



DIVIDING WHOLE NUMBERS

5.NBT.6

CONTENTS

The types of documents contained in the unit are listed below. Throughout the unit, the documents are arranged by lesson.

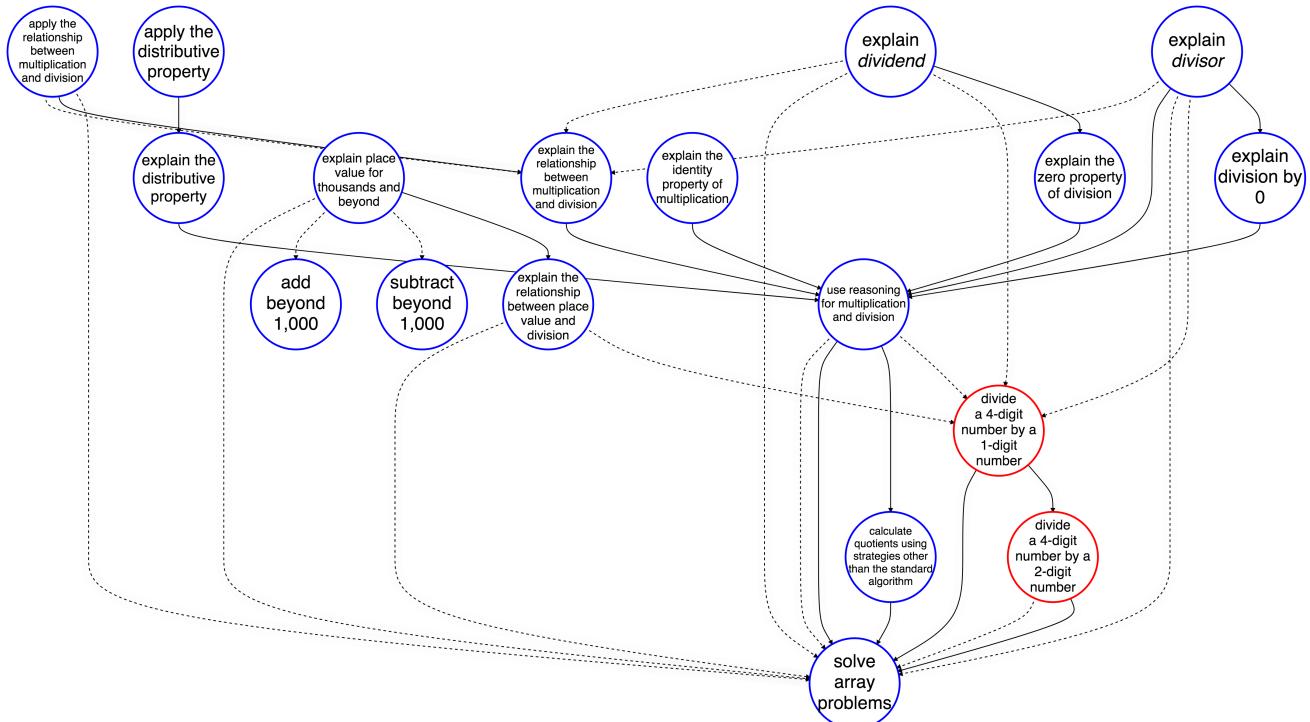
LEARNING MAP INFORMATION	An overview of the standards, the learning map section, and the nodes addressed in this unit
TEACHER NOTES	A brief discussion describing the progression depicted in the learning map section with research-based recommendations for focusing instruction to foster student learning and an introduction to the unit's lessons
OVERVIEW OF INSTRUCTIONAL ACTIVITIES	A table highlighting the lesson goals and nodes addressed in each lesson of this unit
INSTRUCTIONAL ACTIVITY	A detailed walkthrough of the unit
INSTRUCTIONAL ACTIVITY STUDENT HANDOUT	A handout for the guided activity, intended to be paired with the Instructional Activity
INSTRUCTIONAL ACTIVITY SUPPLEMENT	A collection of materials or activities related to the Instructional Activity
STUDENT ACTIVITY	A work-alone activity for students
STUDENT ACTIVITY SOLUTION GUIDE	A solution guide for the work-alone activity with example errors, misconceptions, and links to the learning map section

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LEARNING MAP INFORMATION

STANDARDS

5.NBT.6 Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.



*Learning map model of 5.NBT.6

Node Name	Node Description
ADD BEYOND 1,000	Demonstrate combinations of addition with numbers of 1,000 and beyond with objects, drawings, equations, etc.
APPLY THE DISTRIBUTIVE PROPERTY	Apply the distributive property as a strategy for multiplication. For example, 7×18 can be calculated as $(7 \times 10) + (7 \times 8)$.
APPLY THE RELATIONSHIP BETWEEN MULTIPLICATION AND DIVISION	Use arrays or equal groups to model multiplication or division problems.
CALCULATE QUOTIENTS USING STRATEGIES OTHER THAN THE STANDARD ALGORITHM	Demonstrate division or other strategies such as partitioning or fair-sharing.
DIVIDE A 4-DIGIT NUMBER BY A 1-DIGIT NUMBER	Divide a four-digit number by a one-digit number, using any method to determine the quotient.
DIVIDE A 4-DIGIT NUMBER BY A 2-DIGIT NUMBER	Divide a four-digit number by a two-digit number, using any method to determine the quotient.
EXPLAIN DIVIDEND	Make known your understanding that the dividend is the number being divided.
EXPLAIN DIVISION BY 0	Make known your understanding that zero can never be the divisor in a division problem.
EXPLAIN DIVISOR	Make known your understanding that the divisor is the number that divides another number. It is also known as a factor.
EXPLAIN PLACE VALUE FOR THOUSANDS AND BEYOND	Make known your understanding that the value of a digit is determined by its position in the number. For example, a digit in the thousands place is worth that many thousands, and a digit in the ten-thousands place is worth that many ten thousands.
EXPLAIN THE DISTRIBUTIVE PROPERTY	Make known your understanding that when multiplying a number over a sum, you distribute the factor to each addend (i.e., multiply the factor by each addend and then add).
EXPLAIN THE IDENTITY PROPERTY OF MULTIPLICATION	Make known your understanding that the product of any number and one is the original number.
EXPLAIN THE RELATIONSHIP BETWEEN MULTIPLICATION AND DIVISION	Make known your understanding that division can be interpreted as an unknown-factor problem. For example, find $20 \div 5$ by finding the number that makes 20 when multiplied by five.
EXPLAIN THE RELATIONSHIP BETWEEN PLACE VALUE AND DIVISION	Make known your understanding that a digit in ones place represents $\frac{1}{10}$ of what it represents in the place to its left.
EXPLAIN THE ZERO PROPERTY OF DIVISION	Make known your understanding that the quotient of any division with zero as the dividend is zero.
SOLVE ARRAY PROBLEMS	Use multiplication and division to solve word problems in situations involving arrays. Know that, in a multiplication expression, the first factor represents the number of rows modeled in the corresponding array, and the second factor represents the number of columns modeled in the corresponding array.
SUBTRACT WITHIN 1,000	Demonstrate combinations of subtraction within 1,000 with objects, drawings, equations, etc.
USE REASONING FOR MULTIPLICATION AND DIVISION	Solve multiplication and division problems strategically.

ADDITIONAL NODES RELATED TO THIS UNIT OF INSTRUCTION

Node Name	Node Description	Related Node
DEMONSTRATE THE CONCEPT OF DIVISION	Demonstrate division by splitting a set into a number of fair shares or equal sets.	Prerequisite of EXPLAIN DIVIDEND (through EXPLAIN DIVISION PROBLEMS)
DEMONSTRATE THE CONCEPT OF MULTIPLICATION	Demonstrate multiplication by combining multiple sets of the same quantities together to form a new set.	Prerequisite of APPLY THE RELATIONSHIP BETWEEN MULTIPLICATION AND DIVISION (through MULTIPLY BY 7)
EXPLAIN MULTIPLICATION PROBLEMS	Make known your understanding that, in a multiplication problem, the first factor describes the number of groups, and the second factor describes the number of elements in each group.	Prerequisite of APPLY THE RELATIONSHIP BETWEEN MULTIPLICATION AND DIVISION (through MULTIPLY BY 7 and REPRESENT MULTIPLICATION WITH EQUATIONS and EXPLAIN THE MEANING OF THE MULTIPLICATION SIGN)
MATHEMATIZE CONTEXTUAL SITUATIONS INVOLVING DIVISION	Organize information from contextual situations to write mathematical problems involving division.	Prerequisite of SOLVE ARRAY PROBLEMS
REPRESENT DIVISION WITH EQUATIONS	Use an equation to represent a division problem. For example, $10 \div 2 = 5$.	Postrequisite of CALCULATE QUOTIENTS USING STRATEGIES OTHER THAN THE STANDARD ALGORITHM
SOLVE MEASUREMENT QUANTITIES PROBLEMS	Use multiplication and division to solve word problems in situations involving measurement quantities such as equal measures, rate, etc.	Postrequisite of EXPLAIN DIVISOR and EXPLAIN DIVIDEND
SUBTRACT BEYOND 1,000	Demonstrate combinations of subtraction with numbers of 1,000 and beyond with objects, drawings, equations, etc.	Prerequisite of SUBTRACT WITHIN 1,000

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TEACHER NOTES

This unit includes the following documents:

- ▶ Learning Map Information
- ▶ Instructional Activity (four lessons)
- ▶ Instructional Activity Student Handout (for Lessons 1 – 4)
- ▶ Instructional Activity Supplement (for Lesson 2)
- ▶ Student Activity (Word version)
- ▶ Student Activity Solution Guide

Throughout this unit, students will apply their understanding of the inverse relationship between multiplication and division to successfully solve various division problems. The National Council of Teachers of Mathematics (NCTM) states, “Division is defined by its inverse relationship with multiplication” (NCTM, 2013, p. 55). A deeper understanding of division is attained when students are presented with multiplicative situations. Because multiplicative situations involve a specified number of groups of equal size and a total, students can create an understanding of division in terms of dividing the total into either a specified number of equal size groups or groups of a specific size. Furthermore, because multiplicative situations can lead to both multiplication and division based on their inverse relationship, multiplicative situations are crucial for understanding the concept of division (NCTM, 2013). Students should have opportunities to work with division in a variety of contexts using a variety of models and methods. These lessons and activities require students to solve division problems using a variety of strategies including area models, the distributive property, and the standard algorithm.

Richardson (2012, p. 174), states, “When we multiply, we know the number of groups and the size of the groups, and need to determine the total. When we divide, we know the total and we need to determine either the number of groups or the size of the groups, depending on the situation.” Teachers should present students with both concepts of division situations: measurement (grouping) and sharing (partitive). These situations should be used in a variety of practice settings and compared to each other as problem situations.

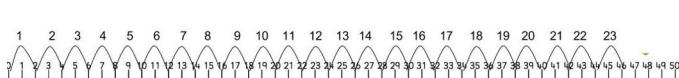
The measurement concept of division, also known as grouping division, provides students with the total and the amount in each group. This context requires students to determine the number of groups in a given situation. The sharing concept of division, also known as partitive division, provides students with the total and the number of groups. This context requires students to determine the amount in each group.

Richardson (2012) stresses the importance of students’ understanding of the meaning of the quotient in a division situation. Recognition of what the quotient represents indicates a student’s understanding that the situation requires division, as well as an understanding of *what* is being asked in the division situation (Richardson, 2012). If the quotient is five, students should be able to identify whether five describes the number of items in each group (e.g., five tiles in each group) or the number of equal-size groups (e.g., five equal-size groups of tiles).

AN EXAMPLE

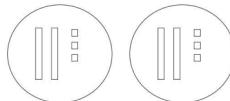
MEASUREMENT

A number line is one good visual representation of the measurement concept of division. Since the student is provided with the total and the size of the groups, the number of groups needs to be determined. The measurement concept of division can also be represented using a set model by dividing the dividend into multiple groups based on the value of the divisor. For example, $46 \div 2$ can be modeled as:



SHARING

Sharing division is best represented with a set model, since the size of the groups is unknown. For the expression $46 \div 2$, two groups are created and the dividend is then divided between the two groups.



According to Fuson (2003, p. 303), two aspects of the standard algorithm for division are problematic for students: that “students must determine the exact number of maximum copies of a divisor that can be taken from a dividend”, and that students do not gain a sense of the size of the quotients that they are writing because they are always multiplying by single digits to determine the quotient. By presenting students with models and accessible methods to determine quotients for long division, teachers help students gain an understanding of the concept of division, learn to provide appropriate estimates to determine quotients or partial quotients, and better understand the relationship between place value and division. According to NCTM (2011), “...understanding involves much more than just computational fluency.” Students can learn and understand the process of division with scaffolds, such as a multiplication table to assist with computational fluency; inversely, a student can be very proficient with computational fluency and not understand the process of calculating quotients using the standard algorithm.

In addition to providing students with division experiences prior to being exposed to the standard algorithm, teachers must attend to the role of language as they promote conceptual understanding of the standard division algorithm. “Language plays an enormous role in thinking conceptually about the standard division algorithm” (Van de Walle et al., 2006, p. 191). Using language such as “goes into” (e.g., “How many times does 4 go into 20?”) can be confusing for students. Likewise, when different teachers use different language or terms to refer to the same concept, it can be confusing for students. For example, students will likely become confused if one teacher refers to “turn-arounds” and another teacher uses the term “commutative” to refer to the commutative property (NCTM, 2011). Confusion generated from inconsistent or imprecise

language may cause students to use the concept incorrectly (NCTM, 2011). Also according to NCTM (2011, p. 75-76), “Mathematical reasoning and discourse are enhanced when teachers use correct terms and help students use definitions that their previous work has helped them to understand.” The idea of division should be communicated in terms that students understand (e.g., sharing equally or creating equal groups). NCTM (2011) identifies vague and ambiguous statements to avoid (e.g., “suppose a group of apples is divided in half”) and phrases that can be interpreted in different ways (e.g., “6 is divided into 12”, “goes into evenly”, and “no remainder”). Vague phrases such as “suppose a group of apples is divided in half” can lead to misconceptions because there is not a sufficient amount of information to determine what the question is asking. The student does not know whether a group of apples is divided into two equal sets or if the apples are cut into halves. The phrase “goes into evenly” creates student misunderstandings because “even” refers to numbers divisible by two, however it is often incorrectly used when the quotient is a whole number that has a zero remainder. A common misconception is that if a quotient is a whole number, there is no remainder. This, however, is untrue because there is always a remainder; if the quotient is a whole number, then the remainder is zero (NCTM, 2011).

Fuson (2003, p. 301) states, “the twenty-first century requires a greater focus on a wider range of problem-solving experiences and a reduced focus on learning and practicing by rote a large body of standard calculation methods.” This wider range of problem-solving experiences encompasses not only using a variety of strategies to calculate quotients, but also an understanding of what problem situations are asking, as well as requiring students to determine the most suitable strategy to calculate the quotient and thus answer the problem situation.

The learning map section for this sequence of activities focuses on students understanding and applying the relationship between multiplication and division, as well as the relationship between place value and division. Early in their experiences with division, students use their understanding of multiplication to determine quotients. For example, to determine the missing value in the equation $6 \div 3 = \square$, students can think of the equation as $3 \times \square = 6$. Emphasizing the idea of common, precise language, the learning map section requires students to explain the terms “dividend” and “divisor”. For students to use reasoning with both multiplication and division, they need to be able to calculate quotients using strategies other than the standard algorithm, including an array.

The activities in this unit are designed to build on students’ prior knowledge and gradually increase in complexity. Each lesson presents division either as a problem situation or in relation to multiplication. For example, the student may be presented with the equation $n \times 13 = 624$ and must use the inverse relationship of multiplication to determine the missing factor. The first lesson focuses on establishing a common vocabulary and determining quotients using arrays and number lines. The second lesson draws on students’ understanding of the area model, previously used to determine products, in order to use the same area model to determine a quotient. The third lesson expands on students’ understanding of the relationship between multiplication and division, established in Lessons 1 and 2, by requiring students to use the distributive property to calculate quotients. The final lesson focuses on student use of both accessible division algorithms and the standard algorithm, continuing students’ awareness of place value as they operate with the standard division algorithm. The activities in this unit allow students to utilize a variety of strategies in response to various situations, enhancing students’ ability to reason and calculate quotients to respond to a given problem situations.

REFERENCES

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- Van de Walle, J.A., Karp, K.S., Lovin, L.H., & Bay-Williams, J.M. (2006). *Teaching Student-Centered Mathematics*. Upper Saddle River, NJ: Pearson Professional Development.

DIVIDING WHOLE NUMBERS

OVERVIEW OF INSTRUCTIONAL ACTIVITIES

Lesson	Learning Goal	Nodes Addressed
Lesson 1	<p>Students will find whole-number quotients of whole numbers with up to three-digit dividends and one-digit divisors, using the relationship between multiplication and division. Students will illustrate and explain their calculation using area models and number lines.</p>	<ul style="list-style-type: none"> ▶ EXPLAIN DIVIDEND ▶ EXPLAIN DIVISOR ▶ CALCULATE QUOTIENTS USING STRATEGIES OTHER THAN THE STANDARD ALGORITHM ▶ EXPLAIN THE ZERO PROPERTY OF DIVISION ▶ EXPLAIN DIVISION BY 0 ▶ EXPLAIN THE IDENTITY PROPERTY OF MULTIPLICATION ▶ SOLVE ARRAY PROBLEMS ▶ EXPLAIN THE RELATIONSHIP BETWEEN MULTIPLICATION AND DIVISION ▶ APPLY THE RELATIONSHIP BETWEEN MULTIPLICATION AND DIVISION
Lesson 2	<p>Students will find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value and the relationship between multiplication and division. Students will illustrate and explain their calculation using area models.</p>	<ul style="list-style-type: none"> ▶ CALCULATE QUOTIENTS USING STRATEGIES OTHER THAN THE STANDARD ALGORITHM ▶ USE REASONING FOR MULTIPLICATION AND DIVISION ▶ DIVIDE A 4-DIGIT NUMBER BY A 1-DIGIT NUMBER
Lesson 3	<p>When presented with situations involving up to four-digit dividends and two-digit divisors, students will determine whole-number quotients using strategies based on the distributive property.</p>	<ul style="list-style-type: none"> ▶ EXPLAIN THE DISTRIBUTIVE PROPERTY ▶ APPLY THE DISTRIBUTIVE PROPERTY
Lesson 4	<p>Students will determine whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value. Students will illustrate and explain the calculation by using equations.</p>	<ul style="list-style-type: none"> ▶ CALCULATE QUOTIENTS USING STRATEGIES OTHER THAN THE STANDARD ALGORITHM ▶ EXPLAIN THE RELATIONSHIP BETWEEN PLACE VALUE AND DIVISION ▶ DIVIDE A 4-DIGIT NUMBER BY A 2-DIGIT NUMBER

DIVIDING WITH WHOLE NUMBER QUOTIENTS

INSTRUCTIONAL ACTIVITY

Lesson 1

LEARNING GOAL

Students will find whole-number quotients of whole numbers with up to three-digit dividends and one-digit divisors, using the relationship between multiplication and division. Students will illustrate and explain their calculation using area models and number lines.

