VARIABLE EXPRESSIONS AND ORDER OF OPERATIONS

6.EE.2.a and 6.EE.2.c

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CONTENTS

The types of documents contained in the unit are listed below. Throughout the unit, the documents are arranged by lesson.

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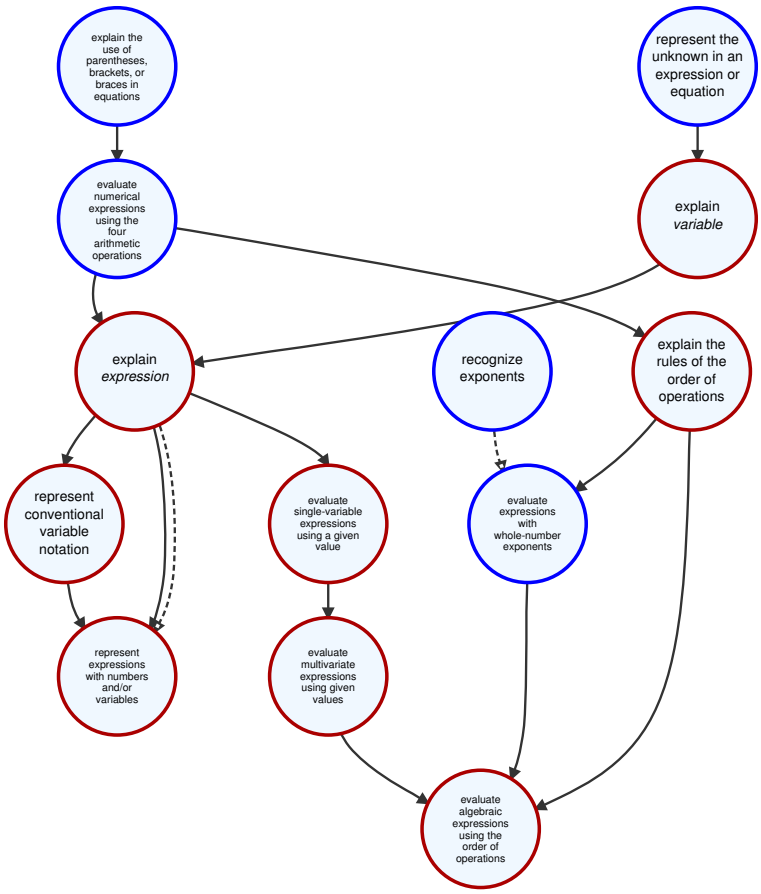
VARIABLE EXPRESSIONS AND ORDER OF OPERATIONS

LEARNING MAP INFORMATION

STANDARDS

6.EE.2.a Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation "Subtract* y *from 5" as 5 –* y.

6.EE.2.c Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). *For example, use the formulas* $V = s^3$ *and* $A = 6s^2$ *to find the volume and surface area of a cube with sides of length* $s = \frac{1}{2}$.



**See saved map* 6.EE.2.a and 6.EE.2.c *in the learning map software.*

Node Name	Node Description
EVALUATE ALGEBRAIC EXPRESSIONS USING THE ORDER OF OPERATIONS	Given an expression with multiple operations and with variables and given values for all variables, demonstrate the order of operations to find the value of the expression.
EVALUATE SINGLE-VARIABLE EXPRESSIONS USING A GIVEN VALUE	Given a specific value for the variable, find the total value of the expression with one variable. Use both mathematical expressions and real-world formulas (e.g., the formula for the area of a square, $A = s^2$).
EVALUATE EXPRESSIONS WITH WHOLE-NUMBER EXPONENTS	Given an expression with a whole-number exponent, find the value of the expression.
EVALUATE MULTIVARIATE EXPRESSIONS USING GIVEN VALUES	Given specific values for the variables, find the total value of an expression with more than one variable, including both general mathematical expressions and real-world formulas (e.g., the formula for the area of a rectangle, $A = lw$).
EVALUATE NUMERICAL EXPRESSIONS USING THE FOUR ARITHMETIC OPERATIONS	Given an expression involving multiple operations and grouping symbols (e.g., parentheses) but without variables, apply the order of operations to find the value of the expression.
EXPLAIN *EXPRESSION*	Make known your understanding through words, drawings, manipulatives, etc., that an expression is a mathematical phrase that includes numbers and/or variables that may include operations but no equal sign (e.g., $x + 5$ or y).
EXPLAIN THE RULES OF THE ORDER OF OPERATIONS	Make known your understanding through words, drawings, manipulatives, etc., that the rules for order of operations are that you complete operations in the following order: parentheses/brackets/braces, exponents, multiplication/division, and addition/subtraction.
EXPLAIN THE USE OF PARENTHESES, BRACKETS, OR BRACES IN EQUATIONS	Make known your understanding through words, drawings, manipulatives, etc., that parentheses, brackets, and braces are used in mathematic expressions to indicate which operations should be done first. Parentheses group operations together; brackets group operations and parentheses together when parentheses are used; and braces group operations, brackets, and parentheses together when brackets are used.
EXPLAIN *VARIABLE*	Make known your understanding through words, drawings, manipulatives, etc., that a variable is a symbol that can stand for an unknown quantity or a varying quantity.
RECOGNIZE EXPONENTS	Identify or describe an exponent as a real number that indicates how many times a number (i.e., the base) is to be multiplied by itself.
REPRESENT CONVENTIONAL VARIABLE NOTATION	Through writing or an appropriate assistive technology, represent conventional variable notation (e.g., $3p$ instead of $3 \times p$ and $7/y$ instead of $7 \div y$).
REPRESENT EXPRESSIONS WITH NUMBERS AND/OR VARIABLES	Through writing or an appropriate assistive technology, represent expressions using the four basic operations, exponents, and grouping symbols with numbers and/or variables (e.g., express "subtract k from 12" as $12 - k$ or "11 squared plus 4" as $11^2 + 4$).
REPRESENT THE UNKNOWN IN AN EXPRESSION OR EQUATION	Through writing or an appropriate assistive technology, represent the unknown in an expression or equation using a symbol or letter.

VARIABLE EXPRESSIONS AND ORDER OF OPERATIONS

TEACHER NOTES

This unit includes the following documents:

- Learning Map Information
- Instructional Activity (includes three lessons)
- Instructional Activity Student Handout (for Lesson 2 and Lesson 3)
- Instructional Activity Supplement (for optional use)
- Student Activity Set (one activity per lesson)
- Student Activity Solution Guide

In this unit, students will draw on their ability to simplify numerical expressions following the order of operations and will learn to simplify and to evaluate algebraic expressions with multiple variables using the order of operations. Once students have evaluated expressions with variables, they will be introduced to real-world formulas. As students proceed through the activities, students should progress from simplifying expressions with numbers and operations to evaluating expressions containing variables to representing real-world formulas involving variables from a description. The main focus should be on proper simplification and representation or substitution of variables.

> NOTE: This unit does not specifically address 6.EE.2.b, which focuses on parts of an expression and the associated vocabulary, as well as various understandings of quantities within an expression. The latter can be addressed as students explain the distributive property.

