THE UNIVERSITY OF KANSAS
Achievement & Assessment Institute

ANALYZING ALGEBRAIC PATTERNS AND RELATIONSHIPS

5.RA.A.1, 5.RA.A.2

CONTENTS

The types of documents contained in the unit are listed below. Throughout the unit, the documents are arranged by lesson.

LEARNING MAP INFORMATION	An overview of the standards, the learning map section, and the nodes addressed in this unit
TEACHER NOTES	A brief discussion describing the progression depicted in the learning map section with research-based recommendations for focusing instruction to foster student learning and an introduction to the unit's lessons
OVERVIEW OF INSTRUCTIONAL ACTIVITIES	A table highlighting the lesson goals and nodes addressed in each lesson of this unit
INSTRUCTIONAL ACTIVITY	A detailed walkthrough of the unit
INSTRUCTIONAL ACTIVITY STUDENT HANDOUT	A handout for the guided activity, intended to be paired with the Instructional Activity
INSTRUCTIONAL ACTIVITY SUPPLEMENT	A collection of materials or activities related to the Instructional Activity
STUDENT ACTIVITY	A work-alone activity for students
STUDENT ACTIVITY SOLUTION GUIDE	A solution guide for the work-alone activity with example errors, misconceptions, and links to the learning map section

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Analyzing Algebraic Patterns and Relationships

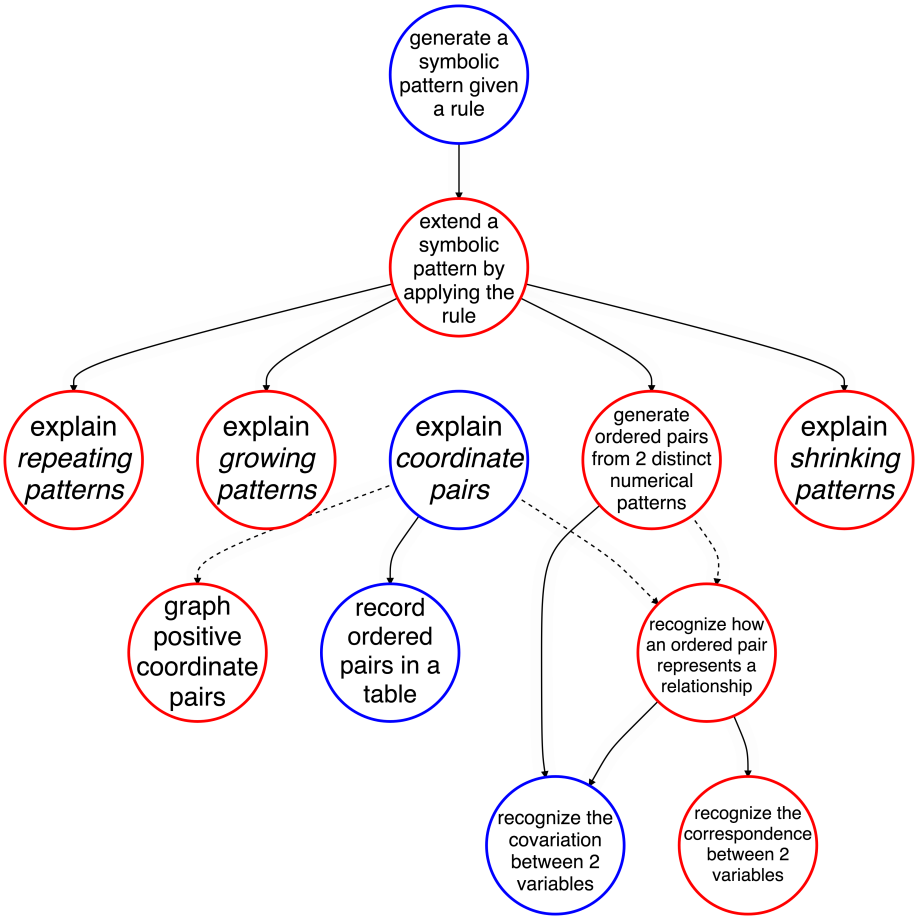
Learning Map Information

Standards

5.RA.A.1 Investigate the relationship between two numeric patterns.

- **5.RA.A.1a** Generate two numeric patterns given two rules.
- **5.RA.A.1b** Translate two numeric patterns into two sets of ordered pairs.
- **5.RA.A.1c** Graph numeric patterns on the Cartesian coordinate plane.
- **5.RA.A.1d** Identify the relationship between two numeric patterns.

5.RA.A.2 Write a rule to describe or explain a given numeric pattern.



**Learning map model of* 5.OA.3

Node Name	Node Description
EXPLAIN *COORDINATE PAIRS*	Make known your understanding that a coordinate pair (ordered pair) is a set of numbers used to show the position on a graph. The first number a in the coordinate pair (a, b) indicates the horizontal distance from zero, and the second number b in the coordinate pair (a, b) indicates the vertical distance from zero.
EXPLAIN *GROWING PATTERNS*	Make known your understanding of the rule in growing patterns and how the successive term is obtained from the preceding term.
EXPLAIN *REPEATING PATTERNS*	Make known your understanding that the core unit in repeating patterns is repeated over and over again.
EXPLAIN *SHRINKING PATTERNS*	Make known your understanding the rule in a shrinking pattern and how each successive term is obtained from the preceding term.
EXTEND A SYMBOLIC PATTERN BY APPLYING THE RULE	Analyze a given symbolic pattern to determine the core unit or the pattern rule, and use the core unit or pattern rule to extend the pattern.
GENERATE A SYMBOLIC PATTERN GIVEN A RULE	Given a rule, generate a symbolic pattern involving either numbers or letters.
GENERATE ORDERED PAIRS FROM 2 DISTINCT NUMERICAL PATTERNS	Represent two distinct patterns using ordered pairs, where the x-value follows one numerical pattern and the y-value follows a different numerical pattern.
GRAPH POSITIVE COORDINATE PAIRS	Through writing or an appropriate assistive technology, graph positive coordinate pairs in real-world or mathematical problems.
RECOGNIZE HOW AN ORDERED PAIR REPRESENTS A RELATIONSHIP	Make known your understanding that in the ordered pair (x, y), x represents the input of a function or relation and y represents the output.
RECOGNIZE THE CORRESPONDENCE BETWEEN 2 VARIABLES	Recognize the correspondence view of a relation as mapping of elements in the domain (input) to elements in the range (output). Given a two-column table or a graph of a function or a relation, describe how to calculate the outputs from the inputs or vice versa.
RECOGNIZE THE COVARIATION BETWEEN 2 VARIABLES	Recognize the covariation view of a relation or a function as the relationship between the two patterns displayed by the independent and dependent variables. Given a two-column table or a graph of a function or a relation, describe how the dependent variable changes in response to changes in the independent variable. For example, as the independent variable increases by two, the dependent variable decreases by four.
RECORD ORDERED PAIRS IN A TABLE	Given a list of ordered pairs or points on a graph, record the x- and y-coordinates in a table.

ADDITIONAL NODES RELATED TO THIS UNIT OF INSTRUCTION

Node Name	Node Description	Related Node
RECOGNIZE THE PATTERN RULE IN A GROWING PATTERN	Identify the pattern rule in a growing pattern by determining how each step in the pattern differs from the preceding step.	Prerequisite of EXPLAIN *GROWING PATTERNS*
EXTEND A PICTORIAL PATTERN BY APPLYING THE RULE	Analyze a given pictorial pattern to determine the core unit or the pattern rule, and use the core unit or pattern rule to extend the pattern.	Prerequisite of EXPLAIN *GROWING PATTERNS*
RECOGNIZE *REPEATING PATTERNS*	Recognize a pattern that has a core unit repeated over and over.	Prerequisite of EXPLAIN *REPEATING PATTERNS*

Analyzing Algebraic Patterns and Relationships

TEACHER NOTES

This unit includes the following documents:

- Learning Map Information
- Instructional Activity (includes three lessons)
- Instructional Activity Student Handout (for Lessons 1 – 3)
- Student Activity
- Student Activity Solution Guide

In this unit, students will learn to identify, analyze, and extend patterns and recognize both covariation and correspondence relationships within a given sequence or situation.

RESEARCH

Burns (2000) states that, "Patterns are key factors in understanding mathematical concepts. The ability to create, recognize, and extend patterns is essential for making generalizations, seeing relationships, and understanding the order and logic of mathematics" (p. 144). Analyzing geometric growing patterns is a concrete introduction into functions. Students should first identify a pattern, including the core unit of repeating patterns. Then, students should focus on extending a specific pattern before they can predict an unknown value in a given sequence. Once students have attended to one pattern, the same understanding should be applied to multiple patterns, in which students identify covariation and correspondence relationships, create rules to identify predicted values, and later create functions to represent given patterns. Students should analyze how patterns grow and change progressively, beginning by analyzing visual patterns, then advancing to numerical relationships, and finally extending patterns to an unknown (n^{th}) step or term (Van de Walle et al., 2014). Students should be able to systematically organize pattern information in order to make predictions about the extension of the pattern and should be able to share this pattern information verbally, numerically, graphically, and symbolically (NCTM, 2000).

According to Schliemann, et al. (2003), viewing "algebra as generalized arithmetic of numbers and quantities" requires a "shift from thinking about relations among particular numbers toward thinking about relations among sets of numbers", as well as a shift from simply "computing numerical answers to describing and representing relations among variables" (p. 128). An example is shifting from $3x = 6$ where $x = 2$ to a situation such as $3x = y$ where the following sets of numbers would satisfy the equation: (2, 6), (3, 9), (4, 12), etc. This shift likewise transitions from simply solving for x to determining the relationship between the variables. For example, in the equation $3x = y$, each x-value is one-third the y-value, or inversely, each y-value is three times the x-value.

When generalizing activities build on what students already know about number operations, these activities can provide a strong connection between number operations and algebra (Lannin, 2003). In addition, Lannin (2003) suggests that these activities could also "develop a deeper understanding of formal algebraic symbols" (p. 342). Students hold misconceptions about the meaning of letters standing for variables and have limited interpretations of the equal sign (Schliemann, et al., 2003). These limited interpretations often result in students viewing the equal sign as an indicator to provide an answer, as opposed to a relational understanding of the equal sign, in which students recognize that quantities on either side of the equal sign are equivalent. Furthermore, students are also limited in their understanding of the equal sign in thinking that an answer or result should appear on the right side of the equal sign, and the problem or operation requiring a solution should appear to the left side of the equal sign. Students should be aware that numbers, operations, and/or variables can appear on either or both sides of the equal sign (Karp, et al., 2015). In addition, students should develop an awareness that the equal sign represents equivalence. They should understand that the values on either side of the equal sign are equivalent or balanced, even if they cannot be simplified to appear identical.

Instructional activities should support student understanding of the use of variables (i.e. letters, empty boxes, or symbols) as place holders for varying quantities, rather than abbreviations, labels, or specific values. In addition to understanding that a variable may stand for a single value, students should also recognize that variables may represent many different possible values (Schliemann, et al., 2003). As students begin to work with symbols representing a single number or sets of numbers, they are beginning to gain an understanding of one meaning of the term "variable".

AN EXAMPLE

VARIABLES

$3 + x$ | $3 + ? = 5$ | $3 + \square = 5$

THE EQUAL SIGN

Students should understand that the equal sign means that the quantities on both sides are equivalent. The following are alternatives for representing 3 + 2 = 5.

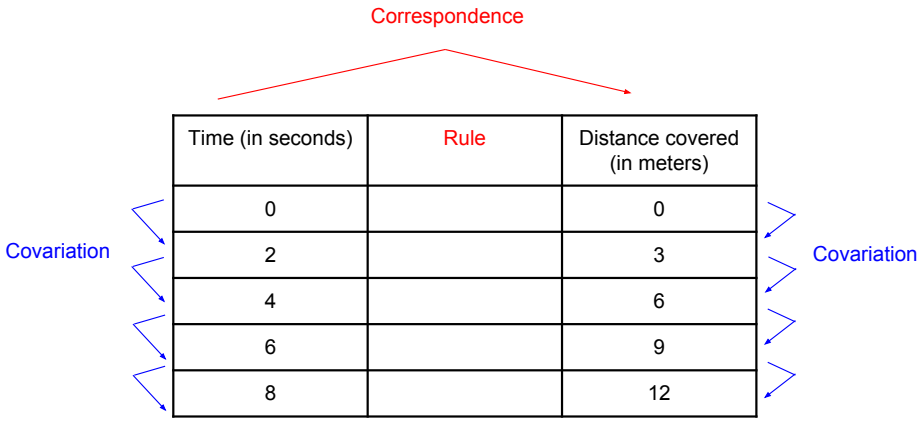
$3 + 2 = 4 + 1$ | $5 = 2 + 3$ | $2 + x = 5$ | $5 = \square + 3$

In this unit, students will combine their knowledge of identifying and analyzing patterns with their new understanding of algebraic relationships to recognize both covariation and correspondence between two sets of values. Using variables and an understanding of the equal sign, students will be able to represent, symbolically, rules to determine unknown values in and across given patterns. Experiences and conversations with algebraic reasoning "provide rich contexts for advancing mathematical understanding and are also an

important precursor to the more formalized study of algebra in the middle and secondary grades" (NCTM, 2000, p. 159). Identifying relationships between patterns in fifth grade leads into understanding of functional relationships in later grades (Leinhardt et al., 1990). Students should be given opportunities to explore relationships within and among numerical patterns by analyzing models (i.e. diagrams and tables) and graphical representation, as well as opportunities to express those relationships both verbally and symbolically (Panorkou, et al., 2014). Panorkou, et al. (2014) describes covariation as the relationship between two patterns in which the quantity in one pattern changes at the same time as the quantity in a second pattern changes, while correspondence is the relationship between the corresponding pairs of values in the two patterns through a rule. For example, in the following diagram, covariation is shown by the change in both time and distance; as the time increases by two seconds, the distance covered increases by three meters. Correspondence is shown in the following diagram through the rule that exists between the time and the distance: in order to obtain the distance covered (in meters), multiply the time (in seconds) by one and one half or 1.5. Students should be able to identify the covariation relationships before they can proceed to identifying the correspondence relationship.

AN EXAMPLE

The following is a diagram of covariation and correspondence relationships represented in a table (Panorkou, et al., 2014, p. 6).



LEARNING MAP INFORMATION

The learning map section for this sequence of activities begins with generating, identifying, and extending symbolic patterns in order to demonstrate understanding of repeating, growing, and shrinking patterns. Once students are capable of extending patterns, they should be ready to generate ordered pairs from two distinct patterns. These ordered pairs can then either be recorded in a table and/or graphed on a coordinate grid. It is important for students to identify that an ordered pair is a result of applying a rule which has been established

by a pattern. These skills precede students' ability to analyze ordered pairs to determine both covariation and correspondence relationships.

INSTRUCTIONAL ACTIVITIES

The activities in this unit are designed to establish and build on students' foundational understanding of patterns and algebraic reasoning. In the first lesson, students will explore and extend geometric patterns, recording numerical data in tables to identify covariation relationships. The second lesson extends the understanding of covariation relationships, as students generate ordered pairs for corresponding terms recorded in tables and graph those pairs on a coordinate grid. In the third lesson, students analyze tables and graphs to develop an understanding of correspondence by identifying the rule or relationship between two given patterns.

REFERENCES

Bay-Williams, J. M. (2001). What is Algebra in elementary school?. *Teaching children mathematics*, *8*(4), 196.

Burns, M. (2000). *About teaching mathematics: A K-8 resource*. Math Solutions Publications, Marilyn Burns Education Associates, 150 Gate 5 Road, Suite 101, Sausalito, CA 94965.

Karp, K. S., Bush, S. B., & Dougherty, B. J. (2015). 12 Math Rules That Expire in the Middle Grades. *Mathematics Teaching in the Middle School*, *21*(4), 208-215.

Lannin, J. K. (2003). Developing algebraic reasoning through generalization. *Mathematics Teaching in the Middle School*, *8*(7), 342.

Leinhardt, G., Zaslavsky, O., & Stein, M. K. (1990). Functions, graphs, and graphing: Tasks, learning, and teaching. *Review of Educational Research, 60*, 1-64.

Markworth, K. A. (2012). Growing patterns: seeing beyond counting. *Teaching Children's Mathematics*, *19*(4), 254-262.

National Council of Teachers of Mathematics (NCTM). *Principles and Standards for School Mathematics*. Reston, VA: NCTM, 2000.

Panorkou, N., Maloney, A. P., & Confrey, J. Expressing Covariation and Correspondence relationships in elementary schooling.

Schliemann, A., Carraher, D., Brizuela, B., Earnest, D., Goodrow, A., Lara-Roth, S., & Peled, I. (2003). Algebra in Elementary School. *International Group for the Psychology of Mathematics Education*, *4*, 127-134.

Tanisli, D., & Ozdas, A. (2009). The Strategies of Using the Generalizing Patterns of the Primary School 5th Grade Students. *Educational Sciences: Theory and Practice*, *9*(3), 1485-1497.

Van de Walle, J.A., Karp, K.S., Lovin, L.H., & Bay-Williams, J.M. (2006). *Teaching Student-Centered Mathematics*. Upper Saddle River, NJ: Pearson Professional Development.

