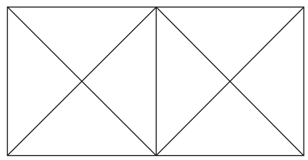
Example Error	Misconception	Missing Knowledge
The student selects shapes that are not partitioned into equal-size pieces. *or* The student does not include all of the shapes that are partitioned correctly.	does not recognize parts that have the same area	RECOGNIZE *EQUAL PART*
The student does not answer the question.	does not understand what the question is asking *or* does not recognize that three of the shapes are partitioned into equal-size parts	RECOGNIZE *EQUAL PART*

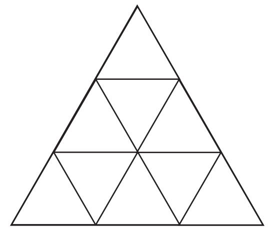
1.c.



CORRECT ANSWER

The following shapes should be circled:





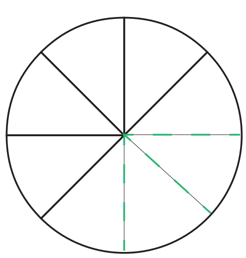
ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student selects shapes that are not partitioned into equal-size pieces. *or* The student does not include both of the shapes that are partitioned correctly.	does not recognize parts that have the same area	RECOGNIZE *EQUAL PART*
The student does not answer the question.	does not understand what the question is asking *or* does not recognize that two of the shapes are partitioned into equal-size parts	RECOGNIZE *EQUAL PART*

2. Finish dividing the shapes into equal-size parts.

2.a.

CORRECT ANSWER

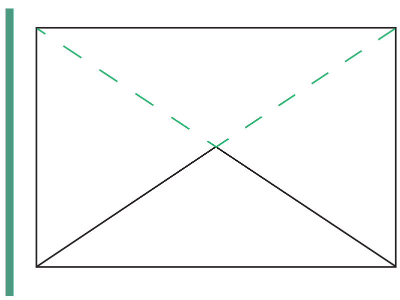


ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student partitions the remaining half of the shape into two equal parts.	does not recognize that the whole circle should be partitioned into eight equal parts	MODEL *EQUAL PART*
The student partitions the remaning half of the shape into parts that are not equal in size.	does not recognize that parts of the whole need to be equal in size	RECOGNIZE *EQUAL PART* and MODEL *EQUAL PART*
The student does not answer the question.	does not understand what the question is asking	RECOGNIZE *EQUAL PART* and MODEL *EQUAL PART*

2.b.

CORRECT ANSWER

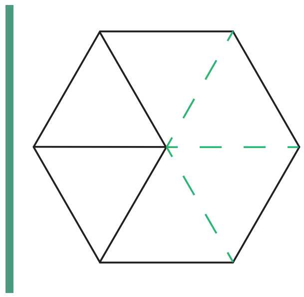


ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student partitions the remaining half of the shape into two equal parts by drawing one line segment.	does not recognize that the whole rectangle should be partitioned into four equal parts	MODEL *EQUAL PART*
The student partitions the remaning portion of the shape into parts that are not equal in size.	does not recognize that parts of the whole need to be equal in size	RECOGNIZE *EQUAL PART* and MODEL *EQUAL PART*
The student does not answer the question.	does not understand what the question is asking	RECOGNIZE *EQUAL PART* and MODEL *EQUAL PART*

2.c.

CORRECT ANSWER

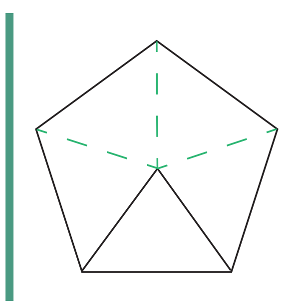


ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student partitions the remaining half of the shape into two or four equal parts.	does not recognize that the whole hexagon should be partitioned into six equal parts	MODEL *EQUAL PART*
The student partitions the remaning half of the shape into parts that are not equal in size.	does not recognize that parts of the whole need to be equal in size	RECOGNIZE *EQUAL PART* and MODEL *EQUAL PART*
The student does not answer the question.	does not understand what the question is asking	RECOGNIZE *EQUAL PART* and MODEL *EQUAL PART*

2.d.

