THE UNIVERSITY OF KANSAS
Achievement & Assessment Institute

SOLVING EQUATIONS AND DEVELOPING THE FOUNDATION FOR PROOFS

6.EE.6 and 6.EE.7

CONTENTS

The types of documents contained in the unit are listed below. Throughout the unit, the documents are arranged by lesson.

LEARNING MAP INFORMATION	An overview of the standards, the learning map section, and the nodes addressed in this unit
TEACHER NOTES	A brief discussion describing the progression depicted in the learning map section with research-based recommendations for focusing instruction to foster student learning and an introduction to the unit's lessons
OVERVIEW OF INSTRUCTIONAL ACTIVITIES	A table highlighting the lesson goals and nodes addressed in each lesson of this unit
INSTRUCTIONAL ACTIVITY	A detailed walkthrough of the unit
INSTRUCTIONAL ACTIVITY STUDENT HANDOUT	A handout for the guided activity, intended to be paired with the Instructional Activity
INSTRUCTIONAL ACTIVITY SUPPLEMENT	A collection of materials or activities related to the Instructional Activity
STUDENT ACTIVITY	A work-alone activity for students
STUDENT ACTIVITY SOLUTION GUIDE	A solution guide for the work-alone activity with example errors, misconceptions, and links to the learning map section

Enhanced Learning Maps developed these materials under a grant from the Department of Education, PR/Award # S368A150013. Contents do not necessarily represent the policy of the Department of Education, and you should not assume endorsement by the Federal Government.

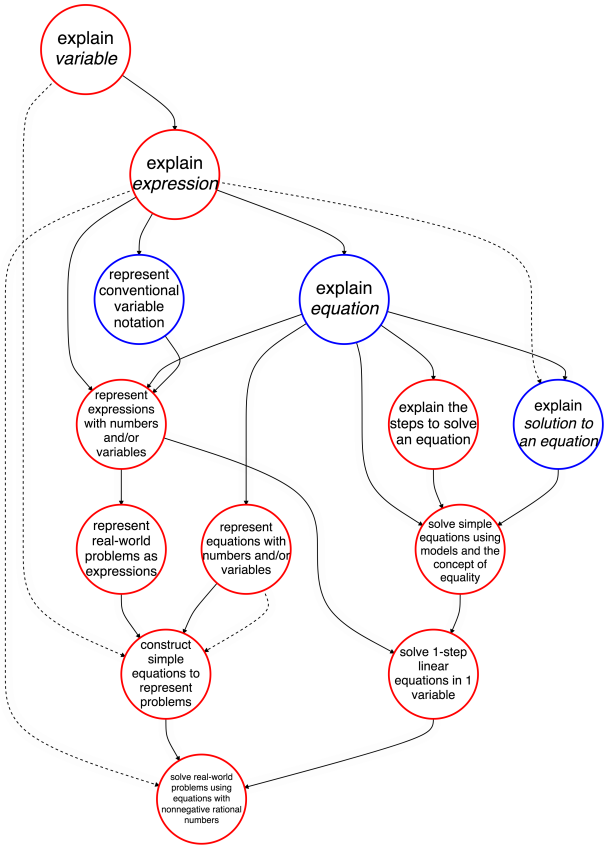
SOLVING EQUATIONS AND DEVELOPING THE FOUNDATION FOR PROOFS

LEARNING MAP INFORMATION

STANDARDS

6.EE.6 Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.

6.EE.7 Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q, and x are all nonnegative rational numbers.



**Learning map model of* 6.EE.6 and 6.EE.7

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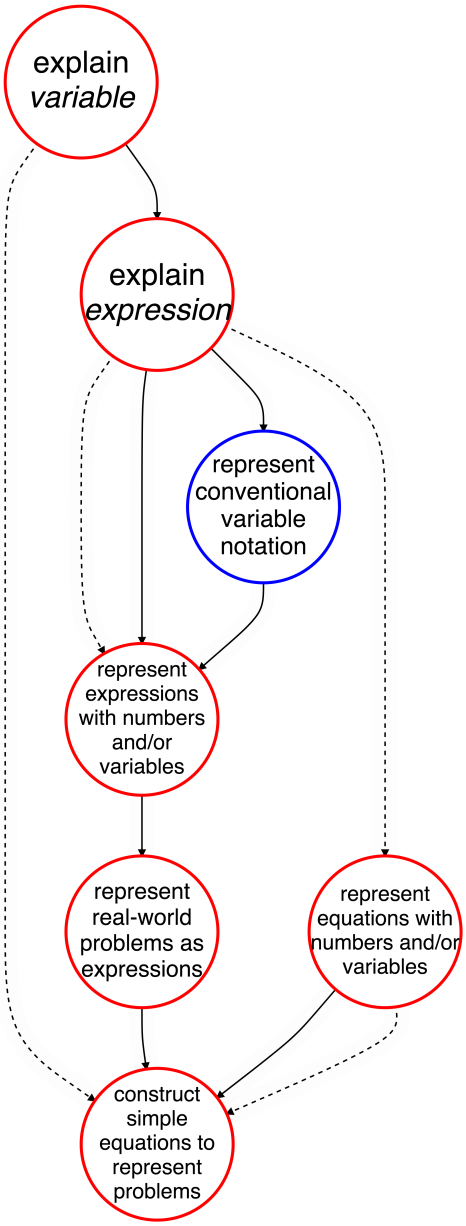
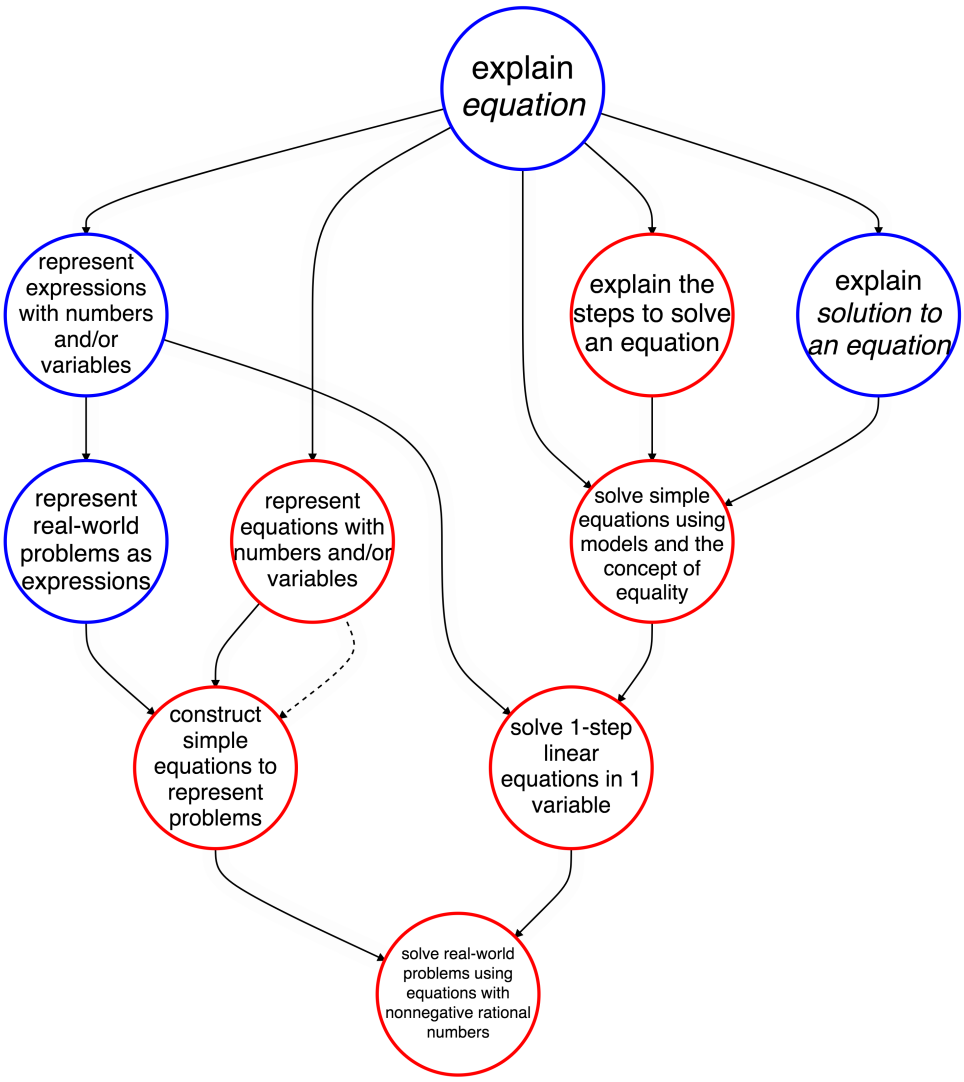


