ADDING AND SUBTRACTING FRACTIONS WITH LIKE DENOMINATORS

4.NF.B.4-6

CONTENTS

The types of documents contained in the unit are listed below. Throughout the unit, the documents are arranged by lesson.

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ADDING AND SUBTRACTING FRACTIONS WITH LIKE DENOMINATORS

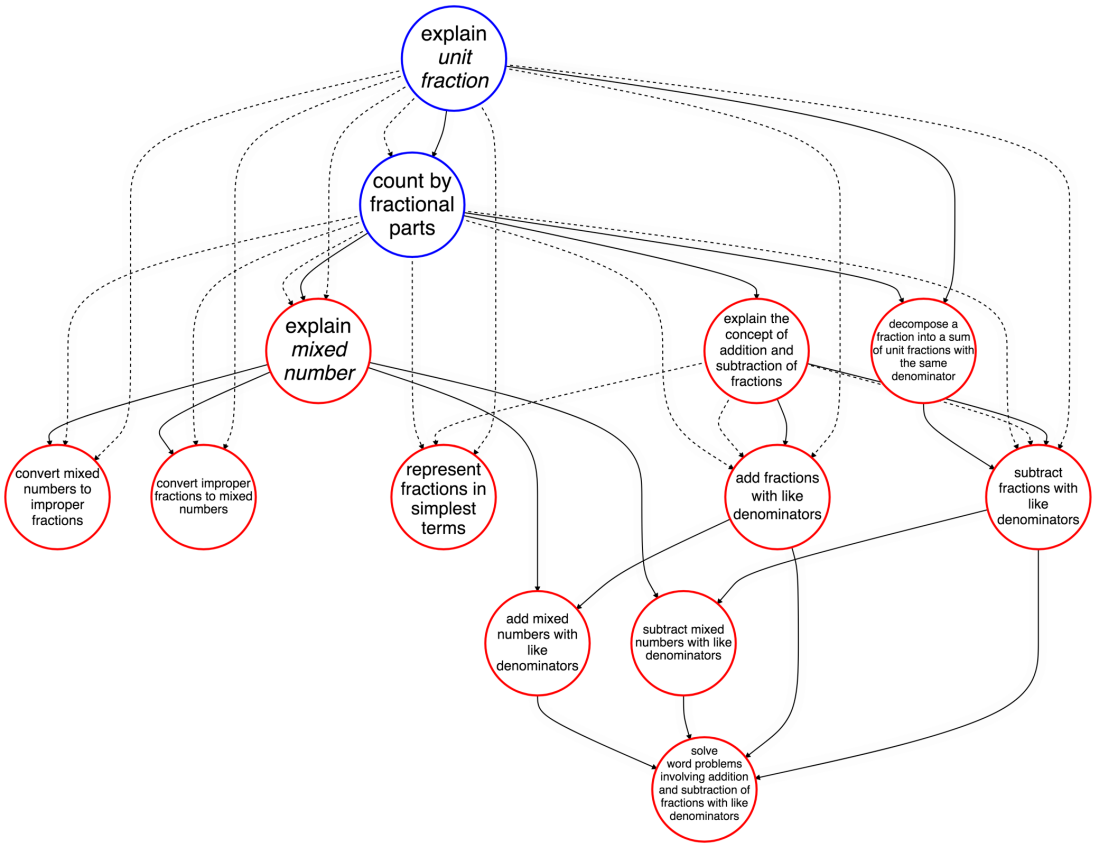
LEARNING MAP INFORMATION

STANDARDS

4.NF.B.4 Understand addition and subtraction of fractions as joining/composing and separating/decomposing parts referring to the same whole.

4.NF.B.5 Decompose a fraction into a sum of fractions with the same denominator and record each decomposition with an equation and justification.

4.NF.B.6 Solve problems involving adding and subtracting fractions and mixed numbers with like denominators.



**Learning map model of* 4.NF.B.4-6

Node Name	Node Description
ADD FRACTIONS WITH LIKE DENOMINATORS	Add two fractions with the same denominator.
ADD MIXED NUMBERS WITH LIKE DENOMINATORS	Add two mixed numbers with the same denominator.
CONVERT IMPROPER FRACTIONS TO MIXED NUMBERS	Change the given improper fraction to a mixed number.
CONVERT MIXED NUMBERS TO IMPROPER FRACTIONS	Change the given mixed number to an improper fraction.
COUNT BY FRACTION PARTS	Fractionally count by fractional parts (e.g., one-half, one, one and one-half, two, two and one-half, three, etc.).
DECOMPOSE A FRACTION INTO A SUM OF UNIT FRACTIONS WITH THE SAME DENOMINATOR	Represent fractions as a decomposition of fractions with the same denominator.
EXPLAIN *MIXED NUMBER*	Make known your understanding that a mixed number consists of a whole number followed by a proper fraction.
EXPLAIN THE CONCEPT OF ADDITION AND SUBTRACTION OF FRACTIONS	Understand addition and subtraction of fractions as joining and separating parts of the same whole. These operations can be used directly on fractional pieces that are the same size (i.e., pieces resulting from subdividing wholes into equal numbers of parts, where each part is the same size). For example, thirds can be added together or subtracted from each other, whereas a third and fourth cannot be directly added together or subtracted.
EXPLAIN *UNIT FRACTION*	Make known your understanding that a unit fraction, $\frac{1}{n}$, is the quantity formed by one part when a whole is partitioned into n equal parts.
REPRESENT FRACTIONS IN SIMPLEST TERMS	Through writing or an appropriate assistive technology, represent a given fraction in simplest terms.
SOLVE WORD PROBLEMS INVOLVING ADDITION AND SUBTRACTION OF FRACTIONS WITH LIKE DENOMINATORS	Use addition and subtraction to solve word problems involving fractions with the same denominator.
SUBTRACT FRACTIONS WITH LIKE DENOMINATORS	Subtract two fractions with the same denominator.
SUBTRACT MIXED NUMBERS WITH LIKE DENOMINATORS	Subtract two mixed numbers with the same denominator.

ADDING AND SUBTRACTING FRACTIONS WITH LIKE DENOMINATORS

TEACHER NOTES

This unit includes the following documents:

- Learning Map Information
- Instructional Activity (four lessons)
- Instructional Activity Student Handout (for Lessons 2, 3, and 4)
- Student Activity
- Student Activity Solution Guide

In this unit, students will extend their understanding of fractions and the meaning of fractions to include computation of fractions with like denominators. Students will use equivalent fraction expressions and combine such expressions to develop an understanding of the process and meaning of adding and subtracting fractions. Students will use fraction models and manipulatives in their decomposition and thinking. The final goal of this unit is for students to solve real-world problem situations using the addition and subtraction of fractions and mixed numbers with common denominators.

RESEARCH

As students develop their understanding of fractions, they begin with the meaning of fractions. Instruction often begins with the use of concrete materials and manipulatives. Classroom activities should build upon students' previous understanding as students extend their knowledge of fractions (Burns, 2007). To help students develop a deep understanding of fractions, student should encounter both pictorial and symbolic representations of fractions (Chval, Lannin, & Jones, 2013). An important component of students' early exposure to fractions is often through the use of manipulatives, which help students create pictorial and symbolic representations of fractions. These manipulatives should be interwoven through the development of fractional concepts to help students solidify their understanding of the relationship between fraction models and the fraction concepts. Additionally, students need a variety of ways to work with and comprehend fractions such as concrete materials, models and diagrams, and symbolic notation (Burns, 2007). Teachers should strive to incorporate multiple modeling techniques into instruction for students. These multiple representations of fractions allow students to develop concepts of fractions that move beyond only identifying fractional parts to include more complex processes of fraction computations. As students work with fractions, they should begin to see the patterns that develop in fraction computations and fraction relationships.

To become fluent in addition and subtraction of fractions, students need not only simple procedural fluency but also conceptual understanding of fractions. Students should develop essential understandings of fractions, including the notion of equivalency, before exploring computation with fractions (Chval, Lannin, & Jones, 2013). Another conceptual understanding of fractions involves unit fractions, as students must understand

how any fraction can be composed from multiple unit fractions. To help move students from the conceptual understanding of fractions to understanding addition and subtraction of fractions, student should continue to work with concrete materials and fraction modeling. Instruction that includes purposeful visual modeling and reasoning strategies can lead to both a procedural and conceptual understanding of addition and subtraction of fractions. To prevent hindering students' number and operational sense, students should not be too quickly introduced to algorithms concerning fractions before they have developed a solid conceptual understanding of fractions (Petit, Laird, Marsden, & Ebby, 2016). Additionally, the use of visual representations helps students see how adding like fractions is related to adding like items (Small, 2014).

