UNDERSTANDING FRACTIONS AS NUMBERS

3.NF.A.1 – 3

CONTENTS

The types of documents contained in the unit are listed below. Throughout the unit, the documents are arranged by lesson.

LEARNING MAP INFORMATION	An overview of the standards, the learning map section, and the nodes addressed in this unit
TEACHER NOTES	A brief discussion describing the progression depicted in the learning map section with research-based recommendations for focusing instruction to foster student learning and an introduction to the unit's lessons
OVERVIEW OF INSTRUCTIONAL ACTIVITIES	A table highlighting the Lesson Goals and Nodes Addressed in each lesson in this unit
INSTRUCTIONAL ACTIVITY	A detailed walkthrough of the unit
INSTRUCTIONAL ACTIVITY STUDENT HANDOUT	A handout for the guided activity, intended to be paired with the Instructional Activity
INSTRUCTIONAL ACTIVITY SUPPLEMENT	A collection of materials or activities related to the Instructional Activity
STUDENT ACTIVITY	A work-alone activity for students
STUDENT ACTIVITY SOLUTION GUIDE	A solution guide for the work-alone activity with example errors, misconceptions, and links to the learning map section

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Understanding Fractions as Numbers

LEARNING MAP INFORMATION

STANDARDS

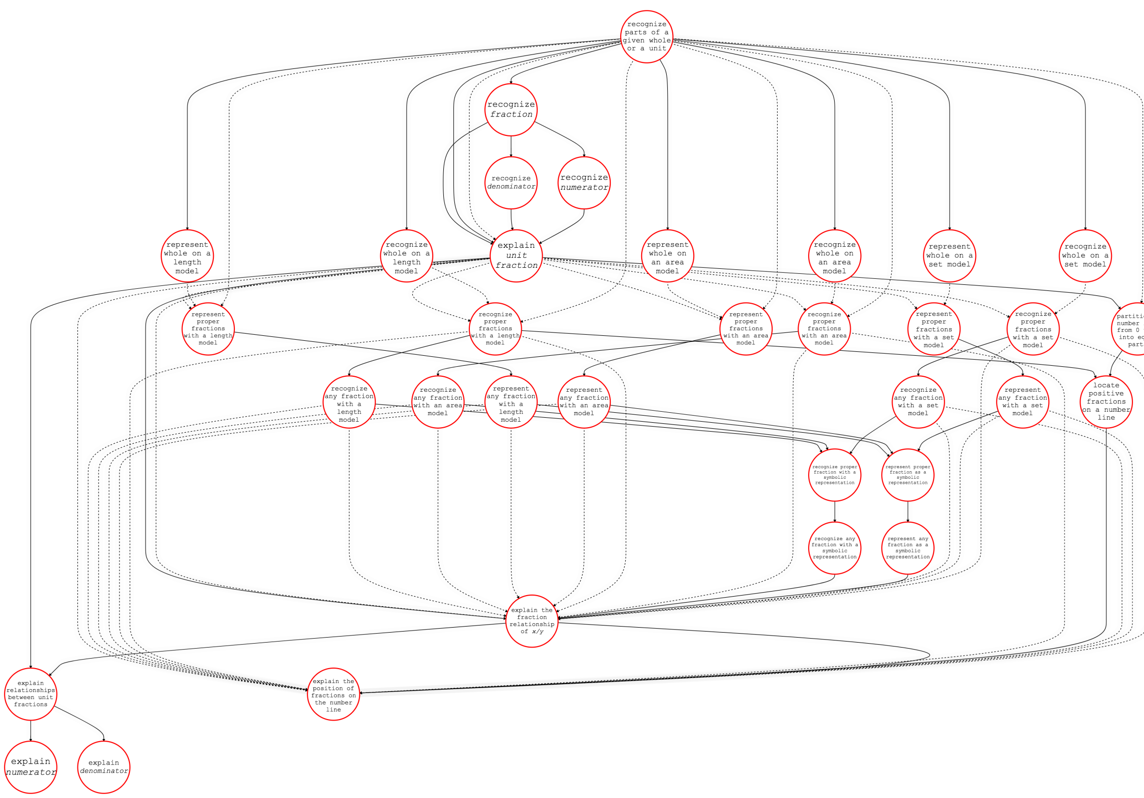
3.NF.A.1 Understand a unit fraction as the quantity formed by one part when a whole is partitioned into equal parts.

3.NF.A.2 Understand that when a whole is partitioned equally, a fraction can be used to represent a portion of the whole.

- **3.NF.A.2a** Describe the numerator as representing the number of pieces being considered.
- **3.NF.A.2b** Describe the denominator as the number of pieces that make the whole.

3.NF.A.3 Represent fractions on a number line.

- **3.NF.A.3a** Understand the whole is the interval from 0 to 1.
- **3.NF.A.3b** Understand the whole is partitioned into equal parts.
- **3.NF.A.3c** Understand a fraction represents the endpoint of a length a given number of partitions from 0.



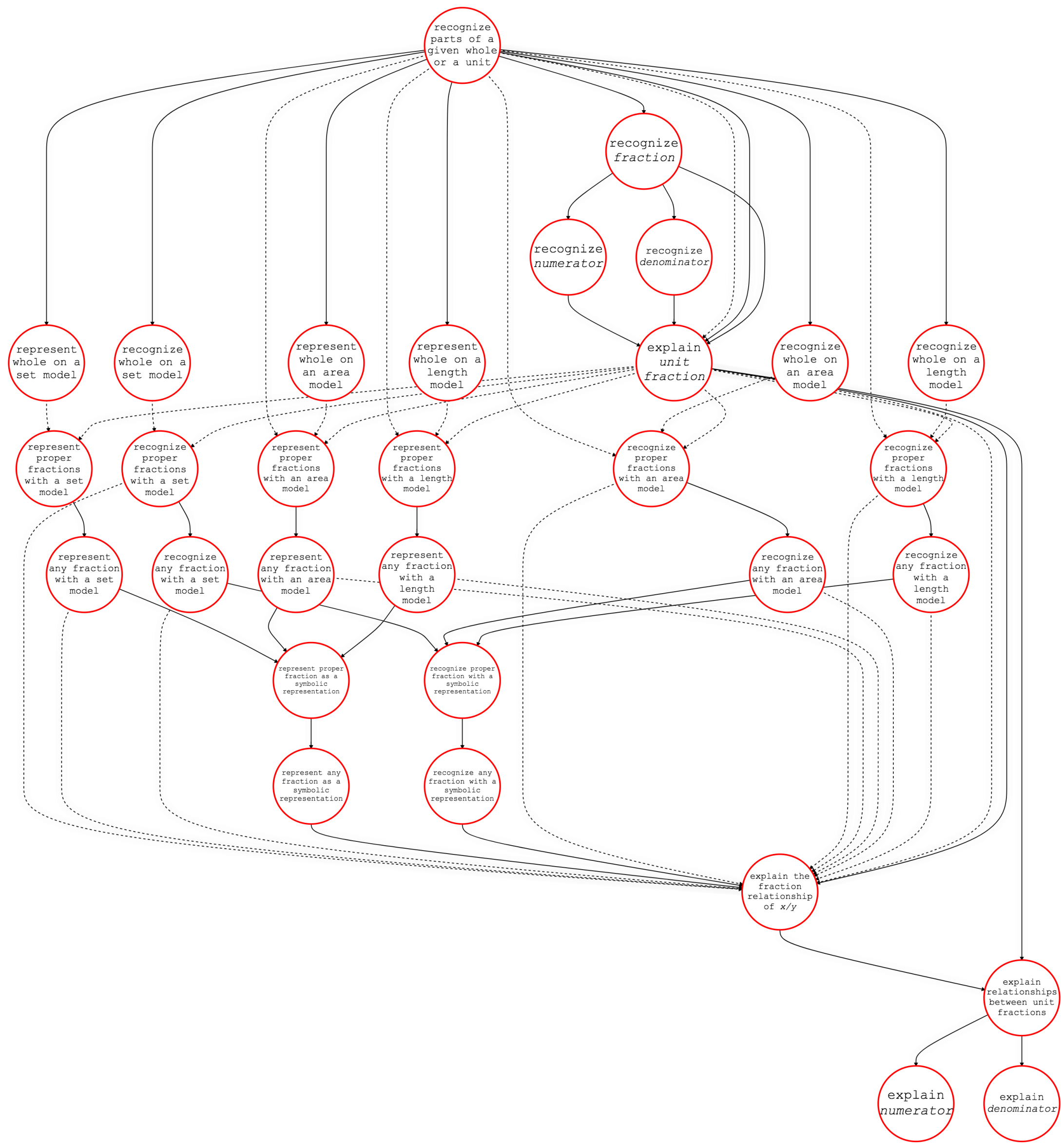
**Learning map model of* 3.NF.1,2

3.NF.A.1 Understand a unit fraction as the quantity formed by one part when a whole is partitioned into equal parts.

3.NF.A.2 Understand that when a whole is partitioned equally, a fraction can be used to represent a portion of the whole.

3.NF.A.2a Describe the numerator as representing the number of pieces being considered.

3.NF.A.2b Describe the denominator as the number of pieces that make the whole.



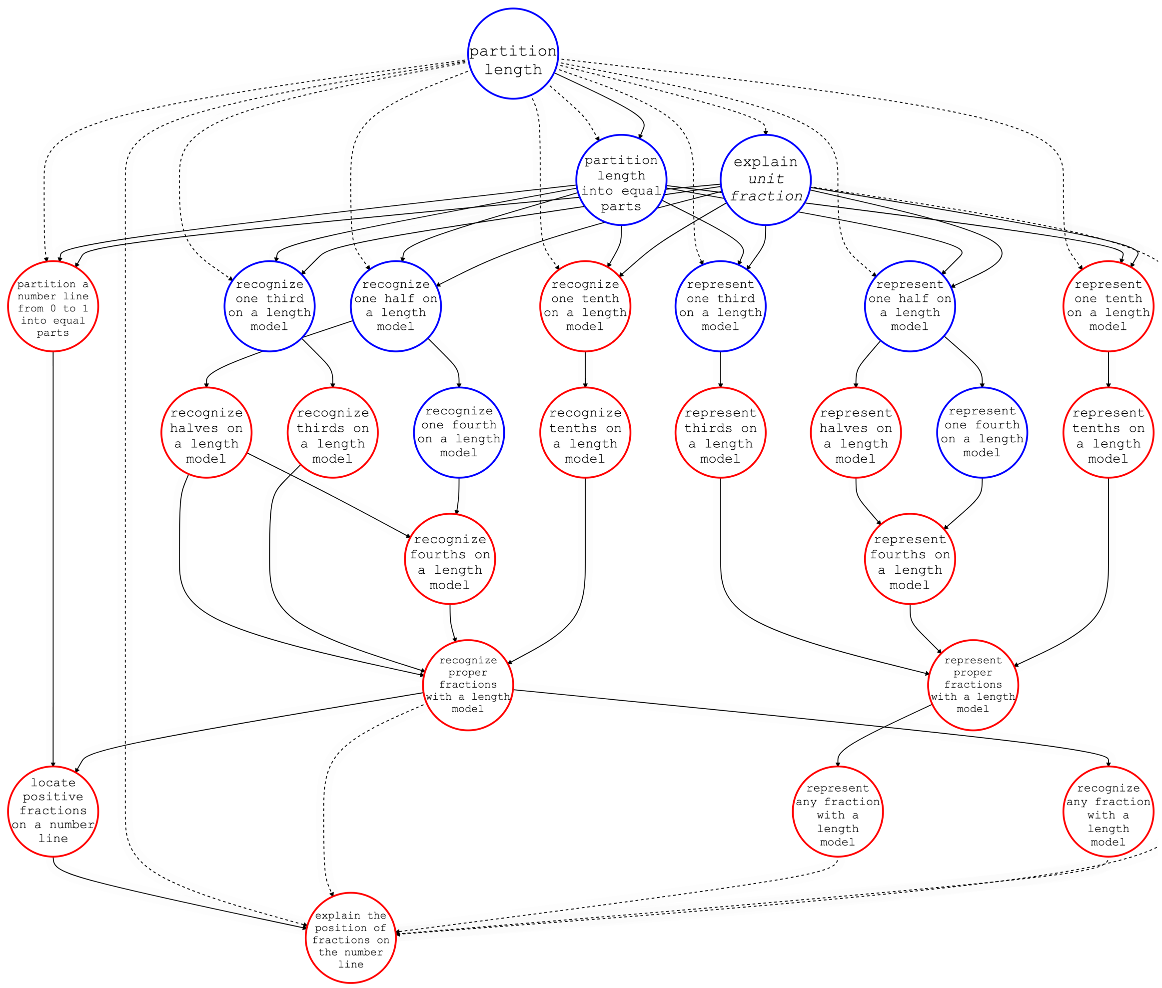
**Learning map model of* 3.NF.1

3.NF.A.3 Represent fractions on a number line.

3.NF.A.3a Understand the whole is the interval from 0 to 1.

3.NF.A.3b Understand the whole is partitioned into equal parts.

3.NF.A.3c Understand a fraction represents the endpoint of the length a given number of partitions from 0.



**Learning map model of* 3.NF.2

Node Name	Node Description
EXPLAIN *DENOMINATOR*	Make known your understanding that the term "denominator" refers to the number below the fraction bar and represents how many equal-size parts make up the whole.
EXPLAIN *NUMERATOR*	Make known your understanding that the term "numerator" refers to the number above the fraction bar and represents x parts of a whole partitioned into y equal-size parts.
EXPLAIN RELATIONSHIPS BETWEEN UNIT FRACTIONS	Make known your understanding through words, drawings, manipulatives, etc. that a whole partitioned into fewer equal parts results in larger parts, and the same whole partitioned into more equal parts results in smaller parts.
EXPLAIN THE FRACTION RELATIONSHIP OF *X/Y*	Make known your understanding that the symbolic fraction $\frac{x}{y}$ represents x parts of a whole that is partitioned into y equal-size parts. The whole could represent a length, a shape, or a collection of objects (i.e., a set).
EXPLAIN THE POSITION OF FRACTIONS ON THE NUMBER LINE	Make known your understanding that, when given a fractional relationship, $\frac{x}{y}$ (when x and y are both positive), the location of $\frac{x}{y}$ on the number line is the endpoint after iterating the unit fraction $(\frac{1}{y})$ x times.
EXPLAIN *UNIT FRACTION*	Make known your understanding that a unit fraction, $\frac{1}{n}$, is the quantity formed by one part when a whole is partitioned into n equal parts.
LOCATE POSITIVE FRACTIONS ON A NUMBER LINE	When given a positive fraction, locate the number on a number line.
PARTITION A NUMBER LINE FROM 0 TO 1 INTO EQUAL PARTS	Divide the interval "zero to one" into equal parts on a number line.
RECOGNIZE ANY FRACTION WITH A LENGTH MODEL	When presented with a group of different length models, select the length model that represents any specified proper or improper fraction.
RECOGNIZE ANY FRACTION WITH A SET MODEL	When presented with a group of different set models, select the set model that represents any specified proper or improper fraction.
RECOGNIZE ANY FRACTION WITH A SYMBOLIC REPRESENTATION	When presented with a group of different symbolic representations, select the symbolic representation that represents any specified proper or improper fraction.
RECOGNIZE ANY FRACTION WITH AN AREA MODEL	When presented with a group of different area models, select the area model that represents any specified proper or improper fraction.
RECOGNIZE *DENOMINATOR*	Identify or name the denominator.
RECOGNIZE *FRACTION*	Identify or name fraction as the numerical representation of a part-whole relationship.
RECOGNIZE *NUMERATOR*	Identify or name the numerator.
RECOGNIZE PARTS OF A GIVEN WHOLE OR A UNIT	Make known your understanding that a part-whole relationship indicates that one unit or whole is made up of one or more parts.
RECOGNIZE PROPER FRACTIONS WITH A LENGTH MODEL	When presented with a group of different length models, select the length model that represents a specified proper fraction. A proper fraction is a fraction that contains a whole number numerator that is less than the whole number denominator.

RECOGNIZE PROPER FRACTIONS WITH A SET MODEL	When presented with a group of different set models, select the set model that represents a specified proper fraction. A proper fraction is a fraction that contains a whole number numerator that is less than the whole number denominator.
RECOGNIZE PROPER FRACTIONS WITH A SYMBOLIC REPRESENTATION	When presented with a group of different symbolic representations, select the symbolic representation that represents a specified proper fraction. A proper fraction is a fraction that contains a whole number numerator that is less than the whole number denominator.
RECOGNIZE PROPER FRACTIONS WITH AN AREA MODEL	When presented with a group of different area models, select the area model that represents a specified proper fraction. A proper fraction is a fraction that contains a whole number numerator that is less than the whole number denominator.
RECOGNIZE WHOLE ON A LENGTH MODEL	Identify or describe a length model or number line that represents a whole (i.e., unit, one).
RECOGNIZE WHOLE ON A SET MODEL	Identify or describe a set model or collection of objects that represents a whole (i.e., unit).
RECOGNIZE WHOLE ON AN AREA MODEL	Identify or describe an area model or figure that represents a whole (i.e., unit, one).
REPRESENT ANY FRACTION AS A SYMBOLIC REPRESENTATION	Through writing or an appropriate assistive technology, represent any fraction as a symbolic representation.
REPRESENT ANY FRACTION WITH A LENGTH MODEL	Through writing or an appropriate assistive technology, represent any fraction with a length model.
REPRESENT ANY FRACTION WITH A SET MODEL	Through writing or an appropriate assistive technology, represent any fraction with a set model.
REPRESENT ANY FRACTION WITH AN AREA MODEL	Through writing or an appropriate assistive technology, represent any fraction with an area model.
REPRESENT PROPER FRACTIONS AS A SYMBOLIC REPRESENTATION	Through writing or an appropriate assistive technology, represent proper fractions as a symbolic representation (e.g., $\frac{2}{3}$).
REPRESENT PROPER FRACTIONS WITH A LENGTH MODEL	Through writing or an appropriate assistive technology, represent proper fractions with a length model.
REPRESENT PROPER FRACTIONS WITH A SET MODEL	Through writing or an appropriate assistive technology, represent proper fractions with a set.
REPRESENT PROPER FRACTIONS WITH AN AREA MODEL	Through writing or an appropriate assistive technology, represent proper fractions with an area model.
REPRESENT WHOLE ON A LENGTH MODEL	Through writing or an appropriate assistive technology, represent whole on a length model.
REPRESENT WHOLE ON A SET MODEL	Through writing or an appropriate assistive technology, represent whole on a set model.
REPRESENT WHOLE ON AN AREA MODEL	Through writing or an appropriate assistive technology, represent whole on an area model.

Understanding Fractions as Numbers

TEACHER NOTES

This unit includes the following documents:

- Learning Map Information
- Instructional Activity (4 lessons)
- Instructional Activity Student Handout (for Lessons 2 – 4)
- Instructional Activity Supplement (for Lessons 1, 2, & 4)
- Student Activity
- Student Activity Solution Guide

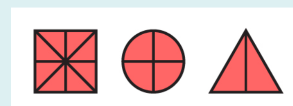
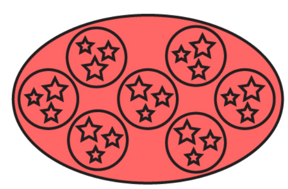
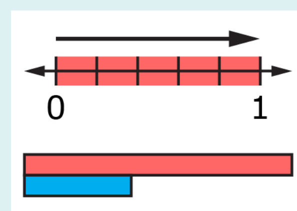
In this unit, students will develop a conceptual understanding of fractions to build a solid foundation for operations with fractions and later experiences with rational numbers. Students will engage with fractions using all three levels of representation, concrete models (manipulatives), semi-concrete models or pictorial models (drawings and images), and symbolic representations (numeric). In addition, students will understand fractions as representative of part-whole relationships and as comparisons.

RESEARCH

Students should work with concrete models over extended periods of time to support their initial conceptions of fractions and as they develop more sophisticated fraction sense. These initial experiences with concrete models form the bases for students' mental images of fractions, which they can incorporate as they work with pictorial and symbolic models. It is important for students to develop a conceptual understanding of fractions prior to working with formal fraction symbols (Bamberger, Oberdorf, & Schultz-Ferrell, 2010; Chval, Lannin, & Jones, 2013; Cramer, Behr, Post, & Lesh, 1997; Van de Walle, Karp, Lovin, Bay-Williams, 2012; Watanabe, 2002). When students do not have a solid conceptual foundation for understanding fractions, it can lead to misconceptions that impact later fraction instruction.

In order for students to effectively use fraction models, they must first be aware of the characteristics of each model. For instance, Small (2014) states that the type of model used dictates how the whole is represented. The three types of fraction models that will be utilized in these lessons are set models (or discrete models), area models, and length models. The whole of a set model is represented by a collection of things, and the equal-size parts are represented by groups or subsets. For example, a box of pencil-top erasers could be a whole, and a group of five erasers would be a "part" or subset of the whole. The whole of an area model is represented by a shape or single area, and the equal-size parts are represented by dividing the shape into pieces (e.g., when a circle is divided into four equal-size parts by drawing two diameters that intersect at the center of the circle at a 90 degree angle). The whole of a length model is often represented as the distance between zero and one on a number line, and the equal parts are represented by drawing line segments to divide the number line into equal-size segments (e.g., a number line labeled zero to one that has been divided into four equal-length segments). It is important to be aware that strip models (such as fraction bars or single column/single row rectangles) are often seen as area models but are actually length models, because a fraction shown with a strip model represents the comparison of a specified length with the length of the whole. It is important for students to recognize the different representations of a whole when making use of fraction models.

Fraction understanding begins with a well-defined idea of *one whole* and an awareness of how equipartitioning is represented in various fraction models (i.e. set models, measurement models, and area models). Before students can identify the number of equal-size parts or equal-size lengths, they must first identify the whole. It is imperative to be explicit about what represents a whole (e.g., one circle, four apples, one foot, one square, etc.). Some resources are vague or imply a whole, which can lead to student confusion (Chval et al., 2013). In addition, when referring to the whole in a set model, it is beneficial to use and emphasize collective nouns repeatedly (e.g., *group of* pencils, *stack of* cards, *collection of* books, etc.), so there is no question or confusion in regards to identifying the whole (Colomb & Kennedy, 2005; Stump, 2003). Stump (2003) states that in order for students to successfully equipartition a whole, they must first understand and identify the whole. The following table displays the different representations of fraction models and their respective wholes (Petit et al., 2016).

	The Whole	**Example**	**Pictorial Representation**
Area Model	Area of a defined region	Shapes: squares, triangles, quadrilaterals, circles, pentagons, hexagons, etc.	
Set Model (Discreet Model)	Definition of what is in a defined set	A collection of marbles, a group of counters, a stack of cards, a container of hard candies, etc.	
Length Model	Continuous unit of distance or length	The distance between any two, consecutive whole numbers on a number line or a ruler, a fraction bar, Cuisenaire rod, length of interlocking cubes	

It is important to note that when using a set model, the value of the whole changes as the set number of objects that form a whole or entire set changes.

FOR EXAMPLE: Joey has a jar of 24 marbles. If the jar of marbles is the whole, then the value of the set or *whole* is 24 marbles. However, if Joey only

> wants to consider the 14 blue and yellow marbles, then the value of the set or *whole* has changed and is now 14 marbles, not 24 marbles.

It is critical, especially with set models, to be explicit about what constitutes a whole to avoid possible student confusion. Likewise, it is significant to recognize that multiple wholes in a measurement model are continuous, whereas multiple wholes in a set or area model are physically separated (Petit et al., 2016).

Once the concept of the whole has been established, students can begin equipartitioning and viewing single equal-size parts of the whole as unit fractions. Students need to appreciate the unique relationship between the size of a part and the denominator of the corresponding unit fraction. This supports their meaningful consideration of and comparisons among different unit fractions; that is, a student with sufficient understanding of how thirds and fourths are created will successfully acknowledge that thirds are larger than fourths, provided they are taken from the same whole.

Without sufficient understanding of the equivalence of parts, students become vulnerable to common difficulties in using fraction models (Petit et al., 2015). Specifically, that some students only consider the number of parts and not the size of the parts when considering fraction models (Petit et al., 2015). This could be due to the fact that fraction area models are often only represented as circles or rectangles, and/or that students often only work with pre-partitioned models, which have been partitioned into congruent parts (Petit et al., 2015). It is recommended that students' experiences involve different visual models, including differently shaped wholes (e.g., hexagons), number lines, and sets, and that equipartitioning is not implemented in a rote way, which leads students to become dependent on pre-partitioned wholes (Petit et al., 2015). The use of pre-partitioned wholes, and eventually student-partitioned wholes, should consistently be tied to the mathematical concept of equal-area parts, equal-size divisions on a number line, or equal-size sets in a set model (Petit et al., 2015). Yoshida and Swano (2002) found that students gain a stronger understanding of fractions when equipartitioning is taught prior to or in conjunction with traditional fraction content.

Creating wholes by repeatedly adding unit fractions (i.e. iterating unit fractions) succeeds partitioning wholes and is as important to building conceptual understanding of fractions as equipartitioning. Both iterating a unit fraction and equipartitioning highlight a fraction's part-whole relationship. As opposed to dividing up or partitioning a whole into equal-size parts, iterating a unit fraction requires building a whole utilizing equal-size parts. Iterating equal-size pieces or unit fractions can support student understanding of counting fractional parts by linking the process to counting other objects. For example, when a student is aware of the kind of part they are counting (*sixths*), they will know they have reached one whole when the number of parts they have counted is equal to the kind of part they are counting (*six-sixths or six one-sixths*); the student can then continue to two wholes (*twelve-sixths or twelve one-sixths*), and so on (Van de Walle et al., 2012). When instructional strategies involve iterating unit fractions, Van de Walle and colleagues (2012) state that students begin to develop the understanding that "the numerator *counts* and the denominator tells what fractional part is being counted" (p. 213).

Iterating unit fractions is also a meaningful way to introduce students to improper fractions; due to confusion and a lack of understanding of fractions as a single value, mixed numbers represented symbolically should be avoided during early fraction instruction. When counting fractional parts, students should be able to identify when they have counted more parts than necessary to complete one whole, resulting in excess pieces that belong to a second whole. For example, counting seven one-fourth parts on a number line will result in seven-fourths. Students will recognize that the fractional parts are greater than one whole for any of the following reasons: no more parts will fit into the shape that represents the whole, more than one set of items is needed to make groups or subsets, they have surpassed *1* (or another whole) on the number line, or the numerator is greater than the denominator; this is an understanding which supports a later transition into mixed number representations.

AN EXAMPLE

Siebert and Gaskin (2006) write that equipartitioning and iterating have often been cited as concepts that are important to developing fraction understanding. In addition, Siebert and Gaskin (2006) state that "powerful fraction reasoning involves both images" (p. 397).

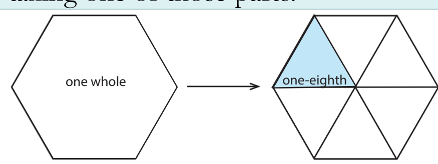
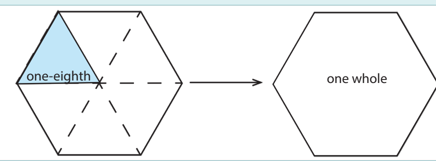
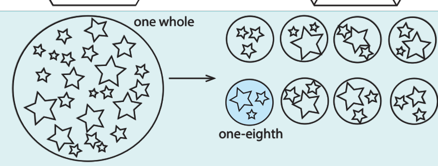
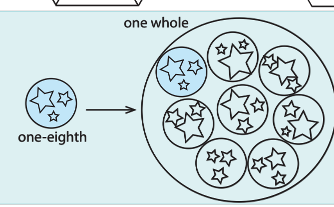
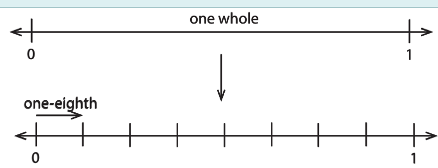
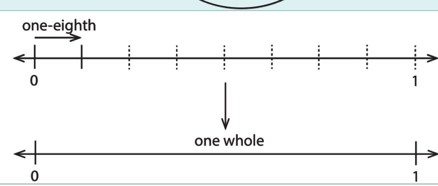
	Partitioning	**Iterating**
	$\frac{1}{8}$ is the amount we get by *dividing* a whole into eight equal parts and taking one of those parts.	$\frac{1}{8}$ is the amount such that eight copies of that amount, *put together*, make a whole.
Area Model		
Set Model		
Measurement Model (Length Model)		

Table modified from Siebert & Gaskin, 2006 p. 396.