**Learning map model of* 6.EE.6

6.EE.7 Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q, and x are all nonnegative rational numbers.



**Learning map model of* 6.EE.7

Node Name	Node Description
CONSTRUCT SIMPLE EQUATIONS TO REPRESENT PROBLEMS	Construct simple equations (one-step or two-step) to represent real-world or mathematical problems.
EXPLAIN *EQUATION*	Make known your understanding that an equation is a mathematical sentence involving two equivalent numerical or algebraic expressions and an equal sign.
EXPLAIN *EXPRESSION*	Make known your understanding that an expression is a mathematical phrase that includes numbers, variables, or both and that may include operations but no equal sign (e.g., $x + 5$ or y).
EXPLAIN *SOLUTION TO AN EQUATION*	Make known your understanding that solutions to an equation with one or more variables are the variable values that make the equation true.
EXPLAIN THE STEPS TO SOLVE AN EQUATION	For a particular equation, explain how to select inverse operations and to apply them to both sides to preserve equivalence and to solve the equation.
EXPLAIN *VARIABLE*	Make known your understanding that a variable is a symbol that can stand for an unknown quantity or a varying quantity.
REPRESENT CONVENTIONAL VARIABLE NOTATION	Through writing or an appropriate assistive technology, represent conventional variable notation (e.g., $3p$ instead of $3 \times p$ and 7/y instead of $7 \div y$).
REPRESENT EQUATIONS WITH NUMBERS AND/OR VARIABLES	Through writing or an appropriate assistive technology, represent equations with numbers, variables, or both.
REPRESENT EXPRESSIONS WITH NUMBERS AND/OR VARIABLES	Through writing or an appropriate assistive technology, represent expressions using the four basic operations, exponents, and grouping symbols with numbers, variables, or both (e.g., express "subtract k from 12" as $12 - k$ or "11 squared plus 4" as $11^2 + 4$).
REPRESENT REAL-WORLD PROBLEMS AS EXPRESSIONS	Through writing or an appropriate assistive technology, represent real-world problems as expressions.
SOLVE 1-STEP LINEAR EQUATIONS IN 1 VARIABLE	Solve one-step linear equations that only have a single variable and nonnegative rational numbers.
SOLVE REAL-WORLD PROBLEMS USING EQUATIONS WITH NONNEGATIVE RATIONAL NUMBERS	Use equations with nonnegative rational numbers to solve real-world problems.
SOLVE SIMPLE EQUATIONS USING MODELS AND THE CONCEPT OF EQUALITY	Use concrete or semi-concrete (pictorial) models to solve simple (one-step or two-step) equations by performing operations that maintain equality.

Solving Equations and Developing the Foundation for Proofs

TEACHER NOTES

This unit includes the following documents:

- Learning Map Information
- Instructional Activity (three lessons)
- Instructional Activity Student Handout (for Lesson 3)
- Instructional Activity Supplement (for Lesson 3)
- Student Activities (Word Version)
- Student Activity Solution Guide

In this unit, students will learn how to represent and to solve a simple one-step equation with nonnegative rational numbers from concrete models and from real world problems. The concept of a variable is reinforced, as well as the idea of equality and of keeping an equation balanced while operating to solve it.

> An option for introducing the idea of balance and equilibrium to students prior to the unit is to ask students to find synonyms for *balance*, *equilibrium*, or *equal*. Students should come up with words such as *equivalent*, *equitable*, *equalize*, *stabilize*, *steady*, etc. They could then draw a representation of these words, define these words, or use these words in a sentence to establish their meaning before they proceed through the lessons.

Once students are comfortable solving a one-step equation, they will proceed to representing a real-world situation using an equation and solving that equation to answer the presented question. Students are asked to justify their work using the Addition, Subtraction, Multiplication, and Division Properties of Equality to emphasize the rationale behind the steps to solve and to develop the foundation for proofs, which will be expanded as they enter high-school-level math courses. In chapter one of their book, Ellis, Bieda, and Knuth (2012) recommended that proofs should be incorporated regularly into every student's mathematical experience.

Properties of Equality

Addition Property of Equality	If $a = b$, then $a + c = b + c$.
Subtraction Property of Equality	If $a = b$, then $a - c = b - c$.
Multiplication Property of Equality	If $a = b$, then $a \cdot c = b \cdot c$.
Division Property of Equality	If $a = b$ and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$.

The learning map section for this sequence of activities reflects students proceeding through various representations and explanations of the components of an equation. Initially, students must explain the

concept of a variable to progress to a complete understanding of an expression. Understanding of expressions is beneficial for students learning about equations because it helps the student move beyond early understandings of the equal sign as a symbol meaning "provide an answer" to an understanding of the equal sign as a relational symbol. The most basic level of understanding is the student's ability to explain what an equation is. This understanding, along with knowledge of inverse operations, allows a student to move into situations where they are able to solve a simple, one-step equation with the use of concrete models. Students should then be able to progress beyond the use of models to justify solutions using the properties of equality and to use inverse operations without a concrete model. Once this ability is in place, students can begin representing a simple real-world problem with an equation and solving that equation to answer a question based on the context of the problem.