Beyond working with fractions, students should also develop an understanding of mixed numbers. Students' understanding of unit fractions—and that any fraction can be composed from unit fractions—serves as a foundational concept to working with improper fractions and mixed numbers (Petit, Laird, Marsden, & Ebby, 2016). When modeling fractions, students should not be limited to working with fractions less than one. Instead, they should also encounter improper fractions and become familiar with the concept that a mixed number is composed from the sum of a whole number and a fraction (Small, 2014), and from there students can move into adding and subtracting with mixed numbers.

LEARNING MAP INFORMATION

The learning map section for this sequence of activities begins with explaining unit fractions and counting by fractional parts. These concepts then lead to explaining the concepts of adding and subtracting fractions, decomposing fractions into a sum of unit fractions, and explaining mixed numbers. From mixed numbers, the concepts move to the conversion between improper fractions and mixed numbers. After explaining addition and subtraction of fractions, then the process of adding and subtracting fractions and mixed numbers occurs with a final goal of applying fraction addition and subtraction to word problems.

INSTRUCTIONAL ACTIVITIES

The activities in this unit are designed to allow students to explore and learn how to decompose a fraction or mixed number into an equivalent fraction expression. These fraction expressions will then be used to move students into an understanding of the process and meaning of adding and subtracting fractions. First, students will decompose a single fraction into multiple equivalent fraction expressions. Next, students will explore converting between improper fractions and mixed numbers by modeling the initial value and then expressing the value using the related number form. Third, students will model the addition and subtraction of fractions and mixed numbers using a diagram. Using a decomposed fraction and models, students will rearrange fraction expressions and compose new values to determine final sums and differences. Finally, students will add and subtract fractions and mixed numbers to solve word problems.

REFERENCES

Burns, M. (2007). *About Teaching Mathematics: A K - 8 Resource.* Sausalito, California: Math Solutions.

Chval, K., & J. Lannin, & D. Jones. (2013). *Putting Essential Understanding of Fractions Into Practice in Grades 3-5*. Reston, VA: The National Council of Teachers of Mathematics.

Petit, M., & R. Laird, & E. Marsden, & C. Ebby. (2016). *A Focus on Fractions: Bringing Research to the Classroom*. New York: Routledge.

Small, M. (2014). *Uncomplicating Fractions to Meet the Common Core Standards in Math, K-7*. New York: Teachers College Press.

Adding and Subtracting Fractions with Like Denominators

Overview of Instructional Activities

Lesson	Learning Goal	Nodes Addressed
Lesson 1	Students will describe different ways to compose a fraction from smaller fractions. Students will represent the composition using an equation.	▶ DECOMPOSE A FRACTION INTO A SUM OF UNIT FRACTIONS WITH THE SAME DENOMINATOR
Lesson 2	Students will convert between mixed numbers and fractions.	▶ EXPLAIN MIXED NUMBER ▶ CONVERT IMPROPER FRACTIONS TO MIXED NUMBERS ▶ CONVERT MIXED NUMBERS TO IMPROPER FRACTIONS
Lesson 3	Students will model the addition and subtraction of fractions and mixed numbers by decomposing and rearranging proper fractions into expressions.	▶ ADD FRACTIONS WITH LIKE DENOMINATORS ▶ ADD MIXED NUMBERS WITH LIKE DENOMINATORS ▶ SUBTRACT FRACTIONS WITH LIKE DENOMINATORS ▶ SUBTRACT MIXED NUMBERS WITH LIKE DENOMINATORS
Lesson 4	Students will apply their understanding of adding and subtracting fractions and mixed numbers to real-world problem situations.	▶ SOLVE WORD PROBLEMS INVOLVING ADDITION AND SUBTRACTION OF FRACTIONS WITH LIKE DENOMINATORS

ADDING AND SUBTRACTING FRACTIONS WITH LIKE DENOMINATORS

INSTRUCTIONAL ACTIVITY

Lesson 1

LEARNING GOAL

Students will describe different ways to compose a fraction from smaller fractions. Students will represent the composition using an equation.

PRIMARY ACTIVITY

Students will draw cards to create proper fractions. In small groups, students will take turns describing ways to compose the given fraction.

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

- Unit fraction
- Proper fraction
- Numerator
- Denominator
- Like denominator
- Equation
- Fraction model

MATERIALS

- Deck of cards with face cards removed (Recommend one deck for every three to four students.)
- Paper and pencil
- Fraction circles, squares, strips, bars, towers, or other fraction manipulatives (optional)

IMPLEMENTATION

Review the use of math mountains, or number bonds, to compose numbers using addition and subtraction. **Explain** to students that they will be composing fractions in similar ways, focusing on composing the numerator in multiple ways.

Explore the fraction $\frac{3}{4}$. Students may need to use or see a model of the fraction in their exploration depending on their comfort level with fractions. Students should be able to describe the meaning of the numerator as the number of pieces and the denominator as the size of the pieces when a whole is divided into fractional parts. **Lead** students in a discussion to see that $\frac{3}{4}$ can be thought of as a single portion of the whole composed of three parts, three individual fourths added together $\left(\frac{3}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4}\right)$, two portions as a two-fourth piece with a one-fourth piece $\left(\frac{3}{4} = \frac{2}{4} + \frac{1}{4}\right)$, or a whole minus one-fourth $\left(\frac{3}{4} = 1 - \frac{1}{4}\right)$. In this discussion, students should recognize that the size of the pieces used to compose $\frac{3}{4}$ are always $\frac{1}{4}$ sized, no matter how many pieces or in what combination the pieces are used. Students need to realize that the denominator will always be "4", as the fractional pieces in the compositions are always fourths. Be careful of students trying to add both the numerators and the denominators. **Guard** against and **watch** for students making this common error.

> NOTE: Students should focus on composing the fraction using like denominator fractions. An extension for some students would be to use equivalent unlike fractions, when possible. In this case, some students may see $\frac{3}{4}$ as one-half and one-fourth $\left(\frac{3}{4} = \frac{1}{2} + \frac{1}{4}\right)$. To help students who struggle to see how a fraction can be decomposed into other combinations of fractions, allow students to physically model, split, and rearrange the fractions into groups to show the different compositions.

Arrange students into small groups containing three or four students. One student will turn over two cards and make a proper fraction using the card values, reading Ace as 1 and the other numbers as given. If the two cards are the same, students will work with a fraction equivalent to 1, such as $\frac{5}{5}$, yet should still focus on the given denominator when composing their fractions. Taking turns, each student will describe a way to compose the fraction, draw a model to represent the composition, and write an equation to represent the composition. Students will continue taking turns until the group can think of no more ways to compose the fraction. Students may use fraction manipulatives such as fraction circles, squares, strips, bars, towers, or cubes to help visualize and decompose the fraction into other equivalent fraction expressions.

Each group should create and write compositions for 10 different fractions.

GUIDING QUESTIONS

Elicit student thinking:

- How can you describe the fraction?
- What is known about the whole?
- How close to one whole is the fraction?
- Can you draw or represent a model for the fraction?

Determine if the student can DECOMPOSE A FRACTION INTO A SUM OF UNIT FRACTIONS WITH THE SAME DENOMINATOR:

- What unit fraction is this fraction related to?
- How many parts are there?
- How can we create the numerator?
- Can you think of more than one way to create the numerator?

At the end of the activity, teachers should give students a visual model of a fraction, such as a fraction bar representing $\frac{7}{8}$. Students should then write equations to model different compositions of fractions that are equivalent to $\frac{7}{8}$. Students should also draw fraction models to represent the equations written.

Adding and Subtracting Fractions with Like Denominators

INSTRUCTIONAL ACTIVITY

Lesson 2

LEARNING GOAL

Students will convert between mixed numbers and fractions.

PRIMARY ACTIVITY

Students will use fraction manipulatives to model and compose mixed numbers from improper fractions and improper fractions from mixed numbers. Through this modeling, students will explore the equivalence of mixed numbers and improper fractions.

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

- Numerator
- Denominator
- Proper fraction
- Improper fraction
- Mixed number
- Like denominators
- Equivalent fractions
- Fraction model

MATERIALS

- Fraction manipulatives such as fraction circles, fraction squares, fraction towers, or fraction strips
- Instructional Activity Student Handout

IMPLEMENTATION

Similar to LESSON 1, students will be composing improper fractions using unit fractions and other fractions. Students will then combine smaller fractions to create whole numbers and fractional parts to write mixed numbers.

Remind students of LESSON 1, where they used composition of fractions to create larger fractions. **Revisit** different ways to compose a fraction such as $\frac{9}{10}$. Then **discuss** how a fraction like $\frac{11}{10}$ would look. **Ask** students what they notice about the fraction $\frac{11}{10}$ versus $\frac{9}{10}$. **Discuss** what the larger numerator means to the fraction value. Manipulatives can be used to model how 11 tenths would exceed the 10 pieces needed to create one whole. **Confirm** that students know the term *improper fraction* and can correctly apply it to improper fractional amounts.