The learning map section for this sequence of activities reflects students drawing on their knowledge of the order of operations with the four basic operations and their understanding of variables to work with algebraic expressions containing variables and exponents. Prior to the introduction of variables and exponents, students should demonstrate they can fluently execute the order of operations with basic expressions. The addition of exponents and variables creates situations in which students must pay close attention to detail while forming connections among their understanding of variables, of exponents, and of the order of operations. A thorough ability to explain variables and their meaning in an expression is beneficial to moving forward in this learning progression. Too often, incomplete understandings of variables create misconceptions that follow students into high school coursework. Emphasizing that variables are letters representing varying quantities (rather than abbreviations, labels, or specific values) in early grades and requiring students to explain their understanding of variables prior to moving forward are critical to students' success with subsequent concepts. Detailed and thorough questioning can help identify and correct misconceptions early (Asquith, Stephens, Knuth, & Alibali, 2007).

Additionally, attention to precision is a critical practice as students evaluate an expression following the order of operations. It is highly likely that a student would be able to explain the order of operations correctly but would make a mistake when performing the order of operations in a complicated expression. Therefore, it is important to require students to explain the steps they should take and their reasoning for those steps in

order to determine whether an inability to correctly evaluate an expression is due to a lack of understanding of the process or an inability to attend to precision.

AN EXAMPLE

The difference between students who are unable to explain the order of operations versus students struggling to attend to precision can be illustrated with the expression below.

$$2 \cdot 5^2 + 7 \cdot 3$$

A student struggling to explain the order of operations would not be able to indicate the correct order of operations. They may indicate that the problem would be simplified from left to right or that it doesn't matter in which order the operations are simplified or may give an inaccurate order.

These misunderstandings can be shown in student work in a number of ways. One example is

$$\begin{aligned} &2 \cdot 5^2 + 7 \cdot 3 \\ &10^2 + 7 \cdot 3 \quad \text{(incorrectly multiplied before applying the exponent)} \\ &100 + 7 \cdot 3 \\ &107 \cdot 3 \quad \text{(incorrectly added before multiplying)} \\ &321 \end{aligned}$$

A student struggling to attend to precision would correctly indicate the exponent should be simplified first, then multiplication, then addition.

However, when asked to simplify, this student may make the following error:

$$\begin{aligned} &2 \cdot 5^2 + 7 \cdot 3 \\ &10^2 + 7 \cdot 3 \quad \text{(incorrectly multiplied before applying the exponent)} \\ &100 + 7 \cdot 3 \\ &100 + 21 \\ &121 \end{aligned}$$

After students can explain variables and the order of operations, they should proceed to evaluate algebraic expressions by substituting particular values for each variable and simplifying the result. Students should work with expressions involving one or more variables or should create expressions given a description of a relationship among quantities. Descriptions and expressions should be representative of both mathematical and real-world problems.

The activities in this unit are designed to provide consistency as students move from expressions without variables into expressions containing multiple variables. The first lesson reviews and combines prior knowledge of the order of operations and of exponents, while the second lesson involves elements of critical thinking, planning, logical reasoning, and competition, in addition to the skills specified in this learning progression. The final lesson is intended to give students an introduction to representations with variables

involving real-world situations that may be more familiar and interesting to them. The descriptions lead students through the construction of a formula a step at a time and should promote dialogue among students, teacher-student interactions, and questions.

REFERENCES

Asquith, P., Stephens, A. C., Knuth, E. J., & Alibali, M. W. (2007). Middle school mathematics teachers' knowledge of students' understanding of core algebraic concepts: Equal sign and variable. *Mathematical Thinking and Learning, 9*, 249–272. doi:10.1080/10986060701360910

VARIABLE EXPRESSIONS AND ORDER OF OPERATIONS

INSTRUCTIONAL ACTIVITY

Lesson 1

LEARNING GOAL

Students will simplify expressions involving the four basic operations and exponents using the order of operations. The critical outcome of this lesson is for students to accurately simplify expressions with exponents using the order of operations.

> NOTE: Your students may have previous experience simplifying exponents in expressions requiring attention to the order of operations. If so, please continue to LESSON 2.

PRIMARY ACTIVITY

Students will roll number cubes to determine values to substitute into expressions and then will simplify those expressions according to the order of operations.

OTHER VOCABULARY

Students will need to know the meaning of

- Expression
- Order of Operations

MATERIALS

- Whiteboards and markers or blank paper

IMPLEMENTATION

This activity introduces the idea that values in expressions or equations do not have to be given fixed values, but rather can vary (using a variable as in LESSON 2).

Review the order of operations with the students, and introduce questions requiring them to consider the order of operations in expressions involving exponents. Students should have seen these concepts separately but may need to work on simplifying exponents within a problem that requires carefully following the order of operations.

Write out an expression that includes exponents. After providing the students with the expression, ask them (a) to describe where in the expression they would begin evaluating and (b) to justify their reasoning using the order of operations (e.g., in the absence of grouping symbols, begin with the exponent).

GUIDING QUESTIONS

Determine if the student can EXPLAIN THE RULES OF THE ORDER OF OPERATIONS:

- What should you simplify first in an expression? Why?
- Once you have simplified grouping symbols and exponents, should you multiply or divide first? Why?
  (Students should indicate to move from left to right to simplify multiplication and division.)
- Once you have simplified grouping symbols, exponents, multiplication, and division, should you add or subtract first? Why?
  (Students should indicate to move from left to right to simplify addition and subtraction.)

Practice. Once students have discussed where to begin, provide them with the following two expressions:

$$4 \cdot 3^2 - 13 + 5$$

$$(10 - 3 \cdot 2)^2 + \frac{6}{2 \cdot 3}$$

Require students to work in pairs or groups of three to simplify the expressions on whiteboards or paper. Here are some examples with correct responses and incorrect responses reflecting common misconceptions.

1) $4 \cdot 3^2 - 13 + 5$

CORRECT ANSWER

28

COMMON MISTAKES AND MISCONCEPTIONS

Example Error	Misconception
136	The student multiplies before evaluating the exponent *(the student does not follow the order of operations, does not understand conventional variable notation, or does not understand when grouping symbols are implied or present).*
18	The student adds 13 and 5 before subtracting *(the student does not understand that addition and subtraction should occur from left to right).*
126	The student multiplies before evaluating the exponent **and** adds 13 and 5 before subtracting.
16	The student thinks 3^2 is 6 *(the student may be unable to recognize exponents).*

2) $(10 - 3 \cdot 2)^2 + \frac{6}{2 \cdot 3}$

CORRECT ANSWER

17

COMMON MISTAKES AND MISCONCEPTIONS

Example Error	Misconception
25	The student divides 6 by 2 and then multiplies by 3 *(the student does not understand when grouping symbols are implied or present).*
–1	The student evaluates the exponent before the grouping symbols, *(the student does not simplify according to the order of operations).*
197	The student does not follow the order of operations within parentheses *(the student does not know or remember to multiply prior to subtraction).*

Present the equation below and ask students to individually determine the number that should replace the box in order to obtain the desired result as an exit ticket for the day.

$$\square \cdot 5^2 + 4(7-1) = 74$$

A student who understands the topic will put a 2 in the box.

