THE UNIVERSITY OF KANSAS
Achievement & Assessment Institute

TRANSFORMATIONS AND CONGRUENCE

8.GM.A.1, 8.GM.A.2, and 8.GM.A.3

CONTENTS

The types of documents contained in the unit are listed below. Throughout the unit, the documents are arranged by lesson.

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TRANSFORMATIONS AND CONGRUENCE

LEARNING MAP INFORMATION

STANDARDS

8.GM.A.1 Verify experimentally the congruence properties of rigid transformations.

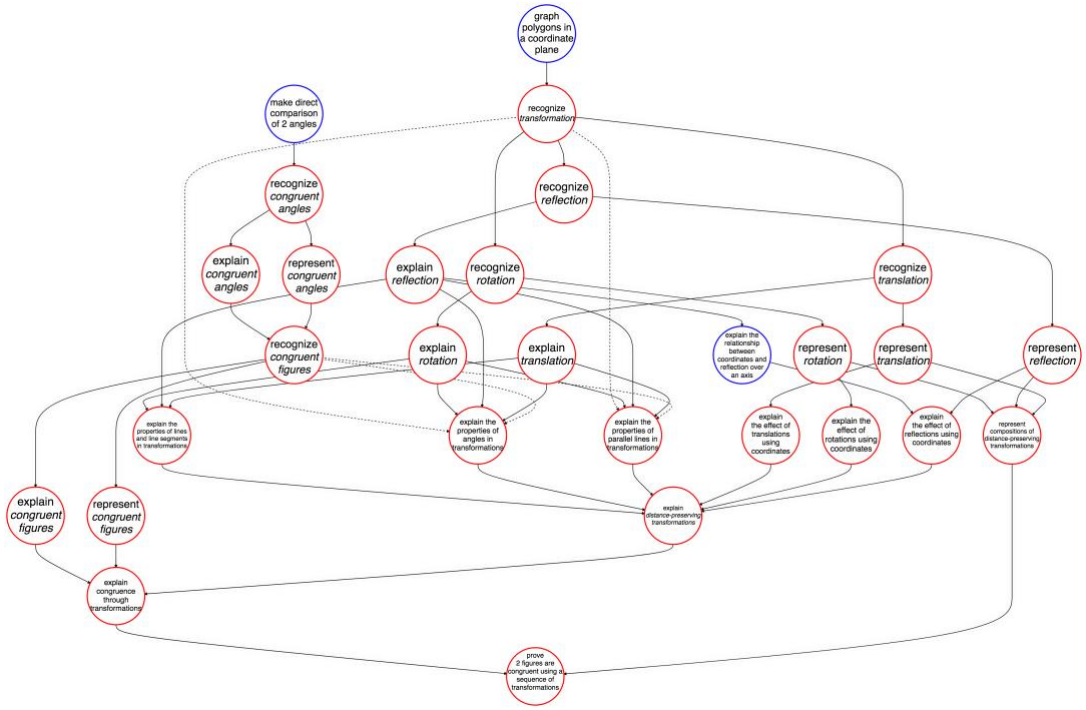
8.GM.A.1.a Verify that angle measure, betweenness, collinearity and distance are preserved under rigid transformations.

8.GM.A.1.b Investigate if orientation is preserved under rigid transformations.

8.GM.A.2 Understand that two-dimensional figures are congruent if a series of rigid transformations can be performed to map the pre-image to the image.

8.GM.A.2.a Describe a possible sequence of rigid transformations between two congruent figures.

8.GM.A.3 Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

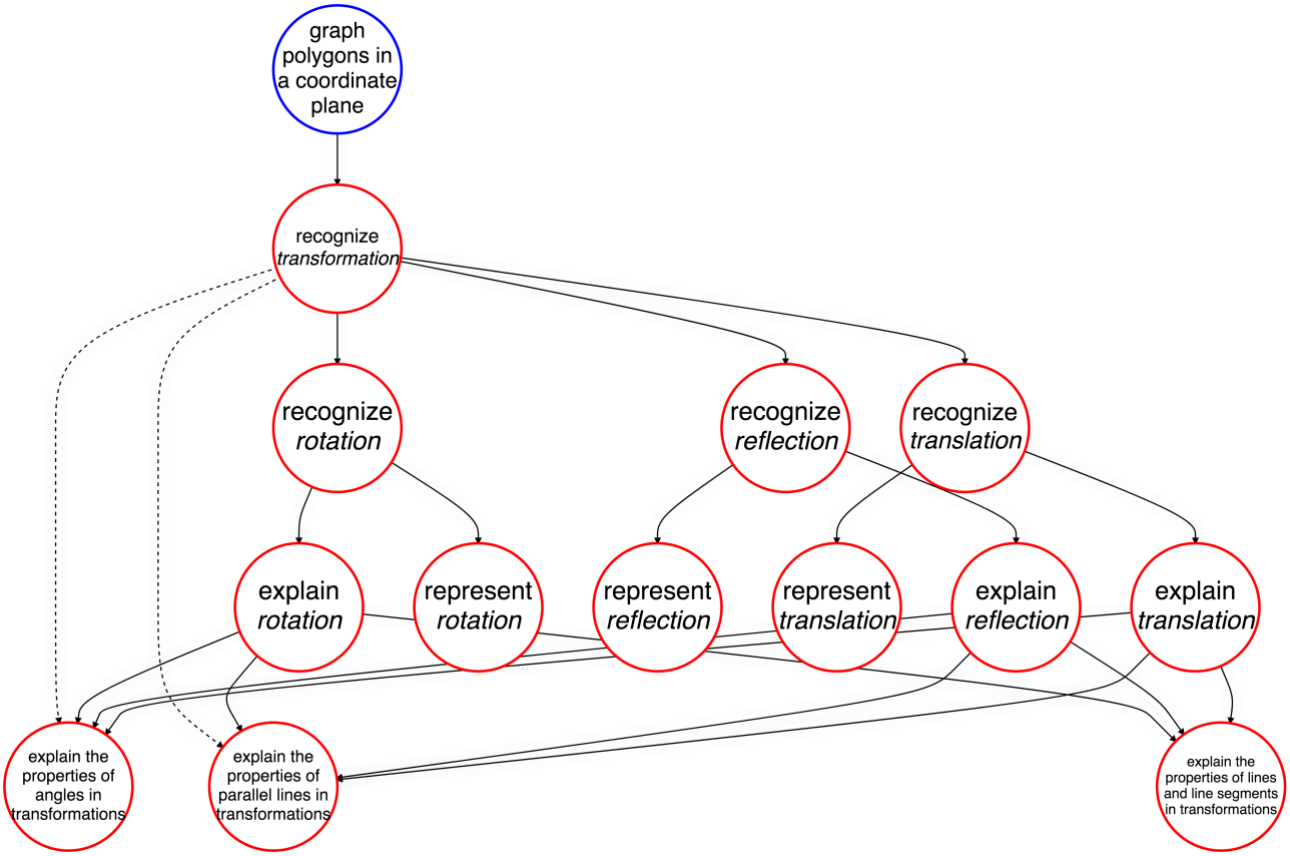


**Learning Map Model of* 8.G.1-3

8.GM.A.1 Verify experimentally the congruence properties of rigid transformations.

8.GM.A.1.a Verify that angle measure, betweenness, collinearity and distance are preserved under rigid transformations.

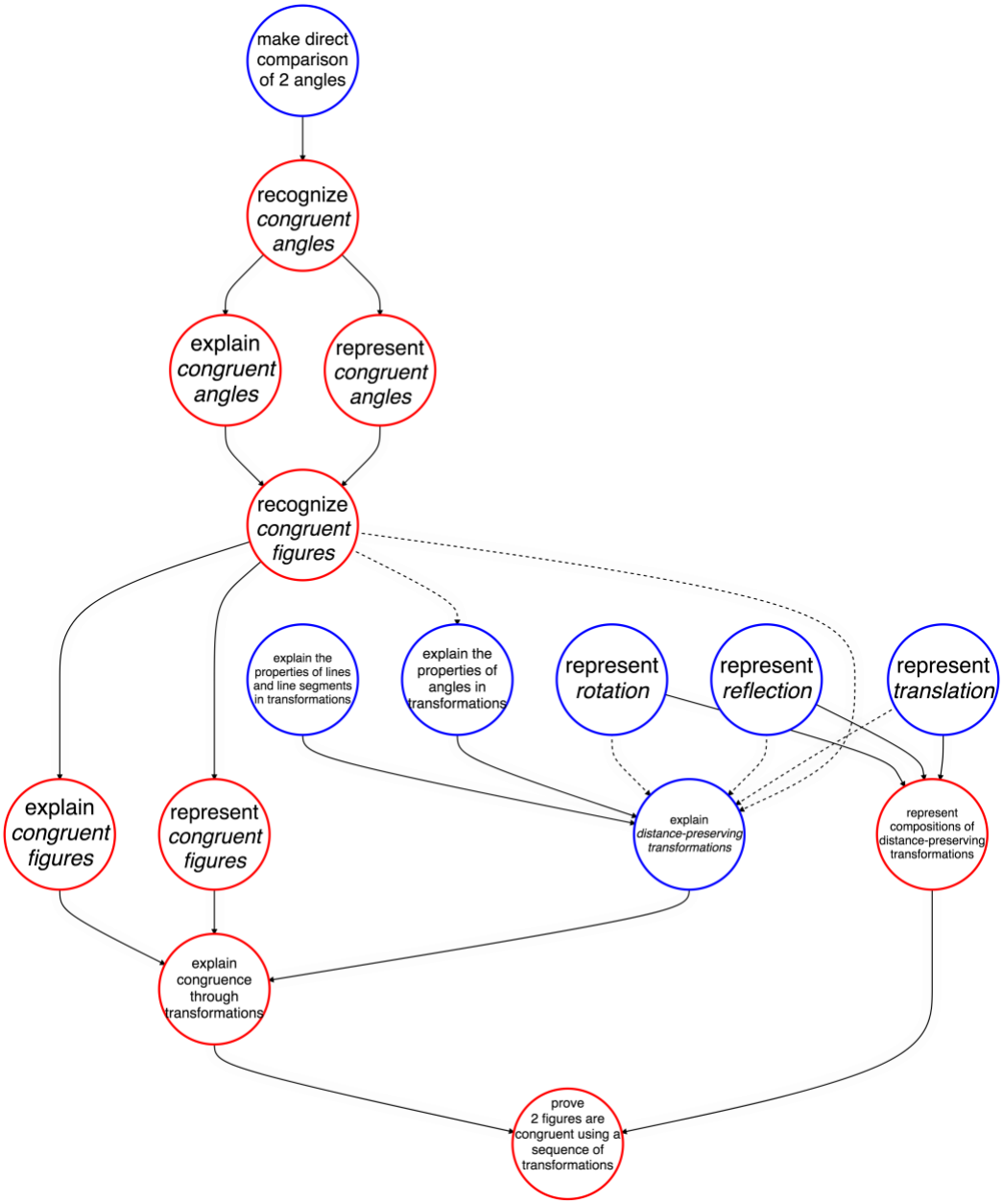
8.GM.A.1.b Investigate if orientation is preserved under rigid transformations.



**Learning Map Model of* 8.G.1

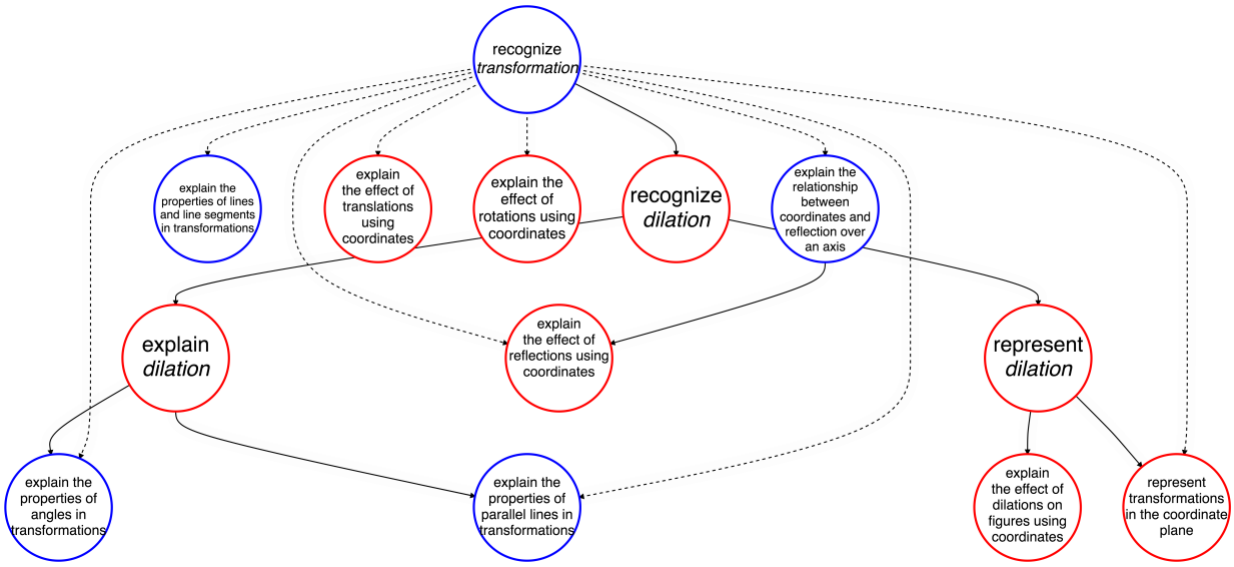
8.GM.A.2 Understand that two-dimensional figures are congruent if a series of rigid transformations can be performed to map the pre-image to the image.

8.GM.A.2.a Describe a possible sequence of rigid transformations between two congruent figures.



**Learning Map Model of* 8.G.2

8.GM.A.3 Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.



**Learning Map Model of* 8.G.3

Node Name	Node Description
EXPLAIN CONGRUENCE THROUGH TRANSFORMATIONS	Make known your understanding through words, drawings, manipulatives, etc., that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations.
EXPLAIN *CONGRUENT ANGLES*	Make known your understanding through words, drawings, manipulatives, etc., that congruent angles are angles that have the same measure.
EXPLAIN *CONGRUENT FIGURES*	Make known your understanding through words, drawings, manipulatives, etc., that congruent figures have the same shape and size.
EXPLAIN *DISTANCE-PRESERVING TRANSFORMATIONS*	Make known your understanding through words, drawings, manipulatives, etc., that reflections, rotations, and translations preserve distances, angle measures, size, and shape.
EXPLAIN *REFLECTION*	Make known your understanding through words, drawings, manipulatives, etc., that a reflection is a transformation that flips a plane containing points, lines, or figures across a line.
EXPLAIN *ROTATION*	Make known your understanding through words, drawings, manipulatives, etc., that a rotation is a transformation that turns a plane containing points, lines, or figures around a point.
EXPLAIN THE EFFECT OF REFLECTIONS USING COORDINATES	For a specified figure and reflection, find the coordinates of the image given the coordinates of the pre-image.
EXPLAIN THE EFFECT OF ROTATIONS USING COORDINATES	For a specified figure and rotation, find the coordinates of the image given the coordinates of the pre-image.
EXPLAIN THE EFFECT OF TRANSLATIONS USING COORDINATES	For a specified figure and translation, find the coordinates of the image given the coordinates of the pre-image.
EXPLAIN THE PROPERTIES OF ANGLES IN TRANSFORMATIONS	Make known your understanding through words, drawings, manipulatives, etc., that in rotations, reflections, translations, and dilations, angles remain angles of the same measure.
EXPLAIN THE PROPERTIES OF LINES AND LINE SEGMENTS IN TRANSFORMATIONS	Make known your understanding through words, drawings, manipulatives, etc., that in rotations, reflections, and translations, lines remain lines and line segments remain line segments of the same length.
EXPLAIN THE PROPERTIES OF PARALLEL LINES IN TRANSFORMATIONS	Make known your understanding through words, drawings, manipulatives, etc., that in rotations, reflections, translations, and dilations, parallel lines remain parallel lines.
EXPLAIN THE RELATIONSHIP BETWEEN COORDINATES AND REFLECTION OVER AN AXIS	Make known your understanding through words, drawings, manipulatives, etc., that coordinate pairs that only differ by signs are reflections across an axis.
EXPLAIN *TRANSLATION*	Make known your understanding through words, drawings, manipulatives, etc., that a translation is a transformation that slides a plane containing points, lines, or figures.
GRAPH POLYGONS IN A COORDINATE PLANE	Through writing or appropriate assistive technology, represent polygons in a coordinate plane given vertices.
MAKE DIRECT COMPARISON OF 2 ANGLES	Directly compare two angles in relation to angle measurement and describe the difference using informal language like greater or less.
PROVE 2 FIGURES ARE CONGRUENT USING A SEQUENCE OF TRANSFORMATIONS	Perform a sequence of distance-preserving transformations that map one figure precisely onto the second figure to prove the two figures are congruent.

RECOGNIZE *CONGRUENT ANGLES*	Identify or name congruent angles.
RECOGNIZE *CONGRUENT FIGURES*	Identify two figures as congruent if the second figure can be obtained from the first figure by a sequence of translations, reflections, and rotations.
RECOGNIZE *REFLECTION*	Identify a reflection as a transformation that flips a plane containing points, lines, or figures across a line of reflection.
RECOGNIZE *ROTATION*	Identify a rotation as a transformation in which a plane containing points, lines, or figures is turned.
RECOGNIZE *TRANSFORMATION*	Identify a transformation (i.e. translation, rotation, reflection, dilation, etc.) as a manipulation of a plane containing points, lines, or figures.
RECOGNIZE *TRANSLATION*	Identify a translation as a transformation in which the position of a plane containing points, lines, or figures is changed.
REPRESENT COMPOSITIONS OF DISTANCE-PRESERVING TRANSFORMATIONS	Through writing or appropriate assistive technology, represent a sequence of distance-preserving transformations on a plane containing points, lines, or figures.
REPRESENT *CONGRUENT ANGLES*	Through writing or appropriate assistive technology, represent congruent angles.
REPRESENT *CONGRUENT FIGURES*	Through writing or appropriate assistive technology, represent congruent figures.
REPRESENT *REFLECTION*	Through writing or appropriate assistive technology, represent a reflection of a plane containing points, lines, or figures.
REPRESENT *ROTATION*	Through writing or appropriate assistive technology, represent a rotation of a plane containing points, lines, or figures.
REPRESENT *TRANSLATION*	Through writing or appropriate assistive technology, represent translations (vertical, horizontal, or diagonal) of a plane containing points, lines, or figures.

TRANSFORMATIONS AND CONGRUENCE

TEACHER NOTES

This unit includes the following documents:

- Learning Map Information
- Instructional Activity (four lessons)
- Instructional Activity Student Handout (for Lessons 2 – 4)
- Instructional Activity Supplement (for Lesson 1)
- Student Activity
- Student Activity Solution Guide

In this unit, students will explore transformations including those represented through technology, with increasing attention to precise language, vocabulary, definitions, and notation.

RESEARCH

Viewing geometry through the lens of transformations provides students with the opportunity to consider mathematics as an interconnected, coherent discipline and engage in higher-level reasoning with multiple representations (Hollebrands, 2003). Transformations can be introduced visually and graphically and can serve to expand a student's understanding of functions with single variable domains and ranges to those with two-dimensional domains and ranges. Thus, experiences with transformations in the context of geometrical figures support students as they study algebraic relations and functions in subsequent courses. Students can use the conceptions they hold of geometric transformations as the basis for the ideas they construct about functions more generally, including meaningful understanding of function notation.

As students gain understanding of transformations, they typically develop increasingly sophisticated ideas (Hollebrands, 2003). Students who have a relatively immature *action conception* can produce an image given a pre-image and information about the transformation. Higher levels of understanding require students to reason about the properties of geometric figures, which implies that students need to operate at the third level of van Hiele's Levels of Geometric Thinking (1986). Specifically, students need to be able to examine and compare the shape, size, and aspects of pre-images and corresponding images to predict or detect effects of transformations. Students who have a *process conception* can imagine a transformed figure without carrying out the transformation concretely. Students who have an *object understanding* view a transformation as a function, mapping points in the plane to points in the plane. These students can reason about composed transformations and the properties that are preserved under one or more transformations.

VAN HIELE'S LEVELS OF GEOMETRIC THINKING

The van Hiele's Levels of Geometric Thinking were developed by Dina van Hiele-Geldof and Pierre van Hiele in order to describe a hierarchy of geometric understanding (Mason, 1998).

Level	Title	Description
1	Visualization	Students recognize figures based on appearance and compare them to a known prototype, such as identifying an object as a circle because it looks like the sun. Students perceive shapes holistically and therefore do not perceive the properties of a figure. Because their classifications are based on appearance, students will consider a rhombus as different from other parallelograms.
2	Analysis	Students see figures as collections of properties and can recognize and list properties of geometric figures. However, the properties are viewed in isolation of each other, and the student cannot discern a relationship between them.
3	Abstraction	Students perceive the relationship between properties and between figures. The student is able to order the properties and it is understood that one property or set of properties can imply another property. However, there is no knowledge of the role of definitions and axioms, and the ability to follow a complex argument is absent. Therefore, students at this level cannot understand or write geometric proofs.
4	Deduction	Students can construct proofs and understand the role of axioms and definitions. However, their understanding is limited to objects in the Euclidian plane and they are not yet able to conceive of non-Euclidian geometry.
5	Rigor	Students are able to understand non-Euclidian geometry and can reason with indirect proof and proof by contrapositive. They can establish and compare geometric and other mathematical systems.

The language we use with students can help or hinder their developing notions of transformations. Usiskin (2015) recommended teachers avoid describing transformations with words indicating motion (e.g., rigid motions). Whereas transformations provide *mathematical models* of physical motions, transformations themselves are not motions. A recommended practice is to reserve informal names (e.g., slide, flip, turn) to describe the physical motion that is modeled by a transformation but to use the mathematical names (i.e., translation, reflection, rotation) to discuss the relationships between the pre-image and image of such a transformation. For example, a teacher might model a translation by "sliding" a triangle drawn on a piece of

patty paper from one position to another position on a coordinate system. This sliding of the patty paper raises an important point to emphasize during instruction: transformations affect the entire plane.

Students need to recognize that transformations affect the entire plane even when students are focused on a particular figure (Usiskin, 1975). Teachers should strive to help students move beyond the perspective that transformations apply only to specific figures and toward the perspective that transformations apply to entire planes:

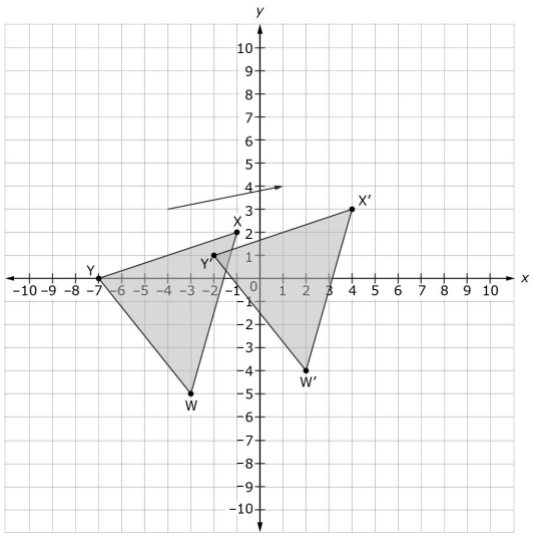
- from the plane as an empty, invisible background to the plane as a set of points
- from geometric figures lying on top of the plane to geometric figures as subsets of points in the plane
- from transformations as physical motions of figures on the plane to transformations as mappings of every point in the plane (DiMatteo, Nikula, & Egan, 2007).

With these guidelines in mind, teachers can consider how to define transformations for students at different levels of knowledge. Early in their experience (i.e., middle school grades), students should recognize that a transformation is a one-to-one correspondence between a set of points making up the pre-image and a set of points making up the image (Usiskin, 2015). Note the set of points should not be restricted to the points defining the shape (e.g., the three vertices of a triangle), but should encompass the plane containing them. Middle school students should also be able to conduct sequential transformations (i.e., composition). Careful instruction with composed transformations can support students' understanding when they compose algebraic functions in subsequent courses. In high school, teachers can help students deepen their understanding by describing transformations as functions that map the plane containing the pre-image (i.e., domain) onto the plane containing the image (i.e., range) (Sinclair, Pimm, & Skelin, 2012). Furthermore, using algebraic language in the study of transformations (e.g., domain, range) allows students to foster connections between pre-image and domain and between image and range. These cognitive connections can help students clarify their conceptions of identity and inverse functions, as well as composition of functions (Usiskin, 2015).

Students should develop an understanding of increasingly formal definitions for each type of transformation. Early experiences can focus on geometric or descriptive definitions, but these need to be replaced by definitions acknowledging the parameters needed for each transformation. For example, translations can first be defined as rules for determining an image a certain distance and direction away from the pre-image. A more sophisticated way to define a translation is to use a vector, which inherently describes a distance and direction.

AN EXAMPLE

The image below illustrates the translation of triangle XYZ according to the distance and direction provided by the vector in the coordinate plane.



Students who hold *object conceptions* of transformations should define transformations in terms of these parameters:

Transformation	Defining Information and Parameters
Translation	Pre-image, vector
Rotation	Pre-image, center of rotation, angle of rotation
Reflection	Pre-image, line of reflection
Dilation	Pre-image, center of dilation, scale factor

Throughout instruction, teachers should draw attention to defining information and parameters for each distance-preserving transformation to support students' conceptual development of transformations. In order to assess the level of understanding students hold, questions have been included in the STUDENT ACTIVITY to address an *object conception*.

Student understanding of congruence can develop parallel to basic understandings of transformations, however both types of knowledge are required in order to relate distance-preserving transformations and congruence. The learning map section for this sequence of activities depicts students' ability to recognize congruent angles and figures. These concepts are shown alongside students' ability to first recognize that a transformation has occurred and then represent and explain each distance-preserving transformation. Once students understand translations, reflections, and rotations, they are able to focus on specific properties of

figures under these transformations. The ability to analyze these properties, combined with students' knowledge of congruence, provides the foundation for students to explain congruence through transformations and, ultimately, to prove two figures are congruent through a series of transformations. Similarly, students can apply their knowledge of transformations, including dilations, to support their understanding of similarity. Ultimately, students should create more formal definitions of transformations and apply transformational reasoning to their work with algebraic functions.

LEARNING MAP INFORMATION

The learning map section for this sequence of activities progresses from a recognition of the different types of transformations to an ability to represent them. Further, students will progress to explaining the transformations and their corresponding properties. For example, after students can recognize a rotation they are then able to represent a rotation, first as an informal "turn" and then as a more precisely calculated effect upon the coordinates of a figure and the plane containing it. After a strong visual foundation is developed through both recognizing and representing the transformations, students proceed to explaining the different features of transformations, such as their distance-preserving properties and that parallel lines remain parallel through transformations.

INSTRUCTIONAL ACTIVITIES

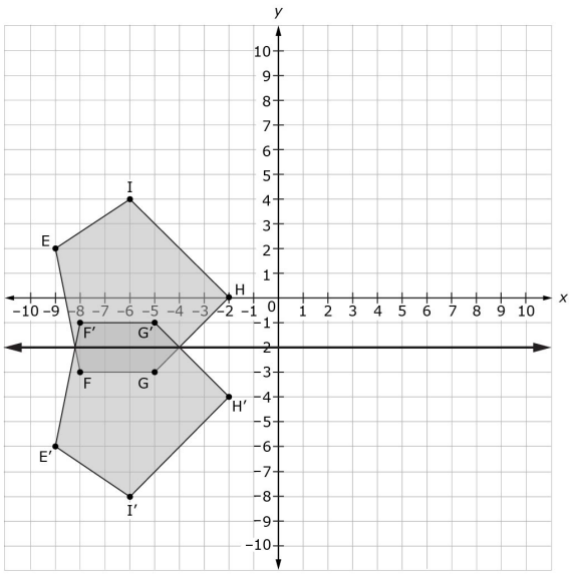
Because of the relatively unfamiliar approach we have taken to teaching transformations, we will provide in the following paragraphs a more detailed explanation of the activities in this unit and their ability to support student understanding of transformations.

The activities in this unit are designed to build on recommendations from research studies investigating student learning. Students need practice examining relationships among aspects of pre-images, images, and the parameters of translations. Specifically, students should attend to lengths, angle measures, areas, parallelism, perpendicularity, collinearity, and orientation of pre-images and images to consider which aspects are preserved under particular transformations and which are not. Additionally, students should practice identifying the effects of parameters on images.

Teachers are wise to have their students work with asymmetrical shapes, particularly for reflections. Students should practice reflections over oblique lines from the start to avoid the misconception that the only lines of reflection are horizontal or vertical (Ronau, Meyer, & Crites, 2015). Students also should practice reflecting drawings that cross over the line of reflection to help them develop their skill in "seeing" the image in spite of its overlap with the pre-image.

AN EXAMPLE

The image below illustrates the reflection of an asymmetric pentagon that crosses over the line of reflection.



