INTERPERT AND REPRESENT MULTIPLICATION AND DIVISION IN CONTEXT

3.OA.1–3

CONTENTS

The types of documents contained in the unit are listed below. Throughout the unit, the documents are arranged by lesson.

LEARNING MAP INFORMATION	An overview of the standards, the learning map section, and the nodes addressed in this unit
TEACHER NOTES	A brief discussion describing the progression depicted in the learning map section with research-based recommendations for focusing instruction to foster student learning and an introduction to the unit's lessons
OVERVIEW OF INSTRUCTIONAL ACTIVITIES	A table highlighting the lesson goals and nodes addressed in each lesson of this unit
INSTRUCTIONAL ACTIVITY	A detailed walkthrough of the unit
INSTRUCTIONAL ACTIVITY STUDENT HANDOUT	A handout for the guided activity, intended to be paired with the Instructional Activity
INSTRUCTIONAL ACTIVITY SUPPLEMENT	A collection of materials or activities related to the Instructional Activity
STUDENT ACTIVITY	A work-alone activity for students
STUDENT ACTIVITY SOLUTION GUIDE	A solution guide for the work-alone activity with example errors, misconceptions, and links to the learning map section

Enhanced Learning Maps developed these materials under a grant from the Department of Education, PR/Award # S368A150013. Contents do not necessarily represent the policy of the Department of Education, and you should not assume endorsement by the Federal Government.

INTERPRET AND REPRESENT MULTIPLICATION AND DIVISION IN CONTEXT

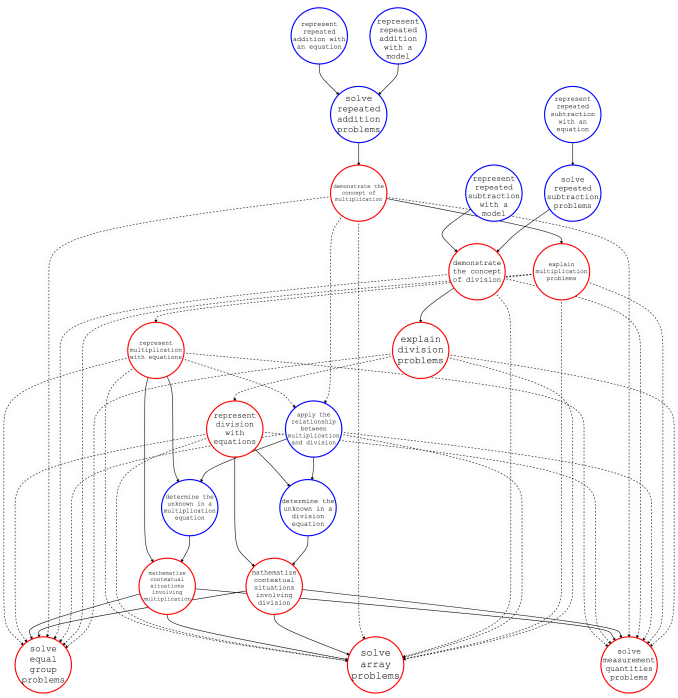
LEARNING MAP INFORMATION

STANDARDS

3.OA.1 Interpret products of whole numbers (e.g., interpret 5×7 as the total number of objects in 5 groups of 7 objects each). *For example, show objects in rectangular arrays or describe a context in which a total number of objects can be expressed as* 5×7.

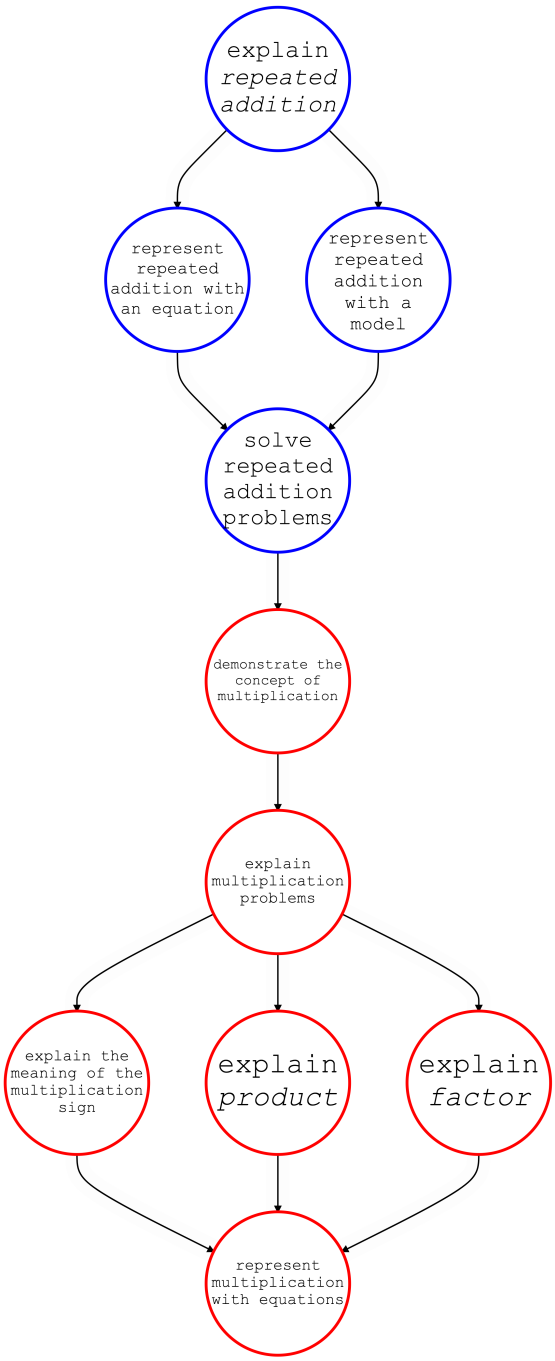
3.OA.2 Interpret whole-number quotients of whole numbers (e.g., interpret $56 \div 8$ as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each). *For example, deconstruct rectangular arrays or describe a context in which a number of shares or a number of groups can be expressed as* $56 \div 8$.

3.OA.3 Use multiplication and division numbers up to 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities (e.g., by using drawings and equations with a symbol for the unknown number to represent the problem).



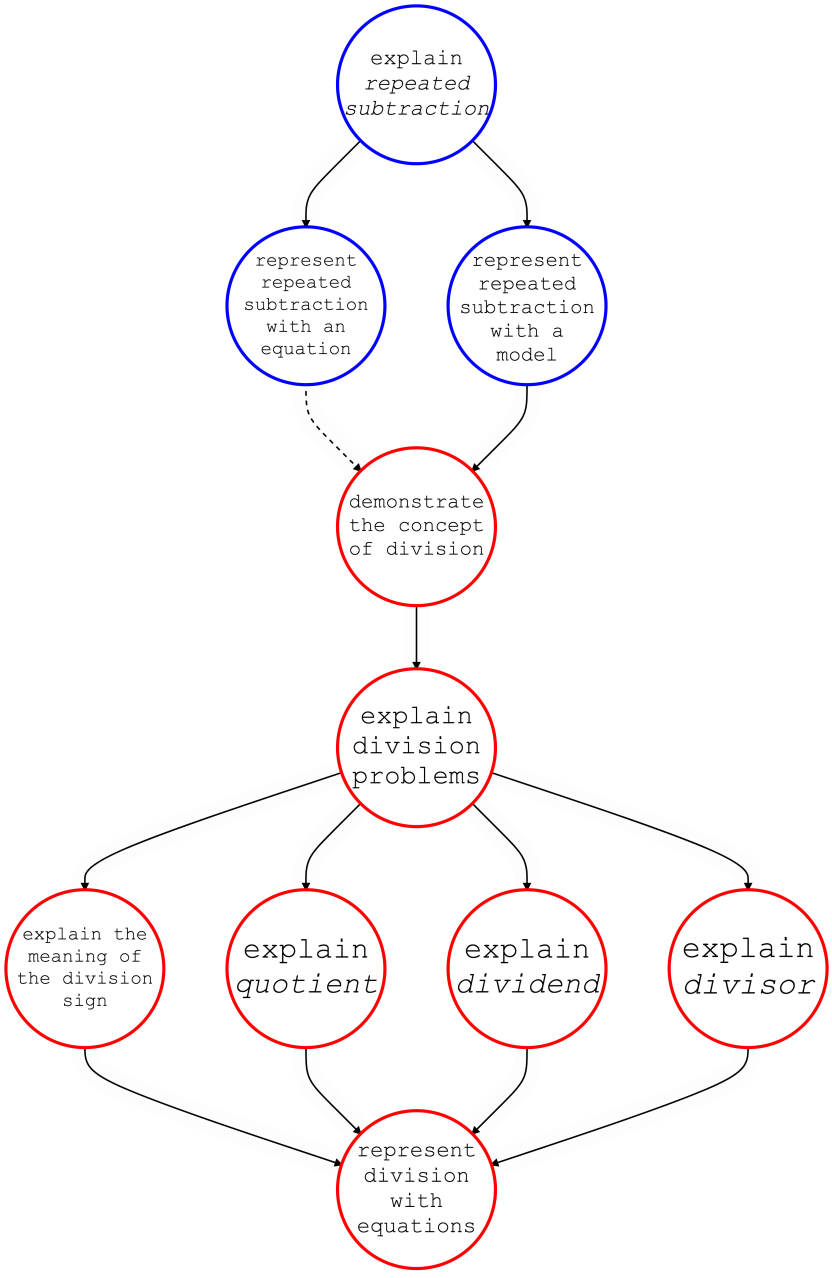
**Learning map model of* 3.OA.1–3

3.OA.1 Interpret products of whole numbers (e.g., interpret 5×7 as the total number of objects in 5 groups of 7 objects each). *For example, show objects in rectangular arrays or describe a context in which a total number of objects can be expressed as* 5×7.



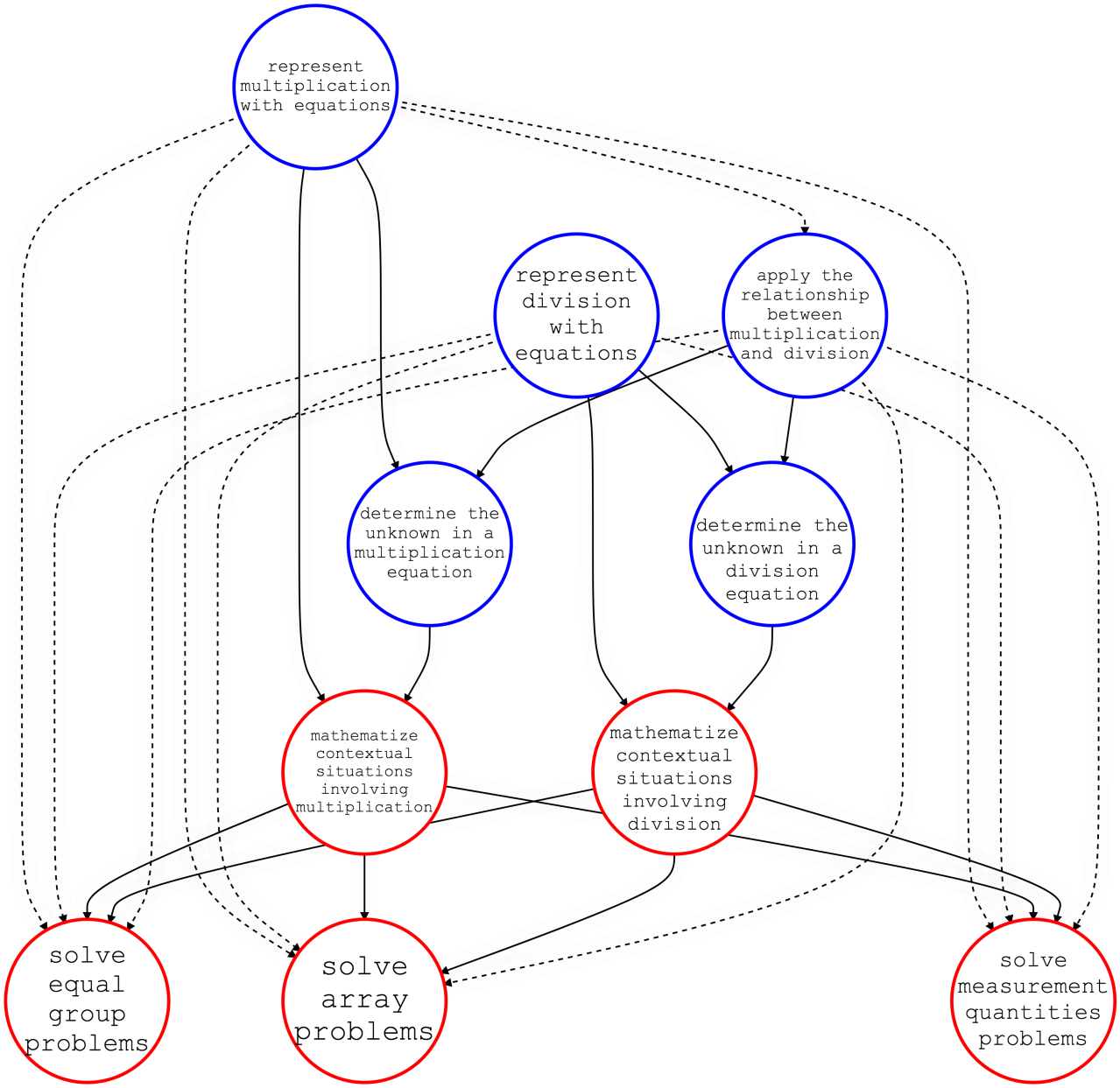
**Learning map model of* 3.OA.1

3.OA.2 Interpret whole-number quotients of whole numbers (e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each). *For example, deconstruct rectangular arrays or describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.*



**Learning map model of* 3.OA.2

3.OA.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using diagrams and equations with a symbol for the unknown number to represent the problem.



**Learning map model of* 3.OA.3

Node Name	Node Description
DEMONSTRATE THE CONCEPT OF MULTIPLICATION	Demonstrate multiplication by combining multiple sets of the same quantities together to form a new set.
EXPLAIN MULTIPLICATION PROBLEMS	Make known your understanding that, in a multiplication problem, the first factor describes the number of groups, and the second factor describes the number of elements in each group.
EXPLAIN THE MEANING OF THE MULTIPLICATION SIGN	Make known your understanding that the multiplication sign represents that the two numbers are multiplied together.
EXPLAIN *PRODUCT*	Make known your understanding that a product is the answer when two or more numbers are multiplied together.
EXPLAIN *FACTOR*	Make known your understanding that a factor is a number you multiply by another to get a product.
REPRESENT MULTIPLICATION WITH EQUATIONS	Use an equation to represent a multiplication sentence. For example, $2 \times 3 = 6$.
DEMONSTRATE THE CONCEPT OF DIVISION	Demonstrate division by splitting a set into a number of fair shares or equal sets.
EXPLAIN DIVISION PROBLEMS	Make known your understanding that division involves partitioning a set to form a specified number of groups that are equal in size.
EXPLAIN THE MEANING OF THE DIVISION SIGN	Make known your understanding that the division sign represents that the previous number is to be divided by the following number.
EXPLAIN *QUOTIENT*	Make known your understanding that a quotient is the number obtained on dividing one number by another.
EXPLAIN *DIVIDEND*	Make known your understanding that the dividend is the number being divided.
EXPLAIN *DIVISOR*	Make known your understanding that the divisor is the number that divides another number. It is also known as a factor.
REPRESENT DIVISION WITH EQUATIONS	Use an equation to represent a division problem. For example, $10 \div 2 = 5$.
APPLY THE RELATIONSHIP BETWEEN MULTIPLICATION AND DIVISION	Use arrays or equal groups to model multiplication or division problems.
DETERMINE THE UNKNOWN IN A MULTIPLICATION EQUATION	Given a mathematical equation involving multiplication operation, determine the unknown factor or product.
DETERMINE THE UNKNOWN IN A DIVISION EQUATION	Given a mathematical equation involving division operation, determine the unknown factor or quotient.
MATHEMATIZE CONTEXTUAL SITUATIONS INVOLVING MULTIPLICATION	Organize information from contextual situations to write mathematical problems involving multiplication.
MATHEMATIZE CONTEXTUAL SITUATIONS INVOLVING DIVISION	Organize information from contextual situations to write mathematical problems involving division.
SOLVE EQUAL-GROUP PROBLEMS	Use multiplication and division to solve word problems in situations involving equal sets.
SOLVE ARRAY PROBLEMS	Use multiplication and division to solve word problems in situations involving arrays. Know that, in a multiplication expression, the first factor represents the number of rows modeled in the corresponding array, and the second factor represents the number of columns modeled in the

	corresponding array.
SOLVE MEASUREMENT-QUANTITIES PROBLEMS	Use multiplication and division to solve word problems in situations involving measurement quantities such as equal measures, rate, etc.
REPRESENT REPEATED ADDITION WITH AN EQUATION	Use equations to represent repeated addition problems.
REPRESENT REPEATED ADDITION WITH A MODEL	Show repeated addition by using a model such as an array, set, number line, etc.
SOLVE REPEATED-ADDITION PROBLEMS	Solve problems involving repeated addition.
REPRESENT REPEATED SUBTRACTION WITH A MODEL	Show repeated subtraction by using a model such as array, set, number line, etc.
REPRESENT REPEATED SUBTRACTION WITH AN EQUATION	Through writing or an appropriate assistive technology, show repeated subtraction with an equation. For example, $15 - 5 - 5 - 5 = 0$.
SOLVE REPEATED-SUBTRACTION PROBLEMS	Use manipulatives or equations to solve problems involving repeated subtraction.

