COMPARING LINEAR AND NONLINEAR FUNCTIONS

8.F.A.2 and 8.F.A.3

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CONTENTS

The types of documents contained in the unit are listed below. Throughout the unit, the documents are arranged by lesson.

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COMPARING LINEAR AND NONLINEAR FUNCTIONS

LEARNING MAP INFORMATION

STANDARDS

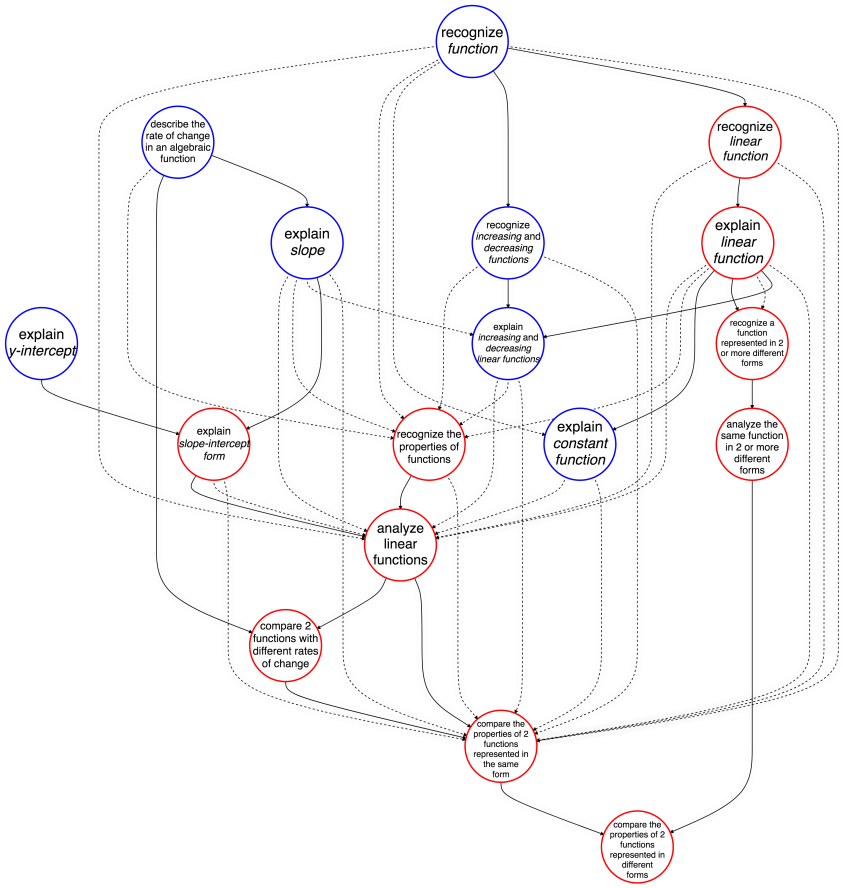
8.F.A.2 Compare characteristics of two functions each represented in a different way.

8.F.A.3 Investigate the differences between linear and nonlinear functions.

a. Interpret the equation $y = mx + b$ as defining a linear function, whose parameters are the slope (m) and the y-intercept (b).

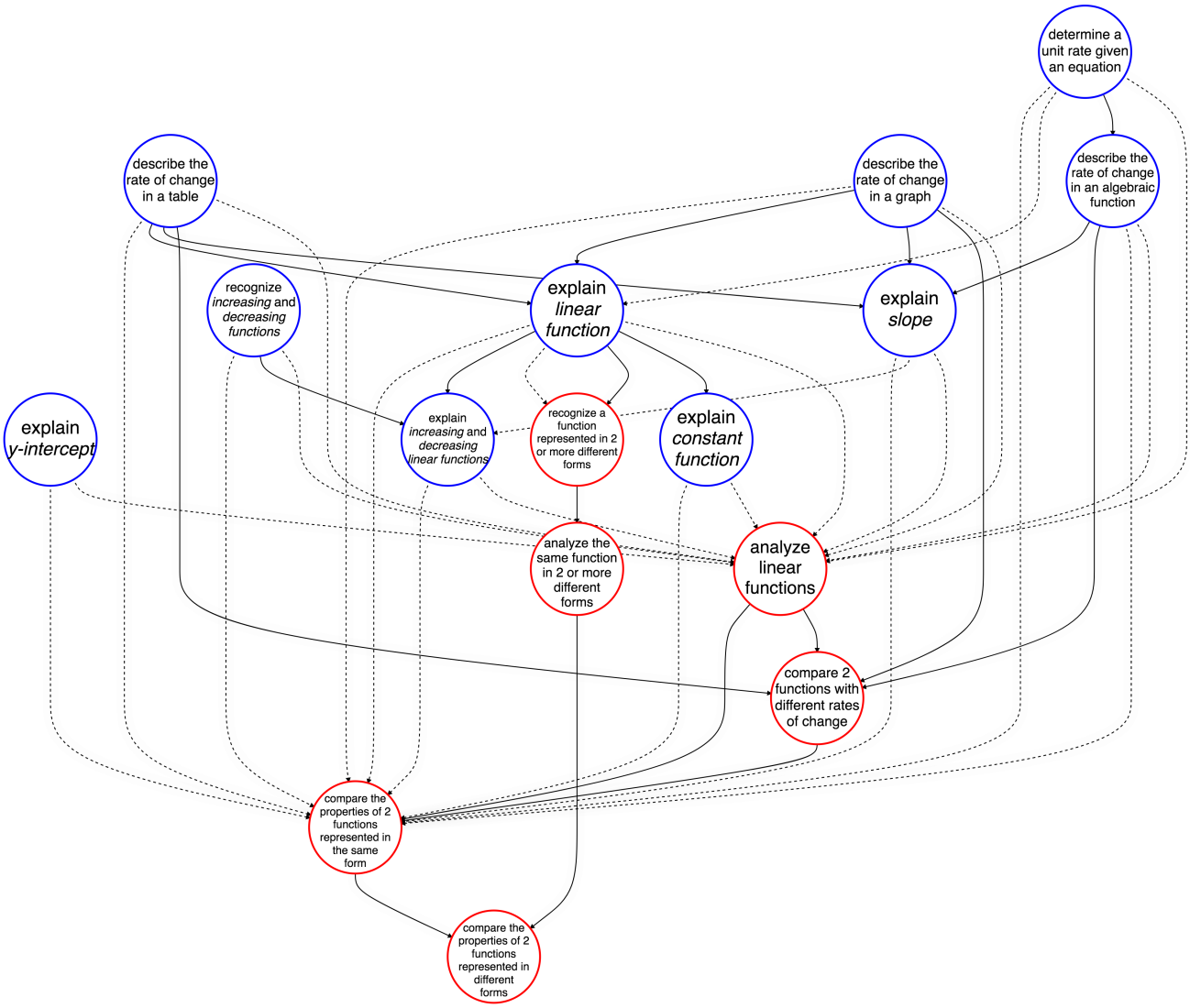
b. Recognize that the graph of a linear function has a constant rate of change.

c. Give examples of nonlinear functions.



**Learning map model of* 8.F.2 and 8.F.3

8.F.A.2 Compare characteristics of two functions each represented in a different way.



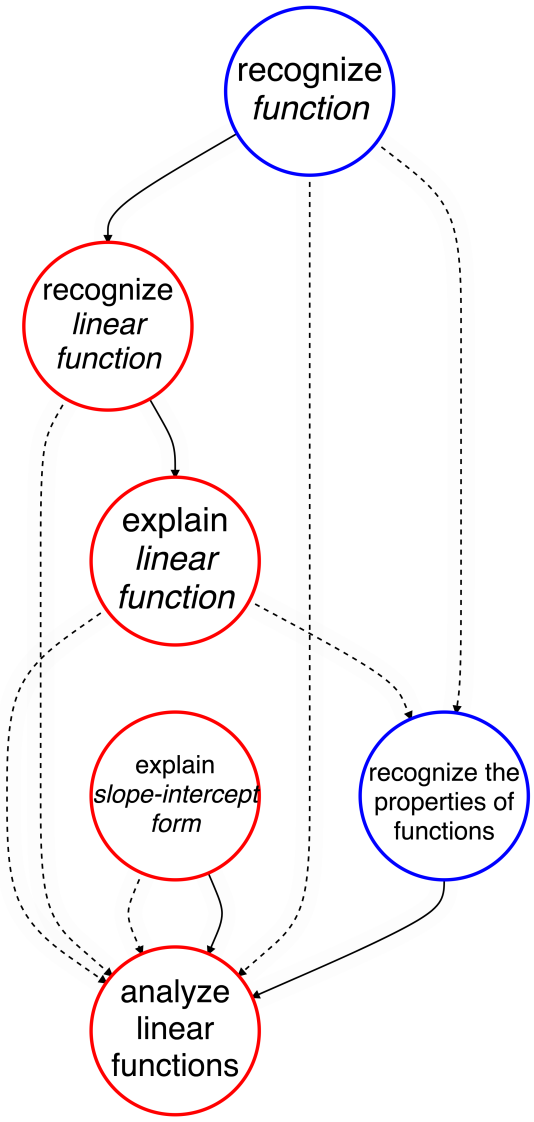
**Learning map model of* 8.F.2

8.F.A.3 Investigate the differences between linear and nonlinear functions.

a. Interpret the equation $y = mx + b$ as defining a linear function, whose parameters are the slope (m) and the y-intercept (b).

b. Recognize that the graph of a linear function has a constant rate of change.

c. Give examples of nonlinear functions.



**Learning map model of* 8.F.3

Node Name	Node Description
ANALYZE LINEAR FUNCTIONS	Analyze a linear function in any representation (e.g., a graph, a table, an algebraic equation, or a description) by identifying and explaining local properties (e.g., intercepts and function values at given points) and global properties (e.g., the slope/rate of change and the direction of covariation).
ANALYZE THE SAME FUNCTION IN 2 OR MORE DIFFERENT FORMS	Analyze the properties of one function represented in different forms (e.g., a graph, a table, an algebraic equation, or a description).
COMPARE THE PROPERTIES OF 2 FUNCTIONS REPRESENTED IN DIFFERENT FORMS	Compare the properties (e.g., domain, range, rate of change, and overall shape) of two functions represented in different ways (e.g., algebraically, graphically, numerically in tables, or verbally in descriptions).
COMPARE THE PROPERTIES OF 2 FUNCTIONS REPRESENTED IN THE SAME FORM	Compare the properties (e.g., rate of change, intercepts, direction of a covariation, or overall shape) of two functions represented in the same way (e.g., algebraically, numerically in tables, graphically, or verbally in descriptions).
COMPARE 2 FUNCTIONS WITH DIFFERENT RATES OF CHANGE	Compare two functions with different rates of change to communicate which function is faster or slower or is higher or lower.
DESCRIBE THE RATE OF CHANGE IN A GRAPH	Describe the rate of change in a function graph by quantifying the covariation between two variables.
DESCRIBE THE RATE OF CHANGE IN A TABLE	Describe the rate of change in a function table by quantifying the covariation between two variables.
DESCRIBE THE RATE OF CHANGE IN AN ALGEBRAIC FUNCTION	Describe the rate of change in an algebraic function by quantifying the covariation between two variables.
DETERMINE A UNIT RATE GIVEN AN EQUATION	Transform a given equation to determine the rate at which one variable changes in terms of one unit of another variable.
EXPLAIN *CONSTANT FUNCTION*	Make known your understanding that a constant linear function has a zero slope and its graph is a straight, horizontal line.
EXPLAIN *INCREASING* AND *DECREASING LINEAR FUNCTIONS*	Make known your understanding that an increasing linear function has a positive slope and a decreasing linear function has a negative slope.
EXPLAIN *LINEAR FUNCTION*	Make known your understanding that a linear function changes at a constant rate and its graph is a straight line.
EXPLAIN *SLOPE*	Make known your understanding that the slope of a line is the steepness of the line as a ratio. Describe *slope* as rise over run or the change in y divided by the change in x.
EXPLAIN *SLOPE-INTERCEPT FORM*	Make known your understanding that a linear function can be described by its slope-intercept form ($y = mx + b$), which consists of the slope, m, and the y-intercept, b.
EXPLAIN Y-*INTERCEPT*	Make known your understanding that the y-intercept is a coordinate pair where the graph of a function intersects the y-axis.
RECOGNIZE A FUNCTION REPRESENTED IN 2 OR MORE DIFFERENT FORMS	Make known your understanding through words, drawings, manipulatives, etc., that a function can be modeled in two or more different forms.
RECOGNIZE *FUNCTION*	Recognize a function as a set of ordered pairs where there exists a relationship between x- and y-coordinates and there are no two ordered pairs with the same input and different outputs.

Node Name	Node Description
RECOGNIZE *INCREASING* AND *DECREASING FUNCTIONS*	Identify or name an increasing function as a function with a graph that rises from left to right over the entire domain. Identify or name a decreasing function as a function with a graph that falls from left to right over the entire domain.
RECOGNIZE *LINEAR FUNCTION*	Identify or name a linear function, and recognize when a function is nonlinear.
RECOGNIZE THE PROPERTIES OF FUNCTIONS	Determine and understand the local properties (e.g., intercepts and max/min) and global properties (e.g., domain, direction of covariance, and end behavior) of a function represented in any form.

ADDITIONAL NODES RELATED TO THIS UNIT OF INSTRUCTION

Node Name	Node Description	Related Node
RECOGNIZE *COVARIATION*	Recognize the change that occurs in the dependent variable when the independent variable is changed.	Prerequisite of EXPLAIN *SLOPE* (through RECOGNIZE THE DIRECTION OF A COVARIATION)
RECOGNIZE *Y-INTERCEPT*	Identify or name a *y*-intercept as a point on the *y*-axis.	Prerequisite of EXPLAIN *Y-INTERCEPT*

COMPARING LINEAR AND NONLINEAR FUNCTIONS

TEACHER NOTES

This unit includes the following documents:

- Learning Map Information
- Instructional Activity (includes four lessons)
- Instructional Activity Student Handout (for Lesson 1)
- Instructional Activity Supplement (for Lessons 1–4)
- Student Activity (Word Version)
- Student Activity Solution Guide

RESEARCH

Students need to have robust understanding of the function concept (Carlson & Oehrtman, 2005), including viewing functions as means for transforming an entire set of values (i.e., the domain) to yield another entire set of values (i.e., the range; Leinhardt, Zaslavsky, & Stein, 1990). Students' early experiences with evaluating functions serve as a basis for interpreting and constructing quantitative relationships among particular, often isolated, values. This process of interpreting and constructing relationships between pairs of corresponding values is identified as a local perspective (Leinhardt et al., 1990). At this stage, students may focus on the correspondence between a particular element in the domain with its image in the range. Students need to move beyond a local focus to a global perspective, where they consider the behavior of a function over its entire domain, describe the function qualitatively, and sketch a graph to capture the general trend of the function (Leinhardt et al., 1990). At this stage, students will attend to the nature of the covariation that governs the relationship between the two quantities. This type of understanding prepares students for concepts such as inverse functions and special properties (i.e., one-to-one, onto).

Instruction that focuses on algebraic procedures provides students with limited opportunities to develop authentic understanding of how functions represent relationships among quantities. Learning experiences that focus on evaluating functions at particular input values to calculate specific outputs are insufficient for developing an understanding of functions as predictable relationships between quantities (Leinhardt et al., 1990). Alternatively, students need learning experiences to reinforce the idea that functions accept inputs and produce outputs according to predictable rules, such as a rate of change, as is the case for linear functions. They also need opportunities to consider how functions act on entire intervals so they may develop global, rather than local, perspectives.

Students' understanding is deepened through activities that require them to work with the same function in multiple representations, such as symbolic, graphical, numerical, or verbal forms (Carlson & Oehrtman, 2005; Leinhardt et al., 1990; Lloyd, Herbel-Eisenmann, & Star, 2011; Ronau, Meyer, Crites, & Dougherty, 2014). As students examine the characteristics of a function in its different representations, they should be able to articulate how each characteristic can be shown in different ways. For example, the y-intercept appears as (1) an initial value in a verbal description, (2) an ordered pair of the form (0, #) in numerical representation, (3) a

point lying on the *y*-axis of a graph, and (4) the value of an algebraic expression evaluated at $x = 0$. The skills addressed by the activities in this module include distinguishing linear and nonlinear functions; recognizing increasing, decreasing, and constant function behavior; comparing *y*-intercepts and slopes; and distinguishing steepness from slope. After working with the characteristics of one function at a time, students need opportunities to compare functions in the same and different representations by examining the features of each function while translating among different representations. Making these comparisons involves identifying similarities and differences, a skill that has been shown to effectively strengthen understanding and provide opportunities for students to form cognitive connections (Marzano, Pickering, & Pollock, 2001).

INSTRUCTIONAL ACTIVITIES

In this unit, lessons 1 and 2 require students to focus on one function at a time. In the first lesson, students will work exclusively with a small number of linear functions, and in the second lesson students will analyze more functions, including nonlinear examples. The functions are provided in multiple representations for the activity "Guess My Function," during which students will construct questions about and analyze properties of functions. After analyzing the properties of a single function in any representation, students will compare two functions in the same representation in lesson 3. Students will use comparison mats and descriptions to organize and to qualitatively compare two functions. This sequence of activities requires students to continue analyzing the properties of functions in various representations while incorporating comparisons of the common and different features of two functions. The final lesson requires students to compare two functions in different representations, where linear and nonlinear functions are represented in equations, tables, graphs, and real-world descriptions.

REFERENCES

Carlson, M., & Oehrtman, M. (2005). Research sampler: 9. Key aspects of knowing and learning the concept of function. Mathematical Association of America.

Leinhardt, G., Zaslavsky, O., & Stein, M. K. (1990). Functions, graphs, and graphing: Tasks, learning, and teaching. *Review of Educational Research, 60*, 1–64. doi:10.3102/00346543060001001

Lloyd, G. M., Herbel-Eisenmann, B. A., & Star, J. R. (2011). *Developing essential understanding of expressions, equations, and functions for teaching mathematics in grades 6–8*. Reston, VA: National Council of Teachers of Mathematics.

Marzano, R. J., Pickering, D. J., & Pollock, J. E. (2001). *Classroom instruction that works: Research-based strategies for increasing student achievement.* Alexandria, VA: Association for Supervision & Curriculum Development.

Ronau, R. N., Meyer, D., Crites, T., & Dougherty, B. J. (2014). *Putting essential understanding of functions into practice in grades 9–12.* Reston, VA: National Council of Teachers of Mathematics.