Students should practice composing transformations and discover how certain combinations produce identities or other transformations. For example, the image produced by the composition of two reflections over parallel lines is the same as the image of a translation. Similarly, the image produced by the composition of two reflections over intersecting lines is the same as the image of a rotation. To build an understanding about symmetry, teachers should introduce symmetry through reflections. Distinguish between the process of reflecting or rotating about a particular line or point as compared to describing symmetries that exist within a figure.

In this unit, students will learn about translations, rotations, and reflections. These are three distance-preserving transformations that are also known as isometries.

> Dilations, another type of transformation where distances are not preserved, will be addressed in a separate instructional activity.

The two main formats for exploring transformations are on blank canvases or using coordinate grids. The materials in this unit provide teachers and students with opportunities to work in both formats. In the first lesson, students will explore translations, reflections, and rotations using concrete materials on a blank canvas, then increase their initial understanding of each distance-preserving transformation through the use of technology. In the second lesson, students will practice performing each transformation on the coordinate

plane with the use of patty paper to represent the plane containing the points and figures that are transformed. In both lessons, there is an emphasis on congruence through transformations in an effort to help students understand that angle measures and line segments are preserved during translations, reflections, and rotations.

The third lesson builds on students' knowledge of translations and reflections by prompting students to notice patterns in the coordinates as particular transformations occur. In the fourth lesson, students will perform compositions of transformations and note specific compositions of reflections which could be defined by a different transformation. These experiences culminate in opportunities for students to use reasoning about series of transformations to prove two figures congruent (or not).

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TRANSFORMATIONS AND CONGRUENCE

OVERVIEW OF INSTRUCTIONAL ACTIVITIES

Lesson	Learning Goal	Nodes Addressed
Lesson 1	Students will discover distance-preserving transformations and become acquainted with related vocabulary.	▶ RECOGNIZE *CONGRUENT ANGLES* ▶ EXPLAIN *CONGRUENT ANGLES* ▶ RECOGNIZE *CONGRUENT FIGURES* ▶ EXPLAIN *CONGRUENT FIGURES* ▶ RECOGNIZE *TRANSLATION* ▶ EXPLAIN *TRANSLATION* ▶ RECOGNIZE *REFLECTION* ▶ EXPLAIN *REFLECTION* ▶ RECOGNIZE *ROTATION* ▶ EXPLAIN *ROTATION*
Lesson 2	Students will learn the definition and properties of distance-preserving transformations, practice performing distance-preserving transformations, and describe distance-preserving transformations that have already occurred.	▶ RECOGNIZE *TRANSLATION* ▶ REPRESENT *TRANSLATION* ▶ EXPLAIN *TRANSLATION* ▶ EXPLAIN THE PROPERTIES OF ANGLES IN TRANSFORMATIONS ▶ EXPLAIN THE PROPERTIES OF LINES AND LINE SEGMENTS IN TRANSFORMATIONS ▶ EXPLAIN THE PROPERTIES OF PARALLEL LINES IN TRANSFORMATIONS ▶ EXPLAIN THE EFFECT OF TRANSLATIONS USING COORDINATES ▶ RECOGNIZE *REFLECTION* ▶ REPRESENT *REFLECTION* ▶ EXPLAIN *REFLECTION* ▶ EXPLAIN THE EFFECT OF REFLECTIONS USING COORDINATES ▶ RECOGNIZE *ROTATION* ▶ REPRESENT *ROTATION* ▶ EXPLAIN *ROTATION* ▶ EXPLAIN THE EFFECT OF ROTATIONS USING COORDINATES
Lesson 3	Students will discover the impact of translations and reflections on the coordinates of a figure, then generalize the rule for any figure under the described transformation.	▶ REPRESENT *TRANSLATION* ▶ EXPLAIN *TRANSLATION* ▶ EXPLAIN THE EFFECT OF TRANSLATION USING COORDINATES ▶ REPRESENT *REFLECTION* ▶ EXPLAIN *REFLECTION* ▶ EXPLAIN THE EFFECT OF REFLECTIONS USING COORDINATES ▶ REPRESENT *ROTATION* ▶ EXPLAIN *ROTATION* ▶ EXPLAIN THE EFFECT OF ROTATIONS USING COORDINATES
Lesson 4	Students will perform a series of transformations and explain that two figures are congruent when there is a series of distance-preserving transformations that maps one figure onto the other.	▶ EXPLAIN CONGRUENCE THROUGH TRANSFORMATIONS ▶ PROVE 2 FIGURES ARE CONGRUENT USING A SEQUENCE OF TRANSFORMATIONS

TRANSFORMATIONS AND CONGRUENCE

INSTRUCTIONAL ACTIVITY

Lesson 1

LEARNING GOAL

Students will discover distance-preserving transformations and become acquainted with related vocabulary.

PRIMARY ACTIVITY

Students will use cut-out shapes, patty paper, and technology to discover rotations, reflections, and translations.

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

- Congruent
- Corresponding
- Rotation
- Reflection
- Translation

MATERIALS

- INSTRUCTIONAL ACTIVITY SUPPLEMENT (Recommend one copy for every student.)
- Patty paper or tracing paper
- Spaghetti, straws, or another straight, rigid object
- Tape
- Protractors
- Internet access

IMPLEMENTATION

NOTE: To create modified graphs or additional graphs to supplement any of the lessons the following websites can be helpful.

Online Charts:
https://www.onlinecharttool.com/

Kid's Zone: Learning with NCES:
https://nces.ed.gov/nceskids/graphing/classic/

To begin the lesson, students will need to cut along the black lines in the INSTRUCTIONAL ACTIVITY SUPPLEMENT. This will result in five rectangles, each containing a figure. Each rectangle containing a figure represents a plane, though it should be noted that planes continue infinitely whereas the paper is unable to do so.

Students will first model a translation of a plane containing a figure.

Direct students to select the plane containing the rectangle from the shapes they cut out and place it on their desks. **Ask** students to place a piece of patty paper over the figure and trace the figure onto the patty paper. Then **ask** students to slide the patty paper containing the figure and representing the plane to a corner of their desks without changing the orientation or rotating the plane. After students slide the plane, **ask** them to describe the movement using words such as "up," "down," "left," and "right." The original plane represents the pre-image, while the patty paper plane represents the image.

Ask students to slide the plane to a different corner of their desk without rotating it, maintaining the orientation, and again describing the movement in words.

Repeat this activity with each plane students cut out.

Inform students that this movement of a plane containing a figure is called a *translation*.

Ask students if the size or shape of the figure changes in a translation. Students should indicate that neither one does.

GUIDING QUESTIONS

Elicit student thinking:

- What can you say about the planes before and after the translation?
- What changes about the figure? What stays the same?

Determine if the student can RECOGNIZE *CONGRUENT ANGLES*:

- Has the appearance of corresponding angles changed in the translation?
- Are there any angles that are the same size in the pre-image and the image?
- Are the corresponding angles of a figure before and after a translation congruent?

Determine if the student can EXPLAIN *CONGRUENT ANGLES*:

- What does it mean for angles to be congruent?

Determine if the student can RECOGNIZE *CONGRUENT FIGURES*:

- Has the shape of the figure changed during the translation?
- Has the size of the figure changed during the translation?
- Is each figure before a translation congruent to the corresponding figure after the translation?

Determine if the student can EXPLAIN *CONGRUENT FIGURES*:

- What does it mean for figures to be congruent?
- Could you use translations to show figures are congruent? If so, how? If not, why not?

Determine if the student can RECOGNIZE *TRANSLATION*:

- [Provide an example if necessary] What type of transformation occurs when you slide a plane containing a figure?
- How do you expect the appearance of a figure to change in a translation?

Determine if the student can EXPLAIN *TRANSLATION*:

- How would you describe a translation?
- What information do you need to perform a specific translation?

Next, students will model a reflection. Similar to the previous activity with translations, patty paper models the plane in the following activity.

Direct students to draw an "L" on one side of their patty paper. **Ask** students to fold the patty paper and, while the paper is folded, trace the "L" on the other side of the fold.

Ask students to unfold the patty paper and note the distance from the original "L" to the crease in the paper that is acting as the line of reflection and the distance from the traced "L" to the crease in the paper. Students should note that the distances are the same from corresponding parts of the "L" to the line of reflection.

Repeat this activity with a variety of figures. Have students fold so the line of reflection is vertical, horizontal, and diagonal.

Inform students that this movement is called a *reflection* and the crease in the patty paper is the *line of reflection*. The line of reflection splits the patty paper into two planes, and one plane "flips" over onto the other.

Ask students if the size or shape of the figure changes in a reflection. Students should indicate that neither one does.

GUIDING QUESTIONS

Elicit student thinking:

- What can you say about the planes before and after the reflection?
- What is the same about the two figures?
- What is different about the two figures?

Determine if the student can RECOGNIZE *CONGRUENT ANGLES*:

- Has the appearance of corresponding angles changed in the reflection?
- Are there any angles that are the same size in the pre-image and the image?
- Are the corresponding angles of a figure before and after a reflection congruent?
- Can you point to two congruent angles on your piece of patty paper?

Determine if the student can EXPLAIN *CONGRUENT ANGLES*:

- What does it mean for angles to be congruent?

Determine if the student can RECOGNIZE *CONGRUENT FIGURES*:

- Has the shape of the figure changed during the reflection?
- Has the size of the figure changed during the reflection?
- Is each figure before a reflection congruent to the corresponding figure after the reflection?

Determine if the student can EXPLAIN *CONGRUENT FIGURES*:

- What does it mean for figures to be congruent?
- Could you use translations to show figures are congruent? If so, how? If not, why not?

Determine if the student can RECOGNIZE *REFLECTION*:

- [Provide an example if necessary] What type of transformation occurs when you flip a figure across a line?
- How do you expect the image of a figure to be produced in a reflection?

Determine if the student can EXPLAIN *REFLECTION*:

- How would you describe a reflection?
- What information do you need to perform a specific reflection?

Finally, students will model a rotation using the planes containing figures used for translations and patty paper.

Direct students to select the plane containing the figure "E" and place it on their desks. Similar to translations, have students place a piece of patty paper over the plane and trace the figure. **Ask** students to place a finger on one corner or side of the "E" and hold that point stationary.

Ask students to turn only the patty paper plane clockwise and counterclockwise, at varying angle measures. Angle measures can be approximated using right angles as a general reference.

Repeat this activity with each plane students cut out, noting that you can rotate around any point on or in a figure.

Follow the rotations about points within or on the figure to a rotation about a point in the plane outside.

Similar to translations, **require** students to either place a piece of patty paper over a plane containing a figure and trace the figure, or to use the patty paper tracings they have already created and superimpose the plane containing the figure with the piece of patty paper containing the same figure.

Students can then tape the piece of patty paper to the end of the straight object (e.g., a piece of uncooked spaghetti or a straw). While holding the end of the straight object opposite the patty paper stationary, students can rotate the patty paper plane (the image) clockwise and counterclockwise, at varying angle measures, using a protractor for reference, while maintaining the position of the original plane (the pre-image).

Ask students what they notice about the distance between the figure and the point they are rotating about. Students should indicate that this distance is maintained throughout the rotation.

Repeat this activity with various planes and with varying lengths of the straight object used to rotate the plane.

Inform students that this movement is called a *rotation*, and that the point they rotate around is called the *center of rotation*.

Ask students if the size or shape of the figure changes in a rotation. Students should indicate that neither one does.

GUIDING QUESTIONS

Elicit student thinking:

- What can you say about the planes before and after the rotation?
- What is the same about the two figures?
- What is different about the two figures?

Determine if the student can RECOGNIZE *CONGRUENT ANGLES*:

- Has the appearance of corresponding angles changed in the rotation?
- Are there any angles that are the same size in the pre-image and the image?
- Are the corresponding angles of a figure before and after a rotation congruent?

Determine if the student can EXPLAIN *CONGRUENT ANGLES*:

- What does it mean for angles to be congruent?

Determine if the student can RECOGNIZE *CONGRUENT FIGURES*:

- Has the shape of the figure changed during the rotation?
- Has the size of the figure changed during the rotation?
- Is each figure before a rotation congruent to the corresponding figure after the rotation?

Determine if the student can EXPLAIN *CONGRUENT FIGURES*:

- What does it mean for figures to be congruent?
- Could you use translations to show figures are congruent? If so, how? If not, why not?

Determine if the student can RECOGNIZE *ROTATION*:

- [Provide an example if necessary] What type of transformation occurs when you turn a figure around a specific point?
- What motion is modeled by a rotation?

Determine if the student can EXPLAIN *ROTATION*:

- How would you describe a rotation?
- What information do you need to perform a specific rotation?

Once students have physically translated, reflected, and rotated shapes on their desk, have them experiment with technology to gain a deeper, more precise understanding of each distance-preserving transformation.

Share an overview video from Khan Academy using this link.

Learn how to complete a translation on Khan Academy using this link.

Practice translations using this link.

Students should follow the directions and answer the questions provided until they correctly complete five in a row.

Direct students to the National Library of Virtual Manipulatives website (on a browser that supports Java) to experiment with reflections by completing the activity in the right panel of the website.

Discuss the students' responses to all three questions to develop an understanding of the behavior of reflections.

> NOTE: Students must have knowledge of how to graph a line in slope-intercept form as well as how to graph horizontal and vertical lines in order to participate in the Khan Academy lessons related to reflections. This is not required knowledge for this particular unit of study or the related standards, but it incorporates elements of students' algebraic knowledge into this geometry topic. If you choose to have students complete the following reflection materials on Khan Academy noted with asterisks, students may need to review these concepts. If you do not wish to include graphing lines in performing reflections, skip the items with asterisks and continue to rotations.

*** Learn** how to complete a reflection on Khan Academy using this link.

*** Practice** reflections using this link.

***** Students should follow the directions and answer the questions provided until they correctly complete five in a row.

Learn how to complete a rotation on Khan Academy using this link.

Practice rotations using this link. Note that as students rotate, the tool rotates in 15° increments.

Students should follow the directions and answer the questions provided until they correctly complete five in a row.

At the end of the activity, teachers should ask students to describe each distance-preserving transformation and whether the size or shape of the object changes under that transformation.

Plane cutouts:

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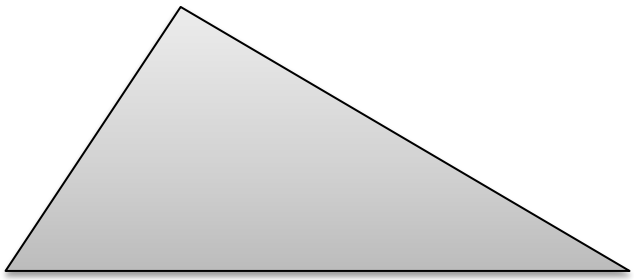


TRANSFORMATIONS AND CONGRUENCE

INSTRUCTIONAL ACTIVITY

Lesson 2

LEARNING GOAL

Students will learn the definition and properties of distance-preserving transformations, practice performing distance-preserving transformations, and describe distance-preserving transformations that have already occurred.

PRIMARY ACTIVITY

Students will use patty paper and the coordinate plane to translate, reflect, and rotate figures and describe translations, reflections, and rotations.

NOTE: This lesson will likely require more than one class period to complete.

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

- Translation
- Reflection
- Rotation
- Pre-image
- Image
- Plane
- Vector
- Line of reflection
- Center of rotation
- Angle of rotation
- Congruent

MATERIALS

- Patty paper or tracing paper
- Colored pencils/markers/pens
- INSTRUCTIONAL ACTIVITY STUDENT HANDOUT

IMPLEMENTATION

This lesson begins with translations, emphasizing more precise language, vocabulary, definitions, and notation than was introduced in LESSON 1.

Ask students if they remember from the first lesson the name of the transformation that models "sliding" a figure. Once students identify this transformation as a translation, **show an example** of a translation by labeling the vertices of the pre-image (e.g., ABC) and the image (e.g., A'B'C') in addition to labeling the figures with the words *pre-image* (figure before the translation) and *image* (figure after the translation). Require students to practice this strategy of labeling and naming their transformations so they can follow their work.

Define translation as a distance-preserving transformation that slides a plane containing a figure while maintaining the size, shape, and orientation of the figure.

Ask what aspects of the figures in the pre-image and image are different and what aspects are the same. Students should identify that the locations of the pre-image and image are different, but that the size, shape, and orientation remain the same.

Introduce or review that the word *congruent*, as defined through transformations, states two figures, α and β, are *congruent* if and only if there is a distance-preserving transformation which maps α onto β. Note the difference between the words *congruent* and *equal*. Figures can be congruent, while numbers can be equal. It is inaccurate to state two figures are *equal*. This attention to vocabulary addresses the Standard for Mathematical Practice "attend to precision."

Ask students if the pre-image and image are congruent in a translation. Students should indicate that the pre-image and image are congruent.

Discuss that it is not just the pre-image, but the entire plane containing the pre-image, that translates. Students will demonstrate this idea using patty paper to represent the plane in relation to a coordinate grid in the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

Require students to practice Tasks 1 and 2 on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT. It may be beneficial to complete the first task as a class and have students practice the second task in pairs or individually.

NOTE: The tasks in the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT require students to plot points to graph the pre-image on the coordinate plane, though the focus of the activity is on the transformations. A student who is struggling may need to address concepts related to plotting points on a coordinate plane rather than concepts related to transformations.

Review Tasks 1 and 2 before introducing the idea that a vector can replace a verbal description of a translation. Return to Task 1 and have students draw an arrow (vector) from each vertex of the pre-image to the corresponding vertex of the image.

Ask students what they notice about all the arrows they drew (students should note that the arrows are parallel or have the same rate of change/slope/direction and that they are all the same length).

Ask students how these arrows are related to the translation they just performed. If necessary, **explain** that this is the vector representing "down five units" on the coordinate plane and it can be drawn anywhere on the coordinate plane (it does not have to connect corresponding vertices, because the entire plane translates). Have students draw another vector with the same direction and magnitude and explain that even in the new location, the vector indicates that you are going to translate the plane containing the figure down five units.

Ask students what types of arrows would indicate translations "right three units" and "up six units." Check that students generalize their recognition of the length of the arrow to indicate the distance of a translation and the orientation of the arrow to indicate the direction of the translation. **Ask** students how they would use one or more arrows to indicate a translation "up two units" and "right three units." Does this translation require two arrows? Why? Why not?

For Task 2, have students repeat the process described for the first task. Students should first connect corresponding vertices with vectors and then draw an additional vector representing "up seven units and right three units" on the coordinate plane.

For Task 3, there is a vector already drawn on the coordinate plane. **Ask** students to describe and write, in words, the direction the vector indicates for the translation. Once students have correctly responded with the direction of the translation for Task 3, have them complete Tasks 3 and 4 in pairs or individually.

Review Tasks 3 and 4 to ensure accuracy. Then, have students describe the translations that have occurred in Tasks 5 through 8 with words and with a vector.

GUIDING QUESTIONS

Elicit student thinking:

- What are we doing with patty paper in this activity?
- What does the patty paper represent?

Determine if the student can RECOGNIZE *TRANSLATION*:

- [Point to an example if necessary.] What type of transformation occurs when you create an image of a figure by sliding the pre-image?
- How do you expect the image of a figure to be produced in a translation?
- Which transformation describes the movement of the plane using direction and distance (e.g., right two and down five)?

Determine if the student can REPRESENT *TRANSLATION*:

- How would you translate this plane if you are provided the direction left three and up one?
- What direction does this vector tell you to translate the plane?

Determine if the student can EXPLAIN *TRANSLATION*:

- How would you describe a translation?
- What information do you need to perform a specific translation?
- Do the points on the figure translate? Do the points inside the figure translate? Do the points outside the figure translate?

Determine if the student can EXPLAIN THE PROPERTIES OF ANGLES IN TRANSFORMATIONS:

- What happens to the angle measures when a figure translates?
- Are the angles in the pre-image congruent to the angles in the image?

Determine if the student can EXPLAIN THE PROPERTIES OF LINES AND LINE SEGMENTS IN TRANSFORMATIONS:

- How would you describe the line segments in the image compared to the line segments in the pre-image?
- Do the line segments change length when they are translated?
- Are the line segments in the pre-image congruent to the line segments in the image?

Determine if the student can EXPLAIN THE PROPERTIES OF PARALLEL LINES IN TRANSFORMATIONS:

- Were there parallel lines in the pre-image?
- Are there parallel lines in the image?
- Do the parallel lines in the pre-image correspond to the parallel lines in the image?

Determine if the student can EXPLAIN THE EFFECT OF TRANSLATIONS USING COORDINATES:

- What are the coordinates of the pre-image? What are the coordinates of the image?
- When translating up or down, which coordinate changes? Why?
- When translating left or right, which coordinate changes? Why?

This lesson continues with reflections, again emphasizing more precise language, vocabulary, definitions, and notation than was introduced in LESSON 1.

Ask students if they remember which transformation they created by folding patty paper and tracing the figure in the first lesson. Once students identify this transformation as a reflection, **show an example** of a reflection by labeling the vertices of the pre-image (e.g., ABCD) and the image (e.g., A'B'C'D') in addition to labeling the figures with the words *pre-image* (figure before the reflection) and the *image* (figure after the reflection).

Define *reflection* as a distance-preserving transformation that generates an image of a plane containing a figure (pre-image) across a fixed line of reflection, thereby modeling the motion of flipping the

plane containing the pre-image over the line of reflection. **Note** the difference between a reflection across a line of reflection and lines of symmetry, which students likely have experience with from earlier grades. A line of reflection is used to generate an image in the plane given a pre-image in the plane, whereas a line of symmetry divides a figure in half, where the parts of the figure lying on each side of the line of symmetry are mirror reflections of each other.

Ask what aspects of the figures in the pre-image and image are different and what aspects are the same. Students should identify that the location and orientation of the pre-image and image are different, but the size and shape remain the same.

Review that the word *congruent*, as defined through transformations, states two figures, α and β, are *congruent* if and only if there is a distance-preserving transformation which maps α onto β.

Ask students if the pre-image and image are congruent in a reflection. Students should indicate that the pre-image and image are congruent.

Discuss that it is not just the pre-image, but the entire plane containing the pre-image, that reflects. This means the distance between the coordinates on the pre-image and the line of reflection should equal the distance between the corresponding coordinates on the image and the line of reflection. Students will represent this using patty paper (or adapted for students to use a Mira instead of patty paper) on a coordinate plane in the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

Require students to practice Tasks 9 through 12 on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT. It may be beneficial to complete the first task as a class and have students practice the second task in pairs or individually.

Review Tasks 9 through 12 to ensure accuracy, then have students draw the line of reflection for Tasks 13 through 16.

GUIDING QUESTIONS

Elicit student thinking:

- What are we doing with patty paper in this activity?
- What does the patty paper represent?

Determine if the student can RECOGNIZE *REFLECTION*:

- [Point to an example if necessary.] What type of transformation occurs when you create an image of a figure across a line?
- How do you expect the image of a figure to be produced in a reflection?

Determine if the student can REPRESENT *REFLECTION*:

- How would you reflect this plane across a vertical line? Across a horizontal line? Across a diagonal line?
- How would you describe the difference between reflecting across a line that goes through the figure compared to a line that is outside the figure?

Determine if the student can EXPLAIN *REFLECTION*:

- How would you describe a reflection?
- What information do you need to perform a specific reflection?
- Do the points on the figure reflect? Do the points inside the figure reflect? Do the points outside the figure reflect?

Determine if the student can EXPLAIN THE PROPERTIES OF ANGLES IN TRANSFORMATIONS:

- What happens to the angle measures when a figure reflects?
- Are the angles in the pre-image congruent to the angles in the image?

Determine if the student can EXPLAIN THE PROPERTIES OF LINES AND LINE SEGMENTS IN TRANSFORMATIONS:

- How would you describe the line segments in the image compared to the line segments in the pre-image?
- Do the line segments change length when they are reflected?
- Are the line segments in the pre-image congruent to the line segments in the image?

Determine if the student can EXPLAIN THE PROPERTIES OF PARALLEL LINES IN TRANSFORMATIONS:

- Were there parallel lines in the pre-image?
- Are there parallel lines in the image?
- Do the parallel lines in the pre-image correspond to the parallel lines in the image?

Determine if the student can EXPLAIN THE EFFECT OF REFLECTIONS USING COORDINATES:

- What are the coordinates of the pre-image? What are the coordinates of the image?
- When reflecting across a vertical line, which coordinate changes? Why?
- When reflecting across a horizontal line, which coordinate changes? Why?

This lesson concludes with rotations, emphasizing more precise language, vocabulary, definitions, and notation than was introduced in LESSON 1.

Ask students if they remember from the first lesson the name of the transformation that models "turning" a figure. Once students identify this transformation as a rotation, **show an example** of a rotation by labeling the vertices of the pre-image (e.g., SRT) and the image (e.g., S'R'T') in addition to labeling the figures with the words *pre-image* (figure before the rotation) and the *image* (figure after the rotation).

Define a *rotation* as a distance-preserving transformation that turns the plane containing a figure about a fixed point.

Ask what aspects of the figures in the pre-image and image are different and what aspects are the same. Students should identify that the locations of the pre-image and image are different, but that the size, shape, and orientation within the shape remain the same. Note: this usage of the word "orientation" refers to the points that define the shape and their relative position in relation to each other.

Review that the word *congruent*, as defined through transformations, states two figures, α and β, are *congruent* if and only if there is a distance-preserving transformation which maps α onto β.

Ask students if the pre-image and image of a rotation are congruent. Students should indicate that the pre-image and image are congruent.

Discuss that it is not just the figure depicted in the pre-image, but the entire plane containing the pre-image, that rotates. Students will represent this using patty paper on a coordinate plane in the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

Require students to practice Tasks 17 through 20 on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT. It may be beneficial to complete Task 17 as a class and have students practice Tasks 18 through 20 in pairs or individually.

Review Tasks 17 through 20 to ensure accuracy, then have students draw in the center of rotation for Tasks 21 through 24.

GUIDING QUESTIONS

Elicit student thinking:

- What are we doing with patty paper in this activity?
- What does the patty paper represent?

Determine if the student can RECOGNIZE *ROTATION*:

- [Point to an example if necessary.] What type of transformation models the motion of turning a figure around a point?
- How do you expect the image of a figure to be produced in a rotation?
- What motion is modeled by a rotation?

Determine if the student can REPRESENT *ROTATION*:

- How would you rotate this plane 90° counterclockwise around the origin? 180° counterclockwise around the origin? 270° counterclockwise around the origin?
- What clockwise rotation about the origin produces the same result as a 270° rotation counterclockwise about the origin?
- How would you describe the difference between rotating around a point that is in or on a figure and rotating around a point that is outside a figure?

Determine if the student can EXPLAIN *ROTATION*:

- How would you describe a rotation?
- What information do you need to perform a specific rotation?
- Do the points on the figure rotate? Do the points inside the figure rotate? Do the points outside the figure rotate?

Determine if the student can EXPLAIN THE PROPERTIES OF ANGLES IN TRANSFORMATIONS:

- What happens to the angles when a figure rotates?
- Are the angles in the pre-image congruent to the angles in the image?

Determine if the student can EXPLAIN THE PROPERTIES OF LINES AND LINE SEGMENTS IN TRANSFORMATIONS:

- How would you describe the line segments in the image compared to the line segments in the pre-image?
- Do the line segments change length when they are rotated?
- Are the line segments in the pre-image congruent to the line segments in the image?

Determine if the student can EXPLAIN THE PROPERTIES OF PARALLEL LINES IN TRANSFORMATIONS:

- Were there parallel lines in the pre-image?
- Are there parallel lines in the image?
- Do the parallel lines in the pre-image correspond to the parallel lines in the image?

Determine if the student can EXPLAIN THE EFFECT OF ROTATIONS USING COORDINATES:

- What are the coordinates of the pre-image? What are the coordinates of the image?
- When rotating at any angle less than 360°, does the *x*–coordinate change? Does the *y*–coordinate change?

Students should be required to be precise, complete, and accurate with their labels and figures for each transformation.

At the end of the activity, provide students with three blank coordinate planes and direct them to draw a figure and label it the "pre-image" on each coordinate plane (the figure can be the same on each coordinate plane or it can vary).

Require students to label one of the coordinate planes "translation," perform a translation on the plane, and label the result of the translation the "image." **Require** students to describe the translation in words and with a vector.

Require students to label another coordinate plane "reflection," draw a line of reflection (preferably not horizontal or vertical), reflect the plane, and label the result of the reflection the "image."

On the third coordinate plane, **require** students to label it "rotation," plot a point (preferably not on the figure) to serve as the center of rotation, rotate the figure 90° counterclockwise, and label it the "image."

For each transformation, **ask** students to write whether the pre-image and image are congruent and to explain their reasoning.