Analyzing Algebraic Patterns and Relationships

Overview of Instructional Activities

Lesson	Learning Goal	Nodes Addressed
Lesson 1	Students will identify the relationship between a pattern (with shapes and/or numbers) and a specific rule (function).	▶ EXPLAIN *GROWING PATTERNS* ▶ EXPLAIN *REPEATING PATTERNS* ▶ EXPLAIN *SHRINKING PATTERNS*
Lesson 2	Students will expand their understanding of algebraic relationships and patterns by completing tables based on given rules, recognizing that corresponding terms can be represented as coordinate pairs, and by plotting those coordinate pairs in the first quadrant of a coordinate grid.	▶ GENERATE A SYMBOLIC PATTERN GIVEN A RULE ▶ EXTEND A SYMBOLIC PATTERN BY APPLYING THE RULE ▶ EXPLAIN *COORDINATE PAIRS* ▶ GENERATE ORDERED PAIRS FROM 2 DISTINCT NUMERICAL PATTERNS ▶ GRAPH POSITIVE COORDINATE PAIRS
Lesson 3	Students will analyze tables and graphs to ascertain relationships between corresponding terms.	▶ RECOGNIZE THE CORRESPONDENCE BETWEEN 2 VARIABLES ▶ RECORD ORDERED PAIRS IN A TABLE ▶ RECOGNIZE THE COVARIATION BETWEEN 2 VARIABLES ▶ RECOGNIZE THE CORRESPONDENCE BETWEEN 3 VARIABLES ▶ RECOGNIZE HOW AN ORDERED PAIR REPRESENTS A RELATIONSHIP

Analyzing Algebraic Patterns and Relationships

INSTRUCTIONAL ACTIVITY

Lesson 1

LEARNING GOAL

Students will identify the relationship between a pattern (with shapes and/or numbers) and a specific rule (function).

PRIMARY ACTIVITY

Students will work cooperatively in groups of two or three to identify and continue patterns using different shapes in order to predict a future design in a given sequence.

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

- Variable
- Corresponding terms
- Growing pattern
- Repeating pattern
- Shrinking pattern

MATERIALS

- Pattern blocks (or paper shapes, see Instructional Activity Supplement)
- Instructional Activity Student Handout
- Instructional Activity Supplement

IMPLEMENTATION

Place students in groups of two or three. Hand out the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT. Make triangle pattern blocks (or paper shapes) available.

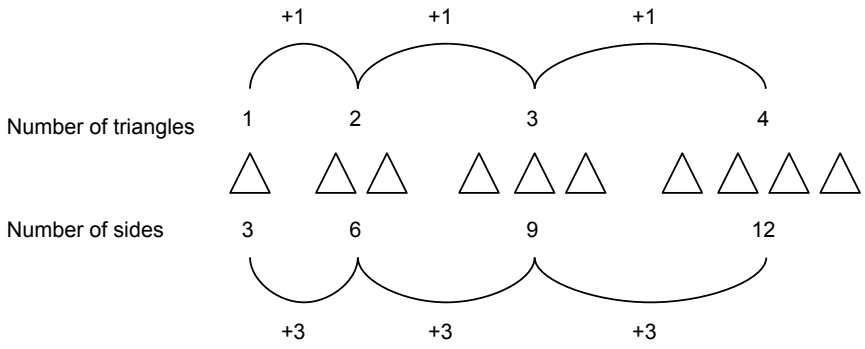
> NOTE: This lesson provides an opportunity to enhance students' understanding of two-dimensional shapes by focusing on the names and attributes of the different shapes used in the patterns. In addition, incorporating irregular representations of shapes is encouraged to reinforce the idea that regular polygons are not the only representation of a given shape.

Require student groups to identify the pattern and create the next two designs in the pattern sequence for Question 1 on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

Require one group to display their continuation of the pattern.

Discuss the pattern as a whole group. Focus the discussion not only on the pattern found with the number of triangles, but also on the pattern found with the number of sides.

Encourage students to understand that there can be more than one pattern within a given sequence.



Direct students' attention to Question 2 and the table on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

Display the table on the board, then as a class complete the number of sides for the first three triangles.

Refer back to the visual sequence of the triangles from Question 1 as you complete the table. Students should finish the table with their group. Use the guiding questions to support student discussions.

Number of Triangles	1	2	3
Number of sides	3	6	9

GUIDING QUESTIONS

Elicit student thinking:

- Can you discuss what you see?
- [Point to a triangle.] What is this figure? How do you know?
- What is a pattern you have seen before?

Determine if the student can EXPLAIN *GROWING PATTERNS*:

- [Point to the sequence of triangles.] What is happening with these triangles?
- [Point to the sequence of triangles.] How many triangles will come next? How do you know?
- What happens with each design? Is it growing, shrinking, or repeating? How do you know?
- How many sides will there be with 10 triangles? How do you know?
- Is there a connection between the number of triangles and the number of sides? Explain.

Students should complete Question 3 independently and then discuss their responses in their groups.

Discuss the pattern of triangles, **requiring** two or three students to share their answers for Question 3 (the number of sides is three times the number of triangles).

Require students to explain and defend what type of pattern this is (growing, shrinking, or repeating). First **require** students to share with an elbow partner, then **select** one or two students to share with the class.

Make quadrilateral pattern blocks (or paper shapes) available for use.

Repeat the same process listed previously using quadrilaterals (rectangles, squares, kites, trapezoids, or a combination). Students should create the sequence given in Question 4 using quadrilateral pattern blocks and then complete Questions 4 and 5 of the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT in groups of two or three.

x	2	4	6	8	10	12
y	8	16	24	32	40	48

where x = the number of quadrilaterals, and y = the number of sides

GUIDING QUESTIONS

Elicit student thinking:

- Have you used tables in math before? Explain.
- [Point to a quadrilateral.] What shape is this? How do you know?
- [Point to a quadrilateral.] What can you tell me about this shape?

Determine if the student can EXPLAIN *GROWING PATTERNS*:

- [Point to the sequence of quadrilaterals.] What is happening with these quadrilaterals?
- [Point to the sequence of quadrilaterals.] How many quadrilaterals will come next? How do you know?
- What happens with each design? Is it growing, shrinking, or repeating? How do you know?
- How many sides will there be with 10 quadrilaterals?
- Is there a connection between the number of quadrilaterals and the number of sides? Explain.

Make a variety of pattern blocks (or paper shapes) available for use. Students will need circles, triangles, quadrilaterals, pentagons, and hexagons.

In groups of two or three, students should complete Questions 6 – 13 on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

Use the guiding questions to support student thinking and discussion.

GUIDING QUESTIONS

Elicit student thinking:

- What does a table look like?
- Is there only one correct way to make a table? Explain.
- [Point to a shape.] What can you tell me about this shape?

Determine if the student can EXPLAIN *REPEATING PATTERNS*:

- [Point to Question 7.] What will be the next shape in the pattern? How do you know?
- What does *repeating* mean?
- [Point to Question 7.] What is happening with this pattern? Is it growing, shrinking, or repeating? How do you know?
- What do you notice about the number of sides in the table on Question 8?
- What does a repeating pattern look like? How do you know it is repeating?

Determine if the student can EXPLAIN *SHRINKING PATTERNS*:

- [Point to Question 12.] What will be the next shape in the pattern? How do you know?
- What does *shrinking* mean?
- [Point to Question 12.] What is happening with this pattern? Is it growing, shrinking, or repeating? How do you know?
- What do you notice about the number of sides in the table on Question 13?
- What does a shrinking pattern look like? How do you know it is shrinking?

At the end of the activity, students should show their understanding of patterns by completing Question 14 on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

Name_____

Analyzing Algebraic Patterns and Relationships

Lesson 1

1. Identify the pattern and draw the next two designs in the sequence.

2. Complete the table.

Number of Triangles	1	2	3	
Number of Sides				12

7	10

3. Describe in words the pattern that is represented in Questions 1 and 2.

Name_____

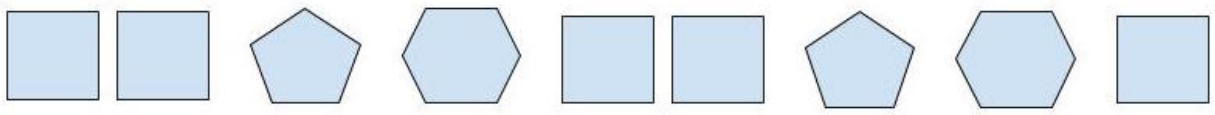
4. Complete the table.

x	2	4	6	8	10	12
y						

where x = the number of quadrilaterals, and y = the number of sides

5. Describe in words the pattern that is represented in Question 5 between corresponding pairs.

6. Draw the next four shapes in the pattern.



Name_____

7. Complete the table using the pattern in Question 6.

Number in the pattern	Number of sides
1	4
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	

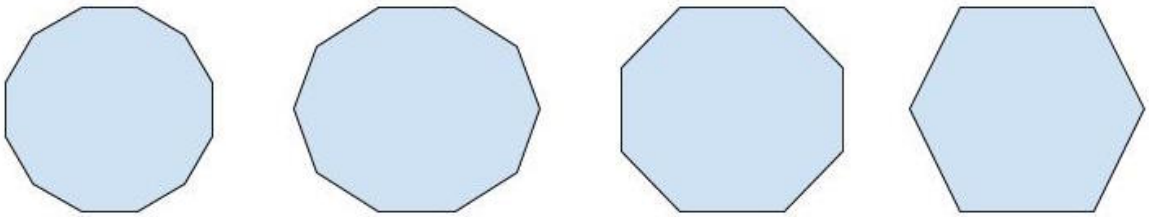
8. Describe in words the pattern that is represented in Question 6.

Name_____

9. Explain how the pattern from Question 6 is different from the pattern in Question 1.

10. Without using pattern blocks to show the pattern, how many sides will the 20th shape have? How do you know?

11. Draw the next shape in the pattern.



Name_____

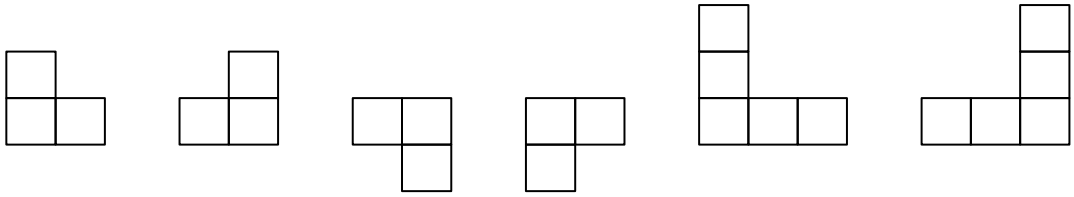
12. Complete the table using the pattern in Question 11.