PRIMARY ACTIVITY

Students will find whole number quotients using the relationship between multiplication and division, dividing number lines, and creating and interpreting rectangular arrays.

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

- ▶ Dividend
 - ▶ Divisor
 - ▶ Quotient
 - ▶ Remainder
 - ▶ Factor
 - ▶ Multiple
-

MATERIALS

- ▶ [INSTRUCTIONAL ACTIVITY STUDENT HANDOUT](#)
 - ▶ Word version [INSTRUCTIONAL ACTIVITY STUDENT HANDOUT](#)
 - ▶ A bag of at least 30 counters (Recommend one bag for every four students.)
 - ▶ A bag of at least 120 beans (Recommend one bag for every four students.)
-

IMPLEMENTATION

Arrange students in groups of four and give each group of students a bag of counters.

Require students to divide the counters among the members of the groups, giving each student an equal portion of the counters. Have students place the remaining counters off to the side.

Ask students the following questions once all of the counters have been divided:

- ▶ Into how many equal-size groups did you divide the counters?
- ▶ How many counters are in each equal group?
- ▶ What is the total number of counters?

Require students to discuss the questions within their groups, then have each group share their answers one at a time. As each group shares with the class, **write** on the board the multiplication and division equations for their situation.

Remind students that the first factor represents the *number of equal groups* and the second factor represents the *amount in each group*. For example:

$$\begin{array}{c} \text{four equal groups, 13 counters in each group, 52 counters total} \\ 4 \times 13 = 52 \\ 52 \div 4 = 13 \end{array}$$

Explain for each situation *how many groups*, *how many in each group*, and *the total number of counters*. Be sure to use the language “equal groups” and “each group.” For the previous example, you would say, “There are four equal groups, and each equal group has 13 counters, therefore the total number of counters is 52. If you have 52 counters to share among four students, each student will get 13 counters.”

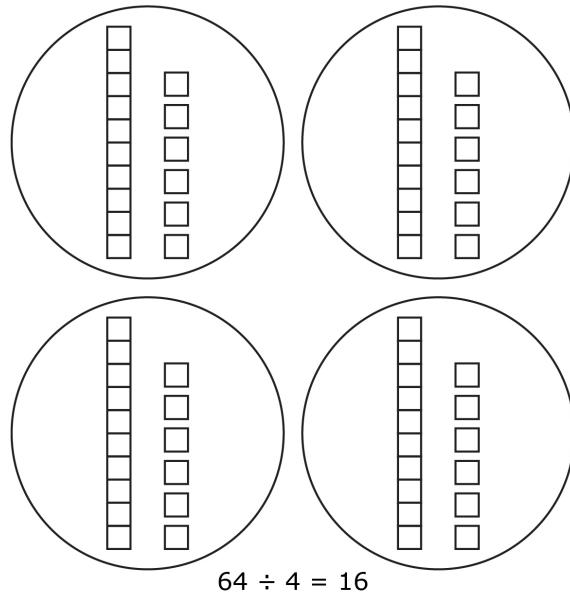
Point to the different parts of the equation as you explain each situation.

Explain that there are two division situations: “sharing” (or partitive) and “measurement” (or grouping).

NOTE: The *sharing* concept of division divides the total (dividend) into the given number of groups (divisor) in order to determine the amount in each equal size group (quotient).

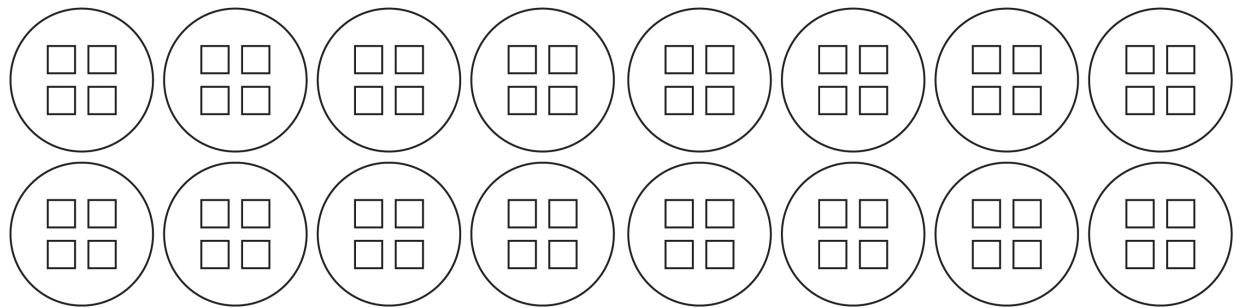
The *measurement* concept of division divides the total (dividend) into groups of a given size (divisor) in order to determine how many groups there are (quotient).

Write on the board $64 \div 4 = 16$ as a set group and as an equation. **Ask** students, “Do you think this is sharing division or measurement division? Why?”



Explain that this is sharing division because the dividend, 64, is divided into four equal size groups. **Ask** students, “How would you represent this situation as a multiplication equation?” ($4 \times 16 = 64$)

Write on the board $64 \div 4 = 16$ as a set group and as an equation. **Ask** students, “Do you think this is sharing division or measurement division? Why?”



Explain that this is measurement division because the dividend, 64, is divided into groups of four. **Ask** students, “How would you represent this situation as a multiplication equation?” ($16 \times 4 = 64$)

Collect the bags of counters and **distribute** a bag of beans to each group.

Repeat the aforementioned process for dividing out and discussing the groups of beans.

Use the following guiding questions to scaffold the discussion about the groups of beans.

GUIDING QUESTIONS

Elicit student thinking:

- ▶ What does it mean to divide?

Determine if the student is can **EXPLAIN DIVIDEND**:

- ▶ What is a dividend?
- ▶ What is being divided in $72 \div 8$? How do you know?
- ▶ If your group has 132 beans and six people, what is the dividend? How do you know?

Determine if the student is can **EXPLAIN DIVISOR**:

- ▶ What is a divisor?
- ▶ For the problem $220 \div 5$, the dividend is being divided into how many equal size groups? How do you know?
- ▶ If your groups has 132 beans and six people, what is the divisor? How do you know?

Determine if the student is can **CALCULATE QUOTIENTS USING STRATEGIES OTHER THAN THE STANDARD ALGORITHM**:

- ▶ How did you determine how many counters (or beans) each person would get?
- ▶ How did you know how many groups to divide the counters (or beans) into?

To elicit student thinking about the zero property of division, **ask** students the following questions:

- ▶ “If you were creating five equal groups with zero beans, how many beans would be in each group?” (*There would be zero beans in each group, because you have zero beans to divide into groups.*)
- ▶ “How could you represent that situation as a division problem?” ($0 \div 5$)

Model the division problem $0 \div 5 = 0$.

Ask students, “What would be the corresponding multiplication equation for this problem?” ($0 \times 5 = 0$) “Is this true?” (Yes, anything times zero equals zero.)

To further student understanding of the zero property of division, **ask** the following questions:

- ▶ “Now, use six beans and create zero equal groups. (*Students should determine that this is not possible.*)
- ▶ “Why does this not work?” (*Because you have to have at least one group of six beans.*)
- ▶ “How could you represent this situation as a division problem?” ($6 \div 0$)

Model on the board the division problem $6 \div 0$ and explain that this cannot be simplified because it is unidentified. **Explain** that this is true because there is no value that would make the statement true. For example:

$$\begin{aligned} 6 \div 0 &\neq a \\ a \times 0 &\neq 6 \end{aligned}$$

Review the identity property of multiplication. **Ask** students:

- ▶ “What is the product of 48×1 ?” (48)
- ▶ “How do you know this is true?” (*Because any time you multiply a number times one, the answer is that number.*)
- ▶ “How would you represent $48 \times 1 = 48$ as a division problem?” ($48 \div 1 = 48$)

Model the identity property and **explain** that any number divided by one is itself. For example, 48 coins divided into one group will be one group of 48 coins ($48 \div 1 = 48$).

Likewise, any number divided by itself is one. For example, 48 coins divided into 48 groups will have one coin in each group ($48 \div 48 = 1$).

GUIDING QUESTIONS

Determine if the student can EXPLAIN THE ZERO PROPERTY OF DIVISION:

- ▶ What is the zero property of division?
- ▶ If you have 25 groups and zero counters, how many counters will be in each group?

Determine if the student can EXPLAIN DIVISION BY 0:

- ▶ If you have 25 cookies and you want to divide them into zero groups, how many cookies will be in each group? Can you simplify this problem? Why or why not?
- ▶ Why are you unable to divide a number by zero? Can you use numbers to support your explanation?

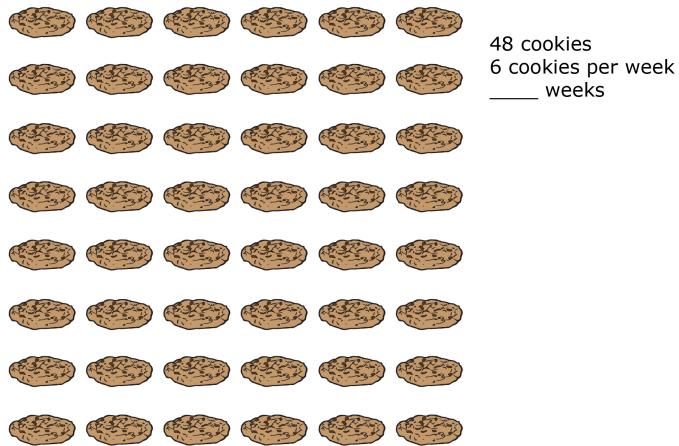
Determine if the student can EXPLAIN THE IDENTITY PROPERTY OF MULTIPLICATION:

- ▶ What is the identity property of multiplication?
- ▶ How is the identity property related to division?
- ▶ If you had 32 pencils, how many pencils would be in each group if you divided them in 32 groups? How many pencils would be in each group if you divided them into one group?

Collect the bags of beans and pass out the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT. **Display** the following problems on the board. **Require** students to draw a rectangular array to solve the problem situations.

Model the first problem situation and array. **Ask** students, “What type of division is the problem situation presenting, sharing or measurement?” (*measurement*)

- ▶ 1. Ben’s mom bought four packages of cookies when she went to the store. There is a total of 48 cookies in all four packages. If Ben eats six cookies each week, how many weeks will the cookies last? (*48 cookies total; six cookies per week; eight weeks; $8 \times 6 = 48$; $48 \div 6 = 8$*)

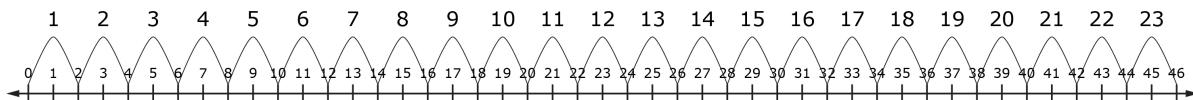


Discuss each problem situation and array model before presenting the next problem situation. Use the guiding questions to direct the discussions.

- ▶ 2. Ms. Baxter was working with six students at her round table. To complete a math problem, Ms. Baxter gave each student a group of counters. She had a total of 84 counters at her round table. How many counters did each student get? ($6 \times 14 = 84$; $84 \div 6 = 14$; *the number in the blanks represents the number of counters each student will get*) [Check student array for accuracy.]
- ▶ 3. Jules and Brett were having a Fourth of July party. They invited 11 of their friends to eat hamburgers and hotdogs. For dessert, everyone received a bag of red, white, and blue candy. If Jules bought 91 pieces of candy, how many pieces will each person get? Be sure to include Jules and Brett. ($13 \times 7 = 91$; $91 \div 13 = 7$; *the number in the blanks represents the number of pieces of candy each person will get; the number 13 represents the number of people that will get candy*) [Check student array for accuracy.]

Model Question 4 from the [INSTRUCTIONAL ACTIVITY STUDENT HANDOUT](#) using a number line. Use the guiding questions to direct the discussions.

- ▶ 4. Daniel enjoys drinking one iced tea during his lunch. The tea from the vending machine costs \$2. Daniel budgeted to spend \$46 on iced teas from the vending machine. How many days can Daniel buy one iced tea for lunch? Use the number line to complete the following blanks. ($23 \times 2 = 46$; $46 \div 2 = 23$; *the number 46 represents the amount of money that Daniel has saved for tea; the number 2 represents the cost of each tea; the number 23 represents the number of days that Daniel can buy tea*) [Check student number lines for accuracy.]



GUIDING QUESTIONS

Determine if the student is can **SOLVE ARRAY PROBLEMS**:

- ▶ How many groups are in this array? What does this number represent?
- ▶ What is the size of each group? What does this number represent?
- ▶ What is the total (product) shown in the array? How do you know?

Determine if the student is can **CALCULATE QUOTIENTS USING STRATEGIES OTHER THAN THE STANDARD ALGORITHM**:

- ▶ How do you use the number line to determine a quotient?
- ▶ In a measurement division situation, how large are each of the equal groups in $46 \div 2$? How is this displayed on the number line?
- ▶ In a sharing division situation, how large are each of the equal groups in $46 \div 2$? How is this displayed as a set group?
- ▶ In the measurement division problem $117 \div 13$, how large are each of the equal groups? What would that look like on a number line?
- ▶ In the sharing division problem $117 \div 13$, how large are each of the equal groups? What would that look like as a set group?

Determine if the student is can **EXPLAIN THE RELATIONSHIP BETWEEN MULTIPLICATION AND DIVISION**:

- ▶ What do multiplication and division have in common? How are they different?

Determine if the student is can **APPLY THE RELATIONSHIP BETWEEN MULTIPLICATION AND DIVISION**:

- ▶ Six times what number is close to or equal to 84? How would you create a division problem from this information?
- ▶ Jules and Brett are having a Fourth of July party. They want each of the 13 people attending to get a bag with seven pieces of candy. How many pieces of candy will Jules and Brett need to buy? How is this problem situation the same as Question 3? How is it different?

Have students finish the **INSTRUCTIONAL ACTIVITY STUDENT HANDOUT** independently, in groups of two, or with their original large groups.

- ▶ 5. Madison was cleaning her bathroom and found 57 hair ties. She placed them in groups based on their color. When she was finished, she had groups of 19 hair ties. How many different colors did Madison have? ($3 \times 19 = 57$; $57 \div 19 = 3$; *the number 57 represents the total number of hair ties; the number 19 represents the number of equal groups of hair ties; the number 3 represents the number of different colors of the hair ties.*)
- ▶ 6. Use the array to complete the blanks. Then, create a problem situation for the division equation. ($13 \times 8 = 104$; $104 \div 8 = 13$ or $104 \div 13 = 8$; *the number 104 represents the total number; the number 13 represents the number of equal size groups, the rows; the number 8 represents the size of the equal groups, the columns*) [Check student problem situation for accuracy.]
- ▶ 7. Explain the difference between the two arrays in terms of a division equation. (*The difference between the two arrays is that, with the sharing concept of division, the array on the left represents $35 \div 7 = 5$, and the array on the right represents $35 \div 5 = 7$.*)
- ▶ 8. Juan drew this number line to model a division problem in his math class. Use the information given with the number line to fill in all of the blanks. ($15 \times 6 = 90$; $90 \div 6 = 15$.)
- ▶ 9. Xavier was organizing his sports equipment and decided to store it in different tubs based on the type of sport. He has footballs, soccer balls, lacrosse balls, volleyballs, basketballs, and tennis balls. He has the same number of each type of ball, and a total of 72 balls. How many balls will be placed in each tub? ($72 \div 6 = 12$; *the number 72 represents the total number of balls Xavier has; the number 6 represents how many groups or different types of balls Xavier has; the number 12 represents the number of each type of ball Xavier has*) [Check student array for accuracy.]
- ▶ 10. Eliza and Zac created the following set models to represent the equation $84 \div 7 = 12$. Which answer do you agree with and why? (*Both students are correct. Eliza's model represents the sharing conception of division and Zac's model represents the measurement conception of division.*)

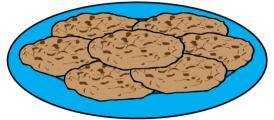
At the end of the activity, teachers should arrange students in groups of four and give each group a bag of counters (from the beginning of the lesson). **Require** the students to divide all the counters from the bag into groups of nine (move the remaining counters to the side). Each group should share their answers to the following questions:

- ▶ Into how many equal groups did you divided the counters?
- ▶ What is the total number of counters?
- ▶ What is a multiplication equation for your counters?
- ▶ What is a division equation for your counters?
- ▶ What is a division problem situation for the counters?

DIVIDING WHOLE NUMBERS

Lesson 1

1. Ben's mom bought four packages of cookies when she went to the store. There is a total of 48 cookies in all four packages. If Ben eats six cookies each week, how many weeks will the cookies last?