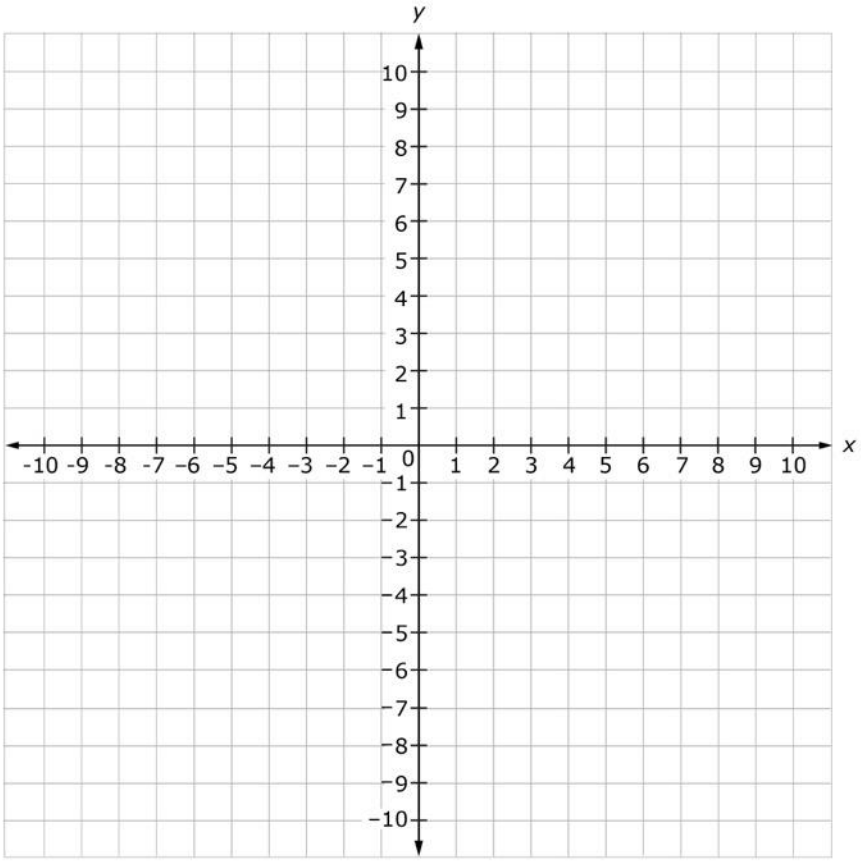
Name_____

TRANSFORMATIONS AND CONGRUENCE

Lesson 2

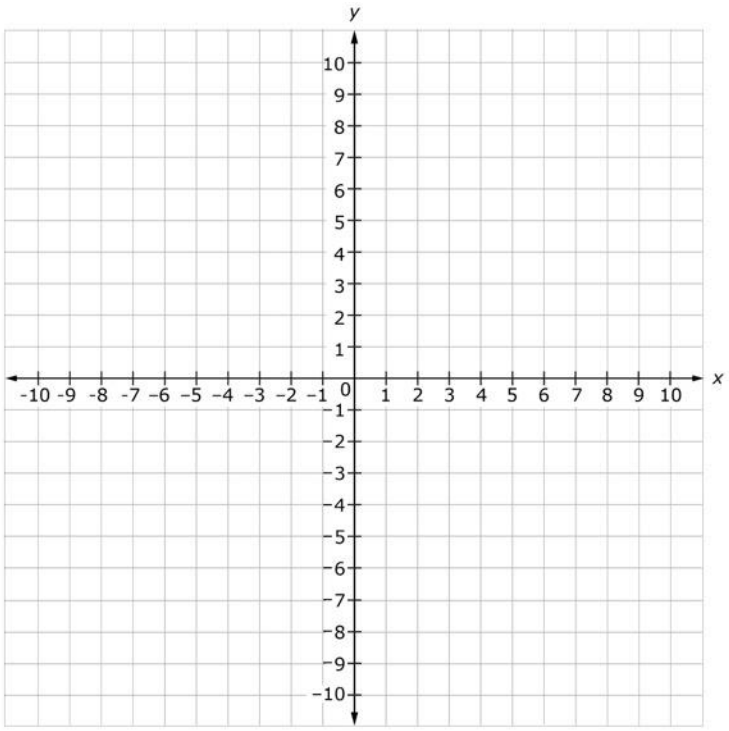
For each translation in Questions 1 through 4, plot the points of the pre-image on the coordinate plane, then place patty paper over the coordinate plane and trace the pre-image. Perform the requested translation by sliding the plane (represented by patty paper), noting the coordinates of the image, and recording the image in a different color on the coordinate plane. Be sure to label the vertices of the pre-image and image.

1. Translate the plane containing rectangle ABCD with vertices A(4, 4), B(1, 4), C(4, 0), and D(1, 0) down 5 units.

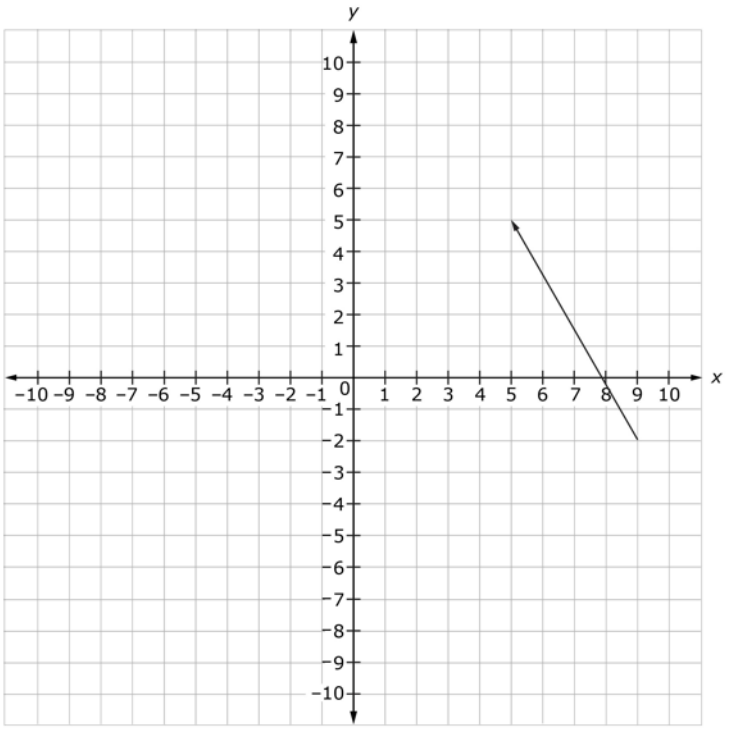


Name_____

2. Translate the plane containing triangle RST with vertices R(–8, –2), S(–6, –5), and T(–1, –1) up 7 units and right 3 units.

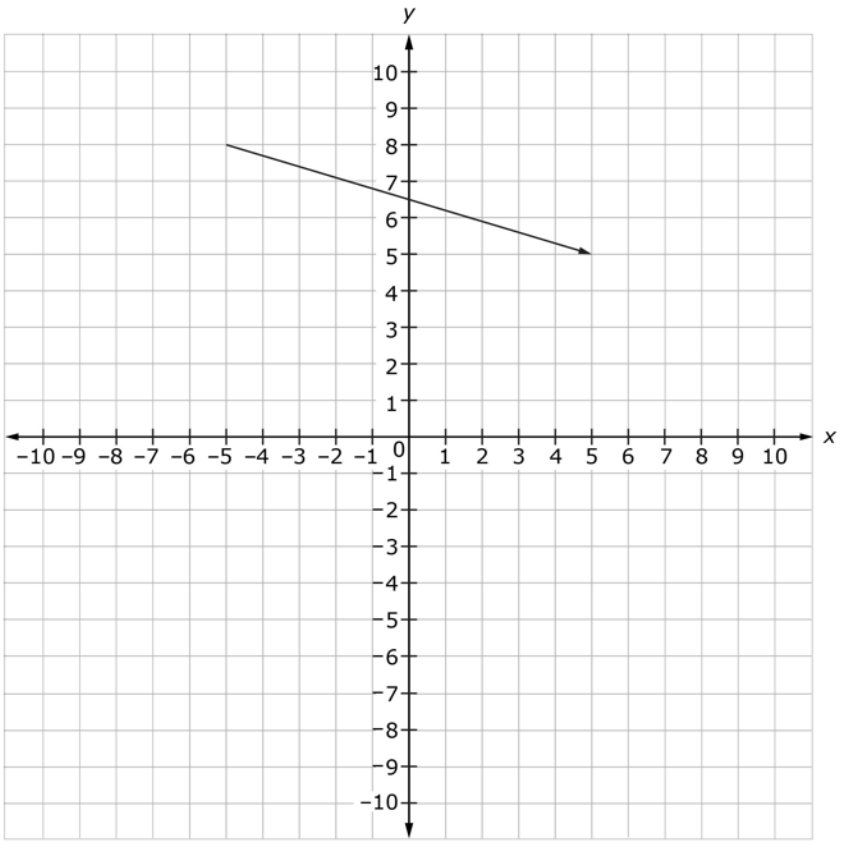


3. Translate the plane containing triangle XYZ with vertices X(3, –5), Y(4, –2), and Z(7, –8) according to vector provided.



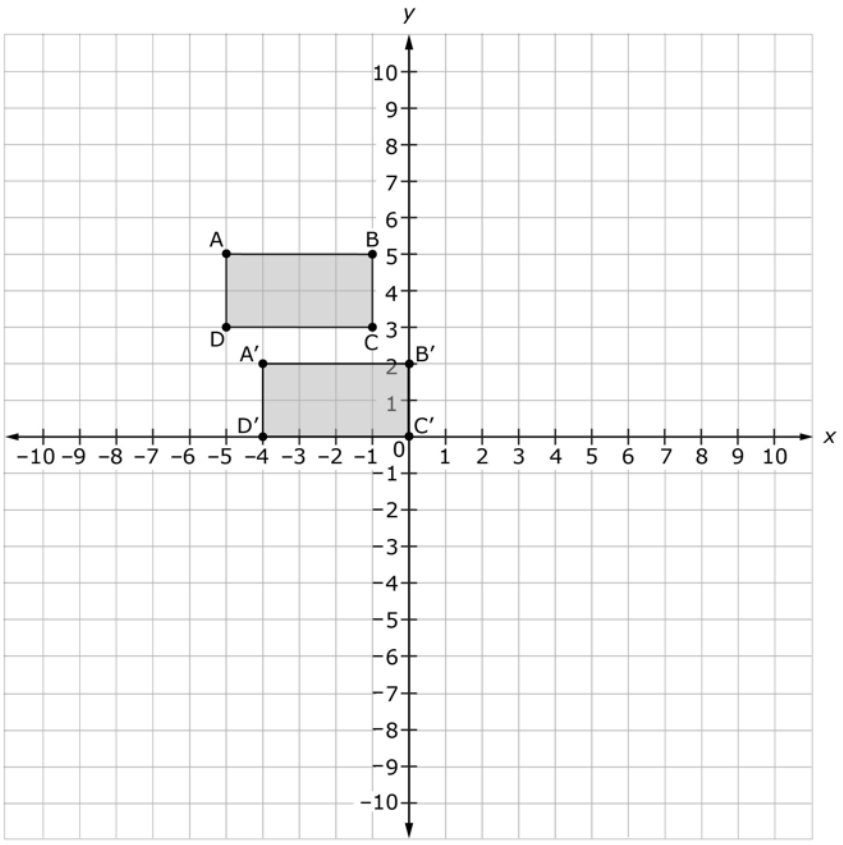
Name_____

4. Translate the plane containing rectangle LMNO with vertices L(–9, 7), M(–9, 2), N(–7, 2), and O(–7, 7) according to vector provided.



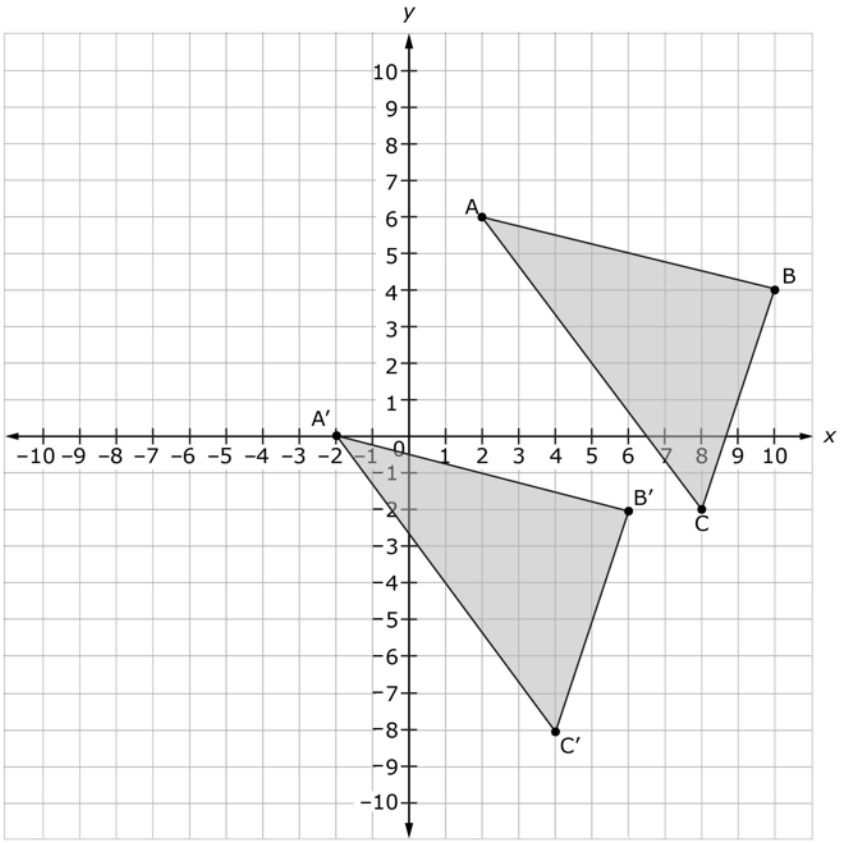
Name_____

5. Describe the translation that has occurred using words (e.g., left 3, up 2) and a vector on the coordinate plane.



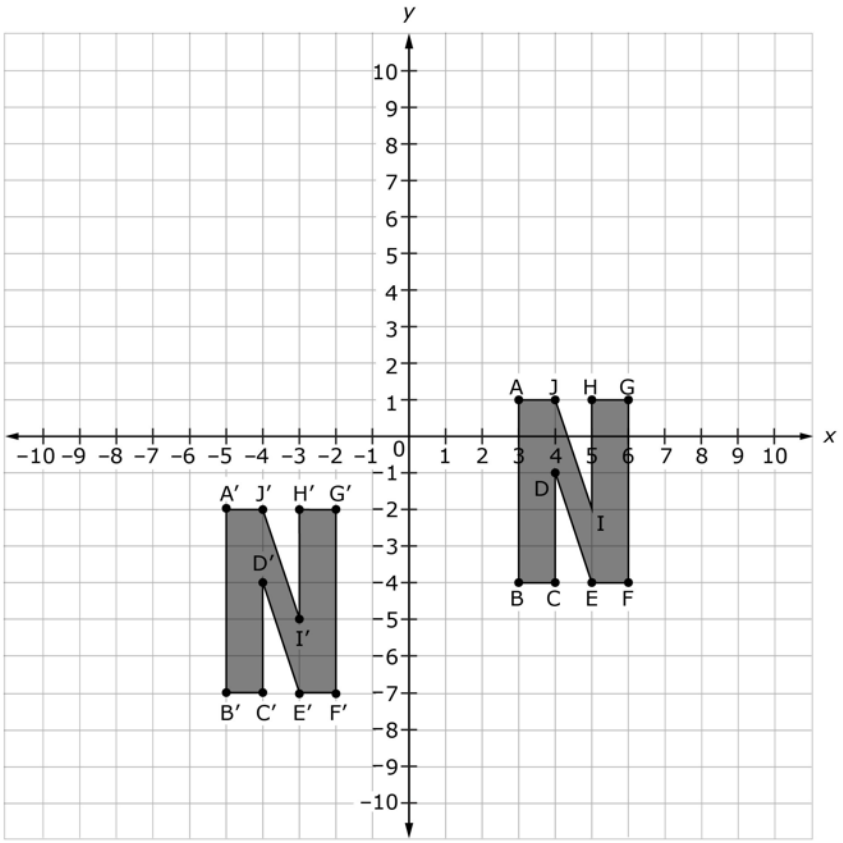
Name_____

6. Describe the translation that has occurred using words and a vector on the coordinate plane.



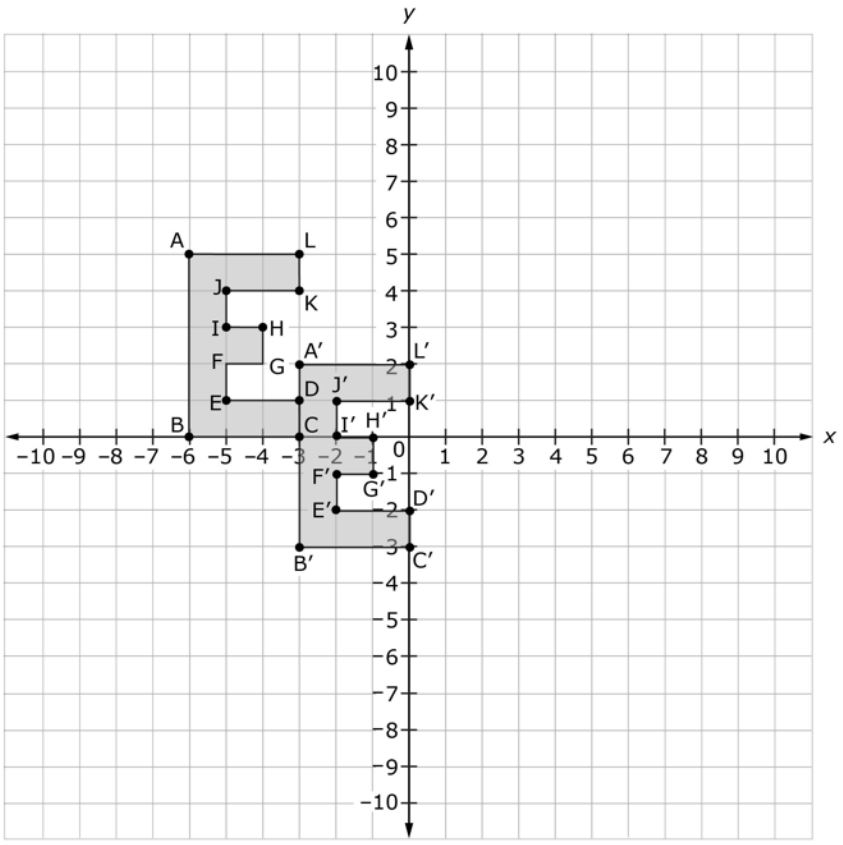
Name_____

7. Describe the translation that has occurred using words and a vector on the coordinate plane.



Name_____

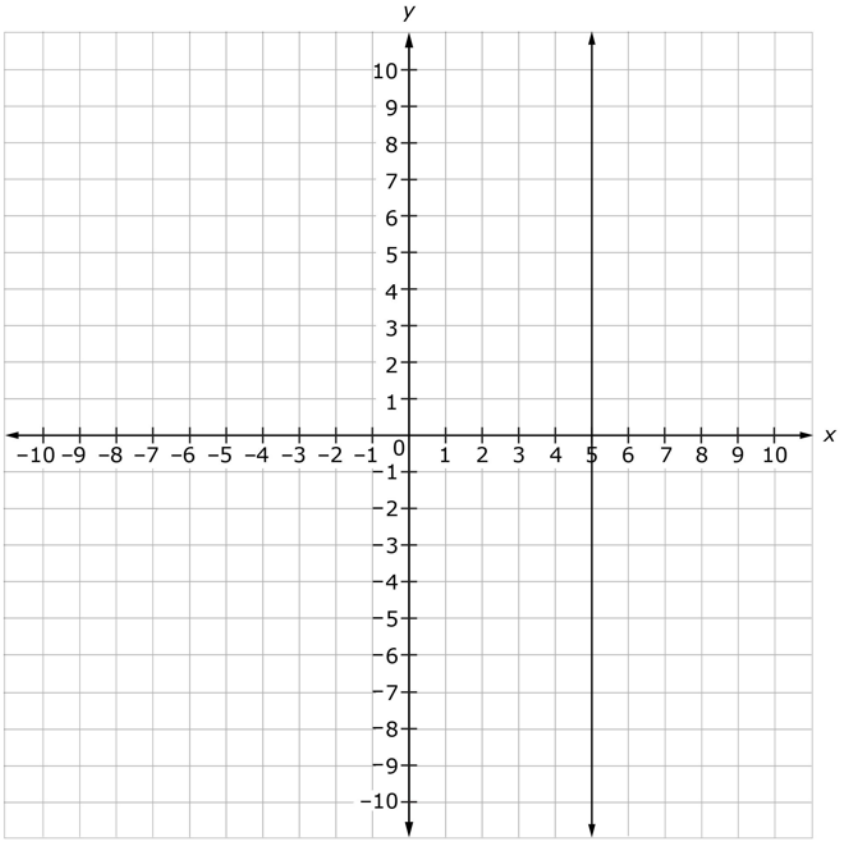
8. Describe the translation that has occurred using words and a vector on the coordinate plane.



Name_____

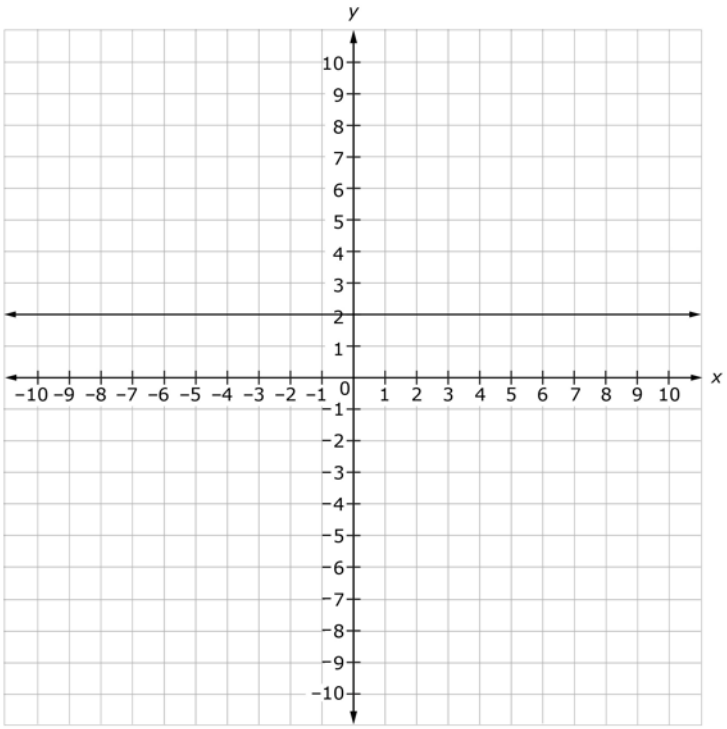
For each reflection in Questions 9 though 12, plot the points of the pre-image on the coordinate plane. Then, place patty paper over the coordinate plane and trace the pre-image and the line of reflection. Perform the requested reflection by flipping the patty paper over the line of reflection on the coordinate plane (ensuring the line on the patty paper lines up exactly with the line of reflection on the coordinate plane after the reflection). Note the coordinates of the image on the patty paper, then record the image in a different color on the coordinate plane. Be sure to label the vertices of the pre-image and image.

9. Reflect the plane containing rectangle ABCD with vertices A(4, 4), B(1, 4), C(4, 0), and D(1, 0) across the line of reflection.

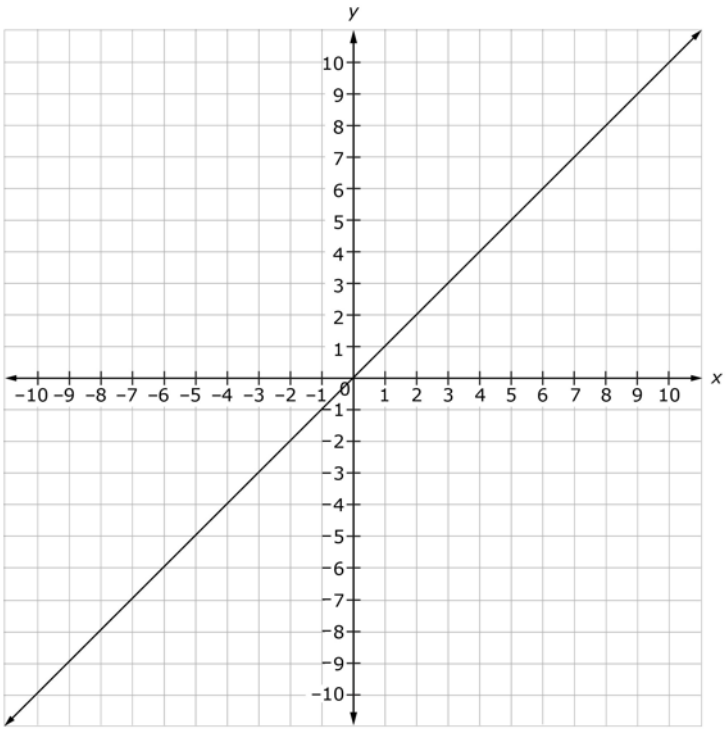


Name_____

10. Reflect the plane containing triangle RST with vertices R(–8, –2), S(–6, –5), and T(–1, –1) across the line of reflection.

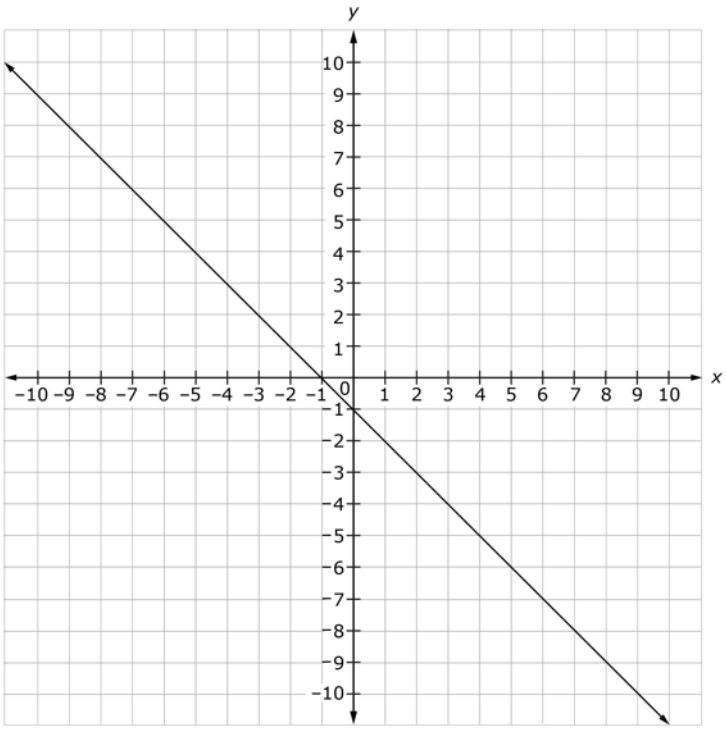


11. Reflect the plane containing triangle XYZ with vertices X(3, –5), Y(4, –2), and Z(7, –8) across the line of reflection.

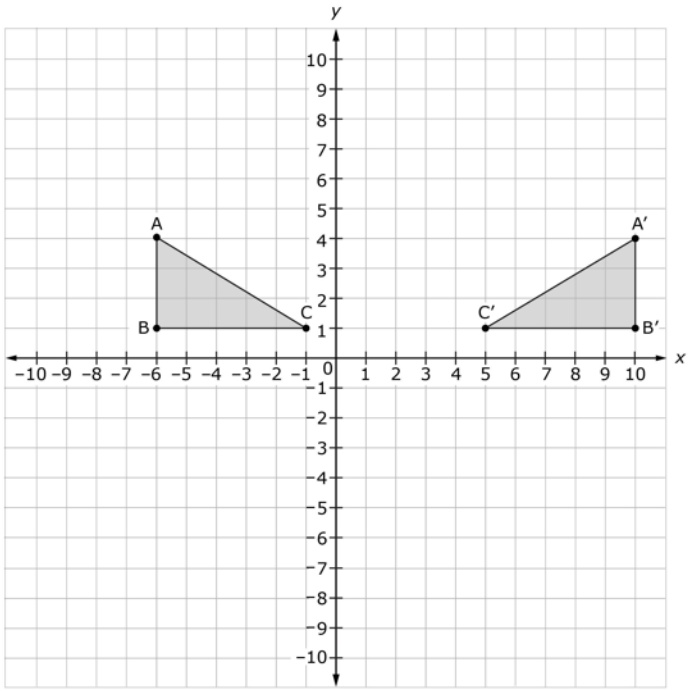


Name_____

12. Reflect the plane containing rectangle LMNO with vertices L(–9, 7), M(–9, 2), N(–7, 2), and O(–7, 7) across the line of reflection.

