RECOGNIZE AND MEASURE VOLUME WITH UNIT CUBES

5.MD.3,4

CONTENTS

The types of documents contained in the unit are listed below. Throughout the unit, the documents are arranged by lesson.

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RECOGNIZE AND MEASURE VOLUME WITH UNIT CUBES

LEARNING MAP INFORMATION

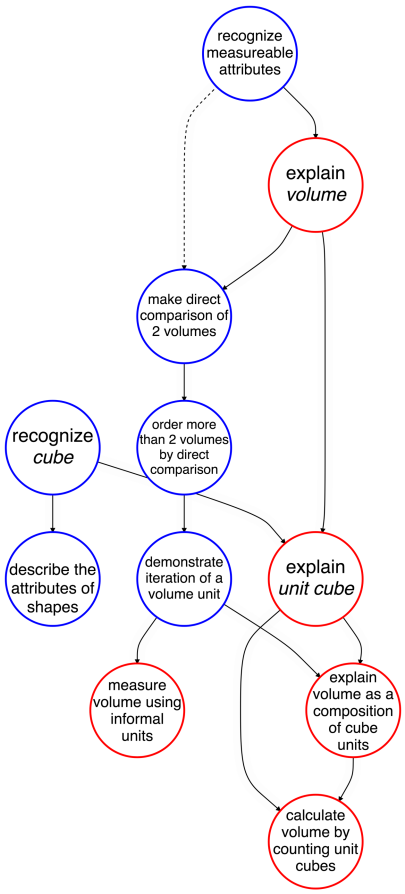
STANDARDS

5.MD.3 Recognize volume as an attribute of solid figures and understand concepts of volume measurement.

5.MD.3a A cube with side length 1 unit, called a "unit cube," is said to have "one cubic units" of volume, and can be used to measure volume.

5.MD.3b A solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units.

5.MD.4 Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft. or non-standard cubic units.



**Learning map model of* 5.MD.3, 4

5.MD.3 Recognize volume as an attribute of solid figures and understand concepts of volume measurement.

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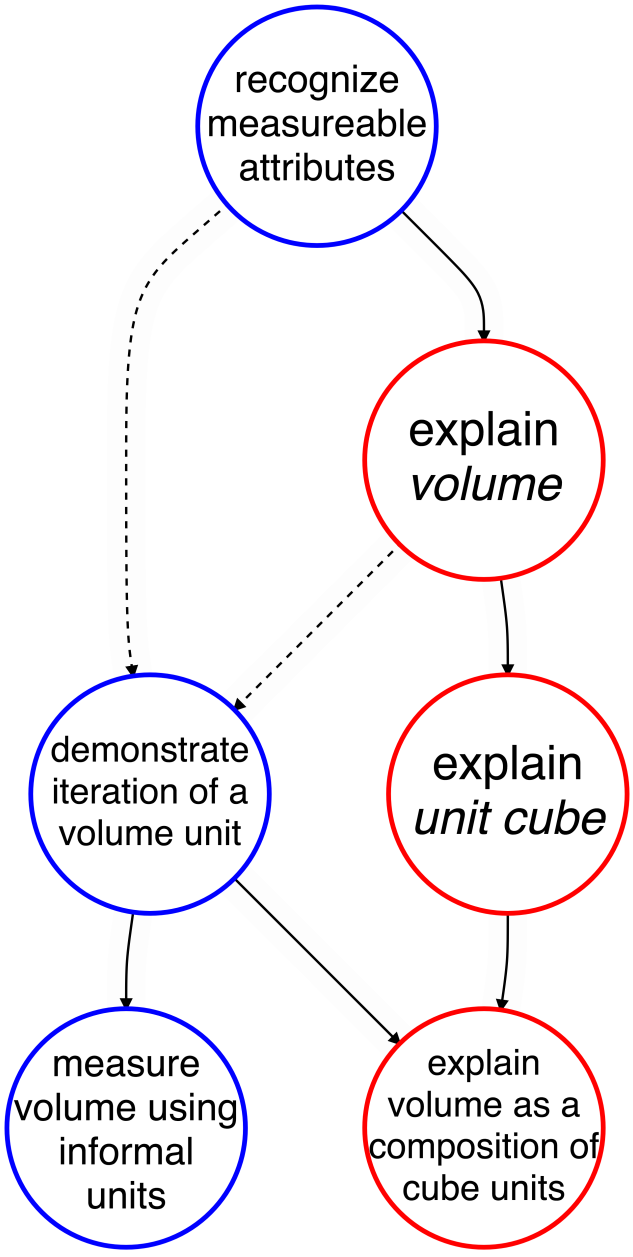
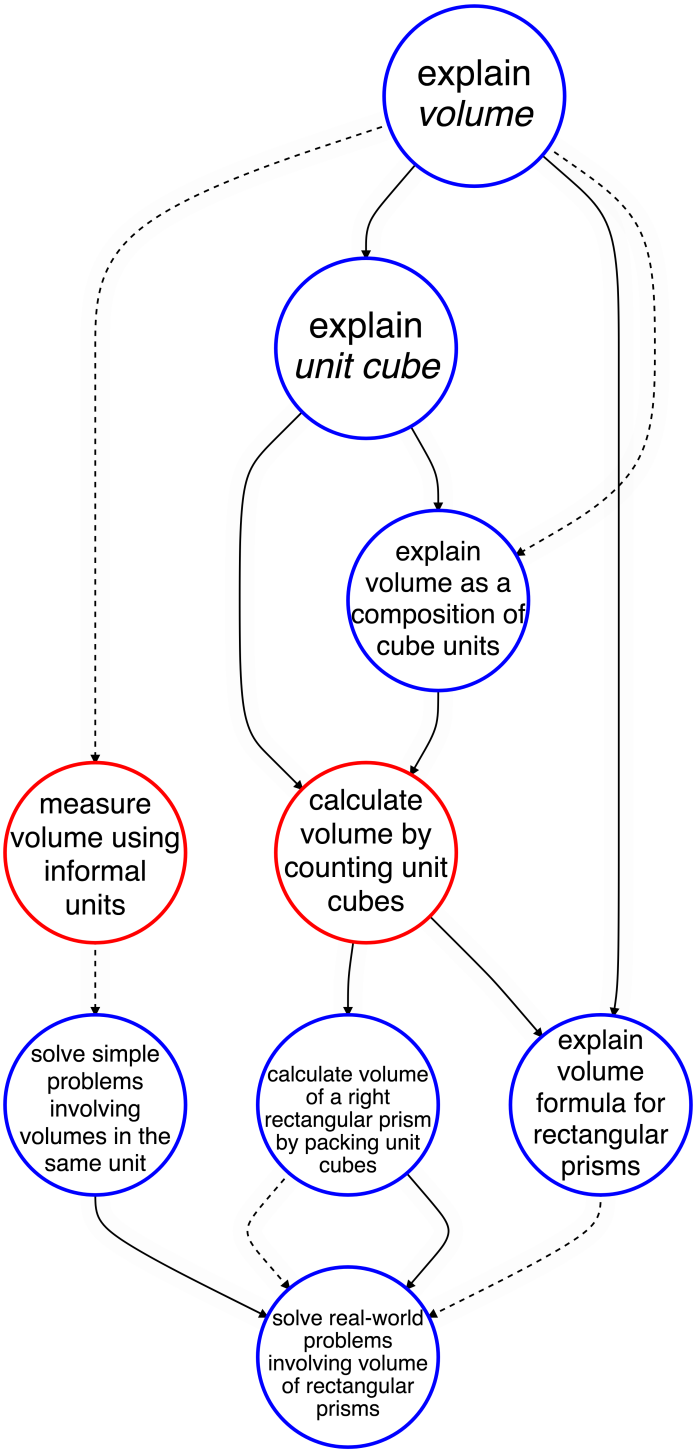


**Learning map model of* 5.MD.3

5.MD.4 Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.



**Learning map model of* 5.MD.4

Node Name	Node Description
DESCRIBE THE ATTRIBUTES OF SHAPES	Describe different attribute values of shapes. For instance, identify how many sides a shape has.
RECOGNIZE *CUBE*	When presented a set of different shapes, select the cube.
RECOGNIZE MEASUREABLE ATTRIBUTES	When shown a certain shape, correctly communicate the name of an attribute that can be measurable.
EXPLAIN *VOLUME*	Make known your understanding that volume is the amount of space enclosed by a shape or an object.
MAKE DIRECT COMPARISON OF 2 VOLUMES	Directly compare two objects in relation to volume and describe the difference using informal full, fuller, etc.
ORDER MORE THAN 2 VOLUMES BY DIRECT COMPARISON	Directly compare more than two objects in relation to volume and order them from least full to fullest or vice versa.
DEMONSTRATE ITERATION OF A VOLUME UNIT	Measure the volume of an object by filling it with multiple copies of a smaller object leaving minimal or no gaps or spaces. For example, use a scoop repeatedly to fill a container with sand.
MEASURE VOLUME USING INFORMAL UNITS	Measure volume of an object using non-formal units of measurement.
EXPLAIN *UNIT CUBE*	Make known your understanding that a unit cube has edge lengths of one unit and has a volume of one cubic unit.
CALCULATE VOLUME BY COUNTING UNIT CUBES	Calculate the volume of an object by counting the number of unit cubes drawn to fill the space inside the object.
EXPLAIN VOLUME AS A COMPOSITION OF CUBE UNITS	Make known your understanding that the volume of a solid figure can be determined by filling the figure with cube units and counting the number of cube units.

ADDITIONAL NODES RELATED TO THIS UNIT OF INSTRUCTION

Node Name	Node Description	Related Node
RECOGNIZE LESS AMOUNT	When presented two or more continuous amounts, select the amount that is least.	Prerequisite of MAKE DIRECT COMPARISON OF 2 VOLUMES
RECOGNIZE MORE AMOUNT	When presented two or more continuous amounts, select the amount that is greatest.	Prerequisite of MAKE DIRECT COMPARISON OF 2 VOLUMES

RECOGNIZE AND MEASURE VOLUME WITH UNIT CUBES

TEACHER NOTES

This unit includes the following documents:

- Learning Map Information
- Instructional Activity (three lessons)
- Instructional Activity Student Handout (for Lessons 1 – 3)
- Instructional Activity Supplement (for Lessons 2 – 3)
- Student Activity
- Student Activity Solution Guide

In this unit, students will explore the concept of volume and unit cubes, then determine the volume of irregular figures and rectangular prisms.

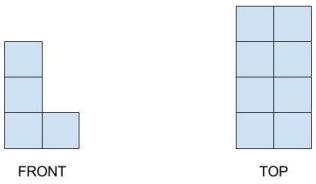
RESEARCH

In order for students to conceptualize and determine volume, it is essential that they understand the difference between two-dimensional figures, specifically squares, and three-dimensional figures, specifically cubes. A common misconception among students who struggle with volume is that they consider an image of a rectangular prism to be a two-dimensional figure (Ben-Haim et al., 1985). According to Battista and Clements (1996), students may have an understanding of the difference between a cube and a square, but they may still see images of cubes as squares and even refer to images of cubes as squares. In a study conducted by Battista and Clements (1996), some students used the terms "cube" and "square" interchangeably when referencing a cube. Student identification of the distinction between square and cube must be established prior to counting unit cubes or calculating volume.