Number in the pattern	Number of sides
1	
2	
3	
4	
5	
6	

13. Describe in words the pattern that is represented in Question 11.

Name_____

14. Describe in words the following pattern.

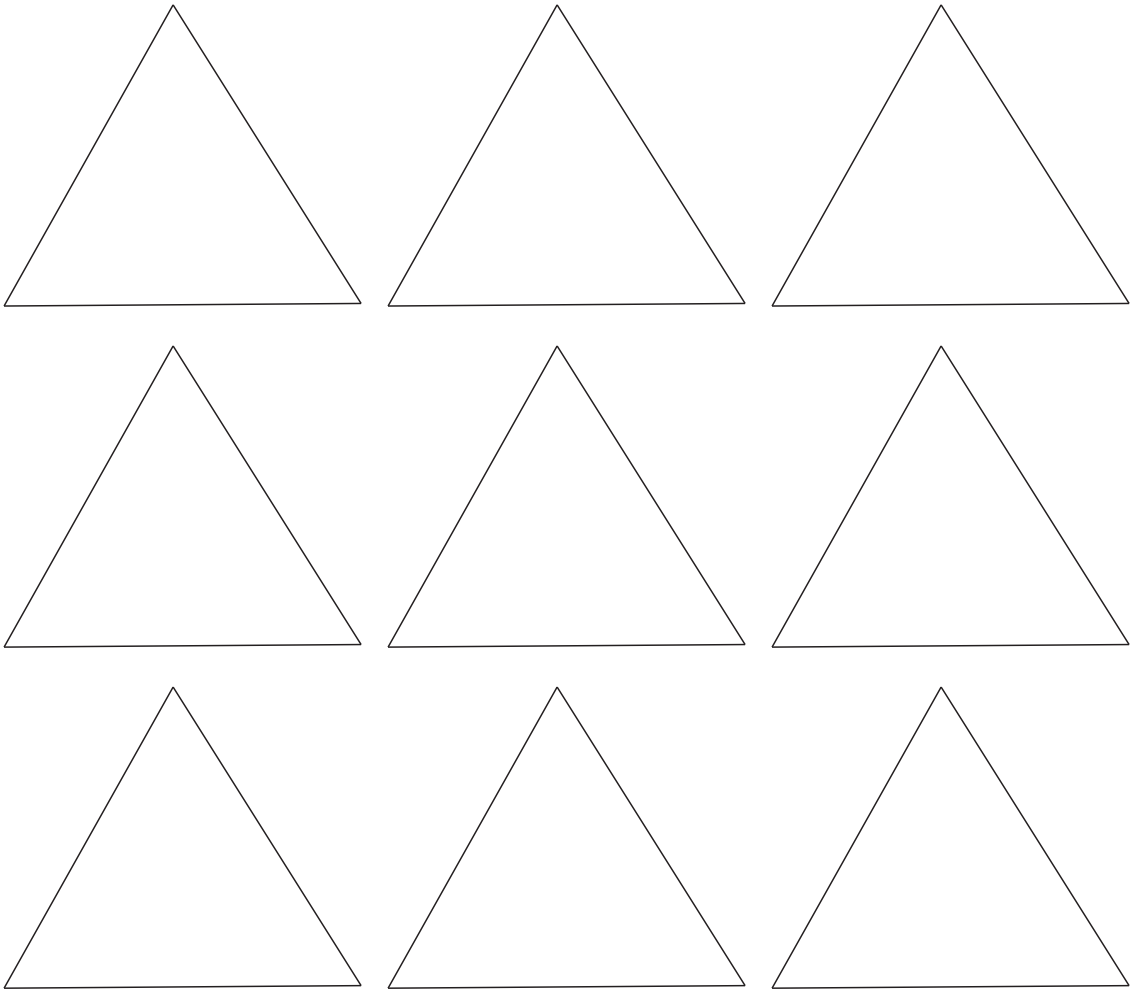


Analyzing Algebraic Patterns and Relationships

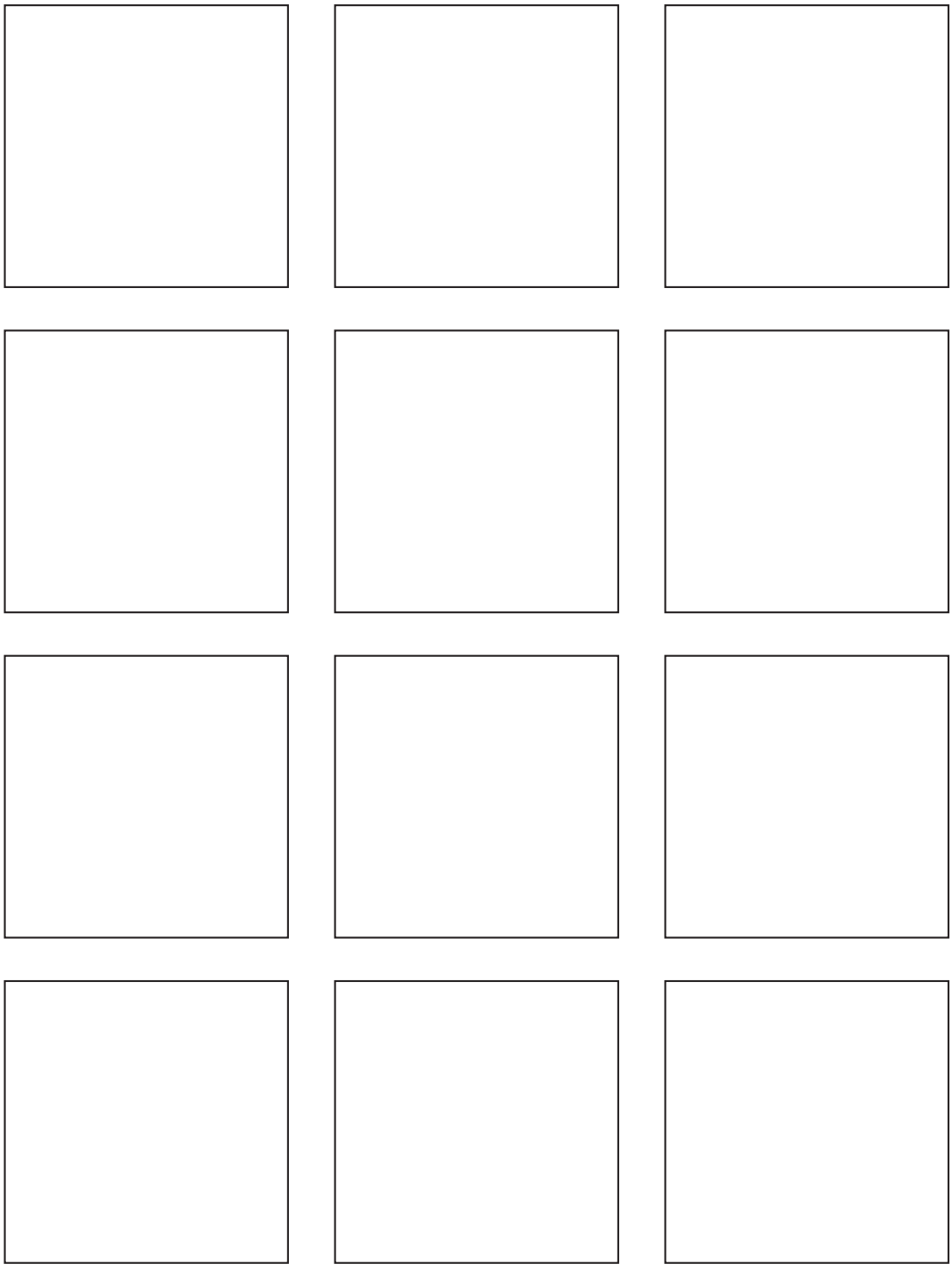
INSTRUCTIONAL ACTIVITY SUPPLEMENT

Lesson 1

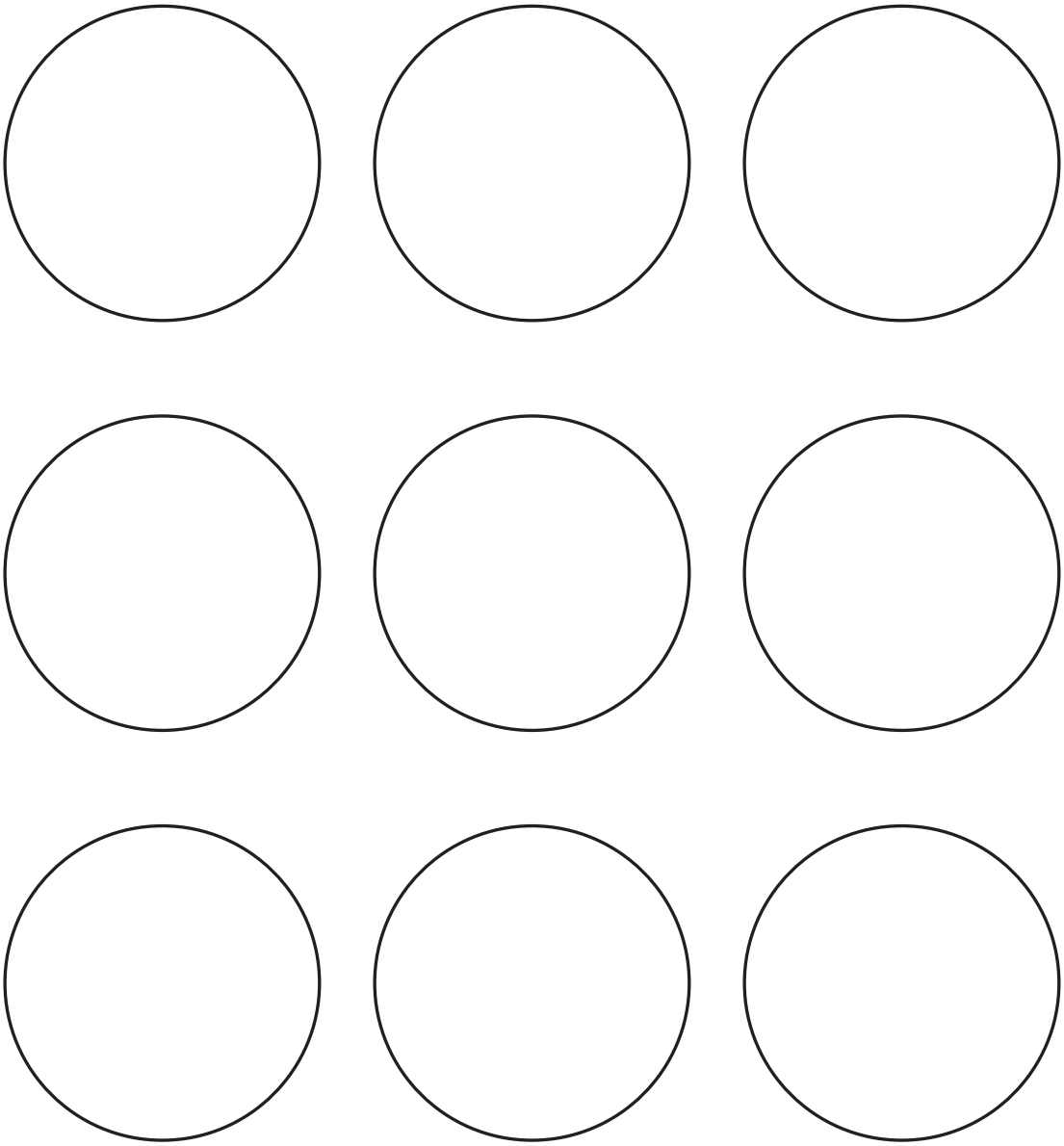
Triangles



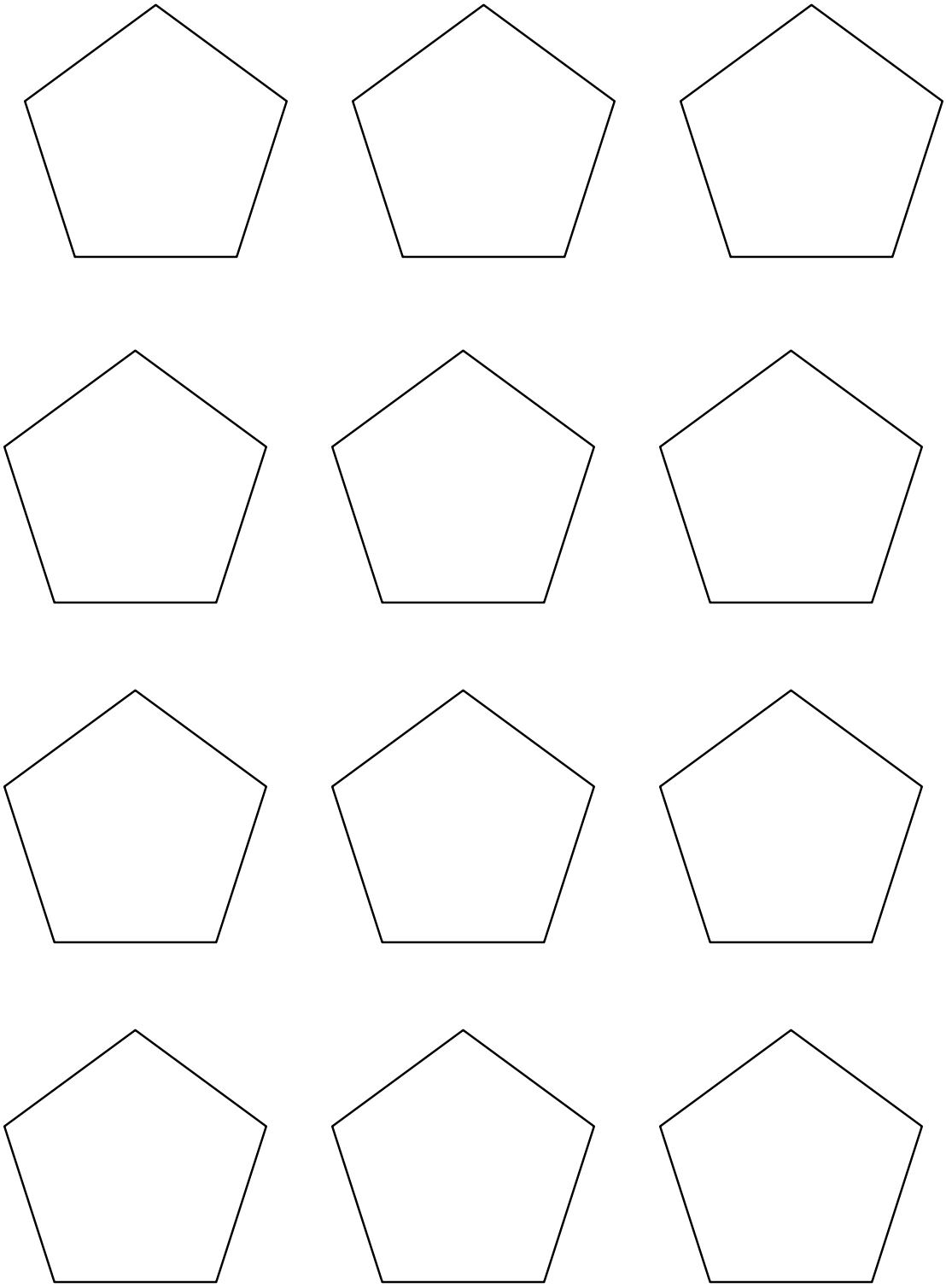
Quadrilaterals



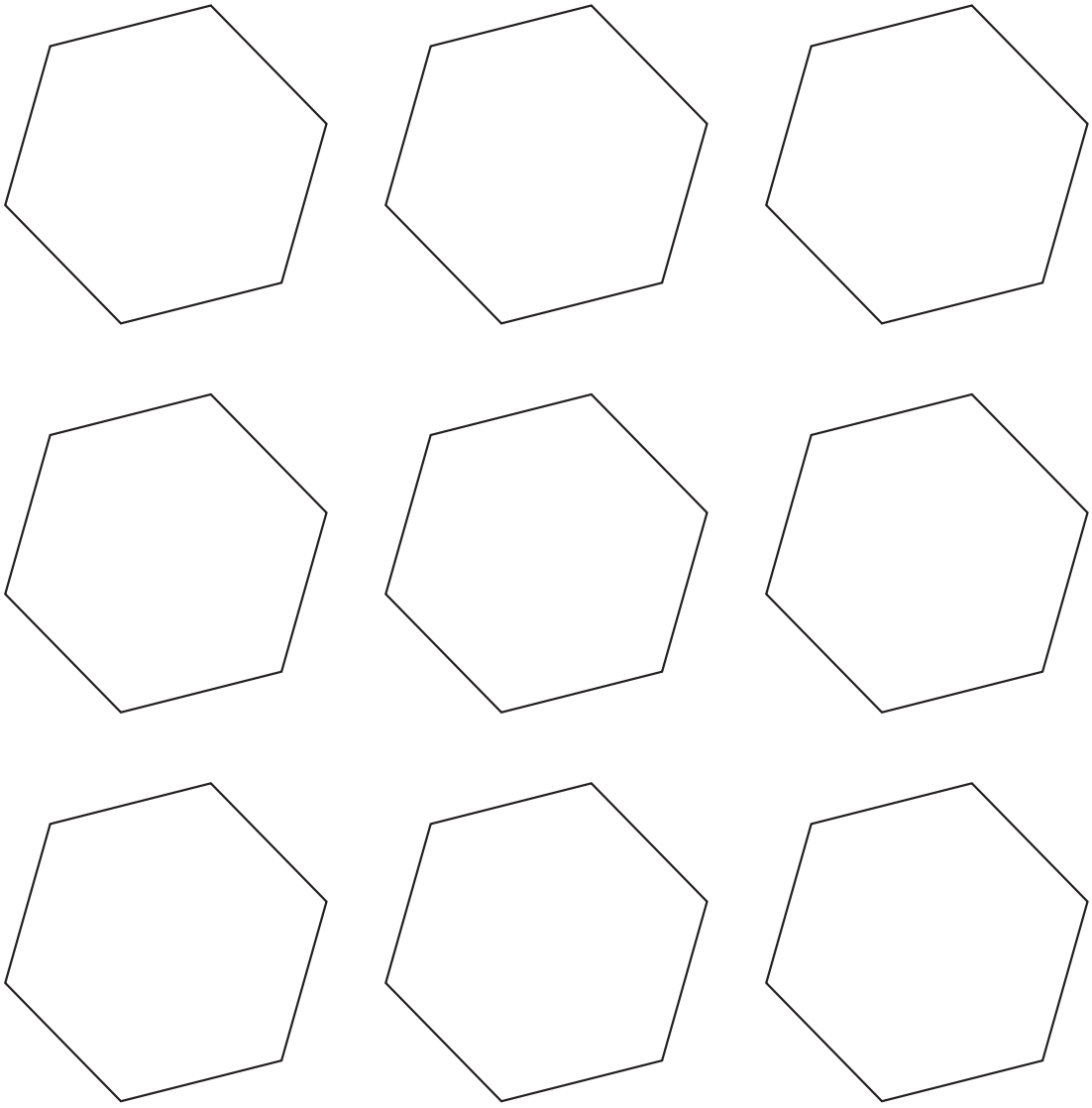
Circles



Pentagons



Hexagons



Analyzing Algebraic Patterns and Relationships

INSTRUCTIONAL ACTIVITY

Lesson 2

LEARNING GOAL

Students will expand their understanding of algebraic relationships and patterns by completing tables based on given rules, recognizing that corresponding terms can be represented as coordinate pairs, and by plotting those coordinate pairs in the first quadrant of a coordinate grid.

PRIMARY ACTIVITY

Students will work cooperatively in groups of two or three to complete tables based on information given in problem situations and then plot the corresponding terms as coordinate pairs.

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

- Rule
- Corresponding terms
- Coordinate pair (ordered pair)
- Coordinate grid
- *x*-axis
- *y*-axis
- Origin

MATERIALS

- Instructional Activity Student Handout
- Instructional Activity Supplement (Recommend one set for every two students.)

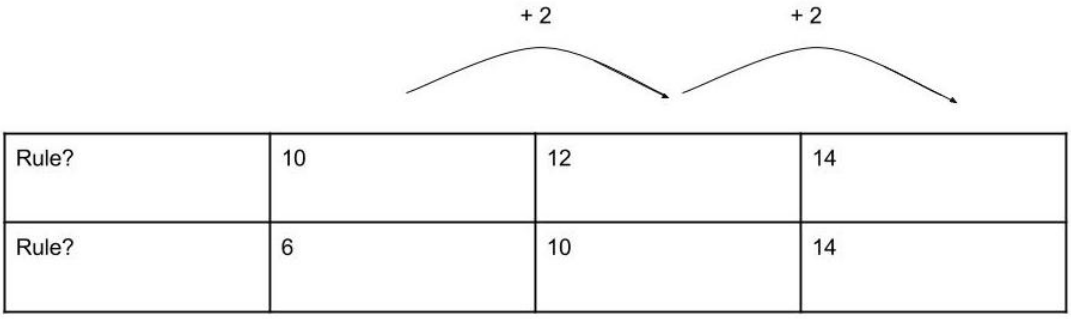
IMPLEMENTATION

Place students in groups of two or three. Display the table below.

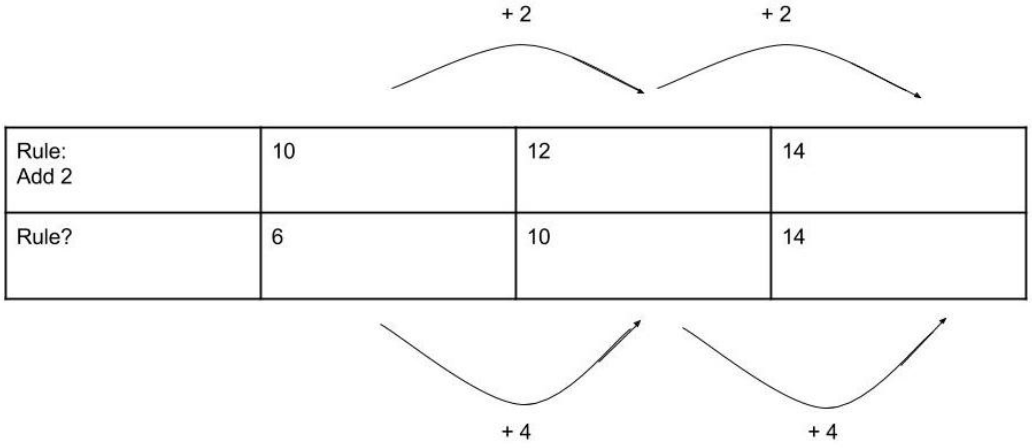
Rule?	10	12	14
Rule?	6	10	14

Require student groups to discuss the patterns for both rows and to create a possible rule for each row.

Select one or two groups to share their pattern and rule for the first row. **Discuss** the possible patterns and rules for the first row. (Rule: add two)



Select one or two groups to share their pattern and rule for the second row. **Discuss** the possible patterns and rules for the second row. (Rule: add four)



Split the student groups into two sets: set A and set B. **Require** all the student groups in set A to determine the next four terms in the first row, and all the student groups in set B to determine the next four terms in the second row. Students should still work in their groups of two or three (not as a whole set).

- First row: 16, 18, 20, 22
- Second row: 18, 22, 26, 30

Distribute the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT. **Require** students to complete Questions 1 – 4 in groups of two or three. Use the guiding questions to support student learning.

GUIDING QUESTIONS

Elicit student thinking:

- What is a table?
- When have you used a table in math?
- Does a table have to look a certain way? Explain.

Determine if the student can GENERATE A SYMBOLIC PATTERN GIVEN A RULE:

- [Point to Question 1.] How do you know what term comes first in the second row?
- What does the rule tell you about the terms in the pattern?
- [Point to a question on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.] Can you explain what the rule means?
- If a pattern starts with the term "35", and the rule is "subtract five", what does that mean? What will be the second term in the pattern? The third term?

Determine if the student can EXTEND A SYMBOLIC PATTERN BY APPLYING THE RULE:

- [Point to Question 3.] How do you know what term is after 14? Explain.
- [Point to Question 3.] How do you know what term is after 20? Explain.
- If the pattern is "12, 22, 32, 42…", what is the rule for the pattern? How do you know what the tenth term will be? Explain.

