SYSTEMS OF LINEAR EQUATIONS IN MULTIPLE REPRESENTATIONS

8.EE.8

CONTENTS

The types of documents contained in the unit are listed below. Throughout the unit, the documents are arranged by lesson.

Learning Map Information	An overview of the standards, the learning map section, and the nodes addressed in this unit
Teacher Notes	A brief discussion describing the progression depicted in the learning map section with research-based recommendations for focusing instruction to foster student learning and an introduction to the unit's lessons
Overview of Instructional Activities	A table highlighting the lesson goals and nodes addressed in each lesson of this unit
Instructional Activity	A detailed walkthrough of the unit
Instructional Activity Student Handout	A handout for the guided activity, intended to be paired with the Instructional Activity
Instructional Activity Supplement	A collection of materials or activities related to the Instructional Activity
Student Activity	A work-alone activity for students
Student Activity Solution Guide	A solution guide for the work-alone activity with example errors, misconceptions, and links to the learning map section

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Systems of Linear Equations in Multiple Representations

LEARNING MAP INFORMATION

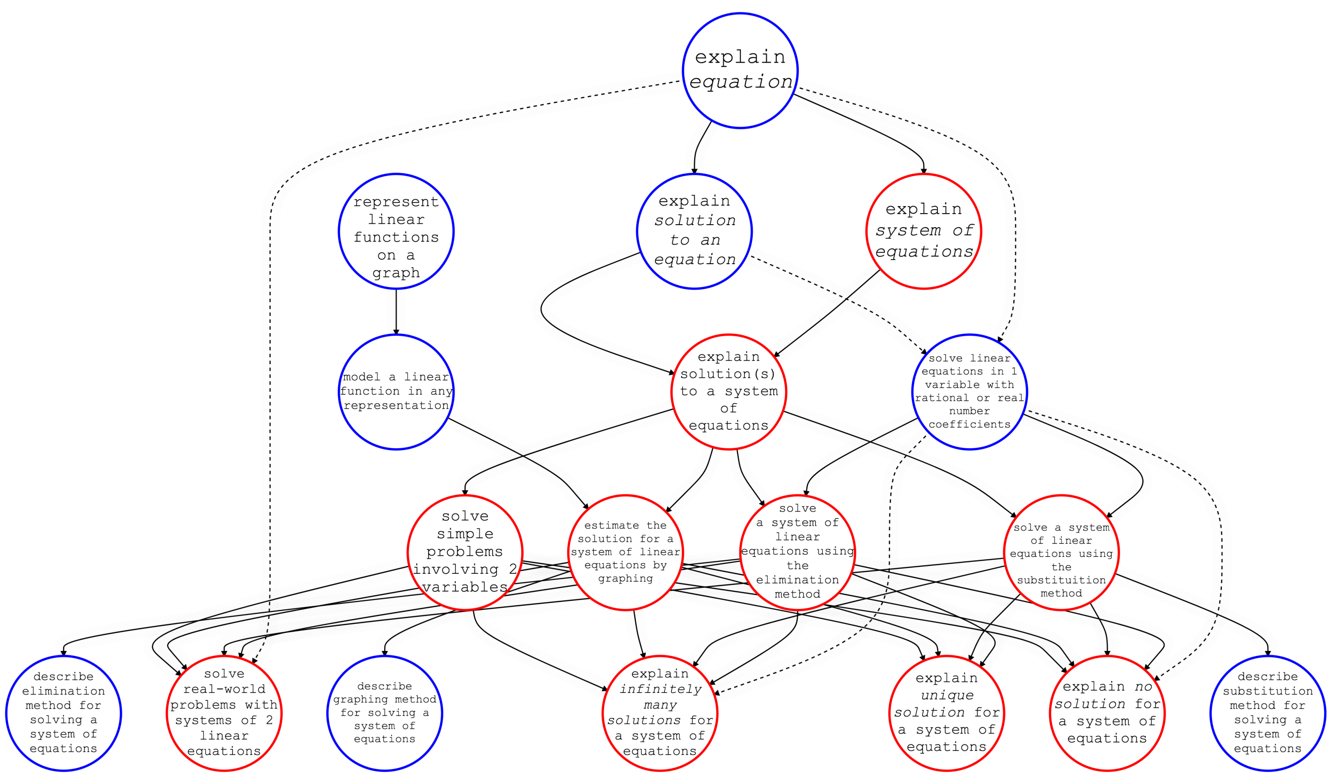
STANDARDS

8.EE.8 Analyze and solve pairs of simultaneous linear equations.

8.EE.8.a Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.

8.EE.8.b Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.*

8.EE.8.c Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*



**Learning map model of* 8.EE.8

Node Name	Node Description
DESCRIBE ELIMINATION METHOD FOR SOLVING A SYSTEM OF EQUATIONS	Describe how the elimination method can be used to rewrite a system of equations in one variable instead of two variables and how that equation can be used to solve the system of equations.
DESCRIBE GRAPHING METHOD FOR SOLVING A SYSTEM OF EQUATIONS	Describe how the graphing method can be used to determine the point(s) of intersection and to solve the system of equations.
DESCRIBE SUBSTITUTION METHOD FOR SOLVING A SYSTEM OF EQUATIONS	Describe how the substitution method can be used to rewrite a system of equations in one variable instead of two variables and how that equation can be used to solve the system of equations.
ESTIMATE THE SOLUTION FOR A SYSTEM OF LINEAR EQUATIONS BY GRAPHING	Graph two linear equations to estimate the solution to both equations.
EXPLAIN *EQUATION*	Make known your understanding that an equation is a mathematical sentence involving two equivalent numerical or algebraic expressions and an equal sign.
EXPLAIN *INFINITELY MANY SOLUTIONS* FOR A SYSTEM OF EQUATIONS	Make known your understanding that for a system of two linear equations in two variables with infinitely many solutions, every solution for one equation is a solution for the other equation. On such a graph, both equations represent the same line.
EXPLAIN *NO SOLUTION* FOR A SYSTEM OF EQUATIONS	Make known your understanding that for a particular system of two linear equations in two variables with no solution, there is no solution for one equation that is also a solution for the other equation. On such a graph, the equations represent parallel lines.
EXPLAIN *SOLUTION TO AN EQUATION*	Make known your understanding that solutions to an equation with one or more variables are the variable values that make the equation true.
EXPLAIN SOLUTION(S) TO A SYSTEM OF EQUATIONS	Make known your understanding that a solution to a system of equations satisfies all equations simultaneously.
EXPLAIN *SYSTEM OF EQUATIONS*	Make known your understanding that a system of equations consists of two or more equations to be considered simultaneously.
EXPLAIN *UNIQUE SOLUTION* FOR A SYSTEM OF EQUATIONS	Make known your understanding that for a system of two linear equations in two variables with one unique solution, there is one solution for one equation that is also a solution for the other equation. On such a graph, the two lines intersect once (not parallel or identical), and the solution is the coordinate pair where the lines intersect.
MODEL A LINEAR FUNCTION IN ANY REPRESENTATION	Model a linear function in any representation (table, graph, algebraic) given a function in a different representation.
REPRESENT LINEAR FUNCTIONS ON A GRAPH	Through writing or an appropriate assistive technology, represent linear functions on a graph.
SOLVE A SYSTEM OF LINEAR EQUATIONS USING THE ELIMINATION METHOD	Using the elimination method, determine the solution(s) for a system of linear equations.
SOLVE A SYSTEM OF LINEAR EQUATIONS USING THE SUBSTITUTION METHOD	Using the substitution method, determine the solution(s) for a system of linear equations.
SOLVE LINEAR EQUATIONS IN 1 VARIABLE WITH RATIONAL OR REAL NUMBER COEFFICIENTS	Solve linear equations with rational or real number coefficients, including equations with solutions that require expanding expressions using the distributive property and collecting like terms.
SOLVE REAL-WORLD PROBLEMS WITH SYSTEMS OF 2 LINEAR EQUATIONS	Solve real-world and mathematical problems leading to two linear equations in two variables.
SOLVE SIMPLE PROBLEMS INVOLVING 2 VARIABLES	Solve simple problems in which two variables are related in two ways, using a numerical approach. Such problems may not necessarily need to be represented by algebraic systems of equations.

Systems of Linear Equations in Multiple Representations

TEACHER NOTES

This unit includes the following documents:

- Learning Map Information
- Instructional Activity (three lessons)
- Instructional Activity Student Handout (for Lessons 2 and 3)
- Instructional Activity Supplement (for Lessons 1 and 3)
- Student Activity
- Student Activity Solution Guide

In this unit, students will learn how to solve systems of linear equations using multiple representations and solution methods. The focus of the lessons is on students' understanding of the logic and intuition involved in systems of linear equations, rather than the rote procedures (e.g., graphing, substitution, and elimination) often taught when students are learning about systems of equations. These procedures are addressed, but in the context of real-world problems. While it is important for students to have access to the standard methods for solving systems of equations, rather than learning one way by rote each day, it is preferable that as students encounter systems of equations, they focus on determining the point of intersection (Van de Walle et al., 2014) and explaining its meaning both in terms of the pure mathematics and in relation to the problem's context and definition of the variables. It is likely that students will need additional practice with these three methods, which they should be able to complete after the second lesson of this unit.

RESEARCH

With traditional teaching methods, students who are asked the purpose of solving systems of equations can, at best, express that the goal is to solve for x and y (Van del Walle et al., 2014). Additionally, as students gain algebraic knowledge, this can cause students to lose sight of the logical structure of problems (Amit & Klass-Tsirulnikov, 2005). Recommendations for teaching systems of equations include exposing students to multiple representations, including real-world contexts, and introducing a nonsymbolic approach that leads to more formal algebraic reasoning.

AN EXAMPLE: A NONSYMBOLIC APPROACH

Consider the following scenario.

- You are in charge of buying snacks for a class party. After going to the store, you determine that purchasing 20 bags of chips and 10 packages of fruit snacks would cost $23. Thinking this may not be enough, you also determine that 20 bags of chips and 20 packages of fruit snacks would cost $29. How much is each bag of chips and each package of fruit snacks?

Students should eventually realize the second option is an increase of 10 fruit snacks and $6 compared to the first, therefore dividing $6 by 10 fruit snacks would give the price per package of fruit snacks ($0.60).

Once the price of fruit snacks is established, students should be able to use either the first or second option to determine the cost of a bag of chips is $0.85.

There are several benefits to working with multiple representations of systems of equations. Transitioning among representations helps students make more connections among mathematical ideas, encourages students to see the big picture surrounding the problem, and promotes students' appreciation of multiple approaches to solving problems (Cleaves, 2008). Additionally, when the same problem is illustrated through multiple representations, the accessibility of the algebraic concepts increases, allowing students to evaluate the advantages and disadvantages of each representation (Cleaves, 2008). Sufficient time to work with multiple representations helps students better understand the connections among representations and inherently increases students' opportunities to consider and reflect on the material they are learning (Kalman, 2008; Preston & Garner, 2003). Therefore, it is important to provide students with high quality mathematical tasks that allow them to reason and discuss ideas in order to foster mathematical thinking, communication, and reasoning about systems of equations (Cai & Kenney, 2000; NCTM, 2014).

As students build their ability to reason with systems of equations through multiple representations, it becomes possible and natural for students to connect their informal algebraic thinking and reasoning to formal algebraic symbols, equations, and processes. It is important for students to be able to relate variables to a context, and approaching algebraic problems in nonsymbolic ways can help students gain an understanding of values that do not change in a problem (i.e., constants) and values that do change (i.e., variables), thereby supporting the development of algebraic intuition that can be applied to representing these problems with formal equations (Van de Walle et al., 2014; Kalman, 2008). Learning how to make sense of problems nonsymbolically, students develop the ability to reason in the context of two variables, which promotes their intuitive understanding of bivariate expressions and equations. These understandings prepare students to skillfully apply an algebraic approach to a system of equations and reduce the interference among algebraic symbols and the logic within the problem (Kalman, 2008). Solving the same problem using logic, then algebraically using symbols and the same logic, can help students make connections among the representations and begin the transition to algebraic solution methods (Kalman, 2008). As students begin working with systems of equations modeled algebraically, it is important for students to try solving by observation, particularly in special cases where patterns and observations regarding the values in the equations may shed light on the solution (Van de Walle et al., 2014).

AN EXAMPLE: NO SOLUTION BY OBSERVATION

Consider the following systems of equations.

1. $\begin{cases} y = 3x + 4 \\ y = 3x - 1 \end{cases}$

2. $\begin{cases} 2x + y = 3 \\ 2x + y = 5 \end{cases}$

Within each system of equations, the coefficients on the variables are identical, yet the constant is different from the first equation to the second equation. In the first example, students may think of the graphs of the lines: the *y*-intercept is different, yet the slope or rate of change is the same, therefore these lines will be parallel, never intersecting to provide a solution. In the second example, the equations imply that the same addends can somehow produce a different sum, which is not possible. The same would be true if the coefficients on the variables were in the same ratio, but the constant was not.

The following are additional examples of systems of equations with no solution, where the coefficients on the variables are in the same ratio, while the constants are not.

3. $\begin{cases} 4y = 8x + 3 \\ y = 2x - 1 \end{cases}$

4. $\begin{cases} 2x + y = 3 \\ 10x + 5y = 10 \end{cases}$

AN EXAMPLE: INFINITELY MANY SOLUTIONS BY OBSERVATION

Consider the following systems of equations.

1. $\begin{cases} y = 3x + 4 \\ y = 3x + 4 \end{cases}$

2. $\begin{cases} 2x + y = 3 \\ 2x + y = 3 \end{cases}$

In each of these systems of equations, the coefficients on the variables are identical, as are the constants. In the first example, students may think of the graphs of the lines: the *y*-intercept is the same, as is the slope or rate of change. Therefore, these lines will overlap, intersecting infinitely many times and creating infinitely many solutions. In the second example, the equations have the same addends and the same sum, therefore any values that satisfy the first equation will also satisfy the second. The same would be true if the coefficients on the variables and the constants were in the same ratio.

The following are additional examples of systems of equations with infinitely many solutions, where the coefficients on the variables and the constants are in the same ratio.

3. $\begin{cases} 4y = 8x - 12 \\ y = 2x - 3 \end{cases}$

4. $\begin{cases} 2x + y = 3 \\ 10x + 5y = 15 \end{cases}$

LEARNING MAP INFORMATION

The learning map section for this sequence of activities begins with students' understanding of equations and their solutions. Once students are able to explain a single equation, they can consider two or more equations simultaneously (or systems of equations). It is important for students to first understand the goal of solving a system of equations before looking for solutions to a particular system of equations. That is, the goal is to find a pair of numbers that satisfy both equations at the same time. There are various methods students can use to determine the solution to a system of equations. These methods include an informal, nonsymbolic approach for simple systems of equations, estimation of the solution(s) on a graph, algebraic substitution to rewrite the system in one variable instead of two variables, and elimination using equations. Upon solving using any one of these four approaches, students should be able to explain the different types of solutions a system of equations can have, and to apply their knowledge to more complex, real-world problems with systems of two linear equations.