During this unit, students will be responsible for counting the volume of irregular figures, particularly figures that have hidden cubes or cubes that are not visible. Presenting these images and requiring students to determine the volume by building and counting cubes will help students visualize and account for the unseen cube(s). Two of the most common errors identified by Ben-Haim et al. (1985) are students not counting hidden cubes and counting each cube face shown in a diagram, which results in counting single cubes more than once; both errors indicate a student's lack of spatial structuring.

Spatial structuring, as explained by Battista and Clements (1996, p. 282) is the "mental act of constructing an organization or form for an object or set of objects." This includes identifying a cubic unit and recognizing a subset of the established cubic unit (i.e. a layer) which can be repeated to create a whole set (Battista and Clements, 1996, p. 282). Spatial structuring supports a student's ability to perceive images of three-dimensional figures with perspective. Students first view images without a specific perspective, then they view images as an assortment of viewpoints, then finally they coordinate each of the viewpoints. These single viewpoints or perspectives are also referred to as orthogonal views. In an orthogonal view, only one face of the figure is visible (Battista and Clements, 1998). The following are examples of orthogonal views.

AN EXAMPLE



Ben-Haim et al. (1985, p. 390) recommends that "instruction may need to be complemented by introducing concrete experiences with solid objects even in the middle school" because students struggle to transfer what is seen in a two-dimensional image to a three-dimensional figure. While concrete experiences with three-dimensional figures provide important foundational understanding, Battista and Clements argue that it is not enough to increase students' understanding of spatial structuring. Battista and Clements (1998, p. 262) state that "having students merely make boxes and fill them with cubes does not promote nearly as much student reflection because (*a*) opportunities for cognitive conflict arising from discrepancies between predicted and actual answers are greatly curtailed and (*b*) students' attention is focused on physical activity rather than on their own thinking." The emphasis needs to be focused on student thinking if we wish to develop students' mental models and spatial structuring.

Atkins (1999, p. 290) extends the idea of student reflection to classroom discussions, stating that students should "respond to one another's comments instead of having their comments filtered through the teacher." Giving students the opportunity to share and discuss how they arrived at an answer makes the student more thoughtful and reflective. In addition, observing these conversations gives the teacher an understanding of a student's thought process and thinking. Teacher questioning should require students to reconsider their thinking and draw out student misconceptions and errors (Atkins, 1999).

AN EXAMPLE

Ben-Haim et al. (1985) identified four types of misconceptions that lead to student errors in fifth through eighth grades. The following examples reference this figure, which has a volume of 18 cubic units.

COUNTING CUBE FACES SHOW IN THE DIAGRAM

The student counts a volume of 21 cubic units. The student sees the image as a two-dimensional object and does not visualize the unseen cubes. The cubes on an edge that display two or three faces are double or triple counted.

COUNTING CUBE FACES SHOWN IN DIAGRAM AND DOUBLING

The student counts a volume of 42 cubic units. The student still sees the figures as a two-dimensional figure but is aware that there are unseen cubes. The cubes on an edge that display two faces are double counted.

COUNTING THE NUMBER OF CUBES SHOWN IN DIAGRAM

The student counts a volume of 14 cubic units. The student recognizes the figure as three-dimensional since they are counting cubes, however they are unaware that there are unseen cubes.

COUNTING THE NUMBER OF CUBES SHOWN IN DIAGARAM AND DOUBLING

The student counts a volume of 28 cubic units. The student recognizes the figure as three-dimensional since they are counting cubes, and they are aware that there are unseen cubes, but they are incapable of determining the number of unseen cubes, so the student doubles the number of visible cubes.

LEARNING MAP INFORMATION

The learning map section for this sequence of activities begins with an understanding of shapes and their measureable attributes. This leads directly into recognizing and explaining not only cubes and unit cubes but volume as well. After students have an understanding of measurable attributes, their ability to recognize a cube and explain volume allows them to explain the appearance and volume of a unit cube. Once students have a conceptual understanding of volume, they can make comparisons of two volumes and order two or more volumes using either formal or informal units. Students should be able to show that volume is the iteration of a unit (formal or informal) before they can explain volume as the iteration of a unit cube, a

specified formal unit. Lastly, a student should be able to transfer these understandings to determine volume by counting unit cubes.

INSTRUCTIONAL ACTIVITIES

> NOTE: It is recommended that prior to LESSON 1 students engage in a discussion of two-dimensional and three-dimensional figures. Depending on students' level of understanding, they may need to research the two types of shapes, create a graphic describing the similarities and differences between 2-D and 3-D shapes, compare and contrast area and volume, and define what is meant by "dimension".

The activities in this unit are designed to build and reinforce students' spatial structuring to support the coordination of transferring two-dimensional images into three-dimensional figures. In LESSON 1, students explore the concept of volume by observing and filling rectangular prisms with informal units. LESSON 2 builds on LESSON 1 by comparing the use of informal units with that of formal units, specifically the unit cube. Students will identify the differences between cubes and squares, focusing on the attribute of dimensionality. Students will also increase their spatial structuring by determining the volume of irregular figures which contain cubes that are not visible in the two-dimensional representation. LESSON 3 requires students to apply their understandings of volume, unit cubes, and spatial structuring to determine the volume of rectangular prisms. Students are required to hypothesize the number of cubes in an image of a rectangular prism before they construct the rectangular prism using concrete materials. Students will then discuss their thoughts and findings in small groups. It is important to understand that the intent of these lessons is to establish students' understanding of volume before they are exposed to the standard formula, which has no meaning to students unless they have had experiences similar to those described in this unit.

REFERENCES

Atkins, Sandra L. (1999). Listening to Students: The Power of Mathematical Conversations. *Teaching Children Mathematics*. 5 (5), 289-295.

Battista, Michael and Clements, Douglas H. (1996). Students' Understanding of Three-Dimensional Rectangular Arrays of Cubes. *Journal for Research in Mathematics Education*. 27 (3), 258-292.

Battista, Michael and Clements, Douglas H. (1998). Finding the Number of Cubes in Rectangular Cube Buildings. *Teaching Children Mathematics*. 4 (5), 258-264.

Ben-Haim, D., Lappan, G., & Houang, R. T. (1985). Visualizing Rectangular Solids Made of Small Cubes: Analyzing and Effecting Students' Performance. *Educational Studies in Mathematics*. 16 (4), 389-409.

Bruni, James V. and Silverman, Helene. (1974). Using Cubes. *The Arithmetic Teacher*. 21 (8), 654-658.

RECOGNIZE AND MEASURE VOLUME WITH UNIT CUBES

OVERVIEW OF INSTRUCTIONAL ACTIVITIES

Lesson	Learning Goal	Nodes Addressed
Lesson 1	Students will determine the meaning of volume and measure volume using informal units.	▶ EXPLAIN *VOLUME* ▶ DEMONSTRATE ITERATION OF A VOLUME UNIT ▶ MEASURE VOLUME USING INFORMAL UNITS ▶ MAKE COMPARISON OF TWO VOLUMES
Lesson 2	Students will explain the meaning of volume and explain a unit cube.	▶ RECOGNIZE *CUBE* ▶ DESCRIBE THE ATTRIBUTES OF SHAPES ▶ EXPLAIN *VOLUME* ▶ EXPLAIN *UNIT CUBE* ▶ EXPLAIN VOLUME AS A COMPOSITION OF CUBE UNITS ▶ MAKE DIRECT COMPARISON OF 2 VOLUMES ▶ CALCULATE VOLUME BY COUNTING UNIT CUBES
Lesson 3	Students will determine the volume of rectangular prisms by counting unit cubes.	▶ EXPLAIN *VOLUME* ▶ EXPLAIN VOLUME AS A COMPOSITION OF CUBE UNITS ▶ EXPLAIN *UNIT CUBE* ▶ CALCULATE VOLUME BY COUNTING UNIT CUBES

RECOGNIZE AND MEASURE VOLUME WITH UNIT CUBES

INSTRUCTIONAL ACTIVITY

Lesson 1

LEARNING GOAL

Students will determine the meaning of volume and measure volume using informal units.

PRIMARY ACTIVITY

Students will fill objects with various items to explore volume concepts and compare and contrast the amount two different rectangular prisms can hold.

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

- Volume
- Rectangular prism
- Compare
- Measure
- Two-dimensional
- Three-dimensional

MATERIALS

- Two identical clear, plastic boxes for the teacher (e.g., plastic kitchen storage containers)
- A collection of clear, plastic boxes of different sizes (Recommend two boxes for every two students.)
- A collection of informal measuring tools of different sizes, such as plastic cups, scoops, small containers, spoons (Recommend one or two tools for every two students.)
- Formal measuring tools such as measuring cups or measuring spoons (Recommend one tool for every two students.)
- Candies or other small objects such as cereal, marbles, buttons, etc. (Recommend enough to fill one clear, plastic container.)

- Two different materials or objects that have different particle sizes (Recommend enough for every two students to completely fill a given box.)
- INSTRUCTIONAL ACTIVITY STUDENT HANDOUT

IMPLEMENTATION

Show students two identical clear boxes. **Explain** that one box will be filled with sand and the other box will be filled with candies (or a similar item).

Ask students, "How can we determine the amount of sand or candies required to fill each box?"

Model measuring the volume of the two boxes. **Fill** one of the boxes with sand using a scoop or plastic cup. **Fill** the second box with candies using the same scoop or plastic cup. Be sure to count the number of scoops as the sand or candies are added to the box.

Emphasize that, regardless of what the box is being filled with, the number of scoops required will remain the same, since the size of the scoop is the same and the boxes are the same size and therefore have the same volume.

Discuss the meaning of *volume* (the amount of space a three-dimensional figure occupies). Students should understand that the volume of an object is measured by reporting the amount that is needed to fill the object without gaps or space. Volume can be measured using different units of measurement, but the space the object occupies does not change. **Refer** back to the clear boxes with sand and candies.

GUIDING QUESTIONS

Elicit student thinking:

- What do you think of when you hear the term "box"?
- What mathematical term could be used to describe an object like a box?
- What do you think is the purpose of a box?
- Why do you think boxes come in different shapes and sizes?

Determine if the student is ready to EXPLAIN *VOLUME*:

- Is this box [reference one of the clear boxes] two-dimensional or three-dimensional? How do you know?
- How would you measure the amount a box or a container can hold?
- How do you know if a container is big enough to hold a given amount?

Determine if the student can DEMONSTRATE ITERATION OF A VOLUME UNIT:

- What was used to measure the volume of each box?
- What do you notice about the way the candies fill the box?
- How is the way the sand fills the box different than the way the candies fill the box? Why do you think they are different?
- If you were to count the number of grains of sand compared to the number of candies, would there be more grains of sand, more candies, or the same amount for both? How do you know?

Place students in small groups of two or three. **Distribute** two clear, plastic boxes of different sizes to each group of students. **Instruct** students to label their boxes with their group name as well as "Box 1" and "Box 2".

Explain that their task is to find the volumes of their boxes using the two provided materials.