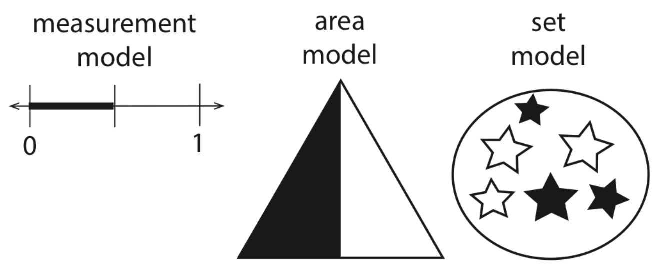
It is significant to note that models and manipulatives do not "speak" for themselves; a student does not inherently understand fractions from looking at or manipulating a model (Smith III, 2002). In addition, it is important to be aware that different models are more appropriate for some contexts than others (Petit et al., 2016). When students are presented with fraction situations they should realize that the context can be representative of different methods (e.g., part-whole, comparison, part-part, etc.) when using fraction models or manipulatives.

AN EXAMPLE

In early fraction instruction, students should have experiences with two representations of fractions: part-whole representations and comparison representations. When utilizing models and manipulatives in the classroom, identify that some models and manipulatives are better representations of certain situations than others (Petit et al., 2016).

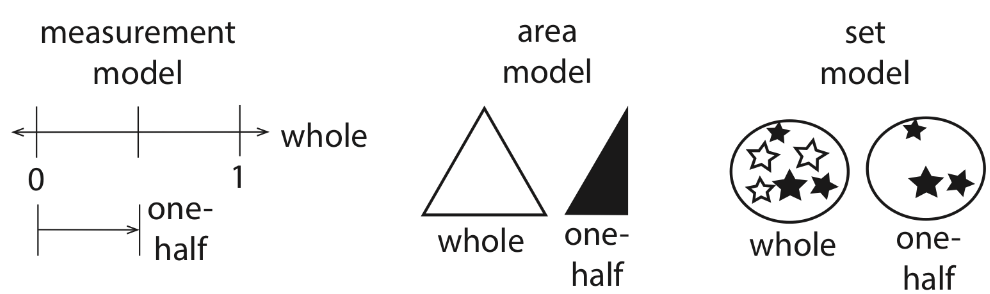
PART-WHOLE

The part-whole method represents the fractional part and the whole simultaneously; the fractional part is embedded in the whole (Watanabe, 2002).



COMPARISION

The comparison method represents the fractional part and the whole separately. The fractional part is identified by its relationship with the whole when the two are compared side by side (Watanabe, 2002).



Students should be made aware that not all fraction situations and models or manipulatives are best suited to both methods of fraction representation. Part-whole situations are often *best* represented by area models and set models, but that does not mean that part-whole situations cannot be represented with a measurement model. However, teachers should be aware that some students may struggle with area models when considering the part-whole relationship because they need to simultaneously consider both the size of the whole and the size of the part (Watanabe, 2002). In addition, Watanabe (2002) questions the use of area models as the best choice for representing fractions because students' "understanding of two-dimensional shapes and their area measurement is still being developed" (p. 459). These possible struggles with area models are further complicated when using concrete area models, because when a student removes a part from the whole, the whole no longer exists, and this may lead to some students comparing the two parts to each other as opposed to comparing the part to the whole (Watanabe, 2002).

In comparison, Charalambous & Pitta-Pantazi (2007) identify a possible struggle for students utilizing number lines to model fractions as students counting the number of partition marks on a number line instead of counting the number of intervals, thereby pinpointing the wrong unit, especially if the number line has a length of two or more units. However, this does not mean that either the area model or the measurement model should be avoided during instruction. Students benefit greatly when they experience working with the different types of fraction models and manipulatives in a variety of situations so that they can then decipher how the problem was represented and solved. These experiences will support students as they become fluent with the different models and manipulatives so that they can independently select an appropriate representation for a specific problem context. This awareness links back to the idea that students do not inherently understand fractions when given a model.

Utilizing manipulatives and visual representations in the classroom can be an "extraordinarily effective teaching technique to develop students' understanding", however, teachers must recognize that "for manipulatives to work effectively, activities must engage students and activate their prior knowledge and intuition" (Gould, 2011, p. 498). Gould points out that students must not only be engaged in their hands-on learning experiences, but the experiences must also tie into previous understanding, which relates back to providing students with explicit instruction on how, why, and when fraction manipulatives and visual representations work. For example, when students learn about "half", they can relate that learning to sharing a sandwich or a cookie with a friend or sibling; this is a way of connecting new experiences with old in order to assimilate the new knowledge into existing knowledge. In addition, Van de Walle and colleagues (2012) state that it is often useful to repeat the same activity with different fraction representations because they offer different opportunities to learn. To further enhance this point of explicitly and authentically incorporating concrete manipulatives and visual representations into everyday fractions instruction, Petit and colleagues (2016) write that, "the use of visual models (both teacher and student generated) should permeate instruction; not just be an incidental experience, but a way of thinking and learning for students" (p. 6). Van de Walle and colleagues (2012) state that the link between physical and visual models leading to the use of mental models is critical in building students' understanding of fractions. It is these early experiences with manipulatives and models that support students' mental representations of fractions and set the foundation for later success when considering fractions symbolically and eventually when operating with rational numbers.

CAUTIONS AND MISCONCEPTIONS

An early introduction to fractions written symbolically can cause confusion and lead to student misconceptions (Bamberger, Oberdorf, & Schultz-Ferrell, 2010). Students should first experience fraction names spoken aloud in conjunction with an image or a concrete representation, which will help emphasize the notion that fractions are *one* number that represent a quantity or how much of something, or an *amount* (Small, 2014). Then, students should view fraction names in word form, again in conjunction with an image or a concrete representation. Finally, after students have had many experiences with different representations of fractions visually and concretely, students should be exposed to fraction names written symbolically using numerals. According to Watanabe (2002), "Introducing the standard notation in the primary grades simultaneously with fraction concepts may be too demanding for many students." For example, when students begin learning about addition, they first comprehend finding a sum by using manipulatives (putting blocks together) before they are shown the symbolic representation (3 + 4).

With fractions comes an additional challenge to view two numerals, which previously each carried independent meaning, as a combination that has a new, joined meaning. Not only must they understand the concept of fractions, but they must also realize that familiar numerals are playing two different roles" (p. 462). Furthermore, Bamberger and colleagues (2010) identify possible student confusion as a result of a lack of understanding of what the numerator and denominator represent as well as what each of those numbers *mean*. To combat this confusion, students should be "introduced to the vocabulary of fractional parts after being exposed to a fractional situation. In the early grades, just name the parts; don't introduce the symbolism" (Bamberger et al., 2010, p. 39). This notion of understanding the role of each numeral in a fraction is connected with the idea expressed by Van de Walle and colleagues (2012) that the numerator *counts* the number of parts and the denominator *names* the part. Even if students are not introduced to the formal names *numerator* and *denominator*, they should be familiar with the role of each numeral. However, teachers should refrain from referring to the numerator as the "top number" and the denominator as the "bottom number", as this reinforces the notion that fractions are two numbers and not a single value.

In addition, Lewis et al. (2015) state that fraction vocabulary and notation are not intuitive, meaning it is difficult for students to transition from a whole number mindset to a mindset that includes fractional understanding. Therefore, Empson (2002) asserts that it is essential to associate clear and concise language with partitioned shapes and fractional models. Students should often be reminded or asked for the type of part (e.g., thirds), how many parts there are, and what the referent whole is. In addition, Small (2014) states the importance of presenting students with word representations of partitions and fractions before symbols; this emphasizes the concept of a fraction as one number, not a two-part number or two numbers separated by a fraction bar. Siebert and Gaskin (2006) reinforce this recommendation by writing that vocabulary has a powerful impact on students' conceptualization of fractions, and that teachers should avoid using language that encourages students to think of fractions as two-part numbers (e.g., "What is that piece called?" and "Three out of five equal pieces"). Instead, teachers should utilize questioning that emphasizes the part-whole relationship of fractions, such as, "How much brownie does one person get?", and "How many of that piece would fit into the whole?" (Empson, 2002).

LEARNING MAP INFORMATION

The learning map section for this sequence of activities begins with students recognizing and representing wholes in each of the identified models (i.e. measurement, set, and area). Once students are able to identify parts of a whole and the given wholes, they should proceed to recognizing and representing proper fractions within each of the identified models. An alternate pathway for students once they recognize parts of a whole would be through understanding fraction parts (i.e. numerator and denominator) and unit fractions; these understandings can then lead to recognizing and representing proper fractions with the identified models.

Corresponding to the available research, once students have an understanding of unit fractions and representing and recognizing proper fractions shown in different models, they can extend their understanding to recognizing and representing improper fractions using each of the identified models.

NOTE: The dashed connections between RECOGNIZE PARTS OF A GIVEN WHOLE OR A UNIT (and each of the recognize/represent whole on a model nodes) and the "recognize" or "represent proper fractions with a model" nodes contain nodes for "recognizing" and "representing" the unit fractions and equipartitions of halves, thirds, fourths and tenths.

The learning progression then extends to recognizing and representing proper, and later improper, fractions symbolically. The culmination of understanding for this map model ends with the student's ability to explain the meaning of a fraction (i.e. the relationship between the numerator and the denominator) as well as the meaning of the parts and wholes within a given context.

INSTRUCTIONAL ACTIVITIES

The activities in this unit are designed to build students' conceptual understanding of fractions prior to exposure to fractions in symbolic representations. Students will explore fractions by manipulating and creating proper and improper fractions using each of the identified models (i.e. measurement, set, and area). During the first lesson (which may take more than one class period to implement), students will begin to develop the idea that fractions can be numbers that describe distances along a line, through a large, whole-class number line activity. During each lesson, students will interact with the same number line to continue to build the understanding that fractions are numbers. Students will also use fraction bags, with a variety of model manipulatives, to investigate the idea of a whole as well as the concept of equal parts making up a whole (Colomb & Kennedy, 2005).

The second lesson will extend students' understanding of recognizing and representing wholes as a collection of equal parts to identify and iterate unit fractions in order to create proper and improper fractions. Students will be exposed to a variety of fraction models during this lesson, including rope or string (Zhang, Clements, Ellerton, 2015). Students will begin to develop an awareness of the relationship between unit fractions of a same size/shape whole: that the larger the size of an equal-size part, the fewer parts there will be, and vice versa (the smaller the size of the equal-size part, the more parts there will be). This lesson will also transition students from working with concrete manipulatives to images of models. Students will continue to use fraction names in word form during this lesson.

The third lesson continues to focus on building fraction understanding by creating and identifying fraction models and connecting the model representations to symbolic representations. During this lesson, students will explore measurement problem situations that include sharing glasses of a beverage, as well as listening and responding to a reading of the text, *Give Me Half!* by Stuart J. Murphy (Kikosicki & Prekeges, 2012; Zhang, Clements, Ellerton, 2015).

During the final lesson, students will continue to consider fractions in their symbolic form with the emphasis on the relationship between the numerator and denominator and the role of each part (i.e. the numerator as the "counting" number and the denominator as the fractional parts being counted). Students will continue to utilize fraction models as they extend their understanding to include improper fractions. The unit will finish by revisiting the large, whole-class number line to reinforce the idea of fractions as numbers.

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Understanding Fractions as Numbers

OVERVIEW OF INSTRUCTIONAL ACTIVITIES

Lesson	Learning Goal	Nodes Addressed
Lesson 1	Students will identify or represent a given whole using set models, length models, and area models. Students will also identify or model equipartitions for halves and fourths using set models, length models, and area models.	▶ RECOGNIZE PARTS OF A GIVEN WHOLE ▶ RECOGNIZE WHOLE ON A SET MODEL ▶ RECOGNIZE WHOLE ON A LENGTH MODEL ▶ REPRESENT WHOLE ON A SET MODEL ▶ REPRESENT ONE-HALF ON A SET MODEL
Lesson 2	Students will increase their conceptual understanding of fractions by equipartitioning and iterating equal-size parts using a variety of manipulatives as well as length, set, and area models. In addition, students will count unit fractions to identify and create models of proper and improper fractions.	▶ RECOGNIZE PROPER FRACTIONS WITH AN AREA MODEL ▶ RECOGNIZE PROPER FRACTIONS WITH A SET MODEL ▶ RECOGNIZE PROPER FRACTIONS WITH A LENGTH MODEL ▶ REPRESENT WHOLE ON AN AREA MODEL ▶ REPRESENT WHOLE ON A SET MODEL ▶ REPRESENT WHOLE ON A LENGTH MODEL ▶ PARTITION A NUMBER LINE FROM 0 TO 1 INTO EQUAL PARTS ▶ REPRESENT PROPER FRACTIONS WITH AN AREA MODEL ▶ REPRESENT PROPER FRACTIONS WITH A SET MODEL ▶ REPRESENT PROPER FRACTIONS WITH A LENGTH MODEL ▶ EXPLAIN *UNIT FRACTION*
Lesson 3	Students will informally add unit fractions of a given whole to identify and create provided fractions. In addition, students will informally subtract unit fractions from a given whole to identify and create provided fractions. Students will also connect their understanding of fraction names with their understanding of pictorial representations and models in order to identify the relationship that exists between the numerator and denominator and to represent fractions symbolically.	▶ RECOGNIZE *DENOMINATOR* ▶ RECOGNIZE *NUMERATOR* ▶ EXPLAIN *UNIT FRACTION* ▶ REPRESENT PROPER FRACTIONS WITH A LENGTH MODEL ▶ RECOGNIZE PROPER FRACTIONS WITH A SYMBOLIC REPRESENTATION ▶ REPRESENT PROPER FRACTIONS AS A SYMBOLIC REPRESENTATION ▶ EXPLAIN THE POSITION OF FRACTIONS ON THE NUMBER LINE ▶ EXPLAIN THE FRACTION RELATIONSHIP OF X/Y ▶ EXPLAIN RELATIONSHIPS BETWEEN UNIT FRACTIONS
Lesson 4	Students will consider different representations of proper and improper fractions in order to conceptualize the relationship between numerators and denominators.	▶ RECOGNIZE ANY FRACTION WITH A LENGTH MODEL ▶ RECOGNIZE ANY FRACTION WITH AN AREA MODEL ▶ RECOGNIZE ANY FRACTION WITH A SET MODEL ▶ RECOGNIZE PROPER FRACTIONS WITH A SYMBOLIC REPRESENTATION ▶ RECOGNIZE ANY FRACTION WITH A SYMBOLIC REPRESENTATION ▶ REPRESENT ANY FRACTION AS A SYMBOLIC REPRESENTATION ▶ EXPLAIN THE FRACTION RELATIONSHIP OF X/Y ▶ EXPLAIN *NUMERATOR* ▶ EXPLAIN *DENOMINATOR*

Understanding Fractions as Numbers

INSTRUCTIONAL ACTIVITY

Lesson 1

LEARNING GOAL

Students will identify or represent a given whole using set models, length models, and area models. Students will also identify or model equipartitions for halves and fourths using set models, length models, and area models.

PRIMARY ACTIVITY

Students will use fraction bags which contain set model manipulatives, partitioned images, and length model manipulatives to identify and model wholes. In addition, students will use the manipulatives from the fraction bags to discuss and model equipartitioning sets, number lines, and area models in order to represent halves and fourths.

NOTE: This lesson will likely require more than one class period to complete.

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

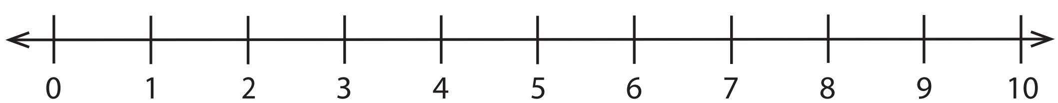
- Whole
- Fraction
- Equal-size groups
- Partition
- Set model
- Length model
- Area model
- Halves
- One-half
- Fourths
- One-fourth

MATERIALS

- Fraction bags (Recommend one bag for every two students.):
  - One sandwich or quart-size baggie
  - Five counters, any color
  - Eight interlocking cubes, any color
  - Six beans
  - Two images (a star and a house) from page 1 of the INSTRUCTIONAL ACTIVITY SUPPLEMENT – cut each image along the dashed line to create one equipartitioned figure and one figure with unequal-size partitions
  - Two paper number lines, labeled with "zero" and "one" and divided into four equal parts
  - Cuisenaire rods or fraction strips, one whole and four one-fourths
- Larger paper number line for display, commercial or homemade
- Highlighters or markers
- Scissors
- INSTRUCTIONAL ACTIVITY SUPPLEMENT (Recommend one copy of pages 1 and 4 for every student.)

IMPLEMENTATION

Display the large paper number line, which has been pre-labeled with whole numbers 0 – 10. Ensure that there is enough room between each whole number to add in fractions as the unit progresses.



Review the number system and number lines by discussing students' previous experiences.

Ask students the following questions while referencing the number line:

- What value is labeled between 3 and 5?
- Are there any values between 8 and 10? How do you know?
- Are there any values between 0 and 1? Explain.
- Can you have less than one whole?

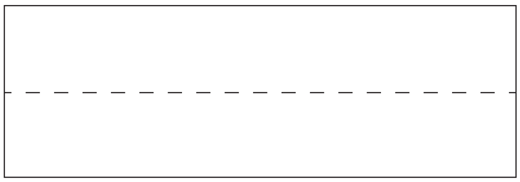
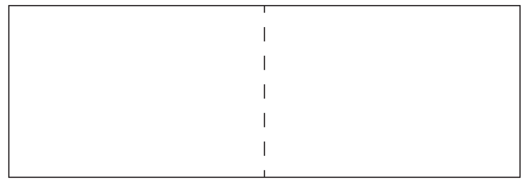
Display an everyday object that can easily be divided, such as a candy bar.

Ask students the following questions while referencing the candy bar:

- What is this?
- How many are there? How do you know?
- Can that number be found on the number line?
- Where on the number line is the number?

Explain that you are going to share the candy bar equally with one other person. **Ask** students how many equal-size parts you need to share with one other person.

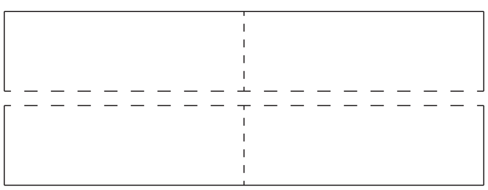
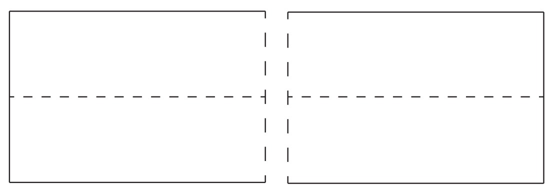
Divide the candy bar into two equal-size parts.

Ask students the following questions:

- [Show the divided candy bar.] Is this still a candy bar? Have I changed it in any way? How do you know?
- How many are there now? How do you know?
- Is there one whole candy bar or two whole candy bars? How do you know?
- How much of the whole candy bar will I get? How do you know?
- Can that number be located on the number line?
- Where on the number line is the number?

Divide the candy bar into four equal-size parts by dividing each half in half.

Repeat the questioning process.

Ask students what they think fractions and wholes are. **Explain** that one section of the candy bar is a fraction of the whole candy bar.

Refer back to the divided candy bar. **Ask** students, "What is the whole?" **Emphasize** that the whole, in the case of the candy bar, is the original candy bar (the whole candy bar).

Refer to the number line. **Ask** students, "What do you think represents one whole on the number line?" **Emphasize** that the numbers are whole numbers, and therefore one whole is the distance between one value and the next (i.e. the distance between 0 and 1 or between 4 and 5, etc.).

Arrange students into groups of two and distribute one fraction bag to each group.

Students should remove all the objects from the bag.

Use the following guiding questions to lead a discussion. Students should respond in groups of two first, then as a whole group.

GUIDING QUESTIONS

Elicit student thinking:

- What is in the fraction bag?
- Have you used any of these tools before? If so, when and how?
- How could these tools relate to fractions?

Determine if the student can RECOGNIZE PARTS OF A GIVEN WHOLE:

- If you consider the items in the fraction bag as a whole set, how many parts (pieces) make up the whole? How do you know?
- What are the parts (pieces) that make up the whole? How do you know?
- Group the interlocking cubes together. If you consider this set of cubes as the whole set, how many parts (pieces) make up the whole? How do you know?
- Group the counters together. If you consider this set of counters as the whole set, how many parts (pieces) make up the whole? How do you know?
- Group the beans together. If you consider this set of beans as the whole set, how many parts (pieces) make up the whole? How do you know?
- If you consider the image of the house as one whole, how many parts (pieces) make up the whole? How do you know?
- If you consider the distance from 0 to 1 on the number line one whole, how many parts (pieces) make up the whole? How do you know?

Direct students to group all of the beans together and put the other tools back in the fraction bag.

Ask students how many beans make up the whole set. **Write** on the board, "The whole set of beans is six beans."

NOTE: **Refrain** from using numerals during lesson one, **emphasize** the word forms and visual representations for whole numbers and fractions so the students focus on the number names and not the symbolic notations.

Tell students to divide the beans into two equal groups. **Ask** students how many beans are in each group. **Write** on the board, “There are two groups of three beans.”

Tell students to group the beans in one large group. **Direct** students to divide the group of beans in half. **Ask** students how many groups they made and how many beans are in each half. **Write** on the board, “One-half of six beans is three beans.”

Ask students, “Is dividing the group of beans in half the same or different than dividing the group of beans into two equal groups? Explain.”

Direct students to place the beans to the side and remove the counters and interlocking cubes from the fraction bag.

Require students to practice making a whole group and then dividing the whole group in half and dividing the whole group into two equal-size groups using the counters and the interlocking cubes.

Note there are five counters, providing an opportunity to observe how students handle a scenario in which there is not an even “fair share” and students must imagine halving a single object.

Use the following guiding questions to support students’ understanding as students divide the groups of counters and interlocking cubes.

GUIDING QUESTIONS

Elicit student thinking:

- When and why have you used counters before?
- When and why have you used interlocking cubes before?
- When have you had to share something with someone else?
- When have you had to divide something into equal-size groups?

Determine if the student can RECOGNIZE PARTS OF A GIVEN WHOLE:

- Group the counters together. If you consider this set of counters as the whole set, how many parts (pieces) make up the whole? How do you know?
- Group the interlocking cubes together. If you consider this set of interlocking cubes as the whole set, how many parts (pieces) make up the whole? How do you know?
- Do you have to connect the interlocking cubes to represent them as a whole? Why or why not?

Determine if the student can RECOGNIZE WHOLE ON A SET MODEL:

- [Draw four stars.] What is the whole for the set of stars? How do you know?
- [Create a group with several interlocking cubes and counters.] What is the whole for the set of cubes and counters? How do you know?
- [Create a group with several interlocking cubes and counters.] What is the whole for the set of interlocking cubes? How do you know?
- [Create a group with several interlocking cubes and counters.] What is the whole for the set of counters? How do you know?
- [Display several counters composed of two different colors.] What is the whole for the set of counters? How do you know?
- [Display several counters composed of two different colors.] What is the whole for the (choose one color) counters? How do you know?

Determine if the student can REPRESENT WHOLE ON A SET MODEL:

- Show me the whole for the set of counters. How do you know this is the whole?
- Show me the whole for the set of interlocking cubes. How do you know this is the whole?
- If a whole is represented as a set of three counters, what does that look like? How do you know?
- Show me a whole that has six interlocking cubes and three counters. How do you know this is the whole? How many parts make up the whole?

Determine if the student can REPRESENT ONE-HALF IN A SET MODEL:

- Show me the whole for the set of counters. How many counters are in one-half of the set of cubes? How do you know?
- Show me the whole for the set of interlocking cubes. How many interlocking cubes are in one-half? How do you know?
- Show me one-half of a set of four counters. How do you know this is one-half?
- Show me one-half of a set of seven interlocking cubes. How do you know this is one-half? What did you do with the seventh cube? Why?

Direct students to place the counters and interlocking cubes to the side with the beans. Bring student attention back to the whole group.

Brainstorm as a class examples of "real world" wholes (sets) that can be divided up (e.g., marbles, pencils, candies, paper clips, etc.). Try to encourage everyday items in addition to food items.

NOTE: Focusing only on food examples could inadvertently lead to student misconceptions that fractions only relate to situations that involve food.

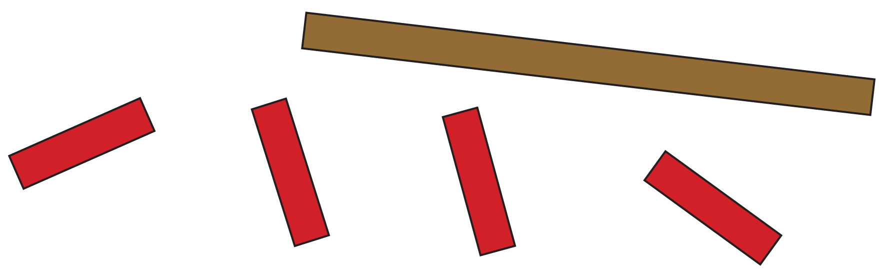
Direct students to take the Cuisenaire rods (or fraction bars) out of the fraction bag.

Ask students how the Cuisenaire rods (or fraction bars) are the same as and different than the beans, interlocking cubes, and counters.

Ask students which rod (or bar) they think represents the whole for the Cuisenaire rods (or fraction bars). Students should discuss with their partners and then share with one other group of two. After discussing, **select** one pair of students to share their answer and reasoning.

Draw attention to the fact that when you divided or partitioned the sets, you were able to physically group the pieces and that you can do that with Cuisenaire rods or fraction strips as well. However, **emphasize** that with the Cuisenaire rods or fraction strips, the pieces will be grouped by same-size pieces (same color) and will not always yield the same number of pieces in each group. With set models, the groups must always be the same size, but the pieces do not need to be the same size.

Display the brown rod (or one-whole strip) and the red rods (or one-fourth bars).



Require students to use their Cuisenaire rods (fraction strips) to show how the red rods (or one-fourth bars) and the brown rod (or one-whole strip) can be the same length.

Use the following guiding questions to lead a class discussion.

GUIDING QUESTIONS

Elicit student thinking:

- Have you used these tools before? If so, when and why?
- How is the rod (or bar) the same or different than the number line?
- How is the rod (or bar) the same or different than the set model?
- How do you think we will partition (divide) the rod (or bar) into equal-size parts?

Determine if the student can RECOGNIZE WHOLE ON A LENGTH MODEL:

- Which color Cuisenaire rod (fraction strip) do you think represents one whole? Why?
- Can the red rod (one-fourth strip) ever represent one whole? Why or why not?
- When using a length model like these rods (strips), how do you know what represents the whole?

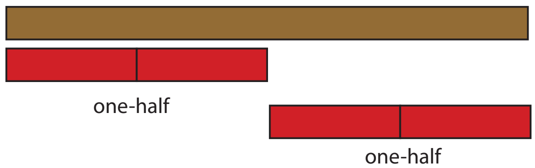
Ask students how many red rods (or one-fourth bars) are the same length as one brown rod (or one-whole bar). **Require** students to explain their thinking.

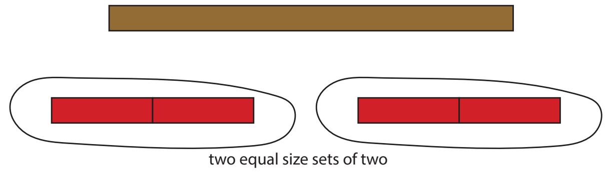
Explain that because four red rods (or one-fourth bars) are the same length as one brown (or one-whole bar), which represents a whole, each of the red rods (or one-fourth bars) are called *fourths*. **Emphasize** the link between the *four* red rods (or one-fourth bars) and the fraction name, *fourths*.

