THE UNIVERSITY OF KANSAS
Achievement & Assessment Institute

RATIOS AND EQUIVALENT RATIOS

6.RP.1,3.a

CONTENTS

The types of documents contained in the unit are listed below. Throughout the unit, the documents are arranged by lesson.

Learning Map Information	An overview of the standards, the learning map section, and the nodes addressed in this unit
Teacher Notes	A brief discussion describing the progression depicted in the learning map section with research-based recommendations for focusing instruction to foster student learning and an introduction to the unit's lessons
Overview of Instructional Activities	A table highlighting the lesson goals and nodes addressed in each lesson of this unit
Instructional Activity	A detailed walkthrough of the unit
Instructional Activity Student Handout	A handout for the guided activity, intended to be paired with the Instructional Activity
Instructional Activity Supplement	A collection of materials or activities related to the Instructional Activity
Student Activity	A work-alone activity for students
Student Activity Solution Guide	A solution guide for the work-alone activity with example errors, misconceptions, and links to the learning map section

RATIOS AND EQUIVALENT RATIOS

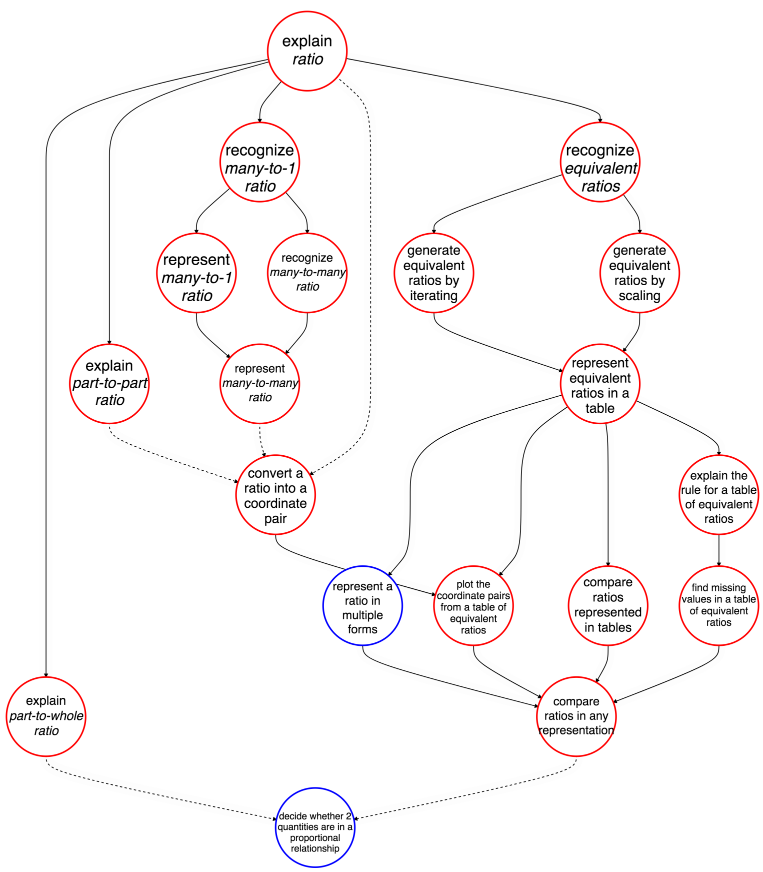
LEARNING MAP INFORMATION

STANDARDS

6.RP.1 Write and describe the relationship in real life context between two quantities using ratio language. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."*

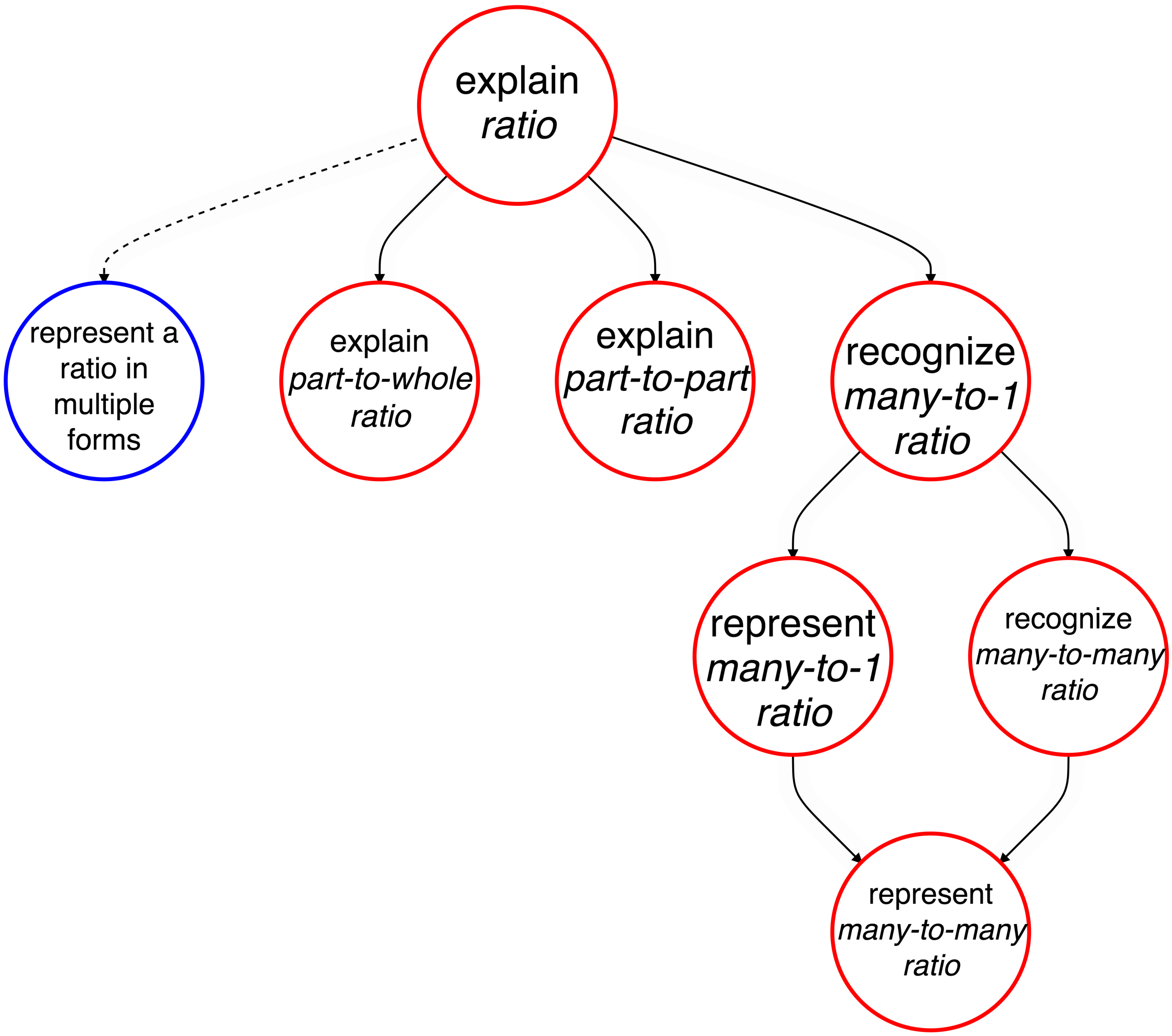
6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems (e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations).

> **6.RP.3.a** Make tables of equivalent ratios relating quantities with whole number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios, and understand equivalencies.



