MEASUREMENT OF ANGLES

4.MD.7,8

CONTENTS

The types of documents contained in the unit are listed below. Throughout the unit, the documents are arranged by lesson.

LEARNING MAP INFORMATION	An overview of the standards, the learning map section, and the nodes addressed in this unit
TEACHER NOTES	A brief discussion describing the progression depicted in the learning map section with research-based recommendations for focusing instruction to foster student learning and an introduction to the unit's lessons
OVERVIEW OF INSTRUCTIONAL ACTIVITIES	A table highlighting the lesson goals and nodes addressed in each lesson of this unit
INSTRUCTIONAL ACTIVITY	A detailed walkthrough of the unit
INSTRUCTIONAL ACTIVITY STUDENT HANDOUT	A handout for the guided activity, intended to be paired with the Instructional Activity
INSTRUCTIONAL ACTIVITY SUPPLEMENT	A collection of materials or activities related to the Instructional Activity
STUDENT ACTIVITY	A work-alone activity for students
STUDENT ACTIVITY SOLUTION GUIDE	A solution guide for the work-alone activity with example errors, misconceptions, and links to the learning map section

MEASUREMENT OF ANGLES

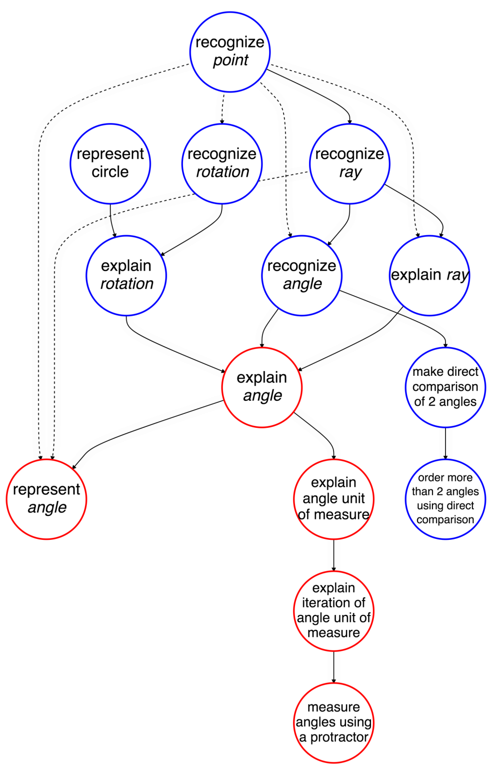
LEARNING MAP INFORMATION

STANDARDS

4.MD.7 Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand the following concepts of angle measurement:

> **4.MD.7.a** An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure angles.
>
> **4.MD.7.b** An angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees.

4.MD.8 Measure and draw angles in whole-number degrees using a protractor. Estimate and sketch angles of specified measure.



**Learning map model of* 4.MD.7,8

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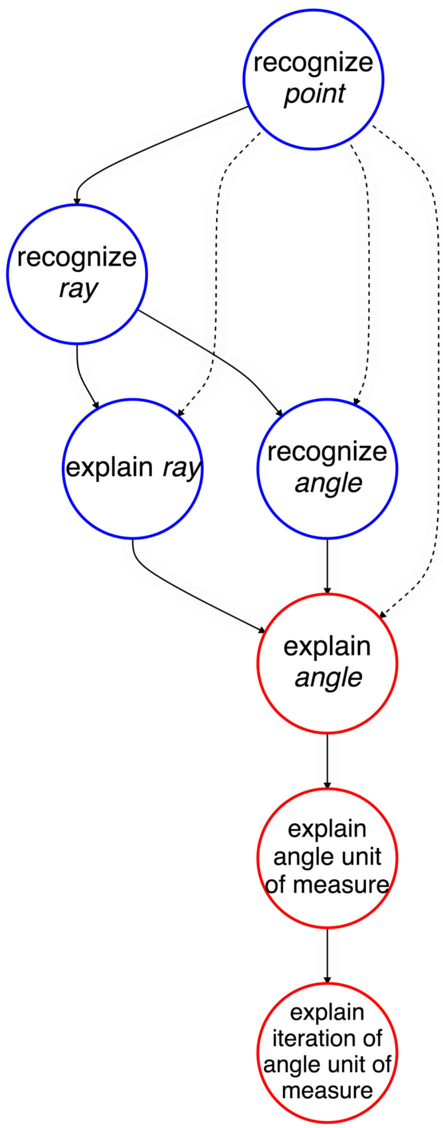
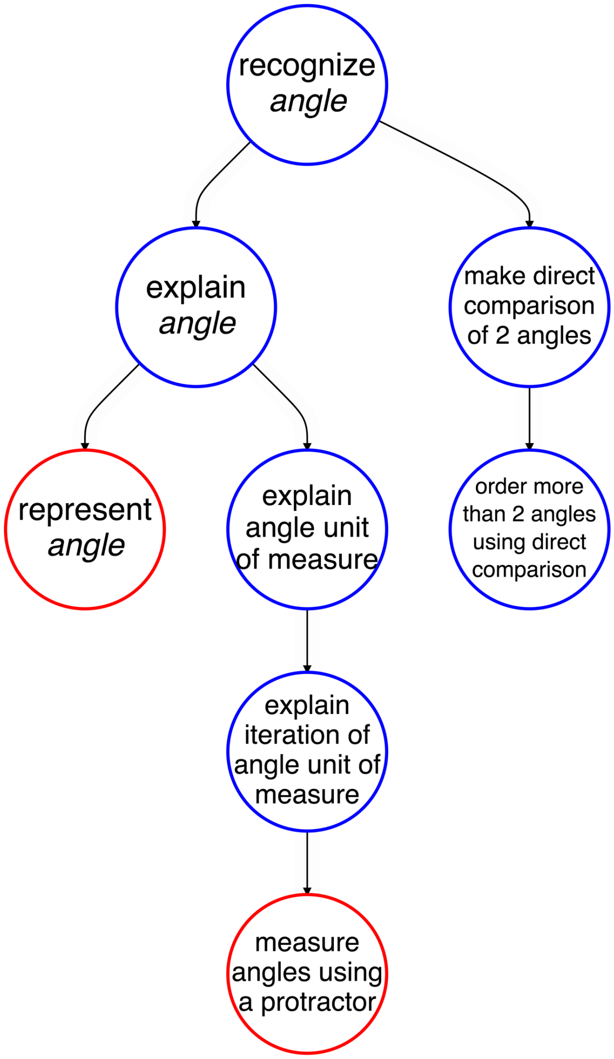


**Learning map model of* 4.MD.7

4.MD.8 Measure and draw angles in whole-number degrees using a protractor. Estimate and sketch angles of specified measure.



**Learning map model of* 4.MD.8

Node Name	Node Description
EXPLAIN *ANGLE*	Make known your understanding that an angle is either a figure formed by two rays with a common endpoint called a *vertex* (static definition) or a rotation of one ray away from another ray with a common endpoint called a *vertex* (dynamic definition).
EXPLAIN ANGLE UNIT OF MEASURE	Make known your understanding that angles are fractions of a circle, and that an angle that turns through $\frac{1}{360}$ of a circle is called a one-degree angle.
EXPLAIN ITERATION OF ANGLE UNIT OF MEASURE	Make known your understanding that an angle unit of measure iterates and accumulates, and that an angle that turns through n one-degree angles is said to have an angle measure of n degrees.
EXPLAIN *RAY*	Make known your understanding that a ray is part of a line that begins at an endpoint and extends infinitely in one direction.
EXPLAIN *ROTATION*	Make known your understanding that a rotation is a transformation that turns a plane containing points, lines, or figures around a point.
MAKE DIRECT COMPARISON OF 2 ANGLES	Directly compare two angles in relation to angle measurement and describe the difference using informal language like greater or less.
MEASURE ANGLES USING A PROTRACTOR	Use a protractor as a tool for measuring angles.
ORDER MORE THAN 2 ANGLES USING DIRECT COMPARISON	Directly compare more than two angles in relation to angle measure and put them in order.
RECOGNIZE *ANGLE*	Identify or name an angle.
RECOGNIZE *POINT*	Identify or name a point.
RECOGNIZE *RAY*	Identify or name a ray.
RECOGNIZE *ROTATION*	Identify a rotation as a transformation in which a plane containing points, lines, or figures is turned.
REPRESENT *ANGLE*	Through writing or an appropriate assistive technology, represent an angle.
REPRESENT CIRCLE	Through drawing or an appropriate assistive technology, represent a circle.

