MULTIPLYING FRACTIONS AND WHOLE NUMBERS

4.NF.4

CONTENTS

The types of documents contained in the unit are listed below. Throughout the unit, the documents are arranged by lesson.

LEARNING MAP INFORMATION	An overview of the standards, the learning map section, and the nodes addressed in this unit
TEACHER NOTES	A brief discussion describing the progression depicted in the learning map section with research-based recommendations for focusing instruction to foster student learning and an introduction to the unit's lessons
OVERVIEW OF INSTRUCTIONAL ACTIVITIES	A table highlighting the lesson goals and nodes addressed in each lesson of this unit
INSTRUCTIONAL ACTIVITY	A detailed walkthrough of the unit
INSTRUCTIONAL ACTIVITY STUDENT HANDOUT	A handout for the guided activity, intended to be paired with the Instructional Activity
INSTRUCTIONAL ACTIVITY SUPPLEMENT	A collection of materials or activities related to the Instructional Activity
STUDENT ACTIVITY	A work-alone activity for students
STUDENT ACTIVITY SOLUTION GUIDE	A solution guide for the work-alone activity with example errors, misconceptions, and links to the learning map section

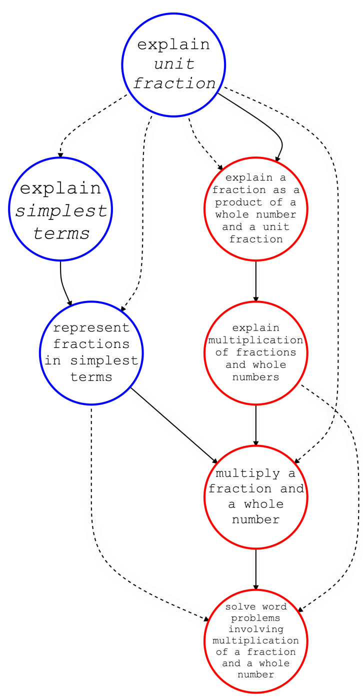
MULTIPLYING FRACTIONS AND WHOLE NUMBERS

LEARNING MAP INFORMATION

STANDARDS

4.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.

> **4.NF.4a** Understand a fraction a/b as a multiple of $1/b$. *For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).*
>
> **4.NF.4b** Understand a multiple of a/b as a multiple of $1/b$, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b).*
>
> **4.NF.4c** Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?*



**Learning map model of* 4.NF.4

Node Name	Node Description
EXPLAIN A FRACTION AS A PRODUCT OF A WHOLE NUMBER AND A UNIT FRACTION	Make known your understanding that a fraction $\frac{a}{b}$ is a multiple of $\frac{1}{b}$. For example, $\frac{5}{4}$ is $5 \times \frac{1}{4}$.
EXPLAIN MULTIPLICATION OF FRACTIONS AND WHOLE NUMBERS	Make known your understanding that when you multiply a fraction and whole number, you must write the whole number as a fraction with a denominator of one, and then multiply the numerators and multiply the denominators.
EXPLAIN *SIMPLEST TERMS*	Make known your understanding that a fraction is in simplest terms when it cannot be written as an equivalent fraction with a smaller denominator.
EXPLAIN *UNIT FRACTION*	Make known your understanding that a unit fraction, $\frac{1}{n}$, is the quantity formed by one part when a whole is partitioned into *n* equal parts.
MULTIPLY A FRACTION AND A WHOLE NUMBER	Multiply a fraction by a whole number. For example, $3 \times \frac{2}{5}$.
REPRESENT FRACTIONS IN SIMPLEST TERMS	Through writing or an appropriate assistive technology, represent a given fraction in simplest terms.
SOLVE WORD PROBLEMS INVOLVING MULTIPLICATION OF A FRACTION AND A WHOLE NUMBER	Use multiplication to solve word problems involving a fraction and a whole number.

MULTIPLYING FRACTIONS AND WHOLE NUMBERS

TEACHER NOTES

This unit includes the following documents:

- Learning Map Information
- Instructional Activity (five lessons)
- Instructional Activity Student Handout (Lessons 3 – 5)
- Instructional Activity Supplement (Lessons 1 – 2)
- Student Activity (WORD VERSION)
- Student Activity Solution Guide

In this unit, students will explore several properties of fractions in order to multiply fractions and whole numbers. Students learn to deconstruct a fraction into a whole number multiplied by a unit fraction. First, students build procedural fluency by transforming fractions into a whole number times a unit fraction (e.g., $\frac{7}{8} = 7 \times \frac{1}{8}$) and vice versa. Students then extend this knowledge to non-unit fractions, concluding that in general, $n \times \frac{a}{b} = \frac{n \times a}{b}$. Students will connect this new form of multiplication to their prior multiplication knowledge through the use of models and repeated addition. Throughout the learning process, students will work with visual fraction models in order to develop a well-rounded mental model of fractions.

RESEARCH

Students learning how to perform operations with fractions must do so from a deeply conceptual standpoint (Petit, Laird, Marsden & Ebby, 2016; Son, 2012; Van de Walle, Karp, Lovin, & Bay-Williams, 2014; Wu, 2001). Students who have not developed a conceptual understanding will struggle to differentiate between the different processes for performing operations with fractions (i.e. addition and subtraction versus multiplication versus division). Students without conceptual understanding will forget when a common denominator is necessary, or when to use the reciprocal, because the processes have no meaning beyond memorization. Multiplication with fractions should not be taught as a memorized algorithm, or associated with a keyword signaling its use, such as "of" (Ott, 1990; Van de Walle, et al., 2014; Witherspoon, 2014; Wu, 2001).

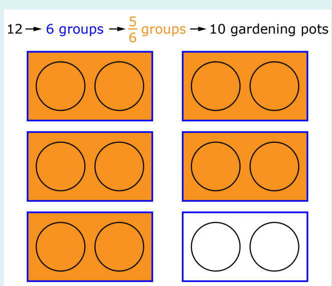
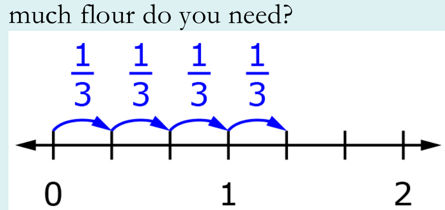
To aid in the development of a conceptual understanding of fractions, students should be presented with contextual problems as they are introduced to the idea of multiplication with fractions (Petit, et al., 2016; Stump, 2007; Van de Walle, et al., 2014; Wu, 2001). A "problem-based, number sense approach" will give students time to comprehend the meaning of multiplying fractions, and it will give students time develop their own intuitive structures before being introduced to a formal algorithm (Van de Walle, et al., pg. 232). Meaningful and relevant scenarios allow students to feel more comfortable making an attempt at solving problems, and students become more creative with their approaches and solutions (Britton, 2006; Stump, 2007; Van de Walle, et al., 2014; Wu, 2001). Using real-world contexts requires students to interpret answers in meaningful ways, providing another opportunity for the development of students' number sense. Students without a conceptual understanding of multiplying fractions can often correctly answer non-contextual problems, but they are unable to interpret those answers in an accurate conceptual sense. However, when students are presented with real-world scenarios, they are forced to think of the context of the situation and the reasonableness of numbers throughout the process.

Students can foster the development and use of mental models when they are required to estimate during the process of multiplying a whole number by a fraction (Petit, et al., 2016; Van de Wall, et al., 2014; Wyberg, Whitney, Cramer, Monson & Leavitt, 2012). Students should estimate the size of the fraction(s) in the problem situation as well as the approximate size of their answers. Having students estimate the size of their product before calculations and reflect on the size of their product after calculations will help students develop number sense. Estimation reinforces the idea that these are not calculations to be performed in isolation, but rather meaningful values that can be approximated and which follow the same customs as more familiar whole numbers.

The use of manipulatives and visual models will further solidify students' mental models for operations with fractions (Petit, et al., 2016; Witherspoon, 2014; Wyberg, et al., 2012). Fraction manipulatives and number lines are beneficial as students begin to grapple with multiplication of whole numbers and fractions. However, these tools should be used as a means to an end and should direct students' learning to the development of their own abstract mental models. A significant intermediate strategy for students is to draw pictures of their thinking in order to represent a problem this creates a bridge between physical representations and abstract thinking. In fact, using a variety of strategies improves students' procedural fluency (Son, 2012). Students should defend their answers using a variety of models to indicate that they fully understand the procedure and solution. The links between representations build into a greater mental model as well as a more robust comprehension of the generalized algorithm.

AN EXAMPLE

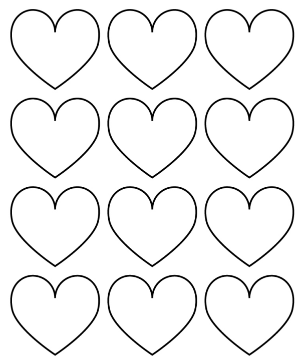
Students often struggle with operations involving fractions (Petit, Laird, Marsden & Ebby, 2016). The following strategies are suggested for student success in developing the ability to reason abstractly and develop number sense with fractions.

<table>
<tr><th></th><th>fraction × whole number</th><th>whole number × fraction</th></tr>
<tr><td>Use of Contextual Tasks</td><td>You have 12 shirts and $\frac{3}{4}$ of them are red. How many shirts are red?</td><td>You have 11 cookies and you want to share with three friends. How many does each person get?</td></tr>
<tr><td>Use of Estimation</td><td colspan="2">• Which is greater, $476\times\frac{3}{3}$ or $476\times\frac{1}{3}$?
• Will $\frac{2}{3}\times 12$ be greater or less than 6?
If you quintuple a recipe that calls for $\frac{3}{4}$ cup of flour, and you only have 5 cups of flour, do you have enough?</td></tr>
<tr><td>Use of Visual Models</td><td>Allie has 12 gardening pots and wants to fill $\frac{5}{6}$ of them with vegetables. How many of the gardening pots will she fill with vegetables?

</td><td>Your recipe calls for $\frac{1}{3}$ cup of flour, and you want to quadruple the recipe. How much flour do you need?

</td></tr>
<tr><td>Address Common Misconceptions</td><td colspan="2">• Does multiplication always create a larger number? Why or why not?
• Does multiplication by a fraction always create a lesser number? Why or why not?
• Why is it incorrect to multiply a whole number by both the numerator and denominator of a fraction?</td></tr>
</table>

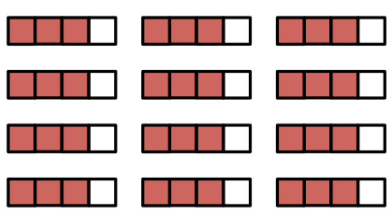
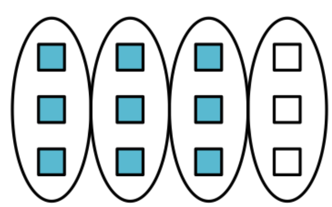
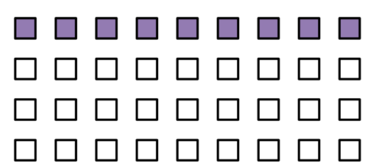
Multiplication of fractions should be developed from students' existing knowledge of multiplying whole numbers (Tsankova & Pjanic, 2010; Wu, 2001). Students' prior knowledge of multiplication of whole numbers is typically visualized as an array, which can be interpreted as a number of groups times the size of each group (Ott, 1990). For example, 4×3 is four groups of three, which lends itself easily to the idea of repeated addition: three added to itself four times, 3 + 3 + 3 + 3 (see Figure 1). When students encounter multiplication with fractions, this idea of grouping is often only applied when the whole number is the first factor (Son, 2012). For example, $10 \times \frac{1}{3}$ is thought of as "10 groups of $\frac{1}{3}$", however $\frac{1}{3} \times 10$ is often presented as "$\frac{1}{3}$ *of* 10", instead of "$\frac{1}{3}$ group of 10". To be consistent, Ott (1990) suggests a unified approach for all multiplication, including those in which one or both factors is a fraction. This approach states that the first factor is referred to as the number of groups needed (e.g., the number of rows in an array), and the second factor is referred to as the size of each group (e.g., the number of units in *each* row or the number of columns).