INSTRUCTIONAL ACTIVITIES

The activities in this unit are designed to expose students to real-world scenarios that can be modeled using systems of equations, and to encourage students to acknowledge and understand multiple mathematical representations which will be relevant to more formal algebraic processes. In Lesson 1, students participate in a jigsaw activity where each group is provided with a different representation of the same problem. Students work within their groups to respond to the questions provided using intuition and logic, then regroup to examine the different representations other groups were provided to examine and solve the same problem. After discussing advantages and disadvantages for each representation, groups solve a new problem using any mathematical reasoning or model they wish. In Lesson 2, students begin with problems in real-world contexts which provide a motivating scenario for transitioning from nonsymbolic solving process to more formal algebraic solution processes (e.g., graphing, substitution, and elimination). Students also consider real-world contexts that result in no solution and infinitely many solutions. In Lesson 3, students determine through observation whether a system of equations will have one solution, no solution, or infinitely many solutions based on patterns among the values in the equations. Students then consider additional systems of equations on the coordinate plane, connecting the appearance of the graph to the solution(s).

REFERENCES

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Systems of Linear Equations in Multiple Representations

OVERVIEW OF INSTRUCTIONAL ACTIVITIES

Lesson	Learning Goal	Nodes Addressed
Lesson 1	Students will consider a system of equations in multiple representations and discuss advantages and disadvantages of information displayed in each representation.	▶ SOLVE SIMPLE PROBLEMS INVOLVING 2 VARIABLES
Lesson 2	Students will consider systems of equations verbally and algebraically. Students will then compare their algebraic solutions to a graphical representation of the solution, including systems of equations with no solution or infinitely many solutions.	▶ EXPLAIN *SYSTEM OF EQUATIONS* ▶ EXPLAIN SOLUTION(S) TO A SYSTEM OF EQUATIONS ▶ ESTIMATE THE SOLUTION FOR A SYSTEM OF LINEAR EQUATIONS BY GRAPHING ▶ SOLVE SIMPLE PROBLEMS INVOLVING 2 VARIABLES ▶ SOLVE REAL-WORLD PROBLEMS WITH SYSTEMS OF 2 LINEAR EQUATIONS ▶ SOLVE A SYSTEM OF LINEAR EQUATIONS USING THE SUBSTITUTION METHOD ▶ SOLVE A SYSTEM OF LINEAR EQUATIONS USING THE ELIMINATION METHOD ▶ EXPLAIN *NO SOLUTION* FOR A SYSTEM OF EQUATIONS ▶ EXPLAIN *INFINITELY MANY SOLUTIONS* FOR A SYSTEM OF EQUATIONS
Lesson 3	Students determine whether a system of linear equations has one solution, no solution, or infinitely many solutions from equations and from graphs.	▶ EXPLAIN *UNIQUE SOLUTION* FOR A SYSTEM OF EQUATIONS ▶ EXPLAIN *NO SOLUTION* FOR A SYSTEM OF EQUATIONS ▶ EXPLAIN *INFINITELY MANY SOLUTIONS* FOR A SYSTEM OF EQUATIONS ▶ ESTIMATE THE SOLUTION FOR A SYSTEM OF LINEAR EQUATIONS BY GRAPHING

Systems of Linear Equations in Multiple Representations

INSTRUCTIONAL ACTIVITY

Lesson 1

LEARNING GOAL

Students will consider a system of equations in multiple representations and discuss advantages and disadvantages of information displayed in each representation.

PRIMARY ACTIVITY

Groups of students will receive a real-world system of equations displayed in words and/or pictures, in a table, or on a graph. In their groups, students will answer questions posed to their group before regrouping to compare information and solutions with members of other groups using the jigsaw strategy. Upon comparing solution methods and representations, mixed groups will solve a new problem.

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

- Equal
- Initial
- Representation
- Total

MATERIALS

- Paper
- Instructional Activity Supplement (Recommend one copy for every four students.)

IMPLEMENTATION

Begin class by posing the following problem to students:

- You are in charge of buying snacks for a class party. After going to the store, you determine that purchasing 20 bags of chips and 10 packages of fruit snacks would cost $23. Thinking this may not be enough, you also determine that 20 bags of chips and 20 packages of fruit snacks would cost $29. How much is each bag of chips and each package of fruit snacks?

Allow students to work in pairs or small groups to determine the cost of each item.

Students should eventually realize the second option is an increase of 10 fruit snacks and $6 compared to the first, therefore dividing $6 by 10 fruit snacks would give the price per package of fruit snacks ($0.60).

Once the price of fruit snacks is established, students should be able to use either the first or second option to determine that the cost of a bag of chips is $0.85.

When students have arrived at the cost of each item, **require** them to confirm the quantities and costs described in the scenario by substituting the values they found into both scenarios described in the problem, to ensure they satisfy both scenarios simultaneously.

Note that because students are considering both situations at the same time, their solutions must satisfy both situations simultaneously.

NOTE: This thought process supports student reasoning about systems of equations and allows students to use their natural intuition and problem solving with numbers and contexts, without clouding their ability to reason through a problem with algebraic symbols and notation.

GUIDING QUESTIONS

Elicit student thinking:

- What is the same about the two sets of information provided? What is different?
- Can you describe this situation in your own words?
- Show me how you got started.

Determine if the student can SOLVE SIMPLE PROBLEMS INVOLVING 2 VARIABLES:

- What do you notice regarding the information provided about chips?
- What do you notice regarding the information provided about fruit snacks?
- What do you notice regarding the information provided about the total cost?
- How does the increase in packages of fruit snacks impact the price? How many fruit snacks were added to cause this price change?
- If an increase of 10 packages of fruit snacks caused a price increase of $6, can you determine the price of one package of fruit snacks?
- Once you know how much a package of fruit snacks costs, how could you determine how much one bag of chips costs?
- Do the prices you found work in both sets of information? How do you know?
- What is the answer to the question posed in this problem?

Next, place students in small groups of three to four to participate in a jigsaw activity. Because it is recommended that students work in groups of three or four, it may be necessary to have more than one Group A, more than one Group B, and so forth. It is preferable to have the same number of students considering each problem representation.

Hand out the same problem representation from the INSTRUCTIONAL ACTIVITY SUPPLEMENT to each student within a group and direct them to work together to answer the questions posed.

Note that students can use any mathematical representation they can think of that may be useful for solving the problem posed.

Use the Guiding Questions to scaffold group and individual student understanding.

Each group should come to the conclusion that if there are 40 students, the costs for the two options are equal. If there are fewer than 40 students, Mike's Mini-Golf is cheaper, and if there are more than 40 students, Izzy's Ice Skating is cheaper.

GUIDING QUESTIONS

Elicit student thinking:

- What do you know about the pricing for Mike's Mini-Golf?
- What do you know about the pricing for Izzy's Ice Skating?
- What should you consider as you respond to these questions?

Determine if the student can SOLVE SIMPLE PROBLEMS INVOLVING 2 VARIABLES:

- How does the initial price of the two places differ?
- How do the initial prices of the two places change?
- Is one place always more expensive than the other? How do you know?
- Are the costs at the two places ever equal for an equal number of students? For how many students? What is the corresponding price? How does this appear on a graph?
- Which place is a better deal? Mike's Mini-Golf or Izzy's Ice Skating? Explain your reasoning.
- What mathematical representation(s) did you use to respond to the question?

Once each group has come to the understanding that the costs are dependent on the number of students and that if there are 40 students, then the costs for each option are equal, students should regroup so there is at least one person from Groups A, B, C, and D in each new group.

Once students have regrouped, **ask** each student to share with the group their representation of the problem situation and the approach they took to respond to the questions.

Require students to discuss the advantages and disadvantages of each representation.

Ask students how their solution process varied based on the initial information they were provided and the way the information was represented.

Once students have considered each representation, **provide** students with the following problem scenario to respond to:

- You are shopping for a new cell phone and have pricing from two different stores, Cellular City and Mobile Mart. Cellular City sells a phone for $300 and charges $45.50 per month for service. Mobile Mart sells the same phone for $260 and charges $48 per month for service. Which store is the better choice based on cost? Is there a point in time at which the total costs would be equal for both stores? If so, when would this occur and what would the total cost be?

Require students to work the problem provided on paper in their groups, using whatever mathematical representations they feel are beneficial to respond to the questions posed. Students should show their work to explain their thinking.

At the end of the activity, teachers should collect each group's solution to the problem to check student understanding.

GROUP A

You are a member of the student council at your school and are planning a field trip for your class. You are considering two different options: Mike's Mini-Golf and Izzy's Ice Skating. After some research, you find that Mike's Mini-Golf charges $10 per person for all you can putt mini-golf for three hours. Izzy's Ice Skating charges $200 plus $5 per person for the same amount of time.

- Which option is the better choice based on cost?
- Is there a number of people for which the costs would be equal?
- Use any mathematics or representations that are useful to your decision making process.

GROUP B

You are a member of the student council at your school and are planning a field trip for your class. You are considering two different options: Mike's Mini-Golf and Izzy's Ice Skating. You see the following ads in the newspaper:

- Which option is the better choice based on cost?
- Is there a number of people for which the costs would be equal?
- Use any mathematics or representations that are useful to your decision making process.

GROUP C

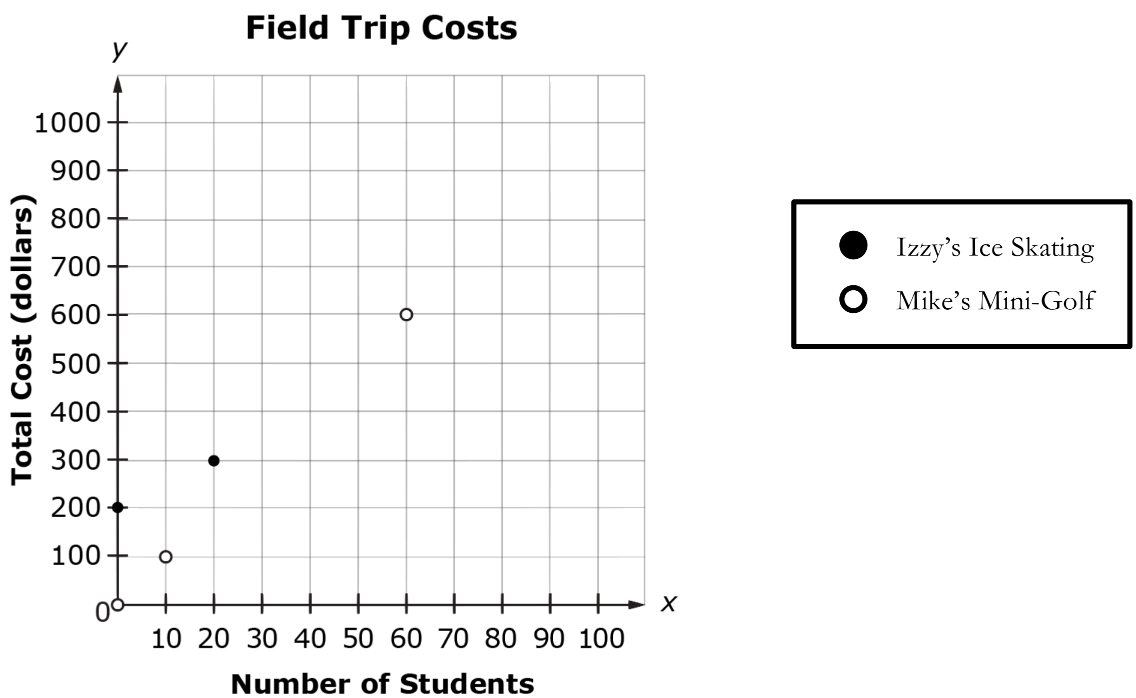
You are a member of the student council at your school and are planning a field trip for your class. You are considering two different options: Mike’s Mini-Golf and Izzy’s Ice Skating. The following table provides information regarding the total costs of each option based on the number of people.

NUMBER OF STUDENTS							
	0	10	20	30	40	50	60
MIKE’S MINI-GOLF	\$0	\$100					\$600
IZZY’S ICE SKATING	\$200		\$300				

- Which option is the better choice based on cost?
- Is there a number of people for which the costs would be equal?
- Use any mathematics or representations that are useful to your decision making process.

GROUP D

You are a member of the student council at your school and are planning a field trip for your class. You are considering two different options: Mike’s Mini-Golf and Izzy’s Ice Skating. The following graph provides information regarding the total costs of each option based on the number of people.



- Which option is the better choice based on cost?
- Is there a number of people for which the costs would be equal?
- Use any mathematics or representations that are useful to your decision making process.

Systems of Linear Equations in Multiple Representations

INSTRUCTIONAL ACTIVITY

Lesson 2

LEARNING GOAL

Students will consider systems of equations verbally and algebraically. Students will then compare their algebraic solutions to a graphical representation of the solution, including systems of equations with no solution or infinitely many solutions.

PRIMARY ACTIVITY

Students will continue considering real-world scenarios that describe a system of equations. They will begin by solving a problem without equations and variables, focusing on the logic and problem solving skills involved. Students will then be guided through an algebraic solution process that mimics their informal solution process, establishing the substitution and elimination methods of solving. Finally, students will graph the system of equations in order to relate the solution with the behavior of the graph.

NOTE: This lesson will likely require more than one class period to complete.

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

- Substitution
- Elimination
- Equation
- Variable
- System of equations
- One solution
- No solution
- Infinitely many solutions
- Intersection
- Parallel (lines)
- Overlapping (lines)

MATERIALS

- Paper
- Graph paper
- Instructional Activity Student Handout

IMPLEMENTATION

Begin class by posing the following problem to students:

- Brad and Kyle have started new jobs and decided they will start saving some of the money they make. Brad has a balance of $20 in his bank account and decides he will save $4 per week. Kyle doesn't have any money in his bank account, so he decides he will save $6 per week. Will Brad and Kyle's bank accounts ever have the same balance? If yes, after how long? What will the balance be? If no, why not?

Allow students to work in pairs or small groups to work through the problem.

Students should notice that although Brad's savings account starts with more than Kyle's, Brad does not save as much per week as Kyle, and therefore Kyle's account will eventually equal and then surpass Brad's.

Students should then notice that Kyle saves $2 more per week than Brad, therefore in order to "catch up" to the $20 "head start" Brad had, it will take 10 weeks. This may be discovered through a table of values or division.

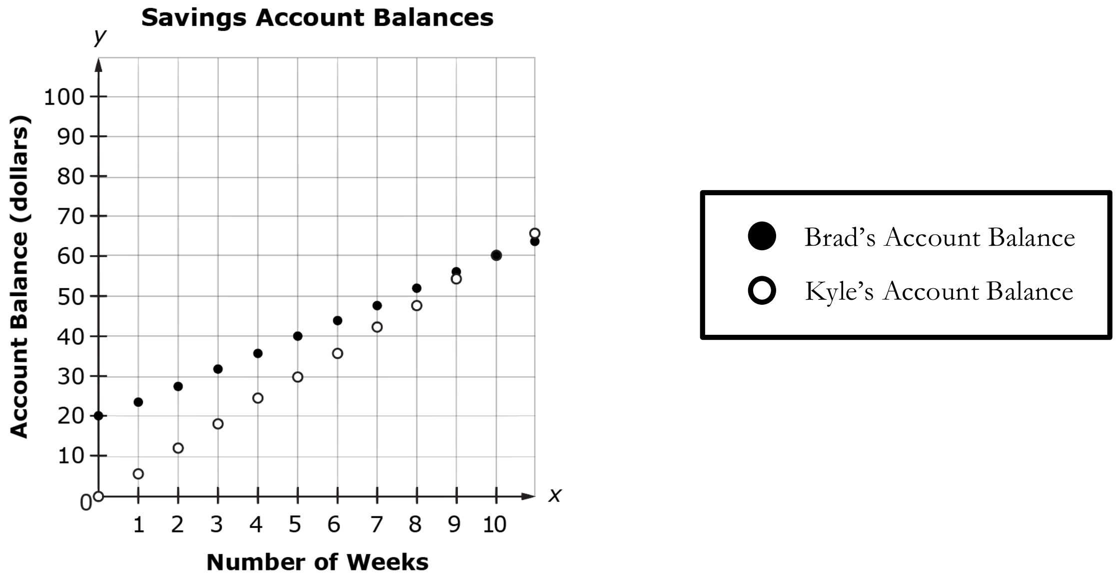
Once students establish that the bank accounts will have the same balance after 10 weeks, they can use either Brad's or Kyle's information to determine that the balance after 10 weeks will be $60.