COMPARING LINEAR AND NONLINEAR FUNCTIONS

OVERVIEW OF INSTRUCTIONAL ACTIVITIES

Lesson	Learning Goal	Nodes Addressed
Lesson 1	Students will compare multiple representations of the same function. The critical outcome of this activity is for students to be able to find the rate of change and the *y*-intercept of a function given in any representation and to match different representations of the same functions.	▶ DESCRIBE THE RATE OF CHANGE IN AN ALGEBRAIC FUNCTION ▶ EXPLAIN *Y-INTERCEPT*
Lesson 2	Students will analyze the properties of linear and nonlinear functions given in different representations. The critical outcome of this activity is that students will be able to fluently describe and determine the rate of change and the *y*-intercept of any linear function and to identify and to describe representations of nonlinear functions.	▶ DESCRIBE THE RATE OF CHANGE IN AN ALGEBRAIC FUNCTION ▶ EXPLAIN *Y-INTERCEPT* ▶ RECOGNIZE *LINEAR FUNCTION* ▶ EXPLAIN *LINEAR FUNCTION* ▶ EXPLAIN *INCREASING* AND *DECREASING LINEAR FUNCTIONS*
Lesson 3	Students will analyze and compare linear and nonlinear functions given in the same representation. The critical outcome of this activity is for students to be able to analyze the properties of the functions separately and then to compare the functions to each other when the functions are represented in the same way.	▶ RECOGNIZE THE PROPERTIES OF FUNCTIONS ▶ ANALYZE LINEAR FUNCTIONS ▶ COMPARE 2 FUNCTIONS WITH DIFFERENT RATES OF CHANGE ▶ COMPARE THE PROPERTIES OF 2 FUNCTIONS REPRESENTED IN THE SAME FORM
Lesson 4	Students will analyze and compare linear and nonlinear functions given in different representations. The critical outcome of this activity is for students to be able to analyze the properties of the functions separately and then to compare them to each other when the functions are represented in different ways.	▶ RECOGNIZE THE PROPERTIES OF FUNCTIONS ▶ ANALYZE LINEAR FUNCTIONS ▶ COMPARE 2 FUNCTIONS WITH DIFFERENT RATES OF CHANGE ▶ COMPARE THE PROPERTIES OF 2 FUNCTIONS REPRESENTED IN DIFFERENT FORMS

COMPARING LINEAR AND NONLINEAR FUNCTIONS

INSTRUCTIONAL ACTIVITY

Lesson 1

LEARNING GOAL

Students will compare multiple representations of the same function. The critical outcome of this activity is for students to be able to find the rate of change and the *y*-intercept of a function given in any representation and to match different representations of the same functions.

PRIMARY ACTIVITY

Students will create a table and a graph for a given equation and then will match the function to a corresponding description of a real-world situation.

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

- Rate of change
- *y*-intercept

MATERIALS

- INSTRUCTIONAL ACTIVITY STUDENT HANDOUT
- INSTRUCTIONAL ACTIVITY SUPPLEMENT (Recommend either one copy for the class *or* one copy for every two to three students.)
- Word Version INSTRUCTIONAL ACTIVITY STUDENT HANDOUT
- Word Version INSTRUCTIONAL ACTIVITY SUPPLEMENT

IMPLEMENTATION

Review rate of change, *y*-intercept, constructing tables, and graphs of linear functions as needed based on the students' experience.

Distribute the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT. Students should work in pairs or groups of three. The handout begins with an equation (these may be written in function notation if students are familiar with function notation) and requires students to construct a table and a graph for that equation.

The students will construct a table and graph for each function represented by an equation. Once students have constructed all three representations, they should match the function to a corresponding description of a real-world situation.

Repeat the activity until students have completed this process for a total of six equations.

GUIDING QUESTIONS

Elicit student thinking:

- How will you construct a table for this function?
- Why is it useful to have a table to construct a graph of a function?

Determine if the student can DESCRIBE THE RATE OF CHANGE IN AN ALGEBRAIC FUNCTION:

- What part of an equation in the form $y = mx + b$ represents the rate of change of a function?
- How can you determine a function's rate of change from a table?
- What process would you use to determine a function's rate of change given a graph?
- Given a description of a real-world problem, what key words indicate the rate of change?

Determine if the student can EXPLAIN *Y-INTERCEPT*:

- What is a phrase that describes the *y*-intercept?
- What part of an equation in the form $y = mx + b$ represents the *y*-intercept of a function?
- How can you determine the *y*-intercept of a function from a table?
- What should you look for to determine the *y*-intercept of a function given a graph?
- Given a description of a real-world problem, what key words indicate the *y*-intercept?

Have the students show all representations of each function on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

Choose a problem from the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT and ask students for the rate of change. Once a student responds with the correct rate of change, **ask** that student which representation they used to answer the question and how they obtained the correct rate of change for that representation.

Ask the class how the rate of change appears in each of the other three function representations. When examining the rate of change, **discuss** with the students how the *x*- and *y*-values change together rather than focusing on the values of one variable at a time.

Repeat this type of questioning for the *y*-intercept and for the other problems on the student handout.

Collaborative activity using the INSTRUCTIONAL ACTIVITY SUPPLEMENT:

Give each student a card with one representation of a linear function from the LESSON 1 section of the INSTRUCTIONAL ACTIVITY SUPPLEMENT. For each function, all four representations of that function are present in the cards.

Ask the students to find and to stand with other students who have the same function but in a different representation. At the end of this activity, students should be in groups of four (or fewer if the number of students in the class does not allow for even groups of four). Require each group to read their cards or to copy their cards onto the board to display to the class so the class (and you) can confirm they are grouped correctly.

Another option would be to create multiple sets of cards and give all cards to groups of students. Each student group would then create groups of cards that display the same function in different representations.

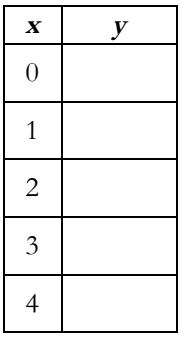
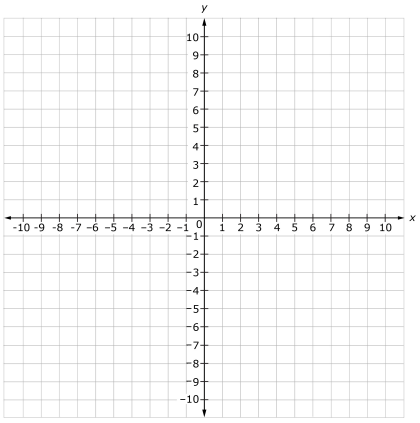
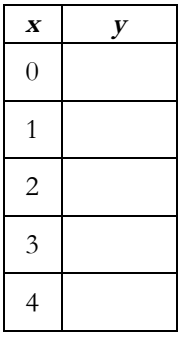
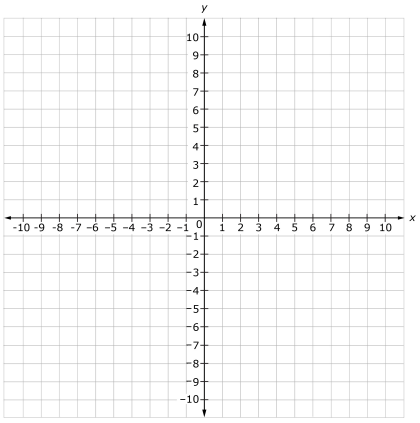
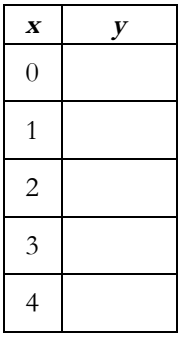
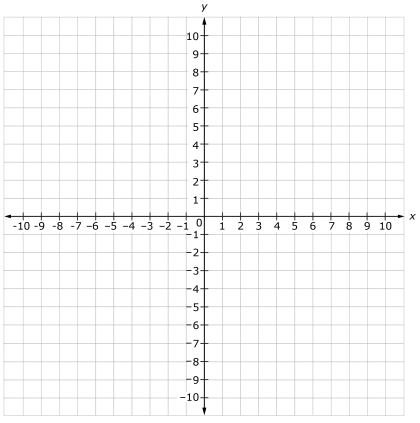
NOTE: Matching graphs to descriptions provides an opportunity to discuss intervals on the graphs that are impossible in terms of the problem situation. For example, in the scenario "You are 5 miles away from home. You start walking home at a rate of 3 miles per hour. How far are you from home after x hours?" it would not be realistic to discuss how far away you are from home after 10 hours.

Name_____

Comparing Linear and Nonlinear Functions

Lesson 1

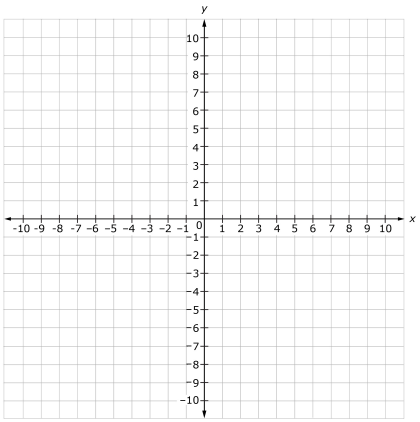
For the following equations, construct a table and a graph. Then match a description from the Description Word Bank to each equation.

Equation	Table	Graph	Description
1) $y = 2x$			
2) $y = -2x + 5$			
3) $y = x + 6$			

Name_____

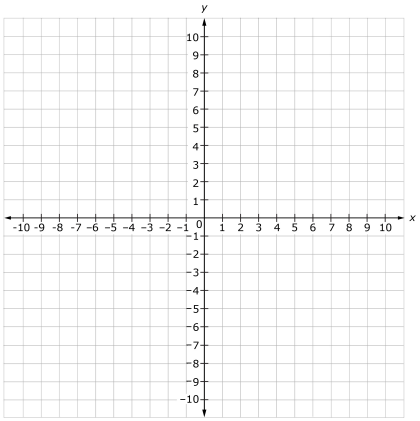
4) $y = \frac{1}{2}x + 1$

x	*y*
0	
1	
2	
3	
4	



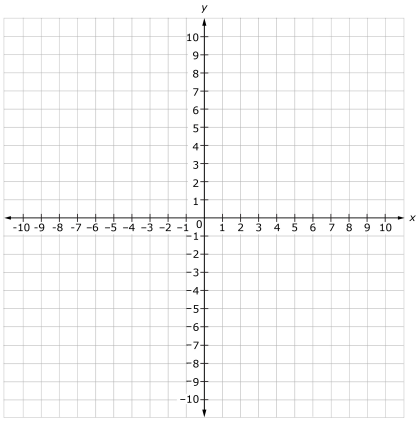
5) $y = -x + 10$

x	*y*
0	
1	
2	
3	
4	



6) $y = -3x + 5$

x	*y*
0	
1	
2	
3	
4	



Name_____

DESCRIPTION WORD BANK

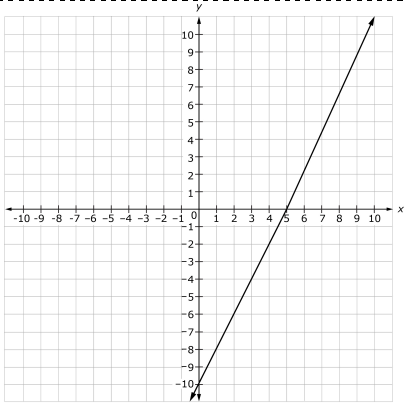
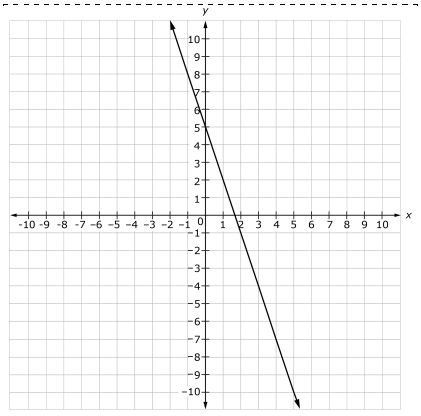
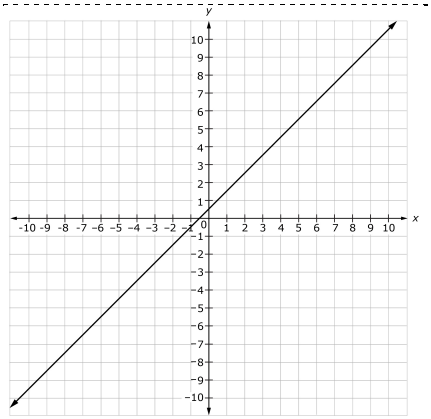
Cut out the function descriptions below and match them to their corresponding equation, table, and graph in questions 1–6.

A bucket has 10 gallons of water in it. The water drains at a rate of 1 gallon per minute. How much water is in the bucket after x minutes?	A kitten weighs 1 pound when it is born. The kitten gains ½ pound per month. How much does the kitten weigh after x months?	You download a new music application on your phone. Each song is $2. How much do you pay for x songs?
A puppy currently weighs 6 pounds. The puppy gains 1 pound per week. How much does the puppy weigh after x weeks?	You are 5 miles away from home. You start walking home at a rate of 3 miles per hour. How far are you from home after x hours?	A paint bucket has 5 gallons of paint. The painters use paint at a rate of 2 gallons per hour. How much paint is in the bucket after x hours?

Comparing Linear and Nonlinear Functions

INSTRUCTIONAL ACTIVITY SUPPLEMENT

Lesson 1

$y = -10 + 2x$	<table><tr><th>x</th><th>y</th></tr><tr><td>0</td><td>–10</td></tr><tr><td>1</td><td>–8</td></tr><tr><td>2</td><td>–6</td></tr><tr><td>3</td><td>–4</td></tr><tr><td>4</td><td>–2</td></tr></table>		The current outside temperature is –10°F. If the temperature increases by 2°F per hour, what is the temperature after x hours?
$y = 5 - 3x$	<table><tr><th>x</th><th>y</th></tr><tr><td>0</td><td>5</td></tr><tr><td>1</td><td>2</td></tr><tr><td>2</td><td>–1</td></tr><tr><td>3</td><td>–4</td></tr><tr><td>4</td><td>–7</td></tr></table>		You are 5 miles away from home. You start walking home at a rate of 3 miles per hour. How far are you from home after x hours?
$y = x + \frac{1}{2}$	<table><tr><th>x</th><th>y</th></tr><tr><td>0</td><td>$\frac{1}{2}$</td></tr><tr><td>1</td><td>$1\frac{1}{2}$</td></tr><tr><td>2</td><td>$2\frac{1}{2}$</td></tr><tr><td>3</td><td>$3\frac{1}{2}$</td></tr><tr><td>4</td><td>$4\frac{1}{2}$</td></tr></table>		You are $\frac{1}{2}$ a mile from home. You walk away from home at a rate of 1 mile per hour. How far are you away from home after x hours?

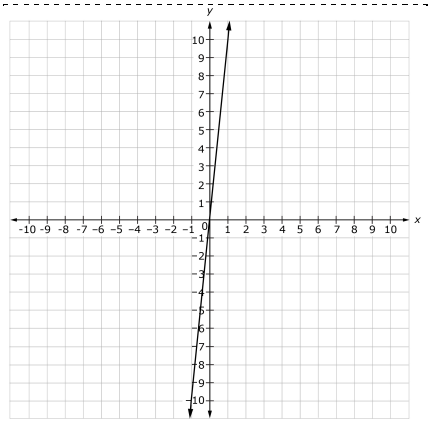
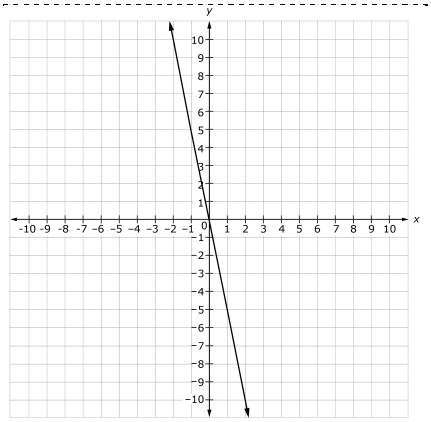
$y = 5x$			A swimming pool is empty. You begin filling the pool at a rate of 5 gallons per minute. How many gallons of water are in the pool after x minutes?
$y = -2x + 10$			You have a bus card with \$10. Each bus trip costs \$2. How much money is on the card after x bus trips?
$y = x$			You are having a bake sale. Cookies cost \$1 each. How much money do you collect if you sell x cookies at the bake sale?
$y = 8 - 3x$			You have been hired to edit an 8-page paper. You are able to edit 3 pages per hour. How many pages still need to be edited after x hours?

x	***y***
0	0
1	5
2	10
3	15
4	20

x	***y***
0	10
1	8
2	6
3	4
4	2

x	***y***
0	0
1	1
2	2
3	3
4	4

x	***y***
0	8
1	5
2	2
3	–1
4	–4

$y = 10x$	**x** | **y** 0 | 0 1 | 10 2 | 20 3 | 30 4 | 40		You have been hired to edit a novel. You charge \$10 to edit each page. How much do you charge to edit x pages?
$y = -5x$	**x** | **y** 0 | 0 1 | –5 2 | –10 3 | –15 4 | –20		A submarine starts at sea level. The submarine descends into the water at a rate of 5 meters per minute. What is the depth of the submarine after x minutes?

COMPARING LINEAR AND NONLINEAR FUNCTIONS

INSTRUCTIONAL ACTIVITY

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Lesson 2

LEARNING GOAL

Students will analyze the properties of linear and nonlinear functions given in different representations. The critical outcome of this activity is that students will be able to fluently describe and determine the rate of change and the *y*-intercept of any linear function and to identify and to describe representations of nonlinear functions.

PRIMARY ACTIVITY

Students will construct and answer questions describing the properties of linear and nonlinear functions given in different representations.

OTHER VOCABULARY

Students will need to know the meaning (and appearance) of the following terms:

- Rate of change
- *y*-intercept
- Increasing function
- Decreasing function
- Constant function
- Linear function
- Nonlinear function

MATERIALS

- INSTRUCTIONAL ACTIVITY SUPPLEMENT (Recommend one copy for every two students.)
- Word Version INSTRUCTIONAL ACTIVITY SUPPLEMENT

- Dry-erase markers (optional)
- Construction paper squares (optional)

IMPLEMENTATION

Prior to the activity, **review** the differences between linear and nonlinear functions in all representations. In pairs, students will play "Guess My Function" using the game boards and card deck provided in the INSTRUCTIONAL ACTIVITY SUPPLEMENT.

> Each student should draw a card from the deck without showing his or her partner. The card shows the function their partner will try to identify through a series of yes-or-no questions. Questions should include mathematically accurate vocabulary. Suggestions are provided in the INSTRUCTIONAL ACTIVITY SUPPLEMENT and can be provided to students or discussed beforehand.
>
> Students will take turns asking their partner a question to narrow down the options for the function their partner has. As students eliminate possibilities, they should cross them off on their game board. (Laminating the game boards will allow students to play multiple times using dry-erase markers.) Each board has only one representation of each function. This will eliminate questions like "Do you have a graph?" and promote conceptual questions and specific vocabulary, such as "Does your function have a positive rate of change?" The student who determines their partner's function first wins.

Below are the directions provided for the activity in the INSTRUCTIONAL ACTIVITY SUPPLEMENT:

> On the following pages, there are four different versions of the same "Guess My Function" game board. Each board has the same functions represented (24 total), but in different representations. For example, the equation $y = 3x - 2$ may appear on board 1, its table on board 2, its graph on board 3, and a corresponding description on board 4.
>
> If the boards are laminated, students can use a dry-erase marker to cross off eliminated functions and then reuse the board to play again. If the boards are not laminated, make additional copies or provide small construction paper squares to cover eliminated functions to allow students to play more than one game.
>
> After the game boards, additional pages display all representations of all functions. These are the cards students will draw from. The student will try to determine the function on their partner's card by asking a series of questions. Students should use mathematically correct vocabulary when asking questions.

Encourage students to ask questions similar to:

- Is your function linear/nonlinear?
- Does your function have a constant/variable rate of change?
- Is your function only increasing/only decreasing/both increasing and decreasing/constant?
- Does your function have a positive/negative/zero rate of change?
- Does your function have a positive/negative *y*-intercept?
- Is the point (x, y) on the graph of your function?