Figure 1

4 x 3 = four groups of three



However, $\frac{1}{3}$ group of 10 can be difficult for students to conceptualize, and students may find it difficult to represent or reason with one-third of a group. Son (2012) states that students can learn two different meanings of fraction and whole number multiplication: repeated addition and operator, or part of a whole. When the multiplication is in the form *whole number* × *fraction*, it is thought of as repeated addition, and when the multiplication is in the form *fraction* × *whole number*, it is thought of as an operator, or a part of a whole.

whole number × fraction	***fraction × whole number***	***fraction × whole number***
$\mathbf{12 \times \frac{3}{4}}$	$\frac{3}{4} \times 12$	$\frac{3 \times 12}{4}$
		
12 groups of $\frac{3}{4}$	**3 groups of 12 ÷ 4**	**1 group of 36 ÷ 4**

While multiplying a *whole number* × *fraction* can be a simple iterative process, according to Son (2012), multiplying a *fraction* × *whole number* requires students to understand two ideas. First, you are partitioning the whole number into as many groups as the denominator. Second, you are multiplying that result by the value of the numerator. For example $\frac{1}{4}\times12$ (one-fourth of 12) is equivalent to one group of $\frac{12}{4}$, or $12 \div 4$. Students should then extend this knowledge to multiply $\frac{3}{4}\times12$ by thinking of it as three groups of $\frac{12}{4}$, or $12 \div 4$. Symbolically, this is $3\times\frac{1}{4}\times12$, or $3\times(12 \div 4)$. In other words, multiplication becomes not just an iterative process, but a partitioning *and* iterative process (Wyberg, et al., 2012).

MISCONCEPTIONS

Teachers should be aware of the common misconceptions that students are likely to hold. Misconceptions should be addressed head-on through discussions, which is when students typically notice their mistakes and make corrections (Petit, et al., 2016; Van de Walle, et al., 2104; Wyberg, et al., 2012). Teachers should create a variety of experiences for students to explore various strategies and engage in meaningful discussions about why some strategies are more appropriate or more efficient than others. Often students over-apply whole number reasoning when performing operations with fractions. This results in students adding denominators when adding fractions or multiplying a whole number by the numerator and denominator of a fraction. Because students are operating with fractions for the first time and are prone to making errors as they develop new schema, just as much emphasis should be placed on incorrect strategies as on correct strategies.

Multiplying fractions has a deceivingly easy algorithm, but students often do not comprehend why you are able to multiply across numerators and multiply across denominators. The act of teaching "answer-getting calculations" to memorize or a keyword to indicate which operation to use "leaves huge gaps in the foundations of" student learning (Van de Walle, et al., 2014). Students should generate their own mental models to be further developed through discussion into generalized algorithms that work for all problems (Mann, 2005; Witherspoon, 2014).

LEARNING MAP INFORMATION

The learning map section for this sequence of activities requires that students be able to explain *unit fraction* before learning to explain that any fraction can be rewritten as a product of a whole number and a unit fraction. Once students have mastered multiplying unit fractions and whole numbers, they are ready to extend their understanding and develop the algorithm for multiplying fractions and whole numbers in general. This should be approached with the use of concrete models (i.e. manipulatives), semi-concrete models (i.e. visual representations), and reasoning with numbers, as well as estimation. Once the algorithm is developed through a variety of strategies and discussions, students are ready to multiply fractions and whole numbers symbolically. Students should be exposed to in-context problem situations throughout the learning sequence, and therefore they should be comfortable solving word problems involving multiplication of a fraction and a whole number.

INSTRUCTIONAL ACTIVITIES

The activities in this unit are designed to allow students to reason how multiplication of fractions and whole numbers is represented contextually.

In Lessons 1 and 2, students are presented with a scenario in which multiplication can be used in place of repeated addition. Students connect the idea of multiplying a whole number by a fraction to their previously learned methods for multiplication with whole numbers (e.g., by representing the multiplication with area models, fraction strips, repeated grouping, and skip counting on a number line). Then, students explore the idea that fraction multiplication represents finding a portion of a whole. In these situations, the whole is represented by the whole number, and the fraction represents a given portion. This interpretation of multiplication is new for students and is connected to students' prior knowledge of multiplication through the use of models. Lesson 1 uses unit fractions, while Lesson 2 broadens students abilities to include proper fractions.

Lessons 3 and 4 transition students from the use of models to more formal arithmetic algorithms. Students are given opportunities to explore different arithmetic strategies for solving multiplication of a whole number and a fraction. Then, through group discussions, students establish which strategies are appropriate and/or efficient and which are not. Lesson 3 develops the standard algorithm utilizing unit fractions, and Lesson 4 extends student understanding to proper fractions.

Lastly, in Lesson 5, students are confronted with a real-world scenario that requires them to set up and solve several situations involving multiplication of a whole number and a fraction. Students are required to interpret their solutions within the context of the problem.

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Multiplying Fractions and Whole Numbers

OVERVIEW OF INSTRUCTIONAL ACTIVITIES

Lesson	Learning Goal	Nodes Addressed
Lesson 1	Students will multiply whole numbers by unit fractions and represent the operation with multiple models.	▶ EXPLAIN A FRACTION AS A PRODUCT OF A WHOLE NUMBER AND A UNIT FRACTION ▶ MULTIPLY A FRACTION AND A WHOLE NUMBER ▶ SOLVE WORD PROBLEMS INVOLVING MULTIPLICATION OF A FRACTION AND A WHOLE NUMBER
Lesson 2	Students will multiply whole numbers by fractions and represent the operation with multiple models.	▶ EXPLAIN A FRACTION AS A PRODUCT OF A WHOLE NUMBER AND A UNIT FRACTION ▶ MULTIPLY A FRACTION AND A WHOLE NUMBER ▶ EXPLAIN MULTIPLICATION OF FRACTIONS AND WHOLE NUMBERS
Lesson 3	Students will multiply whole numbers and unit fractions by transitioning from representing the situation with models to developing an algorithm.	▶ EXPLAIN MULTIPLICATION OF FRACTIONS AND WHOLE NUMBERS ▶ MULTIPLY A FRACTION AND A WHOLE NUMBER ▶ SOLVE WORD PROBLEMS INVOLVING MULTIPLICATION OF A FRACTION AND A WHOLE NUMBER ▶ EXPLAIN A FRACTION AS A PRODUCT OF A WHOLE NUMBER AND A UNIT FRACTION
Lesson 4	Students will multiply whole numbers and fractions by transitioning from representing the situation with models to developing an algorithm.	▶ MULTIPLY A FRACTION AND A WHOLE NUMBER ▶ SOLVE WORD PROBLEMS INVOLVING MULTIPLICATION OF A FRACTION AND A WHOLE NUMBER ▶ EXPLAIN A FRACTION AS A PRODUCT OF A WHOLE NUMBER AND A UNIT FRACTION
Lesson 5	Students will apply their previous knowledge of multiplying fractions and whole numbers to represent real-world scenarios in a variety of models.	▶ MULTIPLY A FRACTION AND A WHOLE NUMBER ▶ SOLVE WORD PROBLEMS INVOLVING MULTIPLICATION OF A FRACTION AND A WHOLE NUMBER

Multiplying Fractions and Whole Numbers

INSTRUCTIONAL ACTIVITY

Lesson 1

LEARNING GOAL

Students will multiply whole numbers by unit fractions and represent the operation with multiple models.

PRIMARY ACTIVITY

Students will explore several ways to represent the problem $5 \times \frac{1}{2}$ in the context of a real-world situation. Students are guided through an example of how to represent a whole number multiplied by a unit fraction with an array, repeated grouping, and skip counting on a number line. The multiplication arises from an in-context problem.

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

- Numerator
- Denominator
- Multiply
- Whole Number
- Unit fraction

MATERIALS

- Instructional Activity Supplement (Recommend one copy for every student.)
- Word version Instructional Activity Supplement

IMPLEMENTATION

Present the following situation to students and display it for students to see.

Jack is making cookies for the bake sale. In order to have enough cookies to sell, he is making five batches of his normal cookie recipe. The recipe calls for two teaspoons of baking powder. How much total baking powder will he need?

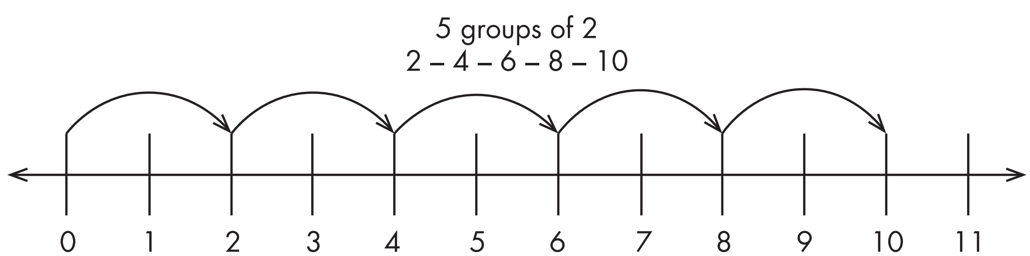
Allow time for students to think about the answer. **Call on** a student to give the answer.

Record "10 teaspoons of baking powder" on the board.

Call on other students to explain how they got their answer. **Allow** students to share addition and multiplication strategies.

Record $2 + 2 + 2 + 2 + 2 = 5 \times 2 = 10$. **Ask**, "Which is more efficient, $2 + 2 + 2 + 2 + 2$ or 5×2?" **Conclude** that the multiplication was more efficient.

Tell students to draw a number line and show how to skip count 5×2.

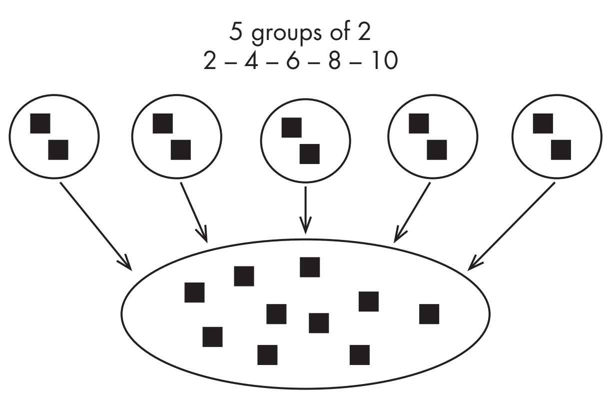


Draw (or **require** a student volunteer to draw**)** the number line and skip counting for students to see, and **tell** students to verify their own number lines.

Ask, "How does this skip counting represent the multiplication of 5×2?"

Call on two or three students to explain the relationship between skip counting and the multiplication sentence.

Repeat this exercise with repeated groupings.



Distribute the INSTRUCTIONAL ACTIVITY SUPPLEMENT.

Read the following out loud to the class while students read along from their INSTRUCTIONAL ACTIVITY SUPPLEMENT.

