USING REPEATED ADDITION TO SOLVE ARRAY PROBLEMS

2.OA.4

CONTENTS

The types of documents contained in the unit are listed below. Throughout the unit, the documents are arranged by lesson.

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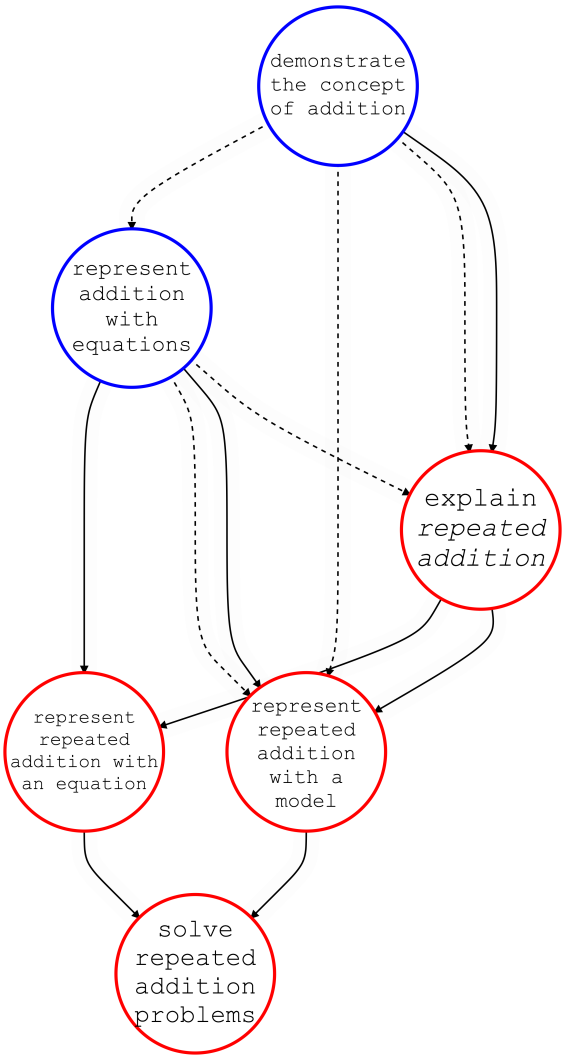
Using Repeated Addition to Solve Array Problems

Learning Map Information

Standards

2.OA.4 Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.



**Learning map model of* 2.OA.4

Node Name	Node Description
DEMONSTRATE THE CONCEPT OF ADDITION	Demonstrate through actions, fingers, mental images, drawings, sounds, acting out situations, verbal explanations, expressions, or use of concrete manipulatives that addition means to put objects together to form a set.
REPRESENT ADDITION WITH EQUATIONS	Use equations to represent addition problems.
EXPLAIN *REPEATED ADDITION*	Make known your understanding, using words or concrete models, that the concept of repeated addition concerns adding the same addend three or more times.
REPRESENT REPEATED ADDITION WITH A MODEL	Show repeated addition by using a model such as an array, set, number line, etc.
REPRESENT REPEATED ADDITION WITH AN EQUATION	Use equations to represent repeated-addition problems.
SOLVE REPEATED-ADDITION PROBLEMS	Solve problems involving repeated addition.

USING REPEATED ADDITION TO SOLVE ARRAY PROBLEMS

TEACHER NOTES

This unit includes the following documents:

- Learning Map Information
- Instructional Activity (two lessons)
- Instructional Activity Student Handout (for Lesson 2)
- Instructional Activity Supplement (for Lesson 2)
- Student Activity
- Student Activity Solution Guide

In this unit, students will model repeated-addition situations as an array and represent them symbolically in order to find the sum.

RESEARCH

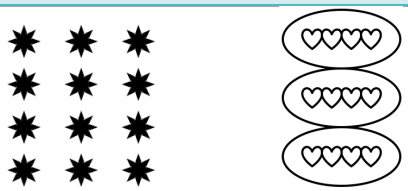
Repeated addition, like other computational skill, should be learned in the context of real-world situations, and students should initially represent the situations with physical manipulatives and models (Jones, 2012). Students' number skills and computation strategies are often more visible when solving word problems than with non-contextual problems (Van de Walle, Karp, Lovin, Bay-Williams, 2014). After students have thoroughly explored repeated addition through these concrete and semi-concrete representations, they are ready to move toward more abstract representations.

As students learn about addition, they will progress through three stages of learning: direct modeling, counting strategies, and derived facts (Van de Walle, et al., 2014). Because students will have previously learned addition concepts and strategies, they may already be ready to process repeated-addition problems at the counting-strategies level. Therefore, teachers should look for indications that students are ready to apply symbolic notation to their counting-strategies thinking. Teachers should model how to represent a students' thinking symbolically by listening to a student describe their strategy and demonstrating how to write a corresponding equation. This helps guide students to think about the concepts more abstractly.

Repeated-addition equations and their corresponding physical representations, arrays, provide the foundation for future learning of multiplication, and students must understand the relationship between the two representations. Specifically, students should recognize the rows of an array as the number of addends, and the columns as the value of each addend. While it is also possible to consider the columns to be the number of groups (i.e. addends) in repeated addition, multiplication will require students to define the rows as the number of groups; therefore, it is beneficial to guide students toward this interpretation in order to be consistent with future learning.

AN EXAMPLE

Students should become fluent in translating between repeated-addition equations and arrays. Note that it is recommended to consider the number of addends to be the number of rows, as is shown in the array with hearts. Additionally, grouping the items by circling them can help students visualize the groups as they relate to the repeated-addition equation, but it is not necessary.