Interpret and Represent Multiplication and Division in Context

TEACHER NOTES

This unit includes the following documents:

- Learning Map Information
- Instructional Activity (six lessons)
- Instructional Activity Student Handout (for Lessons 1, 2, 4)
- Instructional Activity Supplement (for Lessons 1, 2, 3, 5)
- Student Activities
- Student Activity Solution Guide

In this unit, students will learn to recognize and interpret multiplication and division in context. Students will first develop a concept of multiplication as it relates to repeated addition. They will represent multiplication with manipulatives, arrays, repeated grouping drawings, and by skip counting. Students will then extend their understanding of multiplication to the concept of division and will use manipulatives and models to solve division story problems. Finally, students will exhibit their knowledge by writing story problems to represent multiplication and division sentences.

RESEARCH

Multiplicative reasoning takes time for students to develop, and much of that development takes place in students' minds. Therefore, teachers need to first create experiences for students to express their thinking and then utilize those experiences to access students' developing strategies. Hodges, Rose, and Hicks (2012) recommend having students write explanations for their answers in addition to conducting interviews with students in order to access their mathematical reasoning. This is helpful for both students and teachers; as students develop better communication skills and solidify their understanding, teachers can determine what level of understanding each student has attained.

Questioning and Student Discourse

Lannin, Chval, and Jones (2015) recommend using strategic questioning to guide student learning. These strategic questions should deliberately target the differences between additive and multiplicative reasoning. For example, give students a list of problem scenarios and ask which of them require additive reasoning and which require multiplicative reasoning. Students should be required to explain their thinking, with teachers asking follow-up questions, such as "What if the groups in the scenario were unequal? Can we still use multiplicative reasoning?" Additionally, show students an object that is about three times the size of a second object, and have students describe the relationship between the objects. Students with an understanding of multiplicative reasoning will describe the relationship as more than just "bigger" or "three inches longer"; they will describe the larger object as a multiple of the smaller object.

Teachers should use the insights gained from students' thinking to guide instruction (Lannin, Chval, & Jones, 2015). Once students have mastered using physical manipulatives to represent and explain their thinking, guide them to draw a model to illustrate their thought process. Continually prompt students to make connections between the different representations that can portray a multiplication or division problem. For instance, ask students to identify how the multiplicand is represented in an array, in repeated grouping, with skip counting and symbolically. Students' reasoning should become more sophisticated over time, and their procedural fluency should develop from their experiences with multiple physical and visual representations.

Polly and Ruble (2009) and Whitin and Whitin (2008) suggest having students discuss their thinking with their peers. The benefits of this strategy are twofold: it allows teachers to observe the students' knowledge and offers students the opportunity to hear the strategies of their classmates. Many researchers consider discussion to be a vital method for acquainting students with varied ideas and strategies.

Real-World Contexts and Story Problems

A leading strategy that allows teachers to assess students' conceptual knowledge of multiplication and division is tasking students with writing their own story problems to represent a given multiplication or division sentence (Drake & Barlow, 2007; Jong & Magruder, 2014; Lannin et al., 2015; Polly & Ruble, 2009; Sullivan & McDuffie, 2009; Taber & Canonica, 2008; Whitin & Whitin, 2008). Many students are able to solve basic multiplication facts but will create conceptually incorrect problems to represent a given multiplication fact. This indicates that students' understanding does not go beyond memorization of facts and that they lack conceptual understanding of multiplication and division, which reinforces the idea that teachers must focus as much—if not more—attention on student work as they do on student answers (Van de Walle et al., 2014).

Writing story problems connects mathematics to writing instruction, which is an important skill that students need to develop (Polly & Ruble, 2009). Students will not inherently know how to write a story problem so teachers must provide scaffolding and examples for students. Identify the common components of story problems with students, such as the character, setting, and action. These story elements can be either missing or hard to find in some math problems. For example, the problem "Three piles of rocks each contain four rocks. How many rocks total?" would be an acceptable math problem illustrating multiplication; however, it does not include an action and the character is rocks, which is rather dull. Teachers should encourage students to incorporate the story elements to create more illustrative problems, such as "Trevor is collecting shells on the beach. He divides them into three piles. Each pile contains four shells. How many shells did Trevor collect?" The addition of a character and an action creates a richer story problem, and the setting allows students to visualize the problem more easily. Students should practice incorporating these components into their own word problems. It is also helpful for students to work from examples or to use a graphic organizer to develop their story problems.

In general, most of the problems that students encounter should be rooted in a real-world context in order to make problems meaningful, accessible, and engaging. However, "word problems" are often challenging for students, so experiences solving contextual math problems should be supported through scaffolding (Van de Walle et al., 2014). Teachers should model problem-solving strategies in order to help students develop their own approaches and methods for persevering through and solving word problems.

If a student is struggling with a problem, ask them to restate the problem in their own words. This shows whether or not the student is able to make sense of the problem. To make the problem more accessible to

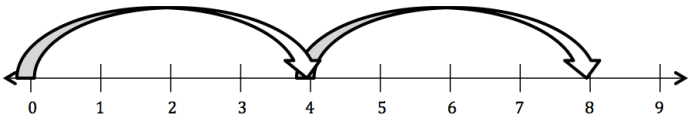
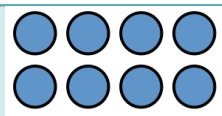
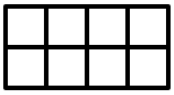
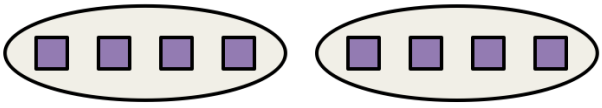
students, encourage them to try thinking of and solving a simpler word problem similar to the given one. Smaller values can be substituted for larger ones so that the problem seems more approachable, and manipulatives, visual aids, and models provide greater accessibility for students. Differentiate strategies and instructions for students by allowing the use of a calculator for the calculations if necessary. Require students to write their answers in complete sentences and to decide if their answer makes sense.

Early in the development of multiplicative reasoning, it is helpful for students to work with physical manipulatives such as square tiles or counters (Battista, 1999). In fact, students' first instinct in solving multiplication problems is often to use physical manipulatives (Lannin et al., 2015). Manipulatives allow a student to make a conjecture and then check it using a concrete representation. Another benefit of the use of manipulatives is that it gives students the grounds for developing their own visual and mental models. After working with manipulatives, students should represent multiplication and division by drawing their own models. These models include linear representations (e.g., number lines and tape diagrams), arrays, and repeated grouping of discrete objects. Students should then internalize these models in order to develop a more abstract understanding of multiplication and division.

AN EXAMPLE

Students can model multiplication problems in several ways.

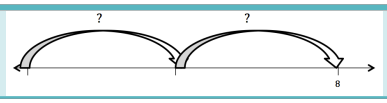
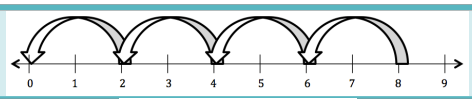
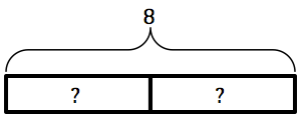
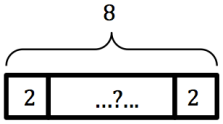
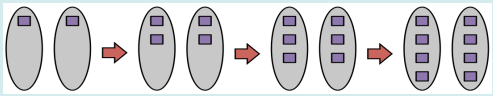
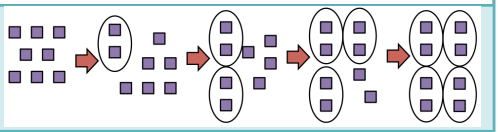
The following table gives examples of how students can model 2×4.

Model	Example
Number Line	
Tape/Bar Diagram	
Array	
Area	
Repeated Grouping	

AN EXAMPLE

Students can model division problems in several ways.

The following table gives examples of how students can model $8 \div 2$.

Method	Partitive Division	Measurement Division
Number Line		
Bar Method		
Direct Modeling: "Dealing"		

It is important for students to develop fluency between all representations and models of multiplication and division. It is possible for students to develop the ability to solve problems in one context or with one model, but also to lack the capability to solve problems outside of that context. Therefore, students should be asked to explain the similarities and differences between all representations of multiplication and division, and they should be able to identify how the components (e.g., the factors, product, quotient, dividend, etc.) are represented amongst the representations. Students should also begin to identify which strategies are appropriate for different types of problems. For example, using a number line is more helpful when solving measurement division problems than partitive division problems.

Students in third grade are expected to learn equal groups, area, and array problem types, and they should be able to solve these problem types with discrete objects as well as measurement quantities. For example, a measurement quantity problem for equal groups would involve length, and a measurement quantity problem for an array would involve area. Measurement quantities are often more difficult for students, so problem types with discrete objects should be introduced first (Common Core State Standards Initiative, 2010). When the number of groups and the size of each group is known, the product is therefore unknown, and the situation represents a multiplication problem. However, students will encounter two types of division: one in which the size of each group is unknown (partitive division), and one in which the number of groups is unknown (measurement division).

AN EXAMPLE

Multiplication and division as represented in equal groups, area, and array problems with both discrete objects and as measurement quantities.

Equal Groups		
	Discrete Objects	**Measurement Quantity**
Product Unknown: Multiplication	There are five bags each containing 12 cookies. How many total cookies are there?	You need five lengths of string, each 12 inches long. How much string will you need altogether?
Size of Groups Unknown: Partitive Division	If 60 cookies are being divided equally into five bags, how many cookies will be in each bag?	A string 60 inches long is being divided equally into five segments. How long will each segment be?
Number of Groups Unknown: Measurement Division	A teacher is dividing 60 cookies into bags. The teacher wants to put 12 cookies in each bag. How many bags are needed?	A string 60 inches long is being cut into 12-inch-long segments. How many segments of string can be cut?

Array		
	Discrete Objects	**Measurement Quantity**
Product Unknown: Multiplication	There are five rows of apples with 12 apples in each row. How many apples are there?	What is the area of a five centimeter by 12 centimeter rectangle?
Size of Groups Unknown: Partitive Division	If 60 apples are arranged into five equal rows, how many apples will be in each row?	A rectangle has area 60 square centimeters. If one side is five centimeters long, how long is a side next to it?
Number of Groups Unknown: Measurement Division	If 60 apples are arranged into equal rows of 12 apples, how many rows will there be?	A rectangle has area 60 square centimeters. If one side is 12 centimeters long, how long is a side next to it?

While partitive division is the most common type of division that students encounter, teachers should make sure that students are familiar with both types of division. Partitive division may make intuitive sense for students, because they are used to sharing objects with siblings, classmates, etc. and therefore can more easily picture "sharing" equal amounts between a given number of groups. Measurement division (sometimes called *quotative* division), which is when the number of groups is unknown, is helpful for understanding division of fractions in later grades.

Array problems provide a bridge between repeated-grouping and area problems, because arrays can be thought of as both repeated groups as well as an area model. An array represents repeated-grouping problems

because it contains rows of equal size. Arrays also represent area problems because the rectangular arrangement of rows and columns provides dimensions that can be multiplied to determine the total area. Area problems are specific types of array problems in which the objects are squares that "have been pushed together so that there are no gaps or overlaps" (Common Core State Standards Initiative, 2010).

When students first explore multiplication and division, it is important to relate new information to familiar concepts such as addition, subtraction, and counting strategies. Additionally, manipulatives and models provide a natural entry point for students. The difference between additive reasoning and multiplicative reasoning should be stressed as students develop their multiplicative understanding, and students should encounter multiplication and division through real-world scenarios in order to give meaning to the problems. Students will engage with the curriculum through these accessible paths and develop a deep conceptual understanding when given time to process and discuss the different strategies and models.

CAUTIONS AND MISCONCEPTIONS

For all word problems, students should be encouraged to think about how to solve the problem before performing any calculations. Doing so avoids mindless "number crunching", which happens when students guess what operation to use based on the "concept of the day". Present the situation to students with numbers covered to engage their problem-solving skills instead of their answer-getting skills. Students may have been taught to look for keywords in order to determine which operation to use, but this "strategy" should be discouraged, as it removes all conceptual thinking from the process and can lead students to incorrect solutions. Ask students guiding questions, such as "What information will the answer give us?" and "Is there any information provided that we don't need?" Students should be encouraged to ask themselves these questions when solving problems independently.

Anticipate the following common errors that arise when students write their own story problems for a given multiplication sentence, $a \times b = c$:

- Not specifying that the groups are of equal size (Jong & Magruder, 2014; Polly & Ruble, 2009)
- Writing the solution in the story problem (Polly & Ruble, 2009)
- Writing a story problem that represents $b \times a$ instead of the given expression $a \times b$ (Drake & Barlow, 2007; Jong & Magruder, 2014; Sullivan & McDuffie, 2009)
- Writing a story problem that represents $c \div a = b$ or $c \div b = a$ (Drake & Barlow, 2007)
- Writing an unrealistic, dull, or artificial story problem (Drake & Barlow, 2007; Sullivan & McDuffie, 2009)
- Writing a story problem in which it is unclear whether the narrator is included in the partitioning (Jong & Magruder, 2014)
- Writing an addition problem $a + b$ instead of a multiplication problem (Sullivan & McDuffie, 2009), and similarly writing a subtraction problem instead of a division problem (Lannin, Chval, & Jones, 2015, Polly & Ruble, 2009)

Many of these errors derive from missing conceptual knowledge of the true meaning of multiplication and division. The simple task of writing a story problem encourages students to confront misconceptions or a

lack of understanding in a way that non-conceptual problems cannot. In this way, multiplication and division are taught *through* problem solving as opposed to being taught independently of it.

LEARNING MAP INFORMATION

The learning map section for this sequence of activities indicates that students will be able to demonstrate the concept of multiplication after connecting that knowledge to the idea of repeated addition. Students must develop the idea that multiplication represents combining multiple sets of the same quantity to create a new set. From this understanding, students can explain multiplication problems and subsequently its individual components: factors (i.e. multiplicand and multiplier), multiplication sign, and product. Once students are equipped with a conceptual understanding of the components of a multiplication problem, they can represent multiplication with an equation.

The development of division knowledge follows a similar path. Students have the ability to solve repeated-subtraction problems and therefore can be introduced to the concept of division. Once the general concept of division is mastered, the conceptual understanding of each component of a division problem can be established. From there the students can represent division equations.

Students need to be able to distinguish which operation is necessary to solve a variety of problems; therefore, the connection and distinction between multiplication and division should be well established. Students will be expected to know how to mathematize situations involving either multiplication or division before developing the ability to solve array problems, equal-group problems, and measurement-quantities problems.

INSTRUCTIONAL ACTIVITIES

The activities in this unit are designed to develop students' multiplicative reasoning, which extends into a conceptual understanding of multiplication and division. Initially students will connect multiplication to the concept of repeated addition through the idea of "groups of groups". Eventually students should develop a sense of multiplication in its own right (i.e. that it is of copies of multiplicative units). Students first explore the "groups of groups" with concrete manipulatives to develop their internal visual models and then graduate to creating written models to represent the multiplication, such as number lines and arrays. A graphic organizer is provided for each problem type to support students' initial interaction with the models.

Students will focus on repeated groups and array multiplication problems and will become comfortable applying these problems to discrete objects as well as measurement quantities. After students have seen and solved examples of these types of problems, they will then attempt to write a story problem of their own to represent a given multiplication sentence. Students then examine a variety of story problems for common errors, such as stating the answer in the problem or writing an addition problem instead of a multiplication problem. Students then make revisions to their original story problems to address any errors that were exhibited in their first draft and write a new story problem for any multiplication sentence of their choosing.