Require students to predict which of their boxes will have a greater volume. Students should write their prediction and reasoning on Part 1 of the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

Focus students' attention on the shape and size of the boxes. **Discuss** the relationship between the size and shape of each box. Students should notice that larger dimensions of a box lead to a greater volume. However, boxes of different dimensions (shape) can also share the same volume measurement.

GUIDING QUESTIONS

Elicit student thinking:

- How do these boxes compare to each other?
- What is different about these boxes?
- What is the same about these boxes?

Determine if the student can EXPLAIN *VOLUME*:

- Which box looks larger? What can you hypothesize about the volume of the larger box?
- Which of your boxes looks smaller? What can you hypothesize about the volume of the smaller box?
- Will each box hold the same amount regardless of what you are filling your box with? Explain your answer.

Determine if the student can MEASURE VOLUME USING INFORMAL UNITS:

- What are you using to measure volume?
- Using a scoop and sand, how would you determine how much space is inside of the box?
- What other items could you use to measure the amount of space inside the box? How would you measure the space inside the box with these items?

Give each group a different informal measuring tool.

Require students to fill their boxes using the informal measuring tools and the smaller material (one that will leave little or no gaps between objects when the box is full) to determine which box has the greater volume. Students will record their findings on Part 2 of the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

GUIDING QUESTIONS

Elicit student thinking:

- How can you tell how much a box can hold?
- What tool(s) could be used to determine volume?

Determine if the student can DEMONSTRATE ITERATION OF A VOLUME UNIT:

- What are you using to fill your box?
- Does using a different size scoop affect your measurement? If so, how?
- What if you used a different size scoop each time you added sand (or another material) to the box? Would that affect your volume measurement?
- Would the volume be the same if you filled the box with rocks instead of sand? How do you know?

Determine if the student can MAKE DIRECT COMPARISON OF TWO VOLUMES:

- Which box has more sand in it? How do you know?
- Which box has less sand in it? How do you know?
- How does the amount of sand in your box compare to the other groups? Why do you think it is different (or the same)?

Lead students in a discussion about the volume of the boxes.

Ask the students if an accurate comparison can be made across groups who used different measuring tools. Students should realize that unless all groups are using the same size measuring tool, then an accurate comparison cannot be made.

Guide students to think about how will they determine a precise, consistent measurement for volume comparison. Students should determine the need for a standardized measurement tool.

Introduce the formal measuring tools (either a measuring cup or measuring spoons). **Demonstrate** how to determine the volume of one of the original boxes using a measuring cup or measuring spoon, counting each measurement as it is added to the box.

Require students to guess how many cups (or other formal form of measurement chosen) they think their box will hold, then record their predictions on Part 3 of the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

Instruct students to use the chosen formal measuring tool to fill their boxes with one of the provided materials. Students should record the actual amount on Part 3 of the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

Instruct students to empty their boxes and predict the amount of the other provided material the box will hold. Students should then record their predictions on Part 3 of the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

Require students to use the same formal measuring tool to measure the provided material as they fill their box. **Remind** students to record the actual amount on Part 3 of the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

Record the volumes of both boxes from each group on the board and **require** students to work together to order all groups' boxes by their approximate volume from least to greatest. **Emphasize** again how important a standardized formal unit is when comparing and ordering different volumes.

GUIDING QUESTIONS

Determine if the student can DEMONSTRATE ITERATION OF A VOLUME UNIT:

- Do all the objects fill the space in the boxes the same way?
- How do you know when to stop adding the material (sand, dirt, rocks) to the box?
- Does the volume measurement change based on the object used to fill the box?

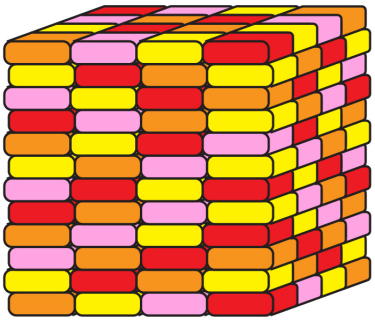
Determine if the student can EXPLAIN *VOLUME*:

- Is there a relationship between the size of a box and the volume of the box?
- Does every three-dimensional figure have a single volume measurement?

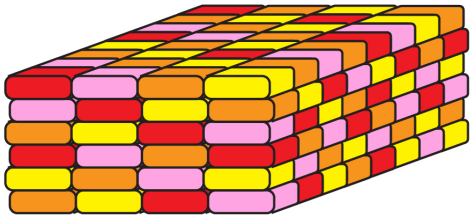
Discuss how the shape of an object impacts the number of that object that fits in each box. Students can conclude that the number of items a box can hold depends on the size and shape of the object that is used to fill the box. For example, because a grain of sand is smaller than a rock, more grains of sand are required than rocks to fill the same size box. However, the volume, or the space enclosed by the box, remains the same regardless of what it is filled with.

Discuss which objects filled the boxes better. It is important for students to understand that to best represent the volume of an object, it should be filled without spaces or gaps. Items like sand are mostly uniform in size and small, so it takes the shape of the box easily with little unfilled space. Liquids such as water leave no gaps or space and fill a container completely. Items such as rocks or candies are irregular in shape and size and therefore leave the most unfilled space.

Present two rectangular prisms made out of cubes or candies that have a cube or rectangular prism shape. One prism should be $4 \times 4 \times 12$, and the other prism should be $4 \times 8 \times 6$; both prisms have a volume of 192 candies.



4 × 4 × 12



4 × 8 × 6

Ask students:

- "If you could choose one prism of candies to keep, which one would you choose? Why?"
- "Which prism do you think has the greater volume? Why?"
- "How can we determine the volume of the prisms?"

Disassemble the rectangular prisms, counting the candies layer by layer as you do. Do not make known any correlation between counting the area of one layer and then multiplying by the number of layers. Students may discover this strategy on their own, but it is not necessary to encourage it.

Explain that the two rectangular prisms have the same volume but different dimensions. For example, one was tall and narrow, whereas the other was short and wide.

Provide each student with 12 candies or cubes.

Require each student to create at least three different rectangular prisms using all 12 candies or cubes. Students should record each structure by drawing a representation on Part 4 of the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

Demonstrate the same concept with two containers that have the same volume but different dimensions. **Fill** one container with water, and then pour the water into the other container to show the volumes are the same.

As an extension activity, **show** students two boxes with different dimensions. **Ask** students which box they think would have a greater volume. **Direct** students first to investigate to confirm or deny their predictions and then to write a summary of their findings.

At the end of the activity, students should complete Part 5 of the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

Name_____

Recognize and Measure Volume with Unit Cubes

Lesson 1

Part 1:

My prediction is that Box _____ will have a greater volume because:

Part 2:

Measuring tool used: _____

	Box 1	Box 2
Number of scoops		

Which box has the greater volume? Explain your reasoning.

Name_____

Part 3:

Measuring tool used: _____

Material				
	Box 1	Box 2	Box 1	Box 2
Predicted Amount				
Actual Amount				

Part 4:

Prism 1:

Prism 2:

Prism 3:

Part 5:

1) What does it mean for an object to have volume?

Name_____

2) How does the volume of Box 1 compare to the volume of Box 2? Explain your reasoning.

3) Explain why smaller, more uniform items will fill a container more fully than items that are more irregular in shape.

4) What is one new thing you learned today?

5) What is one thing you still have a question about or do not understand?

RECOGNIZE AND MEASURE VOLUME WITH UNIT CUBES

INSTRUCTIONAL ACTIVITY

Lesson 2

LEARNING GOAL

Students will explain the meaning of volume and explain a unit cube.

PRIMARY ACTIVITY

Students will identify a unit cube and measure the volume of irregular figures using unit cubes.

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

- Unit cube
- Volume
- Rectangular prism
- Cube
- Square

MATERIALS

- A collection of cubes of different sizes (Recommend one set for the teacher.)
- Square (Recommend one for the teacher.)
- Two clear boxes (Recommend using the identical boxes from the beginning of LESSON 1.)
- INSTRUCTIONAL ACTIVITY STUDENT HANDOUT
- Cubes – sets of the cubes of the same size (Recommend 15 cubes for every student.)
- INSTRUCTIONAL ACTIVITY SUPPLEMENT (Recommend one copy for every two students.)

IMPLEMENTATION

Display a square.

Ask students the following set of questions:

- "What is the name of this figure?" [Write "Square" on the board.]
- "What can you tell me about a square? How do you know this is a square?" [Write the correct characteristics mentioned on the board underneath the word "Square."]

Display a cube.

Ask students the following set of questions:

- "What is the name of this figure?" [Write "Cube" on the board.]
- "What can you tell me about a cube? How do you know this is a cube?" [Write the correct characteristics mentioned on the board underneath the word "Cube."]
- "How are a square and a cube the same?"
- "How are a square and a cube different?" (Focus on two-dimensional versus three-dimensional.)

Present an empty, clear box, preferably one used in LESSON 1.

Ask students, "What would you use to determine the volume of this box, the square or the cube? Why?" (*Cube, because they are three-dimensional.*)

Fill the clear box with cubes, counting the number of cubes as you add them to the box. If the cubes do not fit exactly, explain that instead of using a fraction of a cube, you will be estimating the volume. **Give** the volume of the box as (number of cubes) cubes.

Present the second clear box, filled with rocks (or another irregularly shaped/sized object that would leave unfilled space) from LESSON 1.

Ask students the following set of questions:

- "What do you notice about how the rocks fill the box?"
- "What do you notice about how the cubes fill the box?"
- "How are the two the same?"
- "How are the two different?"
- "Which object would you use to measure the volume of the clear box? Why?"

Display the collection of different size cubes.

Ask students the following set of questions:

- "How are these figures the same?"

- "How are they different?"
- "Do you think it is important to use the same size cubes to determine volume? Why or why not?"

Emphasize that when determining volume, it is important that the cubes being used are all the exact same size.

Clarify that it does not matter what size cubes are being used, but that all cubes used must be the same size.

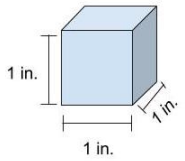
Explain that this is important because if different-size cubes were used, there would be no formal unit of measurement to report the volume. You could not provide a formally measured volume without a specific formal unit inches, feet, millimeters, centimeters, etc.

Hand out the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

Ask students, "What do you know about a square that would help determine the side lengths of a cube?" (*All the side lengths of a square are equivalent, a cube is a three-dimensional figure composed of six congruent square faces, and therefore all the side lengths of a cube are equivalent as well.*)

Explain the definition of a unit cube and have students write the definition and draw a representation on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

A unit cube is a cube with side lengths of one unit. For example, one millimeter, one centimeter, one inch, one foot, etc. The following is an example of a unit cube with a volume of one cubic inch.



GUIDING QUESTIONS

Elicit student thinking:

- How would you describe this figure?
- What do you notice about this figure?

Determine if the student can RECOGNIZE *CUBE*:

- What is a cube?
- What is an example of a cube?
- [Pointing to a cube] How do you know this is a cube?
- [Pointing to a figure/shape other than a cube] How do you know this is not a cube?

Determine if the student can DESCRIBE THE ATTRIBUTES OF SHAPES:

- [Present a rectangular prism] What makes a cube different from a rectangular prism?
- How are a cube and a rectangular prism the same?

Determine if the student can EXPLAIN *VOLUME*:

- What is volume?
- How do you know if a figure has volume?
- What is one way to find the volume of the clear box?