Model	Example
Repeated-Addition Equation (Symbolic)	4 + 4 + 4 = 12
Array	

Students may first rely on counting by ones as a strategy for finding the sum in repeated-addition problems (Van de Walle, et al., 2014). In order to encourage students to move away from a "count all" strategy and develop the idea of a *composite whole*, teachers should withhold information or refrain from showing full diagrams. For example, a tile floor can be shown with a rug covering part of the rectangular tile array, thus requiring students to develop or rely on the idea that the covered rows of tile are equal to the visible rows of tile.

Students may naturally begin to use skip counting as a strategy, which provides another bridge to future multiplicative understanding. Students are usually better at skip counting by certain numbers like two, three, and five, and less proficient at skip counting by numbers such as seven or nine (Carpenter, Fennema, Franke, Levi, Empson, 2015). Additionally, some students only know the first few numbers in a skip counting sequence and rely on counting by ones to continue. The idea of a "unit" being more than one object is the essential idea of multiplication, and therefore repeated addition should provide an opportunity for a conversation about what a "unit" represents. This is a difficult idea for students to master, so time and care should be given to developing it. Mastering this idea prepares a student for success in multiplicative understanding, which is a pivotal concept in mathematics.

LEARNING MAP INFORMATION

The learning map section for this sequence of activities shows a progression from an understanding of addition and an ability to write addition equations to understanding and explaining repeated addition.

Students then transition to representing repeated addition with equations and models. Finally, students are able to use the equations and models to solve repeated-addition problems.

INSTRUCTIONAL ACTIVITIES

The activities in this unit are designed to build on students' existing knowledge of counting and addition. The first lesson will provide students the opportunity to work with concrete manipulatives in order to create arrays and write repeated-addition equations representing them. Students will focus on learning and using new vocabulary such as "addend" and "array" and will reinforce their previous knowledge of "sum", "row" and "column".

In both LESSON 1 and LESSON 2, students represent repeated addition with equations and arrays in the context of a real-world situation. LESSON 2 provides additional practice for students to draw semi-concrete representations of arrays based on a description of a real-world scenario, and to write a corresponding repeated-addition equation to find the total number of objects.

REFERENCES

Carpenter, T. P., Fennema, E., Franke, M. L., Levi, L., & Empson, S. B. (1999). *Children's mathematics: Cognitively guided instruction*. Heinemann, 361 Hanover Street, Portsmouth, NH 03801-3912.

Jones, J. C. (2012). *Visualizing elementary and middle school mathematics methods*. Hoboken, NJ: John Wiley & Sons.

Van de Walle, J., Lovin, L., Karp, K., Bay-Williams, J. (2014). *Teaching student-centered mathematics: Developmentally appropriate instruction for grades pre-k–2* (2nd ed.). Boston, MA: Pearson Education.

Using Repeated Addition to Solve Array Problems

OVERVIEW OF INSTRUCTIONAL ACTIVITIES

Lesson	Learning Goal	Nodes Addressed
Lesson 1	Students will develop a sense of repeated addition in order to calculate the total number of objects in a rectangular array.	▶ DEMONSTRATE THE CONCEPT OF ADDITION ▶ EXPLAIN REPEATED ADDITION ▶ REPRESENT REPEATED ADDITION WITH AN EQUATION ▶ REPRESENT REPEATED ADDITION WITH A MODEL ▶ SOLVE REPEATED-ADDITION PROBLEMS
Lesson 2	Students will practice representing repeated addition with arrays and equations.	▶ REPRESENT REPEATED ADDITION WITH AN EQUATION ▶ REPRESENT REPEATED ADDITION WITH A MODEL ▶ SOLVE REPEATED-ADDITION PROBLEMS

Using Repeated Addition to Solve Array Problems

INSTRUCTIONAL ACTIVITY

Lesson 1

LEARNING GOAL

Students will develop a sense of repeated addition in order to calculate the total number of objects in a rectangular array.

PRIMARY ACTIVITY

Students are introduced to groups of equal size, and then connect the concept of counting by ones to the idea of repeated addition. Students practice arranging counters in an array in order to make sense of the structure and connect the visual model to the repeated-addition equation.

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

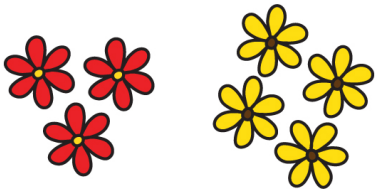
- Row
- Column
- Total
- Addend
- Sum
- Equation
- Equal sign

MATERIALS

- Counting manipulatives (square tiles, beans, etc.) (Recommend 25 per student.)
- Whiteboards
- Dry-erase markers
- Instructional Activity Student Handout (Recommend one teacher copy.)

IMPLEMENTATION

As a warm up, **show** students an image of three red flowers and four yellow flowers.



Distribute approximately 10 counters to each student.

Tell the students to use the counters to show the two groups of flowers. Students should model one group of three counters and one group of four counters. **Tell** students to find the total number of flowers. **Attend** to student strategies as they work.

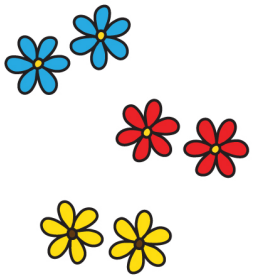
Ask, "How many total flowers are there? How do you know?"

Select two or three students to share how they determined the total. **Scaffold** the experience so that more basic strategies are shared first (i.e. counting by ones) and more abstract strategies are shared after. Students may respond that they counted by ones (1, 2, 3, 4, 5, 6, 7), knew from memory that 3 + 4 = 7, or that they considered 4 as 3 + 1, and therefore worked out 3 + 3 + 1 = 6 + 1 = 7.

Discuss the strategies for counting as a class so that all students consider strategies different from their own. **Ask** students which strategy seemed the most efficient. **Ask** which strategy seemed the most inefficient.

> NOTE: This may be a student's first introduction to the idea of efficient and inefficient strategies. The vocabulary may need to be explained or substituted as necessary for students to understand. It may also be appropriate to have a conversation about the benefits of efficient strategies.