**Learning map model of* 6.RP.1,3.a

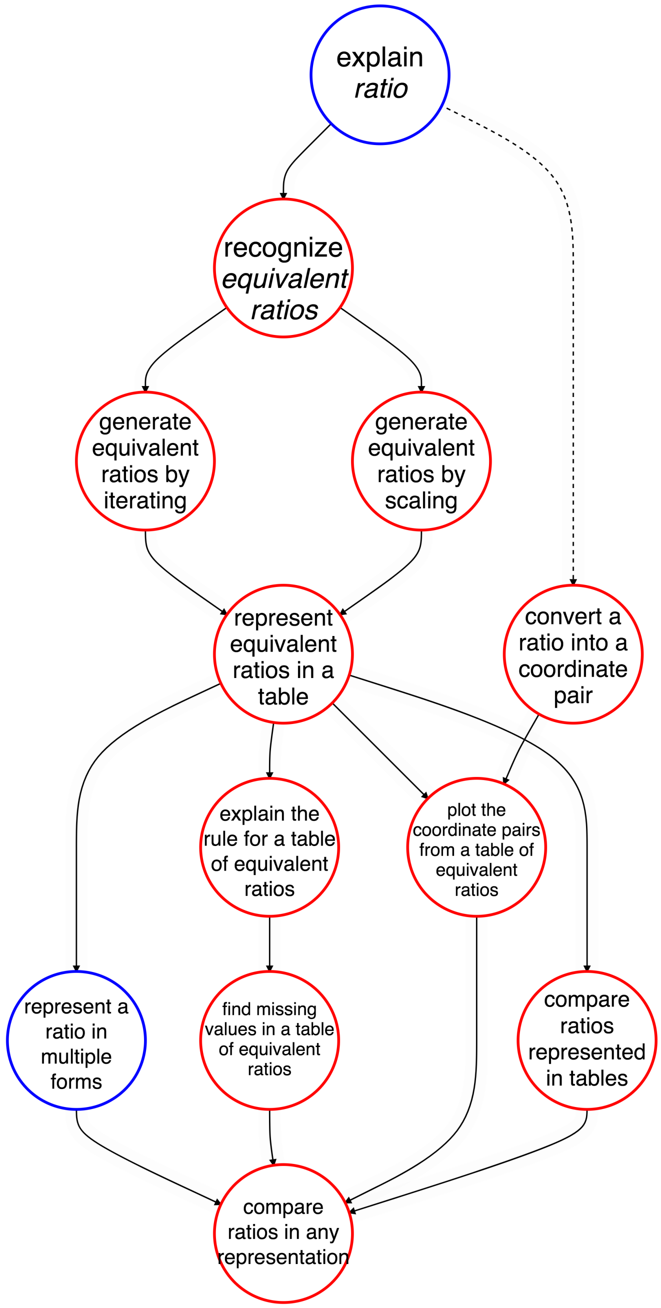
6.RP.1 Write and describe the relationship in real life context between two quantities using ratio language. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."*



**Learning map model of* 6.RP.1

6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems (e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations).

> **6.RP.3.a** Make tables of equivalent ratios relating quantities with whole number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios, and understand equivalencies.



**Learning map model of* 6.RP.3.a

Node Name	Node Description
COMPARE RATIOS IN ANY REPRESENTATION	Compare ratios represented in any representation such as tables or fractions.
COMPARE RATIOS REPRESENTED IN TABLES	Use two equivalent ratio tables with two columns or two rows as a tool for making comparisons between ratios.
CONVERT A RATIO INTO A COORDINATE PAIR	Convert a given ratio (e.g., 3/4) into a coordinate pair (4, 3).
DECIDE WHETHER 2 QUANTITIES ARE IN A PROPORTIONAL RELATIONSHIP	Decide whether two quantities are in a proportional relationship using graphs, tables, etc.
EXPLAIN *PART-TO-PART RATIO*	Make known your understanding that a ratio can express a comparison between two different types of quantities. These ratios are composed of two different quantities that may be measured in different units (e.g., the ratio of the number of chocolate candies to the number of students in a class).
EXPLAIN *PART-TO-WHOLE RATIO*	Make known your understanding that ratios can express a comparison between the number of parts to a given whole. These ratios are composed of two quantities measured in the same unit of measurement. For example, in the ratio of zebras to total animals in the zoo, the unit of measurement is one animal.
EXPLAIN *RATIO*	Make known your understanding through words, drawings, manipulatives, etc., that a ratio represents a multiplicative comparison of two quantities or the joining of two quantities into a composed unit. For example, the ratio of eyes to nose on a person is 2:1, because for every two eyes there is one nose.
EXPLAIN THE RULE FOR A TABLE OF EQUIVALENT RATIOS	Make known your understanding the rule for a table of equivalent ratios.
FIND MISSING VALUES IN A TABLE OF EQUIVALENT RATIOS	Given a table of equivalent ratios, find missing values to complete the table.
GENERATE EQUIVALENT RATIOS BY ITERATING	Generate equivalent ratios using additive reasoning (iterating units). For example, 2:3 is equivalent to 4:6 because two iterations of 2:3 yields 4:6.
GENERATE EQUIVALENT RATIOS BY SCALING	Generate equivalent ratios using multiplicative reasoning to scale up (e.g., doubling, tripling, etc.) or partition (e.g., halving, thirds, etc.).
PLOT THE COORDINATE PAIRS FROM A TABLE OF EQUIVALENT RATIOS	Plot the coordinate pairs from a table of equivalent ratios on a coordinate plane.
RECOGNIZE *EQUIVALENT RATIOS*	Identify ratios that specify the same relationship between two quantities. For example, the ratio 3 buttons to 5 candies is equivalent to the ratio 6 buttons to 10 candies.
RECOGNIZE *MANY-TO-1 RATIO*	Identify or name a ratio that describes a one-to-many or many-to-one multiplicative relationship (e.g., 3:1, 1:4, etc.).
RECOGNIZE *MANY-TO-MANY RATIO*	Identify or name a ratio that describes a many-to-many multiplicative relationship (e.g., 3:3, 4:6, etc.).
REPRESENT A RATIO IN MULTIPLE FORMS	Through writing or an appropriate assistive technology, represent a ratio in multiple forms, including as a fraction, with a colon, or using the word "to" (e.g., 2/1, 2:1, 2 to 1).
REPRESENT EQUIVALENT RATIOS IN A TABLE	Through writing or an appropriate assistive technology, make a table to show equivalent ratios.
REPRESENT *MANY-TO-1 RATIO*	Through writing or an appropriate assistive technology, represent a many-to-one ratio.
REPRESENT *MANY-TO-MANY RATIO*	Through writing or an appropriate assistive technology, represent a many-to-many ratio.

Ratios and Equivalent Ratios

TEACHER NOTES

This unit includes the following documents:

- Learning Map Information
- Instructional Activity (three lessons)
- Instructional Activity Student Handout (for Lesson 2 & 3)
- Instructional Activity Supplement (for Lesson 3)
- Student Activity
- Student Activity Solution Guide

In this unit, students will learn about ratios and equivalent ratios.

RESEARCH

Sixth grade mathematics is the first time for many students to formally explore concepts of ratio and proportionality. Ratios, defined as the comparison of two quantities, are difficult for students because they must pay attention to two different quantities and the relationship between the two quantities. The majority of students' earlier mathematics experiences focused on single quantities, so students benefit from opportunities to explore contexts involving ratios formed by two quantities. Students need clear and explicit problem solving practice to hone their skills with ratios (Ellis, 2013). Through relevant, comprehensive practice, students can begin to understand and master concepts of ratios and proportionality.

Although students have some familiarity of ratios from their daily lives, several misconceptions remain. One common mistake made by students is that they often incorrectly apply additive reasoning when multiplicative reasoning is appropriate (Miller & Fey, 2000). Students need to explore ratios and their equivalent values through iterating and scaling to develop a solid understanding of multiplicative relationships. Modeling techniques—such as ratio tables, graphs, or even concrete manipulatives—may help students recognize multiplicative relationships.

AN EXAMPLE

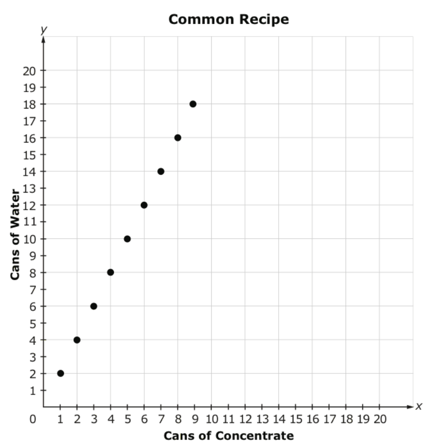
The following are modeling techniques for representing multiplicative relationships. Each model represents a common recipe of two cans of water for every one can of concentrate.

RATIO TABLE

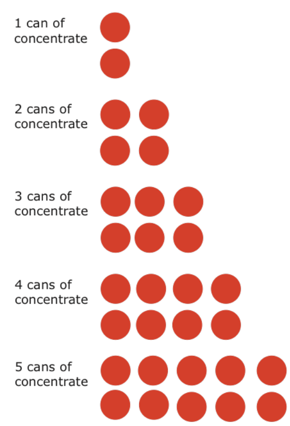
Common Recipe

Cans of concentrate	1	2	3	4	5	6	7	8	9
Cans of water	2	4	6	8	10	12	14	16	18

GRAPH



CONCRETE MANIPULATIVES



A second ratio concept students should focus on is making a meaningful comparison. Students need to attend to the units in a problem situation to make sure that comparisons made in ratios are meaningful (Lo, Watanabe, & Cai, 2004). As students work with part-to-part ratios, a variety of comparisons can be written as ratios, but not all ratios will have meaning. Students should focus on the meaning of ratios to develop a solid sense of proportional reasoning to be later applied to problem solving situations. Although understanding of ratios supports students' later work with forming and solving proportions, students also need to understand how ratios model different relationships between quantities. In particular, students should recognize ratios that model part-part relationships and part-whole relationships (Lobato & Ellis, 2010). They also should develop an understanding of rates as they relate to ratios. For example, if a certain paint combination requires three parts of Color A and four parts of Color B, then the composed unit, Color C, contains three parts of Color A and four parts of Color B, regardless of the total amount. In this example, Color C is composed of Colors A and B. On the other hand, slope is a rate determined by some vertical distance divided by some horizontal distance. A thorough understanding of different types of ratios can serve as the foundation for students succeeding with topics including linear functions, algebra, scaling geometric figures, and probability and statistics.