Ask students how many *fourths* it would take to represent one whole. **Require** students to model and explain their thinking using the Cuisenaire rods (or fraction bars).

Ask students how to represent one-half of the whole, the brown rod (or the one-whole bar), using fourths. **Require** students to explain their thinking.

For students who appear to struggle, have them think of the four red rods (or one-fourth bars) as a set, and **ask** them what would be one-half of the set. Or, **ask** them how they could divide the set of four red rods (or one-fourth bars) into two equal-size groups.



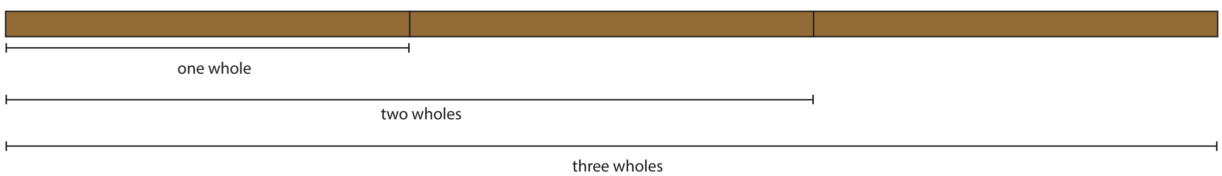


Ask students how the Cuisenaire rods (or fraction bars) are the same as or different than the set models.

Model lining up three brown Cuisenaire rods (or fraction bars) in a row, end to end.

Ask students how many wholes have been displayed, and if the brown rod (or one-whole bar) represents one whole. **Require** students to explain their thinking.

Emphasize that the length of the rod (or bar) is the whole, and therefore the length from the beginning of the row to the end of the row represents three wholes. The lengths from the beginning of the row to end of the second rod (or bar) represents two wholes, and the length from the beginning of the row to the end of the first rod (or bar) represents one whole.

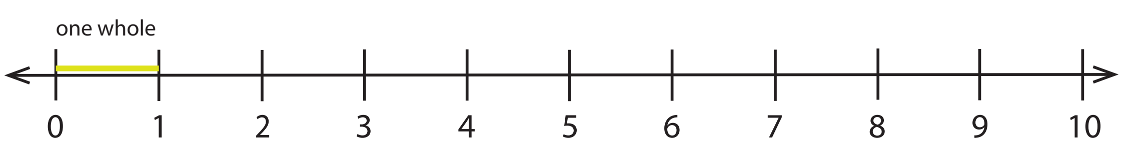


Ask one student to show the class how many *fourths* are in one whole using the Cuisenaire rods (or fraction bars). Be sure to leave all three brown rods (or one-whole bars) visible.

Direct student attention back to the large number line. **Ask** students if they know what the whole is on the number line.

Identify that the whole is a single unit (the distance from 0 to 1 or any other set of consecutive values). **Emphasize** that the unit is the distance from one *whole value* to the next *whole value*, not a single point on the number line.

Highlight the distance from 0 to 1 on the large number line and label the distance "one whole" or "one whole unit."



Discuss the definition of a whole by displaying the number lines from page 2 of the INSTRUCTIONAL ACTIVITY SUPPLEMENT and asking the following guiding questions.

GUIDING QUESTIONS

Elicit student thinking:

- When and why have you used a number line before?
- What is a tool we use in or out of school that looks like a number line?
- What do you notice about these number lines?
- How are these number lines the same? How are they different?

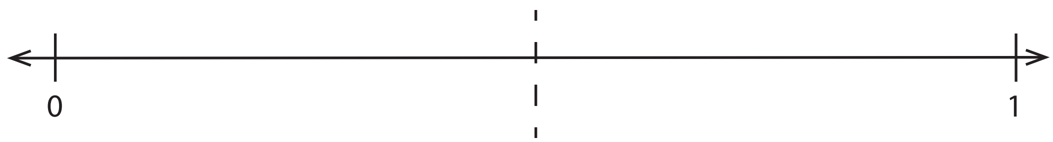
Determine if the student can RECOGNIZE WHOLE ON A LENGTH MODEL:

- [Display the number line labeled 0 through 4.] How many units are shown on this number line? If one unit is a whole, how many wholes are on the number line? How do you know?
- [Display the number line labeled 0 through 4. Highlight the distance between 2 and 3.] Can this be one whole? Why or why not?
- [Display the number line labeled 5 through 10.] How many units are shown on this number line? If one unit is a whole, how many wholes are on the number line? How do you know?
- [Display the number line labeled 5 through 10. Highlight the distance between 9 and 10.] Can this be one whole? Why or why not?

If necessary, continue this type of questioning with number lines of various lengths.

Direct students to place the Cuisenaire rods (or fraction bars) to the side and remove the number lines from the fractions bags. There is one number line for each student.

Ask students how to partition the number line into two equal-size lengths. Students should recognize that the paper number line can be folded in half to create two equal-size lengths. **Require** one student to model for the class how they partitioned their number line into two equal-size lengths.



Ask students how to divide the number line in half. **Follow up** by asking if dividing the number line in half is the same or different as partitioning the number line into two equal-size lengths.

Direct students to partition the number line into four equal-size lengths. **Allow** students to explore and determine how to partition the number line into four equal-size lengths. **Encourage** students to collaborate and work together if they struggle before you assist them.

Use the following guiding questions to support student understanding.

GUIDING QUESTIONS

Elicit student thinking:

- When and why have you used a number line before?
- What is a tool we use in or out of school that looks like a number line?

Determine if the student can PARTITION A NUMBER LINE FROM 0 TO 1 INTO EQUAL PARTS:

- [Point to the student's number line.] How did you partition (divide) this number line into four equal-size parts?
- How can you determine if the parts of a number line are the same length?
- [Point to the partitions on the student's number line.] What does it mean to say that these parts are equal sizes?
- [Draw a number line with unequal-size parts.] Are these parts the same size? How do you know?
- [Draw a number line with equal-size parts.] Are these parts the same size? How do you know?

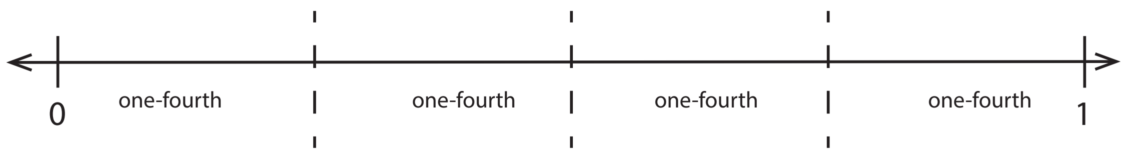
Require a student to model how to create four segments of the same length on the number line.

Ask students whether they would get the same result or a different result if they divided the number line into fourths. Students should discuss with a partner before sharing with the class.

Select two or three students to share their responses.

Emphasize that *fourths* imply *four equal-size parts*, therefore when the number line was partitioned into four equal-size lengths, it was divided into fourths.

Model for students writing three segments on the number line where the folds are, if they have not already. **Write** "one-fourth" in each segment. As you write, **say** the fraction name aloud.



NOTE: **Refrain** from using numerals during lesson one. **Emphasize** the word forms and visual representations for whole numbers and fractions so the students focus on the number names and not the symbolic notations.

Ask students how representing fractions with a number line is the same as or different than using Cuisenaire rods (or fraction bars). **Require** students to explain their thinking.

Ask students how representing fractions with a number line is the same as or different than using counters, beans, and interlocking cubes. **Require** students to explain their thinking.

Ask students if partitioning the number line is a familiar activity. What else have they partitioned in or out of school?

Direct students to place the number lines to the side with the other manipulatives and to remove the two partitioned images from the fraction bag (the images from page 1 of the INSTRUCTIONAL ACTIVITY SUPPLEMENT).

There are four pieces altogether: two pieces that create the image of a star (which have been equipartitioned) and two pieces that create the image of a house (which are not equipartitioned).

Ask students to discuss the following questions in their partner groups before sharing with the whole group:

- How many parts (or pieces) are there? How do you know?
- How many wholes are there? How do you know?
- Each whole is divided into how many parts (or pieces)? How do you know?
- Are the parts (or pieces) of each whole the same size? Explain.
- Do the parts (or pieces) from a single whole have to be the same size? Why or why not?
- If there are two parts (or pieces) from a single whole that are the same size, how can each part be named? Explain.

Ask students whether both partitioned images represent one-half. If they respond that just one partitioned image represents one-half, ask them which one and why.

Emphasize that only one of the partitioned images represents one-half, because when an area is partitioned to represent fractions, each part (or piece) *must* be the same size (have the same area), though they do not necessarily need to be the same shape.

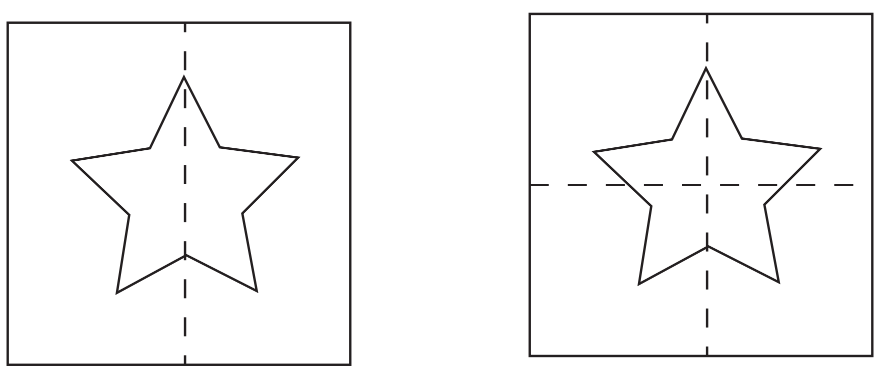
Distribute scissors so that each partner group has at least one pair.

Ask students if they can cut the equipartitioned image into four equal-size parts. Let students explore this possibility in the partner groups before intervening. If necessary, suggest that each student cut one half, and remind students that each half must be cut into two equal-size parts.

Ask students if they have created four equal-size parts by cutting each half in half, and **require** them to explain their thinking.

Ask students to identify the whole they have been asked to cut into equal-size parts.

Emphasize the importance of understanding *what* the whole is by **explaining** that <u>if the image is the whole</u>, they did *not* cut the image into four equal-size parts (or pieces). Therefore, each part is *not* one-fourth of the whole. However, <u>if the whole is the entire piece of paper the image is printed on</u>, then they did indeed cut the whole into four equal-size parts (or pieces), and each part (or piece) represents *one-fourth* of the whole.



two equal-size parts of the star

two equal-size parts of the square paper

four different-size parts of the star

four equal-size parts of the square paper

Direct students to place the scissors and partitioned images to the side with the other manipulatives. Display page 3 from the INSTRUCTIONAL ACTIVITY SUPPLEMENT so students can see the images of the set model, length model, and area model.

Select a few students to identify each of the fraction models displayed.

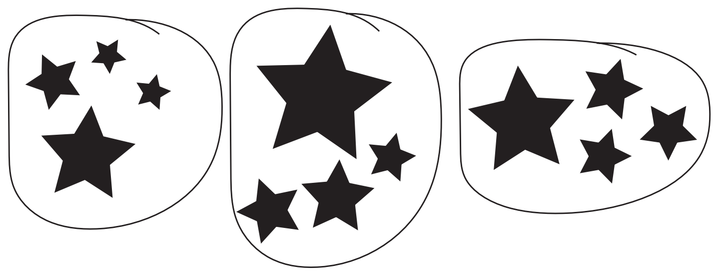
For each model, **ask** students the following questions:

- What do you notice about the model?

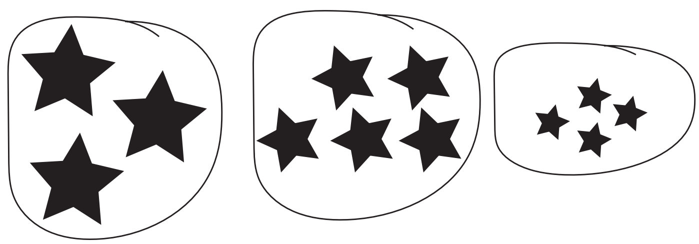
- What is the whole that is represented by the model? How do you know?
- How many equal parts (or pieces) or groups are represented by the model? How do you know?

After discussing each model, make **explicit** that the size of the items in the subset on the set model does not matter; it is the size of the groups that *must* be equal. In addition, the size of the parts (or pieces) on the length and area model *must* be the same size. **Draw** images similar to the following images to highlight and demonstrate the distinction.

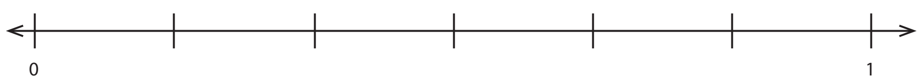
Correct Set Model: Twelve stars grouped into three groups of four.



Incorrect Set Model: Twelve stars grouped into three groups of different sizes, one group of three, one group of five, and one group of four.

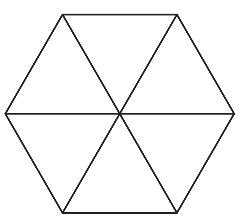


Correct Length Model: One whole partitioned into six segments of equal length.

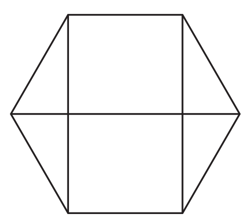


Incorrect Length Model: One whole partitioned into six segments of different lengths.

Correct Area Model: A whole partitioned into six equal size parts.



Incorrect Area Model: A whole partitioned into six different size parts.



Display two area models (or circle manipulatives) of the same size, each equipartitioned into a different number of parts (e.g., one model equipartitioned into fourths and another same-size-and-shape model equipartitioned into sixths).

Present the two models in a context, such as two clock faces, two plates, two pizzas, or something that is the same shape as the equipartitioned wholes.

Ask students to share what they notice about the two equipartitioned shapes, and to describe how the equipartitions are the same and how they are different.

Lead students, through strategic questioning and discussion, to the understanding that the greater number of equipartitions, the smaller each individual partition will be.

Present more examples using set models and number lines, each time discussing that the greater number of partitions leads to smaller-size partitions.

Distribute one copy of page 4 of the INSTRUCTIONAL ACTIVITY SUPPLEMENT to each student. The page is titled *Fraction Representations*.

Require students to review the content from this lesson by completing the first column of the *Fraction Representations* sheet. When students are finished, either collect the paper, have them keep it somewhere secure, or glue it into an interactive notebook. **Note**: Students will add more information to this sheet as they gain more understanding of fractions during the course of the unit.

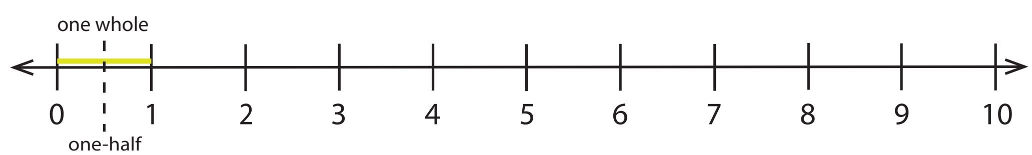
At the end of the activity, direct student attention back to the large number line labeled 0 through 10.

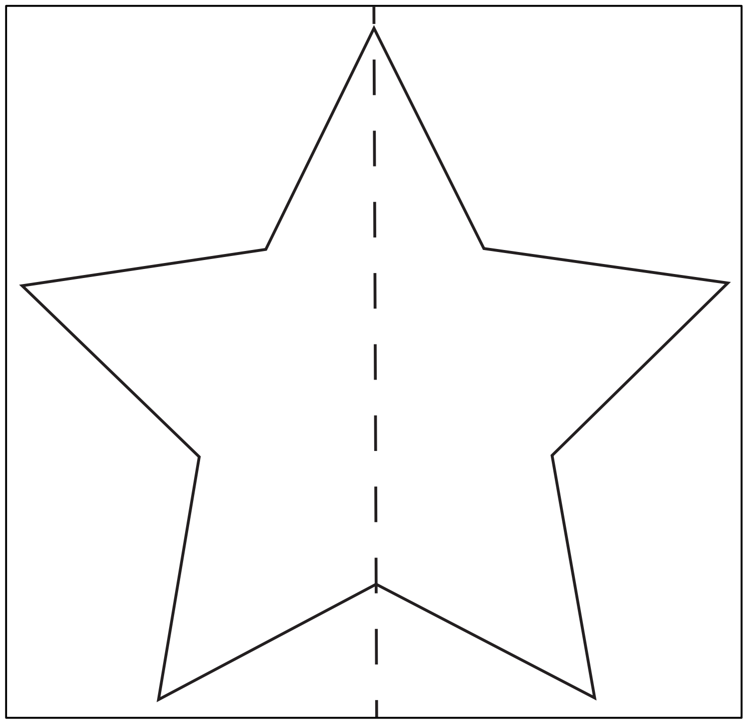
Ask students to discuss with a partner how to represent one whole on the large number line. Then, **select** one or two students to share with the whole group.

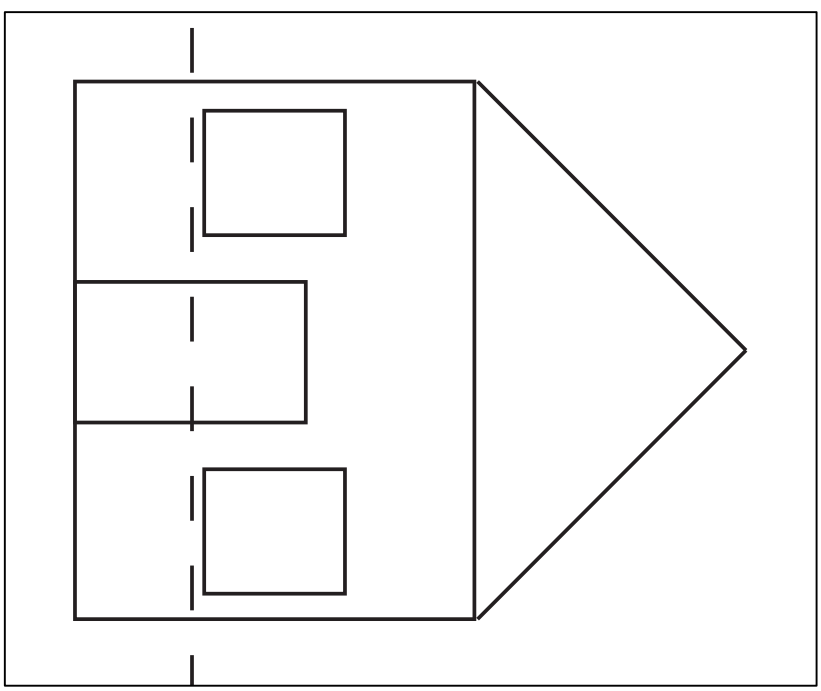
Ask students to discuss with a partner where they think one-half belongs on the large number line. Then, **select** one or two students to share with the whole group.

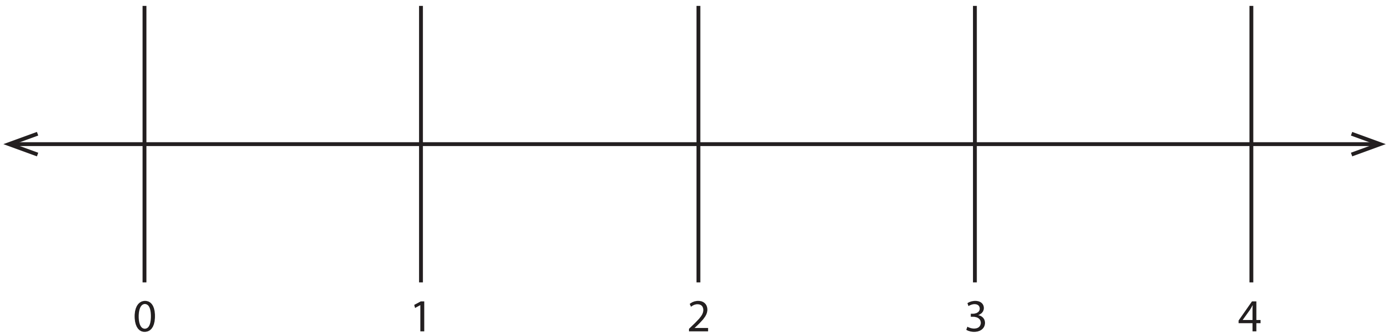
If a student suggests that one-half belongs where the numeral "5" is on the number line, redirect the class back to consider what represents one whole on the number line.

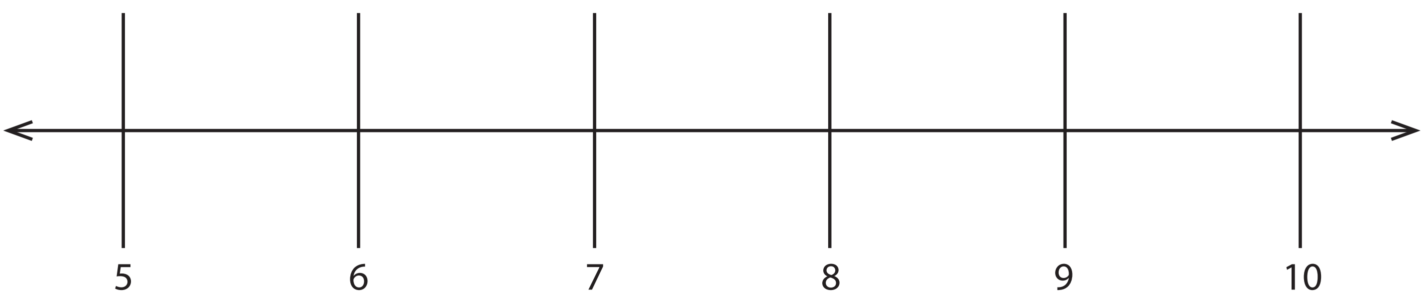
Model that one-half is between the numerals "0" and "1" by drawing a small line segment halfway between the two values.



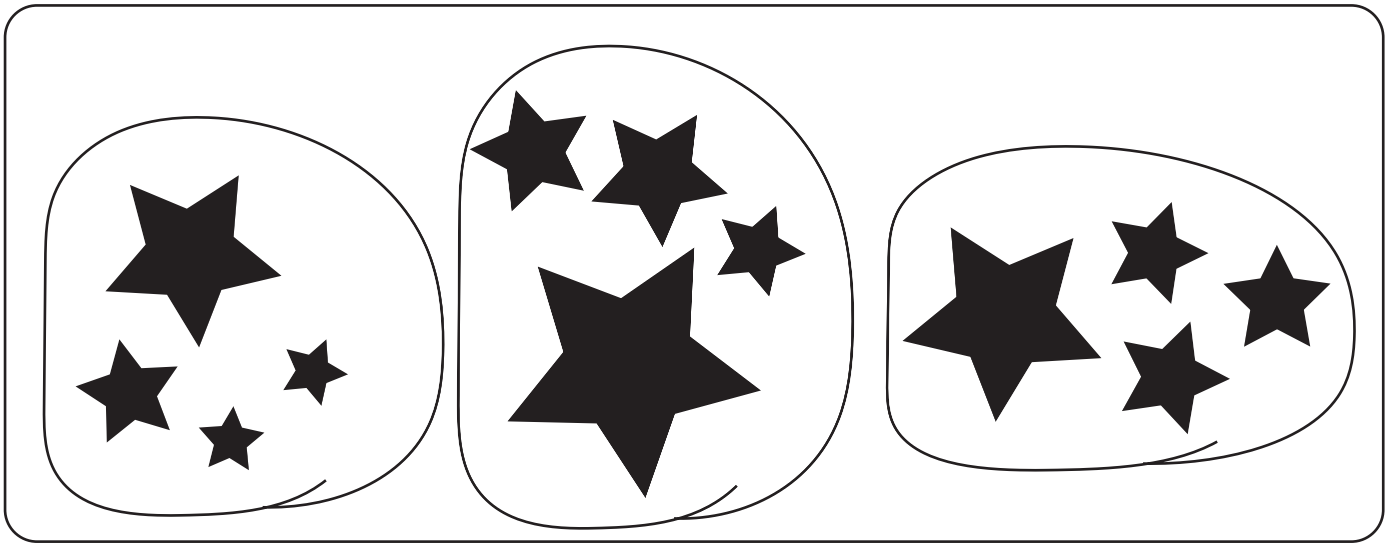


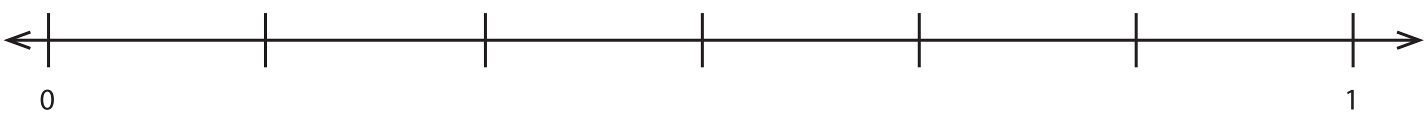


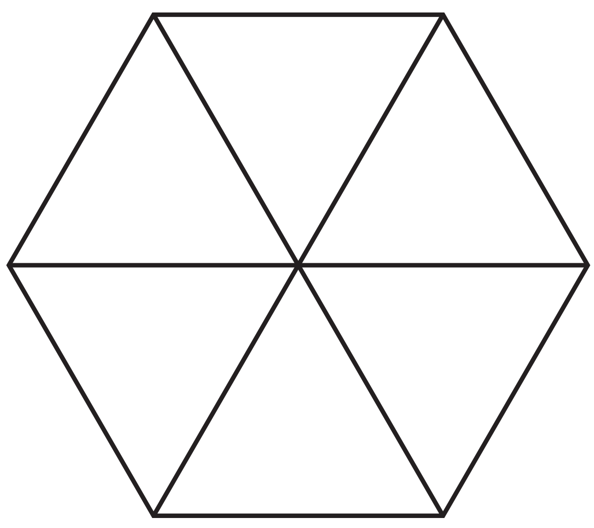

0
1
2
3
4


5
6
7
8
9
10

Fraction Models







Name ___________________________________

Fraction Representations

	Drawing of one whole	Drawing of a proper fraction	Word name of the proper fraction	Symbolic name of the fraction	Drawing of an improper fraction	Symbolic name of the improper fraction
Set Model						
Length Model						
Area Model						

Understanding Fractions as Numbers

INSTRUCTIONAL ACTIVITY

Lesson 2

LEARNING GOAL

Students will increase their conceptual understanding of fractions by equipartitioning and iterating equal-size parts using a variety of manipulatives as well as length, set, and area models. In addition, students will count unit fractions to identify and create models of proper and improper fractions.

PRIMARY ACTIVITY

Students will engage in equipartitioning and iterating groups of students, lengths of rope or string, Cuisenaire rods, interlocking cubes, number lines, pattern shapes, images of shapes, and sets of counters.

NOTE: This lesson will likely require more than one class period to complete.

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

- Whole
- Fraction
- Equal-size groups
- Partition
- Set model
- Length model
- Area model

MATERIALS

- Large number line for display – from Lesson 1
- Rope or string (a length of two and a half units long based on the large paper number line from Lesson 1.)