_____ cookies total _____ cookies per week

_____ weeks

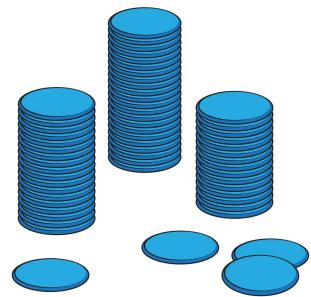
$$\text{_____} \times \text{_____} = \text{_____}$$

$$\text{_____} \div \text{_____} = \text{_____}$$

Array

2. Ms. Baxter was working with six students at her round table. To complete a math problem, Ms. Baxter gave each student a group of counters. She had a total of 84 counters at her round table. How many counters did each student get?

$$6 \times \underline{\hspace{2cm}} = 84$$

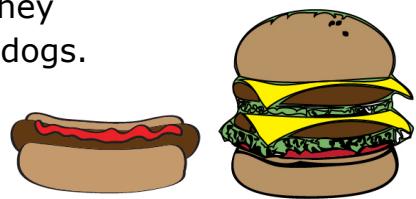


$$84 \div 6 = \underline{\hspace{2cm}}$$

The number in the blanks represents:

Array

3. Jules and Brett were having a Fourth of July party. They invited 11 of their friends to eat hamburgers and hotdogs. For dessert, everyone received a bag of red, white, and blue candy. If Jules bought 91 pieces of candy, how many pieces will each person get? Be sure to include Jules and Brett.



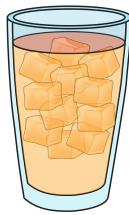
$$13 \times \underline{\hspace{2cm}} = 91 \qquad 91 \div 13 = \underline{\hspace{2cm}}$$

The number in the blanks represents:

The 13 represents:

Array

4. Daniel enjoys drinking one iced tea during his lunch. The tea from the vending machine costs \$2. Daniel budgeted to spend \$46 on iced teas from the vending machine. How many days can Daniel buy one iced tea for lunch? Use the number line to complete the following blanks.



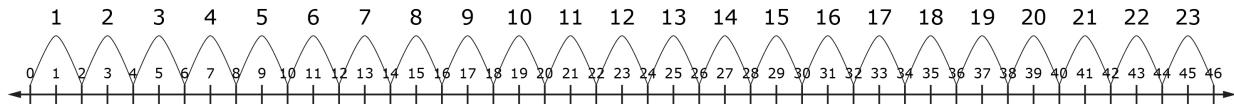
$$\underline{\hspace{2cm}} \times \underline{\hspace{2cm}} = 46$$

$$46 \div \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

The number 46 represents:

The number _____ represents:

The number _____ represents:



5. Madison was cleaning her bathroom and she found 57 hair ties. She placed them in groups based on their color. When she was finished, she had groups of 19 hair ties. How many different colors did Madison have?

$$\underline{\hspace{2cm}} \times \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

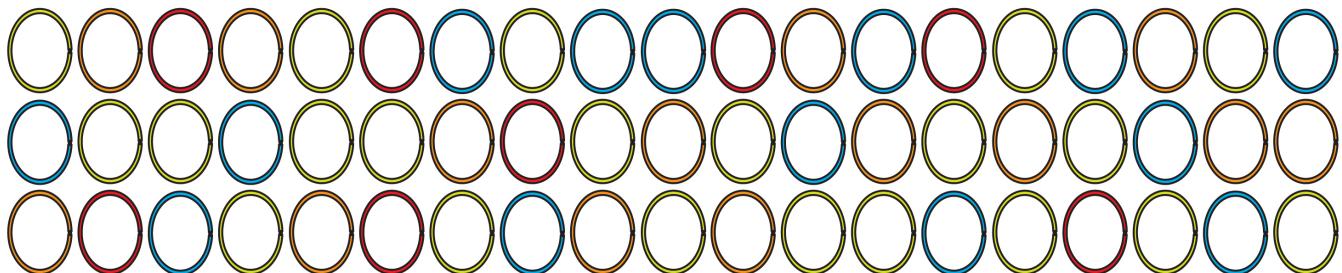
$$\underline{\hspace{2cm}} \div \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

The number _____ represents:

The number _____ represents:

The number _____ represents:

Array



6. Use the array to complete the blanks. Then, create a problem situation for the division equation.

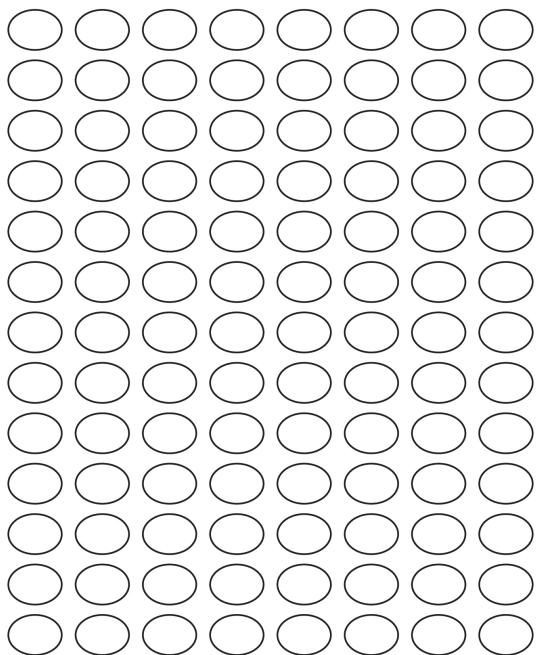
$$\underline{\hspace{2cm}} \times \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

$$\underline{\hspace{2cm}} \div \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

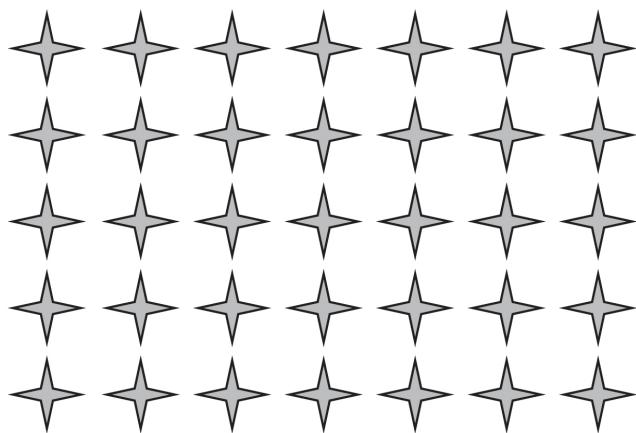
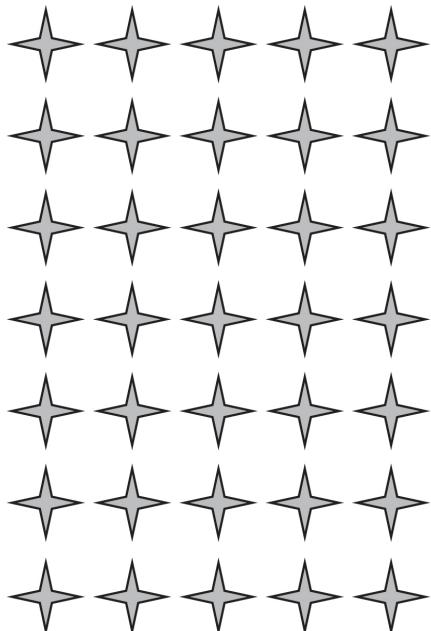
The number _____ represents:

The number _____ represents:

The number _____ represents:

Array**Problem Situation**

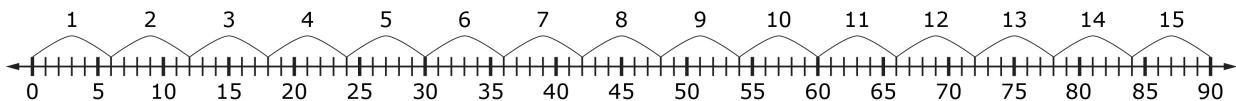
7. Explain the difference between the two arrays in terms of a division equation.



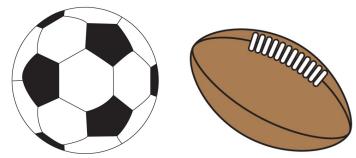
8. Juan drew this number line to model a division problem in his math class. Use the information given with the number line to fill in all of the blanks.

$$\underline{\hspace{2cm}} \times \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

$$\underline{\hspace{2cm}} \div \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$



9. Xavier was organizing his sports equipment and decided to store it in different tubs based on the type of sport. He has footballs, soccer balls, lacrosse balls, volleyballs, basketballs, and tennis balls. He has the same number of each type of ball, and a total of 72 balls. How many balls will be placed in each tub?



$$\underline{\hspace{2cm}} \div \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

The number _____ represents:

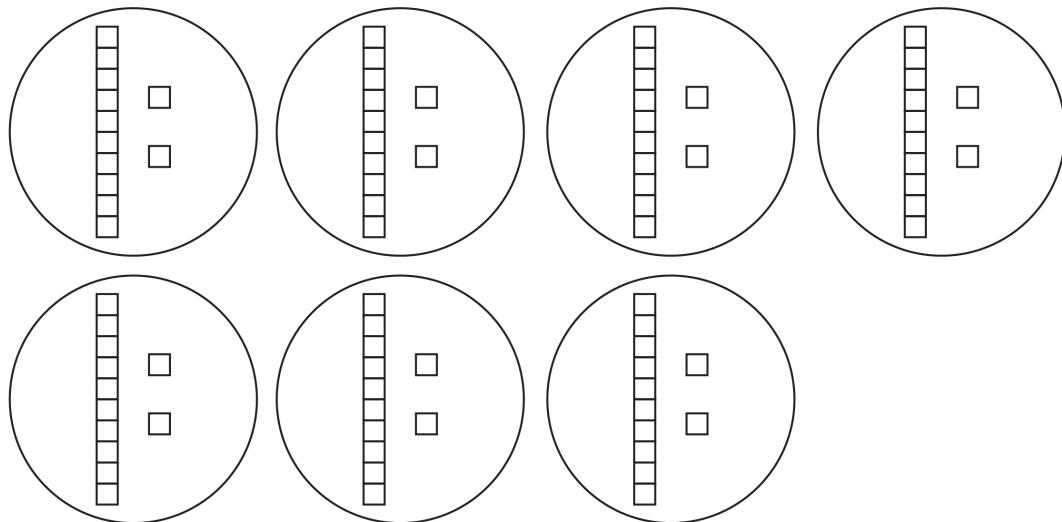
The number _____ represents:

The number _____ represents:

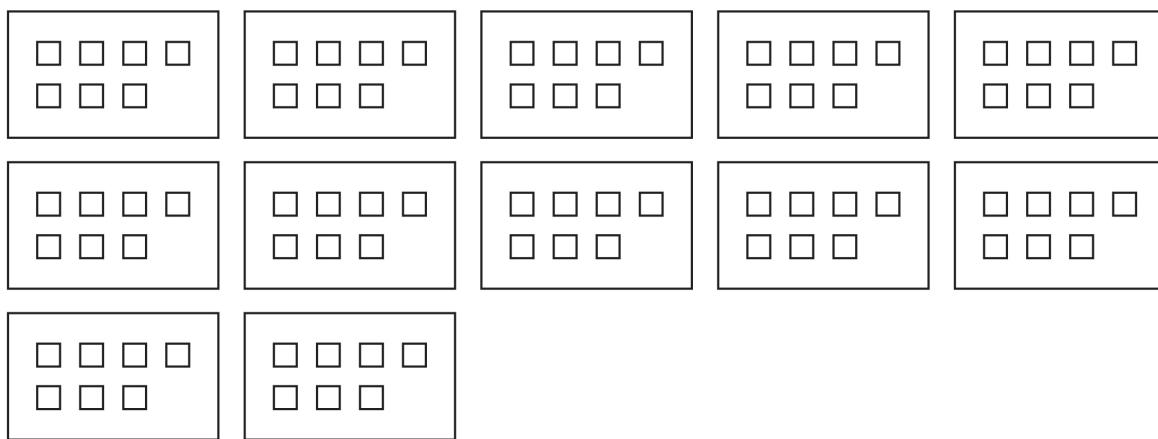
Array

10. Eliza and Zac created the following set models to represent the equation, $84 \div 7 = 12$. Which answer do you agree with and why?

Eliza's Work



Zac's Work



DIVIDING WHOLE NUMBERS

INSTRUCTIONAL ACTIVITY

Lesson 2

LEARNING GOAL

Students will find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value and the relationship between multiplication and division. Students will illustrate and explain their calculation using area models.

PRIMARY ACTIVITY

Students will find whole number quotients using base-ten blocks and area models. Students will finish the lesson by playing a division game.

NOTE: This lesson will likely require more than one class period to complete.

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

- ▶ Dividend
 - ▶ Divisor
 - ▶ Quotient
 - ▶ Remainder
 - ▶ Factor
 - ▶ Multiple
-

MATERIALS

- ▶ Base-ten blocks
 - ▶ [INSTRUCTIONAL ACTIVITY STUDENT HANDOUT](#)
 - ▶ [INSTRUCTIONAL ACTIVITY SUPPLEMENT](#) (Recommend one copy for every two to three students.)
-

- Division Game: board (best if printed on tag board/cardstock), cards (best if printed on tag board/cardstock), area model sheets, answer sheet, directions
 - ▶ Counters (Recommend a different color for each student in groups of two or three.)
 - ▶ Dice (Recommend one for every two to three students.)
 - ▶ Word version [INSTRUCTIONAL ACTIVITY STUDENT HANDOUT](#)
 - ▶ Word version [INSTRUCTIONAL ACTIVITY SUPPLEMENT](#)
-

IMPLEMENTATION

Group students into pairs and give each pair a set of base-ten blocks.

Read and/or display the following problem situation:

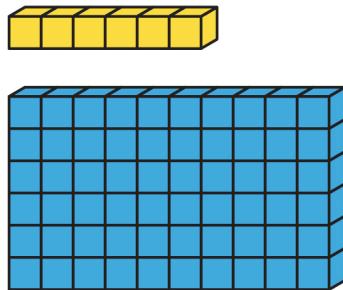
You bought three tickets for the raffle at the fair and won second place. You won a bag with 68 pieces of candy. You want to make your candy last so you divide it into separate bags. If you put four pieces of candy in each bag, how many bags will you need?

Ask students the following questions:

- ▶ What is divided into groups in this situation?
- ▶ What is the total amount we are dividing into equal groups? (68)
- ▶ What concept of division does this problem situation represent, sharing or measurement? (*measurement*)

Require pairs to count out 68 blocks with base-ten blocks.

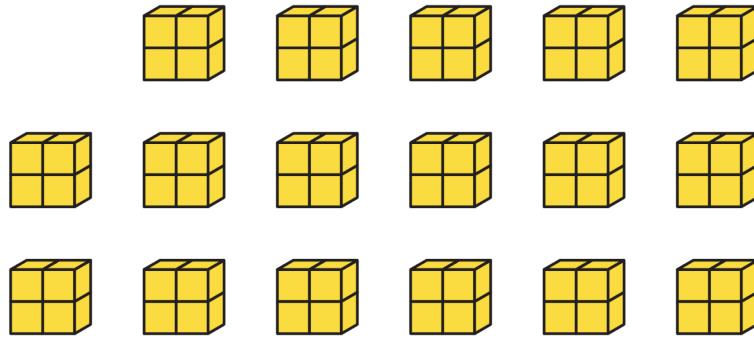
Observe how students count out and group the blocks to reach 68. Do they use ones blocks only or tens and ones? Do they count the blocks one at a time or group by two or five? This will help you determine a student's understanding of multiplicity.



Ask students, “How are the 68 pieces of candy being divided?” (*The 68 pieces of candy are being divided into groups of four.*)

Require pairs to divide the 68 base-ten blocks into groups of four. **Emphasize** they are not dividing 68 into four groups, but groups *of* four.

Observe how students divide the blocks into groups of four. Did they trade out tens blocks for ones blocks? Did they group in a different way keeping the tens blocks? Did they treat tens and ones as having equal values when making groups of four?



Ask students, “How many bags will you need for all of your candy?” (*17 bags*)

Draw attention to the misconception of four bags with 17 pieces in each bag. **Redirect** students' attention back to the question.

Read the problem situation again, now that students have worked through the problem, and **ask** students the following questions:

- ▶ What is the question asking?
- ▶ Does the quotient you found answer the question?
- ▶ Does the quotient make sense with what the question is asking?
- ▶ What is the equation that represents the question? ($68 \div 4 = 17$)

GUIDING QUESTIONS

Elicit student thinking:

- ▶ What does it mean to divide?
- ▶ How are measurement division and sharing division different? How are they the same?

Determine if the student can **CALCULATE QUOTIENTS USING STRATEGIES OTHER THAN THE STANDARD ALGORITHM**:

- ▶ How many blocks do you need to model this division? How do you know?
- ▶ When working with a question that models the sharing concept of division, how many equal groups do you need? How do you know?
- ▶ When working with a question that models the measurement concept of division, what size groups do you need? How do you know?

Determine if the student can **USE REASONING FOR MULTIPLICATION AND DIVISION**:

- ▶ What is this question asking? What operation could help you answer the question?

Repeat the same process with one or both of the following questions:

-
- ▶ Jenny's grandfather is 63 years old. He is seven times older than Jenny. How old is Jenny? (*Jenny is nine years old.*)
 - ▶ Sam's Burger Shack orders ketchup packets two times each month. On the first day of March, Sam ordered a carton of 272 ketchup packets. There is a ketchup packet container on each table. If the whole carton of ketchup packets is divided evenly among each table, and each container has 16 packets, how many tables are in the dining room at Sam's Burger Shack? (*There are 17 tables at Sam's Burger Shack.*)
-

Collect or have students return the base-ten blocks.

Explain that when working with larger numbers, it is time consuming to count out all of the base-ten blocks. Additionally, larger numbers may require more base-ten blocks than you have.

Explain that, like multiplication, you can use an area model to divide.

Read/post the problem situation below and **model** how to determine the quotient using an area model. **Emphasize** what each number represents, as in [LESSON 1](#).

Thomas likes to pack pretzels in his school lunch. The 16-ounce bag his mom purchased contains 1,008 pretzels. If Thomas packs eight pretzels each day, how many days can he have pretzels in his lunch before his mom has to buy a new bag?

Ask students the following questions as you demonstrate how to use an area model with division.