Discuss and **model** with students how to use the denominator of a fraction to determine how many pieces are needed to make one whole. **Practice** as necessary with students until they can identify the number of pieces in the whole. **Connect** the idea of how many pieces in the whole to the idea of the unit fraction, which students should be previously familiar with from previous lessons.

Expand upon the composition of fractions covered in LESSON 1 to the concepts of improper fractions and mixed numbers. **Model** and **discuss** the addition of fractions resulting in an improper fraction using the following example.

$$\frac{7}{10}+\frac{5}{10}$$

Remind students that only the numerators will combine in the addition to result in $\frac{12}{10}$. When drawing a model of $\frac{12}{10}$, students should notice that the 12 pieces are more than the 10 pieces needed for one whole. Thus $\frac{12}{10}$ could be written as the mixed number $1\frac{2}{10}$ or remain an improper fraction $\frac{12}{10}$.

Model and **discuss** the addition of a mixed number and fraction resulting in another, larger mixed number using the following example.

$$1\frac{2}{3}+\frac{2}{3}$$

As before, **use** manipulatives to show that the fractional pieces result in an improper fraction that is greater than one whole. Additionally, students should notice that the whole from the mixed number combines with another whole in the improper fraction to result in the final sum of $2\frac{1}{3}$.

GUIDING QUESTIONS

Elicit student thinking:

- Why are improper fractions called "improper"?
- Are improper fractions actually bad?
- Why are improper fractions sometimes called "top-heavy fractions"?
- Can an improper fraction be used to accurately describe a situation?
- What does it mean to say a fraction is "proper"?
- How are proper fractions different from improper fractions in terms of size?

Determine if the student can EXPLAIN MIXED NUMBER:

- What is a mixed number?
- Why are mixed numbers considered more proper than an improper fraction?
- What information does a mixed number give more clearly than an improper fraction?
- Can you describe a situation where you might encounter a mixed number?

Direct students' attention to how the composition of two fractions through addition can be used to compose an improper fraction or mixed number. Students will be expected to do a similar composition as they convert between improper fractions and mixed numbers. Students should focus on the required number of pieces to make one whole and use that number in their composition to count the wholes in the mixed number.

Give each student an INSTRUCTIONAL ACTIVITY STUDENT HANDOUT. Students may work in pairs or small groups to complete the exploration of mixed numbers and improper fractions using the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT. **Provide** fraction manipulatives to help students model the improper fractions and mixed numbers. **Direct** students' focus as necessary to the denominator in each problem to guide them in forming the whole values.

GUIDING QUESTIONS

Elicit student thinking:

- What does it mean to say that fractions are equivalent?
- Can any fraction be rewritten as an equivalent fraction or composition of fractions?
- How can a mixed number be equivalent to a fraction?
- Are all fractions equivalent to a mixed number? Why or why not?

Determine if the student can CONVERT IMPROPER FRACTIONS TO MIXED NUMBERS:

- What makes an improper fraction equivalent to a mixed number?
- Why can't an improper fraction be equivalent to a proper fraction?
- How can you tell if a fraction can be rewritten as a mixed number?
- Are all improper fractions equivalent to a mixed number?
- Can an improper fraction be equivalent to a whole number?

Determine if the student can CONVERT MIXED NUMBERS TO IMPROPER FRACTIONS:

- How do you change a mixed number to an improper fraction?
- Why does a mixed number convert to an improper fraction instead of a proper fraction?
- When might an improper fraction be more useful than a mixed number?
- Can all mixed numbers be rewritten as an improper fraction?

Students should be required to draw a diagram to model the conversion between improper fractions and mixed numbers.

At the end of the activity, teachers should have students write a short paragraph explaining the relationship between improper fractions and mixed numbers. Students should be instructed to focus on the equivalence and conversion between these two number forms.

Name_____

Adding and Subtracting Fractions with Like Denominators

Lesson 2

1) Use the fraction manipulatives to show that the improper fractions and mixed numbers are equivalent. Then, draw a picture of the manipulatives.

a. $\frac{7}{2} = 3\frac{1}{2}$

b. $\frac{9}{4} = 2\frac{1}{4}$

c. $\frac{13}{8} = 1\frac{5}{8}$

Name_____

2) Use the fraction manipulatives to convert the improper fractions into mixed numbers. Then, draw a picture of the manipulatives.

a. $\frac{13}{3}$

b. $\frac{11}{6}$

c. $\frac{16}{5}$

Name_____

3) Use your fraction manipulatives to convert the mixed numbers into improper fractions. Then, draw a picture of the manipulatives.

a. $2\frac{1}{8}$

b. $3\frac{3}{4}$

c. $1\frac{1}{10}$

Name_____

4) Why are there multiple ways to write a mixed number as an improper fraction? Explain.

5) Convert the following improper fractions to mixed numbers.

a. $\frac{10}{4}$

b. $\frac{16}{6}$

c. $\frac{34}{8}$

d. $\frac{47}{10}$

Name_____

6) Convert the following mixed numbers to improper fractions.

a. $2\frac{3}{4}$

b. $4\frac{1}{6}$

c. $2\frac{2}{5}$

d. $1\frac{3}{10}$

Adding and Subtracting Fractions with Like Denominators

INSTRUCTIONAL ACTIVITY

Lesson 3

LEARNING GOAL

Students will model the addition and subtraction of fractions and mixed numbers by decomposing and rearranging proper fractions into expressions.

PRIMARY ACTIVITY

Students will model the addition and subtraction of fraction and mixed numbers. Through the use of models, students will explore regrouping fractions and mixed numbers.

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

- Numerator
- Denominator
- Unit fraction
- Proper fraction
- Improper fraction
- Mixed number
- Like denominator
- Equivalent fractions
- Evaluate
- Fraction model

MATERIALS

- Fraction manipulatives such as fraction circles, fraction squares, fraction towers, or fraction strips
- Colored pencils or markers
- Instructional Activity Student Handout

IMPLEMENTATION

Revisit the composition of fractions and mixed numbers from LESSON 1 and LESSON 2. Students should understand how to combine fractions with common denominators after using that method to rewrite different compositions of fractions in LESSON 1. Additionally, **revisit** converting between improper fractions and mixed numbers as explored in LESSON 2. Students will use these skills to add and subtract fractions and mixed numbers during this lesson.

Explore with students the following addition examples. Students should use models or manipulatives to represent the addition. If using drawn models, **provide** students with extra models so that they have to think about and choose how many whole and partial units will be represented in the sum, and **encourage** students to use two different colors to represent each addend in the problem. This will allow students to see how the two fractions combine together to compose the sum.

$$\frac{1}{7}+\frac{3}{7}$$

$$4\frac{2}{8}+\frac{5}{8}$$

$$3\frac{2}{5}+1\frac{4}{5}$$

Discuss with students how in the second example the fractional parts can be combined together separately from the whole number in the first addend.

In the third example, students should notice that the fractions sum to an improper fraction, thus requiring them to carry an additional whole number into the final sum. Some students may see that the fraction in the first addend requires three more parts to compose a whole unit. Thus, three of the four fifths in the second addend will be used to move $3\frac{2}{5}$ up to 4, and then 4 is added to the remaining $1\frac{1}{5}$ for a final sum of $5\frac{1}{5}$. This decomposition and resulting composition may be best seen using models to represent the two mixed numbers.

GUIDING QUESTIONS

Elicit student thinking:

- How can you describe the fractions in the problem?
- What is known about the whole unit related to the problem?
- What unit fraction are these fractions related to?
- How can you draw a model to represent each fraction or mixed number in the problem?

Determine if the student can ADD FRACTIONS WITH LIKE DENOMINATORS:

- How many parts are in each addend?
- How far from one whole is each addend?
- When adding the fractions, will the sum give more parts than needed for one whole unit?
- How can you use the models to add the fractions?
- What happens to the model when the two fractions are added together?

Determine if the student can ADD MIXED NUMBERS WITH LIKE DENOMINATORS:

- How do the whole numbers relate in the addition expression?
- How do the fractions relate in the addition expression?
- How can you use the models to add the mixed numbers?
- What happens to the model when the two mixed numbers are added together?

Explore with students the following subtraction examples. Students should use models or manipulatives to represent the subtraction. If using drawn models, **provide** students with extra models so that they have to think and choose how many whole and partial units will be represented in the problem. **Guide** students to represent the first fraction or mixed number and then to mark off or remove the quantity subtracted.

$$\frac{5}{6} - \frac{3}{6}$$

$$4\frac{7}{10} - \frac{4}{10}$$

$$5\frac{1}{4} - 3\frac{1}{4}$$

Similar to the addition examples, **discuss** with students how in the second example the fractional parts can be subtracted separately from the whole number in the first mixed number. In the third example, **assist** students in regrouping $5\frac{1}{4}$ to $4\frac{5}{4}$ so that the fractional parts can be subtracted.