The activities in this unit are designed to introduce students to solving equations through concrete and algebraic representations. Students will first solve equations using concrete models, then solve without the use of a model, and finally represent and solve word problems. At each level, students are expected to justify the chosen operation using a property of equality and to maintain the concept of balance within the equation they are working with.

REFERENCES

Ellis, A., Bieda, K., & Knuth, E. (2012). *Developing essential understanding of proof and proving for teaching mathematics in grades 9–12* (R. M. Zbiek, Series Ed.). Reston, VA: National Council of Teachers of Mathematics.

Solving Equations and Developing the Foundation for Proofs

Overview of Instructional Activities

Lesson	Learning Goal	Nodes Addressed
Lesson 1	Students will solve simple equations using concrete models and the concept of balance. The critical outcome of this activity is for students to model and to solve equations in the form $x + p = q$ and $x - p = q$ and to justify operations used to solve these equations with the Addition Property of Equality or the Subtraction Property of Equality.	▶ EXPLAIN *VARIABLE* ▶ EXPLAIN *EQUATION* ▶ EXPLAIN THE STEPS TO SOLVE AN EQUATION ▶ EXPLAIN *SOLUTION TO AN EQUATION*
Lesson 2	Students will solve simple equations using concrete models and the concept of balance. The critical outcome of this activity is for students to model and solve equations in the form $px = q$ and $x \div p = q$ and to justify operations used to solve these equations with the Multiplication Property of Equality or the Division Property of Equality.	▶ EXPLAIN *VARIABLE* ▶ EXPLAIN *EQUATION* ▶ EXPLAIN THE STEPS TO SOLVE AN EQUATION ▶ EXPLAIN *SOLUTION TO AN EQUATION*
Lesson 3	Students will represent and solve real-world problems using one-step equations. The critical outcome of this activity is for students to represent and to solve real-world equations in the form $x + p = q$, $x - p = q$, $px = q$, and $x \div p = q$ with justification for each step.	▶ EXPLAIN *VARIABLE* ▶ EXPLAIN *EXPRESSION* ▶ EXPLAIN *EQUATION* ▶ REPRESENT EXPRESSIONS WITH NUMBERS AND/OR VARIABLES ▶ CONSTRUCT SIMPLE EQUATIONS TO REPRESENT PROBLEMS ▶ EXPLAIN THE STEPS TO SOLVE AN EQUATION ▶ EXPLAIN *SOLUTION TO AN EQUATION*

SOLVING EQUATIONS AND DEVELOPING THE FOUNDATION FOR PROOFS

INSTRUCTIONAL ACTIVITY

Lesson 1

LEARNING GOAL

Students will solve simple equations using concrete models and the concept of balance. The critical outcome of this activity is for students to model and to solve equations in the form $x + p = q$ and $x - p = q$ and to justify operations used to solve these equations with the Addition Property of Equality or the Subtraction Property of Equality.

PRIMARY ACTIVITY

Students use concrete models and the concept of balance to represent and to solve equations of the forms $x + p = q$ and $x - p = q$.

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

- Variable
- Solution
- Addition Property of Equality
- Subtraction Property of Equality

MATERIALS

- Materials to make balances (binder clip, ruler, yarn/string, cups/bowls, and a pencil/pen); see FIGURE 1
- Brown paper bags
- Unit cubes or an equivalent item
- Items for students to use on balances (nickels, pennies, paper clips, number cubes, pencils, etc.)

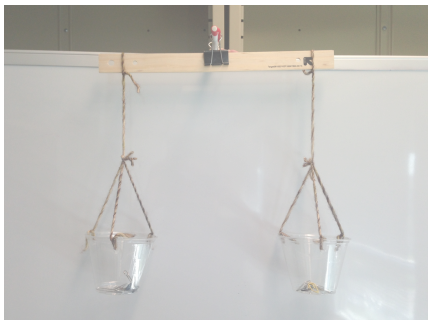


Figure 1. Example balance

IMPLEMENTATION

The activity's introduction can be represented using physical objects or an online version of a comparable situation.

Introduce the idea of a variable representing an unknown quantity by writing an "x" or another variable on the outside of a brown paper bag filled with a predetermined quantity of unit cubes or equivalent items.

Create an equation to allow students to determine how many items are in the bag. For example, if there are 4 unit cubes in the bag, write an equation such as $3 + x = 7$ or $7 = 3 + x$ on the board.

Explain that we know the equation will be balanced if one side of the balance has three unit cubes and the bag with the unknown number of cubes and the other side of the balance has seven unit cubes.

Model this equation on a balance.

GUIDING QUESTIONS

Determine if the student can EXPLAIN *VARIABLE*:

- Why do we use a letter on the outside of the bag instead of a number?
- What does the letter mean?
- What is the letter called?

Determine if the student is ready to EXPLAIN *EQUATION*:

- What does the word "balance" mean?
- What symbol do mathematicians use to show balance?
- Is the equation $3 + x = 7$ equivalent to the equation $7 = 3 + x$? Use your understanding of equations to explain your thinking.

Discuss that the goal is to isolate the bag with the unknown number of items but that we must keep the scale balanced in doing so.

Ask students what they would like to do to isolate the paper bag. Once they request one cube to be removed, take the cube away from the one side only (the scales are now unbalanced).

GUIDING QUESTIONS

Determine if the student can EXPLAIN THE STEPS TO SOLVE AN EQUATION:

- What can be done to restore balance?
- How do you know how much must be removed from the other side of the balance?

Students should indicate a cube from the other side of the balance needs to be removed. Once this is done, balance should be restored. Continue with this process until the bag is the only thing left on one side of the balance. (Students may request to only remove one cube at a time, or they may request that all 3 cubes are removed in the same step.) At this point, there should be 4 unit cubes on the other side of the balance.

Show the students that there are 4 cubes in the bag as well. Indicate the mathematical property that allows students to remove an equal number of items from each side of the equation is the Subtraction Property of Equality.

Model with algebra the steps that were used to solve the equation.

$$\begin{array}{rl} 3 + x = 7 & \text{given equation} \\ \underline{-3 \quad\quad -3} & \text{Subtraction Property of Equality} \\ x = 4 & \end{array}$$

GUIDING QUESTIONS

Determine if the student can EXPLAIN THE STEPS TO SOLVE AN EQUATION:

- Why did we subtract 3?
- Why must we subtract 3 on both sides of the equation?
- How does this relate to what we did on the balance?

Determine if the student can EXPLAIN *SOLUTION TO AN EQUATION*:

- What does the solution mean?