Once a function has been guessed, students may draw a new function and start a new game.

GUIDING QUESTIONS

Elicit student thinking:

- What do you notice about this function?
- What information helps you match different representations of functions?

Determine if the student can DESCRIBE THE RATE OF CHANGE IN AN ALGEBRAIC FUNCTION:

- How do you know the rate of change is positive/negative in this representation of a function?
- What is true about the relative behavior of the two quantities in a function with a positive rate of change?
- What is true about the relative behavior of the two quantities in a function with a negative rate of change?

Determine if the student can EXPLAIN Y-*INTERCEPT*:

- How do you know if the *y*-intercept is positive (or negative) in this representation of a function?
- How would you identify the *y*-intercept on a graph? In a table? In an equation? In a description?

Determine if the student can RECOGNIZE *LINEAR FUNCTION*:

- [Show the student a graph/equation/table/description of a linear and a nonlinear function.] Which of these functions is linear?

Determine if the student can EXPLAIN *LINEAR FUNCTION*:

- [Show the student a graph/equation/table/description of a linear function.] What characteristics of this function indicate that it is linear?
- [Show the student a graph/equation/table/description of a nonlinear function.] What characteristics of this function indicate that it is not linear/nonlinear?

Determine if the student can EXPLAIN *INCREASING* AND *DECREASING LINEAR FUNCTIONS*:

- When you are trying to determine if a function is increasing or decreasing, in which direction should you read the graph? (The student should indicate that the graph should be read from left to right.)

Students should be required to use proper vocabulary in their discussions and questions with their partners.

Monitor vocabulary use as you walk around the room during the game. Also monitor that students are correctly answering their partners' questions. If students are struggling to answer or ask questions, **remind** students of the previous day's work with identifying the rate of change and the *y*-intercept of linear functions given in any representation. Students can play the game again by drawing a new card, either with the same partners or different partners.

Ask students what questions were most effective in narrowing down their partner's function and why this was the case. Require students to use proper vocabulary when they provide a question.

Comparing Linear and Nonlinear Functions

INSTRUCTIONAL ACTIVITY SUPPLEMENT

Lesson 2

On the following pages, there are four different versions of the same "Guess My Function" game board. Each board has the same functions represented (24 total), but in different representations. For example, the equation $y = 3x - 2$ may appear on board 1, its table on board 2, its graph on board 3, and a corresponding description on board 4.

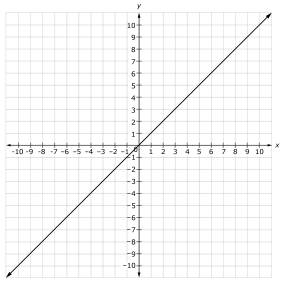
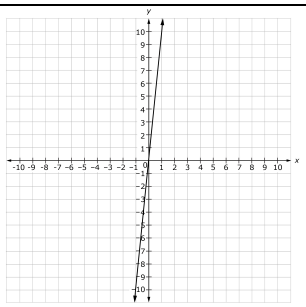
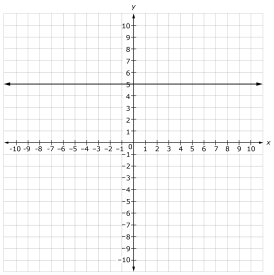
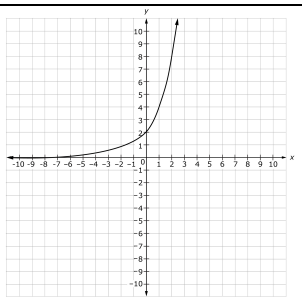
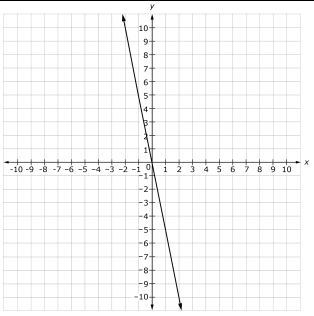
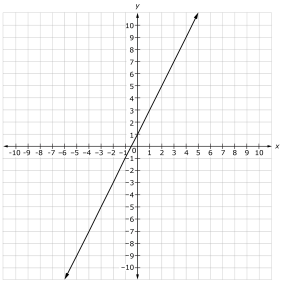
If the boards are laminated, students can use a dry-erase marker to cross off eliminated functions and then reuse the board to play again. If the boards are not laminated, make additional copies or provide small construction paper squares to cover eliminated functions to allow students to play more than one game.

After the game boards, additional pages display all representations of all functions. These are the cards students will draw from. The student will try to determine the function on their partner's card by asking a series of questions. Students should use mathematically correct vocabulary when asking questions.

Encourage students to ask questions similar to:

- Is your function linear/nonlinear?
- Does your function have a constant/variable rate of change?
- Is your function only increasing/only decreasing/both increasing and decreasing/constant?
- Does your function have a positive/negative/zero rate of change?
- Does your function have a positive/negative y–intercept?
- Is the point (x, y) on the graph of your function?

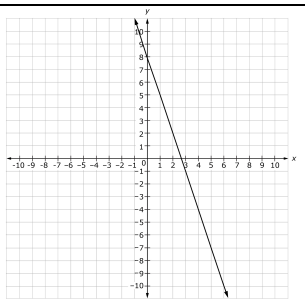
Once a function has been guessed, students may draw a new function and start a new game.

A puppy currently weighs 6 pounds. The puppy gains 1 pound per week. How much does the puppy weigh after x weeks?	$y = 5x$		<table><tr><th>x</th><th>y</th></tr><tr><td>0</td><td>0</td></tr><tr><td>1</td><td>3</td></tr><tr><td>2</td><td>4</td></tr><tr><td>3</td><td>3</td></tr><tr><td>4</td><td>0</td></tr></table>
$y = x + \frac{1}{2}$		<table><tr><th>x</th><th>y</th></tr><tr><td>0</td><td>1</td></tr><tr><td>1</td><td>$1\frac{1}{2}$</td></tr><tr><td>2</td><td>2</td></tr><tr><td>3</td><td>$2\frac{1}{2}$</td></tr><tr><td>4</td><td>3</td></tr></table>	You have 4 meters of yarn for a project. You cut the yarn in half, then half again, and so on. How many meters long is each piece of yarn after x cuts?
<table><tr><th>x</th><th>y</th></tr><tr><td>0</td><td>5</td></tr><tr><td>1</td><td>2</td></tr><tr><td>2</td><td>–1</td></tr><tr><td>3</td><td>–4</td></tr><tr><td>4</td><td>–7</td></tr></table>	A bird flies for 4 seconds, starting at an elevation of 7 feet, descending to an elevation of 3 feet, then rising back up to 7 feet. What is the elevation of the bird after x seconds?	$y = -x - 3$	
$y = 8\left(\frac{1}{2}\right)^x$	<table><tr><th>x</th><th>y</th></tr><tr><td>0</td><td>1</td></tr><tr><td>1</td><td>3</td></tr><tr><td>2</td><td>9</td></tr><tr><td>3</td><td>27</td></tr><tr><td>4</td><td>81</td></tr></table>		The current outside temperature is –10°F. If the temperature increases by 2°F per hour, what is the temperature after x hours?
	$y = -x + 10$	You download a new music application on your phone. Each song is $2. How much do you pay for x songs?	<table><tr><th>x</th><th>y</th></tr><tr><td>0</td><td>8</td></tr><tr><td>1</td><td>5</td></tr><tr><td>2</td><td>2</td></tr><tr><td>3</td><td>–1</td></tr><tr><td>4</td><td>–4</td></tr></table>
<table><tr><th>x</th><th>y</th></tr><tr><td>0</td><td>–1</td></tr><tr><td>1</td><td>–1</td></tr><tr><td>2</td><td>–1</td></tr><tr><td>3</td><td>–1</td></tr><tr><td>4</td><td>–1</td></tr></table>	You have a bus card with $10. Each bus trip costs $2. How much money is on the card after x bus trips?	$y = -\frac{1}{3}x^2 + 2x + 3$	

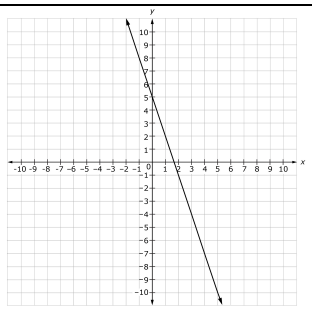
x	y
0	0
1	10
2	20
3	30
4	40

$y = 4\left(\frac{1}{2}\right)^x$

An empty box weighs 1 pound. Each can you place in the box weighs 2 pounds. What is the weight with x cans?

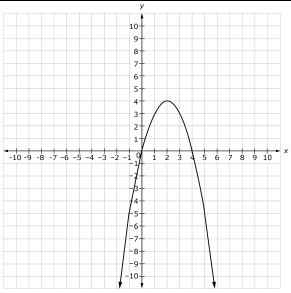


You plan to breed 2 rabbits. Assume that your rabbits will breed every month and the population will double each time. How many rabbits will there be after x months?



x	y
0	3
1	$4\frac{2}{3}$
3	6
5	$4\frac{2}{3}$
6	3

$y = -2x + 10$



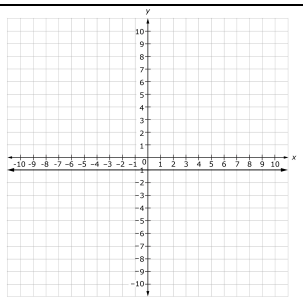
$y = \frac{1}{2}x + 1$

A submarine starts at sea level. The submarine descends into the water at a rate of 5 meters per minute. What is the depth of the submarine after x minutes?

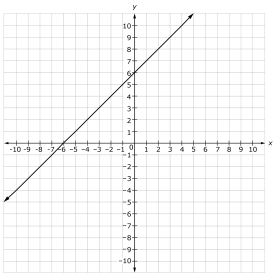
x	y
0	–3
1	–4
2	–5
3	–6
4	–7

$y = 2x$

x	y
0	0
1	5
2	10
3	15
4	20



You are $\frac{1}{2}$ a mile from home and walk away from home at a rate of 1 mile per hour. How far are you away from home after x hours?

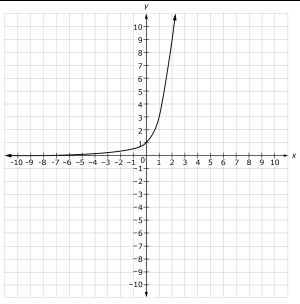


A fish tank has 5 gallons of water. The water level remains the same as time passes. How much water is in the tank after x minutes?

$y = -10 + 2x$

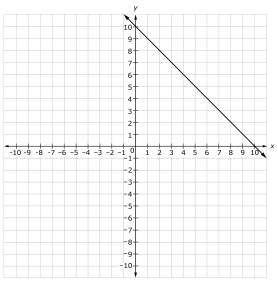
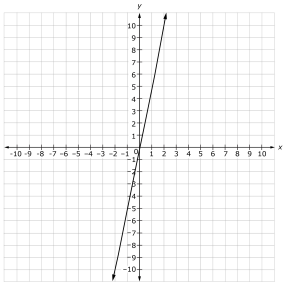
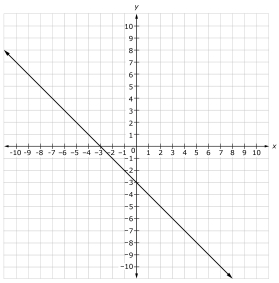
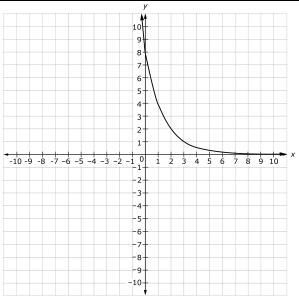
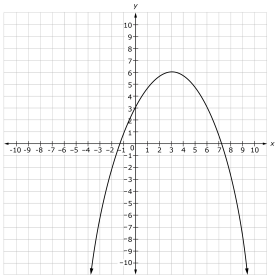
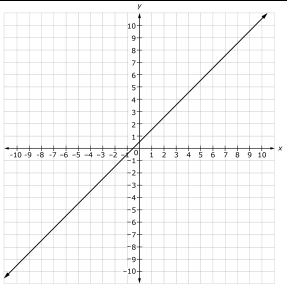
x	y
0	8
1	4
2	2
3	1
4	$\frac{1}{2}$

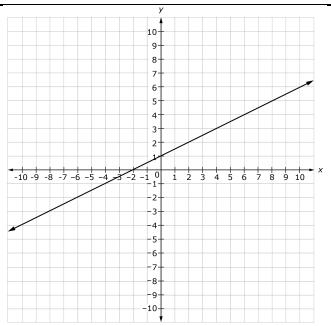
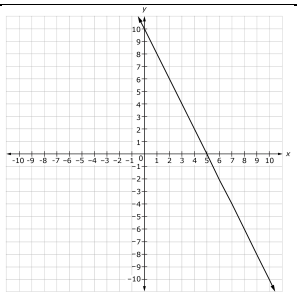
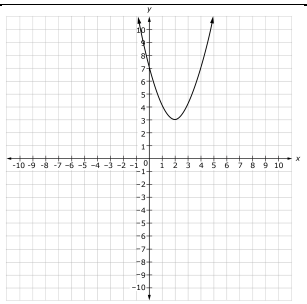
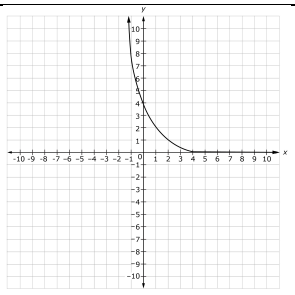
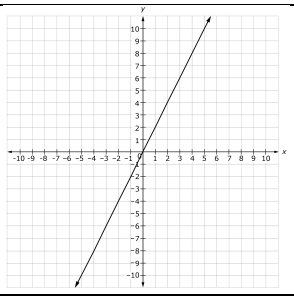
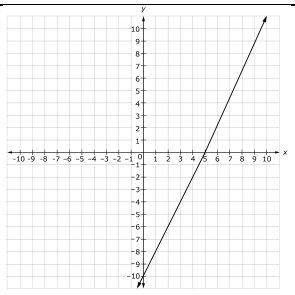
$y = x^2 - 4x + 7$



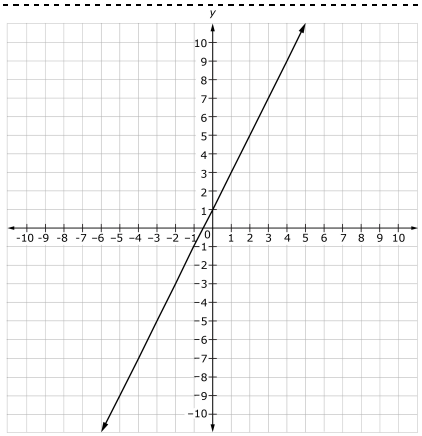
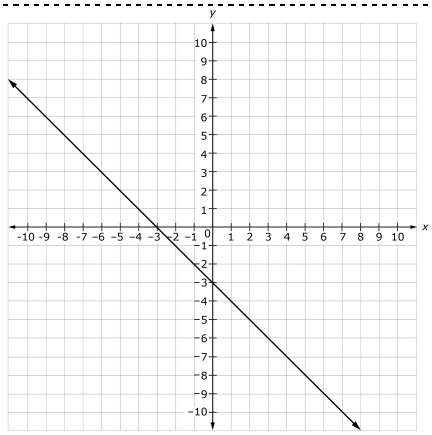
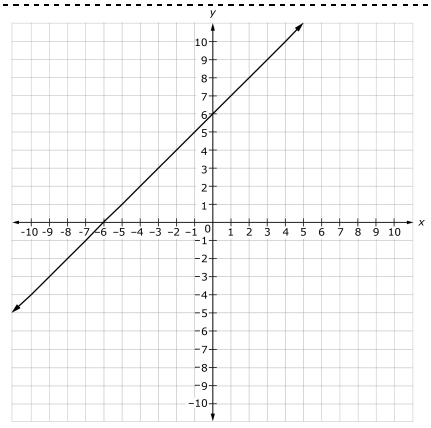
x	y
0	10
1	9
2	8
3	7
4	6

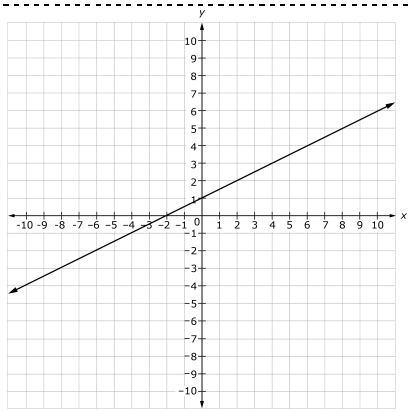
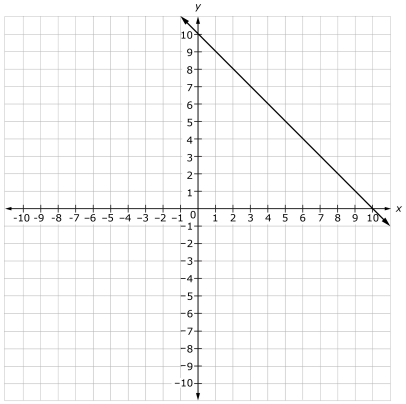
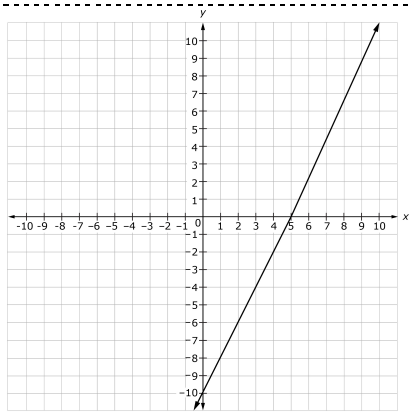
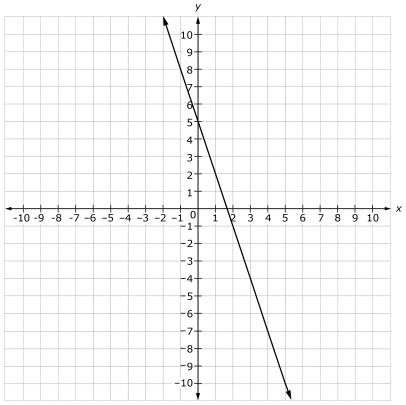
You are having a bake sale. Cookies cost \$1 each. How much money do you collect if you sell x cookies at the bake sale?