GUIDING QUESTIONS

Elicit student thinking:

- How can you describe the fractions in the problem?
- What is known about the whole unit related to the problem?
- What unit fraction are these fractions related to?
- How can you draw a model to represent each fraction or mixed number in the problem?

Determine if the student can SUBTRACT FRACTIONS WITH LIKE DENOMINATORS:

- How many parts are in each number?
- How can you use the models to subtract the fractions?
- What happens to the model when the two fractions are subtracted?

Determine if the student can SUBTRACT MIXED NUMBERS WITH LIKE DENOMINATORS:

- How do the whole numbers relate in the subtraction expression?
- How do the fractions relate in the subtraction expression?
- How can you use the models to subtract the mixed numbers?
- What happens to the model when the two mixed numbers are subtracted?

Next, **give** each student a copy of the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT. Students should complete the handout individually, in pairs, or small groups. **Instruct** students to use the models given to show their thinking. **Inform** students that they will not always use all of the shapes given, but instead they need to think carefully about how to display their model to represent the given expression. Students may use a counting-up or counting-down technique with the models to help them add and subtract the different values.

Students should be required to explain how their model represents the given problem and how their thinking is shown using the model.

At the end of the activity, teachers should have students choose one problem to share their solution and reasoning with a neighbor. Students should ask clarifying questions while engaging in mathematical discourse.

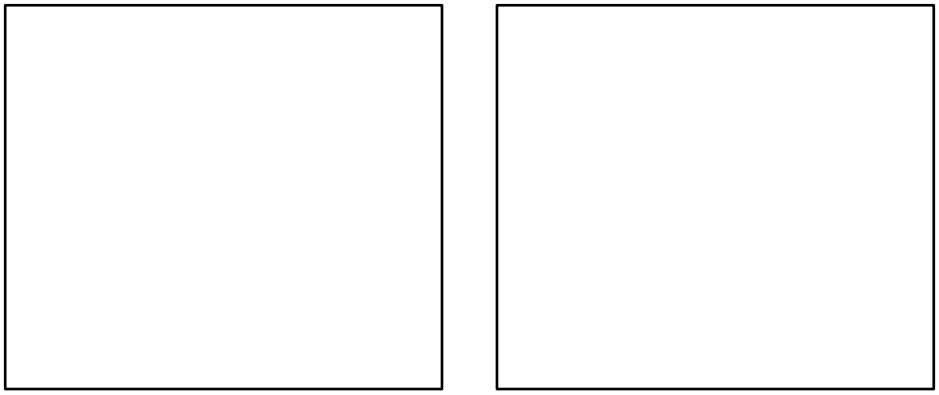
Name_____

Adding and Subtracting Fractions with Like Denominators

Lesson 3

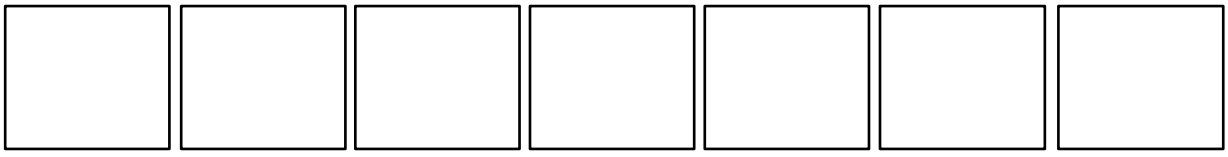
Use the given model to evaluate each expression.

1) $\frac{3}{10} + \frac{8}{10}$



2) $1\frac{1}{3} + 2\frac{1}{3}$

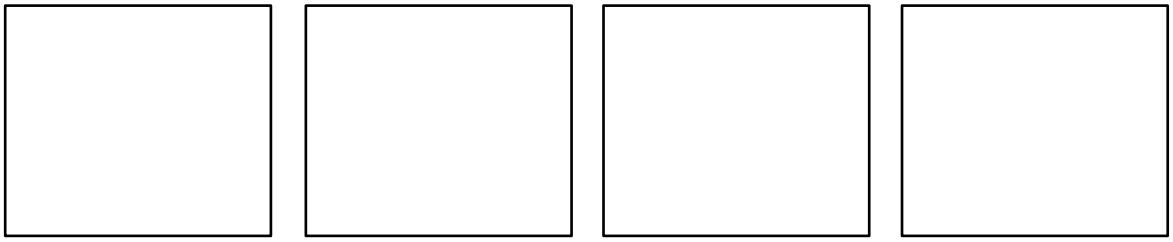
3) $3\frac{1}{4} + 2\frac{3}{4}$



Name_____

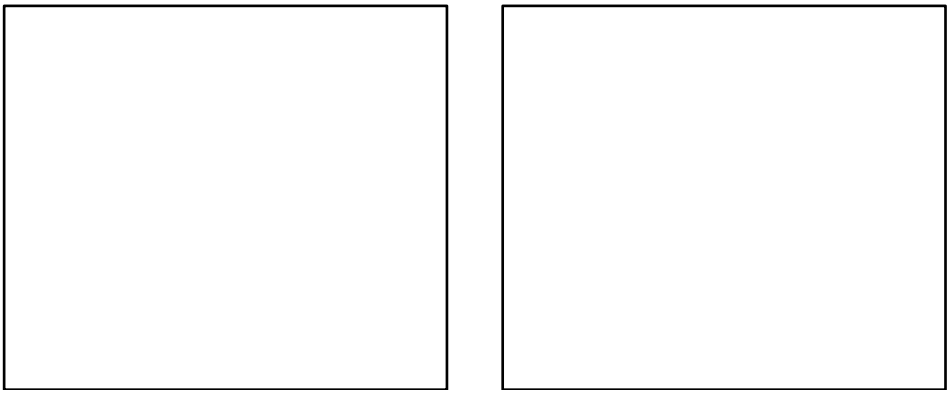
4) $1\frac{1}{5}+\frac{3}{5}$

5) $\frac{4}{6}+2\frac{3}{6}$



6) $1\frac{3}{8}+2\frac{7}{8}$

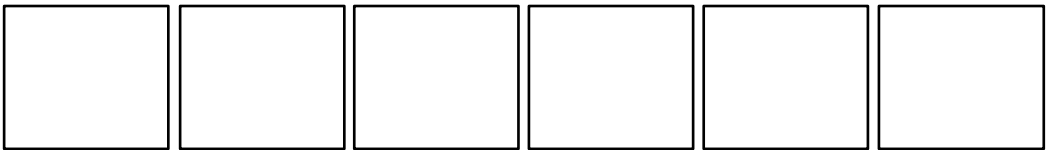
7) $\frac{7}{9}-\frac{2}{9}$



Name_____

8) $3\frac{2}{3} - \frac{1}{3}$

9) $4\frac{1}{5} - 1\frac{4}{5}$



10) $5\frac{1}{2} - 3\frac{1}{2}$

11) $3\frac{5}{8} - \frac{7}{8}$

12) $6\frac{3}{4} - 3\frac{1}{4}$

13) $6\frac{2}{6} - 3\frac{4}{6}$

Adding and Subtracting Fractions with Like Denominators

INSTRUCTIONAL ACTIVITY

Lesson 4

LEARNING GOAL

Students will apply their understanding of adding and subtracting fractions and mixed numbers to real-world problem situations.

PRIMARY ACTIVITY

Students will read and solve fraction and mixed number word problems. Then students will match a card's word problem with another card's solution until a complete chain is formed.

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

- Proper fraction
- Improper fraction
- Mixed number
- Like denominator
- Equivalent fractions
- Fraction model

MATERIALS

- Instructional Activity Student Handout
- Scissors
- Fraction circles, squares, strips, bars, towers, or other fraction manipulatives (optional)

IMPLEMENTATION

Remind students about LESSON 3, when they worked to add and subtract fractions and mixed numbers. Students will use the same procedures to solve the real-world problems in this lesson. Students can use modeling techniques or fraction manipulatives as they work to solve the problems.

Give students, individually or in pairs, a copy of the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT. Students need to cut apart the cards. Beginning with any card, students will solve the application problem. Then students will find the matching solution, thus beginning the chain. The next problem to be solved will be found on the card with the previous problem's solution. Students will solve that problem, find the solution, and continue the chain. This process will repeat until the final card links back to the original card.

GUIDING QUESTIONS

Elicit student thinking:

- What do you know about the situation given?
- What are you asked to do?
- Can you explain mathematically the information given?
- Do you have all of the information needed to solve the problem?
- Is there extra information given?
- What are the fractions or mixed numbers in the problem?
- What do the fractions or mixed numbers represent in the situation?

