INTEGERS, RATIONAL NUMBERS, AND SYMMETRY

6.NS.C.5,6

CONTENTS

The types of documents contained in the unit are listed below. Throughout the unit, the documents are arranged by lesson.

Learning Map Information	An overview of the standards, the learning map section, and the nodes addressed in this unit
Teacher Notes	A brief discussion describing the progression depicted in the learning map section with research-based recommendations for focusing instruction to foster student learning and an introduction to the unit's lessons
Overview of Instructional Activities	A table highlighting the lesson goals and nodes addressed in each lesson of this unit
Instructional Activity	A detailed walkthrough of the unit
Instructional Activity Supplement	A collection of materials or activities related to the Instructional Activity
Student Activity	A work-alone activity for students
Student Activity Solution Guide	A solution guide for the work-alone activity with example errors, misconceptions, and links to the learning map section

Enhanced Learning Maps developed these materials under a grant from the Department of Education, PR/Award # S368A150013. Contents do not necessarily represent the policy of the Department of Education, and you should not assume endorsement by the Federal Government.

INTEGERS, RATIONAL NUMBERS, AND SYMMETRY

LEARNING MAP INFORMATION

STANDARDS

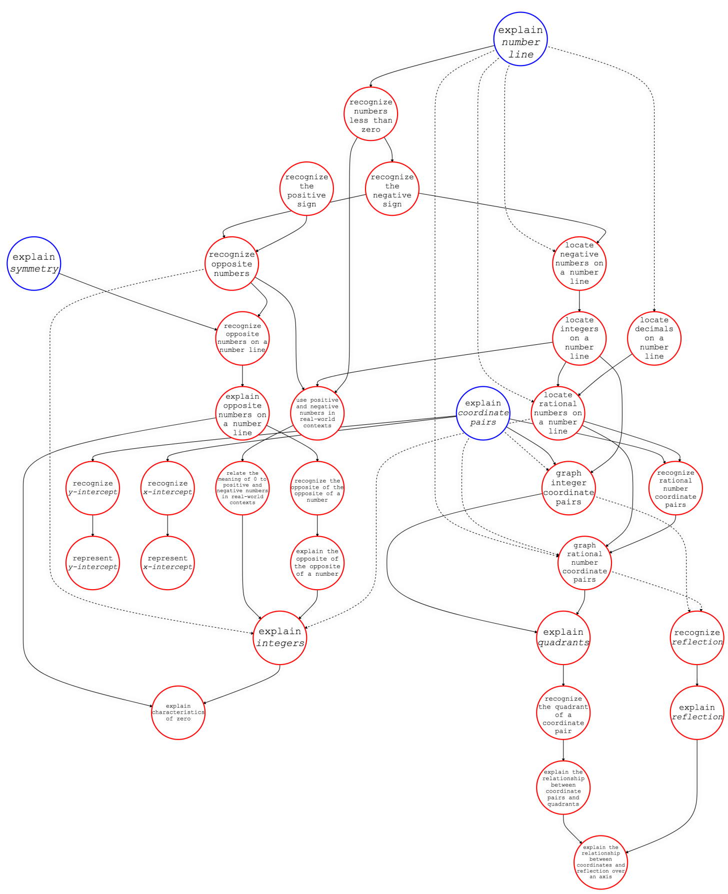
6.NS.C.5 Use positive and negative numbers to represent quantities.

6.NS.C.6 Locate a rational number as a point on the number line.

6.NS.C.6.a Locate rational numbers on a horizontal or vertical number line.

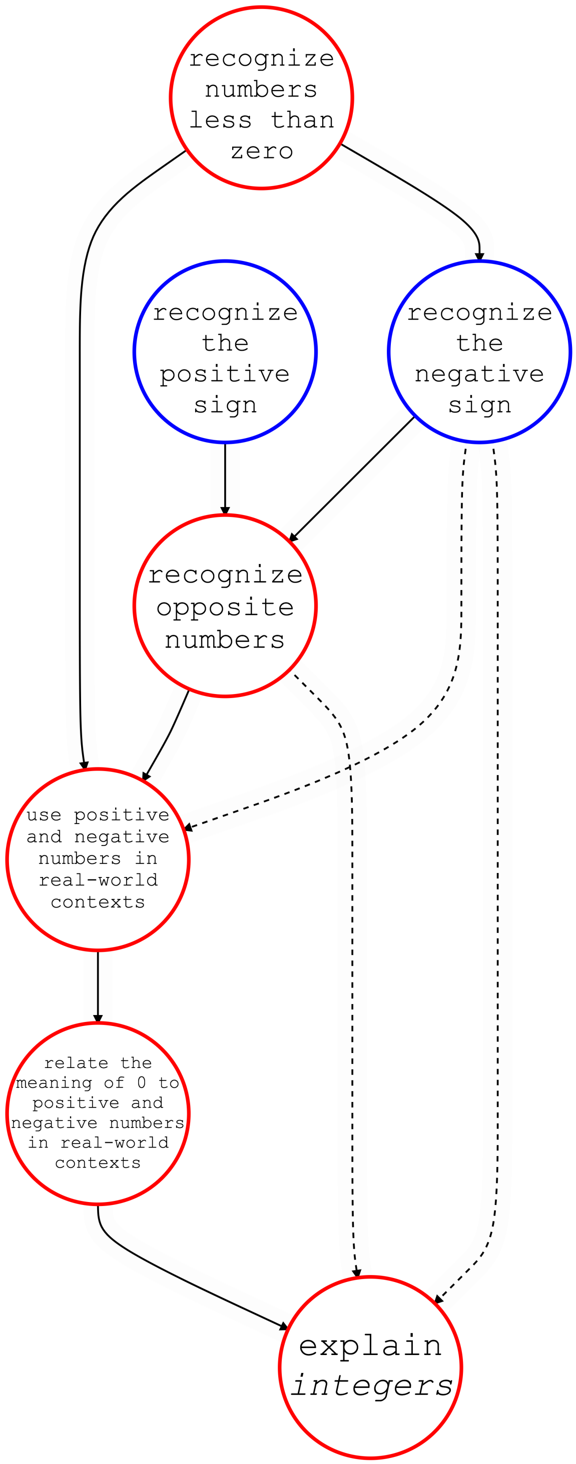
6.NS.C.6.b Write, interpret, and explain problems of ordering of rational numbers.

6.NS.C.6.c Understand that a number and its opposite (additive inverse) are located on opposite sides of zero on the number line.



**Learning map model of* 6.NS.5,6

6.NS.C.5 Use positive and negative numbers to represent quantities.



Learning map model of 6.NS.5

6.NS.C.6 Locate a rational number as a point on the number line.

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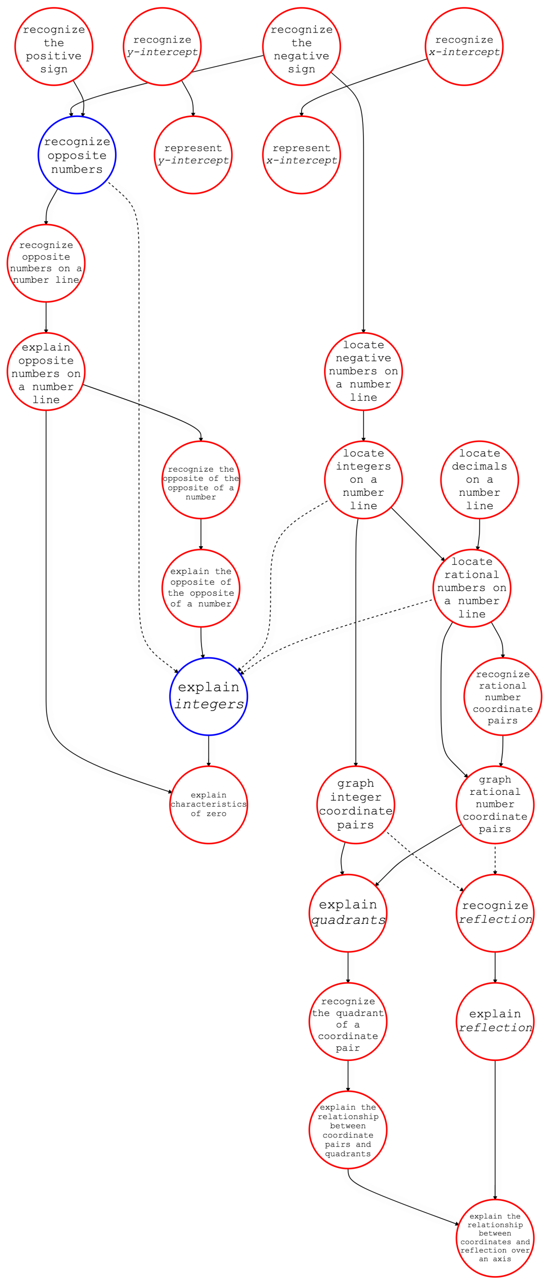

**Learning map model of* 6.NS.6

Node Name	Node Description
EXPLAIN CHARACTERISTICS OF ZERO	Make known your understanding that zero is neither positive nor negative and the opposite of zero is zero.
EXPLAIN *COORDINATE PAIRS*	Make known your understanding that a coordinate pair (ordered pair) is a set of numbers used to show the position on a graph or give the input and output of a relation or function. The first number *a* in the coordinate pair (*a*, *b*) represents the input or *a* units left or right on the *x*-axis, and the second number *b* in the coordinate pair (*a*, *b*) represents the output or *b* units up or down on the *y*-axis.
EXPLAIN *INTEGERS*	Make known your understanding that integers include the whole numbers and their opposites. Each integer possesses both magnitude and direction, and the set of integers is symmetric around zero.
EXPLAIN *NUMBER LINE*	Make known your understanding that a number line is a graphic representation of numbers; a straight line is partitioned into equal segments representing iteration of quantity. Lesser values are positioned to the left of greater values on the number line.
EXPLAIN OPPOSITE NUMBERS ON A NUMBER LINE	Make known your understanding that *x* and –*x* are opposite numbers because they are on different sides of zero and are equal distances from zero on the number line.
EXPLAIN *QUADRANTS*	Make known your understanding that the *x*–axis and *y*–axis divide the coordinate plane into four *quadrants*. In order to label the quadrants, we start from the top right (Quadrant I) and go counter clockwise.
EXPLAIN *REFLECTION*	Make known your understanding that a reflection is a transformation that flips a plane containing points, lines, or figures across a line.
EXPLAIN *SYMMETRY*	Make known your understanding that symmetric figures can be divided into congruent parts, where each of the parts is a mirror image of the other.
EXPLAIN THE OPPOSITE OF THE OPPOSITE OF A NUMBER	Make known your understanding that the opposite of the opposite of a number is the number itself. For example, – (–5) = 5.
EXPLAIN THE RELATIONSHIP BETWEEN COORDINATE PAIRS AND QUADRANTS	Make known your understanding that the signs of a given coordinate pair identify what quadrant it is in. For example, if the coordinate pair is (positive, positive) then it is in the first quadrant.
EXPLAIN THE RELATIONSHIP BETWEEN COORDINATES AND REFLECTION OVER AN AXIS	Make known your understanding that coordinate pairs that only differ by signs are reflections across one or both axes.
GRAPH INTEGER COORDINATE PAIRS	Through writing or an appropriate assistive technology, plot or represent integer coordinate pairs on coordinate plane.
GRAPH RATIONAL NUMBER COORDINATE PAIRS	Through writing or an appropriate assistive technology, plot or represent rational number coordinate pairs on a coordinate plane.
LOCATE DECIMALS ON A NUMBER LINE	When given a decimal, locate the number on a number line.
LOCATE INTEGERS ON A NUMBER LINE	When given an integer, locate the number on a number line.
LOCATE NEGATIVE NUMBERS ON A NUMBER LINE	When given a negative number, locate the number on a number line.
LOCATE RATIONAL NUMBERS ON A NUMBER LINE	When given a rational number, locate the number on a number line.
RECOGNIZE NUMBERS LESS THAN ZERO	Recognize numbers less than zero as they occur in real world contexts. For example, if the temperature is 7 degrees and drops by 10 degrees, the result

	is a temperature that is less than zero. At this point, students need not recognize these as "negative" numbers.
RECOGNIZE OPPOSITE NUMBERS	Given a number, identify or name the opposite of that number. For example, given 7, identify or name –7 as the opposite.
RECOGNIZE OPPOSITE NUMBERS ON A NUMBER LINE	Given a number, identify or name the opposite of that number on a number line. For example, given 17, identify or name –17 as the opposite on a number line.
RECOGNIZE RATIONAL NUMBER COORDINATE PAIRS	Identify or name rational number coordinate pairs on a coordinate plane.
RECOGNIZE *REFLECTION*	Identify a reflection as a transformation that flips a plane containing points, lines, or figures across a line of reflection.
RECOGNIZE THE NEGATIVE SIGN	Identify or name the negative sign and interpret it to indicate a value that is less than zero. Distinguish this use of the "–" symbol from the use of the same symbol to indicate subtraction.
RECOGNIZE THE OPPOSITE OF THE OPPOSITE OF A NUMBER	Given a number, identify or name the opposite of that number's opposite as the number itself. For example, the opposite of the opposite of 9 is 9.
RECOGNIZE THE POSITIVE SIGN	Identify or name the positive sign and interpret it to indicate a value that is greater than zero.
RECOGNIZE THE QUADRANT OF A COORDINATE PAIR	Given a coordinate pair, identify or name the quadrant it is located in.
RECOGNIZE *X-INTERCEPT*	Identify or name an x-intercept as a point on the x-axis.
RECOGNIZE *Y-INTERCEPT*	Identify or name a y-intercept as a point on the y-axis.
RELATE THE MEANING OF 0 TO POSITIVE AND NEGATIVE NUMBERS IN REAL-WORLD CONTEXTS	Make known your understanding the meaning of zero in a real-world context. For example, an elevation of 0 means at sea level.
REPRESENT *X-INTERCEPT*	Through writing or an appropriate assistive technology, represent x-intercepts on a graph or as a coordinate pair.
REPRESENT *Y-INTERCEPT*	Through writing or an appropriate assistive technology, represent y-intercepts on a graph or as a coordinate pair.
USE POSITIVE AND NEGATIVE NUMBERS IN REAL-WORLD CONTEXTS	Reason about positive and negative numbers in real-world contexts (e.g., temperature, elevation, credits and debts, etc.).

Integers, Rational Numbers, and Symmetry

TEACHER NOTES

This unit includes the following documents:

- Learning Map Information
- Instructional Activity (three lessons)
- Instructional Activity Supplement (for Lessons 1 – 3)
- Student Activity
- Student Activity Solution Guide

RESEARCH

Understanding, graphing, and operating with integers comprise important middle school mathematics topics, but these topics challenge students' prior conceptions about numbers and their representations. When introducing integer concepts, students benefit from discussing familiar situations involving assets and debts, sea level, or temperature to help them form a conceptual basis for why negative numbers exist and how they relate to things they already know (Kent, 2000). The learning map section, therefore, promotes the use of contextual situations to provide connections between concrete examples of negative quantities to their symbolic representations.

Working with integers challenges students who try to apply their whole number schemes to integers (Bishop et al., 2014). Even when students experience integers initially through familiar real-world contexts, such opportunities may not require students to acknowledge that negative numbers possess both magnitude and direction, because negative values in context can be labeled differently rather than assigned a negative sign (Peled & Carraher, 2007). Thus teachers should incorporate language that describes how far these numbers are from zero (i.e., distance, magnitude) and whether they are positive or negative (i.e., direction) to help students develop their appreciation for these two aspects of integers.

Integer notation also causes confusion because the symbols used for addition and subtraction operations with whole numbers gain new meanings when used with integers. The learning map model depicts how students must expand their understanding particularly of the "–" sign to incorporate its meaning as negative, opposite, or minus (Lamb et al., 2012). Moreover, the meanings of positive and negative signs can change within a problem, requiring students to incorporate their understanding of signs with their knowledge of operations on integers.

AN EXAMPLE

Teachers should be aware that there are three meanings of "–", which can cause confusion for students (Lamb et al., 2012).

Example	Meaning
5 – 8	Subtraction as a binary operation
– 2	A symbolic representation for a negative number
– (– 4) or – x	The *opposite of*, a unary operation

This table was adapted from (Lamb et al., 2012).

The number line model poses additional challenges to students learning to understand and perform operations with integers. Students initially tend to separate the number line at zero (Peled, Mukhopadhay, & Resnick, 1989). They first view the number line to the left of zero as having similar rules to the number line to the right of zero, and they struggle to coordinate these divided number lines into a continuous number line. These aspects challenge students as they struggle to understand integers as numbers with magnitude and direction and as a set that is symmetric around zero.

LEARNING MAP INFORMATION

The sequences in the learning map section reflect these recommendations, beginning with students' ability to recognize the existence of values that are less than zero. This is followed by students' ability to recognize positive and negative symbols to indicate whether a value is greater than or less than zero, distinguishing these instances from their use to indicate addition and subtraction. The learning map model then progresses to recognizing opposite values and locating integers and rational numbers on the number line. Students should then be able to use positive and negative numbers in real-world contexts and explain opposites of opposites, integers, and characteristics of zero. In addition, the ability to locate numbers on the number line also leads to students' ability to work on the coordinate plane, which includes: recognizing and representing x– and y–intercepts, graphing coordinate pairs, explaining the quadrant of a coordinate pair, and recognizing when a reflection of a point has occurred in the coordinate plane based on the coordinates of the two points in consideration.