Bring students' attention back to the clear box filled with cubes. **Ask** students, "Does anyone remember how many cubes were needed to fill this box?" [Write the number on the board.]

Explain that if the volume of an object does not have a specified unit, the reported volume is relative. For example, the same box could have a volume of 30 or a volume of 10, depending on what is used to measure volume, but those values do not mean anything without a label of measurement.

[Pick up one of the cubes from the clear box.] **State** the unit of the cube (inches, centimeters, etc.) and **explain** that because the box is filled with those cubes, the label is cubic (the unit of the cube).

Identify (or **ask** students for) the relationship between the term *cubic* (as in cubic units) and a *cube*. (*Cubes are used to measure volume since cubes are three-dimensional and volume is the measure of the amount of space a three-dimensional object occupies, and each of the dimensions of a cube are the same measurement, therefore cubic units is the unit used to measure volume.*)

Bring students' attention back to the clear box filled with cubes and the number on the board that represents the number of cubes in the clear box. **Ask** students, "What is the label for the volume of this clear box?" (*cubic followed by the unit of the cube, inches or centimeters*)

[Write the unit after the number on the board.] **Make** note that if no formal unit measurement is given, then volume is labeled as cubic units, because the unit measure is unknown.

Direct students to write on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT how the volume of an object is labeled and why.

Hand out the sets of cubes (one set per student). If there are not enough cubes, student pairs may share the cube sets.

Direct students to look at the image on Question 1 of the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT and write what they think the volume of the figure is.

Require students to create the figure using cubes. **Observe** how students attempt to build the figure, especially the part of the figure that has a "hidden" block.

Ask students for the actual volume of the figure (five cubic units), which may or may not be what students wrote as the volume before building the figure.

Require students to explain how they determined the volume and how they knew there were five cubes even though you can only see four cubes.

Bring students' attention back to the clear box filled with cubes. **Explain** that even though you cannot see all of the cubes used to determine the volume, they are still there.

Make **explicit** that when you count cubes on paper, you do not count every face of the cubes you see. Refer to a cube in the clear box, and ask students, "When you counted this cube as part of the volume did you count it as one or as six (the number of faces)?" (*one*)

Require students to answer Questions 2 – 5 on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT, either independently or in pairs. For each question, students should hypothesize the volume of the pictured figure before they build the figure using blocks to determine the volume. **Ensure** that students use a correct label for each question.

Use the guiding questions to scaffold student learning.

GUIDING QUESTIONS

Determine if the student can EXPLAIN *UNIT CUBE*:

- Show me a unit cube. How do you know this is a unit cube?
- [Refer to Question 2 or 5 on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT] How large is the unit cube? How do you know?
- [Present two different-size cubes] Could these both be unit cubes? Why or why not?
- Why do we need to understand the size of the unit cube?

Determine if the student can EXPLAIN VOLUME AS A COMPOSITION OF CUBE UNITS:

- How can you determine volume using cubes?
- If you cannot see the cubes in the image/figure, how do you know whether or not they are there?
- [Refer to any of the Questions 1 – 5 on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT] How did you determine (student's answer) to be the volume? How do you know what unit measurement label to use?

Pass out a set of task cards from the INSTRUCTIONAL ACTIVITY SUPPLEMENT to pairs of students.

Direct students in pairs to use the cube sets to build the irregular figures on the task cards and determine the volume for each irregular figure.

GUIDING QUESTIONS

Determine if the student can MAKE DIRECT COMPARISON OF 2 VOLUMES:

- [Refer to two task cards with different irregular figures] Which figure has a larger volume? How do you know?
- [Refer to two task cards with different irregular figures] Which figure has a smaller volume? How do you know?

Determine if the student can CALCULATE VOLUME BY COUNTING UNIT CUBES:

- [Refer to a task card/student's model] What is the volume of this figure? How do you know?
- What is the volume of one cube? What is the volume of a figure with five cubes?

Students should be required to build each irregular figure from the image on the card using unit cubes to determine the volume.

At the end of the activity, have students turn to a partner and discuss the following questions:

- What is a unit cube?
- What is volume?
- How can you determine the volume of an irregular figure?
- What label is used when you determine the volume of a figure?

Name_____

Recognize and Measure Volume with Unit Cubes

Lesson 2

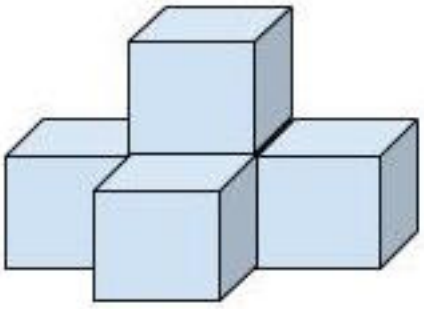
A unit cube is _____.

My drawing of a unit cube:

The volume of an object is labeled as _____, because

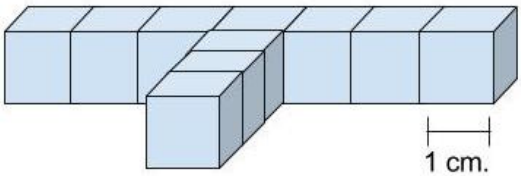
Name_____

1. I think the volume is _____.



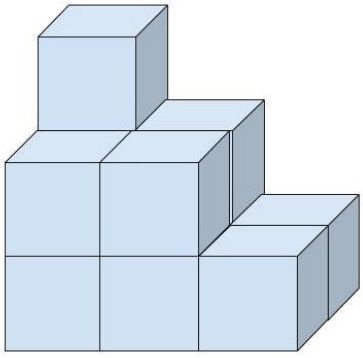
The actual volume is _____.

2. I think the volume is _____.



The actual volume is _____.

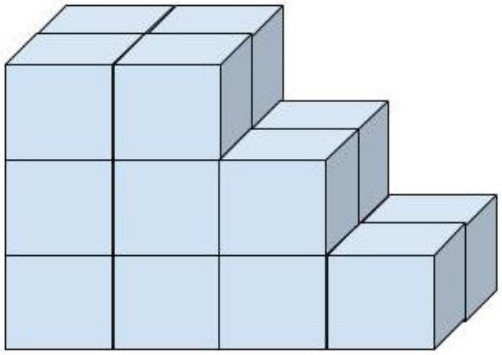
3. I think the volume is _____.



Name_____

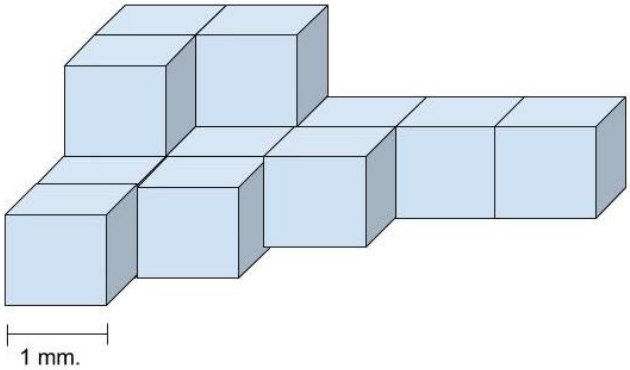
The actual volume is _____.

4. I think the volume is _____.



The actual volume is _____.

5. I think the volume is _____.



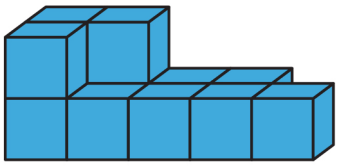
The actual volume is _____.

Recognize and Measure Volume with Unit Cubes

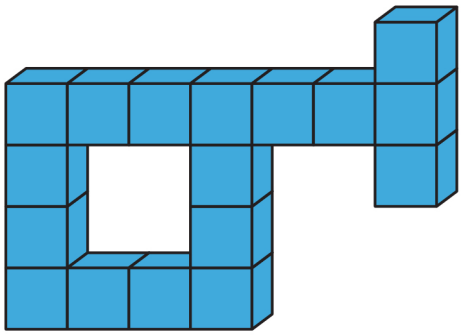
INSTRUCTIONAL ACTIVITY SUPPLEMENT

Lesson 2

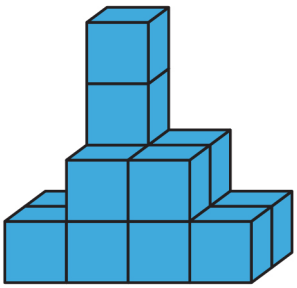
DETERMINE THE VOLUME OF THE FIGURE.



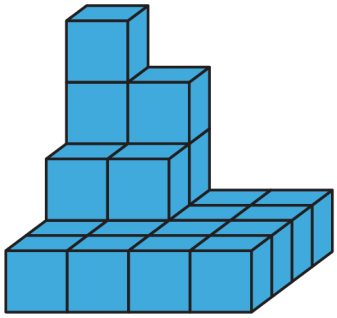
DETERMINE THE VOLUME OF THE FIGURE.



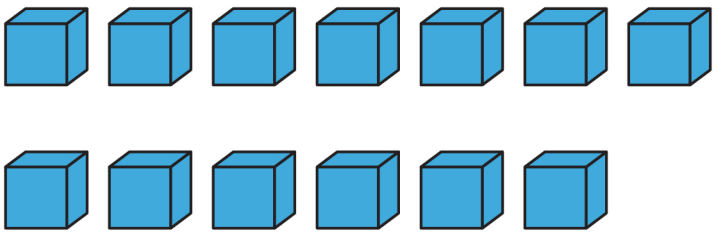
DETERMINE THE VOLUME OF THE FIGURE.



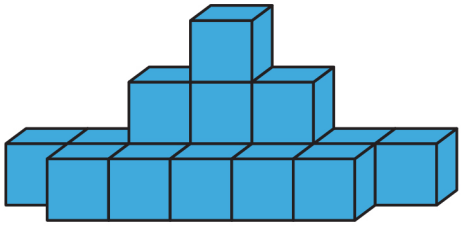
DETERMINE THE VOLUME OF THE FIGURE.



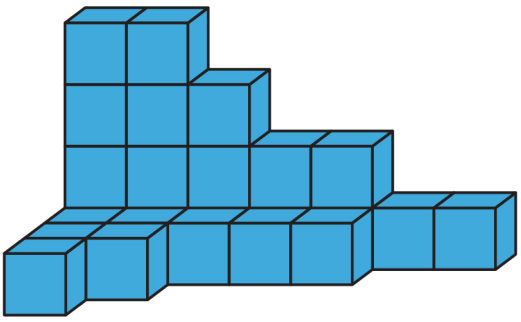
CREATE A FIGURE WITH A VOLUME OF 13 CUBIC UNITS.



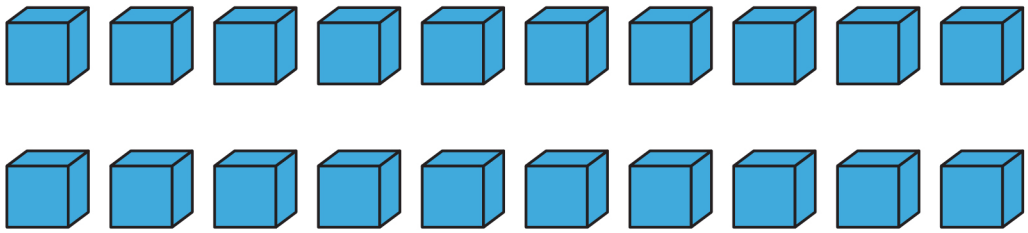
DETERMINE THE VOLUME OF THE FIGURE.