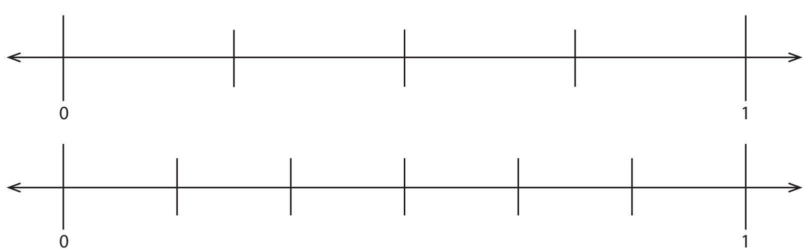
- A variety of manipulatives:
  - Beans
  - Counters
  - Interlocking cubes
  - Fraction circles
  - Fraction bars
  - Paper strips
  - Number lines
- Two sheets of notebook or printer paper (Fill one sheet with a large "0" and the other sheet with a large "1".)
- Markers or pens
- 8.5" × 2" Paper strips – cut sheets of printer paper into two-inch-wide strips (Recommend six strips for each student.)
- Glue sticks or glue bottles (Recommend one glue stick or glue bottle for every one to two students.)
- Scissors
- *Fraction Representations* from the LESSON 1 INSTRUCTIONAL ACTIVITY SUPPLEMENT
- INSTRUCTIONAL ACTIVITY STUDENT HANDOUT
- INSTRUCTIONAL ACTIVITY SUPPLEMENT (Recommend one copy for each student.)

IMPLEMENTATION

Redistribute or **require** students to **retrieve** the *Fraction Representations* sheet from the INSTRUCTIONAL ACTIVITY SUPPLEMENT in LESSON 1.

Review the content from LESSON 1 by selecting several students to share and explain their responses for each row of the first column on the *Fraction Representations* sheet.

Display two number lines of the same length (labeled from 0 to 1), the first equipartitioned into fourths and the second equipartitioned into sixths.

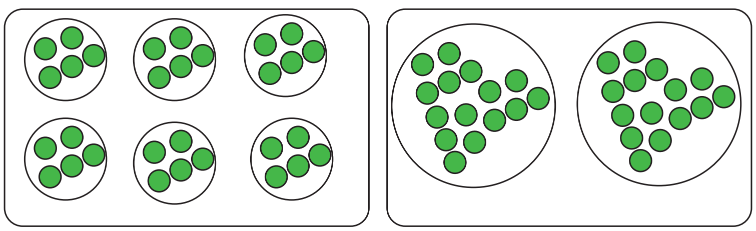


Ask students to share what they notice about the two number lines, to describe how they are the same, and to describe how they are different.

Ask students to discuss the following questions first in partners, then with the whole group.

- How many equal-size parts has the first number line been partitioned into? How do you know?
- How many equal-size parts has the second number line been partitioned into? How do you know?
- Which number line has more equal-size parts? How do you know?
- Which number line has been equipartitioned into lengths that are larger in size? How do you know?
- If you were doing a one-mile fun run where the 0 on the number line was the start and the 1 was the finish line, would you rather stop for water every one-fourth of the run (first number line) or every one-sixth of the run (second number line)? Why?
- On which number line would you stop more often for water? How do you know?
- On which number line would you have to run less between each water break? How do you know?

Place the number line off to the side for later comparison, and do not erase it. **Display** two set models. The models should have the same objects and the same size whole, but different-size sets.



Ask students to share what they notice about the two set models, to describe how the two sets are the same and how they are different.

Ask students to discuss the following questions first in partners, then with the whole group.

- How many equal-size groups has the first set been partitioned into? How do you know?
- How many equal-size groups has the second set been partitioned into? How do you know?
- Which set has more equal-size groups? How do you know?
- Which number line has been equipartitioned into groups that are larger in size? How do you know?
- If you were sharing a bag of candies, and each set was a full bag, would you rather share with five other people and get one-sixth of the bag (first set model) or one other person and get one-half of the bag (second set model)? Why?
- Which set model would allow you to have more candy if you were sharing with other people? How do you know?
- Which set model would allow you to share the bag of candy with more friends? How do you know?

Distribute a pair of scissors, a glue bottle or glue stick, and one copy of page 1 of the INSTRUCTIONAL ACTIVITY SUPPLEMENT to each student.

Require students to cut out the two cards on the INSTRUCTIONAL ACTIVITY SUPPLEMENT.

Direct students to finish the sentence on each card. **Ensure** each student understands that a greater number of equipartitions means that each partition is smaller in size when the wholes are the same size (and shape).

One card should read, "If the wholes are the same size, and the equal-size parts are larger, then the *number* of equal-size parts is <u>fewer</u>." And the second card should read, "If the wholes are the same size, and the equal-size parts are smaller, then the *number* of equal-size parts is <u>greater</u>."

Attend to the vocabulary used so that students connect *large* and *small* with the parts or pieces and not the number of parts or pieces. Likewise, students should connect the words *fewer* and *greater* with the number of parts or pieces.

Direct students to glue the two cards together, so there is writing on each side (if desired, the cards can be kept separate as well).

Collect the glue and scissors.

Project or **display** a pair of identical wholes equipartitioned into a different number of parts.

Ask students to hold up a card for either the left or right (top or bottom) model. For example, "The model on the right (point to the model) is __________ than the model on the left (point to the model)."

Students should then hold up the appropriate card. **Select** a few students to share their thinking for why they selected the card they did.

Repeat the activity using a variety of models as many times as necessary.

Collect the cards or have students store them in a safe place for future use.

Bring students' attention to the large number line from LESSON 1.

Point to the one-half mark (line segment) on the number line. **Ask** students what the segment represents. **Emphasize** that one-half is represented by the distance from 0 to the line segment, not just the position of the line segment.

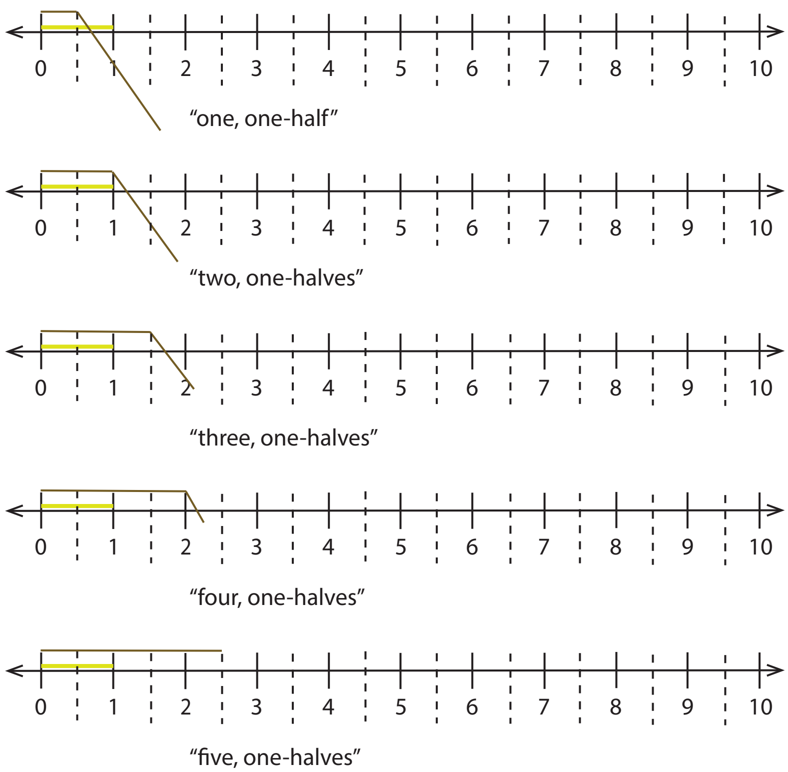
Identify that the number line is equipartitioned into single units. **Ask** students how to equipartition *each* unit into halves. **Select** students to add segments to the number line that represent halves in each unit on the number line. **Ensure** that each segment is exactly half way between each unit, reminding students that equipartitions *must* be the same size.

Ask students how many equal-size parts are between 0 and 1. Follow up with several other units, such as how many equal-size parts are between 5 and 6 or between 3 and 4.

Display the rope/string that is two and a half units long. **Ask** students how long they think the rope/string is. **Require** students to explain their responses.

Emphasize that we use fractions when we measure length, and therefore fractions are part of our number system.

Model how to measure the rope/string using the large number line from LESSON 1 as a "ruler." **Count** by one-half as you measure, and do not use the whole numbers. Refer to the following images as examples.



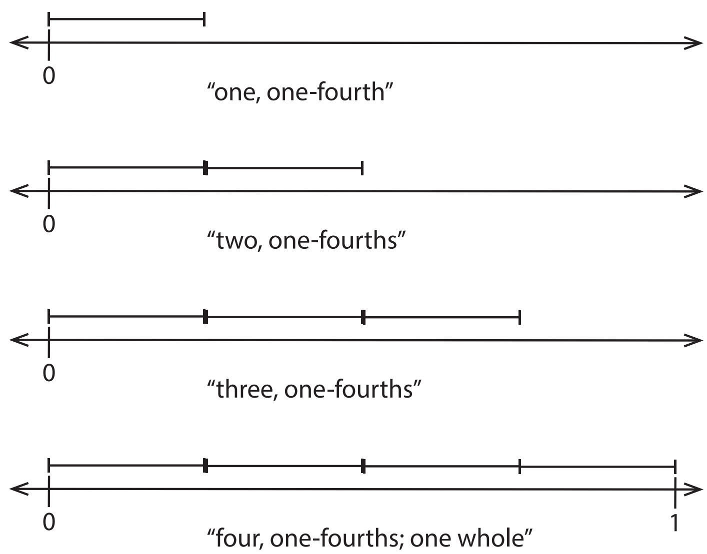
Explain that in addition to dividing a whole to create equal-size parts, you can iterate a single part or unit fraction to create a whole or a fraction.

Emphasize that a unit fraction is a single part of a whole. Therefore, the name or number of equal-size parts required to create one whole is established by the name of the unit fraction. For example, one-fourth requires four equal-size parts to create one whole.

Display a blank number line on the interactive white board or projector. **Draw** or **place** a length at the far left of the number line, at the 0. **Specify** that the length is one-fourth.

Ask students how many copies of the *one-fourth* are needed to create one whole. Students should identify that four copies of equal-size parts (one-fourths) are needed.

Duplicate or **draw** three additional one-fourth size segments to model iterating a whole. As you add each one-fourth part, be sure to count them as you did with string and the large number line. See the following images for examples.



Require students to use area model and set model manipulatives to further explore iterating one-fourths using different types of models.

Ensure that a variety of manipulatives are accessible for students to select from and use.

Use the following guiding questions to support student understanding as they explore iterating manipulatives.

GUIDING QUESTIONS

Elicit student thinking:

- When have you used an area model like this before?
- When have you used a set model like this before?

Determine if the student can REPRESENT PROPER FRACTIONS WITH AN AREA MODEL:

- Show me one one-fourth. How do you know this is one-fourth?

Determine if the student can REPRESENT WHOLE ON AN AREA MODEL:

- How many copies of one-fourth do you have here in your model of one whole? Why did you use that many?
- Would three (or one or two) copies of one-fourth make one whole? Why or why not?
- How many copies of one-fourth are needed to make one whole? How do you know?

Determine if the student can REPRESENT WHOLE ON A SET MODEL:

- How many copies of one-fourth do you have here in your model of one whole? Why did you use that many?
- Would three (or one or two) copies of one-fourth make one whole? Why or why not?
- How many copies of one-fourth are needed to make one whole? How do you know?

Determine if the student can REPRESENT PROPER FRACTIONS WITH A SET MODEL:

- Show me one one-fourth. How do you know this is one-fourth?

Select several students who correctly iterated one-fourths with different area and set models to explain and model for the class how they completed the task.

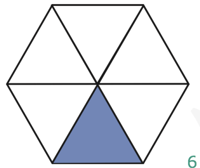
Distribute the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT to each student.

Review the directions and expectations by doing the first question together as a class.

Be sure to **write** the word form of the fraction name (e.g., six copies of one-sixth).

Reference the following image for the expected responses for Question 1.

1.



Unit (equal size parts):

sixths

Shade the unit fraction blue.

How many copies of the unit fraction make one whole:

6 copies of one-sixth OR 6 one-sixths

Require students to complete Questions 2 – 6 either independently or with a partner. Keep the manipulatives available for any students that may require them to physically manipulate the given model.

Use the following guiding questions to support student understanding as they work on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

GUIDING QUESTIONS

Elicit student thinking:

- [Point to any model on Questions 2 – 6.] What type of model is this? How do you know?
- When have you used/seen a model like this?
- [Point to any model on Questions 2 – 6.] How is this model different than the manipulatives we used in LESSON 1?
- [Point to any model on Questions 2 – 6.] How is this model the same as the manipulatives we used in LESSON 1?

Determine if the student can EXPLAIN *UNIT FRACTION*:

- [Point to any model on Questions 2 – 6.] What is the unit fraction for this model? How do you know?
- If you had two copies of the fraction one-fifth, would you have a unit fraction? Why or why not?
- How many copies of a fraction are needed to have a unit fraction? How do you know?
- Show me a unit fraction on your model (or with your manipulatives). How do you know this is a unit fraction?

Determine if the student can REPRESENT WHOLE ON A SET MODEL:

- [Point to a set model on Questions 2 or 6.] How many copies of the unit fraction are needed to create one whole? How do you know?
- [Point to a set model on Questions 2 or 6.] What represents one whole in this model? How do you know?

Determine if the student can REPRESENT WHOLE ON AN AREA MODEL:

- [Point to the area model on Question 3.] How many copies of the unit fraction are needed to create one whole? How do you know?
- [Point to the area model on Question 3.] What represents one whole in this model? How do you know?

Determine if the student can PARTITION A NUMBER LINE FROM 0 TO 1 INTO EQUAL PARTS:

- [Point to Question 5.] How do you know these are equal-size parts?
- [Point to Question 5.] How else can you partition this number line without iterating a unit fraction?
- [Draw a number line, labeled from 0 to 1.] Show me how to partition this number line into fourths. How do you know these are equal-size parts?

Review the answers to Questions 2 – 6 by selecting different students who have correct work to share and explain their solutions.

Ask students how to identify or represent a fraction that is not a unit fraction using a model.

Require students who share to explain their thinking. If necessary, reference Question 4 on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT, which models the fraction two-thirds on a number line.

Select 10 students to stand together in a group.

State that the 10 students represent a whole set. **Ask** students the following questions:

- How many equal-size parts make up the whole set of students? How do you know?
- What is the unit fraction for the set of students? How do you know?
- How many students from the whole set represent the unit fraction? How do you know?

[Move three students together, slightly away from but still part of the whole set. They represent a subset within the whole set.]

- Has the whole set changed? Why or why not?
- How many equal-size parts make up the whole set of students? How do you know? [**Emphasize** that even though the three students are now their own subset, they are still a part of the whole set of students. The number of objects (students) in the set has not changed.]
- [Draw attention to one student in the subset.] What fraction of the whole set of students does this student represent? How do you know?
- [Draw attention to a second student in the subset.] What fraction of the whole set of students does this student represent? How do you know?
- [Draw attention to the third student in the subset.] What fraction of the whole set of students does this student represent? How do you know?

Require all students to count aloud the three students in the subset (e.g., "one, one-tenth, two, one-tenths, three, one-tenths").

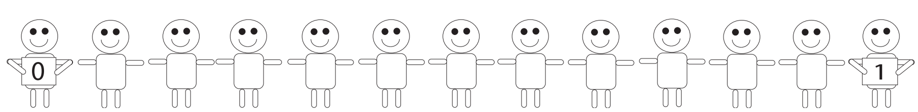
Require the three students in the subset to rejoin the whole set of ten students.

- What fraction of the whole set of students are wearing athletic shoes? How do you know?
- What fraction of the whole set of students have hair longer than their shoulders? How do you know?
- What fraction of the whole set of students are wearing long pants? How do you know?

Allow the set of 10 students to return to their seats.

Select a new group of 13 students. Line the students in a row, equally spaced apart like a number line.

Require the student on the far left to hold the sheet of paper with the "0" and the student on far right hold the sheet of paper with the "1".



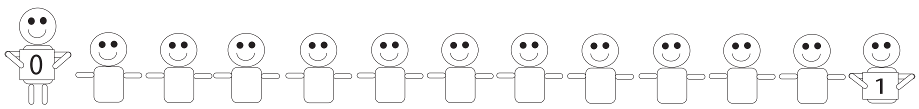
Bring students' attention to the fact that there are 12 equal-size spaces between each student.

Ask students how many equal-size divisions are between the 0 and the 1.

Ask students what the unit fraction for the number line is or the length of the distance between 0 and the next student to the right.

Continue **asking** students questions similar to those asked with the whole set of ten students to elicit the understanding that the space between students in the number line represents a specific distance (i.e. *one-twelfth*, *two-twelfths*, *three-twelfths*, etc.) from the "zero" student.

Require all students, except the "zero" student, to crouch/kneel/sit down in their spot on the number line.

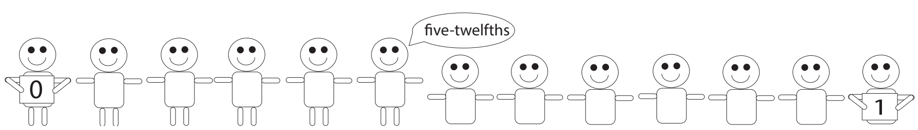


Ask students what the whole is on the number line, and **emphasize** that the whole is the *distance* from 0 to 1. The student with the "1" can stand up to demonstrate this concept if necessary.

Call out different fractions, starting with basic fractions such as *one-twelfth* or *two-twelfths* and proceeding to more difficult fractions such as *nine-twelfths*. Each time you call out a fraction, the students in the number line within that distance should stand up.

An alternative would be to have the students count off out loud as they stand up (e.g., "one-twelfth" as the student directly to the right of the "0" student stands up, or "five-twelfths" as the fifth student to the right of the "0" student says as they stand up *after* the fourth student).

After each fraction, have the students (with the exception of the "0" student) crouch/kneel/sit back down, so that each fraction starts at 0, **emphasizing** that the fraction represents the distance from 0.



Select students that are not part of the number line to call out fractions to be represented with the student number line.

Consider exploring fractions such as *one-half* or *three-fourths* as well. This will give you an opportunity to see how students in the number line and those not in the number line respond to equivalent fractions.

NOTE: Equivalent fractions are not part of this unit, however students could consider the number line (excluding the "0" student) as a set and create two or four equal-size groups to determine *one-half*, *two-halves*, *one-fourth*, *two-fourths*, etc. In the example of one-half, students could connect the idea that one-half of the students (excluding the "0" student) are equivalent to six-twelfths of the students.

Emphasize through questioning that the students did not move or change; the whole stayed fixed and did not change. For example, ask students, "Did the students ever change their position on the number when their fraction was called? Why or why not?"

Require all students to return to their seats and distribute the following supplies to each student:

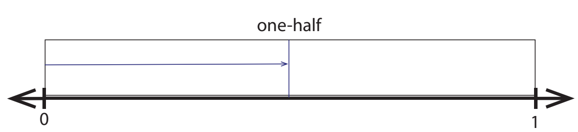
- One copy of the INSTRUCTIONAL ACTIVITY SUPPLEMENT pages 2 and 3
- Six paper strips (8.5" × 2")
- Markers or pens
- Glue sticks or glue bottles

The INSTRUCTIONAL ACTIVITY SUPPLEMENT pages should be printed as two separate pages to make gluing and writing easier for students.

Complete the following folding activity with students for *each* strip of paper; each time the strip will represent a different fraction size.

NOTE: It is recommended that three strips for each student are pre-folded into thirds to alleviate student inaccuracy and frustration.

- Fold the strip in half.
- Trace the fold line segment with marker or pen to make it stand out.
- Ask students how many equal-size parts the strip has been folded into and what the name of the unit fraction is.
- Using the marker or pen, draw an arrow from the left edge of the strip, so that the arrow stops at the fold line segment.
- Glue the strip on the first number line on the INSTRUCTIONAL ACTIVITY SUPPLEMENT so that the ends of the strip align with the line segments on the number line.
- Ask students what the left side of the number line should be labeled, then write "0" under the line segment.
- Ask students what the right side of the number line should be labeled, then write "1" under the line segment.
- Ask students what distance is represented on the number line, then write "one-half" in word form above the paper strip on the INSTRUCTIONAL ACTIVITY SUPPLEMENT.



Repeat the activity using the next consecutive number lines and paper strips. Proceed in the following order, each time writing the unit fraction in word form above the number line.

- One-third (pre-folded into thirds)
- One-fourth
- One-sixth (pre-folded into thirds)

- One-eighth
- One-twelfth (pre-folded into thirds)

Use the following guiding questions to lead a whole class discussion, or have students discuss in small groups or partners.

GUIDING QUESTIONS

Elicit student thinking:

- When or where have you seen number lines outside of school?
- When have you heard of or used fractions outside of school?

Determine if the student can EXPLAIN UNIT FRACTION:

- What is a unit fraction?
- Is seven-tenths a unit fraction? Why or why not?
- Is one-twentieth a unit fraction? Why or why not?
- What is an example of a unit fraction? How do you know that is a unit fraction?

Determine if the student can REPRESENT PROPER FRACTIONS WITH A LENGTH MODEL:

- How many copies of one-fourth would you need to represent three-fourths on your number line? How do you know? How would you model this on the number line?
- How many copies of one-eighth would you need to represent six-eighths? How do you know? How would you model this on the number line?
- How many copies of two-fourths are needed to make one whole? How do you know? How would you model this on the number line?

The pages are meant to be a resource for students during future work. If desired, the pages can be collected to check for student understanding and then returned.

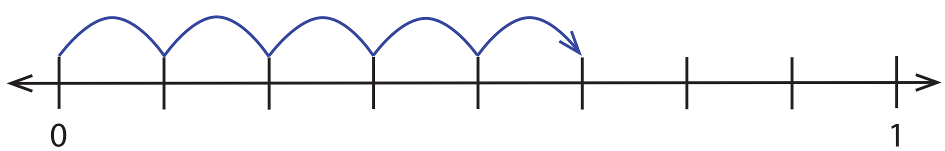
Require students, in partners or independently, to complete Question 7 on the Instructional Activity Student Handout.

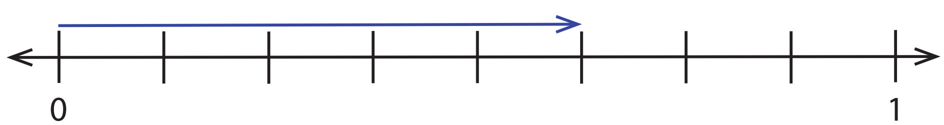
Review student work by selecting students to share their number line and explain their process. **Select** students that possess greater understanding of the process to share last, so that each student selected shares more advanced understanding than the last.

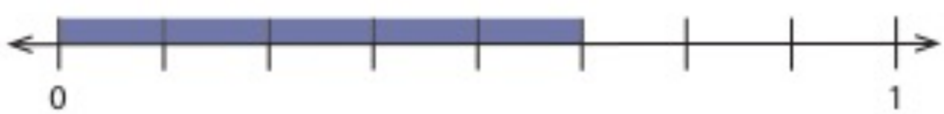
Ensure that students have done the following:

- Labeled the number line, marked the far-left line segment "0" and the far-right line segment "1".
- Divided the number line into eight *equal-size* sections. If necessary refer students to reference the paper number line for eighths they just made.
- Shown the distance from the "0" line segment to the line segment that represents five-eighths with a ray, multiple looping rays showing each unit-fraction, or a shaded bar.

Emphasize that five-eighths represents the distance from 0 on the number line, not the line segment where the ray stops. This can be modeled in different ways, as shown in the following images.







Require students to complete Questions 8 – 10 on the Instructional Activity Student Handout, either independently or with a partner.

Use the following guiding questions to support student understanding as the work on Questions 8 – 10 on the Instructional Activity Student Handout.

GUIDING QUESTIONS

Elicit student thinking:

- What type of model is this? How do you know?
- When have you seen a model like this before?
- When have you used a model like this before?
- Have you seen/used a model like this before outside of school? Explain.
- [Point to the set model on Question 9.] What shapes do you see here? Does it matter that the shapes in the set are different? Why or why not?

Determine if the student can REPRESENT PROPER FRACTIONS WITH AN AREA MODEL:

- [Point to the rectangle on the right in Question 8.] How many equal-size parts do you need to create? How do you know?
- [Point to the rectangle on the right in Question 8.] Why did you divide this rectangle into this many equal-size parts?
- [Point to the rectangle on the right in Question 8.] How many copies of one-tenth did you shade in to represent eight-tenths? Why?
- [Point to the rectangle on the right in Question 8.] Show me one one-tenth. How do you know this is one-tenth?
- How many copies of one-tenth would you need to represent four-tenths? How do you know?
- [Point to Question 8.] If the question asked you to represent seven-eighths using the rectangle instead, how many equal-size parts would you need to create? Why?
- [Point to Question 8.] If the question asked you to represent seven-eighths using the rectangle instead, how many copies of one-eighth would you need to shade? Why?

Determine if the student can REPRESENT PROPER FRACTIONS WITH A SET MODEL:

- [Point to the set model on Question 9.] What represents the whole set? How do you know?
- [Point to the set model on Question 9.] If each individual shape represents one equal-size part of the set, how many equal-size parts can the set be divided into? How do you know?
- [Point to the set model on Question 9.] If each individual shape is an equal-size part, what fraction of the whole set do the circles represent? How do you know?
- [Point to the set model on Question 9.] How many shapes are in the whole set? How do you know? How could you create three equal-size groups from the whole set?
- [Point to the set model on Question 9.] How many copies of one-third would you shade to represent one-third? Two-thirds? Three-thirds? How do you know?

Determine if the student can RECOGNIZE PROPER FRACTIONS WITH A SET MODEL:

- [Point to the set model on Question 10.] What represents the whole set? How do you know?
- [Point to the set model on Question 10.] How many equal-size groups has the whole set been divided into? How do you know?
- [Point to the set model on Question 10.] What does *one* of the equal-size groups represent? How do you know?
- [Point to the set model on Question 10.] How many copies of one-fifth have been identified? How do you know?
- [Point to the set model on Question 10.] What fraction is represented by the set model? How do you know?

Determine if the student can RECOGNIZE PROPER FRACTIONS WITH AN AREA MODEL:

- [Point to the circle area model on Question 10.] How many equal-size parts are represented? How do you know?
- [Point to the circle area model on Question 10.] What does *one* of the equal-size parts represent? How do you know?
- [Point to the circle area model on Question 10.] How many copies of one-eighth have been shaded? How do you know?
- [Point to the circle area model on Question 10.] What fraction of the whole circle do the shaded equal-size parts represent? How do you know?
- [Point to the pentagon area model on Question 10.] How many equal-size parts are represented? How do you know?
- [Point to the pentagon area model on Question 10.] What does *one* of the equal-size parts represent? How do you know?
- [Point to the pentagon area model on Question 10.] How many copies of one-fifth have been shaded? How do you know?
- [Point to the pentagon area model on Question 10.] What fraction of the whole pentagon do the shaded equal-size parts represent? How do you know?

Determine if the student can RECOGNIZE PROPER FRACTIONS WITH A LENGTH MODEL:

- [Point to the bar length model on Question 10.] How many equal-size parts are represented? How do you know?
- [Point to the bar length model on Question 10.] What does *one* of the equal-size parts represent? How do you know?
- [Point to the bar length model on Question 10.] How many copies of one-fifth have been shaded? How do you know?
- [Point to the bar length model on Question 10.] What fraction of the whole bar length model do the shaded equal-size parts represent? How do you know?
- [Point to the number line length model on Question 10.] How many equal-size parts are represented? How do you know?
- [Point to the number line length model on Question 10.] What does *one* of the equal-size parts represent? How do you know?
- [Point to the number line length model on Question 10.] How many copies of one-fourth have been represented? How do you know?
- [Point to the number line length model on Question 10.] What fraction is represented by the number line model? How do you know?

Review the answers to Questions 8 – 10 by selecting students to share their responses and to explain and/or demonstrate their thinking.

Bring students' attention back to the large number line from LESSON 1 and the beginning of this lesson.

Ask students the following questions:

- What does the space between 0 and 1 represent? How do you know?
- What does the space between 7 and 8 represent? How do you know?
- Where on this number line would you mark six wholes and one-half? Why?

Model or have the student who answered model how to count out six wholes and one-half by moving your finger along the length of each whole number as you count.

- How many one-halves are in six wholes? How do you know?
- How many one halves are in four wholes and one-half? How do you know?