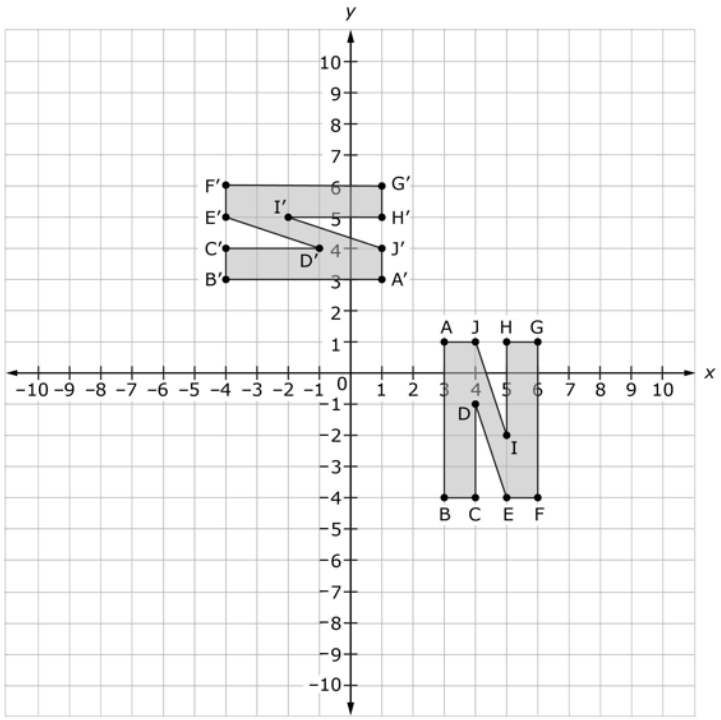


13. The pre-image has been reflected to create the image in the figure below. Draw the line of reflection on the coordinate plane.

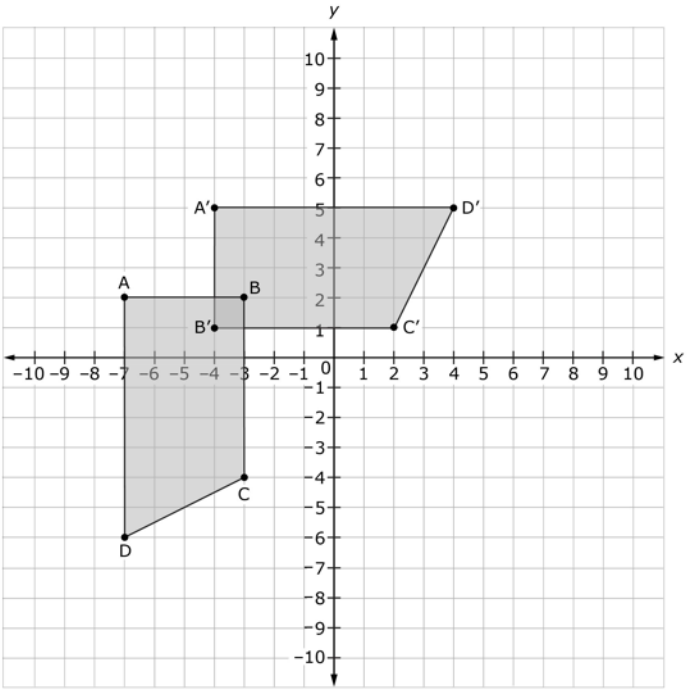


Name_____

14. The pre-image has been reflected to create the image in the figure below. Draw the line of reflection on the coordinate plane.

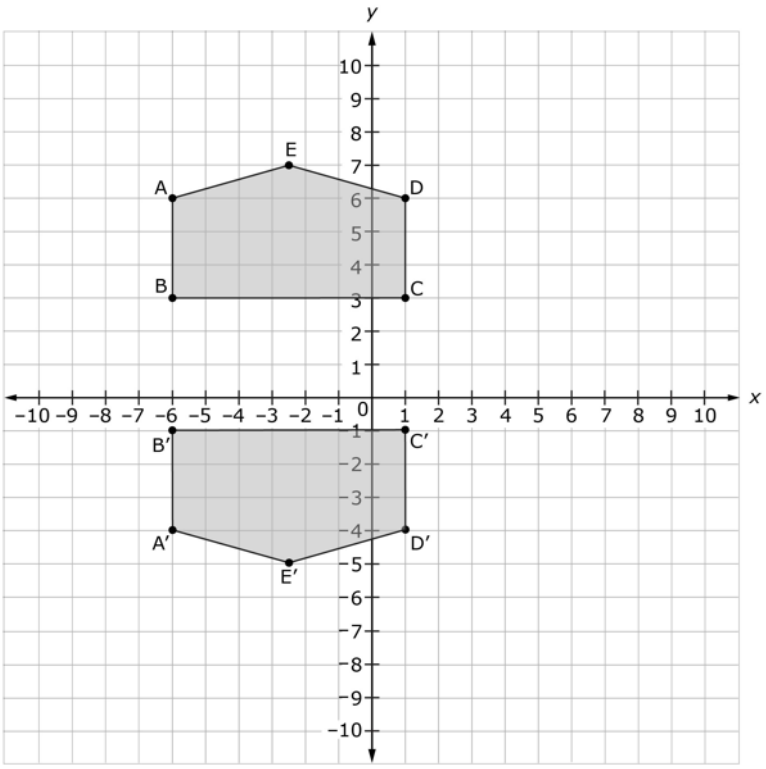


15. The pre-image has been reflected to create the image in the figure below. Draw the line of reflection on the coordinate plane.



Name_____

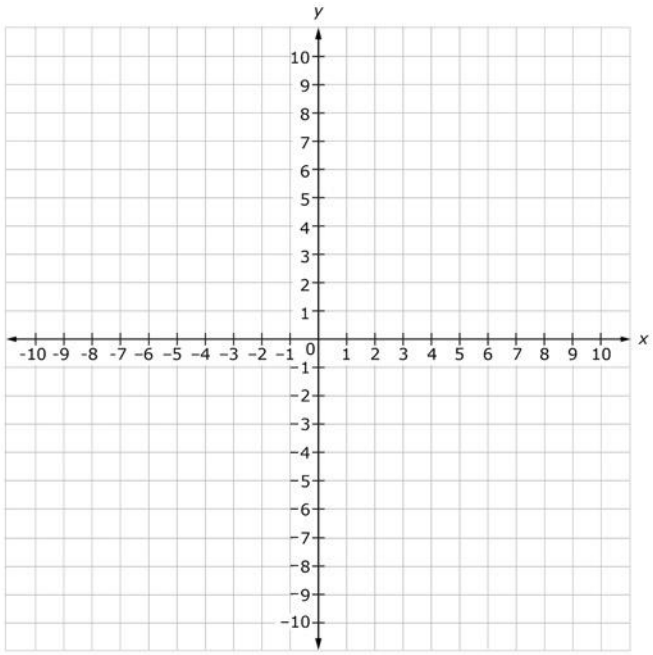
16. The pre-image has been reflected to create the image in the figure below. Draw the line of reflection on the coordinate plane.



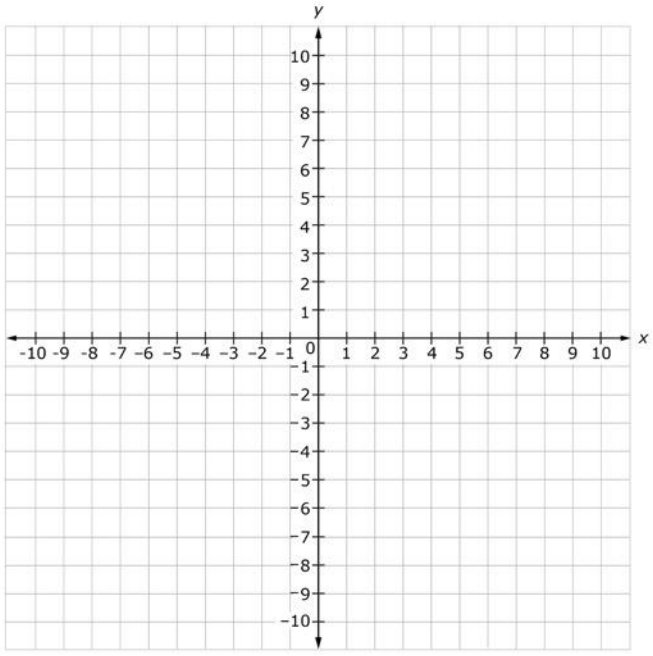
For each rotation in Questions 17 though 20, plot the points of the pre-image on the coordinate plane and plot the center of rotation. Then, place patty paper over the coordinate plane, trace the pre-image, and trace a vertical arrow above the center of rotation to use as a reference when rotating. Perform the requested rotation by rotating the patty paper around the center of rotation by the specified amount on the coordinate plane (ensuring the point representing the center of rotation remains stationary using a pencil). Note the coordinates of the image on the patty paper, then record the image in a different color on the coordinate plane. Be sure to label the vertices of the pre-image and image.

Name_____

17. Rotate the plane containing rectangle ABCD with vertices A(4, 4), B(1, 4), C(4, 0), and D(1, 0) 90° clockwise around the point (0, 0).

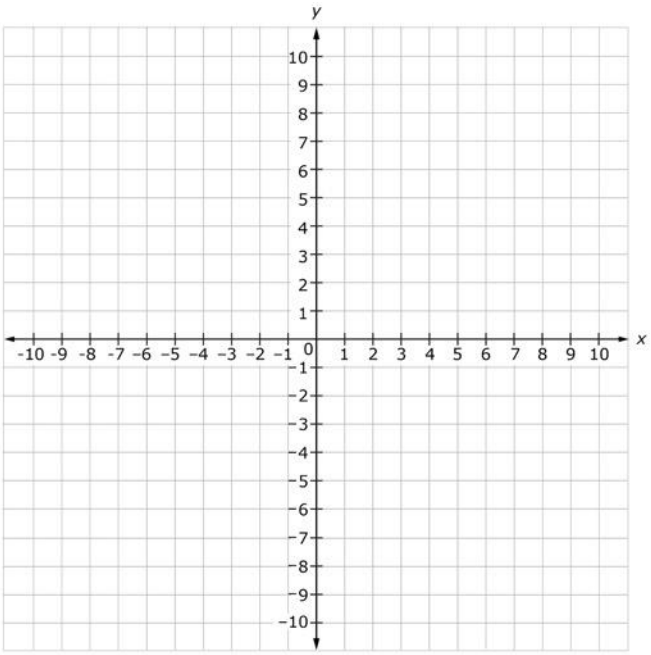


18. Rotate the plane containing triangle RST with vertices R(−8, −2), S(−6, −5), and T(−1, −1) 90° counterclockwise around the point (0, 0).

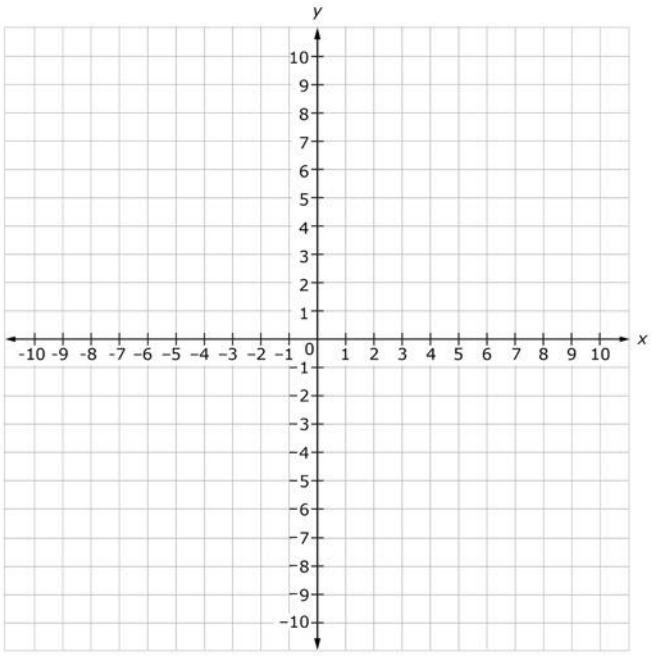


Name_____

19. Rotate the plane containing triangle XYZ with vertices X(3, –5), Y(4, –2), and Z(7, –8) 180° clockwise around the origin.

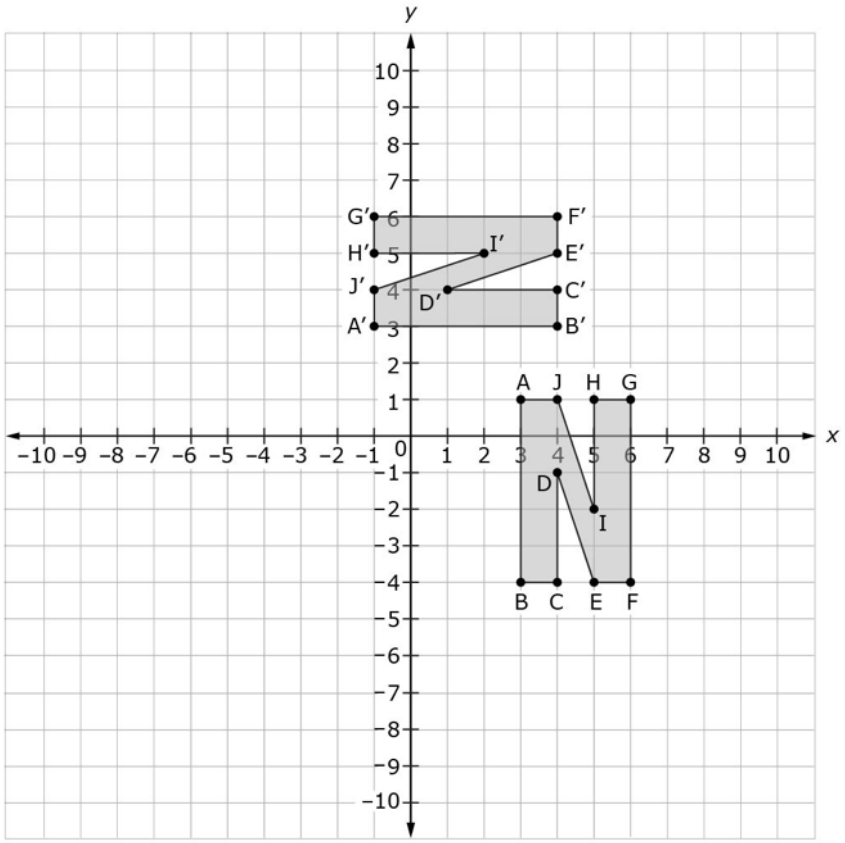


20. Rotate the plane containing rectangle LMNO with vertices L(–9, 7), M(–9, 2), N(–7, 2), and O(–7, 7) 270° clockwise around the point (–8, 3).



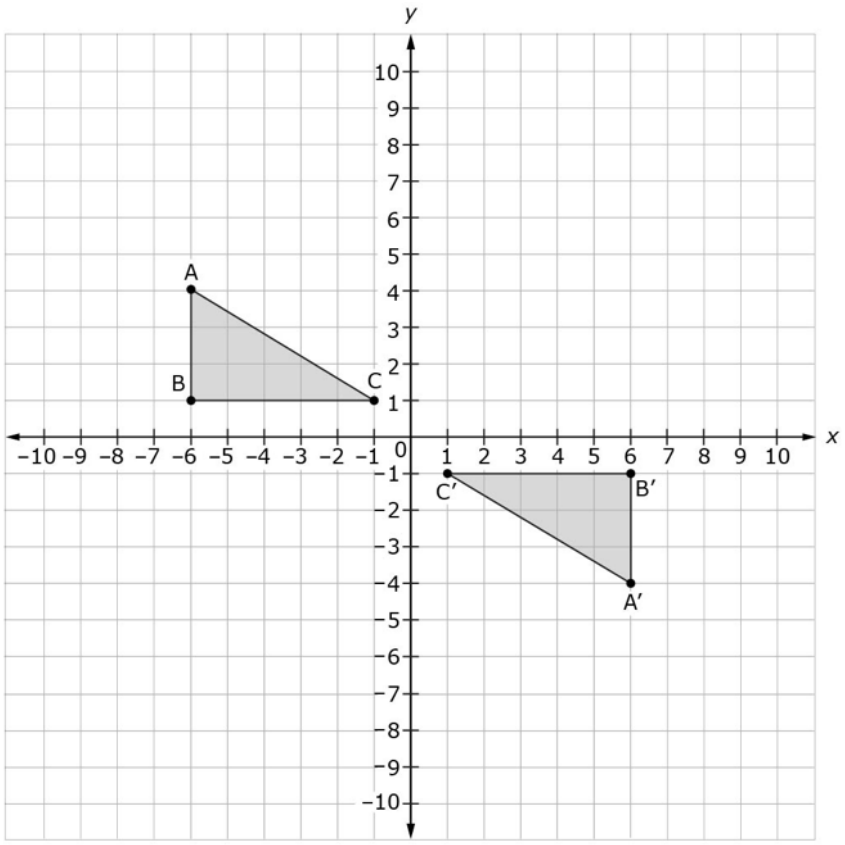
Name_____

21. Describe the rotation that has occurred using words (e.g., 90° clockwise around the point (3, –4)) and place the center of rotation on the coordinate plane.



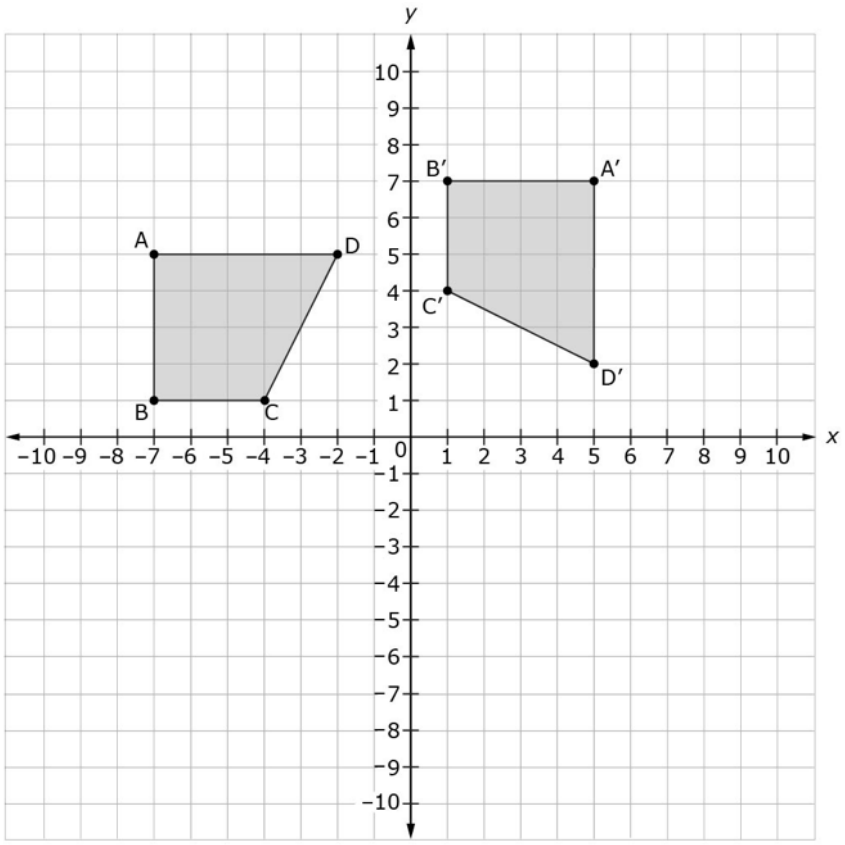
Name_____

22. Describe the rotation that has occurred using words and place the center of rotation on the coordinate plane.



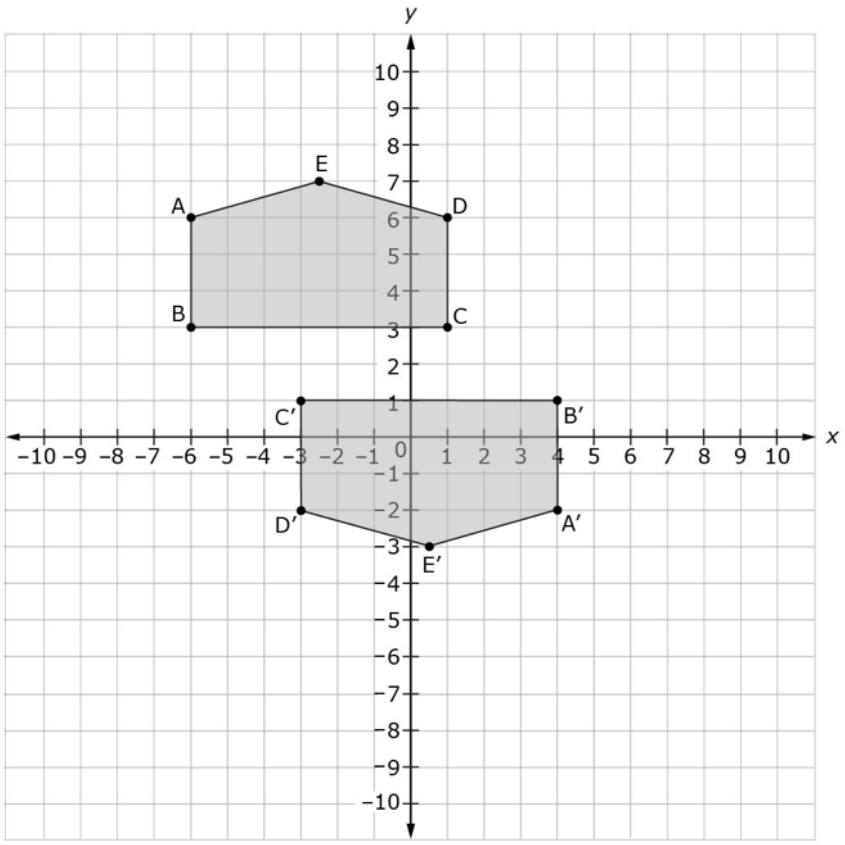
Name_____

23. Describe the rotation that has occurred using words and place the center of rotation on the coordinate plane.



Name_____

24. Describe the rotation that has occurred using words and place the center of rotation on the coordinate plane.



TRANSFORMATIONS AND CONGRUENCE

INSTRUCTIONAL ACTIVITY

Lesson 3

LEARNING GOAL

Students will discover the impact of translations and reflections on the coordinates of a figure, then generalize the rule for any figure under the described transformation.

PRIMARY ACTIVITY

Students will perform various translations and reflections on the coordinate plane in order to determine a pattern in the coordinates of a figure for a given transformation. Students will then use the pattern they discover to perform transformations on coordinates without using a coordinate plane.

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

- Congruent
- Translation
- Vector
- Reflection
- Line of reflection
- Rotation
- Center of rotation
- Angle of rotation
- Clockwise/counterclockwise

MATERIALS

- Patty paper or tracing paper
- INSTRUCTIONAL ACTIVITY STUDENT HANDOUT
- Internet access

IMPLEMENTATION

In this lesson, students will continue performing distance-preserving transformations on the coordinate plane. The focus will be on particular transformations in order to demonstrate patterns in how the coordinates of the pre-image compare to coordinates of the image under these transformations. Students will use a coordinate plane (with or without patty paper) and note the coordinates of the vertices for pairs of pre-images and images in the tables provided in the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

Direct students to work through the first three tasks in the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT in pairs or small groups. The translations are a review from LESSON 2, but the tasks following the translations ask students to look for patterns in the coordinates.

Observe student conversations and responses as they work, responding to questions and redirecting as needed.

Review the first three tasks, particularly Task 3, as a class to check for accuracy. Students should discover translating to the right produces a greater x–coordinate, translating to the left produces a lesser x–coordinate, translating up produces a greater y–coordinate, and translating down produces a lesser y–coordinate.

GUIDING QUESTIONS

Elicit student thinking:

- What do you notice about the coordinates before and after a translation?

Determine if the student can REPRESENT *TRANSLATION*:

- How will you translate the pre-image given the description of the translation?
- What "slides" in a translation?

Determine if the student can EXPLAIN *TRANSLATION*:

- How would you describe a translation?
- What information do you need to perform a specific translation?

Determine if the student can EXPLAIN THE EFFECT OF TRANSLATIONS USING COORDINATES:

- How will the x–coordinate change under this translation?
- How will the y–coordinate change under this translation?
- Can you write the coordinates of the image without plotting the points on the coordinate plane if you know the coordinates of the pre-image and the direction of the translation? If not, then what additional information do you need?
- Given the coordinates of a pre-image, what information do you need about a translation to write the coordinates of the image?

Direct students to work through Tasks 4 through 9 in the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT in pairs or small groups. The reflections are across horizontal and vertical lines and are a review from LESSON 2, but the questions following the reflections ask students to look for patterns in the coordinates. Students can use patty paper or a Mira to assist with the reflections as needed.

Observe student conversations and responses as they work, responding to questions and redirecting as needed.

Review Tasks 4 through 9, particularly Tasks 6 and 9, as a class to check for accuracy. Students should discover that, when reflecting across the x-axis, $(x, y) \rightarrow (x, -y)$ and when reflecting across the y-axis, $(x, y) \rightarrow (-x, y)$.

GUIDING QUESTIONS

Elicit student thinking:

- How would you compare reflecting over the x–axis to reflecting over a different horizontal line? What is similar? What is different?
- How would you compare reflecting over the y–axis to reflecting over a different vertical line? What is similar? What is different?

Determine if the student can REPRESENT *REFLECTION*:

- [For any task where students will reflect a plane across an axis] Which quadrant(s) will the image of this reflection be in if you reflect across the x–axis? If you reflect across the *y*–axis?
- What "flips" in a reflection?
- [Point to a coordinate on the figure] Where will the image of this point lie as a result of a reflection across the x–axis? After a reflection across the *y*–axis?

Determine if the student can EXPLAIN *REFLECTION*:

- How would you describe a reflection?
- What information do you need to perform a specific reflection?

Determine if the student can EXPLAIN THE EFFECT OF REFLECTIONS USING COORDINATES:

- How will the x–coordinate change under this reflection?
- How will the *y*–coordinate change under this reflection?
- Can you write the coordinates of the image without plotting the points on the coordinate plane if you know the coordinates of the pre-image and the line of reflection?
- Given the coordinates of a pre-image, what information do you need about a reflection to write the coordinates of the image?
- What is the general "rule" for identifying the coordinates of the image when a figure is reflected across the x–axis?
- What is the general "rule" for identifying the coordinates of the image when a figure is reflected across the *y*–axis?

Direct students to work through Tasks 10 through 15 in the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT in pairs or small groups. The reflections are across diagonal lines and are a review from LESSON 2, but the questions following the reflections ask students to look for patterns in the coordinates. Students can use patty paper or a Mira to assist with the reflections as needed.

Observe student conversations and responses as they work, responding to questions and redirecting as needed.

Review Tasks 10 through 15, particularly Tasks 12 and 15, as a class to check for accuracy. Students should discover that, when reflecting across the line $y = x$, $(x, y) \rightarrow (y, x)$ and when reflecting across the line $y = -x$, $(x, y) \rightarrow (-y, -x)$.

GUIDING QUESTIONS

Elicit student thinking:

- How would you compare reflecting over a diagonal line to reflecting over a horizontal or vertical line? What is similar? What is different?

Determine if the student can REPRESENT *REFLECTION*:

- [For any task where students will reflect a plane across $y = x$ or $y = -x$] Which quadrant(s) will the image of this reflection be in if you reflect across the line $y = x$? If you reflect across the line $y = -x$?
- What "flips" in a reflection?
- [Point to a coordinate on the figure] Where will the image of this point lie as a result of a reflection across the line $y = x$? After a reflection across the line $y = -x$?

Determine if the student can EXPLAIN *REFLECTION*:

- How would you describe a reflection?
- What information do you need to perform a specific reflection?

Determine if the student can EXPLAIN THE EFFECT OF REFLECTIONS USING COORDINATES:

- How will the x–coordinate change under this reflection?
- How will the y–coordinate change under this reflection?
- Can you write the coordinates of an image without plotting the points on the coordinate plane if you know the coordinates of the pre-image and the line of reflection?
- Given the coordinates of a pre-image, what information do you need about a reflection to write the coordinates of the image?
- What is the general "rule" for identifying the coordinates of the image when a figure is reflected across the line $y = x$?
- What is the general "rule" for identifying the coordinates of the image when a figure is reflected across the line $y = -x$?

Direct students to work through Tasks 16 and 21 in the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT in pairs or small groups. The rotations are about the origin are a review from LESSON 2, but the questions following the rotations ask students to look for patterns in the coordinates. Students can use patty paper to assist with the rotations as needed.

Observe student conversations and responses as they work, responding to questions and redirecting as needed.

Review Tasks 16 and 17 as a class to check for accuracy. Students should discover that, when rotating 90° counterclockwise about the origin, $(x, y) \rightarrow (-y, x)$ and when rotating 180° counterclockwise about the origin, $(x, y) \rightarrow (-x, -y)$.

GUIDING QUESTIONS

Elicit student thinking:

- How would you compare rotating 90° counterclockwise about the origin and rotating 180° counterclockwise about the origin? What is similar? What is different?

Determine if the student can REPRESENT *ROTATION*:

- [For any task where students will rotate a plane counterclockwise about the origin] Which quadrant(s) will the image of this rotation be in if you rotate 90° counterclockwise about the origin? If you rotate 180° counterclockwise about the origin?
- What "turns" in a rotation?
- [Point to a coordinate on the figure] Where will the image of this point lie as a result of a rotation 90° counterclockwise rotation about the origin? After a 180° counterclockwise rotation about the origin?

Determine if the student can EXPLAIN *ROTATION*:

- How would you describe a rotation?
- What information do you need to perform a specific rotation?