Ask groups of students to share their solution process and **discuss** their approach and logic as a class. Identify, if possible, groups of students whose processes are slightly different.

If groups of students have made errors in their reasoning or solution process, these errors would be valuable for students to consider, discuss, and correct.

Some students may share a graphical representation of the savings accounts as weeks pass. If they have, use the graph to discuss how the solution appears on the graph. If a graph has not yet been presented, **require** students to use graph paper to create a graph to represent the scenario.

Graphs should resemble the following:



Discuss the patterns displayed in the graph regarding Brad's account and Kyle's account separately, then compared to each other. Students should notice that the points overlap at the coordinate (10, 60).

Ask students what this point on the graph means and how it relates to their previous work. Students should notice this point represents both savings accounts reaching a balance of $60 after 10 weeks.

Note that the coordinate pair (10, 60), representing $60 in the bank account after 10 weeks, is true for both Brad and Kyle's bank accounts. Therefore, these are the values that satisfy both described conditions at the same time.

Discuss that the graph is *discrete* (not connected) because the deposits are only made once a week. Therefore, plotting values or drawing a line between points does not accurately represent the problem situation.

GUIDING QUESTIONS

Elicit student thinking:

- What information in this problem is important?
- How would you go about solving this problem?
- Show me how you started.
- What information does this graph provide?

Determine if the student can EXPLAIN *SYSTEM OF EQUATIONS*:

- How many people are you considering in this scenario?
- Do you think of Brad and Kyle separately or together in this scenario?

Determine if the student can EXPLAIN SOLUTION(S) TO A SYSTEM OF EQUATIONS:

- What must be true about the solution you found to this problem?
- How could you check to be sure your answer is correct?

Determine if the student can ESTIMATE THE SOLUTION FOR A SYSTEM OF LINEAR EQUATIONS BY GRAPHING:

- What point is significant on the graph in order to solve this problem about Brad and Kyle's bank accounts? Why is this point significant?
- At what point do these graphs intersect? What does that point mean in terms of the problem?

Determine if the student can SOLVE SIMPLE PROBLEMS INVOLVING 2 VARIABLES:

- How can you use the information to start solving this problem?
- Will Brad and Kyle ever have the same amount of money in their bank accounts at the same time? How do you know?
- How can you use this information to determine when the bank account balances will be equal?
- After how long will Kyle and Brad have the same amount of money in their bank accounts?
- What will the balance of the bank accounts be when they are equal?
- How is this information represented on a graph?

Once groups of students have shared their work, begin relating the logic and problem solving processes they used to an algebraic approach with variables and equations.

Ask students to identify in their work which values were set or fixed and which values were unknown and changing.

Students should be able to identify that Brad's initial balance of \$20 was fixed, and that the rate at which he saved, \$4 per week, was constant. They should also be able to identify that the rate at which Kyle saved, \$6 per week, was constant. Values that could change were the number of weeks that had passed and the balance of each bank account.

Note that when there are values that change, these values are represented by variables in equations. In this scenario, because there are two values that are changing (the number of weeks that have passed and the balance of the bank account), there are two variables required to represent the scenario described.

Guide students to write equations similar to the following to represent the balance of each bank account. **Note** that students do not need to use the variables x and y. Any variables students choose, as long as they are two different variables and are clearly defined, are acceptable.

Brad: $y = 4x + 20$

Kyle: $y = 6x$

where x represents the number of weeks that have passed, and y represents the balance of the bank account

Discuss with students that because they are trying to determine if the balances are ever the same, they must consider the two equations together. When two or more equations are considered together, they create a *system of equations*.

Scaffold the solution process for this system of equations with the reasoning students used to solve the problem before it was written as a system of equations.

First, students are looking for the point at which the balance of the accounts will be equal. Therefore, the expressions that represent the balance of the savings accounts can be written as equal to each other. This is also known as *substitution*; because the variable y is equal to $6x$ in one equation, y can be replaced with $6x$ in the other equation.

$$6x = 4x + 20$$

Next, students likely noticed that by subtracting \$4 from \$6, Kyle was saving \$2 more per week than Brad was. This can be represented algebraically through subtraction.

$$6x = 4x + 20$$

$$\underline{-\,4x} \quad \underline{-\,4x}$$

$$2x = 20$$

Next, students likely noticed, through division or a table of values, that because Kyle saves $2 more per week, it will take him 10 weeks to "make up" the $20 head start Brad had. This can be represented algebraically through division.

$$\frac{2x}{2} = \frac{20}{2}$$

$$x = 10$$

In 10 weeks, Brad and Kyle's accounts will have the same balance.

Lastly, once students figured it would take 10 weeks for Brad and Kyle to have the same balance, they could use either equation to determine the account balance after 10 weeks.

Using the equation to represent Brad's savings:

$$y = 4x + 20$$
$$y = 4(10) + 20$$
$$y = 40 + 20$$
$$y = 60$$

In 10 weeks, Brad's bank account will have a balance of $60.

Using the equation to represent Kyle's savings:

$$y = 6x$$
$$y = 6(10)$$
$$y = 60$$

In 10 weeks, Kyle's bank account will have a balance of $60.

Ask students how the problem changes if Kyle starts with $12 in his bank account, but everything else remains the same.

Students should be able to determine that this means Brad only has an $8 head start. Kyle catches up at a rate of $2 per week, therefore it will take four weeks for Brad and Kyle to have the same balance in their bank accounts, but this time the balance will be $36.

Model this slightly different problem using a system of equations and an algebraic approach.

Ask students how the equation representing Kyle's bank account will change if he starts with a balance of $12. **Ask** students if the equation representing Brad's bank account will change.

Students should establish the following equations.

Brad: $y = 4x + 20$

Kyle: $y = 6x + 12$

where x represents the number of weeks that have passed, and y represents the balance of the bank account

Remind students that because they are looking for the point at which the balance of the accounts will be equal, the expressions that represent the balance of the savings accounts can be written as equal to each other. Continue to connect each step in the solution process back to the reasoning processes students experienced without algebraic symbols and variables.

$$\begin{aligned} 6x + 12 &= 4x + 20 \\ \underline{-12\quad} &\quad \underline{-12} \\ 6x &= 4x + 8 \\ \underline{-4x\quad} &\quad \underline{-4x} \\ \frac{2x}{2} &= \frac{8}{2} \\ x &= 4 \end{aligned}$$

In four weeks, Brad and Kyle's accounts will have the same balance.

Brad: $y = 4(4) + 20 = 36$
Kyle: $y = 6(4) + 12 = 36$
In four weeks, both Brad Kyle's bank accounts will have a balance of $36.

Ask students what the graph would look like (e.g., whether the graphs would intersect, where the graphs would intersect, and what this point means in terms of the problem situation) if they were to graph the equations for Brad and Kyle's bank accounts.

GUIDING QUESTIONS

Elicit student thinking:

- What information in this problem is important?
- How would you go about solving this problem?
- Can you describe this situation in your own words?
- What do the variables represent?

Determine if the student can EXPLAIN *SYSTEM OF EQUATIONS*:

- How many people are you considering with the equations you wrote?
- Do you think of the equations separately or together?

Determine if the student can EXPLAIN SOLUTION(S) TO A SYSTEM OF EQUATIONS:

- What must be true about the solution you found to this system of equations?
- How could you check to be sure your answer is correct?

Determine if the student can ESTIMATE THE SOLUTION FOR A SYSTEM OF LINEAR EQUATIONS BY GRAPHING:

- What point is significant on the graph in order to solve this problem about Brad and Kyle's bank accounts? Why is this point significant?
- At what point do these graphs intersect? What does that point mean in terms of the problem?

Determine if the student can SOLVE REAL-WORLD PROBLEMS WITH SYSTEMS OF 2 LINEAR EQUATIONS:

- How can you use the information to start solving this problem?
- How can you represent this problem with equations?
- Will Brad and Kyle ever have the same amount of money in their bank accounts at the same time? How do you know?
- How can you use this information to determine when the bank account balances will be equal?
- After how long will Kyle and Brad have the same amount of money in their bank accounts?
- What will the balance of the bank accounts be when they are equal?
- How is this information represented on a graph?

Determine if the student can SOLVE A SYSTEM OF LINEAR EQUATIONS USING THE SUBSTITUTION METHOD:

- Can you describe when substitution occurs in this solution process?
- How does substitution help you arrive at a solution for the problem?

Next, students will consider a problem situation that lends itself to the elimination or simultaneous problem solving process.

Pose the following problem to students:

- You are looking to adopt a dog and a cat at the local animal shelter. Your friend Sarah recently adopted two dogs and one cat and paid \$130. Your friend Max recently adopted one dog and two cats and paid \$110. Assuming the cost to adopt a cat and a dog does not change, how much does it cost to adopt one dog? How much does it cost to adopt one cat?

Allow students to work in pairs or small groups to work through the problem.

Students may try making tables or doubling the information about one of the friends so that the number of cats or dogs matches from one scenario to the next. This would allow students to isolate the cost of the other animal and work their way towards the cost of one dog and one cat.

Doubling strategies could resemble the following:

- If you double the number of dogs, the number of cats, and the price of Sarah's adoption, you would have four dogs and two cats, which would cost \$260. Compared to Max, this is the same number of cats, three more dogs, and \$150 more. This means that each dog costs \$50, which means each cat would have to cost \$30.

or

- If you double the number of dogs, the number of cats, and the price of Max's adoption, you would have two dogs and four cats which would cost \$220. Compared to Sarah, this is the same number of dogs, three more cats, and \$90 more. This means that each cat costs \$30, which means each dog would have to cost \$50.

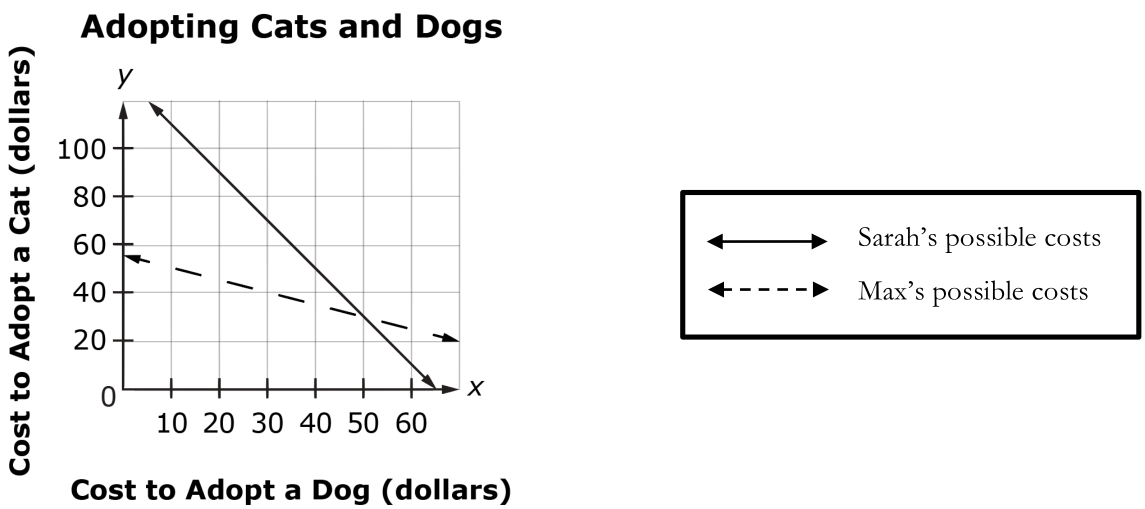
Ask groups of students to share their solution process and **discuss** their approach and logic as a class.

If possible, **identify** groups of students whose processes were slightly different and **ask** them to share their work. If students have not used the doubling strategy, suggest this as another approach and help them make sense of the logic used.

If groups of students have made errors in their reasoning or solution process, these errors would be valuable for students to consider, discuss, and correct.

Some students may share a graphical representation of the savings accounts as weeks pass. If a graph has not yet been presented, students should use graph paper to create a graph to represent the scenario. A strategy to graph this scenario is to consider the different possible combinations of costs for dogs and cats for Sarah, then consider the different possible combinations of costs for dogs and cats for Max. The point that overlaps between the two graphs indicates the solution to the problem. If they have, use the graph to discuss how the solution appears on the graph.

Graphs should resemble the following. Note that the *x*– and *y*–axes can be reversed.



Discuss the patterns displayed in the graph regarding the cost options for each friend separately, then compared to each other. Students should notice that the lines intersect at the coordinate (50, 30).

Ask students what this point on the graph means and how it relates to their previous work. Students should notice this point represents where the cost options for both friends reach the same amount.

Note that the coordinate pair (50, 30), representing a cost of $50 to adopt a dog and $30 to adopt a cat, is true for both friends' adoption scenarios. Therefore, these are the values that satisfy both described conditions at the same time.

Discuss that the graph is *continuous* (connected) because the cost combinations in between intervals are possible in the context of the problem situation. For example, it is possible Max could spend $20.10 for a dog and $44.95 for each cat.

GUIDING QUESTIONS

Elicit student thinking:

- What information in this problem is important?
- How would you go about solving this problem?
- What information does this graph provide?

Determine if the student can EXPLAIN *SYSTEM OF EQUATIONS*:

- How many people are you considering in this scenario?
- Do you think of Sarah and Max separately or together in this scenario?

Determine if the student can EXPLAIN SOLUTION(S) TO A SYSTEM OF EQUATIONS:

- What must be true about the solution you found to this problem?
- How could you check to be sure your answer is correct?

Determine if the student can ESTIMATE THE SOLUTION FOR A SYSTEM OF LINEAR EQUATIONS BY GRAPHING:

- What point is significant on the graph in order to solve this problem adoption costs for cats and dogs? Why is this point significant?
- At what point do these graphs intersect? What does that point mean in terms of the problem?

Determine if the student can SOLVE SIMPLE PROBLEMS INVOLVING 2 VARIABLES:

- How can you use the information to start solving this problem?
- Is it possible that Sarah and Max paid the same amount of money to adopt each cat and each dog? How do you know?
- How can you use this information to determine how much each cat and each dog costs to adopt?
- How much does each cat cost to adopt?
- How much does each dog cost to adopt?
- How is this information represented on a graph?

Once groups of students have shared their work, begin relating the logic and problem solving processes they used to an algebraic approach with variables and equations.

Ask students to identify, in their work, which values were set or fixed and which values were unknown and changing.

Students should be able to identify the number of dogs and cats each friend adopted and that the price they paid is fixed. Values that could change were the cost to adopt a cat and the cost to adopt a dog.

Note that when there are values that change, these values are represented by variables in equations. In this scenario, since there are two values that are changing (the cost to adopt a cat and the cost to adopt a dog), there are two variables required to represent the scenario described.