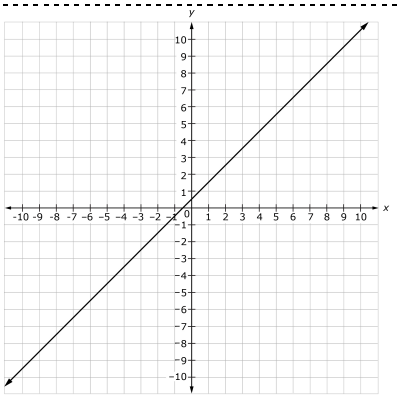
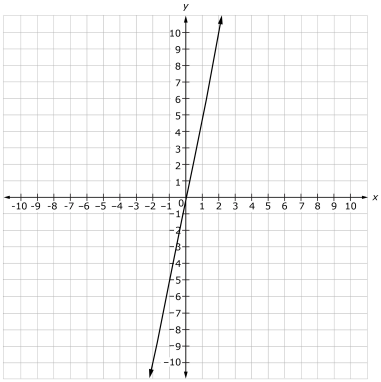
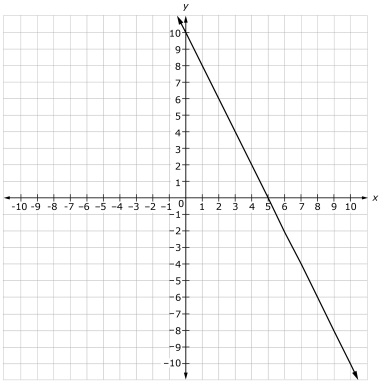
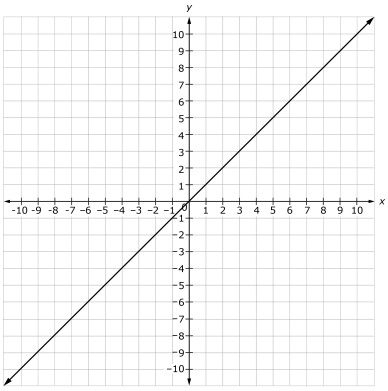
Your freezer is set at –1°F. The temperature does not change over time. What is the temperature after x hours?	$y = x + 6$	<table><tr><th>x</th><th>y</th></tr><tr><td>0</td><td>–10</td></tr><tr><td>1</td><td>–8</td></tr><tr><td>2</td><td>–6</td></tr><tr><td>3</td><td>–4</td></tr><tr><td>4</td><td>–2</td></tr></table>	
You have been hired to edit an 8-page paper. You are able to edit 3 pages per hour. How many pages still need to be edited after x hours?		$y = -5x$	<table><tr><th>x</th><th>y</th></tr><tr><td>0</td><td>7</td></tr><tr><td>1</td><td>4</td></tr><tr><td>2</td><td>3</td></tr><tr><td>3</td><td>4</td></tr><tr><td>4</td><td>7</td></tr></table>
<table><tr><th>x</th><th>y</th></tr><tr><td>0</td><td>0</td></tr><tr><td>1</td><td>2</td></tr><tr><td>2</td><td>4</td></tr><tr><td>3</td><td>6</td></tr><tr><td>4</td><td>8</td></tr></table>	$y = 5$		Your friend lets you borrow money and says you must pay \$1 interest right away. The amount of interest triples every week after that. How much do you owe in interest after x weeks?
	$y = 2(2)^x$	You are 5 miles away from home. You start walking home at a rate of 3 miles per hour. How far are you from home after x hours?	<table><tr><th>x</th><th>y</th></tr><tr><td>0</td><td>10</td></tr><tr><td>1</td><td>8</td></tr><tr><td>2</td><td>6</td></tr><tr><td>3</td><td>4</td></tr><tr><td>4</td><td>2</td></tr></table>
<table><tr><th>x</th><th>y</th></tr><tr><td>0</td><td>1</td></tr><tr><td>1</td><td>3</td></tr><tr><td>2</td><td>5</td></tr><tr><td>3</td><td>7</td></tr><tr><td>4</td><td>9</td></tr></table>	A soccer ball is kicked from the ground. It reaches a maximum height of 4 meters and returns to the ground 4 meters from where it was kicked. How high is the soccer ball x meters from where it was kicked?	$y = 10x$	
$y = x$	<table><tr><th>x</th><th>y</th></tr><tr><td>0</td><td>4</td></tr><tr><td>1</td><td>2</td></tr><tr><td>2</td><td>1</td></tr><tr><td>3</td><td>$\frac{1}{2}$</td></tr><tr><td>4</td><td>$\frac{1}{4}$</td></tr></table>		A kitten weighs 1 pound when it is born. The kitten gains $\frac{1}{2}$ pound per month. How much does the kitten weigh after x months?

$y = -1$	You have been hired to edit a novel. You charge \$10 to edit each page. How much do you charge to edit x pages?	<table><tr><th>x</th><th>y</th></tr><tr><td>0</td><td>0</td></tr><tr><td>1</td><td>1</td></tr><tr><td>2</td><td>2</td></tr><tr><td>3</td><td>3</td></tr><tr><td>4</td><td>4</td></tr></table>	
The temperature is currently –3°F and it is dropping 1°F per hour. What is the temperature after x hours?		<table><tr><th>x</th><th>y</th></tr><tr><td>0</td><td>5</td></tr><tr><td>1</td><td>5</td></tr><tr><td>2</td><td>5</td></tr><tr><td>3</td><td>5</td></tr><tr><td>4</td><td>5</td></tr></table>	$y = 8 - 3x$
$y = 1(3)^x$	You study 8 grams of an element with a half-life of one day (half the element is left after one day). How much of the element is left after x days?		<table><tr><th>x</th><th>y</th></tr><tr><td>0</td><td>6</td></tr><tr><td>1</td><td>7</td></tr><tr><td>2</td><td>8</td></tr><tr><td>3</td><td>9</td></tr><tr><td>4</td><td>10</td></tr></table>
	<table><tr><th>x</th><th>y</th></tr><tr><td>0</td><td>2</td></tr><tr><td>1</td><td>4</td></tr><tr><td>2</td><td>8</td></tr><tr><td>3</td><td>16</td></tr><tr><td>4</td><td>32</td></tr></table>	$y = 2x + 1$	A swimming pool is empty. You begin filling the pool at a rate of 5 gallons per minute. How many gallons of water are in the pool after x minutes?
$y = -x^2 + 4x$	A bucket has 10 gallons of water in it. The water drains at a rate of 1 gallon per minute. How much water is in the bucket after x minutes?	<table><tr><th>x</th><th>y</th></tr><tr><td>0</td><td>$\frac{1}{2}$</td></tr><tr><td>1</td><td>$1\frac{1}{2}$</td></tr><tr><td>2</td><td>$2\frac{1}{2}$</td></tr><tr><td>3</td><td>$3\frac{1}{2}$</td></tr><tr><td>4</td><td>$4\frac{1}{2}$</td></tr></table>	
	<table><tr><th>x</th><th>y</th></tr><tr><td>0</td><td>0</td></tr><tr><td>1</td><td>–5</td></tr><tr><td>2</td><td>–10</td></tr><tr><td>3</td><td>–15</td></tr><tr><td>4</td><td>–20</td></tr></table>	A water balloon starts at a height of 3 feet. It is thrown and reaches a maximum height of 6 feet before it is caught a few feet away at a height of 3 feet. How high is the water balloon x feet after it was thrown?	$y = 5 - 3x$

ALL REPRESENTATIONS OF ALL FUNCTIONS (CARDS STUDENTS WILL DRAW FROM)

$y = 2x + 1$	<table><tr><th>x</th><th>y</th></tr><tr><td>0</td><td>1</td></tr><tr><td>1</td><td>3</td></tr><tr><td>2</td><td>5</td></tr><tr><td>3</td><td>7</td></tr><tr><td>4</td><td>9</td></tr></table>		An empty box weighs 1 pound. Each can you place in the box weighs 2 pounds. What is the weight with x cans?
$y = -x - 3$	<table><tr><th>x</th><th>y</th></tr><tr><td>0</td><td>–3</td></tr><tr><td>1</td><td>–4</td></tr><tr><td>2</td><td>–5</td></tr><tr><td>3</td><td>–6</td></tr><tr><td>4</td><td>–7</td></tr></table>		The temperature is currently –3°F and it is dropping 1°F per hour. What is the temperature after x hours?
$y = x + 6$	<table><tr><th>x</th><th>y</th></tr><tr><td>0</td><td>6</td></tr><tr><td>1</td><td>7</td></tr><tr><td>2</td><td>8</td></tr><tr><td>3</td><td>9</td></tr><tr><td>4</td><td>10</td></tr></table>		A puppy currently weighs 6 pounds. The puppy gains 1 pound per week. How much does the puppy weigh after x weeks?

$y = \frac{1}{2}x + 1$	<table><tr><th>*x*</th><th>*y*</th></tr><tr><td>0</td><td>1</td></tr><tr><td>1</td><td>$1\frac{1}{2}$</td></tr><tr><td>2</td><td>2</td></tr><tr><td>3</td><td>$2\frac{1}{2}$</td></tr><tr><td>4</td><td>3</td></tr></table>		A kitten weighs 1 pound when it is born. The kitten gains $\frac{1}{2}$ pound per month. How much does the kitten weigh after x months?
$y = -x + 10$	<table><tr><th>*x*</th><th>*y*</th></tr><tr><td>0</td><td>10</td></tr><tr><td>1</td><td>9</td></tr><tr><td>2</td><td>8</td></tr><tr><td>3</td><td>7</td></tr><tr><td>4</td><td>6</td></tr></table>		A bucket has 10 gallons of water in it. The water drains at a rate of 1 gallon per minute. How much water is in the bucket after x minutes?
$y = -10 + 2x$	<table><tr><th>*x*</th><th>*y*</th></tr><tr><td>0</td><td>–10</td></tr><tr><td>1</td><td>–8</td></tr><tr><td>2</td><td>–6</td></tr><tr><td>3</td><td>–4</td></tr><tr><td>4</td><td>–2</td></tr></table>		The current outside temperature is –10°F. If the temperature increases by 2°F per hour, what is the temperature after x hours?
$y = 5 - 3x$	<table><tr><th>*x*</th><th>*y*</th></tr><tr><td>0</td><td>5</td></tr><tr><td>1</td><td>2</td></tr><tr><td>2</td><td>–1</td></tr><tr><td>3</td><td>–4</td></tr><tr><td>4</td><td>–7</td></tr></table>		You are 5 miles away from home. You start walking home at a rate of 3 miles per hour. How far are you from home after x hours?

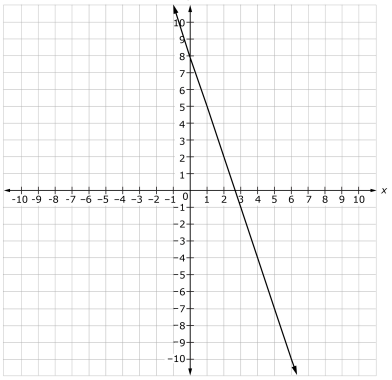
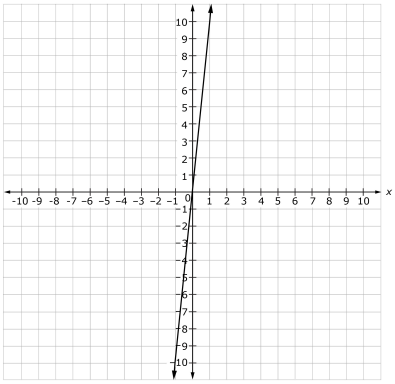
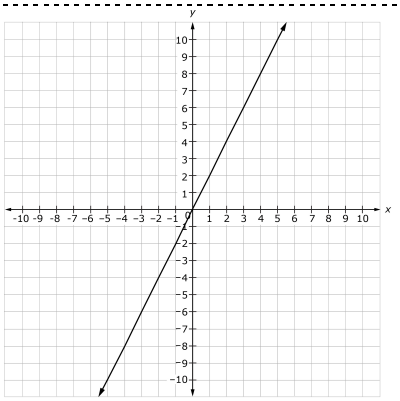
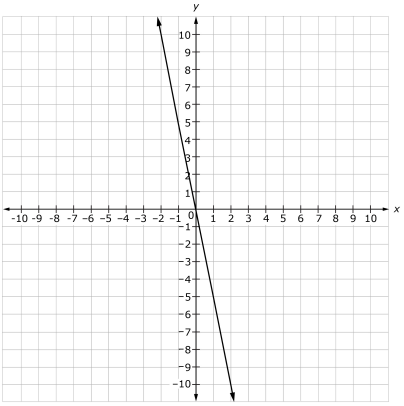
$y = x + \frac{1}{2}$			You are $\frac{1}{2}$ a mile from home and walk away from home at a rate of 1 mile per hour. How far are you away from home after x hours?
$y = 5x$			A swimming pool is empty. You begin filling the pool at a rate of 5 gallons per minute. How many gallons of water are in the pool after x minutes?
$y = -2x + 10$			You have a bus card with \$10. Each bus trip costs \$2. How much money is on the card after x bus trips?
$y = x$			You are having a bake sale. Cookies cost \$1 each. How much money do you collect if you sell x cookies at the bake sale?

x	y
0	$\frac{1}{2}$
1	$1\frac{1}{2}$
2	$2\frac{1}{2}$
3	$3\frac{1}{2}$
4	$4\frac{1}{2}$

x	y
0	0
1	5
2	10
3	15
4	20

x	y
0	10
1	8
2	6
3	4
4	2

x	y
0	0
1	1
2	2
3	3
4	4

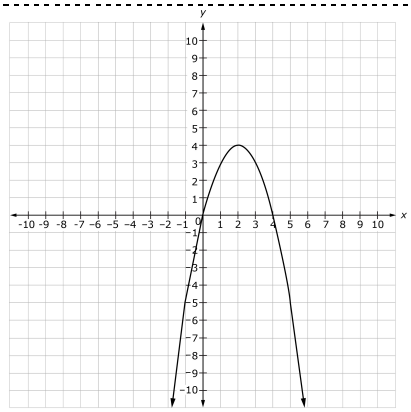
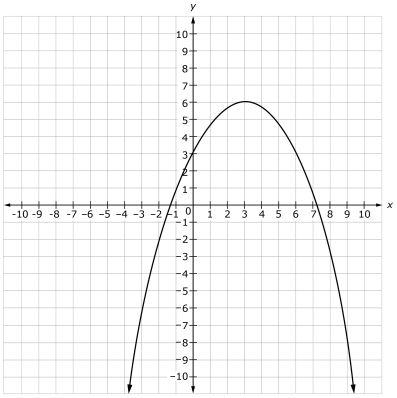
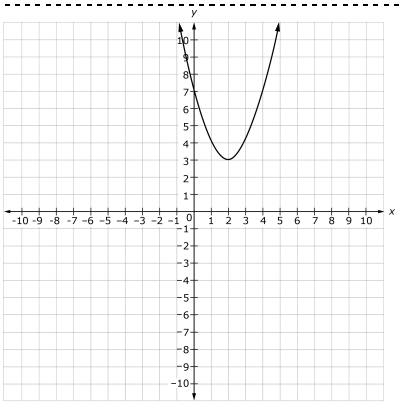
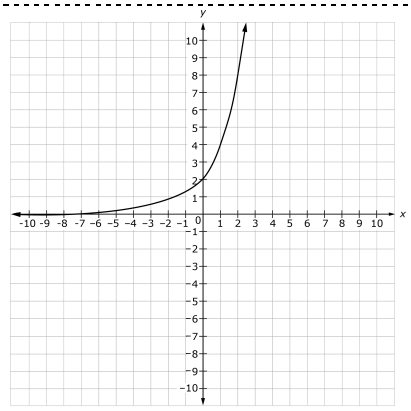
$y = 8 - 3x$			You have been hired to edit an 8-page paper. You are able to edit 3 pages per hour. How many pages still need to be edited after x hours?
$y = 10x$			You have been hired to edit a novel. You charge \$10 to edit each page. How much do you charge to edit x pages?
$y = 2x$			You download a new music application on your phone. Each song is \$2. How much do you pay for x songs?
$y = -5x$			A submarine starts at sea level. The submarine descends into the water at a rate of 5 meters per minute. What is the depth of the submarine after x minutes?

x	y
0	8
1	5
2	2
3	–1
4	–4

x	y
0	0
1	10
2	20
3	30
4	40

x	y
0	0
1	2
2	4
3	6
4	8

x	y
0	0
1	–5
2	–10
3	–15
4	–20

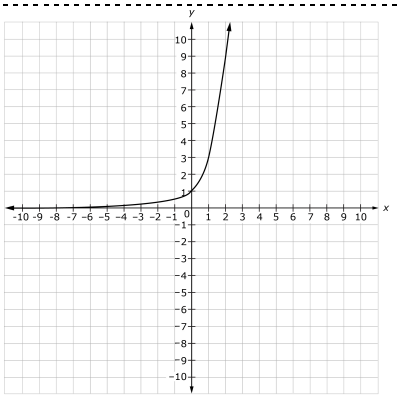
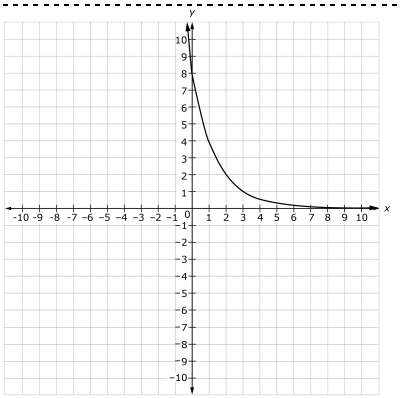
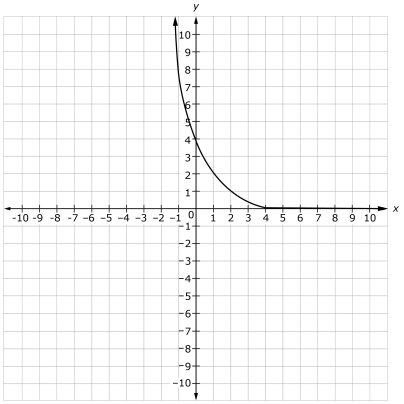
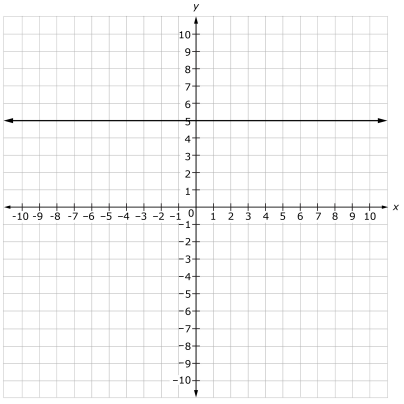
$y = -x^2 + 4x$			A soccer ball is kicked from the ground. It reaches a maximum height of 4 meters and returns to the ground 4 meters from where it was kicked. How high is the soccer ball x meters from where it was kicked?
$y = -\frac{1}{3}x^2 + 2x + 3$			A water balloon starts at a height of 3 feet. It is thrown and reaches a maximum height of 6 feet before it is caught a few feet away at a height of 3 feet. How high is the water balloon x feet after it was thrown?
$y = x^2 - 4x + 7$			A bird flies for 4 seconds, starting at an elevation of 7 feet, descending to an elevation of 3 feet, then rising back up to 7 feet. What is the elevation of the bird after x seconds?
$y = 2(2)^x$			You plan to breed 2 rabbits. Assume that your rabbits will breed every month and the population will double each time. How many rabbits will there be after x months?

x	y
0	0
1	3
2	4
3	3
4	0

x	y
0	3
1	$4\frac{2}{3}$
3	6
5	$4\frac{2}{3}$
6	3

x	y
0	7
1	4
2	3
3	4
4	7

x	y
0	2
1	4
2	8
3	16
4	32

$y = 1(3)^x$			Your friend lets you borrow money and says you must pay \$1 interest right away. The amount triples every week after that. How much do you owe in interest after x weeks?
$y = 8\left(\frac{1}{2}\right)^x$			You study 8 grams of an element with a half–life of one day (half the element is left after one day). How much of the element is left after x days?
$y = 4\left(\frac{1}{2}\right)^x$			You have 4 meters of yarn for a project. You cut the yarn in half, then half again, and so on. How many meters long is each piece of yarn after x cuts?
$y = 5$			A fish tank has 5 gallons of water. The water level remains the same as time passes. How much water is in the tank after x minutes?

x	***y***
0	1
1	3
2	9
3	27
4	81

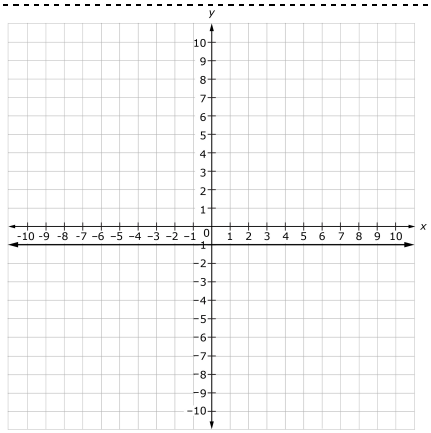
x	***y***
0	8
1	4
2	2
3	1
4	$\frac{1}{2}$

x	***y***
0	4
1	2
2	1
3	$\frac{1}{2}$
4	$\frac{1}{4}$

x	***y***
0	5
1	5
2	5
3	5
4	5

$y = -1$			Your freezer is set at –1°F. The temperature does not change over time. What is the temperature after x hours?

x	***y***
0	–1
1	–1
2	–1
3	–1
4	–1



Comparing Linear and Nonlinear Functions

INSTRUCTIONAL ACTIVITY

Lesson 3

LEARNING GOAL

Students will analyze and compare linear and nonlinear functions given in the same representation. The critical outcome of this activity is for students to be able to analyze the properties of the functions separately and then to compare the functions to each other when the functions are represented in the same way.