Determine if the student can EXPLAIN THE EFFECT OF ROTATIONS USING COORDINATES:

- How will the *x*–coordinate change under this rotation?
- How will the *y*–coordinate change under this rotation?
- Can you write the coordinates of the image without plotting the points on the coordinate plane if you know the coordinates of the pre-image and the degree measure and direction of rotation about the origin?
- Given the coordinates of a pre-image, what information do you need about a rotation to write the coordinates of the image?
- What is the general "rule" for identifying the coordinates of the image when a figure is rotated 90° counterclockwise about the origin?
- What is the general "rule" for identifying the coordinates of the image when a figure is rotated 180° counterclockwise about the origin?

Students should be required to be specific in their descriptions of patterns in the coordinates under each distance-preserving transformation and to be precise as they perform the transformations by hand. Access to patty paper may be helpful for some students, though some may no longer need to use patty paper as an aid for any or all questions.

At the end of the activity, teachers should provide students with a description of one of the distance-preserving transformations from this lesson and ordered pairs on the pre-image. Ask students to provide the corresponding ordered pairs of the resulting image without graphing.

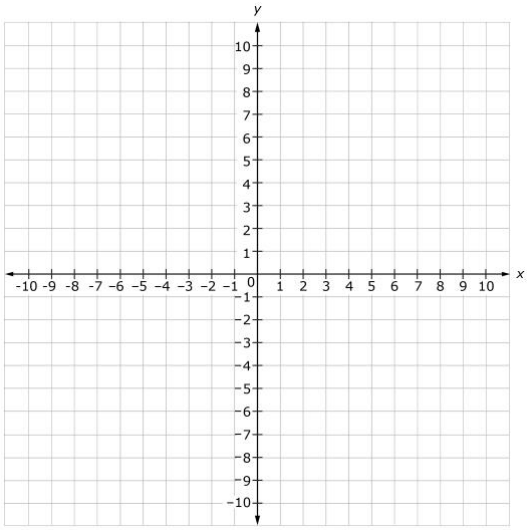
Name_____

TRANSFORMATIONS AND CONGRUENCE

Lesson 3

For each transformation, plot and label the points of the pre-image on the coordinate plane. Perform the requested transformation with or without the use of patty paper to represent the plane. Record the coordinates of the image next to the corresponding coordinates of the pre-image, then answer the corresponding questions.

1. Translate the plane containing rectangle ABCD with vertices A(–4, 5), B(–1, 5), C(–4, –2), and D(–1, –2) to the right 4 units.



Pre-image Coordinate	Image Coordinate
A(–4, 5)	A′(,)
B(–1, 5)	B′(,)
C(–4, –2)	C′(,)
D(–1, –2)	D′(,)

What do you notice about the *x*–coordinates of the pre-image compared to the corresponding *x*–coordinates of the image?

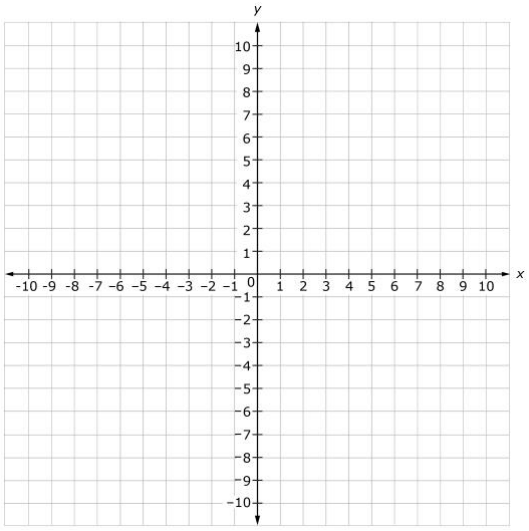
How does this relate to the translation you performed?

Name_____

What do you notice about the *y*–coordinates of the pre-image compared to the corresponding *y*–coordinates of the image?

How does this relate to the translation you performed?

2. Translate the plane containing triangle LMN with vertices L(–2, 1), M(1, 6), and N(4, 0) left 1 unit and down 6 units.



Pre-image Coordinate	Image Coordinate
L(–2, 1)	L'(,)
M(1, 6)	M'(,)
N(4, 0)	N'(,)

What do you notice about the *x*–coordinates of the pre-image compared to the corresponding *x*–coordinates of the image?

How does this relate to the translation you performed?

What do you notice about the *y*–coordinates of the pre-image compared to the corresponding *y*–coordinates of the image?

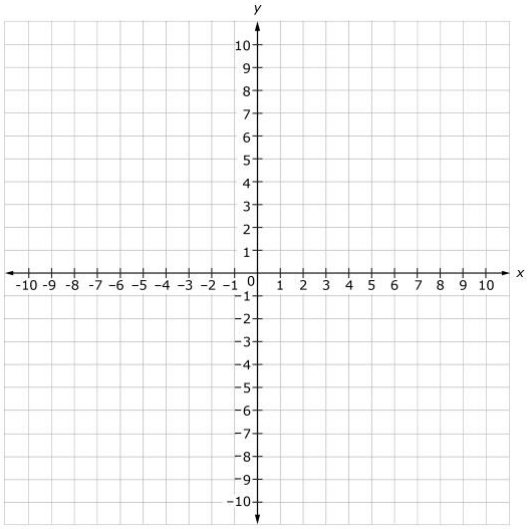
How does this relate to the translation you performed?

Write a general rule for how to determine the coordinates of the image given a translation description.

3. Without graphing, translate the plane containing square WXYZ with vertices W(–3, 4), X(0, 7), Y(3, 4), and Z(0, 1) left 1 unit and down 6 units.

Name_____

4. Reflect the plane containing rectangle ABCD with vertices A(–4, 5), B(–1, 5), C(–4, –2), and D(–1, –2) across the *x*–axis.



Pre-image Coordinate	Image Coordinate
A(–4, 5)	A′(,)
B(–1, 5)	B′(,)
C(–4, –2)	C′(,)
D(–1, –2)	D′(,)

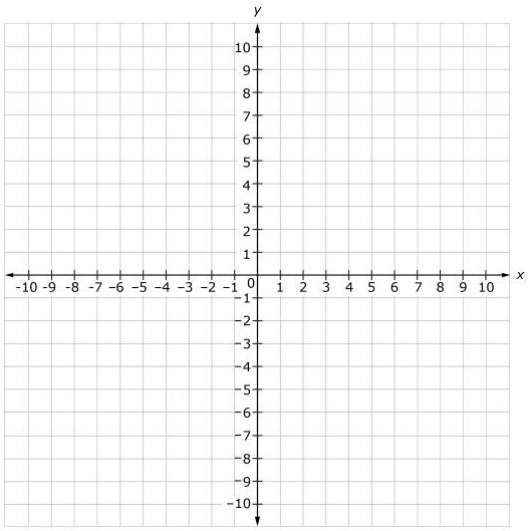
What do you notice about the *x*–coordinates of the pre-image compared to the corresponding *x*–coordinates of the image?

What do you notice about the *y*–coordinates of the pre-image compared to the corresponding *y*–coordinates of the image?

Describe how this makes sense based on the reflection you performed.

Name_____

5. Reflect the plane containing triangle LMN with vertices L(–2, 1), M(1, 6), and N(4, 0) across the *x*–axis.



Pre-image Coordinate	Image Coordinate
L(–2, 1)	L'(,)
M(1, 6)	M'(,)
N(4, 0)	N'(,)

What do you notice about the *x*–coordinates of the pre-image compared to the corresponding *x*–coordinates of the image?

What do you notice about the *y*–coordinates of the pre-image compared to the corresponding *y*–coordinates of the image?

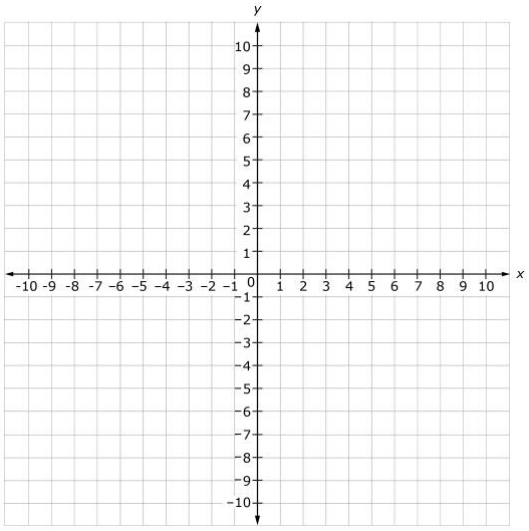
Describe how this makes sense based on the reflection you performed.

Name_____

Write a general rule for how to determine the coordinates of the image given a reflection across the *x*–axis.

6. Without graphing, reflect the plane containing square WXYZ with vertices W(–3, 4), X(0, 7), Y(3, 4), and Z(0, 1) across the *x*–axis.

7. Reflect the plane containing rectangle ABCD with vertices A(–4, 5), B(–1, 5), C(–4, –2), and D(–1, –2) across the *y*–axis.



Pre-image Coordinate	Image Coordinate
A(–4, 5)	A′(,)
B(–1, 5)	B′(,)
C(–4, –2)	C′(,)
D(–1, –2)	D′(,)

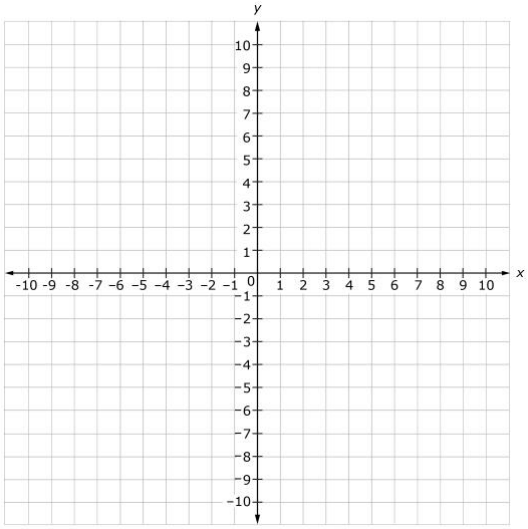
What do you notice about the *x*–coordinates of the pre-image compared to the corresponding *x*–coordinates of the image?

Name_____

What do you notice about the *y*–coordinates of the pre-image compared to the corresponding *y*–coordinates of the image?

Describe how this makes sense based on the reflection you performed.

8. Reflect the plane containing triangle LMN with vertices L(–2, 1), M(1, 6), and N(4, 0) across the *y*–axis.



Pre-image Coordinate	Image Coordinate
L(–2, 1)	L'(,)
M(1, 6)	M'(,)
N(4, 0)	N'(,)

What do you notice about the *x*–coordinates of the pre-image compared to the corresponding *x*–coordinates of the image?

Name_____

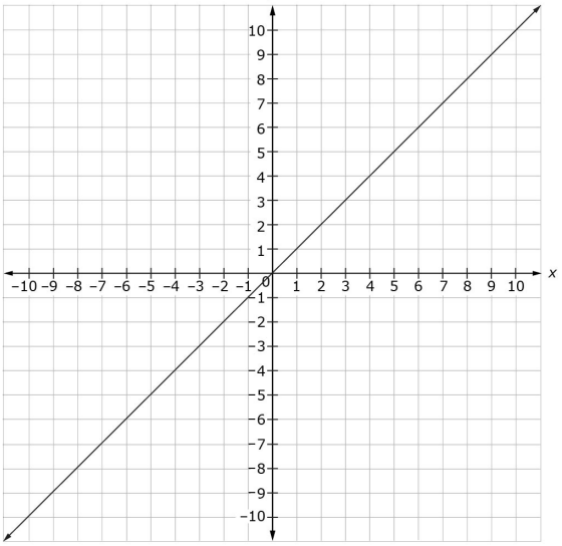
What do you notice about the *y*–coordinates of the pre-image compared to the corresponding *y*–coordinates of the image?

Describe how this makes sense based on the reflection you performed.

Write a general rule for how to determine the coordinates of the image given a reflection across the *y*–axis.

9. Without graphing, reflect the plane containing square WXYZ with vertices W(–3, 4), X(0, 7), Y(3, 4), and Z(0, 1) across the *y*–axis.

10. Reflect the plane containing rectangle ABCD with vertices A(–4, 5), B(–1, 5), C(–4, –2), and D(–1, –2) across the line $y = x$.



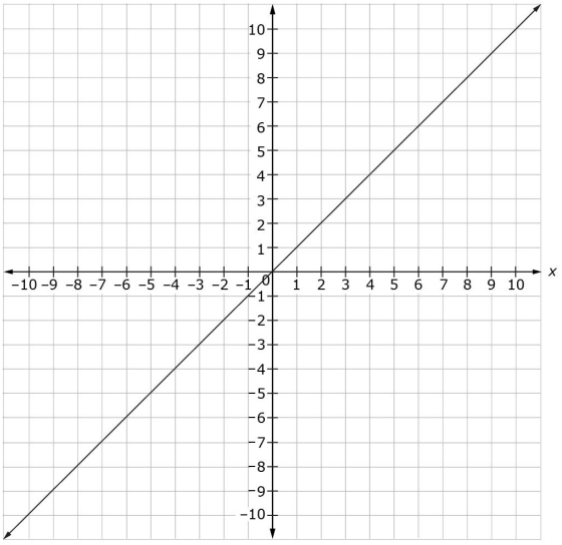
Pre-image Coordinate	Image Coordinate
A(–4, 5)	A′(,)
B(–1, 5)	B′(,)
C(–4, –2)	C′(,)
D(–1, –2)	D′(,)

Name_____

What do you notice about the *x*–coordinates of the pre-image compared to the corresponding *y*–coordinates of the image?

What do you notice about the *y*–coordinates of the pre-image compared to the corresponding *x*–coordinates of the image?

11. Reflect the plane containing triangle LMN with vertices L(–2, 1), M(1, 6), and N(4, 0) across the line $y = x$.



Pre-image Coordinate	Image Coordinate
L(–2, 1)	L'(,)
M(1, 6)	M'(,)
N(4, 0)	N'(,)

What do you notice about the *x*–coordinates of the pre-image compared to the corresponding *y*–coordinates of the image?

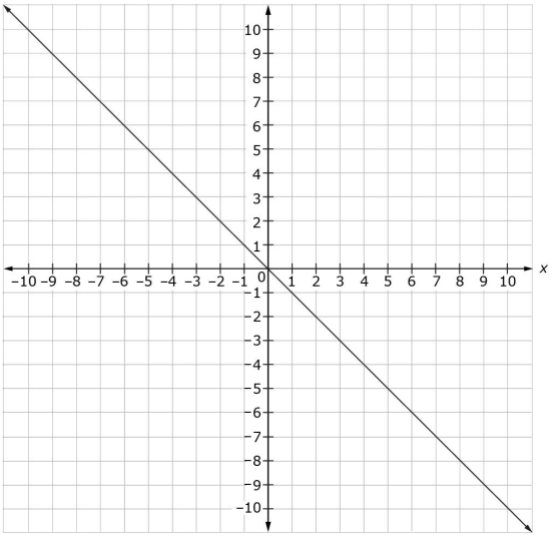
Name_____

What do you notice about the *y*–coordinates of the pre-image compared to the corresponding *x*–coordinates of the image?

Write a general rule for how to determine the coordinates of the image given a reflection across the line $y = x$.

12. Without graphing, reflect the plane containing square WXYZ with vertices W(–3, 4), X(0, 7), Y(3, 4), and Z(0, 1) across the line $y = x$.

13. Reflect the plane containing rectangle ABCD with vertices A(–4, 5), B(–1, 5), C(–4, –2), and D(–1, –2) across the line $y = -x$.



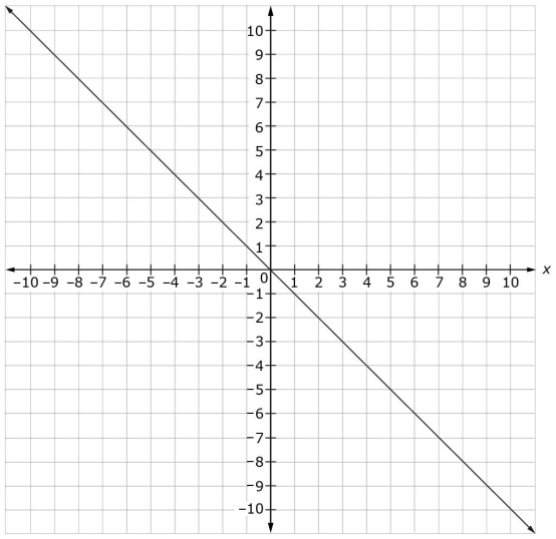
Pre-image Coordinate	Image Coordinate
A(–4, 5)	A′(,)
B(–1, 5)	B′(,)
C(–4, –2)	C′(,)
D(–1, –2)	D′(,)

Name_____

What do you notice about the *x*–coordinates of the pre-image compared to the corresponding *y*–coordinates of the image?

What do you notice about the *y*–coordinates of the pre-image compared to the corresponding *x*–coordinates of the image?

14. Reflect the plane containing triangle LMN with vertices L(–2, 1), M(1, 6), and N(4, 0) across the line $y = -x$.



Pre-image Coordinate	Image Coordinate
L(–2, 1)	L'(,)
M(1, 6)	M'(,)
N(4, 0)	N'(,)

What do you notice about the *x*–coordinates of the pre-image compared to the corresponding *y*–coordinates of the image?

What do you notice about the y–coordinates of the pre-image compared to the corresponding x–coordinates of the image?

Write a general rule for how to determine the coordinates of the image given a reflection across the line $y = -x$.

15. Without graphing, reflect the plane containing square WXYZ with vertices W(–3, 4), X(0, 7), Y(3, 4), and Z(0, 1) across the line $y = -x$.

TRANSFORMATIONS AND CONGRUENCE

INSTRUCTIONAL ACTIVITY

Lesson 4

LEARNING GOAL

Students will perform a series of transformations and explain that two figures are congruent when there is a series of distance-preserving transformations that maps one figure onto the other.

PRIMARY ACTIVITY

Students will use patty paper and technology to perform distance-preserving transformations on a figure to determine if one figure is congruent to another figure.

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

- Congruent
- Translation
- Vector
- Reflection
- Line of reflection
- Rotation
- Center of rotation
- Angle of rotation
- Clockwise/counterclockwise

MATERIALS

- Patty paper or tracing paper
- INSTRUCTIONAL ACTIVITY STUDENT HANDOUT
- Internet access

IMPLEMENTATION

Students will start the lesson by performing a series of transformations on a single figure as directed by the teacher. This could be accomplished in a "Simon Says" format, if desired.

Provide students with at least one copy of the large coordinate plane provided in the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

Require students to plot points to create a figure, then perform a transformation (either a translation, reflection, or rotation) using specific details about the transformation (e.g., distance, direction, etc.). Once students perform the first transformation, they should work from the second figure (the image) and perform a second transformation using the image from the first transformation as the pre-image for the second transformation (either a translation, reflection, or rotation).

Requiring students to use different colors or line types for each transformation will help when evaluating student work in this lesson as well as in the STUDENT ACTIVITY. In addition, this is a good strategy to assist students in following their own work.

Repeat this activity with a new figure and a different series of transformations.

Include a composition of two consecutive reflections across parallel lines. **Ask** students if there is an alternate transformation that could transform the pre-image to the image. Students should indicate that a translation could accomplish the same result as a composition of reflections across parallel lines.

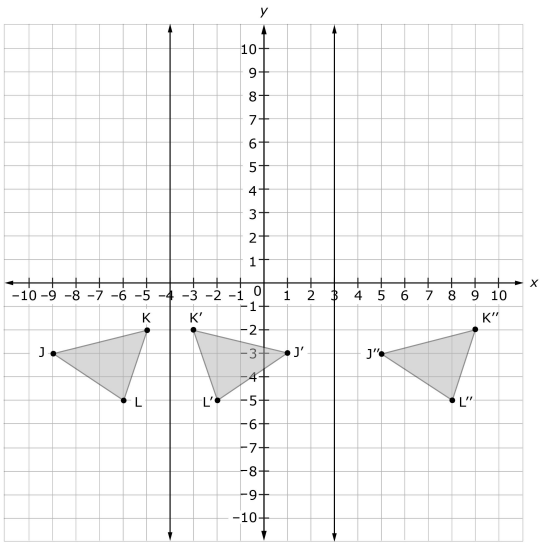


Figure 1: Consecutive reflections across parallel lines.

Include a composition of two consecutive reflections across intersecting lines. **Ask** students if there is an alternate transformation that could transform the pre-image into the image. Students should indicate that a rotation could accomplish the same result as a composition of reflections across intersecting lines.

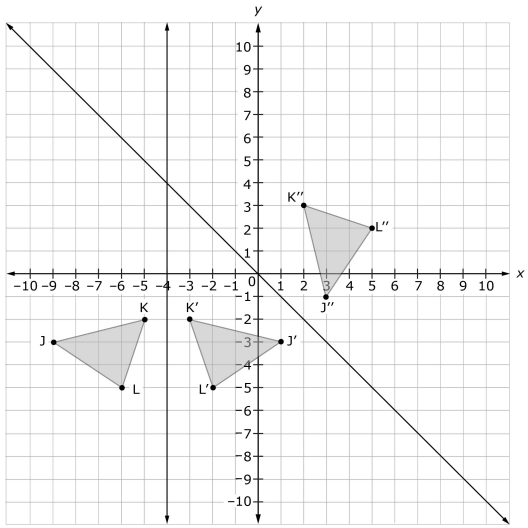


Figure 2: Consecutive reflections across intersecting lines.

As an extension, students could perform more than two consecutive transformations on a single figure.

After a few examples as a class, require students to practice identifying compositions of transformations (with intermediate steps shown) on Tasks 1 through 4 in the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

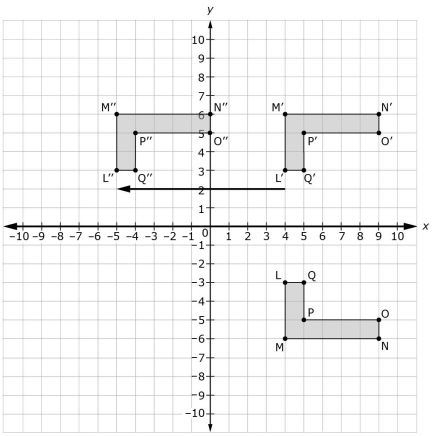
NOTE: Task 4 and 5 in the Instructional Activity Student Handout have multiple correct answers. To differentiate instruction, advanced students could be required to provide more than one intermediate transformation. Additionally, a conversation can be held to address how many intermediate transformations are implied by the use of "double prime" for the images vertex coordinates.

Now that students have practice performing and evaluating a series of transformations, students should individually perform their own series of transformations on a figure of their choice.

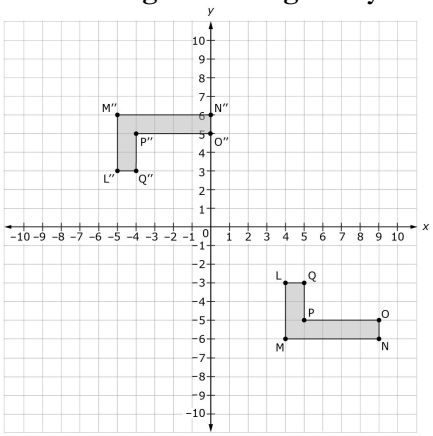
Once students have completed their individual transformations, have students copy only the pre-image and the final image onto a new coordinate plane, then trade with a partner.

EXAMPLE IMAGES

Initial Work



Pre-image and Image Only



The goal of this activity is for the partner to recreate the series of transformations that carried the pre-image onto the image.

Once a student's partner determines how to map the pre-image onto the image through a series of transformations, they should compare their work to their partner's original work. The sequence of transformations may or may not match, since there are always at least two ways to perform a sequence of transformations and end with the same result. If the transformations are different, students should check to ensure both methods are accurate.

Repeat this activity as needed to provide practice for students.

Students will use their new knowledge of multiple transformations to determine, through transformation, whether two figures are congruent.

Show students a Khan Academy video to introduce the concept behind transforming a figure to determine whether it is congruent to a second figure.

Provide students an opportunity to practice on Khan Academy until they get at least three problems correct.

GUIDING QUESTIONS

Elicit student thinking:

- If you perform a transformation on a figure, could you perform a second transformation to "revert" to the original figure? How would you accomplish this for a translation? For a reflection? For a rotation?
- What do you think would happen if you reflected a pre-image across a line and then reflected the image of the first reflection across a line parallel to the first line of reflection? Could this be described as a different transformation?

Determine if the student can EXPLAIN CONGRUENCE THROUGH TRANSFORMATIONS:

- How can you show two figures are congruent using transformations?
- What has to be true about congruent figures?

Determine if the student can PROVE 2 FIGURES ARE CONGRUENT USING A SEQUENCE OF TRANSFORMATIONS:

- What could you do to prove two figures are or are not congruent?
- Which transformation(s) show these figures are congruent?
- Is there an alternate series of transformations that can also show the figures are congruent?

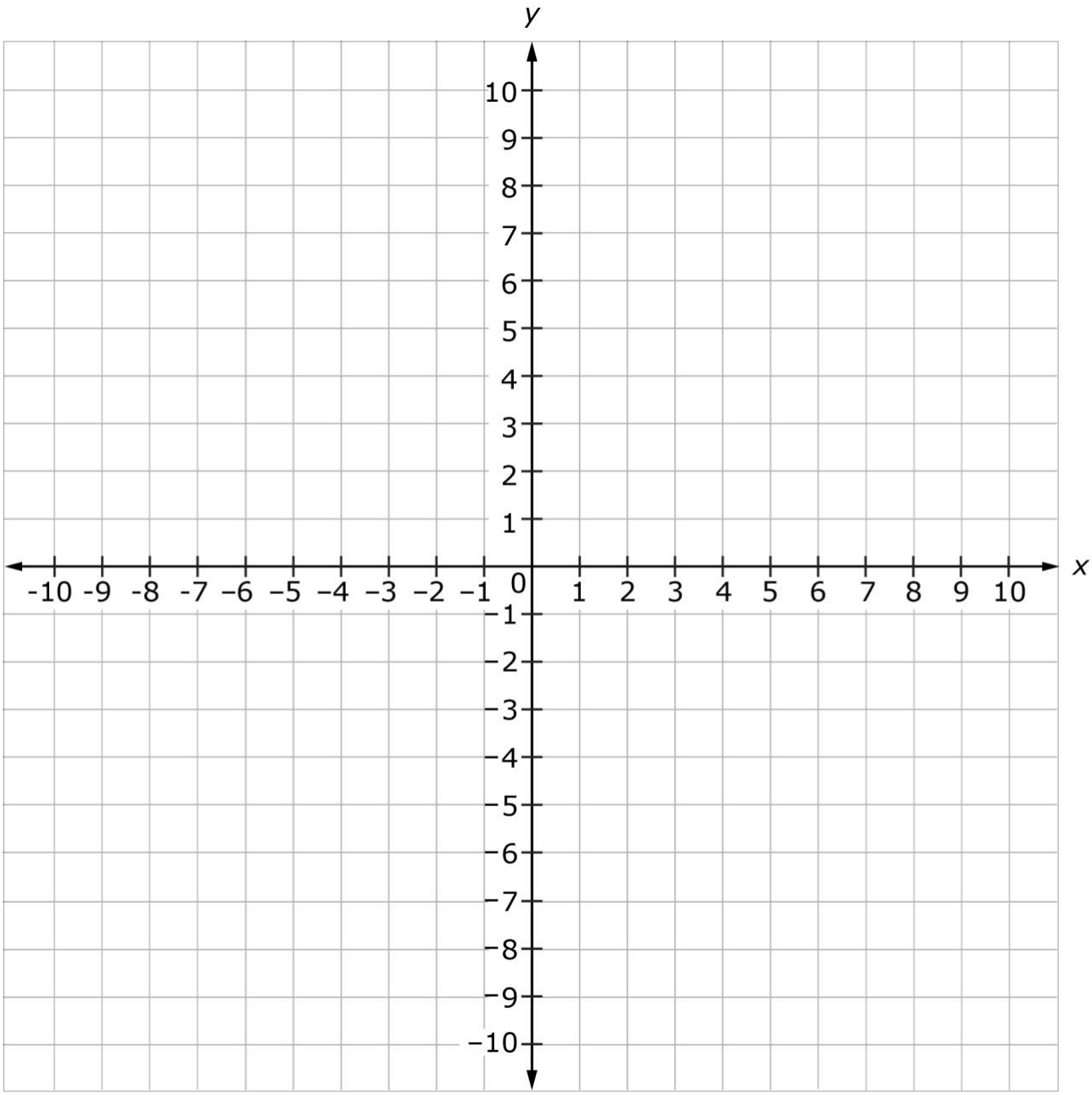
Students should be required to show all their work when they are working on paper and be able to describe their reasoning when they work on paper or with technology.

At the end of the activity, teachers should provide students with two figures on a coordinate plane. Students should, through a series of transformations, show whether or not the figures are congruent and explain how they know the figures are or are not congruent.