Once students have been given time to develop an understanding of multiplication and show their understanding by writing word problems, they are introduced to the idea of division as a multiplication sentence with a missing factor. Students practice both partitive and measurement division and then make their first attempt at writing a division story problem.

After their initial attempt at writing a division story problem, students again are shown various word problems which illustrate common errors made in word problems representing division. After being given a chance to revise their first draft, students write another division story problem for a division sentence of their choosing.

The students' story problems are then used as practice problems in a culminating activity to assess students' overall understanding of the concepts of multiplication and division.

REFERENCES

Battista, M. (1999). The Importance of Spatial Structuring in Geometric Reasoning. *Teaching Children Mathematics, 6*(3), 170-178. Retrieved from http://www.jstor.org.www2.lib.ku.edu/stable/41199100

Drake, J., & Barlow, A. (2007). Assessing Students' Levels of Understanding Multiplication through Problem Writing. *Teaching Children Mathematics, 14*(5), 272-277. Retrieved from http://www.jstor.org.www2.lib.ku.edu/stable/41199905

Hodges, T. E., Rose, T. D. & Hicks, A. D. (2012). Interviews as RtI Tools. *Teaching Children Mathematics, 19*(1), 30-36. doi:10.5951/teacchilmath.19.1.0030

Jong, C. & Magruder, R. (2014). Beyond Cookies: Understanding Various Division Models: Reflect and Discuss. *Teaching Children Mathematics, 20*(6), 366-373. doi:10.5951/teacchilmath.20.6.0366

Moyer, P. S. (2000). A remainder of one: Exploring partitive division. *Teaching Children Mathematics*, *6*(8), 517.

National Governors Association Center for Best Practices, & Council of Chief State School Officers. (2010). *Common Core State Standards for mathematics: Mathematics glossary Table 2*. Washington DC: Author.

Lannin, J. K., Chval, K. B., & Jones, D. (2015) *Putting essential understanding of multiplication and division into practice in grades 3-5*. Reston, VA: The National Council of Teachers of Mathematics, Inc.

Polly, D. (2012) Halloween Treats. *Teaching Children Mathematics, 19*(3), 139-143. doi:10.5951/teacchilmath.19.3.0139

Polly, D. (2012). Sorting Rectangles. *Teaching Children Mathematics, 19*(4), 219-224. doi:10.5951/teacchilmath.19.4.0219

Polly, D., & Ruble, L. (2009). Learning to Share Equally. *Teaching Children Mathematics, 15*(9), 558-563. Retrieved from http://www.jstor.org.www2.lib.ku.edu/stable/41199351

Sullivan, A., & McDuffie, A. (2009). Connecting Multiplication to Contexts and Language. *Teaching Children Mathematics, 15*(8), 502-512. Retrieved from http://www.jstor.org.www2.lib.ku.edu/stable/41199340

Sweda, J., Knotts, L., Moyer-Packenham, P., & Swedab, J. (2004). Mr. Greenthumb's Garden. *Teaching Children Mathematics, 11*(4), 217-225. Retrieved from http://www.jstor.org.www2.lib.ku.edu/stable/41199805

Taber, S., & Canonica, M. (2008). Sharing "Cat Games" and Cookies: "Special Education Students Investigate Division". *Teaching Children Mathematics, 15*(1), 55-61. Retrieved from http://www.jstor.org.www2.lib.ku.edu/stable/41199210

Van de Walle, J., Lovin, L., Karp, K., Bay-Williams, J. (2014). *Teaching student-centered mathematics: Developmentally appropriate instruction for grades Pre-K – 2*. Boston, MA: Pearson.

Wallace, A., & Gurganus, S. (2005). Teaching for Mastery of Multiplication. *Teaching Children Mathematics, 12*(1), 26-33. Retrieved from http://www.jstor.org.www2.lib.ku.edu/stable/41198648

Whitin, P., & Whitin, D. (2008). Learning to Solve Problems in Primary Grades. *Teaching Children Mathematics, 14*(7), 426-432. Retrieved from http://www.jstor.org.www2.lib.ku.edu/stable/41199182

Wickett, M. (2000). Amanda Bean and the Gator Girls: Writing and Solving Multiplication Stories. *Teaching Children Mathematics, 6*(5), 282-303. Retrieved from http://www.jstor.org.www2.lib.ku.edu/stable/41197422

Interpret and Represent Multiplication and Division in Context

Overview of Instructional Activities

Lesson	Learning Goal	Nodes Addressed
Lesson 1	Students will explore situations with groups of objects in order to develop multiplicative reasoning.	▶ EXPLAIN *FACTOR* ▶ EXPLAIN THE MEANING OF THE MULTIPLICATION SIGN ▶ REPRESENT MULTIPLICATION WITH EQUATIONS ▶ EXPLAIN MULTIPLICATION PROBLEMS
Lesson 2	Students will discover how arrays represent multiplication and write their own word problems to represent multiplication in a real-world context.	▶ DEMONSTRATE THE CONCEPT OF MULTIPLICATION ▶ EXPLAIN MULTIPLICATION PROBLEMS ▶ MATHEMATIZE CONTEXTUAL SITUATIONS INVOLVING MULTIPLICATION
Lesson 3	Students will create story problems for a chosen multiplication expression to demonstrate their ability to interpret the meaning of multiplication.	▶ EXPLAIN MULTIPLICATION PROBLEMS ▶ EXPLAIN THE MEANING OF THE MULTIPLICATION SIGN ▶ MATHEMATIZE CONTEXTUAL SITUATIONS INVOLVING MULTIPLICATION ▶ REPRESENT MULTIPLICATION WITH EQUATIONS ▶ SOLVE EQUAL-GROUP PROBLEMS
Lesson 4	Students will explore situations that can be modeled with division.	▶ DEMONSTRATE THE CONCEPT OF DIVISION ▶ EXPLAIN DIVISION PROBLEMS ▶ EXPLAIN *QUOTIENT* ▶ EXPLAIN *DIVIDEND* ▶ EXPLAIN *DIVISOR* ▶ EXPLAIN THE MEANING OF THE DIVISION SIGN ▶ MATHEMATIZE CONTEXTUAL SITUATIONS INVOLVING DIVISION ▶ SOLVE EQUAL-GROUP PROBLEMS
Lesson 5	Students will create story problems for a given division expression to demonstrate their ability to interpret the meaning of division.	▶ EXPLAIN DIVISION PROBLEMS ▶ EXPLAIN THE MEANING OF THE DIVISION SIGN ▶ MATHEMATIZE CONTEXTUAL SITUATIONS INVOLVING DIVISION ▶ REPRESENT DIVISION WITH EQUATIONS ▶ SOLVE EQUAL-GROUP PROBLEMS ▶ SOLVE ARRAY PROBLEMS
Lesson 6	Students will solve multiplication and division story problems.	▶ REPRESENT DIVISION WITH EQUATIONS ▶ REPRESENT MULTIPLICATION WITH EQUATIONS ▶ SOLVE ARRAY PROBLEMS ▶ SOLVE EQUAL-GROUP PROBLEMS ▶ SOLVE MEASUREMENT-QUANTITIES PROBLEMS

Interpret and Represent Multiplication and Division in Context

INSTRUCTIONAL ACTIVITY

Lesson 1

LEARNING GOAL

Students will explore situations with groups of objects in order to develop multiplicative reasoning.

PRIMARY ACTIVITY

Students begin by brainstorming ideas of things that come in groups. Then they begin to ask mathematical questions about the objects and notice the similarities between their questions. Finally, students are introduced to the symbolic notation of multiplication.

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

- Size of group
- Number of groups
- Product
- Factor
- Multiplier
- Multiplicand

MATERIALS

- Prepared images of items that come in groups adhered onto index cards, included in the Instructional Activity Supplement (Recommend one class set.)
- Instructional Activity Student Handout

IMPLEMENTATION

Begin by asking students to brainstorm a list of items that come in groups. Organize the list by the number of items in each group to begin the idea of classifying these groups mathematically. For example, make a list of things that come in twos—such as eyes, ears, twins, chopsticks, etc.—and continue for groups of three, four, and so forth.

Instruct students to brainstorm in pairs, then have students share their lists round-robin style to create a class list.

Show students the index cards with pictures of items that come in groups. Add any new groups of items to the class list.

Ask students to think of a few observations that they notice about the groups of objects. **Call on** three to four students to share their observations. Students may say that they see multiple objects, or that there are a large number of total objects.

Group students into pairs and **distribute** one index card to each pair of students.

Model how to write a mathematical question by choosing an index card and writing a mathematical question about it. Give two examples of mathematical questions (e.g., how many total legs are there?) and two examples of non-mathematical questions (e.g., what color is the person's shirt?).

Tell students to write two mathematical questions about their picture on the back of the index card, and to include the solutions.

Have students share their questions with other students by using a class-building activity.

- Each pair of students finds another pair and shares the questions on their index card. Each partner should share one of the questions they wrote.
- Students then trade their index card with the other pair of students and find a new set of partners to share questions with.
- Students should repeat this activity for three to four minutes. A timer should be displayed so students know how long they have to share.

After sharing their questions, students should think about the kinds of questions that were shared.

Ask students, "What did you notice about the questions that were written about the images?"

Call on students and **create** a list of observations that students make.

Conclude that the questions often referred to how many groups there were, how many items were in the groups, or how many total objects there were.

Introduce the idea that all the images contained *groups of groups*.

Choose a few of the questions that students wrote to answer as a class. **Note** strategies that students use, such as skip counting or repeated addition.

Introduce the notation of counting by equal groups: $number\ of\ groups \times number\ of\ items\ per\ group = total\ number\ of\ items$. **Tell** students that the symbol "×" represents multiplication.

Define these terms as $multiplier \times multiplicand = product$. The two numbers that are multiplied together are also called *factors*. Therefore, it could also be seen as $factor \times factor = product$. The result should be a poster or area of the board for students to reference that looks like this:

_____	x	_____	=	_____
# of groups	x	# of elements per group	=	# of total elements
multiplier	x	multiplicand	=	product
factor	x	factor	=	product

Model this idea by writing the repeated addition equation as well as the multiplication equation for two to three of the images. **Model** finding the number of feet in the duckling picture and the number of blades in the wind turbine picture.

GUIDING QUESTIONS

Elicit student thinking:

- What is the multiplier in this situation? What does it represent?
- What is the multiplicand in this situation? What does it represent?
- What does the product represent?

Determine if the student can EXPLAIN *FACTOR*:

- How did you know which two numbers to choose as the factors?
- What do we call the first factor?
- What do we call the second factor?

Determine if the student can EXPLAIN THE MEANING OF THE MULTIPLICATION SIGN:

- What does the "×" symbol represent?
- What kinds of scenarios can be represented with the "×" symbol?

Determine if the student is ready to REPRESENT MULTIPLICATION WITH EQUATIONS:

- Which factor is written first? How do you know?
- Which factor is written second? How do you know?

Ask students how to find the product of 3×2 in relation to the duckling photo.

Tell students to turn to their partner and discuss how to show 3×2 with skip counting on a number line. **Select** one or two students to illustrate or model their thinking. If no students volunteer, **demonstrate** how to skip count on a number line to find 3×2.

Tell students to turn to their partner and discuss how to draw 3×2 with grouping. **Call on** one or two students to illustrate or model their thinking. If no students volunteer, **demonstrate** how to draw the repeated grouping of 3×2.

Repeat this for the wind turbine picture.

GUIDING QUESTIONS

Elicit student thinking:

- How do these two models help you find the product?

Determine if the student can EXPLAIN MULTIPLICATION PROBLEMS:

- How do you know how many groups to draw?
- How do you know how many objects are in each group?
- How do you know how many "jumps" to make?
- How do you know what distance to "jump"?

Determine if the student can REPRESENT MULTIPLICATION WITH EQUATIONS:

- Which factor represents the number of groups? How do you know?
- Which factor represents the size of each group? How do you know?

Students should be required to write the addition and multiplication sentences, as well as the skip-counting and repeated-grouping models, for the two scenarios on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

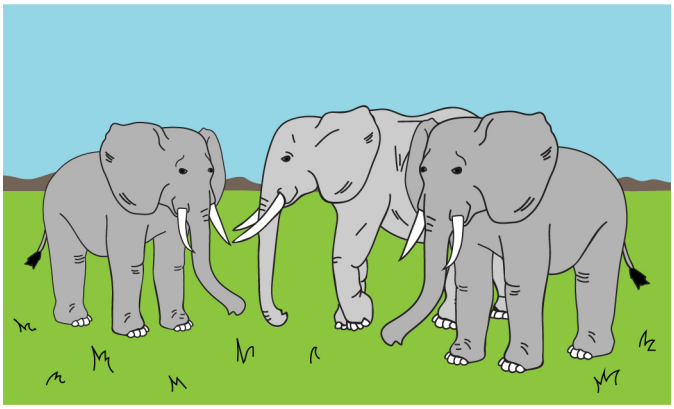
At the end of the activity, teachers should collect and review students' equations and models in the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

Name_____

Interpret and Represent Multiplication and Division in Context

Lesson 1

1. Write the repeated-addition equation to represent the total number of tusks on all three elephants.



_____ + _____ + _____ = _____

number of tusks on one elephant | number of tusks on one elephant | number of tusks on one elephant | total tusks for all three elephants

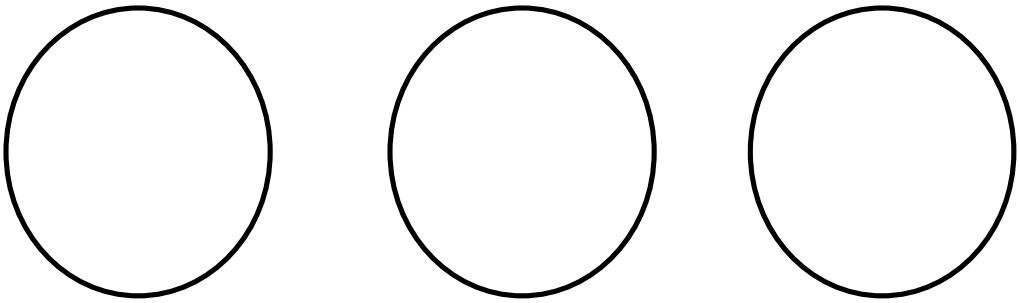
2. Write the multiplication equation to represent the total number of tusks on all three elephants.

_____ × _____ = _____

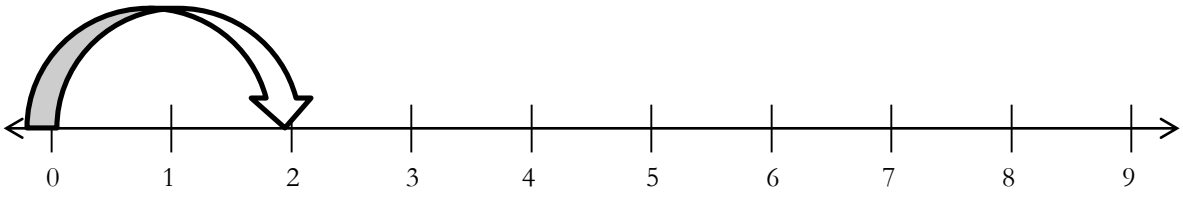
number of elephants | number of tusks on one elephant | total tusks for all three elephants

Name_____

3. Model the total number of tusks on all three elephants with repeated grouping.



4. Use the number line to model the total number of tusks on all three elephants with skip counting.



Name_____

5. Write the repeated-addition equation to represent the total number of toes on all three surfers.

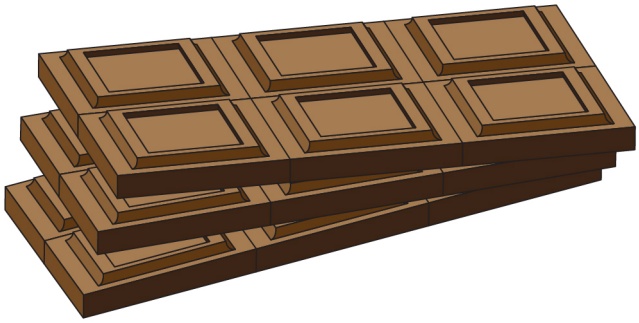
6. Write the multiplication equation to represent the total number of toes on all three surfers.