Name_____

Variable Expressions and Order of Operations

Lesson 1

1. Use the following expression to answer questions (a) through (c):

$$6 + 2 \cdot 3^2 - 5 + 10$$

1.a. Which operation should occur first? Explain your reasoning.

1.b. In what order should addition and subtraction occur in this expression? Explain your reasoning.

1.c. Simplify the expression, showing each step.

2. Use the following expression to answer questions (a) through (c):

$$6 \cdot 2^3 - 4 \cdot 7$$

2.a. Which operations should occur first? Explain your reasoning.

Name_____

2.b. In what order should multiplication and subtraction occur in this expression? Explain your reasoning.

2.c. Simplify the expression, showing each step.

3. Use the following expression to answer questions (a) through (c):

$$12 \div (2 \cdot 3) - 1 + 8$$

3.a. Which operation should occur first? Explain your reasoning.

3.b. In what order should addition and subtraction occur in this expression? Explain your reasoning.

3.c. Simplify the expression, showing each step.

VARIABLE EXPRESSIONS AND ORDER OF OPERATIONS

STUDENT ACTIVITY SOLUTION GUIDE

Lesson 1

1. Use the following expression to answer questions (a) through (c):

$$6 + 2 \cdot 3^2 - 5 + 10$$

1.a. Which operation should occur first? Explain your reasoning.

CORRECT ANSWER

3^2 should be simplified first. Because there aren't any grouping symbols, the exponent should be evaluated first (before multiplication and division or addition and subtraction).

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Addition, because it is first in the expression.	evaluates expressions from left to right	EXPLAIN THE RULES OF THE ORDER OF OPERATIONS

1.b. In what order should addition and subtraction occur in this expression? Explain your reasoning.

CORRECT ANSWER

Addition and subtraction should occur from left to right. Therefore, once the exponent and multiplication have been evaluated, the operations would be addition, subtraction, and then addition.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Addition, because addition has to come before subtraction.	thinks addition always occurs before subtraction, regardless of order in the expression	EXPLAIN THE RULES OF THE ORDER OF OPERATIONS

1.c. Simplify the expression, showing each step.

CORRECT ANSWER

$6 + 2(9) - 5 + 10$
$6 + 18 - 5 + 10$
$24 - 5 + 10$
$19 + 10$
29

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
3^2 is not simplified to 9	does not evaluate exponent accurately	EVALUATE EXPRESSIONS WITH WHOLE-NUMBER EXPONENTS
3^2 not simplified first	does not evaluate the exponent first	EXPLAIN THE RULES OF THE ORDER OF OPERATIONS
$2(9)$ not simplified before +/–.	evaluates operations out of order	EVALUATE NUMERICAL EXPRESSIONS USING THE FOUR ARITHMETIC OPERATIONS
Note: If a student is able to explain the steps in the order of operations but does not perform them properly, they may know the order of operations but struggle with computation or attention to precision in their work.		

2. Use the following expression to answer questions (a) through (c):

$$6 \cdot 2^3 - 4 \cdot 7$$

2.a. Which operation should occur first? Explain your reasoning.

CORRECT ANSWER

2^3 should occur first. Because there aren't any grouping symbols, the exponent should be evaluated first (before multiplication and division or addition and subtraction).

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Multiplication, because it is first in the expression.	evaluates expressions from left to right	EXPLAIN THE RULES OF THE ORDER OF OPERATIONS

2.b. In what order should multiplication and subtraction occur in this expression? Explain your reasoning.

CORRECT ANSWER

Multiplication should occur before subtraction based on the order of operations. Therefore, once the exponent and has been evaluated, the operations would be multiplication, multiplication, and then subtraction.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Multiplication, then subtraction, then multiplication because that is the order the operations appear in.	evaluates expressions from left to right	EXPLAIN THE RULES OF THE ORDER OF OPERATIONS

2.c. Simplify the expression, showing each step.

CORRECT ANSWER

$6(8) - 4(7)$
$48 - 28$
20

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
2^3 is not simplified to 8.	does not evaluate exponent accurately	EVALUATE EXPRESSIONS WITH WHOLE-NUMBER EXPONENTS
2^3 is not simplified first.	does not evaluate exponent first	EXPLAIN THE RULES OF THE ORDER OF OPERATIONS
6(8) and 4(7) are not simplified before −.	evaluates operations out of order	EVALUATE NUMERICAL EXPRESSIONS USING THE FOUR ARITHMETIC OPERATIONS
Note: If a student is able to explain the steps in the order of operations but does not perform them properly, they may know the order of operations but struggle with computation or attention to precision in their work.		

3. Use the following expression to answer questions (a) through (c):

$$12 \div (2 \cdot 3) - 1 + 8$$

3.a. Which operation should occur first? Explain your reasoning.

CORRECT ANSWER

> Multiplication of 2 and 3 should occur first because it is inside a set of grouping symbols (parentheses).

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Division because it is first in the expression.	evaluates expressions from left to right	EXPLAIN THE RULES OF THE ORDER OF OPERATIONS

3.b. In what order should addition and subtraction occur in this expression? Explain your reasoning.

CORRECT ANSWER

Once multiplication and division have been completed, subtraction should occur before addition because the order of operations tells us to add and subtract from left to right.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Addition because addition has to come before subtraction.	thinks addition occurs before subtraction, regardless of order in the expression	EXPLAIN THE RULES OF THE ORDER OF OPERATIONS

3.c. Simplify the expression, showing each step.

CORRECT ANSWER

$12 \div 6 - 1 + 8$
$2 - 1 + 8$
$1 + 8$
9

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
$2 \cdot 3$ not simplified first.	does not evaluate inside parentheses first	EVALUATE NUMERICAL EXPRESSIONS USING THE FOUR ARITHMETIC OPERATIONS
The student evaluates addition $(1 + 8)$ before subtraction $(2 - 1)$.	evaluates operations out of order	EVALUATE NUMERICAL EXPRESSIONS USING THE FOUR ARITHMETIC OPERATIONS
Note: If a student is able to explain the steps in the order of operations but does not perform them properly, they may know the order of operations but struggle with computation or attention to precision in their work.		

VARIABLE EXPRESSIONS AND ORDER OF OPERATIONS

INSTRUCTIONAL ACTIVITY

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Lesson 2

LEARNING GOAL

Students will simplify algebraic expressions involving the four basic operations, exponents, and grouping symbols using the order of operations. The critical outcome of this activity is for students to accurately evaluate an expression given specified values for the variables using the order of operations.

PRIMARY ACTIVITY

Students will roll number cubes to determine values to substitute into expressions and then will simplify those expressions according to the order of operations.

OTHER VOCABULARY

Students will need to know the meaning of

- Variable
- Expression
- Evaluate
- Order of Operations
- Substitution

MATERIALS

- Whiteboards and markers or blank paper
- six-sided number cube (standard number cube or custom number cube from the INSTRUCTIONAL ACTIVITY SUPPLEMENT) (recommend one copy for every group of 2 – 3 students)
- INSTRUCTIONAL ACTIVITY STUDENT HANDOUT

IMPLEMENTATION

Review the order of operations with the students, and include questions that require them to consider the order of operations in expressions involving exponents.

