REWRITING EXPRESSIONS

7.EE.1,2

CONTENTS

The types of documents contained in the unit are listed below. Throughout the unit, the documents are arranged by lesson.

Learning Map Information	An overview of the standards, the learning map section, and the nodes addressed in this unit
Teacher Notes	A brief discussion describing the progression depicted in the learning map section with research-based recommendations for focusing instruction to foster student learning and an introduction to the unit's lessons
Overview of Instructional Activities	A table highlighting the lesson goals and nodes addressed in each lesson of this unit
Instructional Activity	A detailed walkthrough of the unit
Instructional Activity Student Handout	A handout for the guided activity, intended to be paired with the Instructional Activity
Instructional Activity Supplement	A collection of materials or activities related to the Instructional Activity
Student Activity	A work-alone activity for students
Student Activity Solution Guide	A solution guide for the work-alone activity with example errors, misconceptions, and links to the learning map section

Enhanced Learning Maps developed these materials under a grant from the Department of Education, PR/Award # S368A150013. Contents do not necessarily represent the policy of the Department of Education, and you should not assume endorsement by the Federal Government.

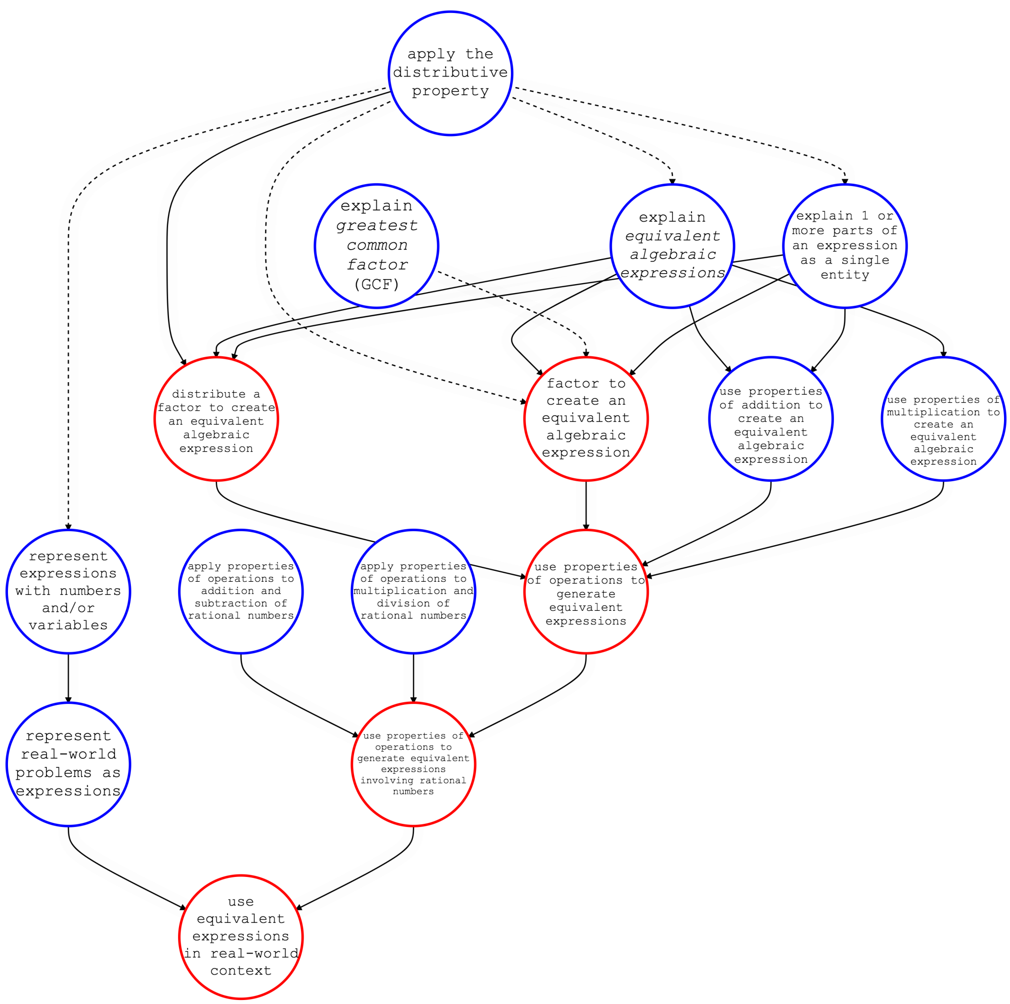
REWRITING EXPRESSIONS

LEARNING MAP INFORMATION

STANDARDS

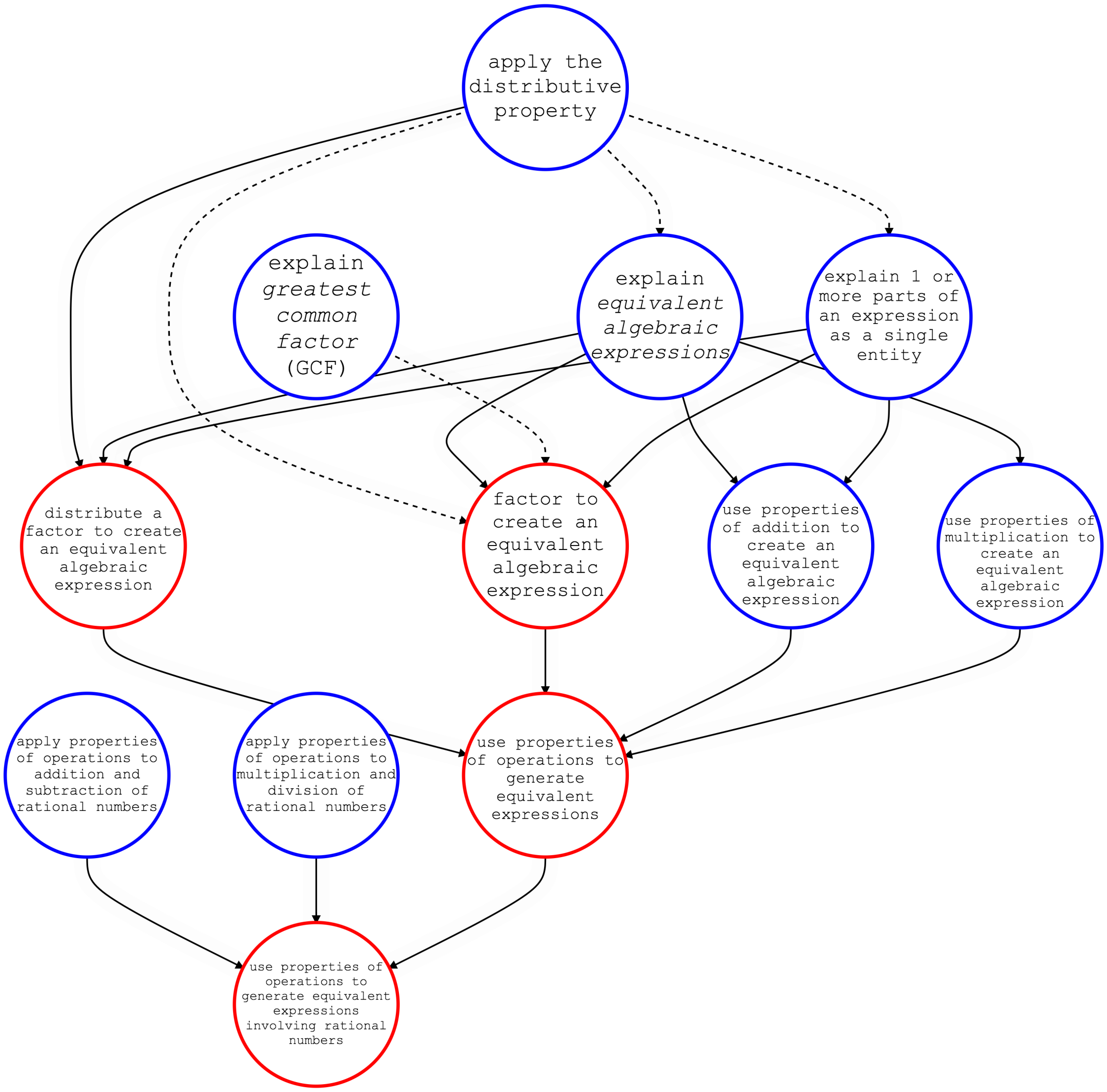
7.EE.1 Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.

7.EE.2 Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example, a + 0.05a = 1.05a means that "increase by 5%" is the same as "multiply by 1.05."*



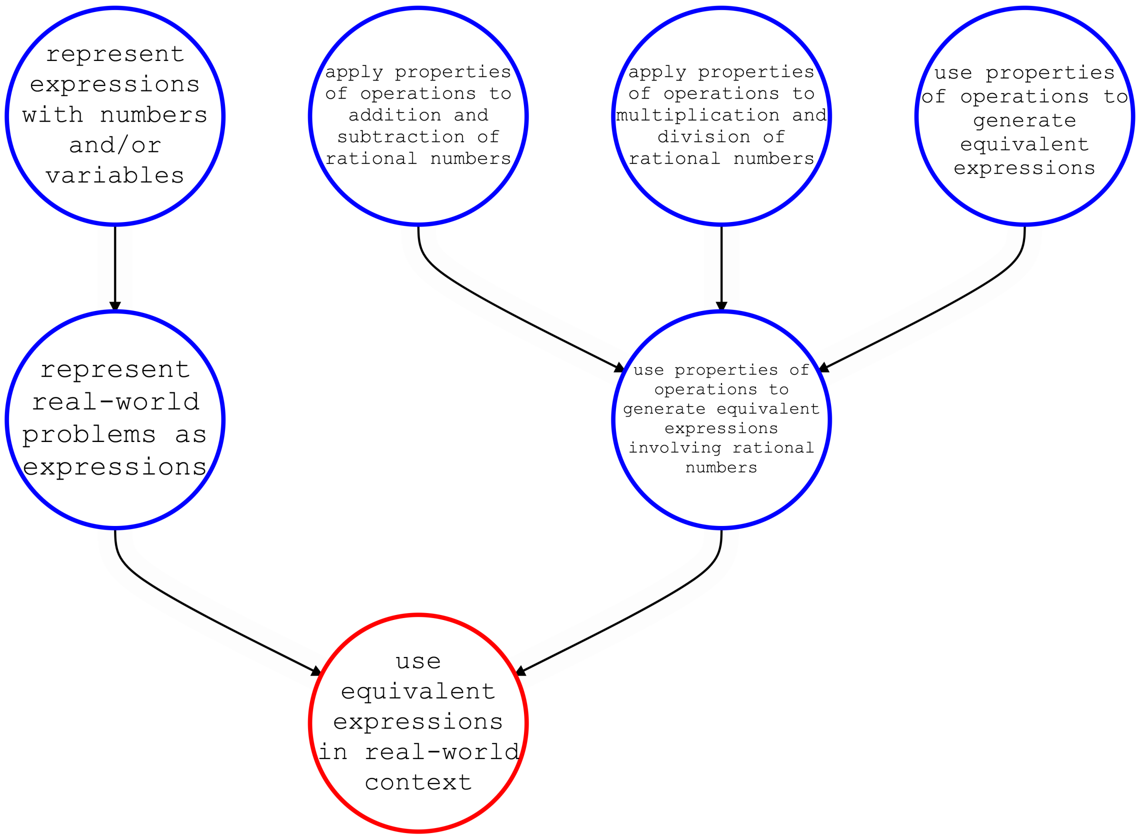
**Learning map model of* 7.EE.1,2

7.EE.1 Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.