Redistribute or **require** students to **retrieve** the *Fraction Representations* sheet from the INSTRUCTIONAL ACTIVITY SUPPLEMENT in LESSON 1.

Review the content from this lesson by requiring each student to complete the second and third columns of the *Fraction Representations* sheet.

When students are finished, either collect the paper, have them keep it somewhere secure, or glue it into an interactive notebook.

At the end of the activity, teachers should collect the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT to analyze for student understanding.

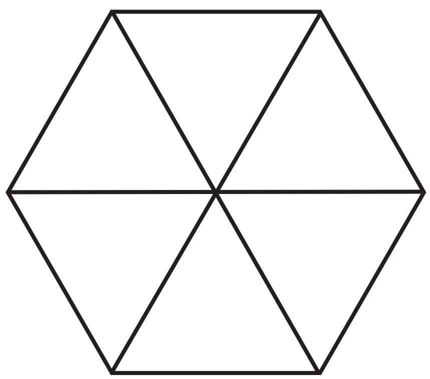
Name________________________________

Understanding Fractions as Numbers

Lesson 2

Use the following models to answer the questions.

1.



Unit (equal-size parts):

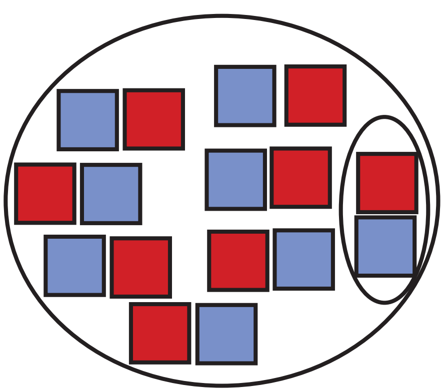
____

Shade the unit fraction blue.

How many copies of the unit fraction make one whole?

____

2.



Circle each copy of the unit fraction.

Unit (equal-size groups):

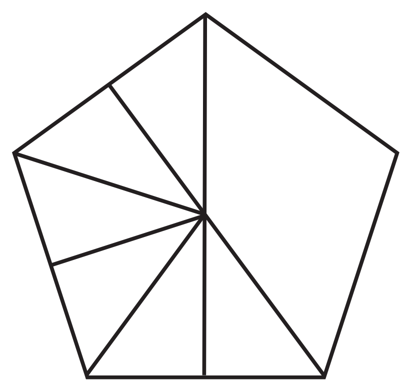
____

How many copies of the unit fraction make one whole?

____

Name______________________________

3.



Finish dividing the pentagon into equal-size parts.

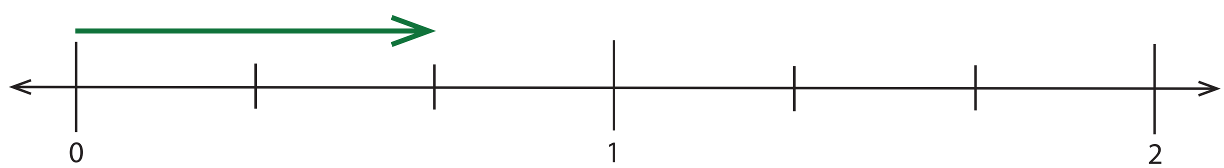
Unit (equal-size parts):

____

How many copies of the unit fraction make one whole?

____

4.



Unit (equal-size lengths): ______________________________

How many unit fractions are represented by the green arrow?

____

How many copies of the unit fraction make up the whole?

____

Name____________________________

5.

Iterate the line segment to find the number of equal-size lengths.

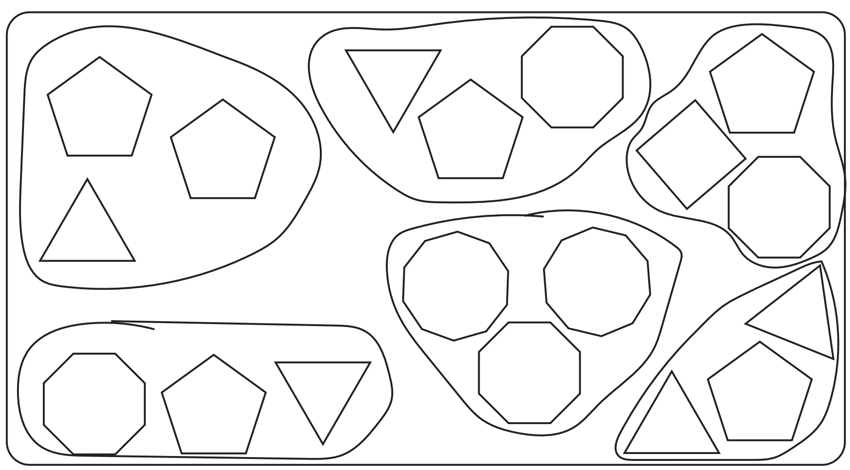
Unit (equal-size lengths): ___________________________

Draw an orange arrow to represent one unit fraction.

How many copies of the unit fraction make up the whole?

____

6.



Units (equal-size groups): ___________________________

Shade the unit fraction purple.

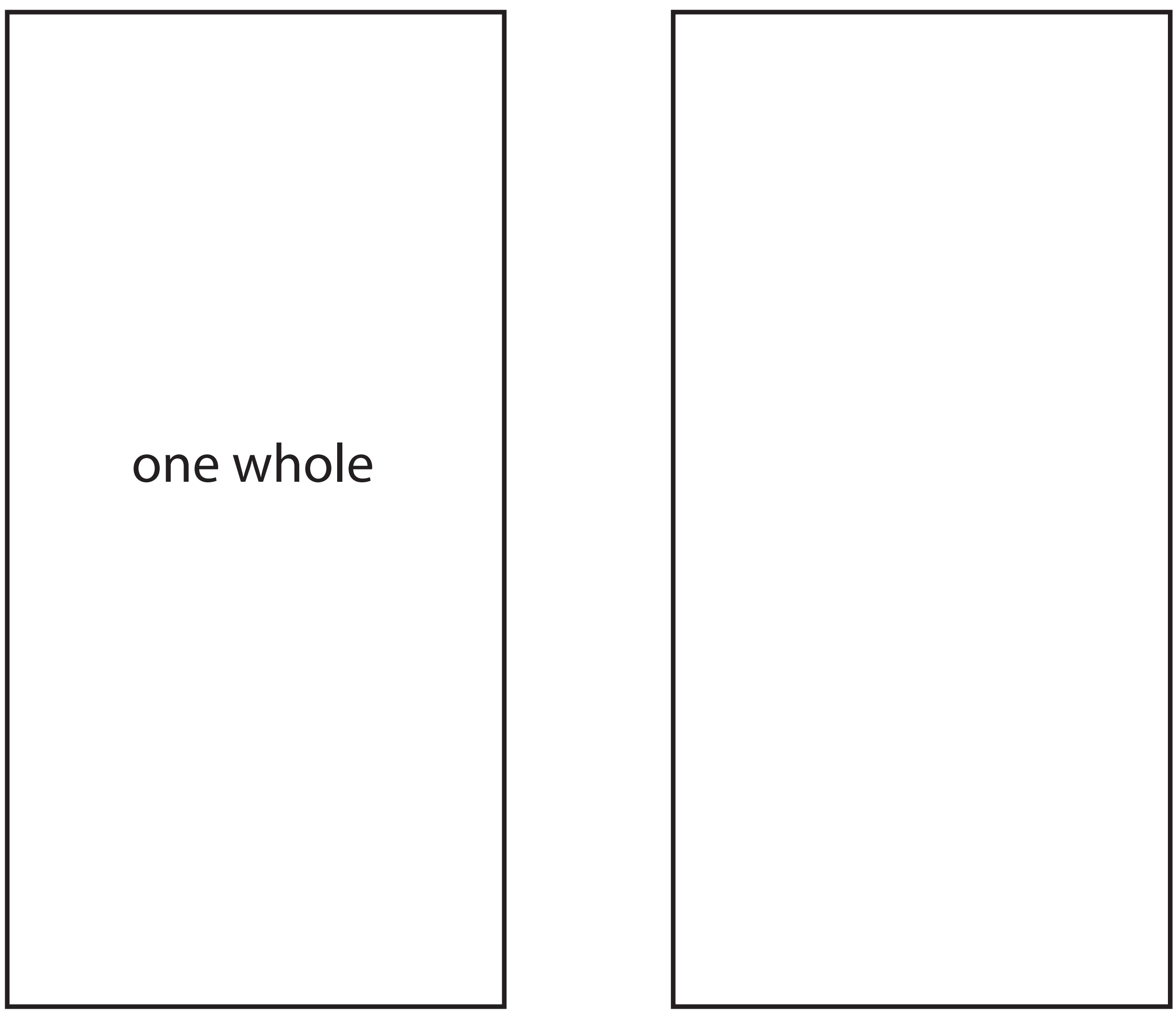
How many copies of the unit fraction make up the whole?

____

Name______________________________

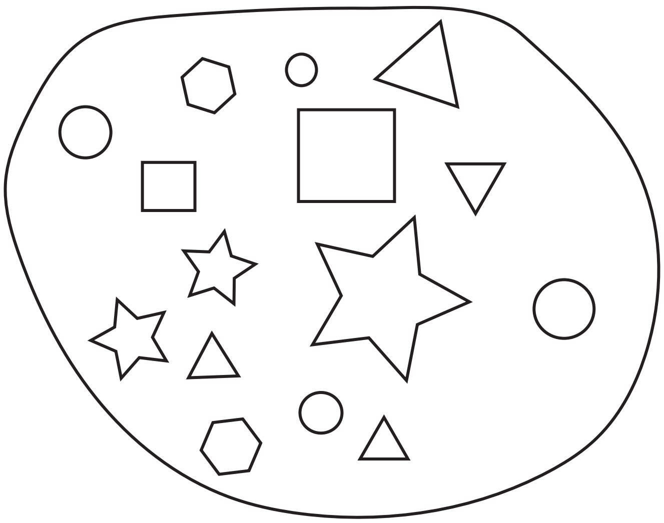
7. Represent five-eighths on the number line.

8. If the rectangle on the left displays one whole, use the rectangle on the right to represent eight-tenths.



Name______________________________

9. Use the set of shapes to answer the following questions.



set of shapes

9.a. How many shapes make up the whole set?

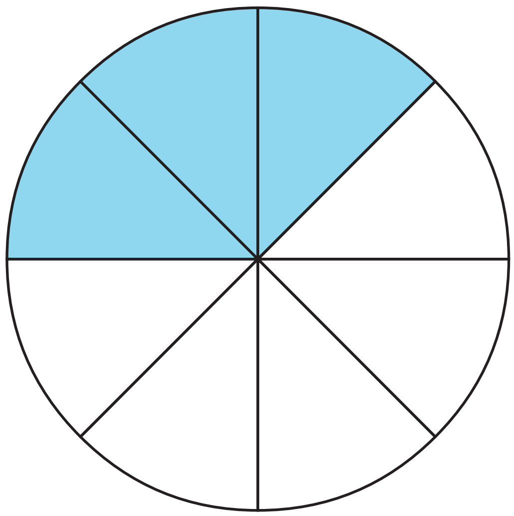
9.b. What fraction of the whole set of shapes are circles?

9.c. Redraw the set of shapes divided into thirds. How many shapes are in one-third?

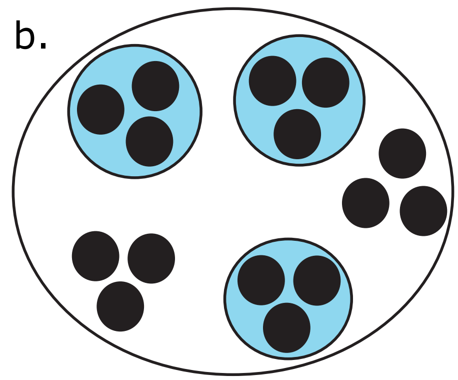
Name________________________________

10. Circle the models that are shaded to represent three-fifths.

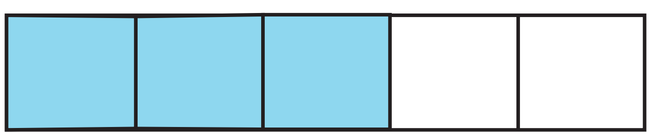
a.



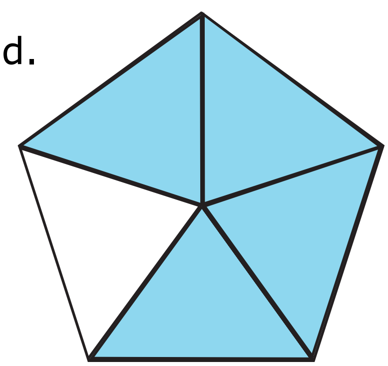
b.



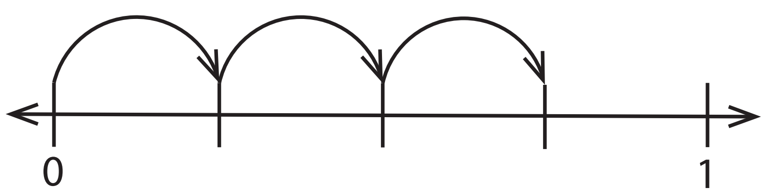
c.



d.



e.



If the wholes are the same size,

and the equal-size parts are **larger**,

then the *number* of equal-size parts is

______________________________.

If the wholes are the same size,

and the equal-size parts are **smaller**,

then the *number* of equal-size parts is

______________________________.

UNDERSTANDING FRACTIONS AS NUMBERS

INSTRUCTIONAL ACTIVITY

Lesson 3

LEARNING GOAL

Students will informally add unit fractions of a given whole to identify and create provided fractions. In addition, students will informally subtract unit fractions from a given whole to identify and create provided fractions. Students will also connect their understanding of fraction names with their understanding of pictorial representations and models in order to identify the relationship that exists between the numerator and denominator and to represent fractions symbolically.

PRIMARY ACTIVITY

Students will interact with a variety of manipulatives and pictorial representations of fraction models to increase their understanding of the relationship between the numerator and denominator. Students will continue to add (or iterate) unit fractions, and then extend that understanding to subtract unit fractions from a given whole to identify fractions. Students will develop this understanding through problem situations that involve measurement situations, such as sharing a drink between several glasses, which will be modeled with a pitcher of water and glasses. Students will also listen to a picture book read aloud and use pictorial representations of different fraction models (length, set, and area) to identify and explain the represented fraction symbolically.

NOTE: This lesson will likely require more than one class period to complete.

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

- Whole
- Fraction
- Equal groups
- Partition
- Set model
- Length model
- Area model

MATERIALS

- Large pitcher, filled with water
- Six clear cups or glasses, all the same size and dimensions
- Large number line for display (from LESSON 1)
- A variety of manipulatives:
    - Beans
    - Counters
    - Interlocking cubes
    - Fraction circles
    - Fraction bars
    - Paper strips
    - Number line
- *Fraction Representations* from the LESSON 1 INSTRUCTIONAL ACTIVITY SUPPLEMENT
- Rope or string (a length of two units long based on the large paper number line from LESSON 1, similar to the one used in LESSON 2.)
- Rope or string, three feet long (Recommend one rope/string for every two to three students.)
- Markers (Recommend one red, one blue, and one green marker for every two to three students.)
- *Answer Number Line* (Create a number line on paper. The length between 0 and 1 should be three feet long. Using the recommend colors, identify half, thirds, and fourths.)
- INSTRUCTIONAL ACTIVITY STUDENT HANDOUT

IMPLEMENTATION

Redistribute or **require** students to **retrieve** the *Fraction Representations* sheet from the INSTRUCTIONAL ACTIVITY SUPPLEMENT in LESSON 1. **Review** each row for the second and third columns by selecting several students to share and explain their responses on the *Fraction Representations* sheet.

Arrange students into groups of two or three. **Distribute** the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT, markers (red, blue, and green), and one piece of three-feet-long rope/string to each group.

> **Explain** the following directions to students.
>
> **Tell** students that they will use the markers to make marks on the string that represent lengths from one end of the string. For example, students will mark the location that represents the distance from one end of the string to one-half the length of the whole string. **Emphasize** that the fraction is *not* the location of the mark, but rather the distance from one end (or the "zero" point) of the string to the mark.
>
> **Tell** students that once they have marked all of their lengths, they will check their string against the *answer number line*. Be sure to display the *answer number line* somewhere prominent for easy student access.

Answer number line: A number line on large butcher paper or a large dry erase board, where the space between the tick marks representing zero and one is three-feet long. **Label** the left end "*zero*" and the right end "*one*". **Identify** one-half using the color red, one-third and two-thirds using the color blue, and one-fourth, two-fourths, and three-fourths using the color green.

Require students to complete all parts of Question 1 on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

Use the following guiding questions to support student understanding as they work through Question 1 of the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

GUIDING QUESTIONS

Elicit student thinking:

- What would you use to measure string (or rope)? Why?
- Why would you have to measure string (or rope)?
- When you measure string (or rope), do you always use a whole unit (e.g., one foot, two feet, etc.)? Why or why not?
- If you do not have a whole unit, how do you know what fraction of the unit you do have?

Determine if the student can EXPLAIN *UNIT FRACTION*:

- Show me one-half of the whole string. How do you know that is one-half?
- Show me one-third of the whole string. How do you know that is one-third?
- Show me one-fourth of the whole string. How do you know that is one-fourth?
- What is a unit fraction? How do you know?
- What is an example of a unit fraction? How do you know that is a unit fraction?
- Is three-tenths a unit fraction? Why or why not?

Determine if the student can REPRESENT PROPER FRACTIONS WITH A LENGTH MODEL:

- Using your string, show me two-thirds of the whole string. How do you know this is two-thirds?
- How many copies of one-fourth would you need to represent two-fourths? How do you know?
- If you wanted a piece of string that was three-fourths of your whole string, where would you cut the string into two pieces? Why? (How do you know that is three-fourths?)
- If you were going to cut the whole string to share it equally with each person in your group, show me how you would cut the string. Why would you cut the string that way?

Review student work by facilitating a discussion and asking student groups to share how they decided where to make the different fraction marks and how their fraction marks matched or did not match up with the *answer number line*.

Encourage students to use their strings to model their thinking and explanations.

Collect the strings back from each student group.

Bring students' attention to the large number line from LESSON 1.

Point to the one-half mark (line segment) on the number line.

Ask students what the segment represents. **Emphasize** that one-half is not the segment, but rather the distance from 0 to the segment. Therefore, the segment represents the point on the number line where the distance from 0 is one-half of the whole on the number line.

Identify that the number line is equipartitioned into single units.

Share the following problem situation with students.

> Mrs. Clark needs six-fourths units of string for a science activity she is planning for her third-grade class. Help Mrs. Clark measure and cut the string so that it is correct length.

Ask students what information they know from the problem situation about the rope/string and the number line. **Write** the information students provide on a piece of chart paper, a large white board, or an interactive white board.

Students should identify that the number line is partitioned into halves, and it needs to be partitioned into fourths. If students do not make the connection, **ask** strategic questions to lead them to the understanding that the number line needs to be equipartitioned into fourths.

Ask students how to equipartition the single unit (the distance from 0 to 1) into fourths. Use strategic questions to guide students to use the existing equipartitions for one-half as a starting point.

Select students to add line segments to the number line using a different color than the segments marking halves. **Ensure** that each segment is exactly half way between each unit, and remind students that equipartitions *must* be the same size.

Ask students how many equal-size parts are between 0 and 1. Follow up with several other units, such as how many equal-size parts are between 5 and 6 or between 3 and 4.

Display the rope/string that is two units long. **Ask** students how to determine how long the rope/string is. **Require** students to explain their responses.

Model measuring the rope/string using the number line.

Refer to lengths longer than 1 as improper fractions or a number of copies of the unit fraction (e.g., *four, one-thirds*). **Avoid** using whole numbers or referring to the total distance as a mixed number.

NOTE: If a student identifies that eight-fourths is the same as two units, have them explain how they know, but do not explicitly teach the concept, as equivalent fractions is not the intended focus of the activity and some students may not be ready to conceptually understand that two whole units is the same as eight copies of one-fourth. This understanding of wholes as multiple copies of a unit fraction will be important later when students are required to regroup while adding and subtracting fractions and mixed numbers.

Ask students the following questions:

- How long is the string/rope in fourths? How do you know?
- How long does Mrs. Clark need the string/rope to be? How do you know?
- What needs to happen to the string/rope so that it is the length Mrs. Clark needs? How do you know?
- How can we measure to make sure to remove (cut off) the correct amount of string/rope?

If students do not suggest measuring and removing the excess by subtracting unit fractions (i.e. "I have eight-fourths and I need seven fourths, so how many fourths do I need to cut off?"), use strategic questions to guide them to consider the idea of subtracting unit fractions from a given fraction to create the desired fraction.

Model how to subtract one-fourth from the string, repeating the subtraction until the string/rope is the correct length. **Emphasize** the subtraction of unit fractions (i.e. one-fourth then one-fourth again) as opposed to subtracting once (i.e. two-fourths).

Share the following problem situation with students.

> Jackson is sharing cookies and milk with four of his friends. There is enough milk to fill one full glass. If Jackson divides the milk equally, what fraction of the full glass of milk will Jackson and each of his friends get?

Ask students to think about what the question is asking, then **require** students to share with a partner what they think the question is asking. Finally, **select** students to explain what they think the question is asking.

Fill one cup with water from the large container of water. **Tell** students that the cup of water represents Jackson's glass full of milk.

Ask students how many glasses are needed to answer the question. **Require** students to explain their thinking.

Display either five empty glasses (one for each person) or the full glass and four empty glasses.

Confirm that because there are four friends *and* Jackson, there will need to be five total glasses with milk in order for each person to have milk with their cookies.

Select a student to demonstrate how the water could be divided among the five glasses. When the student is finished, **discuss** what the student did as a class. Bring attention to the amount of water in each glass and the steps the student took to distribute the water.

Ask students whether or not the water was distributed equally. If it was not, **select** another student with a different strategy to demonstrate how the water could be divided. Be sure to pour all the water back into one glass to start.

Continue selecting students to demonstrate different strategies and discuss the results as a class as needed.

Through questioning and discussion, **lead** students to the connection between repeated subtraction of unit fractions and sharing equal-size portions of the water between five glasses.

Require students to complete all parts of Question 2 on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

Use the following guiding questions to support student understanding as they work through Question 2 of the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

GUIDING QUESTIONS

Elicit student thinking:

- Describe a time when you have had to share something with someone else. Were your shares equal (even/same size)? How do you know?
- Have you ever had "half" of something and it was not actually one-half of the whole? Explain.
- When you measure string (or rope), do you always use a whole unit (e.g., one foot, two feet, etc.)? Why or why not?

Determine if the student can EXPLAIN *UNIT FRACTION*:

- How many partially filled glasses of milk represents one-fifth of the whole glass of milk? How do you know?
- If Jackson gave himself more milk than his friends, would his glass of milk represent a unit fraction of the whole glass of milk? Would one of the glasses of milk he gave to a friend? Explain.
- What is a unit fraction? How do you know?
- What is an example of a unit fraction? How do you know that it is a unit fraction?
- Is two-fifths a unit fraction? Why or why not?

Determine if the student can EXPLAIN THE FRACTION RELATIONSHIP OF *X/Y*:

- If each person gets one-fifth of the whole glass of milk, what does that mean? What does the one mean (represent)? What does the five mean (represent)?

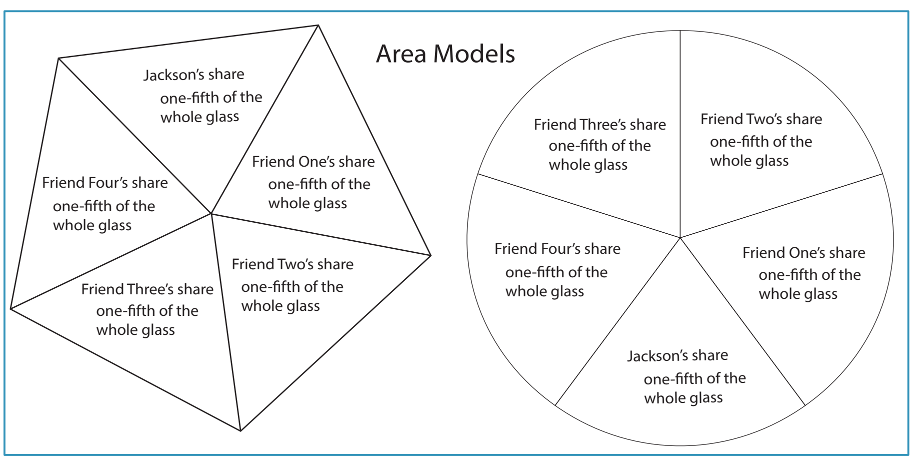
Determine if the student can EXPLAIN RELATIONSHIPS BETWEEN UNIT FRACTIONS:

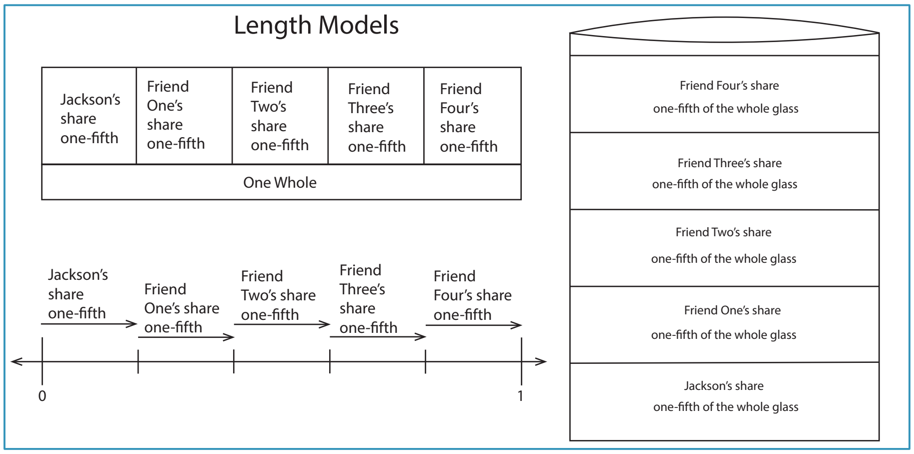
- How is the amount of milk that each person gets different from the amount of milk in the whole glass? Explain.
- If Jackson were to share the milk in one whole glass equally with only two friends, would each person get more or less milk than when he shares the same amount of milk equally with four friends? Explain.
- If you like a lot of milk with your cookies, which would you rather have a glass of milk that is one-third of the whole glass, or a glass of milk that is one-sixth of the whole glass? Why?

Review student responses by selecting students to share their "before and after" model representations from Question 2 with the class. Select students and order them intentionally, ensuring you have chosen students that have accurate representations on their models (i.e. the glasses are consistently sized, as is the amount of "milk" represented).

Begin with students that use a set model drawing of cups of milk, then proceed to include students that utilized a variety of models—not just a set model drawing of glasses, but perhaps an area model or a length model. If no students use alternate representations that are more abstract in the context of the problem situation, ask students how they could represent the problem as an area model or length model.

The following images are examples to display if students do not offer ideas.





After discussing a variety of models, **require** students to share the problem situations they wrote for Question 2.c. on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

Introduce the book *Give Me Half!* by Stuart J. Murphy. If you do not have access to a copy of the book, it is available as a video online (https://www.youtube.com/watch?v=hVaxiJB6Fls).

Read aloud or watch the book *Give Me Half!* once straight through, without stopping for questions.

Review all parts of Question 3 on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT by reading them aloud and clarifying any questions students may have about the questions provided.

Then, **read aloud** or watch the book again, but **require** students to answer all parts of Question 3 on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT as you read/listen or directly after.

Require students to discuss their responses to the different parts of Question 3 on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

Display page 9 or page 15 from the book, or pause the video at the appropriate screen (note: the page numbers will appear on the screen). **Require** students to describe to a partner how the fractions are represented and how the representations are the same and different.

Discuss student descriptions by **selecting** several students to share their responses with the whole class. Note that there are four different representations (i.e. pictorial, word form, symbolic, and as an equation). **Ensure** that each representation is discussed.