Measurement of Angles

OVERVIEW OF INSTRUCTIONAL ACTIVITIES

Lesson	Learning Goal	Nodes Addressed
Lesson 1	Students will be able to describe what is being measured by an angle and will understand the attribute of "angle".	▶ EXPLAIN *ROTATION* ▶ EXPLAIN *RAY* ▶ EXPLAIN *ANGLE* ▶ MAKE DIRECT COMPARISON OF 2 ANGLES ▶ EXPLAIN ANGLE UNIT OF MEASURE ▶ EXPLAIN ITERATION OF ANGLE UNIT OF MEASURE
Lesson 2	Students will measure angles in whole number degrees using a protractor and will sketch angles of specified measure.	▶ EXPLAIN ANGLE UNIT OF MEASURE ▶ EXPLAIN ITERATION OF ANGLE UNIT OF MEASURE ▶ MEASURE ANGLES USING A PROTRACTOR ▶ REPRESENT ANGLE

Measurement of Angles

TEACHER NOTES

This unit includes the following documents:

- Learning Map Information
- Instructional Activity (two lessons)
- Instructional Activity Student Handout (for Lessons 1 and 2)
- Instructional Activity Supplement (for Lesson 1)
- Student Activity
- Student Activity Solution Guide

In this unit, students will learn the attribute of angle, how to measure angles, and how to sketch angles given a measurement.

RESEARCH

Geometric understanding, according to Battista (2007) and van Hiele (1999), is constructed through a series of experiences as students transition through different levels of geometric thinking. According to the van Hiele theory, there are five levels of geometric thought. Students in fourth grade will be at Level 0 or 1, moving towards Level 2 (Van de Walle, Karp, Lovin, Bay-Williams, 2014). The following table outlines Levels 0 through 2 of van Hiele's theory (Van de Walle, et al., 2014).

Level	*Description*	*Example*
Level 0 Visualization	Thought focuses on shapes and what the shapes look like. Students start to recognize how shapes are alike and different.	"This is a triangle, because it looks like a triangle." "I will put these shapes together in a group because they are all 'skinny'" (or "long" or "pointy").
Level 1 Analysis	Thought focuses on classes of shapes as opposed to individual shapes. Students become familiar with properties of individual shapes.	"All squares have four sides of equal length and four right angles." "This is a triangle because all triangles are closed figures with three sides."
Level 2 Informal Deduction	Thought focuses on expanding understanding of properties among different shapes. Students begin to consider relationships among different properties of shapes.	"If it has four sides, and there are two sets of parallel sides, it must be a parallelogram. If it is a square, it must have four sides and two sets of parallel sides. If it is a square, it must be a parallelogram."

Van Hiele (1999) states that instructional activities should follow a sequence of five phases in order to move students between levels of geometric understanding. The first two phases focus on student experiences. Phase one is exploratory, and phase two involves tasks that present characteristic structures gradually (van Hiele, 1999). The teacher takes a more active role in the third phase, introducing and encouraging students to use correct terminology (van Hiele, 1999). The fourth and fifth phases begin to stretch students' understanding and encourage critical thinking. Tasks that have multiple solutions or processes are introduced in the fourth phase, and in the fifth phase, students are encouraged to make connections between their learning and create their own activities (van Hiele, 1999). Building rich visual representations that transition from concrete (e.g., physical manipulatives) to semi-concrete (e.g., drawings and images) to abstract (e.g., internalized images, visualizing in your head) is integral to creating lasting understanding in mathematics, as is creating opportunities for rich and descriptive discussions; cycling through van Hiele's (1999) five phases provides these important experiences.

It is important to be aware that these levels are not age dependent, but they are sequential (Van de Walle, et al., 2014). For example, a third-grade student may be operating at a Level while a fourth grader may be operating at a Level 0, and in order for either student to move forward in their geometric understanding, they must do so one level at a time. Furthermore, a student must begin at Level 0 and progress through each level in order to move through the levels of geometric thought, (Van de Walle, et al., 2014). For students to transition from Level 1 to Level 2, they must have a solid understanding of angles, because angle measurement is one of the properties students will use to compare and classify shapes.

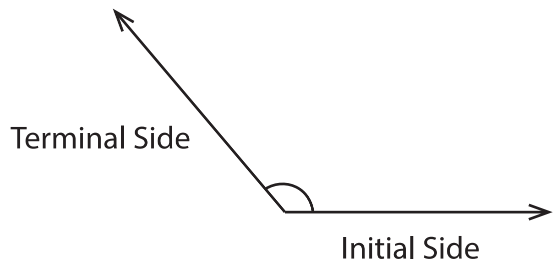
Early angle-measurement instruction should focus on the concept of angle instead of the technical vocabulary (e.g., "degree") and utilizing protractors. This will support the development of solid foundational understanding of the topic (Browning et al., 2007; Millsaps, 2012). Too often students and teachers describe the term *angle* simply by its unit of measure (i.e., degree); teachers should carefully introduce what angles are (i.e. the spread of two rays or line segments) and make known that angles are measured by unit angles. According to Van de Walle, et al. (2014), the concept of angle measurement is the same as the concept of measuring length or area. Much like a unit length or square unit, a unit angle is used to measure the spread of an angle. Developing this concept is critical to the introduction of a degree and how it is used to measure angles. Focusing on degrees too early during instruction reinforces a static definition of an angle and restricts student understanding of a more dynamic definition. Once students have developed a clear understanding of the concept of measuring an angle by iterating unit angles, they are prepared to begin considering angle-measurement tools and degrees.

Measurement Tools

Rather than using a protractor to measure angles at the outset, students should be given early instruction that promotes a rich understanding of angles and their relationship with circles. These early experiences may limit confusion students have with understanding how protractors work and how to read them. Developing the idea of an angle as a turn supports students' understanding of how to use a protractor and allows them to visualize the turn occurring as they place the protractor over the rays that form the angle. Students should also be familiar with standard reference angles (e.g., 45°, 90°, 180°, 270°, and 360°) before they are introduced to protractors (Browning et al., 2007; Millsaps, 2012; St. Julien, 2010; Van de Walle et al., 2014).

This knowledge supplies benchmarks for estimating angle measures, which in turn supports good decisions when students use protractors and helps them identify their errors. Angle rulers are also an alternative to or a step toward protractors. Without the scale running opposite directions on an angle ruler, as it does on a protractor (with numbers running both clockwise and counterclockwise), the measurements are less likely to be misinterpreted. However, utilizing an angle ruler still requires understanding of how to place the tool on the angle and read the scale (Van de Walle, et al., 2014).

When using tools such as a protractor, it is helpful for students—as well as teachers—to be aware of the *initial side* and the *terminal side*. While students do not need to be familiar with the technical language, they do need to be aware of the meaning. When an angle is in standard position (i.e. the vertex is at the origin of a coordinate plane), the initial side is the ray or line segment that is on the positive side of the x- axis, and the other ray or line segment is the terminal side. When utilizing a protractor, the midpoint should be placed over the vertex, with the initial side of the angle in alignment with the zero line of the protractor.



MISCONCEPTIONS

A common theme in the research is both students' and teachers' confusion about what an *angle* is (Browning, Garza-Kling, & Sundling, 2007; Millsaps, 2012; Van de Walle, et al., 2014). It is important to ensure that students do not confuse the unit used to measure angles (i.e. degree) with the concept of angle. Exactly what does an *angle* measure? The static definition of *angle* is "two rays that share a common endpoint", and this static definition limits a person's conception of the more abstract understanding of what an angle is. In contrast, thinking of an angle as a turn, rotation, or wedge grounds a person's understanding in the dynamic idea that an angle measures some amount of circular rotation around a point. The use of real-world applications and in-depth content helps develop a deeper understanding of the concept (Curtis, 2001; Hynes, Dixon & Adams, 2002; Tzur & Clark, 2006).

