THE UNIVERSITY OF KANSAS
Achievement & Assessment Institute

SCALE DRAWINGS AND MEASUREMENT

7.G.1

CONTENTS

The types of documents contained in the unit are listed below. Throughout the unit, the documents are arranged by lesson.

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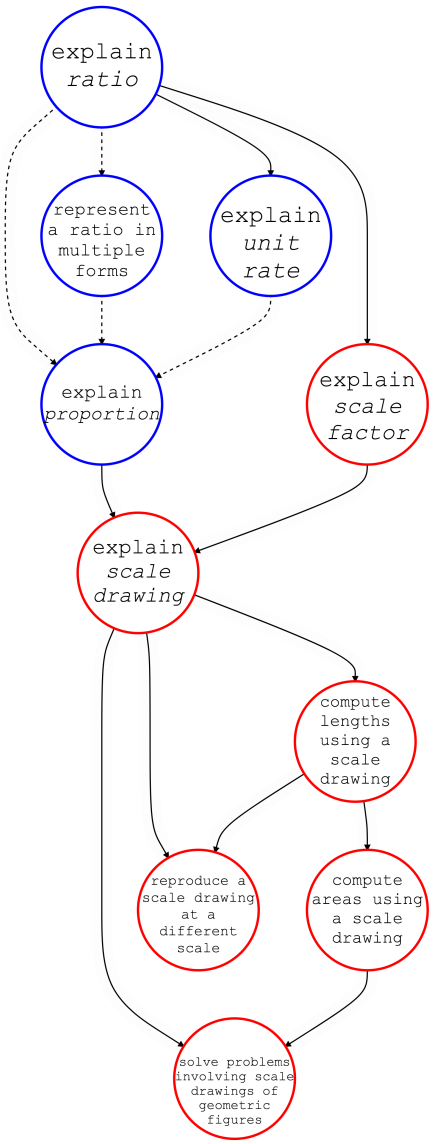
SCALE DRAWINGS AND MEASUREMENT

LEARNING MAP INFORMATION

STANDARDS

7.G.1 Solve problems involving scale drawings of geometric figures, such as computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.



**Learning map model of* 7.G.1

Node Name	Node Description
COMPUTE AREAS USING A SCALE DRAWING	Use your understanding of proportions to compute actual areas or scaled areas using a scale drawing, recognizing that areas, which are two-dimensional measurements, change by the square of the scale factor.
COMPUTE LENGTHS USING A SCALE DRAWING	Use your understanding of proportions to compute actual lengths or scaled lengths using a scale drawing.
EXPLAIN *PROPORTION*	Make known your understanding that a proportion is a statement of equality among two or more ratios; the numerical value of the ratio remains constant even as the amount of the quantities within the ratio changes.
EXPAIN *RATIO*	Make known your understanding that a ratio represents a multiplicative comparison of two quantities or the joining of two quantities into a composed unit. For example, the ratio of eyes to noses on a person is 2:1, because for every two eyes there is one nose.
EXPLAIN *SCALE DRAWING*	Make known your understanding that a scale drawing is a drawing with dimensions at a specific ratio relative to the actual size of the drawing.
EXPLAIN *SCALE FACTOR*	Make known your understanding that in an enlargement or a reduction, the scale factor is the ratio of the lengths in the image to the corresponding lengths in the pre-image. Understand that scale factors greater than one represent an enlargement, while scale factors less than one and greater than zero represent a reduction.
EXPLAIN *UNIT RATE*	Make known your understanding that a unit rate expresses a multiplicative comparison as a quantity per one. For example, 60 miles per hour.
REPRESENT A RATIO IN MULTIPLE FORMS	Through writing or an appropriate assistive technology, represent a ratio in multiple forms, including as a fraction, with a colon, or using the word "to" (e.g., $\frac{2}{1}$, 2:1, 2 to 1).
REPRODUCE A SCALE DRAWING AT A DIFFERENT SCALE	Use your understanding of proportions to reproduce a scale drawing at a different scale.
SOLVE PROBLEMS INVOLVING SCALE DRAWINGS OF GEOMETRIC FIGURES	Use your understanding of proportions to solve problems involving scale drawings of geometric figures.

SCALE DRAWINGS AND MEASUREMENT

TEACHER NOTES

This unit includes the following documents:

- Learning Map Information
- Instructional Activity (four lessons)
- Instructional Activity Student Handout (for Lessons 2 & 3)
- Instructional Activity Supplement (for Lessons 1 – 3)
- Student Activity
- Student Activity Solution Guide

In this unit, students will learn about the multiplicative nature of ratios, building on their intuitive conception of proportions and confronting misconceptions.

RESEARCH

The concept of proportion can be developed from students' intuitive understanding of a unit rate and the algebraic rule $y = mx$ (Cramer & Post, 1993). Students are able to conceptualize unit rate by asking "how many/much for one?" and often apply this strategy when asked to solve proportions (Singh, 2000). Students also use the factor-of-change strategy, or "times as many", where students recognize one quantity being multiplied by the scale factor and apply that multiplication to the other quantity (Cramer & Post, 1993).

Only after students have developed a conceptual understanding of how and why proportions are solved can algorithmic knowledge, such as cross multiplication, be introduced (Dwyer et al. 2003). Even though students may know how to correctly cross multiply, they often do not know when it is appropriate to apply the procedure (Cramer & Post 1993). Student understanding of intuitive methods of solving proportions should help alleviate this issue.