Direct student attention to the table from the beginning of the lesson. **Explain** that corresponding terms are two or more terms that appear in the same position in given patterns. **Ask** students to identify the corresponding terms in the table.

Rule: Add 2	10	12	14
Rule: Add 4	6	10	14

Explain that corresponding terms can be represented as ordered pairs, which can be plotted on a coordinate grid.

Model writing the ordered pairs next to the table.

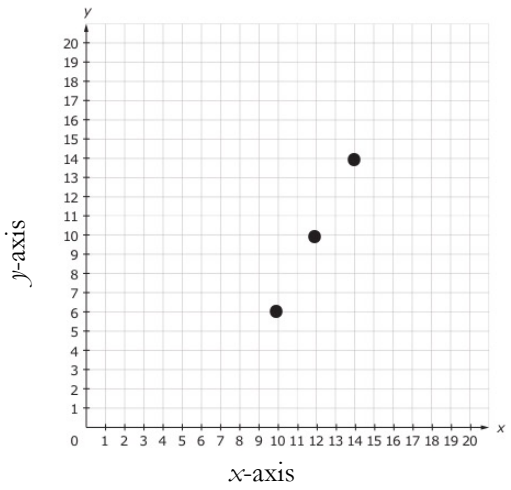
Rule: Add 2	10	12	14
Rule: Add 4	6	10	14
	(10, 6)	(12, 10)	(14, 14)

Explain that the first row typically represents the *x*-coordinates and the second row typically represents the *y*-coordinates. **Encourage** students to use the row or column that appears first in the table (top or left) to represent the movement on the *x*-axis and the second row or column in the table (bottom or right) to represent the movement on the *y*-axis.

> NOTE: When there is not a dependent relationship between the corresponding terms, either row/column can represent movement on either axis. For example, the rule "add two" can be labeled either on the *x*-axis or the *y*-axis because the second rule, "add four", is not impacted by it.

Review Quadrant I, the *x*-axis, and the *y*-axis by **asking** students where to plot each coordinate pair. **Direct** student attention to the labels for both the *x*-axis and the *y*-axis.

Emphasize that the first term in the coordinate pair represents the movement on the *x*-axis, since that is the *x*-value from the table, and the second term in the coordinate pair represents the movement on the *y*-axis, since that is the *y*-value from the table.



Share the following problem situation.

- Maggie and Sam both ride their bikes after school. Maggie rides her bike three miles each day. Sam rides his bike two miles each day. How many miles will Maggie and Sam both ride between Monday and Friday?

Display a table on the board. **Select** students to help complete the table. Students should **identify** the rules for both Maggie and Sam, as well as the number of miles ridden through the week.

Maggie Rule: add three	Sam Rule: add two
3	2
6	4
9	6
12	8
15	10

Ask students to identify the coordinate pairs for the corresponding terms. These should be written on the board next to the table.

Maggie Rule: add three	Sam Rule: add two	
3	2	(3, 2)
6	4	(6, 4)
9	6	(9, 6)
12	8	(12, 8)
15	10	(15, 10)

GUIDING QUESTIONS

Determine if the student can EXPLAIN *COORDINATE PAIRS*:

- What is an ordered pair?
- How do you represent an ordered pair?
- In an ordered pair, which term represents movement along the *x*–axis? How do you know?
- In an ordered pair, which term represents movement along the *y*–axis? How do you know?
- Is (9, 6) the same coordinate pair as (6, 9)? How do you know?

Determine if the student can GENERATE ORDERED PAIRS FROM 2 DISTINCT NUMERICAL PATTERNS:

- [Point to the first column.] What can you tell me about the terms in this column?
- [Point to the second column.] What can you tell me about the terms in this column?
- How are the table and the ordered pairs the same? How are they different?
- [Point to the first row in the table.] Which term represents movement along the *x*–axis? How do you know?
- [Point to the first row in the table.] Which term represents movement along the *y*–axis? How do you know?

Display a blank coordinate grid on the board that represents only Quadrant I.

Ask student to identify the labels for the *x*-axis and the *y*-axis. **Require** students to refer back to the table to explain why "Miles Maggie Rides" is the label for the *x*-axis and why "Miles Sam Rides" is the label for the *y*-axis. (Students should identify that Maggie's miles are represented in the table in the "*x*" column, and that Sam's miles are represented in the "*y*" column.)

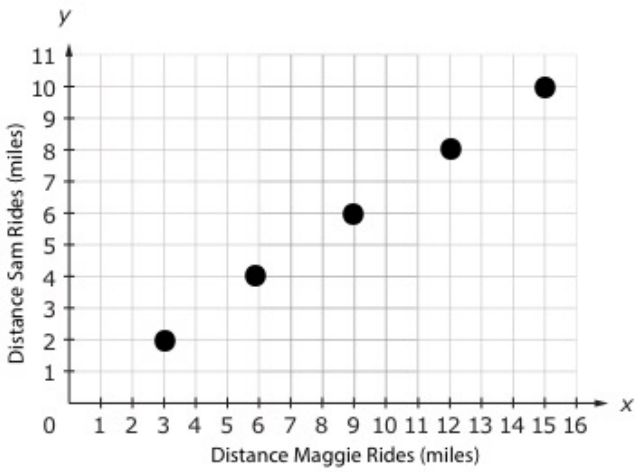
One or two students should **label** the axes on the coordinate grid.

Ask students how to scale the axes (how the axes should be equipartitioned). **Refer** students back to the table and ask them the following questions.

- The point at which the axes meet is the origin. What is the coordinate pair that represents the origin? (0, 0)
- Since the labels on the axes begin at the origin, would it make sense to extend both axes to 50 or 100? Why? (No, because the values do not go that high, the largest value is 15 on the *x*-axis and 10 on the *y*-axis.)
- Looking at the table, what would be a good distance to extend each axis? (Ideally no longer than 20 on either axis, preferably 16 on the *x*-axis and 11 on the *y*-axis.)

One or two students should **label** the divisions on the axes. **Ask** students to explain to an elbow partner how to plot the coordinate pairs on the coordinate grid. (Begin at the origin and move along the *x*-axis first, then move up the *y*-axis and mark where the two intersect.) **Require** one or two students to share their thinking.

Select five students to each **identify** one of the coordinate pairs on the displayed coordinate grid.



GUIDING QUESTIONS

Elicit student thinking:

- What can you tell me about graphs?
- What does a graph tell you?
- Does a graph have to look a certain way? Explain.

Determine if the student can GRAPH POSITIVE COORDINATE PAIRS:

- [Point at the coordinate pair (3, 2).] Where would you plot this point on the coordinate grid? Why?
- Where on this coordinate grid would you plot the coordinate pair (6, 4)? Why?
- Where on the coordinate grid would you plot (9, 6)? Is that the same location where you would plot (6, 9)? Explain.

Refer back to the problem situation. **Require** a student to answer the problem situation and **model** analyzing the answer to ensure it answers the question, makes sense, and is reasonable.

Require students to record at least five task card responses (from the INSTRUCTIONAL ACTIVITY SUPPLEMENT) on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT. Students should identify the rule, complete the table, create coordinate pairs from corresponding terms, and complete a graph for *each* task card, even if the table or graph is given. Use the guiding questions to support student learning.

GUIDING QUESTIONS

Elicit student thinking:

- How are tables and graphs the same? How are they different?
- Why would you create a graph or a table?
- How can graphs and tables help you solve problem situations?

Determine if the student can EXTEND A SYMBOLIC PATTERN BY APPLYING THE RULE:

- [Point to the table.] How do you know what term comes next?
- [Point to a task card.] What is the rule for this situation? How do you determine what term will come next?

Determine if the student can GENERATE ORDERED PAIRS FROM 2 DISTINCT NUMERICAL PATTERNS:

- [Point to the first row in the table.] Which term represents movement along the *x*–axis? How do you know?
- [Point to the first row in the table.] Which term represents movement along the *y*–axis? How do you know?
- How are the table and the ordered pairs the same? How are they different?
- [Point to a table where the terms are listed horizontally.] How do you know which set of terms represent the *x*–values and which set of terms represent the *y*–values? Explain.

Determine if the student can EXPLAIN *COORDINATE PAIRS*:

- How do you represent an ordered pair?
- In an ordered pair, which term represents movement along the *x*–axis? How do you know?
- In an ordered pair, which term represents movement along the *y*–axis? How do you know?

Determine if the student can GRAPH POSITIVE COORDINATE PAIRS:

- [Point at a coordinate pair on a task card.] Where would you graph this pair? Why?
- Where would you plot the ordered pair (2, 14)? Why?
- [Point to a coordinate pair on a student's graph.] What is this coordinate pair? Why do you plot this pair here?

At the end of the activity, students should write a letter to a friend explaining how to represent corresponding terms in a table as ordered pairs and how to create a graph using those ordered pairs.

Name_____

Analyzing Algebraic Patterns and Relationships

Lesson 2

1. Complete the following table.

Rule: add 4	13	17				34			
Rule: add 1			13		15			18	

2. Identify the rule for each column, then complete the table.

Rule:	Rule:
45	24
41	
	32
	36
25	

Name_____

3. Shaun and Tammi are both saving money for new bikes. Shaun starts with $14 in his account, and he adds $3 every week. Tammi starts with $20 in her account, and she adds $1 every week. How much money will Shaun and Tammi each have after five weeks? Identify each rule and complete the table to determine the amount of money Shaun and Tammi will have in their accounts.

Shaun	Rule:	14					
Tammi	Rule:	20					

4. Marcus and Mitchell are reading a book for their book club. Marcus reads 10 pages a night, and Mitchell reads 14 pages a night. How many pages will both Marcus and Mitchell have read after eight nights? Identify each rule and complete the table to determine the number of pages Marcus and Mitchell will have read after eight days.

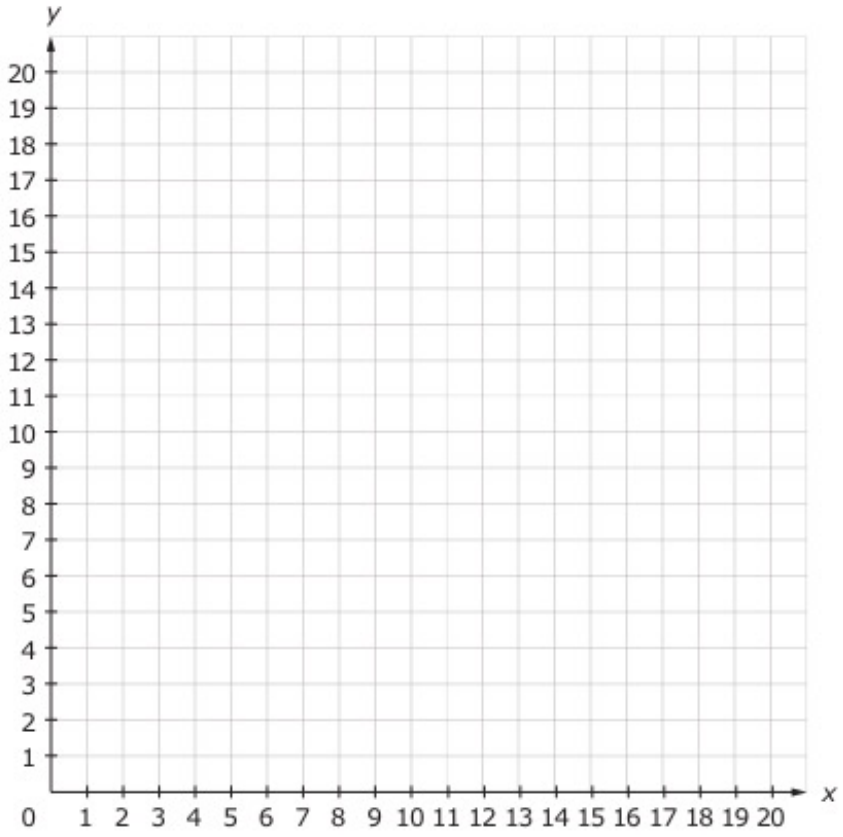
	Marcus	Mitchell
	Rule:	Rule:
Day One	10	14
Day Two		
Day Three		
Day Four		
Day Five		
Day Six		
Day Seven		
Day Eight		

Name_____

Task Card Number: _____

Rule:	Rule:

Coordinate Pairs

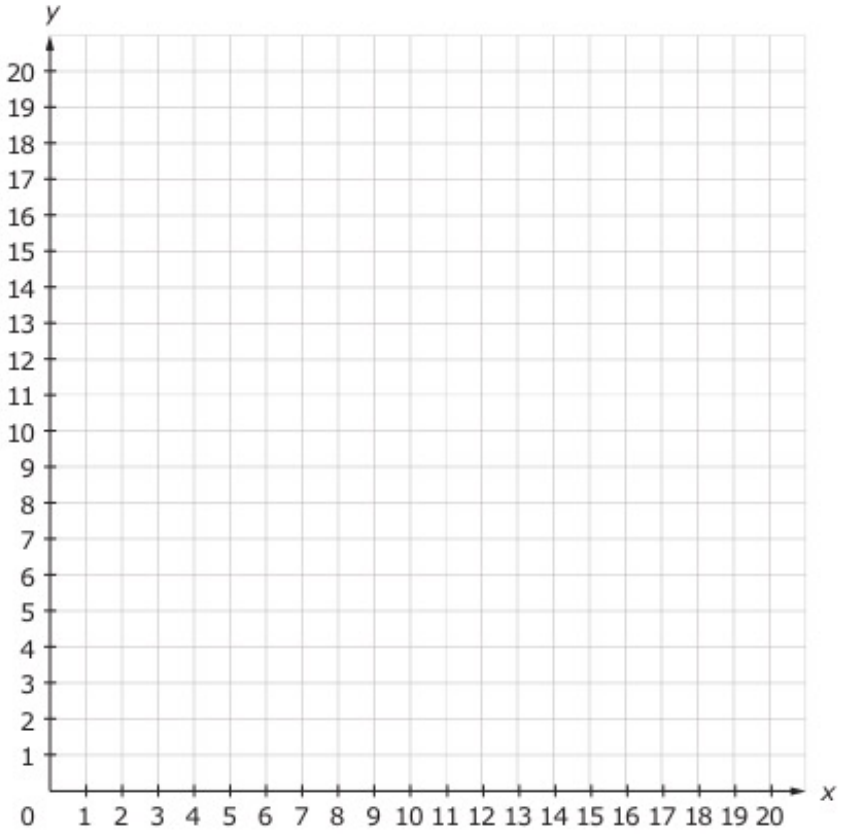


Name_____

Task Card Number: _____

Rule:	Rule:

Coordinate Pairs

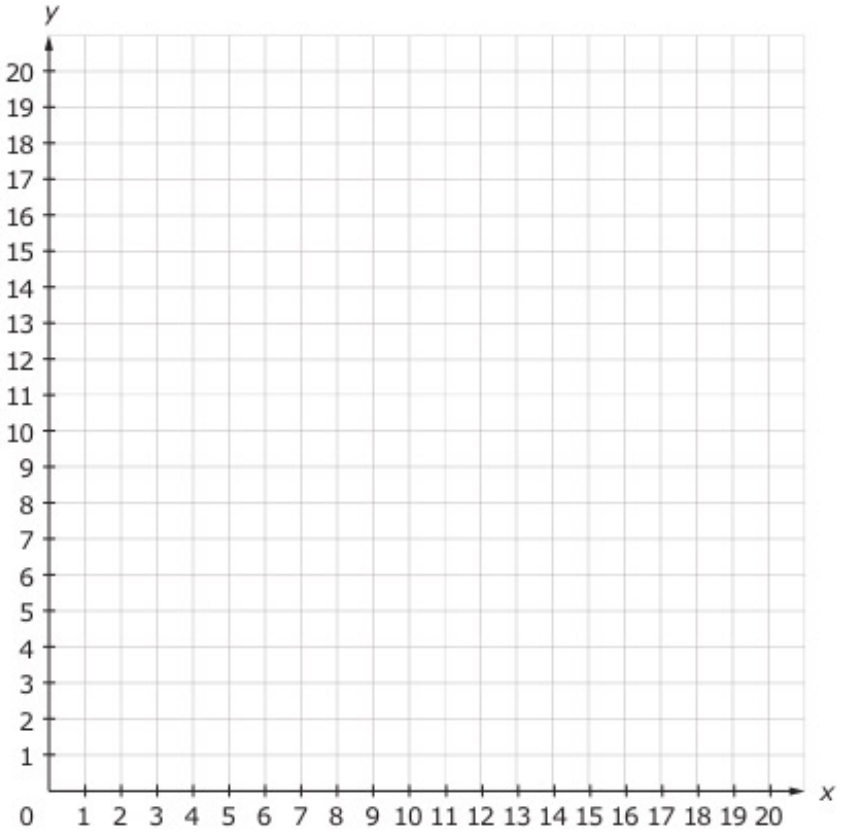


Name_____

Task Card Number: _____

Rule:	Rule:

Coordinate Pairs

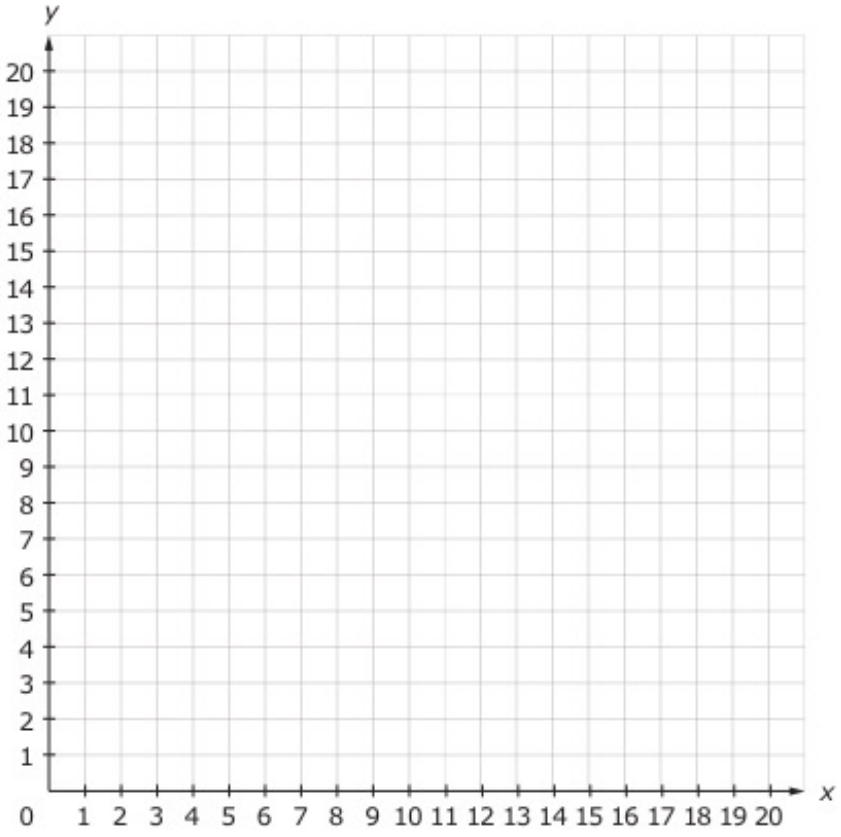


Name_____

Task Card Number: _____

Rule:	Rule:

Coordinate Pairs

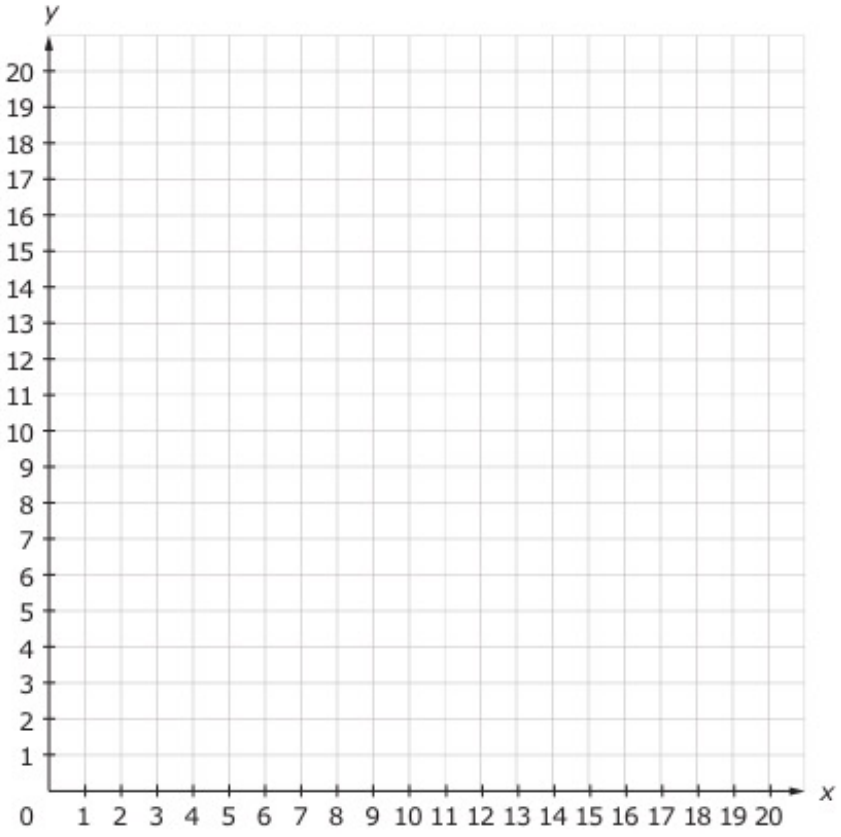


Name_____

Task Card Number: _____

Rule:	Rule:

Coordinate Pairs



Analyzing Algebraic Patterns and Relationships

INSTRUCTIONAL ACTIVITY SUPPLEMENT

Lesson 2

TASK CARD 1

Adam and Mark are saving money every week for new video games. Adam starts with $10 and saves $1 each week. Mark starts with $5 and saves $3 each week. How much money will Adam and Mark each have after four weeks?

Identify each rule, then finish the table and complete a graph to determine the answer.

Adam	10				
Mark	5				
Week	0	1	2	3	4

TASK CARD 2

The Corner Market sells fruit baskets with apples and bananas. Each basket contains four apples and three bananas. How many apples and bananas are there in five fruit baskets?

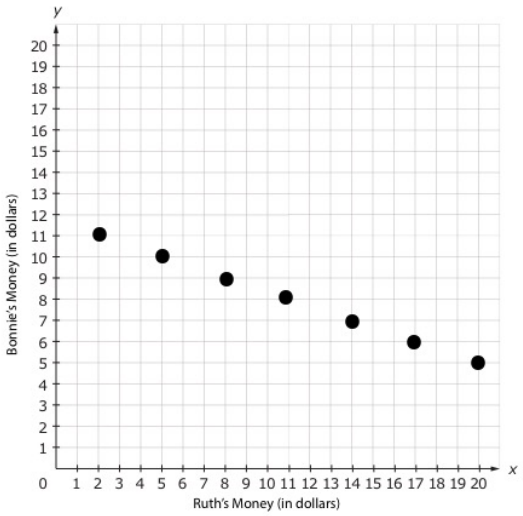
Identify each rule, then finish the table and complete a graph to determine the answer.

Apples	4				
Bananas	3				

TASK CARD 3

On Monday, Ruth has $20 and Bonnie has $5. Ruth spends $3 every day, and Bonnie saves $1 every day. After six days, how much money will Ruth and Bonnie have?

Use the information in the problem situation and the graph to identify the rule and complete the table.



TASK CARD 4

Use the table to identify the rule and complete a graph. Explain how you determined each rule.

Rule:	1	2	3	4	5
Rule:	18	14	10	6	2

TASK CARD 5

Kerri loves to collect colorful socks. Kerri uses four drawers on her dresser to store her colorful sock collection. Each drawer contains 4 pairs of socks. How many pairs of colorful socks does Kerri have? Kerri uses an additional drawer to store her 4 pairs of formal socks. How many pairs of colorful socks and formal socks does Kerri own altogether?

Identify each rule, then finish the table and complete a graph to determine the answer.

Number of Drawers	1				
Pairs of Socks	4				

TASK CARD 6

The Cupcake Club uses 2 cups of powdered sugar and 1 cup of flower for one dozen cupcakes. How many cups of powdered sugar and cups of flower will they have used for seven dozen cupcakes?

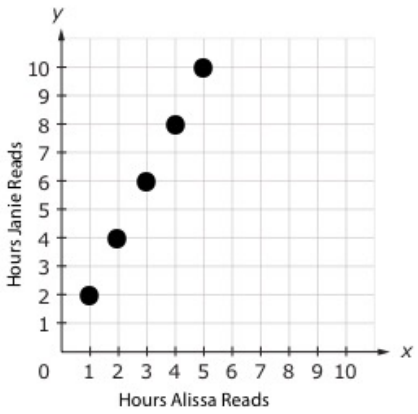
Identify each rule, then complete a table and a graph to determine the answer.

Part Two: Analyze each rule and explain if each rule represents a growing pattern or a shrinking pattern.

TASK CARD 7

Alissa and Janie are participating in the reading challenge at school. Alissa reads for one hour each night, and Janie reads for two hours each night. How many hours will Alissa and Janie have read in five days?

Use the information in the problem situation and the graph to identify the rule and complete the table.



TASK CARD 8

Rachel and Jashawn each have $20 for school lunches. Rachel spends $3 every day on the regular lunch. Jashawn spends $4 every day on the regular lunch plus two extra items: yogurt and cookies. How much money will Rachel and Jashawn have left after five days?

Identify each rule, then complete a table and a graph to determine the answer.

Part Two: Analyze each rule and explain if each rule represents a growing pattern or a shrinking pattern.

Analyzing Algebraic Patterns and Relationships

INSTRUCTIONAL ACTIVITY

Lesson 3

LEARNING GOAL

Students will analyze tables and graphs to ascertain relationships between corresponding terms.

PRIMARY ACTIVITY

Students will play a game called "Memory: Match the Rule" in which they are required to match a rule to a corresponding table, graph, or problem situation.

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

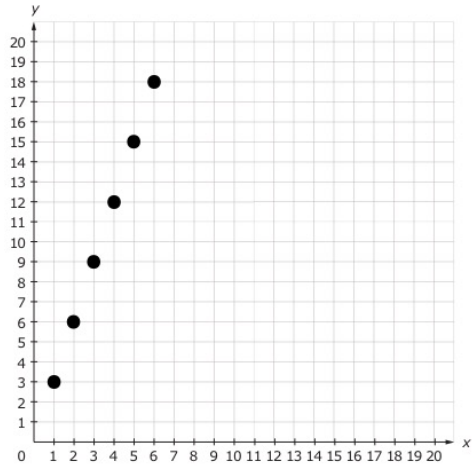
- Rule
- Corresponding terms
- Coordinate pair (ordered pair)
- Coordinate grid
- *x*-axis
- *y*-axis
- Origin
- Independent variable
- Dependent variable

MATERIALS

- Instructional Activity Student Handout
- Instructional Activity Supplement (Recommend one set for every two students.)

IMPLEMENTATION

Place students in groups of two or three. Display the following graph.



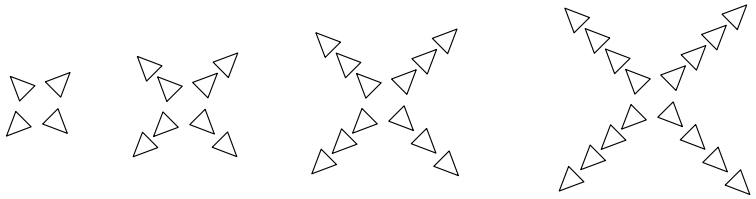
Require students to create a table using the coordinate pairs, starting with the pair that has an *x*-value closest to the origin. Students' tables should resemble the following.

x	y
1	3
2	6
3	9
4	12
5	15
6	18

Ask students to identify a pattern or rule between the corresponding terms. In other words, how could you determine the *y*-values using the *x*-values or vice versa? (For example, the *y*-value is three times the *x*-value, the *x*-value is one-third the *y*-value, $3x = y$, or $x = \frac{1}{3}y$.)

Select one or two groups to share their thinking. **Require** the group(s) to defend their answer by explaining why they believe their pattern or rule is correct.

Present the following triangle sequence.



Ask students to identify a pattern or rule between the stages of the sequence and the number of triangles. **Select** one or two groups to share their thinking. **Require** the groups to defend their answer by explaining why they believe their pattern or rule is correct.

Model analyzing the coordinating terms.

Stage number	1 ↓ × 4	2 ↓ × 4	3 ↓ × 4	4 ↓ × 4
Number of triangles	↑ ÷ 4 4	↑ ÷ 4 8	↑ ÷ 4 12	↑ ÷ 4 16

Ask students the following questions.

- How would you determine the number of sides for the 50th stage? The 100th stage?
- Would it be efficient to complete a table that went to 50 or 100?

Explain (or **reiterate**) that to efficiently determine the 50th or 100th stage, a rule should be created. **Emphasize** that the correspondence between the two sets of terms can either be $4x = y$ or $y \div 4 = x$.

Model how to write an equation that will be the rule to determine the 50th and 100th stages. **Relate** the rule to an input/output situation.

Explain that the *independent variable* is the value that is input into the rule. In the triangle sequence, the independent variable is the stage number. **Explain** that because the stage number can vary, it is represented by a variable, and because it is the independent variable (not dependent on anything else), it is identified as "x".

Explain that the *dependent variable* is the value that is output after the rule has been applied. In the triangle sequence, the dependent variable is the number of triangles in the stage. **Explain** that because the number of triangles varies and depends on the stage number, it is also represented by a variable, and it will be represented by the letter "y".

Ask students to work in their groups to generate an equation to determine how many triangles are in the 50th stage.

Identify the equation $50 \cdot 4 = y$ and **require** students determine the value for y ($y = 200$ sides). **Explain** that the rule for the triangle situation is $x \cdot 4 = y$ or $4x = y$.

Emphasize that the equal sign means that the values on both sides are equivalent. Therefore, when writing an equation, it doesn't matter if the y is before the equal sign or after. For example, $x \cdot 4 = y$ is equivalent to $y = x \cdot 4$.

GUIDING QUESTIONS

Elicit student thinking:

- Why are letters present in algebraic equations?
- What is an example of a pattern using numbers?

Determine if the student can RECOGNIZE THE CORRESPONDENCE BETWEEN 2 VARIABLES:

- What happens to the number of sides when the number of triangles is increased?
- If the same pattern was applied with quadrilaterals, what would be the relationship between the number of quadrilaterals and the number of sides? Explain.
- What is the relationship between the number of triangles and the number of sides?

Determine if the student can RECORD ORDERED PAIRS IN A TABLE:

- [Point to a value in the student's *x* column.] Why did you write this value here and not in the other column?
- [Point to a coordinate pair graphed on the coordinate grid.] What is the *x*-value for this pair? How do you know?
- [Point to a coordinate pair graphed on the coordinate grid.] What is the *y*-value for this pair? How do you know?
- [Point to the column for the *x* terms.] Which values will you put in this column? Why?
- [Point to the column for the *y* terms.] Which values will you put in this column? Why?
- [Point to the first set of corresponding terms in the table.] Why did you begin your table with this coordinate pair (set of terms)?

Hand out the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

Place students in new groups of two or three. Students should complete Question 1 on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

Require each group of two or three to pair up with another group of two or three to create groups of four to six. Students should share the answer to Question 1 within their groups. Then **select** one or two groups to share their work and solution.

Week	Number of Can Tabs
1	12
2	24
3	36
4	48
5	60
6	72

Rule: The week (x) times the number of tabs per week (12) equals the total number of tabs.

Equation:

$x \cdot 12 = y$

$12x = y$

Emphasize the relationship between the corresponding x- and y-values (that the y value is 12 times larger than the x-value).

Ask students which value is the independent value (x, the week) and which is the dependent value (y, the number of can tabs). **Refer** to the previous example with the triangles, if necessary, to support student thinking.

Read aloud Question 2 from the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT. **Discuss** and determine the solution as a whole group (144 can tabs).

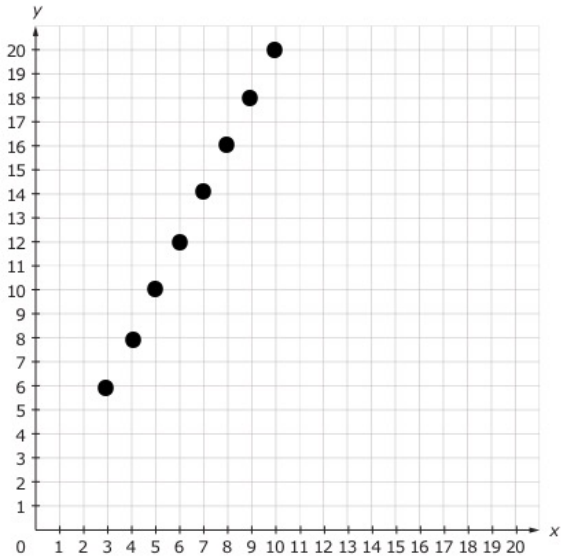
Read aloud Question 3 from the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT. In new groups of two or three, students should complete the problem. **Direct** student attention to the graph to determine the relationship between the x-values and the y-values.

Select one or two groups to share their thinking. **Guide** student thinking in creating a rule for the problem situation ($x \cdot 2 + 15 = y$ or $2x + 15 = y$).

> NOTE: Because of the communitive property of multiplication, the order can be $2x + 15$ or $15 + 2x$. Students may interpret the rule this way because Marcus starts with \$15.

Require students to answer Question 4 from the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT. **Require** each group of two or three to pair up with another group of two or three to create groups of four to six. Students should share the answer to Question 4 within their groups.

Display the following graph.



Ask students the following questions.

- What is the pattern for the *x*-values? (increase by one)
- What is the pattern for the *y*-values? (increase by two)
- What do you notice about the relationship between the *x*-values and the *y*-values? (*x* times two is *y*, *y* is two times *x*, *x* is one-half of *y*, or *x* is *y* divided by two)
- Would it be helpful to organize the coordinate pairs in a table in order to identify the relationship?

Students should participate in writing the coordinate pairs in a table on the board. **Ask** students the previous questions referencing the table instead of the graph.

Display the following rule:

$$y = x \cdot 5 - 4$$

Arrange students in groups of two or three and **require** them to complete the table on Question 5 using the the given rule and *x*-values of their choice. Then, students should complete Question 6 by generating coordinate pairs from the corresponding terms and graphing the pairs on a coordinate grid. **Require** each group to share their table and graph either with the whole class or with one or two other groups.