PRIMARY ACTIVITY

Students will use comparison cards to compare functions given in the same representation.

OTHER VOCABULARY

Students will need to know the meaning (and appearance) of the following terms:

- Rate of change
- *y*-intercept
- Increasing function
- Decreasing function
- Constant function
- Linear function
- Nonlinear function
- Steepness

MATERIALS

- Instructional Activity Supplement (Recommend one copy for every two students.)
- Word Version Instructional Activity Supplement

IMPLEMENTATION

Prior to the activity, **review** increasing, decreasing, and constant functions as well as the difference between steepness and the rate (or slope) when analyzing linear functions. This unit uses steepness to describe the absolute value of the rate. For example, $y = 2x + 5$ and $y = -2x + 5$ have the same steepness, while $y = 2x + 5$ has a greater rate of change. This activity assumes students are familiar with these terms and provides students with an opportunity to strengthen their understanding.

Review the vocabulary required for this activity with the students.

Provide pairs of students with the deck of cards from the INSTRUCTIONAL ACTIVITY SUPPLEMENT. The students should create separate, smaller decks for each type of function representation (i.e., one for equations, one for graphs, one for tables, and one for descriptions). In this lesson, students should work with one small deck at a time, eventually using all four small decks.

For each small deck, students should turn over two cards to reveal two different functions. They should then align the property cards as appropriate for the functions on the cards using the function mat provided. Properties that apply to only one of the functions should be placed below that function. Properties that apply to both functions should be placed in between the functions (similar to a Venn diagram). Students will not use all comparison cards but should use all that are appropriate.

> NOTE: Students should only use *greater rate of change*, *lesser rate of change*, *equal rate of change*, *graph is steeper*, *graph is less steep,* and *graphs are equally steep* comparisons when comparing two linear functions.

An example function comparison mat is shown below:

Function 1	Common Properties of Functions 1 and 2	Function 2
$y = 2x + 1$	increasing function	$y = \frac{1}{2}x + 1$
	equal y-intercept	
graph is steeper	linear function	lesser rate of change
greater rate of change	constant rate of change	graph is less steep

Once students have placed the feature cards, they should take a picture or record their work on paper and then choose two new cards from the same deck. Once the first deck is complete, students should follow the same process with the remaining decks.

Circulate the room and require students to verbally justify their reasoning as they place the comparison cards. For example, **ask** "Why is the function decreasing?" or "Why does this function have a greater rate of change?"

GUIDING QUESTIONS

Elicit student thinking:

- How do different representations of the same function show you details about the function?
- Which function representation do you like to think about first? Why?

Determine if the student can RECOGNIZE THE PROPERTIES OF FUNCTIONS:

- Looking at this function only, what is its *y*-intercept?
- Is this function increasing, decreasing, constant, or both increasing and decreasing?
- Is this function linear or nonlinear?
- Is this function changing at a constant rate or at a variable rate?

Determine if the student can ANALYZE LINEAR FUNCTIONS:

- What is the rate of change of this linear function?

Determine if the student can COMPARE 2 FUNCTIONS WITH DIFFERENT RATES OF CHANGE:

- Which of these linear functions is steeper?
- Which of these linear functions is less steep?

Determine if the student can COMPARE THE PROPERTIES OF 2 FUNCTIONS REPRESENTED IN THE SAME FORM:

- Now that you have identified the properties of each function separately, which properties do the functions share? Which properties are different?

NOTE: An alternative but similar activity is to first select and place comparison strips, then ask students to determine two functions that fit the specified comparisons. Note that the comparison strips must be carefully chosen so it is possible for students to determine two functions that fit the specified characteristics or comparisons.

Students should be required to record their work through pictures or written notes. You may or may not require this for every pair; however, students should record enough to see trends in the functions. Additionally, when asked to provide justification for their comparisons, students should be able to explain their reasoning using mathematically correct vocabulary.

Check for understanding.

Show the whole class a pair of cards that display two functions in the same representation (any of the four representations). **Ask** students to compare the two functions without the guidance of the comparison cards.

Comparing Linear and Nonlinear Functions

INSTRUCTIONAL ACTIVITY SUPPLEMENT

Lesson 3 and Lesson 4

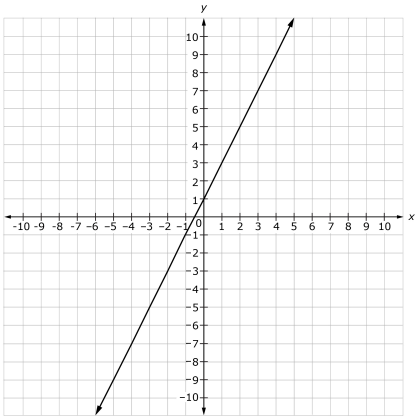
COMPARISON CARDS

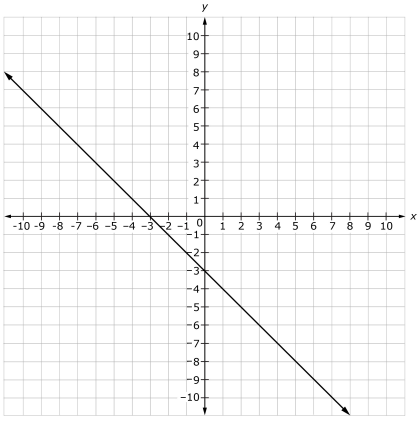
$y = 2x + 1$			An empty box weighs 1 pound. Each can you place in the box weighs 2 pounds. What is the weight with x cans?
$y = -x - 3$			The temperature is currently –3°F and it is dropping 1°F per hour. What is the temperature after x hours?
$y = x + 6$			A puppy currently weighs 6 pounds. The puppy gains 1 pound per week. How much does the puppy weigh after x weeks?

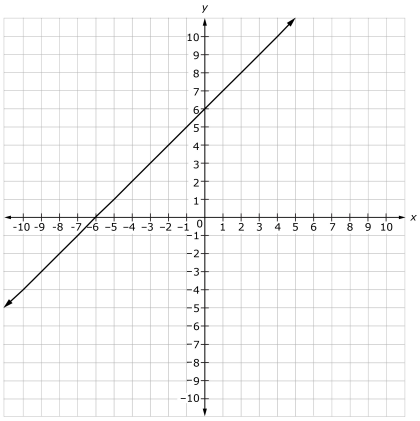
x	y
–3	–5
–2	–3
–1	–1
0	1
1	3

x	y
–1	–2
0	–3
1	–4
2	–5
3	–6

x	y
–2	4
–1	5
0	6
1	7
2	8

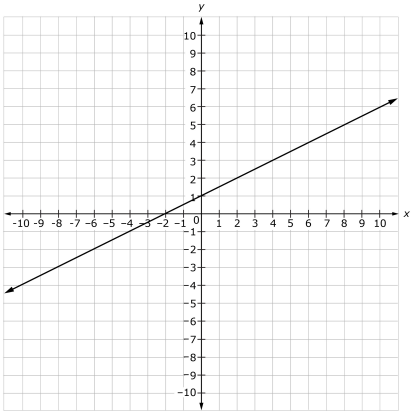




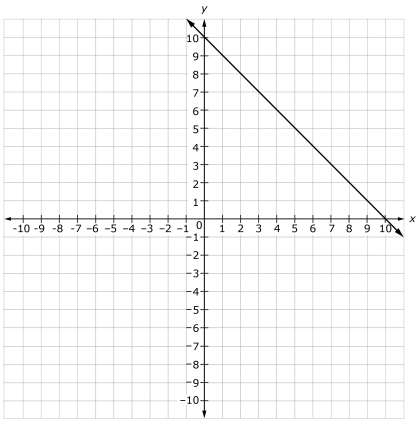


$y = \frac{1}{2}x + 1$			A kitten weighs 1 pound when it is born. The kitten gains $\frac{1}{2}$ pound per month. How much does the kitten weigh after x months?
$y = -x + 10$			A bucket has 10 gallons of water in it. The water drains at a rate of 1 gallon per minute. How much water is in the bucket after x minutes?
$y = -10 + 2x$			The current outside temperature is –10°F. If the temperature increases by 2°F per hour, what is the temperature after x hours?
$y = 5 - 3x$			You are 5 miles away from home. You start walking home at a rate of 3 miles per hour. How far are you from home after x hours?

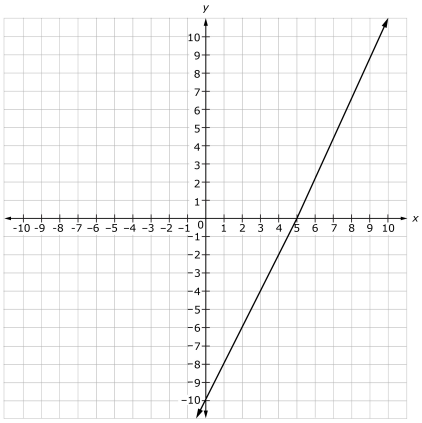
x	y
0	1
1	$1\frac{1}{2}$
2	2
3	$2\frac{1}{2}$
4	3



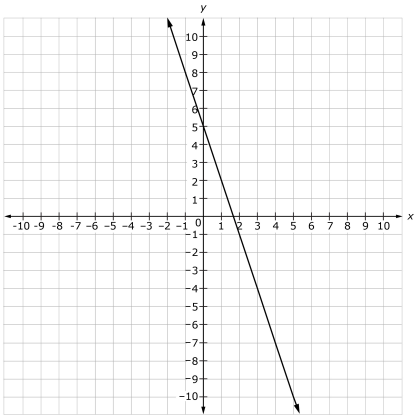
x	y
–1	11
0	10
1	9
2	8
3	7

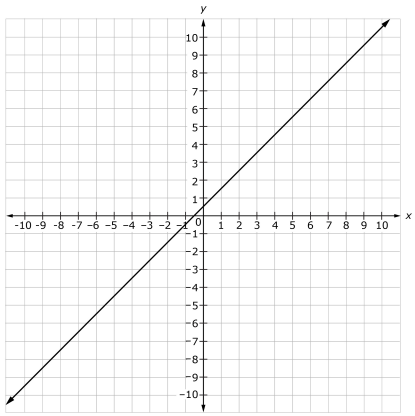
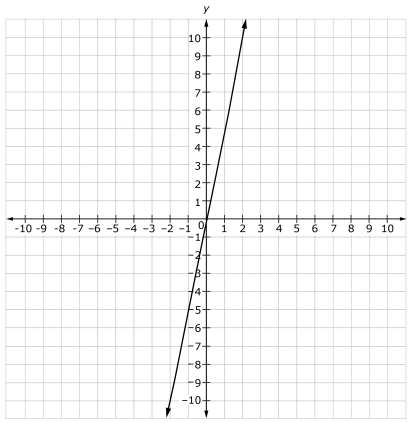
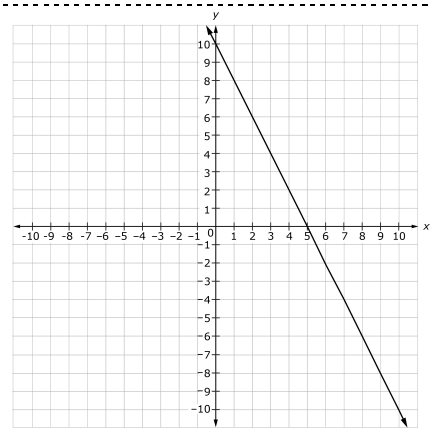
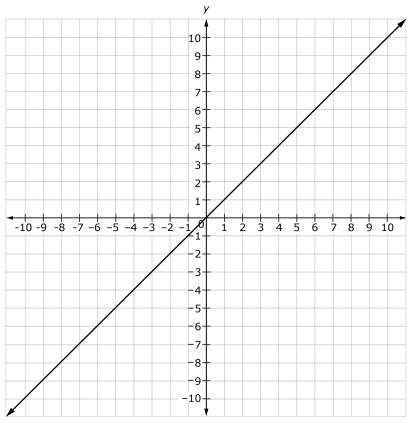


x	y
–2	–14
–1	–12
0	–10
1	–8
2	–6



x	y
–4	17
–3	14
–2	11
–1	8
0	5



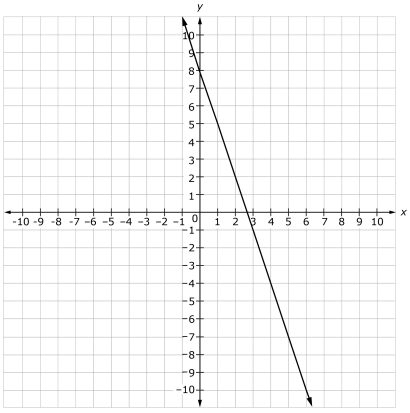
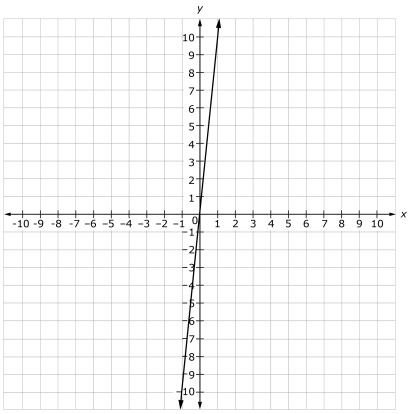
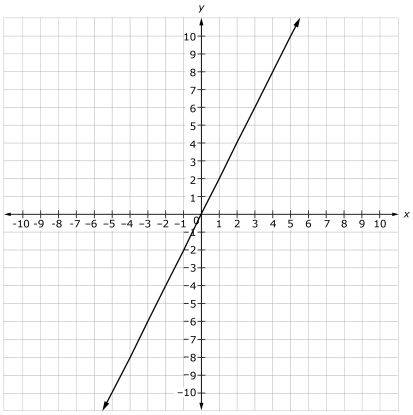
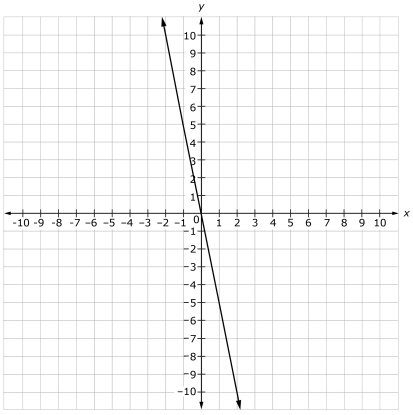
$y = x + \frac{1}{2}$			You are $\frac{1}{2}$ a mile from home and walk away from home at a rate of 1 mile per hour. How far are you away from home after x hours?
$y = 5x$			A swimming pool is empty. You begin filling the pool at a rate of 5 gallons per minute. How many gallons of water are in the pool after x minutes?
$y = -2x + 10$			You have a bus card with \$10. Each bus trip costs \$2. How much money is on the card after x bus trips?
$y = x$			You are having a bake sale. Cookies cost \$1 each. How much money do you collect if you sell x cookies at the bake sale?

x	y
–1	$-\frac{1}{2}$
0	$\frac{1}{2}$
1	$1\frac{1}{2}$
2	$2\frac{1}{2}$
3	$3\frac{1}{2}$

x	y
–3	–15
–2	–10
–1	–5
0	0
1	5

x	y
–1	12
0	10
1	8
2	6
3	4

x	y
–2	–2
–1	–1
0	0
1	1
2	2

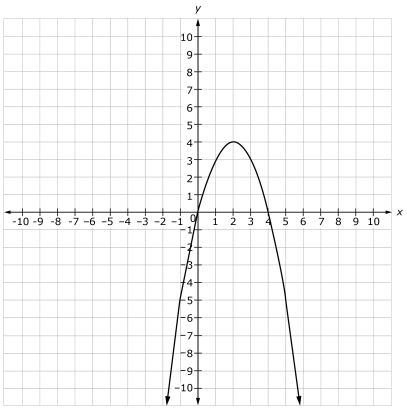
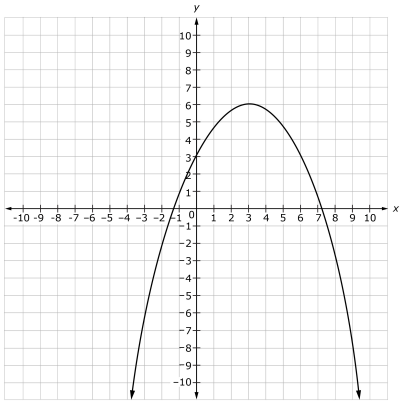
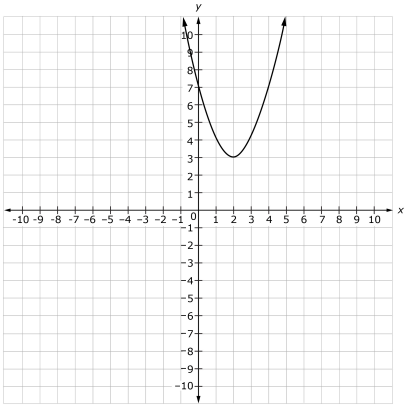
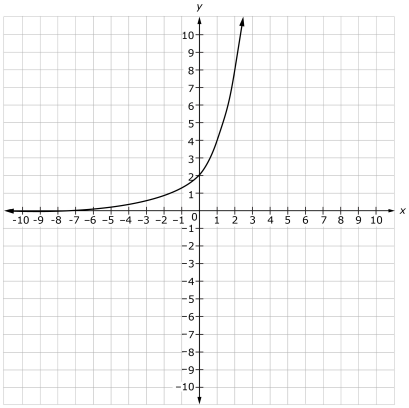
$y = 8 - 3x$			You have been hired to edit an 8-page paper. You are able to edit 3 pages per hour. How many pages still need to be edited after x hours?
$y = 10x$			You have been hired to edit a novel. You charge \$10 to edit each page. How much do you charge to edit x pages?
$y = 2x$			You download a new music application on your phone. Each song is \$2. How much do you pay for x songs?
$y = -5x$			A submarine starts at sea level. The submarine descends into the water at a rate of 5 meters per minute. What is the depth of the submarine after x minutes?