7. Model the total number of toes on all three surfers with repeated grouping.

Name_____

8. Use the number line to model the total number of toes on all three surfers with skip counting.

9. Write an equation representing the following picture.

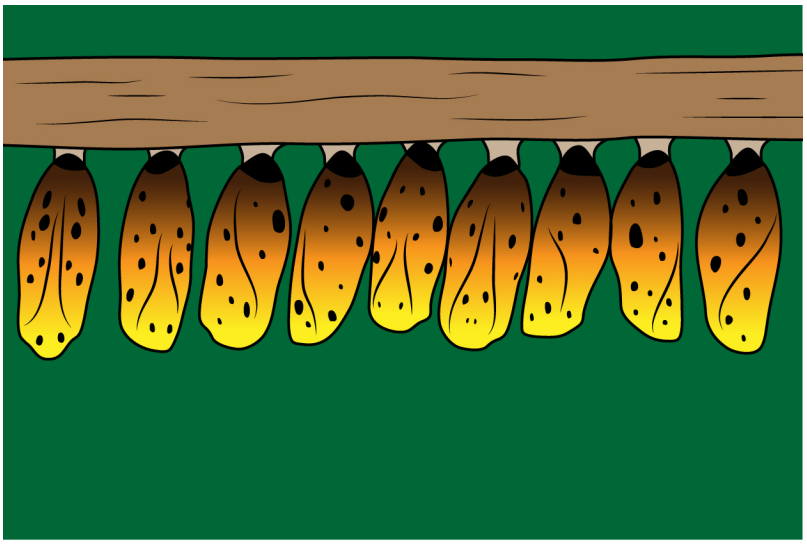


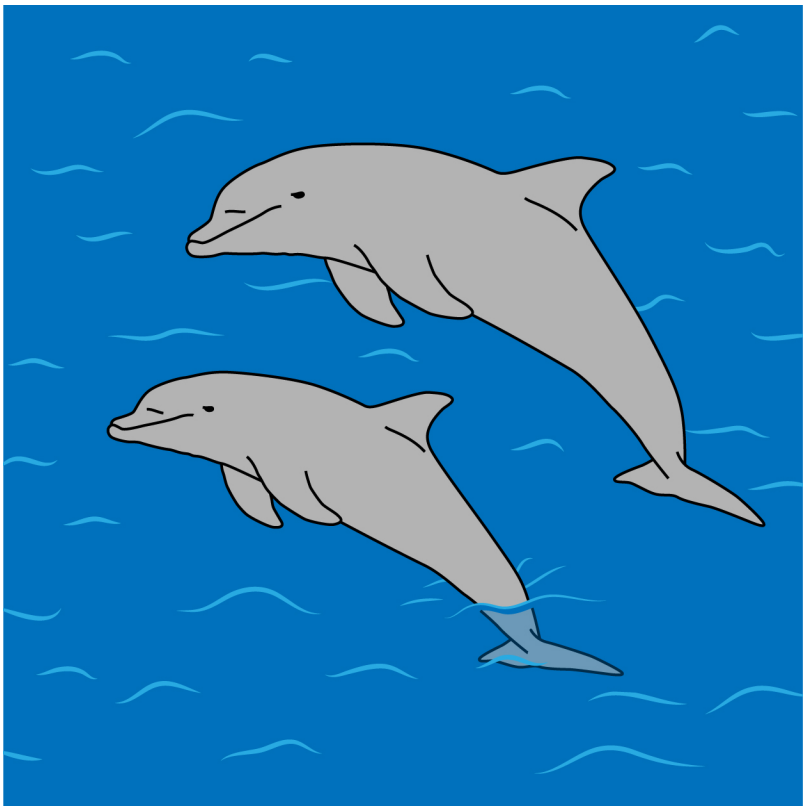
10. Draw a picture to represent 4×3.

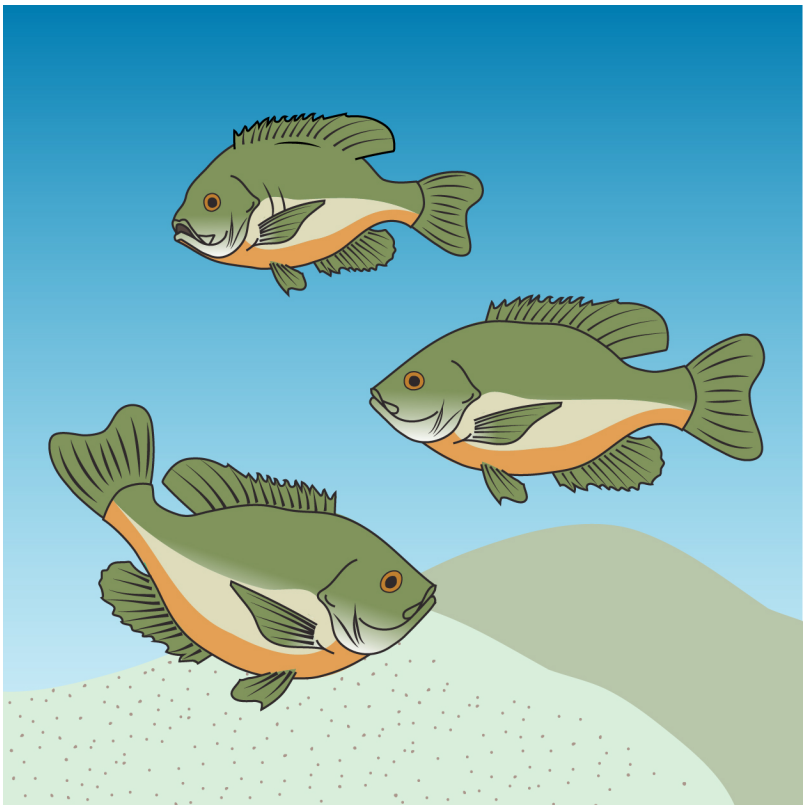
Interpret and Represent Multiplication and Division in Context