INSTRUCTIONAL ACTIVITIES

In this unit, students start the first lesson with a real-world scenario related to temperature. Based on an understanding that temperatures can drop below zero, students establish the need for and the structure of the negative portion of the number line. Emphasis is placed on the symmetry of the number line with respect to zero and the need for symbols to distinguish values that are less than zero from values that are greater than zero. Using concepts of symmetry, direction, and magnitude, students explore opposites as well as opposites of opposites, symbolically and graphically using the number line. Lastly, students plot a variety of rational numbers on the number line and use their understanding that smaller values fall to the left on a horizontal number line and at the bottom of a vertical number line to order numbers from least to greatest.

The second lesson in this unit allows students to explore positive and negative numbers to describe elevation. In addition to exploring an example as a class, students are asked to come up with their own example of a story where positive and negative numbers occur. Teachers are encouraged to look for a variety of real-world contexts among student stories and have students share their stories with the class.

Finally, the third lesson moves students from the symmetry on the number line to the symmetry on the coordinate plane. Students should be familiar with Quadrant I of the coordinate plane and use their understanding of symmetry in number lines to establish the second quadrant of the coordinate plane, then the third and fourth quadrants through folding. Once the coordinate plane is established, students review plotting points in the first quadrant and on the positive x– and y–axes. Students then participate in an activity to discover the patterns in the signs of the second, third, and fourth quadrants and practice plotting points where at least one coordinate is a negative value. Once students are able to plot points comfortably in any quadrant of the coordinate plane, they explore reflections of points across one or both axes and describe the pattern in the coordinates when a reflection occurs.

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Integers, Rational Numbers, and Symmetry

OVERVIEW OF INSTRUCTIONAL ACTIVITIES

Lesson	Learning Goal	Nodes Addressed
Lesson 1	Students will use symmetry and real-world situations to establish the negative side of the number line and use the continuous number line to plot and order rational numbers.	▶ RECOGNIZE THE POSITIVE SIGN ▶ RECOGNIZE THE NEGATIVE SIGN ▶ EXPLAIN *NUMBER LINE* ▶ EXPLAIN *SYMMETRY* ▶ EXPLAIN *INTEGER* ▶ RECOGNIZE OPPOSITE NUMBERS ▶ RECOGNIZE OPPOSITE NUMBERS ON A NUMBER LINE ▶ EXPLAIN OPPOSITE NUMBERS ON A NUMBER LINE ▶ EXPLAIN CHARACTERISTICS OF ZERO ▶ RECOGNIZE THE OPPOSITE OF THE OPPOSITE OF A NUMBER ▶ EXPLAIN THE OPPOSITE OF THE OPPOSITE OF A NUMBER ▶ LOCATE NEGATIVE NUMBERS ON A NUMBER LINE ▶ LOCATE DECIMALS ON A NUMBER LINE ▶ LOCATE INTEGERS ON A NUMBER LINE ▶ LOCATE RATIONAL NUMBERS ON A NUMBER LINE
Lesson 2	Students will provide real-world contexts to demonstrate positive and negative numbers and discuss the meaning of zero in each context.	▶ USE POSITIVE AND NEGATIVE NUMBERS IN REAL-WORLD CONTEXTS ▶ RELATE THE MEANING OF 0 TO POSITIVE AND NEGATIVE NUMBERS IN REAL-WORLD CONTEXTS
Lesson 3	Students will use symmetry to establish the second, third, and fourth quadrants of the coordinate plane and use the coordinate plane to plot ordered pairs with rational numbers in any quadrant and to look for patterns in the locations of coordinate pairs based on signs.	▶ RECOGNIZE THE POSITIVE SIGN ▶ RECOGNIZE THE NEGATIVE SIGN ▶ EXPLAIN *SYMMETRY* ▶ GRAPH INTEGER COORDINATE PAIRS ▶ RECOGNIZE RATIONAL NUMBER COORDINATE PAIRS ▶ GRAPH RATIONAL NUMBER COORDINATE PAIRS ▶ RECOGNIZE *X-INTERCEPT* ▶ RECOGNIZE *Y-INTERCEPT* ▶ EXPLAIN *QUADRANTS* ▶ RECOGNIZE THE QUADRANT OF A COORDINATE PAIR ▶ EXPLAIN THE RELATIONSHIP BETWEEN COORDINATE PAIRS AND QUADRANTS ▶ RECOGNIZE *REFLECTION* ▶ EXPLAIN *REFLECTION* ▶ EXPLAIN THE RELATIONSHIP BETWEEN COORDINATES AND REFLECTION OVER AN AXIS

INTEGERS, RATIONAL NUMBERS, AND SYMMETRY

INSTRUCTIONAL ACTIVITY

Lesson 1

LEARNING GOAL

Students will use symmetry and real-world situations to establish the negative side of the number line and use the continuous number line to plot and order rational numbers.

PRIMARY ACTIVITY

Students will use a real-world situation, folding, and symmetry ideas to develop the negative portion of the number line and establish notation for negative numbers. Once students have developed the continuous number line, they will plot points to order rational numbers and investigate opposites on the number line.

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

- Positive
- Negative
- Number line
- Opposite
- Symmetry
- Integer
- Fraction
- Decimal

MATERIALS

- INSTRUCTIONAL ACTIVITY SUPPLEMENT (Recommend one copy for every student.)
- Patty paper/tracing paper (optional)

IMPLEMENTATION

Start the lesson by focusing students' attention on a situation where negative numbers surface as a result of a familiar context. Temperature on a thermometer will be used in the following examples, although elevation, lending and borrowing money among friends and family, and bank accounts are also viable contexts.

Because thermometers are often vertical, **suggest** that students orient the number line in the INSTRUCTIONAL ACTIVITY SUPPLEMENT vertically.

Require students to label "0" in the middle of the number line and **ask** them to count by ones in order to label the portion of the number line that is above, or greater than, zero.

Note that because the intervals are evenly spaced, students should use the distance between tick marks regularly. In addition, **emphasize** to students that the space or interval *between* tick marks represents the unit, which must be regular within a number line.

> NOTE: A common misconception among students is that the tick marks represent the unit instead of the space between the tick marks.

Discuss whether it is possible for the temperature to drop below zero. Students should indicate that this is possible.

Indicate a change in temperature on the thermometer by starting at one temperature and moving to a new temperature. **Ask** students whether the resulting temperature is warmer or colder than the original temperature. **Repeat** this activity a few times to **establish** that higher on the thermometer means warmer temperatures and lower on the thermometer means colder temperatures.

Ask students if they know which temperature is colder: negative one degree Fahrenheit (one degree below zero) or negative fifty degrees Fahrenheit (fifty degrees below zero). Students should be able to indicate that negative fifty degrees is colder than negative one degree.

Relate the location on the number line to the size of the number. For example, eight (degrees) is greater (warmer) than two (degrees) and therefore eight is located above two on a vertical number line.

Ask students which value they think will be higher on the number line: negative one or negative fifty. Students should indicate that negative one will be higher on the number line than negative fifty because it is warmer.

Require students to fold the number line onto itself so the top portion of the number line lines up with the bottom portion of the number line and the fold is perpendicular to the number line through the position of zero. If desired, students could first trace the existing number line onto patty paper or tracing paper in order to allow them to see through the paper more easily.

Note that the number line is symmetric and the fold acts as the line of symmetry.

In a different color, have students label each value on the negative side of the number line according to the positive value it lines up with. Students can copy the positive value first and add the negative sign after the digit has been positioned.

Tell students that the values they have used to label their number line are called *integers*. Integers are whole numbers and their opposites which have both magnitude and direction and are symmetric around zero.

Discuss the fact that the positive and negative signs distinguish between values with the same magnitude and opposite direction from zero by indicating their direction from zero. Because the direction is different, there cannot be two different points on the number line labeled "3", therefore the smaller or lesser of the two is distinguished with a negative (–) sign and the larger or greater of the two is distinguished with a positive (+) sign.

Note that the positive sign isn't always written but is acceptable to include. Additionally, it is important to discuss and clarify the words "smaller", "larger", "lesser", and "greater" with students. Their previous experiences with larger and smaller may have been related to the size of physical objects and measurement rather than the direction and magnitude of a number. Here, in some ways, the numbers are the same "size", but their values are not the same. This is an example of a situation when mathematics vocabulary is very precise and needs to be discussed and clarified for students.

GUIDING QUESTIONS

Elicit student thinking:

- Other than a thermometer, where have you seen something that looks like a number line?

Determine if the student can RECOGNIZE NUMBERS LESS THAN ZERO:

- If the temperature was nine degrees and then it dropped by fifteen degrees, how does the resulting temperature compare to zero?
- On a vertical number line labeled with zero and values greater than zero located above zero, what can you say about the values located below zero?
- Can you give an example of a situation in which numbers could go below zero?

Determine if the student can RECOGNIZE THE POSITIVE SIGN:

- What is the symbol in front of the numbers above zero or on the right side of zero?
- What does the positive sign indicate regarding the value of the numbers to the right of zero or above zero on the number line?
- Where else have you seen the positive sign?
- Can you provide examples of different ways you can use the "+" symbol?

Determine if the student can RECOGNIZE THE NEGATIVE SIGN:

- What is the symbol in front of the numbers below zero or on the left side of zero?
- What does the negative sign indicate regarding the value of the numbers to the left of zero or below zero on the number line?
- Where else have you seen the negative sign?
- Can you provide examples of different ways you can use the "–" symbol?

Determine if the student can EXPLAIN *NUMBER LINE*:

- What is true about the positive and negative sides of the number line?
- What must be true about the intervals on a number line?
- How would you describe the structure of the number line?

Determine if the student can EXPLAIN *SYMMETRY*:

- How would you describe symmetry in a number line?
- Where is the line of symmetry in the number line? How do you know?

Determine if the student can EXPLAIN *INTEGERS*:

- What are integers?
- How are integers related to symmetry?
- Can you give me an example of a number that is an integer?
- Can you give me an example of a number that is not an integer?

Discuss that values that are symmetric with respect to zero (e.g., +6 and –6) are opposites. **Ask** students for additional pairs of numbers that are opposites. **Start** with values the students have included on their number lines, but move to values that are not visible on their number lines. **Include** fractions and decimals to emphasize that all rational numbers have opposites. Additionally, unlabeled number lines are provided in the INSTRUCTIONAL ACTIVITY SUPPLEMENT for student use as needed. These number lines are oriented horizontally and provide students an opportunity to practice labeling the number line and to plot pairs of opposites.

Ask students to consider the value zero. Have them think about whether they would classify zero as a positive number, a negative number, or neither, and what its opposite would be. After sharing students' opinions with their classmates, **conduct** a class discussion about the value zero. Through the discussion, **establish** that zero is neither positive nor negative (positive numbers are defined as values greater than zero and negative numbers are defined as values less than zero, and zero is not greater than or less than itself), and that the opposite of zero is zero (zero falls on the line of symmetry, therefore its mirror image is itself).

Emphasize that opposites have the same magnitude relative to zero, but in opposite directions. For example, one half and negative one half are both one half of a unit from zero, but in opposite directions.

Discuss the same concept with additional pairs of opposites, using the number line to demonstrate distance from zero and direction.

GUIDING QUESTIONS

Elicit student thinking:

- What do you know about opposites?
- What is an example of a pair of opposites that does not involve numbers? How is that example similar to opposites that are numeric? How is it different?

Determine if the student can RECOGNIZE OPPOSITE NUMBERS:

- What is the opposite of 3.5?
- What is the opposite of $-\frac{1}{3}$?
- What do you look for when you decide whether two values are opposites?
- When given a value, how do you determine its opposite?

Determine if the student can RECOGNIZE OPPOSITE NUMBERS ON A NUMBER LINE:

- Can you point to two values on the number line that are opposites?
- [Point to opposites on the number line] Are these values opposites?
- [Point to values that are not opposites on the number line] Are these values opposites?

Determine if the student can EXPLAIN OPPOSITE NUMBERS ON A NUMBER LINE:

- How do you know when two values on the number line are opposites? What do you look for?
- [Point to or plot a pair of opposites on the number line] How do you know these values are opposites?
- [Point to values that are not opposites on the number line] How do you know these values are not opposites?

Determine if the student can EXPLAIN *SYMMETRY*:

- How would you describe symmetry in a number line?
- Where is the line of symmetry in the number line? How do you know?
- How are opposite values symmetric?

Determine if the student can EXPLAIN CHARACTERISTICS OF ZERO:

- What do you know about the number zero?
- Would you consider zero to be positive? Why or why not?
- Would you consider zero to be negative? Why or why not?
- What is the opposite of zero?

Provide students with the following expressions to emphasize the different meanings the "–"symbol can hold: opposite, negative, and minus. **Encourage** students to realize that the minus sign can have multiple meanings in a context, though some interpretations are more productive than others.

Require students to practice reading the expression using an appropriate word for the "–" symbol based on the context. Recommended responses that will help students interpret various uses of the minus sign are provided in parentheses.

- – 4 (either "opposite of four" or "negative four")
- 6 – 3 ("six minus three")
- – (– 1) ("opposite of negative one" or "the opposite of the opposite of one" is preferable)
- – *x* ("opposite of *x*" is preferable because it allows students to understand this value could be positive or negative)
- *x* – 11 ("*x* minus eleven")

Focus students on the example – (– 1) and the interpretation "the opposite of the opposite of one". To support this interpretation, the expression could be written as – (– (1)).

With number lines available for reference, **cover** both negative symbols and **ask** students to locate one. Then, **uncover** the negative symbol closest to one and **ask** what the opposite of one is. Students should indicate the opposite of one is negative one.

Replace "the opposite of one" in the original phrase with "negative one" since those are equivalent values. The phrase should now read "the opposite of negative one".

Have students locate negative one on the number line. **Uncover** the outside negative symbol and **ask** students what the opposite of negative one is. They should respond that the opposite of negative one is one.

Review the fact that you began with one, found its opposite, then the opposite of the opposite, and ended back at one.

In addition, **point out** to students that this sequence reflects how we operate when following the order of operations to simplify an expression. When parentheses exist in an expression, students should start inside the parentheses and work their way out, which is exactly the process that was used in this example.

Record the example in a table similar to the example table below and add additional examples as they are used in class. This should help students discover, after a few examples, that the opposite of the opposite of a number is always the original number.

SYMBOLIC PROBLEM	WORDS FOR THE SYMBOLIC FORM	SIMPLIFIED FORM	WORDS FOR THE SIMPLIFIED FORM
– (– (1))	the opposite of the opposite of one	1	one
– (– (–3))	the opposite of the opposite of negative three	–3	negative three

Repeat this questioning with different values, including the opposite of the opposite of a negative value (e.g., – (– (–3))). After a few examples, **ask** students if they think the opposite of the opposite of a number will always be the number itself. Give students a chance to think, then discuss their thoughts with a partner.

Ask for volunteers to respond, establishing through the discussion that the opposite of the opposite of a number is always the original number.

GUIDING QUESTIONS

Elicit student thinking:

- What do you know about the symbol "–"?

Determine if the student can RECOGNIZE THE NEGATIVE SIGN:

- Can you write an expression where you would read the "–" symbol as "minus"?
- Can you write an expression where you would read the "–" symbol as "negative"?
- Can you write an expression where you would read the "–" symbol as "opposite of"?

Determine if the student can RECOGNIZE THE OPPOSITE OF THE OPPOSITE OF A NUMBER:

- What is the opposite of the opposite of 4?
- What is the opposite of the opposite of –7?

Determine if the student can EXPLAIN THE OPPOSITE OF THE OPPOSITE OF A NUMBER:

- How would you read – (–3)? What is an equivalent expression?
- What is always true about the opposite of the opposite of a number?

Determine if the student can EXPLAIN OPPOSITE NUMBERS ON A NUMBER LINE:

- How would you demonstrate finding the opposite of the opposite of negative 6 on a number line?
- Given the expression – (– 4.5), how would you demonstrate simplifying using the number line? Can you explain your thinking?

Lastly, connect students' knowledge of the structure of the number line to locating and ordering rational numbers on the number line.

Start by having students draw a number line with a range of at least –10 to 10.

Remind students that intervals must be equally spaced for values that are equidistant from each other. For example, since two and three are one unit apart and negative four and negative three are one unit apart, the distance between two and three on the number line should be the same as the distance between negative four and negative three on the number line.

Provide students with two or more values to plot on the number line. **Start** with integers, then progress to fractions and decimals. For each set of values, **guide** students to correctly locate the value on the number line with a point. For fractions and decimals that fall in between two values on the number line, students should subdivide the number line to allow for an accurate and clearly defined placement of that value on the number line.

Once all values in the set are positioned on the number line, **ask** students to order the values from least to greatest, using the number line to assist them. Students should note that smaller values are always to the left or bottom of the number line, depending on orientation, and larger values are always to the right or top of the number line, depending on orientation.

Following are example sets of values that are sequenced according to the recommendations provided, though students will likely need additional examples with fractions and decimals.