**Learning map model of* 7.EE.1

7.EE.2 Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example, $a + 0.05a = 1.05a$ means that "increase by 5%" is the same as "multiply by 1.05."*



**Learning map model of* 7.EE.2

Node Name	Node Description
APPLY PROPERTIES OF OPERATIONS TO ADDITION AND SUBTRACTION OF RATIONAL NUMBERS	Apply properties of operations such as the commutative, associative, and distributive properties to addition and subtraction of rational numbers.
APPLY PROPERTIES OF OPERATIONS TO MULTIPLICATION AND DIVISION OF RATIONAL NUMBERS	Apply properties of operations such as the commutative, associative, and distributive properties to multiplication and division of rational numbers.
APPLY THE DISTRIBUTIVE PROPERTY	Apply distributive property as a strategy for multiplication. For example, 7×18 can be calculated as $(7 \times 10) + (7 \times 8)$.
DISTRIBUTE A FACTOR TO CREATE AN EQUIVALENT ALGEBRAIC EXPRESSION	Use the distributive property to distribute a factor and create an equivalent expression. For example, $4(3 + b)$ can be written as $12 + 4b$.
EXPLAIN *EQUIVALENT ALGEBRAIC EXPRESSIONS*	Make known your understanding that expressions are equivalent when they name the same number regardless of which value is substituted into them.
EXPLAIN *GREATEST COMMON FACTOR* (GCF)	Make known your understanding that a greatest common factor is a number that is the largest factor for two or more numbers.
EXPLAIN 1 OR MORE PARTS OF AN EXPRESSION AS A SINGLE ENTITY	Make known your understanding that one or more parts of an expression is a single entity and part of the expression at the same time. For example, the expression $7(2 + 3)$ is the product of two factors where $(2 + 3)$ is both a single entity and a sum of two terms.
FACTOR TO CREATE AN EQUIVALENT ALGEBRAIC EXPRESSION	Use the distributive property to factor a common number or variable and create an equivalent expression. For example, $10 + 15c$ can be written as $5(2 + 3c)$.
REPRESENT EXPRESSIONS WITH NUMBERS AND/OR VARIABLES	Through writing or an appropriate assistive technology, use the four basic operations, exponents, and grouping symbols with numbers and/or variables to represent expressions (e.g., express "subtract k from 12" as "$12 – k$", or "11 squared plus 4" as "$11^2 + 4$").
REPRESENT REAL-WORLD PROBLEMS AS EXPRESSIONS	Through writing or an appropriate assistive technology, represent real-world problems as expressions.
USE EQUIVALENT EXPRESSIONS IN A REAL-WORLD CONTEXT	Write an equivalent expression to more clearly represent a given real-world problem.
USE PROPERTIES OF ADDITION TO CREATE AN EQUIVALENT ALGEBRAIC EXPRESSION	Apply commutative and associative properties of addition to generate equivalent numerical expressions.
USE PROPERTIES OF MULTIPLICATION TO CREATE AN EQUIVALENT ALGEBRAIC EXPRESSION	Apply commutative and associative properties of multiplication to generate equivalent numerical expressions.
USE PROPERTIES OF OPERATIONS TO GENERATE EQUIVALENT EXPRESSIONS	Use the properties of operations such as the commutative, associative, and distributive properties to generate equivalent expressions.
USE PROPERTIES OF OPERATIONS TO GENERATE EQUIVALENT EXPRESSIONS INVOLVING RATIONAL NUMBERS	Use the properties of operations such as the commutative, associative, and distributive properties to generate equivalent expressions involving rational numbers.

Rewriting Expressions

TEACHER NOTES

This unit includes the following documents:

- Learning Map Information
- Instructional Activity (two lessons)
- Instructional Activity Supplement (for Lessons 1 & 2)
- Instructional Activity Student Handout (for Lesson 2)
- Student Activity
- Student Activity Solution Guide

In this unit, students first learn what it means for two expressions to be equivalent and then justify their equivalence using the appropriate property. Additionally, students relate expressions in multiple representations. Lastly, students determine what information is emphasized or apparent when an expression representing a problem situation is rewritten in different forms.

RESEARCH

Algebraic reasoning is an essential skill in advanced mathematics, several careers, and everyday life; rewriting expressions and recognizing equivalent expressions are vital components of working algebraically (Ketterlin-Geller, Jungjohann, Chard, & Baker, 2007; Van de Walle, Lovin, Karp, & Bay-Williams, 2014). Basic algebra and arithmetic skills are closely related, and students should have a strong conceptual understanding of arithmetic in order to develop algebraic reasoning (Ketterlin-Geller, Jungjohann, Chard, & Baker, 2007). Teachers who model a variety of concrete examples promote students' ability to build on their knowledge of arithmetic and begin to think algebraically. Teachers' verbalization during modeling should foster conceptual understanding of the topic rather than rote procedures to be memorized. Discussing contexts where it would be advantageous to rewrite an algebraic expression, considering the options available to rewrite an expression, and describing the difference in information that is apparent in one form of an expression compared to another all help students develop a deeper conceptual understanding of algebraic expressions.

In addition, students and teachers should discuss the properties of operations as they relate to numerical values as well as algebraic expressions. Examples should require (or prompt) students to confront misconceptions and common points of confusion, such as distributing a negative value or subtracting a negative value (Gregg & Yackel, 2002). Because students often learn these concepts for the first time in middle school, it is important to promote conceptual understanding to serve as a foundation from which students can continue their learning. Students benefit from seeing these concepts illustrated in multiple representations, such as algebraic symbols, visual drawings, and written descriptions (Driscoll, 2010). A particularly helpful representation is a real-world scenario, because students can check the validity of their answers in the context of the problem situation.