LEARNING MAP INFORMATION

The learning map section for this sequence of activities begins with explaining ratios and then recognizing different types of ratios (i.e., part-part, part-whole, one-many, and many-many). This structure suggests that teachers and students should focus on different types of comparisons among real, tangible quantities. Without the application of ratios to actual objects, students may not develop the depth of understanding needed for proportional reasoning, which is an essential gateway for higher mathematics (Lamon, 2007). Students should have multiple opportunities to represent ratios and generate equivalent ratios. Finally, students should be able to organize, graph, and compare ratios in any representation.

INSTRUCTIONAL ACTIVITIES

The activities in this unit are designed to allow students to identify, describe, and construct ratios. First, students will identify, construct, and explain a variety of ratios. Then students will explore given ratios and create tables of equivalent ratios. From those equivalent ratios, students will make comparisons and draw conclusions to analyze the products of different recipes. Students will also graph the values from the ratio table on the coordinate plane to visually compare ratios. Finally, students will apply their knowledge of ratios and equivalent ratios to determine a missing value in a given collection to match the ratios. Through an assortment of observations, students will determine which houses fit given ratios and design a neighborhood using ratio reasoning and the observations provided.

REFERENCES

Ellis, A. (2013). Teaching Ratio and Proportion in the Middle Grades. Retrieved June 13, 2016, from http://www.nctm.org/Research-and-Advocacy/research-brief-and-clips/Ratio-_-Proportion/

Lamon, S. J. (2007). Rational numbers and proportional reasoning: Toward a theoretical framework for research. In F. K. Lester, Jr. (Ed.), Second handbook of research on mathematics teaching and learning (pp. 629-668). NC: Information Age Publishing.

Lo, J., Watanabe, T., & Cai, J. (2004). Developing Ratio Concepts: An Asian Perspective. *Mathematics Teaching in the Middle School, 9*(7), 362-367.

Lobato, J., & Ellis, A. B. (2010). *Essential understandings: Ratios, proportions, and proportional reasoning.* In R. M. Zbiek (Series Ed.), Essential understandings. Reston, VA: National Council of Teachers of Mathematics.

Miller, J., & Fey, J. (2000). Take Time For Action: Proportional Reasoning. *Mathematics Teaching in the Middle School, 5*(5), 310-313.

Ratios and Equivalent Ratios

OVERVIEW OF INSTRUCTIONAL ACTIVITIES

Lesson	Learning Goal	Nodes Addressed
Lesson 1	Students will use ratios to describe the relationship between two quantities.	▶ EXPLAIN *RATIO* ▶ REPRESENT *MANY-TO-MANY RATIO* ▶ EXPLAIN *PART-TO-WHOLE RATIO* ▶ EXPLAIN *PART-TO-PART RATIO*
Lesson 2	Students will find and apply equivalent ratios and compare ratios.	▶ EXPLAIN *RATIO* ▶ EXPLAIN THE RULE FOR A TABLE OF EQUIVALENT RATIOS ▶ PLOT THE COORDINATE PAIRS FROM A TABLE OF EQUIVALENT RATIOS ▶ COMPARE RATIOS IN ANY REPRESENTATION
Lesson 3	Students will identify and use equivalent ratios to describe a collection of items.	▶ REPRESENT EQUIVALENT RATIOS IN A TABLE ▶ RECOGNIZE *EQUIVALENT RATIOS* ▶ INSERT NODE NAME

Ratios and Equivalent Ratios

INSTRUCTIONAL ACTIVITY

Lesson 1

LEARNING GOAL

Students will use ratios to describe the relationship between two quantities.

PRIMARY ACTIVITY

Students will play an "I Spy" game using ratios. During the game, students will identify and describe a ratio from a given set of items, as well as guess what items other students have identified and described in their ratios.

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

- Ratio
- Part
- Whole

MATERIALS

- A collection of items for each group of students, approximately 15–20 items. Example items: buttons, coins, colored cubes, pencils, small toys, pattern blocks or other small manipulatives, etc.

IMPLEMENTATION

Discuss the meaning of ratio. Many students will have some prior knowledge of ratios but may not be able to define the concept. Make sure students understand that a ratio represents a comparison between two quantities. To identify a ratio means to count and name the amounts for the different items being compared. For example, the ratio of boys to girls describes and relates how many boys and how many girls are in a particular group.

Discuss the different ways to represent and describe a ratio. Students need to be familiar with the written forms using words such as "to", "per", "in", "for", "for each", "for every", "out of every", "out of", etc. Additionally, students should be familiar with the notation using a colon or written to look like a fraction.

Students must understand that 2 boys to 4 girls, 2 boys : 4 girls, and $\frac{2\text{ boys}}{4\text{ girls}}$ are all ways to represent the same ratio.

Model the "I Spy" ratio game for students. **Display** a collection of items for students. If, for example, the collection contains 3 red buttons, 5 black buttons, 4 pennies, 2 white buttons, and 3 dimes, **say** "I spy with my little eye, a ratio of 3 to 5." Then students will try to guess what items you have used to identify your ratio (3 red buttons to 5 black buttons). There may be more than one possible group of items that makes the same ratio (3 dimes to 5 black buttons, for example), but students must identify the specific items you chose.

Give hints as needed. Students should explain their answer when they guess by explaining their understanding of what a 3 to 5 ratio means. Additionally, students need to pay attention to the order of the ratio. Students may give the correct items but have the ratio reversed.

Discuss the importance of paying attention to the order of a ratio as a vital part of precision in identifying ratios. Students need to understand that the order of the quantities will affect equivalence and units used to describe a ratio.

Students may be able to use the given collection of items to describe multiple ratios using different subsets of items. If students offer a different subset of items to match the given ratio, they should be ready to defend their answer. For example, a student may offer the answer of 3 dimes to 5 black buttons and defend their answer by saying the original 3 to 5 ratio relates a group of 3 items to a group of 5 items, just as their ratio does with 3 dimes to 5 black buttons. However, in a general game of "I Spy", many items may fit the given color condition, and the challenge is to find the specific items spied. That same rule of identifying the specific items spied for the given ratio applies to the "I Spy" ratio game.

GUIDING QUESTIONS

Elicit student thinking:

- Why did you choose the items for your answer?
- Describe the collection of items.

Determine if the student can EXPLAIN *RATIO*:

- What does it mean to say a ratio of 3 to 5?
- What information do you know based on the description of a 3 to 5 ratio?
- How would you write the ratio 3 to 5?
- Are there other groups of items that could describe the 3 to 5 ratio other than what I chose?
- Is a 5 to 3 ratio that same as a 3 to 5 ratio? What similarities and differences are there between these two ratios?
- How does a ratio relate to the collection of items?

Determine if the student can REPRESENT *MANY-TO-MANY RATIO*:

- Using this collection of items, can you name a ratio that uses the buttons and coins?
- Can your ratio describe any other group of items?
- Can you describe another ratio that compares two different groups of items?

Assign students to small groups. **Distribute** to each group a collection of items. These items should be placed in the center of the group, randomly arranged. Each collection of items does not need to be identical for all groups.

Students will take turns "spying" and describing a ratio. The remaining students in the group will try to guess which items were chosen for the ratio and defend their answers. The student that described the ratio should be able to check the accuracy of the items other students guessed by identifying the ratio applied by the guess.

Repeat until all students have had a chance to spy a ratio, or as time allows.

Monitor student groups during the activity. **Assist** students to see how other subsets of items might be a correct ratio, even if not the particular items "spied".

GUIDING QUESTIONS

Elicit student thinking:

- What other items could have been chosen to describe the same ratio?
- How does the given ratio describe the set of items?

Determine if the student can EXPLAIN *PART-TO-WHOLE RATIO*:

- How does the whole collection compare to one set of items?
- How do the individual sets fit into the total group?
- How do you use a ratio to describe only one small set from the total collection?
- How are part-to-whole ratios similar to fractions?