- 5, –7, –1
- –8, 4, 0, –1, 6
- $\frac{1}{3}$, $-\frac{1}{2}$, 2, –3
- –4.5, 3.6, –0.1, –1.7, 2, –4

GUIDING QUESTIONS

Elicit student thinking:

- What do you know about the structure of the number line?
- Can you describe a number line?

Determine if the student can LOCATE NEGATIVE NUMBERS ON A NUMBER LINE:

- Can you point to where negative numbers are located on the number line?
- Where are negative numbers located relative to zero on the number line?
- Where is [provide a negative integer] located on the number line?

Determine if the student can LOCATE DECIMALS ON A NUMBER LINE:

- Can you describe how you would determine where to locate a decimal on the number line?
- Where is [provide a positive decimal] located on the number line?

Determine if the student can LOCATE INTEGERS ON A NUMBER LINE:

- How would you describe the position of integers on a number line?
- Where is [provide a positive integer] located on the number line?
- Where is [provide a negative integer] located on the number line?

Determine if the student can LOCATE RATIONAL NUMBERS ON A NUMBER LINE:

- Where is [provide a positive decimal] located on the number line? Why does this location make sense?
- Where is [provide a negative decimal] located on the number line? Why does this location make sense?
- Where is [provide a positive fraction] located on the number line? Why does this location make sense?
- Where is [provide a negative fraction] located on the number line? Why does this location make sense?

At the end of the activity, teachers should require students to create a labeled number line from –10 to 10 without referring to a previously drawn number line. Require students to plot a pair of opposites as well as a fraction and a decimal. Have students describe how they know the values they chose as opposites are, in fact, opposites, using the words "magnitude" and "direction" in their description. Then, have students list the four values on their number line in order from smallest to largest.

Integers, Rational Numbers, and Symmetry

INSTRUCTIONAL ACTIVITY

Lesson 2

LEARNING GOAL

Students will provide real-world contexts to demonstrate positive and negative numbers and discuss the meaning of zero in each context.

PRIMARY ACTIVITY

Students will create stories to convey their knowledge of positive numbers, negative numbers, and zero in real-world contexts.

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

- Positive
- Negative
- Zero
- Number line

MATERIALS

- Instructional Activity Supplement (Recommend one copy for every student.)

IMPLEMENTATION

Start the lesson by telling a story where positive and negative numbers surface as a result of a familiar context. Hiking and elevation will be used in the following example, though temperature and bank accounts are also viable contexts.

Set up the scenario of people hiking in the mountains, therefore the elevation of the hikers will vary depending on their location. Have students determine which way is most beneficial to orient the number line in the INSTRUCTIONAL ACTIVITY SUPPLEMENT.

Tell students that all hikers on this mountain park their car in a certain parking lot, which is somewhere in the middle of the mountain, and from the parking lot there is one path that leads hikers up the mountain and another path that leads hikers down the mountain. Therefore, hikers can choose to either increase or decrease their elevation from their initial location, the parking lot.

Guide students to consider the hikers' initial positions as zero and plot this on the number line (advise locating zero near the middle of the number line because the hikers' elevation can change in either direction).

Encourage students to count by 10 feet in order to label the number line to allow room for a realistic elevation change.

Discuss the need for math models to allow us to consider a wide range of realistic situations. In this situation our model has to go up to 90 feet and down to –105 feet. If we used ones, our number line would be enormous. Therefore, we can *scale* the number line and use 10 between each pair of tick marks.

Note that because the intervals are evenly spaced, students should use the distance between tick marks regularly.

Check that students have properly labeled their number line by tens.

Ask students to place the following hikers at the elevation provided on the number line, labeled with their name.

- Sally: 90 feet
- John: –35 feet
- Jim: 10 feet
- Sue: –105 feet
- Alex: 45 feet

Ask students who is at the highest elevation, who is at the lowest elevation, and other questions about where the hikers are located relative to each other.

GUIDING QUESTIONS

Elicit student thinking:

- How does the number line relate to the hikers and the mountain?

Determine if the student can USE POSITIVE AND NEGATIVE NUMBERS IN REAL-WORLD CONTEXTS:

- What do you know about a hiker if they are at a positive elevation?
- If a hiker left the parking lot and hiked up the mountain, would they have a positive or negative elevation?
- What do you know about a hiker if they are at a negative elevation?
- If a hiker left the parking lot and hiked down the mountain, would they have a positive or negative elevation?

Determine if the student can RELATE THE MEANING OF 0 TO POSITIVE AND NEGATIVE NUMBERS IN REAL-WORLD CONTEXTS:

- If a hiker were at zero, what would that tell you about their position on the mountain?
- How would a hiker at a positive elevation return to an elevation of zero?
- How would a hiker at a negative elevation return to an elevation of zero?

Once students have considered the context of hikers at various elevations and answered questions about their positions, students will write their own story involving a real-world context and positive and negative values.

> **Guide** students to think of a context where positive and negative numbers exist. **Encourage** students to think of a context other than the example provided previously for the whole group.
>
> **Require** students to write a story about at least one negative number and at least one positive number, and use a new number line to represent the values. **Emphasize** that students should be able to explain what zero means in the context of the story they write.

NOTE: The hiker scenario is also found in 6.NS.7, which is an introduction to ordering and taking the absolute value of numbers. As students explore elevations and write their own story problems they may naturally begin to wonder how negative elevations relate to a positive distance traveled. Therefore, this lesson concerning negative numbers could be taught as a prelude to 6.NS.7.

Once students are done writing their stories, have them share stories with a partner and allow time for the partners to ask each other questions about the stories. **Observe** these conversations and **ask** students questions about their stories, the positive and negative values they provide, and the meaning of zero in their stories' contexts.

Select a variety of students to share their stories with the class. It is preferable to **identify** students who have provided a variety of values in their stories and different real-world contexts in which to situate their stories, so that the class is exposed to a variety of real-world situations where positive and negative numbers surface naturally.

GUIDING QUESTIONS

Elicit student thinking:

- How does your story relate to a number line?
- Why did you choose this story to discuss positive and negative numbers?

Determine if the student can USE POSITIVE AND NEGATIVE NUMBERS IN REAL-WORLD CONTEXTS:

- What do positive numbers mean in the context of your story?
- What do negative numbers mean in the context of your story?

Determine if the student can RELATE THE MEANING OF 0 TO POSITIVE AND NEGATIVE NUMBERS IN REAL-WORLD CONTEXTS:

- What does zero mean in the context of your story?

At the end of the activity, teachers should create a new story involving positive and negative numbers. Contexts could include temperatures above and below zero, elevations above and below sea level, credits and debts, or positive and negative electric charges. Provide students with at least two positive and two negative values in the story. Require students to represent the values described using points on a number line and describe the meaning of zero in the story.

Integers, Rational Numbers, and Symmetry

INSTRUCTIONAL ACTIVITY

Lesson 3

LEARNING GOAL

Students will use symmetry to establish the second, third, and fourth quadrants of the coordinate plane and use the coordinate plane to plot ordered pairs with rational numbers in any quadrant and to look for patterns in the locations of coordinate pairs based on signs.

PRIMARY ACTIVITY

Students will use folding and symmetry ideas to develop the second, third, and fourth quadrants of the coordinate plane from the first quadrant. Once students have developed the complete coordinate plane, they will plot ordered pairs with rational numbers in any quadrant and investigate the impact of the signs on the quadrant the ordered pair is in and its relationship to other ordered pairs.

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

- Positive
- Negative
- Coordinate plane
- Opposite
- Symmetry
- Quadrant
- x–coordinate
- y–coordinate
- x–axis
- y–axis
- Origin
- Reflection

MATERIALS

- Instructional Activity Supplement (Recommend one copy for every student.)
- Patty paper/tracing paper (recommended)

IMPLEMENTATION

Start the lesson by focusing students' attention on the first quadrant of the coordinate plane.

Encourage students to notice the origin is labeled "0" and **ask** them to count by ones in order to label the axes of the first quadrant of the coordinate plane as shown in **FIGURE 1**.

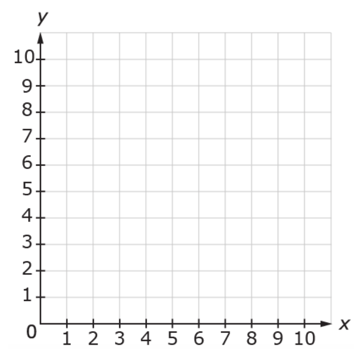


FIGURE 1

Note that because the intervals are evenly spaced, students should use the distance between tick marks regularly.

Ask students, based on their work with number lines in Lesson 1 and Lesson 2, whether they believe they could graph ordered pairs with negative values as the *x*–coordinate. If necessary, remind students about how they folded the number line to generate the negative side of the number line.

Ask students if they have any ideas about how they could go about generating the negative side of the *x*–axis on the coordinate plane. Students hopefully will suggest folding the coordinate plane along the *y*–axis (or at 0 on the *x*–axis) in order to generate and trace the negative side of the *x*–axis.

If tracing paper or patty paper is available, students should first trace the first quadrant of the coordinate plane onto the top right portion of the patty paper or tracing paper in order to allow them to see through the paper more easily. **Require** students to fold the paper containing the first quadrant of the coordinate plane along the *y*-axis to generate the second quadrant of the coordinate plane.

Note that the *x*–axis is symmetric and that the fold acts as the line of symmetry.

Require students to trace the negative side of the *x*–axis while the paper is folded and to label each value on the negative side of the *x*–axis according to the positive value it lines up with. Students can copy the positive value first and add the negative sign after the digit has been positioned. An example of student work is shown in **FIGURE 2**.

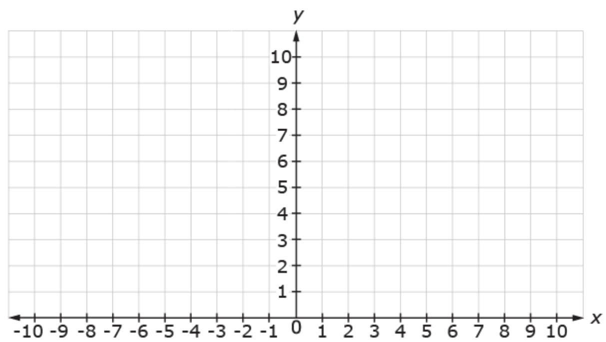


FIGURE 2

Remind students that the positive and negative signs distinguish between values with the same magnitude and opposite direction from zero by indicating their direction from zero. Because the direction is different, there cannot be two different points on an axis labeled "7", therefore the smaller or lesser of the two is distinguished with a negative (–) sign and the larger or greater of the two is distinguished with a positive (+) sign. **Note** that the positive sign isn't always written but is acceptable to include.

GUIDING QUESTIONS

Elicit student thinking:

- What is similar about a coordinate plane and a number line? What is different?

Determine if the student can RECOGNIZE THE POSITIVE SIGN:

- What is the symbol in front of the numbers to the right of zero on the *x*–axis?
- What does the positive sign indicate regarding the value of the numbers to the right of zero on the *x*–axis?
- Where else have you seen the positive sign?
- Can you provide examples of different ways you can use the "+" symbol?

Determine if the student can RECOGNIZE THE NEGATIVE SIGN:

- What is the symbol in front of the numbers to the left of zero on the *x*–axis?
- What does the negative sign indicate regarding the value of the numbers to the left of zero on the *x*–axis?
- Where else have you seen the negative sign?
- Can you provide examples of different ways you can use the "–" symbol?

Determine if the student can EXPLAIN *SYMMETRY*:

- How would you describe symmetry in these two quadrants of the coordinate plane?
- Where is a line of symmetry in the coordinate plane? How do you know?
- How is the *x*–axis symmetric?
- How are opposite values symmetric?

Ask students whether they believe they could graph ordered pairs with negative values as the *y*–coordinate. They should indicate that this is possible because they can have a negative *x*–coordinate (students have experience with horizontal and vertical number lines in LESSON 1 and LESSON 2).

Ask students if they have any ideas about how they could go about generating the negative side of the *y*–axis on the coordinate plane. Students hopefully will suggest folding the coordinate plane along the *x*–axis (or at 0 on the *y*–axis) in order to generate and trace the negative side of the *y*–axis.

Continue with the same piece of paper or tracing paper and **require** students to fold the paper containing the first and second quadrants of the coordinate plane along the *x*-axis to generate the third and fourth quadrants of the coordinate plane.

Note that the *y*–axis is symmetric and that the fold along the *x*–axis acts as the line of symmetry.

Require students to trace the negative side of the *y*–axis while the paper is folded and to label each value on the negative side of the *y*–axis according to the positive value it lines up with. Students can copy the positive value first and add the negative sign after the digit has been positioned. An example of student work is shown in **FIGURE 3**.

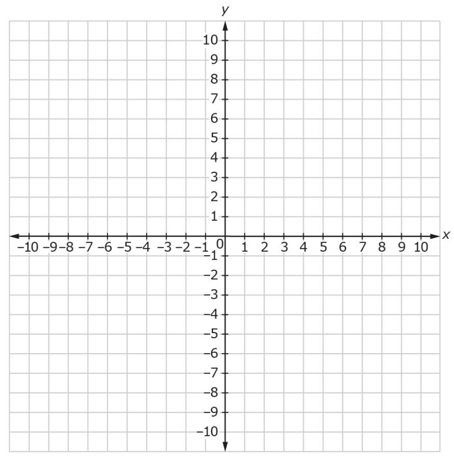


FIGURE 3

Remind students that the positive and negative signs distinguish between values with the same magnitude and opposite direction from zero by indicating their direction from zero. Because the direction is different, there cannot be two different points on an axis labeled "7", therefore the smaller or lesser of the two is distinguished with a negative (–) sign and the larger or greater of the two is distinguished with a positive (+) sign. **Note** that the positive sign isn't always written but is acceptable to include.

GUIDING QUESTIONS

Elicit student thinking:

- What is similar about a coordinate plane and a number line? What is different?

Determine if the student can RECOGNIZE THE POSITIVE SIGN:

- What is the symbol in front of the numbers above zero on the *y*–axis?
- What does the positive sign indicate regarding the value of the numbers above zero on the *y*–axis?
- Where else have you seen the positive sign?
- Can you provide examples of different ways you can use the "+" symbol?

Determine if the student can RECOGNIZE THE NEGATIVE SIGN:

- What is the symbol in front of the numbers below zero on the y–axis?
- What does the negative sign indicate regarding the value of the numbers below zero on the y–axis?
- Where else have you seen the negative sign?
- Can you provide examples of different ways you can use the "–" symbol?

Determine if the student can EXPLAIN *SYMMETRY*:

- How would you describe symmetry in the coordinate plane?
- Where is a line of symmetry in the coordinate plane? How do you know?
- How is the x–axis symmetric? How is the y–axis symmetric?
- How are opposite values symmetric?

Students should now have all four quadrants of the coordinate plane established. Provide students with the complete coordinate plane page from the INSTRUCTIONAL ACTIVITY SUPPLEMENT so they have a larger, clean copy of the coordinate plane to work on. First, focus on Quadrant I.

Review plotting and labeling points in the first quadrant and on the positive x– and y–axes (e.g., (3, 0) and (0, 7)), starting with coordinate pairs with whole number coordinates.

> NOTE: It is recommended to color code the x- and y-values of the coordinate pairs consistently throughout this lesson in order to provide a visual model for students and to emphasize the relationship between coordinates in each quadrant.

Note that coordinate pairs which fall on the x–axis are called x–intercepts, and that coordinate pairs which fall on the y–axis are called y–intercepts. **Relate** the word *intercept* to the word *intersect* and **discuss** how these terms describe the behavior of these coordinate pairs.

Ensure that students know the difference between the x–coordinate and the y–coordinate, and that the x–coordinate indicates movement along or parallel to the x–axis while the y–coordinate indicates movement along or parallel to the y–axis.

Students should have experience plotting coordinate pairs with positive x– and y–coordinates when presented with the first quadrant of the coordinate plane, but they need practice transferring this knowledge to the complete coordinate plane before considering coordinate pairs with negative coordinates.

Add examples of coordinate pairs that include fractions and decimals for students to plot and label. **Begin** with coordinate pairs where one coordinate is a whole number and the other coordinate is a positive fraction or decimal. **Progress** to coordinate pairs where both coordinates are positive fractions or decimals.

Discuss with students how to approximate, as accurately as possible, the location of a coordinate pair that does not have whole number coordinates throughout the examples.

Require students to label the first quadrant "Quadrant I" on their coordinate plane. After indicating that the quadrants are labeled counterclockwise, **require** students to label Quadrants II, III, and IV. Figure 1 shows an example of the labeled quadrants in a coordinate plane. An example of the labeled quadrants is shown in **FIGURE 4**.

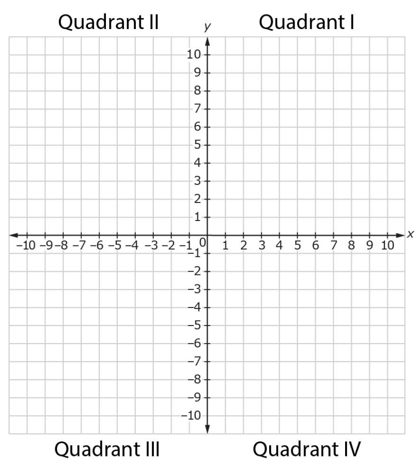


FIGURE 4

Next, shift students' focus to all four quadrants.