Guide students to write equations similar to the following to represent the adoption scenarios for each friend. **Note** that students do not need to use the variables x and y. Any variables students choose, as long as they are two different variables and are clearly defined, are acceptable.

Sarah: $2x + y = 130$

Max: $x + 2y = 110$

where x represents the cost to adopt a dog, and y represents the cost to adopt a cat

Discuss with students that because they are trying to determine how much each friend paid to adopt a cat and a dog, and the adoption costs were the same for both friends, they must consider the two equations together. Because two or more equations are considered together, this is also considered a *system of equations.*

Scaffold the solution process for this system of equations using one of the doubling strategies discussed previously, before it was written as a system of equations.

It is possible to double either the equation that represents Sarah's adoption scenario or the equation that represents Max's adoption scenario. Doubling Sarah's adoption scenario while keeping Max's adoption scenario the same produces the following equations.

Sarah: $4x + 2y = 260$

Max: $x + 2y = 110$

Considering these two equations together, students should now notice that Sarah's equation represents adopting the same number of cats, but three more dogs compared to Max. The extra cost associated with this adoption is $150. This reasoning is a result of subtracting the equation representing Max's adoption from the doubled equation representing Sarah's adoption. Algebraically, this is represented as follows.

$$\begin{array}{c} 4x + 2y = 260 \\ \underline{-(x + 2y = 110)} \\ 3x = 150 \end{array}$$

This is also known as *elimination*, because the variable y is eliminated when the second equation is subtracted from the first.

The remaining equation represents that adopting three dogs costs $150. Through division, students can determine the cost of adopting one dog.

$$\frac{3x}{3} = \frac{150}{3}$$
$$x = 50$$

It costs $50 to adopt a dog.

Once students figure that it costs $50 to adopt a dog, they can use either of the original equations to determine how much it costs to adopt a cat.

Using the equation to represent Sarah's adoption:

$$\begin{aligned} 2x + y &= 130 \\ 2(50) + y &= 130 \\ 100 + y &= 130 \\ \underline{-100 \qquad} & \underline{\qquad -100} \\ y &= 30 \end{aligned}$$

It costs $50 to adopt a dog and $30 to adopt a cat.

Using the equation to represent Max's adoption:

$$\begin{aligned} x + 2y &= 110 \\ 50 + 2y &= 110 \\ \underline{-50 \qquad} & \underline{\qquad -50} \\ \frac{2y}{2} &= \frac{60}{2} \\ y &= 30 \end{aligned}$$

It costs $50 to adopt a dog and $30 to adopt a cat.

GUIDING QUESTIONS

Elicit student thinking:

- What information in this problem is important?
- How would you go about solving this problem?
- Can you describe this situation in your own words?
- What do the variables represent?

Determine if the student can EXPLAIN *SYSTEM OF EQUATIONS*:

- How many people are you considering with the equations you wrote?
- Do you think of the equations separately or together?

Determine if the student can EXPLAIN SOLUTION(S) TO A SYSTEM OF EQUATIONS:

- What must be true about the solution you found to this system of equations?
- How could you check to be sure your answer is correct?

Determine if the student can ESTIMATE THE SOLUTION FOR A SYSTEM OF LINEAR EQUATIONS BY GRAPHING:

- Which point is significant on the graph in order to solve this problem about adoption costs for cats and dogs? Why is this point significant?
- At which point do these graphs intersect? What does that point mean in terms of the problem?

Determine if the student can SOLVE REAL-WORLD PROBLEMS WITH SYSTEMS OF 2 LINEAR EQUATIONS:

- How can you use the information to start solving this problem?
- How can you represent this problem with equations?
- Is it possible that Sarah and Max paid the same amount of money to adopt each cat and each dog? How do you know?
- How can you use this information to determine how much each cat and each dog costs to adopt?
- How much does each cat cost to adopt?
- How much does each dog cost to adopt?
- How is this information represented on a graph?

Determine if the student can SOLVE A SYSTEM OF LINEAR EQUATIONS USING THE ELIMINATION METHOD:

- Can you describe when elimination occurs in this solution process?
- How does elimination help you arrive at a solution for the problem?

Hand out the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT to each student to practice solving systems of equations in Questions 1 and 2. Students can work either individually or in small groups. Use the following guiding questions to assess student understanding as they work.

GUIDING QUESTIONS

Elicit student thinking:

- What information in this problem is important?
- How would you go about solving this problem?
- Can you describe this situation in your own words?
- What do the variables represent?

Determine if the student can EXPLAIN *SYSTEM OF EQUATIONS*:

- How many people are you considering in this problem?
- Do you think of the people's information separately or together?

Determine if the student can EXPLAIN SOLUTION(S) TO A SYSTEM OF EQUATIONS:

- What must be true about the solution you found to this system of equations?
- How could you check to be sure your answer is correct?

Determine if the student can SOLVE REAL-WORLD PROBLEMS WITH SYSTEMS OF 2 LINEAR EQUATIONS:

- What reasoning can you use based on the information provided?
- What options do you have for solving this problem?
- How can you represent this problem with equations?

Determine if the student can ESTIMATE THE SOLUTION FOR A SYSTEM OF LINEAR EQUATIONS BY GRAPHING:

- What point is significant on the graph in order to solve this problem? Why is this point significant?
- At what point do these graphs intersect? What does that point mean in terms of the problem?

Determine if the student can SOLVE A SYSTEM OF LINEAR EQUATIONS USING THE ELIMINATION METHOD:

- [As students work on Question 1] Can you describe when elimination occurs in this solution process?
- [As students work on Question 1] How does elimination help you arrive at a solution for the problem?
- How can you connect the elimination method to a logical solution process that does not use a formal, algebraic solution method?

Determine if the student can SOLVE A SYSTEM OF LINEAR EQUATIONS USING THE SUBSTITUTION METHOD:

- [As students work on Question 2] Can you describe when substitution occurs in this solution process?
- [As students work on Question 2] How does substitution help you arrive at a solution for the problem?
- How can you connect the substitution method to a logical solution process that does not use a formal, algebraic solution method?

Once students have completed Question 1 and 2 on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT, **discuss** the solution methods used and how their solutions appear on a graph.

Emphasize that the point of intersection on the graph represents the values that satisfy both situations at the same time.

Next, students will consider a system of linear equations with no solution.

Pose the following problem to students:

- Sam and Sue decide to start saving money. Sam starts with $30 in his savings account and Sue starts with $45 in her savings account. If Sam and Sue both save $5 per week, after how many weeks will they have the same amount of money in their savings accounts? How much will they have?

Allow students to work in pairs or small groups to work through the problem.

Students should eventually notice that Sam and Sue will never have the same amount of money in their accounts because Sue had more to begin with and they are saving at the same rate. This means Sue will always have $15 more in her savings account than Sam.

Guide students through the development of the equations and graph that represent the scenario. The equations should be as follows and the graph should display parallel lines.

Sam: $y = 5x + 30$

Sue: $y = 5x + 45$

where x represents the number of weeks, and y represents the amount of money in the savings account

Ask students how they could tell from the equations that Sam and Sue would never have the same amount in their savings accounts. Students should notice that the rate is the same, but the initial values are different.

Ask students how they could tell from the graph that Sam and Sue would never have the same amount in their savings accounts. Students should notice that the lines never intersect, therefore they never have the same amount of money.

Discuss that this scenario is said to have *no solution* because there is no set of values that satisfy both equations/situations simultaneously.

GUIDING QUESTIONS

Elicit student thinking:

- What information in this problem is important?
- How would you go about solving this problem?
- Can you describe this situation in your own words?
- What do the variables represent?

Determine if the student can EXPLAIN *NO SOLUTION* FOR A SYSTEM OF EQUATIONS:

- Why are you unable to find a solution for this system of equations?
- What does *no solution* mean in the context of a graph?
- How can you tell there is no solution to this situation based on the equations? What "gives away" the answer?
- Can you describe, in your own words, what it means when there isn't a

Finally, students will consider a system of linear equations with infinitely many solutions.

Pose the following problem to students:

- A baseball coach buys two batting helmets and three baseballs for \$60 at the beginning of the season. He quickly realizes he needs more of both items, so he goes back to the same store and buys four batting helmets and six baseballs for \$120. How much does each batting helmet cost? How much does each baseball cost?

Allow students to work in pairs or small groups to work through the problem.

Students should eventually notice it is impossible to tell the exact price of a batting helmet and a baseball. In setting up a table or a graph, they should be able to come up with several possible combinations of prices that satisfy both scenarios. For example, batting helmets could cost \$10 and baseballs could cost \$15, or batting helmets could cost \$17.50 and baseballs could cost \$10.

Guide students through the development of the equations and the graph that represent the scenario. The equations should be as follows and the graph should show overlapping lines.

First purchase: $2x + 3y = 65$

Second purchase: $4x + 6y = 130$

where x represents the cost of a batting helmet, and y represents the cost of a baseball

Ask students how they could tell from the equations that there were many different possibilities for the cost of a batting helmet and the cost of a baseball. Students should notice that the second purchase is exactly two times the first purchase.

Ask students how they could tell from the graph that there were many different possibilities for the cost of a batting helmet and the cost of a baseball. Students should notice that the lines intersect in infinitely many locations, therefore there are (infinitely) many solutions.

Discuss that this scenario is said to have *infinitely many solutions* because there are infinitely many values that satisfy both equations/situations simultaneously.

GUIDING QUESTIONS

Elicit student thinking:

- What information in this problem is important?
- How would you go about solving this problem?
- Can you describe this situation in your own words?
- What do the variables represent?

Determine if the student can EXPLAIN *INFINITELY MANY SOLUTIONS* FOR A SYSTEM OF EQUATIONS:

- Why are you able to find many solutions for this system of equations?
- What does *infinitely many solutions* mean in the context of a graph?
- What does *infinitely many solutions* mean in the context of the problem?
- How can you tell there are infinitely many solutions to this situation based on the equations? What "gives away" the answer?

Students should be required to explain their solution methods and reasoning at all times, and to continually relate solutions to the context of the problem and the information provided in a graph.

Students will likely need additional practice solving systems with one solution, no solutions, and infinitely many solutions.

At the end of the activity, students should provide and label a system of equations with no solution and a corresponding situation, and they should provide and label a system of equations with infinitely many solutions and a corresponding situaion. Students should hand this in, along with the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT, so teachers can check for understanding among individual students.

Name________________________________

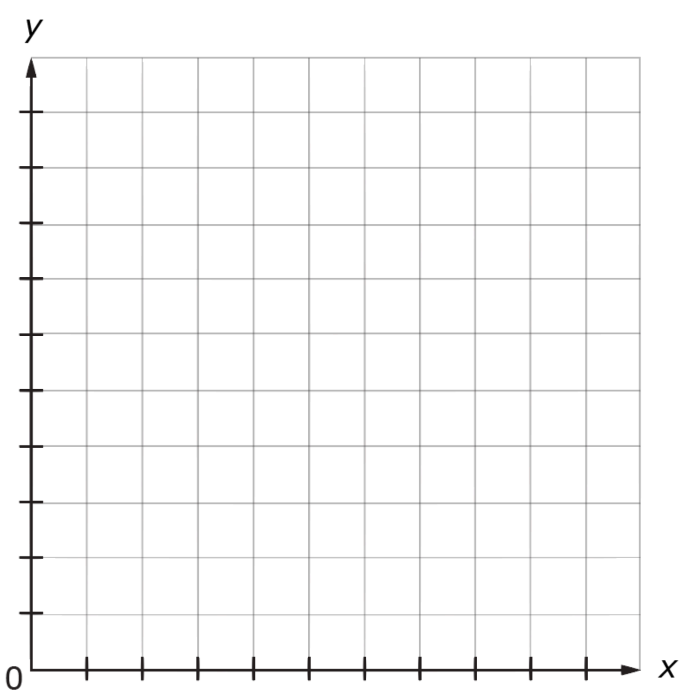
Systems of Linear Equations in Multiple Representations

Lesson 2

1. You are buying flowers at the garden store when you see your neighbor buying flowers as well. You purchase three daffodils and one pansy for $11.00 total. Your neighbor purchases one daffodil and two pansies for $7.00 total.

    a. How much does each daffodil and each pansy cost? Show your reasoning.

    b. Graph the scenario on the following coordinate plane. How does the solution appear on the graph?

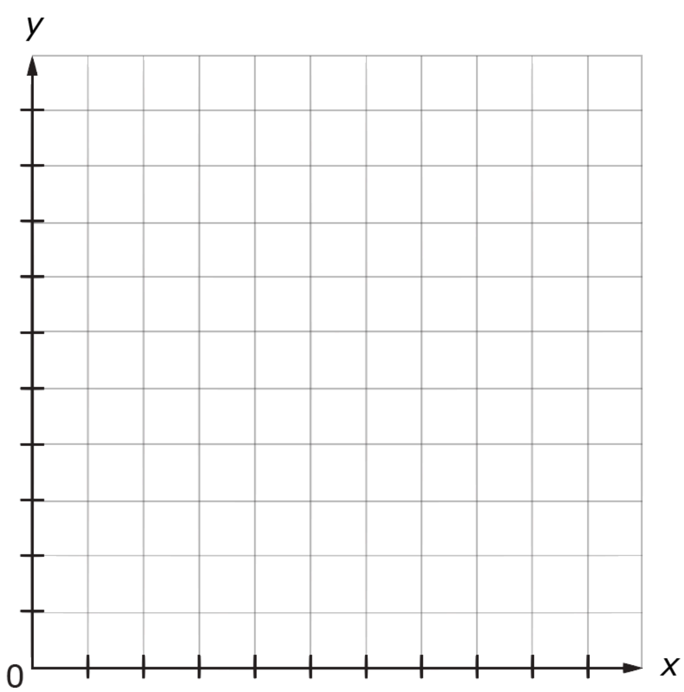


Name_______________________________

2. Molly and James just got new jobs. Molly's job pays $7 per shift plus $3 per item sold. James's job pays $15 per shift plus $1 per item sold.

   a. How many items would Molly and James each have to sell in order to make the same amount of money in a shift? How much money would they make if they sold that number of items? Show your reasoning.

   b. Graph the scenario on the following coordinate plane. How does the solution appear on the graph?



Name________________________________

Systems of Linear Equations in Multiple Representations

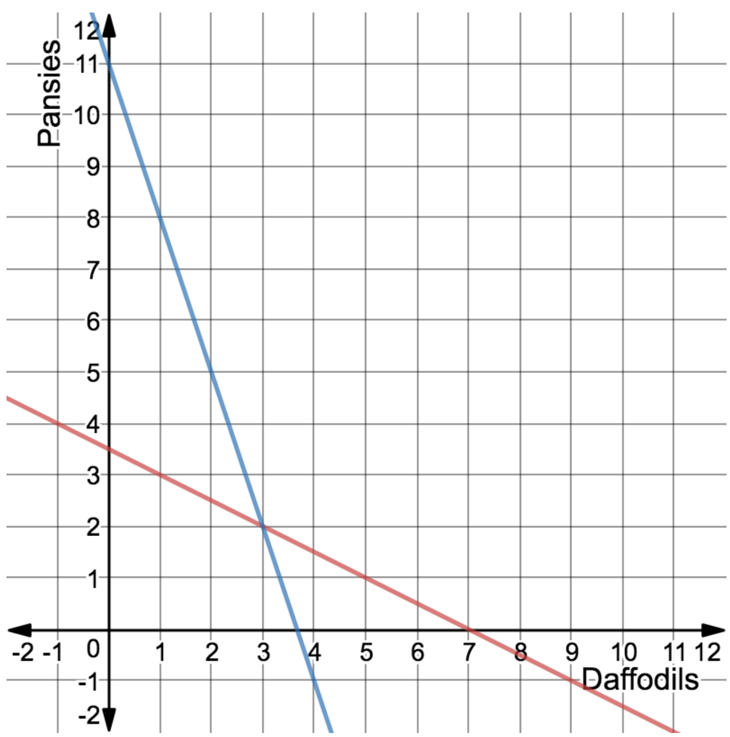
Lesson 2

1. You are buying flowers at the garden store when you see your neighbor buying flowers as well. You purchase three daffodils and one pansy for $11.00 total. Your neighbor purchases one daffodil and two pansies for $7.00 total.
   a. How much does each daffodil and each pansy cost? Show your reasoning.