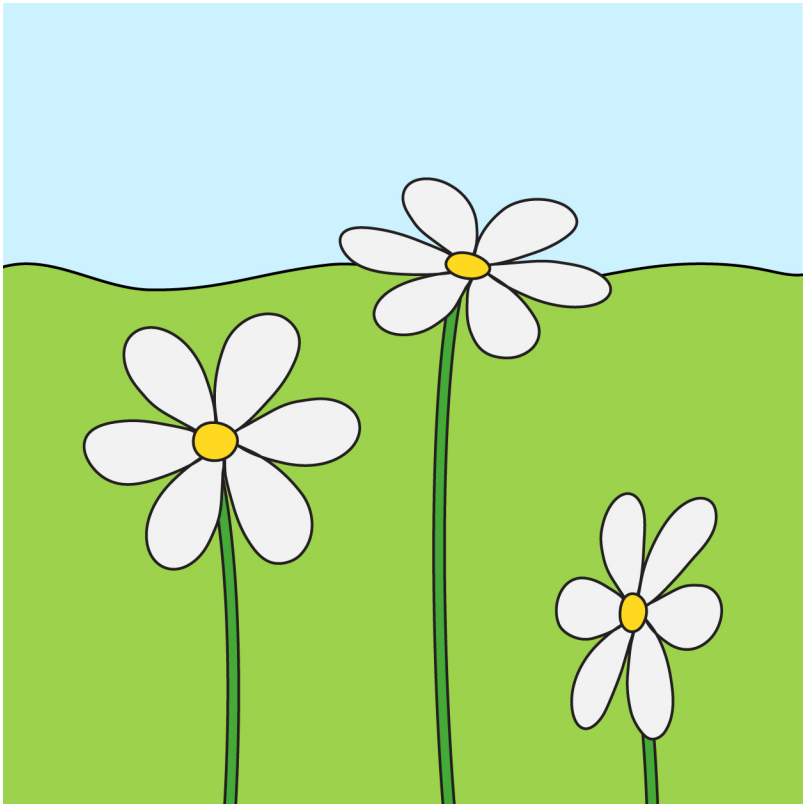
INSTRUCTIONAL ACTIVITY SUPPLEMENT

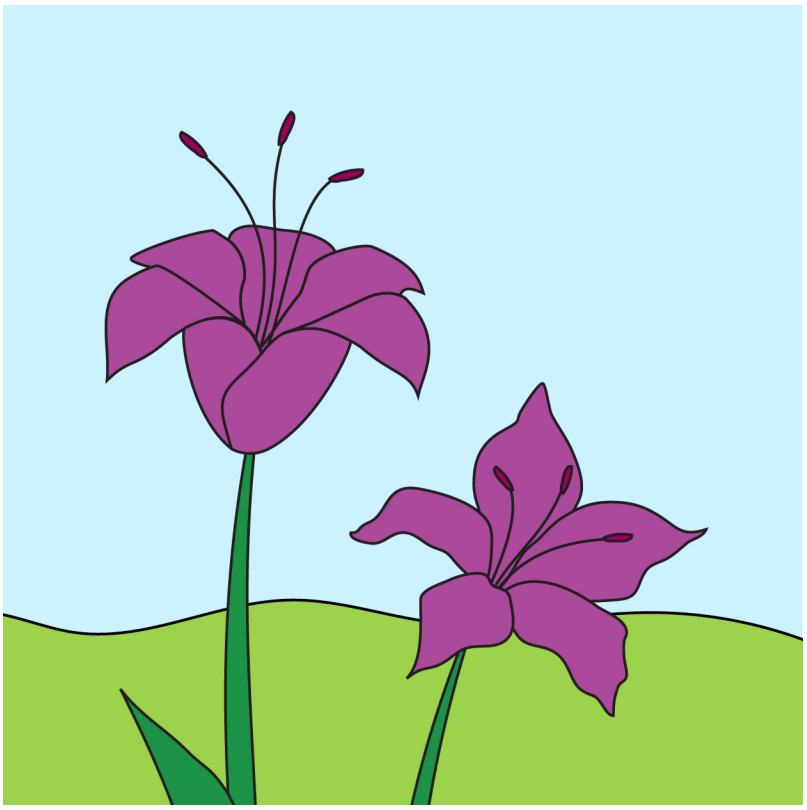
Lesson 1

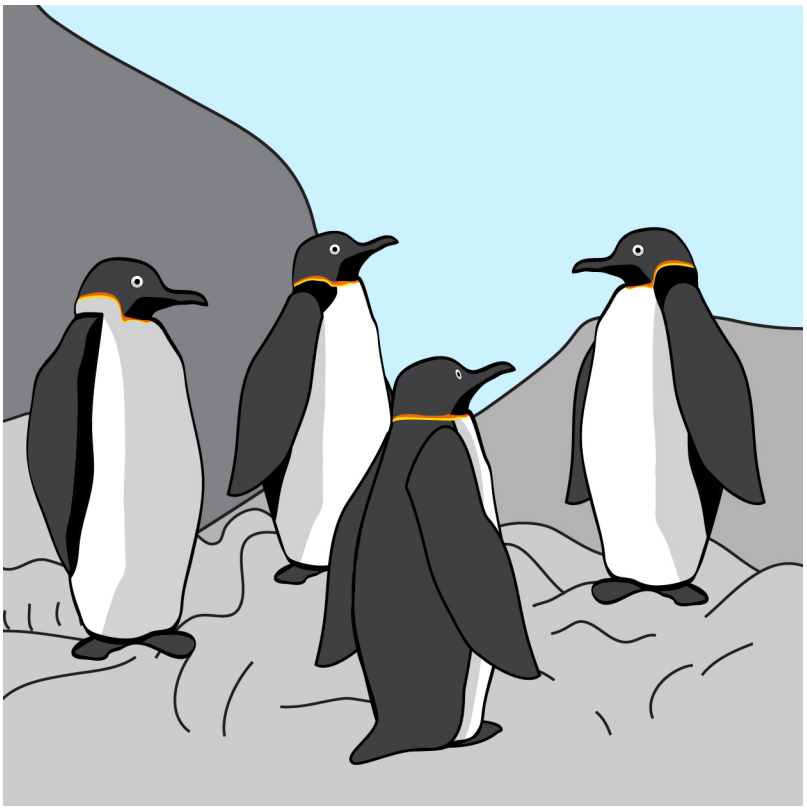


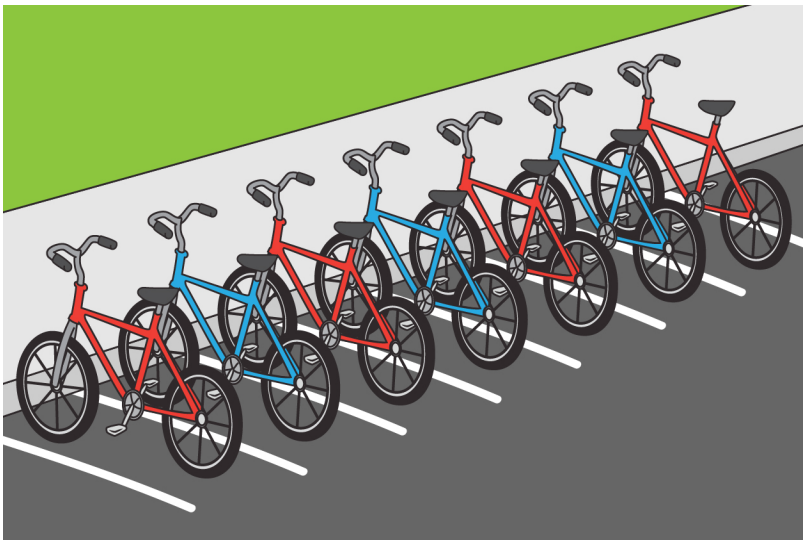


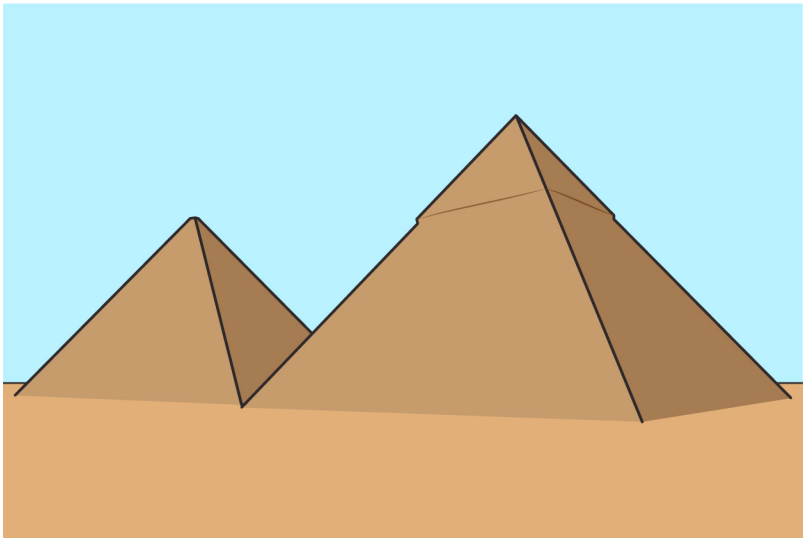


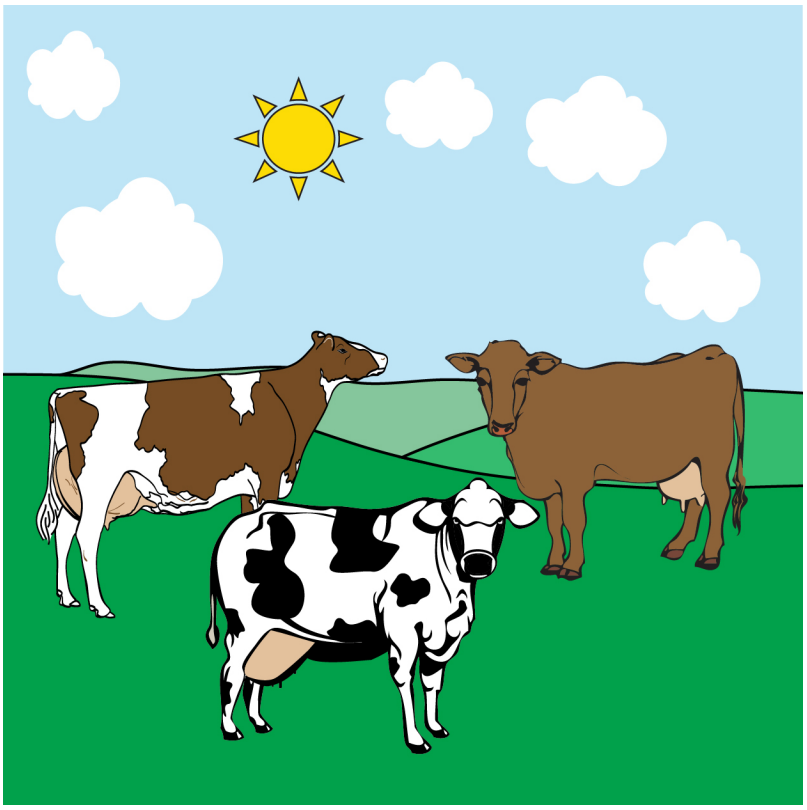


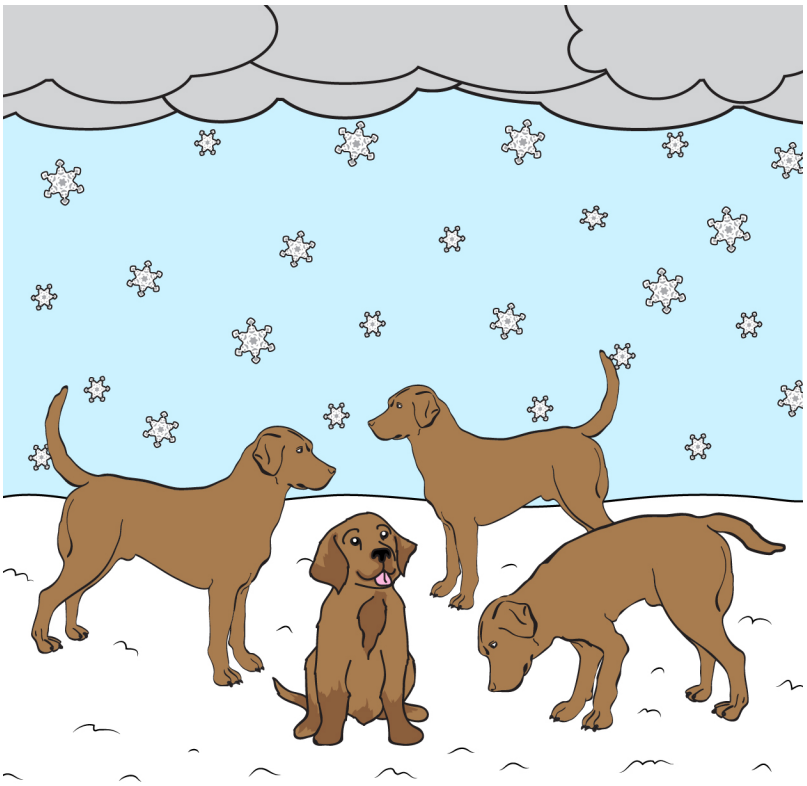


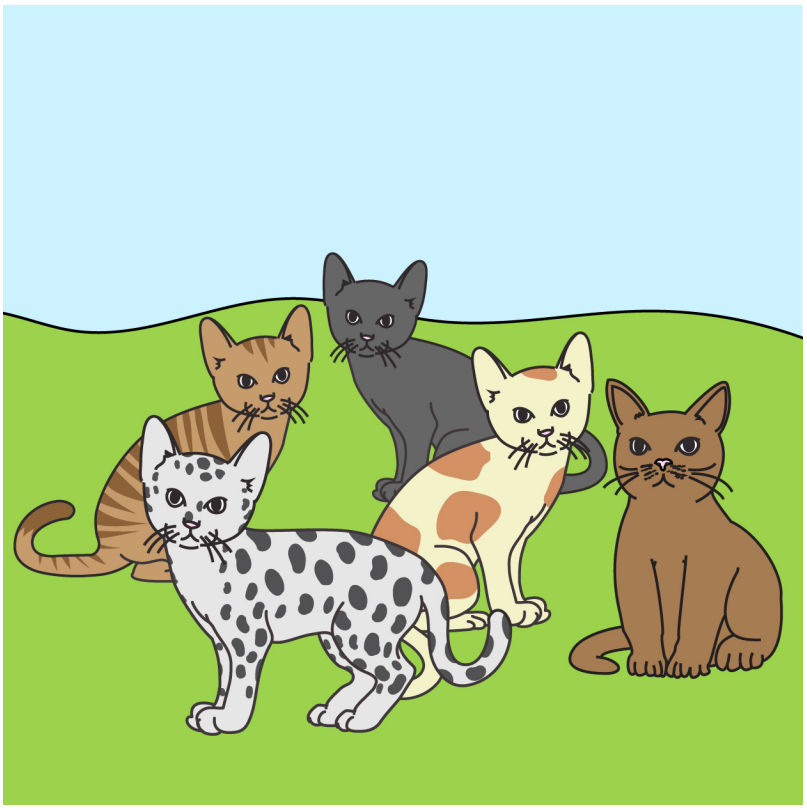


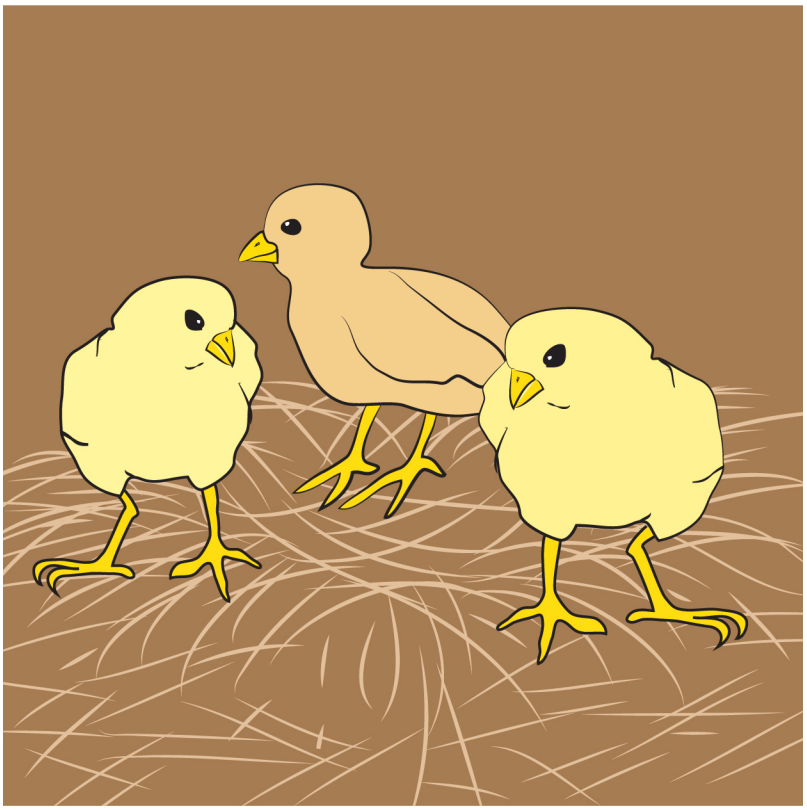


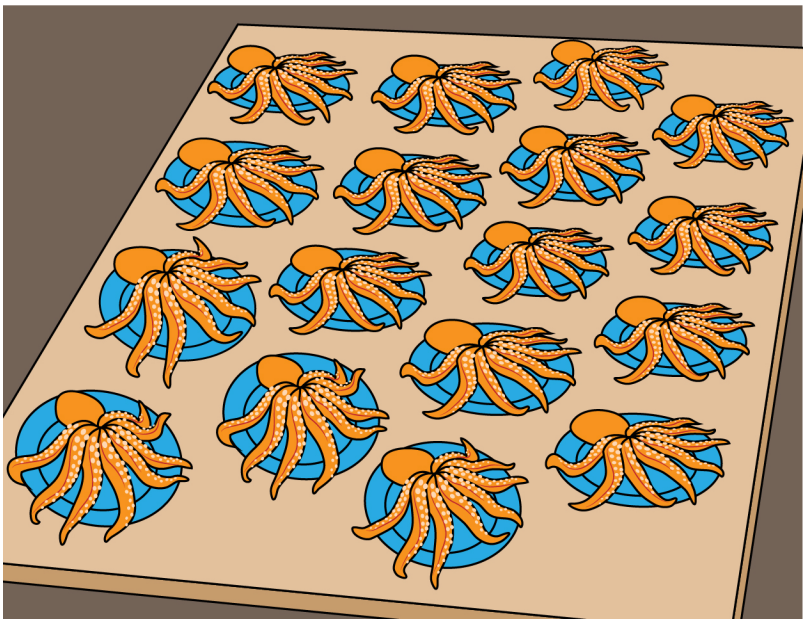


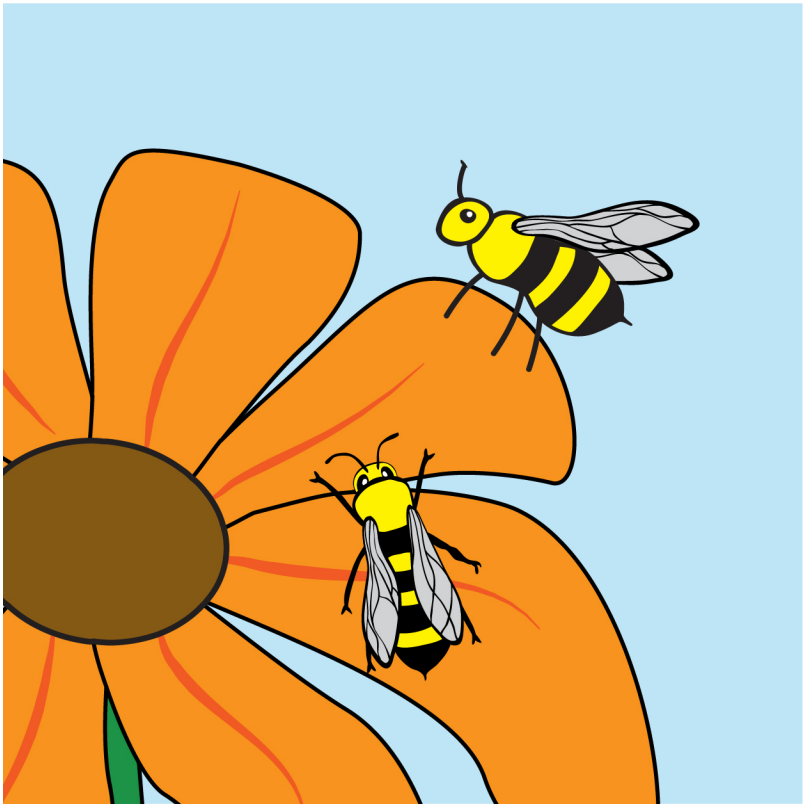












INTERPRET AND REPRESENT MULTIPLICATION AND DIVISION IN CONTEXT

INSTRUCTIONAL ACTIVITY

Lesson 2

LEARNING GOAL

Students will discover how arrays represent multiplication and write their own word problems to represent multiplication in a real-world context.

PRIMARY ACTIVITY

Students explore multiplication models with physical manipulatives and then learn a general format for writing story problems in order to write story problems that represent given multiplication sentences.

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

- Skip counting
- Array
- Repeated grouping
- Multiplier
- Multiplicand
- Product
- Factor
- Story problem
- Character
- Event
- Setting
- Problem

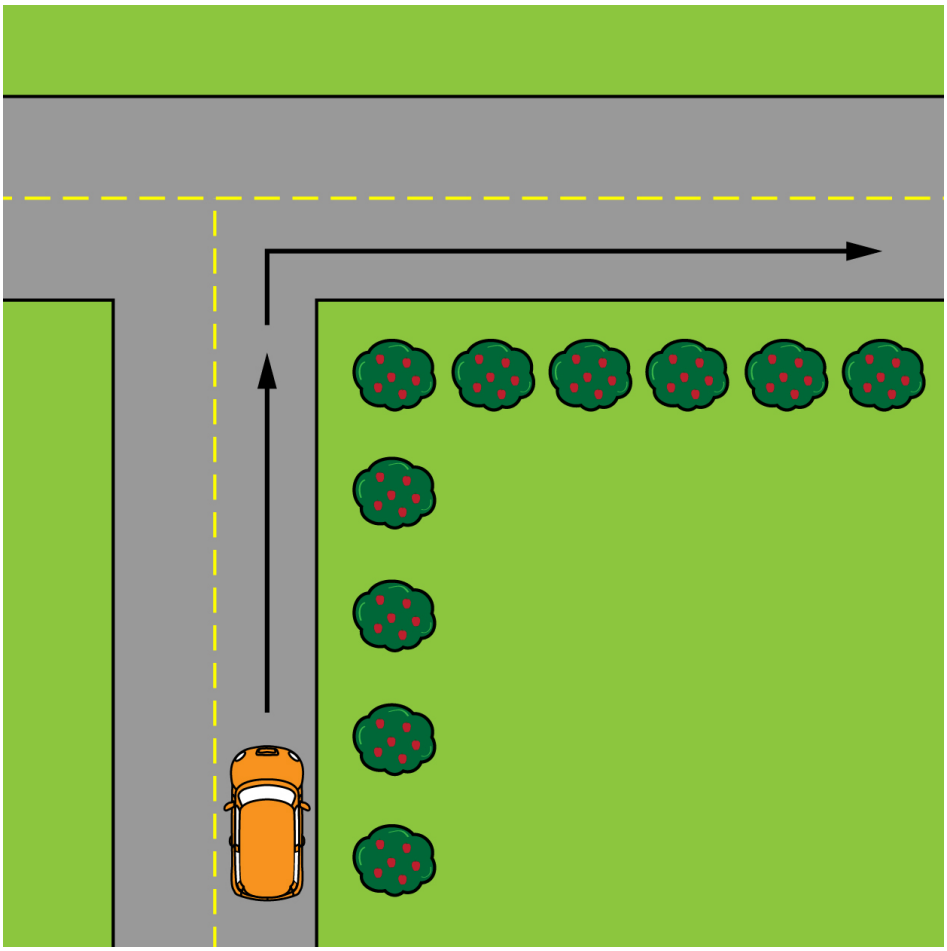
MATERIALS

- Square tiles or counters (Recommend 30 tiles or counters for each student.)
- Highlighters (Recommend one yellow, green, pink, and orange highlighter for each student.)
- INSTRUCTIONAL ACTIVITY SUPPLEMENT (Recommend one copy for each student.)
- INSTRUCTIONAL ACTIVITY STUDENT HANDOUT

IMPLEMENTATION

Present the following situation: You drive past an orchard and count five rows of apple trees. You then turn right at the corner and count six apple trees in the first row. The farmer has planted equal rows and columns. How many trees are in the orchard?

Show the following image.



Distribute the tile squares/counters to students and ask them to show the known edges of the orchard with the manipulatives.

Tell students to use the manipulatives to find the total number of trees in the orchard.

Circulate while students work on this task.

Support struggling students by helping them discover how many trees should be in each row and how many rows there are. **Assist** them in organizing their manipulatives to represent the problem scenario. **Ask** if there is a way to count the total number of trees without counting each individual tree.

Ask students who finish easily to represent their thinking symbolically (e.g., with an addition or multiplication sentence), with another model (e.g., repeated groupings or skip counting), or by writing an explanation in words. To further their thinking, ask division related questions, such as "If there are 30 trees total to be divided into 30 rows, how many trees will there be per row? What if there were five rows?"

Discuss as a class the strategies students used to find the total number of trees in the orchard.

Have a class discussion in order to conclude that the orchard represents groups of groups (i.e. a multiplication situation similar to those explored in LESSON 1).

GUIDING QUESTIONS

Elicit student thinking:

- Is there a way to find the total number of trees without counting each individual one?
- How is this situation similar to those in LESSON 1?
- What do you notice about the orchard?
- What do you notice about the rows and columns?

Determine if the student can DEMONSTRATE THE CONCEPT OF MULTIPLICATION:

- Are the rows of equal size?
- How many equal-size rows are there?
- Show me the tree orchard using the manipulatives. [Point to the "trees" not visible on the image.] How do you know these trees go here?

Determine if the student is ready to EXPLAIN MULTIPLICATION PROBLEMS:

- How many groups are there?
- How many trees are in each group?
- Which factor should be written first in the multiplication sentence?
- What is the question asking you to determine?

Define the orchard arrangement as an *array*, which is an arrangement of objects in equal rows and columns. **Tell** students that this is another way to model a multiplication situation, like repeated grouping and skip counting.

Display the elephant and surfer images from the LESSON 1 INSTRUCTIONAL ACTIVITY STUDENT HANDOUT with the corresponding multiplication expression.

Model how an array can be drawn for each multiplication situation, and how the total number of squares in the array represents the total number of objects or the product of the multiplication sentence.

Distribute the INSTRUCTIONAL ACTIVITY SUPPLEMENT, which gives another situation: Three muffin tins are being baked in the oven. Each muffin tin contains 12 muffins. How many muffins will there be in total?

Highlight the phrase "three muffin tins" in yellow. **Instruct** students to do the same on their INSTRUCTIONAL ACTIVITY SUPPLEMENT. **Ask**, "Which of the words in the word bank describes the information given by the highlighted words?" Students should choose "character" from the word bank. **Explain** that the character of a story problem is "doing" or receiving the action. In this case the *muffins* are being baked, which makes them the character.

Highlight the phrase "are being baked" in green. **Instruct** students to do the same on their INSTRUCTIONAL ACTIVITY SUPPLEMENT. **Ask**, "Which of the words in the word bank describes the information given by the highlighted words?" Students should choose "event/action" from the word bank.