Ask them (a) to describe where in the expression they would begin evaluating and (b) to justify their reasoning using the order of operations (e.g., in the absence of grouping symbols, begin with the exponent).

GUIDING QUESTIONS

Elicit student thinking:

- Why do we need rules for evaluating expressions?
- Can you create an expression that would result in different values with and without the order of operations?

Determine if the student can EXPLAIN THE RULES OF THE ORDER OF OPERATIONS:

- What should you simplify first in an expression?
- Once you have simplified grouping symbols and terms with exponents, should you multiply or divide first?
  (Students should indicate to move from left to right to simplify multiplication and division.)
- Once you have simplified grouping symbols, terms with exponents, multiplication, and division, should you add or subtract first?
  (Students should indicate to move from left to right to simplify addition and subtraction.)

Practice. Once students have discussed where to begin, require them to work the problem on whiteboards or paper and to provide a response. Here are some examples with correct responses and incorrect responses reflecting common misconceptions.

1) $2 \cdot 5^2 - 10 + 8$

CORRECT ANSWER

48

COMMON MISTAKES AND MISCONCEPTIONS

Example Error	Misconception
98	The student multiplies first then evaluates the exponent *(the student does not follow the order of operations, does not understand conventional variable notation, or does not understand when grouping symbols are implied/present).*
32	The student adds 10 and 8 before subtracting *(the student does not understand that addition and subtraction should occur from left to right).*
82	The student multiplies first then evaluates the exponent **and** adds 10 and 8 before subtracting.

2) $\frac{16}{2 \cdot 4} + 5 \cdot 3 - 1$

CORRECT ANSWER

16

COMMON MISTAKES AND MISCONCEPTIONS

Example Error	Misconception
46	The student divides 16 by 2 then multiplies by 4 *(the student does not understand when grouping symbols are implied or present).*
20	The student simplifies the fraction then operates from left to right *(the student does not know or remember to multiply prior to simplifying addition and subtraction).*
14	The student simplifies the fraction then completes addition and subtraction before multiplication *(the student does not know or remember to multiply prior to simplifying addition and subtraction).*

Practice. Once these examples have been checked and discussed, students will practice evaluating expressions using the order of operations in groups of two to three students. Each group will need the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT and a six-sided number cube. This can either be a standard six-sided number cube or a custom number cube created by you or your students. (Options for a custom number cube as an extension to the activity could include using numbers that must add to 20, numbers that are all even or all odd, prime numbers, negative numbers, or fractions.)

> The students will roll the number cube to determine the first number to place in any one of the boxes on the first problem, then roll again to determine the second number they place, and continue in this manner until all boxes are full.
>
> The goal is to strategically place the numbers to create the largest result when simplified. Students will repeat this process for the remaining expressions and evaluate following the order of operations.
>
> When they are finished, students will find the total value of all expressions.

Ask the groups that believe they have the highest total to report their scores.

Identify the group(s) with the highest total. **Write** their expressions on the board so other groups of students can check their work. If the total the group gave was incorrect, they are disqualified.

Repeat the process by identifying the group that claims to have the next highest total. This ending to the activity allows you to check for understanding in the class as a whole. The critical outcome of this activity is for students to accurately use the order of operations to evaluate expressions.

GUIDING QUESTIONS

Elicit student thinking:

- What is your strategy in this activity?

Determine if the student can EXPLAIN THE RULES OF THE ORDER OF OPERATIONS:

- What should you simplify first in an expression?
- Once you have simplified grouping symbols and terms with exponents, should you multiply or divide first?
- Once you have simplified grouping symbols, terms with exponents, multiplication, and division, should you add or subtract first?

Determine if the student can EVALUATE NUMERICAL EXPRESSIONS USING THE FOUR ARITHMETIC OPERATIONS:

- Now that the term with the exponent is simplified, what should you do first?
- After another simplification according to the order of operations, how does your expression change?

Determine if the student can RECOGNIZE EXPONENTS:

- Which portion of this expression is the exponent?
- What does an exponent mean?
- How could you write this expression without an exponent?

Determine if the student can EVALUATE AN EXPRESSION WITH WHOLE-NUMBER EXPONENTS:

- How would you go about simplifying the term with the exponent?
- What does the exponent simplify to?

Once students are comfortable working with the format where they fill in boxes, they should begin working with algebraic expressions. The activity provided in the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT is similar in structure to the first activity in this lesson.

Practice. Students will evaluate expressions with variables using the order of operations either individually or in groups of two or three students. Each student or group will need the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT and a standard or custom six-sided die. (Options for a custom number cube as an extension to the activity could include using numbers that must add to 20, numbers that are all even or all odd, prime numbers, negative numbers, or fractions.)

The students will roll the six-sided number die to determine the first number to replace any of the variables on the first problem, then roll again to determine the second number they place, and continue to do so until numbers have been substituted for all variables.

NOTE: When there is a duplicate variable, the same number must be substituted for each instance of that variable.

The goal is to strategically place the numbers to create the largest number when simplified. Students will repeat this process for the remaining expressions and evaluate following the order of operations.

When they are finished, students will find the total value of all expressions.

GUIDING QUESTIONS

Elicit student thinking:

- What do you know about the word "variable"?

Determine if the student can EXPLAIN *VARIABLE*:

- What do the letters stand for?
- If you are given a value for a variable, how does that value impact the expression?
- When substituting numbers for variables, what do you do if there are repeated variables (i.e., the same letter appearing multiple times)?

Determine if the student can EXPLAIN THE RULES OF ORDER OF OPERATIONS:

- How do you think we should start to simplify the expression?
- What are some things we need to be careful about when we simplify?
- Once you have simplified grouping symbols and exponents, what should you look at next?

Determine if the student can EVALUATE ALGEBRAIC EXPRESSIONS USING THE ORDER OF OPERATIONS:

- How many variables do you have in the expression?
- How many numbers do you need to roll based on the number of variables in the expression?
- What will the expression look like once the numeric values are substituted?

Ask the groups that believe they have the highest total to report their scores.

Identify the group(s) with the highest total. **Write** their expressions on the board so other groups of students can check their work. If the total the group gave was incorrect, they are disqualified.

Repeat the process by identifying the group that claims to have the next highest total. This ending to the activity allows you to check for understanding in the class as a whole.

An alternative to the described activities would be for students to find the smallest result instead of the largest. A follow-up activity would be for students to use number cubes with fractions and negative numbers or build their own expressions with variables using the cutouts in the LESSON 2 EXTENSION portion of the INSTRUCTIONAL ACTIVITY SUPPLEMENT document. You would need to create guidelines for building expressions using the LESSON 2 EXTENSION nets.

Name_____

Variable Expressions and Order of Operations

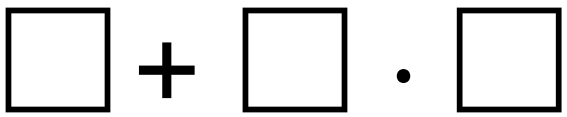
Lesson 2, Activity 1

The objective for this activity is to have the largest possible result after evaluating the expression following the order of operations.