- ▶ What is being divided? What is the total that we are dividing into equal groups? (1,008)
- ▶ Where was the product on the area model when we multiplied? How are the product and dividend the same? (*The product was written on the inside of the area model. The product and the dividend both represent the total amount.*)

Emphasize that because the product and dividend both represent the total amount, the dividend will be written inside the area model where the product was written during multiplication. [Write “1,008” inside the area model.]



1,008

- ▶ How are the 1,008 pretzels divided? Are we determining how many equal groups there are or how many pretzels are in each equal group? (*We are determining how many equal groups there are. We know that there are eight pretzels in each group.*)
- ▶ Since the divisor represents a part of the equal groups, much like a factor in a multiplication expression, where could we write the divisor? Where were the factors on the area model when we multiplied? (*We could write the divisor outside the area model on the left. The factors on an area model are written outside the area model, one factor to the left of the model and one factor above the model.*)

Emphasize that since the divisor represents the size of one equal-size group, like factors in a multiplication expression, we are going to write the divisor outside the area model. [Write “8” on one side of the area model.]

$$\begin{array}{r} 8 \\ \times 1,008 \\ \hline \end{array}$$

- ▶ What is missing on the area model? What are we trying to determine? What does the quotient represent? (*The quotient is missing from the area model. We are trying to determine the quotient. The quotient represents how many days Thomas can have pretzels in his lunch.*)

Emphasize that when you multiply, the two factors that are multiplied are located on the outside of the area model. When you divide, the goal is to determine the missing factor, which indicates how many groups of eight are in 1,008.

Note that there is one group of eight in 10, therefore there must be a minimum of 100 groups of eight in 1000, because 1000 is 100 times more than 10, and 100 is 100 times more than one.

Model writing “100” above the area model, where the second factor would be on the area model if you were multiplying.

$$\begin{array}{r} 100 \\ \times 8 \\ \hline 1,008 \end{array}$$

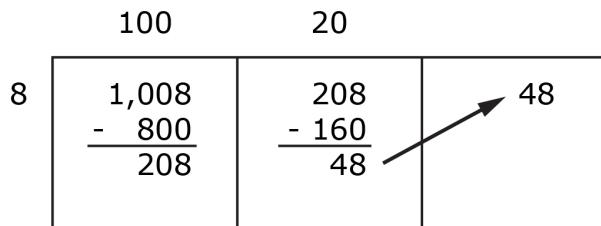
Model multiplying 100 times eight and writing the product inside the area model, underneath the “1,008”. Link this back to the area model for multiplication. Then draw a vertical line in the area model and carry over the remaining after you subtract 800 from 1,008. **Avoid** saying, “How many times does eight go into 1,008?”

$$\begin{array}{r} 100 \\ \times 8 \\ \hline 1,008 \\ - 800 \\ \hline 208 \end{array}$$

Repeat the same process until the difference is zero, meaning there is no remaining value to carry over to the next section.

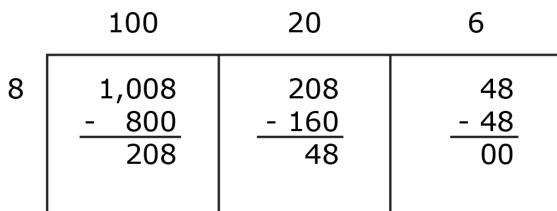
Ask students, “Thinking in terms of multiples of tens only, how many groups of eight make up 208?” (*20 groups of eight*)

Model multiplying 20 times eight and writing the product inside the area model, underneath 208. Link this back to the area model for multiplication. Then draw a vertical line in the area model and carry over the remaining 48 after subtracting 160 from 208. **Avoid** saying, “How many times does eight go into 208?”



Ask students, “How many groups of eight are in 48? Eight times what number equals 48?” (6)

Model multiplying six times eight and writing the product inside the area model, underneath 48. Link this back to the area model for multiplication. **Avoid** saying, “How many times does eight go into 48?”



Explain that the quotient on top of the area model is the decomposed version of the factor in the same position when multiplying using an area model. Therefore, it is necessary to add the partial quotients to determine the total quotient.

$$100 + 20 + 6 = 126$$

Read the problem situation again as well as the full equation, $1,008 \div 8 = 126$.

Ask students the following questions:

- ▶ What is the question asking?
- ▶ Does the quotient you found answer the question?
- ▶ Does the quotient make sense with what the question is asking?

Note that while 126 is the value required to respond to the original question, the context of the question must be included in the student response. Providing only the value is inadequate; students should be required to state that Thomas could take pretzels in his lunch for 126 days before his mom would need to buy a new bag.

Distribute the [INSTRUCTIONAL ACTIVITY STUDENT HANDOUT](#).

Repeat the process for Questions 1 – 4 on the [INSTRUCTIONAL ACTIVITY STUDENT HANDOUT](#).

- ▶ 1. In Room 223, every Friday is “Clean Your Desk Day.” Last Friday, Kelli was taking everything out of her desk and making piles of similar supplies. When she was finished, she ended up with nine piles of different supplies. Kelli noticed that all of her piles had the same number of supplies. When she counted all of her supplies, she had a total of 144 pieces to put back into her desk. How many pieces of each supply did Kelli put back into her desk? (*16 pieces*)
- ▶ 2. Luis is selling candy bars for a school fundraiser. He wants to sell enough to earn the second-place prize, attending a pizza party for lunch. Luis’ goal is to sell 168 candy bars. The candy bars are packaged in boxes of eight. How many boxes of candy bars does Luis have to sell to meet his goal? (*21 boxes*)
- ▶ 3. Gavin and Sam attended a summer sleepaway camp. There are four camp sessions, each two weeks long. There are 1,184 children signed up to attend a session this summer. If the same number of children are in each session, how many children will be in Gavin and Sam’s session, including them? Be sure to write your answer in a complete sentence. (*296 children will be in each session.*)
- ▶ 4. Sarah was given the following problem in her math class: *John is writing a 2,000-word research paper for his science class. He is researching different types of insects in the Amazon rainforest. So far, he has written 1,792 words and has seven sections with subheadings. How many words are in each of John’s sections if all seven sections have the same number of words?* The following is Sarah’s area model [model is shown in Student Handout]. Did she solve the problem correctly? If so, explain how you know. If not, explain what she did wrong and how you know, then provide the correct answer. (*256 words*)

GUIDING QUESTIONS

Determine if the student can **CALCULATE QUOTIENTS USING STRATEGIES OTHER THAN THE STANDARD ALGORITHM**:

- ▶ How do you set up the area model for division?
- ▶ How do you use the area model to determine the quotient?
- ▶ What is a partial quotient?
- ▶ How do you decide which partial quotient to write above the area model?

Determine if the student can **DIVIDE A 4-DIGIT NUMBER BY A 1-DIGIT NUMBER:**

- ▶ What is the dividend? What is the divisor? How do you know?
- ▶ What is the first thing you need to do to simplify the expression $223 \div 9$?
- ▶ Why can you multiply if this is a division problem?
- ▶ Why do you put the first digit of the quotient for $168 \div 8$ in the tens place value and not the hundreds place value?
- ▶ In the expression $1,792 \div 7$, what will be the place value for the first digit of the quotient? How do you know?

Determine if the student can **USE REASONING FOR MULTIPLICATION AND DIVISION:**

- ▶ What is the question asking?
- ▶ How do you know whether or not your answer makes sense?
- ▶ How do you know whether you should divide or multiply when answering a question in a problem situation?

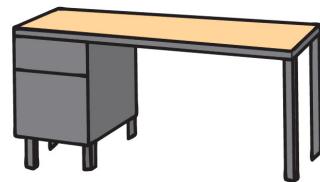
For the Division Game ([INSTRUCTIONAL ACTIVITY SUPPLEMENT](#)), have students play in groups of two or three. Note that the game can also be modified to play in larger groups or whole class. Hand out different color counters, boards, cards, answer sheets, and directions. **Read aloud** and **review** the directions.

At the end of the activity, require students to write a note to an “absent student” describing how to determine whole number quotients using the area model. Provide students with an example expression to scaffold their explanation.

DIVIDING WHOLE NUMBERS

Lesson 2

1. In Room 223, every Friday is "Clean Your Desk Day." Last Friday, Kelli was taking everything out of her desk and making piles of similar supplies. When she was finished, she ended up with nine piles of different supplies. Kelli noticed that all of her piles had the same number of supplies. When she counted all of her supplies, she had a total of 144 pieces to put back into her desk. How many pieces of each supply did Kelli put back into her desk?



Kelli put _____ pieces of each supply back into her desk.

2. Luis is selling candy bars for a school fundraiser. He wants to sell enough to earn the second-place prize, attending a pizza party for lunch. Luis' goal is to sell 168 candy bars. The candy bars are packaged in boxes of eight. How many boxes of candy bars does Luis have to sell to meet his goal?



Luis has to sell _____ boxes of candy bars to meet his goal.

3. Gavin and Sam attended a summer sleepaway camp. There are four camp sessions, each two weeks long. There are 1,184 children signed up to attend a session this summer. If the same number of children are in each session, how many children will be in Gavin and Sam's session, including them? Be sure to write your answer in a complete sentence.



4. Sarah was given the following problem in her math class.

John is writing a 2,000-word research paper for his science class. He is researching different types of insects in the Amazon rainforest. So far, he has written 1,792 words and has seven sections with subheadings. How many words are in each of John's sections if all seven sections have the same number of words?

The following is Sarah's area model. Did she solve the problem correctly? If so, explain how you know. If not, explain what she did wrong and how you know, then provide the correct answer.

100	10	3	
1,792	92	22	100
- 700	- 70	- 21	+ 10
92	22	1	3
7			113

There are 113 words in each of John's seven sections.

DIVIDING WHOLE NUMBERS

INSTRUCTIONAL ACTIVITY SUPPLEMENT

Lesson 2

DIVISION GAME DIRECTIONS

You will need:

- ▶ Division Game board (one per group)
- ▶ Division Game cards (one set per group)
- ▶ Counters or cubes to move around the board (a different color for each student)
- ▶ Answer sheet (one per group)
- ▶ Dice (one per group)

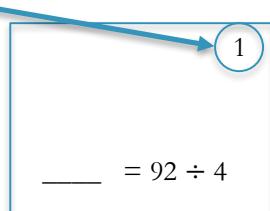
Directions:

1. Place your counter or cube on “Back to School” (the start space).
2. Stack the game cards face down.
3. Roll the die to decide who goes first (highest or lowest number goes first).

When it is your turn:

4. Roll the die. If you answer your problem correctly, the number you rolled is the number of spaces you will move. If you answer incorrectly, do not move any spaces.
5. Take a card from the top of the stack and show your partner. Both of you solve the problem on your sheet of paper using an area model.
6. After you have both finished solving the quotient, check your answers using the answer sheet.
 - a. If you answer correctly, move forward the number of spaces on the board that you rolled with the die. If you answer correctly, your partner does not get to move, even if they answered correctly.
 - b. If you answer incorrectly, do not move. If you answer incorrectly and your partner answers correctly, your partner gets to move your number of spaces.
7. Take turns rolling the die.
8. The first person to reach “Summer Break” (the last space) wins the game.

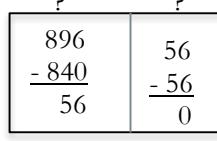
*The small number on the game card matches the number on the answer sheet to make finding the answer easier.



ANSWER SHEET

1.	23	17.	27	33.	56
2.	41	18.	73	34.	77
3.	14	19.	65	35.	36
4.	26 pencils	20.	93	36.	3
5.	16	21.	83 square feet	37.	\$214
6.	33	22.	19	38.	357
7.	240, 24, 4; 44	23.	30; 32	39.	960, 16, 1; 61
8.	68	24.	19	40.	32
9.	51	25.	72	41.	58
10.	231	26.	13	42.	13
11.	19 pictures	27.	85 points	43.	17 fifth-grade classes
12.	16	28.	17	44.	58
13.	38	29.	30, 2; 32	45.	495
14.	24	30.	6	46.	481
15.	82	31.	99	47.	661
16.	16 months	32.	44 times	48.	984 hamburger lunches

<p>1 92 = _____ \times 4</p>	<p>2 $328 \div 8 = _____$</p>	<p>3 _____ = $168 \div 12$</p>	<p>4 Ms. James bought 130 pencils for her fifth-grade class. She gave each student five pencils to start the year. How many students are in Ms. James' class?</p>						
<p>5 $17 \times _____ = 272$</p>	<p>6 $7 \overline{)231}$</p>	<p>7 Finish the area model to determine the quotient.</p> <table style="margin-left: auto; margin-right: auto;"> <tr> <td style="text-align: right;">40</td> <td style="text-align: center;">?</td> </tr> <tr> <td style="text-align: right;">264</td> <td style="text-align: center;">?</td> </tr> <tr> <td style="text-align: right;">- ?</td> <td style="text-align: center;">?</td> </tr> </table>	40	?	264	?	- ?	?	<p>8 Two hundred four divided by three</p>
40	?								
264	?								
- ?	?								
<p>9 _____ \times 9 = 459</p>	<p>10 $4 \overline{)924}$</p>	<p>11 For her Science Fair project, Mari took 456 pictures for her presentation. She only used $\frac{1}{24}$ of the pictures she took. How many pictures did Mari use on her board?</p>	<p>12 $320 \div 20 = _____$</p>						
<p>13 Four hundred ninety-four divided into equal groups of thirteen</p>	<p>14 $144 = 6 \times _____$</p>	<p>15 $11 \overline{)902}$</p>	<p>16 Marcus wants to save \$864 for a new computer. If he saves \$54 a month, how many months will he have to save before he has enough money?</p>						

<p>17</p> <p>$594 \div 22 = \underline{\hspace{2cm}}$</p>	<p>18</p> <p>9 </p>	<p>19</p> <p>Seven hundred eighty divided by twelve</p>	<p>20</p> <p>$8 \times \underline{\hspace{2cm}} = 744$</p>
<p>21</p> <p>Sally was helping her mother make twelve blankets for the school craft fair. They bought 996 square feet of material. How many square feet of material will each blanket need if they are all the same size?</p>	<p>22</p> <p>47 </p>	<p>23</p> <p>Finish the area model to determine the quotient.</p> <p>24</p> <p>$114 = \underline{\hspace{2cm}} \times 6$</p>	
<p>25</p> <p>Two hundred sixteen divided by three</p>	<p>26</p> <p>61 </p>	<p>27</p> <p>Toby and Ashton were competing for points in their online video game. At the end of the first week, Ashton had eleven times more points than Toby. If Ashton had 935 points, how many did Toby have?</p>	<p>28</p> <p>$\underline{\hspace{2cm}} = 731 \div 43$</p>
<p>29</p> <p>Finish the area model to determine the quotient.</p> <p>28</p> <p></p>	<p>30</p> <p>$\underline{\hspace{2cm}} \times 52 = 312$</p>	<p>31</p> <p>9 </p>	<p>32</p> <p>During a science experiment, lab partners needed to record data every twenty-two seconds for 968 seconds during their third-period science class. How many times did the lab partners record data?</p>

<p style="text-align: right;">33</p> $8 \times \underline{\quad} = 448$	<p style="text-align: right;">34</p> $11 \boxed{847}$	<p style="text-align: right;">35</p> <p>Nine hundred seventy-two divided by twenty-seven</p>	<p style="text-align: right;">36</p> $276 \div 92 = \underline{\quad}$
<p style="text-align: right;">37</p> <p>Lucy and her two siblings always received birthday money from their grandparents. If Lucy's grandparents put aside \$642 for birthday money, how much will Lucy and each of her siblings receiving on their birthdays?</p>	<p style="text-align: right;">38</p> $\underline{\quad} \times 2 = 714$	<p style="text-align: right;">39</p> <p>Finish the area model to determine the quotient.</p> $\begin{array}{r} 60 & ? \\ 976 & ? \\ \hline 16 & \underline{\quad} \end{array}$	<p style="text-align: right;">40</p> $\underline{\quad} = 992 \div 31$
<p style="text-align: right;">41</p> <p>Five hundred twenty-two divided into nine equal groups</p>	<p style="text-align: right;">42</p> $75 \boxed{975}$	<p style="text-align: right;">43</p> <p>The school district has five hundred sixty-one fifth-grade math textbooks. Each class receives thirty-three math textbooks. How many fifth-grade classes are in the district?</p>	<p style="text-align: right;">44</p> $26 = 1,508 \div \underline{\quad}$
<p style="text-align: right;">45</p> <p>Finish the area model to determine the quotient.</p> $3 \boxed{1,485} \boxed{\quad} \boxed{\quad}$	<p style="text-align: right;">46</p> $5,772 \div \underline{\quad} = 12$	<p style="text-align: right;">47</p> $4 \overline{)2,644}$	<p style="text-align: right;">48</p> <p>The school cafeteria serves 6 French fries with each hamburger lunch. If the cafeteria orders 5,904 French fries, how many hamburger lunches can they serve?</p>

Back to school	Winter break		
Fall break			
Spring break			

DIVIDING WHOLE NUMBERS

INSTRUCTIONAL ACTIVITY

Lesson 3

LEARNING GOAL

When presented with situations involving up to four-digit dividends and two-digit divisors, students will determine whole-number quotients using strategies based on the distributive property.

PRIMARY ACTIVITY

Students will determine whole number quotients using the distributive property and the relationship between multiplication and division.

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

- ▶ Distributive property
 - ▶ Dividend
 - ▶ Divisor
 - ▶ Quotient
 - ▶ Factor
 - ▶ Multiple
 - ▶ Addend
 - ▶ Sum
-

MATERIALS

- ▶ [INSTRUCTIONAL ACTIVITY STUDENT HANDOUT](#)
 - ▶ Word version [INSTRUCTIONAL ACTIVITY STUDENT HANDOUT](#)
 - ▶ Dry erase boards
 - ▶ Dry erase markers
 - ▶ Pencils
 - ▶ Paper
-

IMPLEMENTATION

Begin the lesson by **asking** students the following questions:

- ▶ “How are multiplication and division related?” (*They are inverse operations.*)
- ▶ “How can multiplication help you divide?”
- ▶ “What is the distributive property?”

Display the expression $6 \times (9 + 3)$.