In their groups of two or three, students will play "Memory: Match the Rule" using the cards provided in the INSTRUCTIONAL ACTIVITY SUPPLEMENT. Students will need to match a rule or an algebraic relationship between *x*-values and *y*-values (e.g., *y*-value is two times greater than *x*-value or $y = 2x$) with a representation of the rule/relationship in the form of a problem situation, table, or graph.

Place cards face down spread out so that they do not overlap. One student should turn over two cards.

- If the cards are an algebraic relationship and a representation that match, the student keeps both cards and goes again.
- If the cards are both algebraic relationships or both representations, then the student turns both cards back face down and it is the next student's turn.
- If the cards are an algebraic relationship and a representation that do not match, then the student turns both cards back face down and it is the next student's turn.

Students should continue to take turns until there are no more cards remaining.

NOTE: There is a representation for each relationship, therefore there are three cards for each relationship. For example, there are three cards that have $y = 2x$, one to match a table, one to match a graph, and one to match a problem situation.

GUIDING QUESTIONS

Elicit student thinking:

- What do you see in this graph?
- What do you see in this table?

Determine if the student can RECOGNIZE THE COVARIATION BETWEEN 2 VARIABLES:

- [Point to the graph.] What pattern do you identify between the *x*–values? Explain.
- [Point to the graph.] What pattern do identify between the *y*–values? Explain.
- What do you notice about the *y*–value as the *x*–value changes? Explain.

Determine if the student can RECOGNIZE THE CORRESPONDENCE BETWEEN 2 VARIABLES:

- [Point to a graph.] What do you notice about the relationship between the *x*–values and the *y*–values on the graph?
- If the *x*–values increase by one and the *y*–values increase by three, what is the relationship between the *x*– and *y*–values? How do you know?
- [Point to a rule.] If you were to input six as an *x*–value, what would the *y*–value be? How do you know?

Determine if the student is ready to RECOGNIZE HOW AN ORDERED PAIR REPRESENTS A RELATIONSHIP:

- If the rule is $x \cdot 5 = y$ and the ordered pair is (4, 20), what part of the rule does the 4 represent? The 20? How do you know?
- What could be a rule for the ordered pair (2, 12)? How did you create this rule?
- Which term in the ordered pair (2, 12) represents the output value? How do you know?

Students should be required to recognize a rule in any representation, a problem situation, a table, and a graph.

At the end of the activity, students should complete Question 7 on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

Name_____

Analyzing Algebraic Patterns and Relationships

Lesson 3

1. Complete the table, determine a rule between the corresponding terms, and create an equation.

Samantha is collecting can tabs for a fundraiser. She plans to collect tabs for six weeks. If Samantha collects 12 tabs each week, how many tabs will Samantha collect in six weeks?

Week	Number of Can Tabs

Rule: Equation:

2. Samantha's fund raiser has been extended to 12 weeks. Using your equation from Question 1, determine how many tabs Samantha will collect if she continues to collect 12 tabs a week.

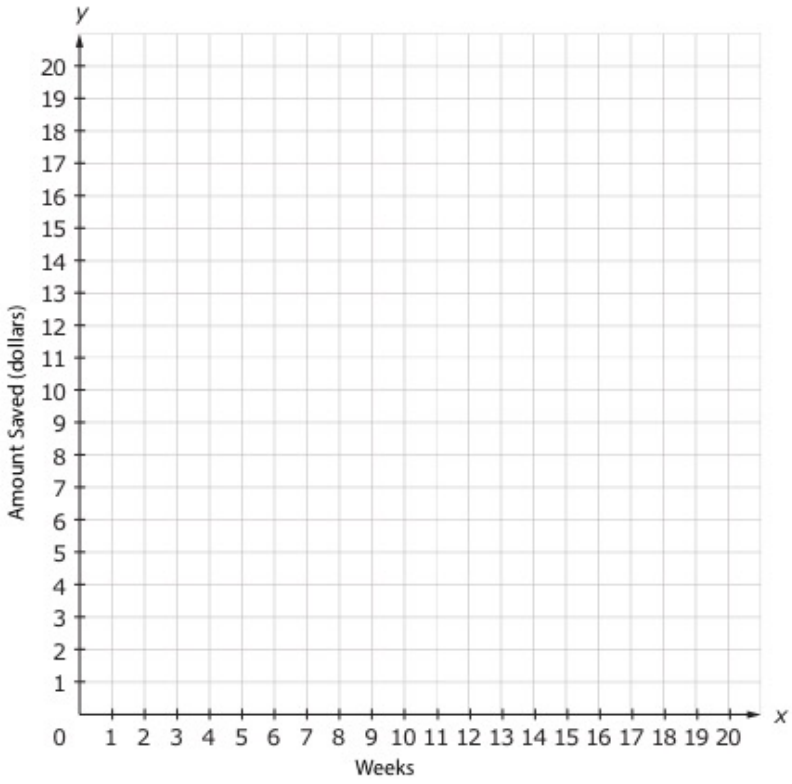
Name_____

3. Complete the table, identify and plot coordinate pairs, and analyze the graph to determine the relationship between the corresponding *x*- and *y*-values.

Marcus is saving money for a new pair of shoes. He has $15. If Marcus saves $2 every week, how much money will he have in four weeks?

Week					
Money Saved					

Coordinate Pairs:



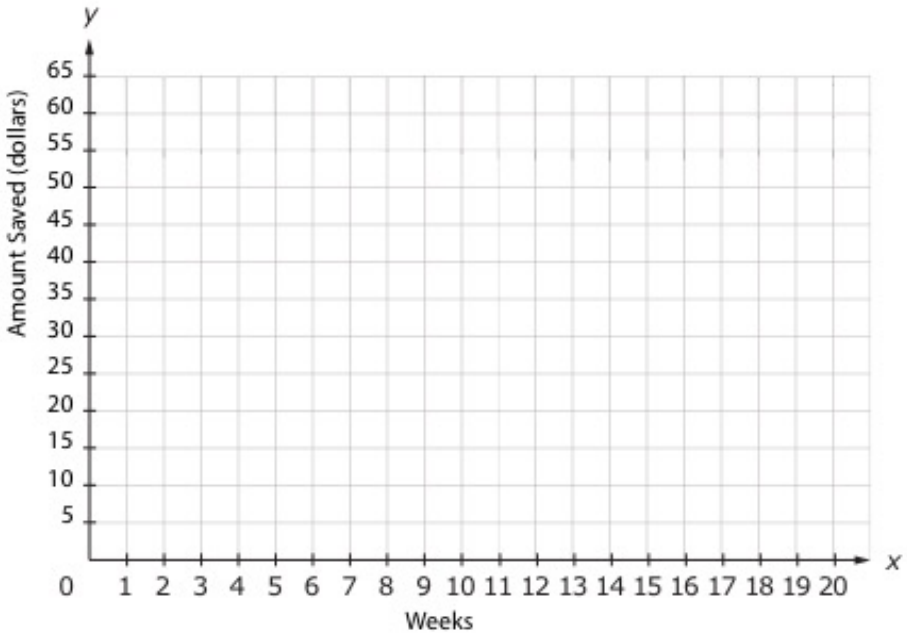
Explain the relationship between the corresponding terms.

Name_____

4. Complete the table, identify and plot coordinate pairs, and analyze the graph to determine the relationship between the corresponding *x*- and *y*-values.

Marcus wants his shoes sooner, so he decides to save $5 every week instead of $2. He has $15. How many weeks will it take him to save $60?

Week	Money Saved



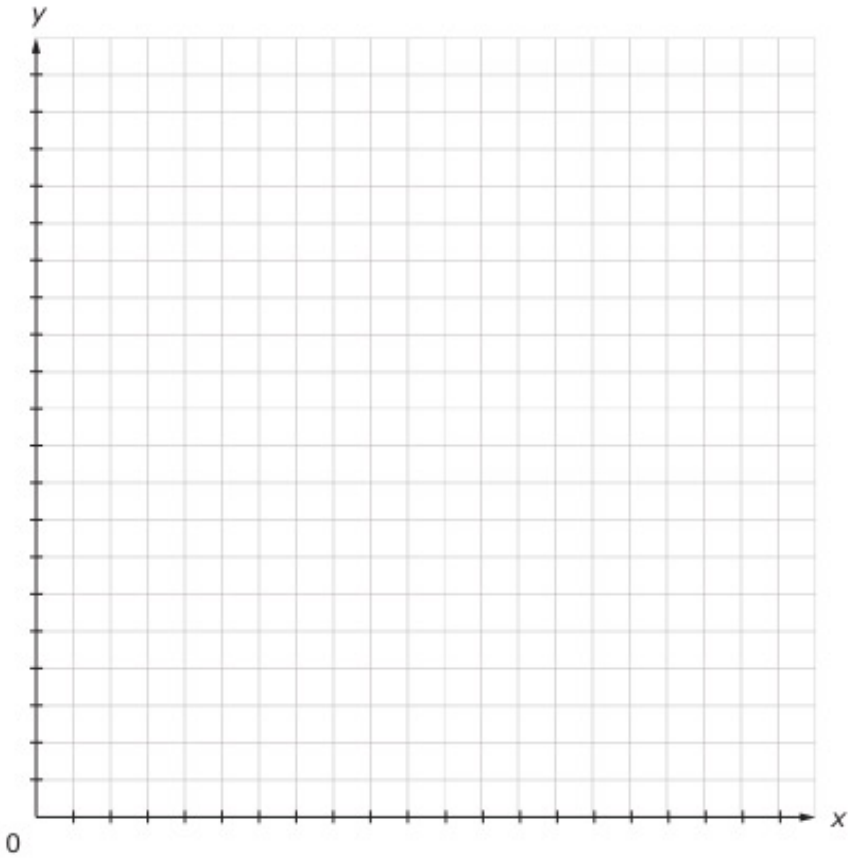
Explain the relationship between the *x*-values and the *y*-values.

Name_____

5. Complete the table using the following rule. $y = x \cdot 5 - 4$

x	y

6. Graph the corresponding terms as coordinate pairs on the coordinate grid.



Coordinate Pairs:

Name_____

7. Answer the following questions based on today's lesson.

a. What is one thing you feel confident about teaching a peer?

b. What is one thing that you are not sure you understand?

c. What is one thing that you do not understand?

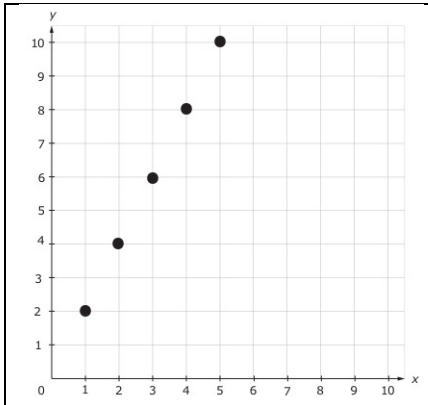
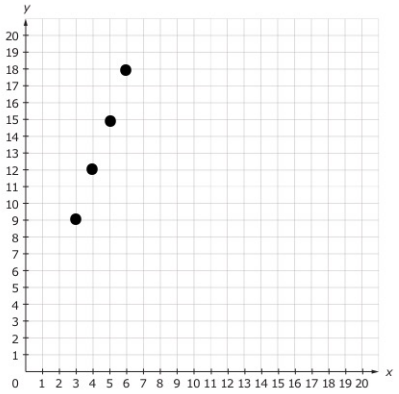
Memory: Match the Rule

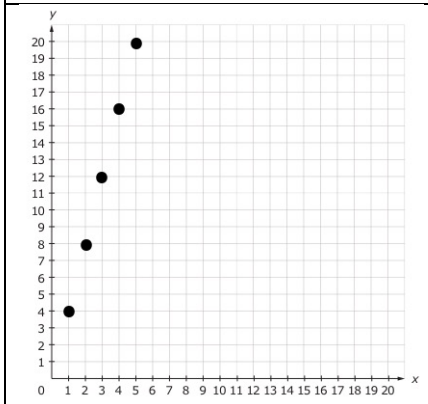
Place cards face down and spread out so that they do not overlap. One student should turn over two cards.

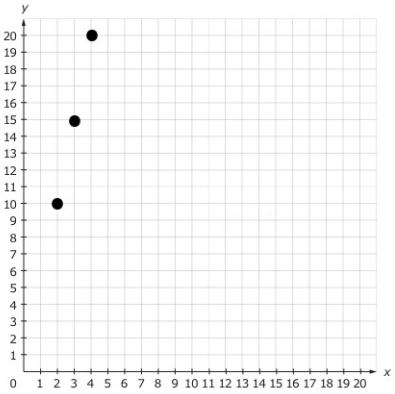
- If the cards are an algebraic relationship and a representation that match, then the student keeps both cards and goes again.
- If the cards are both algebraic relationships or are both representations, then the student turns both cards face down and it is the next student's turn.
- If the cards are an algebraic relationship and a representation that do not match, then the student turns both cards face down and it is the next student's turn.

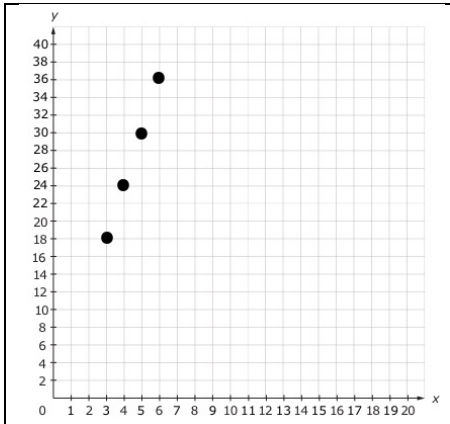
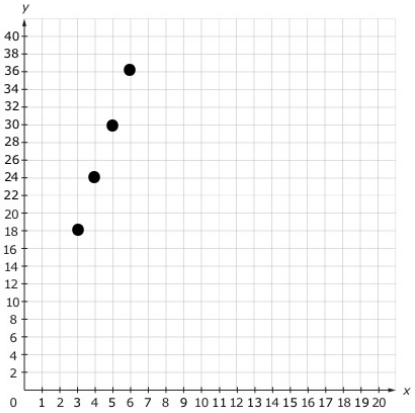
Students should continue take turns until there are no more cards remaining.

$y = 2x$	$2x = y$	$2 \cdot x = y$	x | y 4 | 8 5 | 10 6 | 12 7 | 14

<table>
<tr>
<td> 
</td>
<td>Sam's Market sells fruit baskets. Each basket contains 4 apples, 3 bananas, and 2 oranges. How many oranges are used to make 5 fruit baskets?</td>
<td>$3x = y$</td>
<td>$y = 3x$</td>
</tr>
<tr>
<td>$y = 3 \cdot x$</td>
<td>
<table>
<tr><th>x</th><th>y</th></tr>
<tr><td>1</td><td>3</td></tr>
<tr><td>2</td><td>6</td></tr>
<tr><td>3</td><td>9</td></tr>
<tr><td>4</td><td>12</td></tr>
</table>
</td>
<td> 
</td>
<td>Lucy is saving money for a new phone case. She saves $3 every week. How much money will Lucy have after eight weeks?</td>
</tr>
</table>

$y = 4 \cdot x$	$4 \cdot x = y$	$4x = y$	<table><tr><th>x</th><th>y</th></tr><tr><td>5</td><td>20</td></tr><tr><td>6</td><td>24</td></tr><tr><td>7</td><td>28</td></tr><tr><td>8</td><td>32</td></tr></table>
	Johnny runs four laps at the track every day. How many laps has Johnny run after 10 days?	$y = 5x$	$5x = y$

$5 \cdot x = y$	<table><tr><th>x</th><th>y</th></tr><tr><td>3</td><td>15</td></tr><tr><td>4</td><td>20</td></tr><tr><td>5</td><td>25</td></tr><tr><td>6</td><td>30</td></tr></table>		The elementary school is hosting a change drive. If each class collects $5 in coins, how much money will 23 classes collect?
$y = 6 \cdot x$	$6 \cdot x = y$	$6x = y$	<table><tr><th>x</th><th>y</th></tr><tr><td>1</td><td>6</td></tr><tr><td>2</td><td>12</td></tr><tr><td>3</td><td>18</td></tr><tr><td>4</td><td>24</td></tr></table>

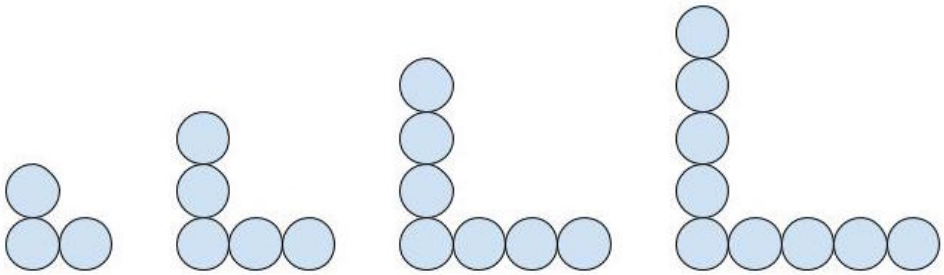
<table>
<tr>
<td>  </td>
<td>Mateo walks six blocks each day (three blocks to school and three blocks home). How many total blocks does Mateo walk in five days?</td>
<td>$y = 7x$</td>
<td>$y = 7 \cdot x$</td>
</tr>
<tr>
<td>$7 \cdot x = y$</td>
<td>
<table>
<tr><th>x</th><th>y</th></tr>
<tr><td>3</td><td>21</td></tr>
<tr><td>4</td><td>28</td></tr>
<tr><td>5</td><td>35</td></tr>
<tr><td>6</td><td>42</td></tr>
</table>
</td>
<td>  </td>
<td>Every month Natalie spends $7 on snacks at school. How much money will Natalie spend on snacks in five months?</td>
</tr>
</table>

Name_____

Analyzing Algebraic Patterns and Relationships

Lessons 1 – 3

1. Use the following pattern to answer the questions.



1.a. Extend the pattern and draw the next three stages.