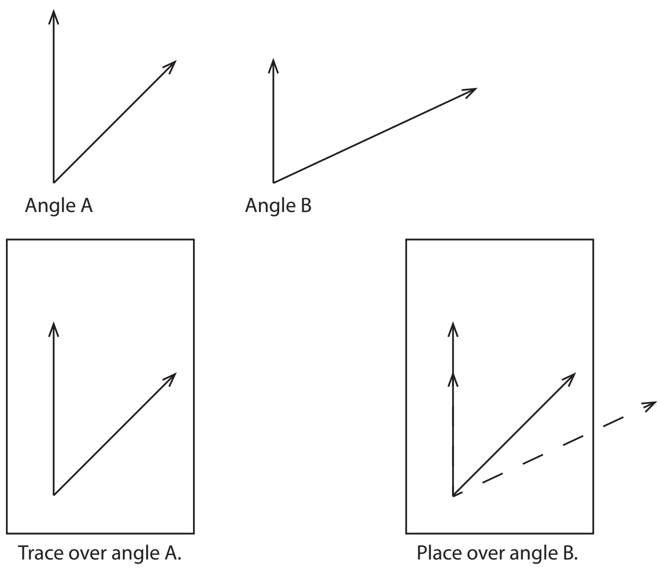
In addition, it is significant to distinguish the difference between the length of the rays or line segments and the distance of the spread. According to Van de Walle and colleagues (2014), "a student might think a wide angle with short rays is less than a narrow angle with long rays" (p. 336). Therefore, students should engage in activities that support their understanding of differentiating between a narrow angle and a wide one, regardless of the length of the rays or line segments. Once students have this foundation, they are ready to begin measuring angles (Van de Wall, et al., 2014).

AN EXAMPLE

In the following activity, two angles are compared to emphasize that the distance of the spread is not related to the length of the rays or line segments (Van de Walle, et al., 2014, p. 336).

ACTIVITY

Compare the two angles by physically placing one angle over another to directly compare them. This can be done by tracing one angle on a piece of tracing paper and then placing that angle over the second angle, making sure to align the vertices.



LEARNING MAP INFORMATION

The learning map section for this sequence of activities strongly relies on students' prior knowledge of vocabulary related to angles. Specifically, students should be able to recognize a point, ray, and angle. To help with developing the dynamic definition of an angle mentioned previously, students should be able to explain what a rotation is. Lastly, students should have some knowledge of reference angles and the ability to directly compare two or more angles.

Students should combine this prior knowledge to form a definition of *angle*, which should lead to the need to measure angles with degrees. Students can then be introduced to a protractor. Once students can define an angle, they can learn to represent an angle, but it is advisable to learn how to use a protractor in order to represent an angle of specified measure.

INSTRUCTIONAL ACTIVITIES

The activities in this unit are designed to provide students with an understanding of the concept of angle prior to utilizing formal measuring tools—such as protractors—to measure angles. The first activities lead students to an understanding of the concept of an angle through an example of when it is used in the real world. Then students explore how to measure and describe angles without using "degrees", which relies too heavily on the static definition of angle. Once students have a solid understanding of how to measure an angle, degrees are introduced and the use of a protractor is taught in order for students to measure given angles and sketch angles of a specified measure.

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Measurement of Angles

OVERVIEW OF INSTRUCTIONAL ACTIVITIES

Lesson	Learning Goal	Nodes Addressed
Lesson 1	Students will be able to describe what is being measured by an angle and will understand the attribute of "angle".	▶ EXPLAIN *ROTATION* ▶ EXPLAIN *RAY* ▶ EXPLAIN *ANGLE* ▶ MAKE DIRECT COMPARISON OF 2 ANGLES ▶ EXPLAIN ANGLE UNIT OF MEASURE ▶ EXPLAIN ITERATION OF ANGLE UNIT OF MEASURE
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Measurement of Angles

INSTRUCTIONAL ACTIVITY

Lesson 1

LEARNING GOAL

Students will be able to describe what is being measured by an angle and will understand the attribute of "angle".

PRIMARY ACTIVITY

Students will first analyze the amount a Ferris wheel has rotated to develop the concept of "angle". Then, students will analyze the difference between narrow and wide angles and the effect that the length of the rays has on an angle.

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

- Rotation/turn
- Ray
- Narrow
- Wide
- Vertex
- Initial (starting) point

MATERIALS

- Ferris wheel image
- Scissors
- Notecards (Recommend one for every student.)
- Patty paper or tracing paper (Recommend one for every student.)
- Instructional Activity Student Handout
- Instructional Activity Supplement (Recommend one copy for every student.)

IMPLEMENTATION

Introduce the lesson by showing an image of a Ferris wheel. Provide background information as necessary for students who may not know how a Ferris wheel works. Specifically, describe the starting position and how the Ferris wheel rotates. For the purpose of this introduction scenario, a starting position and direction of rotation need to be established in order to be consistent throughout the discussion. It is recommended to consider the bottommost point of the Ferris wheel to be the starting point and to rotate counterclockwise.

Tell students to imagine they are on the Ferris wheel. **Ask** students, "How would you describe your location on the Ferris wheel?"

Instruct students to discuss with their shoulder partner how they would describe a location on the Ferris wheel.

Circulate while students are discussing, listening for students who use distance/length or height as a measurement, and for students who use vocabulary such as "turn", "rotation", "angle", or "degree".

Discuss student responses to the question as a class, inviting all students to share their ideas, especially those students who you observed describing with length and those who you observed describing with rotation.

Ask students what advantages and disadvantages there are for each of the methods described.

Guide the discussion toward recognizing that describing your height above ground would be describing two different locations on the Ferris wheel. For example, you could be 10 feet above the ground on the ascent or descent during one rotation of the Ferris wheel.

> NOTE: It is important to refrain from describing the amount of distance traveled around the Ferris wheel. This distance would change based on the size (radius) of the Ferris wheel, and the distance traveled would rely on circumference and ratio understanding, which is not yet developed in fourth grade. Therefore, students should use angle measures to describe the amount that the Ferris wheel has rotated. This approach gives a specific location that can be used for a Ferris wheel of any size, and it is more precise and easier to visualize than "I have traveled 50 meters".

GUIDING QUESTIONS

Elicit student thinking:

- If you were on the Ferris wheel and called someone on the ground, how would you describe your location to them in order for them to know where to look for you?
- Does it help to know if you are more or less than halfway around the circle (rotation)? Why or why not?
- Where would you be if you were halfway around the Ferris wheel? Can you show me how you would get to this location?
- After going around the Ferris wheel once, where would you be? Can you show me your path from start to finish?

Determine if the student can EXPLAIN *ROTATION*:

- What kind of movement is the Ferris wheel making as it moves?
- What is the only part of the Ferris wheel that doesn't move? How do you know?

Next, students will describe the angle formed between the starting position of the Ferris wheel and the position of a person riding the Ferris wheel.

Distribute the INSTRUCTIONAL ACTIVITY SUPPLEMENT to each student.

Develop a definition of what is being measured through discussion with students. Ensure that students understand the focus is on how far the ray (spoke on the Ferris wheel) has rotated from its initial position or starting point.

For the first Ferris wheel, **draw** or **highlight** the two rays that make up the angle from the starting position of the Ferris wheel to the position of a specified person on the Ferris wheel, which further highlights the angle that is being measured.

Lead students to connect the term "angle" with the highlighted image shown on the Ferris wheel.

Encourage students to draw additional angles on the first Ferris wheel on their INSTRUCTIONAL ACTIVITY SUPPLEMENT. One recommendation would be to have students place additional people on the Ferris wheel and draw angles between them, keeping the center of the circle as the vertex.

GUIDING QUESTIONS

Elicit student thinking:

- What are we measuring on the Ferris wheel?

Determine if the student can EXPLAIN *RAY*:

- What do you call the sides of the angle?
- What are the important characteristics of a ray?
- How would you define a ray?

Determine if the student can EXPLAIN *ANGLE*:

- How would you describe an angle in your own words?
- How does an angle appear in a Ferris wheel?
- What happens if we draw longer sides of the angle? What happens if we draw shorter sides of the angle? Does the measure of the angle change?