   > Daffodils cost $3.00 and Pansys cost $2.00.

   b. Graph the scenario on the following coordinate plane. How does the solution appear on the graph?

   > The solution appears as the intersection of the two lines.



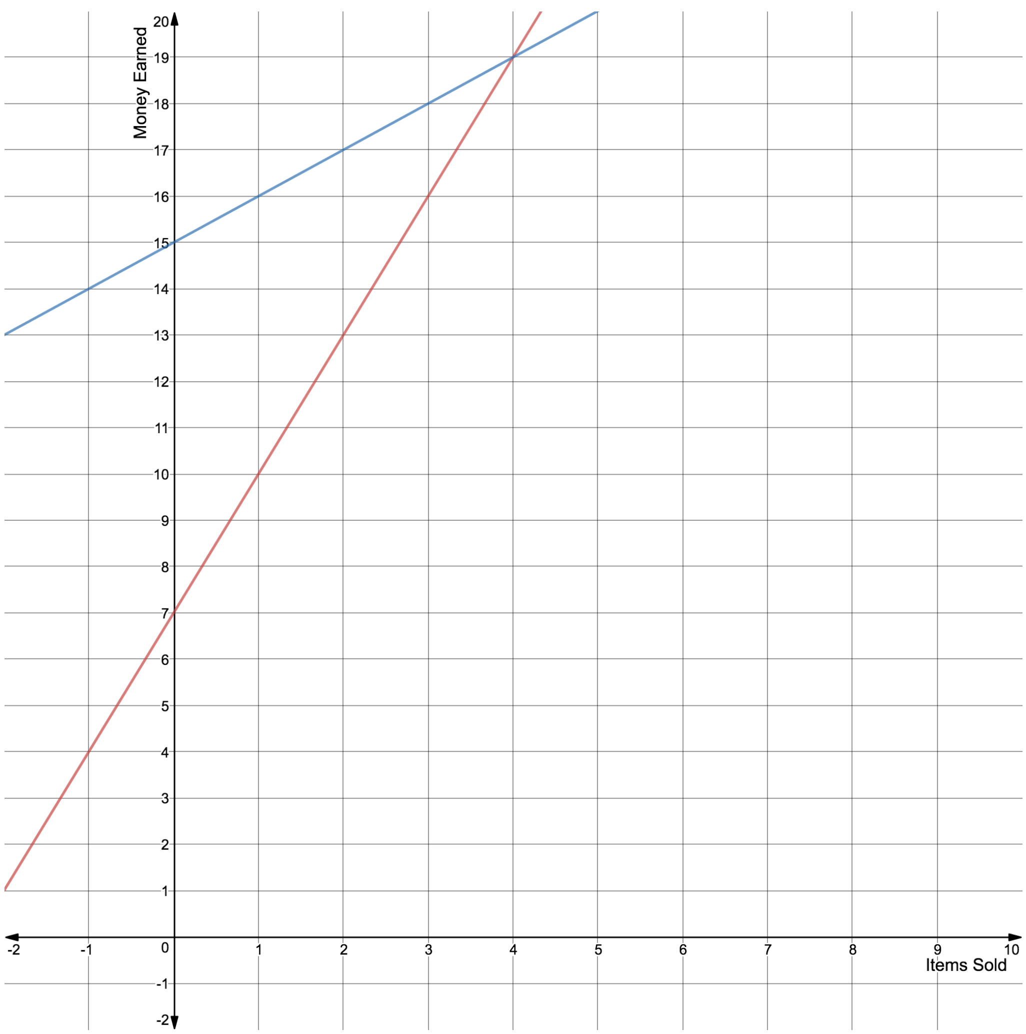
Name______________________________

2. Molly and James just got new jobs. Molly's job pays $7 per shift plus $3 per item sold. James's job pays $15 per shift plus $1 per item sold.
    a. How many items would Molly and James each have to sell in order to make the same amount of money in a shift? How much money would they make if they sold that number of items? Show your reasoning.

    > They will both make $19.00 when they sell 4 items.

    b. Graph the scenario on the following coordinate plane. How does the solution appear on the graph?

    > The solution appears as the intersection of the two lines.



Systems of Linear Equations in Multiple Representations

INSTRUCTIONAL ACTIVITY

Lesson 3

LEARNING GOAL

Students determine whether a system of linear equations has one solution, no solution, or infinitely many solutions from equations and from graphs.

PRIMARY ACTIVITY

Given two equations or two pairs of coordinates, students will determine whether the system of linear equations has one solution, no solution, or infinitely many solutions. If the system of equations has one solution, students should determine the solution.

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

- System of equations
- Coefficient
- Constant
- One solution
- No solution
- Infinitely many solutions
- Intersection
- Parallel (lines)
- Overlapping (lines)

MATERIALS

- Straightedge
- Scissors

- Desmos card sort on Desmos website (optional)
- INSTRUCTIONAL ACTIVITY STUDENT HANDOUT
- INSTRUCTIONAL ACTIVITY SUPPLEMENT (Recommend one copy for every two to three students.)

IMPLEMENTATION

Begin class by reviewing the types of problems and solutions students encountered in LESSON 2 when they were solving problems involving two equations or scenarios. Students should remember that there were some scenarios that led to one solution where the graphs intersected once, a scenario that led to no solution where the graphs did not intersect, and a scenario that led to infinitely many solutions where the graphs overlapped.

Provide students with the following three systems of equations.

1) $\begin{cases} x + 3y = 16 \\ x + 3y = 15 \end{cases}$ 2) $\begin{cases} 3y = 6x + 12 \\ y = -2x + 10 \end{cases}$ 3) $\begin{cases} y = 2x - 3 \\ 4y = 8x - 12 \end{cases}$

Tell students one of the systems has one solution, one of the systems has no solution, and one of the systems has infinitely many solutions.

Require students to work in pairs or small groups to determine the number of solutions each system has by observing patterns among the coefficients and constants in each system of equations.

Guide students to recognize the patterns in the coefficients and constants. For example, in the first system, it is impossible to have identical addends yet result in a different sum. There is no combination of *x*– and *y*–values that can satisfy both equations at the same time. In the third example, the second equation is four times the first equation. When graphed, these lines will overlap each other, producing infinitely many solutions. The second example does not have either of these patterns and will have a unique solution.

Once students establish that the first example has no solution, the second example has one solution, and the third example has infinitely many solutions, **require** students to find the solution for the second example using any method. Students should arrive at the solution (1.5, 7).

Hand out the INSTRUCTIONAL ACTIVITY SUPPLEMENT to pairs of students or small groups.

> NOTE: The INSTRUCTIONAL ACTIVITY SUPPLEMENT cards are available for use on Desmos. In the teacher view, if the cards are sorted correctly they are green and if the cards are sorted incorrectly they are red.
>
> https://teacher.desmos.com/activitybuilder/custom/58aa4074fd477c5c27d4a3e1.

Require students to sort the cards based on the number of solutions the system will have (i.e., one solution, no solution, or infinitely many solutions).

If desired, **require** students to determine the solution for the systems of equations that have one solution.

Once students have sorted the systems of linear equations based on the number of solutions, **discuss** the patterns students used to sort the systems.

Ask several students or groups of students to explain their thinking, as students ensure they have properly sorted the systems through class discussion.

GUIDING QUESTIONS

Elicit student thinking:

- What are you looking for in the equations?
- What patterns do you notice?

Determine if the student can EXPLAIN *UNIQUE SOLUTION* FOR A SYSTEM OF EQUATIONS:

- [Point to a system of equations the student has identified as having one unique solution.] How can you tell this system of equations has one unique solution?
- What does it mean when a system of equations has one unique solution?

Determine if the student can EXPLAIN *NO SOLUTION* FOR A SYSTEM OF EQUATIONS:

- [Point to a system of equations the student has identified as having no solution.] How can you tell this system of equations has no solution?
- What does it mean when a system of equations has no solution?

Determine if the student can EXPLAIN *INFINITELY MANY SOLUTIONS* FOR A SYSTEM OF EQUATIONS:

- [Point to a system of equations the student has identified as having infinitely many solutions.] How can you tell this system of equations has infinitely many solutions?
- What does it mean when a system of equations has infinitely many solutions?

Next, students will plot points on two lines in the coordinate plane and determine or estimate the solution for the system of linear equations.

Hand out the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT for students to work either individually, in pairs, or in small groups.

Circulate among students as they plot pairs of points, draw lines through each pair, and determine or estimate the solution to the simultaneous linear equations.

Students will encounter pairs of lines that intersect once, are parallel, and overlap in order to practice determining or estimating the coordinate of the intersection, interpreting parallel lines to have no solution because there is no intersection, and interpreting overlapping lines to have infinitely many solutions because there are infinitely many points of intersection.

NOTE: Students will need to approximate the point of intersection on Question 4.

Ask students the following guiding questions to determine where they are in their understanding of the relationship between the graph and the solution(s) to the system of equations.

GUIDING QUESTIONS

Elicit student thinking:

- What is the goal of solving a system of equations?
- What information does this graph provide?
- How can you display this information on a graph?

Determine if the student can ESTIMATE THE SOLUTION FOR A SYSTEM OF LINEAR EQUATIONS BY GRAPHING:

- How can you use a graph to solve a system of equations?
- What point(s) on the graph are important when determining the solution to the system of equations?
- Based on this graph, what is the solution?

Determine if the student can EXPLAIN *UNIQUE SOLUTION* FOR A SYSTEM OF EQUATIONS:

- [Point to a pair of lines that intersect at one point.] How many solutions does this system of equations have? How do you know?
- How can you tell from a graph that a system of equations has one (unique) solution?

Determine if the student can EXPLAIN *NO SOLUTION* FOR A SYSTEM OF EQUATIONS:

- [Point to a pair of parallel lines.] How many solutions does this system of equations have? How do you know?
- How can you tell from a graph that a system of equations has no solution?

Determine if the student can EXPLAIN *INFINITELY MANY SOLUTIONS* FOR A SYSTEM OF EQUATIONS:

- [Point to a pair of overlapping lines.] How many solutions does this system of equations have? How do you know?
- How can you tell from a graph that a system of equations has infinitely many solutions?

Students should be required to explain the connection between the appearance of the graph (the number of intersections) and the number of solutions to the system of equations. When a system of equations has one solution, students should be required to determine or estimate the solution with reasonable precision.

At the end of the activity, ask students to draw a graph for a system of linear equations with one solution, a graph for a system of linear equations with no solution, and a graph for a system of linear equations with infinitely many solutions. For each graph, students should explain how the number of solutions are represented on the graph.

Name________________________________

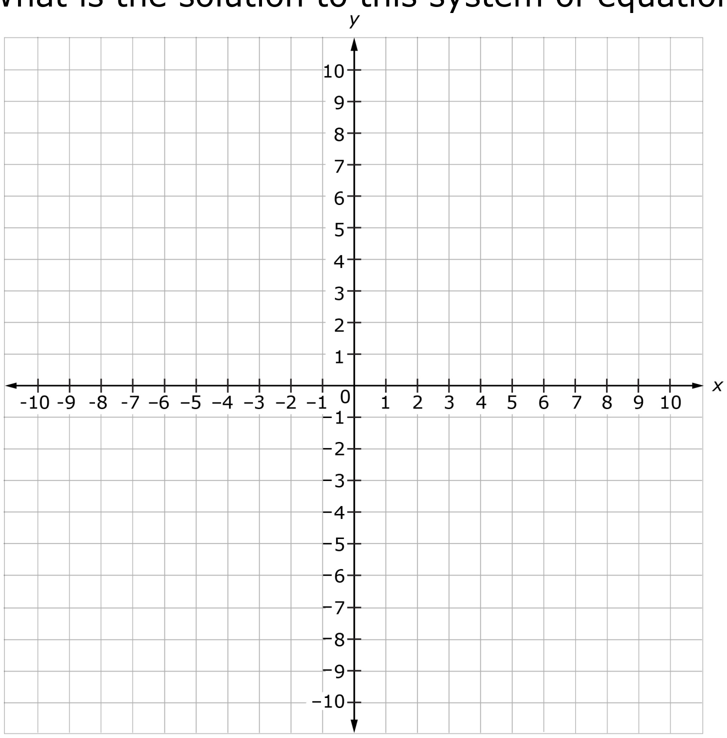
Systems of Linear Equations in Multiple Representations

Lesson 3

For the following questions, plot the first pair of points and draw a line through the points, then plot the second pair of points and draw a line through the points. Use a straightedge to be as precise as possible as you connect each pair of points. Once the lines are graphed, determine (if necessary, estimate) the solution to the system of equations.

1. Line 1 goes through the points (−3, 8) and (−1, 0). Line 2 goes through the points (−2, 4) and (0, −4).

   a. What is the solution to this system of equations?

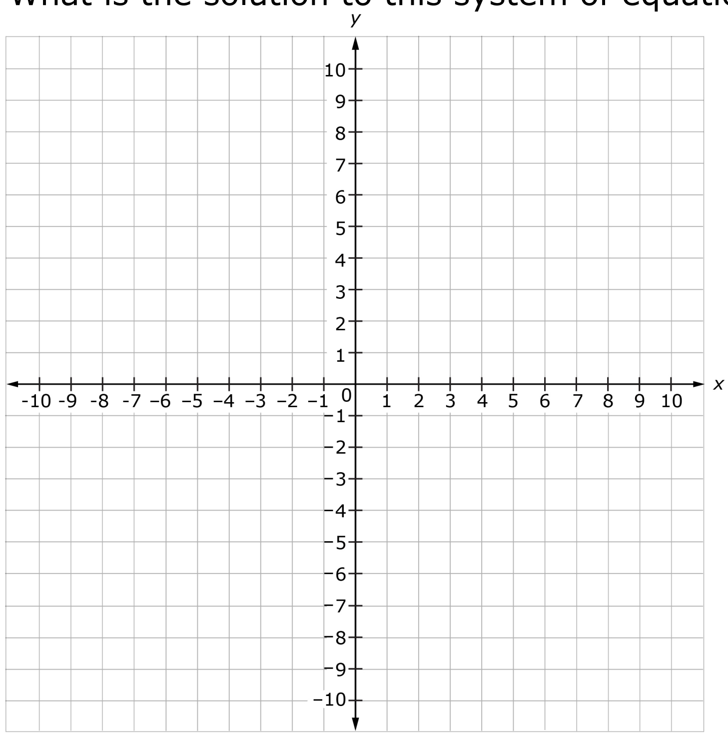


   b. How does the graph represent the solution?

Name______________________________

2. Line 1 goes through the points (–4, 0) and (0, 3). Line 2 goes through the points (–6, 1.5) and (5, 1.5).

   a. What is the solution to this system of equations?