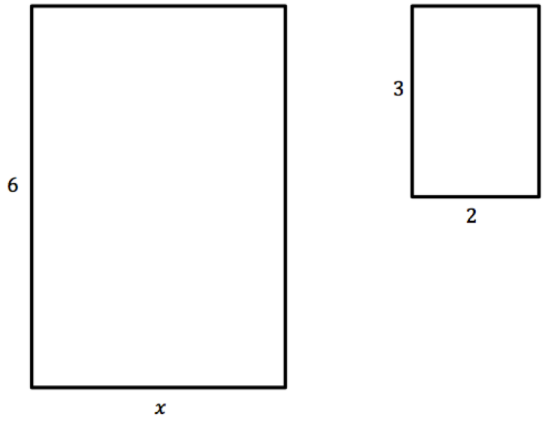
Middle school students may have an awareness of the concept of similarity, however they should develop a more precise understanding through the use of a scale factor, i.e. each corresponding part of similar figures are related by a scale factor (Buhl, Oursland, & Finco, 2003). Their innate knowledge of proportionality, however basic, comes from their experiences with similarity in the context of real-world problems. Therefore, it is important to introduce the concept of a scale drawing with a contextual concrete example (Johnston, 2004). It is also helpful for students to encounter unconventional scales, such as scales that contain fractions, like $\frac{1}{2}$ to 1. Additionally, students should develop an understanding of scale factors represented in all three forms (1 to 2, $\frac{1}{2}$, and 1:2), and understand that ratios written as fractions do not necessarily represent a reduction, such as $\frac{9}{4}$ which shows an enlargement.

CAUTIONS AND MISCONCEPTIONS

The most common misconception related to students' understanding of proportions is their incorrect use of additive reasoning when multiplicative reasoning is required (Dwyer, Causey-Lee & Irby, 2003).

AN EXAMPLE

Consider the following two rectangles:



Students who use additive reasoning may conclude that the long side decreases by three (6 units became 3 units), therefore x decreased by 3 must equal 2. This student would incorrectly conclude that $x = 5$.

To address this misconception directly, show or ask students to draw a simple polygon (a rectangle is simple and can clearly illustrate this concept) and add a constant to each side, in order to make a few additional figures in the pattern (Cox & Edwards, 2012). The more figures the students see in the pattern, the more they should recognize that the figures are not in proportion with one another. This could also be done with a picture or a photograph, to further illustrate additive disproportionality.

LEARNING MAP INFORMATION

The learning map section for this sequence of activities has a foundation in students' understanding of ratios and proportions. Once students are able to work with proportions, they are ready to apply that knowledge to scale drawings. Students should discover that all dimensions in a scale drawing are proportional to the original figure, be able to compute actual lengths and areas from a scale drawing, and redraw the scale drawing at a

different scale. A student's ability to apply these concepts and skills enables them to solve a variety of problems involving scale drawings and geometric figures.

INSTRUCTIONAL ACTIVITIES

The activities in this unit are designed to introduce students to scale factors through work with scale drawings, and to lead students to develop an algebraic understanding of scale factors and proportions. Students will first enlarge a given drawing and begin to see the role scale factors play in enlargements and reductions. Next, students will calculate the dimensions and areas of both the original and scaled figures, in order to finally notice a relationship between scaled dimensions and the scaled areas. The last lesson provides students the opportunity to explore reductions and enlargements through technology. The activities are rooted in real-world contexts, as students work with a floor plan to answer questions about flooring purchases.

REFERENCES

Buhl, D., Oursland, M., & Finco, K. (2003). The Legend of Paul Bunyan: An Exploration in Measurement. *Mathematics Teaching in the Middle School, 8*(8), 441-448.

Cox, D. & Edwards, M. (2012). Sizing up the Grinch's Heart. *Mathematics Teaching in the Middle School, 18*(4), 228-235.

Cramer, K., & Post, T. (1993). Proportional reasoning. *The Mathematics Teacher*, *86*(5), 404-407.

Dwyer, N., Causey-Lee, B., & Irby, N. (2003). Conceptualizing Ratios with Look-Alike Polygons. *Mathematics Teaching in the Middle School, 8*(8), 426-431.

Johnston, D. (2004). Measurement, Scale, & Theater Arts. *Mathematics Teaching in the Middle School, 9*(8), 412-417.

Singh, P. (2000). Understanding the Concepts of Proportion and Ratio Constructed by Two Grade Six Students. *Educational Studies in Mathematics, 43*(3), 271-292.

SCALE DRAWINGS AND MEASUREMENT

OVERVIEW OF INSTRUCTIONAL ACTIVITIES

Lesson	Learning Goal	Nodes Addressed
Lesson 1	Students will distinguish between multiplicative and additive reasoning while creating a scale drawing.	▶ EXPLAIN *PROPORTION* ▶ EXPLAIN *SCALE FACTOR* ▶ EXPLAIN *SCALE DRAWING* ▶ COMPUTE LENGTHS USING A SCALE DRAWING ▶ REPRODUCE A SCALE DRAWING AT A DIFFERENT SCALE
Lesson 2	Students will use a scale drawing of a floor plan to calculate dimensions and areas of rooms.	▶ REPRESENT A RATIO IN MULTIPLE FORMS ▶ EXPLAIN *SCALE FACTOR* ▶ EXPLAIN SCALE DRAWING ▶ COMPUTE LENGTHS USING A SCALE DRAWING ▶ COMPUTE AREAS USING A SCALE DRAWING ▶ SOLVE PROBLEMS INVOLVING SCALE DRAWINGS OF GEOMETRIC FIGURES
Lesson 3	Students will reproduce a scale drawing and use scale drawings to solve problems.	▶ EXPLAIN *SCALE FACTOR* ▶ COMPUTE LENGTHS USING A SCALE DRAWING ▶ COMPUTE AREAS USING A SCALE DRAWING ▶ REPRODUCE A SCALE DRAWING AT A DIFFERENT SCALE
Lesson 4	Students will explore reductions and enlargements through technology.	▶ EXPLAIN *SCALE FACTOR* ▶ COMPUTE LENGTHS USING A SCALE DRAWING ▶ REPRODUCE A SCALE DRAWING AT A DIFFERENT SCALE

SCALE DRAWINGS AND MEASUREMENT

INSTRUCTIONAL ACTIVITY

Lesson 1

LEARNING GOAL

Students will distinguish between multiplicative and additive reasoning while creating a scale drawing.