CORRECT ANSWER

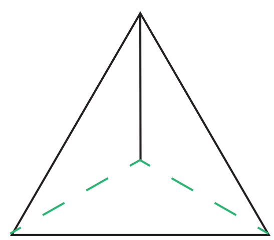


ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student partitions the remaining portion of the shape into two or four equal parts.	does not recognize that the whole pentagon should be partitioned into five equal parts	MODEL *EQUAL PART*
The student partitions the remaning portion of the shape into parts that are not equal in size.	does not recognize that parts of the whole need to be equal in size	RECOGNIZE *EQUAL PART* and MODEL *EQUAL PART*
The student does not answer the question.	does not understand what the question is asking	RECOGNIZE *EQUAL PART* and MODEL *EQUAL PART*

2.e.

CORRECT ANSWER



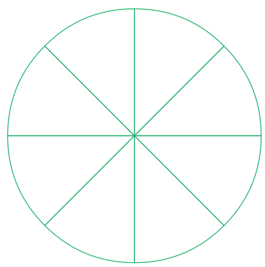
ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student partitions the remaining portion of the shape into two equal parts, dividing the whole triangle into two equal parts. *NOTE:* This solution, although mathematically correct, does not meet the intent of the question.	does not recognize that the whole triangle should be partitioned into three equal parts	MODEL *EQUAL PART*
The student partitions the remaning portion of the shape into parts that are not equal in size.	does not recognize that parts of the whole need to be equal in size	RECOGNIZE *EQUAL PART* and MODEL *EQUAL PART*
The student does not answer the question.	does not understand what the question is asking	RECOGNIZE *EQUAL PART* and MODEL *EQUAL PART*

3. Divide the shapes into the given number of equal-size parts.

3.a. Eighths

CORRECT ANSWER



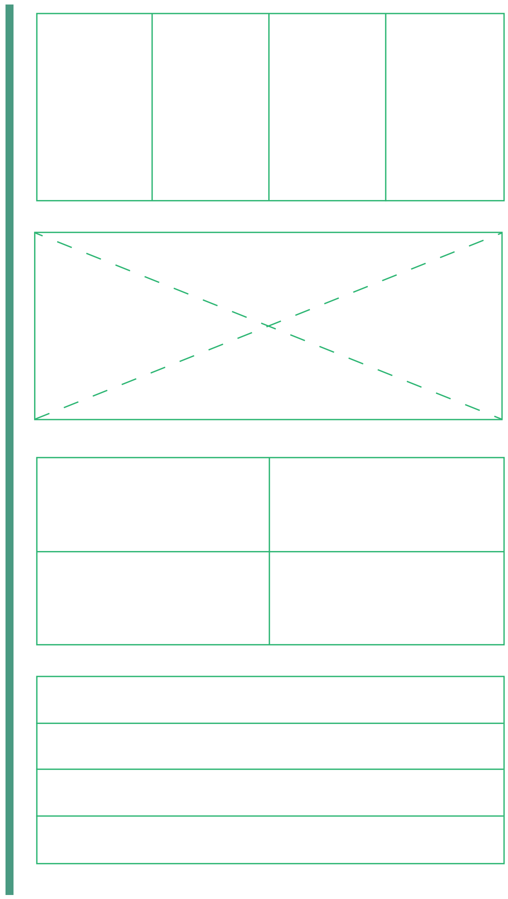
ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student partitions the circle into equal-size parts other than eighths.	does not understand that eighths signify eight equal-size parts	PARTITION ANY SHAPE INTO EQUAL PARTS
The student partitions the circle into eight parts that are not the same size.	does not understand that fractional parts must all be the same size	MODEL *EQUAL PART* and PARTITION ANY SHAPE INTO EQUAL PARTS
The student partitions the circle into a number other than eight and the parts are not equal in size.	does not understand that fractional parts must be the same size and that eighths signify eight equal-size parts	MODEL *EQUAL PART* and PARTITION ANY SHAPE INTO EQUAL PARTS
The student does not answer.	does not understand what the question is asking	PARTITION ANY SHAPE INTO EQUAL PARTS

3.b. Fourths

CORRECT ANSWER

Check student work for understanding. The following are examples of possible student solutions.



ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student partitions the rectangle into equal-size parts other than fourths.	does not understand that fourths signify four equal-size parts	PARTITION ANY SHAPE INTO EQUAL PARTS
The student partitions the rectangle into four parts that are not the same size.	does not understand that fractional parts must all be the same size	MODEL *EQUAL PART* and PARTITION ANY SHAPE INTO EQUAL PARTS
The student partitions the rectangle into a number other than four and the parts are not equal in size.	does not understand that fractional parts must be the same size and that fourths signify four equal-size parts	MODEL *EQUAL PART* and PARTITION ANY SHAPE INTO EQUAL PARTS
The student does not answer.	does not understand what the question is asking	PARTITION ANY SHAPE INTO EQUAL PARTS

3.c. Halves

CORRECT ANSWER

Check student work for understanding. The following are examples of possible student solutions.

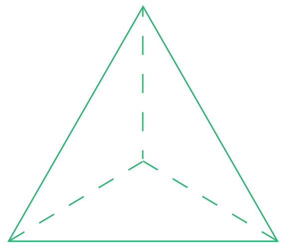
ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student partitions the pentagon into equal-size parts other than halves.	does not understand that halves signify two equal-size parts	PARTITION ANY SHAPE INTO EQUAL PARTS
The student partitions the pentagon into two parts that are not the same size.	does not understand that fractional parts must all be the same size	MODEL *EQUAL PART* and PARTITION ANY SHAPE INTO EQUAL PARTS
The student partitions the pentagon into a number other than two and the parts are not equal in size.	does not understand that fractional parts must be the same size and that halves signify two equal-size parts	MODEL *EQUAL PART* and PARTITION ANY SHAPE INTO EQUAL PARTS
The student does not answer.	does not understand what the question is asking	PARTITION ANY SHAPE INTO EQUAL PARTS

3.d. Thirds

CORRECT ANSWER

Check student work for understanding. The following is an example of a possible student solution.



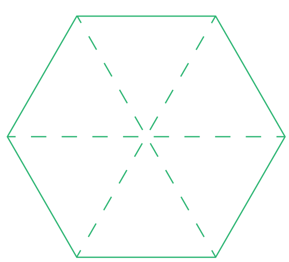
ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student partitions the triangle into equal-size parts other than thirds.	does not understand that thirds signify three equal-size parts	PARTITION ANY SHAPE INTO EQUAL PARTS
The student partitions the triangle into three parts that are not the same size.	does not understand that fractional parts must all be the same size	MODEL *EQUAL PART* and PARTITION ANY SHAPE INTO EQUAL PARTS
The student partitions the triangle into a number other than three and the parts are not equal in size.	does not understand that fractional parts must be the same size and that thirds signify three equal-size parts	MODEL *EQUAL PART* and PARTITION ANY SHAPE INTO EQUAL PARTS
The student does not answer.	does not understand what the question is asking	PARTITION ANY SHAPE INTO EQUAL PARTS

3.e. Sixths

CORRECT ANSWER

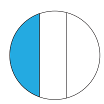
Check student work for understanding. The following is an example of a possible student solution.



ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

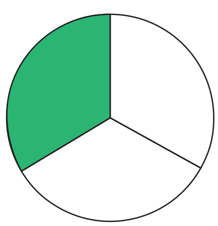
Example Error	Misconception	Missing Knowledge
The student partitions the hexagon into equal-size parts other than sixths.	does not understand that sixths signify six equal-size parts	PARTITION ANY SHAPE INTO EQUAL PARTS
The student partitions the hexagon into six parts that are not the same size.	does not understand that fractional parts must all be the same size	MODEL *EQUAL PART* and PARTITION ANY SHAPE INTO EQUAL PARTS
The student partitions the hexagon into a number other than six and the parts are not equal in size.	does not understand that fractional parts must be the same size and that sixths signify six equal-size parts	MODEL *EQUAL PART* and PARTITION ANY SHAPE INTO EQUAL PARTS
The student does not answer.	does not understand what the question is asking	PARTITION ANY SHAPE INTO EQUAL PARTS

4. Pete has divided the circle into three parts and colored one part to model one-third. Is Pete's work correct? If yes, explain how you know. If no, explain how you know and draw the correct solution.



CORRECT ANSWER

No, Pete's drawing is not correct. The three parts do not have the same area and therefore are not equal-size parts.