Point to one of the symbolic representations of one-half, or write $\frac{1}{2}$.

Ask students to explain what it is, and what they think it means.

NOTE: Avoid writing fractions symbolically using a backslash (e.g., ½), because as students begin to write fractions symbolically, they may misinterpret their own writing of ½ or 1/2 as 112 instead of $\frac{1}{2}$.

Because students have most likely already been exposed to one-half in symbolic form, try to probe deeper using questioning to see if they know anything about numerators, denominators, or if they can identify the relationship between the numerators and the denominators. **Avoid** using terms such as "top number" and "bottom number", as those phrases encourage thinking of fractions as two separate numbers, not one.

Ask students what the whole is and **ensure** that students are specific and concise when describing the whole (i.e. either *the whole pizza* or *the can of juice*, not simply *the pizza* or *the juice*).

Ask students how many equal-size parts the whole has been partitioned (or divided) into.

Ask students how many equal-size parts of the whole the brother and the sister *each* get.

Require students to discuss with a partner how they think this information is related to the symbolic representation of one-half.

Display page 21 from the book or pause the video at the appropriate screen (note: the page number will appear on the screen).

Require students to describe to a partner how the fractions are represented and how the representations are the same and/or different from the representations on either page 9 or page 15.

Select several students to share their responses with the whole class. **Guide** students to think about the fraction models that best illustrate various problem situations. For example, dividing pizza is best illustrated by an area model, while passing out cupcakes to friends is best illustrated by a set model. The goal is to help students visualize the context and choose a model that best represents the problem for them.

Ask students why one-half of the cupcakes from the story is one cupcake and not $\frac{1}{2}$ of a single cupcake. **Require** students to share their response with a partner.

Select one or two students to share their responses with the class. **Emphasize** that it is important to know what the whole is, because it directly affects the amount that one-half represents (e.g., would you rather have one cupcake or $\frac{1}{2}$ of one cupcake?).

Bring student attention back to symbolic representations of fractions by **writing** the fraction $\frac{4}{6}$.

Ask students what number that is. **Ensure** students do not respond using phrases such as "four above six, four over six, or four sixes", and **emphasize** the "sixths" part of the number. In addition, **emphasize** the idea that $\frac{4}{6}$ is a number.

Write the following phrases next to the symbolic representation, and have students provide a response to each.

- The whole:
- Equal-size parts:
- Equal-size parts identified:
- Number of copies of $\frac{1}{6}$:

Ask students to first think on their own about their responses.

Require students to share with a partner after considering their answers. **Discuss** and **record** responses for each of the phrases as a whole group. **Refrain from** telling students the answers. However, if they are struggling, use guided questioning to lead the group to the correct responses.

Select a student to identify a fraction model (i.e. area, set, or length), then require that student to represent one whole using the identified model on the board next to the symbolic representation.

Ask the students to define the whole (e.g., a circle, a number line, a set of stars).

Select a student to equipartition the whole into six equal-size parts. **Ensure** that the student precisely creates six equal-size parts or groups. Provide scaffolding as necessary (e.g., ruler or yard stick for straight edge, identify the center point of a shape, etc.) for students who struggle with precision due to fine motor capabilities.

Select a student to identify the correct number of equal-size parts or groups by shading them in or circling them.

Select a student to explain and show on the model how they know how many copies of $\frac{1}{6}$ have been identified. To show this understanding, students could label each of the equal-size parts in the whole as $\frac{1}{6}$ and point out that four of those unit fractions have been shaded or circled. Students may also use repeated addition to show $\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}=\frac{4}{6}$.

Repeat the process with each of the other two types of fraction models.

Require students to complete Questions 4 – 6 on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT, either independently or in groups of two or three.

Use the following guiding questions to support student understanding as they work through Questions 4 – 6 of the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

GUIDING QUESTIONS

Elicit student thinking:

- What are the different types of fraction models? Can you describe or draw each of them?
- Can a fraction model be divided into parts that are not of equal size? Explain.

Determine if the student can EXPLAIN *UNIT FRACTION*:

- [Point to Question 4.] What is the unit fraction for this number line? How do you know?
- [Point to Question 4.] If the number line was divided into 10 equal parts, what would the unit fraction be? How do you know?
- What is a unit fraction? How do you know?
- What is an example of a unit fraction? How do you know that is a unit fraction?
- [Point to the circle model on Question 5.] What is the unit fraction of this area model? How many unit fractions make up the whole circle? How do you know?
- [Point to the fraction bar model on Question 5.] Is the unit fraction for this model one-sixth or one-eighth? Explain.

Determine if the student can EXPLAIN *DENOMINATOR*:

- [Point to a fraction represented symbolically on Question 5.] What represents how many equal-size parts the whole has been divided into? How do you know?
- In the fraction four-twelfths, how many equal-size parts has the whole been divided into? How do you know?
- [Point to Question 6.] What is the same about the information in the first bullet (the written description) and the last bullet (the symbolic representation)? What is different?

Determine if the student can EXPLAIN *NUMERATOR*:

- [Point to a fraction represented symbolically on Question 5.] What represents the number of equal-size parts (or unit fractions) that have been identified from the whole? How do you know?
- In the fraction four-twelfths, how many equal-size parts (or unit fractions) have been identified? How do you know?
- [Point to Question 6.] What is the same about the information in the second bullet (the written description) and the last bullet (the symbolic representation)? What is different?

Determine if the student can RECOGNIZE FRACTION:

- [Point to the models on Question 5.] Are these symbolic representations of fractions? Why or why not?
- [Point to the fraction names on Question 5.] Which of these fractions are represented symbolically? How do you know?
- [Write a proper fraction is word form.] Is this a symbolic representation of a fraction? Why or why not?
- [Write a symbolic representation of a fraction.] Is this a symbolic representation of a fraction? Why or why not?

Determine if the student can REPRESENT PROPER FRACTIONS AS A SYMBOLIC REPRESENTATION:

- [Point to the number line on Question 4.] Write the symbolic representation of the fraction this number line is modeling. How do you know this is correct?
- Imagine I have a bag of seven candies and I eat four of them. Write the symbolic fraction that represents the how many candies I ate from my whole bag. How do you know this is correct?
- [Point to a fraction model on Question 5.] Write the symbolic representation of this fraction. How do you know this is correct?

Determine if the student can EXPLAIN THE POSITION OF FRACTIONS ON THE NUMBER LINE:

- [Point to Question 4.d.] How do you know this is the symbolic form of the fraction represented on the number line?
- [Point to the numerator written on Question 4.d.] What does this number represent? How do you know?
- [Point to the denominator written on Question 4.d.] What does this number represent? How do you know?

Review the answers to Questions 4 – 6 by selecting students to share their answers by explaining and/or demonstrating their thinking.

Redistribute or **require** students to retrieve the *Fraction Representations* sheet from the INSTRUCTIONAL ACTIVITY SUPPLEMENT in LESSON 1.

Review the content from this lesson by requiring each student to complete the fourth column of the *Fraction Representations* sheet. When students are finished, either collect the paper or have them keep it somewhere secure.

At the end of the activity, teachers should collect the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT to analyze for student understanding.

Name______________________________

UNDERSTANDING FRACTIONS AS NUMBERS

Lesson 3

1. Use your piece of string to complete the following.

- Use the color **red** to mark the string at the length of one-half of the whole rope.
- Use the color **blue** to mark the string at the lengths of one-third and two-thirds of the whole rope.
- Use the color **green** to mark the string at the lengths of one-fourth, two-fourths, and three-fourths.
- Compare your marks with the *Answer Number Line*.

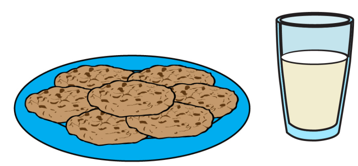
1.a. Which fraction of the string was the easiest to identify? Why?

1.b. Which fraction of the string was the most difficult to identify? Why?

Name______________________________

1.c. Were any of your marks incorrect? If so, which one(s)? Explain how you identified them and what you would do differently next time.

2. Jackson is sharing cookies and milk with four of his friends. There is enough milk to fill one full glass. If Jackson divides the milk equally, what fraction of the full glass of milk will Jackson and each of his friends get?



2.a. Draw a model to represent the milk *before* and *after* it was shared.

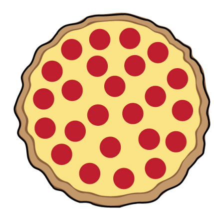
Before: After:

Name______________________________

2.b. What fraction of the one full glass does each person get? How do you know?

2.c. Write another problem situation like Jackson’s, where one whole is divided into five equal-size parts.

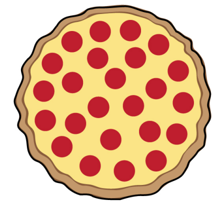
3. Answer the questions using the book *Give Me Half!*



3.a. What does it mean when the parent says the pizza must be cut the same for the brother and sister?

Name______________________________

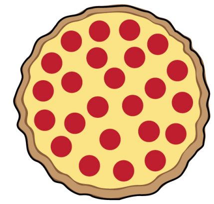
3.b. Why does the pizza need to be cut the same for the brother and sister?

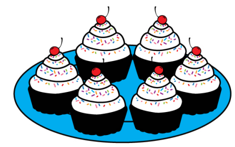


3.c. When the brother and sister share the cupcakes, what represents one whole? How do you know?

Name______________________________

3.d. How are the whole for the pizza and the whole for the cupcakes different? How are they the same?





3.e. What type of fraction model would best represent the pizza? Why?

Name____________________________

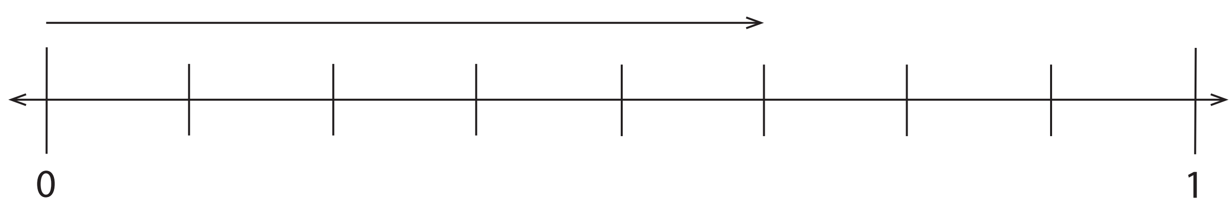
3.f. What type of fraction model would best represent the cupcakes? Why?

3.g. If the brother has three cookies and he shares them equally with the sister, draw how many cookies each child will get. Explain what you did and how you know your answer is correct.

Name______________________________

3.h. Describe how the author represented the fractions.

4. Use the number line to answer the following questions.



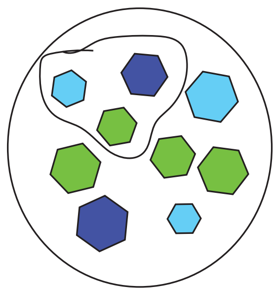
4.a. What represents the whole?

4.b. How many equal-size parts has the whole been divided into?

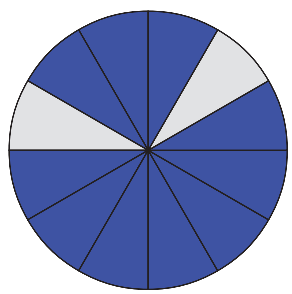
4.c. How many equal-size parts have been identified?

4.d. Use the word form and symbolic form to name the fraction that is represented on the number line.

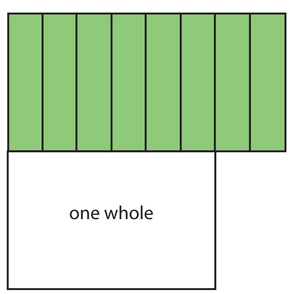
5. Match the fraction model with the correct fraction name. For the area models, consider the fraction that is represented by the shaded parts.



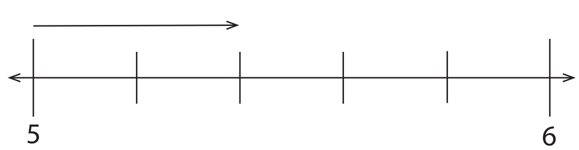
a. eight-sixths



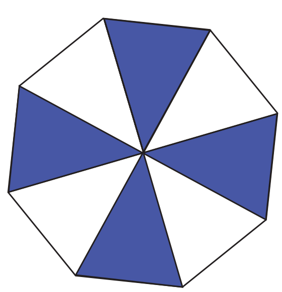
b. $\frac{2}{5}$



c. $\frac{4}{8}$



d. three-ninths



e. $\frac{10}{12}$

Name______________________________

6. Draw a fraction model of your choice to represent the fraction that is described.

- The whole is divided into eight equal-size parts.
- Identify five copies of the unit fraction.
- The symbolic representation of the fraction is $\frac{5}{8}$.

Model:

UNDERSTANDING FRACTIONS AS NUMBERS

INSTRUCTIONAL ACTIVITY

Lesson 4

LEARNING GOAL

Students will consider different representations of proper and improper fractions in order to conceptualize the relationship between numerators and denominators.

PRIMARY ACTIVITY

Students will analyze and compare different representations of fractions to determine the numerator and denominator, and to identify if a fraction is greater than, less than, or equal to one whole using task cards.

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

- Whole
- Equal-size groups
- Partition
- Set model
- Area model
- Length model
- Numerator
- Denominator
- Symbolic representation
- Proper fraction
- Improper fraction
- Unit fraction

MATERIALS

- *Fraction Representations* from the LESSON 1 INSTRUCTIONAL ACTIVITY SUPPLEMENT
- Large number line for display – from LESSON 1

- INSTRUCTIONAL ACTIVITY STUDENT HANDOUT
- INSTRUCTIONAL ACTIVITY SUPPLEMENT (Recommend one for every two to three students.)
- Individual dry erase boards and markers (Recommend one for every student.)
- A variety of manipulatives:
    - Beans
    - Counters
    - Interlocking cubes
    - Fraction circles
    - Fraction bars
    - Paper strips
    - Number line

IMPLEMENTATION

Redistribute or **require** students to **retrieve** the *Fraction Representations* sheet from the INSTRUCTIONAL ACTIVITY SUPPLEMENT in LESSON 1.

Review each row of the fourth column by selecting several students to share and explain their responses on the *Everything I Need to Know About Fraction Representations* sheet.

Bring students' attention to a number line on the board, labeled with "0" and "10".

Ask students if there are any numbers that exist between 0 and 10. **Point out** that there are no written or marked values in the space between 0 and 10 on this number line.

Require students to explain how they know there are numbers between 0 and 10, even though they are not marked on this number line.

Bring students' attention to the large number line from LESSON 1.

Referencing the large number line, **ask** students if any numbers exist between 0 and 10. Students should identify that they do, because they are labeled on the number line.

Ask students if any numbers exist between 0 and 5. **Require** students to explain how they know, even if the numbers are not marked on a number line.

Ask students if any numbers exist between 0 and 1 and, if there are numbers, to explain what those numbers are (i.e. give an example or a name, "fractions", "less than one whole", "a fraction of a whole") and how they know they exist even if those numbers are not marked on a number line.

Ask students what fractions *do* appear on the large number line. Because the number line was partitioned in previous lessons, students should identify halves and fourths.

Ask students if any numbers exist between 0 and $\frac{1}{4}$. **Require** students to explain their thinking.

Ask students for an example of a number that would be located on a number line between 0 and $\frac{1}{4}$ (e.g., fifths, sixths, sevenths). Use questioning, if necessary, to lead students to identify that eighths are smaller than fourths, and therefore that $\frac{1}{8}$ would be between 0 and $\frac{1}{4}$. **Emphasize** that $\frac{1}{8}$ is not the *only* fraction between 0 and $\frac{1}{4}$—it is just the example being used on *this* number line.

Draw or select students to draw line segments representing eighths on the number line using a different color than halves and fourths.

Distribute individual dry erase boards and markers, or have a comparable way for students to show you written work at a glance.

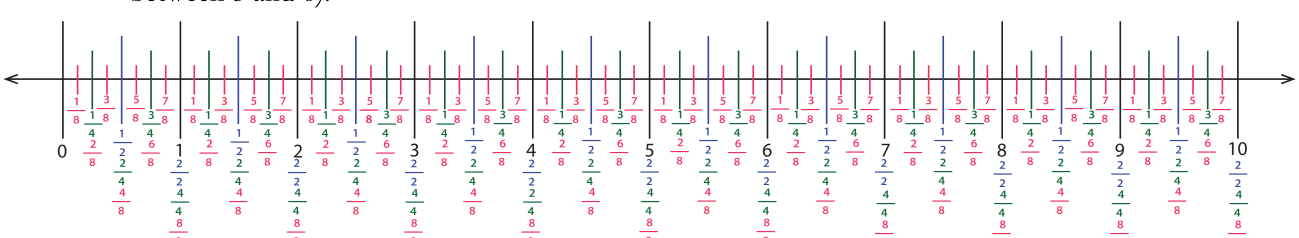
Ask students what color was used to create the line segment for one-half. **Select** a student to explain how they know their response is one-half.

Require students to write one-half symbolically on their dry erase boards. **Ask** one or two students to explain what they wrote.

Select a student to write $\frac{1}{2}$ at every one-half line segment and $\frac{2}{2}$ at every whole, using the same color as the line segment. **Emphasize** that each line segment is one-half the distance of a whole unit, even if the whole unit begins at a whole number other than 0 (e.g., the distance from 2 to 3, 3 to 4, 4 to 5, etc.).

In addition, **explain** that when considered from 0, the lengths are iterated and should be counted like students would count whole numbers. For example, the distance from 0 to $\frac{1}{2}$ is one one-half, whereas the distance from 0 to 1 is two-halves, the distance from 0 to $2\frac{1}{2}$ is five-halves, and so on. **Model** this for students by counting the fractional parts, and **avoid** referencing mixed numbers.

Repeat the preceding steps for fourths and eighths. Each line segment should be labeled as a fraction of a whole unit. **Emphasize** that each segment represents the length of a whole unit (e.g., three-fourths between 5 and 6 will still be labeled $\frac{3}{4}$ because that is the distance of that line segment between 5 and 6).



Ask students the following questions, **requiring** them to share with a partner before discussing with the whole group.

- What do you notice about the number line now that it has been labeled symbolically?
- How is the number line the same as it was before we added the symbolic fractions? Explain.
- How is the number line different than it was before we added the symbolic fractions? Explain.
- Do you think that there are any number(s) between 0 and $\frac{1}{8}$? Why or why not?

> NOTE: Some students may identify equivalent fractions. If so, acknowledge the comment and move on. The emphasis of this lesson is not equivalent fractions, but rather identifying fractions on a number line as a distance from 0 or from the starting location of a unit. Do not discourage any students that recognize equivalent fractions, but do not encourage students to identify equivalent fractions either.

Tell students the definition of a proper fraction.

Require a student to write an example of a proper fraction on the board, both in word form and symbolically.

Point to the numerator and **ask** students what it represents. **Point** to the denominator and **ask** students what it represents. **Require** students to explain their thinking.

Ensure that students are not thinking of fractions as "two-part" numbers, but rather as a single unit.

Introduce and **guide** students to connect the following terms with the correct definitions; the *denominator* names the equal-size parts or equal-size shares, and the *numerator* identifies how many copies of the part.

> NOTE: Understanding the relationship between the numerator and denominator at this stage is more important than memorizing the names. However, it is important that your language *and* the students' language is mathematically accurate and precise. Refrain from using phrases such as "the top number" and "the bottom number", which encourage thinking of fractions as "two-part" numbers.

Ask students what they think an improper fraction is.

Discuss student responses and **direct** students to the understanding that an improper fraction is a fraction greater than one whole, in which the numerator is greater than the denominator. Use strategic questioning and the number line if necessary.

Emphasize that an improper fraction has a numerator that is greater than the denominator. Mixed numbers are not the same as improper fractions, even though they are equivalent representations.

Distribute the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT and **require** students to complete Questions 1 – 3, either independently or in partners.

Use the following guiding questions to support students understanding as they answer Questions 1 – 3.

GUIDING QUESTIONS

Elicit student thinking:

- What do you notice about the models?
- How are the models the same? How are they different?
- Which model do you find easiest to recognize a fraction? Why? Which model do you find hardest to recognize a fraction? Why?
- Which model do you find easiest to represent a fraction? Why? Which model do you find hardest to represent a fraction? Why?

Determine if the student can REPRESENT ANY FRACTION AS A SYMBOLIC REPRESENTATION:

- Write the fraction ten-twelfths symbolically. How do you know that is correct?
- Write the fraction eight-halves symbolically. How do you know that is correct?
- Write the fraction symbolically that has nine copies of the unit fraction one-fifth. How do you know that is correct?
- [Point to Question 1.d., 2.d., or 3.d.] Why did you write this fraction like this?

Determine if the student can EXPLAIN THE FRACTION RELATIONSHIP OF *X/Y*:

- [Write the fraction six-eighths symbolically.] Can you explain what this means?
- [Write the fraction six-eighths symbolically.] What does the six represent? What does the eight represent? How are they related?
- [Point to Question 1.d., 2.d., or 3.d.] Why did you write this fraction? How are the numerator and denominator related?

Determine if the student can RECOGNIZE ANY FRACTION WITH A LENGTH MODEL:

- [Point to Question 1.] What fraction is represented on this number line? How do you know?
- [Point to the right end of the number line on Question 1.] If you added to the number line here so it extended past this point, would that change the fraction shown? Why or why not?
- [Point to Question 1.] Show me where you began counting unit fractions. Why did you begin there?
- [Point to Question 1.] Can you identify this fraction without counting unit fractions? Explain.
- [Point to Question 1.] If I said the fraction two-thirds was represented on this number line, would you agree or disagree with me? Why? What fraction is shown on the number line? How do you know?

Determine if the student can RECOGNIZE ANY FRACTION WITH AN AREA MODEL:

- [Point to Question 2.] What fraction is represented with this area model? How do you know?
- [Point to Question 2.] If you added one more shape that was not shaded in, would that change the fraction shown? Why or why not?
- [Point to Question 2.] Can you identify this fraction without counting unit fractions? Explain.
- [Point to Question 2.] If I said the fraction one-sixth was represented with this area model, would you agree or disagree with me? Why? What fraction is shown with the area model? How do you know?

Determine if the student can RECOGNIZE ANY FRACTION WITH A SET MODEL:

- [Point to Question 3.] What fraction is represented with this set model? How do you know?
- [Point to the unshaded set on the right side of the on Question 3.] If you took this set away, would that change the fraction shown? Why or why not?
- [Point to Question 3.] Show me how you counted the unit fractions. Why did you count them that way?
- [Point to Question 3.] Can you identify this fraction without counting unit fractions? Explain.
- [Point to Question 3.] If I said the fraction thirty-thirds was represented with this set model, would you agree or disagree with me? Why? What fraction is shown with the set model? How do you know?

Review the answers to Questions 1 – 3 on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT by strategically selecting students to share their answers and explain their thinking.

Select students with a weaker grasp of the relationship between the numerators and denominators to share correct answers for questions such as *identifying the unit fraction* or *the number of copies of the unit fraction*. Then, **select** students with a stronger understanding of the relationship between numerators and denominators to share and explain the fraction represented in the model.

Ask students how they know if a fraction is greater than or less than one whole. Students should discuss in partners before sharing with the class. **Require** students to explain or show their thinking using a model of their choice.

Display different representations (i.e. model, word form, symbolic) of fractions, both proper and improper, one at a time and **ask** students if the fraction is greater than or less than one whole. Students can respond in a variety of ways. For example, students could show "thumbs up" for greater than one whole and "thumbs down" for less than one whole, use individual dry erase boards to write greater than one whole or less than one whole, or hold up individual cards where one side says "greater than" and the other side says "less than".

Arrange students into groups of two or three. **Distribute** one copy of the *Greater Than, Less Than, or Equal to One* board and one set of cards from the INSTRUCTIONAL ACTIVITY SUPPLEMENT.

Read aloud and **model** the following directions.

- Stack the cards in a single pile face down.
- One student draws a card, and the group decides if it goes in the *Greater Than One* column, the *Less Than One* column, the *Equal to One* column, or the *Not Sure* column.
- The next student draws a card and the group decides where to place the card.
- Repeat the steps, alternating drawing cards until the pile is gone.

Provide students with adequate time to complete the activity.

Display an answer key for students to self-check, or **review** answers as a class when all students are finished.

Use the following guiding questions to support students understanding.

GUIDING QUESTIONS

Elicit student thinking:

- What fraction representation do you prefer and why?
- Which representation is easiest/hardest for you to recognize a fraction? Why?

Determine if the student can RECOGNIZE ANY FRACTION WITH A SET MODEL:

- [Point to a card showing a set model.] What fraction is represented with this set model? How do you know?
- [Point to a card showing a set model.] Does this set model represent a fraction greater than, less than, or equal to one? How do you know?
- [Point to a card showing a set model.] Looking at this set, describe for me the whole, the unit fraction, and the fraction represented. How do you know you are right?

Determine if the student can EXPLAIN *NUMERATOR*:

- [Point to a card.] What is the numerator for this fraction? How do you know?
- What does the numerator of a fraction represent? Give me an example.

Determine if the student can EXPLAIN *DENOMINATOR*:

- [Point to a card.] What is the denominator for this fraction? How do you know?
- What does the denominator of a fraction represent? Give me an example.

Determine if the student can RECOGNIZE ANY FRACTION WITH A LENGTH MODEL:

- [Point to a card showing a length model.] What fraction is represented with this length model? How do you know?
- [Point to a card showing a length model.] Does this length model represent a fraction greater than, less than, or equal to one? How do you know?
- [Point to a card showing a length model.] Looking at this length model, describe for me the whole, the unit fraction, and the fraction represented. How do you know you are right?

Determine if the student can RECOGNIZE ANY FRACTION WITH AN AREA MODEL:

- [Point to a card showing an area model.] What fraction is represented with this area model? How do you know?
- [Point to a card showing an area model.] Does this area model represent a fraction greater than, less than, or equal to one? How do you know?
- [Point to a card showing an area model.] Looking at this area model, describe for me the whole, the unit fraction, and the fraction represented. How do you know you are right?

Determine if the student can RECOGNIZE PROPER FRACTIONS WITH A SYMBOLIC REPRESENTATION:

- [Point to a card showing a proper fraction symbolically.] What is this fraction? How do you know?
- [Point to a card showing a proper fraction symbolically.] How do you know this fraction is less than “one”?
- [Point to a card showing a proper fraction symbolically.] What is the difference between a proper fraction and an improper fraction? What type of fraction is this? How do you know?

Determine if the student can RECOGNIZE ANY FRACTION WITH A SYMBOLIC REPRESENTATION:

- [Point to a card showing a fraction symbolically.] What is this fraction? How do you know?
- [Point to a card showing a fraction symbolically.] How do you know this fraction is less than "one", greater than "one", or equal to "one"?
- [Point to a card showing a fraction symbolically.] What is the difference between a proper fraction and an improper fraction? What type of fraction is this? How do you know?

Lead a discussion among students by asking the following questions.

- When you had a card with a fraction model, how could you tell if the fraction was less than one whole? Greater than one whole? Equal to one whole?
- When you had a card with a fraction in word form, how could you tell if the fraction was less than one whole? Greater than one whole? Equal to one whole?
- When you had a card with a fraction in symbolic form, how could you tell if the fraction was less than one whole? Greater than one whole? Equal to one whole?
- Were there any fraction cards that you placed in the "Not Sure" column? If so what were they?
- Did you have any fraction cards in the "Not Sure" column that you later moved after placing a few more cards?
- Were there any cards where you and your partner (or group) disagreed about the placement? If so, what were they and how did you resolve the disagreement?
- [Point to the large class number line.] How do you know if a fraction on the number line is greater than one whole?

Bring student attention to the *Fraction Representations* sheet from the INSTRUCTIONAL ACTIVITY SUPPLEMENT in LESSON 1.

Require students to complete each row for the fifth and sixth columns.

Review each row of the fifth and sixth columns by selecting several students to share and explain their responses on the *Everything I Need to Know About Fraction Representations* sheet.