The INSTRUCTIONAL ACTIVITY SUPPLEMENT shows two Ferris wheels of different size with a rotation marked on each.

Ask students whether the angle from the starting point to the specified locations on the first and the second Ferris wheel in the INSTRUCTIONAL ACTIVITY SUPPLEMENT is different or the same.

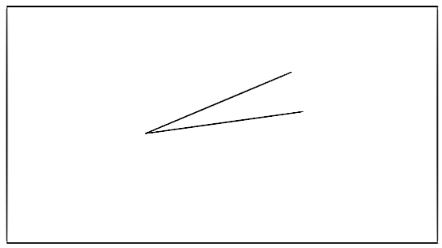
Discussion should indicate that the angle from the starting point to the specified location is the same if the Ferris wheels have turned the same amount, but it needs to be verified whether the Ferris wheels have turned the same amount or not. Students likely will be ready to describe the amount that the Ferris wheel has turned, and the need to quantify the rotation will lead to the idea of measuring how many "little turns" make up the "big turn". The need for a tool to measure the amount of turn naturally arises.

Tell students that they will be making their own tool to measure whether or not the Ferris wheels have turned the same amount.

NOTE: Students may count the spokes of the Ferris wheel in order to determine whether the two angle positions are the same or different. This reasoning is also valid in this situation because there are the same number of spokes on each Ferris wheel and the spokes are evenly spaced. This form of measuring could be introduced for students who are having a hard time visualizing the equivalency of the two angle measures.

Distribute a notecard to each student.

Direct students to draw a small angle in the middle of the notecard.



Model this for students who need more guidance.

Direct students to cut out their angles to be their angle-measuring tool.

Model aligning the vertices of the Ferris wheel angle (center of the circle) and the wedge, while at the same time aligning one side of the wedge with the spoke (ray) on one side of the Ferris wheel angle.

In order to practice measuring angles with their new tool, **direct** students to measure how many of their wedges fit in the turn marked on both Ferris wheels of the INSTRUCTIONAL ACTIVITY SUPPLEMENT.

Circulate while students work and **use** the following guiding questions to check for student understanding as they work through the INSTRUCTIONAL ACTIVITY SUPPLEMENT (and throughout the remainder of the lesson).

GUIDING QUESTIONS

Elicit student thinking:

- Why do you think two students can have different measurements for the same angle on the Ferris wheels when they use the angle they created to make the measurement?
- Do you think that our measurement tools are accurate? What would make them more accurate?

Determine if the student can MAKE DIRECT COMPARISON OF 2 ANGLES:

- [Point to the angles on two different Ferris wheels.] Which of these rotations is larger? Which is smaller? Are they equal?
- Does the size of the Ferris wheel affect the angle measure from the starting point to a specific location on the Ferris wheel?

Determine if the student is ready to EXPLAIN ANGLE UNIT OF MEASURE:

- How have you seen angles measured?
- What units do you hear describing angles?
- Can you tell me how many degrees make up a halfway turn? A quarter turn? A full circle?

Determine if the student is ready to EXPLAIN ITERATION OF ANGLE UNIT OF MEASURE:

- How many total turns did you make with your angle-measuring tool?
- Can you describe how you measured each angle using your angle-measuring tool? Why did you measure this way?

After students have successfully completed the INSTRUCTIONAL ACTIVITY SUPPLEMENT, **call on** two students to demonstrate how they measured the two Ferris wheel rotations with their notecard angle for the class to see. **Call attention to** several details:

- The small and large Ferris wheels had the same number of wedges. Mathematically this means that the angle measure is the same for both the small Ferris wheel and the big Ferris wheel (i.e. the length of the rays doesn't affect the measure of the angle).
- The two students' tools gave different measurements, because their angles are different sizes.
- The number of turns had to be approximated (e.g., this is about seven and a half wedges).

NOTE: Be sure to **emphasize** the first bullet point. In order to illustrate that the size of the Ferris wheel (i.e. the length of the rays of the angle) does not affect the angle measure, trace the smaller Ferris wheel and the identified angle onto patty paper or tracing paper. Align the center of the smaller Ferris wheel on tracing paper with the center of the larger Ferris wheel on the INSTRUCTIONAL ACTIVITY SUPPLEMENT. Position the starting point for each Ferris wheel so they overlap in order to compare the angle measure. Students should notice that the angle measures are the same despite the fact that the Ferris wheels are the different sizes.

Distribute the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

Students should be required to measure several Ferris wheel rotations and stand-alone angles with their angle-measuring tool made of notecard on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

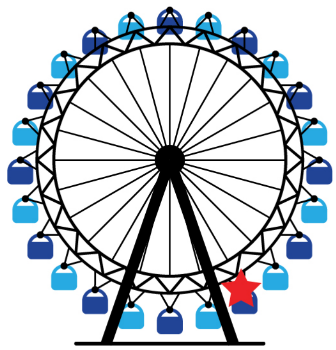
Point out that on the Ferris wheel, the direction of the rotation was specified, therefore it was clear which angle was being referenced and measured. In order to identify the angle to be measured or considered when there is no context, an arc is drawn to connect initial and terminal sides.

The last question on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT requires students to extend the ray on one of the stand-alone angles and explain whether this affects the angle measure. **Attend** to student responses on this question and focus questioning or instruction to solidify student understanding that the length of a ray does not affect angle measure.

At the end of the activity, teachers should close the lesson by requiring students to tape their angle-measuring tool to their INSTRUCTIONAL ACTIVITY STUDENT HANDOUT fort teachers to collect and review for understanding.

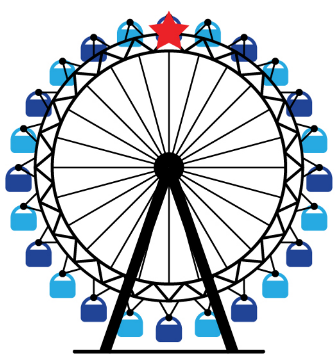
Name______________________________

Use your angle-measuring tool to measure the angle given by the starred location on the Ferris wheel. Assume that the starting position is the bottommost point of the Ferris wheel and that the Ferris wheel rotates counterclockwise. Write the number of wedges under each Ferris wheel.



___________ wedges

___________ wedges

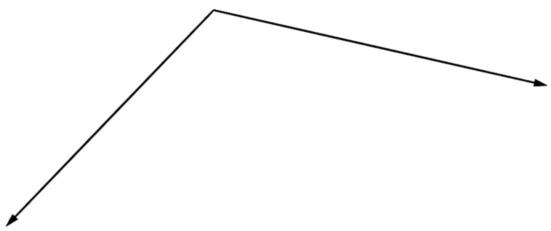


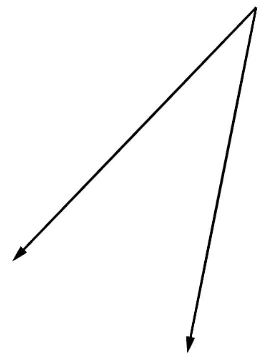
___________ wedges

___________ wedges

Name_____________________________

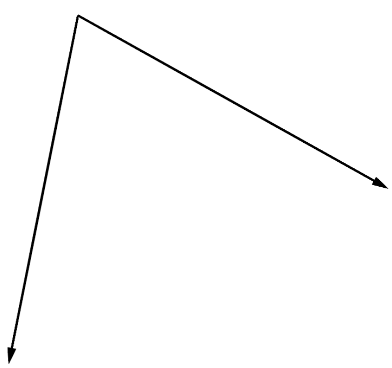
Use your angle-measuring tool that you cut out to measure each angle. Write the number of wedges under each angle.