Display another image for students to work with: two blue flowers, two red flowers, and two yellow flowers.



Tell students to use their counters to model the groups of flowers.

Call on two or three students to explain how they organized their counters. This will likely be in three groups of two. Students will often self-correct any mistakes when strategies and solutions are shared by their peers. After students have shown their three groups of two, **discuss** why the response "two groups of three" is incorrect (say this is an example from a "former student").

Ask, "How many flowers are there altogether?" Students should answer that there are six flowers.

Remind students how they found the total number of flowers in the previous example (3 + 4 = 7), and **ask** how that addition strategy connects to the new example.

Call on two to three students to explain how they found a total of six flowers. Students may respond that they counted by ones (1, 2, 3, 4, 5, 6), or that they added 2 + 2 + 2.

Tell students that the flowers must be equally spaced so that they each get the same amount of sunlight and don't overcrowd each other. **Ask** for a student volunteer to share with the class what is meant by "equally spaced". **Ensure** that all students know what is being asked before moving on.

Tell students to use their counters to brainstorm ways in which the flowers could be arranged to meet this requirement.

Call on two or three students to share how they organized their counters. Students may have created one equally spaced row of six counters, or they might have divided the rows into equal columns.

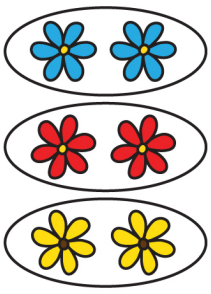
Discuss the different arrangements as a class to determine when one long row would be appropriate to use; for example, if you were planting along a long fence, and when it would be advantageous to split the arrangement into multiple equal rows, such as if you were trying to fit the plants in a specifically sized planter).

Say, "Let's put the flowers in three equal rows, one row for each color. Show me how this would look."

Explain that rows are objects arranged horizontally, and columns are objects arranged vertically. To remember this, students can think of rows of seats in a movie theater and columns that appear in front of buildings. Images of these examples can be shown to reinforce the new vocabulary.

After students have had time to create their own arrays, **model** the array on the board.

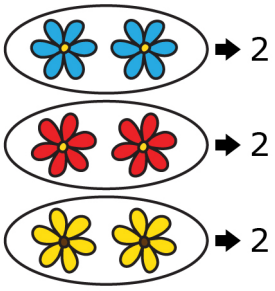
Circle the three rows of flowers.



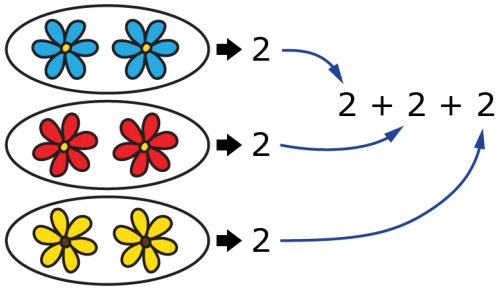
Point to the first row and **ask**, "How many flowers are in this row?" **Write** "2" next to the row.

Point to the second row and **ask**, "How many flowers are in this row?" **Write** "2" next to the row.

Point to the third row and **ask**, "How many flowers are in this row?" **Write** "2" next to the row.



Say, "Two blue flowers, plus two red flowers, plus two yellow flowers is written as the expression "2 + 2 + 2."



Say, "Each row represents a unit of flowers of the same color." **Ask**, "How many rows are there?" Students should answer that there are three rows. **Connect** the number of rows circled with the number of addends in the expression.

Add an equal sign to the right of the expression. It should read "2 + 2 + 2 =".

Ask students what the equal sign means. **Discuss** the meaning of the equal sign, and **define** "sum" for students.

Say, "There are three groups of two. How many total flowers is that?" Students should answer that there are six total flowers.

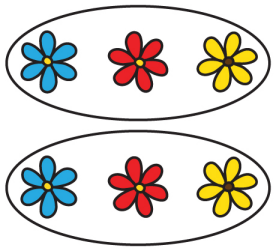
Complete the equation by writing "2 + 2 + 2 = 6".

Write "6 = 2 + 2 + 2" and **explain** that it is acceptable for the sum to be written to the left or the right of the addition sentence.

Ask, "Did the total change when we reorganized the flowers?" Students should answer "no".

Say, "Let's put the plants in three equal columns, one column for each color. Show me how this would look."

Model the arrangement for students. **Circle** the rows and **explain** that the rows of the array will always represent the number of groups.



Say, "Each row represents a unit of one flower of each color." **Ask**, "How many rows are there?"

Ask, "How many flowers are in each row?" **Label** each row with a "3".

Ask, "How many total flowers?" **Call on** a student to write the addition expression.

Explain that the sum could be written to the left or the right of the addition sentence.

Ask, "Did the total change when we rearranged the plants?" Students should answer that it did not.

GUIDING QUESTIONS

Elicit student thinking:

- Why did the total number of flowers stay the same when we rearranged the plants?
- What do you notice about the number of flowers in each color?