Determine if the student can EXPLAIN *PART-TO-PART RATIO*:

- What items in your collection can be compared?
- Can any two groups of items be compared?
- How do the values in the ratio relate to the objects they describe?
- What other information can you get from a part-to-part ratio?

At the end of the activity, teachers should **ask** each group to identify a ratio from their collection of items. The groups should try to stump the class while the class tries to guess the items chosen for the ratio. One way students may attempt to stump each other is to combine smaller subsets of items that share a characteristic, such as buttons of different colors, items that are the same shape, or different coins. Students need to be flexible in looking at the items to identify the different subsets of items that are related within the larger collection.

Ratios and Equivalent Ratios

INSTRUCTIONAL ACTIVITY

Lesson 2

LEARNING GOAL

Students will find and apply equivalent ratios and compare ratios.

PRIMARY ACTIVITY

Students will explore making lemonade from concentrate. Given different ratios of water and concentrate, students will compare the final lemonade. Students will use ratio tables to model and compare the ratios (Noelting, 1980).

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

- Ratio
- Equivalent Ratios
- Ratio Table

MATERIALS

- Water and Lemonade concentrate (optional)
- Blocks or other counting manipulatives (optional)
- Instructional Activity Student Handout

IMPLEMENTATION

This activity focuses on equivalent ratios. Modeling using ratio tables facilitates finding the equivalent ratio values. Equivalent ratios will also be used to compare ratios.

Begin by **discussing** making lemonade from concentrate. Make sure students are familiar with the process and understand the outcome from mixing water and lemonade concentrate.

GUIDING QUESTIONS

Elicit student thinking:

- Why is a recipe important to follow for making lemonade?
- Can lemonade be made from concentrate without knowing a ratio of concentrate to water?
- What happens to the taste of the lemonade if the ratio of lemonade concentrate and water in the recipe is changed?
- What happens to the amount of lemonade concentrate needed if a larger batch of lemonade is made?

Determine if the student can EXPLAIN *RATIO*:

- Explain how ratios apply to making lemonade.
- How do ratios and recipes relate?
- What happens when you change the ratio in a recipe?

Distribute the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT to each student. Students can work in pairs or groups of three to facilitate discussion. Students will benefit from working in small groups as they explore and discuss equivalent ratios and comparisons of ratios.

With their group members, students will discuss and explore the ratios and equivalent ratios on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT. Students may use the counters to represent and model the lemonade concentrate and water in the difference recipes. These manipulatives may help students see how the equivalent ratios are built and related.

When students are asked to graph the pairs of values from the ratio tables, students can choose which variable to use as the independent variable and which variable would then be the dependent variable. Thus, two different graphs would be acceptable because there would not be a specified independent and dependent variable in this context.

Discuss with students the differences in the two graphs. Also **note** the similarities in the meaning of the ratio and how the ratio is represented in either graph. These concepts will later connect to the concept of slope in linear functions, where the independent and dependent variables are specified and must be graphed accordingly.

GUIDING QUESTIONS

Elicit student thinking:

- What information do you see from the ratio table?
- What information would be useful in the ratio table?
- Explain what each row in the ratio table represents.
- Where do you find the ratios in the ratio table? Identify each number in the ratio and what each number means.

Determine if the student can EXPLAIN THE RULE FOR A TABLE OF EQUIVALENT RATIOS:

- How are the values in the ratio table related?
- How can you move from one value to another?
- Is there more than one way to describe movement through the ratio table?

Determine if the student can PLOT THE COORDINATE PAIRS FROM A TABLE OF EQUIVALENT RATIOS:

- How are the points in the graph related?
- Why does that graph match the ratio?
- Can you explain how to match a graph to an unknown ratio?
- How is the ratio modeled in the graphical representation?

Determine if the student is ready to COMPARE RATIOS IN ANY REPRESENTATION:

- How can you tell that Tony's recipe is different from the common recipe?
- How does Tony's recipe change the ratio table compared to the common recipe?
- How does Tony's recipe change the graph compared to the common recipe?
- Can you describe another change to the common recipe that would yield a different ratio table? Describe the change.
- Can you describe another change to the common recipe that would yield a different graph? Describe the change.

Students should be expected to justify their reasoning and defend their answers. Additionally, by working in pairs or groups of three, students will be able to practice productive mathematical discourse.

At the end of the activity, require students to complete a "3-2-1" exit slip. Students should create a list of three things they learned, two questions still in their mind, and one idea or thought that made sense.

REFERENCES

Noelting, G. (1980). The Development of Proportional Reasoning and the Ratio Concept Part II: Problem-Structure at Successive Stages; Problem-Solving Strategies and the Mechanism of Adaptive Restructuring. *Educational Studies in Mathematics*, 11(3), 331-363. Retrieved from http://www.jstor.org/stable/3481806

Name________________________________

Ratios and Equivalent Ratios

Lesson 2

1) Lemonade can be easily made using a concentrate and water. A common recipe using frozen lemonade concentrate requires one can of concentrate mixed with three cans of water.

Write the ratio of lemonade concentrate to water.

What information does the ratio give?

Using the common recipe, fill in the ratio table below.

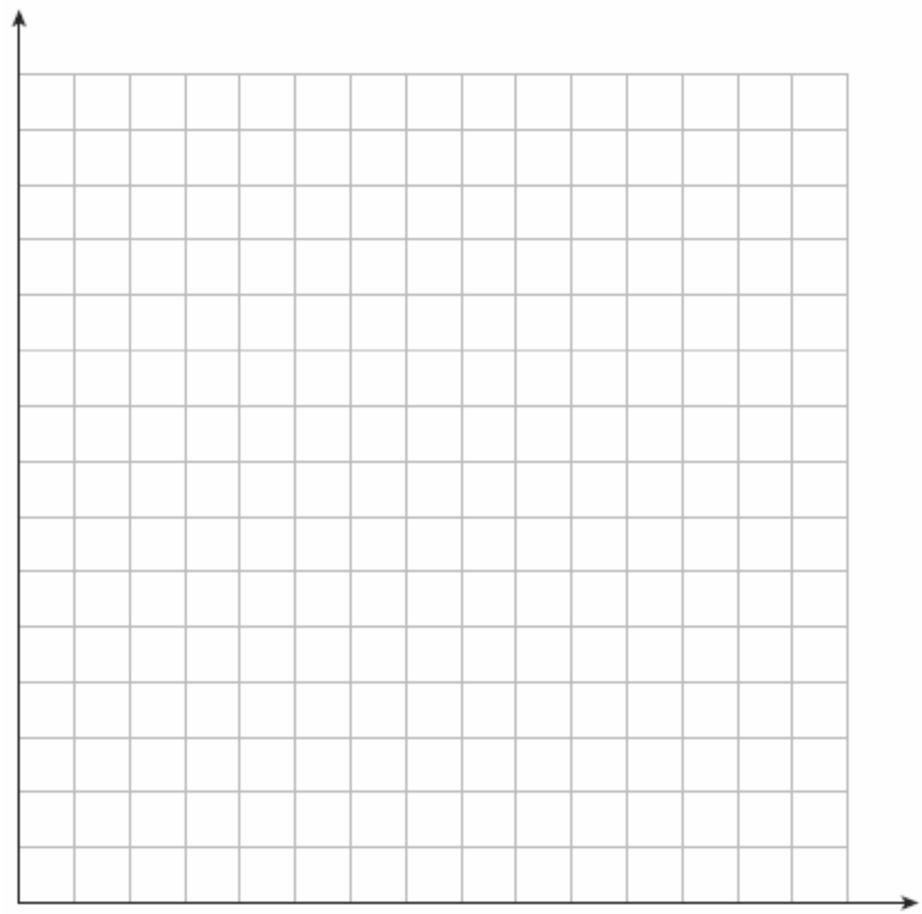
Common Recipe

Cans of concentrate	1	2	3	4	5	6	7	8	9
Cans of water									

What patterns do you notice in the ratio table?

Name______________________________

2) On the grid, graph the ratio table in Question 1 by plotting the pairs of values. Label each axis to show which value is graphed along that axis.

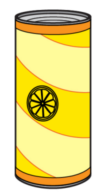


What relationship do you notice about the plotted points?

Name______________________________

How does the graph show the relationship between the corresponding amounts of water and concentrate in the common recipe?

3) Javion is making a batch of lemonade for his soccer tournament. If he uses 10 cans of lemonade concentrate, how many cans of water will Javion need? Justify your reasoning.