> Jack is making cookies for the bake sale. In order to have enough cookies to sell, he is making five batches of his regular cookie recipe. The recipe calls for $\frac{1}{2}$ teaspoon of salt. How much total salt will he need?

Tell students to write down an estimate of the amount of salt Jack will need on their INSTRUCTIONAL ACTIVITY SUPPLEMENT.

Ask, "What is the multiplication sentence you would write to solve this problem? Write it on your INSTRUCTIONAL ACTIVITY SUPPLEMENT. " **Support** students by making a connection with the multiplication sentence 5×2 from the previous scenario and/or showing the corresponding repeated addition sentence.

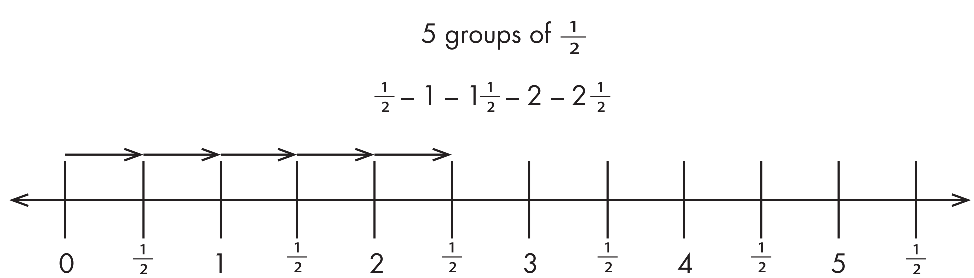
Select a student to share the multiplication sentence they wrote.

Record $5 \times \frac{1}{2}$ for students to see, and correct their own multiplication sentences if necessary.

Tell students that they will represent $5 \times \frac{1}{2}$ using the same models with which they represented 5×2.

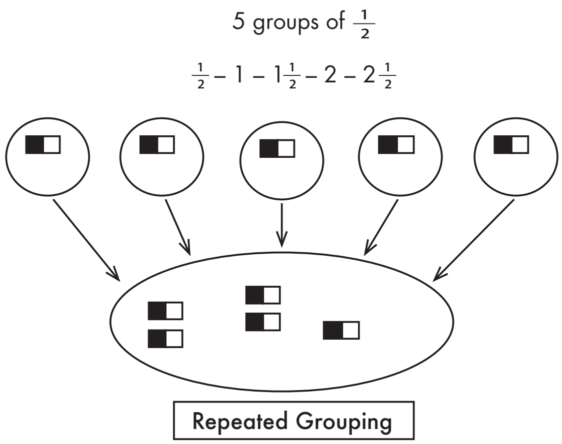
Tell students to use the number line on the INSTRUCTIONAL ACTIVITY SUPPLEMENT to represent $5\times\frac{1}{2}$ by skip counting.

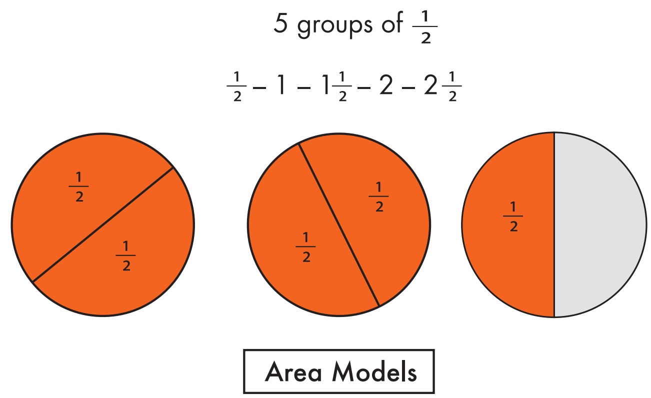
Remind students how 5×2 was represented with skip counting. **Ask**, "How would this look with $5\times\frac{1}{2}$?"



Call on a student to explain and show their skip counting to the class.

Repeat this process with repeated groupings and area models.





Write the multiplication for students to see $5 \times \frac{1}{2}$. **Point** to the 5 and **say**, "There are five groups…", then **point** to the $\frac{1}{2}$ and **say** "…of one-half". **Emphasize** the number of groups, 5, and the size of each equal-size group, $\frac{1}{2}$.

Tell students to find the total amount of salt that Jack needs using any of the models they created. **Call on** students to share their final answer. Accept both $2\frac{1}{2}$ and $\frac{5}{2}$. Discuss the similarities and differences between the two answers.

NOTE: Students may or may not be able to convert between mixed numbers and improper fractions at this time. If students are able to make the connection between $2\frac{1}{2}$ and $\frac{5}{2}$ then a conversation could be had at this point relating the two answers. Students should be able to explain improper fractions, so it is appropriate to anticipate $\frac{5}{2}$ as a solution. Unless otherwise specified, student answers should be accepted in either form since students may not yet have the skills needed to convert between the forms.

Ask, "Between which two whole numbers does your answer lie?" **Call on** a student to determine that the answer is between two and three whole teaspoons.

Ask, "What if the recipe called for $\frac{1}{3}$ teaspoon of salt? How much total salt would he need? Create a model on Question 7 of the INSTRUCTIONAL ACTIVITY SUPPLEMENT to find the answer."

Call on students to share their final answer. Accept both $1\frac{2}{3}$ and $\frac{5}{3}$. Discuss the similarities and differences between the two answers.

Say "Five groups of $\frac{1}{3}$ is $\frac{5}{3}$."

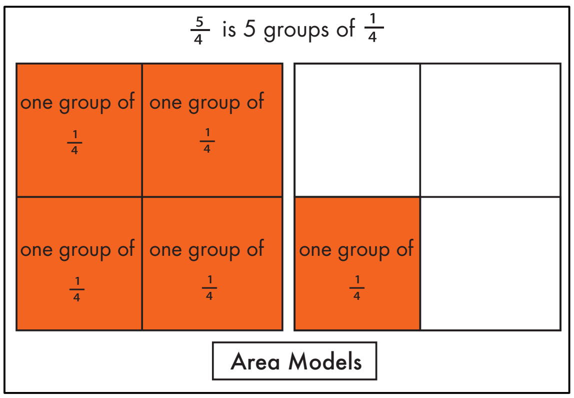
Ask, "Between which two whole numbers does the answer lie?" **Call on** a student to determine that the answer is between one and two whole teaspoons.

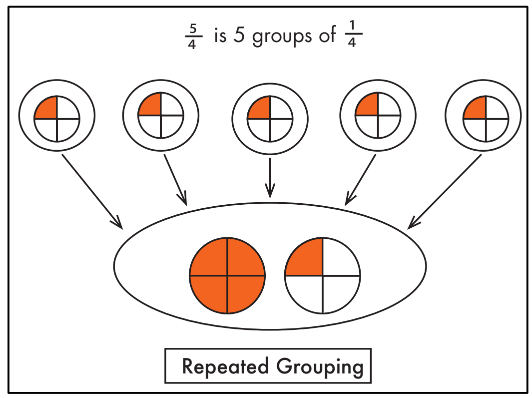
Ask students, "If the total amount of salt used was $\frac{5}{4}$ teaspoons, how much salt is in one batch of cookies? Create a model on Question 9 of the INSTRUCTIONAL ACTIVITY SUPPLEMENT to find the answer."

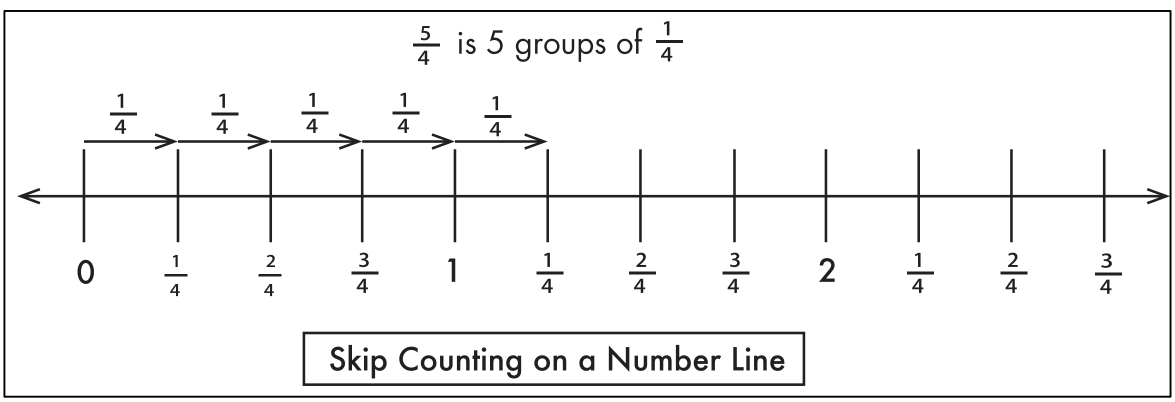
Allow time for students to determine their answer.

Say, "$\frac{5}{4}$ is 5 groups of $\frac{1}{4}$. There is $\frac{1}{4}$ teaspoon of salt in a single batch of cookies."

Show how to model the scenario with area models, repeated groupings, and skip counting on a number line.







GUIDING QUESTIONS

Elicit student thinking:

- Why is the answer to five times one-half less than five?
- How is repeated addition connected related to multiplication?
- [In order to help estimate] Name a number that you know is greater than the amount of salt Jack needs.
- [In order to help estimate] Name a number that you know is less than the amount of salt Jack needs.
- Which is the correct answer, $\frac{5}{2}$ or $2\frac{1}{2}$? If they are both correct, is one *more* correct?
- How are $\frac{5}{2}$ and $2\frac{1}{2}$ the same? How are they different?
- Can you write a whole number in fraction form?

Determine if the student can EXPLAIN A FRACTION AS A PRODUCT OF A WHOLE NUMBER AND A UNIT FRACTION:

- How many halves do you need to represent the salt in all of Jack's cookie batches?
- How many groups of [$\frac{1}{2}$ or $\frac{1}{3}$] do you have? How does that relate to the numerator of your answer?
- There are five groups of what size tablespoon?

Determine if the student is ready to MULTIPLY A FRACTION AND A WHOLE NUMBER:

- Which two numbers were multiplied?
- How does the fraction in your answer relate to the two factors in the multiplication problem?

Students will now expand their previous strategies to another real-world situation.

Present and **read aloud** another situation to students while students read along in their INSTRUCTIONAL ACTIVITY SUPPLEMENT.

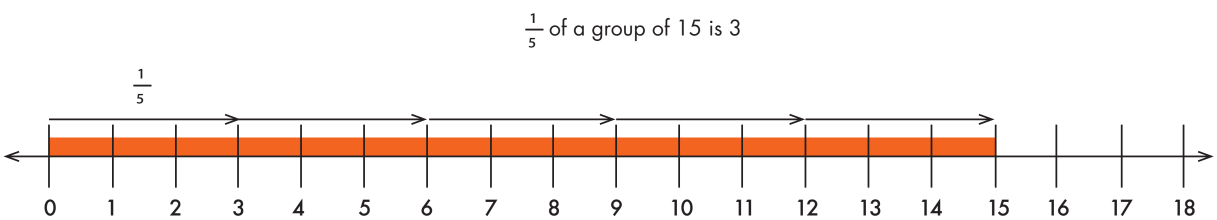
> Lucy has 15 shirts, and $\frac{1}{5}$ of them have long sleeves. How many long-sleeve shirts does Lucy have?

Tell students to write down an estimate of the how many long-sleeve shirts Lucy has on Question 10 of the INSTRUCTIONAL ACTIVITY SUPPLEMENT.