___________ wedges

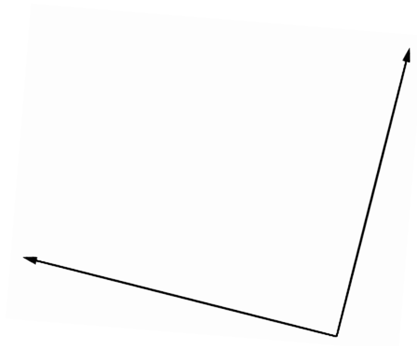
___________ wedges



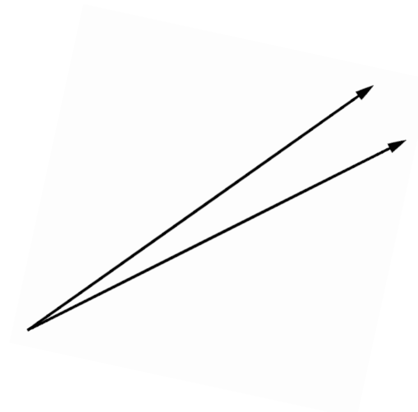
___________ wedges

___________ wedges

Name____________________________



__________ wedges

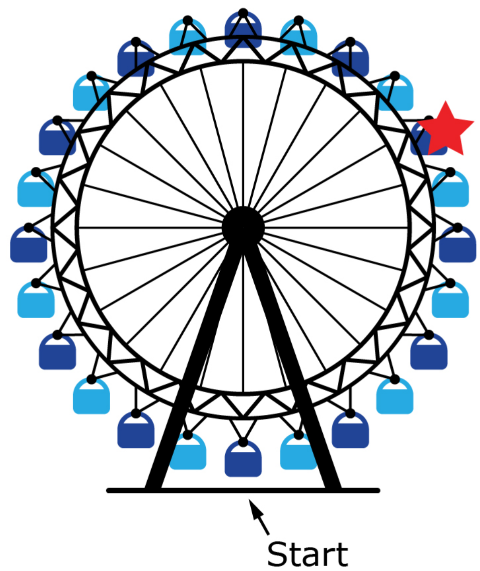


__________ wedges

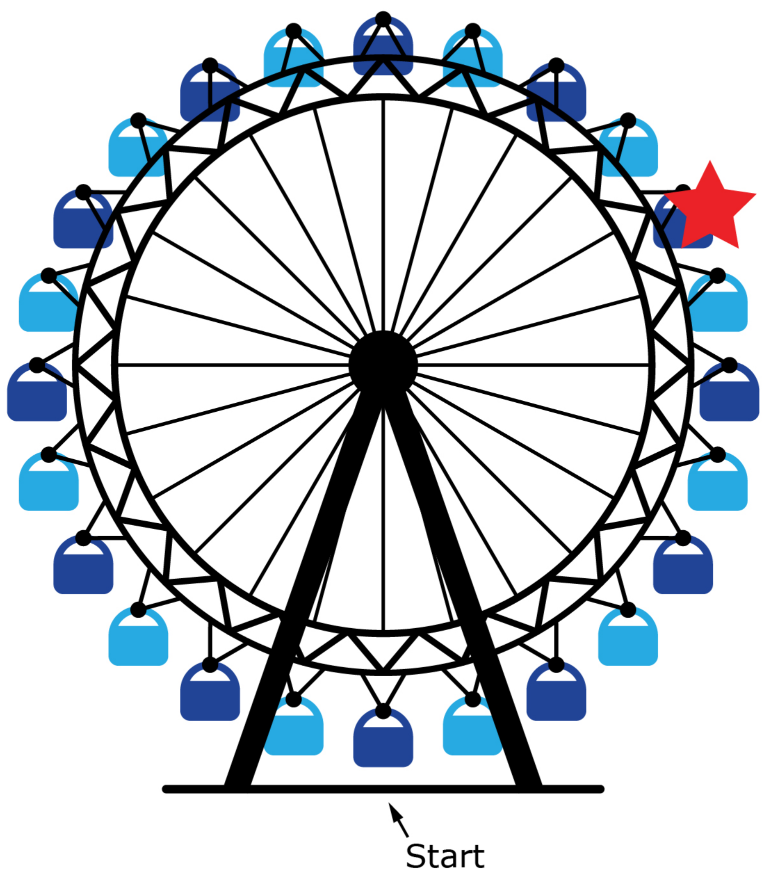
Extend a ray in one of the angles. Use your angle-measuring tool that you cut out to measure that same angle.

How many wedges is the angle after extending the ray?

Did the number of wedges in the angle change after extending the ray? Explain why or why not.



Start



Start

Measurement of Angles

INSTRUCTIONAL ACTIVITY

Lesson 2

LEARNING GOAL

Students will measure angles in whole number degrees using a protractor and will sketch angles of specified measure.

PRIMARY ACTIVITY

Students will use a protractor to measure the same angles of rotation of the Ferris wheels that they measured with their own angle-measuring tool in Lesson 1. Then, students will practice their angle-measurement knowledge using the Instructional Activity Student Handout.

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

- Angle
- Rotation
- Protractor
- Ray
- Vertex
- Initial side/initial ray
- Terminal side/terminal ray
- Reference angle (90^{o}, 180^{o}, 270^{o}, 360^{o})
- Half turn

MATERIALS

- Protractors
- Student angle-measuring tools from Lesson 1
- Instructional Activity Student Handout

IMPLEMENTATION

Remind students that in LESSON 1 they measured angles with the use of their self-made notecard angle measurement tool.

Discuss as a whole class the advantages and disadvantages of using their own measurement tool. If students do not mention it, **guide** them to the realization that their measurement tools had the disadvantage of being imprecise, because they had to approximate some of their measurements. Additionally, their tools do not represent a standard measurement, because their tools are different sizes. Therefore, they reported different measurements for the same-size angle.

Through the discussion, **establish** the need for a unit of measure that is the same for every student and that is small enough to precisely measure angles of any size.

In order to avoid the issues students encountered with their informal measuring tools, **introduce** the protractor.

Display a protractor for students to see, and **distribute** a protractor to each student.

Tell students that a protractor measures an angle by using a very small angle called a *degree*.

Ask students if they know how many degrees are in one complete rotation or circle. **Guide** students toward the understanding that there 360 degrees in one complete rotation or circle, and therefore one degree is $\frac{1}{360}$ of a complete rotation or circle.

NOTE: There is an opportunity to connect student understanding of angle measure to fraction knowledge. Point out that if a circle was partitioned by drawing diameters to create 360 equal-size parts, each one of the equal-size parts would represent $\frac{1}{360}$ of the whole circle or one degree.

Help students realize that measuring with degrees is similar to how they measured the angles in LESSON 1, just with a much smaller angle.

Additionally, **inform** students that all protractors have the same-size degree markings, giving us a standardized unit to measure angles.

GUIDING QUESTIONS

Elicit student thinking:

- What were the advantages of your angle-measuring tool?
- What were the disadvantages of your angle-measuring tool?

Determine if the student can EXPLAIN ANGLE UNIT OF MEASURE:

- Why do we measure angles with degrees?
- What advantages does measuring in degrees offer?

Determine if the student can EXPLAIN ITERATION OF ANGLE UNIT OF MEASURE:

- If you take two one-degree turns, what is your location? Can you draw what this would look like?
- If you take two 90° turns, what is your location? Can you draw what this would look like?

Determine if the student is ready to MEASURE ANGLES USING A PROTRACTOR:

- How do you know which row of numbers on the protractor is accurate for the angle you're measuring?
- Why is there only one number at the 90° marking, but all the other degrees have two markings?

Distribute the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT to each student.

Ask students to write down at the top of their handout what they know about degrees and where they have seen or heard about angles measured in degrees.

While circulating and listening to student conversations, **ask** students to share what they know about degrees with a partner. **Encourage** students to sketch an example and describe the real-world context for each angle they describe.

Finally, **ask** students to share with the class what they know about degrees and what they discussed with their partners.

Ensure that there is mention of a full turn/rotation (360°), a half turn/rotation (180°) and a right angle, or quarter turn/rotation (90°) during the discussion.

Ask students to describe each of these angle measures as they are mentioned and draw an example for the class to see.

If any of these angle measures are not mentioned by students, **guide** their understanding using common examples (e.g., 90° angles can be found in stair cases, windows, traffic intersections, etc.).

Build student knowledge around these reference angles, and refer to them when students are measuring angles to check their work and determine which row of angle measures to look at on the protractor.

Demonstrate how to use a protractor by measuring the first Ferris wheel angle on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

Make particular note of how to align the initial side of the angle with the 0º of the protractor, and how to determine which row of degree measures to use by reminding them of their reference angles (0, 90, 180, 360 degrees). **Note** that either side of the angle can be considered the initial side of the angle.