4) Alyssa is following the common recipe to make an extra large batch of lemonade for a fundraiser. If she uses 42 cans of water, how many cans of lemonade concentrate will Alyssa use? Justify your reasoning.

Name________________________________

5) Tony doesn't follow the common recipe. When his father taught him how to make lemonade, Tony learned a recipe instructing to mix four cans of water with every one can of lemonade concentrate.

What information does the ratio give?

Using Tony's recipe, fill in the ratio table below.

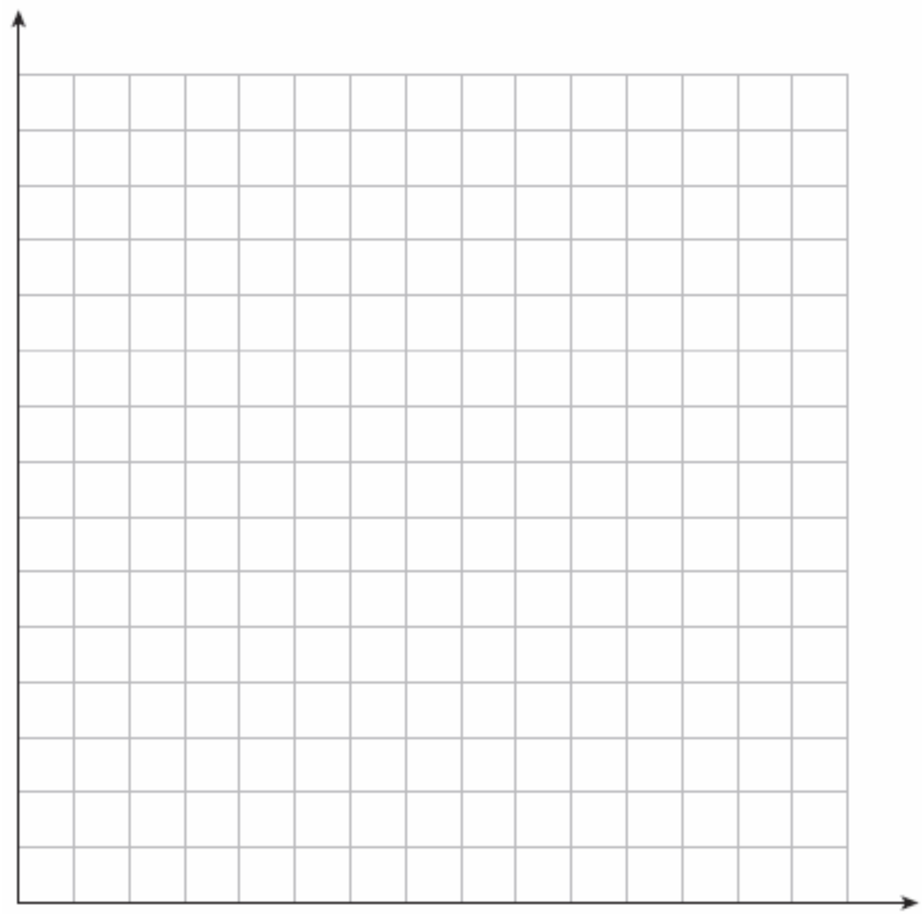
Tony's Recipe

Cans of concentrate	1	2	3	4	5	6	7	8	9
Cans of water									

What patterns do you notice in the ratio table?

Name____________________________

6) On the grid, graph the ratio table in Question 5 by plotting the values. Label each axis to show which value is graphed along that axis.

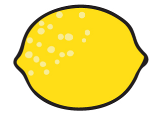


What relationship do you notice about the plotted points?

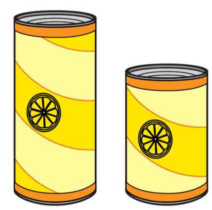
How does the graph show the relationship between the corresponding amounts of water and concentrate in Tony's recipe?

Name______________________________

7) Which lemonade recipe will produce a stronger lemon flavor: the common recipe or Tony's recipe? Justify your reasoning.



8) Flynna and Clarke found two different sized cans of lemonade concentrate in the grocery store: one large and one small. Both cans have instructions to mix one can of concentrate with three cans of water. Flynna thinks that because the two recipes are the same, each batch of lemonade will taste the same, even though the two cans are different sizes. Clarke says that because one can is larger, the lemonade made from the larger can will have a stronger lemon flavor. Who is correct? Justify your reasoning.



Ratios and Equivalent Ratios

INSTRUCTIONAL ACTIVITY

Lesson 3

LEARNING GOAL

Students will identify and use equivalent ratios to describe a collection of items.

PRIMARY ACTIVITY

Students will design a neighborhood based on observations. The observations will contain ratios that describe the houses in the neighborhood. Students will draw items and color the houses to match the given observations.

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

- Ratio
- Equivalent Ratios
- Ratio Tables

MATERIALS

- Colored pencils or markers
- Instructional Activity Student Handout
- Instructional Activity Supplement (Recommended one copy for every group of two or three students)

IMPLEMENTATION

Review the use of equivalent ratios to describe a collection of items. **Pose** the following problem to students. “In a group of 20 students, if 2 in 5 are girls, how many girls are in the entire group?”

GUIDING QUESTIONS

Elicit student thinking:

- What does the ratio 2 in 5 tell you about the group of students, namely about the number of girls and boys in the group?
- What is the role of equivalent ratios in this problem?
- How do you find an equivalent ratio to the 2 girls in 5 students?
- How can you know that ratios are equivalent?

Determine if the student can REPRESENT EQUIVALENT RATIOS IN A TABLE:

- How would you set up a ratio table for the ratio 2 girls in 5 students?
- How can a ratio table be used to identify the number of girls in the group?
- How does a ratio table display equivalent ratios?
- How far can you extend the ratio table given the ratio 2 girls in 5 students and a total group of 20 students?
- How far does the ratio table need to go to find an equivalent ratio for the 2 girls in 5 students in the total group of 20 students?

Determine if the student is ready to RECOGNIZE *EQUIVALENT RATIOS*:

- Identify an equivalent ratio to the given 2 girls in 5 students.
- Identify an equivalent ratio to the given 2 girls in 5 students to determine how many girls are in the group.

Distribute the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT to each student. Students should work in pairs or groups of three. Each pair or group of three needs a copy of the INSTRUCTIONAL ACTIVITY SUPPLEMENT.

With their group members, students will read the observations and decipher the given ratios to decide how many of the houses are represented by the ratio. Then students will color or decorate the correct number of houses to match the observations. Students should have some freedom to choose what their decorations look like and which houses have the given decorations or coloring.

GUIDING QUESTIONS

Elicit student thinking:

- What information does the given ratio tell you?
- How can you use the ratio to decide how many houses match the observation?
- What equivalent ratios can you find to match the ratio in the observation?

Determine if the student can RECOGNIZE *EQUIVALENT RATIOS*:

- Knowing there are 12 houses on Ratio Road, what equivalent ratios would be used with the given observation?
- Describe how you find an equivalent ratio to the ratio given in the observation.
- Is there only one equivalent ratio for each given observation? How do you choose the correct equivalent ratio?