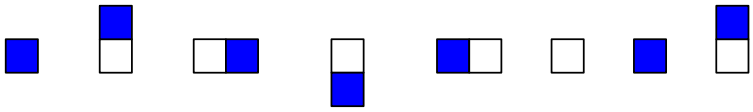
1.b. Describe in words the pattern that is represented.

Name_____

1.c. Use the pattern to complete the following table.

Stage	1	2					
Total Number of Circles	3	5					

2. Use the following pattern to answer the questions.



2.a. What is the next image in the sequence? How do you know?

2.b. Is the sequence a growing, shrinking, or repeating pattern? How do you know?

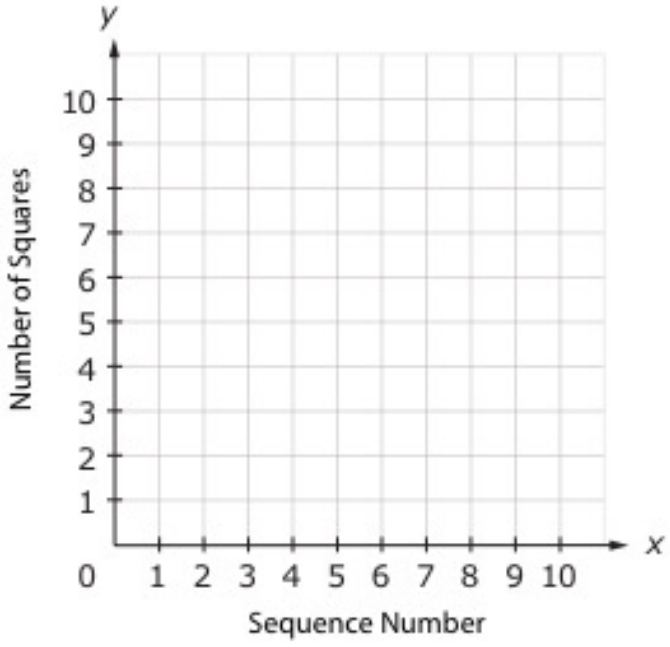
Name_____

2.c. Use the pattern to complete the following table.

Sequence Number	Number of Squares

2.d. Identify the coordinate pairs for the corresponding terms in the table and graph them on the coordinate grid.

Coordinate Pairs:



Name_____

3. Madison and Malaki are saving money. Madison has $10 in her account, and she saves $1 each week. Malaki has $7 in his account, and he saves $2 each week.

   3.a. Identify the rule for Madison and Malaki then complete the table.

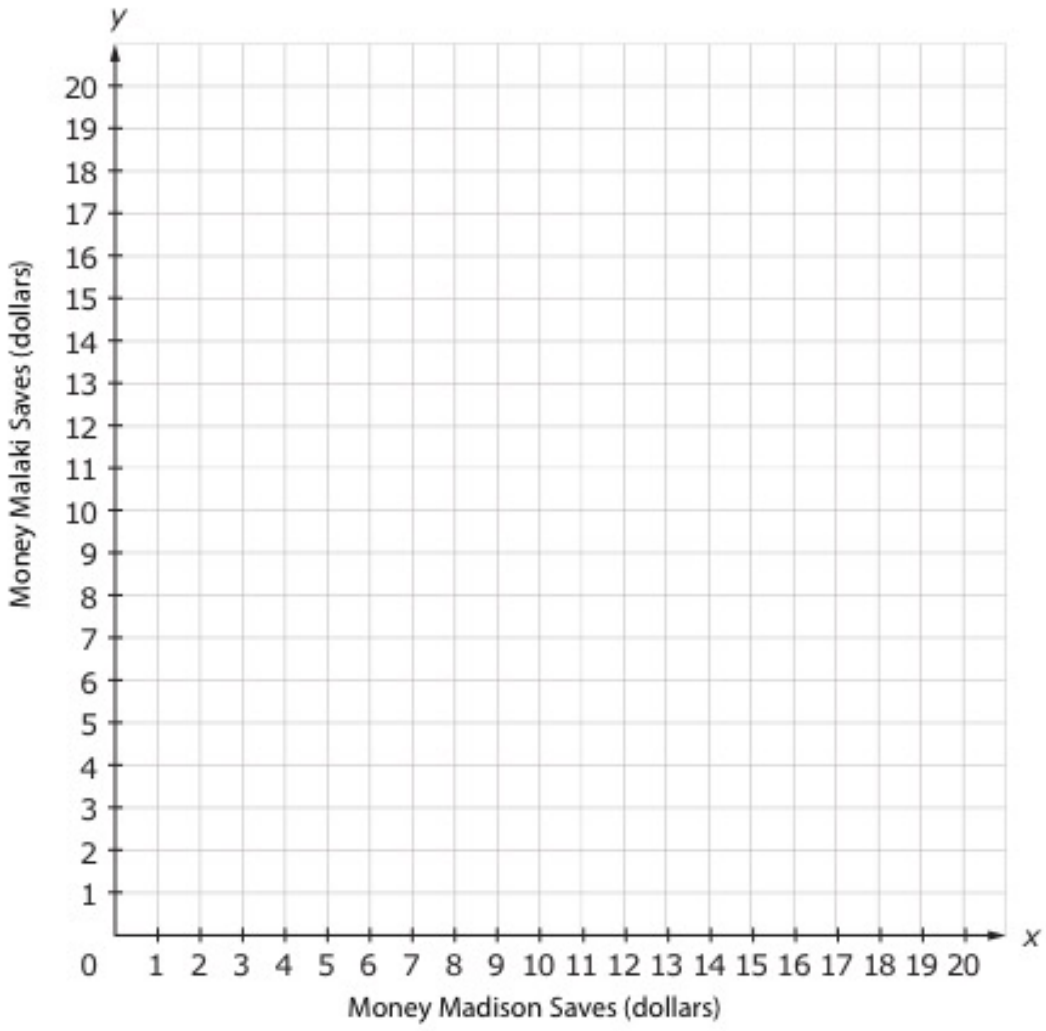
Madison Rule:	10					
Malaki Rule:	7					

   3.b. How much money will Madison and Malaki have after five weeks?

   3.c. List the ordered pairs from the corresponding terms. Note that Madison is labeled on the *x*-axis and Malaki is labeled on the *y*-axis.

Name_____

3.d. Graph the ordered pairs on the coordinate grid.



4. Maggie runs two miles at each soccer practice.

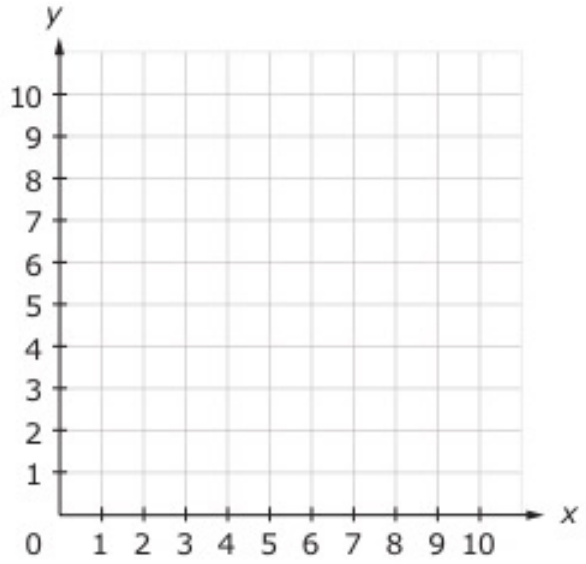
4.a. Complete the table using the problem situation.

Number of practice days					
Total distance run					

4.b. How many miles will Maggie have run after five days of soccer practice?

4.c. Identify and plot coordinate pairs from the table. Be sure to label the *x*- and *y*-axes.

Coordinate Pairs:



4.d. What is the relationship between the *x*- and *y*-values?

Name_____

5. Matthew and Andrew are saving money to buy new video games. Matthew starts with $5 and saves $3 a week. Andrew starts with $2 and saves $4 a week.

   5.a. Complete the table using the problem situation.

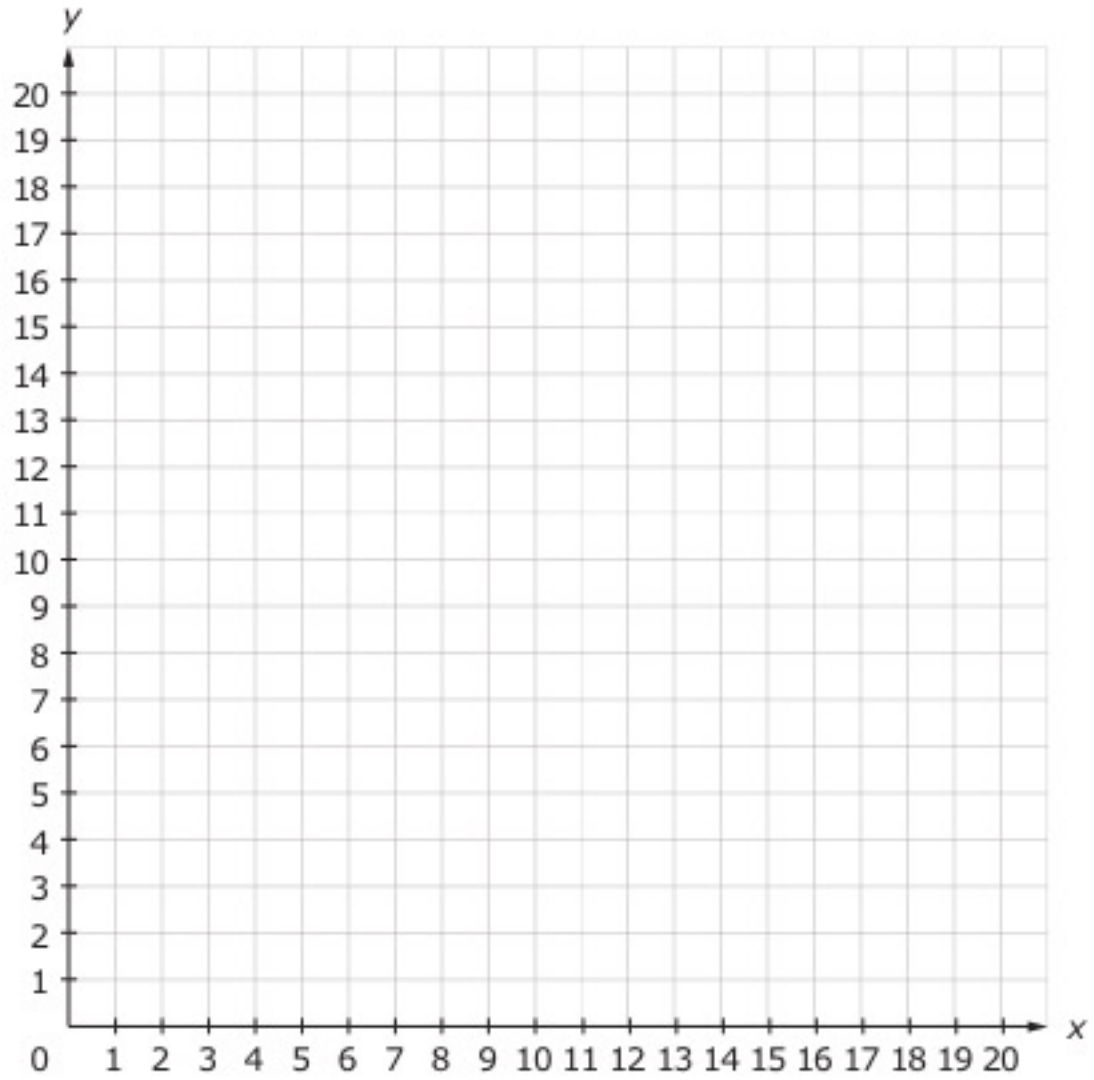
Week	Matthew	Andrew

   5.b. How much money will Matthew and Andrew each have after four weeks?

Name_____

5.c. Identify two sets of coordinate pairs: one for Matthew and one for Andrew. Use two different colors to graph the coordinate pairs on the coordinate grid. Be sure to label the *x*-axis for the time in weeks and the *y*-axis for the amount of money saved in dollars.
Coordinate pairs:

Matthew Andrew



5.d. Describe the relationship between the *x*- and *y*-values for both Matthew and Andrew.

Matthew:

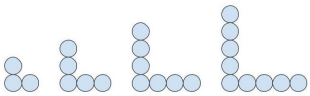
Andrew:

ANALYZING ALGEBRAIC PATTERNS AND RELATIONSHIPS

STUDENT ACTIVITY SOLUTION GUIDE

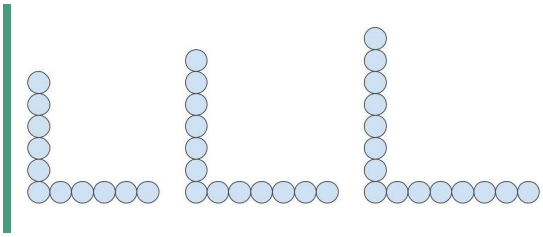
Lessons 1 – 3

1. Use the following pattern to answer the questions.



1.a. Extend the pattern and draw the next three stages.

CORRECT ANSWER



ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student adds more than one circle to each leg in each stage.	does not recognize that in each stage the pattern increases by one circle on each leg	RECOGNIZE THE PATTERN RULE IN A GROWING PATTERN and EXTEND A PICTORIAL PATTERN BY APPLYING THE RULE
The student does not extend the pattern.	does not recognize that in each stage the pattern increases by one circle on each leg	RECOGNIZE THE PATTERN RULE IN A GROWING PATTERN and EXTEND A PICTORIAL PATTERN BY APPLYING THE RULE

1.b. Describe in words the pattern that is represented.

CORRECT ANSWER

[*Check student work for understanding. The following is an example of a written description.*]

The pattern starts with three circles in an "L" shape. For each stage of the pattern, one circle is added to each "arm" of the "L." The pattern increases by two circles each stage.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student identifies the pattern as something other than adding one circle to each leg or adding two total circles to each stage.	does not recognize that in each stage the pattern increases by one circle in each leg	RECOGNIZE THE PATTERN RULE IN A GROWING PATTERN
The student does not answer or leaves the question blank.	cannot describe in words the pattern and/or does not recognize that the pattern increases by one circle in each leg	RECOGNIZE THE PATTERN RULE IN A GROWING PATTERN and EXPLAIN *GROWING PATTERNS*

1.c. Use the pattern to complete the following table.

CORRECT ANSWER

Stage	1	2	3	4	5	6	7
Total Number of Circles	3	5	7	9	11	13	15

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student completes the table, but did not correctly identify the pattern and thus the "Total Number of Circles" terms are incorrect.	does not recognize that in each stage the pattern increases by one circle in each leg	RECOGNIZE THE PATTERN RULE IN A GROWING PATTERN
The student double counts the "corner" circle; for example, in stage three the number of circles written as eight instead of seven.	double counts the corner circle	RECOGNIZE THE PATTERN RULE IN A GROWING PATTERN

2. Use the following pattern to answer the questions.

2.a. What is the next image in the sequence? How do you know?

CORRECT ANSWER

[*Check student work for understanding. The student may provide either a written description or a drawing to extend the sequence. The following is an example of a written description.*]

The next image in the sequence is two squares side by side horizontally, the left square is white and the right square is blue. It is the same as the third image from the left. That is the next image in the sequence because the first image given is one blue square, then the second image is two squares stacked vertically, the top square is blue, the bottom square is white both of these images also appear the end of the sequence, so the next image will be the third from the left.

[*The following is an example of a drawing; see the written description for justification.*]

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student does not correctly identify and explain the next image in the sequence.	does not recognize that the pattern repeats six squares then four squares	RECOGNIZE *REPEATING PATTERNS* and EXTEND A PICTORIAL PATTERN BY APPLYING THE RULE
The student does not answer the question.	does not recognize that the pattern repeats six squares then four squares	RECOGNIZE *REPEATING PATTERNS* and EXTEND A PICTORIAL PATTERN BY APPLYING THE RULE
The student correctly identifies the next image in the sequence but does not provide an explanation or provides an incorrect explanation.	cannot describe or explain in words the pattern	EXPLAIN *REPEATING PATTERN*

2.b. Is the sequence a growing, shrinking, or repeating pattern? How do you know?