Highlight the phrase "in the oven" in pink. **Instruct** students to do the same on their INSTRUCTIONAL ACTIVITY SUPPLEMENT. **Ask**, "Which of the words in the word bank describes the information given by the highlighted words?" Students should choose "setting" from the word bank.

Highlight the question "How many muffins will there be in total?" in orange. **Instruct** students to do the same on their INSTRUCTIONAL ACTIVITY SUPPLEMENT. **Ask**, "Which of the words in the word bank describes the information given by the highlighted words?" Students should choose "problem" from the word bank.

Direct students to use the organizer provided on the INSTRUCTIONAL ACTIVITY SUPPLEMENT to represent the situation with repeated addition, multiplication, repeated grouping, an array, and skip counting.

Distribute the INSTRUCTIONAL ACTIVITY HANDOUT for students to identify the components of each story problem and then to represent each situation with addition, multiplication, repeated grouping, an array, and skip counting.

> NOTE: At this point, students may still heavily rely on the manipulatives to help answer the questions. This strategy is acceptable for students to use at this point in the learning process; however, the ultimate goal is for students to create mental models and reason abstractly instead of relying on physical manifestations of the situation. Help students to develop these mental models through effective questioning.

Circulate and help students.

> NOTE: The first two problems on the INSTRUCTIONAL ACTIVITY HANDOUT are equal-grouping situations, which are now familiar to students. Therefore, all students should complete the first two problems. The subsequent three problem types illustrate multiplication in situations that students are not familiar with. The third problem is a comparison problem, which can be difficult for students to understand initially (Lannin, Chval, & Jones, 2015). Comparison-type problems are covered in 4.OA.1, 2. The last two problems types are area and combination. Typically, the majority of multiplication scenarios that third graders encounter are equal grouping with arrays and areas being introduced as well. All problem types are provided in the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT for students to gain an initial exposure to them and provide a challenge for advanced students.

GUIDING QUESTIONS

Elicit student thinking:

- [Point to a story problem.] What is the problem asking you to find?
- [Point to a story problem.] How is this problem similar to tree orchard problem? How is it different?

Determine if the student can DEMONSTRATE THE CONCEPT OF MULTIPLICATION:

- Are the rows of equal size?
- How many equal-size rows are there?

Determine if the student can MATHEMATIZE CONTEXTUAL SITUATIONS INVOLVING MULTIPLICATION:

- [Point to a number in the story problem.] Where is this represented in your array?
- [Point to a number in the story problem.] Where is this represented in your skip counting?
- [Point to a number in the story problem.] Where is this represented in your addition sentence?
- [Point to a number in the story problem.] Where is this represented in your multiplication sentence?
- [Point to a number in the story problem.] Where is this represented in your repeated grouping model?

Determine if the student is ready to EXPLAIN MULTIPLICATION PROBLEMS:

- How many equal-size groups are there?
- How many objects are in each group?
- Which factor should be written first in the multiplication sentence?

Students should be required to write a story problem that is solved using the multiplication sentence 5×2, and they should remember to use the highlighted elements of story problems.

At the end of the activity, teachers should review the students' story problems and identify examples that illustrate common errors listed in LESSON 3. If students exhibit mistakes not covered in LESSON 3, add those to the list to be covered. Additionally, choose a few well-written story problems to highlight for the class.

Name_____

Interpret and Represent Multiplication and Division in Context

Lesson 2

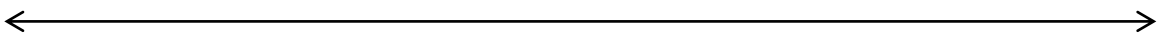
Three siblings—Alyssa, Molly, and Joseph—each picked five apples at the orchard. How many total apples did the siblings pick?

Identify each component of the story problem by filling in the following table with phrases from the story problem.

Character:	Action:
Setting:	Problem:

Name_____

Represent the scenario in the following ways.

Addition Sentence:	Multiplication Sentence:
Repeated grouping:	Array:
Skip Counting: 	

Name_____

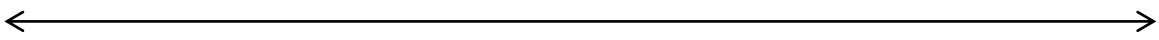
A peach orchard is being planted next door. There will be 10 rows, each containing seven peach trees. How many peach trees are being planted?

Identify each component of the story problem by filling in the following table with phrases from the story problem.

Character:	Action:
Setting:	Problem:

Name_____

Represent the scenario in the following ways.

Addition Sentence:	Multiplication Sentence:
Repeated grouping:	Array:
Skip Counting: 	

Name_____

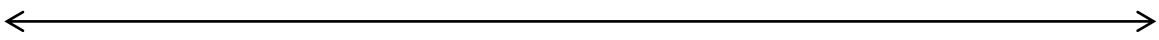
Kayla picked 12 apples at the apple orchard. Chelsea picked three times as many apples. How many apples did Chelsea pick?

Identify each component of the story problem by filling in the following table with phrases from the story problem.

Character:	Action:
Setting:	Problem:

Name_____

Represent the scenario in the following ways.

Addition Sentence:	Multiplication Sentence:
Repeated grouping:	Array:
Skip Counting: 	

Name_____

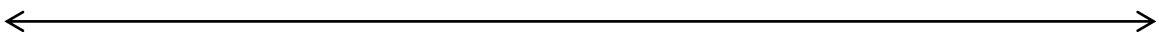
The dimensions of Jason's vegetable garden are 12 feet wide by five feet long. What is the total area of Jason's garden?

Identify each component of the story problem by filling in the following table with phrases from the story problem.

Character:	Action:
Setting:	Problem:

Name_____

Represent the scenario in the following ways.

Addition Sentence:	Multiplication Sentence:
Repeated grouping:	Array:
Skip Counting: 	

Name_____

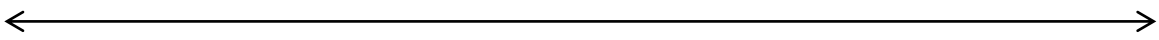
Dean wants to bake a pie with the fruit he picked. He can choose from four different fruits and two different piecrusts. If he only chooses one crust and one pie filling, how many different kinds of pie can he make?

Identify each component of the story problem by filling in the following table with phrases from the story problem.

Character:	Action:
Setting:	Problem:

Name_____

Represent the scenario in the following ways.

Addition Sentence:	Multiplication Sentence:
Repeated grouping:	Array:
Skip Counting: 	

Interpret and Represent Multiplication and Division in Context

INSTRUCTIONAL ACTIVITY SUPPLEMENT

Lesson 2

Three muffin tins are being baked in the oven. Each muffin tin contains 12 muffins. How many muffins will there be in total?

Word Bank:
Problem
Event/Action
Setting
Character

Addition Sentence:	Multiplication Sentence:
Repeated grouping:	Array:
Skip Counting: 	

Interpret and Represent Multiplication and Division in Context

INSTRUCTIONAL ACTIVITY

Lesson 3

LEARNING GOAL

Students will create story problems for a chosen multiplication expression to demonstrate their ability to interpret the meaning of multiplication.

PRIMARY ACTIVITY

Students examine the common errors made in their initial attempts at writing a story problem to represent 5×2. They will revise the 5×2 story problems to address any mistakes and then write a new story problem to represent a different multiplication expression. Students will provide the solution and use different representations to model the multiplication. Finally, students will peer revise their story problems while considering common errors identified at the beginning of the lesson.

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

- Multiplicand
- Multiplier
- Factor
- Product
- Array
- Skip counting
- Repeated grouping

MATERIALS

- Instructional Activity Supplement (Recommend one for each student.)
- Students' previously written story problems from Lesson 2

IMPLEMENTATION

Display a few examples of well-written story problems from LESSON 2 for students to read. At the same time, **distribute** the students' story problems back to them. Each student should receive their own story problem back.

Discuss a few aspects of the displayed story problems that make them good examples.

Conduct a guided class discussion of the common mistakes that are made when writing multiplication story problems. Students will examine their own story problems for any of the common mistakes during the class discussion.

Display this story problem: Jesse's five friends each have two cookies. How many cookies are there total?

Ask, "Does Jesse also have two cookies? Why is it important to be clear about how many people have cookies?" **Discuss** the importance of being specific and what errors can come from ambiguity.

Create a table of "Common Errors", their examples, and a corrected example on poster paper or on the board. Write the following common error next to the displayed story problem it refers to, and then lead the class through a discussion about how to revise the wording to create a correct example.

- Be clear about how many groups there are.

Display this story problem: Coach Jamie has a team of five players and another team of two players. How many total players are there?

Solve this problem by adding $5 + 2 = 7$. **Ask**, "Does this problem represent 5×2? Why or why not?" **Distinguish** between repeated addition, multiplication, and addition.

Add to the table of "Common Errors" and examples by writing the following common errors next to the displayed story problem they refer to, and then lead the class through a discussion about how to revise the wording to create a correct example.

- The question must require multiplication, not addition.
- The groups must be equally sized.

Display this story problem: Jill has two bags of apples. Each bag contains five pounds of apples. How many total pounds of apples does she have?

Solve this problem by multiplying $2 \times 5 = 10$. **Ask**, "How many groups are there? Is that the multiplier or the multiplicand?" **Discuss** the similarities and differences between 2×5 and 5×2.

Add to the table of "Common Errors" and examples by writing the following common error next to the displayed story problem it refers to, and then lead the class through a discussion about how to revise the wording to create a correct example.

- The first factor must represent the number of groups, and the second factor must represent the number of elements in each group.

Display this story problem: There are five groups of birds flying by. Each group has two birds. How many total birds are there?

Ask, "What do you call a group of birds?" **Discuss** the idea of being specific and using the word "flock" instead of "group".

Add to the table of "Common Errors" and examples by writing the following common error next to the displayed story problem it refers to, and then lead the class through a discussion about how to revise the wording to create a correct example.

- Use specific vocabulary (avoid the word "groups").

Display this story problem: Jane lines up her pet dogs in five rows. Each row has two dogs. How many total dogs does she have?

Ask, "How easy do you think it is to get dogs to line up? How many dogs do most people have? How realistic is this story problem?" **Discuss** the importance of having a realistic story problem.

Add to the table of "Common Errors" and examples by writing the following common error next to the displayed story problem it refers to, and then lead the class through a discussion about how to revise the wording to create a correct example.

- The story problem must be realistic.

Display this story problem: Jessa draws five rows of stars. Each row has two stars. How many total stars did she draw?

Ask, "How interesting is it to draw a star? Do you see a lot of people doing this? Have you ever done this?" **Discuss** the importance of having an interesting story problem.

Add to the table of "Common Errors" and examples by writing the following common error next to the displayed story problem it refers to, and then lead the class through a discussion about how to revise the wording to create a correct example.

- The story problem must be interesting.

Display this story problem: Josiah bakes 10 cookies. He made two batches of five cookies. How many total cookies did he bake?

Ask, "Do you see the answer to the problem in the story? Why is it inappropriate to include the total in the story problem?" **Discuss** that there should only be two given values in the story problem because the third value needs to be calculated.

Add to the table of "Common Errors" and examples by writing the following common error next to the displayed story problem it refers to, and then lead the class through a discussion about how to revise the wording to create a correct example.

- The story problem should only give the two factors, not the product.

Tell students first to examine their story problems for any of the listed errors and then to revise their story to fix any errors that they find. **Circulate** while students work and ask the following guiding questions.

GUIDING QUESTIONS

Elicit student thinking:

- Name a quality of a well-written story problem.
- Name a quality of a poorly or badly written story problem.

Determine if the student can EXPLAIN MULTIPLICATION PROBLEMS:

- [Point to a story problem.] Which factor represents the number of groups?
- [Point to a story problem.] Should there be five groups or two groups? Explain how you know.
- [Point to a story problem.] Which factor represents the size of each group?
- [Point to a story problem.] Should there be five elements in each group or two elements in each group? Explain how you know.

Determine if the student can EXPLAIN THE MEANING OF THE MULTIPLICATION SIGN:

- How did you represent the symbol "×" in your story problem?
- What does "×" mean?

Determine if the student can MATHEMATIZE CONTEXTUAL SITUATIONS INVOLVING MULTIPLICATION:

- Where is the multiplicand in your story problem?
- Where is the multiplicand in the expression?
- What information does the multiplicand represent?

Determine if the student is ready to REPRESENT MULTIPLICATIONS WITH EQUATIONS:

- How would you complete the following: $5 \times 2 = ?$

As students finish revising their first story problem, instruct them to write a new story problem for any multiplication sentence under 100. It should be a different context and different multiplication than their first story problem. Provide a multiplication sentence for students who struggle to get started.

> **Distribute** the INSTRUCTIONAL ACTIVITY SUPPLEMENT for them to write their new story problem on. On the back of the INSTRUCTIONAL ACTIVITY SUPPLEMENT, students should fill in the prompts to solve their story problem. Instruct students to only show work and solutions on the reverse side of the INSTRUCTIONAL ACTIVITY SUPPLEMENT so the front can be photocopied for a class set of story problems.

Students who finish writing their story problem should find a partner who is also finished and trade story problems in order to peer-revise them. This activity can be scaffolded by intentionally pairing struggling students with a mathematically strong peer.

GUIDING QUESTIONS

Elicit student thinking:

- Is your story problem clear about how many groups there are? Explicit? Specific?
- Does your story problem use specific vocabulary?
- Is your story problem realistic? Interesting?
- Is your story problem an addition problem or multiplication problem?

Determine if the student can MATHEMATIZE CONTEXTUAL SITUATIONS INVOLVING MULTIPLICATION:

- How is the multiplier represented in the situation?
- How is the multiplicand represented in the situation?

Determine if the student is ready to SOLVE EQUAL-GROUP PROBLEMS:

- How many equal-size groups are in your story problem?
- How many items are in each group in your story problem?

Students should be required to turn in their story problem on the INSTRUCTIONAL ACTIVITY SUPPLEMENT for teacher review.

At the end of the activity, teachers should review the story problems for accuracy.

Interpret and Represent Multiplication and Division in Context

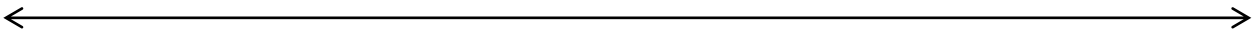
INSTRUCTIONAL ACTIVITY SUPPLEMENT

Lesson 3

Write your story problem here. Include a picture to illustrate the story problem. Do not include the solution on this page.

Represent your story problem with an array.

Represent your story problem with skip counting.



Represent your story problem with repeated grouping.

Represent your story problem with repeated addition.

Represent your story problem with a multiplication equation. Include the solution.

Interpret and Represent Multiplication and Division in Context

INSTRUCTIONAL ACTIVITY SUPPLEMENT

Lesson 3

Story-problem checklist:

☐ It is clear how many groups there are.

☐ The question requires multiplication, not addition.

☐ The groups are equally sized.

☐ The first number represents the number of groups, and the second number represents the size of each group.

☐ Specific vocabulary is used (avoided the word "groups").

☐ The story problem is realistic.

☐ The story problem is interesting.

☐ The story problem does not contain the answer.

Interpret and Represent Multiplication and Division in Context

INSTRUCTIONAL ACTIVITY

Lesson 4

LEARNING GOAL

Students will explore situations that can be modeled with division.

PRIMARY ACTIVITY

Students are introduced to division through a story problem. They will discuss the similarities and differences between multiplication and division situations and will represent the division with manipulatives and appropriate models. After practicing a few division word problems, students will attempt to write their own story problems to represent $10 \div 2$.

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

- Multiplicand
- Multiplier
- Product
- Quotient
- Dividend
- Divisor
- Array
- Skip counting
- Repeated grouping

MATERIALS

- Square tiles or counting manipulatives (Recommend 24 tiles or counters for each student.)
- Instructional Activity Student Handout

IMPLEMENTATION

Present the following situation to students: A parade unit has 24 members. The greatest number of rows that will fit on the parade route is eight. How many people will be in each row?

Distribute the manipulatives to each student. **Tell** students to model the situation with their manipulatives in order to answer the question.

Provide a visual of the first row of eight people to jump start students' thinking if they are struggling.

Look for student strategies to include "dealing" out the manipulatives into each row, which develops into dealing out by larger quantities (e.g., two or three counters at a time), drawing groups of eight, and repeated subtraction of 8 from 24.

Lead a class discussion to determine the number of people in each row and identify student strategies for solving.

Introduce the connection between the multiplication story problems from LESSONS 1-3 and this new situation. The multiplication story problems gave two factors and required finding the product. However, in this new scenario, we know the total product, but we are missing one of the factors.