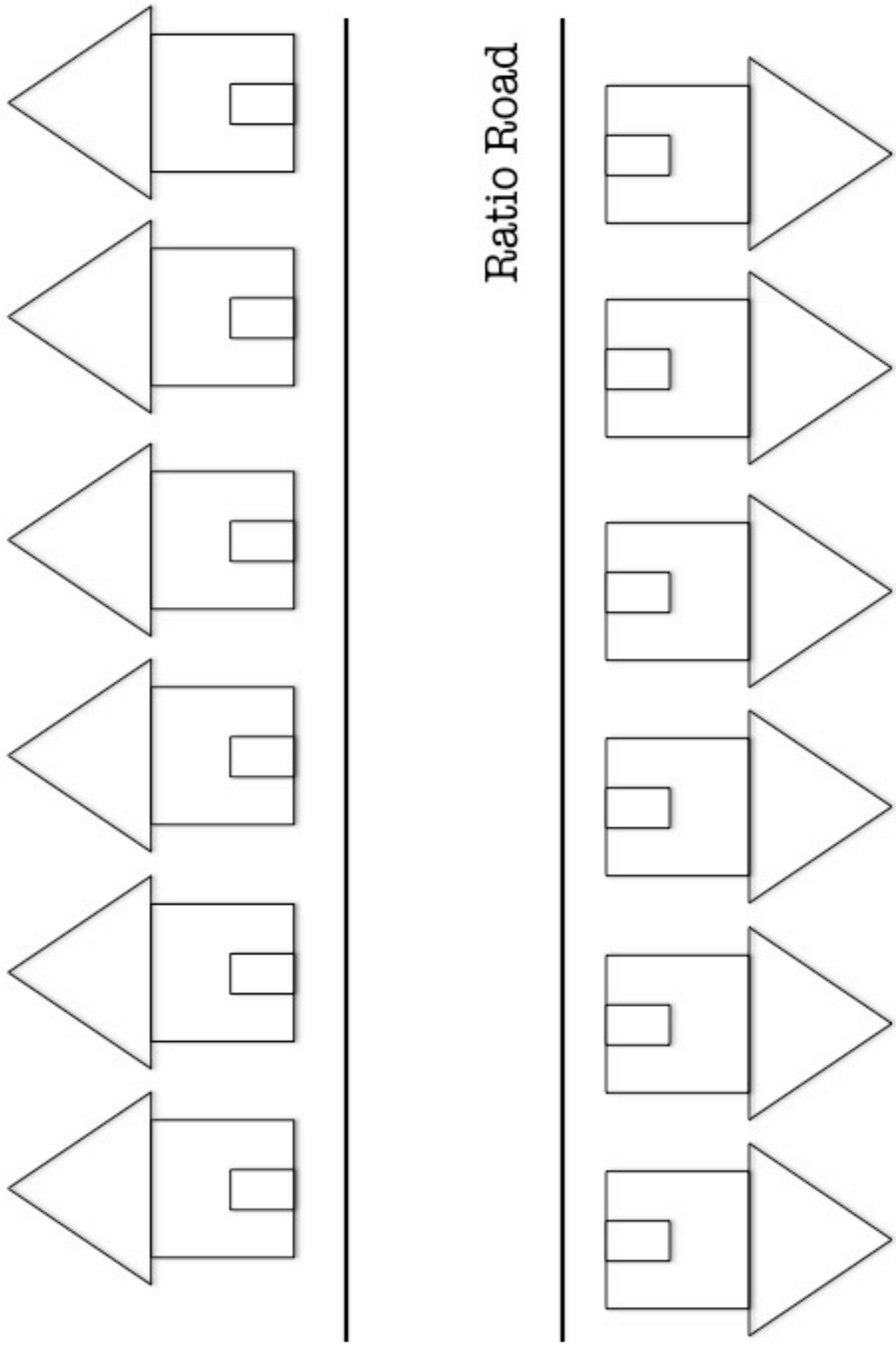
Students should be required to justify their choices as they color and decorate Ratio Road.

At the end of the activity, teachers should **display** students' work. Students should be ready to defend and explain their choices in decorating Ratio Road.

Name________________________________

Ratios and Equivalent Ratios

Lesson 3



Ratio Road is a nice neighborhood filled with interesting sights. On the given picture, draw the details for the houses based on the neighbors' observations below. For each observation, explain how you decided what to draw on your picture.

1) Abbey sees a variety of colored doors in the neighborhood. She sees that 1 in 3 houses have blue doors. Abbey also notices that 1 in every 6 houses have red doors. None of the other doors match.

2) Fred notices that many of the houses are white. In fact, 6 out of the 12 houses are white. All of the other houses are either gray or blue.

3) Roger sees that flamingos are popular on Ratio Road. A ratio of houses equivalent to 1 in 4 have at least one flamingo in the front yard. Two thirds of those houses also have gnomes with their flamingos.

4) Dawn counts that 3 out of every 4 houses have dogs.

5) Many of the houses have cars in front, while the others park their cars in the garage behind the house. 2 out of every 6 houses use their garage.

6) Bernardo always leaves his bike in his front yard, just like 1 out of every 4 of the neighborhood kids.

7) Tamar observes many trees and counts that 5 of 6 houses have trees in the yard.

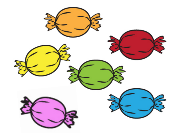
8) Fiorella loves to tend to her flowers. She is not alone, as 2 in 3 houses have flowerbeds in the front.

Name____

Ratios and Equivalent Ratios

Lessons 1 — 3

1. There are 7 red candies, 5 green candies, and 2 white candies in a bag.



1.a. What does the ratio 5:14 represent? Circle one answer and justify your reasoning.

a) The ratio of red candies to green candies

b) The ratio of green candies to the total

c) The ratio of the total to white candies

d) The ratio of red candies to white candies

Justification:

1.b. Is the 5:14 ratio a part-to-part or a part-to-whole ratio? Justify your answer.

1.c. Write a ratio for each of the three answer choices in Problem 1.a that did **not** match the ratio 5:14.

Name____

2. Lorne collects stickers, bracelets, and seashells. Lorne has collected 30 stickers, 18 bracelets, and 12 seashells.

2.a. Write three different part-to-whole ratios comparing the individual items to the total collection.

2.b. Write three different ratios comparing any two different types of items in Lorne's collection.

3. A soccer coach is planning for the next season. Fill in the table to help the coach plan.

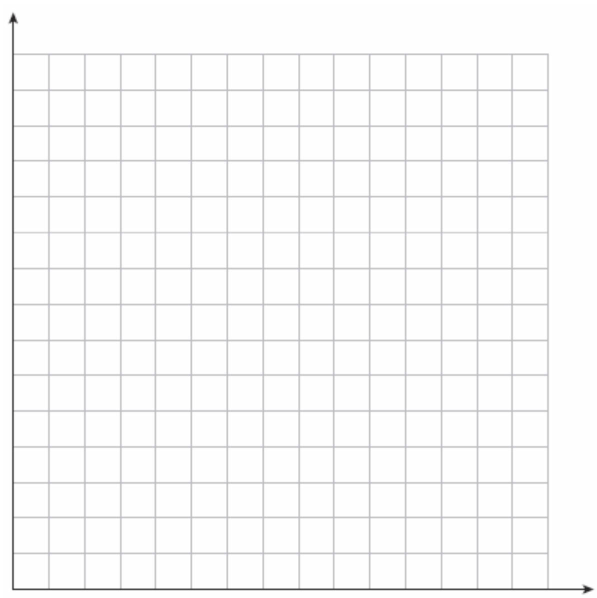
Number of Soccer Balls	1	2	3	4	5
Number of Players	3				

3.a. Explain the relationship between the number of soccer balls and the number of players.

Name____

3.b. If the coach is now planning for the entire league and wants to keep the same ratio of players to soccer balls, how many soccer balls are needed for 120 players? Explain your procedure.

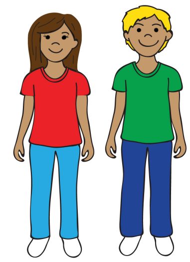
3.c. Graph the values from the table on the coordinate grid. Label each axis to show what value is graphed along that axis.



3.d. What relationship do you notice about the plotted points?

Name____

4. Rita's class has 14 girls and 16 boys.

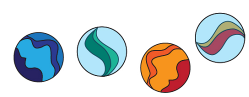


4.a. How does the ratio 14:16 describe Rita's class?

4.b. How does the ratio 14:30 describe Rita's class?

4.c. If Rita's class is to be divided into teams of girls versus boys, which ratio would be most useful? Explain your reasoning.

5. There are blue, green, and red marbles in a bag. The ratio of blue marbles to green marbles is 3 to 2.



5.a. Which answer choice represents the possible marbles in the bag? Circle one answer and explain your choice.

a) B, B, B, G, G, G, G, R, R

b) B, B, B, B, G, G, G, G, G, G, R, R, R, R

c) B, B, B, B, B, B, G, G, G, G, R, R, R, R

d) B, B, B, B, G, G, G, G, R, R, R, R

5.b. Amanda says there is another possible answer that is not listed. What possibility could Amanda have in mind?

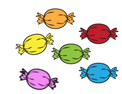
5.c. Mikel agrees with Amanda that there is an answer not listed. Is it possible that Amanda and Mikel are thinking about different sets of marbles, but both of those different sets have a 3:2 ratio of blue to green marbles? Support your decision with reasoning.

Ratios and Equivalent Ratios

STUDENT ACTIVITY SOLUTION GUIDE

Lessons 1—3

1. There are 7 red candies, 5 green candies, and 2 white candies in a bag.



1.a. What does the ratio 5:14 represent? Circle one answer and justify your reasoning.

a) The ratio of red candies to green candies

b) The ratio of green candies to the total

c) The ratio of the total to white candies

d) The ratio of red candies to white candies

CORRECT ANSWER

b) The ratio of green candies to the total

The 5 represents the five green candies and the 14 represents the total number of candies, because 7 + 5 + 2 = 14.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
a) The ratio of red candies to green candies	does not understand the construction of a ratio or attend to the amount of red candies compared to green candies	RECOGNIZE *MANY-TO-MANY RATIO*
c) The ratio of the total to white candies	does not understand the construction of a ratio or attend to the total amount of candies compared to white candies	RECOGNIZE *MANY-TO-MANY RATIO*
d) The ratio of red candies to white candies	does not understand the construction of a ratio or attend to the amount of red candies compared to white candies	RECOGNIZE *MANY-TO-MANY RATIO*

1.b. Is the 5:14 ratio a part-to-part or a part-to-whole ratio? Justify your answer.