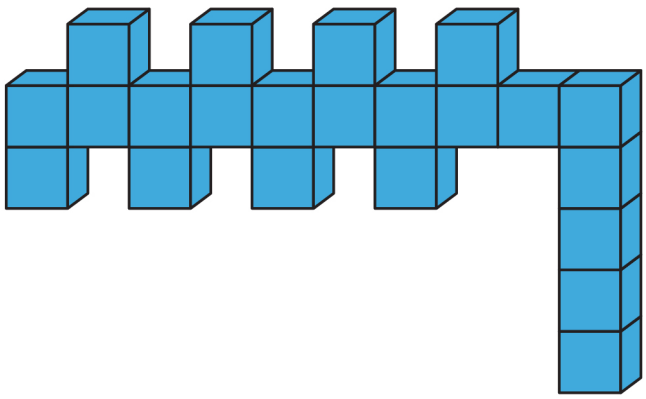
DETERMINE THE VOLUME OF THE FIGURE.



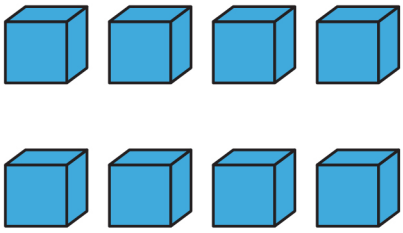
CREATE A FIGURE THAT HAS A VOLUME OF 20 CUBIC UNITS.



DETERMINE THE VOLUME OF THE FIGURE.



CREATE A FIGURE THAT HAS A VOLUME OF EIGHT CUBIC UNITS.



RECOGNIZE AND MEASURE VOLUME WITH UNIT CUBES

INSTRUCTIONAL ACTIVITY

Lesson 3

LEARNING GOAL

Students will determine the volume of rectangular prisms by counting unit cubes.

PRIMARY ACTIVITY

Students will use unit cubes to build and count the volume of rectangular prisms.

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

- Unit cube
- Volume
- Rectangular prism
- Cube
- Square

MATERIALS

- A collection of boxes of different sizes (Recommend one box for every three or four students.)
- Cubes – sets of cubes of the same size (Recommend 25 cubes for every student.)
- Square or rectangular candy assembled into a rectangular prism held together with clear plastic wrap (Recommend two prisms: one $4 \times 4 \times 12$ prism and one $4 \times 8 \times 6$ prism.)
- INSTRUCTIONAL ACTIVITY STUDENT HANDOUT
- INSTRUCTIONAL ACTIVITY SUPPLEMENT

IMPLEMENTATION

Distribute the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT. Display several rectangular prisms of different sizes that demonstrate a variety of dimensions (e.g., shoe box, tissue box, cereal box, model from a three-dimensional set, box for a board game, etc.).

> NOTE: For boxes, ask parents for donations, check the school recycle bins, ask fellow teachers for discarded boxes, check with local business for unused personal size pizza boxes, or discarded shoe boxes of different sizes. Make sure the boxes are not too large; students should be able to fill the boxes completely with unit cubes.

Require students to brainstorm characteristics of rectangular prisms on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT. After two or three minutes, have students share characteristics they brainstormed.

Write on the board, "Characteristics of Rectangular Prisms". Underneath, **write** the correct characteristics students share.

Correct any misconceptions that students share, or present common misconceptions students may have as a "student from last year…" Characteristics should include: three-dimensional; faces are rectangles or squares; bases are rectangles or squares; has six faces. **Instruct** students to write the list of characteristics on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

Ask students, "How would you determine the volume of this rectangular prism?" [Pick up one of the boxes you have on display.] "What could you use to help you determine the volume?" (Fill it with unit cubes.)

Distribute the boxes so that groups of three or four students each have a box. **Provide** each student with a set of the same-size cubes.

Require students to determine the volume of their box using the unit cubes.

Note whether the box has been filled to the top with cubes for each group. If not, **remind** students that the box should be completely full to determine the volume. (Do not consider spaces that require a fraction of a cubic unit as being empty, but **discuss** with students that because we cannot fill the figure exactly, the volume they determine will be an approximate volume.)

Require students to record a description of the box and the volume on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT. Once they are finished, students should return the box to you and acquire a second box.

Students should repeat the process until they have determined the volume of three different boxes.

GUIDING QUESTIONS

Elicit student thinking:

- What everyday objects are rectangular prisms?
- Why would you need to know the volume of an object?

Determine if the student can EXPLAIN *VOLUME*:

- What is volume?
- What are you doing with unit cubes?

Determine if the student can EXPLAIN VOLUME AS A COMPOSITION OF CUBE UNITS:

- How many cubes were required to fill the box?
- Do you need to fill the box entirely to determine the volume?
- How does the number of cubes required to fill the box relate to the volume of the box?

Require students to discuss the volumes of the different boxes with others who were not in their group. Have each group pair up with another group and explain the volume for the boxes they currently have. After five minutes, **require** students to return to their seats and **share** the volume of each of the boxes. **Address** any differences in the volumes. (You can label the different boxes with the volumes and display them in the room.) To aid discussion, use questioning such as the following set of questions:

- Who measured the (type of box) box?
- What did you determine the volume of the box to be?
- Did any other groups get a different volume? (If groups have different volumes for the same box, question further to determine why there were different volumes (e.g., a group's misconception, error, or different size cubes).

Group students into pairs. Each student should still have their set of unit cubes. Using both sets of unit cubes, **require** students to complete the table on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT using the task cards in the INSTRUCTIONAL ACTIVITY SUPPLEMENT. Some figures require more cubes than students will have. Some students will be able to determine the volume of these figures without having

all the cubes, whereas others will need the necessary number of cubes to build the entire figure. To scaffold these problems for students, **provide** them with extra cubes.

GUIDING QUESTIONS

Determine if the student can EXPLAIN *UNIT CUBE*:

- What are you using to measure volume?
- If there is no formal unit, such as inches or feet, how can you identify the unit?
- How do you know you are measuring with a unit cube and not a rectangular prism?

Determine if the student can CALCULATE VOLUME BY COUNTING UNIT CUBES:

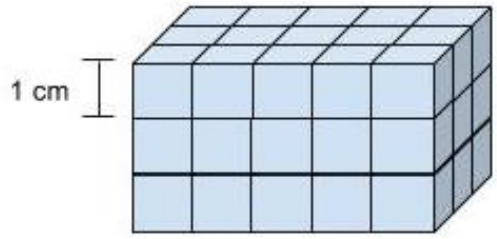
- How many cubes were required to create Figure D? How do you know?
- How do you know there are cubes in the figure if you cannot see them?
- How can you calculate the volume if you do not have enough cubes? Explain.
- [Point to a task card.] Can you make a different rectangular prism that has the same volume?

Solutions to the Task Cards from the Lesson 3 INSTRUCTIONAL ACTIVITY SUPPLEMENT:

- Figure A: 48 cubic inches ($in.^3$)
- Figure B: 27 cubic inches ($in.^3$)
- Figure C: 20 cubic inches ($in.^3$)
- Figure D: 30 cubic meters ($m.^3$)
- Figure E: 24 cubic feet ($ft.^3$)
- Figure F: 24 cubic units ($units^3$)
- Figure G: 48 cubic units ($units^3$)
- Figure H: 12 cubic units ($units^3$)
- Figure I: 160 cubic meters ($m.^3$)
- Figure J: 63 cubic units ($units^3$)

NOTE: Figures I and J require more cubes than students will have in groups of two. These task cards may be used with extra cubes or for students who want an extra challenge.

At the end of the activity, teachers should display the following rectangular prism and require students to write the volume on a sticky note, which should then be stuck to a wall or door for teacher collection.



Name_____

Recognize and Measure Volume with Unit Cubes

Lesson 3

Brainstorm characteristics of rectangular prisms.

Characteristics of Rectangular Prisms:

Box 1: _____

Volume: _____ cubic units

Box 2: _____

Volume: _____ cubic units

Box 3: _____

Volume: _____ cubic units

Name_____

Rectangular Prism 1:

Rectangular Prism 2:

Rectangular Prism 3:

Name_____

Use the task cards to complete the table. Use the figure label (A, B, C, etc.) to identify the task card in the "Figure" column. Explain how you determined the volume of the rectangular prism.