PRIMARY ACTIVITY

Students are given an individual small "puzzle piece" of a picture, which they will enlarge and combine with their classmates' enlarged pieces in order to create a large poster.

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

- Scale factor
- Scale drawing
- Enlargement
- Ratio
- Proportional

MATERIALS

- Copy of a drawing or design
- Puzzle pieces
- 8.5 x 11" computer paper
- Scissors
- Markers/crayons/colored pencils
- INSTRUCTIONAL ACTIVITY SUPPLEMENT (Recommend one copy for every student.)

IMPLEMENTATION

In this lesson, each student will be given a square of the same dimension, cut from a drawing, which is their puzzle piece. The students will enlarge the design on the puzzle pieces by the same factor in order to create a larger version of the original design.

Cut a drawing or design into as many pieces as students in a class.

> NOTE: The drawing should be relatively simple. A team or school logo, or a page from a coloring book will work best. Additionally, the amount of detail in the drawing could be adjusted based on the level of students and their previous experience and understanding of ratios and proportions.

Distribute one puzzle piece and one INSTRUCTIONAL ACTIVITY SUPPLEMENT to each student.

Direct students to read the instructions on the INSTRUCTIONAL ACTIVITY SUPPLEMENT. After students have read the directions, **ask** them to think individually or discuss in pairs how they will enlarge their puzzle piece.

Discuss suggested methods as a class. This is an opportunity to foster multiplicative reasoning and to show how and why additive reasoning will not create a scale drawing.

> NOTE: Students often incorrectly rely on additive reasoning in order to work with ratios and proportions. For example, if the scale factor is four, a student may suggest to "add four" to the lengths of their puzzle piece to create the enlargement. Addressing this misconception before students begin their scale drawings will be beneficial.

Model how adding (or subtracting) a constant does not create a proportional drawing. A visual aid could be premade, or a few drawings could be drawn on the board for the class. For example, draw a five-inch by six-inch rectangle. Then add one inch to each side to create a few more rectangles. The students should begin to see that the rectangles are no longer in proportion with each other.

> NOTE: Students may describe the rectangles as "skinnier" or "shorter" when they are trying to describe a lack of proportionality. This should be used as an opportunity to introduce "proportion" into students' vocabulary.

Provide an appropriate scale factor for the students to enlarge their puzzle pieces by. A scale factor of 4:1 or 5:1, to enlarge the puzzle piece four or five times its original size, is usually acceptable.

Note that the scale factor is always the ratio of the new figure (image) to the original figure (pre-image) and can be written in three different forms (e.g., 4 to 1, 4:1, $\frac{4}{1}$). **Note** that writing the scale factor as a fraction may assist students in using proportional reasoning to determine the necessary lengths in the enlarged puzzle piece.

Direct students to begin drawing their enlarged puzzle piece, and then circulate around the room to assist students and ask guiding questions.

GUIDING QUESTIONS

Elicit student thinking:

- What does it mean to enlarge the image?
- What does the scale factor tell you?

Determine if the student can EXPLAIN *PROPORTION*:

- What do you know about proportions that could help determine the lengths in the enlarged puzzle piece?
- Given the ratio for the scale factor, how could you set up a proportion to determine the corresponding length in your enlarged puzzle piece?

Determine if the student can EXPLAIN *SCALE FACTOR*:

- What is a scale factor?
- What quantities does the scale factor compare?
- How can you tell whether the scale factor describes an enlargement or a reduction?

Determine if the student can EXPLAIN *SCALE DRAWING*:

- Can you describe the difference between a scale drawing and an original drawing?
- What must be true of the dimensions in a scale drawing compared to the dimensions in the original drawing?

Determine if the student can COMPUTE LENGTHS USING A SCALE DRAWING:

- How will you find the dimensions of your enlarged puzzle piece?
- [Point to a line or portion of a student's puzzle piece.] What will the length of this be in your enlarged drawing?

Determine if the student is ready to REPRODUCE A SCALE DRAWING AT A DIFFERENT SCALE:

- How long are the sides of your puzzle piece?
- How long will the sides be on your enlarged piece?

Students should be required to assemble their puzzle pieces as a class to create the enlarged image.

At the end of the activity, **ask** students to calculate the dimensions of the enlarged image, given the dimensions of the original image.

INSTRUCTIONAL ACTIVITY SUPPLEMENT

Lesson 1

Directions:

Study the puzzle piece you have been given. You will need to enlarge the puzzle piece and the image it contains by the scale factor provided. Keep in mind that the thickness of the lines will also need to increase by the given scale factor. When you have recreated the puzzle piece to its enlarged size, its color should be consistent with the original puzzle piece.

SCALE DRAWINGS AND MEASUREMENT

INSTRUCTIONAL ACTIVITY

Lesson 2

LEARNING GOAL

Students will use a scale drawing of a floor plan to calculate dimensions and areas of rooms.