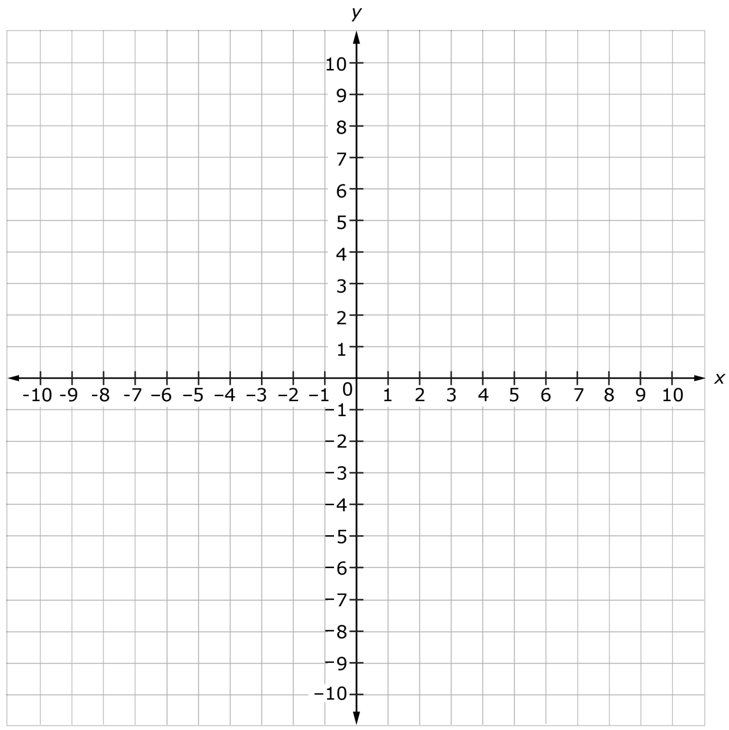


   b. How does the graph represent the solution?

Name_______________________________

3. Line 1 goes through the points (−7, −4) and (−1, 2). Line 2 goes through the points $(-1, -\frac{2}{3})$ and $(3, 3\frac{1}{3})$.

   a. What is the solution to this system of equations?

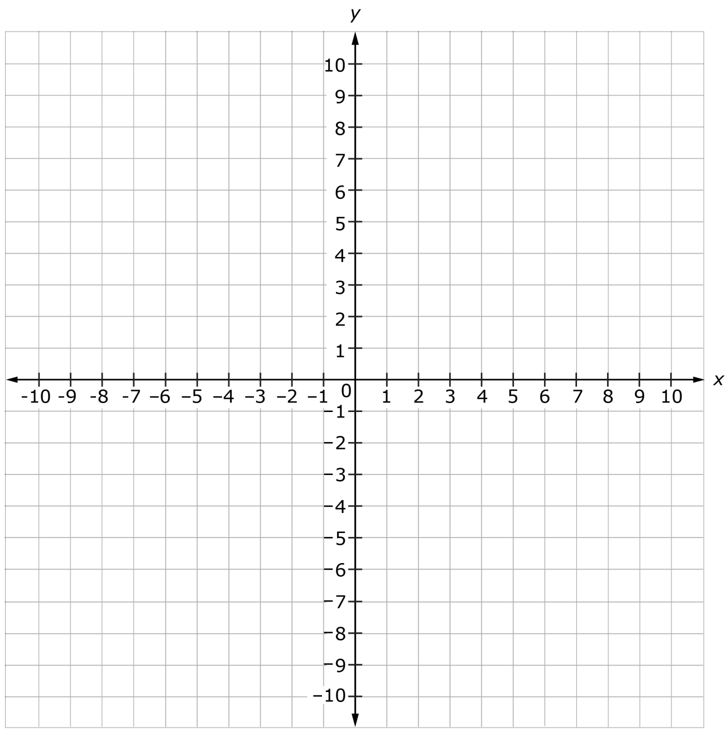


   b. How does the graph represent the solution?

Name______________________________

4. Line 1 goes through the points (2, 7) and (8, −3). Line 2 goes through the points (0, 0) and (10, −2).

   a. What is the solution to this system of equations?



   b. How does the graph represent the solution?

Name________________________________

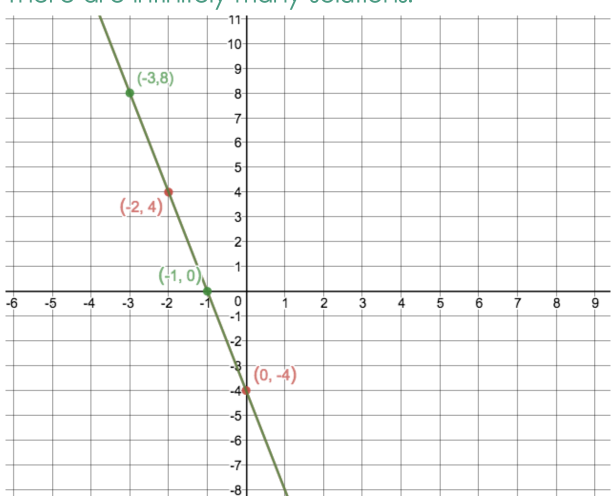
Systems of Linear Equations in Multiple Representations

Lesson 3

For the following questions, plot the first pair of points and draw a line through the points, then plot the second pair of points and draw a line through the points. Use a straightedge to be as precise as possible as you connect each pair of points. Once the lines are graphed, determine (if necessary, estimate) the solution to the system of equations.

1. Line 1 goes through the points (–3, 8) and (–1, 0). Line 2 goes through the points (–2, 4) and (0, –4).
   a. What is the solution to this system of equations?

There are infinitely many solutions.



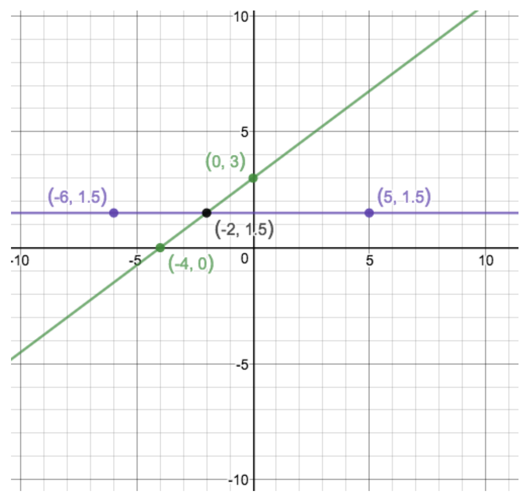
   b. How does the graph represent the solution?

The graph represents the solution with two overlapping lines which intersect infinitely many times.

2. Line 1 goes through the points (–4, 0) and (0, 3). Line 2 goes through the points (–6, 1.5) and (5, 1.5).
   a. What is the solution to this system of equations?

The solution is (-2, 1.5)

Name________________________________



b. How does the graph represent the solution?

The graph represents this solution as the intersection point of the two lines.

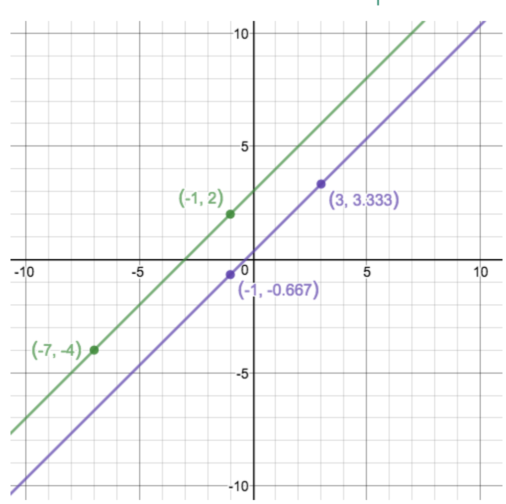
3. Line 1 goes through the points (–7, –4) and (–1, 2). Line 2 goes through the points $(-1, -\frac{2}{3})$ and $(3, 3\frac{1}{3})$.

a. What is the solution to this system of equations?

There are no solutions to this system of equations.

b. How does the graph represent the solution?

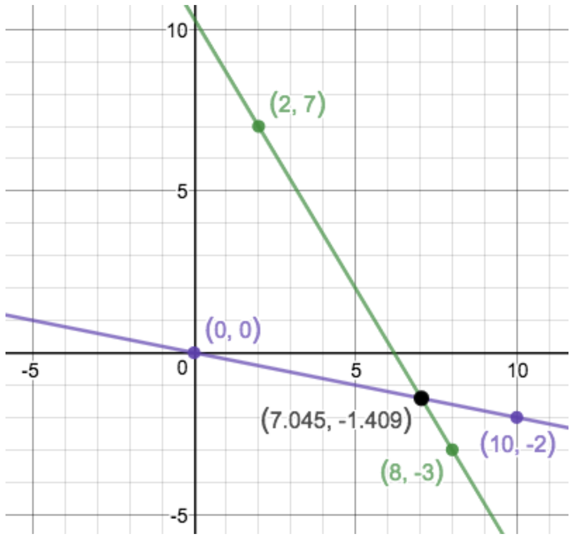
The graph represents this solution by showing that these two lines never cross and have no intersection points.



Name_______________________________

4. Line 1 goes through the points (2, 7) and (8, –3). Line 2 goes through the points (0, 0) and (10, –2).
   a. What is the solution to this system of equations?

The solution is approximately (7, -1.5). A more exact decimal approximation is shown on the graph.



   b. How does the graph represent the solution?

The graph represents this solution as the intersection point of the two lines.

Cut out the following systems of equations and sort them based on whether they have one solution, no solution, or infinitely many solutions.

$\begin{cases} y = 2x + 1 \\ y = 2x - 1 \end{cases}$	$\begin{cases} x + y = \frac{1}{2} \\ 4x + 6y = 3 \end{cases}$	$\begin{cases} y = 2x + 7 \\ y = 2x + 7 \end{cases}$
$\begin{cases} y = -x + 1 \\ y = 5x + 19 \end{cases}$	$\begin{cases} y = 3x - 1 \\ 3y = 9x - 3 \end{cases}$	$\begin{cases} x + y = 5 \\ x + y = 7 \end{cases}$
$\begin{cases} y = \frac{1}{2}x + 1 \\ 2y = -\frac{1}{4}x + 12 \end{cases}$	$\begin{cases} 2x + 3y = 6 \\ 4x + 6y = 11 \end{cases}$	$\begin{cases} 2x + y = 10 \\ 4x + 2y = 20 \end{cases}$
$\begin{cases} 4y = x + 8 \\ y = \frac{1}{4}x - 5 \end{cases}$	$\begin{cases} y - x = 8 \\ 2y - 2x = 16 \end{cases}$	$\begin{cases} y = x \\ 3x + 2y = -20 \end{cases}$
$\begin{cases} \frac{1}{2}y = \frac{1}{4}x + 1 \\ 2y = x + 4 \end{cases}$	$\begin{cases} 4x - 3y = 10 \\ 4x - 3y = 8 \end{cases}$	$\begin{cases} 6x - 9y = -4 \\ 3x + 3y = 3 \end{cases}$
$\begin{cases} 5x - y = 10 \\ x - \frac{1}{5}y = 2 \end{cases}$	$\begin{cases} 4x + 3y = 5 \\ 2x - y = 5 \end{cases}$	$\begin{cases} y = -\frac{1}{2}x - 3 \\ y = -\frac{1}{2}x - 4 \end{cases}$

Systems of Linear Equations in Multiple Representations

INSTRUCTIONAL ACTIVITY SUPPLEMENT

Lesson 3

Cut out the following systems of equations and sort them based on whether they have one solution, no solution, or infinitely many solutions.

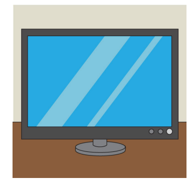
$\begin{cases} y = 2x + 1 \\ y = 2x - 1 \end{cases}$ No Solution	$\begin{cases} x + y = \frac{1}{2} \\ 4x + 6y = 3 \end{cases}$ One Solution	$\begin{cases} y = 2x + 7 \\ y = 2x + 7 \end{cases}$ Infinitely many solutions
$\begin{cases} y = -x + 1 \\ y = 5x + 19 \end{cases}$ One Solution	$\begin{cases} y = 3x - 1 \\ 3y = 9x - 3 \end{cases}$ Infinitely many solutions	$\begin{cases} x + y = 5 \\ x + y = 7 \end{cases}$ No solution
$\begin{cases} y = \frac{1}{2}x + 1 \\ 2y = -\frac{1}{4}x + 12 \end{cases}$ One solution	$\begin{cases} 2x + 3y = 6 \\ 4x + 6y = 11 \end{cases}$ One solution	$\begin{cases} 2x + y = 10 \\ 4x + 2y = 20 \end{cases}$ Infinitely many solutions
$\begin{cases} 4y = x + 8 \\ y = \frac{1}{4}x - 5 \end{cases}$ One solution	$\begin{cases} y - x = 8 \\ 2y - 2x = 16 \end{cases}$ Infinitely many solutions	$\begin{cases} y = x \\ 3x + 2y = -20 \end{cases}$ One solution
$\begin{cases} \frac{1}{2}y = \frac{1}{4}x + 1 \\ 2y = x + 4 \end{cases}$ Infinitely many solutions	$\begin{cases} 4x - 3y = 10 \\ 4x - 3y = 8 \end{cases}$ No solution	$\begin{cases} 6x - 9y = -4 \\ 3x + 3y = 3 \end{cases}$ One solution
$\begin{cases} 5x - y = 10 \\ x - \frac{1}{5}y = 2 \end{cases}$ Infinitely many solutions	$\begin{cases} 4x + 3y = 5 \\ 2x - y = 5 \end{cases}$ One solution	$\begin{cases} y = -\frac{1}{2}x - 3 \\ y = -\frac{1}{2}x - 4 \end{cases}$ No solution

Name____

Systems of Linear Equations in Multiple Representations

Lessons 1 – 3

1. You have just moved to a new home and need to set up cable service. After calling two companies, TV Now and Cable Crew, you find out both companies charge to set up cable service, then charge monthly to use the service. Assuming the cost per month is constant for each company, complete and use the table to answer the following questions.



TOTAL AMOUNT PAID FOR CABLE SERVICE		
Month	**TV Now**	**Cable Crew**
0	\$100	\$70
1	\$165	\$138
2	\$230	\$206
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		

Name____

1.a. Will the costs ever be the same for both companies? If yes, after how many months? How much would you have spent in total? If no, why not?

1.b. Which company should you get cable service from? Explain your reasoning.

Name____

2. Jack and Jill are saving money in their piggy banks. Jack currently has $1.50 in his piggy bank, and Jill currently has $1.80 in her piggy bank. If Jack adds $0.30 to his piggy bank every day and Jill adds $0.25 to her piggy bank every day, will Jack and Jill ever have the same amount of money in their piggy banks? If yes, after how many days? How much money will be in the piggy banks? If no, why not? Explain or show the mathematical reasoning you use.

3. Sam goes to the store to buy fishing equipment with his friend Joe. Sam buys three hooks and two bobbers and pays $3.20. Joe buys two hooks and one bobber and pays $1.85. How much does one hook cost? How much does one bobber cost? Explain or show the mathematical reasoning you use.

Name____

4. Alyssa and Alice are running on the track. Alyssa starts half a lap ahead of Alice. Alyssa runs half a lap every minute. Alice also runs half a lap every minute. Will Alyssa and Alice ever be at the same point on the track at the same time? If yes, after how many minutes? How many total laps will they have run? If no, why not? Explain or show the mathematical reasoning you use.