CORRECT ANSWER

> The 5:14 ratio is a part-to-whole ratio because the five green candies represent a part of the 14 total candies.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
part-to-part ratio	confuses a part of a group with the amount of a total group	EXPLAIN *PART-TO-WHOLE RATIO*

1.c. Write a ratio for each of the three answer choices in Problem 1.a that did **not** match the ratio 5:14.

CORRECT ANSWER

> The ratio of red candies to green candies is 7:5.
> The ratio of the total to white candies is 14:2.
> The ratio of red candies to white candies is 7:2.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The ratio of red candies to green candies is 7:14.	confuses the total amount of candies with the amount of the green candies	REPRESENT *MANY-TO-MANY RATIO*
The ratio of red candies to green candies is 7:2.	confuses the amount of white candies with the amount of the green candies	REPRESENT *MANY-TO-MANY RATIO*
The ratio of red candies to white candies is 7:5.	confuses the amount of green candies with the amount of the white candies	REPRESENT *MANY-TO-MANY RATIO*
The ratio of red candies to white candies is 7:14.	confuses the total amount of candies with the amount of the white candies	REPRESENT *MANY-TO-MANY RATIO*
The ratio of red candies to green candies is 5:7.	does not attend to the order of the ratio given	REPRESENT *MANY-TO-MANY RATIO*
The ratio of the total to white candies is 2:14.	does not attend to the order of the ratio given	REPRESENT *MANY-TO-MANY RATIO*
The ratio of red candies to white candies is 2:7.	does not attend to the order of the ratio given	REPRESENT *MANY-TO-MANY RATIO*

2. Lorne collects stickers, bracelets, and seashells. Lorne has collected 30 stickers, 18 bracelets, and 12 seashells.

2.a. Write three different part-to-whole ratios comparing the individual items to the total collection.

CORRECT ANSWER

1. 30 stickers : 60 total items or 1 sticker : 2 total items
2. 18 bracelets : 60 total items or 3 bracelets : 10 total items
3. 12 seashells : 60 total items or 1 seashell : 5 total items

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
60 total items : 30 stickers or 60 total items : 18 bracelets or 60 total items : 12 seashells	does not attend to the order of the ratio given	REPRESENT *MANY-TO-MANY RATIO*
30 stickers : 18 bracelets or 30 stickers : 12 seashells or 18 bracelets : 12 seashells	does not understand that a part-to-whole ratio must contain the total number of items as one of the quantities	EXPLAIN *PART-TO-WHOLE RATIO*
incorrectly simplifies the ratio, such as 1 sticker : 4 total items	unable to divide out common factors, error in division or finding a common factor	GENERATE EQUIVALENT RATIOS BY SCALING

2.b. Write three different ratios comparing any two different types of items in Lorne's collection.

CORRECT ANSWER

1. 30 stickers : 18 bracelets or 5 stickers : 3 bracelets
2. 30 stickers : 12 seashells or 5 stickers : 2 bracelets
3. 18 bracelets : 12 seashells or 3 bracelets : 2 seashells

Note: as order is not specified for the ratios, the items can be commuted in the ratio.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
30 stickers : 60 total items or 18 bracelets : 60 total items or 12 seashells : 60 total items	does not understand that a part-to-part ratio does not contain the total number of items as one of the quantities	EXPLAIN *PART-TO-PART RATIO*
incorrectly simplifies the ratio, such as 1 sticker : 6 bracelets	unable to divide out common factors, error in division or finding a common factor	GENERATE EQUIVALENT RATIOS BY SCALING

3. A soccer coach is planning for the next season. Fill in the table to help the coach plan.

Number of Soccer Balls	1	2	3	4	5
Number of Players	3				

CORRECT ANSWER

Number of Soccer Balls	1	2	3	4	5
Number of Players	3	6	9	12	15

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

<table>
<tr><th>Example Error</th><th>Misconception</th><th>Missing Knowledge</th></tr>
<tr><td><table><tr><td>Number of Soccer Balls</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr><tr><td>Number of Players</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td></tr></table></td><td>uses an additive relationship between the number of soccer balls and the number of players</td><td>REPRESENT EQUIVALENT RATIOS IN A TABLE</td></tr>
<tr><td><table><tr><td>Number of Soccer Balls</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr><tr><td>Number of Players</td><td>3</td><td>3</td><td>3</td><td>3</td><td>3</td></tr></table></td><td>does not increase the number of players to match an increase in the number of soccer balls</td><td>REPRESENT EQUIVALENT RATIOS IN A TABLE</td></tr>
</table>

3.a. Explain the relationship between the number of soccer balls and the number of players.

CORRECT ANSWER

> For every soccer ball, there are three players.
>
> *or*
>
> For every three players on the team, there is one soccer ball.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
For every player, there are three soccer balls.	reverses the labels from the ratio table	EXPLAIN THE RULE FOR A TABLE OF EQUIVALENT RATIOS
For every soccer ball, there are two more players.	uses an additive relationship between the number of soccer balls and the number of players	EXPLAIN THE RULE FOR A TABLE OF EQUIVALENT RATIOS

3.b. If the coach is now planning for the entire league and wants to keep the same ratio of players to soccer balls, how many soccer balls are needed for 120 players? Explain your procedure.

CORRECT ANSWER

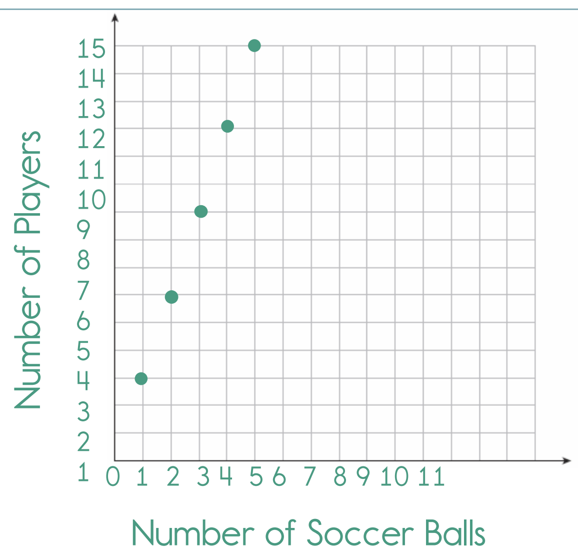
> For 120 players, 40 soccer balls are needed. For every three players on the team, one soccer ball is needed. Then the number of soccer balls needs to be scaled up by 40, because the number of players is scaled up by 40.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
For 120 players, 118 soccer balls are needed. For every soccer ball, there are two more players or two fewer soccer balls than the number of players. So, if there are 120 players, then two fewer soccer balls are needed.	uses an additive relationship between the number of soccer balls and the number of players	EXPLAIN THE RULE FOR A TABLE OF EQUIVALENT RATIOS
For 120 players, 60 soccer balls are needed. For every three players on the team one soccer ball is needed. Then the number of soccer balls needs to be scaled up by 60 because the number of players is scaled up by 60.	miscalculates the scaling value from three players to 120 players	GENERATE EQUIVALENT RATIOS BY SCALING
For 120 players, 360 soccer balls are needed. For every player on the team three soccer balls are needed. Then the number of soccer balls needs to be scaled up by 120 because the number of players is scaled up by 120.	reverses the labels from the ratio table	EXPLAIN THE RULE FOR A TABLE OF EQUIVALENT RATIOS