Remind students that previously they multiplied *number of groups* × *size of each group* = *total number of items*. The parade situation could be represented by substituting in the known information $8 \times ? = 24$.

Introduce the notation $24 \div 8 = 3$ as the notation used when a factor is unknown in a multiplication situation. Tell students that this is a division problem and it is a rearrangement of the multiplication sentence with an unknown factor. **Define** the value 24 as the *dividend*, or the number being divided. **Define** the value 8 as the *divisor*, or the number dividing 24. It could either be the size of one group, or the number of equal-size groups, depending on the problem situation.

Ask students which values were given to them in the parade scenario. They should answer that they knew the number of groups and the total number of people.

Ask students which value was missing. They should answer that the missing value is the number of people in each group.

Write *total number* ÷ *number of groups* = *number in each group*. **Conclude** that there should be three parade members in each group (row).

Ask students what similarities they notice between division and multiplication. Discuss as a class.

Ask students what differences they notice between division and multiplication. Discuss as a class.

Present the following situation to students: A parade unit has 24 members. They want to have six people in each row. How many rows will they need?

Distribute the manipulatives to each student. **Tell** students to model the situation with their manipulatives in order to answer the question.

Call on two to three students to show and explain their thought process with the manipulatives to the class.

Model for students or **call on** student volunteers to show how to skip count backwards on a number line, draw an array, draw a bar diagram, or draw repeated grouping to determine the solution.

Ask, "What two piece of information were you given in the story problem?" Students should answer that they were given the total number and the size of each group.

Write *total number* ÷ *size of groups* = *number of groups* (24 ÷ 6 = 4).

GUIDING QUESTIONS

Elicit student thinking:

- What are the similarities between multiplication and division?
- What are the differences between multiplication and division?

Determine if the student can DEMONSTRATE THE CONCEPT OF DIVISION:

- Show me how to divide 24 counters into eight groups.

Determine if the student can EXPLAIN DIVISION PROBLEMS:

- How do you know how many equal-size groups to create?
- What do you know about the size of all the groups?

Determine if the student can EXPLAIN *QUOTIENT*:

- What do you call the answer to the problem?
- What does the quotient represent?

Determine if the student can EXPLAIN *DIVIDEND*:

- What do you call the total number of people?
- What does the dividend represent?

Determine if the student can EXPLAIN *DIVISOR*:

- What does the divisor represent?

Determine if the student is ready to EXPLAIN THE MEANING OF THE DIVISION SIGN:

- What does ÷ mean?

Distribute the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT, which requires students to analyze and solve two more division problems.

Encourage students to use their physical manipulatives while they reason through the division problems, and to create models such as number lines, bar diagrams, and arrays to show their thinking.

Circulate while students work, and **support** student understanding with the following guiding questions.

GUIDING QUESTIONS

Elicit student thinking:

- How do you know to use division to solve this problem?

Determine if the student can MATHEMATIZE CONTEXTUAL SITUATIONS INVOLVING DIVISION:

- How is the divisor represented in the situation?
- How is the dividend represented in the situation?

Determine if the student is ready to SOLVE EQUAL-GROUP PROBLEMS:

- [Point to the first word problem.] Should all guests get the same number of party favors/candy?
- [Point to the second word problem.] What must be true of all the students' ribbon lengths?

Students should be required to write a story problem to represent $10 \div 2$.

At the end of the activity, teachers should review the students' story problems and identify examples that illustrate common errors listed in LESSON 5. If students exhibit mistakes not covered in LESSON 5, add those mistakes to the list to be covered. Additionally, choose a few well-written story problems to highlight for the class.

Name_____

Interpret and Represent Multiplication and Division in Context

Lesson 4

You have 56 pieces of candy to hand out as favors at your party, and you want to give each person four pieces of candy.

1. What is the *divisor* in this situation? How do you know?

2. What is the *dividend* in this situation? How do you know?

3. Write the division expression that represents this situation.

4. Tell how many people can get party favors. Explain your reasoning with words or a model.

5. Write the division equation with the quotient.

Name_____

Mr. Zimmerman needs to distribute equal-size pieces of ribbon to his students for a class project. He bought a ribbon that is 36 inches long. If there are 18 students in his class, what length of ribbon will each student get?

1. What is the *divisor* in this situation? How do you know?

2. What is the *dividend* in this situation? How do you know?

3. Write the division expression that represents this situation.

4. Tell what length of ribbon each student receives. Explain your reasoning with words or a model.

5. Write the division equation with the quotient.

Interpret and Represent Multiplication and Division in Context

INSTRUCTIONAL ACTIVITY

Lesson 5

LEARNING GOAL

Students will create story problems for a given division expression to demonstrate their ability to interpret the meaning of division.

PRIMARY ACTIVITY

Students examine the common errors made in their initial attempts at writing a story problem to represent $10 \div 2$. They will revise their $10 \div 2$ story problems to address any mistakes and then write a new story problem to represent a different division expression. Students will provide the solution and use different representations to model the division. Finally, students will peer-revise their story problems while considering the common errors identified at the beginning of the lesson.

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

- Dividend
- Divisor
- Quotient
- Array
- Skip counting
- Repeated grouping

MATERIALS

- Instructional Activity Supplement (Recommend one for each student.)

IMPLEMENTATION

Display a few examples of well-written story problems from LESSON 4 for students to read. At the same time, **distribute** the students' story problems back to them. Each student should receive their own story problem back.

Discuss a few aspects of the displayed story problems that make them good examples.

Conduct a guided class discussion of the common mistakes that are made when writing division story problems. Students will examine their own story problems for any of the common mistakes during the class discussion.

Display this story problem: Jesse has 10 cookies that he wants to share with five friends. How many cookies does each person get?

Ask, "Does Jesse get any cookies? Why is it important to be clear about how many people are sharing cookies?" **Discuss** the importance of being specific and what errors can come from ambiguity.

Create a table of "Common Errors", their examples, and a corrected example on poster paper or on the board. **Write** the following common error next to the displayed story problem it refers to, and then **lead** the class through a discussion about how to revise the wording to create a correct example.

- Be clear about how many groups there are.

Display this story problem: Coach Jamie has a team of 10 players and two players leave. How many total players are there?

Solve this problem by subtracting $10 - 2 = 8$. **Ask**, "Does this problem represent $10 \div 2$? Why or why not?" **Distinguish** between repeated subtraction, division, and subtraction.

Add to the table of "Common Errors" and examples by writing the following common error next to the displayed story problem it refers to, and then **lead** the class through a discussion about how to revise the wording to create a correct example.

- The question must require division, not subtraction.

Display this story problem: Jill has two cakes that she wants to share equally with five people. How much cake does everyone get?

Solve this problem by dividing $2 \div 10 = \frac{1}{5}$. **Ask**, "What is the total number of cakes? Is that the dividend or the divisor?" **Discuss** the similarities and differences between $2 \div 10$ and $10 \div 2$.

Add to the table of "Common Errors" and examples by writing the following common error next to the displayed story problem it refers to, and then **lead** the class through a discussion about how to revise the wording to create a correct example.

- The first number must represent the total number of items and the second number must represent either the number of elements in each group or the number of groups.

Display this story problem: Jill has two cakes that she wants to share with five people. How much cake does everyone get?

Solve this problem by partitioning two circles that represent cakes into five unequal shares. **Ask**, "Does this answer the question as written? Does division partition something into unequal groups?" **Discuss** the importance of stating that there are equal-size partitions.

Add to the table of "Common Errors" and examples by writing the following common error next to the displayed story problem it refers to, and then **lead** the class through a discussion about how to revise the wording to create a correct example.

- The size of the groups must be equal.

Display this story problem: There are 10 birds, and they fly in two equal groups. How many birds are in each group?

Ask, "What do you call a group of birds?" **Discuss** the idea of being specific and using the word "flock" instead of "group".

Add to the table of "Common Errors" and examples by writing the following common error next to the displayed story problem it refers to, and then **lead** the class through a discussion about how to revise the wording to create a correct example.

- Use specific vocabulary (avoid the word "groups").

Display this story problem: Jane lines up her 10 dogs in five rows. How many dogs are in each row?

Ask, "How easy do you think it is to get dogs to line up? How many dogs do most people have? How realistic is this story problem?" **Discuss** the importance of having a realistic story problem.

Add to the table of "Common Errors" and examples by writing the following common error next to the displayed story problem it refers to, and then **lead** the class through a discussion about how to revise the wording to create a correct example.

- The story problem must be realistic.

Display this story problem: Jessa draws 10 stars in two rows. How many stars are in each row?

Ask, "How interesting is it to draw a star? Do you see a lot of people doing this? Have you ever done this?" **Discuss** the importance of having an interesting story problem.

Add to the table of "Common Errors" and examples by writing the following common error next to the displayed story problem it refers to, and then **lead** the class through a discussion about how to revise the wording to create a correct example.

- The story problem must be interesting.

Display this story problem: Josiah bakes 10 cookies. He made two batches of five cookies. How many total cookies did he bake?

Ask, "Do you see the answer to the problem in the story? Why is it inappropriate to include the number of batches or the number of cookies per batch in the story problem?" **Discuss** that there should only be two given values in the story problem because the third value needs to be calculated.

Add to the table of "Common Errors" and examples by writing the following common error next to the displayed story problem it refers to, and then **lead** the class through a discussion about how to revise the wording to create a correct example.

- The story problem should only give the total and the number of groups, OR
- The story problem should only give the total and the size of each group.
- The story problem should not include the quotient.

Tell students to examine their story problems for any of the listed errors and to revise their stories to fix any errors they find. **Circulate** while students work and **ask** the following guiding questions.

GUIDING QUESTIONS

Elicit student thinking:

- Name a quality of a well-written story problem.
- Name a quality of a poorly written story problem.

Determine if the student can EXPLAIN DIVISION PROBLEMS:

- [Point to a story problem.] What are we doing with the 10 total objects?
- [Point to a story problem.] What do we know about the size of the groups we are creating?

Determine if the student can EXPLAIN THE MEANING OF THE DIVISION SIGN:

- How did you represent the symbol "÷" in your story problem?
- What does "÷" mean?

Determine if the student can MATHEMATIZE CONTEXTUAL SITUATIONS INVOLVING DIVISION:

- Where is the divisor in your story problem?
- Where is the divisor in the expression?
- What information does the divisor represent?

Determine if the student is ready to REPRESENT DIVISION WITH EQUATIONS:

- How would you complete the following: $10 \div 2 = ?$

As students finish revising their first division story problem, instruct them to write a new story problem for any division sentence under 100. It should be a different context and different division than their first story problem. Provide a multiplication sentence for students who struggle to get started.

> **Distribute** the INSTRUCTIONAL ACTIVITY SUPPLEMENT for them to write their new story problem on. On the back of the INSTRUCTIONAL ACTIVITY SUPPLEMENT, students should fill in the prompts to solve their story problem. Instruct students to only show work and solutions on the reverse side of the INSTRUCTIONAL ACTIVITY SUPPLEMENT so the front can be photocopied for the class book.

Students who finish writing their story problems should find a partner who is also finished and trade story problems in order to peer-revise them. This activity can be scaffolded by intentionally pairing struggling students with a mathematically strong peer.

> NOTE: Students will likely write equal-group-type division problems and may need to be prompted to attempt an array problem.

GUIDING QUESTIONS

Elicit student thinking:

- Is your story problem clear about how many groups there are?
- Does your story problem use specific vocabulary?
- Is your story problem realistic? Is it interesting?
- Is your story problem a subtraction or division problem?

Determine if the student can MATHEMATIZE CONTEXTUAL SITUATIONS INVOLVING DIVISION:

- How is the dividend depicted in the situation?
- How is the divisor depicted in the situation?

Determine if the student is ready to SOLVE EQUAL-GROUP PROBLEMS:

- How many groups are in your story problem?
- What's the total number of items in your story problem?

Determine if the student is ready to SOLVE ARRAY PROBLEMS:

- Could you think of your story problem as an array? Explain.
- How could you rewrite your story problem to be an array?

At the end of the activity, teachers should collect and review the story problems for accuracy.

Make photocopies of the students' multiplication and division story problems and create a packet or book to be used in LESSON 6.

Suggestions for the book creation include:

- One book per pair of students
- One book per student
- One set of index cards with one story problem printed for each card
- A digital book on a class website or using presentation software

Interpret and Represent Multiplication and Division in Context

INSTRUCTIONAL ACTIVITY SUPPLEMENT

Lesson 5

Write your story problem here. Don't include the solution on this page—only provide the answer on the reverse side.

Represent your story problem with an array.

Represent your story problem with skip counting.

Represent your story problem with repeated grouping.

Represent your story problem with a division equation. Show the quotient.

Story problem checklist:

☐ It is clear how many groups there are.

☐ The question requires division, not subtraction.

☐ The groups are equally sized.

☐ The first number represents the total number of items, and the second number represents either the number of groups or the size of each group.

☐ Specific vocabulary is used (avoided the word "groups").

☐ The story problem is realistic.

☐ The story problem is interesting.

☐ The story problem does not contain the answer.

Interpret and Represent Multiplication and Division in Context

INSTRUCTIONAL ACTIVITY

Lesson 6

LEARNING GOAL

Students will solve multiplication and division story problems.

PRIMARY ACTIVITY

Students read through the story problems written by their peers and choose five to solve with a model for mixed practice of division and multiplication.

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

- Factor
- Product
- Multiplier
- Multiplicand
- Quotient
- Divisor
- Dividend
- Array
- Skip counting
- Repeated grouping
- Equation
- Expression

MATERIALS

- Instructional Activity Supplement (Recommend one copy for each student.)

IMPLEMENTATION

Describe the chosen activity for students. Depending on the format of the collection of student-written problems, students could trade index cards, work in teams during a PowerPoint review, or choose story problems to solve from a book. In general, tell students to avoid solving the problems they wrote themselves. For each story problem, they should complete the following tasks on the INSTRUCTIONAL ACTIVITY SUPPLEMENT:

- Draw an array.
- Draw equal grouping.
- Draw skip counting on a number line.
- Draw a picture illustrating the scenario.
- Write the correct multiplication or division equation.

Circulate and help students as they solve the story problems. Anticipate students to struggle with determining which operation to use.

Tell students who finish the first five index cards on their INSTRUCTIONAL ACTIVITY SUPPLEMENT to continue solving the story problems on their own sheet of paper. They may choose the best model to represent the situation instead of drawing all of them, or they may choose to simply solve the story problem mentally and only write the equation representing the situation. To extend student understanding, encourage them to solve in more advanced or abstract ways than they previously did. For example, suggest solving without manipulatives or without a model, understanding that some students may still be working with manipulatives and encouraging them to transfer the concrete awareness to semi-concrete awareness.

GUIDING QUESTIONS

Elicit student thinking:

- How do you know which operation to use?
- [Point to a multiplication story problem.] What is the multiplier and multiplicand?
- [Point to a division story problem.] What is the dividend and divisor?

Determine if the student can REPRESENT DIVISION WITH EQUATIONS:

- [Point to a division story problem.] How do you write the equation representing this situation?

Determine if the student can REPRESENT MULTIPLICATION WITH EQUATIONS:

- [Point to a multiplication story problem.] How do you write the equation representing this situation?

Determine if the student can SOLVE ARRAY PROBLEMS:

- [Point to an array problem.] How does an array represent this problem?
- [Point to an array problem.] How do you know that this is an array problem?

Determine if the student can SOLVE EQUAL-GROUP PROBLEMS:

- [Point to an equal group problem.] How do equal groups represent this problem?
- [Point to an equal group problem.] How do you know that this is an equal-group problem?

Determine if the student can SOLVE MEASUREMENT-QUANTITIES PROBLEMS:

- [Point to a measurement-quantity problem.] How does a measurement quantity represent this problem?
- [Point to a measurement-quantity problem.] How do you know that this is a measurement-quantity problem?

At the end of the activity, teachers should collect and review the INSTRUCTIONAL ACTIVITY SUPPLEMENT to gauge how well students can interpret multiplication and division in context.

Interpret and Represent Multiplication and Division in Context

INSTRUCTIONAL ACTIVITY SUPPLEMENT

Lesson 6

Problem #1

Represent the story problem with an array.

Represent the story problem with repeated grouping.

Represent the story problem with skip counting.

Represent the story problem with an illustration.

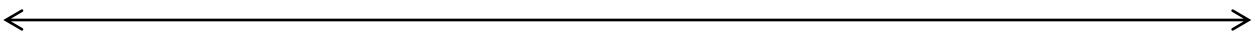
Represent the story problem with an equation. Include the answer.

Problem #2

Represent the story problem with an array.

Represent the story problem with repeated grouping.

Represent the story problem with skip counting.