5. Bob buys painting supplies at the local hardware store. In his first trip to the store, he buys two gallons of paint and three paint rollers for $55.00. Bob runs out of paint and paint rollers, so he goes back to the same hardware store and buys four gallons of paint and six rollers for $110.00. How much does one gallon of paint cost? How much does one paint roller cost? Explain or show the mathematical reasoning you use.

Name____

6. Based on your observation of the patterns in the following systems of linear equations, determine whether there is one solution, no solution, or infinitely many solutions for each system of equations. Explain your reasoning for each problem. If there is one solution, determine the solution.

6.a. $\begin{cases} 2x - y = 8 \\ 6x - 3y = 24 \end{cases}$

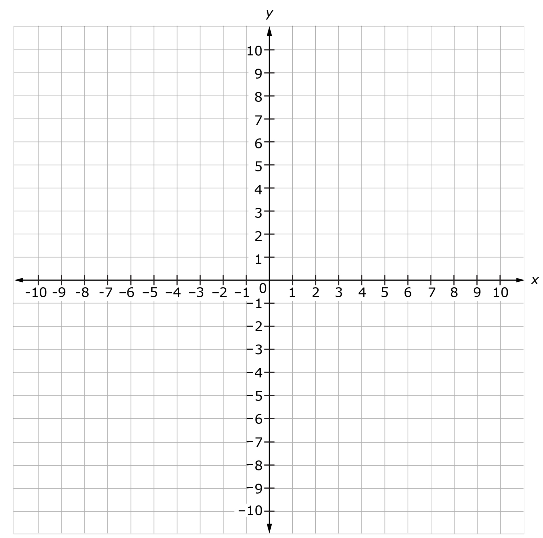
6.b. $\begin{cases} 2y = 8x - 10 \\ y = 2x + 10 \end{cases}$

6.c. $\begin{cases} y = 3x - 3 \\ y = 3x - 6 \end{cases}$

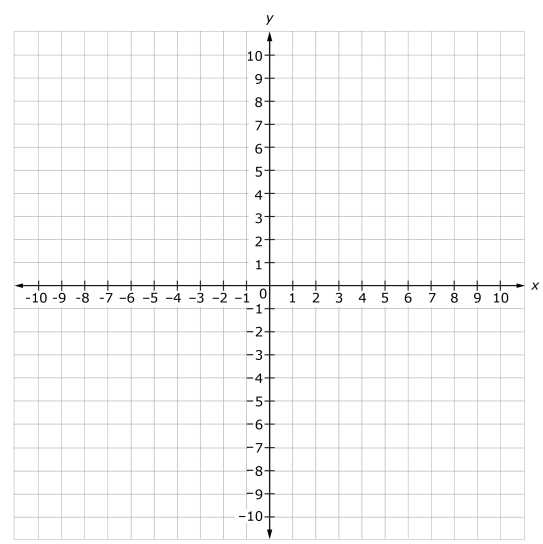
Name____

7. Plot each pair of coordinates and draw a line through each pair of points. Determine whether the system of linear equations has one solution, no solution, or infinitely many solutions. Explain how the number of solutions is related to the lines for each graph. If there is one solution, determine the solution.

   7.a. Line 1 goes through the points (0, 6) and (6, 0). Line 2 goes through the points (–1, 2) and (3, –2).

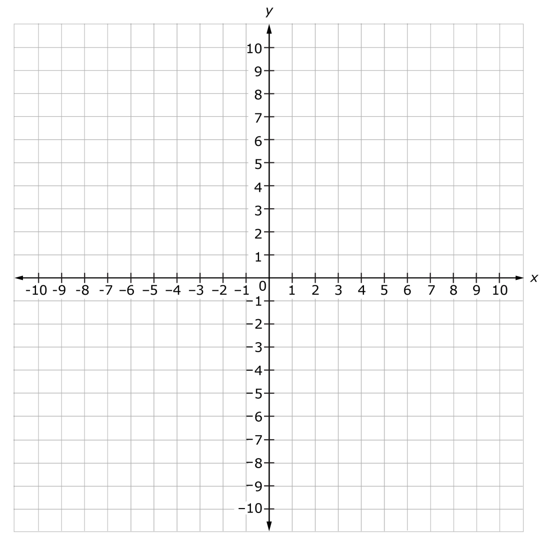


   7.b. Line 1 goes through the points (–8, –1) and (–2, 3). Line 2 goes through the points (1, 5) and (–5, 1).



Name____

7.c. Line 1 goes through the points (0, 7) and (2, 3). Line 2 goes through the points (0, –2) and (4, –1).

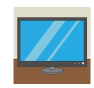


Systems of Linear Equations in Multiple Representations

STUDENT ACTIVITY SOLUTION GUIDE

Lessons 1 – 3

1. You have just moved to a new home and need to set up cable service. After calling two companies, TV Now and Cable Crew, you find out both companies charge to set up cable service, then charge monthly to use the service. Assuming the cost per month is constant for each company, complete and use the table to answer the following questions.



TOTAL AMOUNT PAID FOR CABLE SERVICE		
Month	TV Now	Cable Crew
0	$100	$70
1	$165	$138
2	$230	$206
3	$295	$274
4	$360	$342
5	$425	$410
6	$490	$478
7	$555	$546
8	$620	$614
9	$685	$682
10	$750	$750
11	$815	$818
12	$880	$886

1.a. Will the costs ever be the same for both companies? If yes, after how many months? How much would you have spent in total? If no, why not?

CORRECT ANSWER

Yes, the costs will be the same for TV Now and Cable Crew after 10 months. At this point in time, it will have cost a total of $750 for either company.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
No, the costs will never be the same.	completes the table correctly but does not interpret the row for the tenth month where the total cost is $750 for both companies to indicate the costs are the same for both companies, or completes the table incorrectly based on the information given in the problem, therefore there is no month where the total costs are the same for both companies	SOLVE SIMPLE PROBLEMS INVOLVING 2 VARIABLES
No, the costs will never be the same because TV Now costs more to set up than Cable Crew.	does not consider the rate per month for Cable Crew is more than the rate per month for TV Now, and therefore Cable Crew will not always be cheaper than TV Now	SOLVE SIMPLE PROBLEMS INVOLVING 2 VARIABLES
NOTE: It is possible that a student makes a computation error in the table, causing an incorrect answer that follows from correct reasoning and logic. These errors are less significant than errors in conceptual reasoning or logic.		

1.b. Which company should you get cable service from? Explain your reasoning.

CORRECT ANSWER

It depends how long you plan to have cable service. If you plan to have cable service for less than 10 months, it is cheaper to get service from Cable Crew. If you plan to have cable service for exactly 10 months, either service will cost the same. If you plan to have cable service for more than 10 months, it is cheaper to get service from TV Now.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
You should get cable service from TV Now because it is cheaper each month.	does not consider the set up costs or how the length of time impacts the cost	SOLVE SIMPLE PROBLEMS INVOLVING 2 VARIABLES
You should get cable service from Cable Crew because it is cheaper to set up.	does not consider the monthly costs or how the length of time impacts the cost	SOLVE SIMPLE PROBLEMS INVOLVING 2 VARIABLES
Both companies are the same because they eventually cost the same amount.	does not realize one company is cheaper for the first nine months and the other company is cheaper after ten months; focuses on the fact that both companies cost the same at ten months	SOLVE SIMPLE PROBLEMS INVOLVING 2 VARIABLES

2. Jack and Jill are saving money in their piggy banks. Jack currently has $1.50 in his piggy bank, and Jill currently has $1.80 in her piggy bank. If Jack adds $0.30 to his piggy bank every day and Jill adds $0.25 to her piggy bank every day, will Jack and Jill ever have the same amount of money in their piggy banks? If yes, after how many days? How much money will be in the piggy banks? If no, why not? Explain or show the mathematical reasoning you use.

CORRECT ANSWER

Methods will vary but will likely include reasoning based on the substitution method, creating a table, or drawing a graph. It is also possible to solve this problem using the elimination method.

Response without using equations, tables, or graphs:

Jill has $0.30 more than Jack to begin with, but Jack saves $0.05 more every day. Therefore, Jack's savings will catch up to Jill's savings at a rate of $0.05 per day, making up the $0.30 head start in six days. In six days, both Jack and Jill will have saved $3.30 because $1.80 + 6 ($0.25) = $3.30 and $1.50 + 6 ($0.30) = $3.30.

Response using equations:

Jack's savings can be represented by the equation $y = 0.30x + 1.50$, where x represents the number of days and y represents Jack's total savings.

Jill's savings can be represented by the equation $y = 0.25x + 1.80$, where x represents the number of days and y represents Jill's total savings.

Substitution produces the following equation which can be solved for x.

$$\begin{array}{rcl}
0.25x + 1.80 & = & 0.30x + 1.50 \\
\underline{-0.25x \qquad\quad} & & \underline{-0.25x \qquad\quad} \\
1.80 & = & 0.05x + 1.50 \\
\underline{-1.50} & & \underline{\qquad\quad -1.50} \\
\dfrac{0.30}{0.05} & = & \dfrac{0.05x}{0.05} \\
6 & = & x
\end{array}$$

Substitute 6 into either original question.

$y = 0.25(6) + 1.80 = 3.30$
$y = 0.30(6) + 1.50 = 3.30$

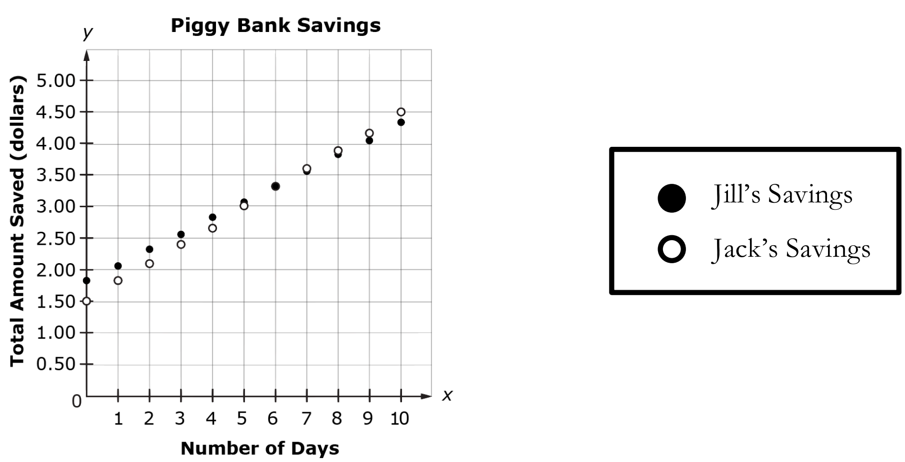
After six days, Jack and Jill will have both saved $3.30.

Response using a table:

Number of Days	Jack's Savings	Jill's Savings
0	$1.50	$1.80
1	$1.80	$2.05
2	$2.10	$2.30
3	$2.40	$2.55
4	$2.70	$2.80
5	$3.00	$3.05
6	$3.30	$3.30

After six days, Jack and Jill will have both saved $3.30.

Response using a graph:



After six days, Jack and Jill will have both saved $3.30.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
No, Jack and Jill will never have the same amount saved in their piggy banks at the same time, because Jill starts with more money than Jack.	does not consider that Jack saves more each day than Jill does, and therefore Jill will not always have more money saved than Jack	SOLVE SIMPLE PROBLEMS INVOLVING 2 VARIABLES
It will take 36 days for Jack and Jill to have the same amount of money saved in their piggy banks.	divides Jill's initial savings (\$1.80) by the difference in the amount saved per day (\$0.05) rather than dividing the difference in Jack and Jill's savings by the difference in the amount saved per day	SOLVE SIMPLE PROBLEMS INVOLVING 2 VARIABLES or SOLVE REAL-WORLD PROBLEMS WITH SYSTEMS OF 2 LINEAR EQUATIONS
It will take 30 days for Jack and Jill to have the same amount of money saved in their piggy banks.	divides Jack's initial savings (\$1.50) by the difference in the amount saved per day (\$0.05) rather than dividing the difference in Jack and Jill's savings by the difference in the amount saved per day	SOLVE SIMPLE PROBLEMS INVOLVING 2 VARIABLES or SOLVE REAL-WORLD PROBLEMS WITH SYSTEMS OF 2 LINEAR EQUATIONS
Student incorrectly substitutes when solving with equations.	does not employ the logic behind the substitution method to solve the system of equations	SOLVE A SYSTEM OF LINEAR EQUATIONS USING THE SUBSTITUTION METHOD
Student incorrectly solves with a table or graph.	does not understand the separate conditions/scenarios or cannot compare two separate conditions/scenarios to each other	SOLVE SIMPLE PROBLEMS INVOLVING 2 VARIABLES or ESTIMATE THE SOLUTION FOR A SYSTEM OF LINEAR EQUATIONS BY GRAPHING
NOTE: It is possible that a student makes a computation error in the solution method they choose, causing an incorrect answer that follows from correct reasoning and logic. These errors are less significant than errors in conceptual reasoning or logic.		

3. Sam goes to the store to buy fishing equipment with his friend Joe. Sam buys three hooks and two bobbers and pays \$3.20. Joe buys two hooks and one bobber and pays \$1.85. How much does one hook cost? How much does one bobber cost? Explain or show the mathematical reasoning you use.

CORRECT ANSWER

> Methods will vary but will likely include reasoning based on doubling Joe's purchase in order to create a scenario where both Sam and Joe buy the same number of bobbers. This allows the student to focus on the cost of a hook, then to go back and figure out the cost of a bobber. It is also possible to solve this problem using the substitution method, by creating a table, or by creating a graph.

Response without using equations:

Doubling Joe's purchase results in four hooks and two bobbers for \$3.70. This is one more hook than Sam bought and the exact same number of bobbers for an additional \$0.50. This means one hook costs \$0.50. Using Joe's original purchase, two hooks would cost \$1.00, so one bobber costs \$0.85. Therefore, one hook costs \$0.50 and one bobber costs \$0.85.

Response using equations:

Sam's purchase can be represented by the equation $3x + 2y = 3.20$, where x represents the cost of a hook and y represents the cost of a bobber.

Joe's purchase can be represented by the equation $2x + 1y = 1.85$, where x represents the cost of a hook and y represents the cost of a bobber.