PRIMARY ACTIVITY

Students are given a scale drawing of a house and asked to find the areas of the room in order to calculate how many square feet of flooring are needed.

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

- Scale factor
- Length
- Width
- Area
- Pre-image
- Image

MATERIALS

- Ruler
- INSTRUCTIONAL ACTIVITY STUDENT HANDOUT (Recommend one copy for every group of two to three students.)
- INSTRUCTIONAL ACTIVITY SUPPLEMENT (Recommend one copy for every group of two to three students.)

IMPLEMENTATION

Review ratios and introduce scale factors and scale drawings to students. Then, require students to find the actual area of several rooms from a scale drawing.

Ask students where they have encountered scale factors in their lives. If students struggle with the term "scale factor", use the terms "reduction" and "enlargement" to connect the information to prior knowledge.

Discuss the suggestions from students and mention common examples, such as blueprints, pupil dilations, and model cars, if students do not suggest them.

Model an example of a scale factor by showing students a model car. **Ask** students how much smaller the model car is than the actual car. Take suggestions, and come to a general consensus (or give the actual answer if known).

Write the decided-on scale factor on the board for students to see, and **define** the term *scale factor*, **emphasizing** that it is the ratio of the *image*, or the scaled object, to the *pre-image*, or the actual object.

For example, if it is decided that the model car is 20 times smaller than the actual car, write 1:20, 1 to 20, and $\frac{1}{20}$ to describe the scale factor (or the ratio) of the model car to the actual car.

> NOTE: Students have trouble with the multiple representations of scale factors (as a fraction, with a colon, and with the word "to"). All representations should be shown to students in order to increase students' familiarity with the different forms.

Discuss the difference between the scale factors 1:20 (a reduction) and 20:1 (an enlargement) in order to help students differentiate between a scale factor that represents an enlargement and a scale factor that represents a reduction.

GUIDING QUESTIONS

Elicit student thinking:

- What differences are there between the scale factor 1:20 and the scale factor 20:1?
- Which scale factors represent an enlargement?
- Which scale factors represent a reduction?

Determine if the student can REPRESENT A RATIO IN MULTIPLE FORMS:

- What would the scale factor 1:20 look like in fraction form?
- What would the scale factor 20:1 look like in fraction form?

Determine if the student can EXPLAIN *SCALE FACTOR*:

- What is a scale factor?
- What quantities does the scale factor compare?
- How can you tell whether the scale factor describes an enlargement or a reduction?

Determine if the student is ready to EXPLAIN SCALE DRAWING:

- What is true of all the dimensions of the model car?

Ask students to determine the *actual* length of the car if the model has a length of 10 inches. Have students share their method for finding the actual length of the car with their neighbors or in groups.

Change the scale factor to two to twenty-one (2:21). **Require** students to work in groups of three or four to find the actual length of the car (still considering the model length to be 10 inches). **Circulate** throughout the class to assist students.

Share/model students' work to show various methods of finding the length of the actual car. Solution methods may include setting up proportions to cross-multiply, reducing the scale factor to the unit rate, and using a common denominator (i.e. writing the fractions with a common denominator and comparing the numerators).

Distribute the INSTRUCTIONAL ACTIVITY SUPPLEMENT and the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT for students to work on in pairs or groups of three if necessary. Students will find the area of each room on the scale drawing in order to find the total square footage of the house.

NOTE: Teachers have a few options for the required flooring research in the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT. Students can perform this research in pairs or groups on school-provided or home technology. If technology is not readily available for individuals/partners, the teacher can lead an Internet search with the whole class. Another option would be to provide the class with samples of flooring from a home improvement store.

GUIDING QUESTIONS

Elicit student thinking:

- [Point to a room on the scale drawing.] What geometric shape is this room? How do you calculate the area of that shape?
- How could you find flooring prices?
- What kinds of qualities would you look for in flooring (carpet, hardwoods, etc.)?

Determine if the student can COMPUTE LENGTHS USING A SCALE DRAWING:

- [Point to a side length on the scale drawing.] How did you determine the length of this wall/side?

Determine if the student can COMPUTE AREAS USING A SCALE DRAWING:

- [Point to a room on the scale drawing.] How did you determine the length of this rectangle/room?

Determine if the student is ready to SOLVE PROBLEMS INVOLVING SCALE DRAWINGS OF GEOMETRIC FIGURES:

- What is the total square footage of the house?
- How much would it cost to carpet the entire house?
- How much would it cost to put down hardwood flooring in the entire house?

Students should be required to determine the total square footage of the house, and to decide the total cost of the flooring required.

At the end of the activity, teachers should instruct students to write a small summary (approximately one or two pages) of their calculations and decisions in response to Question 14 of the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT. The summary should include the total square footage of the house, which rooms they want to have carpet in, which rooms they want to have hardwood floors in, the cost for carpet, the cost for hardwood floors, and the total cost to put flooring throughout the house.