As students first learn and use variables, teachers should attend to the idea that variables are letters representing varying quantities (rather than abbreviations, labels, or specific values) because of students' tendency to use letters as abbreviations instead of variables (Gregg & Yackel, 2002). Students should be exposed to variables that represent varying quantities as well as unknown quantities, where the variables are clearly defined as such, and have conversations about the difference when working with variables (e.g., $2x = 6$ compared to $2x = y$). Any use of letters as shorthand or abbreviation (such as 4*d* to represent 4 dogs, instead of 4*d* representing 4 multiplied by the number of dogs) should be addressed and corrected.

AN EXAMPLE

Consider the following scenario:

> Sam sets up a cookie stand to make some extra money. He sells boxes of cookies in addition to single cookies. In the morning, Sam sells two boxes of cookies and three single cookies. In the afternoon, Sam sells four boxes of cookies and two single cookies. How many boxes of cookies and single cookies did Sam sell during the day?

Students who use letters as abbreviations may use *b* as a label or abbreviation for boxes of cookies, and *s* as a label or abbreviation for single cookies and write the following:

$$\begin{array}{r} 2b + 3s \\ \underline{+\ 4b + 2s} \\ 6b + 5s \end{array}$$

To move students toward interpreting letters as variables signifying a quantity that can change, consider asking students how many total cookies were sold that day. Students should realize that in order to answer this question, they need to know how many cookies are in each box. Provide students with different possibilities for the number of cookies a box can hold (e.g., five cookies, eight cookies, 12 cookies). As students calculate each value, they should be able to generalize their process (multiply the number of cookies in each box by six, then add five). This enables students to establish the expression $6x + 5$, where x represents the number of cookies in each box and is a quantity that can vary (Gregg & Yackel, 2002).

Additionally, if students' initial experiences with variables occur exclusively in the context of solving one-step equations where the variable stands for a specified value, they may struggle to understand that, in other contexts, variables can take on any value (Asquith, Stephens, Knuth, & Alibali, 2007). Insufficient understanding of variables can lead students to conjoin or "finish" expressions by adding or subtracting unlike terms (Tirosh, Even, & Robinson, 1998).

AN EXAMPLE

Students who conjoin or "finish" expressions by adding or subtracting unlike terms may make errors similar to the following:

$3 + 7x = 10x$ (incorrectly adds the digits and keeps the variable)

or

$6 - 2x = 4$ (incorrectly subtracts the digits and does not keep the variable)

LEARNING MAP INFORMATION

The learning map section for this sequence of activities begins with prerequisite skills requiring students to explain equivalent expressions and apply the properties of operations independently of each other. Once students have a strong understanding of how to use the properties of operations independently, they should move on to choosing appropriate properties to strategically rewrite expressions. The learning map model also describes representing expressions with numbers and/or variables as a prerequisite skill to representing real-world problems with algebraic expressions. The ability to represent real-world problems with expressions, combined with an understanding of how to choose an appropriate property to strategically rewrite an algebraic expression, allows students to use equivalent expressions in real-world contexts flexibly. Students should recognize that expressions can be rewritten in advantageous forms, which allows specific information to be more readily recognized.

INSTRUCTIONAL ACTIVITIES

The activities in this unit allow students to manipulate algebraic expressions in mathematical and real-world scenarios. In the first lesson, students rewrite linear expressions by applying the commutative, associative, and distributive properties, as well as by factoring and expanding. Students are expected to justify the equivalence of expressions by citing an appropriate property. In Lesson 2, the students again demonstrate their knowledge of equivalence by identifying groups of equivalent expressions. Students then match the groups of equivalent expressions to a corresponding problem scenario to demonstrate their knowledge of the relationship between algebraic notation and real-world situations. After matching the group of equivalent expressions to a real-world scenario, students choose one expression from the group to be the best representation of the problem situation. This draws attention to the fact that the expressions display equivalent but unique information, and it requires students to choose what information is most important to shed light on. In this lesson, students are also asked to explain what the values and terms in each expression represent, in order to help students understand the components of an algebraic expression and the meaning each component holds.

REFERENCES

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Rewriting Expressions

OVERVIEW OF INSTRUCTIONAL ACTIVITIES

Lesson	Learning Goal	Nodes Addressed
Lesson 1	Students will apply properties of operations to rewrite linear expressions with rational coefficients.	▶ FACTOR TO CREATE AN EQUIVALENT ALGEBRAIC EXPRESSION ▶ DISTRIBUTE A FACTOR TO CREATE AN EQUIVALENT ALGEBRAIC EXPRESSION ▶ USE PROPERTIES OF OPERATIONS TO GENERATE EQUIVALENT EXPRESSIONS
Lesson 2	Students will identify equivalent expressions and match real-world scenarios with their algebraic representations.	▶ USE PROPERTIES OF OPERATIONS TO GENERATE EQUIVALENT EXPRESSIONS INVOLVING RATIONAL NUMBERS ▶ USE EQUIVALENT EXPRESSIONS IN A REAL-WORLD CONTEXT ▶ REPRESENT REAL WORLD PROBLEMS AS EXPRESSIONS

Rewriting Expressions

INSTRUCTIONAL ACTIVITY

Lesson 1

LEARNING GOAL

Students will apply properties of operations to rewrite linear expressions with rational coefficients.

PRIMARY ACTIVITY

Students will work in pairs or groups to rewrite a series of given expressions after observing the teacher model the process. Groups will have 30-60 seconds to write down as many equivalent rewritten expressions as they can think of.

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

- Expression
- Property
- Coefficient
- Term
- Factor

MATERIALS

- Instructional Activity Supplement (Recommend one copy for every group of two to three students.)