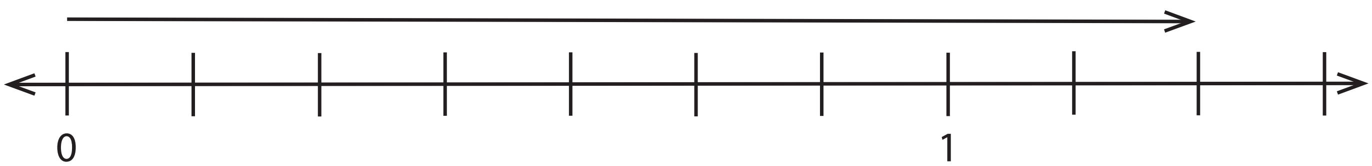
At the end of the activity, teachers should collect and analyze the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT to review student understanding.

Name________________________________

UNDERSTANDING FRACTIONS AS NUMBERS

Lesson 4

1. Use the model to complete the blanks. One whole is the length between 0 and 1. Write all fractions symbolically.



a. Unit (equal-size parts): ______________________________

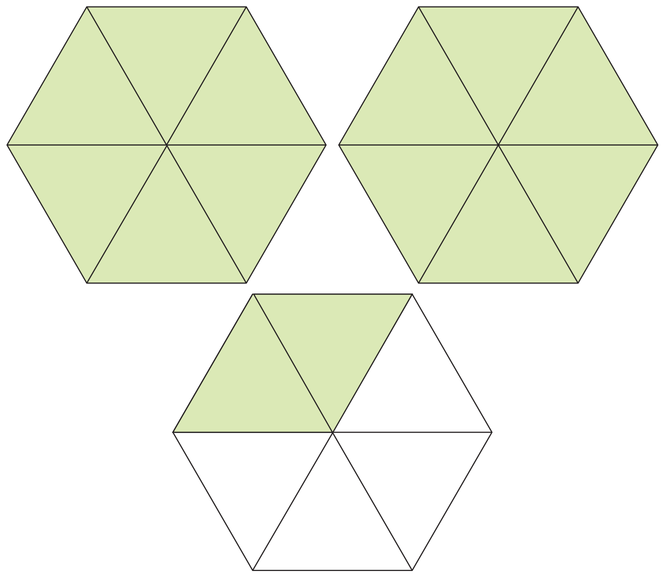
b. Unit fraction: ______________________________________

c. Number of copies of unit fraction identified: ____________

d. Fraction represented by the ray: ______________________

Name____________________________

2. Use the model to complete the blanks. One hexagon (six-sided shape) represents one whole. Write all fractions symbolically.



a. Unit (equal-size parts): ______________________________

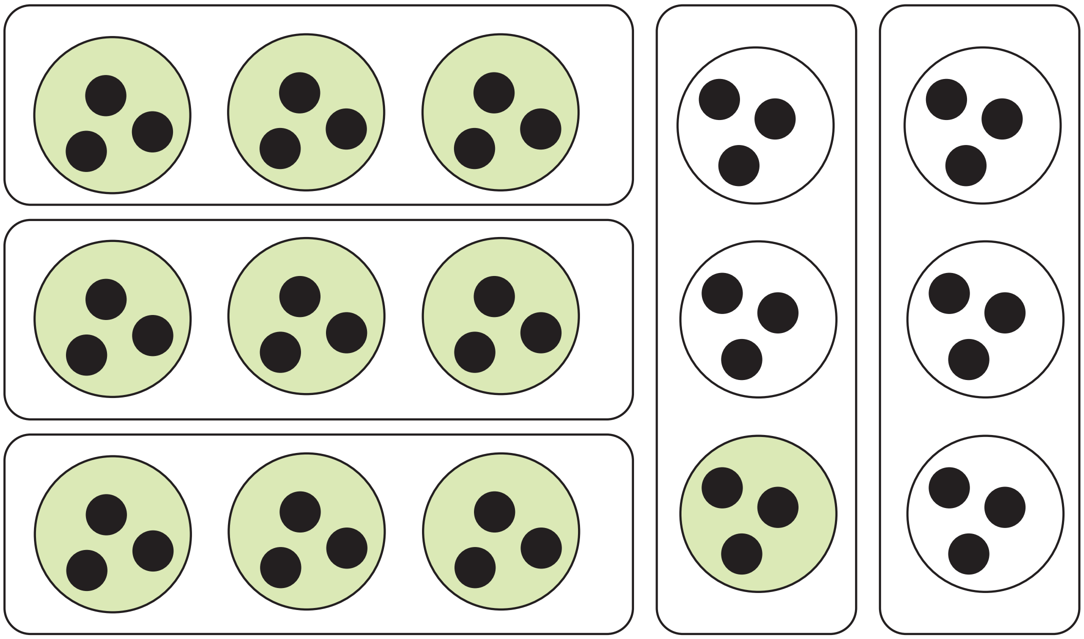
b. Unit fraction: _______________________________________

c. Number of copies of unit fraction shaded: ______________

d. Fraction represented by the shaded parts: _____________

Name______________________________

3. Use the model to complete the blanks. One whole is a set of three large circles with three small circles each. Write all fractions symbolically.

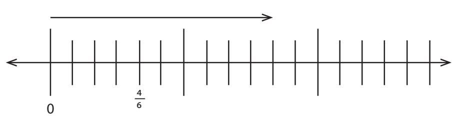
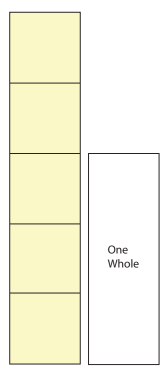
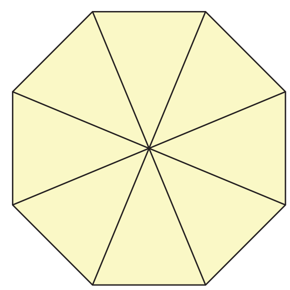


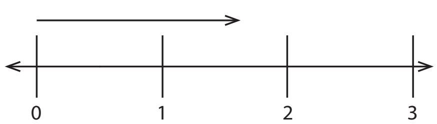
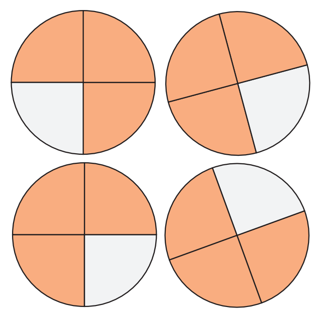
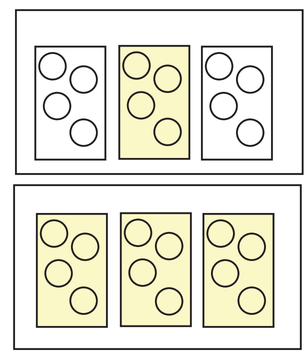
a. Unit (equal-size set): ______________________________

b. Unit fraction: _______________________________________

c. Number of copies of unit fraction shaded: ______________

d. Fraction represented by the shaded sets: _____________

three-fifths	ten-eighths	six-sixths
		 one shape is one whole
$\frac{3}{9}$	$\frac{9}{3}$	$\frac{8}{7}$

six-wholes	three-halves	fifteen-fifths
	 one shape is one whole	 one set is one whole
$\frac{10}{10}$	$\frac{2}{12}$	$\frac{5}{1}$

GREATER THAN, LESS THAN, OR EQUAL TO ONE

GREATER THAN ONE	LESS THAN ONE	EQUAL TO ONE	NOT SURE

Name____

Understanding Fractions as Numbers

1. Draw one whole for each situation using the given representation. Write what the whole represents. *Example: The rectangle is one whole candy bar.*

   1.a. Sam is baking a cherry pie to share at a school party. Draw the pie as an area model.

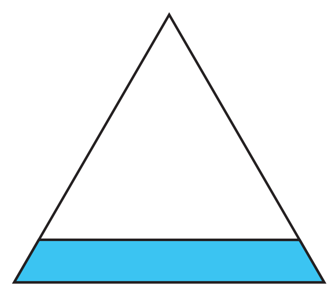
   1.b. Bella has a bag of 12 circle erasers in different colors. Draw the bag of erasers as a set model.

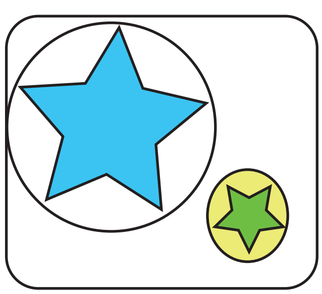
Name________________________________

1.c. Adam is describing the distance he travels to school. Draw a length model that displays one whole unit.

2. Circle the model that represents the given fraction. Explain why you chose the model you circled. Each length between 0 and 1 is one whole, each single shape is one whole, and each set is one whole.

2.a. One-half

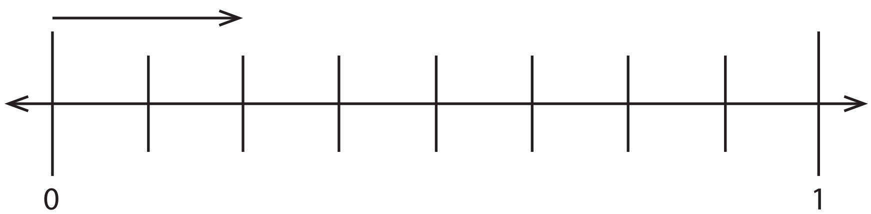


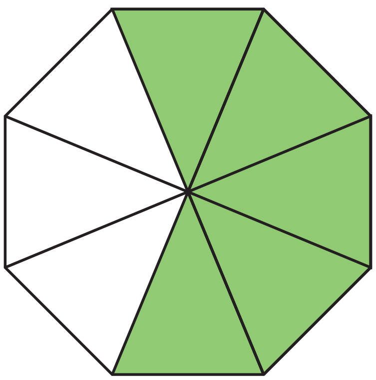


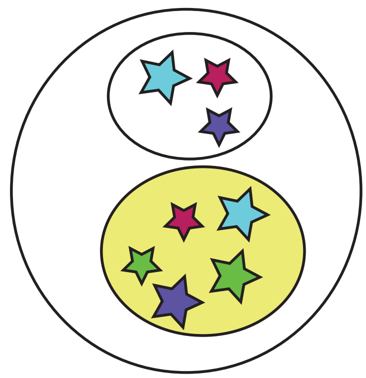
I chose this model because:

Name____

2.b. Two-eighths



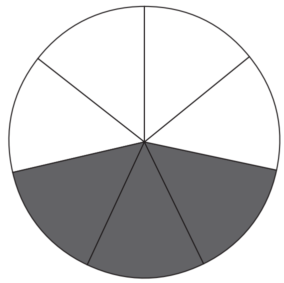


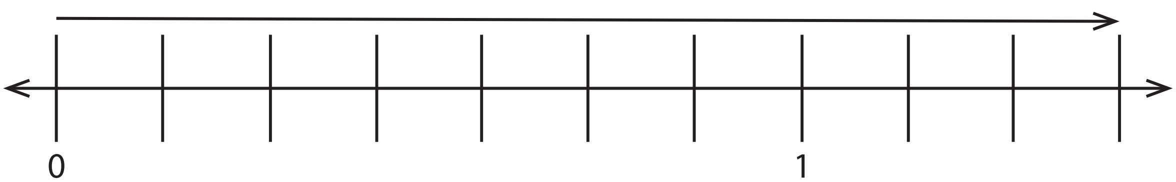


I chose this model because:

Name____

2.c. $\frac{7}{10}$





I chose this model because:

Name______________________________________

3. Draw a model to show the fraction described, then write the fraction symbolically.

   3.a. Alex has a bag with four cookies. Alex and his sister share the cookies so they both get an equal amount. Create a model to show what fraction of the bag of cookies Alex gets.

Alex will get ______________ of the bag of cookies.

Name____

3.b. Bryson has one foot of string. He needs 6 copies of $\frac{1}{9}$ of one foot of string for an art project. Create a model to show what fraction of the foot of string Bryson will need. One whole is equal to one foot.

Bryson will need ______________ of one foot of string.

Name____

3.c. Sam and Terri are sharing a piece of paper. If they divide the paper into 8 equal-size parts and they each take 3 parts, what fraction of the whole piece of paper will be left over?

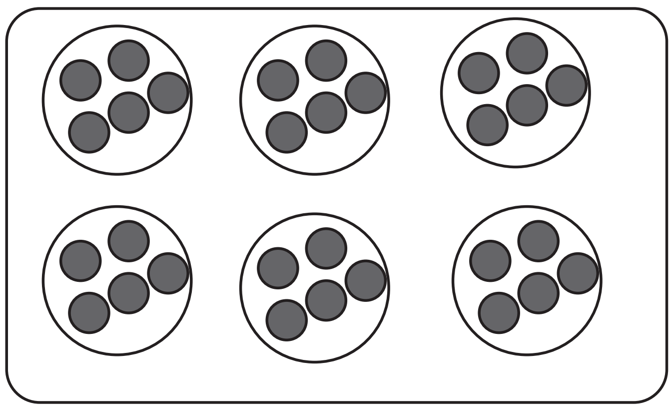
______________ of the whole piece of paper will be left over.

4. What is a unit fraction?

Name____________________________________

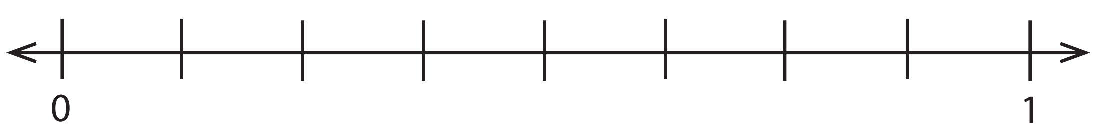
4.a. Use the model to show the unit fraction, then write the unit fraction symbolically.

The unit fraction for this set model is _____________.



4.b. Use the model to show the unit fraction, then write the unit fraction symbolically.

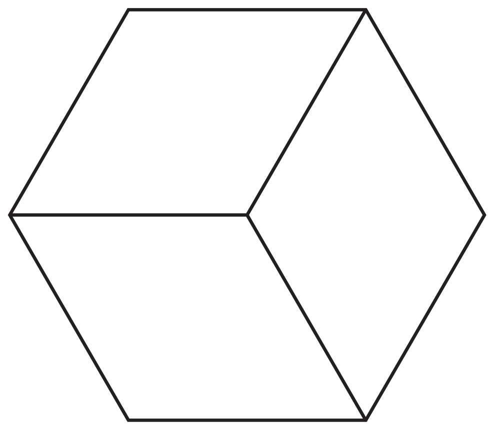
The unit fraction for this length model is ___________.



Name______________________________________

4.c. Use the model to show the unit fraction, then write the unit fraction symbolically.

The unit fraction for this area model is ____________.



5. Circle the unit fraction that has larger equal-size parts in the same whole. Support your answer with words or models.

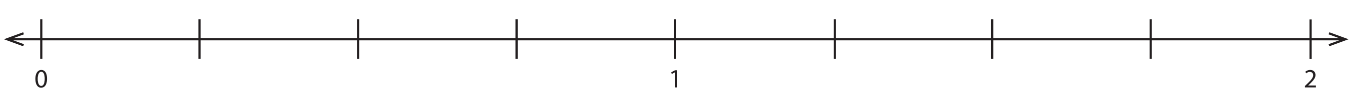
5.a. $\frac{1}{2}$ or $\frac{1}{5}$

Name____

5.b. $\frac{1}{10}$ or $\frac{1}{3}$

6. Identify or represent the given fraction, then circle an answer to indicate whether the fraction is greater than, less than, or equal to one whole. Write all fractions symbolically.

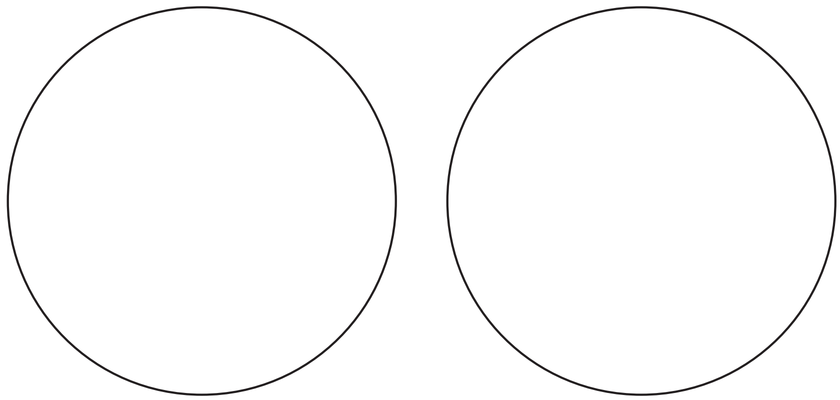
6.a. Show the fraction $\frac{6}{4}$ on the number line. The distance from 0 to 1 is one whole.



$\frac{6}{4}$ is (greater than / less than / equal to) one whole.

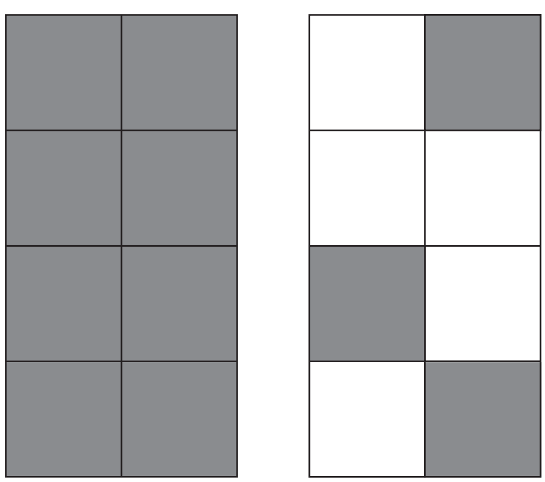
Name____

6.b. Show the fraction eight-eighths on the area model. One circle is equal to one whole.



is (greater than / less than / equal to) one whole.

6.c. One large rectangle is one whole. What is the fraction shown by the shaded parts?



The fraction shown by the shaded parts is:

is (greater than / less than / equal to) one whole.

Understanding Fractions as Numbers

STUDENT ACTIVITY SOLUTION GUIDE

1. Draw one whole for each situation using the given representation. Write what the whole represents.
   Example: The rectangle is one whole candy bar.

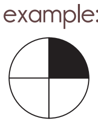
1.a. Sam is baking a cherry pie to share at a school party. Draw the pie as an area model.

CORRECT ANSWER

Check student work for understanding. The drawing should include a shape referenced as "the whole". The following is an example of a possible response:

The circle represents one whole pie.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

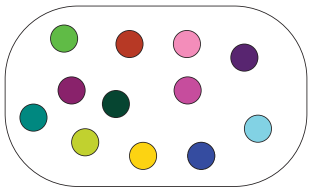
Example Error	Misconception	Missing Knowledge
The student draws a set model or a length model.	does not recognize the difference between the types of fraction models *or* does not understand what an area model looks like	REPRESENT WHOLE ON AN AREA MODEL and REPRESENT WHOLE ON A LENGTH MODEL or REPRESENT A WHOLE ON A SET MODEL
The student draws an area model, paritions, and shades in a fraction. For example: 	does not understand how to represent or consider a whole without a fractional part	REPRESENT WHOLE ON AN AREA MODEL

1.b. Bella has a bag of 12 circle erasers in different colors. Draw the bag of erasers as a set model.

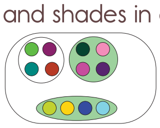
CORRECT ANSWER

Check student work for understanding. The drawing should include a set with 12 circles of different colors referenced as "the whole". The following is an example of a possible response:

The oval represents one whole set of 12 erasers.



ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

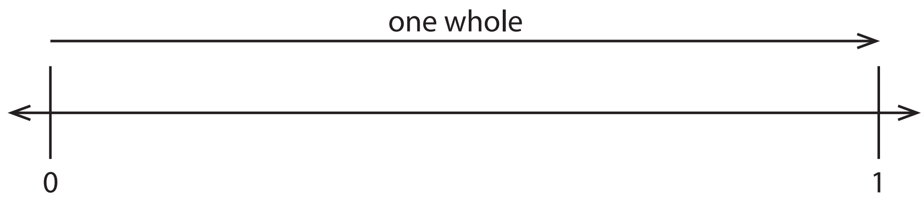
Example Error	Misconception	Missing Knowledge
The student draws an area model or a length model.	does not recognize the difference between the types of fraction models *or* does not understand what a set model looks like	REPRESENT WHOLE ON A SET MODEL and REPRESENT WHOLE ON A LENGTH MODEL or REPRESENT A WHOLE ON AN AREA MODEL
The student draws a set model, paritions, and shades in a fraction. For example: 	does not understand how to represent or consider a whole without a fractional part	REPRESENT WHOLE ON A SET MODEL
The student does not confine the set to represent one whole or does not write that the whole for the set model is a set of 12 circles; the student only draws 12 circles.	does not recognize what it looks like to represent one whole on a set model	REPRESENT WHOLE ON A SET MODEL

1.c. Adam is describing the distance he travels to school. Draw a length model that displays one whole unit.

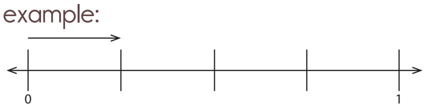
CORRECT ANSWER

Check student work for understanding. The drawing should include a number line or fraction bar labeled "one whole". The following is an example of a possible response:

The length on the number line between 0 and 1 represents one whole.

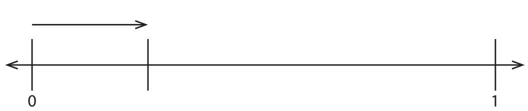


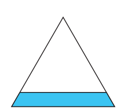
ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

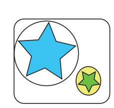
Example Error	Misconception	Missing Knowledge
The student draws an area model or a set model.	does not recognize the difference between the types of fraction models *or* does not understand what a length model looks like	REPRESENT WHOLE ON A LENGTH MODEL and REPRESENT WHOLE ON A SET MODEL or REPRESENT A WHOLE ON AN AREA MODEL
The student draws a length model, paritions, and shades in a fraction. For example: 	does not understand how to represent or consider a whole without a fractional part	REPRESENT WHOLE ON A LENGTH MODEL
The student does not label the number line and just draws a line or line segment.	does not understand how to create a number line	REPRESENT WHOLE ON A LENGTH MODEL

2. Circle the model that represents the given fraction. Explain why you chose the model you circled. Each length between 0 and 1 is one whole, each single shape is one whole, and each set is one whole.

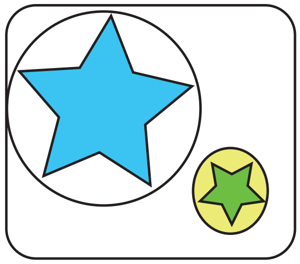
2.a. One-half







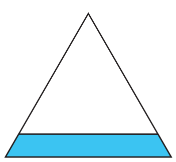
CORRECT ANSWER



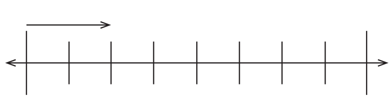
Check student response for understanding. The following is an example of a possible response:

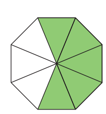
I chose this model because the set has two equal-size groups of one star each. One equal-size group is shaded in (or not shaded in) to show one-half of the whole set.

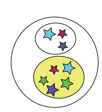
ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
	does not understand that the whole must be partitioned into equal-size parts	RECOGNIZE ONE-HALF ON A LENGTH MODEL and RECOGNIZE *EQUAL PART*
	does not understand that the whole must be partitioned into equal-size parts	RECOGNIZE ONE-HALF ON AN AREA MODEL and RECOGNIZE *EQUAL PART*
The student does not provide an explanation.	does not know why the model was chosen *or* is unable to explain the numerator, denominator, or both	RECOGNIZE ONE-HALF IN A SET MODEL or EXPLAIN *UNIT FRACTION*

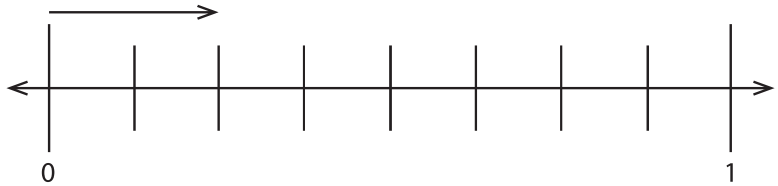
2.b. Two-eighths







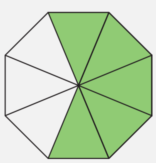
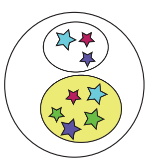
CORRECT ANSWER



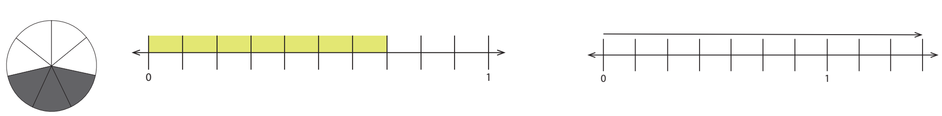
Check student response for understanding. The following is an example of a possible response:

I chose this model because the set has eight equal-size parts. The arrow begins at 0 and is the length of two equal-size parts to show two-eighths of one whole.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
	is not aware that the numerator represents the number of copies of the unit fraction, the number of equal-size parts shaded (or unshaded)	RECOGNIZE PROPER FRACTIONS WITH AN AREA MODEL and EXPLAIN *NUMERATOR*
	does not understand that the whole set must be partitioned into equal-size groups	RECOGNIZE PROPER FRACTIONS WITH A SET MODEL and EXPLAIN *DENOMINATOR*
The student does not provide an explanation.	does not know why the model was chosen *or* is unable to explain that fractions on a number line represent the distance from 0 to a specified location	RECOGNIZE PROPER FRACTIONS WITH A LENGTH MODEL or EXPLAIN THE POSITION OF FRACTIONS ON THE NUMBER LINE

2.c. $\frac{7}{10}$

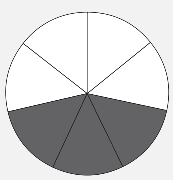
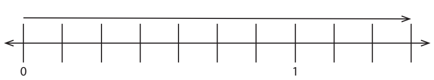


CORRECT ANSWER

Check student response for understanding. The following is an example of a possible response:

I chose this model because the set has 10 equal-size parts. The color band begins at 0 and is the length of seven equal-size parts to show seven-tenths of one whole.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

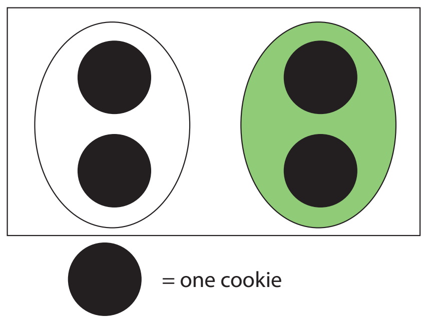
Example Error	Misconception	Missing Knowledge
	mistakes the numerator for the number of equal-size parts	RECOGNIZE PROPER FRACTIONS WITH AN AREA MODEL, EXPLAIN *NUMERATOR*, EXPLAIN *DENOMINATOR* and possibly RECOGNIZE PROPER FRACTIONS WITH A SYMBOLIC REPRESENTATION
	does not understand the different between the numerator and the denominator; believes that the numerator represents the number of equal-size parts and the denominator represents the number of copies of the unit fraction, the number of equal-size parts indicated by the length of the arrow	RECOGNIZE PROPER FRACTIONS WITH A NUMBER LINE, EXPLAIN *NUMERATOR*, EXPLAIN *DENOMINATOR* and possibly RECOGNIZE FRACTIONS WITH A SYMBOLIC REPRESENTATION
The student does not provide an explanation.	does not know why the model was chosen *or* is unable to explain that fractions on a number line represent the distance from 0 to a specified location	RECOGNIZE PROPER FRACTIONS WITH A LENGTH MODEL or EXPLAIN THE POSITION OF FRACTIONS ON THE NUMBER LINE

3. Draw a model to show the fraction described, then write the fraction symbolically.

3.a. Alex has a bag with four cookies. Alex and his sister share the cookies so they both get an equal amount. Create a model to show what fraction of the bag of cookies Alex gets.