Determine if the student can DEMONSTRATE THE CONCEPT OF ADDITION:

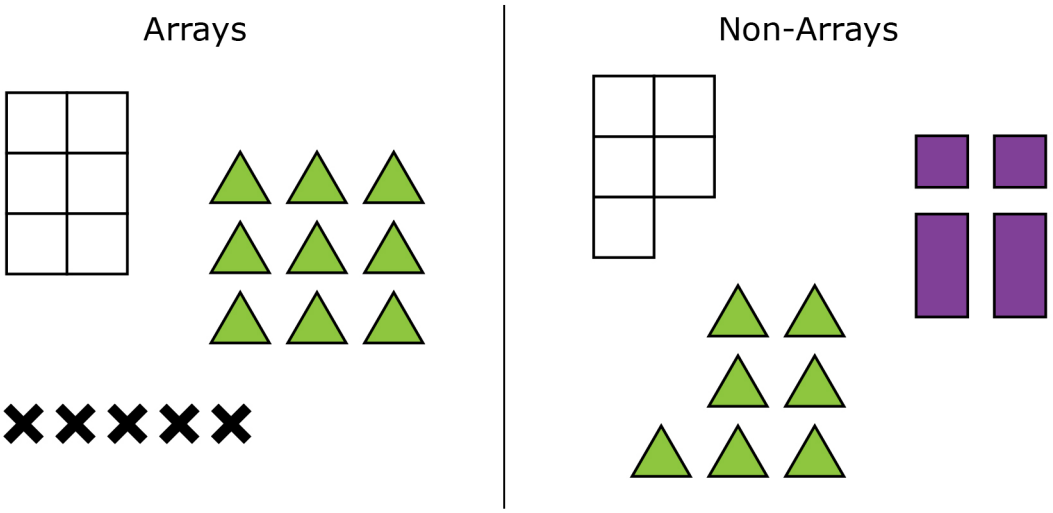
- What operation did you use to find the total number of flowers? Why?

Determine if the student is ready to EXPLAIN REPEATED ADDITION:

- What do you notice about the addends of our addition sentence?
- Why are the addends all the same?
- What do the equal rows tell you about the addends of the addition equation?
- What does one row represent?

Define the arrangements that students have been making as an *array*, which is a rectangular arrangement of objects (or shapes) in equal rows and columns.

Show a few examples and non-examples of arrays. Include area models in which the objects in the array are squares or square tiles that do not overlap and have no gaps between them.



Explain that arrays show copies of a group or unit.

Distribute the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT to students, and **model** how to complete the first problem.

Tell students to continue working on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT, and **circulate** as they work.

Students may first need to create the array with physical manipulatives before drawing the array on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

As appropriate for individual students, **encourage** the transition from using the physical manipulatives first to drawing the array first.

GUIDING QUESTIONS

Elicit student thinking:

- What do you know about addition?
- How did the array help you determine the equation?
- What can you tell me about an array?

Determine if the student can REPRESENT REPEATED ADDITION WITH AN EQUATION:

- How do you know how many addends there are?
- How do you know what the addend is?
- How did you find the sum?

Determine if the student can REPRESENT REPEATED ADDITION WITH A MODEL:

- What does the number of rows/columns represent?
- How do you know how many counters to put in each row/column?

Determine if the student is ready to SOLVE REPEATED-ADDITION PROBLEMS:

- [Point to an equation.] What is the sum of this equation?

Students should be required to complete an exit ticket in which they rearrange a group of 15 objects into an array with three rows and then into an array with three columns.

At the end of the activity, teachers should review the exit tickets for accuracy.

Name_____

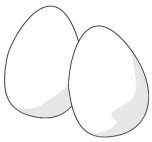
Using Repeated Addition to Solve Array Problems

Lesson 1

For each situation, draw an array, circle the rows, and write the corresponding addition equation.

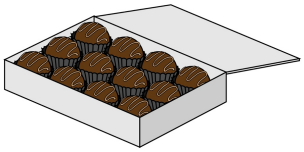
A muffin tin contains 3 rows of 4 muffins.

An egg carton contains 3 rows of 6 eggs.



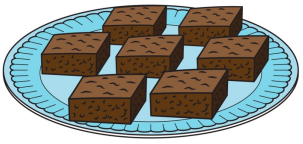
Name_____

A chocolate box contains four rows of three chocolates.



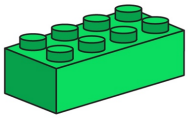
A pack of water bottles contains 5 rows of 3 water bottles.

A pan of brownies is cut into two rows of six brownies.

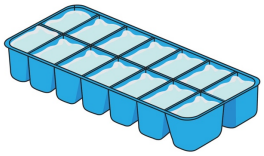


Name_____

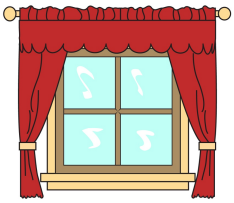
A plastic building brick has 4 rows of 2 pegs.



An ice tray has two rows of five ice cubes.



A window has 6 rows of 3 panes.



Name_____

A solar panel array has 3 rows of 5 panels.

Using Repeated Addition to Solve Array Problems

INSTRUCTIONAL ACTIVITY

Lesson 2

LEARNING GOAL

Students will practice representing repeated addition with arrays and equations.

PRIMARY ACTIVITY

Students practice converting written scenarios into arrays and equations during a stations activity. Students work individually or in pairs in order to complete the stations.

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

- Row
- Column
- Total
- Addend
- Sum

MATERIALS

- Counting manipulatives (square tiles, beans, etc.) (Recommend 25 per student.) *Note: Physical manipulatives should be made available for those students who need them.*
- Whiteboards (optional)
- Dry erase markers (optional)
- Instructional Activity Supplement (Recommend one copy.)
- Instructional Activity Student Handout

IMPLEMENTATION

Review the material covered in LESSON 1. Distribute students' exit tickets back to them, and call on a student to **model** how to solve the problem on the exit ticket.

> **Ask** students to draw an array representing 4 + 4 + 4 + 4 = 16 on scratch paper or whiteboards. **Display** the correct array for students to self-check their own arrays. **Remind** students that the number of groups is always represented by the number of rows. Therefore, the number of addends will be the same as the number of rows. Additionally, the number of columns (i.e. the amount in each row) will be represented by the addend.
>
> **Note** that in LESSON 1 students wrote equations from a given array, but it is also possible to draw an array that represents a given equation, as illustrated by the previous example.
>
> **Tell** students to complete at least one station by drawing the array first, and at least one station by writing the repeated-addition equation first.