IMPLEMENTATION

Review how to apply the properties of operations to generate equivalent expressions, being sure to include examples of the commutative, associative, and distributive properties.

Model/review the process of rewriting an algebraic expression, using the example $10x - 4$. **Prompt** students as to how they could rewrite the given expression in a variety of different ways.

Students may notice that they could factor out a 2, rewrite $10x$ as $5x + 5x$ (or any other combinations of terms with a sum of $10x$), or rewrite -4 as $-2 - 2$ (or any other values whose sum is -4).

Include options students may not consider, such as rewriting $10x$ as $12x - 2x$, as well as the commutative and associative properties.

Address student questions and review as needed.

Distribute the first page of the INSTRUCTIONAL ACTIVITY SUPPLEMENT to each group of two to three students.

Students will rewrite the expression in as many ways as they can come up with. If a group has more than three students in it, it might be beneficial to have students pass the worksheet around and for each student take a turn writing a rewritten form.

As students are working, observe groups around the room and highlight or circle specific examples that students will share with the whole class. These examples should include a variety of strategies including factoring, expanding by addition, expanding by subtraction, use of commutative or associative properties, and any other novel or noteworthy strategies.

Discuss the properties of operations as a class while the highlighted or circled examples are shared. Specifically, **ask** the students to justify why they are able to rewrite the expression in their chosen ways, citing an appropriate property or strategy.

To address student misconceptions, present incorrectly rewritten expressions either from student work or known mistakes that students make.

> NOTE: One common misconception is to "finish" or conjoin an open expression. For example, the expression $3 + 4x$ would mistakenly be rewritten as $7x$ or 7.

Repeat this activity with the additional expressions in the INSTRUCTIONAL ACTIVITY SUPPLEMENT. The activity may be presented as a competition to see which group can come up with the most rewritten expressions.

GUIDING QUESTIONS

Elicit student thinking:

- What do you know about all the expressions that were shared?

Determine if the student can FACTOR TO CREATE AN EQUIVALENT ALGEBRAIC EXPRESSION:

- Does this expression have a greatest common factor (GCF)?
- How did you determine the greatest common factor of the two (or more) terms?

Determine if the student can DISTRIBUTE A FACTOR TO CREATE AN EQUIVALENT ALGEBRAIC EXPRESSION:

- What operation do you perform when distributing a factor?
- How do you know what the sign of the products will be?

Determine if the student is ready to USE PROPERTIES OF OPERATIONS TO GENERATE EQUIVALENT EXPRESSIONS:

- How do you know that all the expressions you wrote are equivalent?
- What does it mean for two expressions to be equivalent?
- What property did you use in this step to rewrite the expression?

At the end of the activity, distribute an exit slip with several expressions listed on it. Some of the expressions should be equivalent to each other, while the rest of the expressions should not be equivalent to any of the other expressions on the page. The students' directions should be to circle all equivalent expressions.

$$20x + 8$$

$$7(x-4)$$

$$\frac{1}{2}x + 9$$

Rewriting Expressions

INSTRUCTIONAL ACTIVITY

Lesson 2

LEARNING GOAL

Students will identify equivalent expressions and match real-world scenarios with their algebraic representations.

PRIMARY ACTIVITY

Students will work in pairs or groups to identify groups of equivalent expressions, and to match expressions with real-world scenarios. Lastly, students will choose the best expression to represent a real-world scenario and provide a justification for their choice.

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

- Expression
- Equivalent
- Property
- Coefficient
- Term
- Factor
- Distribute

MATERIALS

- Instructional Activity Student Handout
- Instructional Activity Supplement (Recommend one copy for every group of two to three students.)

IMPLEMENTATION

Hand out the first two pages of the INSTRUCTIONAL ACTIVITY SUPPLEMENT, which should be cut into individual cards and scrambled, to groups of two to three students.

Direct the students to sort the cards into groups of equivalent expressions.

After students have finished grouping all the expressions, hand out the third page of the INSTRUCTIONAL ACTIVITY SUPPLEMENT, which should also be cut into individual cards. The students will read each description of a real-world scenario on each card and match the appropriate group of expressions to that scenario.

Ask students to choose one of the expressions on the cards to be the best representation of that real-world scenario.

Choose one scenario from the INSTRUCTIONAL ACTIVITY SUPPLEMENT and show its expressions on the board.

Discuss with students the advantages and disadvantages of each expression. For example, discuss as a class what information the expression $0.1x + 0.333x + 0.056x + 0.161x$ describes that the expression $0.65x$ does not and vice versa.

GUIDING QUESTIONS

Elicit student thinking:

- How did you choose the best expression to represent the scenario?
- How do you represent percentages in an algebraic expression?

Determine if the student can USE PROPERTIES OF OPERATIONS TO GENERATE EQUIVALENT EXPRESSIONS INVOLVING RATIONAL NUMBERS:

- [Point to two equivalent expressions] How do you know that one expression is equivalent to the other expression?
- What properties were used to rewrite in this form?
- What is another way to rewrite this expression using a different property of operations? What property did you use?

Determine if the student can REPRESENT REAL-WORLD PROBLEMS AS EXPRESSIONS:

- How do you represent spending money in an expression?
- How do you represent an increase by a given percent in an expression?
- How do you represent a decrease by a given percent in an expression?
- [Point to corresponding scenario] What does "descend" tell you about the rate of change?
- [Point to corresponding scenario] What does "per hour" tell you about the $15?

Determine if the student is ready to USE EQUIVALENT EXPRESSIONS IN A REAL-WORLD CONTEXT:

- [Point to a term] What does this term mean in this context?
- [Point to a numerical value] What does this value mean in this context?
- [Point to the corresponding scenario] To describe this scenario, is it better to have the expression written as $10x + 33.5 + 16.5$ or $10x + 50$?