Tell students to write down the multiplication sentence representing the scenario on Questions 11 of the INSTRUCTIONAL ACTIVITY SUPPLEMENT.

Tell students that they will represent $\frac{1}{5} \times 15$ using the same models with which they represented 5×2 and $5 \times \frac{1}{2}$.

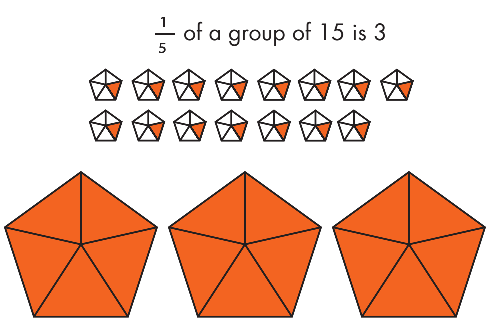
Tell students to use the number line provided on Question 12 in the INSTRUCTIONAL ACTIVITY SUPPLEMENT to represent $\frac{1}{5} \times 15$ with skip counting. **Remind** students how they represented 5×2 by skip counting on a number line. **Ask**, "How would this look with $\frac{1}{5} \times 15$?"



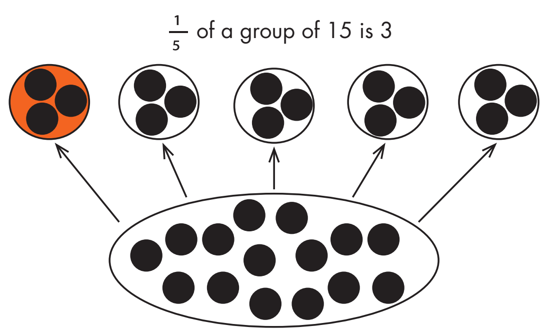
Skip Counting on a Number Line

Call on a student to explain and show their number line to the class.

Repeat this process for repeated groupings and area models.



Area Models



Repeated Grouping

State that multiplication appeared as a strategy for both problem scenarios. Multiplication represented repeated addition in the first situation, and it represents an amount "of" a whole in this situation.

NOTE: Student confusion arises from the fact that sometimes multiplication with fractions is considered repeated addition, and other times it is considered an amount "of" a whole. Ott (1990) suggests that in all multiplication with fractions, the first factor should be thought of as the number of groups, and the second number should be considered the size of each group. This wording is included, however it is also appropriate to refer to the different scenarios as "repeated addition" or "part-whole" problems.

Model the multiplication for students to see $\frac{1}{5} \times 15$. **Point** to $\frac{1}{5}$ and **say** "There is one-fifth of a group…" then **point** to 15 and **say** "…of size 15". **Emphasize** the number of groups, $\frac{1}{5}$, and the size of the group, 15.

Tell students to find the number of long-sleeve shirts that Lucy has using a model on Question 15 of the INSTRUCTIONAL ACTIVITY SUPPLEMENT. **Call on** students to share their final answer.

GUIDING QUESTIONS

Elicit student thinking:

- What does $\frac{1}{5}$ mean?
- What is the "whole" in the situation?

Determine if the student can EXPLAIN A FRACTION AS A PRODUCT OF A WHOLE NUMBER AND A UNIT FRACTION:

- $\frac{15}{5}$ is how many groups of $\frac{1}{5}$?
- How could you write $\frac{3}{4}$ as the product of a whole number and a unit fraction?

Determine if the student is ready to SOLVE WORD PROBLEMS INVOLVING MULTIPLICATION OF A FRACTION AND A WHOLE NUMBER:

- How did you know how many equal-size groups to divide 15 into?
- How did you know how many shirts made up each equal-size group?

Students should be required to complete an exit ticket that says, “Show $6 \times \frac{1}{4}$ with area models, repeated grouping, and on a number line.”

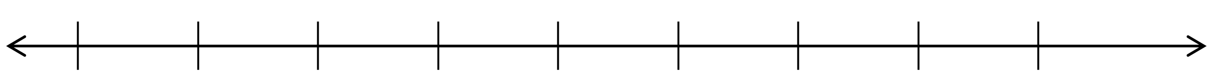
At the end of the activity, teachers should review the exit ticket in order to assess student understanding and determine if there is need for further review of the lesson and activity.

Jack is making cookies for the bake sale. In order to have enough cookies to sell, he is making five batches of his regular cookie recipe. The recipe calls for $\frac{1}{2}$ teaspoon of salt. How much total salt will he need?

1. Write down an estimate for how much total salt Jack needs.

2. Write the multiplication sentence that represents this scenario.

3. Represent $5 \times \frac{1}{2}$ by skip counting on the number line.



4. Draw repeated groupings to represent $5 \times \frac{1}{2}$.

5. Draw fraction strips representing $5 \times \frac{1}{2}$.

6. What is the total amount of salt that Jack needs? Between which two whole numbers does your answer lie?

7. What if the recipe called for $\frac{1}{3}$ teaspoon of salt? How much total salt would he need? Create a model to find the answer.

8. Between which two whole numbers does your answer lie?

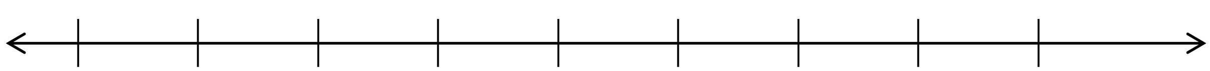
9. If the total amount of salt used was $\frac{5}{4}$ teaspoons, how much salt is in one batch of cookies? Create a model, write the multiplication sentence, and find the answer.

Lucy has 15 shirts, and $\frac{1}{5}$ of them have long sleeves. How many long-sleeve shirts does Lucy have?

10. Write down an estimate of the how many long-sleeved shirts Lucy has.

11. Write the multiplication sentence that represents this scenario.

12. Represent $\frac{1}{5} \times 15$ by skip counting on the number line.



13. Draw repeated groupings to represent $\frac{1}{5} \times 15$.

14. Draw fraction strips representing $\frac{1}{5} \times 15$.

15. How many long-sleeve shirts does Lucy have?

Multiplying Fractions and Whole Numbers

INSTRUCTIONAL ACTIVITY

Lesson 2

LEARNING GOAL

Students will multiply whole numbers by fractions and represent the operation with multiple models.

PRIMARY ACTIVITY

Students will explore several ways to represent the problem $2 \times \frac{3}{4}$ in the context of a real-world situation. Students are guided through an example of how to represent a whole number multiplied by a fraction with an array, repeated grouping, and skip counting on a number line. The multiplication arises from an in-context problem.

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

- Numerator
- Denominator
- Multiply
- Whole Number

MATERIALS

- Instructional Activity Supplement (Recommend one copy for every student.)
- Word version Instructional Activity Supplement

IMPLEMENTATION

Present the following situation to students:

Heather is making brownies for the school bake sale and decides to double her recipe. The original recipe calls for $\frac{3}{4}$ cup of flour. How much total flour does she need?

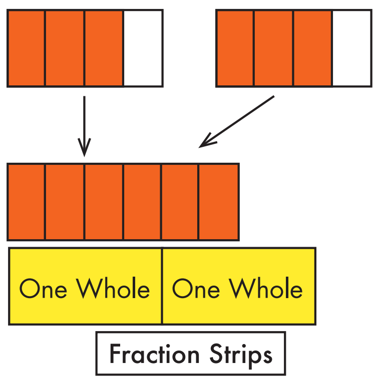
Distribute the INSTRUCTIONAL ACTIVITY SUPPLEMENT.

Tell students to write down an estimate for the amount of flour that Heather will need on Question 1 of their INSTRUCTIONAL ACTIVITY SUPPLEMENT.

Tell students to write the multiplication sentence that represents this scenario on Question 2 of their INSTRUCTIONAL ACTIVITY SUPPLEMENT.

Tell students to draw fractions strips representing $2 \times \frac{3}{4}$ on Question 3 of their INSTRUCTIONAL ACTIVITY SUPPLEMENT.

Draw (or **require** a student volunteer to draw**)** the fraction strips for students to see and to verify their own fraction strips.



Ask, "How do the fraction strips represent the multiplication of $2 \times \frac{3}{4}$?"

Call on two or three students to explain the relationship between the fraction strips and the multiplication sentence.

Repeat this process for repeated groupings and for skip counting on a number line.

Model the multiplication for students to see $2 \times \frac{3}{4}$. **Point** to the 2 and **say**, "There are two groups…" then **point** to the $\frac{3}{4}$ and **say** "…of three-fourths".

Tell students to determine the total amount of flour that Heather needs using any of the models they created on their INSTRUCTIONAL ACTIVITY SUPPLEMENT.

Call on students to share their final answer. **Accept** $1\frac{1}{2}$, $1\frac{2}{4}$, $\frac{6}{4}$ and $\frac{3}{2}$. **Discuss** how the four answers are the same and how they are different.

Ask, “Between which two whole numbers does your answer lie?” **Call on** a student to determine that the answer is between one and two whole cups of flour.

Ask, “What if the recipe called for $\frac{2}{3}$ cup of flour? How much total flour would Heather need? Find the amount of flour that Heather needs using one of the models you created on Question 7 of your INSTRUCTIONAL ACTIVITY SUPPLEMENT.”

Allow time for students to find the answer.

Say, “Two groups of $\frac{2}{3}$ is $\frac{4}{3}$.”

Ask, “Between which two whole numbers does your answer lie? **Call on** a student to determine that the answer is between one and two whole cups of flour.

Ask students, “If the total amount of flour used for the double recipe was $\frac{8}{3}$ cups, how much flour is in one batch of brownies? Use a model to find the answer.”

Allow time for students to find the answer.

Say, “$\frac{8}{3}$ is two groups of $\frac{4}{3}$. There is $\frac{4}{3}$ cup of flour in a single batch of brownies.”

GUIDING QUESTIONS

Elicit student thinking:

- Why is the answer to five times one-half less than five?
- How is repeated addition related to multiplication?
- [In order to help estimate] Name a number that you know is greater than the amount of flour Heather needs.
- [In order to help estimate] Name a number that you know is less than the amount of flour Heather needs.
- Which is the correct answer, $1\frac{1}{2}$, $1\frac{2}{4}$, $\frac{6}{4}$ or $\frac{3}{2}$? If they are all correct, is one *more* correct?
- How are $1\frac{1}{2}$, $\frac{6}{4}$, and $\frac{3}{2}$ the same? How are they different?
- Can you write a whole number in fraction form? Show me.

Determine if the student can EXPLAIN A FRACTION AS A PRODUCT OF A WHOLE NUMBER AND A UNIT FRACTION:

- How many fourths do you need to represent the flour in all of Heather's cookie batches?
- How many groups of [$\frac{3}{4}$ or $\frac{2}{3}$] do you have? How does that relate to the numerator of your answer?
- There are two groups of what size cup of flour?

Determine if the student is ready to MULTIPLY A FRACTION AND A WHOLE NUMBER:

- Which two numbers were multiplied?
- How does the fraction in your answer relate to the two factors in the multiplication problem?

Students will now expand their previous strategies to another real-world situation.

Present the situation to students:

> The bake sale has a total of 30 brownie pans, and $\frac{2}{5}$ of them contain nuts. If you are allergic to nuts, how many brownie pans are you unable to eat?

Tell students to write down an estimate of how many brownie pans contain nuts.

Tell students to write an estimate for how many pans of brownies *do not* contain nuts.

Tell students to write the multiplication sentence representing the scenario on their INSTRUCTIONAL ACTIVITY SUPPLEMENT.