FIGURE	VOLUME	EXPLANATION

Name_____

Recognize and Measure Volume with Unit Cubes

INSTRUCTIONAL ACTIVITY SUPPLEMENT

Lesson 3

FIGURE A

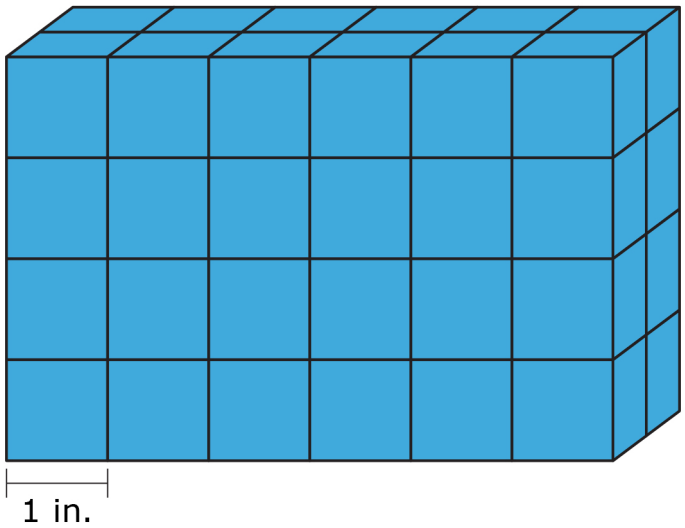


FIGURE B

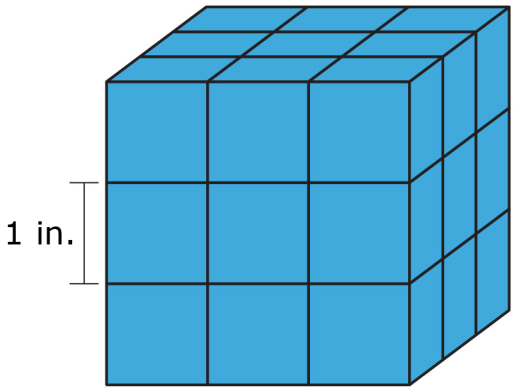


FIGURE C

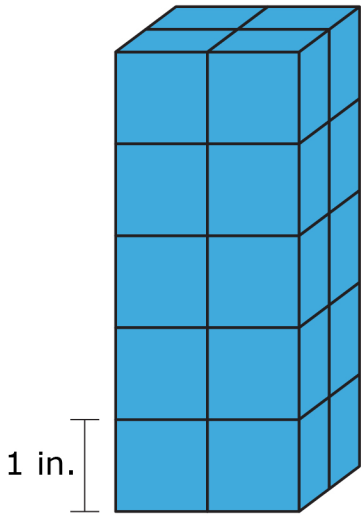


FIGURE D

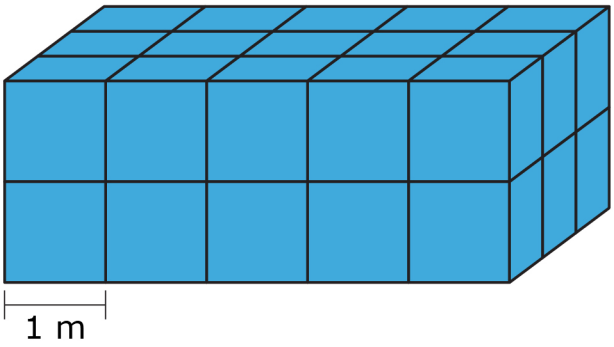


FIGURE E

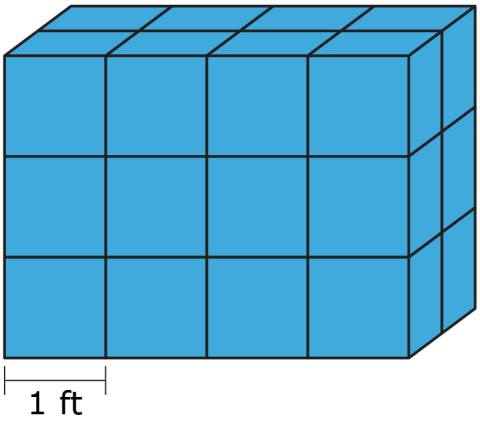


FIGURE F

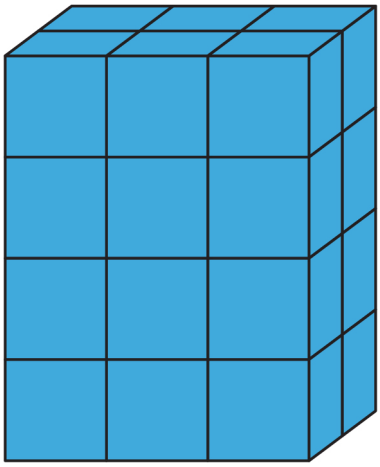


FIGURE G

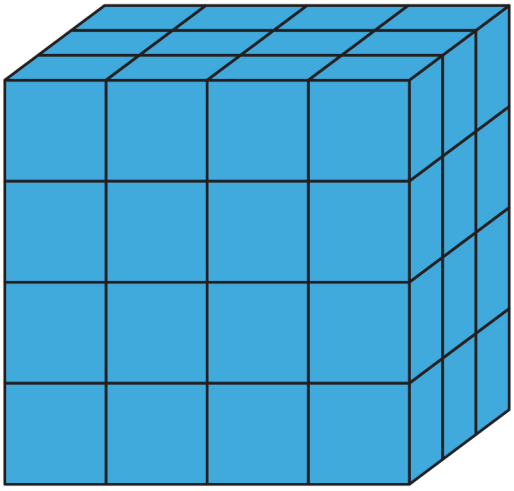


FIGURE H

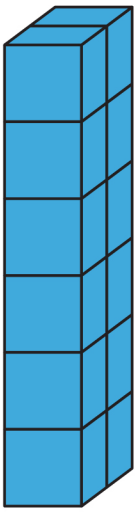


FIGURE I

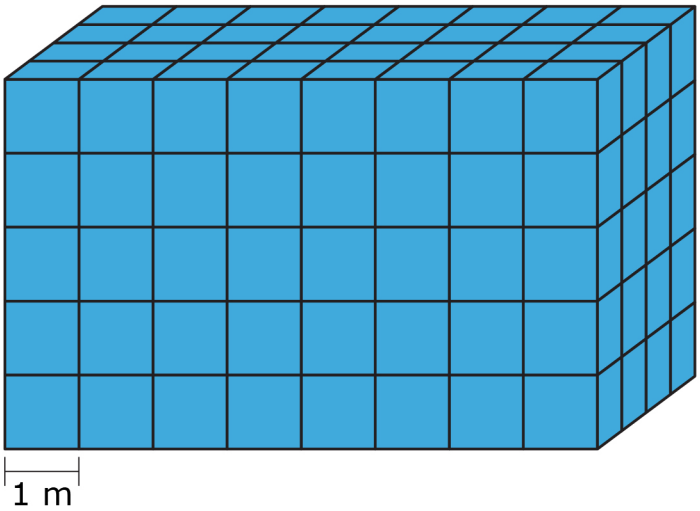
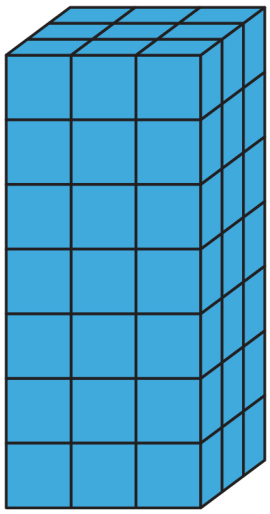


FIGURE J



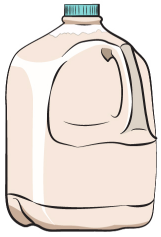
Name_____

Recognize and Measure Volume with Unit Cubes

Lessons 1 – 4

1. Order the containers from least to greatest according to their volume.

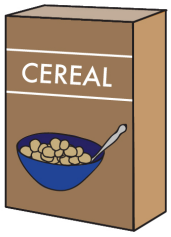
1.a. Less _____ Greater _____



A

B

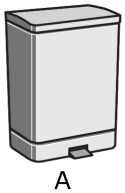
1.b. Less _____ Greater _____



A

B

1.c. Least _____, _____, _____ Greatest



A

B

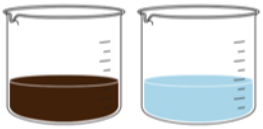
C

Name_____

2. Answer the following questions in complete sentences.

2.a. Sonia's fifth grade science class was planting a school garden. Sonia was in charge of filling one of the planting beds with soil. Sonia used her shovel to scoop and add soil to the planting bed. By the time the planting bed was full, Sonia had filled her shovel 28 times. What is the volume of the planting bed in scoops of soil?

2.b. Frankie and Johnny each have a full beaker for their science experiment. Johnny's beaker has 50 milliliters of water, and Frankie's beaker has 250 milliliters of vinegar. Which beaker has a greater volume? How do you know?

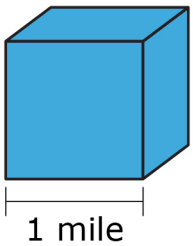


Name_____

2.c. Maggie and Anna were organizing their craft supplies. They had four different-size containers for different materials. They stored the containers on the shelf, with the largest container on the left and the smallest container on the right. One container contained 2 cups of sequins, one container held 4 cups of cotton balls, one container had 3 cups of beads, and the fourth container had 1 cup of glitter. Each container was filled completely. List the containers from greatest to least according to volume, as they would appear on the shelf.

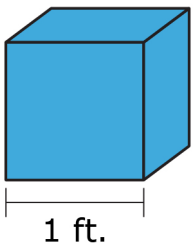
3. Identify the measurement unit for each unit cube.

3.a.



1 cubic _____

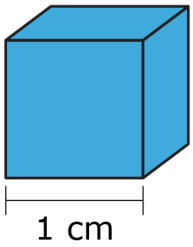
3.b.



1 _____

Name_____

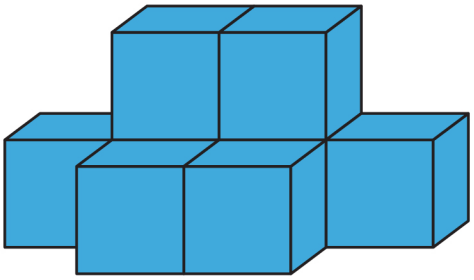
3.c.



_____

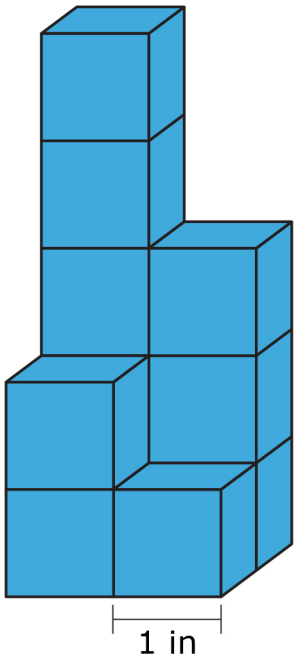
4. Determine the volume of each figure. Be sure to include the unit label.

4.a. _____

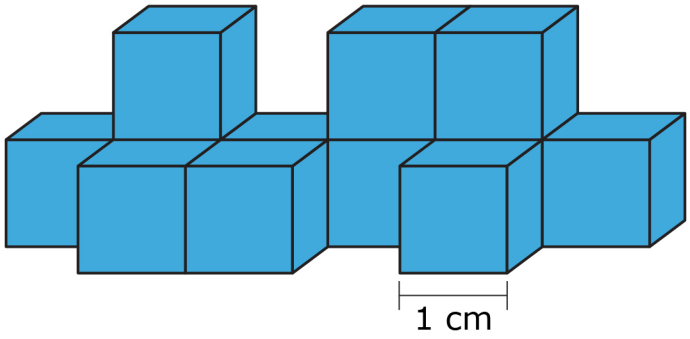


Name_____

4.b. _____

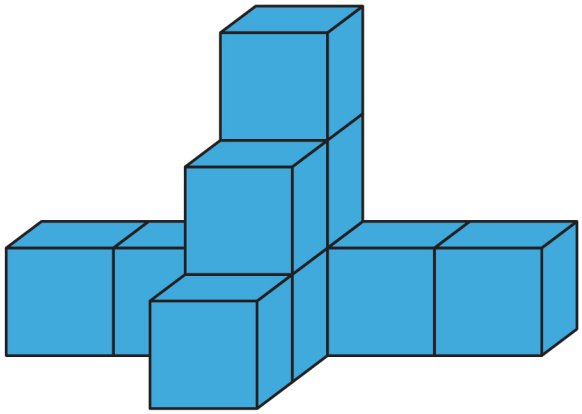


4.c. _____



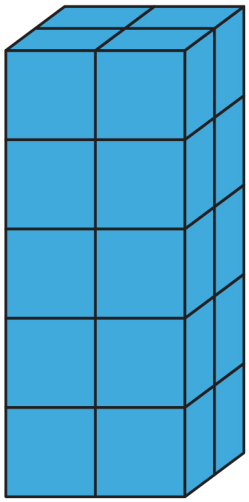
Name_____

4.d. _____



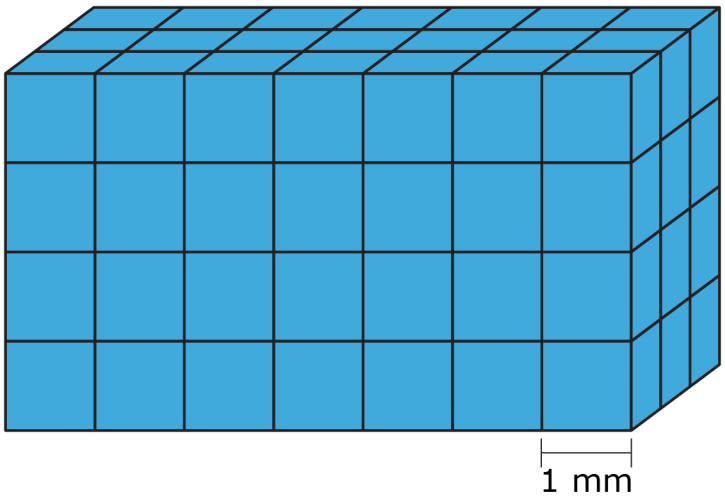
5. Determine the volume of each figure. Be sure to include the unit label.