Ask students, “How would the distributive property help you simplify this expression?” (*Multiply six times nine and six times three, then add the products together. $6 \times 9 + 6 \times 3 = 54 + 18 = 72$*)

Ask students if they could use the distributive property to simplify division expressions. **Allow** students to discuss in partners how they would use the distributive property to determine the quotient to $72 \div 6$.

Select student volunteers to share how the distributive property can be used to simplify $72 \div 6$. **Allow** students to model their thinking on the board.

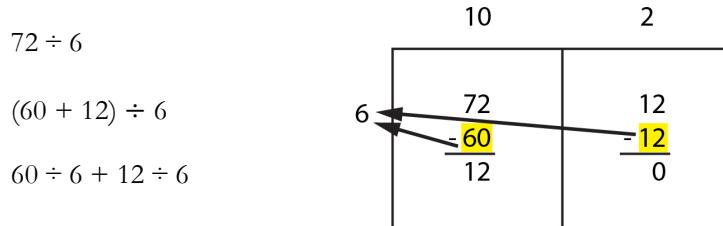
If there are no successful models of how to simplify the expression using the distributive property, **explain** and **model** a correct sequence using an area model and the distributive property.

Break apart 72 into numbers that are easily divided by six. One possibility is $60 + 12$.

Note that multiples of tens, hundreds, thousands, etc. are often easy to divide.

NOTE: The beginning of this lesson would be a timely opportunity to explain that using the distributive property when dividing provides students with a strategy for mentally dividing larger numbers. For example, when a larger number is decomposed into “easier” numbers for the divisor, it enables a student to do the division in their head without the aid of writing it down or using a method requiring visuals or manipulatives.

Model the distributive property for division, rewriting 72 as $60 + 12$ and showing the division of both addends by six.



Ask students to simplify the expression. **Remind** students to attend to the order of operations as they simplify.

$60 \div 6 + 12 \div 6$	10	+	2						
10 + 2	6								
12									
<table border="1" style="margin: auto; border-collapse: collapse; width: fit-content;"> <tr> <td style="padding: 5px; text-align: center;">72</td> <td style="padding: 5px; text-align: center;">12</td> </tr> <tr> <td style="padding: 5px; text-align: center;">- 60</td> <td style="padding: 5px; text-align: center;">- 12</td> </tr> <tr> <td style="padding: 5px; text-align: center;"><hr/>12</td> <td style="padding: 5px; text-align: center;"><hr/>0</td> </tr> </table>				72	12	- 60	- 12	<hr/> 12	<hr/> 0
72	12								
- 60	- 12								
<hr/> 12	<hr/> 0								

NOTE: The area model leads into the standard algorithm in Lesson 4. Modeling the area model alongside the distributive property throughout this lesson is optional to support that transition.

Require students to answer questions to simplify $3,304 \div 14$ using the distributive property as previously modeled. **Write** each step on the board as the questions are answered.

- ▶ “Thinking of multiples of 10, 100, 1,000, what is an addend of 3,304 that could easily divide by 14?” [Students will likely suggest $14 \times 100 = 1,400$. While this is acceptable, it would require more steps to reach the quotient. It is recommended to keep questioning students to lead them to $14 \times 200 = 2,800$.]
- ▶ “What is left to divide?” ($3,304 - 2,800 = 504$)
- ▶ “What is an addend of 504 that could easily divide by 14?” ($14 \times 30 = 420$)
- ▶ “What is left to divide?” ($504 - 420 = 84$)
- ▶ “What is an addend of 84 that could easily divide by 14?” ($14 \times 6 = 84$)
- ▶ “How did we decompose 3,304?” ($2,800 + 420 + 84$)

Require a student to write each step of the equation for the class using the distributive property.

$$\begin{aligned}
 3,304 \div 14 &= (2,800 + 420 + 84) \div 14 \\
 &= 2,800 \div 14 + 420 \div 14 + 84 \div 14 \\
 &= 200 + 30 + 6 \\
 &= 236
 \end{aligned}$$

Require students to complete Question 1 on the [INSTRUCTIONAL ACTIVITY STUDENT HANDOUT](#).

-
1. Brad and Petra were volunteering to clean up a local park. They were in charge of building a sandbox for children to use. The area of the sand box is 2,704 square inches. If the sand box is 26 inches wide, how long is the sand box?”
-

Ask students the following questions:

- ▶ “What division problem is modeled in this situation?” ($2,704 \div 26$) [Write the problem in the first set of blanks.]

- ▶ “What is the total area that is divided by 26?” (2,704) [Write 2,704 in the first blank under the model.]
- ▶ “The dividend will be decomposed into how many addends?” (*two addends, the two blanks separated by an addition sign just under the model*)
- ▶ “Thinking of multiples of 10, 100, and 1,000, what is an addend of 2,704 that could easily divide by 26?” (2,600; $26 \times 100 = 2,600$) [Write the first addend, 2,600, in the appropriate blank.]
- ▶ “What is the remaining portion of the dividend that needs to be divided by 26?” (104)
- ▶ “Can you divide 104 by 26? Or do we need to consider addends of 104?” (*Yes, you can divide 104 by 26; $104 \div 26 = 4$*) [Write the second addend, 104, in the appropriate blank.]
- ▶ “Look at the next line of blanks. Explain how to use the distributive property to fill in the blanks.” ($2,600 \div 26 + 104 \div 26$)
- ▶ “Complete the new line of blanks, simplifying using the distributive property.” (100 + 4)
- ▶ “Now, add the addends, what is the sum?” (104)
- ▶ “Look at the first set of blanks, where you wrote the division problem. What is the quotient for $2,704 \div 26$?” (104) [Write 104 in the space for the quotient.]
- ▶ “What is the corresponding multiplication problem for this set of numbers?” ($26 \times 104 = 2,704$) [Complete the second row of blanks with the multiplication problem.]

Require students to complete Questions 2 – 7 on the [INSTRUCTIONAL ACTIVITY STUDENT HANDOUT](#) using the distributive property. Use the guiding questions to scaffold student learning.

- ▶ 2. Franco’s family was having a yard sale. They lived in the middle of a large neighborhood, and Franco wanted to make sure people didn’t get lost on the way to the yard sale. Their house is 1,551 yards from the neighborhood’s south entrance. Franco wanted to place a direction sign every 33 yards. How many signs will Franco need to make?

$$\begin{aligned} 1,551 \div 33 &= (1,320 + 231) \div 33 \\ &= 1,320 \div 33 + 231 \div 33 \\ &= 40 + 7 \\ &= 47 \end{aligned}$$

- ▶ 3. Fill in the blanks.

$$\begin{aligned} 28 \times 96 &= 2,688 \\ 2,688 \div 96 &= (1,920 + 768) \div 96 \\ &= 1,920 \div 96 + 768 \div 96 \\ &= 20 + 8 \\ &= 28 \end{aligned}$$

- ▶ 4. Ruby is saving her babysitting money to buy a car when she turns 16. So far she has saved \$3,392. If Ruby has been saving for 53 weeks, how much does Ruby earn each week? Show your working using the distributive property. (*\$64 a week*)

- ▶ 5. Justin and his friends went to Game Center on Saturday to celebrate his birthday. Justin wanted to earn 1,000 tickets to trade in for a key chain with his favorite video game character. So far Justin has played 18 games and earned 882 tickets. If Justin earned the same number of tickets each game, how many tickets has he earned per game? How many more games does he need to play to earn 1,000 if he earns the same amount of tickets each game? Show your work using the distributive property. (*49 tickets a game; three more games*)

- ▶ 6. Review Nicole's work and solution to the following problem.

Kai has been saving the same amount of money every month for a new computer. Kai has been saving for 12 months. So far, he has saved \$648. How much money has Kai saved?

Nicole's work:

$$\begin{aligned} a \times 12 &= 648 \\ 648 \div 12 &= a \\ a &= (600 + 48) \div 12 \\ a &= 600 \div 12 + 48 \div 12 \\ a &= 5 + 4 \\ a &= 9 \end{aligned}$$

Kai has saved \$9 so far.

Is Nicole's answer correct? If so, how do you know? If not, what was Nicole's error, and what is the correct answer? (*Nicole's work is incorrect. She wrote that $600 \div 12$ is 5 instead of 50, which made her answer \$9 instead of \$54. Check student explanations for understanding.*)

- ▶ 7. Use the distributive property to answer the following questions.
 - 7.a. How many different ways can you decompose 4,475 in order to divide by 5? (*Check student work for accuracy and understanding.*)
 - 7.b. To divide by 5 using the distributive property, could you decompose 4,475 as $500 + 500 + 500 + 500 + 500 + 500 + 500 + 500 + 200 + 200 + 70 + 5$? Explain. Would this be an efficient way to decompose 4,475 to divide? Why or why not? (*Check student work for understanding. The following is an example answer: Yes, you could decompose 4,475 this way, but no, it would not be an efficient way, because you are dividing 500 by 5 eight times then dividing 200 by 5 two times, and that is a lot of partial quotients to keep track of; overall you would divide 12 times before you added the partial quotients.*)

GUIDING QUESTIONS

Elicit student thinking:

- ▶ How would you describe multiplication?
- ▶ How would you describe division?

Determine if the student can [EXPLAIN THE DISTRIBUTIVE PROPERTY](#):

- ▶ What is the distributive property?
- ▶ What are the benefits of using the distributive property?

Determine if the student can [APPLY THE DISTRIBUTIVE PROPERTY](#):

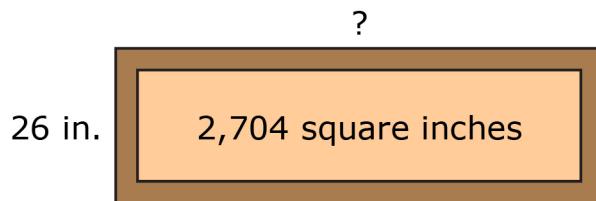
- ▶ If $3 \times (10 + 2) = 30 + 6$, how would you decompose 36 in $36 \div 3$ using distributive property?
- ▶ After you divide each addend by the divisor, what do you do to determine the quotient for the original expression?
- ▶ If $1,551 \div 33 = (1,320 + 231) \div 33$, how would you use the distributive property to simplify the expression?
- ▶ How would you decompose 3,392 so that it is easier to divide by 53? Which multiple of 10 can you multiply by 53 for an addend?

At the end of the activity, teachers should require students to answer Question 8 on the [INSTRUCTIONAL ACTIVITY STUDENT HANDOUT](#) as a check for understanding. Students should circle the choice that best describes how they feel about dividing using the distributive property and the relationship between multiplication and division.

DIVIDING WHOLE NUMBERS

Lesson 3

1. Brad and Petra were volunteering to clean up a local park. They were in charge of building a sandbox for children to use. The area of the sandbox is 2,704 square inches. If the sandbox is 26 inches wide, how long is the sandbox?



$$\underline{\hspace{2cm}} \div \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

$$\underline{\hspace{2cm}} \times \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

$$\underline{\hspace{2cm}} \div 26$$

$$= \underline{\hspace{2cm}} + \underline{\hspace{2cm}} \div 26$$

$$= \underline{\hspace{2cm}} \div \underline{\hspace{2cm}} + \underline{\hspace{2cm}} \div \underline{\hspace{2cm}}$$

$$= \underline{\hspace{2cm}} + \underline{\hspace{2cm}}$$

$$= \underline{\hspace{2cm}}$$

2. Franco's family was having a yard sale. They lived in the middle of a large neighborhood, and Franco wanted to make sure people didn't get lost on the way to the yard sale. Their house is 1,551 yards from the neighborhood's south entrance. Franco wanted to place a direction sign every 33 yards. How many signs will Franco need to make?



$$\underline{\hspace{2cm}} \div \underline{\hspace{2cm}}$$

$$= (\underline{\hspace{2cm}} + \underline{\hspace{2cm}}) \div \underline{\hspace{2cm}}$$

$$= \underline{\hspace{2cm}} \div \underline{\hspace{2cm}} + \underline{\hspace{2cm}} \div \underline{\hspace{2cm}}$$

$$= \underline{\hspace{2cm}} + \underline{\hspace{2cm}}$$

$$= \underline{\hspace{2cm}}$$

3. Fill in the blanks.

$$\underline{\hspace{2cm}} \times 96 = 2,688$$

$$\underline{\hspace{2cm}} \div \underline{\hspace{2cm}}$$

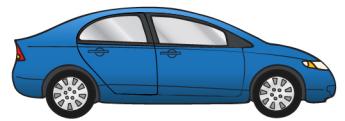
$$= (\underline{\hspace{2cm}} + 768) \div \underline{\hspace{2cm}}$$

$$= \underline{\hspace{2cm}} \div \underline{\hspace{2cm}} + 768 \div \underline{\hspace{2cm}}$$

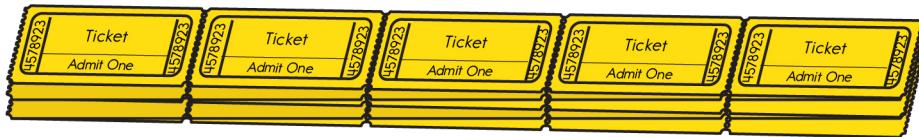
$$= 20 + \underline{\hspace{2cm}}$$

$$= \underline{\hspace{2cm}}$$

4. Ruby is saving her babysitting money to buy a car when she turns 16. So far, she has saved \$3,392. If Ruby has been saving for 53 weeks, how much does Ruby earn each week? Show your working using the distributive property.



5. Justin and his friends went to Game Center on Saturday to celebrate his birthday. Justin wanted to earn 1,000 tickets to trade in for a key chain with his favorite video game character. So far Justin has played 18 games and earned 882 tickets. If Justin earned the same number of tickets each game, how many tickets has he earned per game? How many more games does he need to play to earn 1,000 if he earns the same amount of tickets each game? Show your work using the distributive property.



6. Review Nicole's work and solution to the following problem.

Kai has been saving the same amount of money every month for a new computer. Kai has been saving for 12 months. So far, he has saved \$648. How much money has Kai saved?



Nicole's work:

$$\begin{aligned}a \times 12 &= 648 \\648 \div 12 &= a \\a &= (600 + 48) \div 12 \\a &= 600 \div 12 + 48 \div 12 \\a &= 5 + 4 \\a &= 9\end{aligned}$$

Kai has saved \$9 so far.

Is Nicole's answer correct? If so, how do you know? If not, what was Nicole's error, and what is the correct answer?

7. Use the distributive property to answer the following questions.

7.a. How many different ways can you decompose 4,475 in order to divide by 5?

7.b. To divide by 5 using the distributive property, could you decompose 4,475 as $500 + 500 + 500 + 500 + 500 + 500 + 500 + 500 + 200 + 200 + 70 + 5$? Explain. Would this be an efficient way to decompose 4,475 to divide? Why or why not?

8. Circle the choice that best describes how you feel about dividing whole numbers using the distributive property and the relationship between multiplication and division.

STOP – I am totally confused. I do not understand.

SLOW DOWN – I think I have got it, but I could use some more practice or clarification.

GO! – I've got it! I am ready to move on.

DIVIDING WHOLE NUMBERS

INSTRUCTIONAL ACTIVITY

Lesson 4

LEARNING GOAL

Students will determine whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value. Students will illustrate and explain the calculation by using equations.

PRIMARY ACTIVITY

Students will determine whole number quotients using computation with place value and estimation.

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

- ▶ Dividend
 - ▶ Divisor
 - ▶ Quotient
 - ▶ Remainder
 - ▶ Factor
 - ▶ Multiple
-

MATERIALS

- ▶ [INSTRUCTIONAL ACTIVITY STUDENT HANDOUT](#)
- ▶ Word version [INSTRUCTIONAL ACTIVITY STUDENT HANDOUT](#)
- ▶ Dry erase boards and markers, or paper and pencils

IMPLEMENTATION

Draw an area model on the board as students create an area model on their dry erase board or paper.

Share the following problem situation.

Jessa works part time after school and on weekends at her favorite clothing store. She earns \$14 an hour. If Jessa's most recent paycheck was \$322, how many hours did Jessa work that week?

Require students to set up the area model for the problem situation.

Ask students the following questions and **write** the answers on the board for each question.

- ▶ “What type of expression is required to determine the solution to the problem situation? How do you know?” (*division; because the total money she earns is \$322 and the size of the groups is \$14, you need to determine the number of equal size groups, which is how many hours a week Jessa works*)
- ▶ “What is the division problem we can set up to solve this problem?” ($322 \div 14$)
- ▶ “What is the dividend?” (322) “Where on the area model do I write ‘322?’” (*inside the area model*)
- ▶ “What is the divisor?” (14) “Where on the area model do you write ‘14?’” (*outside the area model on the left side*)

NOTE: When evaluating division with partial quotients, students can use any partial quotients that make sense to them. Some students may use 10s, 5s, and 2s repeatedly to determine the total quotient. As long as students correctly determine the total quotient, the partial quotients can be any combination resulting in the correct total quotient.

Model how to determine partial quotients by creating a list of multiples. To the side of the area model, create a list of the divisor times multiples of 10, since 140 is less than the dividend and 1,400 is larger than the dividend.

Explain to students the process as you proceed through the steps. This list of multiples will help students easier determine a partial quotient. **Emphasize** that you stop the list as soon as your product is greater than the dividend.

14	322	
----	-----	--

$$\begin{aligned}14 \times 10 &= 140 \\14 \times 20 &= 280 \\14 \times 30 &= 420\end{aligned}$$

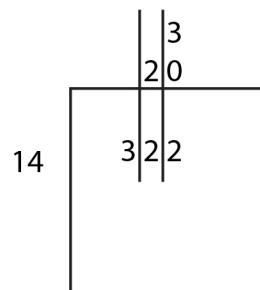
Ask students for a partial quotient: “14 times what number would be close to 322?” (*Students can respond with either 10 or 20, but the example is set up with 20. If the students respond with 10, it is accurate to write ‘10’ and repeat the process twice, resulting in two addends of 10, which simplifies to 20. Alternatively, you could guide students to a partial quotient of 20.*)

Continue to simplify $322 \div 14$ by having the students guide you to the quotient. Check the solution with the problem situation to ensure it answers the question and is reasonable.