Provide students with the coordinate pair cards from the INSTRUCTIONAL ACTIVITY SUPPLEMENT.

Require students to cut apart the cards so they are ready to sort on the coordinate plane.

Inform students that each coordinate pair belongs in one of the four quadrants and that there are multiple coordinate pairs that belong in each quadrant.

Require students to evaluate each card and determine which quadrant the coordinate pair belongs in. Once they decide on a quadrant for a coordinate pair, they should place that card on the coordinate plane in the appropriate quadrant. Students do not need to plot any points yet.

Once students have sorted the coordinate pairs in the coordinate plane, **ask** pairs of students to compare their groups with each other and discuss any differences they have.

Discuss the coordinate pairs students have determined to fall in each quadrant, one at a time, as a class. If there is disagreement among students, **allow** for a conversation regarding the signs of each coordinate and the signs on the axes to help students come to a consensus regarding the correct quadrant for the coordinate pair.

Once students have the coordinates grouped in the correct quadrant, **require** students to look for patterns among the signs of the coordinates for each quadrant. Students should work individually, then share the patterns they found with a partner or small group.

As a class, **discuss** and **establish** patterns for the coordinates in each quadrant. For example, the *x*–coordinate is always negative to the left of the *y*–axis in Quadrants II and III. Allow students to contribute several observations and patterns that apply to one or more quadrants.

Students should establish the following patterns for each quadrant in addition to any other generalizations they describe.

QUADRANT	X-COORDINATE	Y-COORDINATE	COORDINATE PAIRS
I	positive	positive	(positive, positive)
II	negative	positive	(negative, positive)
III	negative	negative	(negative, negative)
IV	positive	negative	(positive, negative)

Now that signs and patterns have been connected to quadrants, require students to plot the points on each of the cards. **Begin** with cards where both coordinates are integers, **move** to cards where one coordinate is an integer and the other is a (non-integer) rational number, then **end** with cards where both coordinates are (non-integer) rational numbers.

It is suggested that some points are plotted as a class, in order to **scaffold** the process of plotting points that are not integers and approximating their location, particularly in quadrants containing negative values. Then **allow** students to practice plotting the remaining points individually or in small groups.

Ask students to describe the location of the coordinate pairs (–5.3, 0) and $\left(0, -\frac{11}{9}\right)$ on the coordinate plane. Students should indicate that (–5.3, 0) falls on the *x*–axis and that $\left(0, -\frac{11}{9}\right)$ falls on the *y*–axis.

Ask students what a coordinate pair that falls on the *x*–axis is called. Students should indicate that a coordinate pair that falls on the *x*–axis is called an *x*–intercept.

Ask students what a coordinate pair that falls on the *y*–axis is called. Students should indicate that a coordinate pair that falls on the *y*–axis is called a *y*–intercept.

Discuss with students that coordinate pairs which fall on the axes are not considered to be in any of the four quadrants because they fall on the boundaries of the quadrants.

Ask students to think of two additional coordinate pairs that are not located in one of the four quadrants. Once students have come up with two coordinate pairs that meet the criteria, have students share their coordinate pairs with a partner or small group. In pairs or groups, students should verify each other's coordinate pairs and determine whether each coordinate pair is an *x*–intercept or a *y*–intercept.

Distinguish the difference between natural number coordinates, whole number coordinates, integer coordinates, and rational number coordinates with students to build their vocabulary surrounding how numbers are classified. When presenting a coordinate pair to students, **ask** for each coordinate's most specific classification (natural number, whole number, integer, and/or rational number). It is appropriate to mention that all natural numbers, whole numbers, and integers are also rational numbers, but that we are generally referring to the most specific classification when we use these terms.

GUIDING QUESTIONS

Elicit student thinking:

- What do you look at first when you are graphing a coordinate pair?
- How does the structure of the coordinate plane help you graph coordinate pairs?
- Why do you think the horizontal axis is named the *x*-axis?
- Why do you think the vertical axis is named the *y*-axis?

Determine if the student can GRAPH INTEGER COORDINATE PAIRS:

- Where would [provide an integer coordinate pair] fall on the coordinate plane?
- Which direction should you move from the origin based on the *x*–coordinate of this coordinate pair?
- Which direction should you move from the origin based on the *y*–coordinate of this coordinate pair?

Determine if the student can RECOGNIZE RATIONAL NUMBER COORDINATE PAIRS:

- [Plot a point with at least one rational number coordinate that is not an integer] What could the coordinate pair be for this point? How do you know one of the coordinates is a non-integer, rational number?
- What is an example of a coordinate pair whose *x*–coordinate is a rational number (but not an integer)?
- What is an example of a coordinate pair whose *y*–coordinate is a rational number (but not an integer)?
- [Provide a rational number coordinate pair and an integer coordinate pair] How do the positions of these coordinate pairs compare to each other? Can you graph these points on the same coordinate plane?

Determine if the student can GRAPH RATIONAL NUMBER COORDINATE PAIRS:

- [Provide the student with a coordinate pair with at least one rational number that is not an integer] Where would this coordinate pair be located on the coordinate plane?
- [Provide the student with a coordinate pair where both coordinates are rational numbers that are not integers] Which two integers would the *x*–coordinate fall between? Which two integers would the *y*–coordinate fall between?

Determine if the student can RECOGNIZE *X-INTERCEPT*:

- [Draw a student's attention to a point on the *x*–axis] What is this point called?
- When there is a coordinate pair that falls on the the *x*–axis, what name do we give that coordinate pair?

Determine if the student can RECOGNIZE *Y-INTERCEPT*:

- [Draw a student's attention to a point on the *y*–axis] What is this point called?
- When there is a coordinate pair that falls on the the *y*–axis, what name do we give that coordinate pair?

Determine if the student can REPRESENT *X-INTERCEPT*:

- Can you plot a point on the coordinate plane that is an *x*–intercept?
- Can you write a coordinate pair for a point that is an *x*–intercept?

Determine if the student can REPRESENT *Y-INTERCEPT*:

- Can you plot a point on the coordinate plane that is a *y*–intercept?
- Can you write a coordinate pair for a point that is a *y*–intercept?

Determine if the student can EXPLAIN *QUADRANTS*:

- How are the quadrants of the coordinate plane separated?
- How are the quadrants labeled?
- What is a quadrant?
- Can you point to the first quadrant?

Determine if the student can RECOGNIZE THE QUADRANT OF A COORDINATE PAIR:

- [Provide the student with any coordinate pair] Can you describe how you would determine which quadrant this coordinate pair belongs in?
- [Provide the student with any coordinate pair] Which quadrant does this coordinate pair belong in?

Determine if the student can EXPLAIN THE RELATIONSHIP BETWEEN COORDINATE PAIRS AND QUADRANTS:

- How can you tell, without graphing, which quadrant a coordinate pair is in?
- If the *x*–coordinate is positive, which quadrant(s) could the coordinate pair be in?
- If the *x*–coordinate is negative, which quadrant(s) could the coordinate pair be in?
- If the *y*–coordinate is positive, which quadrant(s) could the coordinate pair be in?
- If the *y*–coordinate is negative, which quadrant(s) could the coordinate pair be in?
- If both coordinates are positive, which quadrant(s) could the coordinate pair be in?
- If both coordinates are negative, which quadrant(s) could the coordinate pair be in?
- If one coordinate is positive and the other coordinate is negative, which quadrant(s) could the coordinate pair be in?

Now that students are familiar with the four quadrants of the coordinate plane, move students toward looking for patterns among points that are reflections across one or both axes. Students should start with a blank coordinate plane on the INSTRUCTIONAL ACTIVITY SUPPLEMENT.

Provide students with a pair of coordinate pairs that are a reflection across the *y*–axis of each other. For example, (3, 2) and (–3, 2).

Before plotting the points on a coordinate plane, **ask** students which quadrant (3, 2) will fall in. They should respond with Quadrant I. Then, **ask** students which quadrant (–3, 2) will fall in. They should respond with Quadrant II.

Ask students what they notice about the position of each point compared to each other and the *y*–axis. **Scaffold** questions to lead students to notice that the points are reflections across the *y*–axis.

Ask students to compare the *x*– and *y*–coordinates of the coordinate pairs. They should notice that the coordinate pairs have opposite *x*–coordinates and exactly the same *y*–coordinates.

Provide students with a second pair of coordinate pairs that are a reflection across the *y*–axis of each other, this time coordinate pairs that fall in the third and fourth quadrants. For example, (–4, –7) and (4, –7).

Before plotting the points on a coordinate plane, **ask** students which quadrant (–4, –7) will fall in. They should respond with Quadrant III. Then, **ask** students which quadrant (4, –7) will fall in. They should respond with Quadrant IV.

Ask students what they notice about the position of each point compared to each other and the *y*–axis. Students should notice that the points are reflections across the *y*–axis.

Ask students to compare the *x*– and *y*–coordinates of the coordinate pairs. They should notice that the coordinate pairs have opposite *x*–coordinates and exactly the same *y*–coordinates.

Require students individually to write two coordinate pairs that are reflections of each other across the *y*–axis.

Once each student has two coordinate pairs they believe are reflections of each other across the *y*–axis, **ask** students to share with a partner and discuss whether or not the coordinate pairs are reflections of each other across the *y*–axis.

After students have shared in pairs, **have** a few students report to the class. As students share their pair of coordinate pairs, **plot** the points on the coordinate plane, and **ask** students if they agree or disagree with the coordinate pairs. Be sure to include points that fall in the first and second quadrants, as well as points that fall in the third and fourth quadrants.

GUIDING QUESTIONS

Elicit student thinking:

- [Point to any two points on the coordinate plane] What can you tell me about these points?
- [Point to a pair of points that are reflections across the *y*–axis] What can you tell me about these points? How are these points similar to any other pair of points? How are they different?

Determine if the student can GRAPH RATIONAL NUMBER COORDINATE PAIRS:

- [Provide the student with a coordinate pair with at least one rational number that is not an integer] Where would this coordinate pair be located on the coordinate plane?
- [Provide the student with a coordinate pair where both coordinates are rational numbers that are not integers] Which two integers would the *x*–coordinate fall between? Which two integers would the *y*–coordinate fall between?

Determine if the student can EXPLAIN *QUADRANTS*:

- How are the quadrants of the coordinate plane separated?
- How are the quadrants labeled?
- What is a quadrant?
- Can you point to the first quadrant?

Determine if the student can RECOGNIZE *REFLECTION*:

- [Point to a pair of points that are reflections of each other across the *y*–axis] What do you notice about these two points? How could we use one point to obtain the other?
- [Point to a pair of points that are reflections of each other across the *y*–axis] Where is the line of reflection for these two points?

Determine if the student can EXPLAIN *REFLECTION*:

- [Point to a pair of points that are reflections of each other across the *y*–axis] How do you know these two points are reflections of each other?
- What is a reflection?
- Can you provide two points that are reflections of each other across the *y*–axis? How do you know these points are reflections of each other?
- If a point lies on the *y*–axis and you reflect it across the *y*–axis, where will the image of the reflected point fall? How do you know?

Determine if the student can EXPLAIN THE RELATIONSHIP BETWEEN COORDINATES AND REFLECTION OVER AN AXIS:

- When given two coordinate pairs, how would you know whether they are reflections of each other across the *y*–axis without plotting them?
- [Provide the coordinates of a pair of points that are reflections over the *y*–axis] Are these points reflections of each other? How do you know?
- [Provide the coordinates of a pair of points that are not reflections over the *y*–axis] Are these points reflections of each other? How do you know?

Shift students' attention to points that are reflections across the *x*–axis. Students should start with a blank coordinate plane on the INSTRUCTIONAL ACTIVITY SUPPLEMENT.

Provide students with a pair of coordinate pairs that are a reflection across the *x*–axis of each other. For example, (5, 1) and (5, –1).

Before plotting the points on a coordinate plane, **ask** students which quadrant (5, 1) will fall in. They should respond with Quadrant I. Then, **ask** students which quadrant (5, –1) will fall in. They should respond with Quadrant IV.

Ask students what they notice about the position of each point compared to each other and the *x*–axis. **Scaffold** questions to lead students to notice that the points are reflections across the *x*–axis.

Ask students to compare the *x*– and *y*–coordinates of the coordinate pairs. They should notice that the coordinate pairs have exactly the same *x*–coordinates and opposite *y*–coordinates.

Provide students with a second pair of coordinate pairs that are a reflection across the *x*–axis of each other, this time with coordinate pairs that fall in the second and third quadrants. For example, (–6, 3) and (–6, –3).

Before plotting the points on a coordinate plane, **ask** students which quadrant (–6, 3) will fall in. They should respond with Quadrant II. Then, **ask** students which quadrant (–6, –3) will fall in. They should respond with Quadrant III.

Ask students what they notice about the position of each point compared to each other and the *x*–axis. Students should notice that the points are reflections across the *x*–axis.

Ask students to compare the *x*– and *y*–coordinates of the coordinate pairs. They should notice that the coordinate pairs have exactly the same *x*–coordinates and opposite *y*–coordinates.

Require students individually to write two coordinate pairs that are reflections of each other across the x–axis.

Once each student has two coordinate pairs they believe are reflections of each other across the x–axis, **ask** students to share with a partner and discuss whether or not the coordinate pairs are reflections of each other across the x–axis.

After students have shared in pairs, **have** a few students report to the class. As students share their pair of coordinate pairs, **plot** the points on the coordinate plane, and **ask** students if they agree or disagree with the coordinate pairs. Be sure to include points that fall in the first and fourth quadrants, as well as points that fall in the second and third quadrants.

GUIDING QUESTIONS

Elicit student thinking:

- [Point to any two points on the coordinate plane] What can you tell me about these points?
- [Point to a pair of points that are reflections across the x–axis] What can you tell me about these points? How are these points similar to any other pair of points? How are they different?

Determine if the student can GRAPH RATIONAL NUMBER COORDINATE PAIRS:

- [Provide the student with a coordinate pair with at least one rational number that is not an integer] Where would this coordinate pair be located on the coordinate plane?
- [Provide the student with a coordinate pair where both coordinates are rational numbers that are not integers] Which two integers would the x–coordinate fall between? Which two integers would the y–coordinate fall between?

Determine if the student can EXPLAIN *QUADRANTS*:

- How are the quadrants of the coordinate plane separated?
- How are the quadrants labeled?
- What is a quadrant?
- Can you point to the first quadrant?

Determine if the student can RECOGNIZE *REFLECTION*:

- [Point to a pair of points that are reflections of each other across the *x*–axis] What do you notice about these two points? How could we use one point to obtain the other?
- [Point to a pair of points that are reflections of each other across the *x*–axis] Where is the line of reflection for these two points?

Determine if the student can EXPLAIN *REFLECTION*:

- [Point to a pair of points that are reflections of each other across the *x*–axis] How do you know these two points are reflections of each other?
- What is a reflection?
- Can you provide two points that are reflections of each other across the *x*–axis? How do you know these points are reflections of each other?
- If a point lies on the *x*–axis and you reflect it across the *x*–axis, where will the image of the reflected point fall? How do you know?

Determine if the student can EXPLAIN THE RELATIONSHIP BETWEEN COORDINATES AND REFLECTION OVER AN AXIS:

- When given two coordinate pairs, how would you know whether they are reflections of each other across the *x*–axis without plotting them?
- [Provide the coordinates of a pair of points that are reflections over the *x*–axis] Are these points reflections of each other? How do you know?
- [Provide the coordinates of a pair of points that are not reflections over the *x*–axis] Are these points reflections of each other? How do you know?

Now that students have experienced reflections over the *x*–axis and *y*–axis separately, they should consider a composition of these two reflections. Students should start with a blank coordinate plane on the INSTRUCTIONAL ACTIVITY SUPPLEMENT.

Require students to start by plotting the point (8, 3) on the coordinate plane.

Ask students what the *x*– and *y*–coordinates would be after reflecting over the *y*–axis. Students should indicate the result of a reflection over the *y*–axis is the point (–8, 3). Plot this point on the coordinate plane.

Require students now to start with the point (–8, 3). **Ask** students what the *x*– and *y*–coordinates would be after reflecting the point (–8, 3) over the *x*–axis. Students should indicate the result of a reflection over the *x*–axis is the point (–8, –3). Plot this point on the coordinate plane.

Compare the coordinate pair (8, 3) to the coordinate pair (–8, –3). Remind students they have reflected over the *x*–axis *and* the *y*–axis. Students should notice that the coordinate pairs have opposite *x*–coordinates and opposite *y*–coordinates.

Require students to plot another point, (9, –10), on a different coordinate plane.

Ask students what the *x*– and *y*–coordinates would be after reflecting over the *x*–axis. Students should indicate the result of a reflection over the *x*–axis is the point (9, 10). Plot this point on the coordinate plane.

Require students now to start with the point (9, 10). **Ask** students what the *x*– and *y*–coordinates would be after reflecting the point (9, 10) over the *y*–axis. Students should indicate the result of a reflection over the *y*–axis is the point (–9, 10). Plot this point on the coordinate plane.