For each expression on the following page, roll a number cube to determine a number, which you can use to fill in one box. One roll of the number cube allows you to fill in one box. You can put the number into any box within that expression. Complete each expression before moving on the next expression. Once all expressions are evaluated, find the total of all five expressions. The student or group with the highest **accurate** total wins.

EXAMPLE

You have the following expression:

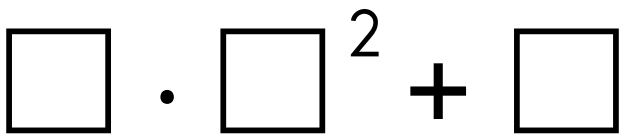


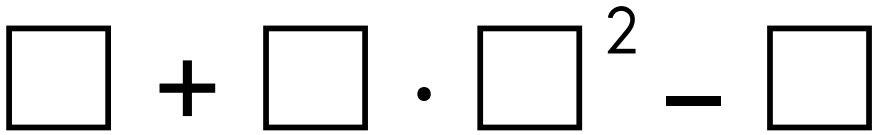
You roll a 4 on the number cube. You can put it into any box, so you choose the middle box.

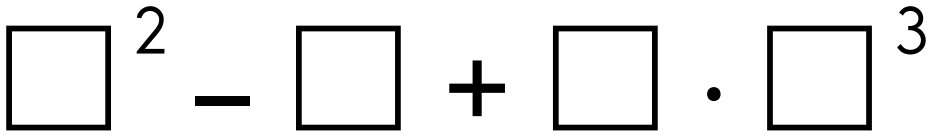
You next roll a 6 on the number cube. You can put it anywhere in the expression. You make your decision and then roll the number cube one more time.

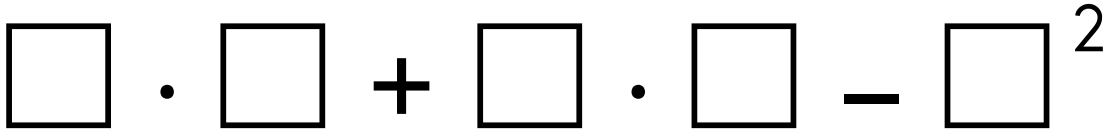
Now, you have this expression completed. You evaluate it and then move on to work on the next expression.

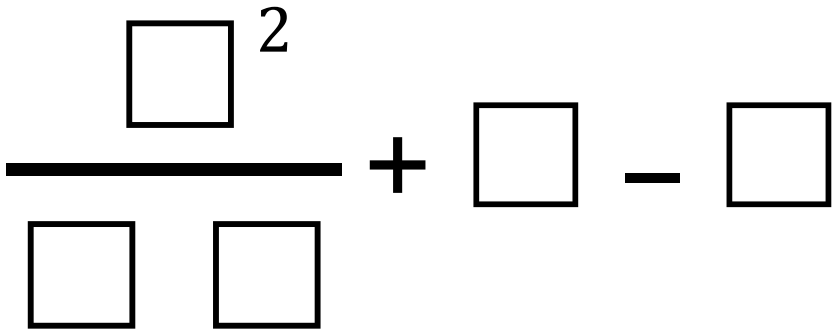
Name_____

1) 

2) 

3) 

4) 

5) 

Total value of all 5 expressions combined: _____

Name_____

VARIABLE EXPRESSIONS AND ORDER OF OPERATIONS

Lesson 2, Activity 2

The objective for this activity is to have the largest possible result after evaluating the expression following the order of operations.

For each expression on the following page, roll a number cube to determine a number, which you can use to substitute for any variable in the expression. When there is a duplicate variable, the same number must be substituted for each instance of that variable in that expression. Complete and evaluate each expression before moving on the next expression. Once all expressions are evaluated, find the total of all five expressions. The student or group with the highest **accurate** total wins.

EXAMPLE

You have the following expression:

$a + b \cdot c$

You roll a 4 on the number cube. You can substitute it for any variable, so you choose b.

$a + 4 \cdot c$

You next roll a 6 on the number cube. You can put it anywhere in the expression. You make your decision and then roll the number cube one more time.

Now, you have this expression completed. You evaluate it and then move on to work on the next expression.

Name_____

1) $a + b^2 + a - c$

2) $a + a \cdot b + b \cdot c - c$

3) $(a^2 - c) + (b^2 - c)$

4) $a^2 + b \div c + (b - c)$

5) $a + b \cdot c - \frac{a}{b \cdot c}$

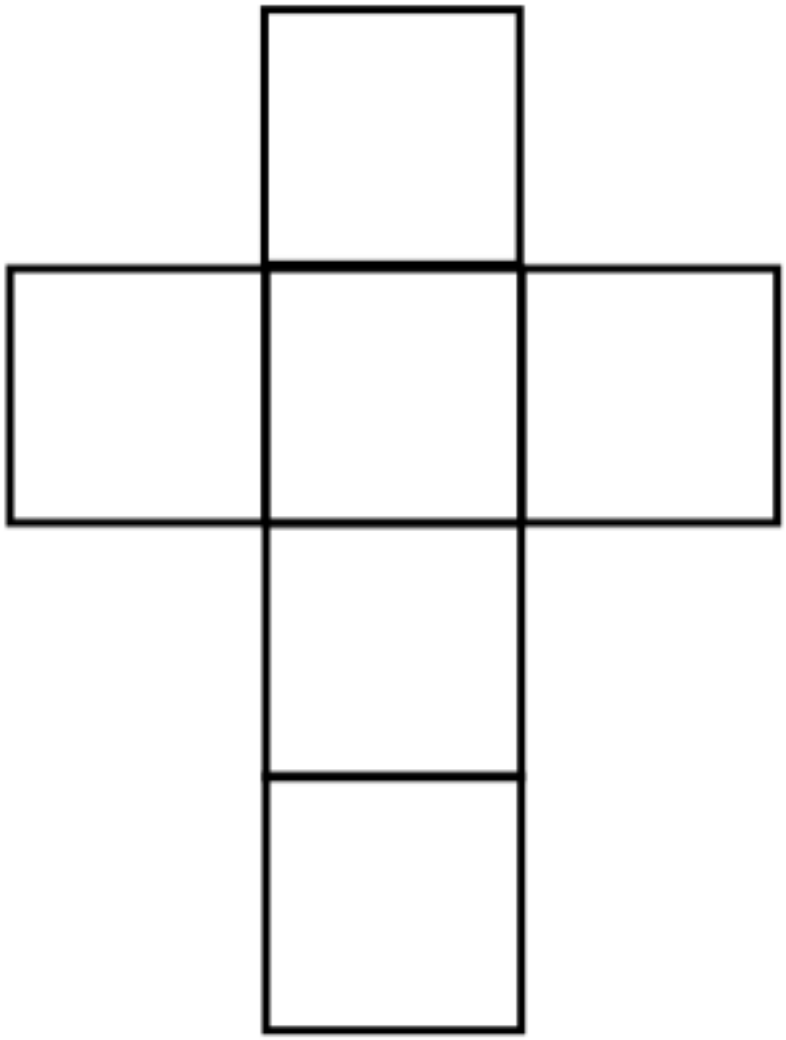
Total value of all 5 expressions combined:_____