Name_____

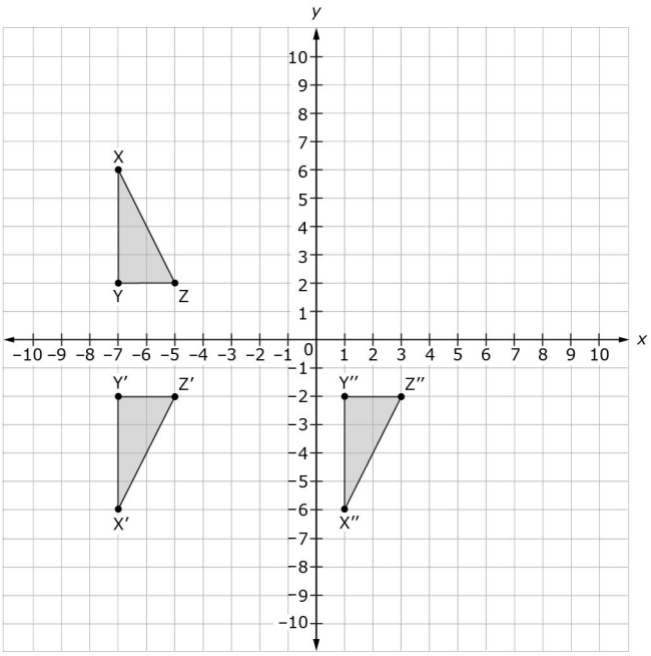
TRANSFORMATIONS AND CONGRUENCE

Lesson 4

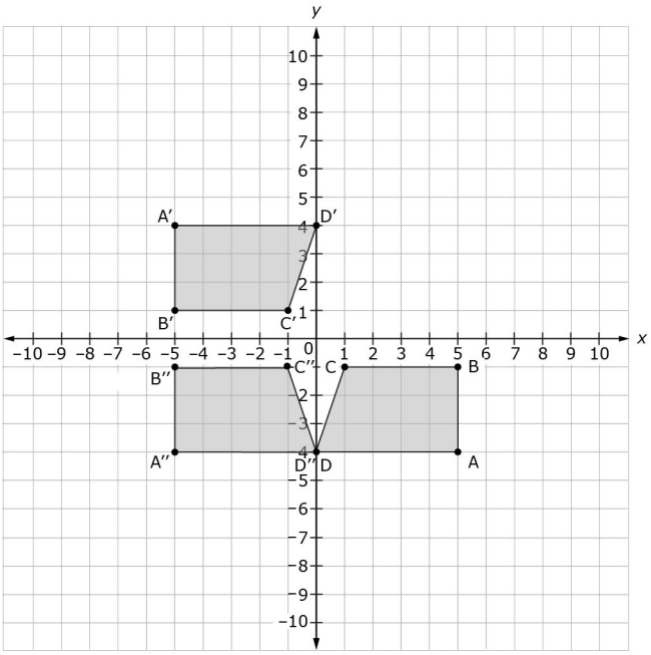


Name_____

1. Describe the transformations that have occurred using words (e.g., translation left 3 and up 2, reflection across the *y*–axis) and draw the vector, line of reflection, and/or center of rotation on the coordinate plane.

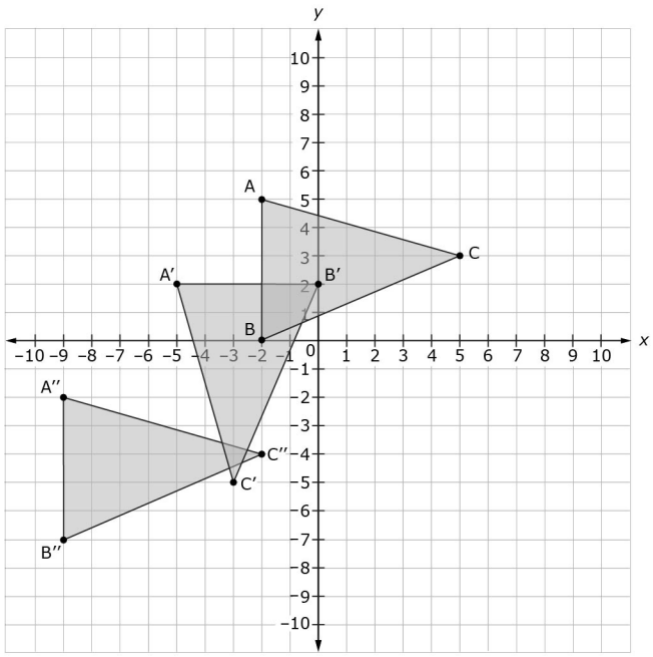


2. Describe the transformations that have occurred using words and draw the vector, line of reflection, and/or center of rotation on the coordinate plane.

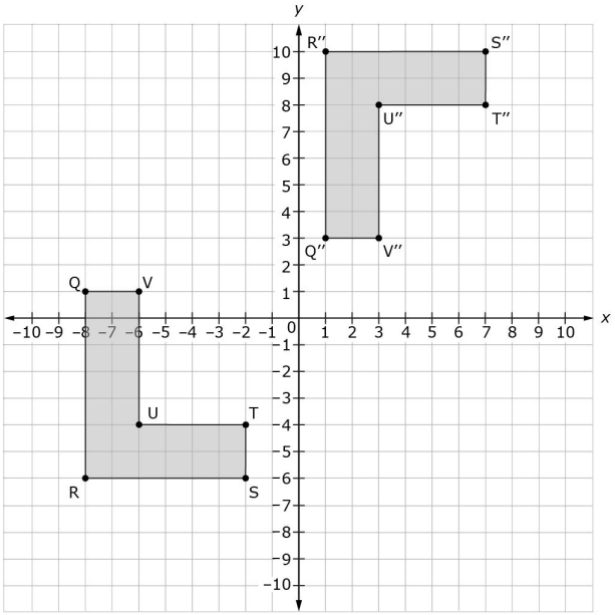


Name_____

3. Describe the transformations that have occurred using words and draw the vector, line of reflection, and/or center of rotation on the coordinate plane.

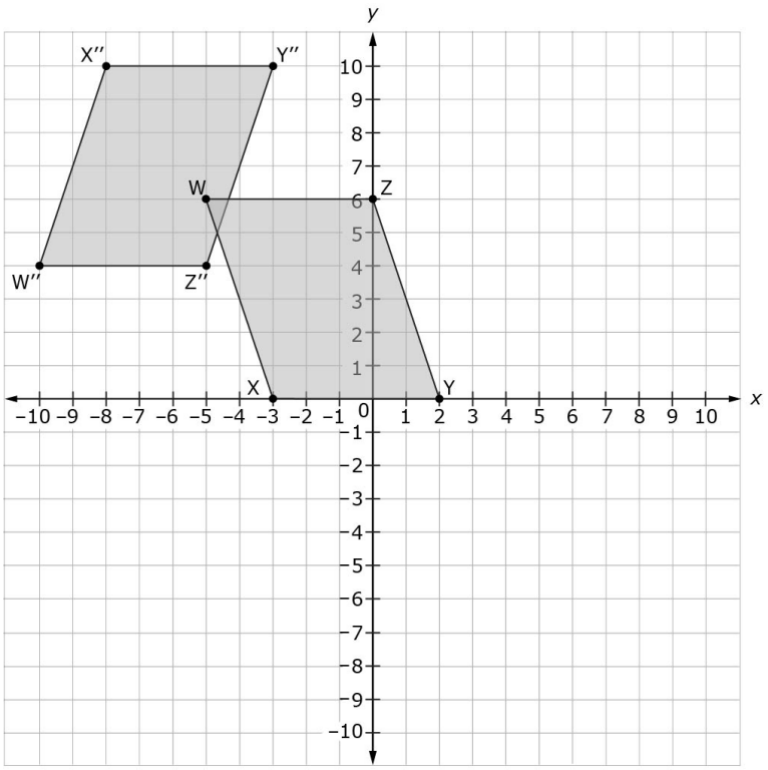


4. Draw and label an intermediate figure used to obtain the final image, then describe the combination of transformations that have occurred using words. Be sure to include any vectors, lines of reflection, and/or centers of rotation on the coordinate plane. Note that there is more than one possible correct answer.



Name_____

5. Draw and label an intermediate figure used to obtain the final image, then describe the combination of transformations that have occurred using words. Be sure to include any vectors, lines of reflection, and/or centers of rotation on the coordinate plane. Note that there is more than one possible correct answer.



Name_____

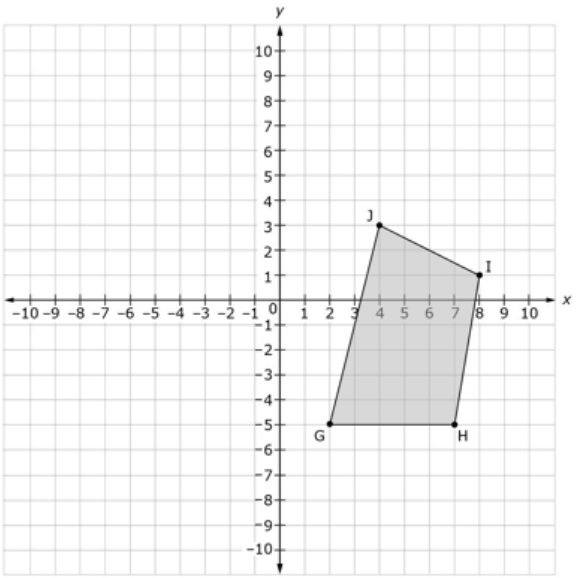
TRANSFORMATIONS AND CONGRUENCE

STUDENT ACTIVITY

Lessons 1 – 4

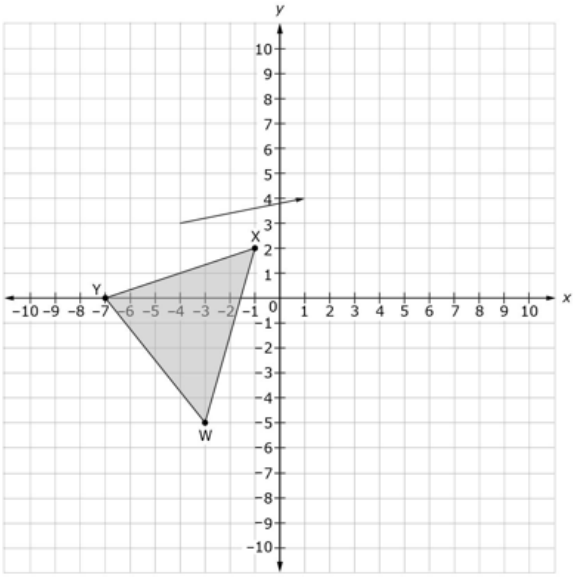
1. Perform the following translations on the coordinate plane. Patty paper may be used if needed. Be sure to label the vertices appropriately and accurately.

1.a. Translate the plane containing quadrilateral GHIJ with coordinates G(2, –5), H(7, –5), I(8, 1), and J(4, 3) to the left 6 units and down 2 units. Is the pre-image congruent to the image? Explain your reasoning.



Name_____

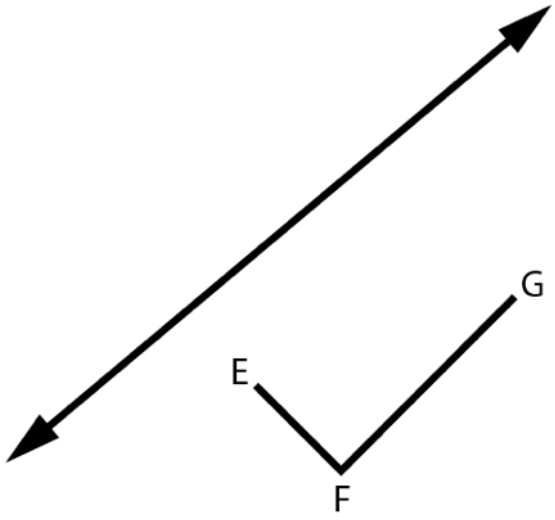
1.b. Translate the plane containing triangle WXY with coordinates W(−3, −5), X(−1, 2), and Y(−7, 0) according to the vector provided on the coordinate plane. Is the pre-image congruent to the image? Explain your reasoning. What information did you use to perform this translation?



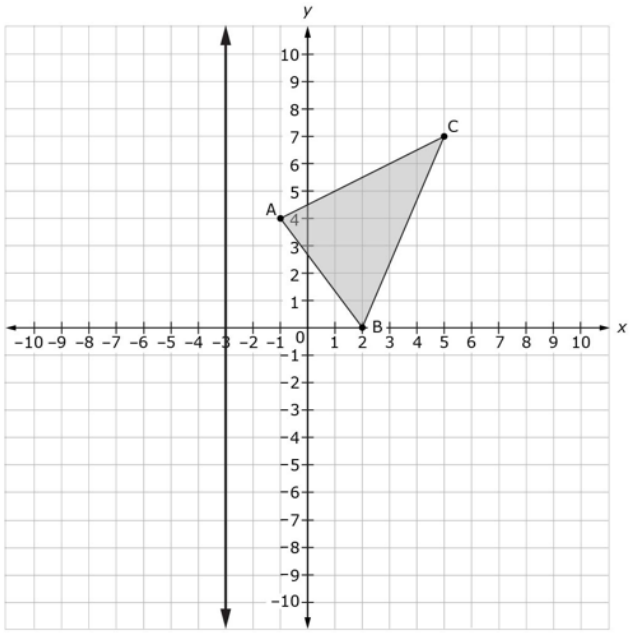
Name_____

2. Perform the following reflections on the coordinate plane. Patty paper may be used if needed. Be sure to label the vertices appropriately and accurately.

2.a. Draw the approximate reflection of the plane containing the figure across the line of reflection provided.

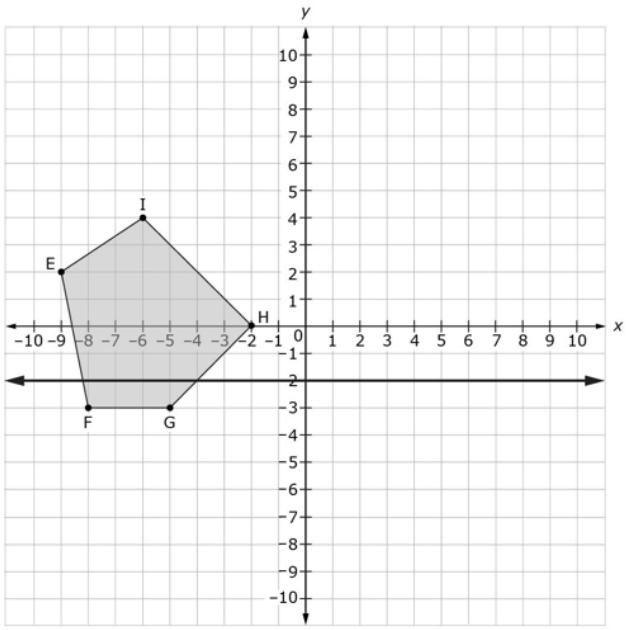


2.b. Reflect the plane containing triangle ABC with coordinates A(−1, 4), B(2, 0), and C(5, 7) across the line of reflection provided on the coordinate plane. Is the pre-image congruent to the image? Explain your reasoning.



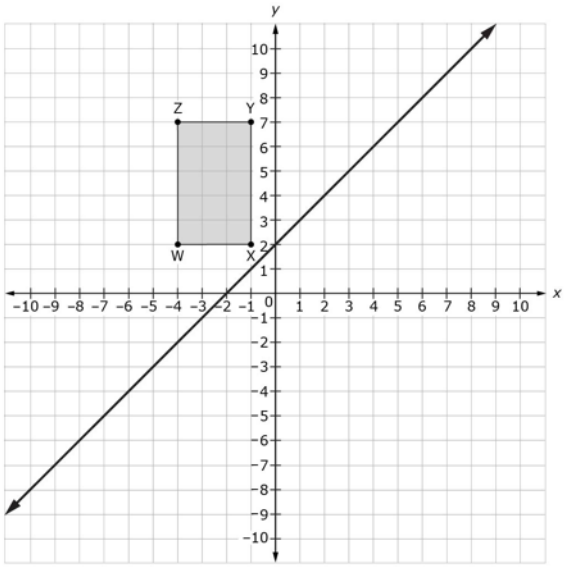
Name_____

2.c. Reflect the plane containing pentagon EFGHI with coordinates E(−9, 2), F(−8, −3), G(−5, −3), H(−2, 0), and I(−6, 4) across the line of reflection provided on the coordinate plane. Is the pre-image congruent to the image? Explain your reasoning.



Name_____

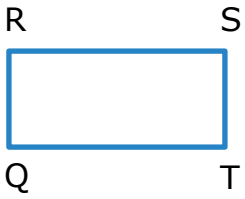
2.d. Reflect the plane containing quadrilateral WXYZ with coordinates W(−4, 2), X(−1, 2), Y(−1, 7), and Z(−4, 7) across the line of reflection provided on the coordinate plane. Is the pre-image congruent to the image? Explain your reasoning. What information did you use to perform this reflection?



Name_____

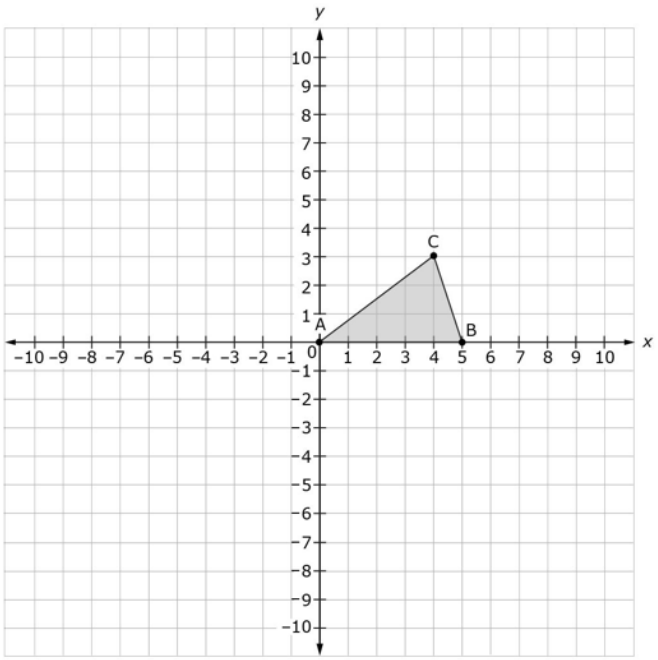
3. Perform the following rotations. Patty paper may be used if needed. Be sure to label the vertices appropriately and accurately.

3.a. Draw the approximate rotation of the plane containing quadrilateral QRST 45° clockwise about the point provided.



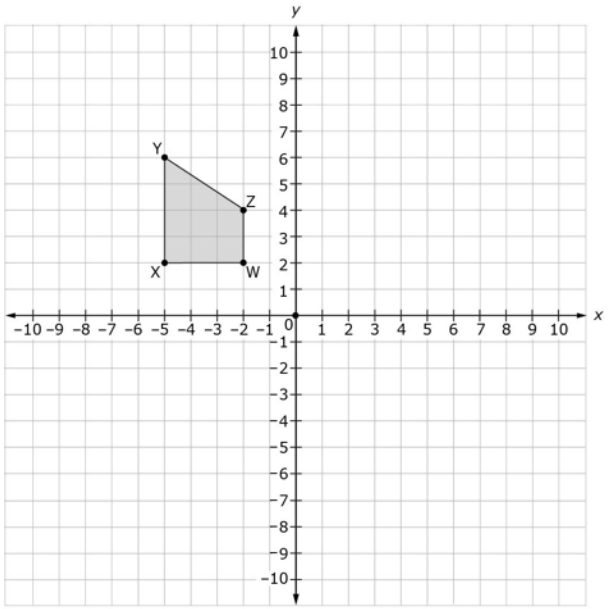
Name_____

3.b. Rotate the plane containing triangle ABC with coordinates A(0, 0), B(5, 0), and C(4, 3) 180° about the point (0, 0) on the coordinate plane. Is the pre-image congruent to the image? Explain your reasoning.



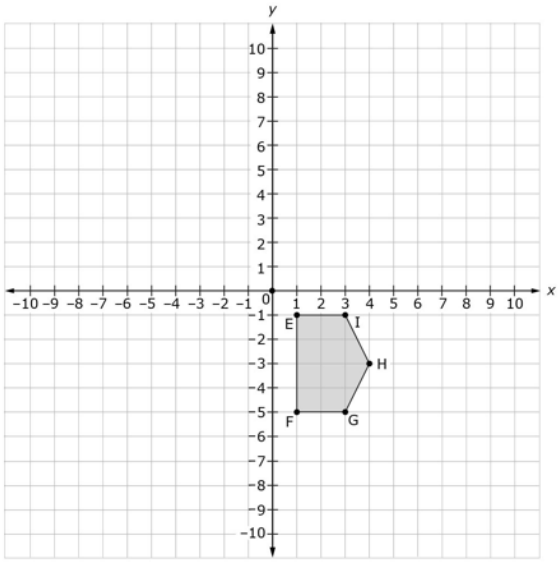
Name_____

3.c. Rotate the plane containing quadrilateral WXYZ with coordinates W(−2, 2), X(−5, 2), Y(−5, 6), and Z(−2, 4) 90° counterclockwise about the origin on the coordinate plane. Is the pre-image congruent to the image? Explain your reasoning. What information did you use to perform this rotation?



Name_____

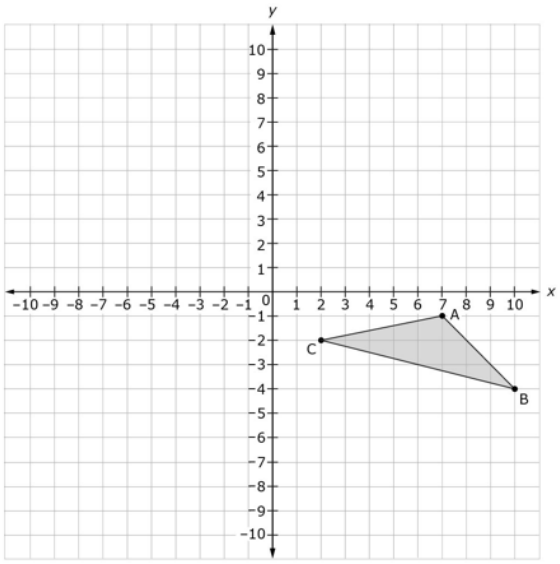
3.d. Rotate the plane containing pentagon EFGHI with coordinates E(1, –1), F(1, –5), G(3, –5), H(4, –3), and I(3, –1) 90° clockwise about the origin on the coordinate plane. Is the pre-image congruent to the image? Explain your reasoning. What is another way to describe the same transformation using a counterclockwise rotation?



Name_____

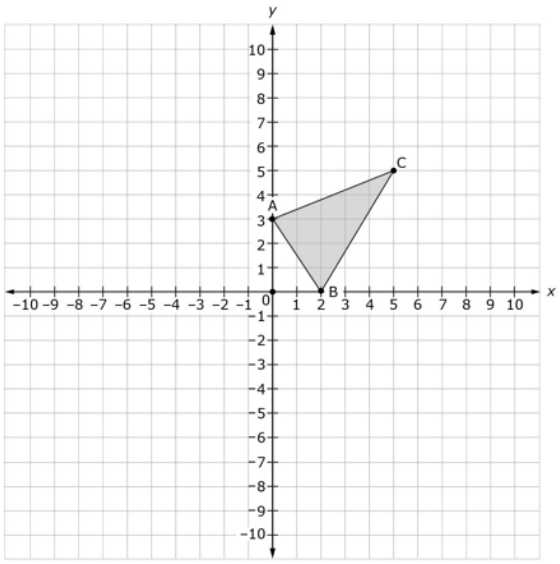
4. Perform the following transformations on the coordinate plane. Patty paper may be used if needed. Be sure to label the vertices appropriately and accurately.

4.a. Translate the plane containing triangle ABC with coordinates A(7, –1), B(10, –4), and C(2, –2) left 3 units and up 5 units, then reflect across the *y*–axis on the coordinate plane. Is the pre-image congruent to the image? Explain your reasoning.



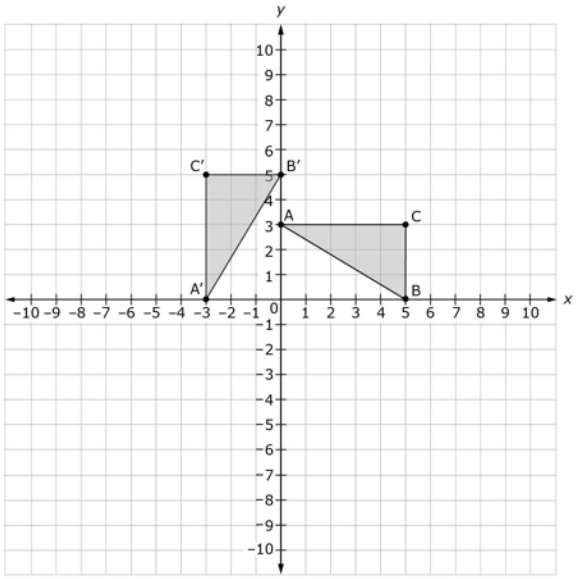
Name_____

4.b. Rotate the plane containing triangle ABC with coordinates A(0, 3), B(2, 0), and C(5, 5) 90° counterclockwise about the origin, then reflect across the *x*–axis, then translate right 4 units and down 1 unit. Is the pre-image congruent to the image? Explain your reasoning.



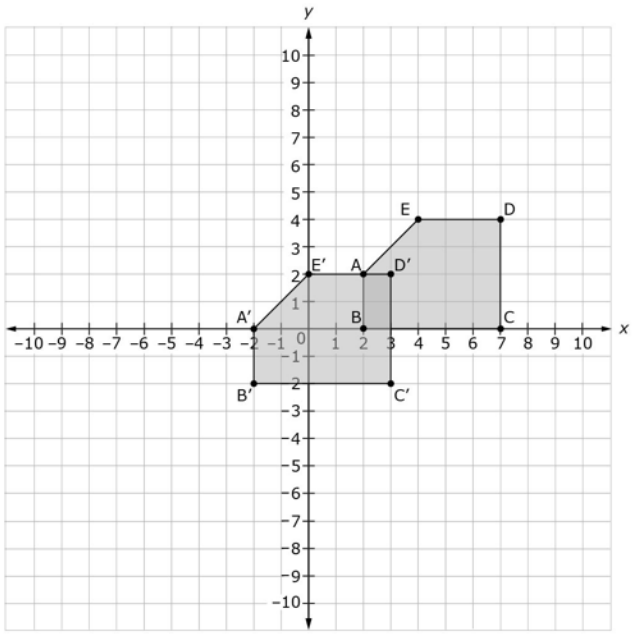
Name_____

5. Describe the transformation(s) that have occurred using words and draw the translation vector, line of reflection, and/or center of rotation on the coordinate plane.



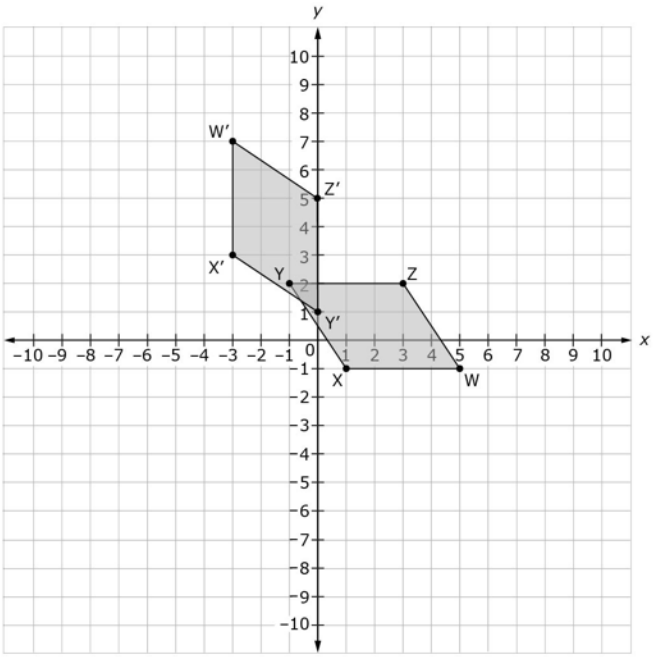
Name_____

6. Describe the transformation(s) that have occurred using words and draw the translation vector, line of reflection, and/or center of rotation on the coordinate plane.



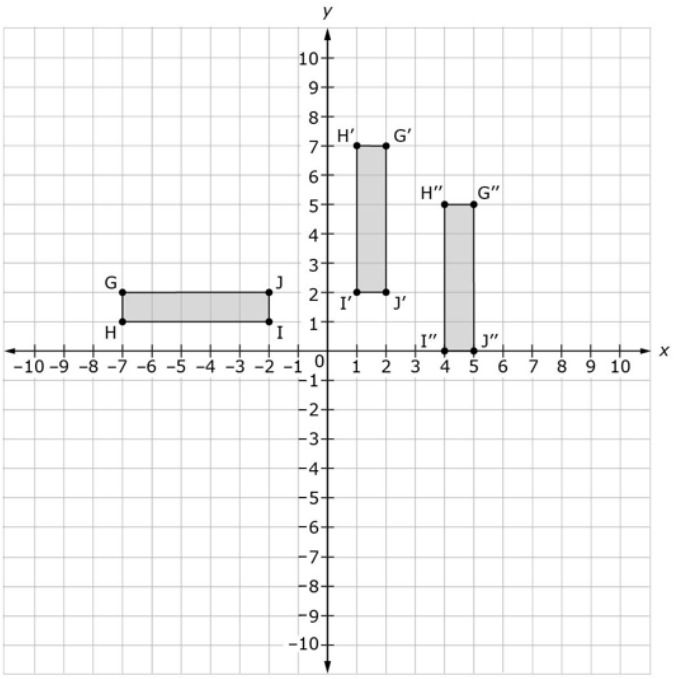
Name_____

7. Describe the transformation(s) that have occurred using words and draw the translation vector, line of reflection, and/or center of rotation on the coordinate plane.