Repeat this activity more than once before asking students to model similar situations in pairs. You could model this activity with objects other than a paper bag with a predetermined number of items. For example, model finding the weight of a pencil in terms of paperclips by creating a one-step equation on the balance. Place a pencil and three paperclips on side of the balance, and then add paperclips to the other side to balance the equation. If it takes 10 paperclips to do so, then the equation to find the weight of a pencil in terms of paperclips would be $x + 3 = 14$ or $14 = x + 3$. Solving for x would indicate that the weight of one pencil is equal to the weight of 11 paperclips.

Write an expression on the board once the students are ready to work in pairs, and **ask** students to represent the expression on one side of the balance (e.g., place a nickel and three pennies on one side of the balance) and to add objects to the other side (e.g., add pennies only to the other side of the balance) to reach equilibrium. **Ask** students to represent and to solve the resulting equation using the balance model and algebraically (with justification) to determine the value of the unknown quantity.

GUIDING QUESTIONS

Determine if the student can EXPLAIN *VARIABLE*:

- What is the goal of this process?

Determine if the student can EXPLAIN THE STEPS TO SOLVE AN EQUATION:

- How do you know how many units to remove?
- How do you make sure your scale remains balanced?

Determine if the student can EXPLAIN *EQUATION*:

- How are the sides of the balance related to each other?
- What symbol acts as our balance in mathematics?

Practice. Students should be required to record their steps and reasoning on paper as they work and to justify the subtraction of units with the Subtraction Property of Equality. Repeat this practice with a variety of equations in the form $x + p = q$. Include examples that are difficult to model with the balance, such as those containing fractions or some negative integers ($x - p = q$).

NOTE: Vary the placement of the variable so students become familiar with equations in the forms $x + p = q$, $p + x = q$, $q = x + p$, $q = p + x$, $x - p = q$, and $q = x - p$.

Write an equation in the form $x + p = q$ or $x - p = q$ at the end of the activity. **Ask** the students to determine whether they will add or subtract (and which property of equality they will use) and to use the idea of balance to equally adjust both sides. Then, **ask** the students what the solution is. **Repeat** this activity with a few different equations, ensuring that equations in the form $x + p = q$ and $x - p = q$ are represented.

SOLVING EQUATIONS AND DEVELOPING THE FOUNDATION FOR PROOFS

INSTRUCTIONAL ACTIVITY

Lesson 2

LEARNING GOAL

Students will solve simple equations using concrete models and the concept of balance. The critical outcome of this activity is for students to model and solve equations in the form $px = q$ and $x \div p = q$ and to justify operations used to solve these equations with the Multiplication Property of Equality or the Division Property of Equality.

PRIMARY ACTIVITY

Students use concrete models and the concept of balance to represent and to solve equations of the forms $px = q$ and $\frac{x}{p} = q$ (or $x \div p = q$).

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

- Variable
- Solution
- Multiplication Property of Equality
- Division Property of Equality

MATERIALS

- Materials to make balances (binder clip, ruler, yarn/string, cups/bowls, and a pencil/pen); see FIGURE 2
- Brown paper bags
- Unit cubes or an equivalent item
- Items for students to use on balances (nickels, pennies, paper clips, number cubes, pencils, etc.)

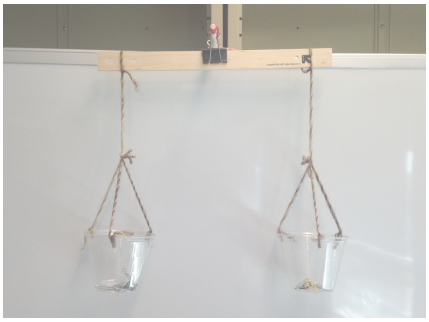


Figure 1. Example balance

IMPLEMENTATION

This lesson builds on students' understanding of equations in the form $x + p = q$ and $x - p = q$ by introducing equations in the forms $px = q$ and $x \div p = q$.

Introduce the concept of division in the same fashion as the introduction to equations in the form $x + p = q$. An introductory example could be $2x = 6$ or $6 = 2x$, where each brown paper bag has three unit cubes. Since there are two brown bags, the students should request to divide the six unit cubes on the other side of the balance into two equal groups. Count the unit cubes in each group, and indicate that each group represents the unknown quantity in each bag.

Model with algebra the steps to solve the equation, including the justification for each step.

$$\begin{array}{rl} \frac{2x}{2} = \frac{6}{2} & \text{given equation} \\ x = 3 & \text{Division Property of Equality} \end{array}$$

GUIDING QUESTIONS

Elicit student thinking:

- What is similar or different between equations having a number added to the x and equations having a number multiplied by the x?

Determine if the student can EXPLAIN THE STEPS TO SOLVE AN EQUATION:

- Why is it that we divide by 2?
- Why must we divide by 2 on both sides of the equation?
- How does this relate to what we did on the balance?

Determine if the student can EXPLAIN *SOLUTION TO AN EQUATION*:

- What does the solution mean?

Repeat this activity a few times before asking students to model similar situations in pairs.

Model this activity with objects other than a paper bag with a predetermined number of items. For example, model finding the weight of a nickel in terms of pennies by creating a one-step equation on the balance. Place three nickels on side of the balance, and then add pennies to the other side to balance the equation. If it takes six pennies to do so, then the equation to find the weight of a nickel

in terms of pennies would be $3x = 6$ or $6 = 3x$. Solving for x would indicate that the weight of one nickel is equal to the weight of two pennies.

Present an expression on the board, and **ask** students to model the expression on one side of the balance (e.g., place four cubes on one side of the balance) and to add objects to the other side (e.g., add pennies only to the other side of the balance) to reach equilibrium.

Ask students to represent and to solve the resulting equation using the balance model and to determine the value of the unknown quantity algebraically (with justification for each step).

GUIDING QUESTIONS

Elicit student thinking:

- How do each of the objects on the balance relate to the numbers and the variable in the equation?
- Why do we try to keep the balance level?
- How do we imitate the balance when we work with an equation?

Determine if the student can EXPLAIN *VARIABLE*:

- What is the goal of this process?

Determine if the student can EXPLAIN THE STEPS TO SOLVE AN EQUATION:

- How do you know how many groups to create?
- What must be true about the groups you created?

Determine if the student can EXPLAIN *EQUATION*:

- How are the sides of the balance related to each other?

Practice. Students should be required to record their reasoning on paper as they work and to justify equal division of units with the Division Property of Equality. **Repeat** this practice with a variety of equations in the form $px = q$ or $q = px$.

Introduce equations in the form $x \div p = q$ and $q = x \div p$. These are more difficult to model concretely with the balance but could be described in a similar fashion. (For example, if there is only half a number cube on one side of the balance, you would have to double that amount to have one whole number cube. Therefore you would double, or multiply by two, the amount on the other side of the balance.)

Present an equation in the form $x + p = q$, $x - p = q$, $px = q$, or $x \div p = q$ (or an equivalent equation). **Ask** students to determine whether they will add, subtract, multiply, or divide (and which property of equality they will use) and how they will use the idea of balance to equally adjust both sides. Then ask students what the solution is.