Relate the way the protractor is positioned to the way they positioned their cut-out angles in LESSON 1.

GUIDING QUESTIONS

Elicit student thinking:

- How many degrees is a three-quarter (or three-fourths of one) turn?
- [Point to or draw a quadrant of the Ferris wheel.] What two numbers will all angles in this region be between?

Determine if the student can MEASURE ANGLES USING A PROTRACTOR:

- Which side of the angle will you use as the initial side? Why?
- Can you tell without measuring if this angle greater than or less than 90°? What does that tell you about which row of numbers to use on the protractor?

Determine if the student is ready to REPRESENT ANGLE:

- How would you draw a quarter turn on the Ferris wheel?
- Can you draw an approximation of a 100° angle based on your knowledge of 90° angles?

After completing the front of the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT, students may naturally begin work on the back of the handout.

If students are struggling to draw the specified angles, first **remind** them how to measure given angles as they did on the front of the handout. Second, **model** the process of aligning the protractor with the initial side, marking the specified degree and drawing the terminal side of the angle. **Remind** all students as necessary to use the straight edge of the protractor to draw both the initial and terminal side of the angles.

Students should be required to successfully complete the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

At the end of the activity, teachers should collect the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT to check for student understanding and ask students to complete an exit ticket requiring them to measure a given angle and draw an angle of specified measure.

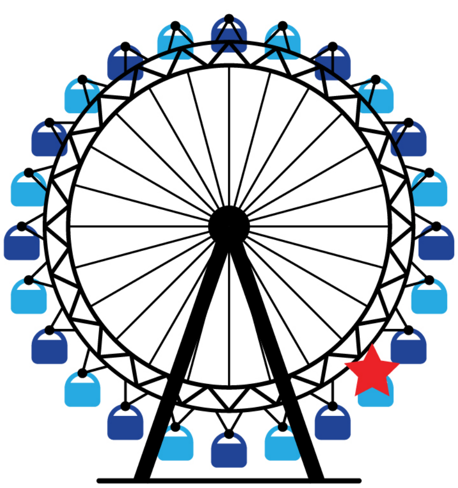
Name______________________________

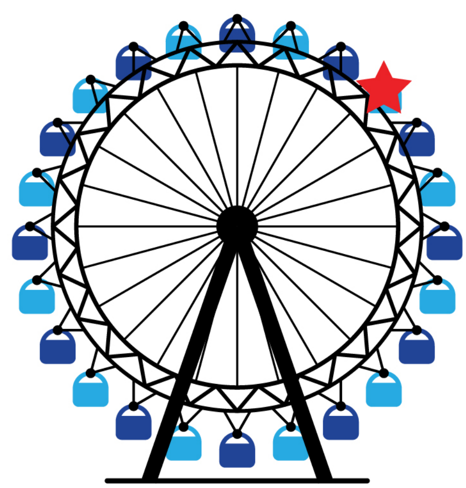
Measurement of Angles

Lesson 2

1. Write what you know about *degrees* in the space below. Provide at least one real-world example that includes an angle measurement in degrees.

2. Using a protractor, measure the angle given by the starred location on the Ferris wheel. Use the bottommost point of the Ferris wheel as the starting point, and assume the Ferris wheel rotates counterclockwise.

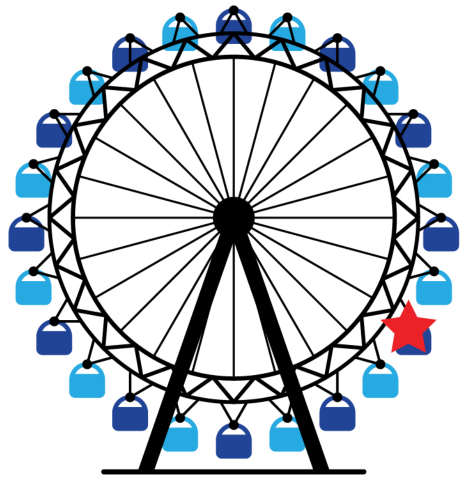




_______ degrees

_______ degrees

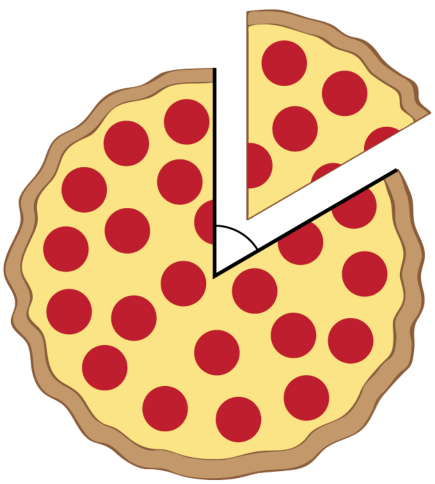
Name______________________________

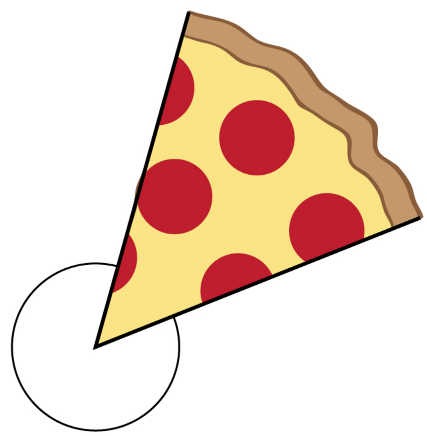


_______ degrees

_______ degrees

3. Using a protractor, measure the angle marked in the slice of pizza.





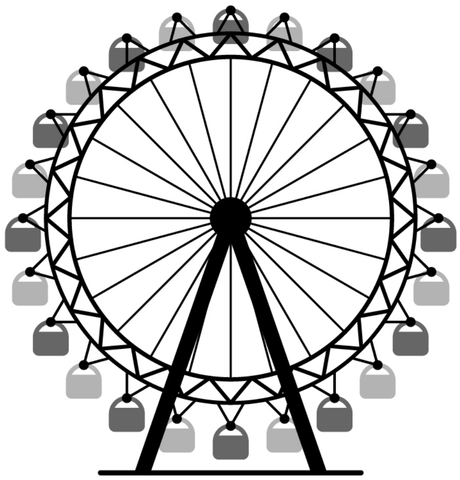
_______ degrees

_______ degrees

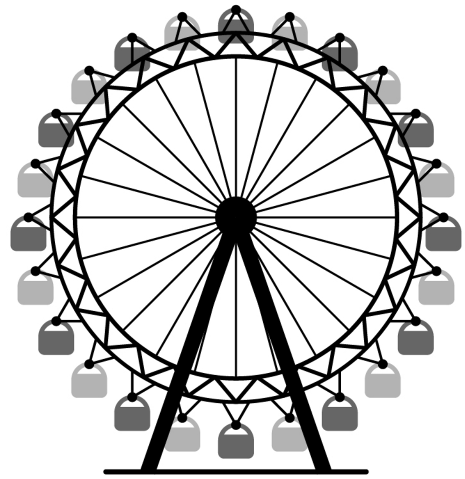
Name____________________________

4. Draw the angle measure indicated for each Ferris wheel.

A 53° turn.

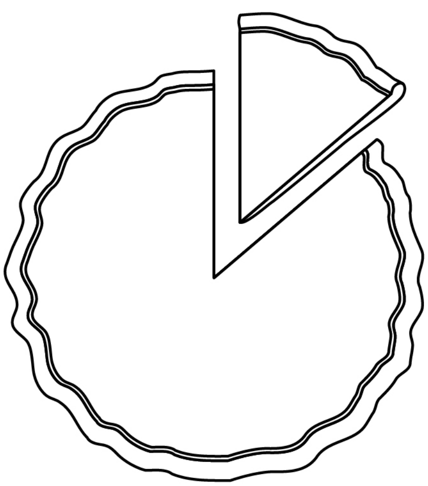


A 78° turn.