State that multiplication is a solution strategy for both problem scenarios. Multiplication represented repeated addition in the first situation, and it represents an amount "of" a whole in this situation.

Tell students that they will represent $\frac{2}{5} \times 30$ using the same models with which they represented $2 \times \frac{3}{4}$.

Tell students to draw fraction strips representing $\frac{2}{5} \times 30$. **Remind** students how $2 \times \frac{3}{4}$ was represented with fraction strips. **Ask**, "How would this look with $\frac{2}{5} \times 30$?"

Call on a student to show their fraction strips to the class.

Repeat this process for repeated groupings and for skip counting on a number line.

Model the multiplication sentence for students to see $\frac{2}{5} \times 30$. **Point** to $\frac{2}{5}$ and **say**, "There are two-fifths of a group…" then **point** to the 30 and **say**, "…of size 30".

Tell students to find the number of brownie pans that contain nuts using one of the models. **Call on** students to share their final answer.

GUIDING QUESTIONS

Elicit student thinking:

- What does $\frac{2}{5}$ mean?
- What is the "whole" in the situation?

Determine if the student can EXPLAIN A FRACTION AS A PRODUCT OF A WHOLE NUMBER AND A UNIT FRACTION:

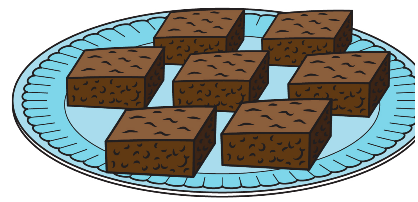
- How could you write $\frac{2}{5}$ as the product of a whole number and a unit fraction?

Determine if the student is ready to EXPLAIN MULTIPLICATION OF FRACTIONS AND WHOLE NUMBERS:

- How did you know how many equal-size groups to divide 30 into?
- How did you know how many brownie pans made up each equal-size group?

Students should be required to complete an exit ticket that says, "Show $6 \times \frac{2}{3}$ with fraction strips, repeated grouping, and on a number line."

At the end of the activity, teachers should review the exit ticket in order to assess student understanding and determine if there is need for further review of the lesson and activity.



Heather is making brownies for the school bake sale and decides to double her recipe. The original recipe calls for $\frac{3}{4}$ cup of flour. How much total flour does she need?

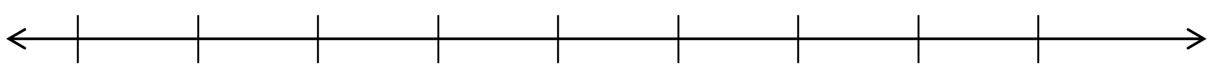
1. Write down an estimate for how much total flour Heather needs.

2. Write the multiplication sentence that represents this scenario.

3. Draw fraction strips representing $2 \times \frac{3}{4}$.

4. Draw repeated groupings to represent $2 \times \frac{3}{4}$.

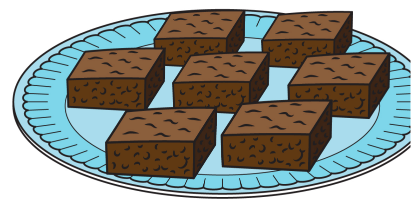
5. Represent $2 \times \frac{3}{4}$ by skip counting on the number line.



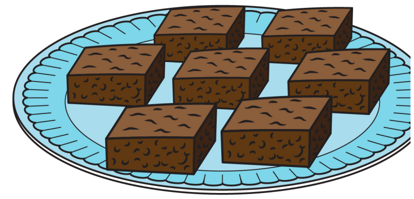
6. What is the total amount of flour that Heather needs? Between which two whole numbers does your answer lie?

7. What if the recipe called for $\frac{2}{3}$ cup of flour? How much total flour would Heather need? Between which two whole numbers does your answer lie?

8. If the total amount of flour used for the double recipe was $\frac{8}{3}$ cups, how much flour is in one batch of brownies? Use a model to answer the question.



The bake sale had a total of 30 brownie pans, and $\frac{2}{5}$ of them contained nuts. If you are allergic to nuts, how many brownie pans are you unable to eat?



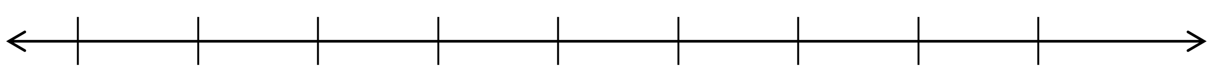
9. Write an estimate for how many brownie pans contain nuts.

10. Write the multiplication sentence representing this scenario.

11. Draw fraction strips representing $2 \times \frac{3}{4}$.

12. Draw repeated groupings to represent $2 \times \frac{3}{4}$.

13. Represent $2 \times \frac{3}{4}$ by skip counting on the number line.



14. How many brownie pans contain nuts?

Multiplying Fractions and Whole Numbers

INSTRUCTIONAL ACTIVITY

Lesson 3

LEARNING GOAL

Students will multiply whole numbers and unit fractions by transitioning from representing the situation with models to developing an algorithm.

PRIMARY ACTIVITY

Students are guided through how a whole number multiplied by a unit fraction can be represented symbolically. The two multiplication examples arise from in-context situations. Students will practice multiplying whole numbers and unit fractions in real-world contexts in the Instructional Activity Student Handout.

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

- Numerator
- Denominator
- Factor
- Multiply
- Whole Number
- Simplify
- Simplest terms

MATERIALS

- Instructional Activity Student Handout
- Word version Instructional Activity Student Handout

IMPLEMENTATION

Students will explore several ways to represent the problem $5 \times \frac{1}{2}$ in the context of a real-world situation.

Remind students of the following situation from LESSON 1: Jack is making cookies for the bake sale. In order to have enough cookies to sell, he is making five batches of his normal cookie recipe. The recipe calls for two teaspoons of baking powder. How much total baking powder will he need?

Record $2 + 2 + 2 + 2 + 2 = 5 \times 2 = 10$. **Ask**, "Which is more efficient, $2 + 2 + 2 + 2 + 2$ or 5×2?" Students should answer that multiplication is more efficient.

Draw or **show** a model representing the situation. **Ask**, "Which is more efficient, drawing a model or calculating 5×2?" Students should answer that multiplication is more efficient.

Remind students of the following situation from LESSON 1 and display it on the board for reference: Jack is making cookies for the bake sale. In order to have enough cookies to sell, he is making five batches of his regular cookie recipe. The recipe calls for $\frac{1}{2}$ teaspoon of salt. How much total salt will he need?

Ask students to recall how much total salt Jack needs. They may redraw a model if needed. **Tell** students to turn to a partner and share how much total salt Jack needs to ensure that all students know that the correct answer is $2\frac{1}{2}$ or $\frac{5}{2}$ teaspoons.

Ask, "Was it efficient to draw a model to solve 5×2? Do you think there is a more efficient way to calculate $5 \times \frac{1}{2}$?"

Tell students to brainstorm ideas for how to calculate $5 \times \frac{1}{2}$ without using a model.

Call on several students to share their ideas. **Prompt** students by asking, "Can we write the whole number in fraction form so that the two factors are more similar?"

When students have finished sharing ideas, **model** how to solve the multiplication sentence by **writing** $5 \times \frac{1}{2} = \frac{5}{1} \times \frac{1}{2} = \frac{5 \times 1}{1 \times 2} = \frac{5}{2}$. **Say**, "Five groups of $\frac{1}{2}$ is $\frac{5}{2}$." **Ask**, "Does this match the answer we got from the models (area models, fraction strips, skip counting, repeated grouping)?"

Confirm that the same answer was found from the algorithm as from the models.

NOTE: Students may or may not be able to convert between mixed numbers and improper fractions at this time. If students are able to make the connection between $2\frac{1}{2}$ and $\frac{5}{2}$, then a conversation could be had at this point relating the two answers. Students should be able to explain improper fractions so it is appropriate to anticipate $\frac{5}{2}$ as a solution. Unless otherwise specified, student answers should be accepted in either form, since students may not yet have the skills needed to convert between the forms.

Ask, "What if the recipe called for $\frac{1}{3}$ teaspoon of salt? How much total salt would he need? Write the multiplication sentence to find the answer."

Students should write $5 \times \frac{1}{3} = \frac{5}{1} \times \frac{1}{3} = \frac{5 \times 1}{1 \times 3} = \frac{5}{3}$. **Say** "Five groups of $\frac{1}{3}$ is $\frac{5}{3}$."

Ask students, "If the total amount of salt used was $\frac{5}{4}$ teaspoons, how much salt is in one batch of cookies? Write a multiplication sentence to find your answer."

Students should write $\frac{5}{4} = \frac{5 \times 1}{1 \times 4} = \frac{5}{1} \times \frac{1}{4} = 5 \times \frac{1}{4}$.

Say, "$\frac{5}{4}$ is five groups of $\frac{1}{4}$. There is $\frac{1}{4}$ teaspoon of salt in a single batch of cookies."

GUIDING QUESTIONS

Elicit student thinking:

- Why is it important to find the most efficient method for solving?
- Which is the correct answer, $\frac{5}{2}$ or $2\frac{1}{2}$? If they are both correct, is one *more* correct?
- How are $\frac{5}{2}$ and $2\frac{1}{2}$ the same? How are they different?

Determine if the student can EXPLAIN MULTIPLICATION OF FRACTIONS AND WHOLE NUMBERS:

- Write the whole number 6 as a fraction.
- How does the fraction in your answer relate to the two factors in the multiplication problem?

Determine if the student can MULTIPLY A FRACTION AND A WHOLE NUMBER:

- Which pairs of numbers were multiplied?
- What is the first step in order to multiply a fraction and a whole number?
- Can we simplify the fraction $\frac{5}{2}$? Can we simplify the fraction $\frac{5}{3}$?

Determine if the student is ready to SOLVE WORD PROBLEMS INVOLVING MULTIPLICATION OF A FRACTION AND A WHOLE NUMBER:

- How did you know to use multiplication to solve this problem?

Students will now expand their previous strategies to another real-world situation.

Remind students of this situation from LESSON 1: Lucy has 15 shirts, and $\frac{1}{5}$ of them are long sleeved. How many long-sleeve shirts does Lucy have?

Ask students to connect the previous example's multiplication sentence to this scenario by writing the multiplication sentence they would use to answer the question.

Connect students' strategies to the previous example by writing $\frac{1}{5} \times 15 = \frac{1 \times 15}{5} = \frac{15}{5}$.

Ask, "Should we leave $\frac{15}{5}$ in its current form? Is there a way to simplify this fraction?"

Students should note that $\frac{15}{5}$ can be simplified to $\frac{3}{1}$ and that $\frac{3}{1}$ is equivalent to three.

Say, "$\frac{1}{5}$ group of size 15 is three, or $\frac{1}{5}$ of 15 is three."

State that multiplication appeared as a strategy for both problem scenarios. Multiplication represented repeated addition in the first situation, and it represents an amount "of" a whole in this situation.

GUIDING QUESTIONS

Elicit student thinking:

- What is the "whole" in the situation?

Determine if the student can EXPLAIN A FRACTION AS A PRODUCT OF A WHOLE NUMBER AND A UNIT FRACTION:

- How could you write $\frac{3}{4}$ as the product of a whole number and a unit fraction?

Determine if the student is ready to SOLVE WORD PROBLEMS INVOLVING MULTIPLICATION OF A FRACTION AND A WHOLE NUMBER:

- How did you know how many equal-size groups to divide 15 into?
- How did you know how many shirts made up each equal-size group?