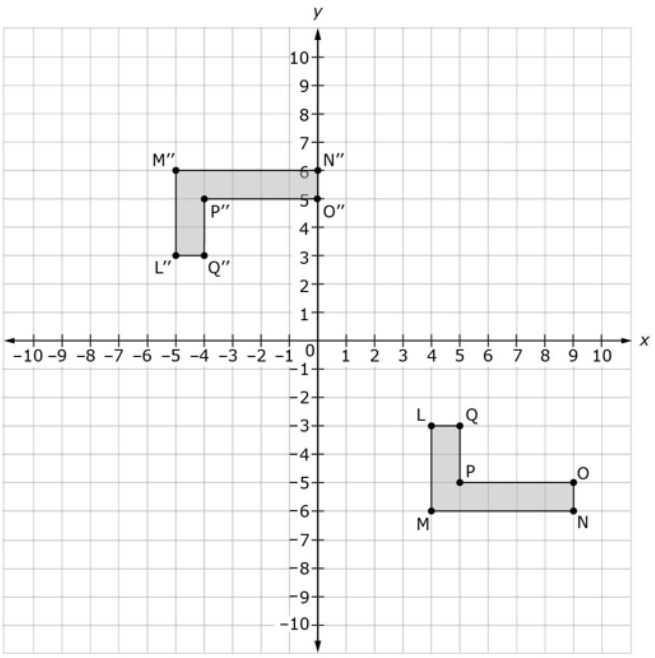
Name_____

8. Describe the transformation(s) that have occurred using words and draw the translation vector, line of reflection, and/or center of rotation on the coordinate plane.



Name_____

9. Determine whether the following figures are congruent. Draw and describe any translation vectors, lines of reflection, and/or centers of rotation you use on the coordinate plane as well as any intermediate figures necessary to support your conclusion.



10. The plane containing triangle EFG with coordinates E(−7, 0), F(−5, −3), and G(1, 1) is translated up 8 units and right 3 units. Without graphing, write the coordinates of the image. Explain how you determined the coordinates of the image.

Name_____

11. The plane containing rectangle LMNO with coordinates L(2, 3), M(5, 3), N(5, –4), and O(2, –4) is translated down 4 units and left 2 units. Without graphing, write the coordinates of the image. Explain how you determined the coordinates of the image.

12. The plane containing quadrilateral QRST with coordinates Q(1, –3), R(4, 0), S(5, 2), and T(2, 3) is reflected across the *x*–axis. Without graphing, write the coordinates of the image. Explain how you determined the coordinates of the image.

13. The plane containing pentagon VWXYZ with coordinates V(–8, 2), W(–7, –4), X(–3, –4), Y(–2, 2) and Z(–5, 4) is reflected across the *y*–axis. Without graphing, write the coordinates of the image. Explain how you determined the coordinates of the image.

Name_____

14. The plane containing triangle JKL with coordinates J(7, 9), K(–1, 8), and L(3, 4) is reflected across the line $y = x$. Without graphing, write the coordinates of the image. Explain how you determined the coordinates of the image.

15. The plane containing rectangle ABCD with coordinates A(–3, 7), B(–3, –1), C(5, –1), and D(5, 7) is reflected across the line $y = -x$. Without graphing, write the coordinates of the image. Explain how you determined the coordinates of the image.

16. How can you use transformations to show two figures are congruent?

17. After a translation, reflection, or rotation, are corresponding angles congruent? Are corresponding line segments congruent?

18. If you have parallel line segments in the pre-image, will the corresponding line segments be parallel in the image?

19. Identify the information that would be needed, in addition to the pre-image, to perform the requested transformation. In your response, explain how you would use this information in order to perform the transformation.

19.a. Reflection

19.b. Translation

19.c. Rotation

TRANSFORMATIONS AND CONGRUENCE

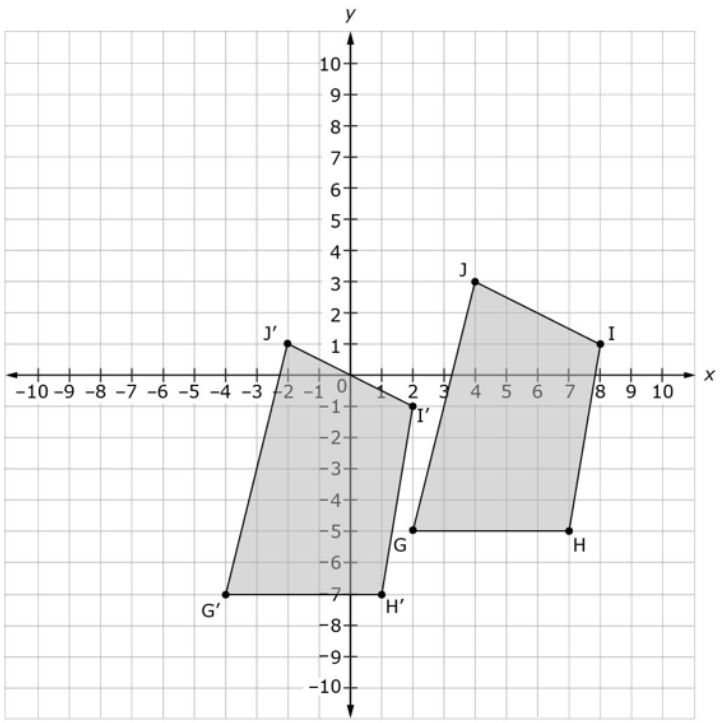
STUDENT ACTIVITY SOLUTION GUIDE

Lessons 1 – 4

1. Perform the following translations on the coordinate plane. Patty paper may be used if needed. Be sure to label the vertices appropriately and accurately.

   1.a. Translate the plane containing quadrilateral GHIJ with coordinates G(2, –5), H(7, –5), I(8, 1), and J(4, 3) to the left 6 units and down 2 units. Is the pre-image congruent to the image? Explain your reasoning.

CORRECT ANSWER



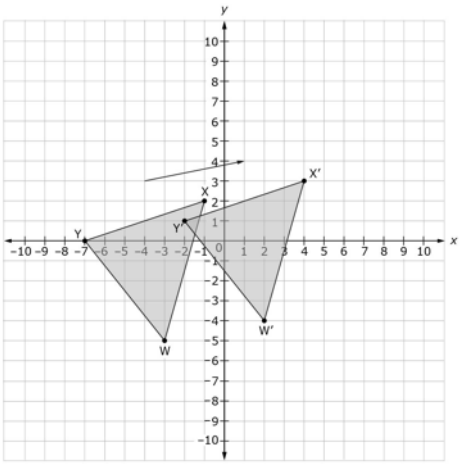
The pre-image is congruent to the image. Translating a figure preserves angle measures and lengths of line segments. Therefore, the size and shape of the figure do not change during the translation.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Student translates some, but not all, points in the figure.	does not understand that every point in the plane translates	REPRESENT *TRANSLATION* and EXPLAIN *TRANSLATION*
Student states that the pre-image and image are not congruent.	is not able to reason about properties that are preserved during a transformation or cannot describe what makes two figures congruent	EXPLAIN *CONGRUENT FIGURES* or EXPLAIN *DISTANCE-PRESERVING TRANSFORMATIONS*
Note: A student who makes a mistake transforming the pre-image may produce an image that is not congruent to the pre-image. In this instance, the student may accurately state the pre-image and image are not congruent. The student may also recall that because translations (and reflection and rotations) produce an image congruent to the pre-image, they must be congruent. Carefullly evaluate, in this question and in subsequent questions, whether a student's understanding of this concept is very rudimentary or if the student was careless in their work.		

1.b. Translate the plane containing triangle WXY with coordinates W(–3, –5), X(–1, 2), and Y(–7, 0) according to the vector provided on the coordinate plane. Is the pre-image congruent to the image? Explain your reasoning. What information did you use to perform this translation?

CORRECT ANSWER



The pre-image is congruent to the image. Translating a figure preserves angle measures and lengths of line segments. Therefore, the size and shape of the figure do not change during the translation. I used the information provided by the pre-image and the translation vector

(which indicates direction and magnitude) in order to perform this translation.

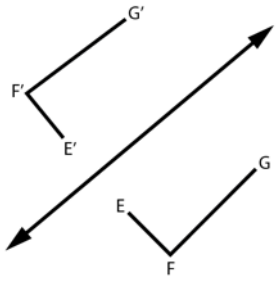
ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Student translates some, but not all, points in the figure.	does not understand that every point in the plane translates	REPRESENT *TRANSLATION* and EXPLAIN *TRANSLATION*
Student translates across the vector instead of in the direction the vector indicates by the given magnitude.	thinks vectors behave similarly to lines of reflection	REPRESENT *TRANSLATION*
Student translates the pre-image to the tail of the vector, then translates according to the direction and magnitude of the vector provided.	thinks the pre-image must have a vertex at the tail of the vector and the image must have the corresponding vertex at the head of the vector	REPRESENT *TRANSLATION*
Student states that the pre-image and image are not congruent.	is not able to reason about properties that are preserved during a transformation or cannot describe what makes two figures congruent	EXPLAIN *CONGRUENT FIGURES* or EXPLAIN *DISTANCE-PRESERVING TRANSFORMATIONS*
Student identifies only the translation vector as the information used to perform the translation.	focuses on the translation vector as the most critical piece of information provided without considering the pre-image	EXPLAIN *TRANSLATION*
Student identifies only the pre-image as the information used to perform the translation.	focuses on the pre-image as the most critical piece of information provided without considering the importance of a translation vector (or description of direction and magnitude)	EXPLAIN *TRANSLATION*

2. Perform the following reflections on the coordinate plane. Patty paper may be used if needed. Be sure to label the vertices appropriately and accurately.

   2.a. Draw the approximate reflection of the plane containing the figure across the line of reflection provided.

CORRECT ANSWER

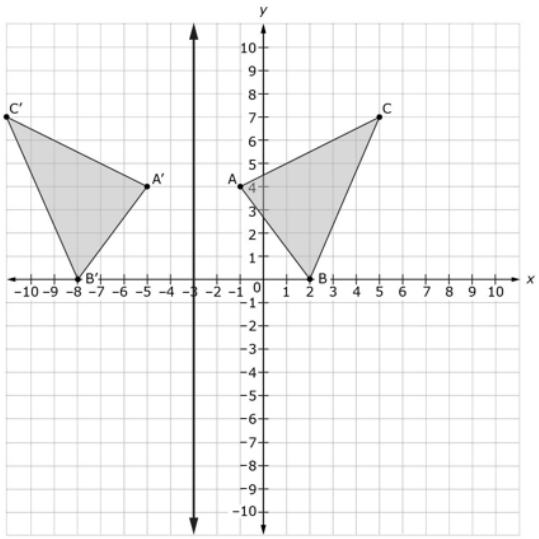


ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Student translates one vertex of the pre-image to the line of reflection, then reflects across the line of reflection.	does not understand that the entire plane containing the figure reflects and therefore the distance between the points on the pre-image and the line of reflection is preserved in the reflection	REPRESENT *REFLECTION* and EXPLAIN *REFLECTION*
Student translates across the line of reflection instead of reflecting.	confuses lines of reflection and translation vectors	REPRESENT *REFLECTION*

2.b. Reflect the plane containing triangle ABC with coordinates A(–1, 4), B(2, 0), and C(5, 7) across the line of reflection provided on the coordinate plane. Is the pre-image congruent to the image? Explain your reasoning.

CORRECT ANSWER



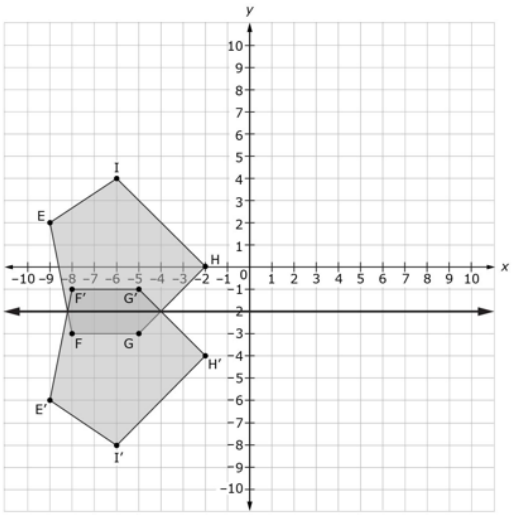
The pre-image is congruent to the image. Reflecting a figure preserves angle measures and lengths of line segments. Therefore, the size and shape of the figure do not change during the reflection, even though the orientation changes.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Student translates so point A is on the line of reflection, then reflects across the line of reflection.	does not understand that the entire plane containing the triangle reflects and therefore the distance between the points on the pre-image and the line of reflection is preserved in the reflection	REPRESENT *REFLECTION* and EXPLAIN *REFLECTION*
Student translates across the line of reflection instead of reflecting.	confuses lines of reflection and translation vectors	REPRESENT *REFLECTION*
Student states that the pre-image and image are not congruent.	is not able to reason about properties that are preserved during a transformation or cannot describe what makes two figures congruent	EXPLAIN *CONGRUENT FIGURES* or EXPLAIN *DISTANCE-PRESERVING TRANSFORMATIONS*

2.c. Reflect the plane containing pentagon EFGHI with coordinates E(–9, 2), F(–8, –3), G(–5, –3), H(–2, 0), and I(–6, 4) across the line of reflection provided on the coordinate plane. Is the pre-image congruent to the image? Explain your reasoning.

CORRECT ANSWER



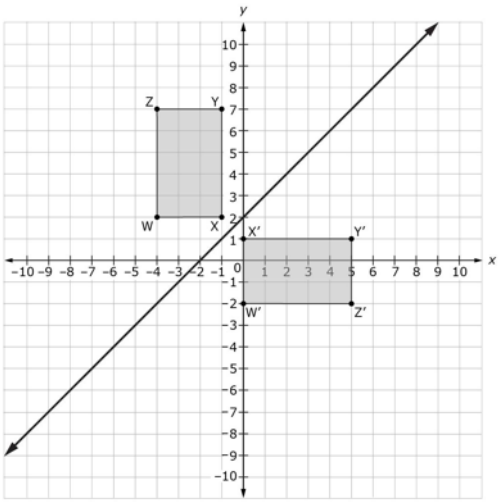
The pre-image is congruent to the image. Reflecting a figure preserves angle measures and lengths of line segments. Therefore, the size and shape of the figure do not change during the reflection, even though the orientation changes.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Student reflects across the line $y = -3$ (or side $\overline{FG}$) instead of the line of reflection.	does not know how to reflect across a line that goes through the figure	REPRESENT *REFLECTION*
Student translates so side $\overline{FG}$ is on the line of reflection, then reflects across the line of reflection.	does not understand that the entire plane containing the pentagon reflects and therefore the distance between the points on the pre-image and the line of reflection is preserved in the reflection	REPRESENT *REFLECTION* and EXPLAIN *REFLECTION*
Student states that the pre-image and image are not congruent.	is not able to reason about properties that are preserved during a transformation or cannot describe what makes two figures congruent	EXPLAIN *CONGRUENT FIGURES* or EXPLAIN *DISTANCE-PRESERVING TRANSFORMATIONS*

2.d. Reflect the plane containing quadrilateral WXYZ with coordinates W(–4, 2), X(–1, 2), Y(–1, 7), and Z(–4, 7) across the line of reflection provided on the coordinate plane. Is the pre-image congruent to the image? Explain your reasoning. What information did you use to perform this reflection?

CORRECT ANSWER



The pre-image is congruent to the image. Reflecting a figure preserves angle measures and lengths of line segments. Therefore, the size and shape of the figure do not change during the reflection, even though the orientation changes. I used the information provided by the pre-image and the line of reflection in order to perform this reflection.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

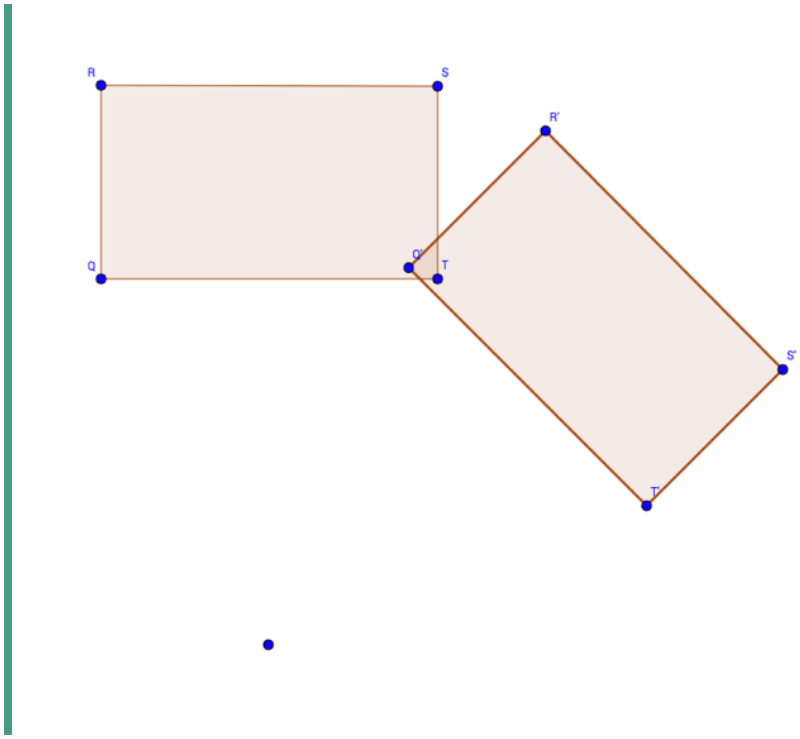
Example Error	Misconception	Missing Knowledge
Student translates across the line of reflection instead of reflecting.	confuses lines of reflection and translation vectors	REPRESENT *REFLECTION*
Student translates so point X is on the line of reflection, then reflects across the line of reflection.	does not understand that the entire plane containing the rectangle reflects and therefore the distance between the points on the pre-image and the line of reflection is preserved in the reflection	REPRESENT *REFLECTION* and EXPLAIN *REFLECTION*
Student reflects vertically or horizontally.	believes reflections must be horizontal or vertical, does not understand that corresponding points in the plane should be the same distance (but on opposite sides) from the line of reflection	REPRESENT *REFLECTION*
Student states that the pre-image and image are not congruent.	is not able to reason about properties that are preserved during a transformation or cannot describe what makes two figures congruent	EXPLAIN *CONGRUENT FIGURES* or EXPLAIN *DISTANCE-PRESERVING TRANSFORMATIONS*

Example Error	Misconception	Missing Knowledge
Student identifies only the line of reflection as the information used to perform the reflection.	focuses on the line of reflection as the most critical piece of information provided without considering the pre-image	EXPLAIN *REFLECTION*
Student identifies only the pre-image as the information used to perform the reflection.	focuses on the pre-image as the most critical piece of information provided without considering the importance of a line of reflection	EXPLAIN *REFLECTION*

3. Perform the following rotations on the coordinate plane. Patty paper may be used if needed. Be sure to label the vertices appropriately and accurately.

3.a. Draw the approximate rotation of the plane containing quadrilateral QRST 45° clockwise about the point provided.

CORRECT ANSWER

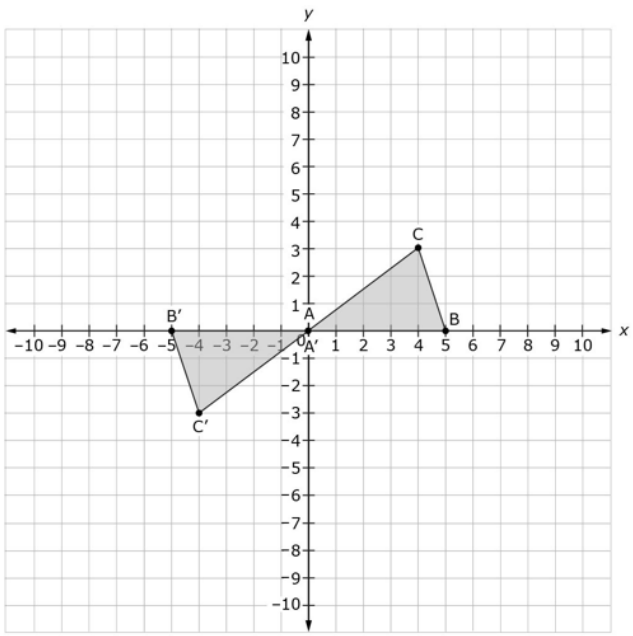


ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Student translates the rectangle to the center of rotation, then rotates.	does not understand that the entire plane containing the rectangle rotates and therefore the distance between the points on the pre-image and the center of rotation are preserved in the rotation	REPRESENT *ROTATION* and EXPLAIN *ROTATION*
Student rotates around one of the vertices instead of the point provided.	does not understand that the center of rotation is significant and impacts the accuracy of the image, or thinks rotations must occur around a vertex instead of a point outside the figure	REPRESENT *ROTATION* and EXPLAIN *ROTATION*
Student accurately rotates one point in the plane 45° clockwise but not all points in the plane in order to maintain the position of the figure.	does not understand that the entire plane containing the rectangle rotates and therefore the distance between all points on the pre-image and the center of rotation is preserved in the rotation	REPRESENT *ROTATION* and EXPLAIN *ROTATION*

3.b. Rotate the plane containing triangle ABC with coordinates A(0, 0), B(5, 0), and C(4, 3) 180° about the point (0, 0) on the coordinate plane. Is the pre-image congruent to the image? Explain your reasoning.

CORRECT ANSWER



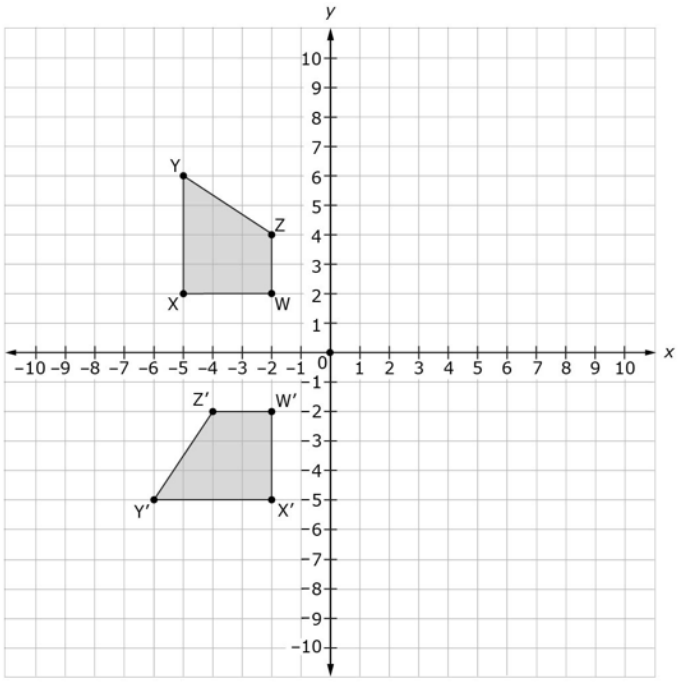
The pre-image is congruent to the image. Rotating a figure preserves angle measures and lengths of line segments. Therefore, the size and shape of the figure do not change during the rotation.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Student rotates around a point in the center of the triangle instead of the origin, keeping the triangle in the first quadrant.	thinks the point of rotation has to be in the figure	REPRESENT *ROTATION* and EXPLAIN *ROTATION*
Student states that the pre-image and image are not congruent.	is not able to reason about properties that are preserved during a transformation or cannot describe what makes two figures congruent	EXPLAIN *CONGRUENT FIGURES* or EXPLAIN *DISTANCE-PRESERVING TRANSFORMATIONS*

3.c. Rotate the plane containing quadrilateral WXYZ with coordinates W(–2, 2), X(–5, 2), Y(–5, 6), and Z(–2, 4) 90° counterclockwise about the origin on the coordinate plane. Is the pre-image congruent to the image? Explain your reasoning. What information did you use to perform this rotation?

CORRECT ANSWER



> The pre-image is congruent to the image. Rotating a figure preserves angle measures and lengths of line segments. Therefore, the size and shape of the figure do not change during the rotation. I used the information provided by the pre-image, the center of rotation, and the angle of rotation in order to perform this rotation.

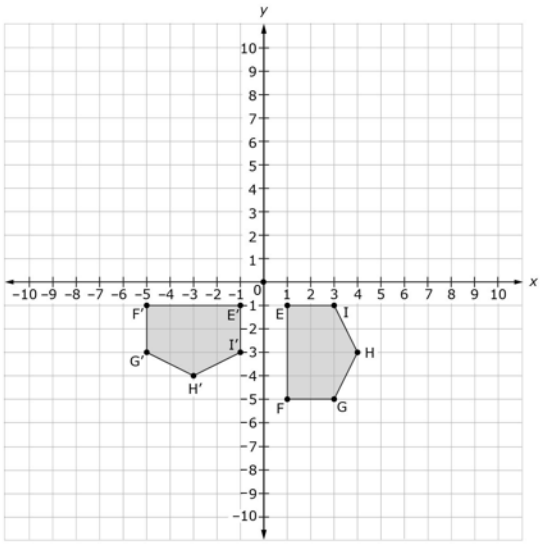
ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Student translates the quadrilateral to the center of rotation, then rotates.	does not understand that the entire plane containing the quadrilateral rotates and therefore the distance between the points on the pre-image and the center of rotation are preserved in the rotation	REPRESENT *ROTATION* and EXPLAIN *ROTATION*
Student rotates around one of the vertices instead of the origin.	does not understand that the center of rotation is significant and impacts the accuracy of the image, or thinks rotations must occur around a vertex instead of a point outside the figure	REPRESENT *ROTATION* and EXPLAIN *ROTATION*
Student accurately rotates one point in the plane 90° counterclockwise but not all points in the plane in order to maintain the position of the figure.	does not understand that the entire plane containing the quadrilateral rotates and therefore the distance between all points on the pre-image and the center of rotation is preserved in the rotation	REPRESENT *ROTATION* and EXPLAIN *ROTATION*
Student states that the pre-image and image are not congruent.	is not able to reason about properties that are preserved during a transformation or cannot describe what makes two figures congruent	EXPLAIN *CONGRUENT FIGURES* or EXPLAIN *DISTANCE-PRESERVING TRANSFORMATIONS*
Student identifies only the center of rotation and/or the angle of rotation as the information used to perform the rotation.	focuses on the center of rotation and/or angle of rotation as the most critical piece of information provided without considering the pre-image	EXPLAIN *ROTATION*
Student identifies only the pre-image as the information used to perform the rotation.	focuses on the pre-image as the most critical piece of information provided without considering the importance of the center of rotation and the angle of rotation	EXPLAIN *ROTATION*

3.d. Rotate the plane containing pentagon EFGHI with coordinates E(1, –1), F(1, –5), G(3, –5), H(4, –3), and I(3, –1) 90° clockwise about the origin on the coordinate plane. Is the pre-image congruent to

the image? Explain your reasoning. What is another way to describe the same transformation using a counterclockwise rotation?

CORRECT ANSWER



The pre-image is congruent to the image. Rotating a figure preserves angle measures and lengths of line segments. Therefore, the size and shape of the figure do not change during the rotation.

To describe the transformation resulting in the same position of the image, the rotation could be 270° counterclockwise about the origin.

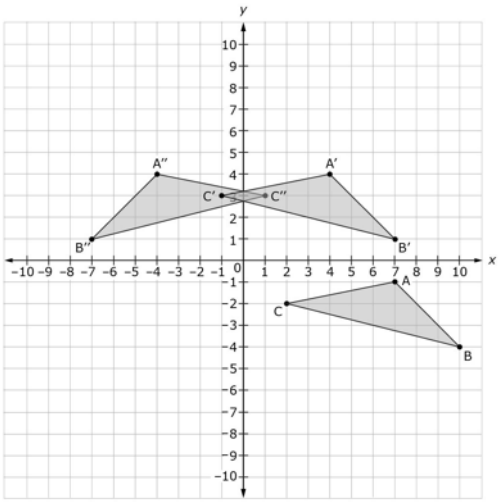
ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Student translates the pentagon to the center of rotation, then rotates.	does not understand that the entire plane containing the pentagon rotates and therefore the distance between the points on the pre-image and the center of rotation are preserved in the rotation	REPRESENT *ROTATION* and EXPLAIN *ROTATION*
Student rotates around one of the vertices instead of the origin.	does not understand that the center of rotation is significant and impacts the accuracy of the image, or thinks rotations must occur around a vertex instead of a point outside the figure	REPRESENT *ROTATION* and EXPLAIN *ROTATION*
Student accurately rotates one point in the plane 90° clockwise but not all points in the plane in order to maintain the position of the figure.	does not understand that the entire plane containing the pentagon rotates and therefore the distance between all points on the pre-image and the center of rotation are preserved in the rotation	REPRESENT *ROTATION* and EXPLAIN *ROTATION*
Student states that the pre-image and image are not congruent.	is not able to reason about properties that are preserved during a transformation or cannot describe what makes two figures congruent	EXPLAIN *CONGRUENT FIGURES* or EXPLAIN *DISTANCE-PRESERVING TRANSFORMATIONS*
Student does not state that a 90° clockwise rotation and a 270° counterclockwise result in the same position for the image.	believes that since the angle of rotation is different, the image will be different	EXPLAIN *ROTATION*

4. Perform the following transformations on the coordinate plane. Patty paper may be used if needed. Be sure to label the vertices appropriately and accurately.