As a review activity, have students individually or in pairs go to stations to read scenarios, draw an array, and write a repeated-addition equation representing the situation on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

GUIDING QUESTIONS

Elicit student thinking:

- Do you prefer to draw an array or write the equation first?

Determine if the student can REPRESENT REPEATED ADDITION WITH AN EQUATION:

- How does the array help you write a repeated-addition equation?
- How is the total number of objects represented in the equation?

Determine if the student is ready to REPRESENT REPEATED ADDITION WITH A MODEL:

- How does the repeated-addition equation help you draw an array?
- How is the sum represented in the array?
- If there are four objects in the first row of an array, how many objects will there be in all rows of the array?

Determine if the student is ready to SOLVE REPEATED-ADDITION PROBLEMS:

- [Point to the station 8 problem.] How can you keep all the brownies equally sized?
- [Point to the station 8 problem.] How did you decide how many rows and columns to create?

Students should be required to answer the questions at all the stations, and then to turn in their INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

At the end of the activity, teachers should require students to create an array on scrap paper, trade arrays with a partner, and write the repeated-addition equation representing their partner's array.

Name_____

Using Repeated Addition to Solve Array Problems

Lesson 2

Station 1

Array:

Equation:

Station 2

Array:

Equation: 4 + ☐ + ☐ = ☐

Name_____

Station 3

Array:

Equation:

Station 4

Array:

Equation:

Name_____

Station 5

Array:

Equation:

Station 6

Array:

Equation:

Name_____

Station 7

Array:

Station 8

Array:

Equation:

Using Repeated Addition to Solve Array Problems

INSTRUCTIONAL ACTIVITY SUPPLEMENT

Lesson 2

The following pages should be printed out for each station.

Station 1

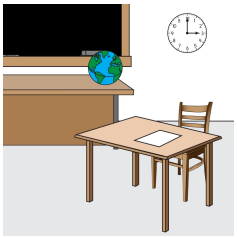
A marching band has three rows. Five people are in each row. Draw an array, and write an equation representing this situation.

Station 2

A muffin tin has three rows of four muffins. Draw an array, and write an equation representing this situation.

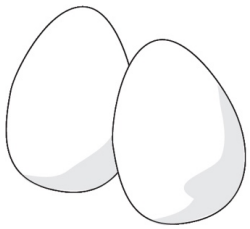
Station 3

A classroom has five rows of five desks. Draw an array, and write an equation representing this situation.



Station 4

An egg carton has two rows of six eggs. Draw an array, and write an equation representing this situation.



Station 5

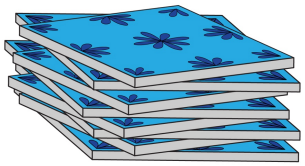
A basketball team is running drills. The players are in 3 lines, and there are 3 players in each line. Draw an array, and write an equation representing this situation.

Station 6

A bookshelf has four shelves, each containing five books. Draw an array, and write an equation representing this situation.

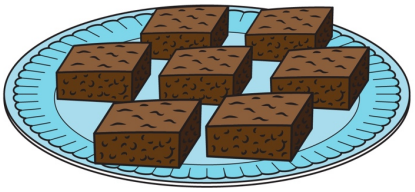
Station 7

A tile installer finds the total number of tiles he laid using the equation 3 + 3 + 3 + 3 + 3 = 15. Draw an array representing this situation.



Station 8

A baker just finished baking a pan of brownies and needs to cut it into twelve equally sized brownies. Show how to cut the brownies by drawing an array, and represent the situation with an equation.



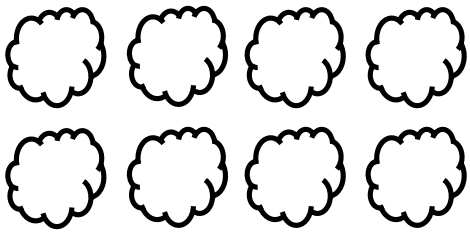
Name_____

Using Repeated Addition to Solve Array Problems

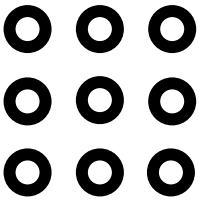
Lesson 1 – 2

1. Write a repeated-addition equation for each array.

1.a.



1.b.

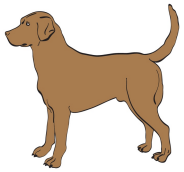


Name_____

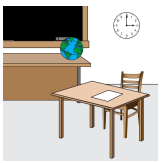
2. Write a repeated-addition equation for each problem.

2.a. Five students each make a stack of three pennies.

2.b. Evan, Ryan, and Lily each have two dogs.



3. Mr. Hill put his students' desks into three rows with four desks in each row.



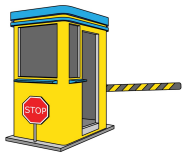
3.a. Draw an array to model this problem.

Name_____

3.b. Circle each **row** in the array, then write the number of desks next to each row to tell how many desks are circled.

3.c. Write the repeated-addition equation for this problem.

4. Four highway tollbooths have two cars in each line.

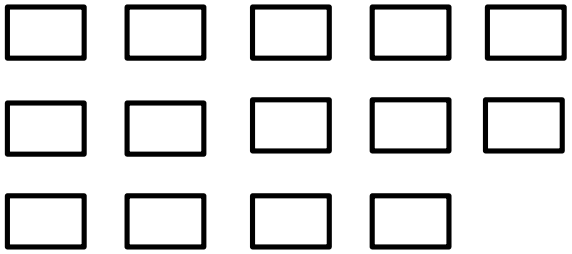


4.a. Draw an array to model this problem.

4.b. Write the repeated-addition equation for this problem.

Name_____

5. You have 14 business cards and set them out into rows, as shown below. Does this make an array? Explain why or why not.