Compare the coordinate pair (9, –10) to the coordinate pair (–9, 10). Remind students they have reflected over the *x*–axis *and* the *y*–axis. Students should notice that the coordinate pairs have opposite *x*–coordinates and opposite *y*–coordinates.

GUIDING QUESTIONS

Elicit student thinking:

- What do you know about reflections on the coordinate plane?
- [Provide students with any coordinate pair] How could you reflect this point? Where would it end up if you did that?

Determine if the student can RECOGNIZE *REFLECTION*:

- [Point to a pair of points that are reflections of each other across both axes] What do you notice about these two points? How could we use one point to obtain the other?
- [Point to a pair of points that are reflections of each other across both axes] How many lines of reflection would be used to obtain one point from the other? What are the lines of reflection?

Determine if the student can EXPLAIN *REFLECTION*:

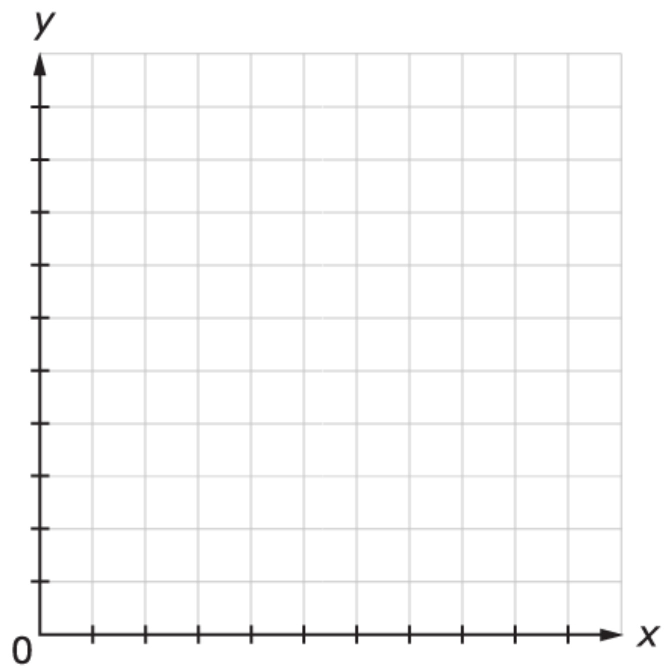
- [Point to a pair of points that are reflections of each other across both axes] How are these points reflections of each other?
- What is a reflection?
- Can you provide two points that are reflections of each other across both axes? How do you know these points are reflections of each other?

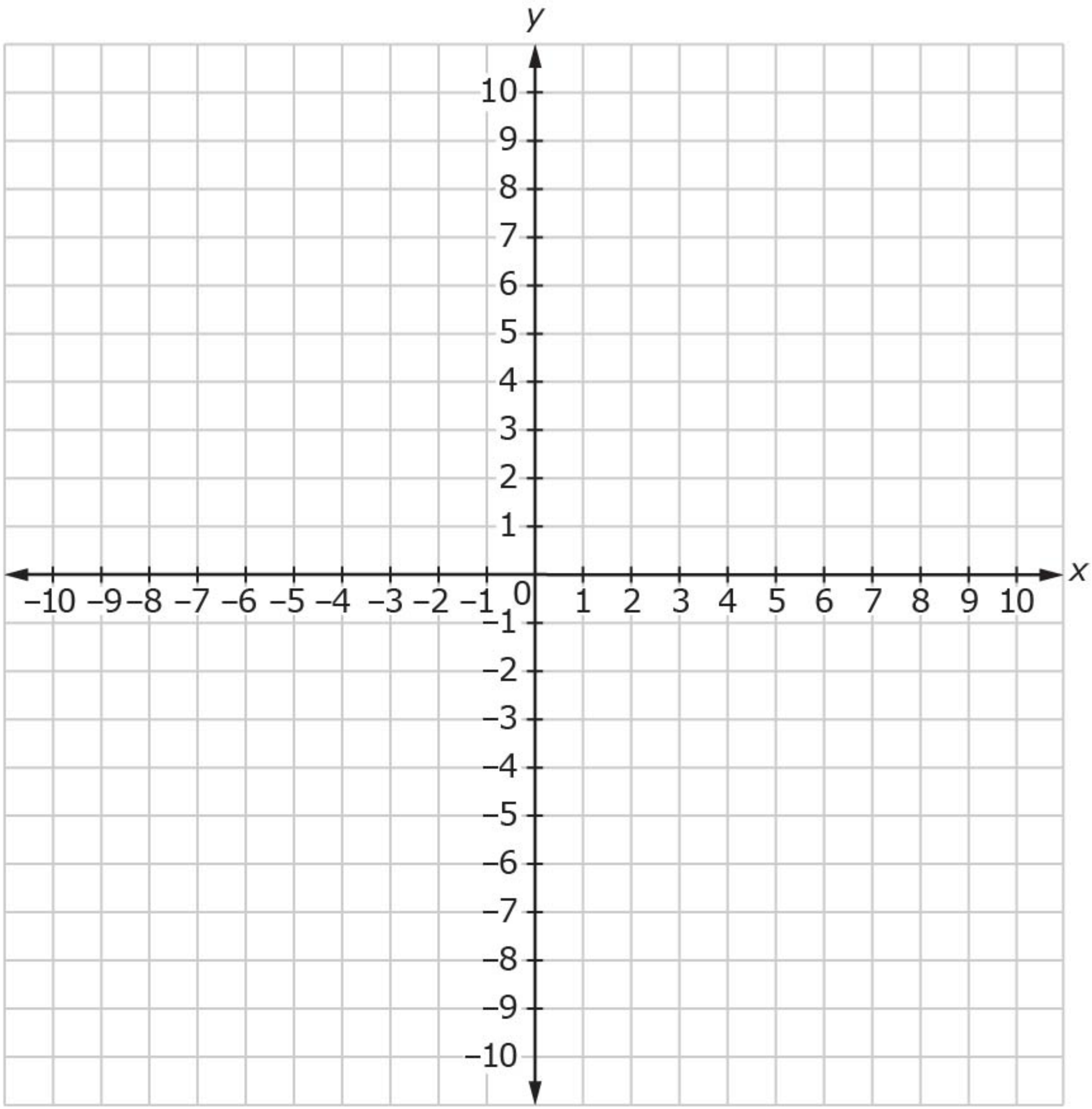
Determine if the student can EXPLAIN THE RELATIONSHIP BETWEEN COORDINATES AND REFLECTION OVER AN AXIS:

- When given two coordinate pairs, how would you know whether they are reflections of each other across both axes without plotting them?
- [Provide the coordinates of a pair of points that are reflections over both axes] Is one of these points the result of reflecting the other point over both axes? How do you know?
- [Provide the coordinates of a pair of points that are not reflections over the x–axis] Is one of these points the result of reflecting the other point over both axes? How do you know?

At the end of the activity, provide students with a coordinate pair and ask them to provide the following:

- the coordinates and quadrant after the original coordinate pair is reflected across the x–axis
- the coordinates and quadrant after the original coordinate pair is reflected across the y–axis
- the coordinates and quadrant after the original coordinate pair is reflected across the x–axis and the y–axis

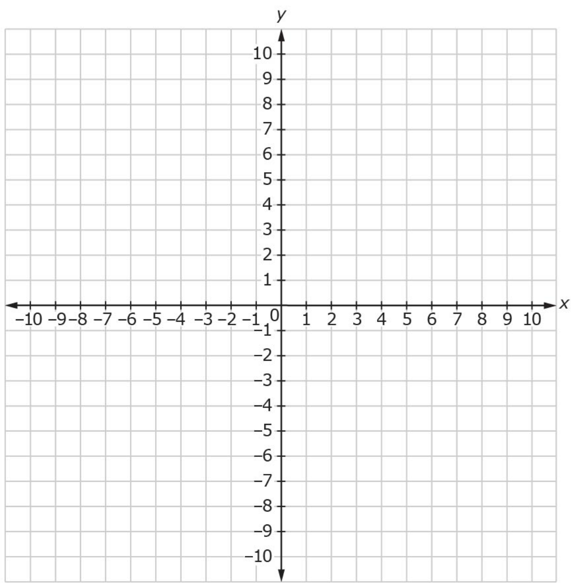
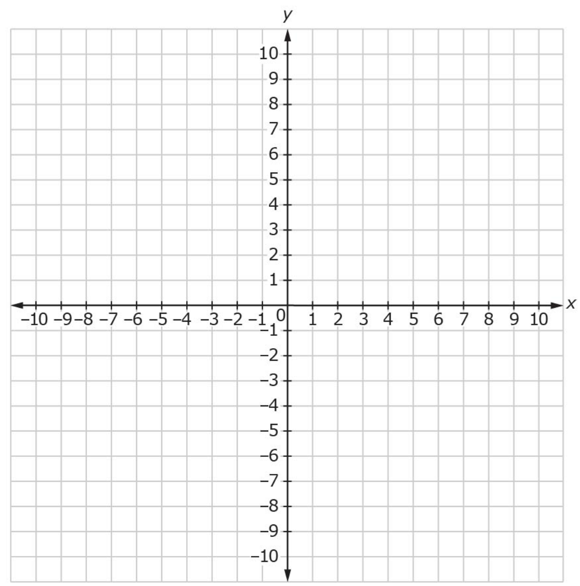
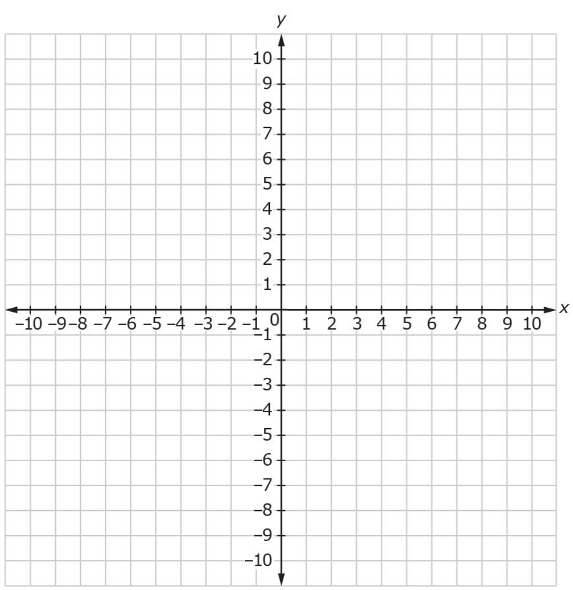
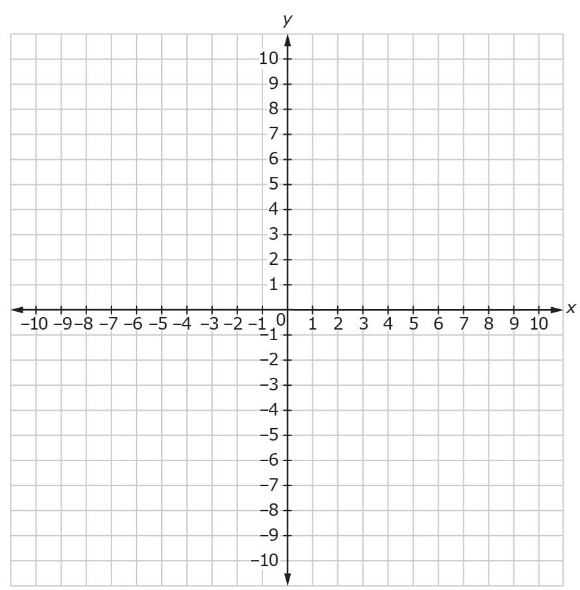

y
0
x



y
x
10
9
8
7
6
5
4
3
2
1
0
−1
−2
−3
−4
−5
−6
−7
−8
−9
−10
−10 −9 −8 −7 −6 −5 −4 −3 −2 −1
1 2 3 4 5 6 7 8 9 10

COORDINATE PAIRS: SORTING ACTIVITY

A(−3, 6)	B(1, 3.5)	C(5, −1.4)
D(−8.5, −3.1)	E$\left(6\frac{1}{4}, 8\frac{3}{4}\right)$	F$\left(-\frac{1}{2}, \frac{1}{2}\right)$
G$\left(-\frac{11}{2}, -6\right)$	H(2.3, 7)	I(−9, 3.1)
J$\left(\frac{1}{2}, -3\right)$	K$\left(-2\frac{1}{3}, -1\right)$	L(−1.9, 4)
M$\left(\frac{7}{4}, \frac{1}{3}\right)$	N(2.8, −7.1)	O(−1, −8)
P$\left(-6, -\frac{9}{2}\right)$	Q(4, 5)	R $\left(\frac{7}{3}, 9\right)$
S$\left(-7, 1\frac{2}{3}\right)$	T$\left(3\frac{3}{4}, -5\frac{1}{2}\right)$	U$\left(9, \frac{9}{2}\right)$
V(−4, −9.9)	W(7, −8.9)	X$\left(-4\frac{1}{3}, 8\right)$
Y(9, −2)	Z$\left(\frac{4}{3}, -6\right)$	

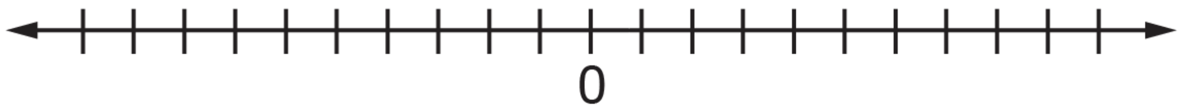





Name____

INTEGERS, RATIONAL NUMBERS, AND SYMMETRY

Lessons 1 – 3

1. Use the following number line to respond to 1.a. through 1.e.



1.a. Label the number line using integers.

1.b. Describe how a number line that displays integers is symmetric with respect to zero. Then name two specific integers and explain why their positions on the number line are symmetric with respect to zero.

Name_______________________________________

1.c. Provide an example of a pair of opposite values, then plot these values on the number line. How do you know these values are opposites?

1.d. Identify the symbol used to indicate a number is less than zero. Describe two additional meanings or different ways the symbol could be used in expressions with numbers.

1.e. Identify the symbol used to indicate values that are greater than zero. Describe an additional meaning or purpose this symbol can have in expressions with numbers.

Name____

2. Describe, in your own words, numbers that are integers. Include examples and non-examples of integers in addition to a general description.

3. Consider the number zero as you respond to the following questions.

   3.a. Is zero positive? Why or why not?

   3.b. Is zero negative? Why or why not?

3.c. What is the opposite of zero? Explain your reasoning.

4. For each symbolic expression, provide a phrase for how the expression should be read. Use an appropriate interpretation of the "–" symbol.

4.a. $8 - 7$

4.b. -5

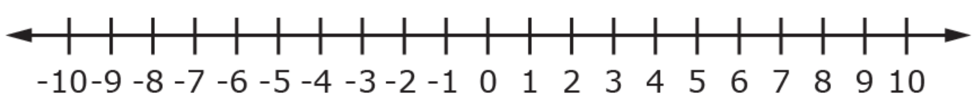
4.c. $-(-(-10))$

Name____

5. What is the opposite of the opposite of −9? Describe how you determined your answer.

6. Locate and label the values provided on the number line, then list the values from least to greatest.

$$7.3, \quad -1\frac{2}{5}, \quad 4, \quad \frac{8}{3}, \quad -5.5$$



Name____

7. A bank teller provides a detailed account history for a customer. Use the information provided to answer the following questions.

TRANSACTION	CREDITS	DEBITS
Sally's Shoe Store		–$34.58
ATM Deposit	+$75.00	
Hank's Hamburgers		–$13.99
Carmen's Coffee Shop		–$2.25
Interest	+$0.23	

7.a. What is the effect of a positive value on the customer's account balance?

7.b. What is the effect of a negative value on the customer's account balance?

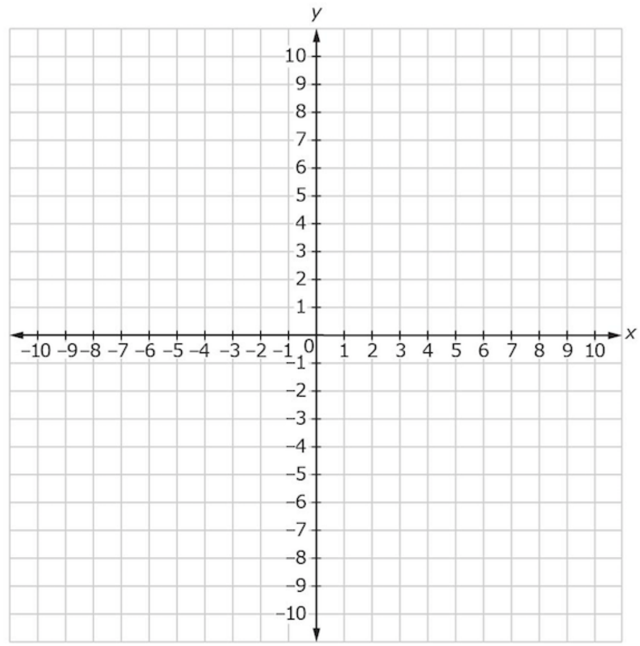
7.c. What does zero represent in this scenario?