Distribute the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT to provide students with additional practice writing expressions from real-world scenarios and considering advantages and disadvantages to representing a scenario as various equivalent expressions.

Ask students the guiding questions as you move around the room to check on individual student understanding.

GUIDING QUESTIONS

Elicit student thinking:

- How did the expression change from part (a) to part (b)?
- [Point to Question 1] What does 0.46 represent in terms of the problem situation?
- [Point to Question 1] What is the difference in meaning between 0.46 and $0.46x$?

Determine if the student can REPRESENT REAL-WORLD PROBLEMS AS EXPRESSIONS:

- How do you represent spending money in an expression?
- How do you represent an increase by a given percent in an expression?
- How do you represent a decrease by a given percent in an expression?

Determine if the student can USE EQUIVALENT EXPRESSIONS IN A REAL-WORLD CONTEXT:

- What are the advantages to combining the like terms?
- What are the disadvantages of combining the like terms?
- Why would you want to use different expressions to represent the same real-world scenario?

Students should be required to generate their own expressions based off a real-world scenario, as indicated in the questions in the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT. Students may need reminders regarding how to write percentages as decimals when writing expressions to describe real-world problem situations. As needed, **scaffold** students' experiences writing expressions for real-world problem situations using the guiding questions.

NOTE: A common misunderstanding of variables is to think of the letter as an abbreviation. For example, in the second scenario in the INSTRUCTIONAL ACTIVITY SUPPLEMENT, students may incorrectly think of $10x$ as ten items, instead of $10 times the number of items sold. Therefore, careful consideration should be given when describing variables, and questioning should be used to elicit student understanding of variables.

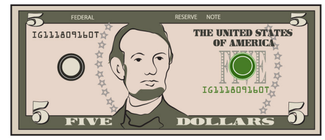
At the end of the activity, write on the board a real-world scenario similar to the ones in the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT and INSTRUCTIONAL ACTIVITY SUPPLEMENT. Hand students a sticky note and have them write two equivalent expressions that represent the scenario written on the board. Have students place their sticky notes on the door on their way out of the classroom, and do a quick visual check that the students wrote appropriate and equivalent expressions.

Name________________________________

Rewriting Expressions

Lesson 2

Each month, Jake spends 4% of his monthly income on transportation, 12% on groceries, and 30% on rent. The following are all equivalent expressions representing this scenario.



a. $0.04x + 0.12x + 0.30x$

b. $0.46x$

c. $0.16x + 0.30x$

1. What does x represent in the expressions?

2. What information does the expression in part (b) give that the expression in part (a) does not?

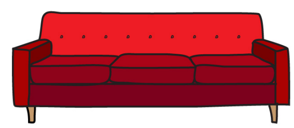
3. What information does the expression in part (a) give that the expression in part (b) does not?

Name______________________________

4. What does “0.46” represent in the expression in part (b)?

5. What would be one reason for writing the expression in part (c)?

Jessie buys an item at a home goods store that costs $\$x$. Sales tax is 8.12%. The expression $1.0812x$ represents the total amount of money she pays for the item.



6. Write an expanded expression that represents the cost of the item and the amount of taxes owed.

7. Write an expression to represent the total cost if Jessie uses a 20%-off coupon.

Name________________________________

Winston receives a 5% raise and puts 6% of his income in an individual retirement account.

8. Write an expression representing Winston’s income after his raise and retirement withdrawal.

9. Rewrite the expression in your answer to Question 8 by combining like terms.

10. Winston wants to talk to his accountant about his annual income. Which of the expressions from Questions 8 and 9 would you recommend he use in his discussion? Why?

Rewriting Expressions

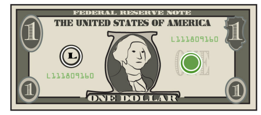
INSTRUCTIONAL ACTIVITY SUPPLEMENT

Lesson 2

$20(170 - 12x)$	$40(-6x + 85)$	$-240x + 3400$	$3400 - 240x$
$10x + 50$	$10(x + 5)$	$5(2x + 10)$	$10x + 33.5 + 16.5$
$20x - 5x - 10y$	$10x + 5x - 9y - y$	$5(3x - 2y)$	$15x - 10y$
$2x(2)$	$4x$	$2x(3 - 1)$	$6x - 2x$
$2[16.1 + (x - 56)]$	$2x - 79.8$	$32.2 + 2x - 112$	$2(16.1) + 2(x - 56)$

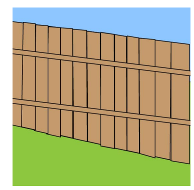
$10x + 33.3x + 56x$	$.1x + .333x + .056x + .161x$
$99.3x$	$.65x$
$43.3x + 56x$	$.1x + .55x$
$16x - 6x + 35x - 1.7x + 100x - 44x$	$.05x + .05x + .333x + .056x + .161x$

Each month, Catherine spends 10% of her monthly income on her electricity bill, 33.3% on rent, 5.6% on her water bill, and 16.1% on groceries.



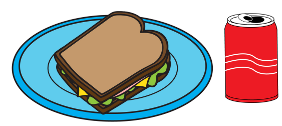
For each item sold, a company gives $10 to the advertising department, $33.30 to the distribution department, and $56 to the production department.

Cindy's backyard has dimensions of 16.1 feet and $x - 56$ feet, and she wants to build a fence around it.

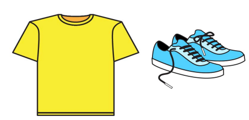


Carol makes $6 per hour babysitting and puts $2 for every hour worked in savings.

Justin makes $15 per hour at his job. He spends $10 a day on lunch.

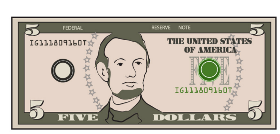


Gabe will be paid $10 per hour at his after school job, and the company will take away a one-time expense for his company shirt and shoes. The shoes cost $33.50, and the shirt costs $16.50.