Students should now practice writing these multiplication sentences for various word problems in the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

Distribute the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT. **Circulate** and **assist** students using the following guiding questions.

GUIDING QUESTIONS

Elicit student thinking:

- [Point to a fraction.] Can you draw a model of this fraction?
- How does the model relate to the multiplication sentence?
- [Point to an answer that is a fraction.] Between which two whole numbers is this fraction?
- How do you know whether or not to simplify your answer?

Determine if the student can MULTIPLY A FRACTION AND A WHOLE NUMBER:

- [Point to a question.] What are the two factors of the multiplication problem? How do you know?
- [Point to a question.] Which factor represents the size of the groups, and which factor represents the number of groups?
- [Point to a question.] How do you know what the denominator of the product should be?
- [Point to a question.] How do you know which whole number to multiply by?

Determine if the student is ready to SOLVE WORD PROBLEMS INVOLVING MULTIPLICATION OF A FRACTION AND A WHOLE NUMBER:

- [Point to a multiplication sentence.] Can you rewrite this multiplication sentence as an addition sentence?
- [Point to a multiplication sentence.] Can you rewrite this multiplication sentence as a division sentence?
- [Point to a question.] How do you know to use multiplication to solve this problem?
- [Point to a question.] How do you know what the denominator of the fraction should be?

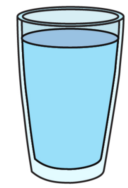
Students should be required to complete the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

At the end of the activity, teachers should review the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT in order to assess student understanding and determine if there is need for further review of the lesson and activity.

Name________________________________

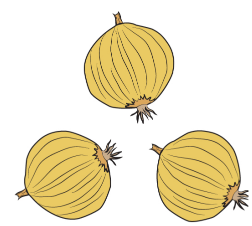
Multiplying Fractions and Whole Numbers

Lesson 3



1. Jessica fills six glasses with $\frac{1}{2}$ liter of water. Fill in the blanks in the following number sentence to show how many total liters of water Jessica used.

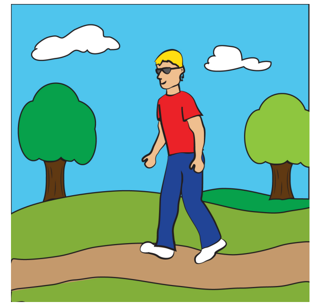
$$\frac{1}{2}\times\_\_ = \frac{\_\_\times 6}{2} = \frac{6}{2} = \_\_$$



2. Katie's mom buys three onions at the store. Each onion weighs $\frac{1}{3}$ of a pound. How many pounds of onions does Katie's mom have? Show a multiplication sentence representing your answer.

Name_______________________________

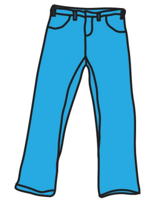
3. George walks $\frac{1}{6}$ of a mile to his friend Michael's house. George and Michael walk $\frac{1}{6}$ of a mile to the park. George and Michael return to Michael's house for dinner, and then George walks back to his own house. How many miles did George walk in total? Draw a model to represent your answer.



4. Buster has a coin collection with a total of 100 coins. $\frac{1}{5}$ of his collection is pennies. How many pennies are in Buster's collection?

Name______________________________

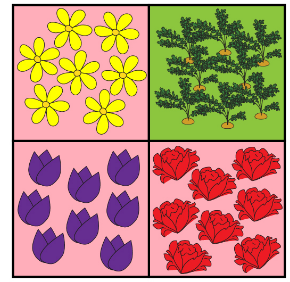
5. Jeans that regularly cost \$20.00 are on sale for $\frac{1}{4}$ off. How much money is being subtracted from the regular price?



6. At the end of the school year, Ms. Jackson has 12 glue bottles that are each $\frac{1}{5}$ full. How many full bottles of glue can she make?

Name_______________________________

7. Gretchen's garden is $\frac{3}{4}$ flowers and $\frac{1}{4}$ vegetables. There are three types of flowers in the garden: roses, daisies, and tulips. What fraction of the garden is made up of tulips?



8. Priscilla is buying fabric for a quilt. She gets $\frac{1}{7}$ of a yard of fabric in five different colors. How many yards of fabric does she buy in total?

Multiplying Fractions and Whole Numbers

INSTRUCTIONAL ACTIVITY

Lesson 4

LEARNING GOAL

Students will multiply whole numbers and fractions by transitioning from representing the situation with models to developing an algorithm.

PRIMARY ACTIVITY

Students are guided through how a whole number multiplied by a fraction can be represented symbolically. The two multiplication examples arise from in-context situations. Students will then practice multiplying whole numbers and fractions in real-world contexts in the Instructional Activity Student Handout.

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

- Factor
- Multiply
- Numerator
- Denominator
- Whole number
- Simplify
- Simplest terms

MATERIALS

- Instructional Activity Student Handout
- Word version Instructional Activity Student Handout
- Whiteboards (Recommend one for each student.)

IMPLEMENTATION

Guide students through a real-world situation in which students will need to multiply a whole number by a non-unit fraction.

Remind students of the following situation from LESSON 2 and display it on the board for students to reference: Heather is making brownies for the school bake sale and decides to double her recipe. The original recipe calls for $\frac{3}{4}$ cup of flour. How much total flour does she need?

Tell students to answer the question by writing a multiplication sentence similar to the ones they wrote in LESSON 3.

Write $2\times\frac{3}{4}$ on the board.

Tell students to solve the multiplication sentence on their whiteboards.

Circulate as they solve and make note of students who use different methods of solving (e.g., doesn't create a denominator of 1).

Call on three or four students to come up to the front and model and explain their solutions. In general, students should see the problem solved in the following ways:

- $2\times\frac{3}{4}=\frac{2}{1}\times\frac{3}{4}=\frac{2\times3}{1\times4}=\frac{6}{4}=\frac{3}{2}$
- $2\times\frac{3}{4}=\frac{2\times3}{4}=\frac{6}{4}=\frac{3}{2}$
- $\cancel{2}\times\frac{3}{4}=1\times\frac{3}{2}=\frac{1\times3}{2}=\frac{3}{2}$ (2 and 4 have been simplified before the multiplication as opposed to after.)

NOTE: As students explain, it is likely that they will catch any mistakes that they have made, so it is OK to call on students that have a valid strategy but who made a mistake in calculations or did not simplify. This is a good opportunity for all students to see why some strategies work and others do not.

Discuss how the solution methods are the same and how they are different.

Ask, "Between which two whole numbers does your answer lie?" **Call on** a student to determine that the answer is between one and two whole cups of flour.

Say, "Two groups of $\frac{3}{4}$ is $\frac{6}{4}$. Six-fourths is equivalent to three-halves."

Ask, "What if the recipe called for $\frac{2}{3}$ cup of flour? How much total flour would Heather need?"

Tell students to estimate the answer and then to write and solve the multiplication sentence.

Students should record $2\times\frac{2}{3}=\frac{2\times2}{3}=\frac{4}{3}$.

Ask, "Between which two whole numbers does your answer lie?"

Say "Two groups of $\frac{2}{3}$ is $\frac{4}{3}$, which is between one and two."

Ask students, "If the total amount of flour used for the double recipe was $\frac{8}{3}$ cups, how much flour is in one batch of brownies?" Before performing any calculations, ask students to estimate the answer.

Say, "Write the multiplication sentence and find the answer."

Students should record $\frac{8}{3}=\frac{2\times4}{3}=2\times\frac{4}{3}$.

Say, "$\frac{8}{3}$ is two groups of $\frac{4}{3}$. There is $\frac{4}{3}$ cup of flour in a single batch of brownies."

GUIDING QUESTIONS

Elicit student thinking:

- Why is the answer to $2\times\frac{3}{4}$ less than two?
- Which is the correct answer, $\frac{6}{4}$ or $\frac{3}{2}$? If they are both correct, is one *more* correct?
- Which is the correct answer, $\frac{3}{2}$ or $1\frac{1}{2}$? If they are both correct, is one *more* correct?
- How are $\frac{3}{4}$ and $1\frac{1}{2}$ the same? How are they different?

Determine if the student can MULTIPLY A FRACTION AND A WHOLE NUMBER:

- Do you have to write the whole number with a denominator of 1?
- There are two groups of what size?

Determine if the student is ready to SOLVE WORD PROBLEMS INVOLVING MULTIPLICATION OF A FRACTION AND A WHOLE NUMBER:

- Which number represents the size of each group? How do you know?
- Which number represents the number of groups? How do you know?
- What does the product represent?

Students will now expand their previous strategies to another real-world situation.

Present the situation to students: The bake sale had a total of 30 brownie pans, and $\frac{2}{5}$ of them contained nuts. If you are allergic to nuts, how many brownie pans are you unable to eat?

Direct students to answer the question by writing and solving a multiplication sentence.

Circulate and **observe** students using different strategies.

Select two or three students to share and explain their solutions.

Discuss the solutions as a class and decide which ones are correct and which ones are incorrect. Decide if there are some that are more correct than others.

Say, "$\frac{2}{5}$ group of 30 is 12."

Clarify that this is the same as 30 groups of $\frac{2}{5}$ by the commutative property of multiplication.

GUIDING QUESTIONS

Elicit student thinking:

- What does $\frac{2}{5}$ group mean?
- What is the "whole" in the situation?
- Estimate the answer before performing any calculations.

Determine if the student can EXPLAIN A FRACTION AS A PRODUCT OF A WHOLE NUMBER AND A UNIT FRACTION:

- How many ways could you write $\frac{10}{4}$ as the product of a whole number and a fraction?

Determine if the student is ready to SOLVE WORD PROBLEMS INVOLVING MULTIPLICATION OF A FRACTION AND A WHOLE NUMBER:

- How did you know how many fractional parts to divide 30 into?
- How did you know how many brownie pans made up each fractional part?
- How many brownie pans can you *not* eat from?

Students should now practice writing these multiplication sentences for various word problems in the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

Distribute the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

Circulate and **assist** students using the following guiding questions.

GUIDING QUESTIONS

Elicit student thinking:

- [Point to a fraction.] Can you draw a model of this fraction?
- How does the model relate to the multiplication sentence?
- [Point to an answer that is a fraction.] Between which two whole numbers is this fraction?
- How do you know whether or not to simplify your answer?

Determine if the student can MULTIPLY A FRACTION AND A WHOLE NUMBER:

- [Point to a question.] What are the two factors of the multiplication problem? How do you know?
- [Point to a question.] Which factor represents the size of the groups, and which factor represents the number of groups?
- [Point to a question.] How do you know what the denominator of the product should be?
- [Point to a question.] How do you know which whole number to multiply by?

Determine if the student is ready to SOLVE WORD PROBLEMS INVOLVING MULTIPLICATION OF A FRACTION AND A WHOLE NUMBER:

- [Point to a multiplication sentence.] Can you rewrite this multiplication sentence as an addition sentence?
- [Point to a multiplication sentence.] Can you rewrite this multiplication sentence as a division sentence?
- [Point to a question.] How do you know to use multiplication to solve this problem?
- [Point to a question.] How do you know what the denominator of the fraction factor should be?

Students should be required to complete the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

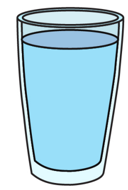
At the end of the activity, teachers should review the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT in order to assess student understanding and determine if there is need for further review of the lesson and activity.