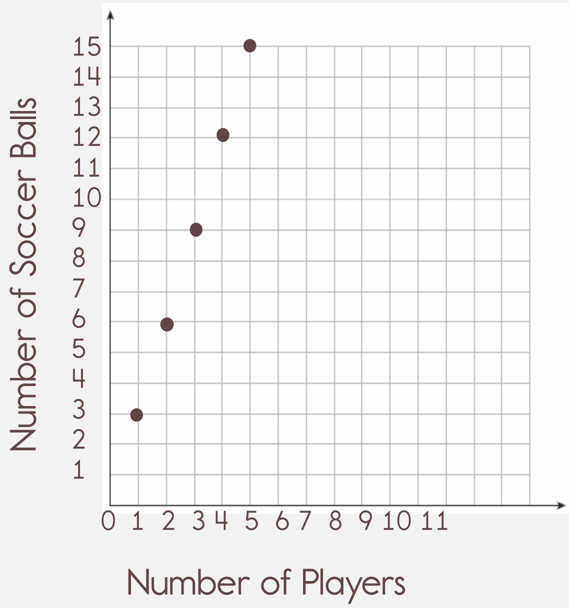
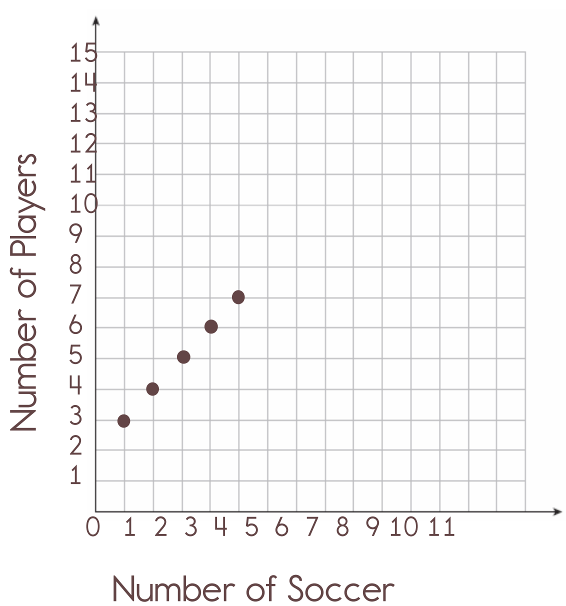
3.c. Graph the values from the table on the coordinate grid. Label each axis to show what value is graphed along that axis.

CORRECT ANSWER



Note: A graph where the axes are reversed is also appropriate because there is not a specified independent and dependent variable in this context.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
	reverses the labels from the ratio table	PLOT THE COORDINATE PAIRS FROM A TABLE OF EQUIVALENT RATIOS
	uses an additive relationship between the number of soccer balls and the number of players	REPRESENT EQUIVALENT RATIOS IN A TABLE

3.d. What relationship do you notice about the plotted points?

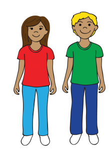
CORRECT ANSWER

> The points fall in a line.
> *or*
> If the points were connected, they would create a line.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Students do not recognize the graph as a straight line.	does not recognize the set of equivalent ratios as a cohesive group	DECIDE WHETHER 2 QUANTITIES ARE IN A PROPORTIONAL RELATIONSHIP
Note: We are trying to lead students towards noticing proportional relationships in the coordinate plane. Students should be familiar with a line and therefore may begin to associate points that fall in a line with a proportional relationship between the variables.		

4. Rita's class has 14 girls and 16 boys.



4.a. How does the ratio 14:16 describe Rita's class?

CORRECT ANSWER

> The ratio 14:16 describes Rita's class by identifying how many girls are in the class compared to how many boys are in the class.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The ratio 14:16 describes Rita's class by identifying how many girls are in the class compared to the total number of students.	confuses a part-to-whole ratio with a part-to-part ratio	EXPLAIN *PART-TO-PART RATIO*
The ratio 14:16 describes Rita's class by identifying how many boys are in the class compared to how many girls are in the class.	confuses the amount of boys with the amount of the girls	RECOGNIZE *MANY-TO-MANY RATIO*

4.b. How does the ratio 14:30 describe Rita's class?

CORRECT ANSWER

> The ratio 14:30 describes Rita's class by identifying how many girls are in the class compared to the total number of students.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The ratio 14:30 describes Rita's class by identifying how many girls are in the class compared to how many boys are in the class.	confuses a part-to-part ratio with a part-to-whole ratio	EXPLAIN *PART-TO-WHOLE RATIO*
The ratio 14:30 describes Rita's class by identifying how many boys are in the class compared to the total number of students.	confuses the amount of boys with the amount of the girls	RECOGNIZE *MANY-TO-MANY RATIO*

4.c. If Rita's class is to be divided into teams of girls versus boys, which ratio would be most useful? Explain your reasoning.

CORRECT ANSWER

> The 14:16 ratio would be most useful because it describes the class in terms of the two groups: girls and boys. This ratio shows exactly how many girls and how many boys are in in the class to be divided into teams.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The 14:30 ratio would be most useful because it describes the class in terms of the girls versus the whole class. From this, the girls can be divided into teams.	does not attend to the need for boys to be divided into teams	EXPLAIN *PART-TO-PART RATIO*

5. There are blue, green, and red marbles in a bag. The ratio of blue marbles to green marbles is 3 to 2.

5.a. Which answer choice represents the possible marbles in the bag? Circle one answer and explain your choice.

a) B, B, B, G, G, G, G, R, R

b) B, B, B, B, G, G, G, G, G, G, R, R, R, R

c) B, B, B, B, B, B, G, G, G, G, R, R, R, R

d) B, B, B, B, G, G, G, G, R, R, R, R

CORRECT ANSWER

c) B, B, B, B, B, B, G, G, G, G, R, R, R, R

The ratio of 3 blue marbles to 2 green marbles is found in the 6 blue marbles and 4 green marbles, because the 3:2 ratio has been scaled up by a factor of 2.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
a) B, B, B, G, G, G, G, R, R The ratio of 3 to 2 is found in the 3 blue marbles and 2 red marbles.	does not attend to the given labels of the ratio	RECOGNIZE *MANY-TO-MANY RATIO*
b) B, B, B, B, G, G, G, G, G, G, R, R, R, R The ratio of 3 to 2 is found in the 6 green marbles and 4 blue marbles, because the 3:2 ratio has been scaled up by 2.	does not attend to the given order of the ratio	RECOGNIZE *MANY-TO-MANY RATIO*
d) B, B, B, B, G, G, G, G, R, R, R, R The ratio of 3 to 2 is found in the 3 blue marbles and 2 green marbles.	does not attend to the relationship between the total number of blue marbles and the total number of green marbles; notices that three blue marbles and two green marbles are present in this option	RECOGNIZE *MANY-TO-MANY RATIO*

5.b. Amanda says there is another possible answer that is not listed. What possibility could Amanda have in mind?

CORRECT ANSWER

Amanda could be thinking about a bag of marbles represented by B, B, B, G, G, R, R.

Note: Any combination of 3 blues marbles and 2 green marbles with any number of red marbles could be a correct answer. In addition, any combination with a scaled amount of 3 blue marbles and 2 green marbles with any number of red marbles could be a correct answer.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Amanda could be thinking about a bag of marbles represented by B, B, B, B, B, B, G, G, R, R. Or any combination that does not list 3 blue marbles and 2 green marbles or a scaled amount of 3 blue marbles and 2 green marbles.	scales the amount of one marble color without changing the amount of the other	GENERATE EQUIVALENT RATIOS BY SCALING
Amanda is mistaken. There is no other possibility other than the answer choice c).	does not recognize that multiple equivalent ratios exist for a single given ratio	RECOGNIZE *EQUIVALENT RATIOS*

5.c. Mikel agrees with Amanda that there is an answer not listed. Is it possible that Amanda and Mikel are thinking about different sets of marbles, but both of those different sets have a 3:2 ratio of blue to green marbles? Support your decision with reasoning.

CORRECT ANSWER

Yes, Amanda could be thinking about a bag of marbles represented by B, B, B, G, G, R, R but Mikel might be thinking about a bag of marbles represented by B, B, B, B, B, B, G, G, G, G, R, R, R. The ratio of blue to green marbles in Amanda's bag is 3:2, while Mikel has a ratio of 6:4, which is equivalent to a 3:2 ratio of blue to green marbles. There are an infinite number of possible combinations for the marbles that can have a 3:2 ratio of blue to green marbles.

Note: any two combinations of 3 blues marbles and 2 green marbles with any number of red marbles could be a correct answer. Or any two combinations with a scaled amount of 3 blue marbles and 2 green marbles with any number of red marbles could be a correct answer.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
No, there is only one possible combination of marbles that Amanda can be thinking of, so Mikel must be thinking of the same combination.	does not recognize that multiple equivalent ratios exist for a single given ratio	RECOGNIZE *EQUIVALENT RATIOS*
Yes, Amanda could be thinking about a bag of marbles represented by B, B, B, G, G, R, R but Mikel might be thinking about a bag of marbles represented by B, B, B, B, B, B, G, G, R, R.	does not correctly identify two equivalent ratios of blue to green marbles	GENERATE EQUIVALENT RATIOS BY SCALING