Repeat this activity with a few different equations, ensuring that equations in the form $x + p = q$, $p + x = q$, $q = x + p$, $q = p + x$, $x - p = q$, $q = x - p$, $px = q$, $q = px$, $x \div p = q$ and $q = x \div p$ are represented.

SOLVING EQUATIONS AND DEVELOPING THE FOUNDATION FOR PROOFS

INSTRUCTIONAL ACTIVITY

Lesson 3

LEARNING GOAL

Students will represent and solve real-world problems using one-step equations. The critical outcome of this activity is for students to represent and to solve real-world equations in the form $x + p = q$, $x - p = q$, $px = q$, and $x \div p = q$ with justification for each step.

PRIMARY ACTIVITY

Students will write expressions and equations using numbers and variables. Students will solve equations written from descriptions of real-world problems.

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

- Expression
- Equation

MATERIALS

- INSTRUCTIONAL ACTIVITY SUPPLEMENT (Recommend one copy for the class.)
- INSTRUCTIONAL ACTIVITY STUDENT HANDOUT
- Word Version INSTRUCTIONAL ACTIVITY SUPPLEMENT
- Word Version INSTRUCTIONAL ACTIVITY STUDENT HANDOUT

IMPLEMENTATION

Begin the lesson with practice representing expressions and equations using numbers and variables representing unknown quantities. For example, "five more than a number" can be represented as "$a + 5$" or "$5 + a$" and "four times as many as a number" can be represented as "$4y$".

Present examples on the board with the class as a whole.

Play "I have.., who has…?" with similar expressions and equations. The cards for this activity are provided in the INSTRUCTIONAL ACTIVITY STUDENT SUPPLEMENT.

GUIDING QUESTIONS

Elicit student thinking:

- Is there more than one way to write an expression for "3 more than twice a number"? Why?
- Is there more than one way to write an expression for "7 less than a number"? Why?

Determine if the student can EXPLAIN *VARIABLE*:

- How do we represent a quantity we do not know in an equation?
- Does it matter what letter represents the unknown number?

Determine if the student can EXPLAIN *EXPRESSION* and EXPLAIN *EQUATION*:

- How do you know when the description does or does not require an equal sign?
- If the description does not require an equal sign, are you representing an expression or an equation?
- If the description does require an equal sign, are you representing an expression or an equation?

Determine if the student can REPRESENT EXPRESSIONS WITH NUMBERS AND/OR VARIABLES:

- Which operation relates the number to the unknown quantity?
- How can you express the relationship between the number and the unknown quantity using symbols rather than words?

Provide small groups of students with descriptions of real-world situations from the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT, and ask them to determine what they are trying to find, to represent the word problem using an equation, to solve the equation, and to identify the property they used to solve the problem. Circulate in the room while students work through the practice problems. Be sure the students are completing all requested tasks for each problem, and not only finding the unknown values.

GUIDING QUESTIONS

Elicit student thinking:

- What patterns do you see in the words used in the problems and the types of equations you model?
- What do you notice about the order of the words in the sentences and the way you write your equations?

Determine if the student can EXPLAIN *VARIABLE*:

- What don't we know?
- What are we trying to determine?
- How do we represent a quantity we do not know in an equation?

Determine if the student can EXPLAIN *EQUATION*:

- What needs to be true about what you write on either side of the equal sign?

Determine if the student can CONSTRUCT SIMPLE EQUATIONS TO REPRESENT PROBLEMS:

- How can you represent the written description using symbols and numbers?

Determine if the student can EXPLAIN THE STEPS TO SOLVE AN EQUATION:

- In order to isolate *x* or to determine the unknown, what should you do?
- How do you ensure the equation remains balanced?

Determine if the student can EXPLAIN *SOLUTION TO AN EQUATION* in terms of a real-world problem:

- What does your solution mean in terms of the problem situation?

Ask the students to write a word problem in the space provided and to trade with their neighbor to solve. They should know the answer to their own problem and should make sure it is reasonable before they trade.

Show one additional word problem, and ask the students to represent, to solve, and to justify their steps independently as an exit ticket for the day.

Name_____

Solving Equations and Developing the Foundation for Proofs

Lesson 3

1. Sara cannot weigh her cat because he will not sit still on the scale. Sara weighs 88 pounds. Sara picks up her cat, steps on the scale, and sees that she and the cat weigh 96 pounds together.

   - What is Sara trying to determine?

   - Represent the situation with an equation.

   - Solve the equation you wrote to determine the answer to Sara's question.

   - Did you use a property of equality to solve the equation? If so, which property did you use?

Name_____

2. Sam and his friends order a pizza for $24. They need to figure out how much each person should pay based on the number of pieces they eat. There are 8 pieces of pizza in all.

   - What are Sam and his friends trying to determine?

   - Represent the situation with an equation.

   - Solve the equation you wrote to determine the answer to Sam's question.

   - Did you use a property of equality to solve the equation? If so, which property did you use?

Name_____

3. Ben has some money. After buying a basketball for $25, Ben has $4.56 left over. Ben wants to calculate how much money he had before he bought the basketball.

   - What is Ben trying to determine?

   - Represent the situation with an equation.

   - Solve the equation you wrote to determine the answer to Ben's question.

   - Did you use a property of equality to solve the equation? If so, which property did you use?

Name_____

4. Sally has a box of pencils. Sally gives 5 pencils to each of her 4 friends. The pencil box is now empty, and Sally is trying to determine how many pencils were in the box before she gave them to her friends.

   - What is Sally trying to determine?

   - Represent the situation with an equation.

   - Solve the equation you wrote to determine the answer to Sally's question.

   - Did you use a property of equality to solve the equation? If so, which property did you use?

Name_____

5. Write your own real-world problem in the space below. Your problem should have a reasonable and possible solution that you have already determined (but not written down).

Trade with the person next to you. They should solve the problem using the space provided. Write answers as complete sentences. Once you are both finished, trade back, and check each other's work. If your partner is unable to complete your question or does so incorrectly, please assist them to ensure they understand the question and the process to solve for the unknown.

- What are you trying to determine?

- Represent the situation with an equation.

- Solve the equation you wrote to determine the answer to the question.

- Did you use a property of equality to solve the equation? If so, which property did you use?

Solving Equations and Developing the Foundation for Proofs

INSTRUCTIONAL ACTIVITY SUPPLEMENT

Lesson 3

I HAVE..., WHO HAS...?

I have $\frac{1}{2}y = 17$. Who has the sum of a number and three?	I have $x + 3$. Who has the product of a number and four?
I have $4a$. Who has the quotient of a number and six?	I have $\frac{x}{6}$. Who has the difference of five and a number?
I have $5 - c$. Who has seven more than a number?	I have $7 + z$. Who has ten times as many as a number?
I have $10n$. Who has one less than a number?	I have $r - 1$. Who has one fourth of a number?
I have $\frac{1}{4}x$. Who has the sum of a number and three is eleven?	I have $11 = a + 3$. Who has four more than a number?
I have $4 + y$. Who has seven less than a number is ten?	I have $n - 7 = 10$. Who has the difference of a number and six is two?