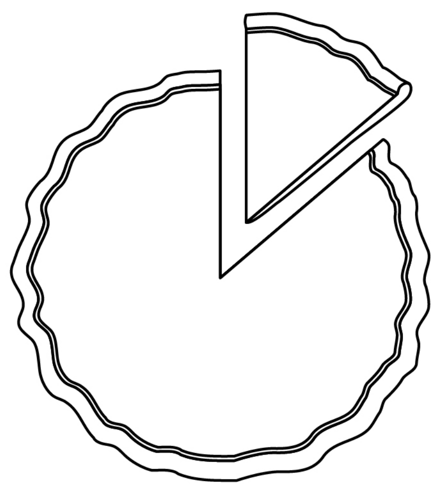


5. Draw the angle measure indicated for each slice of pizza. Use one of the existing cuts as the initial side of the angle.

A 112° slice of pizza



A 177° slice of pizza



Name____________________________

6. In the space provided below, draw a 25° angle.

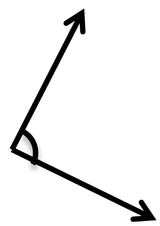
7. In the space provided below, draw a 149° angle.

Name____

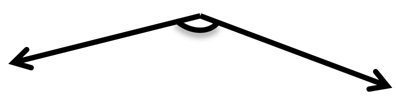
Measurement of Angles

1. Use a protractor to measure each of the following angles.

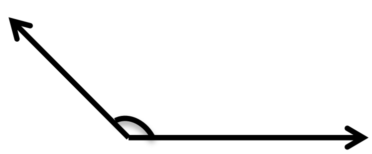
1.a.



1.b.



1.c.



2. Draw an angle with the given degree measure.

2.a. 27^{o}

Name____

2.b. 119°

2.c. 90°

Name_______________________________________

3. Describe each of the following terms.

3.a. Angle

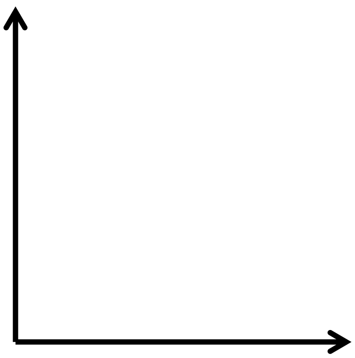
3.b. Degree

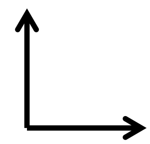
3.c. Ray

4. Why is it important to have a standard unit to measure angles?

Name_______________________________________

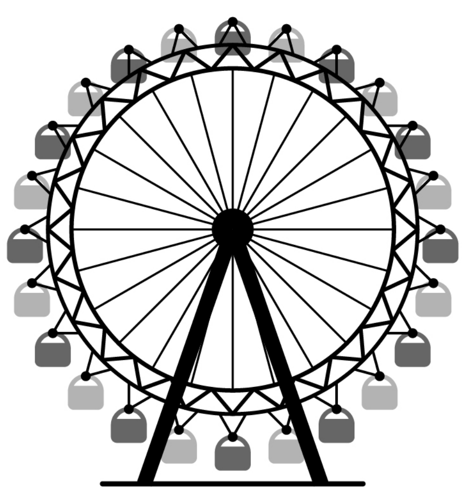
5. Without using a protractor, state whether you believe the angles have the same measure or different measures. Explain your reasoning.





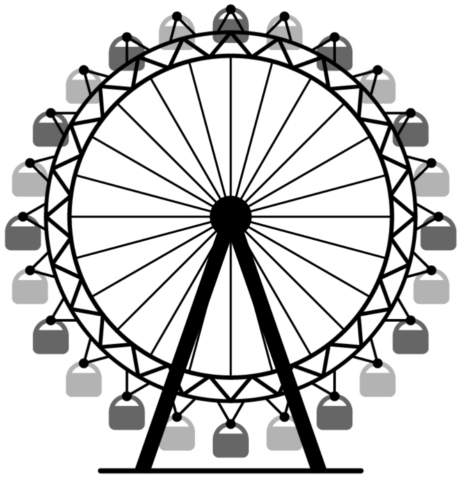
6. Color or shade in the portion of the Ferris wheel between the two angle measures given. Assume that the Ferris wheel starts at the bottom and turns counterclockwise.

6.a. Between 0^{o} and 180^{o}

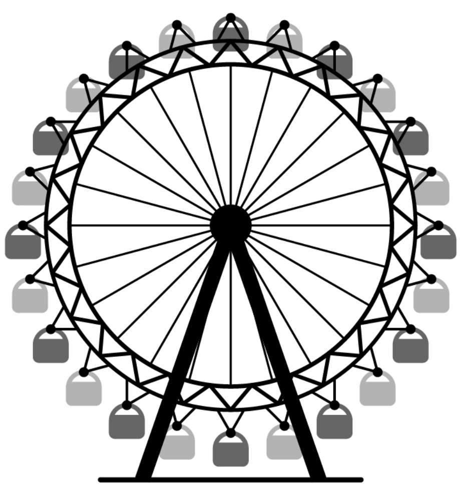


Name____

6.b. Between 180^{o} and 360^{o}



6.c. Between 90^{o} and 180^{o}

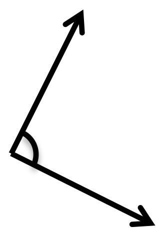


MEASUREMENT OF ANGLES

STUDENT ACTIVITY SOLUTION GUIDE

1. Use a protractor to measure each of the following angles.

1.a.



CORRECT ANSWER

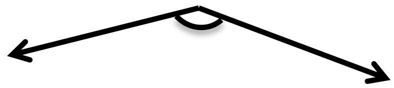
(Answers may vary by a few degrees.)

90°

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	**Misconception**	**Missing Knowledge**
75°	holds the protractor horizontally and measures an angle between the horizontal and the top ray	MEASURE ANGLES USING A PROTRACTOR
115°	holds the protractor horizontally and measures an angle between the horizontal and the top ray but uses the wrong row of numbers on the protractor	MEASURE ANGLES USING A PROTRACTOR
The student provides any other angle measure.	incorrectly positions the protractor to measure the angle	MEASURE ANGLES USING A PROTRACTOR
The student provides a length measurement (e.g., 1 inch or 2 centimeters).	measures the length from the endpoint to the arrowhead on the ray instead of measuring the angle	MEASURE ANGLES USING A PROTRACTOR

1.b.



CORRECT ANSWER

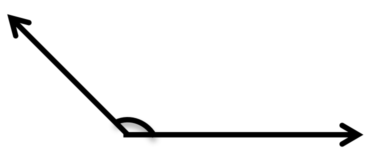
(Answers may vary by a few degrees.)

145°

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
35°	uses the wrong set of numbers on the protractor	MEASURE ANGLES USING A PROTRACTOR
155°	uses the correct set of numbers on the protractor but misreads the angle measure between 140° and 150° as 155°	MEASURE ANGLES USING A PROTRACTOR
The student provides any other angle measure.	incorrectly positions the protractor to measure the angle	MEASURE ANGLES USING A PROTRACTOR
The student provides a length measurement (e.g., 1 inch or 2 centimeters).	measures the length from the endpoint to the arrowhead on the ray instead of measuring the angle	MEASURE ANGLES USING A PROTRACTOR

1.c.



CORRECT ANSWER

(Answers may vary by a few degrees.)

135°

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
45°	uses the wrong set of numbers on the protractor	MEASURE ANGLES USING A PROTRACTOR
145°	uses the correct set of numbers on the protractor but misreads the angle measure between 130° and 140° as 145°	MEASURE ANGLES USING A PROTRACTOR
The student provides any other angle measure.	incorrectly positions the protractor to measure the angle	MEASURE ANGLES USING A PROTRACTOR
The student provides a length measurement (e.g., 1.25 inches or 3 centimeters).	measures the length from the endpoint to the arrowhead on the ray instead of measuring the angle	MEASURE ANGLES USING A PROTRACTOR

2. Draw an angle with the given degree measure.

2.a. 27°

CORRECT ANSWER

(Angle orientation may vary.)