Name________________________________

Multiplying Fractions and Whole Numbers

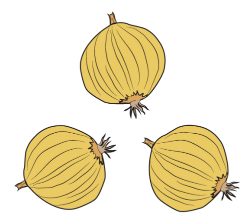
Lesson 4

1. Jessica fills six glasses with $\frac{2}{3}$ liter of water. Fill in the blanks in the following number sentence to show how many total liters of water Jessica used.



$$\frac{2}{3} \times \_\_ = \frac{\_\_ \times 6}{3} = \frac{12}{3} = \_\_$$

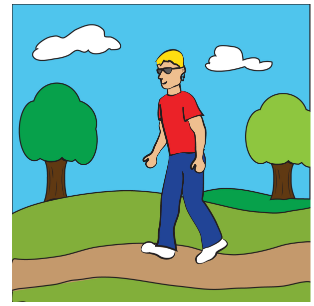
2. Katie's mom buys three onions at the store. Each onion weighs $\frac{3}{5}$ of a pound. How many pounds of onions does Katie's mom have? Show a multiplication sentence representing your answer.



Name______________________________

3. George walks $\frac{5}{6}$ of a mile to his friend Michael's house.

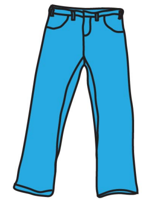
   George and Michael walk $\frac{5}{6}$ of a mile to the park. George and Michael return to Michael's house for dinner, and then George walks back to his own house. How many miles did George walk in total? Draw a fraction model to represent your answer.



4. Buster has a coin collection with 100 total coins. $\frac{3}{5}$ of his collection is pennies. How many pennies are in Buster's collection? Show a multiplication sentence representing your answer.

Name________________________________

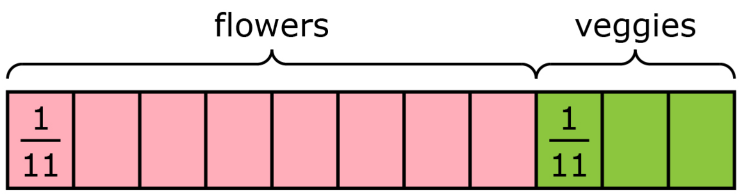
5. Jeans regularly cost $21.00 and are on sale for $\frac{2}{7}$ off. How much money is being subtracted from the regular price? Show a multiplication sentence representing your answer.



6. At the end of the school year, Ms. Jackson has 10 glue bottles that are each $\frac{4}{15}$ full. How many full bottles of glue can she make? Show a multiplication sentence representing your answer, then rewrite it as an addition sentence.

Name________________________________

7. Gretchen's garden is $\frac{8}{11}$ flowers and $\frac{3}{11}$ vegetables. There are 2 types of flowers in the garden: roses and tulips. What fraction of the garden is made up of tulips? Label the area of the garden that is tulips.



8. Priscilla buys fabric for a quilt. She gets $\frac{3}{7}$ of a yard of fabric in 5 different colors. How many yards of fabric does she have in total? Draw a fraction model to represent your answer.

Multiply Fractions and Whole Numbers

INSTRUCTIONAL ACTIVITY

Lesson 5

LEARNING GOAL

Students will apply their previous knowledge of multiplying fractions and whole numbers to represent real-world scenarios in a variety of models.

PRIMARY ACTIVITY

Students translate between different representations of multiplication of fractions and whole numbers.

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

- Multiply
- Numerator
- Denominator
- Whole number
- Simplify
- Simplest terms

MATERIALS

- Instructional Activity Supplement (Recommend one copy for every two students.)
- Word version Instructional Activity Supplement

IMPLEMENTATION

Provide students with an example for how to represent a multiplication problem in a variety of models.

Present the following scenario to students: A virus has wiped out $\frac{3}{4}$ of your emails and you only have three emails left. How many emails were in your inbox before the virus?

Pair students and **distribute** the INSTRUCTIONAL ACTIVITY SUPPLEMENT.

Tell students to answer the questions on the INSTRUCTIONAL ACTIVITY SUPPLEMENT with their partner.

Circulate while students work and use the following questions to guide students.

GUIDING QUESTIONS

Elicit student thinking:

- How many emails represent $\frac{1}{4}$ of the total you started with?
- Should the original amount of emails be more or less than three?
- What fractional part of the original emails does three represent?
- How many fractional parts should your fraction model contain?
- What does the denominator represent?

Determine if the student can MULTIPLY A FRACTION AND A WHOLE NUMBER:

- [Point to an expression in the table in the INSTRUCTIONAL ACTIVITY SUPPLEMENT.] Explain how you know that this expression is equivalent to $8 \times \frac{3}{4}$.
- Explain how you can estimate the result of $8 \times \frac{3}{4}$ without performing any calculations.
- Will the result of $12 \times \frac{1}{4}$ be greater than or less than 12? How do you know?

Determine if the student can SOLVE WORD PROBLEMS INVOLVING MULTIPLICATION OF A FRACTION AND A WHOLE NUMBER:

- [Point to a question.] Explain how you know to multiply in this situation.
- [Point to the first sentence.] Should we multiply $3 \times \frac{3}{4}$ to determine how many emails you had before the virus? Why or why not?

Students should be required to complete the INSTRUCTIONAL ACTIVITY SUPPLEMENT.

At the end of the activity, teachers should review the answers of the INSTRUCTIONAL ACTIVITY SUPPLEMENT with students.

A virus has deleted $\frac{3}{4}$ of your emails, and you only have 3 emails left. How many emails were in your inbox before the virus?

- Draw a fraction model to represent this situation.

- Write a multiplication sentence that represents the number of emails you had **at the start of the day**.

- Use the previous multiplication sentence to represent the situation as a number of equal-size groups.

  __________________ groups of _______________ size.

- Write a multiplication sentence to represent the number of emails that were **deleted** by the virus.

  - Write the previous multiplication sentence as an addition sentence.

  - Tell which is more efficient, the addition sentence or the multiplication sentence.

- Your friend had 8 emails **before** the same virus deleted $\frac{3}{4}$ of her emails. She set up the following multiplication sentence to represent how many emails were lost.

$$8 \times \frac{3}{4}$$

Circle all equivalent representations in the table.

$\frac{8}{1} \times \frac{3}{4}$	$\frac{3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3}{4}$	$\frac{8 \times 3}{8 \times 4}$
$8 \times \frac{4}{3}$	8×3	$\frac{6}{1}$
$\frac{24}{3}$	6	$\frac{8}{4} \times 3$
$\frac{3}{2}$	2×3	$\frac{8 \times 3}{4}$

- Write an email to your email service provider explaining what happened. Tell how many emails you lost, and explain how you know.

Name____

Multiplying Fractions and Whole Numbers

Lesson 1–5

1. Charlie has 15 pairs of shoes, and $\frac{1}{3}$ of the pairs are sneakers.

1.a. Draw a visual representation of all Charlie's shoes. Indicate which shoes are sneakers and which shoes are not sneakers by color-coding your representation.

1.b. Tell how many pairs of sneakers Charlie has.

Name____

1.c. Explain in words how you know.

2. You are buying ice cream for 9 guests at your birthday party. On average, each person will eat $\frac{1}{4}$ of a pint of ice cream.

2.a. Write an addition sentence to represent the situation.

2.b. Write a multiplication sentence to represent the problem.

Name____

2.c. Tell how many total pints the guests will eat by solving the multiplication sentence you wrote in 2.b. Write your answer in simplest fraction form.

2.d. Tell how many pints you must buy at the grocery store if ice cream is only sold by the pint.

3. You are serving soda at your party, and you fill each guest's cup $\frac{4}{5}$ full.

3.a. Write a multiplication sentence representing the total amount of cups filled.

3.b. Use the previous multiplication sentence to represent the situation as a number of equal-size groups.

__________________ groups of size __________________.

3.c. Solve your multiplication sentence from 3.a. Show all work.

Name____

4. Represent $12\times\frac{5}{6}$ in three other equivalent forms.

5. Circle the greater expression, or circle both if they are equal.

5.a. $567\times\frac{2}{3}$ $567\times\frac{3}{2}$

5.b. $100\times\frac{2}{3}$ $200\times\frac{2}{3}$

5.c. $10\times\frac{2}{3}$ 10

Name____

6. You decide to invite 24 people to your birthday.

6.a. You have $4.00 to mail the invitations, and each card costs $\frac{1}{5}$ of a dollar to send. Do you have enough money to send the invitations, or will you need to ask to borrow some money? Estimate to determine your answer.

6.b. Only $\frac{5}{8}$ of those invited can attend. How many people can come to your party? Show your work.

Name____

6.c. How many people invited cannot come to your party? Show your work.

Multiplying Fractions and Whole Numbers

STUDENT ACTIVITY SOLUTION GUIDE

Lesson 1-5

1. Charlie has 15 pairs of shoes, and $\frac{1}{3}$ of the pairs are sneakers.

1.a. Draw a visual representation of all Charlie's shoes. Indicate which shoes are sneakers and which shoes are not sneakers by color-coding your representation.

CORRECT ANSWER

One block represents one pair of shoes. Blue blocks represent sneakers, and green blocks represent non-sneakers.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Draws three pairs of sneakers out of 15 total pairs of shoes.	considers 3 and $\frac{1}{3}$ to be equivalent	SOLVE WORD PROBLEMS INVOLVING MULTIPLICATION OF A FRACTION AND A WHOLE NUMBER
Draws five pairs of sneakers only.	correctly finds the number of sneakers but does not include the total number of pairs of shoes	SOLVE WORD PROBLEMS INVOLVING MULTIPLICATION OF A FRACTION AND A WHOLE NUMBER
Draws 15 pairs of shoes but doesn't distinguish between sneakers and non-sneakers.	does not demonstrate that they can find one-third of fifteen	MULTIPLY A FRACTION AND A WHOLE NUMBER

1.b. Tell how many pairs of sneakers Charlie has.

CORRECT ANSWER

Charlie has five pairs of sneakers.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Charlie has 15 pairs of sneakers.	does not distinguish between the part and the whole	SOLVE WORD PROBLEMS INVOLVING MULTIPLICATION OF A FRACTION AND A WHOLE NUMBER
Charlie has three pairs of sneakers.	considers the denominator to be the size of each part, not the number of parts	SOLVE WORD PROBLEMS INVOLVING MULTIPLICATION OF A FRACTION AND A WHOLE NUMBER

1.c. Explain in words how you know.

CORRECT ANSWER

The 15 pairs of shoes need to be partitioned into three equal-size groups because the denominator of $\frac{1}{3}$ is 3. 15 partitioned into three groups is five groups of three. One of the groups has five pairs of shoes, so he owns five pairs of sneakers.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Leaves the question blank.	cannot explain their thinking or methods in solving	EXPLAIN MULTIPLICATION OF FRACTIONS AND WHOLE NUMBERS
$15 \div 3 = 5$	does not describe in words and only shows the mathematic process	EXPLAIN MULTIPLICATION OF FRACTIONS AND WHOLE NUMBERS
You divide the 15 pairs of shoes by three and you get five.	gives a brief explanation in words but does not provide a justification for the steps	EXPLAIN MULTIPLICATION OF FRACTIONS AND WHOLE NUMBERS

2. You are buying ice cream for 9 guests at your birthday party. On average, each person will eat $\frac{1}{4}$ of a pint of ice cream.

2.a. Write an addition sentence to represent the situation.

CORRECT ANSWER

$\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}$

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
12×4	writes the multiplication sentence instead of the addition sentence	DECOMPOSE A FRACTION INTO A SUM OF UNIT FRACTIONS WITH THE SAME DENOMINATOR
Writes an addition sentence without the correct number of addends (nine).	knows that $\frac{1}{4}$ needs to be added to itself but doesn't pay attention to how many addends there should be	DECOMPOSE A FRACTION INTO A SUM OF UNIT FRACTIONS WITH THE SAME DENOMINATOR

2.b. Write a multiplication sentence to represent the problem.