I have $x - 6 = 2$. Who has seven times a number is fifty-six?	I have $56 = 7a$. Who has twice a number is thirty?
I have $2r = 30$. Who has the quotient of twelve and a number is four?	I have $\frac{12}{y} = 4$. Who has twelve more than a number is eighteen?
I have $18 = c + 12$. Who has three times as much as a number?	I have $3z$. Who has six less than a number?
I have $r - 6$ Who has two more than a number is nine?	I have $s + 2 = 9$. Who has five times a number is forty?
I have $40 = 5n$. Who has the sum of a number and eleven?	I have $a + 11$. Who has the product of a number and seven?
I have $7x$. Who has the quotient of a number and 3?	I have $\frac{x}{3}$. Who has eight less than a number is twenty?
I have $y - 8 = 20$. Who has the quotient of a number and 12 is 3?	I have $\frac{w}{12} = 3$. Who has the product of six and a number?
I have $6c$. Who has two more than a number is thirty?	I have $30 = b + 2$. Who has three times a number is eighteen?
I have $3g = 18$. Who has the quotient of a number and seven?	I have $\frac{p}{7}$. Who has half a number is seventeen?

Name_____

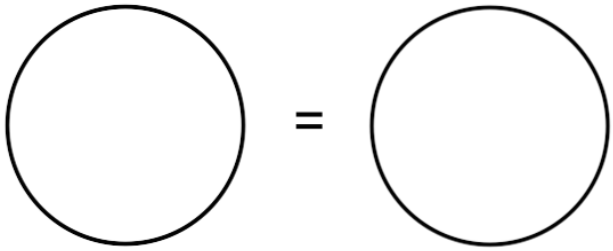
Solving Equations and Developing the Foundation for Proofs

STUDENT ACTIVITY

Lessons 1 – 2

1. Use the equation $x + 2 = 5$ to answer questions (a) through (d).

1.a. Draw a representation of the equation on the paper plates below.



1.b. Describe any steps you would take to solve for x using the representation you just made.

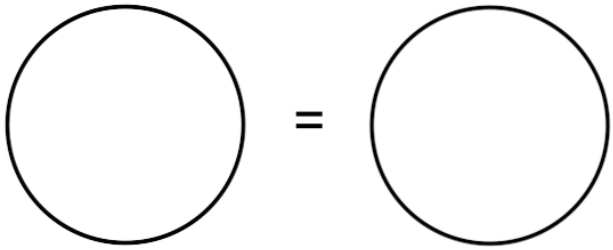
1.c. Which property justifies your description in part (b)?

1.d. Solve $x + 2 = 5$ algebraically, showing each step and the justification for each step or operation used.

Name_____

2. Use the equation $8 = 4x$ to answer questions (a) through (d).

2.a. Draw a representation of the equation on the paper plates below.



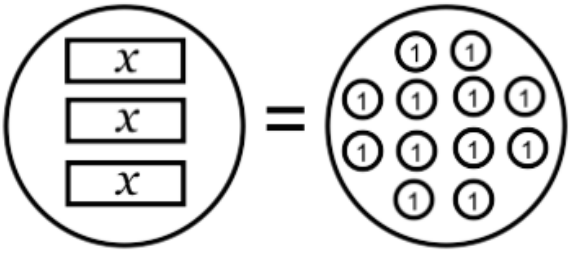
2.b. Describe any steps you would take to solve for x using the representation you just made.

2.c. Which property justifies your description in part (b)?

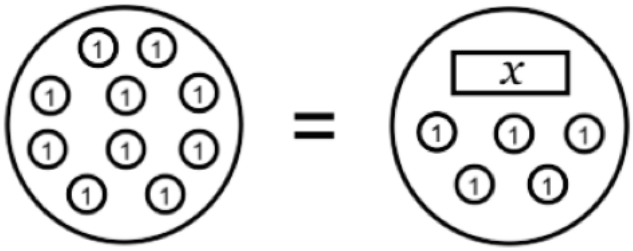
2.d. Solve $8 = 4x$ algebraically, showing each step and the justification for each step or operation.

Name_____

3. Represent the model below using an equation. Then solve the equation, and include justification for each step or operation used to solve it.



4. Represent the model below using an equation. Then solve the equation, and include justification for each step or operation used to solve it.



Name_____

5. Use the equation $x - 3 = 9$ to answer questions (a) through (c).

5.a. Which operation would you use to isolate x? Explain your reasoning.

5.b. Solve the equation $x - 3 = 9$, showing all steps.

5.c. Which property allows you to solve using the operation you chose?

Name_____

6. Use the equation $5 = \frac{a}{7}$ to answer questions (a) through (c).

6.a. Which operation would you use to isolate *a*? Explain your reasoning.

6.b. Solve the equation $5 = \frac{a}{7}$, showing all steps.

6.c. Which property allows you to solve using the operation you chose?

Name_____

Solving Equations and Developing the Foundation for Proofs

STUDENT ACTIVITY

Lesson 3

Represent the situation or problem with an equation that requires you to use one of the Properties of Equality to solve. Solve the equation to answer the question. Show all work, and justify your operation with the corresponding property.

1. Sally adopted a puppy when it weighed 9 pounds. Now the dog is full grown, and it weighs 26 pounds. How much weight has the puppy gained? Write your answer as a complete sentence.

Name_____

2. Ben buys a birthday cake cut into 12 pieces of the same size. Ben pays $36 for the cake. How much does each slice of cake cost? Assume each piece costs the same amount, and write your answer as a complete sentence.

3. Becky starts with a specific amount of money. Then Becky spends $6 on a toy. After buying the toy, Becky now has $5.50. How much money did Becky have before she bought the toy? Write your answer as a complete sentence.

SOLVING EQUATIONS AND DEVELOPING THE FOUNDATION FOR PROOFS

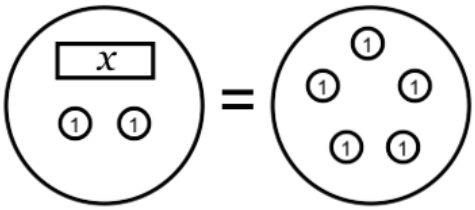
STUDENT ACTIVITY SOLUTION GUIDE

Lessons 1 – 2

1. Use the equation $x + 2 = 5$ to answer questions (a) through (d).

1.a. Draw a representation of the equation on the paper plates below.

CORRECT ANSWER



Note: Students may place the five 1s on the plate on the left and the x and two 1s on the plate on the right, which is also a correct representation of the equation provided.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student places three 1s on the plate that should include an *x* and two 1s.	cannot differentiate between number and variable	EXPLAIN *VARIABLE*
The student places all items on one plate or mixes grouping of items across plates.	does not correctly interpret the equal sign	EXPLAIN *EQUATION* or EXPLAIN *EXPRESSION*

1.b. Describe any steps you would take to solve for x using the representation you just made.