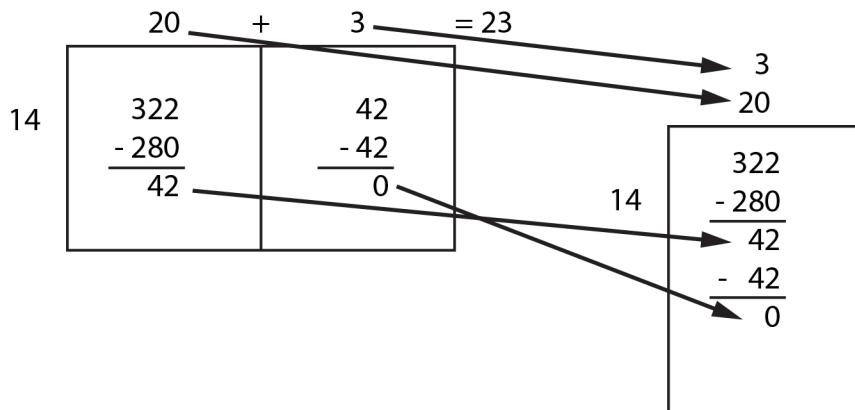
Draw a new area model vertically, so it looks similar to the setup of the standard algorithm.

Explain that like multiplication, there is a more efficient way to calculate the quotient than the horizontal area model.

Emphasize as you complete the vertical area model that the place values are aligned between the quotient and the dividend. The ‘0’ and the ‘3’ are in line with the ones place above the ‘2’ in ‘322’, and the ‘2’ is in line with the tens place above the ‘2’ for ‘20’ in ‘322’. This is important to transition to the standard algorithm.



Make known that instead of adding the partial quotients horizontally, as you would with the area model, the partial quotients stacked vertically will be added to determine the total quotient. This is important as students transition to the standard algorithm and partial quotients will not be written in their entirety, but as digits in a specified place value. At this point, addition will no longer take place above the division bar.



Model the next problem situation using the vertical area model.

Share the following problem situation.

Kamryn's class is learning about Westward Expansion and the Santa Fe Trail in social studies. Each student has to create a diary detailing their experiences as if they were traveling along the trail with their family. The total distance of the route from Fort Leavenworth, Kansas, to Santa Fe, New Mexico, is 834 miles. Kamryn decided that her family would choose to settle six miles outside of Santa Fe, making her traveling distance 828 miles. If Kamryn writes in her diary that her family traveled 12 miles a day. How many days would it have taken Kamryn and her family to travel from Fort Leavenworth to their new homestead?

Ask students [write the answers on the board as you ask each question]:

- ▶ “What operation needs to be performed to determine the answer to the situation? How do you know?” (*division, because you are given the total distance and the distance travelled each day, and the question asks how many days; The total number of days is the number of equal groups, and the distance each day is the size of the group.*)
- ▶ “What is the division problem we need to set up?” ($828 \div 12$)
- ▶ “What is the dividend?” (828) “Where on the area model do I write ‘828?’” (*inside the area model*)
- ▶ “What is the divisor?” (12) “Where on the area model do you write ‘12?’” (*outside the area model on the left side*)

Require students to help complete the model on the board, as they complete the model on their boards/papers. You may leave the area model box open (looking more like the division box used with the standard algorithm for long division) since the length of the subtraction problems is unknown. This will also support the transition from area model to standard algorithm.

Emphasize as you complete the area model that the place values are aligned between the quotient and the dividend.

$$\begin{array}{r}
 & & 9 \\
 & & 6 0 \\
 \boxed{1} & 2 & | & 8 & 2 & 8 \\
 & - & 7 & 2 & 0 \\
 & & & 1 & 0 & 8 \\
 & & - & 1 & 0 & 8 \\
 & & & & & 0
 \end{array}$$

GUIDING QUESTIONS

Elicit student thinking:

- ▶ How do you know if a problem situation is asking you to divide?
- ▶ How are multiplication and division related?
- ▶ How is the area model for multiplication the same as the area model for division? How are they different?

Determine if the student can **CALCULATE QUOTIENTS USING STRATEGIES OTHER THAN THE STANDARD ALGORITHM**:

- ▶ How you would set up an area model for $504 \div 14$?
- ▶ What is a partial quotient? Explain using one of your problem situations.
- ▶ How do you determine the total quotient?

Determine if the student can **EXPLAIN THE RELATIONSHIP BETWEEN PLACE VALUE AND DIVISION**:

- ▶ In the example $828 \div 12$, why is it important to write the partial product 60 above the same place values, 28 from 828, in the dividend?
- ▶ Is the partial quotient six the same as 60? Why or why not?

Hand out the **INSTRUCTIONAL ACTIVITY STUDENT HANDOUT**. **Require** students to complete Questions 1 – 4.

- ▶ 1. A large bag of mini chocolate candies contains 504 individual pieces. Leo wants to split the bag into smaller baggies so he can take some to work each day to share with his coworkers. It took Leo 14 baggies to divide up the large bag of candies equally. How many mini chocolate candies will Leo take to work each day? Show all your work, and write your answer in a complete sentence. (*36 pieces*)
- ▶ 2. Determine the missing value in the equation $\square \times 27 = 891$. Show all your work. (*The missing value is 33.*)
- ▶ 3. Abby and her sister spent the day picking blueberries at their aunt and uncle's farm. They filled up 16 pails total, with the same number of blueberries in each pail. When they returned to the farm house, they cleaned and counted the berries. Together they had picked 1,680 blueberries.

How many blueberries were in each pail? Show all your work, and write your answer in a complete sentence. (*There were 105 blueberries in each pail.*)

- 4. Bob's Fish Shop has a large variety of fish available for purchase. Some of the fish are kept in 22-quart tanks. On cleaning day, Bob uses 1,144 quarts of water to refill all the 22-quart tanks. How many 22-quart tanks does Bob need to refill on cleaning day? Show all your work, and write your answer in a complete sentence. (*Bob needs to refill 52 tanks.*)

To **review** the answers for Questions 1 – 4, have a student share and model how they simplified the problem situation.

Require students to check their answer with the problem situation consistently to make sure their answer makes sense and is reasonable.

Ask students, “Do you think we can make this process more efficient? Think back to multiplication. Did we have to write all of the partial products aligned vertically?”

Explain that, similar to multiplication, you do not have to write out each individual partial quotient.

Share the following problem situation.

Three fifth grade classes took a field trip to the Old Theater in the city to see a play. The theater has 882 seats, and there are 21 seats in each row. How many rows of seats are in the Old Theater?

Ask students:

- “What is the dividend?” (882)
- “What is the divisor?” (21)
- “What is the division problem that needs to be set up?” ($882 \div 21$)

Write the problem on the board twice, using a long division symbol for each problem. The first instance of the problem will be used to complete the division problem as modeled in the previous problems. The second instance will be used to model the standard algorithm and compare the methods of division.

Emphasize the place values of the dividend.

$$\begin{array}{r}
 & \text{H} \mid \text{T} \mid \text{O} \\
 21 & \overline{)882}
 \end{array}$$

Explain that the divisor can be rounded to determine an estimate of approximately how many groups of 21 the dividend, 882, can be divided into.

Ask students, “The divisor could be rounded to what number?” (20)

Explain that 882 can be divided into 80 groups of 10. **Ask** students, “If 20 is twice as much as 10, how many groups of 20 would 882 be divided into?”

Note that if you double one number, you need to halve the other. Therefore, 882 can be divided into 40 groups of 20. Write a ‘4’, representing ‘40’, above the ‘8’ in the tens column.

Continue to simplify the problem by asking, “How many is 40 groups of 21?” (840) Then write “840” under “882”.

Explain that you are trying to determine how many groups of 21 are in 882. If 40 groups of 21 equals 840, how much is left to divide into groups? (42)

Write the subtraction on the board to show that you still need to determine how many groups of 21 42 can be divided into (2). Write “2” in the ones column above the “2” from “882”.

Continue to simplify the problem. **Write** the subtraction to show that 42 is two groups of 21 with no remainder.

Check the solution with the problem situation to ensure it answers the question and is reasonable.

Read Question 5 on the [INSTRUCTIONAL ACTIVITY STUDENT HANDOUT](#).

Four fifth-grade classes went on their weekly walk to the lake to gather plant and water specimens for their science project. There were 42 groups, each with two students. Each group was assigned to bring back the same number of specimens. Once all the classes returned to the school, they went through their collections. Overall there were 672 specimens collected. How many specimens was each group assigned to collect? Show all your work, and write your answer in a complete sentence.

Require students to identify the dividend and divisor, then set up the division problem. ($672; 42; 672 \div 42$)

One strategy to identify how many equal groups of 42 make up 672 is to multiply the divisor by multiples of tens (since multiplying by multiples of 100 would be too large) to determine how many tens to write in the tens column for the quotient.

Require students to determine the greatest multiple of ten without going over the dividend, 672. (1)

$$42 \times 10 = 420$$

$$42 \times 20 = 840 \text{ (too large; } 840 > 672\text{)}$$

Ask students the following questions:

- ▶ “What is the place value of the ‘1’ in 10?” (*tens*)
- ▶ “Where should the ‘1’ from 10 be written in the quotient?” (*in the tens column above the seven of 672*)
- ▶ “What is the total product of 42 groups of 10?” (420)
- ▶ “Where do we represent 420?” (*under 672*)
- ▶ “What is remaining? What still needs to be divided into the 42 groups?” (252)

$$\begin{array}{r}
 42 \\
 - 420 \\
 \hline
 252
 \end{array}$$

Ask students, “What numbers can 42 and 252 be rounded to, in order to determine approximately how many groups of 42 are in 252?” (*40 and 250 or 200*)

Encourage students to determine about how many groups of 40 are in 250 by relating to four and 25. **Note** that if you divide both values by 10, they are easier numbers to consider. [The estimate should be five or six.]

Once students have estimated, **require** them to determine the exact product of five or six and 42. If a student uses five, point out that they have not exceeded 252. Therefore, it is possible that they could add one more group of 42.

Students who used five should try adding another group of 42 to determine the product that is as close to 252 as possible without going over.

When determining partial quotients, students should continue multiplying until they have met or exceeded the dividend or partial dividend. **Bring** students’ attention to the fact that if a product exceeds the divided, then the factor that was multiplied by the divisor is too large and the student must use the next lowest factor as the partial quotient.

Remind students of the inverse relationship between multiplication and division (if the divisor and the quotient are multiplied and the product is larger than the dividend, there is an error). When multiplying, each factor must be equal to the divisor and the quotient of the division equation. Likewise, the product of the multiplication equation must be equal to the dividend of the division equation. For example:

$$\begin{array}{r}
 & 45 \\
 & \swarrow \\
 6 \times & 45 = 270 \\
 & \swarrow \\
 & 6 \quad | \\
 & 270 \\
 \boxed{6 \times 45 \neq 270}
 \end{array}$$

Ask students the following questions:

- ▶ “What is the place value of ‘6’, for the six groups of 42?” (*ones*)
- ▶ “Where is the ‘6’ written in the quotient?” (*in the ones column above the two from 672 in the dividend*)
- ▶ “What is the total product of six groups of 42?” (*252*)
- ▶ “Is there anything remaining?” (*zero*)

42

	1	6	
	6	7	2
-	4	2	0
	2	5	2
-	2	5	2
			0

Ask students if the solution answers the problem situation and if it is reasonable.

GUIDING QUESTIONS

Determine if the student can **DIVIDE A 4-DIGIT NUMBER BY A 2-DIGIT NUMBER:**

- ▶ What is the dividend? What is the divisor? How do you know?
- ▶ What is the first thing you need to do to simplify $9,557 \div 19$?
- ▶ How can rounding help when dividing with a two-digit divisor?
- ▶ [Refer to Question 7 on the [INSTRUCTIONAL ACTIVITY STUDENT HANDOUT](#)] Why do you multiply 57 times a value when determining a quotient?
- ▶ [Refer to Question 7 on the [INSTRUCTIONAL ACTIVITY STUDENT HANDOUT](#)] Why do you subtract $2,052 - 1,710$? If this is a division problem, why do you use subtraction?
- ▶ In the expression $9,557 \div 19$, what will be the place value of the first digit of the quotient? How do you know?

Students should complete Questions 6 – 10 on the [INSTRUCTIONAL ACTIVITY STUDENT HANDOUT](#). Allow students to stack their partial quotients as needed in order to scaffold their learning. Although the standard algorithm is the most efficient method for dividing, it is not necessary that all students divide in this way. It is most important that students are able to conceptualize the meaning behind division processes and divide accurately.

- ▶ 6. Evan researched African elephants for his science report. He researched 19 animal reserves in eight different countries in Africa. Combined, the reserves have a total African elephant population of 9,557. If there are the same number of African elephants at each animal reserve, how many elephants live at each reserve? (*503 elephants live at each reserve.*)

- 7. Complete the division problem by determining the value of each of the question marks. Show all your work.

57

			3	6
	2	0	5	2
-	1	7	1	0
	0	3	4	2
-		3	4	2
		0	0	0

- 8. The fifth grade advanced history class went on a trip to the state capital to see the state government in session. There were 28 students going on the trip. The total cost for the trip, including the bus and the hotel rooms for two nights, was \$2,688. If each student had to pay the same amount, how much did each student need to pay to attend the trip to the state capital? Show all your work, and write your answer in a complete sentence. (*Each student needed to pay \$96.*)
- 9. Determine the missing value in the equation $4,524 = \square \times 58$. Show all your work.

58

			7	8
	4	5	2	4
-	4	0	6	0
	0	4	6	4
-	4	6	4	
	0	0	0	0

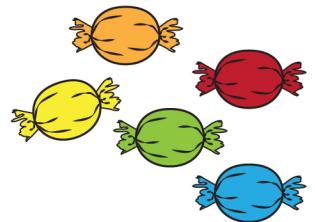
- 10. Mr. Henry is getting his classroom ready for the new school year and needs 200 paper clips for different projects. He goes to the office to get paper clips. The office had ordered one large case containing 83 boxes of paper clips. The large case of paper clips contains 7,802 paper clips. How many paper clips are in each box, assuming all boxes have the same number of paper clips? How many boxes of paper clips will Mr. Henry need for his projects? Show all your work, and write your answer in a complete sentence. (*Each box contains 94 paper clips. Mr. Henry will need three boxes.*)

At the end of the activity, **require** students to write a journal entry, on paper or in a journal, explaining how to evaluate a division expression using the standard algorithm. Use the equation $294 \div 21 = 14$ as a reference for student explanations.

DIVIDING WITH WHOLE NUMBER QUOTIENTS

Lesson 4

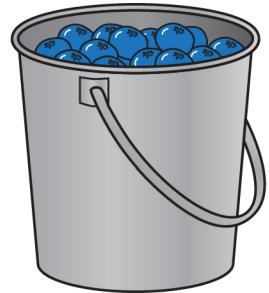
1. A large bag of mini chocolate candies contains 504 individual pieces. Leo wants to split the bag into smaller baggies so he can take some to work each day to share with his coworkers. It took Leo 14 baggies to divide up the large bag of candies equally. How many mini chocolate candies will Leo take to work each day? Show all your work, and write your answer in a complete sentence.



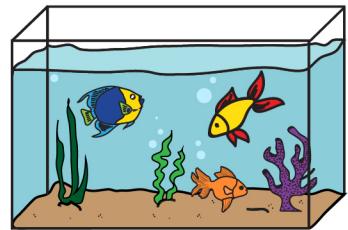
Name_____

2. Determine the missing value in the equation $\square \times 27 = 891$. Show all your work.

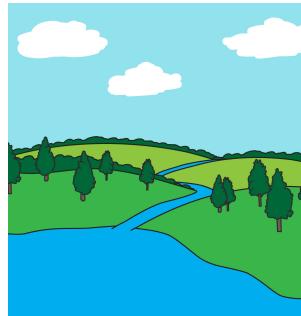
3. Abby and her sister spent the day picking blueberries at their aunt and uncle's farm. They filled up 16 pails total, with the same number of blueberries in each pail. When they returned to the farm house, they cleaned and counted the berries. Together they had picked 1,680 blueberries. How many blueberries were in each pail? Show all your work, and write your answer in a complete sentence.



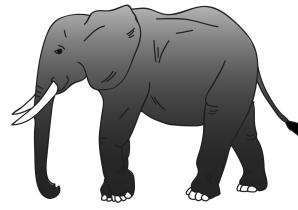
4. Bob's Fish Shop has a large variety of fish available for purchase. Some of the fish are kept in 22-quart tanks. On cleaning day, Bob uses 1,144 quarts of water to refill all the 22-quart tanks. How many 22-quart tanks does Bob need to refill on cleaning day? Show all your work, and write your answer in a complete sentence.



5. Four fifth-grade classes went on their weekly walk to the lake to gather plant and water specimens for their science project. There were 42 groups, each with two students. Each group was assigned to bring back the same number of specimens. Once all the classes returned to the school, they went through their collections. Overall there were 672 specimens collected. How many specimens was each group assigned to collect? Show all your work, and write your answer in a complete sentence.



6. Evan researched African elephants for his science report. He researched 19 animal reserves in eight different countries in Africa. Combined, the reserves have a total African elephant population of 9,557. If there are the same number of African elephants at each animal reserve, how many elephants live at each reserve?



7. Complete the division problem by determining the value of each of the question marks. Show all your work.

57

			?	6
	2	0	5	2
-	1	7	1	0
	?	?	?	?
-		3	4	2
		?	?	?

8. The fifth-grade advanced history class went on a trip to the state capital to see the state government in session. There were 28 students going on the trip. The total cost for the trip, including the bus and the hotel rooms for two nights, was \$2,688. If each student had to pay the same amount, how much did each student need to pay to attend the trip to the state capital? Show all your work, and write your answer in a complete sentence.



Name _____

9. Determine the missing value in the equation $4,524 = \square \times 58$. Show all your work.

10. Mr. Henry is getting his classroom ready for the new school year and needs 200 paper clips for different projects. He goes to the office to get paper clips. The office had ordered one large case containing 83 boxes of paper clips. The large case of paper clips contains 7,802 paper clips. How many paper clips are in each box, assuming all boxes have the same number of paper clips? How many boxes of paper clips will Mr. Henry need for his projects? Show all your work, and write your answer in a complete sentence.