Julia is descending a mountain. She begins at an elevation of 3,400 feet and descends at a rate of 240 feet per hour.

Jaden has $75.00 in a bank account. He is returning two items that cost $5.00 each and another item that costs $12.80.



Name____

REWRITING EXPRESSIONS

Lessons 1 & 2

1. Write three equivalent expressions to the given expression. Be sure to use more than one property or strategy to rewrite.

1.a. $54x - 9$

1.b. $\frac{3}{4}x + 3$

1.c. $5(3 - x)$

Name____

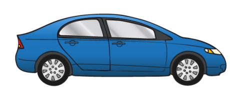
2. Provide a property of operations as justification for each of the following rewritten forms of the expression $7x - 28$.

2.a. $7(x - 4)$

2.b. $5x + 2x - 28$

2.c. $7x - 30 + 2$

3. Hannah is saving up to buy a car. She plans to put 15% of her income in a savings account for the car, and she plans to use an additional 5.4% of her income on gas. She writes the following expression to represent the scenario.



$$0.15x + 0.054x$$

3.a. What does x represent in the problem situation?

Name____

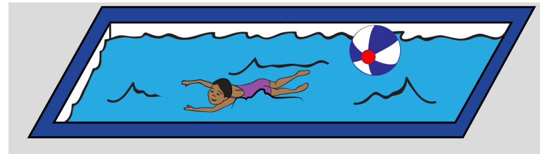
3.b. What does 0.15 represent in the problem situation?

3.c. What information is given in the expression $0.15x + 0.054x$ that is not given in the expression $0.204x$? What information is not readily provided in the expression $0.15x + 0.054x$?

4. What does it mean for two expressions to be equivalent?

Name____

5. Cameron gives swim lessons for $20 a day. She uses $50 of her income to buy a season pool pass.



$20x - 25 - 25$	$30x - 10x + 50$	$20x - 54 + 4$
$10x + 10x + 50$	$19x + x - 50$	$20x - 21 + 29$
$20x - 50$	$50x + 20$	$20x + 50$
$50x - 20$	$-50 + 20x$	$-20 + 50x$

5.a. Circle all the expressions in the table above that represent Cameron's remaining income after working *x* days and purchasing a season pool pass.

5.b. What must be true of all the expressions you circled in part (a)?

5.c. Put a star next to the expression that best represents the problem situation, then provide a justification for your choice.

REWRITING EXPRESSIONS

STUDENT ACTIVITY SOLUTION GUIDE

Lessons 1 & 2

1. Write three equivalent expressions to the given expression. Be sure to use more than one property/strategy to rewrite.

1.a. $54x - 9$

CORRECT ANSWER

Answers will vary. Strategies that students could use include rewriting the variable terms, rewriting the constant term, factoring out a greatest common factor, applying the commutative property, and applying the associative property.

Possible answers include $50x + 4x - 9$, $54x - 10 + 1$, *and* $9(6x - 1)$.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
$45x$	tries to "complete" the expression, i.e. combine unlike terms	USE PROPERTIES OF OPERATIONS TO GENERATE EQUIVALENT EXPRESSIONS
$9(6x - 9)$	struggles when factoring a greatest common factor; only factors from the leading term	FACTOR TO CREATE AN EQUIVALENT ALGEBRAIC EXPRESSION

1.b. $\frac{3}{4}x + 3$

CORRECT ANSWER

Answers will vary. Strategies that students could use include rewriting the variable terms, rewriting the constant term, factoring out a greatest common factor, applying the commutative property, and applying the associative property.

Possible answers include $\frac{3}{4}x + 1 + 1 + 1$, $\frac{1}{2}x + \frac{1}{4}x + 3$, *or* $x - \frac{1}{4}x + 3$.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
$\frac{15}{4}x$, 3.75x, or $3\frac{3}{4}x$	tries to “complete” the expression, i.e. combine unlike terms	USE PROPERTIES OF OPERATIONS TO GENERATE EQUIVALENT EXPRESSIONS

1.c. $5(3 - x)$

CORRECT ANSWER

> *Answers will vary. Strategies that students could use include rewriting the variable terms, rewriting the constant term, distributing a factor, factoring out a greatest common factor, applying the commutative property, and applying the associative property.*
>
> *Possible answers include* $5(-x + 3)$, $-5x + 15$, *and* $15 - 5x$.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
$15 + 5x$ or $-15 + 5x$	has difficulties distributing a positive value to a negative term; confuses which sign each product will have	DISTRIBUTE A FACTOR TO CREATE AN EQUIVALENT ALGEBRAIC EXPRESSION
$15 - x$	distributes the five to the first term in parentheses only	DISTRIBUTE A FACTOR TO CREATE AN EQUIVALENT ALGEBRAIC EXPRESSION
$5(x - 3)$	incorrectly applies the commutative property of addition by mistakenly giving the negative sign to the 3 instead of the x term	APPLY PROPERTIES OF OPERATIONS TO ADDITION AND SUBTRACTION OF RATIONAL NUMBERS
$10x$	tries to “complete” the expression, i.e. combine unlike terms	USE PROPERTIES OF OPERATIONS TO GENERATE EQUIVALENT EXPRESSIONS

2. Provide a property of operations as justification for each of the following rewritten forms of the expression $7x - 28$.

NOTE: ELL and EEN students may benefit from a vocabulary list of properties to choose from when answering questions 2*a*–2*c*.