Variable Expressions and Order of Operations

INSTRUCTIONAL ACTIVITY SUPPLEMENT

Lesson 2

NET: BLANK 6-SIDED NUMBER CUBE

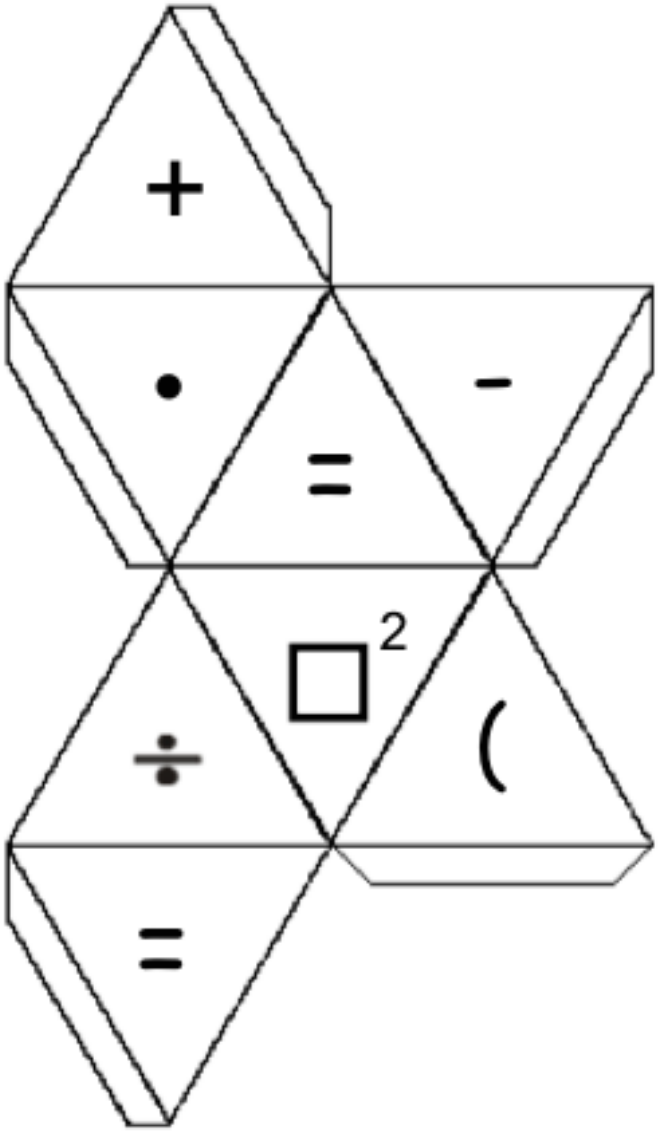


Variable Expressions and Order of Operations

INSTRUCTIONAL ACTIVITY SUPPLEMENT

Lesson 2 Extension

NET: OPERATIONS 8-SIDED NUMBER OCTAHEDRON

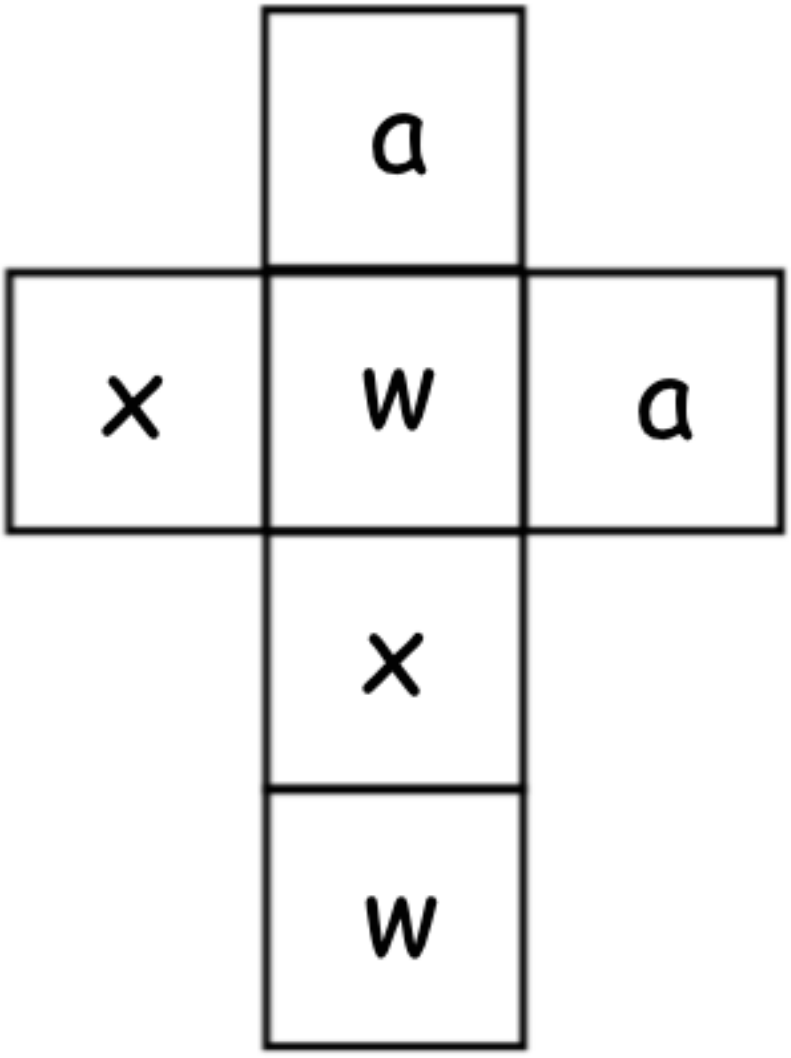


VARIABLE EXPRESSIONS AND ORDER OF OPERATIONS

INSTRUCTIONAL ACTIVITY SUPPLEMENT

Lesson 2 Extension

NET: VARIABLES SIX-SIDED NUMBER CUBE



Name_____

VARIABLE EXPRESSIONS AND ORDER OF OPERATIONS

Lesson 2

1. Use the following expression to answer questions (a) through (d):

$a \cdot b^2 + a \cdot c - b$ for $a = 2$, $b = 4$, $c = 1$

1.a. How many **different** variables are there in the expression? Explain your reasoning.

1.b. Substituting the value of either a, b, or c would allow you to begin simplifying according to the order of operations. Which variable would you have to replace first in order to begin simplifying? Explain your reasoning.

1.c. Rewrite the entire expression using the given variable values.

1.d. Simplify the expression. Show each step.

Name_____

2. Use the following expression to answer questions (a) through (d):

$$x^3 - \frac{w}{yz} + xy \text{ for } w = 8,\ x = 3,\ y = 2,\ z = 4$$

2.a. How many **different** variables are there in the expression? Explain your reasoning.

2.b. Which operation should occur first when evaluating this expression? Be specific and explain your reasoning.

2.c. Rewrite the entire expression using the given variable values.

2.d. Simplify the expression. Show each step.

VARIABLE EXPRESSIONS AND ORDER OF OPERATIONS

STUDENT ACTIVITY SOLUTION GUIDE

Lesson 2

1. Use the following expression to answer questions (a) through (d):

$a \cdot b^2 + a \cdot c - b$ for $a = 2$, $b = 4$, $c = 1$

1.a. How many **different** variables are there in the expression? Explain your reasoning.