Name_____

Scale Drawings and Measurement

Lesson 2

1. Determine the lengths (in inches) of the following sides of the scale drawing:

   a. AB

   b. BC

   c. CD

   d. DE

   e. EF

   f. FG

   g. GH

   h. HI

Name_____

2. Use the given scale factor to determine the actual length of the sides:

a. AB

e. EF

b. BC

f. FG

c. CD

g. GH

d. DE

h. HI

Name_____

3. Describe the geometric shape(s) that make up the following rooms:

    a. Kitchen

    b. Bathroom

    c. Bedroom

    d. Living Room

4. Find the actual area of the kitchen. Show all your work.

5. Find the actual area of the living room. Show all your work.

6. Find the actual area of the bedroom. Show all your work.

7. Find the actual area of the bathroom. Show all your work.

8. Determine the cost per square foot of carpet and write it here:

9. Determine the cost per square foot of hardwood and write it here:

Name_____

10. Suppose your task is to choose the flooring that should be installed in the house. Decide what type of flooring you think should be installed in each room.

a. Living Room:_____

b. Kitchen:_____

c. Bathroom:_____

Bedroom:_____

Name_____

11. Determine the cost to put your chosen flooring in the following rooms:

a. Living Room:_____

b. Kitchen:_____

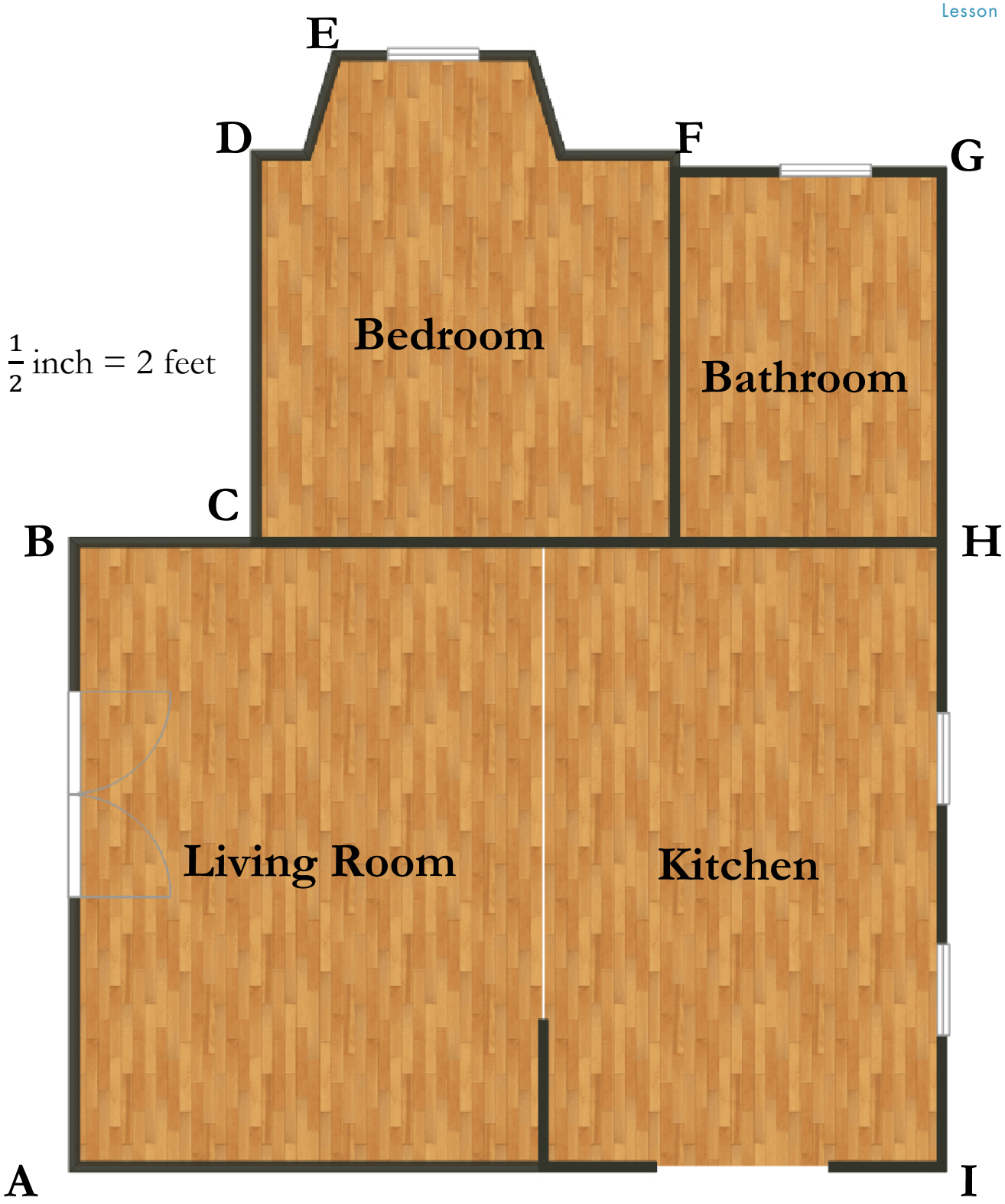
c. Bathroom:_____

d. Bedroom:_____

12. What is the total square footage of the house?

13. What is the total cost of the flooring for all rooms?

14. Write a proposal of your chosen flooring and their associated costs. This should be a summary of what you have calculated on this worksheet, as well as a justification of your choices.



SCALE DRAWING AND MEASUREMENT

INSTRUCTIONAL ACTIVITY

Lesson 3

LEARNING GOAL

Students will reproduce a scale drawing and use scale drawings to solve problems.

PRIMARY ACTIVITY

Students create a larger drawing of the floor plan they were given in LESSON 2. Their scale drawing will show their choices of flooring and will be large enough to present to the class.

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

- Reduction
- Enlargement
- Area
- Square units

MATERIALS

- Poster board
- Construction paper
- Glue
- Scissors
- Colored pencils
- INSTRUCTIONAL ACTIVITY STUDENT HANDOUT
- INSTRUCTIONAL ACTIVITY SUPPLEMENT (Recommend one copy for every two students.)

IMPLEMENTATION

After debriefing from LESSON 2, students create larger scale drawings to present their ideas to the class.

Recall that actual lengths and areas were found from the scale drawing of a floor plan.

Ask students to explain the methods they used to find actual lengths based on scale drawings.