   4.a. Translate the plane containing triangle ABC with coordinates A(7, –1), B(10, –4), and C(2, –2) left 3 units and up 5 units, then reflect across the *y*–axis on the coordinate plane. Is the pre-image congruent to the image? Explain your reasoning.

CORRECT ANSWER



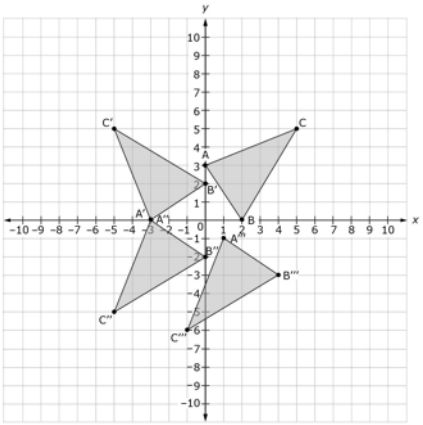
The pre-image is congruent to the image. Translating and reflecting a figure preserves angle measures and lengths of line segments. Therefore, the size and shape of the figure do not change during the translation or reflection, even though the orientation changes.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Student performs the translation and reflection out of order.	does not believe it matters what order the transformations occur in as long as all described transformations are performed	REPRESENT COMPOSITIONS OF DISTANCE-PRESERVING TRANSFORMATIONS
Student translates accurately, then reflects across C′ instead of the *y*–axis because the described line of reflection goes through the figure.	cannot reflect across a line that goes through the figure	REPRESENT *REFLECTION*
Student translates triangle ABC, then reflects triangle ABC instead of A′B′C′.	Student believes the initial figure (pre-image) must always be the figure that is transformed	REPRESENT COMPOSITIONS OF DISTANCE-PRESERVING TRANSFORMATIONS
Student states that the pre-image and image are not congruent.	is not able to reason about properties that are preserved during multiple transformations or cannot describe what makes two figures congruent	EXPLAIN *CONGRUENT FIGURES* or EXPLAIN *DISTANCE-PRESERVING TRANSFORMATIONS*
Note: The example errors provided address possible errors with sequences of transformations, though a variety of additional errors could occur within individual transformations. Please consider all combinations of errors when evalutating student work.		

4.b. Rotate the plane containing triangle ABC with coordinates A(0, 3), B(2, 0), and C(5, 5) 90° counterclockwise about the origin, then reflect across the *x*–axis, then translate right 4 units and down 1 unit. Is the pre-image congruent to the image? Explain your reasoning.

CORRECT ANSWER

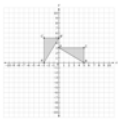


The pre-image is congruent to the image. Rotating, reflecting, and translating a figure preserves angle measures and lengths of line segments. Therefore, the size and shape of the figure do not change during the rotation, reflection, or translation, even though the reflection causes the orientation to change.

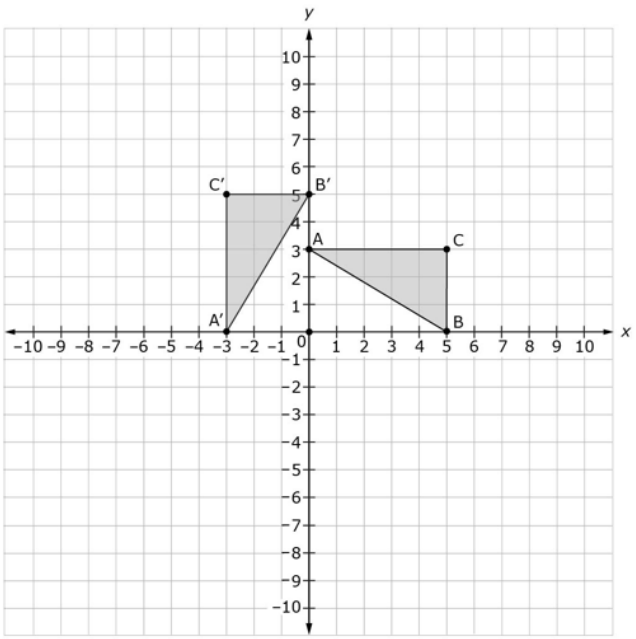
ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Student performs the transformations out of order, resulting in an incorrect image.	does not believe it matters what order the transformations occur in as long as all described transformations are performed	REPRESENT COMPOSITIONS OF DISTANCE-PRESERVING TRANSFORMATIONS
Student begins each transformation with triangle ABC instead of the result of the most recent transformation.	believes the initial figure (pre-image) must always be the figure that is transformed	REPRESENT COMPOSITIONS OF DISTANCE-PRESERVING TRANSFORMATIONS
Student states that the pre-image and image are not congruent.	is not able to reason about properties that are preserved during multiple transformations or cannot describe what makes two figures congruent	EXPLAIN *CONGRUENT FIGURES* or EXPLAIN *DISTANCE-PRESERVING TRANSFORMATIONS*
Note: The example errors provided address possible errors with sequences of transformations, though a variety of additional errors could occur within individual transformations. Please consider all combinations of errors when evalutating student work.		

5. Describe the transformation(s) that have occurred using words and draw the translation vector, line of reflection, and/or center of rotation on the coordinate plane.



CORRECT ANSWER



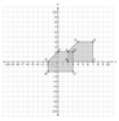
To transform the pre-image to the image, a 90° counterclockwise rotation about the point (0, 0) (or the origin) has occurred.

*Note: It is also possible for a student to rotate the pre-image around a different point and then translate to account for the position or perform two reflections across intersecting lines and result in the same image, though these are less likely responses.

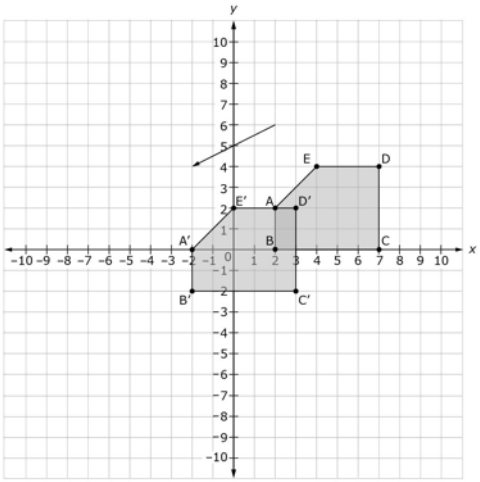
ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student identifies the center of rotation as one of the vertices of triangle ABC.	believes the rotation must be around a vertex on the pre-image	REPRESENT *ROTATION* and EXPLAIN *ROTATION*
The student does not identify the angle of rotation.	determines the center of rotation but does not know how to use the points of the pre-image and the image to find the angle of rotation	REPRESENT *ROTATION*
Student identifies either a single translation or a single reflection instead of a rotation.	does not recognize that the figure has been turned, making a rotation	RECOGNIZE *ROTATION* and EXPLAIN *ROTATION*

6. Describe the transformation(s) that have occurred using words and draw the translation vector, line of reflection, and/or center of rotation on the coordinate plane.



CORRECT ANSWER



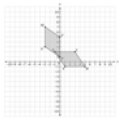
To transform the pre-image to the image, a translation to the left 4 units and down 2 units has occurred.

**Note: It is also possible for a student to perform two reflections across parallel lines and result in the same image, though this is a less likely response.*

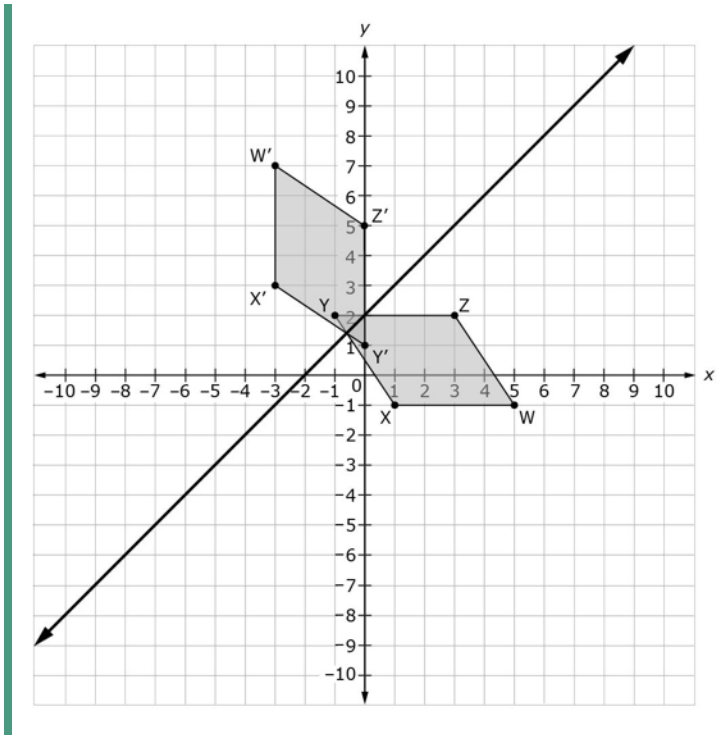
ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Student identifies the translation as "up 2 units, right 4 units".	starts with the image instead of the pre-image	REPRESENT *TRANSLATION*
Student describes the correct translation but does not draw the translation vector on the coordinate plane.	does not understand that the vector mirrors the verbal description of the translation	REPRESENT *TRANSLATION*
Student identifies either a single rotation or a single reflection instead of a translation.	does not recognize that the figure slides to form the image, making a translation	RECOGNIZE *TRANSLATION* and EXPLAIN *TRANSLATION*

7. Describe the transformation(s) that have occurred using words and draw the translation vector, line of reflection, and/or center of rotation on the coordinate plane.



CORRECT ANSWER

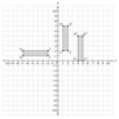


To transform the pre-image to the image, a reflection across a diagonal line ($y = x + 2$) has occurred.

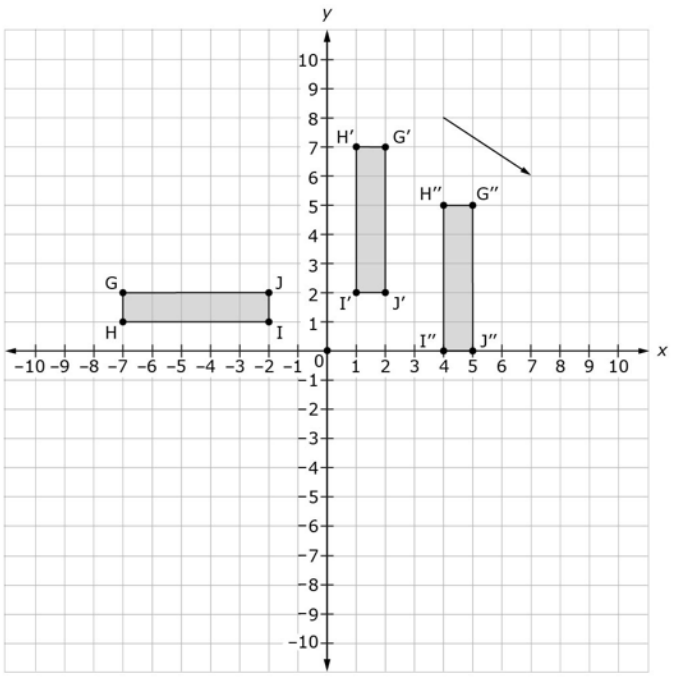
ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Student draws a vertical or horizontal line of reflection.	believes lines of reflection have to be vertical or horizontal	REPRESENT *REFLECTION*
Student draws a diagonal line through vertex Y on the pre-image.	does not think the line of reflection can go through the pre-image	REPRESENT *REFLECTION*
Student identifies either a translation or a rotation instead of a reflection.	does not recognize that the figure flips to form the image, making a reflection	RECOGNIZE *REFLECTION* and EXPLAIN *REFLECTION*

8. Describe the transformation(s) that have occurred using words and draw the translation vector, line of reflection, and/or center of rotation on the coordinate plane.



CORRECT ANSWER

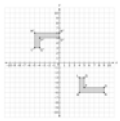


To transform the pre-image to the image, a 90° clockwise rotation about the point (0, 0) (or the origin), then a translation down 2 units and right 3 units have occurred.

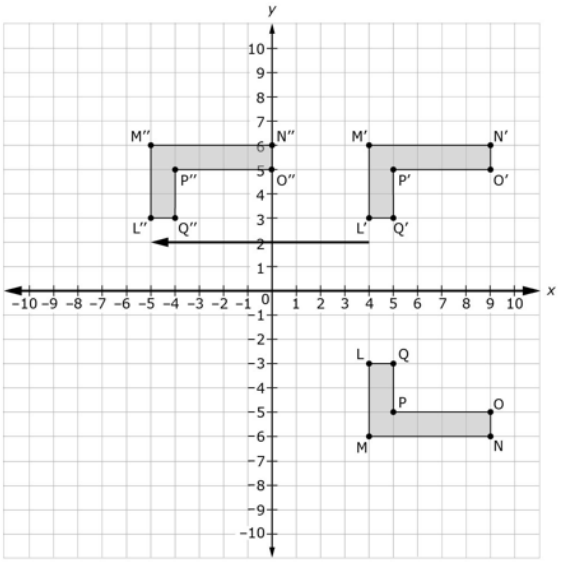
ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Student describes a transformation from rectangle GHIJ to G''H''I''J'' instead of the translation from G'H'I'J' to G''H''I''J''.	believes the initial figure (pre-image) must always be the figure that is transformed	REPRESENT COMPOSITIONS OF DISTANCE-PRESERVING TRANSFORMATIONS
Student places the center of rotation on the positive *y*–axis.	thinks the center of rotation has to be between the two figures	REPRESENT *ROTATION*

9. Determine whether the following figures are congruent. Draw and describe any translation vectors, lines of reflection, and/or centers of rotation you use on the coordinate plane as well as any intermediate figures necessary to support your conclusion.



CORRECT ANSWER



The figures are congruent because a sequence of distance-preserving transformations map the original figure precisely onto the second

figure. In order to transform the pre-image to the image, a reflection across the *x*–axis, then a translation left 9 units have occurred.

**Note: There are other possible correct answers involving a reflection across a horizontal line and a translation.*

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
A rotation of 180° will transform the pre-image to the image.	focuses on points L" and Q" only, rather than all points in the figure/plane	REPRESENT *ROTATION*
Student describes a single reflection across a diagonal line.	thinks a single reflection across a diagonal line can account for both transformations	REPRESENT *REFLECTION*
Student states that the figures are not congruent.	does not know when figures are congruent but may be able to perform transformations	EXPLAIN CONGRUENCE THROUGH TRANSFORMATION
Student does not attempt to superimpose the pre-image onto the image.	does not know how to prove two figures are congruent using transformations but may be able to perform transformations	PROVE 2 FIGURES ARE CONGRUENT USING A SEQUENCE OF TRANSFORMATIONS

10. The plane containing triangle EFG with coordinates E(–7, 0), F(–5, –3), and G(1, 1) is translated up 8 units and right 3 units. Without graphing, write the coordinates of the image. Explain how you determined the coordinates of the image.

CORRECT ANSWER

Since there is a translation to the right 3 units, I added 3 to the *x*–coordinate of each ordered pair. Since there is a translation up 8 units, I added 8 to the *y*–coordinate of each coordinate pair. Therefore, the coordinates of the image are E'(–4, 8), F'(–2, 5), and G'(4, 9).

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
E'(1, 3), F'(3, 0), G'(9, 4)	adds 8 to the *x*–coordinate and 3 to the *y*–coordinate; may not understand horizontal movement affects the *x*–coordinate and vertical movement affects the *y*–coordinate	EXPLAIN THE EFFECT OF TRANSLATIONS USING COORDINATES
E'(–10, –8), F'(–8, –11), G'(–2, –7)	subtracts instead of adds to the *x*– and *y*–coordinates, resulting in a translation left and down instead of right and up	EXPLAIN THE EFFECT OF TRANSLATIONS USING COORDINATES

11. The plane containing rectangle LMNO with coordinates L(2, 3), M(5, 3), N(5, –4), and O(2, –4) is translated down 4 units and left 2 units. Without graphing, write the coordinates of the image. Explain how you determined the coordinates of the image.

CORRECT ANSWER

> Since there is a translation to the left 2 units, I subtracted 2 from the *x*–coordinate of each ordered pair. Since there is a translation down 4 units, I subtracted 4 from the *y*–coordinate of each coordinate pair. Therefore, the coordinates of the image are L'(0, –1), M'(3, –1), N'(3, –8), and O'(0, –8).

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
L'(–2, 1), M'(1, 1), N'(1, –6), O'(–2, –6)	subtracts 4 from the *x*–coordinate and 2 from the *y*–coordinate; may not understand horizontal movement affects the *x*–coordinate and vertical movement affects the *y*–coordinate	EXPLAIN THE EFFECT OF TRANSLATIONS USING COORDINATES
L'(4, 7), M'(7, 7), N'(7, 0), O'(4, 0)	adds instead of subtracts to the *x*– and *y*–coordinates, resulting in a translation right and up instead of left and down	EXPLAIN THE EFFECT OF TRANSLATIONS USING COORDINATES

12. The plane containing quadrilateral QRST with coordinates Q(1, –3), R(4, 0), S(5, 2), and T(2, 3) is reflected across the *x*–axis. Without graphing, write the coordinates of the image. Explain how you determined the coordinates of the image.

CORRECT ANSWER

> Since there is a reflection across the *x*–axis, which is a vertical movement, the *x*–values will stay the same but the *y*–values will change if they are not on the *x*–axis, becoming the opposite of their value in the pre-image. Therefore, I kept the *x*–coordinates the same and found the opposite value for each *y*–coordinate. The coordinates of the image are Q'(1, 3), R'(4, 0), S'(5, –2), and T'(2, –3).

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Q'(–1, –3), R'(–4, 0), S'(–5, 2), T'(–2, 3)	thinks a reflection across the *x*–axis impacts the *x*–coordinate	EXPLAIN THE EFFECT OF REFLECTIONS USING COORDINATES
Q'(1, –3), R'(4, 0), S'(5, –2), T'(2, –3)	thinks a reflection across the *x*–axis makes the *y*–value negative instead of the opposite value	EXPLAIN THE EFFECT OF REFLECTIONS USING COORDINATES
Explanation is limited to "The *y*–coordinate is the opposite."	Does not understand why the *y*–coordinate becomes the opposite value under a reflection across the *x*–axis	EXPLAIN THE EFFECT OF REFLECTIONS USING COORDINATES

13. The plane containing pentagon VWXYZ with coordinates V(–8, 2), W(–7, –4), X(–3, –4), Y(–2, 2) and Z(–5, 4) is reflected across the *y*–axis. Without graphing, write the coordinates of the image. Explain how you determined the coordinates of the image.

CORRECT ANSWER

> Since there is a reflection across the *y*–axis, which is a horizontal movement, the *y*–values will stay the same but the *x*–values will change if they are not on the *y*–axis, becoming the opposite of their value in the pre-image. Therefore, I kept the *y*–coordinates the same and found the

opposite value for each x–coordinate. The coordinates of the image are V'(8, 2), W'(7, –4), X'(3, –4), Y'(2, 2), and Z'(5, 4).

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
V'(–8, –2), W'(–7, 4), X'(–3, 4), Y'(–2, –2), Z'(–5, –4)	thinks a reflection across the y–axis impacts the y–coordinate	EXPLAIN THE EFFECT OF REFLECTIONS USING COORDINATES
V'(–8, 2), W'(–7, –4), X'(–3, –4), Y'(–2, 2), Z'(–5, 4)	thinks a reflection across the y–axis makes the x–value negative instead of the opposite value	EXPLAIN THE EFFECT OF REFLECTIONS USING COORDINATES
Explanation is limited to "The x–coordinate is the opposite."	Does not understand why the x–coordinate becomes the opposite value under a reflection across the y–axis	EXPLAIN THE EFFECT OF REFLECTIONS USING COORDINATES

14. The plane containing triangle JKL with coordinates J(7, 9), K(–1, 8), and L(3, 4) is reflected across the line $y = x$. Without graphing, write the coordinates of the image. Explain how you determined the coordinates of the image.

CORRECT ANSWER

Since there is a reflection across the line $y = x$, which is an increasing diagonal line through the origin, the x–values and the y–values will switch such that the x–coordinate of the pre-image will be the y–coordinate of the image and the y–coordinate of the pre-image will be the x–coordinate of the image. The coordinates of the image are J'(9, 7), K'(8, –1), and L'(4, 3).

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
J'(–7, 9), K'(1, 8), L'(–3, 4)	gives coordinates for a reflection across the y–axis instead of the line $y = x$	EXPLAIN THE EFFECT OF REFLECTIONS USING COORDINATES
J'(7, –9), K'(–1, –8), L'(3, –4)	gives coordinates for a reflection across the x–axis instead of the line $y = x$	EXPLAIN THE EFFECT OF REFLECTIONS USING COORDINATES

Example Error	Misconception	Missing Knowledge
J'(–7, –9), K'(1, –8), L'(–3, –4)	thinks that in a reflection across the line $y = x$, all coordinates become opposite	EXPLAIN THE EFFECT OF REFLECTIONS USING COORDINATES
J'(7, 9), K'(1, 8), L'(3, 4)	thinks that in a reflection across the line $y = x$, all x– and y–coordinates become positive	EXPLAIN THE EFFECT OF REFLECTIONS USING COORDINATES, EXPLAIN THE PROPERTIES OF ANGLES IN TRANSFORMATIONS, and EXPLAIN THE PROPERTIES OF LINES AND LINE SEGMENTS IN TRANSFORMATIONS

15. The plane containing rectangle ABCD with coordinates A(–3, 7), B(–3, –1), C(5, –1), and D(5, 7) is reflected across the line $y = -x$. Without graphing, write the coordinates of the image. Explain how you determined the coordinates of the image.

CORRECT ANSWER

Since there is a reflection across the line $y = -x$, which is a decreasing diagonal line through the origin, the x–values and the y–values will switch and become the opposite such that the opposite of the x–coordinate of the pre-image will be the y–coordinate of the image and the opposite of the y–coordinate of the pre-image will be the x–coordinate of the image. The coordinates of the image are A'(–7, 3), B'(1, 3), C'(1, –5), and D'(–7, –5).

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
A'(3, 7), B'(3, –1), C'(–5, –1), D'(–5, 7)	gives coordinates for a reflection across the y–axis instead of the line $y = -x$; may think the notation indicates that the y–coordinate stays the same and the x–coordinate is the opposite value	EXPLAIN THE EFFECT OF REFLECTIONS USING COORDINATES
A'(–3, –7), B'(–3, 1), C'(5, 1), D'(5, –7)	gives coordinates for a reflection across the x–axis instead of the line $y = -x$	EXPLAIN THE EFFECT OF REFLECTIONS USING COORDINATES
A'(–3, 7), B'(–3, 1), C'(–5, 1), D'(–5, 7)	thinks that in a reflection across the line $y = -x$, all y–coordinates become positive and all x–coordinates become negative	EXPLAIN THE EFFECT OF REFLECTIONS USING COORDINATES and EXPLAIN THE PROPERTIES OF LINES AND LINE SEGMENTS IN TRANSFORMATIONS
A'(7, –3), B'(–1, –3), C'(–1, 5), D'(7, 5)	may think reflecting across the line $y = -x$ is the same as reflecting across the line $y = x$	EXPLAIN THE EFFECT OF REFLECTIONS USING COORDINATES

16. How can you use transformations to show two figures are congruent?

CORRECT ANSWER

If a series of translations, rotations, and reflections can obtain one figure from another, then they are congruent.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Student cannot explain or states there is not a relationship between transformations and congruence.	does not understand that rotations, reflections, and translations preserve size and shape therefore produce congruent figures	EXPLAIN *DISTANCE-PRESERVING TRANSFORMATIONS* and EXPLAIN *CONGRUENT FIGURES*
Student states that rotations and translations produce congruent figures but reflections do not because the orientation of the figure changes.	thinks that if the orientation changes, figures cannot be congruent	EXPLAIN *DISTANCE-PRESERVING TRANSFORMATIONS* and EXPLAIN *CONGRUENT FIGURES*

17. After a translation, reflection, or rotation, are corresponding angles congruent? Are corresponding line segments congruent?

CORRECT ANSWER

> In a translation, reflection, and rotation, angle measures will not change size and line segments will not change length, therefore corresponding angles are congruent and corresponding line segments are congruent.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Student states that only the angles are congruent.	does not know line segments lengths are also preserved in translations, reflections, and rotations	EXPLAIN THE PROPERTIES OF LINES AND LINE SEGMENTS IN TRANSFORMATIONS
Students states that only the segments are congruent.	does not know angle measures are also preserved in translations, reflections, and rotations	EXPLAIN THE PROPERTIES OF ANGLES IN TRANSFORMATIONS
Student states that only translations produce images whose angles and line segments are congruent to corresponding angles and line segments in the pre-image.	thinks changing the position/orientation in rotations and reflections affects the angle measures and line segments	EXPLAIN THE PROPERTIES OF LINES AND LINE SEGMENTS IN TRANSFORMATIONS and EXPLAIN THE PROPERTIES OF ANGLES IN TRANSFORMATIONS
Student states that only translations and rotations produce images whose angles and line segments are congruent to corresponding angles and line segments in the pre-image.	thinks changing the orientation in rotations and reflections affects the angle measures and line segments	EXPLAIN THE PROPERTIES OF LINES AND LINE SEGMENTS IN TRANSFORMATIONS and EXPLAIN THE PROPERTIES OF ANGLES IN TRANSFORMATIONS

18. If you have parallel line segments in the pre-image, will the corresponding line segments be parallel in the image?

CORRECT ANSWER

> In translations, reflections, and rotations, parallel lines in the pre-image will remain parallel in the image.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Student states that parallel lines in the pre-image do not remain parallel in the image.	does not know that parallel lines remain parallel after a transformation	EXPLAIN THE PROPERTIES OF PARALLEL LINES IN TRANSFORMATIONS
Student states that parallel lines in the pre-image only remain parallel in the image under a translation.	thinks changing the position/orientation in rotations and reflections affects the parallel lines	EXPLAIN THE PROPERTIES OF PARALLEL LINES IN TRANSFORMATIONS

19. Identify the information that would be needed, in addition to the pre-image, to perform the requested transformation. In your response, explain how you would use this information in order to perform the transformation.

19.a. Reflection

CORRECT ANSWER

> In addition to the pre-image, a line of reflection is needed in order to perform a reflection. To reflect the plane, I would reflect each point on the pre-image across the line of reflection in order to create the image. Corresponding points on the pre-image and image should be equidistant from and on opposite sides of the line of reflection.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Student identifies a vector, center of rotation, or angle of rotation as the necessary additional information.	confuses information required for reflections with information required for translations or rotations	EXPLAIN *REFLECTION*
Student does not explain how they would use the line of reflection to perform the reflection.	knows or has memorized what information is required for a reflection but cannot explain how it is used	EXPLAIN *REFLECTION*

19.b. Translation

CORRECT ANSWER

> In addition to the pre-image, a translation vector or description of direction and magnitude is needed in order to perform a translation. To translate the plane, I would translate each point on the pre-image according to the direction and magnitude indicated by the translation vector or description in order to create the image.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Student identifies a line of reflection, center of rotation, or angle of rotation as the necessary additional information.	confuses information required for translations with information required for reflections or rotations	EXPLAIN *TRANSLATION*
Student does not explain how they would use the vector or description of direction and magnitude to perform the translation.	knows or has memorized what information is required for a translation but cannot explain how it is used	EXPLAIN *TRANSLATION*

19.c. Rotation

CORRECT ANSWER

> In addition to the pre-image, a center of rotation and angle of rotation are needed in order to perform a rotation. To rotate the plane, I would rotate each point on the pre-image according to the center of rotation and angle of rotation provided in order to create the image. Corresponding points on the pre-image and image should be equidistant from the center of rotation.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Student identifies a vector or a line of reflection as the necessary additional information.	confuses information required for rotations with information required for reflections or translations	EXPLAIN *ROTATION*
Student identifies either a center of rotation or an angle of rotation as the required information, but not both.	does not consider that only information about the center of rotation or the angle of rotation would not fully define a rotation	EXPLAIN *ROTATION*
Student does not explain how they would use the center of rotation or angle of rotation to perform the rotation.	knows or has memorized what information is required for a rotation but cannot explain how it is used	EXPLAIN *ROTATION*