8. Describe how the coordinate plane is symmetric. Include references to the x–axis and the y–axis in your description.

Name____

9. Locate and label the coordinate pairs provided on the coordinate plane.

U(0, −2)
V(3, −6)
W(−4, 0)
X(−1, −9)
Y(10, 4)
Z(−7, 8)



10. Use the following points to respond to 10.a. through 10.c.

A(10, −3) B$\left(\frac{19}{2}, -\frac{10}{3}\right)$

10.a. What is the same about points A and B?

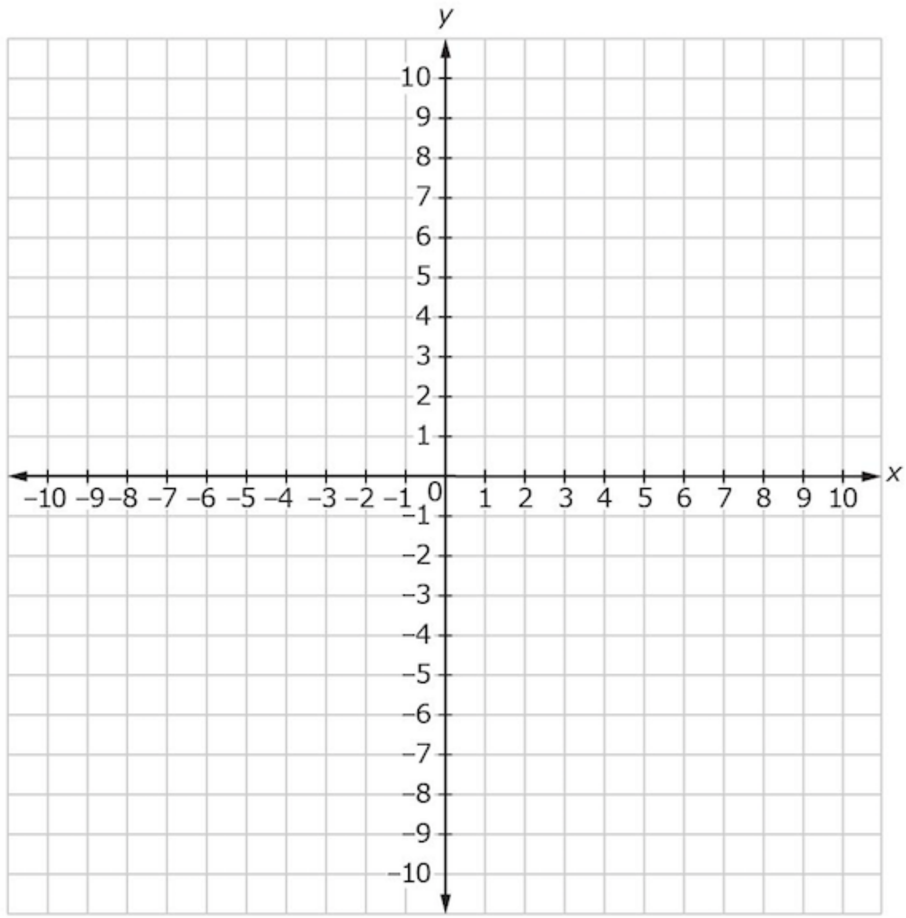
10.b. Describe the position of B$\left(\frac{19}{2}, -\frac{10}{3}\right)$ on the coordinate plane relative to integer values.

Name____

10.c. Describe the position of B$\left(\frac{19}{2}, -\frac{10}{3}\right)$ compared to A(10, −3) on the coordinate plane.

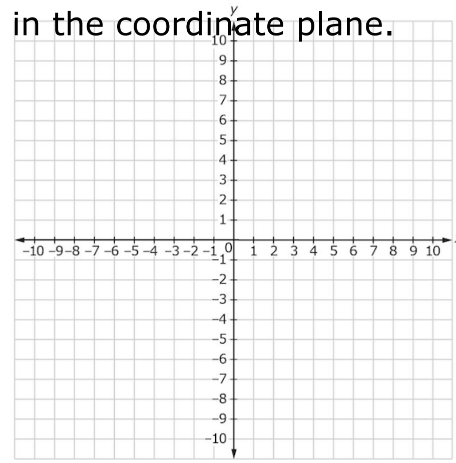
11. Locate and label the coordinate pairs provided on the coordinate plane.

A(3, 6.5)
B$\left(-4\frac{1}{3}, -5\right)$
C(6.8, −1)
D$\left(-1\frac{2}{3}, 8\frac{1}{2}\right)$
E$\left(\frac{19}{2}, -\frac{10}{3}\right)$
F(0, −9.1)



Name____

12. Label Quadrant I, Quadrant II, Quadrant III, and Quadrant IV on the following coordinate plane. Then describe, in your own words, what a quadrant is and how quadrants are separated in the coordinate plane.



13. For each coordinate pair, identify the quadrant the coordinate pair belongs in without plotting the coordinate pair on a coordinate plane. Explain how you determined your response.

13.a. (8, −10)

13.b. (−0.5, −7.1)

13.c. $\left(\frac{9}{4}, \frac{3}{2}\right)$

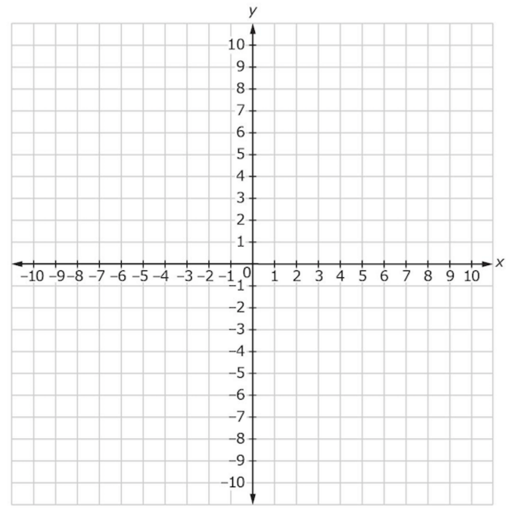
13.d. $\left(-5\frac{1}{3}, 3\frac{1}{2}\right)$

13.e. (6.3, 0)

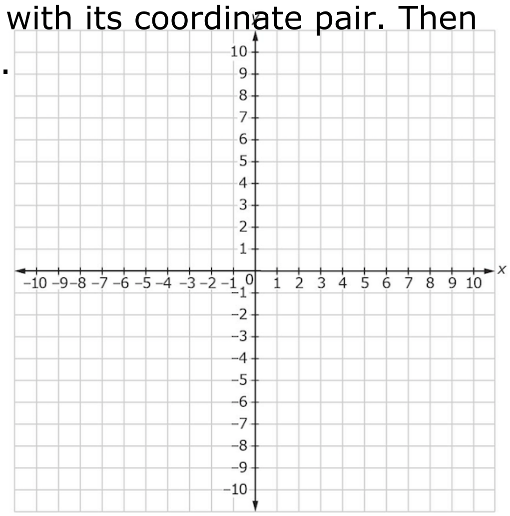
14. Describe how the signs of the *x*– and *y*–coordinates indicate the quadrant the coordinate pair belongs in. Be sure to mention the signs of the coordinates in each quadrant of the coordinate plane.

Name____

15. Use the following coordinate plane to plot a point that would be considered an x–intercept. Label the point with its coordinate pair. Then describe an x–intercept in your own words.



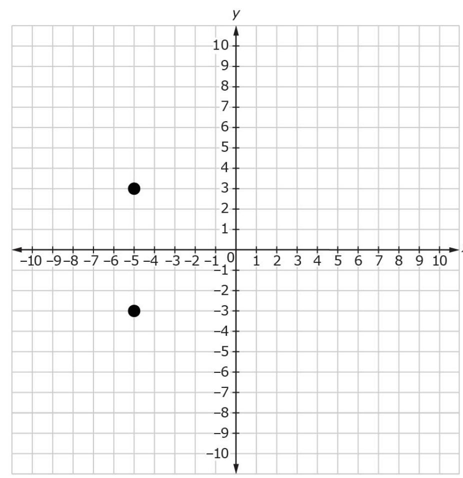
16. Use the following coordinate plane to plot a point that would be considered a y–intercept. Label the point with its coordinate pair. Then describe a y–intercept in your own words.



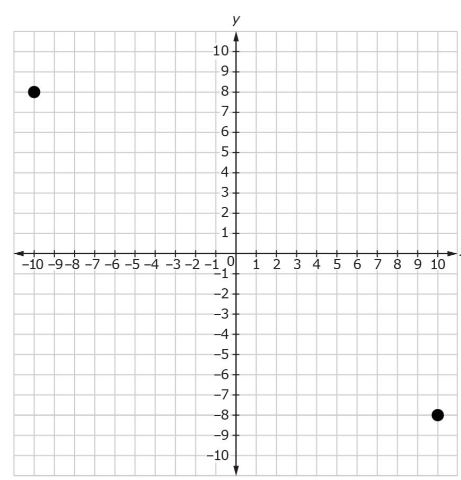
Name____

17. Consider how the points are related in each coordinate plane as you answer the following questions.

17.a. Describe the reflection that would map one of the points provided in the coordinate plane onto the other point.

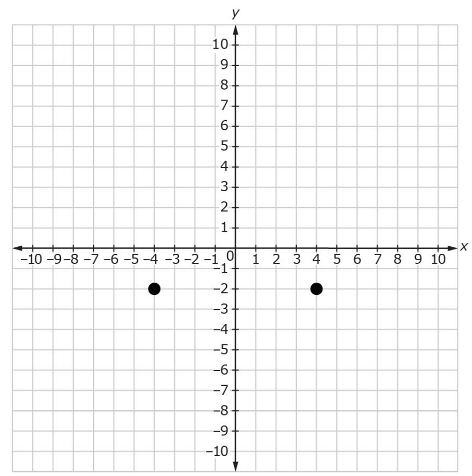


17.b. Describe the reflection that would map one of the points provided in the coordinate plane onto the other point.



Name____

17.c. Describe the reflection that would map one of the points provided in the coordinate plane onto the other point.



18. Explain the relationship between the coordinates of a pair of coordinate pairs that are reflections of each other across the *x*–axis.

19. Explain the relationship between the coordinates of a pair of coordinate pairs that are reflections of each other across both axes.

Name____

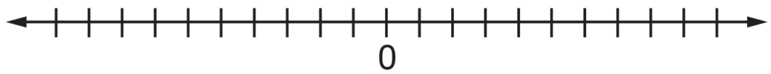
20. Describe how the coordinate pairs (2, –3) and (–2, –3) are reflections of each other on the coordinate plane. Include in your description how you know the coordinate pairs are reflections of each other based on the coordinates, as well as how you know they are reflections of each other based on their locations on a coordinate plane.

INTEGERS, RATIONAL NUMBERS, AND SYMMETRY

STUDENT ACTIVITY SOLUTION GUIDE

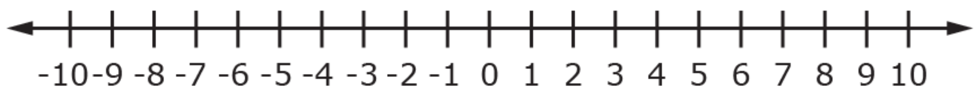
Lessons 1 – 3

1. Use the following number line to respond to 1.a. through 1.e.



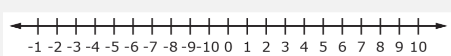
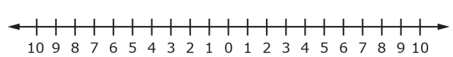
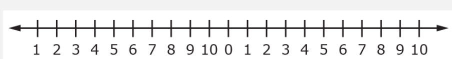
1.a. Label the number line using integers.

CORRECT ANSWER



Note: It is also correct for students to label the number line such that each interval represents more than one unit, but the interval must be regular.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
	knows that the portion of the number line to the left of zero contains negative values, but does not see the symmetry of the values of the number line	EXPLAIN *INTEGERS*
	understands the symmetry of the number line but does not use the negative symbol to indicate values that are less than zero	RECOGNIZE THE NEGATIVE SIGN
	does not understand the symmetry of the number line and does not use the negative symbol to indicate values that are less than zero	EXPLAIN *INTEGERS* and RECOGNIZE THE NEGATIVE SIGN
Student does not use the space between tick marks to represent the same interval.	does not understand that equally spaced tick marks must correspond to equal distance between numbers	EXPLAIN *NUMBER LINE*

1.b. Describe how a number line that displays integers is symmetric with respect to zero. Then name two specific integers and explain why their positions on the number line are symmetric with respect to zero.

CORRECT ANSWER

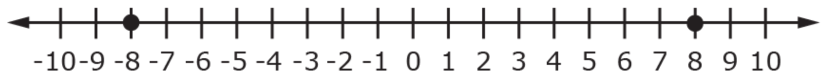
> The number line is symmetric because each number is a reflection across zero of its opposite value on the number line. Opposites have the same magnitude (or are the same distance from zero) but are positioned in different (opposite) directions on the number line, therefore they are symmetric.
>
> For example, four and negative four are symmetric with respect to zero on the number line. Four is four units greater than zero and negative four is four units less than zero. Four and negative four are the same distance from zero but in opposite directions.
>
> *Note: A student may note that values such as two and six are symmetric with respect to four. While this is accurate, it is also important for the student to notice the general symmetry of the integers on the number line with respect to zero.*

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The number line is symmetric because the numbers to the right of zero are the same as the numbers to the left.	does not specify that opposite values are symmetric across zero or that the values to the left of zero are negative and the values to the right of zero are positive	RECOGNIZE THE NEGATIVE SIGN and EXPLAIN OPPOSITE NUMBERS ON A NUMBER LINE
Student does not refer to zero when describing symmetry on the number line.	does not understand that zero plays a critical role in the symmetry of the number line	EXPLAIN SYMMETRY and EXPLAIN OPPOSITE NUMBERS ON A NUMBER LINE

1.c. Provide an example of a pair of opposite values, then plot these values on the number line. How do you know these values are opposites?

CORRECT ANSWER



An example of a pair of opposite values is –8 and 8. I know these values are opposites because they are the same distance from zero, but in opposite directions. In other words, they have the same magnitude, but different directions.

Note: Students can choose any pair of opposite values to describe and plot on the number line.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Student cannot provide a pair of opposites.	does not know what opposite numbers are	RECOGNIZE OPPOSITE NUMBERS
Student provides a pair of opposite values but cannot place them on the number line.	cannot recognize opposite numbers on a number line	RECOGNIZE OPPOSITE NUMBERS ON A NUMBER LINE
Student places opposite values on the number line, but cannot explain how they know they are opposites.	does not know what makes opposite values opposites, only knows how to produce a pair of opposites	EXPLAIN OPPOSITE NUMBERS ON A NUMBER LINE

1.d. Identify the symbol used to indicate a number is less than zero. Describe two additional meanings or different ways the symbol could be used in expressions with numbers.

CORRECT ANSWER

The "–" or negative symbol can be used to indicate values that are less than zero. This symbol can also be used to indicate an opposite or subtraction (which could also be referred to as "take away" or "minus") depending on the context in which it is used.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Student provides an informal symbol (e.g., a box or a circle around a number to indicate it is negative) instead of the "–" symbol.	does not know the "–" is used in front of values that are negative or less than zero	RECOGNIZE THE NEGATIVE SIGN
Student does not recognize "subtraction" as an additional meaning of the "–" symbol.	does not understand that the "–" symbol has several different meanings, depending on the context in which it is used, including subtraction	RECOGNIZE THE NEGATIVE SIGN
Student does not recognize "opposite" as an additional meaning of the "–" symbol.	does not understand that the "–" symbol has several different meanings, depending on the context in which it is used, including to indicate an opposite	RECOGNIZE THE NEGATIVE SIGN

1.e. Identify the symbol used to indicate values that are greater than zero. Describe an additional meaning or purpose this symbol can have in expressions with numbers.

CORRECT ANSWER

> The "+" or positive symbol can be used to indicate values that are greater than zero. In addition, this symbol can be used to indicate "addition", depending on the context in which it is used.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Student provides an informal symbol instead of the "+" symbol.	does not know the "+" can be used in front of values that are positive or greater than zero	RECOGNIZE THE POSITIVE SIGN
Student only identifies the "+" symbol as indicating positive values.	does not understand that the "+" symbol has several different meanings, depending on the context in which it is used	RECOGNIZE THE POSITIVE SIGN

2. Describe, in your own words, numbers that are integers. Include examples and non-examples of integers in addition to a general description.

CORRECT ANSWER

> The set of integers includes whole numbers and their opposites. For example, −3, 8, −5, 0, and 2 are all integers. Examples of non-integers would be decimals such as 6.3 and −1.4 and fractions such as $-\frac{1}{2}$ and $7\frac{2}{3}$.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Student describes integers as positive and negative numbers.	does not exclude fractions and decimals that cannot be written as whole numbers or their opposites in their description	EXPLAIN *INTEGERS*
Student includes decimals and fractions that cannot be written as integers in their examples of integers.	does not know that fractions and decimals that cannot be written as whole numbers or their opposites are not integers; may confuse the terms *integers* and *rational numbers*	EXPLAIN *INTEGERS*
Student does not provide examples of non-integers.	does not consider or think of fractions and decimals that cannot be written as integers to be examples of numbers that are not integers	EXPLAIN *INTEGERS*

3. Consider the number zero as you respond to the following questions.

3.a. Is zero positive? Why or why not?