CORRECT ANSWER

Remove 2 unit cubes from each plate.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student describes adding two unit cubes to each plate.	sees addition and believes they should add to solve	EXPLAIN THE STEPS TO SOLVE AN EQUATION

1.c. Which property justifies your description in part (b)?

CORRECT ANSWER

Subtraction Property of Equality (or a response similar to "When you take the same amount away from both sides, the sides stay the same.")

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student identifies an incorrect property or does not explain the Subtraction Property of Equality adequately.	cannot relate the properties of equality to steps in solving an equation	EXPLAIN THE STEPS TO SOLVE AN EQUATION

1.d. Solve $x + 2 = 5$ algebraically, showing each step and the justification for each step or operation used.

CORRECT ANSWER

$x + 2 = 5$ given equation
$-2 \quad -2$ Subtraction Property of Equality
$x = 3$

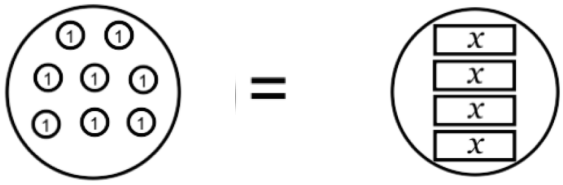
ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
$x + 7$	does not understand the equal sign separates the equation into two expressions that are equal to each other	EXPLAIN *EQUATION*
The student adds 2 to both sides of the equation.	sees addition and believes they should add in order to solve	EXPLAIN THE STEPS TO SOLVE AN EQUATION
The student subtracts 2 from the left side of the equation and adds 2 to the right side of the equation.	thinks they must add to one side of the equation and subtract from the other	EXPLAIN THE STEPS TO SOLVE AN EQUATION

2. Use the equation $8 = 4x$ to answer questions (a) through (d).

2.a. Draw a representation of the equation on the paper plates below.

CORRECT ANSWER



Note: Students may place the five 1s on the plate on the left and the x and two 1s on the plate on the right, which is also a correct representation of the equation provided.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student places four 1s and an *x* on the plate that should include four *x*s.	does not understand conventional variable notation	REPRESENT CONVENTIONAL VARIABLE NOTATION
The student places all items on one plate or mixes grouping of items across plates.	does not correctly interpret the equal sign	EXPLAIN *EQUATION* or EXPLAIN *EXPRESSION*

2.b. Describe any steps you would take to solve for x using the representation you just made.

CORRECT ANSWER

Since there are 4 variables on one paper plate, divide the 8 units on the other plate into 4 equal groups.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student describes multiplying the number of unit cubes by 4.	sees multiplication and believes they should multiply in order to solve	EXPLAIN THE STEPS TO SOLVE AN EQUATION

2.c. Which property justifies your description in part (b)?

CORRECT ANSWER

Division Property of Equality (or a response similar to "When you divide both sides into the same number of equal-size groups, the sides stay the same.")

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student identifies an incorrect property or does not explain the Division Property of Equality adequately.	cannot relate the properties of equality to steps in solving an equation	EXPLAIN THE STEPS TO SOLVE AN EQUATION

2.d. Solve $8 = 4x$ algebraically, showing each step and the justification for each step or operation.

CORRECT ANSWER

$\frac{8}{4} = \frac{4x}{4}$ given equation

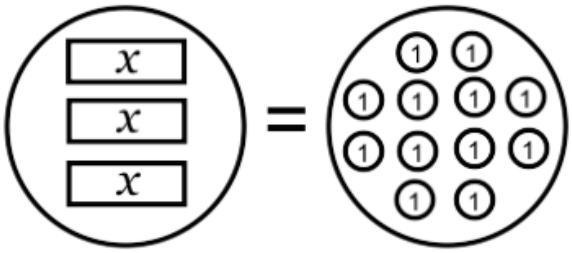
Division Property of Equality

$2 = x$

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
4 *x* 8, simplifying to 32*x*	does not understand the equal sign separates the equation into two expressions that are equal to each other	EXPLAIN *EQUATION*; EXPLAIN *VARIABLE*; REPRESENT CONVENTIONAL VARIABLE NOTATION
4 *x* 8 = 32	reads the variable as an operation	REPRESENT CONVENTIONAL VARIABLE NOTATION
The student multiplies by 4 on both sides of the equation, getting 32 = *x*.	sees multiplication and believes they should multiply in order to solve	EXPLAIN THE STEPS TO SOLVE AN EQUATION
The student divides by 4 on the right side of the equation and multiplies by 4 on the left side of the equation.	thinks they must multiply on one side of the equation and divide on the other	EXPLAIN THE STEPS TO SOLVE AN EQUATION

3. Represent the model below using an equation. Then solve the equation, and include justification for each step or operation used to solve it.



CORRECT ANSWER

$\frac{3x}{3} = \frac{12}{3}$ given equation

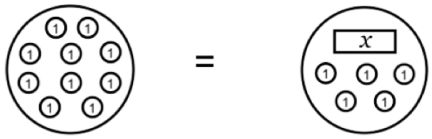
Division Property of Equality

$x = 4$

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
$x + 3$ or 3 (instead of $3x$)	does not understand how to represent variables	REPRESENT EXPRESSIONS WITH NUMBERS AND/OR VARIABLES
The student multiplies by 3 on both sides of the equation, getting $x = 36$.	sees multiplication and believes they should multiply in order to solve	EXPLAIN THE STEPS TO SOLVE AN EQUATION
The student divides by 3 on the left side of the equation and multiplies by 3 on the right side of the equation.	thinks they must multiply on one side of the equation and divide on the other	EXPLAIN THE STEPS TO SOLVE AN EQUATION

4. Represent the model below using an equation. Then solve the equation, and include justification for each step or operation used to solve it.



CORRECT ANSWER

$10 = x + 5$ given equation
$-5 \quad -5$ Subtraction Property of Equality
$5 = x$

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
$5x$ or 6 (instead of $x + 5$ or $5 + x$)	does not understand how to represent variables or does not understand like and different terms	REPRESENT EXPRESSIONS WITH NUMBERS AND/OR VARIABLES
The student adds 5 on both sides of the equation, getting $15 = x$.	sees addition and believes they should add in order to solve	EXPLAIN THE STEPS TO SOLVE AN EQUATION
The student subtracts 5 from the right side of the equation and adds 5 to the left side of the equation.	thinks they must add to one side of the equation and subtract from the other	EXPLAIN THE STEPS TO SOLVE AN EQUATION

5. Use the equation $x - 3 = 9$ to answer questions (a) through (c).

5.a. Which operation would you use to isolate x? Explain your reasoning.