Determine if the student can SOLVE WORD PROBLEMS INVOLVING ADDITION AND SUBTRACTION OF FRACTIONS WITH LIKE DENOMINATORS:

- Does the given situation involve addition or subtraction? How do you know?
- How are the fractions or mixed numbers related to each other?
- What model can you draw to represent this problem?
- Are the fractions and mixed numbers written so that they can be added or subtracted with each other?
- Does your answer make sense?
- What number form makes the most sense to answer the question?
- Did you answer completely the question asked?

Students should be required to justify their reasoning on how they connected their cards.

At the end of the activity, teachers should ask students to choose one card and draw a fraction model to show how they solved the problem. Students should then choose partners and share their models and solutions.

Name_____

Adding and Subtracting Fractions with Like Denominators

Lesson 4

The following pages contain the cards students will use to create a complete chain. Students need to cut apart the cards and arrange them in a chain to link all cards. The cards are labeled for easy checking.

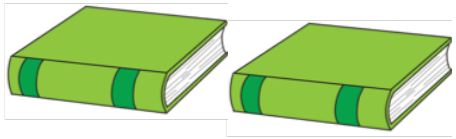
The correct card order is A, F, J, B, G, D, L, C, K, E, H, I.

Note: the chain can begin with any card, but the sequence of cards will remain the same.

Name_____

A

2



Shawn wants to read 2 books over the weekend. She reads $\frac{3}{4}$ of the first book on Friday evening. What fraction of the books does Shawn have left to read on Saturday and Sunday?

B

$\frac{4}{12}$

Miya walked $\frac{1}{3}$ of a mile in the morning and $1\frac{1}{3}$ after school. How much did Miya walk altogether?

C

$\frac{11}{15}$

Adam practices the drums for $\frac{13}{8}$ hours and his guitar for $\frac{4}{8}$ hours more. How many hours does Adam practice his instruments?

D

$\frac{1}{5}$

In their championship win, the Rockets had two leading scorers. Matthew scored $\frac{37}{100}$ of the Rockets' points, and Lukas scored $\frac{42}{100}$ of the Rockets' points. By what fraction did Lukas outscore Matthew?

Name_____

E

$\frac{7}{10}$

Julia is saving to buy a new basketball. She has $\frac{7}{4}$ dollars in her change bank and $\frac{9}{4}$ dollars in her hidden book. How much does Julia have saved towards her basketball?

F

$1\frac{1}{4}$

Patty has $\frac{13}{8}$ cakes left over from her party. After giving cake to her friends to take home, she has $\frac{5}{8}$ of a cake left. How much cake did Patty give to her friends?

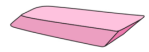
G

$1\frac{2}{3}$

The Thompsons have a full box of cereal. Randy eats $\frac{1}{5}$ of the cereal and Karen eats $\frac{3}{5}$ of the cereal. How much cereal is left?

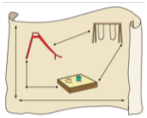
H

4



A $\frac{2}{4}$-inch pencil eraser is placed on a $3\frac{3}{4}$-inch pencil. What is the total length of the pencil and eraser?

Name_____

I  $4\frac{1}{4}$ Danny needs $3\frac{2}{3}$ cups of tomato sauce for his pasta sauce recipe. If a can has $\frac{5}{3}$ cups of tomato sauce, how much more does Danny need?	**J**  1 Riley is preparing soup. He fills a pot $\frac{5}{12}$ full with water and $\frac{3}{12}$ of the pot with vegetables. What fraction of the pot is still empty?
K  $2\frac{1}{8}$ Jessica painted $\frac{3}{10}$ of her room on Saturday and $\frac{2}{10}$ of her room on both Sunday and Monday. How much has Jessica painted so far?	**L**  $\frac{5}{100}$ Leya is riding in a bike race. She has already finished $\frac{4}{15}$ of the race. How much does Leya has left to ride?

Name_____

Adding and Subtracting Fractions with Like Denominators

Lessons 1 – 4

1. Decompose the fraction $\frac{18}{13}$ using a sum of fractions.

1.a. Which of the following sums are equivalent to $\frac{18}{13}$? Choose all that apply.

a) $\frac{3}{13}+\frac{4}{13}+\frac{5}{13}+\frac{6}{13}$	b) $\frac{4}{13}+\frac{5}{13}+\frac{4}{13}+\frac{6}{13}$
c) $\frac{6}{13}+\frac{12}{13}$	d) $\frac{6}{13}+\frac{7}{13}+\frac{8}{13}$
e) $\frac{5}{13}+\frac{3}{13}+\frac{5}{13}$	f) $\frac{5}{13}+\frac{8}{13}+\frac{5}{13}$

1.b. What is another way to write $\frac{18}{13}$ as a sum of two fractions?

Name_____

1.c. What is another way to write $\frac{18}{13}$ as a sum of three fractions?

2. Decompose the fraction $\frac{7}{11}$ using a difference of fractions.

2.a. Which of the following differences are equivalent to $\frac{7}{11}$? Choose all that apply.

a) $\frac{15}{11} - \frac{8}{11}$	b) $\frac{11}{11} - \frac{4}{11}$
c) $\frac{10}{11} - \frac{4}{11}$	d) $\frac{16}{11} - \frac{7}{11} - \frac{2}{11}$
e) $\frac{13}{11} - \frac{3}{11}$	f) $\frac{14}{11} - \frac{7}{11} - \frac{2}{11}$

2.b. What is another way to write $\frac{7}{11}$ as a difference of two fractions?

Name_____

2.c. What is another way to write $\frac{7}{11}$ as a difference of three fractions?

3. Decompose the mixed number $2\frac{2}{5}$ into a fraction expression.

3.a. Which of the following expressions are equivalent to $2\frac{2}{5}$? Choose all that apply.

a) $\frac{5}{5}+\frac{5}{5}+\frac{2}{5}$	b) $2+\frac{2}{5}$
c) $\frac{3}{5}+\frac{3}{5}$	d) $1+\frac{2}{5}$
e) $\frac{6}{5}+\frac{6}{5}$	f) $\frac{15}{5}+\frac{2}{5}$

3.b. What is another way to write $2\frac{2}{5}$ as a fraction expression?

Name_____

4. Consider the fraction $\frac{5}{6}$.

4.a. Cody says the only way to decompose the fraction is using the sum $\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}=\frac{5}{6}$, because it takes five ones to equal five, so it has to take five one-sixths to equal $\frac{5}{6}$. Do you agree or disagree with Cody? Explain your reasoning.

4.b. Sheina says that because $\frac{5}{6}$ is so close to one whole, the only decomposition that can be used is the difference $1-\frac{1}{6}=\frac{5}{6}$. Do you agree or disagree with Sheina? Explain your reasoning.

Name_____

5. Ethan picks $4\frac{3}{4}$ pounds of apples from a tree in his school garden.

5.a. If Ethan gives $1\frac{1}{4}$ pounds of the apples to his teacher Ms. Charm, how many pounds of apples does Ethan have left? Explain your reasoning.

5.b. Of the pounds of apples Ethan has left, he then takes $\frac{3}{4}$ pounds for his mother. How many pounds of apples remain? Explain your reasoning.

Name_____

5.c. Ethan gives the rest of the apples to the school cafeteria to make apple cobbler for the teachers. The recipe requires 3 pounds of apples. How many more pounds of apples does the cafeteria need? Explain your reasoning.

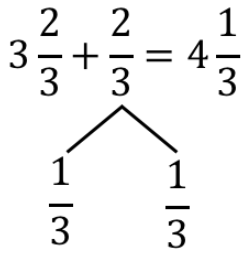
6. Beatrice is making her grandmother's legendary Fruity Tea for a party. After looking through the recipe, Beatrice sees that she will mix $4\frac{5}{8}$ quarts of fruit juice with $7\frac{7}{8}$ quarts of tea.

6.a. As Beatrice is planning, she finds a 14-quart container to serve the Fruity Tea. Is this container large enough to hold Beatrice's batch of Fruity Tea? Explain your reasoning.

6.b. How much Fruity Tea will Beatrice make using this recipe? Explain your reasoning.

Name_____

7. Jae adds $3\frac{2}{3}$ and $\frac{2}{3}$. Jae shows his thinking here.



7.a. Explain the steps Jae takes in his work.

7.b. What is another way to add $3\frac{2}{3}$ and $\frac{2}{3}$? Explain your thinking.

Name_____

8. Micala adds $5\frac{7}{10}$ and $6\frac{9}{10}$. She shows her work here.

$$5\frac{7}{10}+6\frac{9}{10}=11+\frac{16}{10}$$
$$=11+\frac{10}{10}+\frac{6}{10}$$
$$=12\frac{6}{10}$$

8.a. Explain the steps Micala takes in her work.