CORRECT ANSWER

There are three different variables in the expression. Repeated variables are not different variables. Therefore, even though there are five letters in all in the expression, there are only three distinct variables present.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Five since there are five letters in the expression in all.	understands repeated variables as separate entities	EXPLAIN *VARIABLE*

1.b. Substituting the value of either *a*, *b*, or *c* would allow you to begin simplifying according to the order of operations. Which variable would you have to replace first in order to begin simplifying? Explain your reasoning.

CORRECT ANSWER

4 would have to be substituted for *b* first in order to perform the first simplification. Exponents must be evaluated first in the absence of grouping symbols.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
a and b because they appear first and must be multiplied.	evaluates expressions from left to right	EXPLAIN THE RULES OF THE ORDER OF OPERATIONS

1.c. Rewrite the entire expression using the given variable values.

CORRECT ANSWER

$2 \cdot 4^2 + 2 \cdot 1 - 4$

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
$2a \cdot 4b^2 + 2a \cdot 1c - 4b$	does not understand the meaning of *variable*	EXPLAIN *VARIABLE*

1.d. Simplify the expression. Show each step.

CORRECT ANSWER

$2(16) + 2(1) - 4$
$32 + 2 - 4$
$34 - 4$
30

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
$4^2 \neq 16$	does not evaluate exponent accurately	EVALUATE EXPRESSIONS WITH WHOLE-NUMBER EXPONENTS
4^2 not simplified first.	does not evaluate exponent first	EXPLAIN THE RULES OF THE ORDER OF OPERATIONS

Example Error	Misconception	Missing Knowledge
2(16) and 2(1) are not simplified before +/–.	evaluates operations out of order	EVALUATE NUMERICAL EXPRESSIONS USING THE FOUR ARITHMETIC OPERATIONS
Note: If a student is able to explain the steps in the order of operations but does not perform them properly, they may know the order of operations but struggle with computation or attention to precision in their work.		

2. Use the following expression to answer questions (a) through (d):

$$x^3 - \frac{w}{yz} + xy \text{ for } w = 8, x = 3, y = 2, z = 4$$

2.a. How many **different** variables are there in the expression? Explain your reasoning.

CORRECT ANSWER

There are four different variables in the expression. Repeated variables are not different variables. Therefore, even though there are five letters in all in the expression, there are only four distinct variables present.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Six since there are six letters in the expression in all.	understands repeated variables as separate entities	EXPLAIN *VARIABLE*

2.b. Which operation should occur first when evaluating this expression? Be specific and explain your reasoning.

CORRECT ANSWER

The multiplication in the denominator of the fraction should occur first. Since yz is grouped in the bottom of the fraction, you would have to multiply y (2) by z (4) first.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
x^3 because x^3 appears first in the expression.	evaluates expressions from left to right	EXPLAIN THE RULES OF THE ORDER OF OPERATIONS
Note: In this case, it would not produce an incorrect response if a student simplifies x^3 first, though when learning about the order of operations student should look for grouping symbols before evaluating exponents.		

2.c. Rewrite the entire expression using the given variable values.

CORRECT ANSWER

$$3^3 - \frac{8}{2 \cdot 4} + 3 \cdot 2$$

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
$3x^3 - \frac{8w}{2y\ 4z} + 3x\ 2y$	does not understand the meaning of *variable*	EXPLAIN *VARIABLE*

2.d. Simplify the expression. Show each step.

CORRECT ANSWER

$$33 - 8 \div 8 + 3(2)$$
$$27 - 8 \div 8 + 3(2)$$
$$27 - 1 + 3(2)$$
$$27 - 1 + 6$$
$$26 + 6$$
$$32$$

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
3^3 is not simplified to 27.	does not evaluate exponent accurately	EVALUATE EXPRESSIONS WITH WHOLE-NUMBER EXPONENTS
$2(4)$ is not simplified first.	does not evaluate grouping symbols first	EVALUATE NUMERICAL EXPRESSIONS USING THE FOUR ARITHMETIC OPERATIONS
$8 \div 8$ and $3(2)$ are not simplified before +/-.	evaluates operations out of order	EVALUATE NUMERICAL EXPRESSIONS USING THE FOUR ARITHMETIC OPERATIONS
Note: If a student is able to explain the steps in the order of operations but does not perform them properly, they may know the order of operations but struggle with computation or attention to precision in their work.		

VARIABLE EXPRESSIONS AND ORDER OF OPERATIONS

INSTRUCTIONAL ACTIVITY

Lesson 3

LEARNING GOAL

Students will write and evaluate formulas from a description to represent real-world situations. The critical outcome of this activity is for students to be able to represent a real-world description of a formula using proper notation and then evaluate the expression given specified values using the order of operations.

PRIMARY ACTIVITY

Students will write formulas representing real-world problems from written descriptions and evaluate the formulas for given values using the order of operations.

OTHER VOCABULARY

Students will need to know the meaning of

- Expression
- Evaluate
- Order of Operations

MATERIALS

- INSTRUCTIONAL ACTIVITY STUDENT HANDOUT
- Ruler (able to measure in centimeters)

IMPLEMENTATION

Distribute the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT to pairs of students.

Ask the students to read the situation description and answer the questions leading the student towards writing a formula for a real world situation. For the shoe size conversion example, the

student should measure their own foot (in centimeters) and use the formula they produced (i.e., $E = 1.5(f + 1.5)$, where E represents European shoe size and f represents the length of your foot in centimeters) to determine their shoe size in Italy. For the temperature scenario, once the students have determined the formulas for converting between the Fahrenheit and Celsius temperature scales (i.e., $F = 9/5C + 32$ and $C = 5/9(F - 32)$), they should continue with questions that require substitution of a variable and simplification following the order of operations.

GUIDING QUESTIONS

Elicit student thinking:

- How are formulas different from equations?
- What words stand out when you read the description?

Determine if the student can EXPLAIN *VARIABLE*:

- What do the letters stand for?
- If you are given a value for a variable, how does that value impact the expression?

Determine if the student can EXPLAIN *EXPRESSION* and EXPLAIN *EQUATION*:

- What does it mean to represent an expression?
- What is the difference between an expression and an equation?

Determine if the student can REPRESENT CONVENTIONAL VARIABLE NOTATION:

- How would you write "3 times x" as an algebraic expression?

Determine if the student can REPRESENT EXPRESSIONS WITH NUMBERS AND/OR VARIABLES:

- Using the given description, which words indicate operations?
- How are the variables and numbers related to each other?
- Are grouping symbols necessary?

At the end of this activity, give the class either a temperature or foot length in centimeters and the end result (Italian shoe size, Fahrenheit temperature, or Celsius temperature), and ask the students to write the formula they will use to obtain the requested information. Then give a value for the students to plug into their formula and follow the order of operations to answer the question.

Name_____

Variable Expressions and Order of Operations

Lesson 3

The goal of this activity is to create formulas for real-world situations and use them to evaluate a given value.