CORRECT ANSWER

[*Check student work for understanding. The following is an example of a written description.*]

The sequence is a repeating pattern because the images repeat, one blue square, two squares stacked vertically the top is blue the bottom is white, two squares side by side horizontally the left one white the right one blue, two squares stacked vertically the top one white the bottom one blue, two squares side by side horizontally the left one blue the right one white, one white square, then the pattern starts over again with one blue square.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student identifies the sequence as a growing pattern or a shrinking pattern.	does not recognize the sequence as a repeating pattern	RECOGNIZE *REPEATING PATTERNS*
The student does not provide an answer.	does not recognize the sequence as a repeating pattern	RECOGNIZE *REPEATING PATTERNS*
The student correctly identifies the sequence as a repeating pattern but does not provide an explanation.	cannot describe or explain in words the pattern	EXPLAIN *REPEATING PATTERN*

2.c. Use the pattern to complete the following table.

CORRECT ANSWER

Sequence Number	Number of Squares
1	1
2	2
3	2
4	2
5	2
6	1
7	1
8	2
9	2
10	2

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

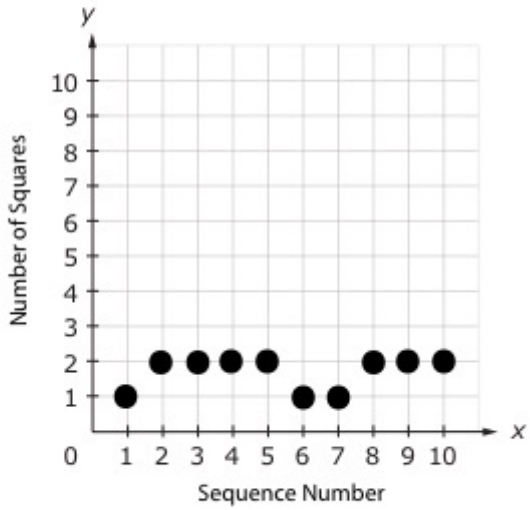
Example Error	Misconception	Missing Knowledge
The student lists the number of squares as a growing pattern and adds each sequence to the previous number; for example, 6, 10, 16, 20, 26, etc.	does not recognize that the pattern does not grow but repeats and/or does not realize that the question is not asking for total number of squares	RECOGNIZE *REPEATING PATTERNS* AND RECOGNIZE *GROWING PATTERNS*
The student does not complete the table past the seventh sequence since that is what is provided in the image.	does not know how to extend the pattern	EXTEND A PICTORIAL PATTERN BY APPLYING THE RULE

2.d. Identify the coordinate pairs for the corresponding terms in the table and graph them on the coordinate grid.

CORRECT ANSWER

Coordinate Pairs:

(1, 1)
(2, 2)
(3, 2)
(4, 2)
(5, 2)
(6, 1)
(7, 1)
(8, 2)
(9, 2)
(10, 2)



ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student reverses the *x*- and *y*-coordinates in the coordinate pairs and when graphing on the coordinate grid.	does not attend to the labels in the table and/or the coordinate grid	GENERATE ORDERED PAIRS FROM 2 DISTINCT NUMERICAL PATTERNS
The student generates the correct coordinate pairs but reverses the *x*- and *y*-coordinates when graphing.	does not recognize that the first term in a coordinate pair is the *x*-value (indicating movement horizontally along or parallel to the *x*-axis) and the second term in a coordinate pair is the *y*-value (indicating movement vertically along or parallel to the *y*-axis)	EXPLAIN *COORDINATE PAIRS* and GRAPH POSITIVE COORDINATE PAIRS

3. Madison and Malaki are saving money. Madison has $10 in her account, and she saves $1 each week. Malaki has $7 in his account, and he saves $2 each week.

3.a. Complete the table.

CORRECT ANSWER

Madison Rule: Add 1	10	11	12	13	14	15
Malaki Rule: Add 2	7	9	11	13	15	17

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student does not generate the pattern from the starting value; for example, lists Madison's amounts as 10, 1, 2, 3, etc.	does not consider the starting amount when creating the pattern	GENERATE A SYMBOLIC PATTERN GIVEN A RULE and EXTEND A SYMBOLIC PATTERN BY APPLYING THE RULE
The student completes the table correctly but does not correctly identify the rule.	cannot generate a rule based on the information presented in the problem situation	RECOGNIZE THE PATTERN RULE IN A GROWING PATTERN

3.b. How much money will Madison and Malaki have after five weeks?

CORRECT ANSWER

After five weeks, Madison will have $15 and Malaki will have $17.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student identifies Madison as having $14 and Malaki as having $15.	does not realize that the starting amount is week 0 and not week 1	GENERATE A SYMBOLIC PATTERN GIVEN A RULE

Example Error	Misconception	Missing Knowledge
The student identifies Madison as having $5 and Malaki as having $10.	answers based on an incorrect table in which does not consider the starting amount when creating the pattern	GENERATE A SYMBOLIC PATTERN GIVEN A RULE and EXTEND A SYMBOLIC PATTERN BY APPLYING THE RULE

3.c. List the ordered pairs from the corresponding terms.

CORRECT ANSWER

(10, 7), (11, 9), (12, 11), (13, 13), (14, 15), (15, 17)

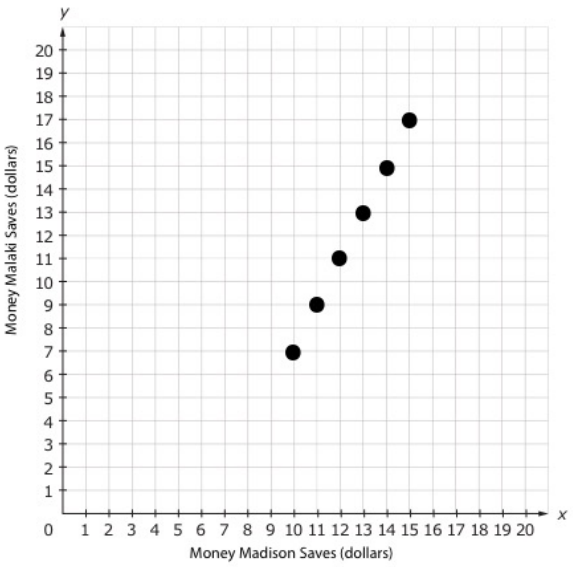
[*NOTE: Because the axes in the graph are already labeled, students must list the ordered pairs as given in the table with the top row as the x-values and the bottom row as the y-values.*]

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student does not list the terms in the first row as the *x*-values in the ordered pairs.	does not attend to the order of the labels in the table and on the graph	GENERATE ORDERED PAIRS FROM 2 DISTINCT NUMERICAL PATTERNS
The student pairs values that appear side by side; for example, (10, 11), (12, 13), (14, 15), etc.	does not recognize that corresponding terms are in columns for the given table and/or does not realize that ordered pairs require one term from each pattern	GENERATE ORDERED PAIRS FROM 2 DISTINCT NUMERICAL PATTERNS

3.d. Graph the ordered pairs on the coordinate grid.

CORRECT ANSWER



ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student reverses the *x*- and *y*-values; for example, plots the first point at (7, 10).	does not recognize that the first term in a coordinate pair is the *x*-value (indicating movement horizontally along or parallel to the *x*-axis) and the second term in a coordinate pair is the *y*-value (indicating movement vertically along or parallel to the *y*-axis)	EXPLAIN *COORDINATE PAIRS* and GRAPH POSITIVE COORDINATE PAIRS
The student correctly graphed incorrect ordered pairs; for example, (10, 11), (12, 13), (14, 15), etc.	does not recognize that corresponding terms are in columns for the given table and/or does not realize that ordered pairs require one term from each pattern	GENERATE ORDERED PAIRS FROM 2 DISTINCT NUMERICAL PATTERNS

4. Maggie runs two miles at each soccer practice.

4.a. Complete the table using the problem situation.

CORRECT ANSWER

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Number of practice days	1	2	3	4	5
Total distance run	2	4	6	8	10

Example Error	Misconception	Missing Knowledge
The student enters the distance ran in the first row and the number of practice days in the bottom row.	does not attend to the labels in the table and/or the information in the problem situation	GENERATE SYMBOLIC PATTERNS GIVEN A RULE
The student does not use the rule *times 2* to determine the distance ran, provides values other than those listed.	does not understand what the problem situation is asking and/or the information being provided	EXTEND A SYMBOLIC PATTERN BY APPLYING THE RULE and RECOGNIZE THE COVARIATION BETWEEN 2 VARIABLES
The student enters a two for "Total distance ran" for each practice day; for example, each entry in the second row is "2".	does not realize the rule is a growing pattern or running total and only enters the number of miles ran each day	EXTEND A SYMBOLIC PATTERN BY APPLYING THE RULE

4.b. How many miles will Maggie have run after five days of soccer practice?

CORRECT ANSWER

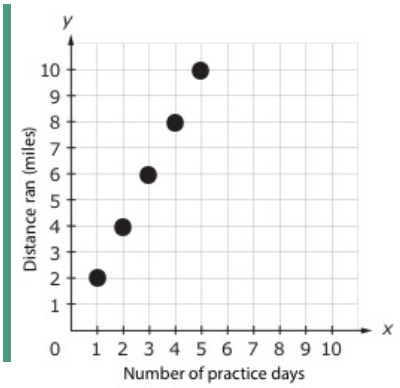
After five days of soccer practice, Maggie will have run 10 miles.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student writes that Maggie ran two miles.	does not understand what the problem situation is asking and/or is answering from an incorrect table in which does not realize the rule is a growing pattern	EXTEND A SYMBOLIC PATTERN BY APPLYING THE RULE and RECOGNIZE THE COVARIATION BETWEEN 2 VARIABLES

4.c. Identify and plot coordinate pairs from the table. Be sure to label the *x*- and *y*-axes.

CORRECT ANSWER



Coordinate Pairs:

(1, 2)
(2, 4)
(3, 6)
(4, 8)
(5, 10)

[NOTE: Because the distance Maggie runs is dependent on the number of days she attends practice, the distances must be the y-values and the number of days must be the x-values.]

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student reverses the *x*- and *y*-values when writing the coordinate pairs.	does not recognize that the first term in a coordinate pair is the *x*-value (indicating movement horizontally along or parallel to the *x*-axis) and the second term in a coordinate pair is the *y*-value (indicating movement vertically along or parallel to the *y*-axis)	EXPLAIN *COORDINATE PAIRS*
The student reverses the *x*- and *y*-values when graphing the coordinate pairs.	does not recognize that the first term in a coordinate pair is the *x*-value (indicating movement horizontally along or parallel to the *x*-axis) and the second term in a coordinate pair is the *y*-value (indicating movement vertically along or parallel to the *y*-axis)	GRAPH COORDINATE PAIRS

4.d. What is the relationship between the *x*- and *y*-values?

CORRECT ANSWER

The *y*-value is two times the *x*-value.

$y = 2x$

[NOTE: The relationship can be described informally or formally, as long as the student understands the relationship that exists between the x- and y-values.]

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student identifies the relationship as the y-value is two more than the x-value or $x + 2 = y$.	does not recognize that two times and two more than are not the same and/or is answering from an incorrect table	RECOGNIZE THE CORRESPONDENCE BETWEEN 2 VARIABLES and EXTEND A SYMBOLIC PATTERN BY APPLYING THE RULE
The student does not provide an answer, or the answer does not make sense for the situation.	does not understand what the problem situation is asking and/or the information being provided	RECOGNIZE THE CORRESPONDENCE BETWEEN 2 VARIABLES
The student states that the y-value is one more than the x-value, then two more than the x-value, then three more than the x-value, and so on.	does not recognize the multiplicative relationship between the two variables	RECOGNIZE THE CORRESPONDENCE BETWEEN 2 VARIABLES

5. Matthew and Andrew are saving money to buy new video games. Matthew starts with $5 and saves $3 a week. Andrew starts with $2 and saves $4 a week.

5.a. Complete the table using the problem situation.

CORRECT ANSWER

Week	Matthew	Andrew
0	5	2
1	8	6
2	11	10
3	14	14
4	17	18

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student starts week one at $5 and $2 instead of $8 and $6.	does not recognize that both boys have a starting balance before they start saving	GENERATE A SYMBOLIC PATTERN GIVEN A RULE
The student does not account for the starting balance and begins the table with week one, $3, and $4.	does not recognize that both boys have a starting balance before they start saving	GENERATE A SYMBOLIC PATTERN GIVEN A RULE
Does not use the correct rules to determine the amounts saved, provides values other than those listed.	does not understand what the problem situation is asking and/or the information being provided	GENERATE A SYMBOLIC PATTERN GIVEN A RULE

5.b. How much money will Matthew and Andrew each have after four weeks?

CORRECT ANSWER

After four weeks, Matthew will have $17 and Andrew will have $18.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student identifies Matthew as having $14 and Andrew as having $14.	does not realize that the starting amount is week 0 and not week 1	GENERATE A SYMBOLIC PATTERN GIVEN A RULE
The student identifies Matthew as having $12 and Andrew as having $16.	answers based on an incorrect table in which does not consider the starting amount when creating the pattern	GENERATE A SYMBOLIC PATTERN GIVEN A RULE and EXTEND A SYMBOLIC PATTERN BY APPLYING THE RULE

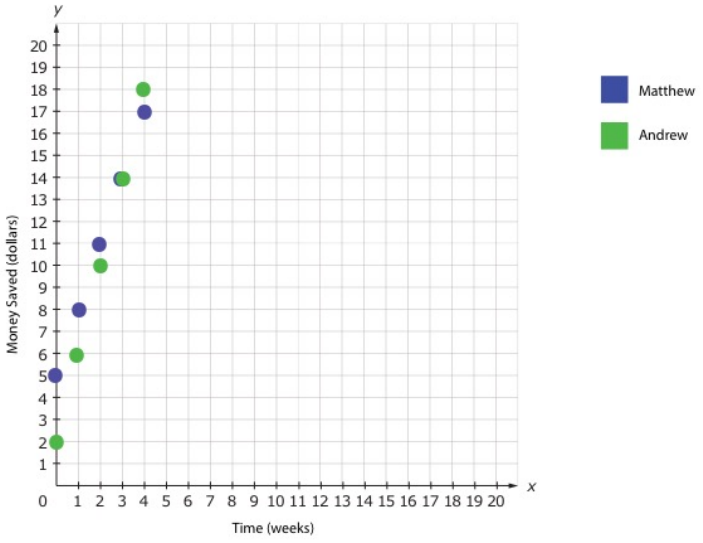
5.c. Identify two sets of coordinate pairs: one for Matthew and one for Andrew. Use two different colors to graph the coordinate pairs on the coordinate grid. Be sure to label the *x*-axis for the time in weeks and the *y*-axis for the amount of money saved in dollars.

CORRECT ANSWER

Coordinate pairs:

Matthew: (0, 5), (1, 8), (2, 11), (3, 14), (4, 17)

Andrew: (0, 2), (1, 6), (2, 10), (3, 14), (4, 18)



> [*NOTE: Because the amount of money saved is dependent on the number of weeks, the amount of money saved must be the y-values and the number of weeks must be the x-values.*]

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student correctly plots the coordinate pairs, but uses incorrect pairs; for example, for Matthew uses the pairs: (1, 5), (2, 8), (3, 11), (4, 17).	created incorrect coordinate pairs as a result of incorrectly completing the table in Question 5.a	GENERATE A SYMBOLIC PATTERN GIVEN A RULE
The student correctly plots the corrdinate pairs, but uses incorrect pairs; for example, for Matthew uses pairs: (1, 3), (2, 6), (3, 9), (4, 12).	created incorrect coordinate pairs as a result of incorrectly completing the table in Question 5.a	GENERATE A SYMBOLIC PATTERN GIVEN A RULE
The student reverses the *x*- and *y*-values when graphing points on the coordinate grid.	*x*-values do not match the label for the *x*-axis and vice versa and/or the *x*-value was identified by counting vertically on the *y*-axis and vice versa	GRAPH POSITIVE COORDINATE PAIRS and EXPLAIN *COORDINATE PAIRS*
The student only uses two of the columns given in the table to create one set of coordinate pairs; for example: (5, 2), (8, 6), (11, 10), (14, 14), (17, 18).	does not attend to the question directions and/or does not recognize the ordered pairs represent a relationship	RECOGNIZE HOW AN ORDERED PAIR REPRESENTS A RELATIONSHIP

5.d. Describe the relationship between the *x*- and *y*-values for both Matthew and Andrew.

CORRECT ANSWER

> Matthew: the relationship is the *x*-value times three plus five equals the *y*-value. Or, $3x + 5 = y$.
>
> Andrew: the relationship is the *x*-value times four plus two equals the *y*-value. Or, $4x + 2 = y$.
>
> [*NOTE: The relationship can be described informally or formally, as long as the student understands the relationship that exists between the x- and y-values.*]

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student identifies Matthew's relationship as $3x = y$ and/or Andrew's relationship as $4x = y$.	does not recognize that both boys have a starting balance before they start saving which needs to be added to the multiplicative expression	RECOGNIZE THE CORRESPONDENCE BETWEEN 2 VARIABLES and GENERATE A SYMBOLIC PATTERN GIVEN A RULE
The student identifies Matthew's relationship as $5 + x = y$ and/or Andrew's relationship as $2 + x = y$.	does not recognize that the patterns "+5" and/or "+2" do not match the input/output values in the table and/or graph but does recognize that they are part of the sequence	RECOGNIZE THE CORRESPONDENCE BETWEEN 2 VARIABLES and RECOGNIZE THE PATTERN RULE IN A GROWING PATTERN
The student does not provide an answer, or the answer does not make sense for the situation.	does not understand what the problem situation is asking and/or the information being provided	RECOGNIZE THE CORRESPONDENCE BETWEEN 2 VARIABLES