8.b. What is another way to add $5\frac{7}{10}$ and $6\frac{9}{10}$? Explain your thinking.

ADDING AND SUBTRACTING FRACTIONS WITH LIKE DENOMINATORS

STUDENT ACTIVITY SOLUTION GUIDE

Lessons 1 – 4

1. Decompose the fraction $\frac{18}{13}$ using a sum of fractions.

1.a. Which of the following sums are equivalent to $\frac{18}{13}$? Choose all that apply.

a) $\frac{3}{13}+\frac{4}{13}+\frac{5}{13}+\frac{6}{13}$	b) $\frac{4}{13}+\frac{5}{13}+\frac{4}{13}+\frac{6}{13}$
c) $\frac{6}{13}+\frac{12}{13}$	d) $\frac{6}{13}+\frac{7}{13}+\frac{8}{13}$
e) $\frac{5}{13}+\frac{3}{13}+\frac{5}{13}$	f) $\frac{5}{13}+\frac{8}{13}+\frac{5}{13}$

CORRECT ANSWER

Three of the options correctly sum to $\frac{18}{13}$:

a) $\frac{3}{13}+\frac{4}{13}+\frac{5}{13}+\frac{6}{13}$

c) $\frac{6}{13}+\frac{12}{13}$

f) $\frac{5}{13}+\frac{8}{13}+\frac{5}{13}$

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Student does not choose all three correct options.	incorrectly adds fractions with like denominators	ADD FRACTIONS WITH LIKE DENOMINATORS
Student chooses option b, d, or e.	incorrectly adds fractions with like denominators	ADD FRACTIONS WITH LIKE DENOMINATORS

1.b. What is another way to write $\frac{18}{13}$ as a sum of two fractions?

CORRECT ANSWER

Many possible solutions exist, such as $\frac{10}{13}+\frac{8}{13}$ *or* $\frac{9}{13}+\frac{9}{13}$ *or* $\frac{17}{13}+\frac{1}{13}$.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
$\frac{10}{13}+\frac{9}{13}$	incorrectly adds numerators of fractions with like denominators	ADD FRACTIONS WITH LIKE DENOMINATORS
$\frac{10}{6}+\frac{8}{7}$	adds both the denominators and the numerators	EXPLAIN THE CONCEPT OF ADDITION AND SUBTRACTION OF FRACTIONS
Student writes a sum of three fractions instead of two fractions, such as $\frac{6}{13}+\frac{6}{13}+\frac{6}{13}$.	does not attend to the directions in the question	N/A

1.c. What is another way to write $\frac{18}{13}$ as a sum of three fractions?

CORRECT ANSWER

Many possible solutions exist, such as $\frac{6}{13}+\frac{6}{13}+\frac{6}{13}$ *or* $\frac{10}{13}+\frac{6}{13}+\frac{2}{13}$ *or* $\frac{9}{13}+\frac{6}{13}+\frac{3}{13}$.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
$\frac{7}{13}+\frac{6}{13}+\frac{6}{13}$	incorrectly adds numerators of fractions with like denominators.	ADD FRACTIONS WITH LIKE DENOMINATORS
$\frac{6}{5}+\frac{6}{5}+\frac{6}{3}$	adds both the denominators and the numerators	EXPLAIN THE CONCEPT OF ADDITION AND SUBTRACTION OF FRACTIONS
Student writes a sum of two fractions instead of three fractions, such as $\frac{10}{13}+\frac{8}{13}$.	does not attend to the directions in the question	N/A

2. Decompose the fraction the fraction $\frac{7}{11}$ using a difference of fractions.

2.a. Which of the following differences are equivalent to $\frac{7}{11}$? Choose all that apply.

a) $\frac{15}{11} - \frac{8}{11}$	b) $\frac{11}{11} - \frac{4}{11}$
c) $\frac{10}{11} - \frac{4}{11}$	d) $\frac{16}{11} - \frac{7}{11} - \frac{2}{11}$
e) $\frac{13}{11} - \frac{3}{11}$	f) $\frac{14}{11} - \frac{7}{11} - \frac{2}{11}$

CORRECT ANSWER

Three of the options are correct decompositions of $\frac{7}{11}$:

a) $\frac{15}{11} - \frac{8}{11}$

b) $\frac{11}{11} - \frac{4}{11}$

d) $\frac{16}{11} - \frac{7}{11} - \frac{2}{11}$

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Student does not choose all three correct options.	incorrectly subtracts fractions with like denominators	SUBTRACT FRACTIONS WITH LIKE DENOMINATORS
Student chooses option c, e, or f.	incorrectly subtracts fractions with like denominators	SUBTRACT FRACTIONS WITH LIKE DENOMINATORS

2.b. What is another way to write $\frac{7}{11}$ as a difference of two fractions?

CORRECT ANSWER

Many possible solutions exist, such as $\frac{17}{11} - \frac{10}{11}$ *or* $\frac{14}{11} - \frac{7}{11}$ *or* $\frac{10}{11} - \frac{3}{11}$.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
$\frac{16}{11} - \frac{10}{11}$	incorrectly subtracts fractions with like denominators	SUBTRACT FRACTIONS WITH LIKE DENOMINATORS
$\frac{17}{15} - \frac{10}{4}$	subtracts both the denominators and the numerators	EXPLAIN THE CONCEPT OF ADDITION AND SUBTRACTION OF FRACTIONS
Student writes a decomposition of three fractions instead of two fractions, such as $\frac{20}{11} - \frac{10}{11} - \frac{3}{11}$.	does not attend to the directions in the question	N/A

2.c. What is another way to write $\frac{7}{11}$ as a difference of three fractions?

CORRECT ANSWER

Many possible solutions exist, such as $\frac{17}{11} - \frac{5}{11} - \frac{5}{11}$ *or* $\frac{20}{11} - \frac{10}{11} - \frac{3}{11}$ *or* $\frac{10}{11} - \frac{1}{11} - \frac{2}{11}$.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
$\frac{15}{11} - \frac{7}{11} - \frac{2}{11}$	incorrectly subtracts fractions with like denominators	SUBTRACT FRACTIONS WITH LIKE DENOMINATORS
$\frac{18}{17} - \frac{10}{4} - \frac{1}{2}$	subtracts both the denominators and the numerators	EXPLAIN THE CONCEPT OF ADDITION AND SUBTRACTION OF FRACTIONS
Student writes a decomposition of two fractions instead of three fractions, such as $\frac{17}{11} - \frac{10}{11}$.	does not attend to the directions in the question	N/A

3. Decompose the mixed number $2\frac{2}{5}$ into a fraction expression.

3.a. Which of the following expressions are equivalent to $2\frac{2}{5}$? Choose all that apply.

a) $\frac{5}{5}+\frac{5}{5}+\frac{2}{5}$	b) $2+\frac{2}{5}$
c) $\frac{3}{5}+\frac{3}{5}$	d) $1+\frac{2}{5}$
e) $\frac{6}{5}+\frac{6}{5}$	f) $\frac{15}{5}+\frac{2}{5}$

CORRECT ANSWER

Three of the options are expressions equivalent to $2\frac{2}{5}$:

a) $\frac{5}{5}+\frac{5}{5}+\frac{2}{5}$

b) $2+\frac{2}{5}$

e) $\frac{6}{5}+\frac{6}{5}$

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Student does not choose all three correct options.	incorrectly adds fractions with like denominators	ADD FRACTIONS WITH LIKE DENOMINATORS
Student chooses option c, d, or f.	incorrectly adds fractions with like denominators	ADD FRACTIONS WITH LIKE DENOMINATORS

3.b. What is another way to write $2\frac{2}{5}$ as a fraction expression?

CORRECT ANSWER

Many possible solutions exist, such as $1+1+\frac{2}{5}$ *or* $1+\frac{5}{5}+\frac{2}{5}$ *or* $\frac{7}{5}+\frac{5}{5}$.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
$\frac{7}{5}+\frac{6}{5}$	incorrectly adds fractions with like denominators	ADD FRACTIONS WITH LIKE DENOMINATORS
$\frac{6}{3}+\frac{6}{2}$	adds both the denominators and the numerators	EXPLAIN THE CONCEPT OF ADDITION AND SUBTRACTION OF FRACTIONS

4. Consider the fraction $\frac{5}{6}$.

4.a. Cody says the only way to decompose the fraction is using the sum $\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}=\frac{5}{6}$ because it takes five ones to equal five, so it has to take five one-sixths to equal $\frac{5}{6}$. Do you agree or disagree with Cody? Explain your reasoning.