Pretend you have just moved to Italy and are trying to adjust to your new home. One of the things you must do is purchase some new walking shoes—but you don't know your European shoe size, which is different than your US shoe size. Your neighbor tells you that it only takes two simple calculations. First you must find the sum of your foot length (in centimeters) and 1.5 centimeters (the extra space needed inside a shoe for a comfortable fit).

- Represent the sum of your foot length, f, and 1.5 centimeters using an algebraic expression.

Then you need to find the product of that sum and the number 1.5 to convert from centimeters to the unit used by European shoe makers.

- Represent the product of your previous algebraic expression and the number 1.5. Make sure to use grouping symbols around the first step to indicate the order of operations.

Now that you have the formula, you will need to figure out your shoe size in Italy. The next step is to measure your foot in centimeters so you can evaluate the expression to determine your specific shoe size. (Only one partner's foot needs to be measured.)

- How long is your foot (in centimeters)?

- Find your Italian shoe size using the formula.

Name_____

Now you are ready to buy shoes. When you leave your home, you notice that it is hot outside. You are curious what the temperature is, but Italians measure temperature using the Celsius scale instead of the Fahrenheit scale you are used to. You go back to your neighbor and ask how to convert back and forth. Follow the directions below to write formulas for changing between the Celsius and Fahrenheit temperature scales.

To change a temperature from the Celsius scale to the Fahrenheit scale, create an algebraic expression by following these directions.

- The temperature in Fahrenheit (F) is 32 degrees more than the product of the temperature in Celsius, C, and 9/5, a conversion factor. Write this sentence as a formula:

  $F =$

- If it is 40 degrees Celsius, what is the temperature in Fahrenheit?

To change a temperature from the Fahrenheit scale to the Celsius scale, write an algebraic expression by following these directions.

- The temperature in Celsius (C) equals the product of 5/9 and the difference between the temperature in Fahrenheit (F) and 32 degrees. Write this sentence as a formula:

  $C =$

- If it is 95 degrees Fahrenheit, what is the temperature in Celsius?

Name_____

Variable Expressions and Order of Operations

Lesson 3

Given the following descriptions of a real world formula, write the described formula. Then, evaluate the formula with the specified values.

1. The area, A, of a triangle is one half the product of the base, b, and the height, h.

   1.a. Write the formula for the area of a triangle.

   1.b. If the base of a triangle is 6 inches and the height is 3 inches, what is the area? Write your answer as a complete sentence.

2. The surface area, A, of a cube is 6 times a side length, s, squared.

   2.a. Write a formula for the surface area of a cube.

   2.b. If a side length of a cube is 4 centimeters long, what is the surface area? Write your answer as a complete sentence.

   2.c. Describe how you used your knowledge of the order of operations to find the surface area of the cube described in part (b).

Name_____

3. The perimeter, *P*, of a rectangle is the sum of two times the length, *l*, and two times the width, *w*.

   3.a. Write a formula for the perimeter of a rectangle.

   3.b. If the length of a rectangle is 5 feet and the width is 3 feet, what is the perimeter? Write your answer as a complete sentence.

   3.c. Describe how you used your knowledge of the order of operations to find the perimeter of the rectangle described in part (b).

VARIABLE EXPRESSIONS AND ORDER OF OPERATIONS

STUDENT ACTIVITY SOLUTION GUIDE

Lesson 3

Given the following descriptions of a real world formula, write the described formula. Then, evaluate the formula with the specified values.

1. The area, A, of a triangle is one half the product of the base, b, and the height, h.

1.a. Write the formula for the area of a triangle.

CORRECT ANSWER

$A = \frac{1}{2} b h$

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
$A \frac{1}{2} b h$ or $A = bh$	does not understand how to represent expressions using numbers and variables	REPRESENT EXPRESSIONS WITH NUMBERS AND/OR VARIABLES

1.b. If the base of a triangle is 6 inches and the height is 3 inches, what is the area? Write your answer as a complete sentence.

CORRECT ANSWER

$A = \frac{1}{2} (6) (3)$
$A = 3(3)$
$A = 9$
The area of the described triangle is 9 square inches.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
$A = \frac{1}{2} 6b \cdot 3h$	does not understand the meaning of *variable*	EXPLAIN *VARIABLE*
$A = 3(1.5)$	evaluates operations incorrectly	EVALUATE NUMERICAL EXPRESSIONS USING THE FOUR ARITHMETIC OPERATIONS

2. The surface area, A, of a cube is 6 times a side length, s, squared.

2.a. Write a formula for the surface area of a cube.

CORRECT ANSWER

$A = 6s^2$

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
$A = 6s$	does not understand how to represent expressions using numbers and variables	REPRESENT EXPRESSIONS WITH NUMBERS AND/OR VARIABLES

2.b. If a side length of a cube is 4 centimeters long, what is the surface area? Write your answer as a complete sentence.

CORRECT ANSWER

$A = 6 \cdot 4^2$
$A = 6(16)$
$A = 96$
The surface area of the described cube is 96 square centimeters.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
$A = 6(4s)^2$	does not understand the meaning of *variable*	EXPLAIN *VARIABLE*
$A = 24^2$	evaluates operations out of order	EXPLAIN THE RULES OF THE ORDER OF OPERATIONS
4^2 is not simplified to 16.	does not evaluate exponent accurately	EVALUATE EXPRESSIONS WITH WHOLE-NUMBER EXPONENTS

2.c. Describe how you used your knowledge of the order of operations to find the surface area of the cube described in part (b).

CORRECT ANSWER

> The order of operations states that exponents must occur before multiplication, so I first simplified 4^2, then multiplied the result by 6.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
I multiplied 6 and 4 because they appeared first in the expression.	evaluates expressions from left to right	EXPLAIN THE RULES OF THE ORDER OF OPERATIONS

3. The perimeter, *P*, of a rectangle is the sum of two times the lenth, *l*, and two times the width, *w*.

3.a. Write a formula for the perimeter of a rectangle.

CORRECT ANSWER

> $p = 2l + 2w$

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
$p = 2lw$ OR $p = l + w$	does not understand how to represent expressions using numbers and variables	REPRESENT EXPRESSIONS WITH NUMBERS AND/OR VARIABLES.

3.b. If the length of a rectangle is 5 feet and the width is 3 feet, what is the perimeter? Write your answer as a complete sentence.

CORRECT ANSWER

$p = 2(5) + 2(3)$
$p = 10 + 2(3)$
$p = 10 + 16$
$p = 26$
The perimeter of the described rectangle is 26 feet.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
$p = 2(5)(l) + 2(3)(w)$ OR $p = l + w$	does not understand the meaning of *variable*	EXPLAIN *VARIABLE*
The student does not evaluate $2(5)$ and $2(3)$ before addition.	evaluates operations out of order	EVALUATE NUMERICAL EXPRESSIONS USING THE FOUR ARITHMETIC OPERATIONS

3.c. Describe how you used your knowledge of the order of operations to find the perimeter of the rectangle described in part (b).

CORRECT ANSWER

The order of operations states that multiplication must occur before addition, so I first simplified 2 times 5 and 2 times 3, then added the results together.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
I multiplied 2 and 5 to get 10, then added 10 and 2, then multiplied by 3.	evaluates expressions from left to right	EXPLAIN THE RULES OF THE ORDER OF OPERATIONS