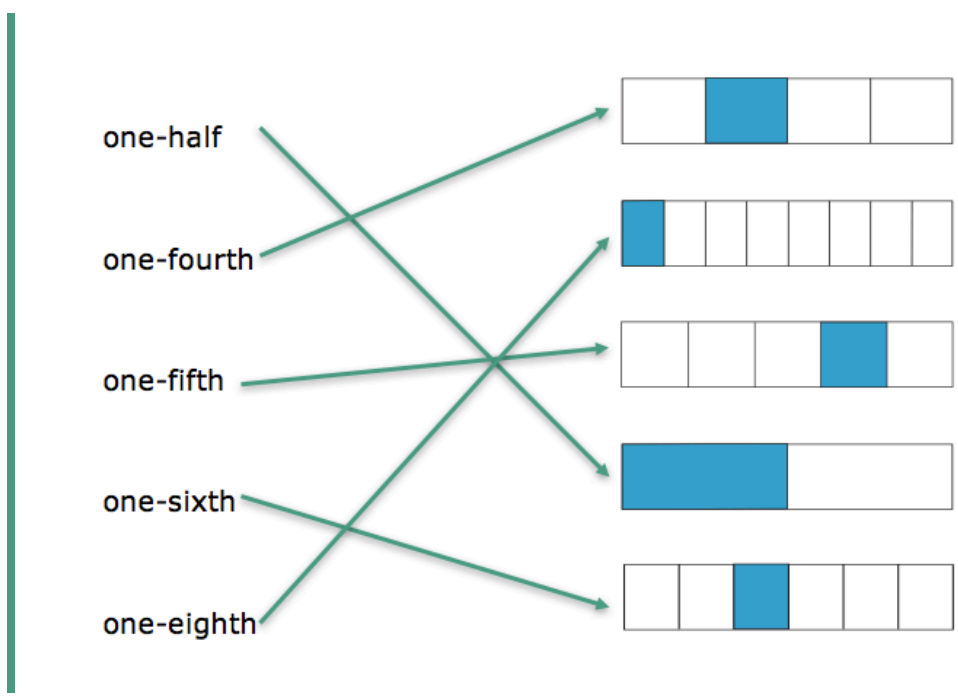


ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student states that Pete's work is correct.	does not understand that the three parts must be the same size or does not recognize that the three parts are not the same size in Pete's drawing	RECOGNIZE *EQUAL PART*
The student identifies that Pete's work is not correct but does not provide a correct model or does not provide a model at all.	does not know how to represent three equal parts with a circle model	PARTITION ANY SHAPE INTO EQUAL PARTS and/or MODEL *EQUAL PART*
The student states that Pete's work is incorrect and provides a correct model but does not include a reason for why Pete's work is incorrect.	does not know how to explain that the model is partitioned into part that are not equal in size	RECOGNIZE *EQUAL PART*
The student does not answer the question.	does not understand what the question is asking	RECOGNIZE *EQUAL PART* and PARTITION ANY SHAPE INTO EQUAL PARTS

5. Match the divided shape with the correct unit fraction.

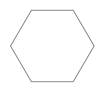
CORRECT ANSWER



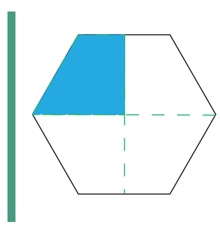
ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student does not connect the correct unit fraction names to the models.	does not recognize the model and/or does not recognize the fraction name	RECOGNIZE *EQUAL PART* and/or EXPLAIN *UNIT FRACTION*
The student does not answer the question.	does not understand what the question is asking	RECOGNIZE *EQUAL PART* and/or EXPLAIN *UNIT FRACTION*

6. Jenny is making a game board for her math class. The board is shaped like a hexagon. Jenny wants to have four equal-size parts, each a different color. One-fourth of the board will be blue. Use the hexagon to show the blue section of Jenny's game board.



CORRECT ANSWER



ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student does not divide the hexagon into four equal-size parts but does color one part blue.	does not understand how to create four equal-size parts with a hexagon	PARTITION ANY SHAPE INTO EQUAL PARTS
The student partitions the shape correctly but does not color one-fourth of the hexagon in blue or colors more than one part of the hexagon blue.	does not understand that only one part of the partitioned hexagon is the unit fraction one-fourth	EXPLAIN *UNIT FRACTION*
The student does not parition the shape correctly and does not color one part blue.	does not understand how to create four equal-size parts and does not understand that one of those parts represents the unit fraction one-fourth	PARTITION ANY SHAPE INTO EQUAL PARTS and EXPLAIN *UNIT FRACTION*
The student does not answer the question.	does not understand what the question is asking	PARTITION ANY SHAPE INTO EQUAL PARTS and EXPLAIN *UNIT FRACTION*