Remind students of the methods used in LESSON 1 to solve proportions.

GUIDING QUESTIONS

Elicit student thinking:

- What are some methods we can use to solve proportions?

Determine if the student can EXPLAIN *SCALE FACTOR*:

- What scale factor did your group choose? How did you decide on this scale factor?
- Does your scale factor represent an enlargement or a reduction? How do you know?

Determine if the student can COMPUTE LENGTHS USING A SCALE DRAWING and COMPUTE AREAS USING A SCALE DRAWING:

- How did you calculate the actual lengths of the rooms?
- How did you calculate the actual areas of the rooms?

Once students have determined a scale factor for their poster-sized scale drawing, they will gather materials and start creating their enlarged scale drawing.

Previously, students chose flooring for each room of the house and calculated the actual square footage needed. **Direct** students to reproduce the scale drawing to be the size of a poster board. The larger size should be a better representation for presentation to a builder or homeowner.

Provide an overview of the task. Students will work with the same partners as in LESSON 2. **Tell** students that their scale drawing should take up at least 75 percent of the poster board, should be

labeled with dimensions, should display the scale factor, and should be colored to show the different types of flooring chosen.

Distribute the INSTRUCTIONAL ACTIVITY SUPPLEMENT to provide full directions.

Ask students if they have questions about the task. **Address** any questions and/or comments, then allow students to begin working.

GUIDING QUESTIONS

Elicit student thinking:

- What is a scale factor that you know creates an image too large for the poster board?
- What is a scale factor that you know creates an image too small for the poster board?

Determine if the student can REPRODUCE A SCALE DRAWING AT A DIFFERENT SCALE:

- How did you choose the scale factor for your poster-sized scale drawing?
- What scale factor would produce a model smaller than the actual size, but larger than the floor plan you were given?
- How do you set up and solve a proportion in order to find the new dimensions of the poster-sized scale drawing?

Students should be required to present their posters to the class and to describe how they determined the scale factor they used, if time allows.

At the end of the activity, **discuss** the patterns students identified between the lengths and the areas in the scale drawing compared to the original (i.e. that lengths increase by a factor of x, areas by a factor of x^2).

Name_____

Scale Drawings and Measurement

Lesson 3

1. Give the scale factor of your scale drawing: _____

2. Calculate and label the dimensions of your scale drawing.

3. Fill out the table for the corresponding lengths and widths of your drawing.

	Length on scale drawing	**Actual length**	**Width on scale drawing**	**Actual width**	**Area on scale drawing**	**Actual area**
Bathroom						
Living Room						
Kitchen						
Increase factor						

Name_____

4. By what factor did the length and width of your scale drawing increase?

5. By what factor did the area of your scale drawing increase? How does this factor compare to the factors in Question 4? Why do you think this is the case?

Instructions:

In order to create a presentation-sized floor plan, you will reproduce the scale drawing of the floor plan from LESSON 2. The new scale drawing will be larger, so that a builder or homeowner could easily see your design.

Requirements:

- The scale drawing should take up at least 75 percent of the poster board.
- The scale factor should be displayed on the poster board.
- All calculated dimensions should be labeled.
- Recreate the flooring in your scale model using construction paper and colored pencils.

Use the space below to show your work calculating the dimensions on the enlarged scale drawing.

SCALE DRAWING AND MEASUREMENT

INSTRUCTIONAL ACTIVITY

Lesson 4

LEARNING GOAL

Students will explore reductions and enlargements through technology.

PRIMARY ACTIVITY

Students will create a simple design in GeoGebra and reproduce it at a given scale factor.

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

- Enlargement
- Reduction
- Scale Factor
- Line Segment

MATERIALS

- Computers with internet connectivity
- Printer

IMPLEMENTATION

Students will choose a design to create on the website GeoGebra. The initial design will allow students to familiarize themselves with the software. The students are then given a scale factor in order to recreate the design as either an enlargement or reduction.

Explain the general idea of what students will be doing in this lesson.

Display a list of designs for students to choose from:

- A house
- A stick figure
- The student's initials
- 3 – 4 basic shapes
- A flower

Tell students that their design must be created with only line segments – no curved elements. Students should choose one of the design options. If students have their own idea for a design, it may be approved; however, the overall design should be fairly simple to create in GeoGebra and easy to recreate.

Direct students to go to www.geogebra.org/geometry and begin to create their chosen design.

Show students how to create line segments of specific length by choosing the  button.

Tell students to keep track of the lengths of the line segments in their design in order to accurately scale the corresponding segments in their scale drawing.

Allow students to explore the software for a while and find the different tools for creating their design. If students find one design difficult to create, they may choose another.

> NOTE: There are several tools in the GeoGebra software that students may use which are outside the scope of 7.G.1 standard but which also may help them in creating their design. For example, the may make rotations, 90-degree angles, or parallel lines. Allow students to explore these other mathematical concepts, but limit or support their use as appropriate. Additionally, students have been limited to only line segments of given lengths in order to require students to compute the lengths of the line segments in the scaled drawing. Allow other curved elements as deemed necessary and appropriate for various levels of student understanding.

Tell students to have their initial design approved by the teacher once they feel it is complete. They will then be given a scale factor to use for redrawing their design as either an enlargement or reduction.

There are a few options for assigning students a scale factor.

- Students may choose a scale factor for their image.
- A jar with several different scale factors written on paper could be prepared for students to choose a scale factor from at random.
- An interview process could be set up, where student and teacher have a discussion to determine an appropriate scale factor. The teacher can question the student in order to assess understanding, for example by asking, "Should you create an enlargement or a reduction? Why? How will your scale factor need to look for it to be an enlargement/reduction?"
- Scale factors can be created using a random number generator.