Elimination produces the following equation which can be solved for x.

$$\begin{array}{lcr} 2(2x + 1y = 1.85) & \longrightarrow & 4x + 2y = 3.70 \\ 3x + 2y = 3.20 & \longrightarrow & \underline{-(3x + 2y = 3.20)} \\ & & 1x = 0.50 \end{array}$$

Substitute 0.50 into either original question.

$$\begin{array}{ll} 2(0.50) + 1y = 1.85 & 3(0.50) + 2y = 3.20 \\ 1.00 + 1y = 1.85 & 1.50 + 2y = 3.20 \\ \underline{-1.00 \qquad -1.00} & \underline{-1.50 \qquad -1.50} \\ \qquad y = 0.85 & \qquad \dfrac{2y}{2} = \dfrac{1.70}{2} \\ & \qquad y = 0.85 \end{array}$$

One hook costs \$0.50 and one bobber costs \$0.85.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Student determines values for the cost of a hook and a bobber that satisfy Sam's purchase only (e.g., hooks cost \$1.00 and bobbers cost \$0.10).	does not consider both purchase scenarios at the same time or ensure the solution satisfies both purchase scenarios	EXPLAIN *SYSTEM OF EQUATIONS* and EXPLAIN SOLUTION(S) TO A SYSTEM OF EQUATIONS
Student determines values for the cost of a hook and a bobber that satisfy Joe's purchase only (e.g., hooks cost \$0.25 and bobbers cost \$1.35).	does not consider both purchase scenarios at the same time or ensure the solution satisfies both purchase scenarios	EXPLAIN *SYSTEM OF EQUATIONS* and EXPLAIN SOLUTION(S) TO A SYSTEM OF EQUATIONS
Student incorrectly eliminates when solving with equations.	does not employ the logic behind the elimination method to solve the system of equations	SOLVE A SYSTEM OF LINEAR EQUATIONS USING THE ELIMINATION METHOD
Five hooks and three bobbers cost \$5.05.	adds Sam and Joe's purchases together; does not determine the price of a single hook or the price of a single bobber	SOLVE SIMPLE PROBLEMS INVOLVING 2 VARIABLES or SOLVE REAL-WORLD PROBLEMS WITH SYSTEMS OF 2 LINEAR EQUATIONS
It is impossible to determine the cost of a hook and a bobber without additional information.	does not consider rewriting or adjusting the equations in a way that creates equal amounts of one item in order to determine the cost of the other item	SOLVE SIMPLE PROBLEMS INVOLVING 2 VARIABLES or SOLVE REAL-WORLD PROBLEMS WITH SYSTEMS OF 2 LINEAR EQUATIONS and SOLVE A SYSTEM OF LINEAR EQUATIONS USING THE ELIMINATION METHOD
NOTE: It is possible that a student makes a computation error in the solution method they choose, causing an incorrect answer that follows from correct reasoning and logic. These errors are less significant than errors in conceptual reasoning or logic.		

4. Alyssa and Alice are running on the track. Alyssa starts half a lap ahead of Alice. Alyssa runs half a lap every minute. Alice also runs half a lap every minute. Will Alyssa and Alice ever be at the same point on the track at the same time? If yes, after how many minutes? How many total laps will they have run? If no, why not? Explain or show the mathematical reasoning you use.

CORRECT ANSWER

> Alyssa and Alice will never be at the same point on the track at the same time. Alyssa has a half-lap head start and both Alyssa and Alice are running at the same rate. Therefore, the distance between them will never change and they will always be half a lap apart.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Alyssa and Alice will always be at the same point on the track and the same time because they are running at the same rate.	does not consider the initial positions of Alyssa and Alice, focuses only on the rate	SOLVE SIMPLE PROBLEMS INVOLVING 2 VARIABLES or SOLVE REAL-WORLD PROBLEMS WITH SYSTEMS OF 2 LINEAR EQUATIONS
Alice will be at the same point on the track as Alyssa after one minute.	thinks that because Alyssa is half a lap ahead of Alice and Alice runs half a lap in a minute, it will take Alice one minute to catch Alyssa; does not consider that Alyssa is also running at a pace of half a lap per minute	SOLVE SIMPLE PROBLEMS INVOLVING 2 VARIABLES or SOLVE REAL-WORLD PROBLEMS WITH SYSTEMS OF 2 LINEAR EQUATIONS

5. Bob buys painting supplies at the local hardware store. In his first trip to the store, he buys two gallons of paint and three paint rollers for $55.00. Bob runs out of paint and paint rollers, so he goes back to the same hardware store and buys four gallons of paint and six rollers for $110.00. How much does one gallon of paint cost? How much does one paint roller cost? Explain or show the mathematical reasoning you use.

CORRECT ANSWER

> There are infinitely many possible costs of a gallon of paint and a paint roller. Because Bob's second purchase was exactly twice as many gallons of paint, twice as many paint rollers, and twice as much cost, any combination of paint cost and paint roller cost that satisfies the first scenario will satisfy the second scenario as well.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Student finds a single solution that works in both equations (e.g., a gallon of paint costs $20.00 and a paint roller costs $5.00).	does not notice there are many other possible combinations of costs that satisfy both equations	SOLVE SIMPLE PROBLEMS INVOLVING 2 VARIABLES and EXPLAIN *INFINITELY MANY SOLUTIONS* FOR A SYSTEM OF EQUATIONS
It is impossible to determine the cost of a gallon of paint or the cost of a paint roller.	does not express an understanding that there are an infinite number of possible cost combinations along with the realization that it is not possible determine the actual cost of a gallon of paint or the cost of a paint roller	EXPLAIN *INFINITELY MANY SOLUTIONS* FOR A SYSTEM OF EQUATIONS

6. Based on your observation of the patterns in the following systems of linear equations, determine whether there is one solution, no solution, or infinitely many solutions for each system of equations. Explain your reasoning for each problem. If there is one solution, determine the solution.

6.a. $\begin{cases} 2x - y = 8 \\ 6x - 3y = 24 \end{cases}$

CORRECT ANSWER

This system of linear equations has infinitely many solutions because the second equation is exactly three times the first equation. If you were to graph these lines, they would overlap. Any x– and y–values that satisfy the first equation will also satisfy the second equation.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
This system of equations has no solution because the coefficients in the second equation are exactly three times the coefficients in the first equation.	does not realize the constant in the second equation is also three times the constant in the first equation	EXPLAIN *INFINITELY MANY SOLUTIONS* FOR A SYSTEM OF EQUATIONS and EXPLAIN *NO SOLUTION* FOR A SYSTEM OF EQUATIONS
This system of equations has one solution because the coefficients in the second equation are different from the coefficients in the first equation.	does not realize the coefficients in the second equation are exactly three times the coefficients in the first equation	EXPLAIN *INFINITELY MANY SOLUTIONS* FOR A SYSTEM OF EQUATIONS and EXPLAIN *UNIQUE SOLUTION* FOR A SYSTEM OF EQUATIONS
This system of equations has infinitely many solutions.	does not explain how they determined the number of solutions	EXPLAIN *INFINITELY MANY SOLUTIONS* FOR A SYSTEM OF EQUATIONS

6.b. $\begin{cases} 2y = 8x - 10 \\ y = 2x + 10 \end{cases}$

CORRECT ANSWER

This system of linear equations has exactly one solution because the coefficients in the first and second equations are not the same in both equations or in the same ratio. For example, the coefficient of y in the first equation is twice the coefficient of y in the second equation, but the coefficient of x in the first equation is four times the coefficient of x in the second equation.

The solution to the system of equations is $(7.5, 25)$ based on the following work. [*Student work may vary.*]

Divide the first equation by two: $y = 4x - 5$.

Substitute the value of y in one equation for the variable y in the other equation: $4x - 5 = 2x + 10$

Solve for x:

$$\begin{array}{l} 4x - 5 = 2x + 10 \\ \underline{-2x + 5 \quad -2x + 5} \\ \dfrac{2x}{2} = \dfrac{15}{2} \\ x = 7.5 \end{array}$$

Determine y:

$$\begin{array}{l} y = 2(7.5) + 10 \\ y = 15 + 10 \\ y = 25 \end{array}$$

The solution to the system of equations is $(7.5, 25)$.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
This system of equations has no solution because the constants are different.	does not look for patterns among the coefficients, only notices the constants are different and interprets that to mean there is no solution	EXPLAIN *UNIQUE SOLUTION* FOR A SYSTEM OF EQUATIONS and EXPLAIN *NO SOLUTION* FOR A SYSTEM OF EQUATIONS
Student states there is one unique solution because the coefficients are not the same in both equations or in the same ratio, but does not determine the solution to the system of equations.	cannot use substitution or elimination to solve a system of equations without a real-world context	SOLVE A SYSTEM OF LINEAR EQUATIONS USING THE ELIMINATION METHOD and SOLVE A SYSTEM OF EQUATIONS USING THE SUBSTITUTION METHOD
Student only states there is one unique solution.	does not explain how they determined the number of solutions	EXPLAIN *UNIQUE SOLUTION* FOR A SYSTEM OF EQUATIONS
Student incorrectly eliminates when solving with equations.	does not employ the logic behind the elimination method to solve the system of equations	SOLVE A SYSTEM OF LINEAR EQUATIONS USING THE ELIMINATION METHOD
Student incorrectly substitutes when solving with equations.	does not employ the logic behind the substitution method to solve the system of equations	SOLVE A SYSTEM OF LINEAR EQUATIONS USING THE SUBSTITUTION METHOD
Student incorrectly solves with a table or graph.	does not understand the separate conditions/scenarios or cannot compare two separate conditions/scenarios to each other	SOLVE SIMPLE PROBLEMS INVOLVING 2 VARIABLES or ESTIMATE THE SOLUTION FOR A SYSTEM OF LINEAR EQUATIONS BY GRAPHING
NOTE: It is possible that a student makes a computation error when solving, causing an incorrect solution that follows from a correct method and process. These errors are less significant than errors in conceptual reasoning or logic.		

6.c. $\begin{cases} y = 3x - 3 \\ y = 3x - 6 \end{cases}$

CORRECT ANSWER

This system of linear equations has no solution because the second equation has exactly the same coefficients as the first equation, but a different constant. Therefore, the rate is the same but the initial values are different. If you were to graph these lines, they would be parallel. Any x– and y–values that satisfy the first equation will never satisfy the second equation.

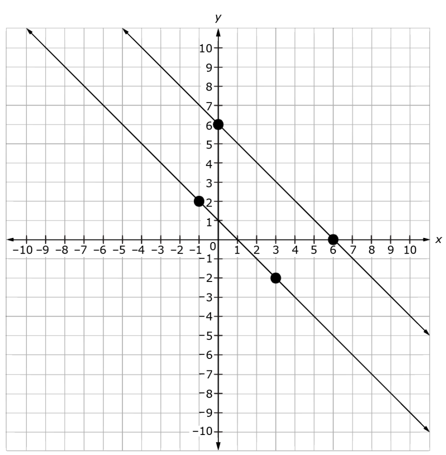
ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
This system of equations has infinitely many solutions because the coefficients in the second equation are exactly the same as the coefficients in the first equation.	does not realize the constant in the second equation is not the same as the constant in the first equation and, in order to have infinitely many solutions when the coefficients are the same, the constants must be the same	EXPLAIN *NO SOLUTION* FOR A SYSTEM OF EQUATIONS and EXPLAIN *INFINITELY MANY SOLUTIONS* FOR A SYSTEM OF EQUATIONS
This system of equations has one solution because the numbers in the first equation are different from the numbers in the second equation.	does not notice that although the constants are different, the coefficients are the same in the first and second equations	EXPLAIN *UNIQUE SOLUTION* FOR A SYSTEM OF EQUATIONS and EXPLAIN *INFINITELY MANY SOLUTIONS* FOR A SYSTEM OF EQUATIONS
Student only states there is no solution.	does not explain how they determined the number of solutions	EXPLAIN *NO SOLUTION* FOR A SYSTEM OF EQUATIONS

7. Plot each pair of coordinates and draw a line through each pair of points. Determine whether the system of linear equations has one solution, no solution, or infinitely many solutions. Explain how the number of solutions is related to the lines for each graph. If there is one solution, determine the solution.

7.a. Line 1 goes through the points (0, 6) and (6, 0). Line 2 goes through the points (–1, 2) and (3, –2).

CORRECT ANSWER



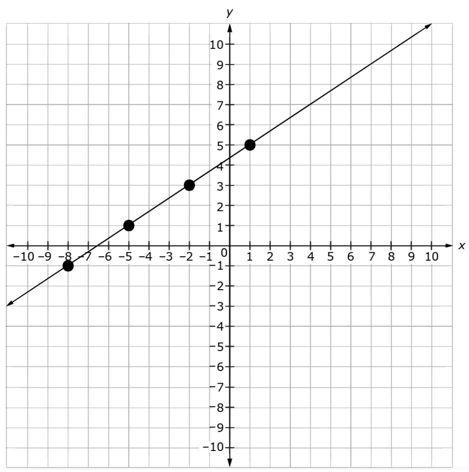
This system of linear equations has no solution because there is not a point on the coordinate plane where the lines will intersect.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Student correctly plots points and draws the lines on the graph, but states there is either one solution or infinitely many solutions.	does not understand that when lines are parallel (do not intersect) there cannot be a solution to the system of equations	EXPLAIN *NO SOLUTION* FOR A SYSTEM OF EQUATIONS and ESTIMATE THE SOLUTION FOR A SYSTEM OF LINEAR EQUATIONS BY GRAPHING
Student does not accurately graph the lines provided, but provides an accurate response based on their graph.	does not understand how to precisely graph the lines on the coordinate plane given two points on each line	REPRESENT LINEAR FUNCTIONS ON A GRAPH

7.b. Line 1 goes through the points (–8, –1) and (–2, 3). Line 2 goes through the points (1, 5) and (–5, 1).

CORRECT ANSWER



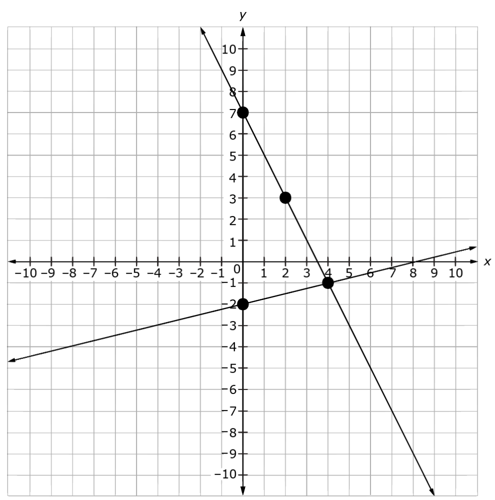
This system of linear equations has infinitely many solutions because the lines overlap each other, and therefore the lines intersect infinitely many times.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Student correctly plots points and draws the lines on the graph but states there is either one solution or no solution.	does not understand that when lines overlap, they intersect infinitely many times and therefore there are infinitely many solutions to the system of equations	EXPLAIN *INFINITELY MANY SOLUTIONS* FOR A SYSTEM OF EQUATIONS and ESTIMATE THE SOLUTION FOR A SYSTEM OF LINEAR EQUATIONS BY GRAPHING
Student does not accurately graph the lines provided but provides an accurate response based on their graph.	does not understand how to precisely graph the lines on the coordinate plane given two points on each line	REPRESENT LINEAR FUNCTIONS ON A GRAPH

7.c. Line 1 goes through the points (0, 7) and (2, 3). Line 2 goes through the points (0, –2) and (4, –1).

CORRECT ANSWER



This system of linear equations has one solution because the lines intersect once. The solution to this system of linear equations is (4, −1) because that is the point where the lines intersect.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Student correctly plots points and draws the lines on the graph but states there is either no solution or infinitely many solutions.	does not understand that when lines intersect once, there is one unique solution to the system of equations	EXPLAIN *UNIQUE SOLUTION* FOR A SYSTEM OF EQUATIONS and ESTIMATE THE SOLUTION FOR A SYSTEM OF LINEAR EQUATIONS BY GRAPHING
Student correctly plots the points and states there is one unique solution but does not provide the solution.	does not identify the solution from the point of intersection on the graph	ESTIMATE THE SOLUTION FOR A SYSTEM OF LINEAR EQUATIONS BY GRAPHING
Student does not accurately graph the lines provided but provides an accurate response based on their graph.	does not understand how to precisely graph the lines on the coordinate plane given two points on each line	REPRESENT LINEAR FUNCTIONS ON A GRAPH