x	y
0	8
1	5
2	2
3	–1
4	–4

x	y
–4	–40
–3	–30
–2	–20
–1	–10
0	0

x	y
–2	–4
–1	–2
0	0
1	2
2	4

x	y
–3	15
–2	10
–1	5
0	0
1	–5

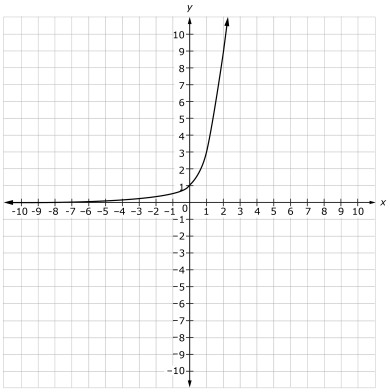
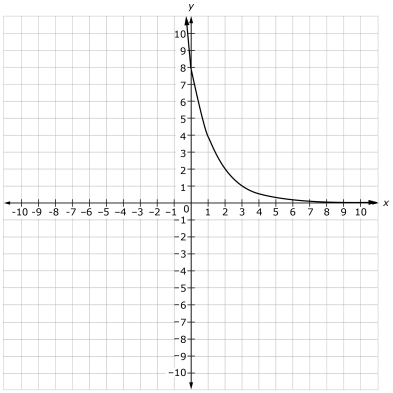
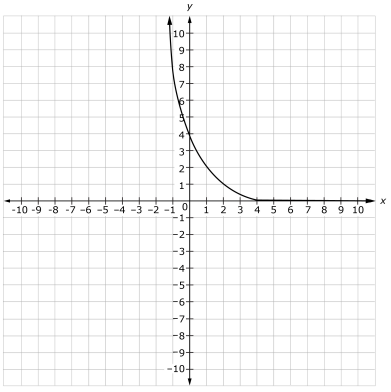
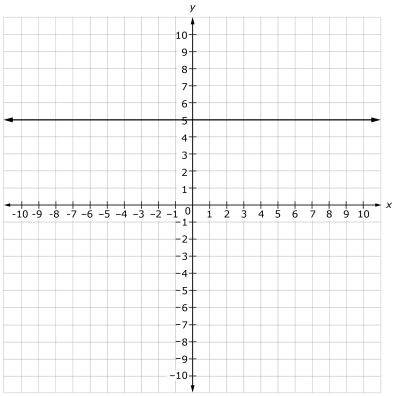
$y = -x^2 + 4x$			A soccer ball is kicked from the ground. It reaches a maximum height of 4 meters and returns to the ground 4 meters from where it was kicked. How high is the soccer ball x meters from where it was kicked?
$y = -\frac{1}{3}x^2 + 2x + 3$			A water balloon starts at a height of 3 feet. It is thrown and reaches a maximum height of 6 feet before it is caught a few feet away at a height of 3 feet. How high is the water balloon x feet after it was thrown?
$y = x^2 - 4x + 7$			A bird flies for 4 seconds, starting at an elevation of 7 feet, descending to an elevation of 3 feet, then rising back up to 7 feet. What is the elevation of the bird after x seconds?
$y = 2(2)^x$			You plan to breed 2 rabbits. Assume that your rabbits will breed every month and the population will double each time. How many rabbits will there be after x months?

x	y
0	0
1	3
2	4
3	3
4	0

x	y
0	3
1	$4\frac{2}{3}$
3	6
5	$4\frac{2}{3}$
6	3

x	y
0	7
1	4
2	3
3	4
4	7

x	y
–1	1
0	2
1	4
2	8
3	16

$y = 1(3)^x$			Your friend lets you borrow money and says you must pay \$1 interest right away. The amount of interest triples every week after that. How much do you owe in interest after x weeks?
$y = 8\left(\frac{1}{2}\right)^x$			You study 8 grams of an element with a half-life of one day (half the element is left after one day). How much of the element is left after x days?
$y = 4\left(\frac{1}{2}\right)^x$			You have 4 meters of yarn for a project. You cut the yarn in half, then half again, and so on. How many meters long is each piece of yarn after x cuts?
$y = 5$			A fish tank has 5 gallons of water. The water level remains the same as time passes. How much water is in the tank after x minutes?

x	y
0	1
1	3
2	9
3	27
4	81

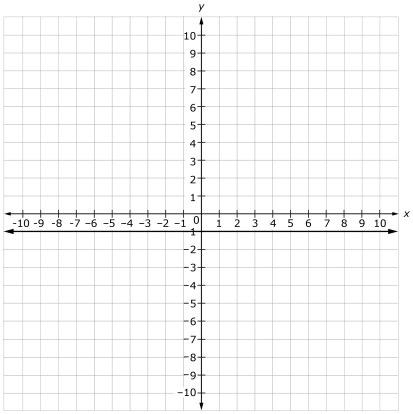
x	y
–2	32
–1	16
0	8
1	4
2	2

x	y
–1	8
0	4
1	2
2	1
3	$\frac{1}{2}$

x	y
–4	5
–3	5
–2	5
–1	5
0	5

$y = -1$			Your freezer is set at –1°F. The temperature does not change over time. What is the temperature after x hours?

x	*y*
–3	–1
–2	–1
–1	–1
0	–1
1	–1



COMPARISON STATEMENTS

equal rate of change	equal *y*–intercept
lesser rate of change	lesser *y*–intercept
greater rate of change	greater *y*–intercept
linear function	nonlinear function
increasing function	decreasing function
increasing and decreasing function	constant function
graph is steeper	graph is less steep
graphs are equally steep	constant rate of change
variable rate of change	

Function 1	Common Properties of Functions 1 and 2	Function 2
Place 1st function here.		Place 2nd function here.

Comparing Linear and Nonlinear Functions

INSTRUCTIONAL ACTIVITY

Lesson 4

LEARNING GOAL

Students will analyze and compare linear and nonlinear functions given in different representations. The critical outcome of this activity is for students to be able to analyze the properties of the functions separately and then to compare them to each other when the functions are represented in different ways.

PRIMARY ACTIVITY

Students will use comparison cards to compare functions given in different representations.

OTHER VOCABULARY

Students will need to know the meaning (and appearance) of the following terms:

- Rate of change
- *y*-intercept
- Increasing function
- Decreasing function
- Constant function
- Linear function
- Nonlinear function

MATERIALS

- Instructional Activity Supplement (Recommend one copy for every two students.)
- Word Version Instructional Activity Supplement

IMPLEMENTATION

Prior to the activity, **review** the previous lesson's comparison of different functions given in the same representation. This will remind students of the need to analyze the properties of the functions separately and then to compare the properties across functions.

> **Provide** pairs of students with a mixed deck of cards from the INSTRUCTIONAL ACTIVITY SUPPLEMENT. This deck contains cards with functions given in different representations (i.e., one deck for equations, one for graphs, one for tables, and one for descriptions). In this lesson, students should work with the entire deck so they are comparing two different functions given in different representations.
>
> Students should turn over two cards from the shuffled deck to reveal two different functions. They should then align the property cards as appropriate for the functions on the cards. Properties that apply to only one of the functions should be placed below that function. Properties that apply to both functions should be placed between the functions (similar to a Venn diagram). Students will not use all comparison cards but should use all that are appropriate.
>
> NOTE: Students can only use *greater rate of change*, *lesser rate of change*, *equal rate of change*, *graph is steeper*, *graph is less steep*, and *graphs are equally steep* comparisons when comparing two linear functions.

An example function comparison mat is shown below:

Function 1	Common Properties of Functions 1 and 2	Function 2

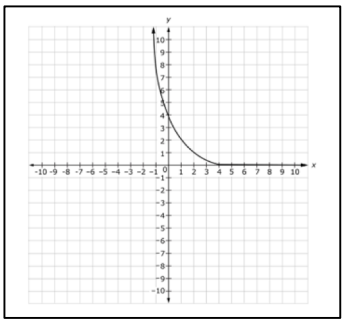
x	***y***
0	0
1	–5
2	–10
3	–15
4	–20

linear function

lesser *y*-intercept

constant rate of change

decreasing function



nonlinear function

greater *y*-intercept

variable rate of change

Once the comparison cards have been placed, students should take a picture or record their work on paper and then choose two new cards to play again.

Circulate the room and require students to verbally justify their reasoning as they place the comparison cards. For example, **ask** "What evidence indicates the function is decreasing?" or "How do you know this function has a greater slope?"

GUIDING QUESTIONS

Elicit student thinking:

- How do different representations of the same function show you details about the function?
- Which function representation do you like to use to identify the *y*-intercept? Why?
- Which function representation do you like to use to identify the rate of change? Why?

Determine if the student can RECOGNIZE THE PROPERTIES OF FUNCTIONS:

- Looking at this function only, what is its *y*-intercept?
- Is this function increasing, decreasing, constant, or both increasing and decreasing?
- Is this function linear or nonlinear?
- Is this function changing at a constant rate or at a variable rate?

Determine if the student can ANALYZE LINEAR FUNCTIONS:

- What is the rate of change of this linear function?

Determine if the student can COMPARE 2 FUNCTIONS WITH DIFFERENT RATES OF CHANGE:

- Which of these linear functions is steeper?
- Which of these linear functions is less steep?

Determine if the student can COMPARE THE PROPERTIES OF 2 FUNCTIONS REPRESENTED IN DIFFERENT FORMS:

- Now that you have identified the properties of each function separately, which properties do the functions share? Which properties are different?
- Can you compare two functions if they are not represented in the same way?
- What strategies did you use to compare these two functions?

NOTE: An alternative but similar activity is to first select and place comparison strips, then ask students to determine two functions that fit the specified comparisons. Note that the comparison strips must be carefully chosen so it is possible for students to determine two functions that fit the specified characteristics or comparisons.

Students should be required to record their work through pictures or written notes. You may or may not require this for every pair; however, students should record enough to see trends in the functions. Additionally, when asked to provide justification for their comparisons, students should be able to explain their reasoning using mathematically correct vocabulary.

Check for understanding.

Show the whole class two cards that display functions in different representations (any of the four representations). **Ask** students to compare the two functions without the guidance of the comparison cards.

Comparing Linear and Nonlinear Functions

INSTRUCTIONAL ACTIVITY SUPPLEMENT

Lesson 3 and Lesson 4

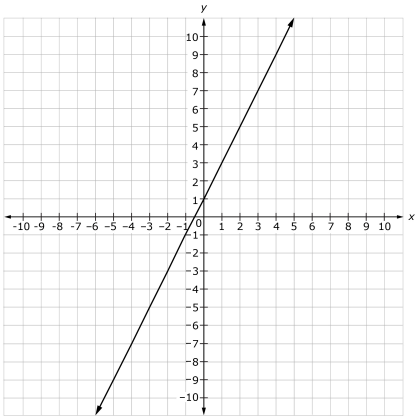
COMPARISON CARDS

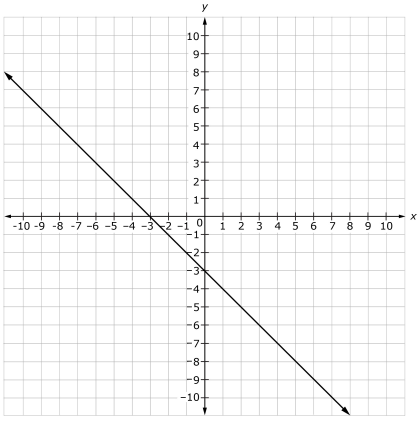
$y = 2x + 1$			An empty box weighs 1 pound. Each can you place in the box weighs 2 pounds. What is the weight with x cans?
$y = -x - 3$			The temperature is currently –3°F and it is dropping 1°F per hour. What is the temperature after x hours?
$y = x + 6$			A puppy currently weighs 6 pounds. The puppy gains 1 pound per week. How much does the puppy weigh after x weeks?

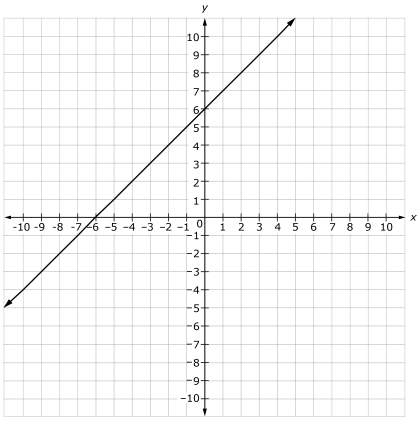
x	y
–3	–5
–2	–3
–1	–1
0	1
1	3

x	y
–1	–2
0	–3
1	–4
2	–5
3	–6

x	y
–2	4
–1	5
0	6
1	7
2	8

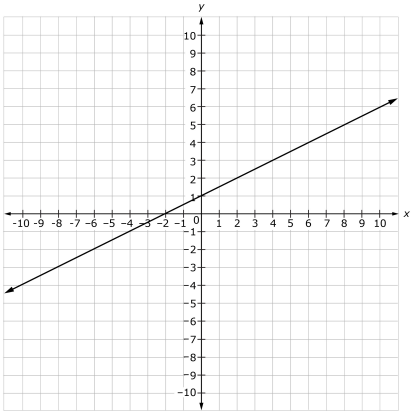




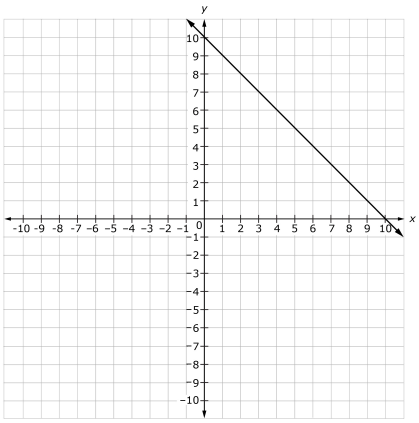


$y = \frac{1}{2}x + 1$			A kitten weighs 1 pound when it is born. The kitten gains $\frac{1}{2}$ pound per month. How much does the kitten weigh after x months?
$y = -x + 10$			A bucket has 10 gallons of water in it. The water drains at a rate of 1 gallon per minute. How much water is in the bucket after x minutes?
$y = -10 + 2x$			The current outside temperature is –10°F. If the temperature increases by 2°F per hour, what is the temperature after x hours?
$y = 5 - 3x$			You are 5 miles away from home. You start walking home at a rate of 3 miles per hour. How far are you from home after x hours?

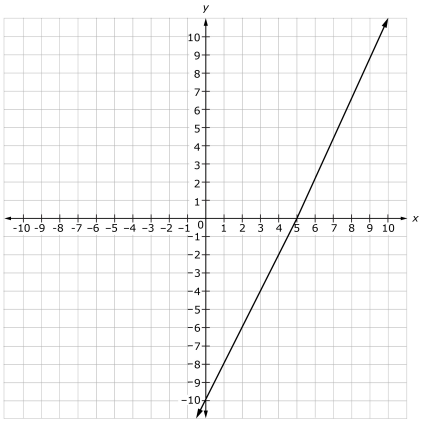
x	y
0	1
1	$1\frac{1}{2}$
2	2
3	$2\frac{1}{2}$
4	3



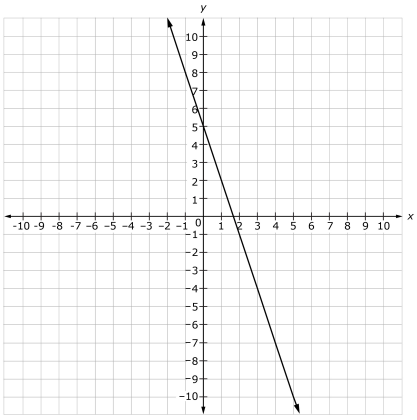
x	y
–1	11
0	10
1	9
2	8
3	7

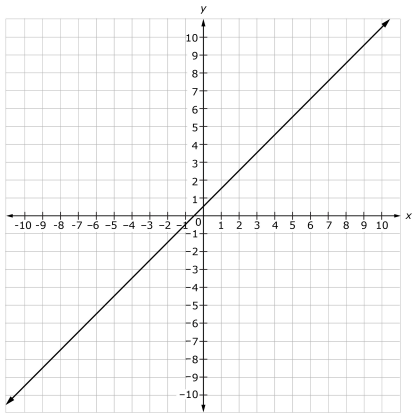
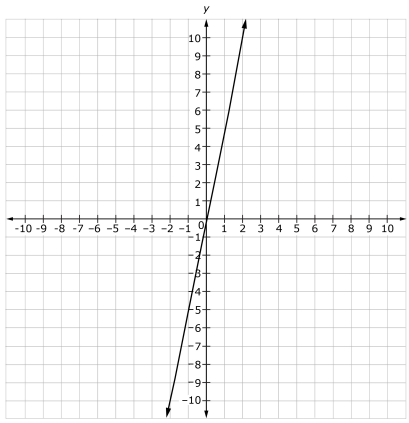
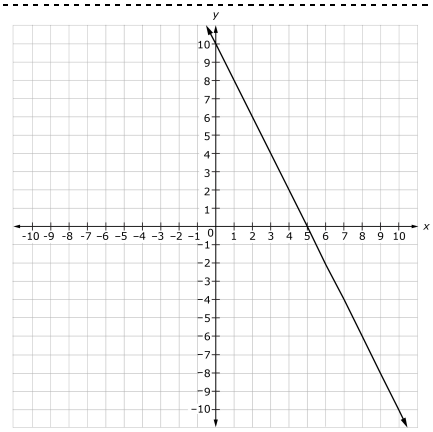
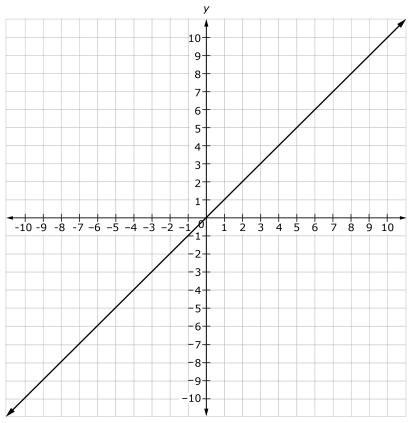


x	y
–2	–14
–1	–12
0	–10
1	–8
2	–6



x	y
–4	17
–3	14
–2	11
–1	8
0	5



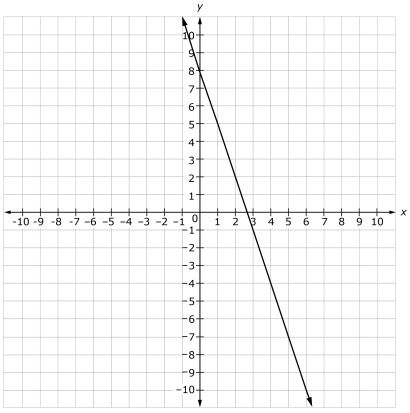
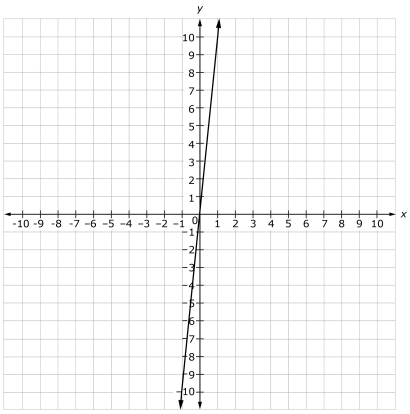
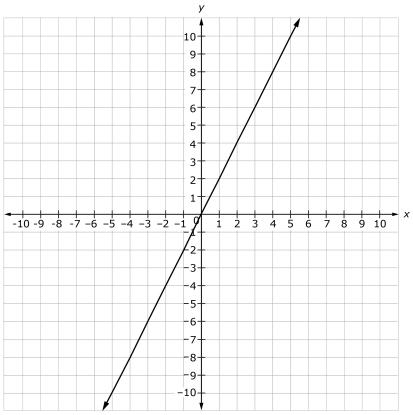
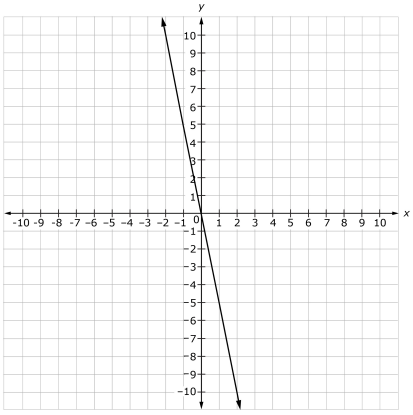
$y = x + \frac{1}{2}$			You are $\frac{1}{2}$ a mile from home and walk away from home at a rate of 1 mile per hour. How far are you away from home after x hours?
$y = 5x$			A swimming pool is empty. You begin filling the pool at a rate of 5 gallons per minute. How many gallons of water are in the pool after x minutes?
$y = -2x + 10$			You have a bus card with \$10. Each bus trip costs \$2. How much money is on the card after x bus trips?
$y = x$			You are having a bake sale. Cookies cost \$1 each. How much money do you collect if you sell x cookies at the bake sale?