CORRECT ANSWER

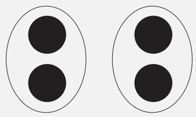
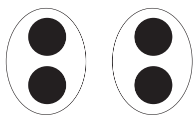
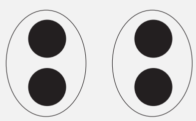
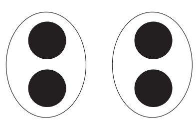
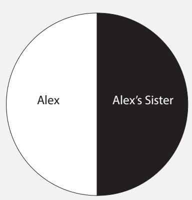
Check student response for understanding. Note: the following is an example response using the model that best represents the scenario, however students may choose to use an alternate model.



The green shaded group (or the unshaded group) represents the cookies Alex will get.

Alex will get $\frac{1}{2}$ of the bag of cookies. ("Alex will get $\frac{2}{4}$ of the four cookies" is also an acceptable answer.)

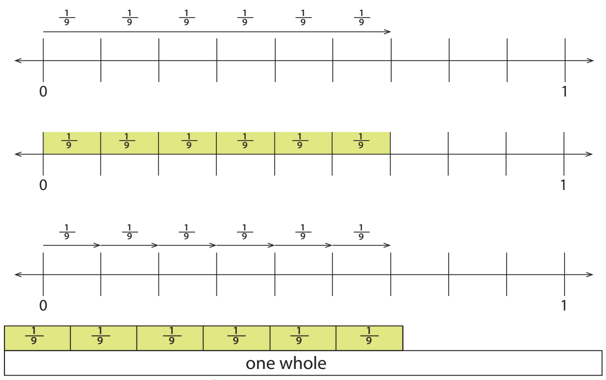
ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
 Alex will get two of the four cookies.	does not follow the directions *or* does not understand that two cookies represents one-half of the whole set of cookies	REPRESENT ONE-HALF IN A SET MODEL and/or REPRESENT PROPER FRACTIONS WITH A SYMBOLIC REPRESENTATION
 Alex will get $\frac{1}{4}$ of the four cookies.	recognizes that Alex gets one equal-size group, but does not understand that the denominator should be 2	REPRESENT ONE-HALF IN A SET MODEL and EXPLAIN *DENOMINATOR*
 Alex will get $\frac{1}{4}$ of the four cookies.	does not understand that there are two equal-size groups, so the denominator should be 2	REPRESENT ONE-HALF IN A SET MODEL and EXPLAIN *DENOMINATOR*
 Alex will get $\frac{2}{2}$ of the four cookies.	does not recognize that two-halves is one whole; recognizes that Alex should get two cookies, but sees each equal-size group as its own whole; does not understand what the numerator and/or the denominator represents	ONE-HALF IN A SET MODEL and RECOGNIZE WHOLE ON A SET MODEL and EXPLAIN *NUMERATOR* and/or EXPLAIN *DENOMINATOR*
	does not understand that the whole is four cookies instead of one cookie	RECOGNIZE A UNIT and REPRESENT WHOLE ON A SET MODEL
The student does not create a set model.	does not understand how to represent the situation as a fraction model	REPRESENT ONE-HALF IN A SET MODEL

3.b. Bryson has one foot of string. He needs 6 copies of $\frac{1}{9}$ of one foot of string for an art project. Create a model to show what fraction of the foot of string Bryson will need. One whole is equal to one foot.

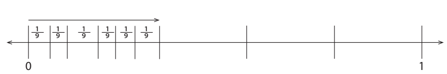
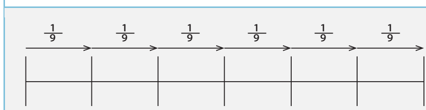
CORRECT ANSWER

Check student response for understanding. Note: the following is an example response using the model that best represents the scenario, however students may choose to use an alternate model.



Bryson will need $\frac{6}{9}$ of one foot of string.

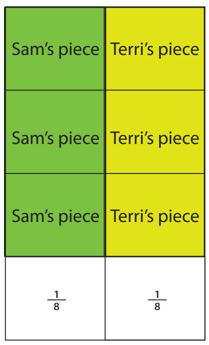
ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

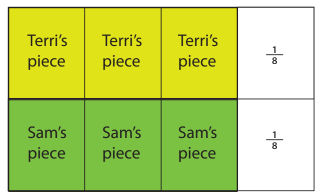
Example Error	Misconception	Missing Knowledge
The student creates a model representing nine-sixths.	does not understand what the numerator and/or the denominator represents	EXPLAIN *NUMERATOR*, EXPLAIN *DENOMINATOR* and/or EXPLAIN THE FRACTION RELATIONSHIP OF *X*/*Y*
The student creates a model represesnting one-ninth.	does not understand that the situation requires six copies of one-ninth	COUNT BY FRACTIONAL PARTS
The student creates six wholes. *or* The student creates six and one-ninth.	does not understand that the situation requires six copies of one-ninth	EXPLAIN *UNIT FRACTION* and COUNT BY FRACTIONAL PARTS
	does not understand that the whole must be partitioned into equal-size parts	PARTITION LENGTH INTO EQUAL PARTS or PARTITION A NUMBER LINE FROM 0 TO 1 INTO EQUAL PARTS
	does not understand that the whole should be represented as part of the fraction model	REPRESENT A WHOLE ON A LENGTH MODEL
The student writes, $\frac{1}{9}+\frac{1}{9}+\frac{1}{9}+\frac{1}{9}+\frac{1}{9}+\frac{1}{9}$ *or* The student does not provide a model.	does not understand how to represent the situation as a fraction model	REPRESENT PROPER FRACTIONS WITH A LENGTH MODEL

3.c. Sam and Terri are sharing a piece of paper. If they divide the paper into 8 equal-size parts and they each take 3 equal-size parts, what fraction of the whole piece of paper will be left over?

CORRECT ANSWER

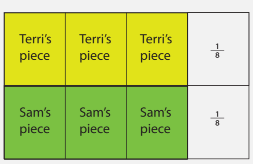
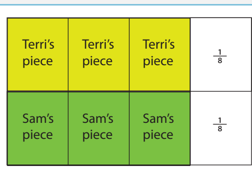
Check student response for understanding. Note: the following is an example response using the model that best represents the scenario, however students may choose to use an alternate model.

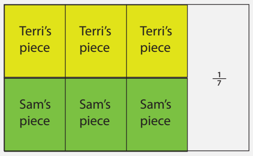
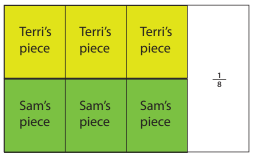
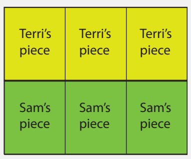
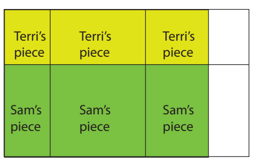
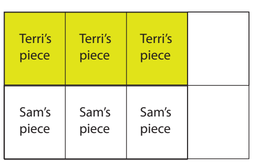
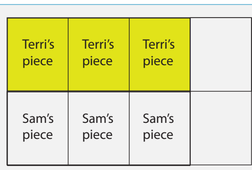




$\frac{2}{8}$ of the whole piece of paper will be left over.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
 $\frac{6}{8}$ of the whole piece of paper will be left over.	does not understand that the situation is asking for the remaining amount of paper, not the fraction of paper that Sam and Terri will take	SOLVE WORD PROBLEMS INVOLVING ADDITION AND SUBTRACTION OF FRACTIONS WITH LIKE DENOMINATORS
 $\frac{2}{2}$ of the whole piece of paper will be left over.	does not recognize that two-halves is one whole; recognizes that two equal-size parts remains, but considers only the remaining portion of the paper as the whole so the denominator is the two	RECOGNIZE PARTS OF A GIVEN WHOLE OR A UNIT

Example Error	Misconception	Missing Knowledge
 $\frac{1}{7}$ of the whole piece of paper will be left over.	does not understand that the whole must be partitioned into equal-size parts; considers the two left-over pieces of the paper as one piece	PARTITION ANY SHAPE INTO EQUAL PARTS
 $\frac{1}{8}$ of the whole piece of paper will be left over.	reads that the situation states eight equal-size parts, but creates a model with seven parts of different-size and considers the left-over paper as one piece	PARTITION ANY SHAPE INTO EQUAL PARTS and REPRESENT PROPER FRACTIONS WITH AN AREA MODEL
 $\frac{3}{6}$ (or $\frac{1}{2}$) of the whole piece of paper will be left over.	does not understand what the question is asking; misinterprets the whole as the amount of paper Terri and Sam are splitting and provides the fraction of the "whole" that each Terri and Sam would get	REPRESENT WHOLE ON AN AREA MODEL and REPRESENT PROPER FRACTIONS WITH AN AREA MODEL
	does not understand that the whole must be partitioned into equal-size parts	MODEL *EQUAL PART* and PARTITION ANY SHAPE INTO EQUAL PARTS
The student writes $\frac{1}{8} + \frac{1}{8}$ *or* The student does not provide a model.	Does not understand how to represent the situation as a fraction model	REPRESENT PROPER FRACTIONS WITH AN AREA MODEL
 $\frac{3}{8}$ of the whole piece of paper will be left over.	does not understand what the situation is asking and combines the two numerical values given in the problem situation to create a fraction	SOLVE WORD PROBLEMS INVOLVING ADDITION and DECOMPOSE A FRACTION INTO A SUM OF UNIT FRACTIONS WITH THE SAME DENOMINATOR
 $\frac{3}{8}$ of the whole piece of paper will be left over.	does not understand what the situation is asking and identifies the fraction for either Terri's pieces of paper or Sam's pieces of paper in relation to the whole	SOLVE WORD PROBLEMS INVOLVING ADDITION and DECOMPOSE A FRACTION INTO A SUM OF UNIT FRACTIONS WITH THE SAME DENOMINATOR

4. What is a unit fraction?

CORRECT ANSWER

Check student work for understanding. The following is an example of a possible response:

A unit fraction is one partition of a whole divided into equal-size parts.

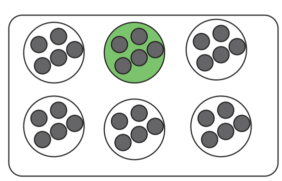
ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student's response is vauge or the definition does not make sense. *or* The student does not provide a definition.	does not understand what a unit fraction is	EXPLAIN *UNIT FRACTION*
A fraction with a numerator and a denominator.	does not understand that a unit fraction is a fraction in which the numerator is one	EXPLAIN *UNIT FRACTION*
A fraction that has a numerator that is larger than the denominator.	does not understand that a unit fraction is a fraction in which the numerator is one; confuses a unit fraction with an improper fraction	EXPLAIN *UNIT FRACTION* and EXPLAIN *IMPROPER FRACTIONS*
A fraction that the numerator and the denominator are the same number.	does not understand that a unit fraction is a fraction in which the numerator is one; confuses a unit fraction with a fraction representation of one whole	EXPLAIN *UNIT FRACTION* and EXPLAIN THE FRACTION RELATIONSHIP OF *X*/*Y*

4.a. Use the model to show the unit fraction, then write the unit fraction symbolically.

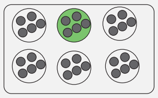
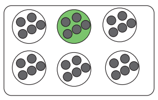
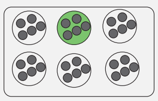
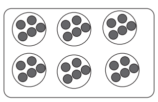
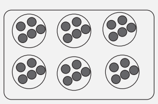
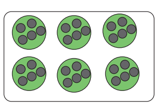
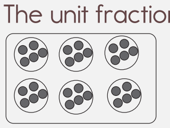
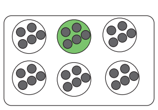
CORRECT ANSWER

The unit fraction for this set model is $\frac{1}{6}$.



Note: Any one of the equal-size groups can be shaded in; the focus is on the fact that only one equal-size group is shaded in.

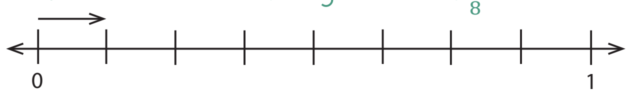
ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

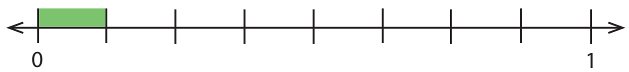
Example Error	Misconception	Missing Knowledge
The unit fraction for this set model is $\frac{5}{30}$. 	does not understand that a unit fraction has a numerator of one and may not understand how to interpret a set model	EXPLAIN *UNIT FRACTION* and possibly COUNT BY FRACTIONAL PARTS
The unit fraction for this set model is $\frac{1}{30}$. 	recognizes that a unit fraction has a numerator of one and has identified one equal-size group on the model, however does not understand that the set has been divided into six equal-size groups, not 30 equal-size groups	EXPLAIN *DENOMINATOR* and RECOGNIZE PROPER FRACTIONS WITH A SET MODEL
The unit fraction for this set model $\frac{5}{6}$. 	does not understand that a unit fraction has a numerator of one, and does not understand how to interpret a set model; incorrectly identifies that the numerator is the number of items in each equal-size group	EXPLAIN *UNIT FRACTION* and RECOGNIZE PROPER FRACTIONS WITH A SET MODEL
The unit fraction for this set model is $\frac{30}{6}$. 	does not understand that a unit fraction has a numerator of one, and does not understand how to interpret or represent a fraction with a set model	EXPLAIN *UNIT FRACTION*, RECOGNIZE PROPER FRACTIONS WITH A SET MODEL and REPRESENT PROPER FRACTIONS WITH A SET MODEL
The unit fraction for this set model is $\frac{30}{6}$. 	does not understand that a unit fraction has a numerator of one, and does not understand that the numerator represents the number of copies of the unit fraction identified and the denominator represents the number of equal-size partitions	EXPLAIN *UNIT FRACTION*, EXPLAIN *NUMERATOR* and EXPLAIN *DENOMINATOR*
The unit fraction for this set model is $\frac{6}{6}$. 	does not understand that a unit fraction has a numerator of one, and does not understand and/or cannot interpret from a model that when the numerator and denominator are the same number, the value of the fraction is one whole	EXPLAIN *UNIT FRACTION* and/or RECOGNIZE WHOLE ON A SET MODEL
The unit fraction for this set model is 6. 	interprets the number of equal-size groups as the unit fraction or thinks that the denominator is the unit fraction; does not recognize that the answer provided is not written symbolically as a fraction but as a whole number	RECOGNIZE *FRACTION* and EXPLAIN *UNIT FRACTION*
The unit fraction for this set model is $\frac{1}{5}$. 	considers the remaining equal-size groups as the denominator because one equal-size group has been identified; thinks that one equal-size group as the numerator and five equal-size groups left as the denominator are combined to create one whole	RECOGNIZE WHOLE ON A SET MODEL, RECOGNIZE PROPER FRACTIONS WITH A SET MODEL and EXPLAIN *DENOMINATOR*

4.b. Use the model to show the unit fraction, then write the unit fraction symbolically.

CORRECT ANSWER

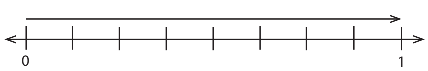
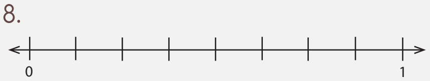
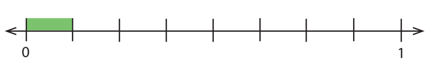
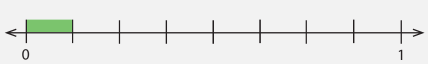
The unit fraction for this length model is $\frac{1}{8}$.





Note: Any one of the equal-size groups can be shaded in; the focus is on the fact that only one equal-size group is shaded in. However, consider following up with any students that do not begin at 0 to ensure their understanding of fractional lengths as a distance on a number from 0.

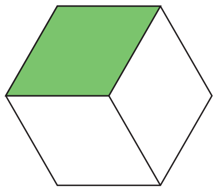
ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The unit fraction for this length model is $\frac{8}{1}$. 	recognizes the need for a one in a unit fraction but does not understand what the numerator and the denominator represent	EXPLAIN THE FRACTION RELATIONSHIP OF *X*/*Y*
The unit fraction for this length model is $\frac{8}{8}$. 	does not understand that a unit fraction has a numerator of one, and does not understand and/or cannot identify a symbolic fraction from a length model	EXPLAIN *UNIT FRACTION* and RECOGNIZE WHOLE ON A LENGTH MODEL
The unit fraction for this length model is 8. 	interprets the number of equal-size groups as the unit fraction or thinks that the denominator is the unit fraction; does not recognize that the answer provided is not written symbolically as a fraction but as a whole number	RECOGNIZE *FRACTION*
The unit fraction for this length model is $\frac{1}{9}$. 	does not understand how to interpret the partitions on a number line; determines the denominator is the number of line segments as opposed to the lengths between the segments	RECOGNIZE *EQUAL PART* and COUNT BY FRACTIONAL PARTS
The unit fraction for this length model is $\frac{1}{7}$. 	considers the remaining equal-size partitions as the denominator because one equal-size length has been identified; thinks that one equal-size length as the numerator and seven equal-size lengths left as the denominator are combined to create one whole	RECOGNIZE PROPER FRACTIONS WITH A LENGTH MODEL and EXPLAIN *DENOMINATOR*

4.c. Use the model to show the unit fraction, then write the unit fraction symbolically.

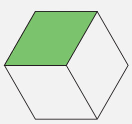
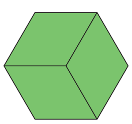
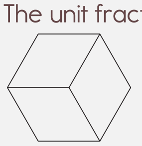
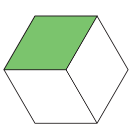
CORRECT ANSWER

The unit fraction for this area model is $\frac{1}{3}$.



Note: Any one of the equal-size parts can be shaded in; the focus is on the fact that only one equal-size part is shaded in.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The unit fraction for this area model is $\frac{3}{1}$. 	recognizes the need for a one in a unit fraction but does not understand what the numerator and the denominator represent	EXPLAIN THE FRACTION RELATIONSHIP OF *X/Y*
The unit fraction for this area model is $\frac{3}{3}$. 	does not understand that a unit fraction has a numerator of one, and does not understand and/or cannot identify a symbolic fraction from an area model	EXPLAIN *UNIT FRACTION* and RECOGNIZE WHOLE ON A AN AREA MODEL
The unit fraction for this area model is 3. 	interprets the number of equal-size parts as the unit fraction or thinks that the denominator is the unit fraction; does not recognize that the answer provided is not written symbolically as a fraction but as a whole number	RECOGNIZE *FRACTION*
The unit fraction for this area model is $\frac{1}{2}$. 	considers the two unshaded equal-size parts as the denominator because one equal-size part has been shaded; thinks that one equal-size part as the numerator and two equal-size parts left as the denominator are combined to create one whole	RECOGNIZE THIRDS ON AN AREA MODEL and EXPLAIN *DENOMINATOR*

5. Circle the unit fraction that has larger equal-size parts in the same whole. Support your answer with words or models.

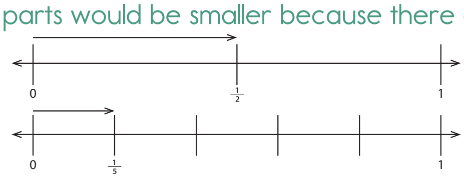
5.a. $\frac{1}{2}$ or $\frac{1}{5}$

CORRECT ANSWER

$\frac{1}{2}$

Check student work for understanding. The following are examples of possible responses:

$\frac{1}{2}$ has larger equal-size parts because there are only two parts in the whole, but if you divide the same whole into five equal-size parts, those parts would be smaller because there are more parts.



Note: If a student provides a model, ensure that both wholes are the same size (and shape if applicable).

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
$\frac{1}{5}$ has larger equal-size parts because five is larger than two.	does not understand that the denominator indicates the number of equal-size parts and the greater the denominator the smaller the equal-size parts; thinks that because five is a larger whole number, the equal-size parts must therefore be larger	EXPLAIN RELATIONSHIPS BETWEEN UNIT FRACTIONS and EXPLAIN DENOMINATOR
The student selects either answer without providing a model or written work to support the slection.	is unable to defend the answer due to a lack of understanding about equipartition sizes and unit fractions	EXPLAIN *UNIT FRACTION* and EXPLAIN RELATIONSHIPS BETWEEN UNIT FRACTIONS
The student selects either answer and provides incorrect models or an incorrect explanation.	is unable to defend the answer due to a lack of understanding about equipartitioning and unit fractions	EXPLAIN *UNIT FRACTION* and PARTITION ANY SHAPE INTO EQUAL PARTS, PARTITION LENGTH INTO EQUAL PARTS or PARTITION SETS INTO EQUAL SUBSETS

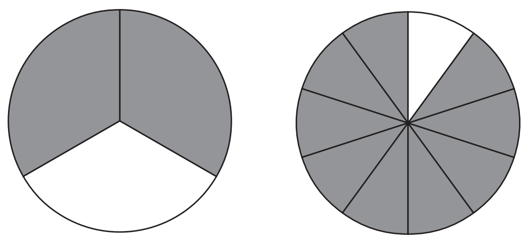
5.b. $\frac{1}{10}$ or $\frac{1}{3}$

CORRECT ANSWER

$\frac{1}{3}$

Check student work for understanding. The following are examples of possible responses:

$\frac{1}{3}$ has larger-equal-size parts because there are three parts in the whole, but if you divide the same whole into 10 equal-size parts, those parts would be smaller because there are more parts.



ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

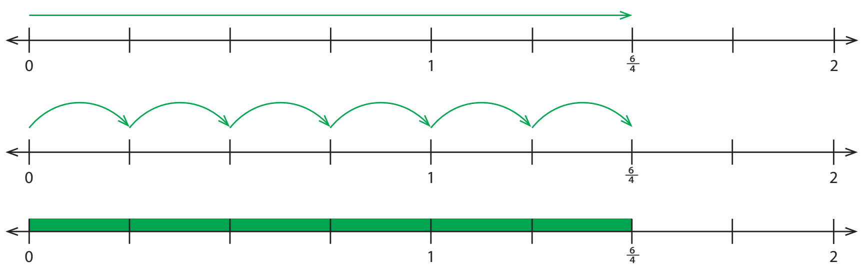
Example Error	Misconception	Missing Knowledge
$\frac{1}{10}$ has larger-equal-size parts because ten is larger than three.	does not understand that the denominator indicates the number of equal-size parts and the greater the denominator the smaller the equal-size parts; thinks that because 10 is a larger whole number, the equal-size parts must therefore be larger	EXPLAIN RELATIONSHIPS BETWEEN UNIT FRACTIONS and EXPLAIN DENOMINATOR
The student selects either answer without providing a model or written work to support the slection.	is unable to defend the answer due to a lack of understanding about equipartition sizes and unit fractions	EXPLAIN *UNIT FRACTION* and EXPLAIN RELATIONSHIPS BETWEEN UNIT FRACTIONS
The student selects either answer and provides incorrect models or an incorrect explanation.	is unable to defend the answer due to a lack of understanding about equipartition sizes and unit fractions	EXPLAIN *UNIT FRACTION* and PARTITION ANY SHAPE INTO EQUAL PARTS, PARTITION LENGTH INTO EQUAL PARTS or PARTITION SETS INTO EQUAL SUBSETS

6. Identify or represent the given fraction, then circle an answer to indicate whether the fraction is greater than, less than, or equal to one whole. Write all fractions symbolically.

6.a. Show the fraction $\frac{6}{4}$ on the number line. The distance from 0 to 1 is one whole.

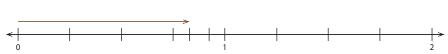
CORRECT ANSWER

Check student work for understanding. The following are examples of possible responses:



$\frac{6}{4}$ is greater than one whole.

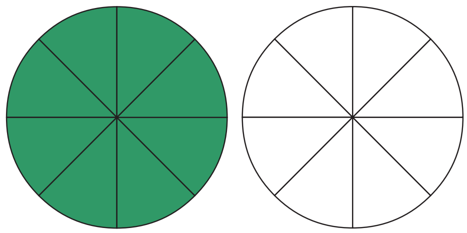
ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
	does not understand the meaning of the numerator and the denominator in a symbolic fraction; confuses the numerator and denominator by identifying four equal-size parts on the number line	REPRESENT ANY FRACTION WITH A LENGTH MODEL and RECOGNIZE ANY FRACTION WITH A SYMBOLIC REPRESENTATION
	does not understand the meaning of the numerator and denominator in a symbolic fraction; adds two line segments to create six parts and identifies four parts on the number line	RECOGNIZE EQUAL PART, REPRESENT ANY FRACTION WITH A LENGTH MODEL and/or RECOGNIZE ANY FRACTION WITH A SYMBOLIC REPRESENTATION
$\frac{6}{4}$ is equal to one whole. *or* $\frac{6}{4}$ is less than one whole.	does not understand or recognize that six copies of one-fourth are greater than one whole because there are more copies than the number of equal-size parts that make up the whole	RECOGNIZE WHOLE ON A LENGTH MODEL and COUNT BY FRACTIONAL PARTS

6.b. Show the fraction eight-eighths on the area model. One circle is equal to one whole.

CORRECT ANSWER

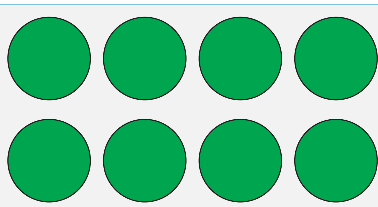
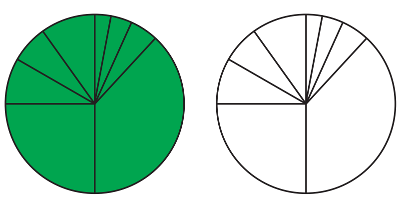
Check student work for understanding. The following are examples of possible responses:



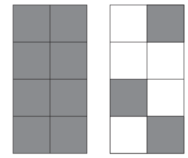
$\frac{8}{8}$ is **equal to** one whole.

Note: It is not required that students partition the second circle. Ensure that the student attempted to equipartition the circle as equally as possible.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
	thinks that eight-eighths means eight wholes	REPRESENT ANY FRACTION WITH AN AREA MODEL and/or EXPLAIN THE FRACTION RELATIONSHIP OF *X*/*Y*
	does not understand that the shape(s) must be partitioned into equal-size parts	PARTITION ANY SHAPE INTO EQUAL PARTS
$\frac{8}{8}$ is greater than one whole. *or* $\frac{8}{8}$ is less than one whole.	does not understand or recognize that eight copies of one-eighth are the equal to one whole	RECOGNIZE WHOLE ON AN AREA MODEL and COUNT BY FRACTIONAL PARTS

6.c. One large rectangle is one whole. What is the fraction shown by the shaded parts?



CORRECT ANSWER

The fraction shown by the shaded part is $\frac{11}{8}$.

$\frac{11}{8}$ is greater than one whole.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The fraction shown by the shaded part is $\frac{11}{16}$.	did not attend to the directions or does not understand that one whole is equipartitioned into eight equal-size parts	RECOGNIZE WHOLE ON AN AREA MODEL and/or RECOGNIZE ANY FRACTION WITH AN AREA MODEL
The fraction shown by the shaded part is $\frac{8}{11}$.	does not understand what the numerator and denominator represent	RECOGNIZE ANY FRACTION WITH AN AREA MODEL
The fraction shown by the shaded part is $\frac{8}{8}$.	only attends to one whole of the area model; disregards the area model that represents three-eighths	RECOGNIZE ANY FRACTION WITH AN AREA MODEL
The fraction shown by the shaded part is $\frac{3}{8}$.	only attends to one whole of the area model; disregards the model that represents eight-eighths	RECOGNIZE ANY FRACTION WITH AN AREA MODEL
$\frac{11}{8}$ is equal to one whole. *or* $\frac{11}{8}$ is less than one whole.	does not understand or recognize that eleven copies of one-eighth are greater than one whole because there are more copies than the number of equal-size parts that make up the whole	RECOGNIZE WHOLE ON AN AREA MODEL and COUNT BY FRACTIONAL PARTS