CORRECT ANSWER

$9 \times \frac{1}{4}$

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
9×4	considers one-fourth to be equivalent to four	SOLVE WORD PROBLEMS INVOLVING MULTIPLICATION OF A FRACTION AND A WHOLE NUMBER
$\frac{1}{4} \times 9$	writes the factors in the multiplication sentence but interprets the number of groups and size of each group incorrectly (or backwards)	SOLVE WORD PROBLEMS INVOLVING MULTIPLICATION OF A FRACTION AND A WHOLE NUMBER
$\frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4}$	confuses the repeated addition with repeated multiplication	DECOMPOSE A FRACTION INTO A SUM OF UNIT FRACTIONS WITH THE SAME DENOMINATOR

2.c. Tell how many total pints the guests will eat by solving the multiplication sentence you wrote in 2.b. Write your answer in simplest fraction form.

CORRECT ANSWER

$$9 \times \frac{1}{4} = \frac{9 \times 1}{4} = \frac{9}{4} = 2\frac{1}{4}$$

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
$\frac{9}{36} = \frac{1}{4}$	thinks that the whole number needs to be multiplied by the numerator and denominator of the fraction	MULTIPLY A FRACTION AND A WHOLE NUMBER
$\frac{91}{4}$ or $\frac{91}{14}$	combines the numerator and denominator instead of multiplying them	MULTIPLY A FRACTION AND A WHOLE NUMBER
$9\frac{1}{4}$	incorrectly attempts to rewrite $\frac{9}{4}$ as a mixed number	CONVERT IMPROPER FRACTIONS TO MIXED NUMBERS

2.d. Tell how many pints you must buy at the grocery store if ice cream is only sold by the pint.

CORRECT ANSWER

You need to buy three pints of ice cream in order to serve everyone, since $\frac{9}{4}$ or $2\frac{1}{4}$ is greater than two but less than three.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
You need two pints.	only considers the whole number in their answer instead of the mixed number as one value	SOLVE WORD PROBLEMS INVOLVING MULTIPLICATION OF A FRACTION AND A WHOLE NUMBER
You need nine pints, one for each guest.	cannot interpret the result of their multiplication in context	SOLVE WORD PROBLEMS INVOVLING MULTIPLICATION OF A FRACTIN AND A WHOLE NUMBER

3. You are serving soda at your party, and you fill each guest's cup $\frac{4}{5}$ full.

3.a. Write a multiplication sentence representing the total amount of cups filled.

CORRECT ANSWER

$9 \times \frac{4}{5}$

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
$\frac{4}{5} \times 9$	does not correctly interpret the factors as the size of each group and the number of groups	SOLVE WORD PROBLEMS INVOLVING MULTIPLICATION OF A FRACTION AND A WHOLE NUMBER
Writes an addition sentence.	relies on prior knowledge of addition instead of adopting the new algorithm for multiplying a fraction and a whole number	SOLVE WORD PROBLEMS INVOLVING MULTIPLICATION OF A FRACTION AND A WHOLE NUMBER
4×5	uses the two given numbers to create a multiplication sentence without interpreting what the values mean in context	SOLVE WORD PROBLEMS INVOLVING MULTIPLICATION OF A FRACTION AND A WHOLE NUMBER
$4 \times \frac{1}{5}$	rewrites the fraction as a product of a whole number and a unit fraction but does not write a multiplication sentence that represents the total amount of soda	SOLVE WORD PROBLEMS INVOLVING MULTIPLICATION OF A FRACTION AND A WHOLE NUMBER

3.b. Use the previous multiplication sentence to represent the situation as a number of equal-size groups.
________ groups of size __________.

CORRECT ANSWER

Nine groups of size $\frac{4}{5}$.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
$\frac{4}{5}$ groups of size nine.	cannot interpret which factor is the size of the groups and which factor is the number of groups	EXPLAIN MULTIPLICATION OF FRACTIONS AND WHOLE NUMBERS
Four groups of size $\frac{1}{5}$.	only considers the fraction as a grouping problem instead of the entire multiplication sentence of a fraction multiplied by a whole number	EXPLAIN MULTIPLICATION OF FRACTIONS AND WHOLE NUMBERS

3.c. Solve your multiplication sentence from 3.a. Show all work.

CORRECT ANSWER

$$9 \times \frac{4}{5} = \frac{9 \times 4}{5} = \frac{36}{5} = 7\frac{1}{5}$$

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
$9 \times \frac{4}{5} = \frac{9 \times 4}{9 \times 5} = \frac{36}{45}$	thinks that the whole number should be multiplied by the numerator and the denominator	MULTIPLY A FRACTION AND A WHOLE NUMBER
$\frac{94}{5}$ or $\frac{94}{95}$	combines the whole number with the numerator (and possibly also the denominator) instead of carrying out the multiplication	MULTIPLY A FRACTION AND A WHOLE NUMBER
Performs the algorithm correctly but makes a mistake in calculations (e.g., $9 \times 4 = 32$).	does not know their multiplication facts	MULTIPLY A FRACTION AND A WHOLE NUMBER

4. Represent $12\times\frac{5}{6}$ in three other equivalent forms.

CORRECT ANSWER

Answers may vary. Some options are:

$\frac{12 \times 5}{6}$, $\frac{60}{6}$, 10

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
$12\times\frac{5}{6}$	provides no equivalent forms and only rewrites the original problem	EXPLAIN A FRACTION AS A PRODUCT OF A WHOLE NUMBER AND A UNIT FRACTION
Only provides one or two equivalent forms.	cannot develop multiple equivalent representations of a fraction	EXPLAIN A FRACTION AS A PRODUCT OF A WHOLE NUMBER AND A UNIT FRACTION
Simplifies incorrectly (e.g., $\frac{12}{6}$ simplified to $\frac{1}{2}$ instead of $\frac{2}{1}$).	cannot correctly simplify fractions	REPRESENT FRACTIONS IN SIMPLEST TERMS

5. Circle the greater expression, or circle both if they are equal.

5.a. $567\times\frac{2}{3}$ $567\times\frac{3}{2}$

CORRECT ANSWER

$567\times\frac{2}{3}$ $567\times\frac{3}{2}$

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Circles both expressions.	sees the two whole numbers as being equivalent and assumes that the two expressions are therefore equivalent	RECOGNIZE EQUIVALENT FRACTIONS IN SYMBOLIC REPRESENTATION and MULTIPLY A FRACTION AND A WHOLE NUMBER
Circles both expressions.	considers the whole numbers to be equivalent as well as $\frac{2}{3}$ and $\frac{3}{2}$ since the fractions contain the same numbers	RECOGNIZE EQUIVALENT FRACTIONS IN SYMBOLIC REPRESENTATION

5.b. $100\times\frac{2}{3}$ $200\times\frac{2}{3}$

CORRECT ANSWER

$100\times\frac{2}{3}$ $200\times\frac{2}{3}$

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Circles $200\times\frac{2}{3}$ but only because they compared the whole numbers and concluded that 200 was larger.	does not consider the entire multiplication sentence	MULTIPLY A FRACTION AND A WHOLE NUMBER
Circles both expressions.	thinks that the two expressions are equivalent because they share the same factor $(\frac{2}{3})$	MULTIPLY A FRACTION AND A WHOLE NUMBER

5.c. $10\times\frac{2}{3}$ 10

CORRECT ANSWER

$10\times\frac{2}{3}$ 10

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Chooses $10\times\frac{2}{3}$.	believes that multiplication always makes a number larger	MULTIPLY A FRACTION AND A WHOLE NUMBER
Chooses both.	sees that they both contain 10 and considers the two expressions equivalent	MULTIPLY A FRACTION AND A WHOLE NUMBER

6. You decide to invite 24 people to your birthday party.

6.a. You have \$4.00 to mail the invitations and each card costs $\frac{1}{5}$ of a dollar to send. Do you have enough money to send the invitations, or will you need to ask to borrow some money?

CORRECT ANSWER

Five invitations will cost $1.00, so $4.00 will only buy 20 invitations. I will need to borrow money to cover the cost of invitations.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
I will need to borrow money because I don't have $24.00.	does not consider the fractional part of a dollar	MULTIPLY A FRACTION AND A WHOLE NUMBER
I don't have enough money.	only guesses and provides no explanation for their thinking	MULTIPLY A FRACTION AND A WHOLE NUMBER

6.b. Only $\frac{5}{8}$ of those invited can attend. How many people can come to your party? Show your work.

CORRECT ANSWER

15 people can attend the party.

$$24 \times \frac{5}{8} = \frac{24 \times 5}{8} = \frac{120}{8} = 15.$$

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Eight people can attend the party.	interprets the denominator of the fraction as a whole number of people who can attend the party	SOLVE WORD PROBLEMS INVOLVING MULTIPLICATION OF A FRACTION AND A WHOLE NUMBER
Five people can attend the party.	interprets the numerator of the fraction as a whole number of people who can attend the party	SOLVE WORD PROBLEMS INVOLVING MULTIPLICATION OF A FRACTION AND A WHOLE NUMBER
$\frac{5}{8}$ of a person can attend the party.	does not interpret the fraction as a part of a whole	SOLVE WORD PROBLEMS INVOLVING MULTIPLICATION OF A FRACTION AND A WHOLE NUMBER
$24 \times \frac{5}{8} = \frac{24 \times 5}{24 \times 8} = \frac{120}{192}$	incorrectly multiplies by multiplying the whole number to both the numerator and denominator	MULTIPLY A FRACTION AND A WHOLE NUMBER
$\frac{245}{8}$ or $\frac{245}{248}$	combines the whole number with the numerator (and possibly also the denominator) instead of multiplying	MULTIPLY A FRACTION AND A WHOLE NUMBER
$24 \times \frac{1}{8} = 3$	finds the unit fraction but does not iterate it five times (only finds the size of each part, does not multiply by the number of parts)	MULTIPLY A FRACTION AND A WHOLE NUMBER

6.c. How many people invited cannot come to your party? Show your work.

CORRECT ANSWER

Nine people cannot come to the party.

$24 - 15 = 9$ or $24 \times \frac{3}{8} = \frac{24 \times 3}{8} = \frac{72}{8} = 9.$

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Three people cannot attend the party.	finds the complement of $\frac{5}{8}$ to be $\frac{3}{8}$ and interprets the numerator of the fraction as a whole number of people who cannot attend the party	SOLVE WORD PROBLEMS INVOLVING MULTIPLICATION OF A FRACTION AND A WHOLE NUMBER
Eight people cannot attend the party.	finds the complement of $\frac{5}{8}$ to be $\frac{3}{8}$ and interprets the denominator of the fraction as a whole number of people who cannot attend the party	SOLVE WORD PROBLEMS INVOLVING MULTIPLICATION OF A FRACTION AND A WHOLE NUMBER
$\frac{3}{8}$ of a person cannot attend the party.	finds the complement of $\frac{5}{8}$ to be $\frac{3}{8}$, but does not find that corresponding part of the whole (24)	SOLVE WORD PROBLEMS INVOLVING MULTIPLICATION OF A FRACTION AND A WHOLE NUMBER
$24 \times \frac{3}{8} = \frac{24 \times 3}{24 \times 8}$	incorrectly multiplies the whole number by the numerator and the denominator	MULTIPLY A FRACTION AND A WHOLE NUMBER