x	y
–1	$-\frac{1}{2}$
0	$\frac{1}{2}$
1	$1\frac{1}{2}$
2	$2\frac{1}{2}$
3	$3\frac{1}{2}$

x	y
–3	–15
–2	–10
–1	–5
0	0
1	5

x	y
–1	12
0	10
1	8
2	6
3	4

x	y
–2	–2
–1	–1
0	0
1	1
2	2

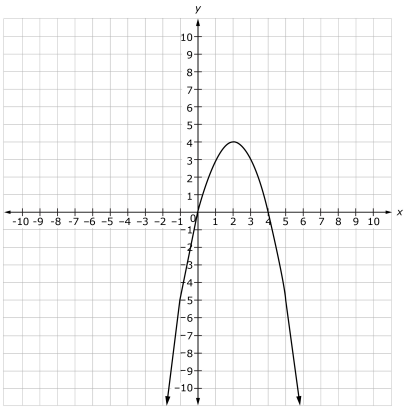
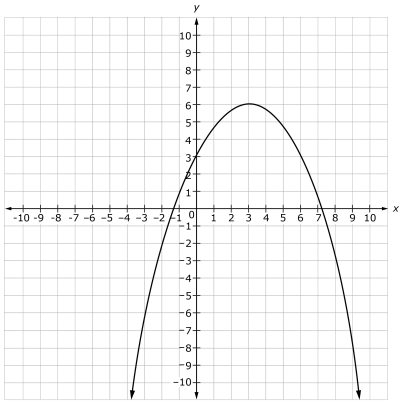
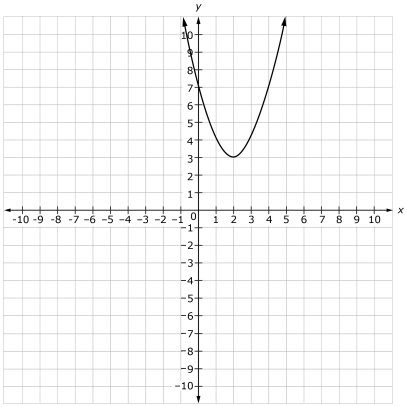
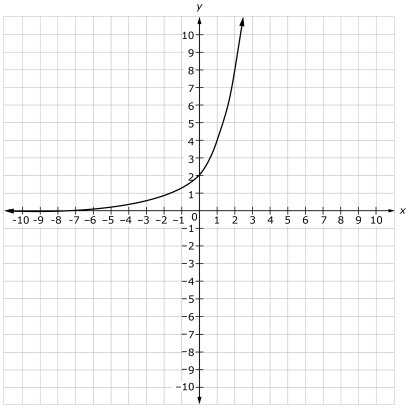
$y = 8 - 3x$			You have been hired to edit an 8-page paper. You are able to edit 3 pages per hour. How many pages still need to be edited after x hours?
$y = 10x$			You have been hired to edit a novel. You charge \$10 to edit each page. How much do you charge to edit x pages?
$y = 2x$			You download a new music application on your phone. Each song is \$2. How much do you pay for x songs?
$y = -5x$			A submarine starts at sea level. The submarine descends into the water at a rate of 5 meters per minute. What is the depth of the submarine after x minutes?

x	y
0	8
1	5
2	2
3	–1
4	–4

x	y
–4	–40
–3	–30
–2	–20
–1	–10
0	0

x	y
–2	–4
–1	–2
0	0
1	2
2	4

x	y
–3	15
–2	10
–1	5
0	0
1	–5

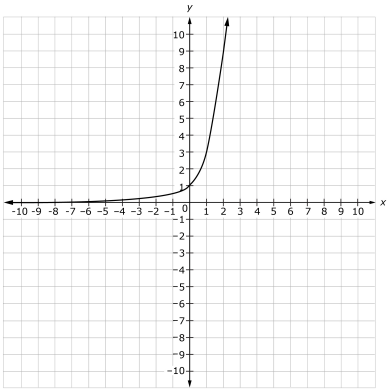
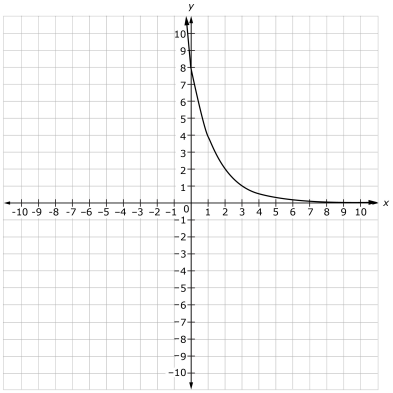
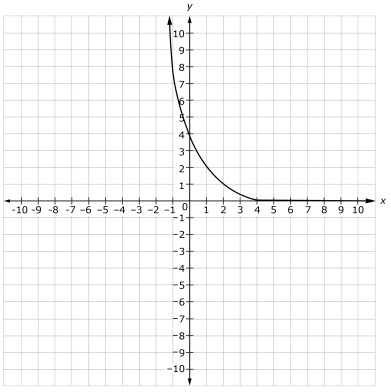
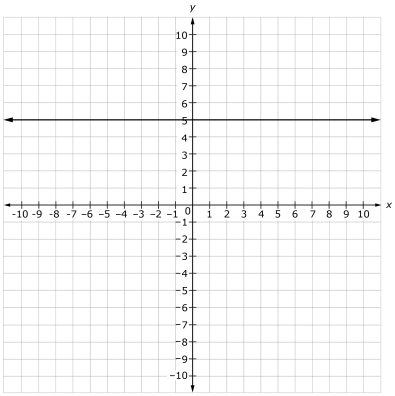
$y = -x^2 + 4x$			A soccer ball is kicked from the ground. It reaches a maximum height of 4 meters and returns to the ground 4 meters from where it was kicked. How high is the soccer ball x meters from where it was kicked?
$y = -\frac{1}{3}x^2 + 2x + 3$			A water balloon starts at a height of 3 feet. It is thrown and reaches a maximum height of 6 feet before it is caught a few feet away at a height of 3 feet. How high is the water balloon x feet after it was thrown?
$y = x^2 - 4x + 7$			A bird flies for 4 seconds, starting at an elevation of 7 feet, descending to an elevation of 3 feet, then rising back up to 7 feet. What is the elevation of the bird after x seconds?
$y = 2(2)^x$			You plan to breed 2 rabbits. Assume that your rabbits will breed every month and the population will double each time. How many rabbits will there be after x months?

x	y
0	0
1	3
2	4
3	3
4	0

x	y
0	3
1	$4\frac{2}{3}$
3	6
5	$4\frac{2}{3}$
6	3

x	y
0	7
1	4
2	3
3	4
4	7

x	y
–1	1
0	2
1	4
2	8
3	16

$y = 1(3)^x$			Your friend lets you borrow money and says you must pay \$1 interest right away. The amount of interest triples every week after that. How much do you owe in interest after x weeks?
$y = 8\left(\frac{1}{2}\right)^x$			You study 8 grams of an element with a half-life of one day (half the element is left after one day). How much of the element is left after x days?
$y = 4\left(\frac{1}{2}\right)^x$			You have 4 meters of yarn for a project. You cut the yarn in half, then half again, and so on. How many meters long is each piece of yarn after x cuts?
$y = 5$			A fish tank has 5 gallons of water. The water level remains the same as time passes. How much water is in the tank after x minutes?

x	y
0	1
1	3
2	9
3	27
4	81

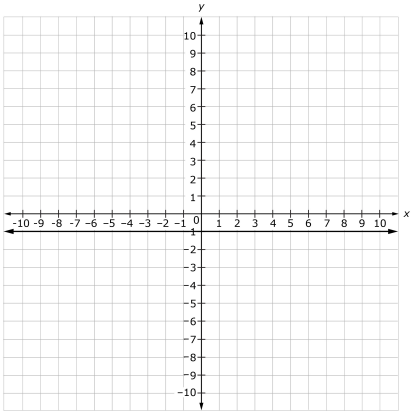
x	y
–2	32
–1	16
0	8
1	4
2	2

x	y
–1	8
0	4
1	2
2	1
3	$\frac{1}{2}$

x	y
–4	5
–3	5
–2	5
–1	5
0	5

$y = -1$			Your freezer is set at –1°F. The temperature does not change over time. What is the temperature after x hours?

x	*y*
–3	–1
–2	–1
–1	–1
0	–1
1	–1



COMPARISON STATEMENTS

equal rate of change	equal *y*–intercept
lesser rate of change	lesser *y*–intercept
greater rate of change	greater *y*–intercept
linear function	nonlinear function
increasing function	decreasing function
increasing and decreasing function	constant function
graph is steeper	graph is less steep
graphs are equally steep	constant rate of change
variable rate of change	

Function 1	Common Properties of Functions 1 and 2	Function 2
Place 1st function here.		Place 2nd function here.

Name_____

Comparing Linear and Nonlinear Functions

STUDENT ACTIVITY

Lessons 1 – 4

1. Use the function $y = -2x + 7$ to complete the following questions.

   1.a. What is the rate of change of the function? Describe how you determined the rate of change from the equation.

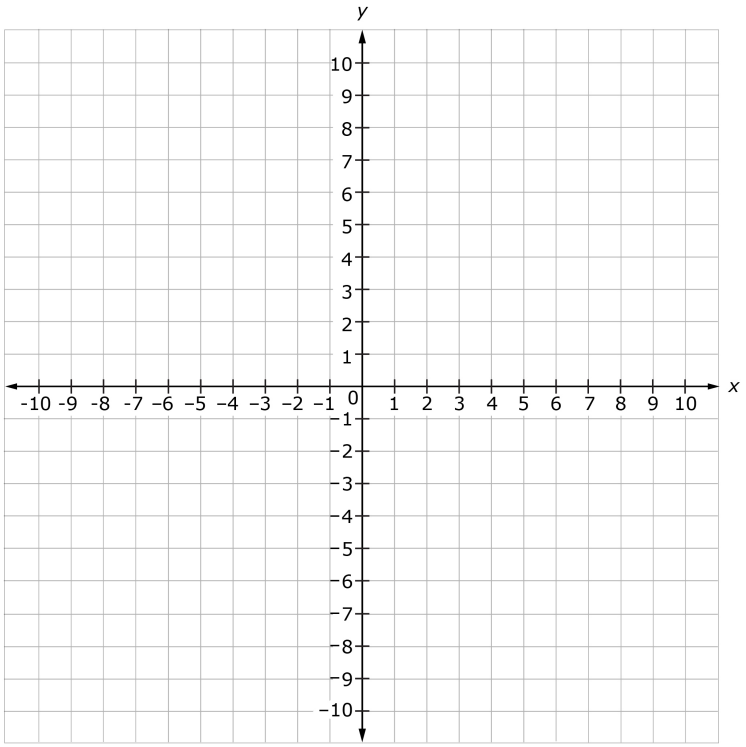
   1.b. What is the y–intercept of the function? Describe how you determined the y–intercept from the equation.

   1.c. Construct a table of values for the equation. How does the rate of change appear in the table? How does the y–intercept appear in the table?

x	***y***
–1	
0	
1	
2	
3	

Name_____

1.d. Construct a graph for the function. How does the rate of change appear on the graph? How does the y–intercept appear on the graph?

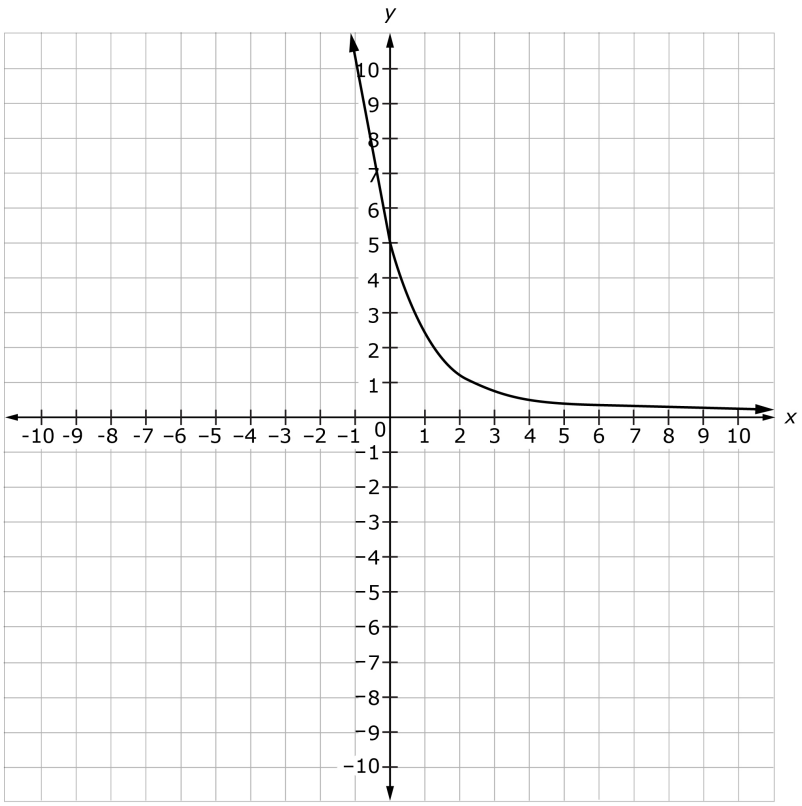


Name_____

WORD BANK

positive rate of change	negative rate of change	zero rate of change
positive *y*–intercept	negative *y*–intercept	zero *y*–intercept
linear function	nonlinear function	constant rate of change
variable rate of change	increasing function	decreasing function
constant function	increasing and decreasing function	

2. Use the word bank to identify the properties of the function shown below. List all that apply.



Name_____

3. Use the word bank to identify the properties of each function shown below. Then list the properties the functions have in common.

WORD BANK

positive rate of change	negative rate of change	zero rate of change	positive y–intercept
negative y–intercept	zero y–intercept	linear function	nonlinear function
constant rate of change	variable rate of change	increasing function	decreasing function
constant function	increasing and decreasing function		

FUNCTION 1	FUNCTION 2
You have \$20 in your savings account. You deposit an additional \$5 per week. How much money is in your savings account after x weeks?	You have \$30 on a bus card. Each trip costs \$3. How much money is on your bus card after x trips?
PROPERTIES OF FUNCTION 1	**PROPERTIES OF FUNCTION 2**
COMMON PROPERTIES OF FUNCTION 1 AND FUNCTION 2	

Name_____

4. Use the word bank to identify the properties of each function shown below. Then list the properties the functions have in common.

WORD BANK

positive rate of change	negative rate of change	zero rate of change	positive y–intercept
negative y–intercept	zero y–intercept	linear function	nonlinear function
constant rate of change	variable rate of change	increasing function	decreasing function
constant function	increasing and decreasing function		

FUNCTION 1

x	y
–2	–6
–1	–3
0	0
1	3
2	6

FUNCTION 2

$y = x - 5$

PROPERTIES OF FUNCTION 1

PROPERTIES OF FUNCTION 2

COMMON PROPERTIES OF FUNCTION 1 AND FUNCTION 2

Name_____

5. Use the word bank to identify the properties of each function shown below. Then list the properties the functions have in common.

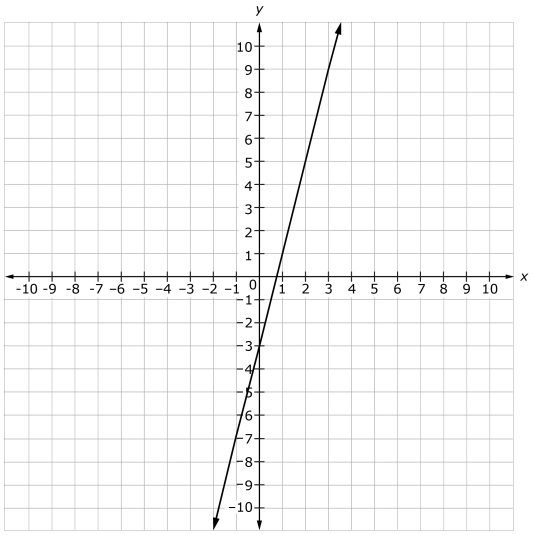
WORD BANK

positive rate of change	negative rate of change	zero rate of change	positive y–intercept
negative y–intercept	zero y–intercept	linear function	nonlinear function
constant rate of change	variable rate of change	increasing function	decreasing function
constant function	increasing and decreasing function		

FUNCTION 1

$y = 2$

FUNCTION 2



PROPERTIES OF FUNCTION 1

PROPERTIES OF FUNCTION 2

COMMON PROPERTIES OF FUNCTION 1 AND FUNCTION 2

Name_____

6. Use the mat and word bank provided to compare the functions.

WORD BANK

greater rate of change	lesser rate of change	equal rate of change	graph is steeper
graph is less steep	graphs are equally steep	greater y–intercept	lesser y–intercept
equal y–intercept	linear function	nonlinear function	constant rate of change
variable rate of change	increasing function	decreasing function	constant function

increasing and decreasing function

FUNCTION 1	FUNCTION 2
$y = 1 + x$	Sam has \$12. Sam buys baseball cards for \$1 each. How much money does Sam have left if he buys x baseball cards?