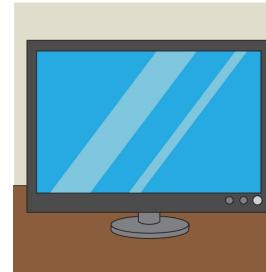


DIVIDING WHOLE NUMBERS

Lessons 1 – 4

1. During the school year, Janie has a TV time limit of 8 hours each week. She is limited to 288 hours of TV time for the school year. How many weeks are in the school year? Create an array in the space provided to show your work.

_____ hours total



_____ hours per week

_____ weeks

$$\underline{\hspace{1cm}} \times \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$$

$$\underline{\hspace{1cm}} \div \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$$

Array

2. Bella had her birthday party at a local park. The picnic tables were hidden in the park, so Bella marked the path from the parking lot to the tables with balloons. Bella placed one balloon every nine yards. The total distance on the path from the parking lot to the tables was 162 yards. How many balloons did Bella need to mark the entire path? Be sure to label and use the number line to show your work.

**Number Line**

_____ distance from parking lot to picnic tables

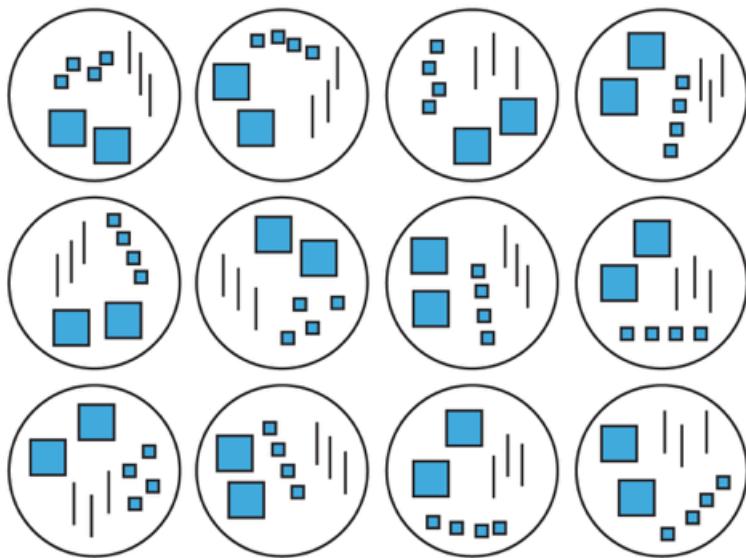
_____ distance between each balloon

_____ number of balloons

$$\underline{\hspace{2cm}} \times \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

$$\underline{\hspace{2cm}} \div \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

3. Use the set group model with base-ten blocks to fill in the blanks.



_____ number of groups

_____ number in each group

_____ total

$$\underline{\hspace{1cm}} \times \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$$

$$\underline{\hspace{1cm}} \div \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$$

4. For his snack day at school, Marco wants to bring carrots and ranch dressing. Marco's mom purchases one bag of baby carrots that contains 200 carrots. Marco determines that he needs 184 carrots for his class. There are 23 students in his class, including Marco. How many baby carrots will each student receive? Create an array in the space provided to show your work.

**Array**

$$\underline{\hspace{2cm}} \div \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

The number _____ represents:

The number _____ represents:

The number _____ represents:

5. Solve the following problems using an area model.

5.a. Mr. Smith plays music for his students while they are working. His music website tracks his listening time. On Monday he checked how much time his class listened to music during five school days in the previous week. The total listening time was 1,200 minutes. How many minutes did Mr. Smith's class listen to music each day last week if they listened to music for the same amount of time each day? Show all your work, and answer in a complete sentence.



5.b. _____ $\times 9 = 5,067$

Name_____

5.c. Fill in the blanks to complete the area model.

2	$\underline{\hspace{2cm}}$ $\underline{\hspace{2cm}}$ $\underline{\hspace{2cm}}$	$\underline{\hspace{2cm}}$ $\underline{\hspace{2cm}}$ $\underline{\hspace{2cm}}$
	$\begin{array}{r} 1\ 3\ 6\ 8 \\ - \quad \quad \quad \quad \\ \hline \end{array}$	$\begin{array}{r} 1\ 6\ 0\ 8 \\ - \quad \quad \quad \quad \\ \hline \end{array}$
		$\begin{array}{r} 8 \\ - 8 \\ \hline 0 \end{array}$

5.d. _____ = $6,816 \div 12$

--

6. Solve the following problems using the distributive property.

6.a. Fill in the blanks.

$$\underline{\hspace{2cm}} \times 24 = 6,192$$

$$\underline{\hspace{2cm}} \div \underline{\hspace{2cm}}$$

$$= (2,400 + \underline{\hspace{2cm}} + \underline{\hspace{2cm}} + 192) \div \underline{\hspace{2cm}}$$

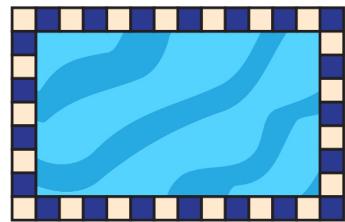
$$= 2,400 \div 24 + \underline{\hspace{2cm}} \div \underline{\hspace{2cm}} + \underline{\hspace{2cm}} \div$$

$$\underline{\hspace{2cm}} + 192 \div 24$$

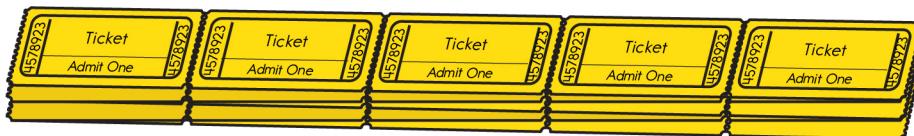
$$= 100 + \underline{\hspace{2cm}} + \underline{\hspace{2cm}} + 8$$

$$= \underline{\hspace{2cm}}$$

6.b. The pool needs a new rectangular cover for when the pool is closed. The area of the pool cover the manager ordered is 3,234 square feet. The width of the pool cover is 21 feet. How long is the pool cover? Will the pool cover fit a rectangular pool that is 150 feet long? Show all your work, and answer in a complete sentence.

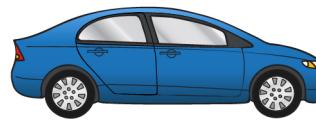


6.c. Every Saturday, Marcie's aunt takes her to Game Center. Marcie loves to play the games that award tickets because she is saving for a large stuffed dog. So far Marcie has saved 2,280 tickets over six weeks. How many tickets does Marcie earn each Saturday if she earns the same number of tickets each week? Show all your work, and answer in a complete sentence.



7. Solve the following problems using the standard algorithm, with or without partial quotients.

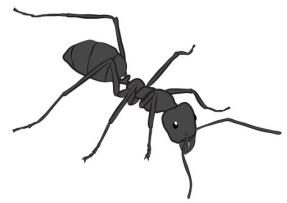
7.a. Jasmine and her family are driving to California for summer vacation. The drive from her home in Ohio to the beach house is 2,624 miles. Every 82 miles, Jasmine asks if they are there yet. How many times does Jasmine ask her parents if they are there yet? Show all your work, and answer in a complete sentence.



82 2624

7.b. Maria read about an ant colony in Europe that covers 3,828 miles underground and is made up of 33 different nests.

How many miles does each nest cover if the nests are equally spaced? Show all your work, and answer in a complete sentence.



33	3 8 2 8
----	------------------

7.c. $58 \times \underline{\hspace{2cm}} = 4,350$

7.d. Simone's work for the expression $4,727 \div 29$ is shown in the following picture. Is Simone's work correct? If so, how do you know? If not, identify her mistake, and provide the correct quotient.

$$\begin{array}{r} 1630 \\ 29 \overline{)4727} \\ -2900 \\ \hline 1827 \\ -1740 \\ \hline 0087 \\ -87 \\ \hline 00 \end{array}$$

DIVIDING WHOLE NUMBERS

STUDENT ACTIVITY SOLUTION GUIDE

-
1. During the school year, Janie has a TV time limit of 8 hours each week. She is limited to 288 hours of TV time for the school year. How many weeks are in the school year? Create an array in the space provided to show your work.



CORRECT ANSWER

288 hours total

eight hours per week

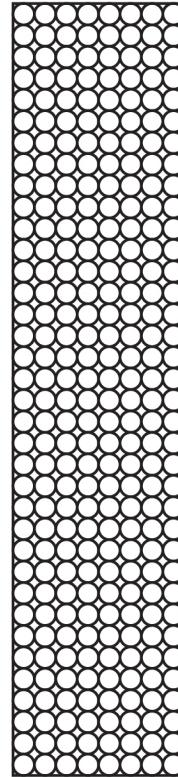
36 weeks

$$\underline{36} \times \underline{8} = \underline{288}$$

$$\underline{288} \div \underline{8} = \underline{36}$$

NOTE: the array must be created vertically to accurately represent 36 groups of eight and a total of 288.

There are 36 weeks in Janie's school year.



ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student does not create an array to solve the problem.	does not understand the grouping organization of an array and its relationship to division and multiplication	SOLVE ARRAY PROBLEMS and APPLY THE RELATIONSHIP BETWEEN MULTIPLICATION AND DIVISION
The student does not provide the correct multiplication and/or division equation for the problem. For example, $288 \times 8 \neq 36$ $36 \div 8 \neq 288$	does not understand what division and/or multiplication problems represent	DEMONSTRATE THE CONCEPT OF DIVISION and/or DEMONSTRATE THE CONCEPT OF MULTIPLICATION
The student does not give the correct order for the multiplication equation, either based on an incorrect array or misinterpreting the array. For example, $8 \times 36 = 288$	does not understand the order of the factors and/or what each factor represents and the relationship of the rows and columns with the order of the factors	SOLVE ARRAY PROBLEMS and/or EXPLAIN MULTIPLICATION PROBLEMS
The student completes the division equation blanks but not the multiplication blanks.	does not recognize or know how to apply the relationship between multiplication and division	EXPLAIN THE RELATIONSHIP BETWEEN MULTIPLICATION AND DIVISION
The student does not correctly identify the parts of the division problem; total hours, weeks, hours per week.	does not understand the different parts of the problem and/or what the problem is asking	USE REASONING FOR MULTIPLICATION AND DIVISION
The student does not create the correct number of rows or columns resulting in an incorrect quotient, and/or the student does not divide the correct total number, uses a number other than 288. This results in a quotient other than 36.	does not understand how to construct an array to solve a division situation and/or does not count the number of rows, columns, or the total correctly	SOLVE ARRAY PROBLEMS

2. Bella had her birthday party at a local park. The picnic tables were hidden in the park, so Bella marked the path from the parking lot to the tables with balloons. Bella placed one balloon every nine yards. The total distance on the path from the parking lot to the tables was 162 yards. How many balloons did Bella need to mark the entire path? Be sure to label and use the number line to show your work.



CORRECT ANSWER

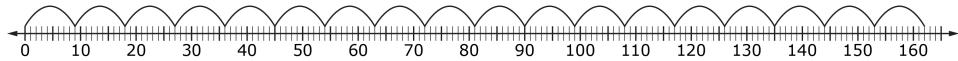
162 yards distance from parking lot to picnic tables

nine yards distance between each balloon

18 number of balloons

$$\underline{18 \times 9 = 162}$$

$$\underline{162 \div 9 = 18}$$

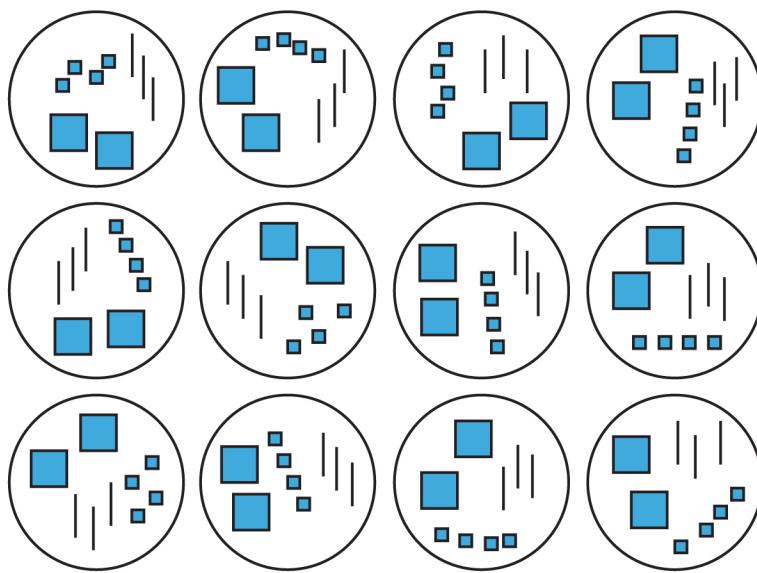


Bella will need 18 balloons to mark the entire path.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student does not create a number line to solve the problem.	does not understand the grouping organization of a number line and its relationship to division and multiplication	CALCULATE QUOTIENTS USING STRATEGIES OTHER THAN THE STANDARD ALGORITHM and APPLY THE RELATIONSHIP BETWEEN MULTIPLICATION AND DIVISION
The student does not provide the correct multiplication and/or division equation for the problem. For example, $9 \times 162 \neq 18$ $18 \div 9 \neq 162$	does not understand what division and/or multiplication problems represent	DEMONSTRATE THE CONCEPT OF DIVISION and DEMONSTRATE THE CONCEPT OF MULTIPLICATION
The student does not give the correct order for the multiplication equation based on the numberline and problem situation. For example, $9 \times 18 = 162$	does not understand the order of the factors and/or what each factor represents (the number of groups times the size of each group)	EXPLAIN MULTIPLICATION PROBLEMS
The student completes the division equation blanks but not the multiplication blanks.	does not recognize or know how to apply the relationship between multiplication and division	EXPLAIN THE RELATIONSHIP BETWEEN MULTIPLICATION AND DIVISION
The student does not correctly identify the parts of the division problem: total distance, distance between balloons, number of balloons.	does not understand the different parts of the problem and/or what the problem is asking	USE REASONING FOR MULTIPLICATION AND DIVISION
The student does not divide and/or label the number line correctly; identifies a quotient other than 18.	does not understand how to solve a division situation using a number line; does not group by nines on the number line	CALCULATE QUOTIENTS USING STRATEGIES OTHER THAN THE STANDARD ALGORITHM and SOLVE MEASUREMENT QUANTITIES PROBLEMS

3. Use the set group model with base-ten blocks to fill in the blanks.



CORRECT ANSWER

12 number of groups

234 number in each group

2808 total

$12 \times \underline{234} = \underline{2808}$

$\underline{2808} \div 12 = \underline{234}$

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student does not provide the correct multiplication and/or division equation.	cannot interpret a model as a division or multiplication equation	REPRESENT DIVISION WITH EQUATIONS and REPRESENT MULTIPLICATION WITH EQUATIONS
The student completes the division equation blanks but not the multiplication blanks.	does not recognize or know how to apply the relationship between multiplication and division	EXPLAIN THE RELATIONSHIP BETWEEN MULTIPLICATION AND DIVISION
The student completes the multiplication equation blanks but not the division blanks.	does not recognize or know how to apply the relationship between multiplication and division	EXPLAIN THE RELATIONSHIP BETWEEN MULTIPLICATION AND DIVISION

4. For his snack day at school, Marco wants to bring carrots and ranch dressing. Marco's mom purchases one bag of baby carrots that contains 200 carrots. Marco determines that he needs 184 carrots for his class. There are 23 students in his class, including Marco. How many baby carrots will each student receive? Create an array in the space provided to show your work.



CORRECT ANSWER

$$\underline{184} \div \underline{23} = \underline{8}$$

NOTE: The answers to "The number ____ represents:" can be completed in any order; the focus is on the fact that the student understands what each number represents.

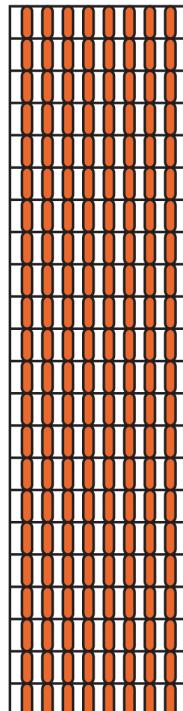
The number 184 represents: the total number of baby carrots

The number 23 represents: the number of students in Marco's class, the number of equal groups

The number 8 represents: the number of carrots each student will receive, the number of parts in each group

NOTE: the array must be created vertically in order to accurately represent 23 groups of 8 and a total of 184.

Each student will receive eight baby carrots.



 ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student does not create an array to solve the problem.	does not understand the grouping organization of an array and its relationship to division and multiplication	SOLVE ARRAY PROBLEMS and APPLY THE RELATIONSHIP BETWEEN MULTIPLICATION AND DIVISION
The student provides the following division equation for the problem: $23 \div 8 = 184$	does not understand what division and/or multiplication problems represent	DEMONSTRATE THE CONCEPT OF DIVISION
The student provides the following division equation for the problem: $184 \div 8 = 23$	does not model the division the question is describes; uses the factors from the multiplication equation in an order that is inconsistent with the problem situation	MATHEMATIZE CONTEXTUAL SITUATIONS INVOLVING DIVISION
The student does not correctly identify and explain the parts of the division problem.	does not understand the different parts of the problem and/or what the problem is asking	USE REASONING FOR MULTIPLICATION AND DIVISION
The student does not use the correct values from the problem (for example, divides 200 carrots instead of 184 carrots).	does not recognize the important information from the question and/or does not recognize what the question is asking; uses the first larger value as the dividend and the first smaller value as the divisor	USE REASONING FOR MULTIPLICATION AND DIVISION
The student does not create the correct number of rows or columns resulting in an incorrect quotient, and/or the student does not divide the correct number, uses a number other than 184. This results in a quotient other than eight.	does not understand how to construct an array to solve a division situation and/or does not count the number of rows, columns, or the total correctly	SOLVE ARRAY PROBLEMS

5. Solve the following problems using an area model.

- 5.a. Mr. Smith plays music for his students while they are working. His music website tracks his listening time. On Monday he checked how much time his class listened to music during five school days in the previous week. The total listening time was 1,200 minutes. How many minutes did Mr. Smith's class listen to music each day last week if they listened to music for the same amount of time each day? Show all your work, and answer in a complete sentence.