CORRECT ANSWER

I disagree with Cody. While Cody has correctly decomposed $\frac{5}{6}$ using his equation, it is not the only way to decompose the fraction $\frac{5}{6}$. Other ways are $\frac{2}{6}+\frac{3}{6}=\frac{5}{6}$ or $\frac{1}{6}+\frac{4}{6}=\frac{5}{6}$.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
I agree with Cody. $\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}=\frac{5}{6}$ is the only way to decompose the fraction $\frac{5}{6}$.	does not see multiple equations to compose a fraction as a sum of fractions	ADD FRACTIONS WITH LIKE DENOMINATORS
I disagree with Cody because $\frac{2}{6}+\frac{3}{6}=\frac{5}{6}$, so that is the only way to decompose the fraction $\frac{5}{6}$.	does not see multiple equations to compose a fraction as a sum of fractions	ADD FRACTIONS WITH LIKE DENOMINATORS

4.b. Sheina says that because $\frac{5}{6}$ is so close to one whole, the only decomposition that can be used is the difference $1 - \frac{1}{6} = \frac{5}{6}$. Do you agree or disagree with Sheina? Explain your reasoning.

CORRECT ANSWER

I disagree with Sheina. While Sheina is correct that $\frac{5}{6}$ is close to one whole and $1 - \frac{1}{6} = \frac{5}{6}$ is a true equation, it is not the only expression showing the decomposition of $\frac{5}{6}$. Other expressions such as $\frac{2}{6} + \frac{3}{6}$ or $\frac{1}{6} + \frac{4}{6}$ are also equivalent to $\frac{5}{6}$ and are valid decompositions of $\frac{5}{6}$.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
I agree with Sheina. $1 - \frac{1}{6} = \frac{5}{6}$, so it is the only way to decompose the fraction $\frac{5}{6}$.	does not see multiple equations to decompose a fraction	SUBTRACT FRACTIONS WITH LIKE DENOMINATORS
I disagree with Sheina because $\frac{2}{6} + \frac{3}{6} = \frac{5}{6}$, so that is the only way to decompose the fraction $\frac{5}{6}$.	does not see multiple equations to compose a fraction as a sum of fractions	ADD FRACTIONS WITH LIKE DENOMINATORS

5. Ethan picks $4\frac{3}{4}$ pounds of apples from a tree in his school garden.

5.a. If Ethan gives $1\frac{1}{4}$ pounds of the apples to his teacher Ms. Charm, how many pounds of apples does Ethan have left? Explain your reasoning.

CORRECT ANSWER

If Ethan starts with $4\frac{3}{4}$ pounds of apples and gives $1\frac{1}{4}$ to his teacher, then the expression $4\frac{3}{4} - 1\frac{1}{4}$ represents this situation. Since $4\frac{3}{4} - 1\frac{1}{4} = 3\frac{2}{4}$, Ethan has $3\frac{2}{4}$ pounds of apples left.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
If Ethan starts with $4\frac{3}{4}$ pounds of apples and gives $1\frac{1}{4}$ to his teacher, then the expression $4\frac{3}{4}+1\frac{1}{4}$ represents this situation. Since $4\frac{3}{4}+1\frac{1}{4}=6$, Ethan has 6 pounds of apples left.	adds instead of subtracts for the apples given away	SOLVE WORD PROBLEMS INVOLVING ADDITION AND SUBTRACTION OF FRACTIONS WITH LIKE DENOMINATORS
If Ethan starts with $4\frac{3}{4}$ pounds of apples and gives $1\frac{1}{4}$ to his teacher, then the expression $4\frac{3}{4}-1\frac{1}{4}$ represents this situation. Since $4\frac{3}{4}-1\frac{1}{4}=3\frac{1}{4}$, Ethan has $3\frac{1}{4}$ pounds of apples left.	incorrectly subtracts mixed numbers with like denominators	SUBTRACT MIXED NUMBERS WITH LIKE DENOMINATORS

5.b. Of the pounds of apples Ethan has left, he then takes $\frac{3}{4}$ pounds for his mother. How many pounds of apples remain? Explain your reasoning.

CORRECT ANSWER

After giving apples to his teacher, Ethan has $3\frac{2}{4}$ pounds of apples. Then, taking $\frac{3}{4}$ pounds for his mother means that $3\frac{2}{4}-\frac{3}{4}=2\frac{3}{4}$. So, Ethan has $2\frac{3}{4}$ pounds of apples left.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Because this question relies on the answer from question 5.a., an incorrect answer for the previous question may effect the answer for this part. If the previous anwer found was 6 pounds of apples, then $6 - \frac{3}{4} = 5\frac{1}{4}$. Thus, Ethan has $5\frac{1}{4}$ pounds of apples left.	incorrectly adds instead of subtracts for the apples given away in previous question, thus creating an incorrect equation to solve in this section	SOLVE WORD PROBLEMS INVOLVING ADDITION AND SUBTRACTION OF FRACTIONS WITH LIKE DENOMINATORS
After giving apples to his teacher, Ethan has $3\frac{2}{4}$ pounds of apples. Then, taking $\frac{3}{4}$ pounds for his mother means that $3\frac{2}{4} + \frac{3}{4} = 4\frac{1}{4}$. So, Ethan has $4\frac{1}{4}$ pounds of apples left.	incorrectly adds instead of subtracts for the apples taken	SOLVE WORD PROBLEMS INVOLVING ADDITION AND SUBTRACTION OF FRACTIONS WITH LIKE DENOMINATORS
After giving apples to his teacher, Ethan has $3\frac{2}{4}$ pounds of apples. Then, taking $\frac{3}{4}$ pounds for his mother means that $3\frac{2}{4} - \frac{3}{4} = 3\frac{1}{4}$. So, Ethan has $3\frac{1}{4}$ pounds of apples left.	keeps the whole number 3 and subtracts the fractional parts in the wrong order: $\frac{3}{4} - \frac{2}{4} = \frac{1}{4}$	SUBTRACT MIXED NUMBERS WITH LIKE DENOMINATORS

5.c. Ethan then gives the rest of the apples to the school cafeteria to make apple cobbler for the teachers. The recipe requires 3 pounds of apples. How many more pounds of apples does the cafeteria need? Explain your reasoning.

CORRECT ANSWER

After taking apples for his mother, Ethan has $2\frac{3}{4}$ pounds of apples to give to the school cafeteria. If the recipe requires 3 pounds of apples, then $3 - 2\frac{3}{4} = \frac{1}{4}$, which means the cafeteria needs $\frac{1}{4}$ pound more apples for the apple cobbler.

Or, a student could reason that $2\frac{3}{4} + \frac{1}{4} = 3$, so the cafeteria needs $\frac{1}{4}$ pound more apples for the apple cobbler.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Because this question relies on the answers from questions 5.a. and 5.b., an incorrect answer for the previous questions may effect the answer for this part. If the previous anwer found was $2\frac{1}{4}$ pounds of apples, then $3 - 2\frac{1}{4} = \frac{3}{4}$. Thus, the cafeteria needs $\frac{3}{4}$ pound more apples more for the apple cobbler.	incorrectly subtracts mixed numbers on the previous part, thus creating an incorrect equation to solve for this section	SUBTRACT MIXED NUMBERS WITH LIKE DENOMINATORS
After taking apples for his mother, Ethan has $2\frac{3}{4}$ pounds of apples to give to the school cafeteria. If the recipe requires 3 pounds of apples, then $3 - 2\frac{3}{4} = 1\frac{3}{4}$, which means the cafeteria needs $1\frac{3}{4}$ pounds more apples for the apple cobbler.	subtracts 3 – 2 and keeps the $\frac{3}{4}$ fractional part	SUBTRACT MIXED NUMBERS WITH LIKE DENOMINATORS
After taking apples for his mother, Ethan has $2\frac{3}{4}$ pounds of apples to give to the school cafeteria. If the recipe requires 3 pounds of apples, then $3 - 2\frac{3}{4} = 1\frac{1}{4}$, which means the cafeteria needs $1\frac{1}{4}$ pounds more apples for the apple cobbler.	subtracts by regrouping, but instead of regrouping 3 to $2\frac{4}{4}$ regroups 3 to $3\frac{4}{4}$ and then subtracts $2\frac{3}{4}$	SUBTRACT MIXED NUMBERS WITH LIKE DENOMINATORS

6. Beatrice is making her grandmother's legendary Fruity Tea for a party. After looking through the recipe, Beatrice sees that she will mix $4\frac{5}{8}$ quarts of fruit juice with $7\frac{7}{8}$ quarts of tea.

6.a. As Beatrice is planning, she finds a 14-quart container to serve the Fruity Tea. Is this container large enough to hold Beatrice's batch of Fruity Tea? Explain your reasoning.

CORRECT ANSWER

Yes, the container will be big enough to hold the batch of Fruity Tea, because $4\frac{5}{8} + 7\frac{7}{8} = 11\frac{12}{8} = 12\frac{4}{8}$. $12\frac{4}{8}$ quarts is less than the 14-quart drink dispenser.