5.a. _____

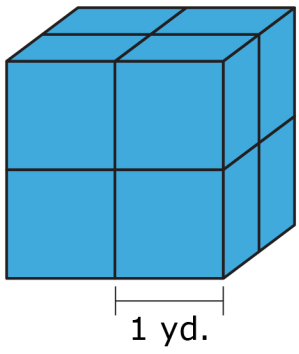


Name_____

5.b. _____

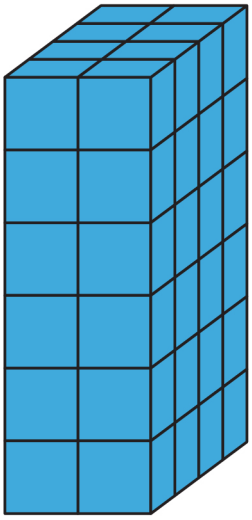


5.c. _____

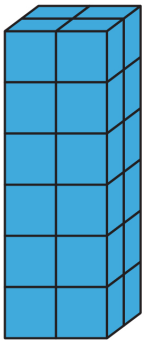


Name_____

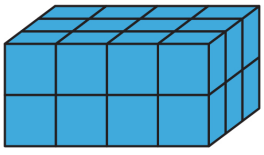
5.d. _____



6. Use figures A and B to answer the following



A



B

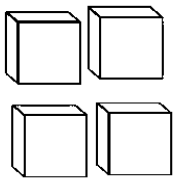
6.a. What is the volume of figure A?

6.b. What is the volume of figure B?

Name_____

6.c. Create and complete a Venn diagram comparing and contrasting figure A and figure B.

7. Walt was packing sugar cubes into a shoe box for his science project. It took 432 sugar cubes to fill the shoe box, and each sugar cube had a side length of one centimeter. What is the volume of the shoe box?



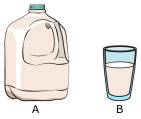
RECOGNIZE AND MEASURE VOLUME WITH UNIT CUBES

STUDENT ACTIVITY SOLUTION GUIDE

Lesson 1 – 4

1. Order the containers from least to greatest according to their volume.

1.a. Less _____ Greater _____



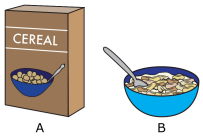
CORRECT ANSWER

Less: B Greater: A

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student identifies figure A as the lesser volume.	does not recognize that a gallon of milk is larger than a glass of milk and/or does not recognize the terms *less* and *greater*	MAKE DIRECT COMPARISON OF 2 VOLUMES and/or RECOGNIZE LESS AMOUNT and RECOGNIZE MORE AMOUNT

1.b. Less _____ Greater _____



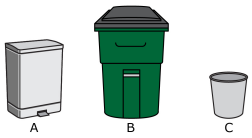
CORRECT ANSWER

Less: B Greater: A

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student identifies figure A as the lesser volume.	does not recognize that a box of cereal is larger than a bowl of cereal and/or does not recognize the terms *less* and *greater*	MAKE DIRECT COMPARISON OF 2 VOLUMES and/or RECOGNIZE LESS AMOUNT and RECOGNIZE MORE AMOUNT

1.c. Least _____, _____, _____ Greatest



CORRECT ANSWER

> Least C, A, B Greatest

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student responds with the order B, A, C, listing the figures from greatest to least.	does not recognize the terms least and greatest and/or does not read or attend to the directions	RECOGNIZE LESS AMOUNT and RECOGNIZE MORE AMOUNT
The student responds A, B, C because they recognize that figure C has the smallest volume, however identifies figure B as the next largest volume and figure C as the largest volume.	recognizes that figure C is the smallest but does not compare figures A and B or does not recognize that figure B is larger than figure C	MAKE DIRECT COMPARISON OF 2 VOLUMES and ORDER MORE THAN 2 VOLUMES BY DIRECT COMPARISON
The student leaves the middle space blank and only completes the blanks next to the terms *least* and *greatest.*	does not recognize the need to order more than two items	ORDER MORE THAN 2 VOLUMES BY DIRECT COMPARISON

2. Answer the following questions in complete sentences.

2.a. Sonia's fifth grade science class was planting a school garden. Sonia was in charge of filling one of the planting beds with soil. Sonia used her shovel to scoop and add soil to the planting bed. By the time the planting bed was full, Sonia had filled her shovel 28 times. What is the volume of the planting bed in scoops of soil?

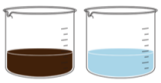
CORRECT ANSWER

> The volume of the planting bed is 28 scoops of soil.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student does not answer the question or leaves the answer blank.	does not understand what the question is asking	DEMONSTRATE ITERATION OF A VOLUME UNIT and MEASURE VOLUME USING INFORMAL UNITS
The student uses a formal unit of measure in the answer. For example, 28 cubic feet.	does not recognize the unit of measurement as an informal unit	MEASURE VOLUME USING INFORMAL UNITS
The student responds with a value other than 28.	does not understand the concept of volume as the amount of soil the planting bed can hold	EXPLAIN *VOLUME* and MEASURE VOLUME USING INFORMAL UNITS

2.b. Frankie and Johnny each have a full beaker for their science experiment. Johnny's beaker has 50 milliliters of water, and Frankie's beaker has 250 milliliters of vinegar. Which beaker has a greater volume? How do you know?



CORRECT ANSWER

> Frankie's beaker has a greater volume because 250 milliliters is larger than 50 milliliters.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student does not answer the question. The students leaves the answer blank.	does not understand what the question is asking	MAKE DIRECT COMPARISON OF 2 VOLUMES
The student responds that Johnny's beaker (50 milliliters) is greater than Frankie's beaker (250 milliliters).	does not recognize the term greater and/or does not read or attend to the directions	RECOGNIZE MORE AMOUNT
The student answers correctly but does not provide an explaination.	cannot explain how they know one volume is greater than the other and/or does not read or attend to the directions	EXPLAIN *VOLUME* and MAKE DIRECT COMPARISON OF 2 VOLUMES

2.c. Maggie and Anna were organizing their craft supplies. They had four different size containers for different materials. They stored the containers on the shelf, with the largest container on the left and the smallest container on the right. One container contained 2 cups of sequins, one container held 4 cups of cotton balls, one container had 3 cups of beads, and the fourth container had 1 cup of glitter. Each container was filled completely. List the containers from greatest to least according to volume, as they would appear on the shelf.

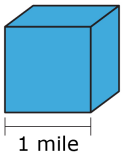
CORRECT ANSWER

> The largest container is the one with the four cups of cotton balls, then the three cups of beads, then the two cups of sequins, and lastly the smallest container is the one with one cup of glitter.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student responds by listing the containers from least to greatest. For example, one cup of glitter, two cups of sequins, three cups of beads, and four cups of cotton balls.	does not recognize the terms least and greatest and/or does not read or attend to the directions	RECOGNIZE LESS AMOUNT and RECOGNIZE MORE AMOUNT
The student does not answer the question. The students leaves the answer blank.	does not understand what the question is asking	ORDER MORE THAN 2 VOLUMES BY DIRECT COMPARISON

3. Identify the measurement unit for each unit cube.

3.a. 

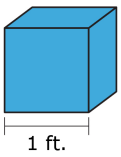
1 cubic _____

CORRECT ANSWER

1 cubic mile

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student responds with the term *unit*.	does not recognize that *unit* is used when a formal unit of measurement is not provided	EXPLAIN *UNIT CUBE*
The student responds with the term *cube*.	does not understand they are supposed to write the unit of measurement and/or does not know what the unit of measurement is	EXPLAIN *UNIT CUBE*
The student responds with the term *square*.	does not recognize the difference between a square and a cube and/or does not know what the unit of measurement is	RECOGNIZE *CUBE* and EXPLAIN *UNIT CUBE*
The student does not answer, leaves the blank empty.	does not understand they are supposed to write the unit of measurement and/or does not know what the unit of measurement is	EXPLAIN *UNIT CUBE*

3.b. 

1 _____

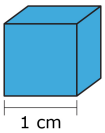
CORRECT ANSWER

1 cubic foot or 1 foot^3

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student responds with the term *cubic unit*.	does not recognize that *unit* is used when a formal unit of measurement is not provided	EXPLAIN *UNIT CUBE*
The student responds with the term *cube*.	does not understand they are supposed to write the unit of measurement and/or does not know what the unit of measurement is	EXPLAIN *UNIT CUBE*
The student responds with the term *square*.	does not recognize the difference between a square and a cube and/or does not know what the unit of measurement is	RECOGNIZE *CUBE* and EXPLAIN *UNIT CUBE*
The student does not include the term *cubic* or the superscript *3*.	does not recognize that volume is a three-dimensional measurement measured in cubic units	EXPLAIN *UNIT CUBE*
The student does not answer, leaves the blank empty.	does not understand they are supposed to write the unit of measurement and/or does not know what the unit of measurement is	EXPLAIN *UNIT CUBE*

3.c.



_____

CORRECT ANSWER

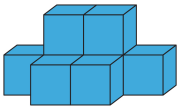
1 cubic centimeter or 1 centimeter^3

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student responds with the term *1 cubic unit*.	does not recognize that *unit* is used when a formal unit of measurement is not provided	EXPLAIN *UNIT CUBE*
The student responds with the term *cube*.	does not understand they are supposed to write the unit of measurement and/or does not know what the unit of measurement is	EXPLAIN *UNIT CUBE*
The student responds with the term *square*.	does not recognize the difference between a square and a cube and/or does not know what the unit of measurement is	RECOGNIZE *CUBE* and EXPLAIN *UNIT CUBE*
The student does not include the term *cubic* or the superscript *3*.	does not recognize that volume is a three-dimensional measurement measured in cubic units	EXPLAIN *UNIT CUBE*
The student identifies the number of faces visible (3)as the value instead of 1.	does not understand that the cube is a single unit, instead counts all the visible faces	EXPLAIN *UNIT CUBE* and RECOGNIZE *CUBE*
The student identifies the number faces in a cube (6) as the value instead of 1.	does not understand that the cube is a single unit, instead identifies all the faces of a cube	EXPLAIN *UNIT CUBE*
The student does not answer, leaves the blank empty.	does not understand they are supposed to write the unit of measurement and/or does not know what the unit of measurement is	EXPLAIN *UNIT CUBE*