CORRECT ANSWER

$$1,200 \div 5 = 240$$

$\begin{array}{r} 200 \\ 5 \overline{)1200} \\ -1000 \\ \hline 200 \end{array}$	$\begin{array}{r} 40 \\ 5 \overline{)200} \\ -200 \\ \hline 000 \end{array}$
---	--

Mr. Smith's class listened to 240 minutes of music each day last week.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student does not use the area model to determine the quotient.	does not understand the process of determining partial quotients to determine a total quotient using the area model	CALCULATE QUOTIENTS USING STRATEGIES OTHER THAN THE STANDARD ALGORITHM
The student uses single-digit partial quotients instead of multi-digit partial quotients (for example, 2 instead of 200 and 4 instead of 40).	does not understand and/or recognize the relationship between division and place value; misunderstands the meaning of each digit in the standard algorithm	EXPLAIN THE RELATIONSHIP BETWEEN PLACE VALUE AND DIVISION
The student does not multiply or subtract correctly, resulting in a quotient other than 240.	does not understand when or how to multiply or subtract when simplifying division	DIVIDE A 4-DIGIT NUMBER BY A 1-DIGIT NUMBER

5.b. _____ $\times 9 = 5,067$

CORRECT ANSWER

$563 \times 9 = 5,067$

NOTE: The area model represented is one solution. There are alternate partial quotients that will lead to the same total quotient. Check student work for accuracy.

	500	60	3
9	$\begin{array}{r} 5067 \\ - 4500 \\ \hline 0567 \end{array}$	$\begin{array}{r} 567 \\ - 540 \\ \hline 027 \end{array}$	$\begin{array}{r} 27 \\ - 27 \\ \hline 00 \end{array}$

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student does not answer the question or multiplies the two given values instead of dividing.	does not recognize that division is required to simplify the equation	EXPLAIN THE RELATIONSHIP BETWEEN MULTIPLICATION AND DIVISION
The student uses single-digit partial quotients instead of multi-digit partial quotients (for example, 5 instead of 500 and 6 instead of 60).	does not understand and/or recognize the relationship between division and place value; misunderstands the meaning of each digit in the standard algorithm	EXPLAIN THE RELATIONSHIP BETWEEN PLACE VALUE AND DIVISION
The student does not use the area model to determine the quotient.	does not understand the process of determining partial quotients to determine a total quotient using the area model	CALCULATE QUOTIENTS USING STRATEGIES OTHER THAN THE STANDARD ALGORITHM
The student does not multiply or subtract correctly, resulting in a quotient other than 563.	does not understand when or how to multiply or subtract when simplifying division	DIVIDE A 4-DIGIT NUMBER BY A 1-DIGIT NUMBER

5.c. Fill in the blanks to complete the area model.

CORRECT ANSWER

2

$\begin{array}{r} 600 \\ \times 2 \\ \hline 1200 \end{array}$	$\begin{array}{r} 80 \\ \times 2 \\ \hline 160 \end{array}$	$\begin{array}{r} 4 \\ \times 2 \\ \hline 8 \end{array}$
---	---	--

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student does not complete all of the blanks.	unsure how to determine each value using division, multiplication, and subtraction	CALCULATE QUOTIENTS USING STRATEGIES OTHER THAN THE STANDARD ALGORITHM and USE REASONING FOR MULTIPLICATION AND DIVISION
The student uses single-digit partial quotients instead of multi-digit partial quotients (for example, 6 instead of 600 and 8 instead of 80).	does not understand and/or recognize the relationship between division and place value; misunderstands the meaning of each digit in the standard algorithm	EXPLAIN THE RELATIONSHIP BETWEEN PLACE VALUE AND DIVISION
The student does not use the correct values in the blanks, miscalculates the partial quotients, or incorrectly multiplies the factors.	miscalculates computation while dividing or multiplying	USE REASONING FOR MULTIPLICATION AND DIVISION

5.d. _____ = $6,816 \div 12$

 CORRECT ANSWER

$$568 = 6,816 \div 12$$

NOTE: The area model represented is one solution. There are alternate partial quotients that will lead to the same total quotient. Check student work for accuracy.

12	500 60 8 <table style="margin-left: auto; margin-right: auto; border-collapse: collapse;"> <tr> <td style="border: 1px solid black; padding: 5px; text-align: center;">6816</td> <td style="border: 1px solid black; padding: 5px; text-align: center;">816</td> <td style="border: 1px solid black; padding: 5px; text-align: center;">96</td> </tr> <tr> <td style="border: 1px solid black; padding: 5px; text-align: center;">- 6000</td> <td style="border: 1px solid black; padding: 5px; text-align: center;">- 720</td> <td style="border: 1px solid black; padding: 5px; text-align: center;">- 96</td> </tr> <tr> <td style="border: 1px solid black; padding: 5px; text-align: center;">0816</td> <td style="border: 1px solid black; padding: 5px; text-align: center;">096</td> <td style="border: 1px solid black; padding: 5px; text-align: center;">00</td> </tr> </table>	6816	816	96	- 6000	- 720	- 96	0816	096	00
6816	816	96								
- 6000	- 720	- 96								
0816	096	00								

 ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student uses single-digit partial quotients instead of multi-digit partial quotients (for example, 5 instead of 500 and 6 instead of 60).	does not understand and/or recognize the relationship between division and place value; misunderstands the meaning of each digit in the standard algorithm	EXPLAIN THE RELATIONSHIP BETWEEN PLACE VALUE AND DIVISION
The student does not use the area model to determine the quotient.	does not understand the process of determining partial quotients to determine a total quotient using the area model	CALCULATE QUOTIENTS USING STRATEGIES OTHER THAN THE STANDARD ALGORITHM

-
6. Solve the following problems using the distributive property.

- 6.a. Fill in the blanks.

$$\begin{aligned}
 & \underline{\hspace{2cm}} \times 24 = 6192 \\
 & \underline{\hspace{2cm}} \div \underline{\hspace{2cm}} = (2400 + \underline{\hspace{2cm}} + \underline{\hspace{2cm}} + 192) \div \\
 & \underline{\hspace{2cm}} = 2400 \div 24 + \underline{\hspace{2cm}} \div \underline{\hspace{2cm}} + \underline{\hspace{2cm}} \div \underline{\hspace{2cm}} \\
 & + 192 \div 24 \\
 & = 100 + \underline{\hspace{2cm}} + \underline{\hspace{2cm}} + 8 \\
 & = \underline{\hspace{2cm}}
 \end{aligned}$$

CORRECT ANSWER

$$\begin{aligned}
 & 258 \times 24 = 6,192 \\
 & 6,192 \div 24 = (2,400 + 2,400 + 1,200 + 192) \div 24 \\
 & = 2,400 \div 24 + 2,400 \div 24 + 1,200 \div 24 + 192 \div 24 \\
 & = 100 + 100 + 50 + 8 \\
 & = 258
 \end{aligned}$$

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student does not decompose the dividend correctly. The addends do not equal the dividend.	does not understand that the sum of the addends is equal to the dividend; miscalculates when decomposing the dividend	APPLY THE DISTRIBUTIVE PROPERTY, SUBTRACT WITHIN 1000, SUBTRACT BEYOND 1000, and/or ADD BEYOND 1000
The student does not answer the question by filling in the blanks.	cannot apply the distributive property to a division problem	APPLY THE DISTRIBUTIVE PROPERTY
The student does not divide correctly when dividing the addends.	miscalculates when dividing the addend by the divisor	DIVIDE A 4-DIGIT NUMBER BY A 2-DIGIT NUMBER

- 6.b. The pool needs a new rectangular cover for when the pool is closed. The area of the pool cover the manager ordered is 3,234 square feet. The width of the pool cover is 21 feet. How long is the pool cover? Will the pool cover fit a rectangular pool that is 150 feet long? Show all your work, and answer in a complete sentence.



CORRECT ANSWER

The length of the pool cover is 154 feet. Yes, the pool cover will fit a pool that is 150 feet long.

NOTE: The following partial quotients are one solution. There are alternate partial quotients that will lead to the same total quotient. Check student work for accuracy.

$$\begin{aligned}
 3,234 \div 21 &= (2,100 + 1,050 + 84) \div 21 \\
 &= 2,100 \div 21 + 1,050 \div 21 + 84 \div 21 \\
 &= 100 + 50 + 4
 \end{aligned}$$

=154

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student does not decompose the dividend correctly. The addends do not equal the dividend.	does not understand that the sum of the addends is equal to the dividend; miscalculates when decomposing the dividend	APPLY THE DISTRIBUTIVE PROPERTY, SUBTRACT WITHIN 1000, SUBTRACT BEYOND 1000, and/or ADD BEYOND 1000
The student does not divide correctly when dividing the addends.	miscalculates when dividing the addend by the divisor	DIVIDE A 4-DIGIT NUMBER BY A 2-DIGIT NUMBER
The student does not answer the question using the distributive property.	unsure of why or how to use the distributive property for division	APPLY THE DISTRIBUTIVE PROPERTY
The student divides 3,234 by 150 instead of by 21.	unsure of what the question is asking and divides by the wrong value	MATHEMATIZE CONTEXTUAL SITUATIONS INVOLVING DIVISION and USE REASONING FOR MULTIPLICATION AND DIVISION
The student multiples 150 × 21 instead of dividing.	unsure of what the question is asking and multiplies instead of divides	MATHEMATIZE CONTEXTUAL SITUATIONS INVOLVING DIVISION and USE REASONING FOR MULTIPLICATION AND DIVISION

- 6.c. Every Saturday, Marcie's aunt takes her to Game Center. Marcie loves to play the games that award tickets because she is saving for a large stuffed dog. So far Marcie has saved 2,280 tickets over six weeks. How many tickets does Marcie earn each Saturday if she earns the same number of tickets each week? Show all your work, and answer in a complete sentence.



CORRECT ANSWER

Marcie earns 380 tickets each Saturday.

NOTE: The following partial quotients are one solution. There are alternate partial quotients that will lead to the same total quotient.

Check student work for accuracy.

$$\begin{aligned} 2,280 \div 6 &= (1,800 + 480) \div 6 \\ &= 1,800 \div 6 + 480 \div 6 \end{aligned}$$

$$\begin{aligned}
 &= 300 + 80 \\
 &= 380
 \end{aligned}$$

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student does not decompose the dividend correctly. The addends do not equal the dividend.	does not understand that the sum of the addends is equal to the dividend; miscalculates when decomposing the dividend	APPLY THE DISTRIBUTIVE PROPERTY, SUBTRACT WITHIN 1000, SUBTRACT BEYOND 1000, and/or ADD BEYOND 1000
The student does not divide correctly when dividing the addends.	miscalculates when dividing the addend by the divisor	DIVIDE A 4-DIGIT NUMBER BY A 2-DIGIT NUMBER
The student does not answer the question using the distributive property.	unsure of why or how to use the distributive property for division	APPLY THE DISTRIBUTIVE PROPERTY
The student multiples instead of divides.	unsure of what the question is asking and multiples instead of divides	MATHEMATIZE CONTEXTUAL SITUATIONS INVOLVING DIVISION and USE REASONING FOR MULTIPLICATION AND DIVISION

7. Solve the following problems using the standard algorithm, with or without partial quotients.

NOTE: Allow students to use partial quotients above the quotient bar as modeled in the Instructional Activity Lesson 4 if needed. Answers in the solution guide will be given in standard algorithm form.

- 7.a. Jasmine and her family are driving to California for summer vacation. The drive from her home in Ohio to the beach house is 2,624 miles. Every 82 miles, Jasmine asks if they are there yet. How many times does Jasmine ask her parents if they are there yet? Show all your work, and answer in a complete sentence.



CORRECT ANSWER

Jasmine asks her parents 32 times if they are there yet.

$$\begin{array}{r}
 32 \\
 \overline{)2624} \\
 24
 \end{array}$$

$$\begin{array}{r}
 -2460 \\
 0164 \\
 -164 \\
 \hline
 000
 \end{array}$$

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student does not write the quotient in the correct place value (for example, the 3 in 32 above the 2 in the thousands place instead of above the 2 in the tens place).	does not understand and/or recognize the relationship between division and place value; misunderstands the meaning of each digit in the standard algorithm	EXPLAIN THE RELATIONSHIP BETWEEN PLACE VALUE AND DIVISION
The student has an incorrect quotient or a quotient with a remainder.	miscalculates computation while dividing	DIVIDE A 4-DIGIT NUMBER BY A 2-DIGIT NUMBER
The student does not answer the question and leaves the space blank.	does not understand what the question is asking	MATHEMATIZE CONTEXTUAL SITUATIONS INVOLVING DIVISION
The student adds the dividend and product instead of subtracts. (for example, $2624 + 2460$.)	does not understand when and/or why to perform operations other than dividing when determining a quotient	DIVIDE A 4-DIGIT NUMBER BY A 2-DIGIT NUMBER
The student does not multiply a partial quotient by the divisor (for example, does not multiply 82×30 to determine a product to subtract from the dividend).	does not understand when and/or why to perform operations other than dividing when determining a quotient	DIVIDE A 4-DIGIT NUMBER BY A 2-DIGIT NUMBER

- 7.b. Maria read about an ant colony in Europe that covers 3,828 miles underground and is made up of 33 different nests. How many miles does each nest cover if the nests are equally spaced? Show all your work, and answer in a complete sentence.



CORRECT ANSWER

$$\begin{array}{r}
 3,828 \div 33 = 116 \\
 \quad \quad \quad 116 \\
 33 \quad \quad \quad \boxed{\begin{array}{r} 3828 \\ - 3300 \\ \hline 0528 \\ - 330 \\ \hline 198 \end{array}}
 \end{array}$$

$$\begin{array}{r} -198 \\ 000 \end{array}$$

Each nest covers 116 miles underground.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student does not write the quotient values in the correct place value (for example, the 1 for 100 in 116 above the 3 in the thousands place instead of above the 8 in the hundreds place).	does not understand and/or recognize the relationship between division and place value; misunderstands the meaning of each digit in the standard algorithm	EXPLAIN THE RELATIONSHIP BETWEEN PLACE VALUE AND DIVISION
The student has an incorrect quotient or a quotient with a remainder.	miscalculates computation while dividing	DIVIDE A 4-DIGIT NUMBER BY A 2-DIGIT NUMBER
The student does not answer the question and leaves the space blank.	does not understand what the question is asking	MATHEMATIZE CONTEXTUAL SITUATIONS INVOLVING DIVISION
The student adds the dividend and product instead of subtracts. For example, 3828 + 3300	does not understand when and/or why to perform operations other than dividing when determining a quotient	DIVIDE A 4-DIGIT NUMBER BY A 2-DIGIT NUMBER
The student does not multiply a partial quotient by the divisor (for example, does not multiply 33×100 to determine a product to subtract from the dividend).	does not understand when and/or why to perform operations other than dividing when determining a quotient	DIVIDE A 4-DIGIT NUMBER BY A 2-DIGIT NUMBER

7.c. $58 \times \underline{\hspace{2cm}} = 4,350$

CORRECT ANSWER

$$58 \times 75 = 4,350$$

$$\begin{array}{r}
 & 75 \\
 \times & 58 \\
 \hline
 & 4350 \\
 - & 4060 \\
 \hline
 & 0290 \\
 - & 290 \\
 \hline
 & 4350
 \end{array}$$

000

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student does not answer the question or multiplies the two given values instead of dividing.	does not recognize that division is required to determine the missing factor	EXPLAIN THE RELATIONSHIP BETWEEN MULTIPLICATION AND DIVISION
The student has an incorrect quotient or a quotient with a remainder.	miscalculates during the computation while dividing	DIVIDE A 4-DIGIT NUMBER BY A 2-DIGIT NUMBER
The student does not write the quotient values in the correct place value (for example, the 7 for 75 above the 4 in the thousands place instead of above the 5 in the tens place).	does not understand and/or recognize the relationship between division and place value; misunderstands the meaning of each digit in the standard algorithm	EXPLAIN THE RELATIONSHIP BETWEEN PLACE VALUE AND DIVISION
The student adds the dividend and product instead of subtracts. For example, $\begin{array}{r} 4350 \\ + 4060 \\ \hline \end{array}$	does not understand when and/or why to perform operations other than dividing when determining a quotient	DIVIDE A 4-DIGIT NUMBER BY A 2-DIGIT NUMBER
The student does not multiply a partial quotient by the divisor (for example, does not multiply 58×70 to determine a product to subtract from the dividend).	does not understand when and/or why to perform operations other than dividing when determining a quotient	DIVIDE A 4-DIGIT NUMBER BY A 2-DIGIT NUMBER

- 7.d. Simone's work for the expression $4,727 \div 29$ is shown in the following picture. Is Simone's work correct? If so, how do you know? If not, identify her mistake, and provide the correct quotient.

$$\begin{array}{r}
 1630 \\
 29 \overline{)4727} \\
 -2900 \\
 \hline
 1827 \\
 -1740 \\
 \hline
 087 \\
 -87 \\
 \hline
 00
 \end{array}$$

CORRECT ANSWER

Simone's work is not correct. The place value in her quotient is incorrect because she has written that there are 1,000 groups of 29, which equals 29,000, and that is larger than the dividend. This error led her to

write a zero at the end of the quotient, making it 10 times larger than the correct quotient. The correct quotient is 163.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student thinks that Simone's work is correct.	does not recognize that the quotient digits are written in the incorrect place value, causing Simone to write a zero to the end of the quotient	EXPLAIN THE RELATIONSHIP BETWEEN PLACE VALUE AND DIVISION
The student answers that Simone's work is incorrect but cannot identify why or provide the correct answer.	does not understand the concept of long division; is unable to identify and/or cannot complete process to simplify a division problem	DIVIDE A 4-DIGIT NUMBER BY A 2-DIGIT NUMBER
The student answers that Simone's work is incorrect but provides an incorrect answer as the correct quotient.	miscalculates during the computation while dividing	DIVIDE A 4-DIGIT NUMBER BY A 2-DIGIT NUMBER