6. The first row of an array is given.

6.a. Complete the array by drawing the next two rows.

_____

_____

Name_____

6.b. Fill in the blanks.

3 rows of _____ = 5 rows of _____

_____ + _____ + _____ = _____

_____ + _____ + _____ + _____ + _____ = _____

6.c. How do you know that the equations in 6.b. show repeated addition?

7. Use the picture to answer the questions.

7.a. Draw circles around groups of three.

Name_____

7.b. Draw the objects in an array with equal **rows** of three.

7.c. Write a repeated-addition equation to match the array.

8. Use the following expression to answer the questions.

3 + 3 + 3 + 3

8.a. How many addends are there?

8.b. What is the value of each addend?

Name_____

8.c. Draw an array for this expression.

8.d. Find the sum. Show your work.

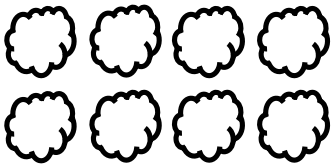
$3 + 3 + 3 + 3 =$ ☐

Using Repeated Addition to Solve Array Problems

STUDENT ACTIVITY SOLUTION GUIDE

Lesson 1 – 2

1. Write a repeated addition-equation for each array.

1.a. 

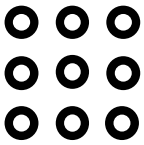
CORRECT ANSWER

2 + 2 + 2 + 2 = 8 *or* 4 + 4 = 8

[Note: As students prepare for multiplication understanding, they should begin to consider the rows as the number of groups. A follow-up with students who write 2 + 2 + 2 + 2 = 4 might be needed.]

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
There are 8 clouds.	finds the total without representing the situation with an equation	REPRESENT REPEATED ADDITION WITH AN EQUATION
1, 2, 3, 4, 5, 6, 7, 8	indicates a counting strategy instead of an addition strategy; cannot write an equation	EXPLAIN *REPEATED ADDITION*, REPRESENT REPEATED ADDITION WITH AN EQUATION
2 + 4 (= 6)	adds the dimensions of the array instead of each group of the array	EXPLAIN *REPEATED ADDITION*, REPRESENT REPEATED ADDITION WITH AN EQUATION
2 + 2 + 2 + 2 *or* 4 + 4	only writes the expression	REPRESENT REPEATED ADDITION WITH AN EQUATION

1.b. 

CORRECT ANSWER

3 + 3 + 3 = 9

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
There are 9 circles.	finds the total without representing the situation with an equation	REPRESENT REPEATED ADDITION WITH AN EQUATION
1, 2, 3, 4, 5, 6, 7, 8, 9	indicates a counting strategy instead of an addition strategy	EXPLAIN *REPEATED ADDITION*, REPRESENT REPEATED ADDITION WITH AN EQUATION
3 + 3 (= 6)	adds the dimensions of the array instead of each group of the array	EXPLAIN *REPEATED ADDITION*, REPRESENT REPEATED ADDITION WITH AN EQUATION
3 + 3 + 3	only writes the expression	REPRESENT REPEATED ADDITION WITH AN EQUATION

2. Write a repeated-addition equation for each problem.

2.a. Five students each make a stack of three pennies. 

CORRECT ANSWER

3 + 3 + 3 + 3 + 3 = 15

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
5 + 3 = 8	adds the two numbers given in the scenario	EXPLAIN *REPEATED ADDITION*, REPRESENT REPEATED ADDITION WITH AN EQUATION
5 + 5 + 5 = 15	switches the number of addends and the value of each addend	REPRESENT REPEATED ADDITION WITH AN EQUATION
3 + 3 + 3 + 3 + 3	only writes the expression	REPRESENT REPEATED ADDITION WITH AN EQUATION
15	only writes the total	REPRESENT REPEATED ADDITION WITH AN EQUATION
3 pennies	gives the total in one stack; does not correctly interpret what "each" means	EXPLAIN *REPEATED ADDITION*
3 + 3 + 3 + 3 (+ 3) = 12 (or 18)	miscounts the number of addends (only represents 4 instead of 5), or writes the correct number of addends but adds incorrectly	SOLVE REPEATED-ADDITION PROBLEMS

2.b. Evan, Ryan, and Lily each have two dogs. 

CORRECT ANSWER

2 + 2 + 2 = 6

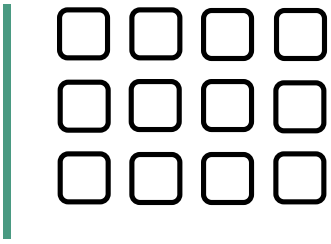
ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
3 + 2 = 5	adds the two numbers given in the scenario	EXPLAIN *REPEATED ADDITION*, REPRESENT REPEATED ADDITION WITH AN EQUATION
3 + 3 = 6	switches the number of addends and the value of each addend	REPRESENT REPEATED ADDITION WITH AN EQUATION
2 + 2 + 2	only writes the expression	REPRESENT REPEATED ADDITION WITH AN EQUATION
6	only writes the sum	REPRESENT REPEATED ADDITION WITH AN EQUATION
2 dogs	gives the number of dogs one person has, does not correctly interpret what "each" means	EXPLAIN *REPEATED ADDITION*
2 + 2 + 2 = 4 *(or other incorrect value)*	writes the correct addition but does not evaluate it correctly	SOLVE REPEATED-ADDITION PROBLEMS

3. Mr. Hill put his students' desks into three rows with four desks in each row.

3.a. Draw an array to model this problem.