CORRECT ANSWER

> Zero is not positive because positive values must be greater than zero by definition, and zero is equal to but not greater than zero.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Zero is positive because it does not have a negative sign.	does not understand the value of zero and relies on the presence of symbols instead	EXPLAIN CHARACTERISTICS OF ZERO
Zero is not positive because it does not have a positive sign.	does not understand the value of zero and relies on the presence of symbols instead	EXPLAIN CHARACTERISTICS OF ZERO

3.b. Is zero negative? Why or why not?

CORRECT ANSWER

> Zero is not negative because negative values must be less than zero by definition, and zero is equal to but not less than zero.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Zero is negative because it does not have a positive sign.	does not understand the value of zero and relies on the presence of symbols instead	EXPLAIN CHARACTERISTICS OF ZERO
Zero is not negative because it does not have a negative sign.	does not understand the value of zero and relies on the presence of symbols instead	EXPLAIN CHARACTERISTICS OF ZERO

3.c. What is the opposite of zero? Explain your reasoning.

CORRECT ANSWER

> The opposite of zero is zero because opposites should have the same magnitude (or distance from zero) in different directions from zero. Since zero is zero units from zero, its opposite would also be zero units from zero.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Zero does not have an opposite.	believes only positive and negative numbers can have opposites	EXPLAIN CHARACTERISTICS OF ZERO
Student does not explain why the opposite of zero is zero.	knows zero is its own opposite but cannot relate direction and magnitude to why this is the case	EXPLAIN OPPOSITE NUMBERS ON A NUMBER LINE

4. For each symbolic expression, provide a phrase for how the expression should be read. Use an appropriate interpretation of the "–" symbol.

4.a. 8 – 7

CORRECT ANSWER

This expression can be read "eight minus seven".

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
eight negative seven	focuses on "–" as negative instead of the appropriate interpretation of "minus" in this context	RECOGNIZE THE NEGATIVE SIGN
eight opposite seven	focuses on "–" as opposite instead of the appropriate interpretation of "minus" in this context	RECOGNIZE THE NEGATIVE SIGN

4.b. – 5

CORRECT ANSWER

This expression can be read either as "negative five" or "the opposite of five".

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
subtract five	focuses on "–" as subtraction, the earliest meaning students learned for this symbol	RECOGNIZE THE NEGATIVE SIGN

4.c. – (–(–10))

CORRECT ANSWER

This expression can be read either as "the opposite of the opposite of negative ten" or "the opposite of the opposite of the opposite of ten".

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
negative negative negative ten	focuses on "–" as negative instead of an interpretation that provides more meaning in this context	RECOGNIZE THE NEGATIVE SIGN
minus minus minus ten	focuses on "–" as minus instead of an interpretation that provides more meaning in this context	RECOGNIZE THE NEGATIVE SIGN

5. What is the opposite of the opposite of –9? Describe how you determined your answer.

CORRECT ANSWER

The opposite of the opposite of negative nine is itself, negative nine. The opposite of the opposite of a number is always the original number.

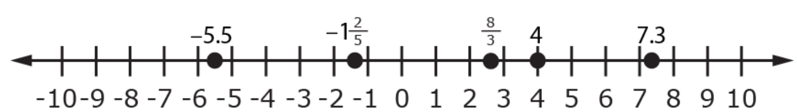
ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The opposite of the opposite of –9 is 9.	finds the opposite of the number	RECOGNIZE THE OPPOSITE OF THE OPPOSITE OF A NUMBER
Student cannot explain why the opposite of the opposite is –9.	determines through a process the opposite of the opposite of –9 is –9, but does not generalize that the opposite of the opposite of a number is always the number itself	EXPLAIN THE OPPOSITE OF THE OPPOSITE OF A NUMBER

6. Locate and label the values provided on the number line, then list the values from least to greatest.

$$7.3, \quad -1\frac{2}{5}, \quad 4, \quad \frac{8}{3}, \quad -5.5$$

CORRECT ANSWER



These values, in order from least to greatest, are –5.5, $-1\frac{2}{5}$, $\frac{8}{3}$, 4, 7.3.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Student locates –5.5 between –5 and –4.	locates –5, then moves half an interval to the right to account for the five tenths as they would with a positive decimal	LOCATE RATIONAL NUMBERS ON A NUMBER LINE
Student locates $-1\frac{2}{5}$ between –1 and 0.	locates –1, then moves two fifths of an interval to the right to account for the two fifths as they would with a positive fraction	LOCATE RATIONAL NUMBERS ON A NUMBER LINE
Student locates $\frac{8}{3}$ at 3, at 8, or at both 3 and 8 on the number line.	does not understand the value of $\frac{8}{3}$ or how to represent fractional amounts on the number line	LOCATE RATIONAL NUMBERS ON A NUMBER LINE
Student locates $\frac{8}{3}$ at $\frac{10}{3}$ on the number line.	knows $\frac{8}{3}$ is $\frac{1}{3}$ away from three, but moves to a number $\frac{1}{3}$ greater than three instead of $\frac{1}{3}$ less than three on the number line	LOCATE RATIONAL NUMBERS ON A NUMBER LINE
Student plots the values on the number line correctly but does not list them in order from least to greatest.	does not understand that lesser values are positioned to the left of greater values on the number line	EXPLAIN *NUMBER LINE*

7. A bank teller provides a detailed account history for a customer. Use the information provided to answer the following questions.

TRANSACTION	CREDITS	DEBITS
Sally's Shoe Store		–\$34.58
ATM Deposit	+\$75.00	
Hank's Hamburgers		–\$13.99
Carmen's Coffee Shop		–\$2.25
Interest	+\$0.23	

7.a. What is the effect of a positive value on the customer's account balance?

CORRECT ANSWER

> Positive values represent amounts that are added to or credited to the bank account. These values cause the balance of the bank account to increase.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Positive values are added.	does not clarify the relationship between the positive values and the context of the bank account	USE POSITIVE AND NEGATIVE NUMBERS IN REAL-WORLD CONTEXTS
Note: A student who understands the concept may give this response, which is not completely wrong, just incompletely expressed. In this instance, it is recommended that the teacher follows up with the student to allow them to more thoroughly explain their response.		

7.b. What is the effect of a negative value on the customer's account balance?

CORRECT ANSWER

> Negative values represent amounts that are removed or taken away from the bank account. These values cause the balance of the bank account to decrease.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Negative values are subtracted.	does not clarify the relationship between the negative values and the context of the bank account	USE POSITIVE AND NEGATIVE NUMBERS IN REAL-WORLD CONTEXTS
Note: A student who understands the concept may give this response, which is not completely wrong, just incompletely expressed. In this instance, it is recommended that the teacher follows up with the student to allow them to more thoroughly explain their response.		

7.c. What does zero represent in this scenario?

CORRECT ANSWER

> Zero represents neither an addition nor removal of money from the account. Zero would result in no change in the balance of the account.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Zero represents a balance of zero dollars.	thinks of zero in terms of the balance of the account instead of the context, which is additions and subtractions of credits and debits	RELATE THE MEANING OF 0 TO POSITIVE AND NEGATIVE NUMBERS IN REAL-WORLD CONTEXTS

8. Describe how the coordinate plane is symmetric. Include references to the x–axis and the y–axis in your description.

CORRECT ANSWER

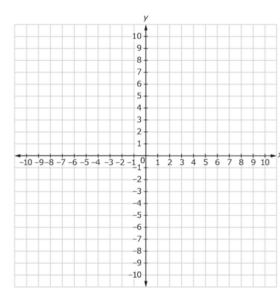
> The coordinate plane is symmetric because both the *x*–axis and *y*–axis are symmetric with respect to zero. On the horizontal *x*–axis, positive values are to the right of zero and negative values are to the left of zero. Opposite *x*–values such as –5 and 5 are the same distance from zero in opposite directions on the *x*–axis. On the vertical *y*–axis, positive values are above zero and negative values are below zero. Opposite *y*–values such as –9 and 9 are the same distance from zero in opposite directions on the *y*–axis.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

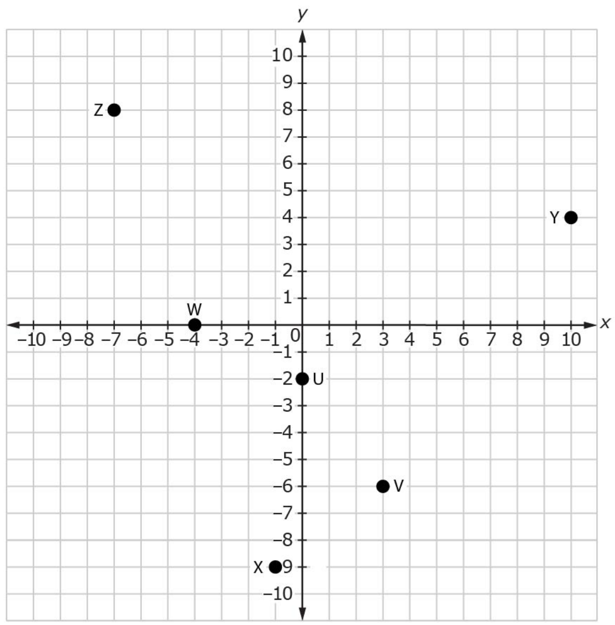
Example Error	Misconception	Missing Knowledge
The student does not explain or know the x–axis and y–axis contain opposite values that are symmetric.	does not see the symmetry of opposite values on the x–axis and/or the y–axis	EXPLAIN *SYMMETRY* and EXPLAIN OPPOSITE NUMBERS ON A NUMBER LINE

9. Locate and label the coordinate pairs provided on the coordinate plane.

U(0, –2) V(3, –6)
W(–4, 0) X(–1, –9)
Y(10, 4) Z(–7, 8)



CORRECT ANSWER



ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student plots the coordinate pairs (−2, 0), (−6, 3), (0, −4), (−9, −1), (4, 10), and (8, −7) instead of the coordinate pairs provided.	switches the positions of the *x*–coordinates and the *y*–coordinates	EXPLAIN *COORDINATE PAIRS*
The student plots the coordinate pairs (0, 2), (3, 6), (4, 0), (1, 9), (10, 4), and (7, 8) instead of the coordinate pairs provided.	plots points as if the negative values are positive	RECOGNIZE THE NEGATIVE SIGN

10. Use the following points to respond to 10.a. through 10.c.

A(10, –3) B$\left(\frac{19}{2}, -\frac{10}{3}\right)$

10.a. What is the same about points A and B?

CORRECT ANSWER

Points A and B are both ordered pairs that can be graphed on the same coordinate plane. Each has an *x*–coordinate and a *y*–coordinate. Based on the signs of the coordinates, both coordinate pairs would be plotted in the fourth quadrant.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Both points can be graphed but you need different coordinate planes because the numbers are different.	does not think rational number coordinate pairs can be graphed on the same coordinate plane as integer coordinate pairs	GRAPH RATIONAL NUMBER COORDINATE PAIRS
Nothing is the same because one point has fractions and the other one does not.	cannot see that both points are ordered pairs that have the same meaning to a graph	RECOGNIZE RATIONAL NUMBER COORDINATE PAIRS

10.b. Describe the position of B$\left(\frac{19}{2}, -\frac{10}{3}\right)$ on the coordinate plane relative to integer values.

CORRECT ANSWER

The *x*–coordinate, $\frac{19}{2}$, is greater than 9 and less than 10, so horizontally this point will fall between 9 and 10 relative to the *x*–axis.

The *y*–coordinate, $-\frac{10}{3}$, is greater than (or less negative than) –4 and less than (or more negative than) –3, so vertically this point will fall between –3 and –4 relative to the *y*–axis.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The *x*–coordinate would be at 19 and the *y*–coordinate would be at 10.	looks at the numerator only in order to determine the position of the point on the coordinate plane	GRAPH RATIONAL NUMBER COORDINATE PAIRS
This point would be graphed in the fourth quadrant.	does not refer to the specific position of the point relative to integer values, relies on knowledge of signs and coordinates to identify the quadrant	GRAPH RATIONAL NUMBER COORDINATE PAIRS

10.c. Describe the position of $B\left(\frac{19}{2}, -\frac{10}{3}\right)$ compared to A(10, –3) on the coordinate plane.

CORRECT ANSWER

Because the *x*–coordinate of point B is less than the *x*–coordinate of point A, point B will be positioned to the left of point A.

Because the *y*–coordinate of point B is less than the *y*–coordinate of point A, point B will be positioned below point A.

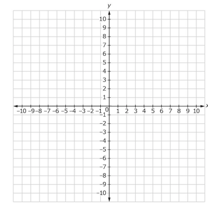
Therefore, point B will be a little to the left and below point A when they are positioned on the coordinate plane.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

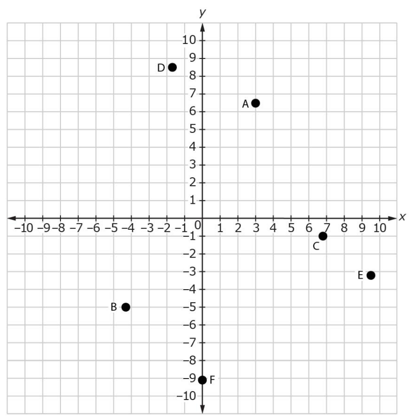
Example Error	Misconception	Missing Knowledge
Point B would be to the right and below point A because 19 is more than 10 and –10 is less than –3.	looks at the numerator only in order to determine the position of point B on the coordinate plane relative to point A	GRAPH RATIONAL NUMBER COORDINATE PAIRS
Point B would be to the left and above point A because $\frac{19}{2}$ is less than 10 and $-\frac{10}{3}$ is greater than –3.	compares $\frac{10}{3}$ and 3 rather than $-\frac{10}{3}$ and –3	RECOGNIZE THE NEGATIVE SIGN

11. Locate and label the coordinate pairs provided on the coordinate plane.

A(3, 6.5) $B\left(-4\frac{1}{3}, -5\right)$ C(6.8, –1) $D\left(-1\frac{2}{3}, 8\frac{1}{2}\right)$
$E\left(\frac{19}{2}, -\frac{10}{3}\right)$ F(0, –9.1)



CORRECT ANSWER

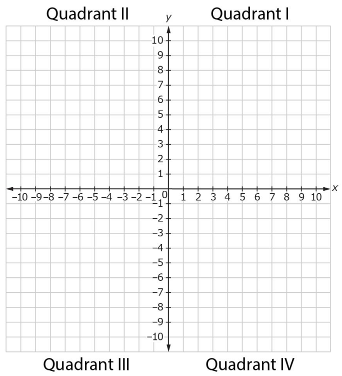


ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student plots the coordinate pairs A(6.5, 3), $B\left(-5, -4\frac{1}{3}\right)$, C(–1, 6.8), $D\left(8\frac{1}{2}, -1\frac{2}{3}\right)$, $E\left(-\frac{10}{3}, \frac{19}{2}\right)$, and F(–9.1, 0) instead of the coordinate pairs provided.	switches the positions of the x–coordinates and the y–coordinates	EXPLAIN *COORDINATE PAIRS*
The student uses only the first quadrant and plots the coordinate pairs A(3, 6.5), $B\left(4\frac{1}{3}, 5\right)$, C(6.8, 1), $D\left(1\frac{2}{3}, 8\frac{1}{2}\right)$, $E\left(\frac{19}{2}, \frac{10}{3}\right)$, and F(0, 9.1).	does not acknowledge the negative sign, plotting all coordinate pairs as if both coordinates are positive	RECOGNIZE THE NEGATIVE SIGN
Student positions point B between –4 and –3 on the x–axis.	locates –4 on the x–axis, then moves one third of an interval to the right to account for the one third as they would with a positive fraction	GRAPH RATIONAL NUMBER COORDINATE PAIRS
Student positions point D between –1 and 0 on the x–axis.	locates –1 on the x–axis, then moves two thirds of an interval to the right to account for the two thirds as they would with a positive fraction	GRAPH RATIONAL NUMBER COORDINATE PAIRS

12. Label Quadrant I, Quadrant II, Quadrant III, and Quadrant IV on the following coordinate plane. Then describe, in your own words, what a quadrant is and how quadrants are separated in the coordinate plane.

CORRECT ANSWER



A quadrant is a portion, a fourth, of the coordinate plane. The four quadrants are separated by the *x*– and *y*–axes, which do not lie in any of the four quadrants.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Student labels the quadrants counterclockwise but does not start with Quadrant I in the top right.	cannot recall where the first quadrant is located	EXPLAIN *QUADRANTS*
Student labels the quadrants clockwise starting with Quadrant I in the top right.	cannot recall which direction to label the quadrants in	EXPLAIN *QUADRANTS*
Student describes quadrants as part of the coordinate plane without mentioning the *x*– and *y*–axes.	does not see the axes as the divisions of the four quadrants of the coordinate plane	EXPLAIN *QUADRANTS*
Student labels the top left quadrant Quadrant I, the top right quadrant Quadrant II, the bottom left quadrant Quadrant III, and the bottom right quadrant Quadrant IV.	labels the quadrants from left to right, starting in the top half of the coordinate plane	EXPLAIN *QUADRANTS*

13. For each coordinate pair, identify the quadrant the coordinate pair belongs in without plotting the coordinate pair on a coordinate plane. Explain how you determined your response.