- Scale factors could be assigned to students ahead of time based on ability level in order to differentiate students' experiences.

NOTE: Students and teachers may choose to adapt this lesson to include finding and reproducing the areas of shapes.

GUIDING QUESTIONS

Elicit student thinking:

- How did you choose your design?
- Is there a scale factor that won't work with your design (i.e. too large or too small)? Why or why not?

Determine if the student can EXPLAIN *SCALE FACTOR*:

- Will your reproduction be smaller or larger than your original drawing?

Determine if the student can COMPUTE LENGTHS USING A SCALE DRAWING:

- What is the length of your original line segment?
- How will you compute the length of the reproduction?
- Will you need to cross multiply in order to find the length of the reproduced line segment?
- What computational strategies will you use to calculate the lengths of the scale drawing?

Determine if the student can REPRODUCE A SCALE DRAWING AT A DIFFERENT SCALE:

- [Point to a line segment in the original design.] Show me how you calculate the length of this line segment in the scale drawing.

Students should be required to print their designs and label the image with the scale factor that their designs represent and whether it is an enlargement or reduction.

At the end of the activity, teachers should review the student work for accuracy and display student images.

Name_____

Scale Drawings and Measurement

Lessons 1 – 4

1. Tell whether each of the following scale factors represents an enlargement or reduction of the original figure. Explain your reasoning.

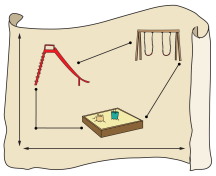
   1.a. 3:4

   1.b. 1 to 2

   1.c. $\frac{9}{5}$

Name_____

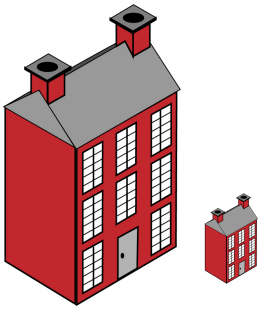
2. A map of a new playground has a scale factor of 1:50.



2.a. If the actual length of the monkey bars is 10 feet, what is the length of the monkey bars on the map? Give your answer in feet and inches.

2.b. If the length of the slide on the map is three inches, what is the actual length of the slide?

3. Alex builds dollhouses that are smaller versions of existing houses. The scale factor for a dollhouse is 1:5.



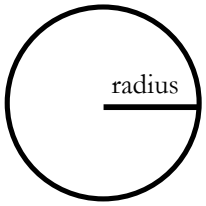
3.a. What is the scale factor for the area of the rooms in the dollhouse?

Name_____

3.b. If a room in the model house has an area of 2.5 square feet, what is the actual area of the room?

3.c. If the length of a kitchen wall on the dollhouse is 19.5 inches, what is the actual length of the wall?

4. By what factor does the area of a circle increase if the radius (the length from the center of the circle to a point on the circle) increases by a factor of 6? Explain your reasoning.



Name_____

5. Use your knowledge of scale factors to complete the following table. Note that the dimensions provided do not apply to the same figure. Therefore, the area of Figure 3 cannot be calculated by multiplying the length of Figure 1 by the width of Figure 2; understanding of scale factor must be used.

Scale Factor	Length of **Figure 1** on Scale Drawing (inches)	Actual Length of **Figure 1** (inches)	Width of **Figure 2** on Scale Drawing (centimeters)	Actual Width of **Figure 2** (centimeters)	Area of **Figure 3** on Scale Drawing ($inches^2$)	Actual Area of **Figure 3** ($inches^2$)
1:5	4			35	20	
2:1		45	3			450
	3	9	13		65	
		4	18	3	108	

SCALE DRAWINGS AND MEASUREMENT

STUDENT ACTIVITY SOLUTION GUIDE

Lessons 1 – 4

1. Tell whether each of the following scale factors represents an enlargement or reduction of the original figure. Explain your reasoning.

1.a. 3:4

CORRECT ANSWER

> Reduction. The scale factor is less than one, but greater than zero, therefore the image is smaller than the pre-image.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Enlargement, because 4 is bigger than 3.	does not understand that scale factor is image : pre-image, and mistakenly thinks that scale factor represents pre-image : image	EXPLAIN *SCALE FACTOR*
Enlargement. The ratio is not a fraction.	mistakenly believes that all scale factors must be written in fraction form, or that the only way to have a reduction is to see a fraction bar in the scale factor	REPRESENT A RATIO IN MULTIPLE FORMS
Reduction.	does not explain answer	EXPLAIN *SCALE FACTOR*

1.b. 1 to 2

CORRECT ANSWER

> Reduction. The scale factor is less than one, but greater than zero, therefore the image is smaller than the pre-image.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Enlargement, because the scale factor says to double.	thinks that the scale factor is representing pre-image to image, or sees the numbers 2 and 1 and assumes that they are representing "twice as much"	EXPLAIN *SCALE FACTOR*
Enlargement. The ratio is not a fraction.	mistakenly believes that all scale factors must be written in fraction form, or that the only way to have a reduction is to see a fraction bar in the scale factor	REPRESENT A RATIO IN MULTIPLE FORMS
Reduction.	does not explain answer	EXPLAIN *SCALE FACTOR*

1.c. $\frac{9}{5}$

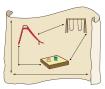
CORRECT ANSWER

> Enlargement. The scale factor is greater than one, therefor the image is larger than the pre-image.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Reduction because it's a fraction.	incorrectly assumes that all fractions are less than one and therefore represent a "small" number, or reduction	REPRESENT A RATIO IN MULTIPLE FORMS
Enlargement, because 9 and 5 are big numbers.	concludes that, because the numbers are larger in part c than part a and b, this must be an enlargement	EXPLAIN *SCALE FACTOR*
Enlargement.	does not explain answer	EXPLAIN *SCALE FACTOR*

2. A map of a new playground has a scale factor of 1:50.



2.a. If the actual length of the monkey bars is 10 feet, what is the length of the monkey bars on the map? Give your answer in feet and inches.