CORRECT ANSWER

I would use addition on both sides of the equation to isolate x because 3 is subtracted from x and addition is the inverse operation of subtraction. In this case, $-3 + 3 = 0$.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Subtract 3 from both sides of the equation.	sees subtraction and believes they should subtract in order to solve	EXPLAIN THE STEPS TO SOLVE AN EQUATION
Add 3 to the left side of the equation.	does not describe maintaining a balanced equation	EXPLAIN *EQUATION*

5.b. Solve the equation $x - 3 = 9$, showing all steps.

CORRECT ANSWER

$$\begin{array}{rcl} x - 3 & = & 9 \\ +3 & & +3 \\ \hline x & = & 12 \end{array}$$

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
$x + 6$	does not understand the equal sign separates the equation into two expressions that are equal to each other	EXPLAIN *EQUATION*
The student subtracts 3 from both sides of the equation.	sees subtraction and believes they should subtract in order to solve	EXPLAIN THE STEPS TO SOLVE AN EQUATION
The student adds 3 to the left side of the equation and subtracts 3 from the right side of the equation.	thinks they must add to one side of the equation and subtract from the other	EXPLAIN THE STEPS TO SOLVE AN EQUATION

5.c. Which property allows you to solve using the operation you chose?

CORRECT ANSWER

> Addition Property of Equality (or satisfactory description of the Addition Property of Equality)

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student identifies an incorrect property or does not explain the Addition Property of Equality adequately.	cannot relate the properties of equality to steps in solving an equation	EXPLAIN THE STEPS TO SOLVE AN EQUATION

6. Use the equation $5 = \frac{a}{7}$ to answer questions (a) through (c).

6.a. Which operation would you use to isolate *a*? Explain your reasoning.

CORRECT ANSWER

> I would use multiplication to isolate *a* because *a* is divided by a number and multiplication is the inverse operation of division. Multiplying by 7 on both sides of the equation will isolate *a*.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Divide by 7 on both sides of the equation.	sees division and believes they should divide in order to solve	EXPLAIN THE STEPS TO SOLVE AN EQUATION
Multiply by 7 on the left side of the equation.	does not describe maintaining a balanced equation	EXPLAIN *EQUATION*

6.b. Solve the equation $5 = \frac{a}{7}$, showing all steps.

CORRECT ANSWER

$$5 = \frac{a}{7}$$
$$7 \cdot 5 = \frac{a}{7} \cdot 7$$
$$35 = a$$

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
$\frac{5a}{7}$	does not understand the equal sign separates the equation into two expressions equal to each other	EXPLAIN *EQUATION*
The student divides by 7 on both sides of the equation.	sees division and believes they should divide in order to solve	EXPLAIN THE STEPS TO SOLVE AN EQUATION
The student multiplies by 7 on the right side of the equation and divides by 7 on the left side of the equation.	thinks they must multiply on one side of the equation and divide on the other	EXPLAIN THE STEPS TO SOLVE AN EQUATION

6.c. Which property allows you to solve using the operation you chose?

CORRECT ANSWER

Multiplication Property of Equality (or satisfactory description of the Multiplication Property of Equality)

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student identifies an incorrect property or does not explain the Multiplication Property of Equality adequately.	cannot relate the properties of equality to steps in solving an equation	EXPLAIN THE STEPS TO SOLVE AN EQUATION

SOLVING EQUATIONS AND DEVELOPING THE FOUNDATION FOR PROOFS

STUDENT ACTIVITY SOLUTION GUIDE

Lesson 3

Represent the situation or problem with an equation that requires you to use one of the Properties of Equality to solve. Solve the equation to answer the question. Show all work, and justify your operation with the corresponding property.

1. Sally adopted a puppy when it weighed 9 pounds. Now the dog is full grown, and it weighs 26 pounds. How much weight has the puppy gained? Write your answer as a complete sentence.

CORRECT ANSWER

$x + 9 = 26$
$-9 \quad -9$ Subtraction Property of Equality
$x = 17$
The dog has gained 17 pounds since Sally adopted it.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
$26 - 9 = x$	cannot represent the equation in a way that illustrates the use of Subtraction Property of Equality to solve	REPRESENT EQUATIONS WITH NUMBERS AND/OR VARIABLES
Note: The student does know what to do but does not understand the concept behind representing a real-world problem with an equation and does not understand why the correct equation and this one are equivalent.		
The student adds 9 on both sides of the equation, getting $x = 35$.	sees addition and believes they should add in order to solve	EXPLAIN THE STEPS TO SOLVE AN EQUATION
The student writes "x is equal to 17" instead of a sentence describing what the solution means regarding the dog's weight.	solves correctly but is not able to explain what their solution means	EXPLAIN *SOLUTION TO AN EQUATION*

2. Ben buys a birthday cake cut into 12 pieces of the same size. Ben pays $36 for the cake. How much does each slice of cake cost? Assume each piece costs the same amount, and write your answer as a complete sentence.

CORRECT ANSWER

$\frac{12x}{12} = \frac{36}{12}$ Division Property of Equality

$x = 3$

Each slice of cake costs $3.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
$36 \div 12 = x$	cannot represent the equation in a way that illustrates the use of the Division Property of Equality to solve	REPRESENT EQUATIONS WITH NUMBERS AND/OR VARIABLES
Note: The student does know what to do but does not understand the concept behind representing a real-world problem with an equation and does not understand why the correct equation and this one are equivalent.		
The student multiplies by 12 on both sides of the equation, getting $x = 432$.	sees multiplication and believes they should multiply in order to solve	EXPLAIN THE STEPS TO SOLVE AN EQUATION
The student writes "x is equal to 3" instead of a sentence describing what the solution means regarding the price of each slice of cake.	solves correctly but is not able to explain what their solution means	EXPLAIN *SOLUTION TO AN EQUATION*

3. Becky starts with a specific amount of money. Then Becky spends $6 on a toy. After buying the toy, Becky now has $5.50. How much money did Becky have before she bought the toy? Write your answer as a complete sentence.

CORRECT ANSWER

$x - 6 = 5.50$

$+6 \quad +6$ Addition Property of Equality

$x = 11.50$

Becky started with $11.50.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
$6 + 5.50 = x$	cannot represent the equation in a way that illustrates the use of the Addition Property of Equality to solve	REPRESENT EQUATIONS WITH NUMBERS AND/OR VARIABLES
Note: The student does know what to do but does not understand the concept behind representing a real-world problem with an equation and does not understand why the correct equation and this one are equivalent.		
The student subtracts 6 on both sides of the equation, getting $x = -0.5$.	sees subtraction and believes they should subtract in order to solve	EXPLAIN THE STEPS TO SOLVE AN EQUATION
The student writes "x is equal to 11.50" instead of a sentence describing what the solution means regarding the amount of money Becky started with.	solves correctly but is not able to explain what their solution means	EXPLAIN *SOLUTION TO AN EQUATION*