PROPERTIES OF FUNCTION 1 COMPARED TO PROPERTIES OF FUNCTION 2

PROPERTIES OF BOTH FUNCTION 1 AND FUNCTION 2

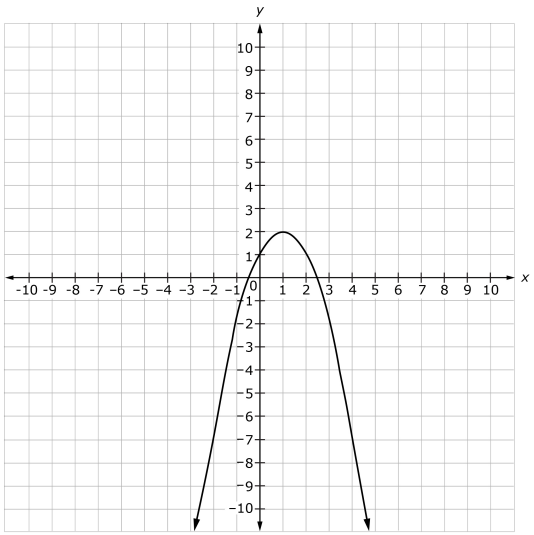
Name_____

7. Use the mat and word bank provided to compare the functions.

WORD BANK

greater rate of change	lesser rate of change	equal rate of change	graph is steeper
graph is less steep	graphs are equally steep	greater y–intercept	lesser y–intercept
equal y–intercept	linear function	nonlinear function	constant rate of change
variable rate of change	increasing function	decreasing function	constant function
increasing and decreasing function			

FUNCTION 1



FUNCTION 2

Beth has $4. Beth earns $2 per week. If Beth does not spend any money, how much will she have after x weeks?

PROPERTIES OF FUNCTION 1 COMPARED TO PROPERTIES OF FUNCTION 2

PROPERTIES OF BOTH FUNCTION 1 AND FUNCTION 2

Name_____

8. Use the mat and word bank provided to compare the functions.

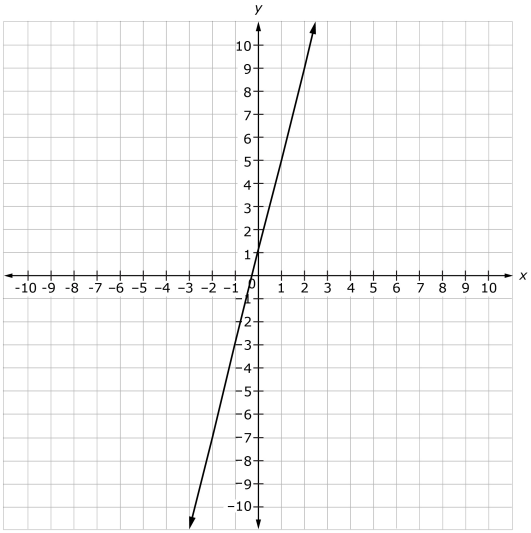
WORD BANK

greater rate of change	lesser rate of change	equal rate of change	graph is steeper
graph is less steep	graphs are equally steep	greater y–intercept	lesser y–intercept
equal y–intercept	linear function	nonlinear function	constant rate of change
variable rate of change	increasing function	decreasing function	constant function
increasing and decreasing function			

FUNCTION 1

x	y
0	3
1	6
2	12
3	24
4	48

FUNCTION 2



PROPERTIES OF FUNCTION 1 COMPARED TO PROPERTIES OF FUNCTION 2

PROPERTIES OF BOTH FUNCTION 1 AND FUNCTION 2

9. What is a common feature of the rate of change of all linear functions?

COMPARING LINEAR AND NONLINEAR FUNCTIONS

STUDENT ACTIVITY SOLUTION GUIDE

Lessons 1 – 4

1. Use the function $y = -2x + 7$ to complete the following questions.

1.a. What is the rate of change of the function? Describe how you determined the rate of change from the equation.

CORRECT ANSWER

> The rate of change is $\frac{-2}{1}$ or –2. As the *x*-value increases by 1, the *y*-value decreases by 2. I was able to determine the rate of change by looking at the coefficient of *x* in the equation in slope-intercept form.

It is also accurate to state "as the *x*-value decreases by 1, the *y*-value increases by 2." However, this description is inconsistent with the convention of reading graphs from left to right, a practice that will benefit students in future coursework.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
7	is unfamiliar or confused about the structure of an equation in slope-intercept form	EXPLAIN *SLOPE-INTERCEPT FORM*
2	does not pay attention to the direction of covariation when identifying the rate	DETERMINE A UNIT RATE GIVEN AN EQUATION

In addition, there may be students who fail to attend to the integer (-2) and mistakenly identify the slope as 2. This error may be caused by an immature understanding of integers rather than flawed understanding of rates.

1.b. What is the *y*-intercept of the function? Describe how you determined the *y*-intercept from the equation.

CORRECT ANSWER

> The *y*-intercept is $(0, 7)$. I was able to determine the *y*-intercept by looking at the constant in the equation in slope-intercept form.
>
> or
>
> The *y*-intercept is $(0, 7)$. I was able to determine the *y*-intercept by evaluating the equation at $x = 0$.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
7	does not understand that the y-intercept is a coordinate pair or a point on the line rather than a single value	EXPLAIN Y-*INTERCEPT*
$(0, -2)$	is unfamiliar or confused about the structure of an equation in slope-intercept form	EXPLAIN *SLOPE-INTERCEPT FORM*

1.c. Construct a table of values for the equation. How does the rate of change appear in the table? How does the *y*-intercept appear in the table?

CORRECT ANSWER

x	*y*
–1	9
0	7
1	5
2	3
3	1

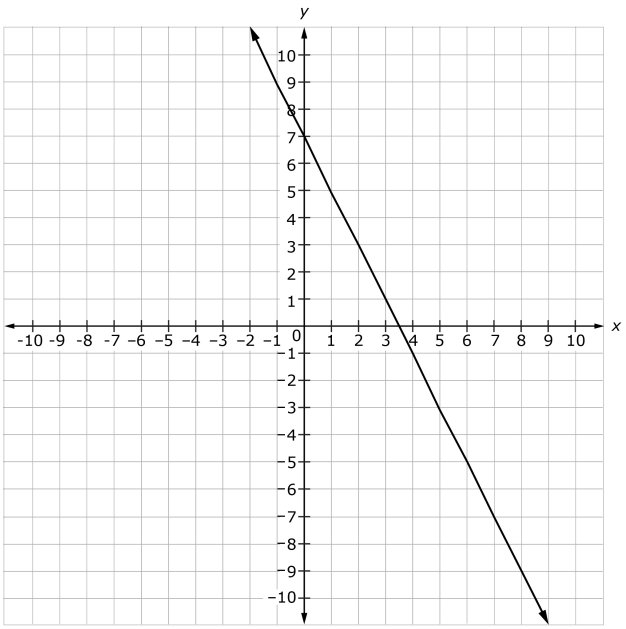
The rate of change can be determined from the table by looking at the change in *y*, –2, as *x* increases by 1.
The *y*-intercept can be determined from the table by locating the *y*-value when the *x*-value is 0.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student only refers to the pattern in *y*-values when describing the rate of change.	only looks at the change in *y*-values and does not consider or reference the change in *x*-values	RECOGNIZE *COVARIATION*
The student does not describe finding the *x*-value of 0 in order to determine the *y*-intercept.	does not relate the *y*-intercept to the coordinate pair where the line crosses the *y*-axis	EXPLAIN Y-*INTERCEPT*

1.d. Construct a graph for the function. How does the rate of change appear on the graph? How does the *y*-intercept appear on the graph?

CORRECT ANSWER

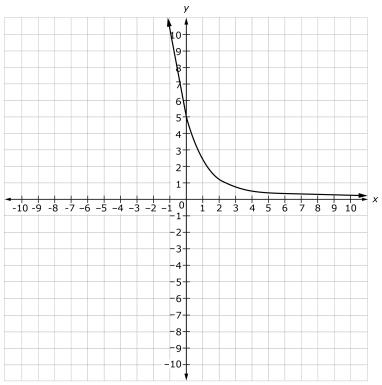


As you read the graph from left to right, the rate of change can be determined by the change in *y* divided by the change in *x* (slope). The slope of this line decreases by 2 as the *x*-values increase by 1.
The *y*-intercept appears on the graph as the point where the line crosses the *y*-axis.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student does not describe how changes in the *x*- and *y*-values are coordinated when describing the rate of change.	only looks at the change in *y*-values and does not consider the change in both variables together	DESCRIBE THE RATE OF CHANGE IN A GRAPH
The student does not refer to the intersection of the graphed line and the *y*-axis as the location of the *y*-intercept.	does not relate the *y*-intercept to the coordinate pair on the *y*-axis	EXPLAIN Y-*INTERCEPT*

2. Use the word bank to identify the properties of the function shown below. List all that apply.



CORRECT ANSWER

decreasing function
negative rate of change
positive *y*-intercept
nonlinear function
variable rate of change

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student does not include *decreasing function*.	may believe *decreasing function* can only describe linear functions	RECOGNIZE *INCREASING AND DECREASING FUNCTIONS*
The student states this is an increasing function.	reads the graph from right to left instead of left to right	RECOGNIZE *INCREASING AND DECREASING FUNCTIONS*
The student identifies the function as a linear function.	may believe the function is linear because it is a decreasing function	RECOGNIZE *LINEAR FUNCTION*

3. Use the word bank to identify the properties of each function shown below. Then list the properties the functions have in common.

CORRECT ANSWER

FUNCTION 1	FUNCTION 2
You have \$20 in your savings account. You deposit an additional \$5 per week. How much money is in your savings account after x weeks?	You have \$30 on a bus card. Each trip costs \$3. How much money is on your bus card after x trips?
PROPERTIES OF FUNCTION 1 increasing function positive rate of change positive *y*-intercept linear function constant rate of change	**PROPERTIES OF FUNCTION 2** decreasing function negative rate of change positive *y*-intercept linear function constant rate of change
COMMON PROPERTIES OF FUNCTION 1 AND FUNCTION 2 positive *y*-intercept linear function constant rate of change	

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student identifies the incorrect rate of change or the incorrect direction of the rate of change.	cannot identify the rate of change from a description of a real-world problem	DESCRIBE THE RATE OF CHANGE IN AN ALGEBRAIC FUNCTION
The student does not identify *positive* y-*intercept* as a property of both functions.	cannot identify the *y*-intercept from a description of a real-world problem	EXPLAIN Y-*INTERCEPT*
The student lists *constant function* as a property.	does not distinguish between *constant function* and *constant rate of change*	EXPLAIN *CONSTANT FUNCTION*
The student does not mention the correct common properties even though their property lists for the individual functions are accurate.	can identify the properties of a single function but cannot compare functions in order to identify common properties	COMPARE THE PROPERTIES OF 2 FUNCTIONS REPRESENTED IN THE SAME FORM

4. Use the word bank to identify the properties of each function shown below. Then list the properties the functions have in common.

CORRECT ANSWER

FUNCTION 1

x	y
–2	–6
–1	–3
0	0
1	3
2	6

FUNCTION 2

$y = x - 5$

PROPERTIES OF FUNCTION 1

positive rate of change
zero y-intercept
linear function
constant rate of change
increasing function

PROPERTIES OF FUNCTION 2

positive rate of change
negative y-intercept
linear function
constant rate of change
increasing function

COMMON PROPERTIES OF FUNCTION 1 AND FUNCTION 2

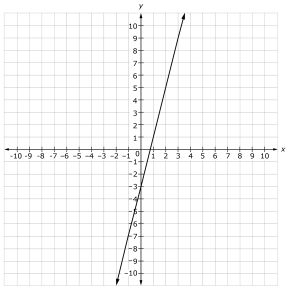
positive rate of change
linear function
constant rate of change
increasing function

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student does not identify *positive rate of change* as a common property of the functions.	is not able to identify the rate of change in algebraic and/or table representation of a function	DESCRIBE THE RATE OF CHANGE IN AN ALGEBRAIC FUNCTION; DESCRIBE THE RATE OF CHANGE IN A TABLE
The student does not identify *zero* y-*intercept* for Function 1 and/or the student does not identify *negative* y-*intercept* for Function 2.	is not able to identify the *y*-intercept in algebraic and/or table representation of a function	RECOGNIZE Y-*INTERCEPT*
The student does not identify *linear function* as a common property of the functions.	cannot recognize linear functions in different representations	RECOGNIZE *LINEAR FUNCTION*
The student does not mention the correct similarities.	can identify properties of a single function but cannot compare functions in order to identify common properties	COMPARE THE PROPERTIES OF 2 FUNCTIONS REPRESENTED IN DIFFERENT FORMS

5. Use the word bank to identify the properties of each function shown below. Then list the properties the functions have in common.

CORRECT ANSWER

FUNCTION 1	FUNCTION 2
$y = 2$	
PROPERTIES OF FUNCTION 1	**PROPERTIES OF FUNCTION 2**
zero rate of change positive *y*-intercept linear function constant rate of change constant function	positive rate of change negative *y*-intercept linear function constant rate of change increasing function
COMMON PROPERTIES OF FUNCTION 1 AND FUNCTION 2	
linear function constant rate of change	

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student does not identify *zero rate of change* for Function 1 and/or the student does not identify *positive rate of change* for Function 2.	is not able to identify the rate of change in algebraic and/or graphical function representations; does not know what a constant function is, cannot recognize the equation of a constant function, or cannot recognize the graph of a constant function.	DESCRIBE THE RATE OF CHANGE IN AN ALGEBRAIC FUNCTION or EXPLAIN *CONSTANT FUNCTION*
The student does not identify *positive* y-*intercept* for Function 1 and/or the student does not identify *negative y-intercept* for Function 2.	is not able to identify the *y*-intercept in algebraic and/or graphical representations of functions	RECOGNIZE Y-*INTERCEPT*
The student does not identify *linear function* as a common property of the functions.	cannot recognize linear functions in different representations	RECOGNIZE *LINEAR FUNCTION*
The student does not identify *constant rate of change* as a common property of the functions.	cannot determine rate of change in algebraic and/or graphical representations of functions or does not distinguish between *constant function* and *constant rate of change*	DESCRIBE THE RATE OF CHANGE IN AN ALGEBRAIC FUNCTION or EXPLAIN *CONSTANT FUNCTION*; EXPLAIN *LINEAR FUNCTION*

6. Use the mat and word bank provided to compare the functions.

CORRECT ANSWER

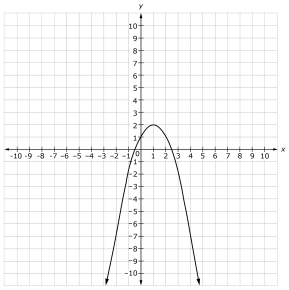
FUNCTION 1	FUNCTION 2
$y = 1 + x$	Sam has \$12. Sam buys baseball cards for \$1 each. How much money does Sam have left if he buys x baseball cards?
PROPERTIES OF FUNCTION 1 COMPARED TO PROPERTIES OF FUNCTION 2	
greater rate of change lesser *y*-intercept increasing function	lesser rate of change greater y-intercept decreasing function
PROPERTIES OF BOTH FUNCTION 1 AND FUNCTION 2	
constant rate of change linear function graphs are equally steep	

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student misplaces or does not identify *greater rate of change* or *lesser rate of change* as properties of the individual functions.	is not able to identify the rate of change in all function representations	DESCRIBE THE RATE OF CHANGE IN AN ALGEBRAIC FUNCTION
The student misplaces or does not identify *greater* y-*intercept* or *lesser* y-*intercept* as properties of the individual functions.	is not able to identify the *y*-intercept in all function representations	EXPLAIN Y-*INTERCEPT*
The student does not identify *equally steep* as a shared property.	does not understand the difference between steepness and the rate of change or slope or does not recognize the effect of the negative sign on the direction of the slope	EXPLAIN *SLOPE*; EXPLAIN THE RATE OF CHANGE IN AN ALGEBRAIC FUNCTION
The student does not identify *linear function* as a shared property.	cannot recognize linear functions in different representations	RECOGNIZE *LINEAR FUNCTION*
The student does not mention the correct similarities.	can identify properties of a single function but cannot compare functions in order to identify common properties	COMPARE THE PROPERTIES OF 2 FUNCTIONS REPRESENTED IN DIFFERENT FORMS

7. Use the mat and word bank provided to compare the functions.

CORRECT ANSWER

FUNCTION 1	FUNCTION 2
	Beth has $4. Beth earns $2 per week. If Beth does not spend any money, how much will she have after *x* weeks?
PROPERTIES OF FUNCTION 1 COMPARED TO PROPERTIES OF FUNCTION 2	
lesser *y*-intercept nonlinear function variable rate of change increasing and decreasing function	greater y-intercept linear function constant rate of change increasing function
PROPERTIES OF BOTH FUNCTION 1 AND FUNCTION 2	
(none)	

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student identifies *equal rate of change*, *greater rate of change*, or *lesser rate of change* as a property of the first function.	does not understand the definition of *rate of change*	DESCRIBE THE RATE OF CHANGE IN AN ALGEBRAIC FUNCTION
The student misplaces or does not identify *greater* y-*intercept* or *lesser* y-*intercept* as properties of the individual functions.	is not able to identify the *y*-intercept in all function representations	EXPLAIN Y-*INTERCEPT*
The student does not identify *nonlinear function* for the first function and/or *linear function* for the second function.	cannot recognize the difference between linear and nonlinear functions	RECOGNIZE *LINEAR FUNCTION*
The student identifies similarities between the functions.	can identify properties of a single function but cannot compare functions in order to identify common properties	COMPARE THE PROPERTIES OF 2 FUNCTIONS REPRESENTED IN DIFFERENT FORMS

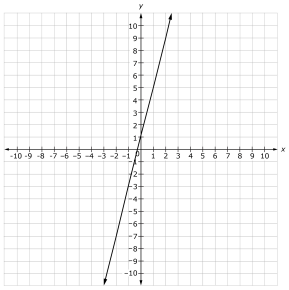
8. Use the mat and word bank provided to compare the functions.

CORRECT ANSWER

FUNCTION 1

x	y
0	3
1	6
2	12
3	24
4	48

FUNCTION 2



PROPERTIES OF FUNCTION 1 COMPARED TO PROPERTIES OF FUNCTION 2

- greater y-intercept
- nonlinear function
- variable rate of change

- lesser y-intercept
- linear function
- constant rate of change

PROPERTIES OF BOTH FUNCTION 1 AND FUNCTION 2

- increasing function

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student identifies *equal rate of change*, *greater rate of change*, or *lesser rate of change* as a property of the first function.	does not understand the definition of *rate of change*	DESCRIBE THE RATE OF CHANGE IN AN ALGEBRAIC FUNCTION
The student misplaces or does not identify *greater y-intercept* or *lesser y-intercept* as properties of the individual functions.	is not able to identify the *y*-intercept in all function representations	EXPLAIN Y-*INTERCEPT*
The student does not identify *nonlinear function* for the first function and/or *linear function* for the second function.	cannot recognize the difference between linear and nonlinear functions	RECOGNIZE *LINEAR FUNCTION*
The student does not identify *increasing function* as a similarity between the functions.	can identify properties of a single function but cannot compare functions in order to identify common properties	COMPARE PROPERTIES OF 2 FUNCTIONS REPRESENTED IN DIFFERENT FORMS

9. What is a common feature of the rate of change of all linear functions?

CORRECT ANSWER

> All linear functions have a constant rate of change.
>
> The rate of change between any two points on a linear function is the same.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student does not relate constant rate of change to linear functions.	does not understand the definition of a linear function	EXPLAIN *LINEAR FUNCTION*