CORRECT ANSWER

> The length of the monkey bars on the map is 0.2 feet, or 2.4 inches.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
0.2	does not label units	SOLVE PROBLEMS INVOVLING SCALE DRAWINGS OF GEOMETRIC FIGURES
500 ft	multiplies by 50 instead of dividing by 50	COMPUTE LENGTHS USING A SCALE DRAWING or EXPLAIN *SCALE FACTOR*

2.b. If the length of the slide on the map is 3 inches, what is the actual length of the slide?

CORRECT ANSWER

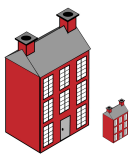
> The actual length of the slide is 12.5 feet, or 150 inches.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
0.06 inches	divides by 50 instead of multiplying by 50	COMPUTE LENGTHS USING A SCALE DRAWING or EXPLAIN *SCALE FACTOR*
0.005	converts to feet, but divides by 50 instead of multiplying by 50	COMPUTE LENGTHS USING A SCALE DRAWING or EXPLAIN *SCALE FACTOR*

3. Alex builds dollhouses that are smaller versions of existing houses. The scale factor for a dollhouse is 1:5.

3.a. What is the scale factor for the area of the rooms in the dollhouse?



CORRECT ANSWER

The scale factor for the area of the rooms is 1:25, 1 to 25, or $\frac{1}{25}$.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
1:5	does not recognize that areas increase by a factor of the unit rate squared	COMPUTE AREAS USING A SCALE DRAWING
25:1	writes the scale factor backwards	EXPLAIN *SCALE FACTOR*

3.b. If a room in the model house has an area of 2.5 square feet, what is the actual area of the room?

CORRECT ANSWER

The actual area of the room is 62.5 square feet.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
0.1 square feet	divides 2.5 by 25 instead of multiplying 2.5 by 25	COMPUTE AREAS USING A SCALE DRAWING or EXPLAIN *SCALE FACTOR*
10 square feet	divides 25 by 2.5 instead of multiplying 2.5 by 25	COMPUTE AREAS USING A SCALE DRAWING
12.5 square feet	multiplies by a factor of 5 instead of 25	COMPUTE AREAS USING A SCALE DRAWING

3.c. If the length of a kitchen wall on the dollhouse is 19.5 inches, what is the actual length of the wall?

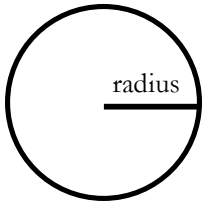
CORRECT ANSWER

The actual length of the wall is 97.5 inches, or 8.125 feet.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
487.5 inches	uses a scale factor of 1:25 instead of 1:5	COMPUTE LENGTHS USING A SCALE DRAWING
3.9 inches	divides 19.5 by 5 instead of multiplying 19.5 by 5	COMPUTE LENGTHS USING A SCALE DRAWING or EXPLAIN *SCALE FACTOR*

4. By what factor does the area of a circle increase if the radius (the length from the center of the circle to a point on the circle) increases by a factor of 6? Explain your reasoning.



CORRECT ANSWER

> If the radius of a circle increases by a factor of 6, then the area of the circle will increase by a factor of 36. Given that a one-dimensional measurement is increased by a factor of k, I know that a two-dimensional measurement will increase by a factor of k^2.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The area increases by a factor of 36.	does not explain	COMPUTE AREAS USING A SCALE DRAWING
The area increases by a factor of 6, because the scale factor is 6:1.	does not understand that area increases by a factor of k^2	COMPUTE AREAS USING A SCALE DRAWING

5. Use your knowledge of scale factors to complete the following table. Note that the dimensions provided do not apply to the same figure. Therefore, the area of Figure 3 cannot be calculated by multiplying the length of Figure 1 by the width of Figure 2; understanding of scale factor must be used.

Scale Factor	Length of **Figure 1** on Scale Drawing (inches)	Actual Length of **Figure 1** (inches)	Width of **Figure 2** on Scale Drawing (centimeters)	Actual Width of **Figure 2** (centimeters)	Area of **Figure 3** on Scale Drawing (inches^2)	Actual Area of **Figure 3** (inches^2)
1:5	4			35	20	
2:1		45	3			450
	3	9	13		65	
		4	18	3	108	

CORRECT ANSWER

Scale Factor	Length of **Figure 1** on Scale Drawing (inches)	Actual Length of **Figure 1** (inches)	Width of **Figure 2** on Scale Drawing (centimeters)	Actual Width of **Figure 2** (centimeters)	Area of **Figure 3** on Scale Drawing (inches^2)	Actual Area of **Figure 3** (inches^2)
1:5	4	**20**	**7**	35	20	**500**
2:1	**90**	45	3	**1.5**	**1800**	450
1:3	3	9	13	**39**	65	**585**
6:1	**24**	4	18	3	108	**3**

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Actual Area Column: 100, 450, 195, 18	uses the scale factor given instead of squaring the given scale factor	COMPUTE AREAS USING A SCALE DRAWING
Actual Length Column: 0.8, 45, 9, 4	calculates a reduction instead of an enlargement	EXPLAIN *SCALE FACTOR*