13.a. (8, –10)

CORRECT ANSWER

The coordinate pair (8, –10) falls in Quadrant IV because the *x*–coordinate is positive and the *y*–coordinate is negative.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The coordinate pair falls in Quadrant II because the *x*–coordinate is negative and the *y*–coordinate is positive.	reverses the *x*– and *y*–coordinates	EXPLAIN *COORDINATE PAIRS*
The coordinate pair falls in Quadrant II because it has a positive *x*–coordinate and a negative *y*–coordinate and would be plotted in the bottom right quadrant of the coordinate plane.	labels the quadrants in the clockwise direction instead of counterclockwise	EXPLAIN *QUADRANTS*
The coordinate pair falls in Quadrant IV.	does not describe how they determined the quadrant the coordinate pair belongs in	EXPLAIN THE RELATIONSHIP BETWEEN COORDINATE PAIRS AND QUADRANTS
The coordinate pair falls in Quadrant I.	does not recognize the negative sign or does not know how it impacts how the coordinate pair is plotted	RECOGNIZE THE NEGATIVE SIGN or GRAPH INTEGER COORDINATE PAIRS
It is impossible to know without graphing which quadrant the coordinate pair falls in.	does not know that the signs of the coordinates in a coordinate pair always indicate the quadrant the coordinate pair will fall in	EXPLAIN THE RELATIONSHIP BETWEEN COORDINATE PAIRS AND QUADRANTS

13.b. (–0.5, –7.1)

CORRECT ANSWER

The coordinate pair (–0.5, –7.1) falls in Quadrant III because both the *x*–coordinate and the *y*–coordinate are negative.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The coordinate pair falls in Quadrant I.	does not recognize the negative sign or does not know how it impacts how the coordinate pair is plotted	RECOGNIZE THE NEGATIVE SIGN or GRAPH INTEGER COORDINATE PAIRS
The coordinate pair falls in Quadrant III.	does not describe how they determined the quadrant the coordinate pair belongs in	EXPLAIN THE RELATIONSHIP BETWEEN COORDINATE PAIRS AND QUADRANTS
It is impossible to know without graphing which quadrant the coordinate pair falls in.	does not know that the signs of the coordinates in a coordinate pair always indicate the quadrant the coordinate pair will fall in	EXPLAIN THE RELATIONSHIP BETWEEN COORDINATE PAIRS AND QUADRANTS

13.c. $\left(\frac{9}{4}, \frac{3}{2}\right)$

CORRECT ANSWER

The coordinate pair $\left(\frac{9}{4}, \frac{3}{2}\right)$ falls in Quadrant I because both the x–coordinate and the y–coordinate are positive.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The coordinate pair falls in Quadrant I.	does not describe how they determined the quadrant the coordinate pair belongs in	EXPLAIN THE RELATIONSHIP BETWEEN COORDINATE PAIRS AND QUADRANTS
It is impossible to know without graphing which quadrant the coordinate pair falls in.	does not know that the signs of the coordinates in a coordinate pair always indicate the quadrant the coordinate pair will fall in	EXPLAIN THE RELATIONSHIP BETWEEN COORDINATE PAIRS AND QUADRANTS

13.d. $\left(-5\frac{1}{3}, 3\frac{1}{2}\right)$

CORRECT ANSWER

The coordinate pair $\left(-5\frac{1}{3}, 3\frac{1}{2}\right)$ falls in Quadrant II because the *x*–coordinate is negative and the *y*–coordinate is positive.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The coordinate pair falls in Quadrant IV because the *x*–coordinate is positive and the *y*–coordinate is negative.	reverses the *x*– and *y*–coordinates	EXPLAIN *COORDINATE PAIRS*
The coordinate pair falls in Quadrant IV because it has a negative *x*–coordinate and a positive *y*–coordinate and would be plotted in the top left quadrant of the coordinate plane.	labels the quadrants in the clockwise direction instead of counterclockwise	EXPLAIN *QUADRANTS*
The coordinate pair falls in Quadrant II.	does not describe how they determined the quadrant the coordinate pair belongs in	EXPLAIN THE RELATIONSHIP BETWEEN COORDINATE PAIRS AND QUADRANTS
The coordinate pair falls in Quadrant I.	does not recognize the negative sign or does not know how it impacts how the coordinate pair is plotted	RECOGNIZE THE NEGATIVE SIGN or GRAPH INTEGER COORDINATE PAIRS
It is impossible to know without graphing which quadrant the coordinate pair falls in.	does not know that the signs of the coordinates in a coordinate pair always indicate the quadrant the coordinate pair will fall in	EXPLAIN THE RELATIONSHIP BETWEEN COORDINATE PAIRS AND QUADRANTS

13.e. (6.3, 0)

CORRECT ANSWER

The coordinate pair $(6.3, 0)$ does not fall in a quadrant because it lies on the *x*–axis and the *x*–axis is not in any of the four quadrants.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The coordinate pair falls in Quadrant I because both coordinates are positive.	believes zero is a positive number	EXPLAIN CHARACTERISTICS OF ZERO
The coordinate pair falls in Quadrant I and Quadrant II.	believes the *x*–axis is located in the quadrants rather than dividing the quadrants	EXPLAIN *QUADRANTS*
It is impossible to know without graphing which quadrant the coordinate pair falls in.	does not know the signs of the coordinates in a coordinate pair always indicate the quadrant the coordinate pair will fall in	EXPLAIN THE RELATIONSHIP BETWEEN COORDINATE PAIRS AND QUADRANTS

14. Describe how the signs of the *x*– and *y*–coordinates indicate the quadrant the coordinate pair belongs in. Be sure to mention the signs of the coordinates in each quadrant of the plane.

CORRECT ANSWER

> In the coordinate plane, ordered pairs (or points) with a positive *x*–coordinate will fall to the right of the *y*–axis in either the first or the fourth quadrant, and coordinates with a negative *x*–coordinate will fall to the left of the *y*–axis in either the second or the third quadrant. In addition, coordinates with a positive *y*–coordinate will fall above the *x*–axis in either the first or the second quadrant, and coordinates with a negative *y*–coordinate will fall below the *x*–axis in either the third or the fourth quadrant. Therefore, the signs in each quadrant are as follows: Quadrant 1 (positive, positive), Quadrant II (negative, positive), Quadrant III (negative, negative), and Quadrant IV (positive, negative).

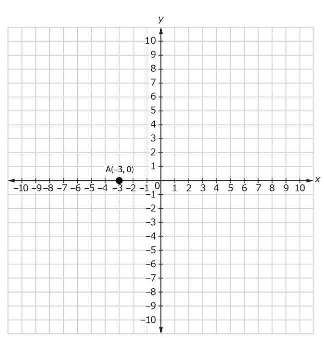
ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The coordinates are positive on the top and negative on the bottom of the coordinate plane.	does not distinguish between the x– and y–coordinates; describes only the y–coordinate but does not indicate they are describing the y–coordinate	EXPLAIN THE RELATIONSHIP BETWEEN COORDINATE PAIRS AND QUADRANTS
The coordinates are positive on the right and negative on the left of the coordinate plane.	does not distinguish between the x– and y–coordinates; describes only the x–coordinate but does not indicate they are describing the x–coordinate	EXPLAIN THE RELATIONSHIP BETWEEN COORDINATE PAIRS AND QUADRANTS
Whether the coordinates are positive or negative tells you which quadrant the coordinate pair is in.	understands to look for positive and negative coordinates, but does not describe specifically how positive and negative coordinates correlate to quadrants in the coordinate plane	EXPLAIN THE RELATIONSHIP BETWEEN COORDINATE PAIRS AND QUADRANTS
Quadrant I has positive coordinates.	only is able to describe the first quadrants signs; does not consider the quadrants containing negative coordinates	EXPLAIN THE RELATIONSHIP BETWEEN COORDINATE PAIRS AND QUADRANTS

15. Use the following coordinate plane to plot a point that would be considered an x–intercept. Label the point with its coordinate pair. Then describe an x–intercept in your own words.

CORRECT ANSWER

For this question, there are several possible correct answers. Students should provide a point plotted on the x–axis and labeled accurately. The following is an example of an appropriate response.



An x–intercept is a point that falls on the x–axis. In other words, it is the coordinate pair (s) where a graph crosses the x–axis.

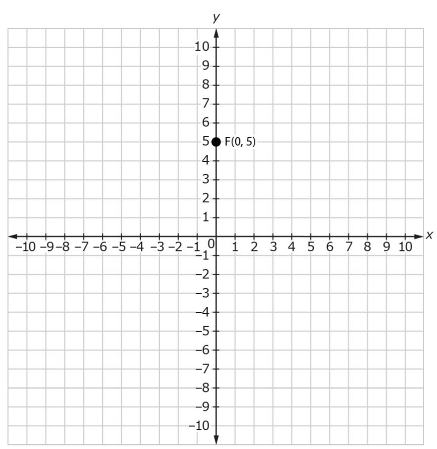
ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Student plots a point on the *y*–axis.	confuses the *x*– and *y*–axes	REPRESENT *X–INTERCEPT*
Student plots a point on the *x*–axis but does not label the point accurately.	knows an *x*–intercept falls on the *x*–axis but does not understand how to write the coordinate pair for the point	REPRESENT *X–INTERCEPT*
Student plots a point on the *x*–axis and labels it (0, *x*–coordinate).	knows an *x*–intercept falls on the *x*–axis and one of the coordinates is zero, but reverses the coordinates when writing the ordered pair	EXPLAIN *COORDINATE PAIRS*
Student describes an *x*–intercept as a coordinate pair that has an *x*–coordinate.	does not consider that all points have *x*–coordinates; may be referring to the fact that *x*–intercepts always have a *y*–coordinate of zero	RECOGNIZE *X–INTERCEPT*

16. Use the following coordinate plane to plot a point that would be considered a *y*–intercept. Label the point with its coordinate pair. Then describe a *y*–intercept in your own words.

CORRECT ANSWER

For this question, there are several possible correct answers. Students should provide a point plotted on the y–axis and labeled accurately. The following is an example of an appropriate response.



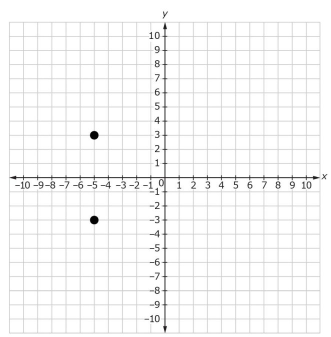
A *y*–intercept is a point that falls on the *y*–axis. In other words, it is the coordinate pair (s) where a graph crosses the y–axis.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Student plots a point on the x–axis.	confuses the x– and y–axes	REPRESENT *Y–INTERCEPT*
Student plots a point on the y–axis but does not label the point accurately.	knows a y–intercept falls on the y–axis but does not understand how to write the coordinate pair for the point	REPRESENT *Y–INTERCEPT*
Student plots a point on the y–axis and labels it $(y\text{–coordinate}, 0)$.	knows a y–intercept falls on the y–axis and one of the coordinates is zero, but reverses the coordinates when writing the ordered pair	EXPLAIN *COORDINATE PAIRS*
Student describes a y–intercept as a coordinate pair that has a y–coordinate.	does not consider that all points have x–coordinates; may be referring to the fact that y–intercepts always have an x–coordinate of zero	RECOGNIZE *Y–INTERCEPT*

17. Consider how the points are related in each coordinate plane as you answer the following questions.

17.a. Describe the reflection that would map one of the points provided in the coordinate plane onto the other point.



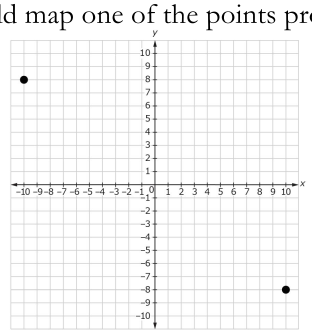
CORRECT ANSWER

A reflection across the x–axis would map one of the points in the coordinate plane onto the other point.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Move the point up (or down) six spaces.	describes a translation that would map one of the points onto the other instead of a reflection; may not be able to recognize a reflection in the coordinate plane	RECOGNIZE *REFLECTION*
a reflection across the *y*–axis	confuses the *x*– and *y*–axes	EXPLAIN *REFLECTION*
a reflection across –5	sees –5 between the two points and uses that value to describe the reflection rather than the *x*–axis; does not know reflections occur across a line	EXPLAIN *REFLECTION*

17.b. Describe the reflection that would map one of the points provided in the coordinate plane onto the other point.



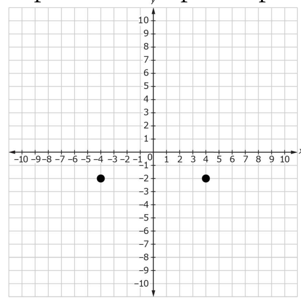
CORRECT ANSWER

> A reflection across the *x*–axis and the *y*–axis (in either order) would map one of the points in the coordinate plane onto the other point.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Move the point left (or right) 20 spaces and up (or down) 16 spaces.	describes a translation that would map one of the points onto the other instead of a reflection; may not be able to recognize a reflection in the coordinate plane	RECOGNIZE *REFLECTION*
a reflection across 0	sees 0 between the two points and uses that value to describe the reflection rather than the axes; does not know reflections occur across a line	EXPLAIN *REFLECTION*

17.c. Describe the reflection that would map one of the points provided in the coordinate plane onto the other point.



CORRECT ANSWER

A reflection across the y–axis would map one of the points in the coordinate plane onto the other point.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Move the point right (or left) eight spaces.	describes a translation that would map one of the points onto the other instead of a reflection; may not be able to recognize a reflection in the coordinate plane	RECOGNIZE *REFLECTION*
a reflection across the x–axis	confuses the x– and y–axes	EXPLAIN *REFLECTION*
a reflection across –2	sees –2 between the two points and uses that value to describe the reflection rather than the y–axis; does not know reflections occur across a line	EXPLAIN *REFLECTION*

18. Explain the relationship between the coordinates of a pair of coordinate pairs that are reflections of each other across the x–axis.

CORRECT ANSWER

Coordinate pairs that are reflections of each other across the x–axis will have exactly the same x–coordinate and y–coordinates that differ only by sign (opposites).

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Coordinate pairs that are reflections of each other across the *x*–axis will have *x*–coordinates that differ only by sign (opposites) and exactly the same *y*–coordinate.	believes the *x*–coordinates are opposites when the reflection is over the *x*–axis	EXPLAIN THE RELATIONSHIP BETWEEN COORDINATES AND REFLECTION OVER AN AXIS
Both the *x*– and the *y*–coordinates will be opposite.	believes both coordinates are opposites in any reflection	EXPLAIN THE RELATIONSHIP BETWEEN COORDINATES AND REFLECTION OVER AN AXIS
The coordinates will be the same but the point will be in a different quadrant.	does not understand that points in different quadrants cannot have the same coordinates	EXPLAIN THE RELATIONSHIP BETWEEN COORDINATE PAIRS AND QUADRANTS

19. Explain the relationship between the coordinates of a pair of coordinate pairs that are reflections of each other across both axes.

CORRECT ANSWER

> Coordinate pairs that are reflections of each other across both axes will have *x*–coordinates and *y*–coordinates that differ only by sign (opposites). For example, (3, –4) and (–3, 4) are coordinate pairs that are reflections of each other across both the *x*–axis and the *y*–axis.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Coordinate pairs that are reflections of each other across both axes will have one coordinate that is opposite and one coordinate that is the same.	may believe either coordinate can be opposite and either coordinate can be the same since the reflection is across both axes	EXPLAIN THE RELATIONSHIP BETWEEN COORDINATES AND REFLECTION OVER AN AXIS
The coordinates will be the same but the point will be in a different quadrant.	does not understand that points in different quadrants cannot have the same coordinates	EXPLAIN THE RELATIONSHIP BETWEEN COORDINATE PAIRS AND QUADRANTS

20. Describe how the coordinate pairs (2, –3) and (–2, –3) are reflections of each other on the coordinate plane. Include in your description how you know the coordinate pairs are reflections of each other based on the coordinates, as well as how you know they are reflections of each other based on their locations on a coordinate plane.

CORRECT ANSWER

> The coordinate pairs (2, –3) and (–2, –3) are reflections of each other across the *y*–axis because the *x*–coordinates are opposites and the *y*–coordinates are exactly the same. This means that from the *y*–axis, the first coordinate pair is two units to the right and the second coordinate pair is two units to the left. This is the same distance, but in opposite directions. Both coordinate pairs will be plotted three units below the *x*–axis. Therefore, reflecting one point across the *y*–axis will result in the other point.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The coordinate pairs are reflections of each other across the *x*–axis since the *x*–coordinates are opposites.	believes the *x*–coordinates are opposites when the reflection is over the *x*–axis	EXPLAIN THE RELATIONSHIP BETWEEN COORDINATES AND REFLECTION OVER AN AXIS
Student does not describe visually how the coordinate pairs are reflections of each other on the coordinate plane based on their location.	cannot picture or describe a reflection as being the same distance in opposite directions from a line of reflection on the coordinate plane	EXPLAIN *REFLECTION*