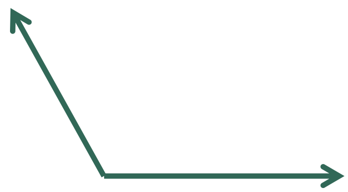
ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student draws a 153° angle.	uses the wrong set of numbers on the protractor	MEASURE ANGLES USING A PROTRACTOR
The student attempts to draw a 27° angle without using a protractor.	understands the approximate size of a 27° angle but does not use the protractor to precisely draw the angle	MEASURE ANGLES USING A PROTRACTOR
The student draws an angle with any other measure.	does not know the approximate size of a 27° angle and cannot use a protractor to draw an angle of a specified measure	MEASURE ANGLES USING A PROTRACTOR

2.b. 119°

CORRECT ANSWER

(Angle orientation may vary.)



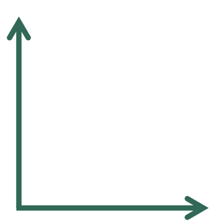
ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student draws a 119° angle.	uses the wrong row of numbers on the protractor	MEASURE ANGLES USING A PROTRACTOR
The student attempts to draw a 61° angle without using a protractor.	understands the approximate size of a 61° angle but does not use the protractor to precisely draw the angle	MEASURE ANGLES USING A PROTRACTOR
The student draws an angle with any other measure.	does not know the approximate size of a 61° angle and cannot use a protractor to draw an angle of a specified measure	MEASURE ANGLES USING A PROTRACTOR

2.c. 90°

CORRECT ANSWER

(Angle orientation may vary.)



ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student attempts to draw a 90° angle without using a protractor.	understands the approximate size of a 90° angle but does not use the protractor to precisely draw the angle	MEASURE ANGLES USING A PROTRACTOR
The student draws an angle with any other measure.	does not know the approximate size of a 90° angle and cannot use a protractor to draw an angle of a specified measure	MEASURE ANGLES USING A PROTRACTOR

3. Describe each of the following terms.

3.a. Angle

CORRECT ANSWER

> An angle is the space between two rays that share a common endpoint. An angle can also be described as the rotation of a ray away from another ray with a common endpoint.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
An angle is a position on a Ferris wheel.	only has a definition in the context of a scenario; does not have an abstract or complete understanding of the concept of an angle	EXPLAIN *ANGLE*
An angle is a corner.	has a static understanding of angle; might only think that angles are 90° or are always located in two-dimensional figures	EXPLAIN *ANGLE*

3.b. Degree

CORRECT ANSWER

> A degree is the measure of a very small angle that is $\frac{1}{360}$ of one complete turn/rotation. Degrees are used to measure other larger angles.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
A degree is one mark on the protractor.	only thinks of degrees in terms of the measurement device	EXPLAIN ANGLE UNIT OF MEASURE
A degree is a small angle.	does not know what portion of a circle one degree is; just knows that degrees are related to angles	EXPLAIN ANGLE UNIT OF MEASURE
A degree is 1°.	only knows the symbolic representation of one degree	EXPLAIN ANGLE UNIT OF MEASURE

3.c. Ray

CORRECT ANSWER

> A ray is a portion of a line that begins at an endpoint and extends infinitely in one direction.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
A ray is a line.	does not have an understanding of the direction of a ray	EXPLAIN *RAY*
A ray is a line segment.	does not have an understanding of the direction of a ray	EXPLAIN *RAY*
A ray is the side of an angle.	provides an example of a ray but does not show understanding of the attributes of a ray	EXPLAIN *RAY*

4. Why is it important to have a standard unit to measure angles?

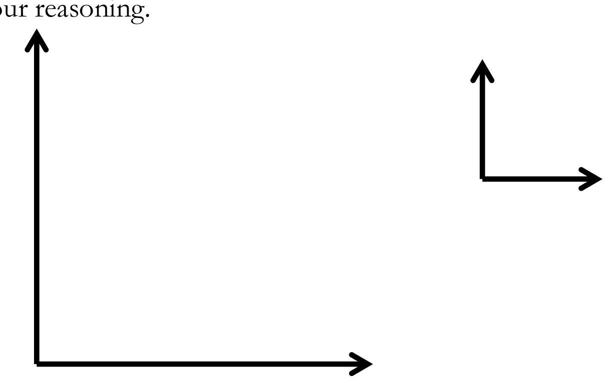
CORRECT ANSWER

> A standard unit is important to measure angles because it allows different people to measure the same angle and always find the same measure. Using degrees gives us a standardized unit that helps us communicate about angles.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
A standard angle measurement is a degree.	understands that degrees are a standard angle unit of measure but does not address the question by explaining why they are important	EXPLAIN ANGLE UNIT OF MEASURE
Standard units of measure help measure angles.	does not understand what a standard unit of measure is; offers a generic or vague response	EXPLAIN ANGLE UNIT OF MEASURE

5. Without using a protractor, state whether you believe the angles have the same measure or different measures. Explain your reasoning.



CORRECT ANSWER

> The two angles have the same measure, because the length of the rays doesn't affect the measure of the angle, and the ray that is positioned vertically in each angle appears to have rotated the same amount from the initial ray.

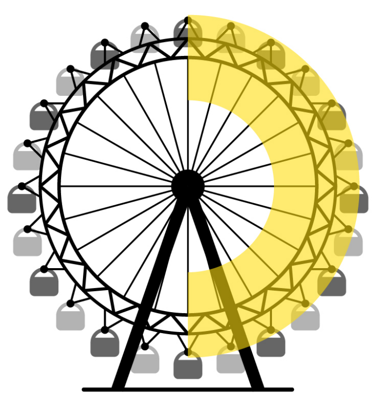
ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The angle on the left has a larger measure.	believes that the lengths of the rays of the angle affect the measure of the angle	MAKE DIRECT COMPARISON OF 2 ANGLES
The angles are the same.	lacks the vocabulary necessary to describe the angle measure and does not provide an explanation for why they believe the angles have the same measure	MAKE DIRECT COMPARISON OF 2 ANGLES

6. Color or shade in the portion of the Ferris wheel between the two angle measures given. Assume that the Ferris wheel starts at the bottom and turns counterclockwise.

6.a. Between 0° and 180°

CORRECT ANSWER

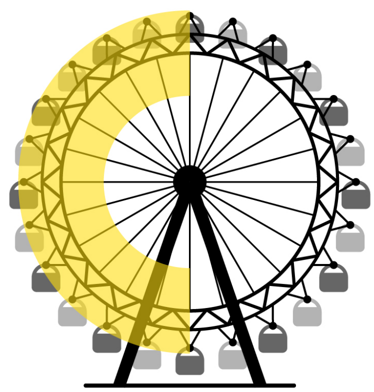


ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student highlights a different 180° portion of the Ferris wheel.	does not consider the starting position as 0°	REPRESENT *ANGLE* and MEASURE ANGLES USING A PROTRACTOR
The student highlights a 90° portion of the circle.	does not understand that there are 180° between 0° and 180°	REPRESENT *ANGLE* and MEASURE ANGLES USING A PROTRACTOR

6.b. Between 180° and 360°

CORRECT ANSWER

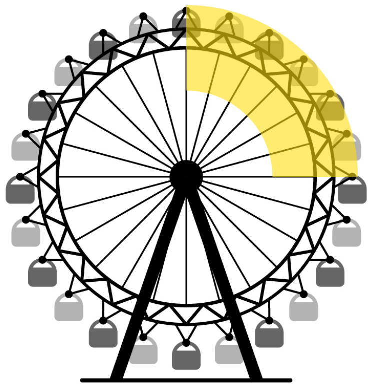


ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student highlights the entire circle.	sees 360° in the directions and believes that the highlighted portion should be 360°	REPRESENT *ANGLE* and MEASURE ANGLES USING A PROTRACTOR
The student highlights a different 180° portion of the Ferris wheel.	does not consider the starting position as 0°	REPRESENT *ANGLE* and MEASURE ANGLES USING A PROTRACTOR

6.c. Between 90° and 180°

CORRECT ANSWER



ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student highlights a different 90° portion of the Ferris wheel.	does not consider the starting position as 0°	REPRESENT *ANGLE* and MEASURE ANGLES USING A PROTRACTOR
The student highlights a 180° portion of the circle.	does not understand that there are 90° between 90° and 180°, or sees 180° in the directions and thinks that the portion of the circle should also be 180°	REPRESENT *ANGLE* and MEASURE ANGLES USING A PROTRACTOR