4. Determine the volume of each figure. Be sure to include the unit label.

4.a. _____



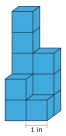
CORRECT ANSWER

8 cubic units or 8 units^3

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student identifies the volume as 6 cubic units.	does not recognize that there are cubes that are unseen, only counts the cubes that are visible	CALCULATE VOLUME BY COUNTING UNIT CUBES
The student identifies the volume as 14 cubic units.	does not recognize the cube as a three-dimensional figure, counts all the faces that are visible	CALCULATE VOLUME BY COUNTING UNIT CUBES and RECOGNIZE *CUBE*
The student does not write the unit label, only writes the value (8). Or the student does not write the correct unit label. For example, 8 units or 8 cubic inches.	does not recognize the necessity to label volume with the appropriate unit measurement and/or does not read or attend to the directions	EXPLAIN VOLUME AS A COMPOSITION OF CUBE UNITS and EXPLAIN *UNIT CUBE*

4.b. _____



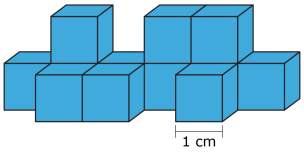
CORRECT ANSWER

11 cubic inches or 11 inches^3

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student identifies the volume as 8 or 9 cubic units.	does not recognize that there are cubes that are unseen, only counts the cubes that are visible	CALCULATE VOLUME BY COUNTING UNIT CUBES
The student identifies the volume as 19 cubic units.	does not recognize the cube as a three-dimensional figure, counts all the faces that are visible	CALCULATE VOLUME BY COUNTING UNIT CUBES and RECOGNIZE *CUBE*
The student does not write the unit label, only writes the value (11). Or the student does not write the correct unit label. For example, 11 units or 11 cubic inches.	does not recognize the necessity to label volume with the appropriate unit measurement and/or does not read or attend to the directions	EXPLAIN VOLUME AS A COMPOSITION OF CUBE UNITS and EXPLAIN *UNIT CUBE*

4.c. _____



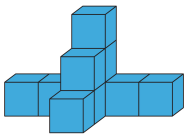
CORRECT ANSWER

12 cubic centimeters or 12 centimeters^3

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student identifies the volume as 9 or 10 cubic units.	does not recognize that there are cubes that are unseen, only counts the cubes that are visible	CALCULATE VOLUME BY COUNTING UNIT CUBES
The student identifies the volume as 23 cubic units.	does not recognize the cube as a three-dimensional figure, counts all the faces that are visible	CALCULATE VOLUME BY COUNTING UNIT CUBES and RECOGNIZE *CUBE*
The student does not write the unit label, only writes the value (12). Or the student does not write the correct unit label. For example, 12 units or 12 cubic inches.	does not recognize the necessity to label volume with the appropriate unit measurement and/or does not read or attend to the directions	EXPLAIN VOLUME AS A COMPOSITION OF CUBE UNITS and EXPLAIN *UNIT CUBE*

4.d. _____



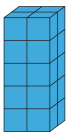
CORRECT ANSWER

10 cubic units or 10 units^3

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student identifies the volume as 7 or 9 cubic units.	does not recognize that there are cubes that are unseen, only counts the cubes that are visible	CALCULATE VOLUME BY COUNTING UNIT CUBES
The student identifies the volume as 20 cubic units.	does not recognize the cube as a three-dimensional figure, counts all the faces that are visible	CALCULATE VOLUME BY COUNTING UNIT CUBES and RECOGNIZE *CUBE*
The student does not write the unit label, only writes the value (10). Or the student does not write the correct unit label. For example, 10 units or 10 cubic inches.	does not recognize the necessity to label volume with the appropriate unit measurement and/or does not read or attend to the directions	EXPLAIN VOLUME AS A COMPOSITION OF CUBE UNITS and EXPLAIN *UNIT CUBE*

5. Determine the volume of each figure. Be sure to include the unit label.



5.a. _____

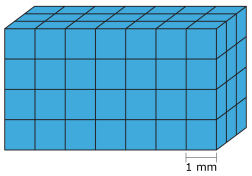
CORRECT ANSWER

20 cubic units or 20 units^3

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student identifies the volume as 10 or 16 cubic units.	does not recognize that there are cubes that are unseen, only counts the cubes that are visible	CALCULATE VOLUME BY COUNTING UNIT CUBES
The student identifies the volume as 24 cubic units.	does not recognize the cube as a three-dimensional figure, counts all the faces that are visible	CALCULATE VOLUME BY COUNTING UNIT CUBES and RECOGNIZE *CUBE*
The student does not write the unit label, only writes the value (20). Or the student does not write the correct unit label. For example, 20 units or 20 cubic inches.	does not recognize the necessity to label volume with the appropriate unit measurement and/or does not read or attend to the directions	EXPLAIN VOLUME AS A COMPOSITION OF CUBE UNITS and EXPLAIN *UNIT CUBE*

5.b. _____



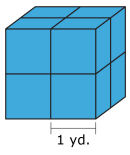
CORRECT ANSWER

84 cubic millimeters or 84 millimeters^3

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student identifies the volume as 28 or 64 cubic units.	does not recognize that there are cubes that are unseen, only counts the cubes that are visible	CALCULATE VOLUME BY COUNTING UNIT CUBES
The student identifies the volume as 76 cubic units.	does not recognize the cube as a three-dimensional figure, counts all the faces that are visible	CALCULATE VOLUME BY COUNTING UNIT CUBES and RECOGNIZE *CUBE*
The student does not write the unit label, only writes the value (84). Or the student does not write the correct unit label. For example, 84 units or 84 cubic inches.	does not recognize the necessity to label volume with the appropriate unit measurement and/or does not read or attend to the directions	EXPLAIN VOLUME AS A COMPOSITION OF CUBE UNITS and EXPLAIN *UNIT CUBE*
The student does not provide an answer, leaves the question blank.	is unable to determine the volume of a larger rectangular prism	CALCULATE VOLUME BY COUNTING UNIT CUBES

5.c. _____



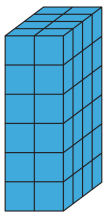
CORRECT ANSWER

8 cubic yards or 8 yards^3

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student identifies the volume as 7 cubic units.	does not recognize that there are cubes that are unseen, only counts the cubes that are visible	CALCULATE VOLUME BY COUNTING UNIT CUBES
The student identifies the volume as 12 cubic units.	does not recognize the cube as a three-dimensional figure, counts all the faces that are visible	CALCULATE VOLUME BY COUNTING UNIT CUBES and RECOGNIZE *CUBE*
The student does not write the unit label, only writes the value (8). Or the student does not write the correct unit label. For example, 8 units or 8 cubic inches.	does not recognize the necessity to label volume with the appropriate unit measurement and/or does not read or attend to the directions	EXPLAIN VOLUME AS A COMPOSITION OF CUBE UNITS and EXPLAIN *UNIT CUBE*

5.d. _____



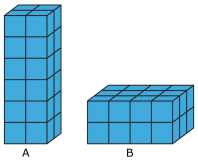
CORRECT ANSWER

48 cubic units or 48 units^3

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student identifies the volume as 33 cubic units.	does not recognize that there are cubes that are unseen, only counts the cubes that are visible	CALCULATE VOLUME BY COUNTING UNIT CUBES
The student identifies the volume as 44 cubic units.	does not recognize the cube as a three-dimensional figure, counts all the faces that are visible	CALCULATE VOLUME BY COUNTING UNIT CUBES and RECOGNIZE *CUBE*
The student does not write the unit label, only writes the value (48). Or the student does not write the correct unit label. For example, 48 units or 48 cubic inches.	does not recognize the necessity to label volume with the appropriate unit measurement and/or does not read or attend to the directions	EXPLAIN VOLUME AS A COMPOSITION OF CUBE UNITS and EXPLAIN *UNIT CUBE*

6. Use figures A and B to answer the following questions.



6.a. What is the volume of figure A?

CORRECT ANSWER

24 cubic units or 24 units^3

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student identifies the volume as 18 cubic units.	does not recognize that there are cubes that are unseen, only counts the cubes that are visible	CALCULATE VOLUME BY COUNTING UNIT CUBES
The student identifies the volume as 28 cubic units.	does not recognize the cube as a three-dimensional figure, counts all the faces that are visible	CALCULATE VOLUME BY COUNTING UNIT CUBES and RECOGNIZE *CUBE*
The student does not write the unit label, only writes the value (24). Or the student does not write the correct unit label. For example, 24 units or 24 cubic inches.	does not recognize the necessity to label volume with the appropriate unit measurement and/or does not read or attend to the directions	EXPLAIN VOLUME AS A COMPOSITION OF CUBE UNITS and EXPLAIN *UNIT CUBE*

6.b. What is the volume of figure B?

CORRECT ANSWER

24 cubic units or 24 units^3

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student identifies the volume as 18 cubic units.	does not recognize that there are cubes that are unseen, only counts the cubes that are visible	CALCULATE VOLUME BY COUNTING UNIT CUBES
The student identifies the volume as 32 cubic units.	does not recognize the cube as a three-dimensional figure, counts all the faces that are visible	CALCULATE VOLUME BY COUNTING UNIT CUBES and RECOGNIZE *CUBE*
The student does not write the unit label, only writes the value (24). Or the student does not write the correct unit label. For example, 24 units or 24 cubic inches.	does not recognize the necessity to label volume with the appropriate unit measurement and/or does not read or attend to the directions	EXPLAIN VOLUME AS A COMPOSITION OF CUBE UNITS and EXPLAIN *UNIT CUBE*

6.c. Create and complete a Venn diagram comparing and contrasting figure A and figure B.

CORRECT ANSWER

[Check student diagrams for accuracy. Students should make note that both figures have the same volume. Also, students should note the different dimensions of the figures as differences. For example, Figure A is six units tall or has six layers. Figure B is two units tall or has two layers.]

[Note: "one is tall and one is short" is not sufficient.]

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student does not identify both figures has having the same volume.	did not calculate the correct volumes for figures A and B	CALCULATE VOLUME BY COUNTING UNIT CUBES
The student does not create a Venn diagram, leaves the question blank.	does not recognize the similarities and differences between the two figures and/or does not read or attend to the directions and/or does not understand what a Venn diagram is	MAKE DIRECT COMPARISON OF 2 VOLUMES and RECOGNIZE MEASUREABLE ATTRIBUTES

7. Walt was packing sugar cubes into a shoe box for his science project. It took 432 sugar cubes to fill the shoe box, and each sugar cube had a side length of one centimeter. What is the volume of the shoe box?

CORRECT ANSWER

> The volume of the shoe box is 432 cubic centimeters or 432 centimeters^3.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student does not answer, leaves the question blank.	does not know what the question is asking and/or does not understand that the amount of space an object takes up is volume which can also be interpreted as the amount an object can hold	EXPLAIN VOLUME AS A COMPOSITION OF CUBE UNITS and CALCULATE VOLUME BY COUNTING UNIT CUBES
The student recognizes that the volume of the shoe box is 432, but does not understand that because each cube is one centimeter cubed, the units must be cubic centimeters. Instead, the student answers 432 cubic units, or 432 units, or 432 centimeters.	does not understand that the sugar cube represents a cubic unit with a measurement of one cubic centimeter	EXPLAIN VOLUME AS A COMPOSITION OF CUBE UNITS and EXPLAIN *UNIT CUBE*
The student gives the answer "one cubic centimeter".	misinterprets the question as asking for the volume of the sugar cube and/or does not thoroughly read what the question is asking	DEMONSTRATE ITERATION OF A VOLUME UNIT