CORRECT ANSWER

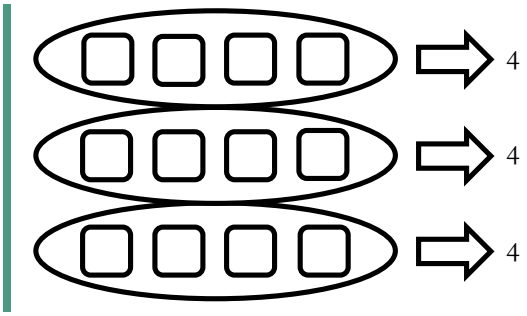


ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Student draws an array with four rows of three desks.	confuses the rows and columns	REPRESENT REPEATED ADDITION WITH A MODEL
Student draws a group of three desks and a group of four desks.	only gives attention to the dimensions of the array	REPRESENT REPEATED ADDITION WITH A MODEL

3.b. Circle each **row** in the array, then write the number of desks next to each row to tell how many desks are circled.

CORRECT ANSWER



ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Student circles each column.	cannot distinguish between rows and columns	EXPLAIN *REPEATED ADDITION*, REPRESENT REPEATED ADDITION WITH A MODEL
Student circles each row but incorrectly labels each row with "3".	confuses the number of elements in each row with the number of rows	EXPLAIN *REPEATED ADDITION*

3.c. Write the repeated-addition equation for this problem.

CORRECT ANSWER

4 + 4 + 4 = 12

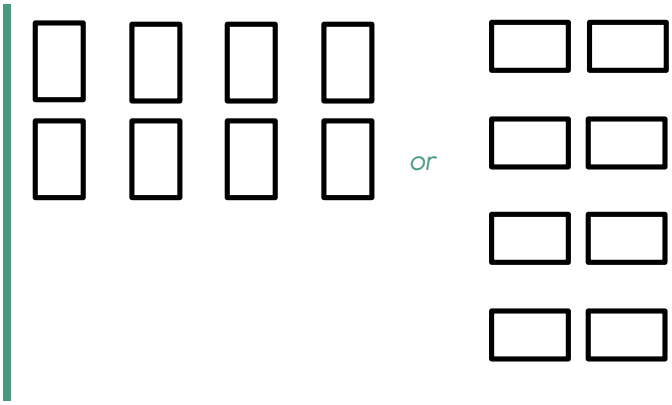
ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
3 + 3 + 3 + 3 = 12	switches the number of addends and the value of each addend	REPRESENT REPEATED ADDITION WITH AN EQUATION
4 + 4 + 4	only writes the expression	REPRESENT REPEATED ADDITION WITH AN EQUATION
12	only writes the total	REPRESENT REPEATED ADDITION WITH AN EQUATION
3 + 4 = 7	adds the dimensions of the array	REPRESENT REPEATED ADDITION WITH AN EQUATION, EXPLAIN *REPEATED ADDITION*

4. Four highway tollbooths have two cars in each line.

   4.a. Draw an array to model this problem. 

CORRECT ANSWER



ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Student draws two cars.	gives the number of cars in one line, does not correctly interpret what "each" means	REPRESENT REPEATED ADDITION WITH A MODEL
Student draws four cars.	sees the number four in the problem and draws that many cars	REPRESENT REPEATED ADDITION WITH A MODEL
Student writes an equation.	does not know how to model repeated addition	REPRESENT REPEATED ADDITION WITH A MODEL
8	gives the total without representing the situation with a model	REPRESENT REPEATED ADDITION WITH A MODEL
Student draws two groups of four cars in repeated grouping, or eight cars with no distinguishable pattern.	does not know how to draw objects in an array	REPRESENT REPEATED ADDITION WITH A MODEL

4.b. Write the repeated-addition equation for this problem.

CORRECT ANSWER

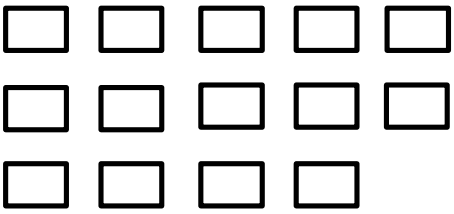
2 + 2 + 2 + 2 = 8 *or* 4 + 4 = 8

[Note: Depending on which array the student drew, either of these equations might show the number of rows representing the number of addends. It is suggested to follow up with students who write repeated-addition sentences based on the number of columns, as this will need to be corrected as they develop multiplication understanding.]

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
2 + 4 (= 6)	adds the dimensions of the array, or adds the two numbers shown in the problem	REPRESENT REPEATED ADDITION WITH AN EQUATION
2 + 2 + 2 + 2	does not know that an addition equation needs a sum	REPRESENT REPEATED ADDITION WITH AN EQUATION
2 + 2 + 2 + 2 = 7 *or* 9	counts individually and "gets off" by one	REPRESENT REPEATED ADDITION WITH AN EQUATION, SOLVE REPEATED-ADDITION PROBLEMS
2 + 2 + 2 + 2	only writes the expression	REPRESENT REPEATED ADDITION WITH AN EQUATION
8	only writes the total	REPRESENT REPEATED ADDITION WITH AN EQUATION
2 + 2 + 2 + 2 = 4	only adds the first two addends	SOLVE REPEATED-ADDITION PROBLEMS

5. You have 14 business cards and set them out into rows, as shown below. Does this make an array? Explain why or why not.



CORRECT ANSWER

No, this does not make an array, because the rows and columns must be equal, and the last column is missing one business card.

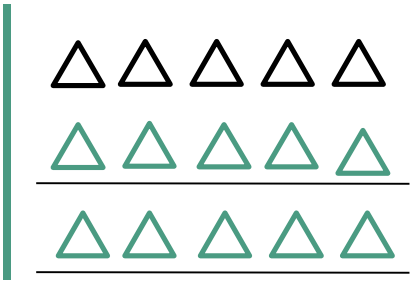
ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Yes.	does not explain their reasoning	REPRESENT REPEATED ADDITION WITH A MODEL
No.	does not explain their reasoning	REPRESENT REPEATED ADDITION WITH A MODEL
Yes, this makes an array because there are rows and columns.	believes that any arrangement of rows is an array	REPRESENT REPEATED ADDITION WITH A MODEL
Yes, because you can just get rid of two business cards.	does not evaluate the shown diagram as necessary to decide whether or not it is an array	REPRESENT REPEATED ADDITION WITH A MODEL

6. The first row of an array is given.

6.a. Complete the array by drawing the next two rows.