Alternative reasoning could be that $4\frac{5}{8}$ quarts is less than 5 quarts and $7\frac{7}{8}$ quarts is less than 8 quarts. Since 5 + 8 = 13, the total batch of Fruity tea will be less than 13 quarts, which will fit in the 14-quart drink dispenser.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Yes, the container will be big enough to hold the batch of Fruity Tea, because $7\frac{7}{8} - 4\frac{5}{8} = 3\frac{2}{8}$. $3\frac{2}{8}$ quarts is less than the 14-quart drink dispenser.	subtracts instead of adds for the combination of ingredients	SOLVE WORD PROBLEMS INVOLVING ADDITION AND SUBTRACTION OF FRACTIONS WITH LIKE DENOMINATORS
No, the container will not be big enough to hold the batch of Fruity Tea, because $4\frac{5}{8} + 7\frac{7}{8} = 11\frac{12}{8} = 12\frac{4}{8}$. $12\frac{4}{8}$ is greater than the 14-quart drink dispenser.	incorrectly compares the mixed number $12\frac{4}{8}$ with 14	COMPARE FRACTIONS WITH THE SAME DENOMINATOR
No, the container will not be big enough to hold the batch of Fruity Tea, because $4\frac{5}{8} + 7\frac{7}{8} = 13\frac{12}{8} = 14\frac{4}{8}$. $14\frac{4}{8}$ is greater than the 14-quart drink dispenser.	incorrectly adds the mixed numbers	ADD MIXED NUMBERS WITH LIKE DENOMINATORS

6.b. How much Fruity Tea will Beatrice make using this recipe? Explain your reasoning.

CORRECT ANSWER

Beatrice will make $12\frac{4}{8}$ (or $12\frac{1}{2}$) quarts of Fruity Tea. $4\frac{5}{8} + 7\frac{7}{8} = 11\frac{12}{8} = 12\frac{4}{8}$.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Beatrice will make $3\frac{2}{8}$ (or $3\frac{1}{4}$) quarts of Fruity Tea. $7\frac{7}{8} - 4\frac{5}{8} = 3\frac{2}{8}$.	subtracts instead of adds for the combination of ingredients	SOLVE WORD PROBLEMS INVOLVING ADDITION AND SUBTRACTION OF FRACTIONS WITH LIKE DENOMINATORS
Beatrice will make $14\frac{4}{8}$ (or $14\frac{1}{2}$) quarts of Fruity Tea. $4\frac{5}{8} + 7\frac{7}{8} = 13\frac{12}{8} = 14\frac{4}{8}$.	incorrectly adds the mixed numbers	ADD MIXED NUMBERS WITH LIKE DENOMINATORS

7. Jae adds $3\frac{2}{3}$ and $\frac{2}{3}$. Jae shows his thinking here.

$$3\frac{2}{3} + \frac{2}{3} = 4\frac{1}{3}$$

$$\frac{1}{3} \quad \frac{1}{3}$$

7.a. Explain the steps Jae takes in his work.

CORRECT ANSWER

Jae decomposed the $\frac{2}{3}$ fraction into $\frac{1}{3}$ and $\frac{1}{3}$ because $3\frac{2}{3} + \frac{1}{3} = 4$, then 4 and the remaining $\frac{1}{3}$ equals $4\frac{1}{3}$.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Jae made an error because he should have also decomposed the $3\frac{2}{3}$ so everything could then be combined together.	does not recognize that $3\frac{2}{3}$ and $\frac{1}{3}$ have like denominators	ADD MIXED NUMBERS WITH LIKE DENOMINATORS
Jae had to add $3\frac{2}{3} + \frac{1}{3}$ because he couldn't add $3\frac{2}{3}$ and $\frac{2}{3}$ since it would be too big.	does not understand how to regroup when adding fractions or mixed numbers	ADD MIXED NUMBERS WITH LIKE DENOMINATORS

7.b. What is another way to add $3\frac{2}{3}$ and $\frac{2}{3}$? Explain your thinking.

CORRECT ANSWER

$3\frac{2}{3}+\frac{2}{3}=3\frac{4}{3}=4\frac{1}{3}$. First, you can add the fractional parts together to get $\frac{4}{3}$ which equals $1\frac{1}{3}$. Then add $1\frac{1}{3}+3=4\frac{1}{3}$.

or

$3\frac{2}{3}+\frac{2}{3}=\frac{11}{3}+\frac{2}{3}=\frac{13}{3}=4\frac{1}{3}$. First, you can convert $3\frac{2}{3}$ into the improper fraction $\frac{11}{3}$. Then add $\frac{11}{3}+\frac{2}{3}=\frac{13}{3}$. Finally, convert $\frac{13}{3}$ into the mixed number $4\frac{1}{3}$.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
$3\frac{2}{3}+\frac{2}{3}=3\frac{4}{3}=3\frac{1}{3}$. First, add the fractional parts together to get $\frac{4}{3}$ which equals $1\frac{1}{3}$. So the final sum is $3\frac{1}{3}$.	does not regroup the whole number from $1\frac{1}{3}$ to the final sum	ADD MIXED NUMBERS WITH LIKE DENOMINATORS
$3\frac{2}{3}+\frac{2}{3}=3\frac{4}{6}$. Because there is only one mixed number, only the fractional parts have to be added. $\frac{2}{3}+\frac{2}{3}=\frac{4}{6}$. Then bring over the 3 for a final sum of $3\frac{4}{6}$.	adds both the denominators and the numerators	EXPLAIN THE CONCEPT OF ADDITION AND SUBTRACTION OF FRACTIONS

8. Micala adds $5\frac{7}{10}$ and $6\frac{9}{10}$. She shows her work here.

$$5\frac{7}{10}+6\frac{9}{10}=11+\frac{16}{10}$$
$$=11+\frac{10}{10}+\frac{6}{10}$$
$$=12\frac{6}{10}$$

8.a. Explain the steps Micala takes in her work.

CORRECT ANSWER

Micala adds the whole numbers and the fractional parts separately. 5 + 6 = 11 and $\frac{7}{10} + \frac{9}{10} = \frac{16}{10}$. Then, $\frac{16}{10}$ can be split because $\frac{10}{10} + \frac{6}{10} = \frac{16}{10}$. Since $\frac{10}{10} = 1$, the equation can be thought of as $11 + 1 + \frac{6}{10}$. Add the 11 and 1 for a sum of 12, then add the $\frac{6}{10}$ for a final sum of $12\frac{6}{10}$.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Micala makes a mistake, because she needs to add the entirety of the mixed numbers together instead of separating the parts.	does not understand that the whole number and fraction parts can be added separately when adding mixed numbers	ADD MIXED NUMBERS WITH LIKE DENOMINATORS
Micala makes a mistake, because you cannot have a fraction like $\frac{16}{10}$ since a fraction always has to have a smaller numerator than denominator.	does not understand the relationship between an improper fraction and a mixed number	CONVERT IMPROPER FRACTIONS TO MIXED NUMBERS

8.b. What is another way to add $5\frac{7}{10}$ and $6\frac{9}{10}$? Explain your thinking.

CORRECT ANSWER

Separate $5\frac{7}{10}$ into $5\frac{6}{10}$ and $\frac{1}{10}$, then the expression can be rewritten as $5\frac{6}{10} + \frac{1}{10} + 6\frac{9}{10}$. Then add $\frac{1}{10} + 6\frac{9}{10} = 7$. Finally, add $5\frac{6}{10} + 7 = 12\frac{6}{10}$.

or

Convert both mixed numbers into improper fractions. Then $5\frac{7}{10} = \frac{57}{10}$ and $6\frac{9}{10} = \frac{69}{10}$. Then the expression can be rewritten as $\frac{57}{10} + \frac{69}{10} = \frac{126}{10} = 12\frac{6}{10}$.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Separate $5\frac{7}{10}$ into $5\frac{6}{10}$ and $\frac{1}{10}$, then the expression can be rewritten as $5\frac{6}{10} + \frac{1}{10} + 6\frac{9}{10}$. Then add $\frac{1}{10} + 6\frac{9}{10} = 61$. Finally, add $5\frac{6}{10} + 61 = 66\frac{6}{10}$.	incorrectly adds the fraction and mixed number	ADD MIXED NUMBERS WITH LIKE DENOMINATORS
Add the whole numbers and the fractional parts separately. 5 + 6 = 11 and $\frac{7}{10} + \frac{9}{10} = \frac{16}{20}$. Then put the whole number and fraction together for a final sum of $11\frac{16}{20}$.	adds both the denominators and the numerators	EXPLAIN THE CONCEPT OF ADDITION AND SUBTRACTION OF FRACTIONS