2.a. $7(x - 4)$

CORRECT ANSWER

Factor the greatest common factor from both terms.
or
"Undo" the multiplication of the factor (seven) using the distributive property.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Distribute a factor.	incorrectly identifies this factored expression as the result of distribution	DISTRIBUTE A FACTOR TO CREATE AN EQUIVALENT ALGEBRAIC EXPRESSION

2.b. $5x + 2x - 28$

CORRECT ANSWER

Expanding the variable term.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Commutative Property of Addition, Associative Property of Addition	may see the addition sign and try to identify the property as any property with "addition" in the title	APPLY PROPERTIES OF OPERATIONS TO ADDITION AND SUBTRACTION OF RATIONAL NUMBERS

2.c. $7x - 30 + 2$

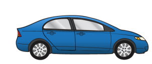
CORRECT ANSWER

Expanding the constant term.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Commutative Property of Addition, Associative Property of Addition	may see the addition sign and try to identify the property as any property with "addition" in the title	APPLY PROPERTIES OF OPERATIONS TO ADDITION AND SUBTRACTION OF RATIONAL NUMBERS

3. Hannah is saving up to buy a car. She plans to put 15% of her income in a savings account for the car, and she plans to use an additional 5.4% of her income on gas. She writes the following expression to represent the scenario.



$$0.15x + 0.054x$$

3.a. What does x represent in the problem situation?

CORRECT ANSWER

x represents the amount of Hannah's income.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
x represents Hannah's savings	applies any "unknown" value to the variable since variables represent unknown quantities; in this example, we do not know her total savings amount, so students may incorrectly believe that the variable represents that amount	EXPLAIN VARIABLE
x represents money	defines variables in vague terms, or thinks that only one word is necessary to define a variable	EXPLAIN VARIABLE

3.b. What does 0.15 represent in the problem situation?

CORRECT ANSWER

> 0.15 represents 15%, which is the percent of Hannah's income that she puts in a savings account for a new car.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
0.15 represents 15%	translates the value into a percent without relating the value to the problem situation	REPRESENT REAL-WORLD PROBLEMS AS EXPRESSIONS

3.c. What information is given in the expression $0.15x + 0.054x$ that is not given in the expression $0.204x$? What information is not readily provided in the expression $0.15x + 0.054x$?

CORRECT ANSWER

> The expression $0.15x + 0.054x$ shows the breakdown of savings by category. It does not readily show the total percent of Hannah's income that she puts towards the car, like the expression $0.204x$ does.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The information shown is the same in both expressions.	cannot interpret the difference between the expressions; does not understand that equivalent expressions highlight different information	USE EQUIVALENT EXPRESSIONS IN A REAL-WORLD CONTEXT

4. What does it mean for two expressions to be equivalent?

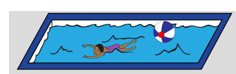
CORRECT ANSWER

> Two expressions are equivalent when they simplify to the same number regardless of which value is substituted for the variable(s).

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	**Misconception**	**Missing Knowledge**
Expressions are equivalent when they equal each other.	shows an understanding of equivalency but does not show an understanding that equivalent expressions are equivalent over all real numbers	EXPLAIN EQUIVALENT ALGEBRAIC EXPRESSIONS
Equivalent expressions are the same.	has basic knowledge of the word "equivalent" and does not attempt to explain equivalence as it applies to expressions, or in a mathematical manner	EXPLAIN EQUIVALENT ALGEBRAIC EXPRESSIONS

5. Cameron gives swim lessons for \$20 a day. She uses \$50 of her income to buy a season pool pass.



$20x - 25 - 25$	$30x - 10x + 50$	$20x - 54 + 4$
$10x + 10x + 50$	$19x + x - 50$	$20x - 21 + 29$
$20x - 50$	$50x + 20$	$20x + 50$
$50x - 20$	$-50 + 20x$	$-20 + 50x$

5.a. Circle all the expressions in the table above that represent Cameron's remaining income after working x days and purchasing a season pool pass.

CORRECT ANSWER

Students should circle $20x - 25 - 25$, $20x - 54 + 4$, $19x + x - 50$, and $20x - 50$.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
$20x + 50$	does not have an understanding of how to relate a real-world problem to its algebraic representation; does not understand that spending money would result in using subtraction	REPRESENT REAL-WORLD PROBLEMS AS EXPRESSIONS
$20x - 21 + 29$	only focuses on the numbers and not the signs/operations	REPRESENT REAL-WORLD PROBLEMS AS EXPRESSIONS

5.b. What must be true of all the expressions you circled in part (a)?

CORRECT ANSWER

All of the circled expressions are equivalent.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
All of the circled expressions represent the problem situation.	does not understand that when multiple expressions represent the same problem situation, then the algebraic expressions are equivalent; might also be misunderstanding the relationship between the various forms of expressions	USE EQUIVALENT EXPRESSIONS IN A REAL-WORLD CONTEXT
The circled expressions are unrelated or different.	cannot recognize equivalent expressions	EXPLAIN EQUIVALENT ALGEBRAIC EXPRESSIONS

5.c. Put a star next to the expression that best represents the problem situation, then provide a justification for your choice.

CORRECT ANSWER

The starred expression should be $20x - 50$. This expression best represents the problem scenario, because it shows the rate of change to be \$20 per day, and a subtracted fee of \$50.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
$50x + 20$	only looks at the values of the problem scenario, not how they relate; does not understand the rate of change in the real-world problem	MATHEMATIZE REAL-WORLD PROBLEMS INVOLVING CONSTANT RATE OF CHANGE
$-20 + 50x$	only looks at the values of the problem scenario and not how they relate	REPRESENT REAL-WORLD PROBLEMS AS EXPRESSIONS
Chooses any of the circled answers. For example, $20x - 25 - 25$, $20x - 54 + 4$, or $19x + x - 50$.	shows knowledge of equivalence but does not understand that a scenario is best represented by one expression, highlighting the specific information from the problem scenario	REPRESENT REAL-WORLD PROBLEMS AS EXPRESSIONS