CORRECT ANSWER



ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Student only draws one additional row.	does not follow all directions or realize that there are two additional rows needed	REPRESENT REPEATED ADDITION WITH A MODEL
Student draws additional triangles in the first row.	does not understand that the first row represents a set number of triangles and no triangles need to be added there	REPRESENT REPEATED ADDITION WITH A MODEL
Student draws rows with a different number of triangles than the first row.	does not show understanding that the rows and columns in an array must be equally sized	REPRESENT REPEATED ADDITION WITH A MODEL

6.b. Fill in the blanks.

3 rows of _____ = 5 rows of _____

_____ + _____ + _____ = _____

_____ + _____ + _____ + _____ + _____ = _____

CORRECT ANSWER

3 rows of __5__ = 5 rows of __3__

__5__ + __5__ + __5__ = __15__

__3__ + __3__ + __3__ + __3__ + __3__ = __15__

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
3 + 3 + 3 = 15 *and/or* 5 + 5 + 5 + 5 + 5 = 15	knows the sum but does not relate the number of addends to the number of corresponding rows/columns	REPRESENT REPEATED ADDITION WITH AN EQUATION
Student gives a sum of 14 or 16.	counts individually and "gets off" by one	SOLVE REPEATED-ADDITION PROBLEMS
Student gives a sum of 10 or 6.	only adds the first two addends	SOLVE REPEATED-ADDITION PROBLEMS

6.c. How do you know that the equations in 6.b.show repeated addition?

CORRECT ANSWER

This scenario represents repeated addition because the addend is the same, meaning the same number is being added to itself. Additionally, three or more groups of the same size were combined.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

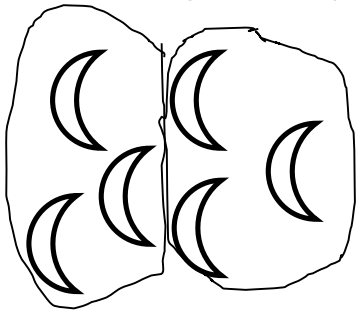
Example Error	Misconception	Missing Knowledge
…because there is a "+".	only associates addition with a symbol	EXPLAIN *REPEATED ADDITION*
…because we combined groups.	understands addition but does not show understanding of *repeated* addition	EXPLAIN *REPEATED ADDITION*

7. Use the picture to answer the questions.

7.a. Draw circles around groups of three.

CORRECT ANSWER

Check student work for understanding and ensure that the student has circled groups of three. The following is an example of a possible response:

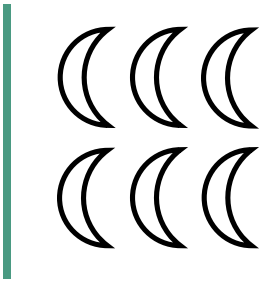


ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Student draws three circles instead of circling groups of three.	confuses the number of groups with the size of each group	COMPOSE NUMBERS UP TO 5
Student circles each object.	reads the command "circle" but cannot correctly interpret the following directions	COMPOSE NUMBERS UP TO 5

7.b. Draw the objects in an array with equal **rows** of three.

CORRECT ANSWER



ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Student draws columns of three.	confuses rows and columns, or does not attend to the directions	REPRESENT REPEATED ADDITION WITH A MODEL
	does not indicate knowledge of what an array is	REPRESENT REPEATED ADDITION WITH A MODEL

7.c. Write a repeated-addition equation to match the array.

CORRECT ANSWER

3 + 3 = 6

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
3 + 2 = 5	adds the dimensions of the array	REPRESENT REPEATED ADDITION WITH AN EQUATION
3 + 3	only writes the expression	REPRESENT REPEATED ADDITION WITH AN EQUATION

8. Use the following expression to answer the questions.

3 + 3 + 3 + 3

8.a. How many addends are there?

CORRECT ANSWER

There are four addends.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
There are three addends.	does not know what an addend is and therefore gives the value of the addend instead of the number of addends	EXPLAIN *ADDEND*
There are 12 addends.	does not know what an addend is and therefore gives the sum instead of the number of addends	EXPLAIN *ADDEND*

8.b. What is the value of each addend?

CORRECT ANSWER

The value of each addend is three.

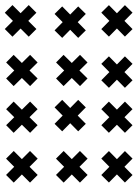
ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The value of each addend is four.	confuses the number of addends with the value of the addend	EXPLAIN *ADDEND*
The value of each addend is 12.	does not know the definition of addend and therefore evaluates the expression to find an answer	EXPLAIN *ADDEND*

8.c. Draw an array for this expression.

CORRECT ANSWER

Check student work for understanding and ensure that the student has created an array with either four columns of three or four rows of three. The following is an example of a possible response:



ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Student creates an array with three rows of four or three columns of four.	interprets the expression to be four added three times instead of three added four times	REPRESENT REPEATED ADDITION WITH A MODEL
Student creates four groups of three that are not arranged in an array.	cannot create an array as requested in the problem	REPRESENT REPEATED ADDITION WITH A MODEL

8.d. Find the sum. Show your work.

3 + 3 + 3 + 3 = ☐

CORRECT ANSWER

3 + 3 + 3 + 3 = **12**

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
= 11 *or* = 13	counted to find the sum and "got off"	SOLVE REPEATED-ADDITION PROBLEMS
= 6 *or* = 9	only added the first two or three addends	SOLVE REPEATED-ADDITION PROBLEMS