Represent the story problem with an illustration.

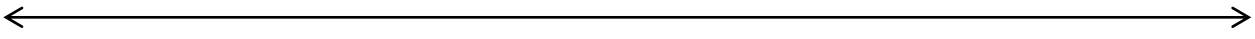
Represent the story problem with an equation. Include the answer.

Problem #3

Represent the story problem with an array.

Represent the story problem with repeated grouping.

Represent the story problem with skip counting.



Represent the story problem with an illustration.

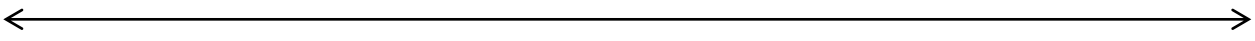
Represent the story problem with an equation. Include the answer.

Problem #4

Represent the story problem with an array.

Represent the story problem with repeated grouping.

Represent the story problem with skip counting.



Represent the story problem with an illustration.

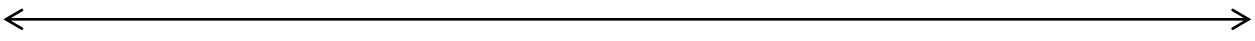
Represent the story problem with an equation. Include the answer.

Problem #5

Represent the story problem with an array.

Represent the story problem with repeated grouping.

Represent the story problem with skip counting.



Represent the story problem with an illustration.

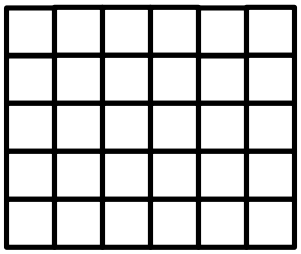
Represent the story problem with an equation. Include the answer.

Name_____

Interpret and Represent Multiplication and Division in Context

Lesson 1-6

1. Consider the following array:



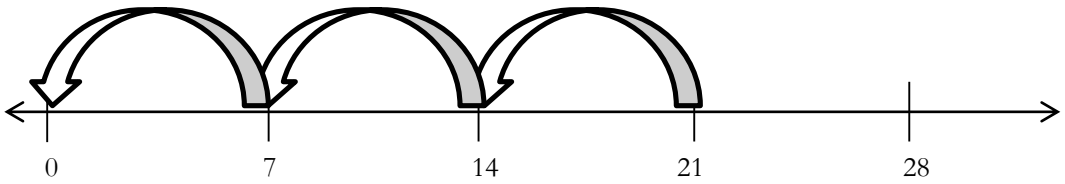
1.a. Write the multiplication equation that the array represents.

1.b. The number of groups in a multiplication problem is called the multiplier. What is the multiplier in this scenario?

1.c. The group size in a multiplication problem is called the multiplicand. What is the multiplicand in this scenario?

Name_____

2. Consider the following number line:



2.a. Write the division equation that the number line represents.

2.b. How many groups does the number line represent?

2.c. What dividend is represented by the number line?

Name_____

3. Four students each bring three bags of candy to the class Halloween party. How many total bags of candy are there? Represent this scenario with the following models.

3.a. An array

3.b. Repeated grouping

3.c. Symbolic form

Name_____

4. Write a story problem to represent 6×2.

Name_____

5. A music teacher has 45 feet of ribbon. She wants to cut it into nine-foot sections for the Maypole dance. How many sections will there be? Represent this scenario with the following models.

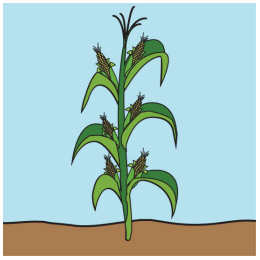
5.a. Repeated subtraction on a number line

5.b. An array

5.c. Symbolic form

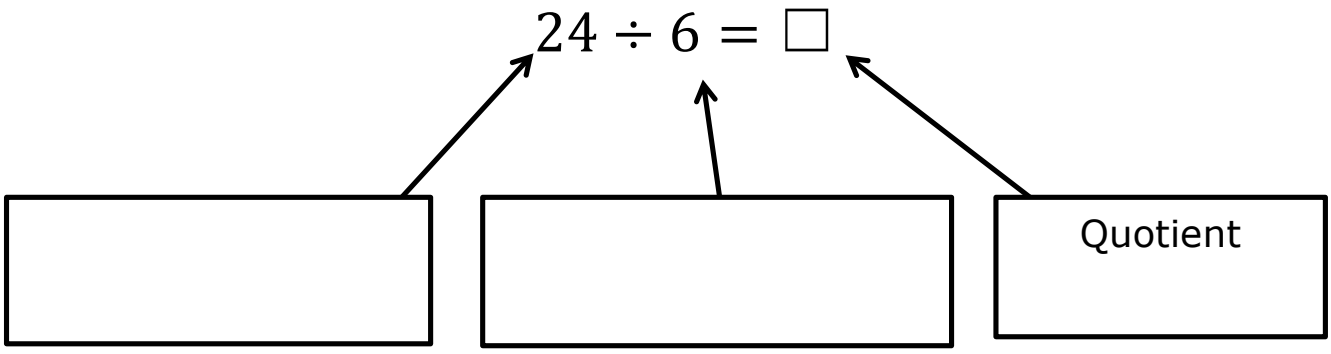
Name_____

6. Mr. McGregor is planting his garden. He has 24 vegetable plants and wants to plant them in four rows.



6.a. How many vegetable plants will be in each row?

6.b. Label and fill in all the missing components of the division problem representing this situation.

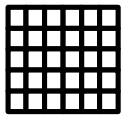


6.c. Mr. McGregor realized that his garden does not have room for six rows. Draw and explain another rectangular arrangement besides six rows for planting his 24 vegetables.

INTERPRET AND REPRESENT MULTIPLICATION AND DIVISION IN CONTEXT

STUDENT ACTIVITY SOLUTION GUIDE

Lesson 1-6

1. Consider the following array: 

1.a. Write the multiplication equation that the array represents.

CORRECT ANSWER

The array represents $5 \times 6 = 30$.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The array represents $6 \times 5 = 30$.	confuses the rows and columns, switches the multiplicand with the multiplier	EXPLAIN MULTIPLICATION PROBLEMS, DETERMINE PRODUCT OF TWO FACTORS USING AN ARRAY
The array represents 30.	only gives the product, or total, which does not indicate knowledge of multiplication	EXPLAIN MULTIPLICATION PROBLEMS, REPRESENT MULTIPLICATION WITH EQUATIONS, DETERMINE PRODUCT OF TWO FACTORS USING AN ARRAY
The array represents $6 + 6 + 6 + 6 + 6 = 30$.	shows additive reasoning instead of multiplicative	DETERMINE PRODUCT OF TWO FACTORS USING AN ARRAY, REPRESENT MULTIPLICATION WITH EQUATIONS, EXPLAIN MULTIPLICATION PROBLEMS
The array represents five rows and six columns.	interprets the dimensions of the array, but does not write a multiplication sentence	REPRESENT MULTIPLICATION WITH EQUATIONS

1.b. The number of groups in a multiplication problem is called the multiplier. What is the multiplier in this scenario?

CORRECT ANSWER

> The multiplier in this scenario is five.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The multiplier in this scenario is six.	confuses the multiplier with the multiplicand	EXPLAIN MULTIPLICATION PROBLEMS
The multiplier is 30.	confuses the multiplier with the product	EXPLAIN *FACTOR*, EXPLAIN *PRODUCT*, EXPLAIN MULTIPLICATION PROBLEMS
The multiplier is this symbol: ×.	thinks that the multiplication symbol is called the "multiplier"	EXPLAIN THE MEANING OF THE MULTIPLICATION SIGN

1.c. The group size in a multiplication problem is called the multiplicand. What is the multiplicand in this scenario?

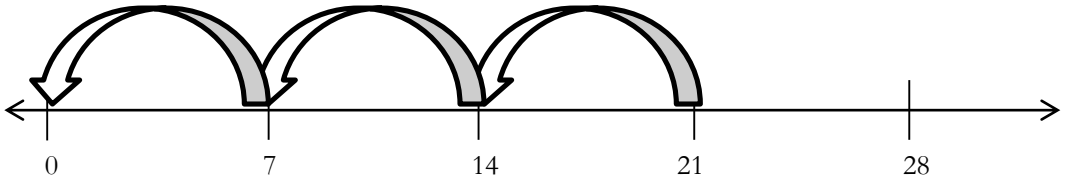
CORRECT ANSWER

> The multiplicand in this scenario is six.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The mutliplicand is five.	confuses the multiplier with the multiplicand	EXPLAIN MULTIPLICATION PROBLEMS
The multiplicand is 30.	confuses the multiplicand with the product	EXPLAIN *FACTOR*, EXPLAIN *PRODUCT*, EXPLAIN MULTIPLICATION PROBLEMS
The multiplier is this symbol: ×.	thinks that the multiplication symbol is called the "multiplicand"	EXPLAIN THE MEANING OF THE MULTIPLICATION SIGN

2. Consider the following number line:



2.a. Write the division equation that the number line represents.

CORRECT ANSWER

> The number line represents $21 \div 7 = 3$.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The number line represents $21 \div 3 = 7$.	confuses the size of each part with the number of parts	EXPLAIN DIVISION PROBLEMS, REPRESENT DIVISION WITH EQUATIONS
The number line represents 3.	does not write a division equation, only writes one of the numbers that they recognize from the number line	REPRESENT DIVISION WITH EQUATIONS
Student draws a model.	cannot write the division in symbolic notation	REPRESENT DIVISION WITH EQUATIONS
$21 - 7 - 7 - 7 = 0$	writes a subtraction equation instead of a division equation	REPRESENT DIVISION WITH EQUATIONS, EXPLAIN DIVISION PROBLEMS

2.b. How many groups does the number line represent?

CORRECT ANSWER

> The number line represents three groups.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The array represents seven groups.	confuses the number of groups with the size of each group	EXPLAIN DIVISION PROBLEMS
The number line represents 21 groups.	confuses the total with the number of groups	EXPLAIN DIVISION PROBLEMS

2.c. What total is represented by the number line?

CORRECT ANSWER

> The total represented by the number line is 21.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

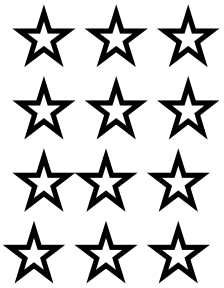
Example Error	Misconception	Missing Knowledge
The total represented by the number line is 28.	chooses the greatest value shown on the number line instead of the total	REPRESENT REPEATED SUBTRACTION WITH A MODEL
The total represented by the number line cannot be found.	cannot interpret the starting value of the repeated subtraction as the total	REPRESENT REPEATED SUBTRACTION WITH A MODEL

3. Four students each bring three bags of candy to the class Halloween party. How many total bags of candy are there? Represent this scenario with the following models.

3.a. An array

CORRECT ANSWER

Check student work for understanding and ensure that the array has four rows and three columns. The following is an example of a possible response:



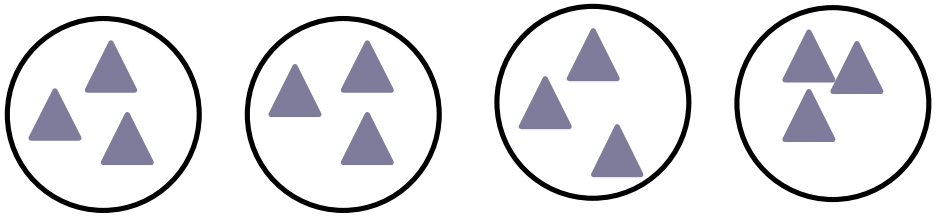
ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Draws an array with three rows of four.	confuses the multiplier and the multiplicand	EXPLAIN MULTIPLICATION PROBLEMS
Draws 12 objects with an incorrect pattern or no pattern.	does not know what an array is	DETERMINE THE PRODUCT OF TWO FACTORS USING AN ARRAY

3.b. Repeated grouping

CORRECT ANSWER

Check student work for understanding and ensure that there are four sets with three elements in each set. The following is an example of a possible response:



ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Draws three groups of four.	confuses the number of groups and the size of each group	REPRESENT REPEATED ADDITION WITH MODELS
Draws 12 objects with no grouping or pattern.	does not know how to represent equal groups	REPRESENT REPEATED ADDITION WITH MODELS

3.c. Symbolic form

CORRECT ANSWER

$4 \times 3 = 12$.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Student draws a model.	does not know what symbolic form means	REPRESENT MULTIPLICATION WITH EQUATIONS
$3 \times 4 = 12$	confuses the number of groups with the size of each group	REPRESENT MULTIPLICATION WITH EQUATIONS, EXPLAIN MULTIPICATION PROBLEMS
$12 \div 4 = 3$	writes a division equation instead of a multiplication equation	REPRESENT MULTIPLICATION WITH EQUATIONS

4. Write a story problem to represent 6×2.

CORRECT ANSWER

Answers will vary. The following is an example of a possible response:

There are six cheetahs looking at a herd of antelope. Each cheetah has two eyes. How many eyes are looking at the antelope?

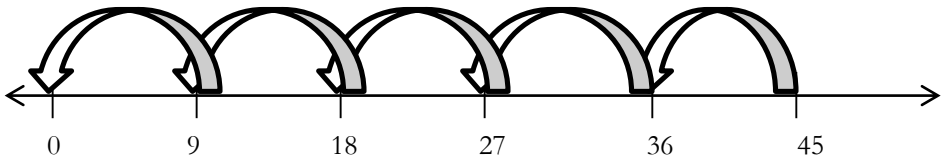
ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Student writes a story representing 2 × 6.	confuses the number of groups with the size of each group	MATHEMATIZE CONTEXTUAL SITUATIONS INVOLVING MULTIPLICATION, EXPLAIN MULTIPLICATION PROBLEMS
Student writes a story representing an addition problem.	does not exhibit multiplicative reasoning	MATHEMATIZE CONTEXTUAL SITUATIONS INVOLVING MULTIPLICATION
Student ncludes the answer/product (12) in the story with the two factors.	cannot identify which information should be included in the story problem and which information is left to be calculated	MATHEMATIZE CONTEXTUAL SITUATIONS INVOLVING MULTIPLICATION
Student finds the product (12) and writes a mutliplication story problem that includes the product and one of the factors.	cannot identify which information should be included in the story problem and which information is left to be calculated	MATHEMATIZE CONTEXTUAL SITUATIONS INVOLVING MULTIPLICATION
Student finds the product (12) and writes a division story problem that includes the product and one of the factors.	cannot identify which information should be included in the story problem and which information is left to be calculated	MATHEMATIZE CONTEXTUAL SITUATIONS INVOLVING MULTIPLICATION

5. A music teacher has 45 feet of ribbon. She wants to cut it into nine-foot sections for the Maypole dance. How many sections will there be? Represent this scenario with the following models.

5.a. Repeated subtraction on a number line

CORRECT ANSWER

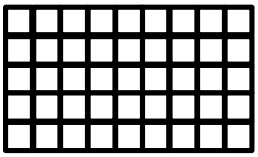


ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Student draws a number line with nine jumps of size five.	confuses the number of groups with the size of each group	DEMONSTRATE CONCEPT OF DIVISION, EXPLAIN DIVISION PROBLEMS, SOLVE MEASUREMENT-QUANTITIES PROBLEMS
Student shows arithmetic instead of a number line.	does not know how to represent skip counting on a number line	DEMONSTRATE CONCEPT OF DIVISION, EXPLAIN DIVISION PROBLEMS, SOLVE MEASUREMENT-QUANTITIES PROBLEMS

5.b. An array

CORRECT ANSWER



ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Student draws an array with nine rows and five columns.	confuses the number of groups with the size of each group, switches the rows and columns	SOLVE ARRAY PROBLEMS, REPRESENT REPEATED SUBTRACTION WITH A MODEL
Student draws repeated grouping or other model.	does not understand what an array is	SOLVE ARRAY PROBLEMS

5.c. Symbolic form

CORRECT ANSWER

$45 \div 5 = 9$

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
$5 \times 9 = 45$	confuses what information is given and what information is calculated	REPRESENT DIVISION WITH EQUATIONS
$45 \div 9 = 5$	switches the divisor and the quotient, mistakes the given information with the calculated information	EXPLAIN DIVISION PROBLEMS
$9 \div 5$ or $5 \div 9$	does not include the total ribbon length	EXPLAIN DIVISION PROBLEMS, REPRESENT DIVISION WITH EQUATIONS
Student draws a model or other illustration.	does not know what symbolic form is	REPRESENT DIVISION WITH EQUATIONS

6. Mr. McGregor is planting his garden. He has 24 vegetable plants and wants to plant them in four rows.

6.a. How many vegetable plants will be in each row?

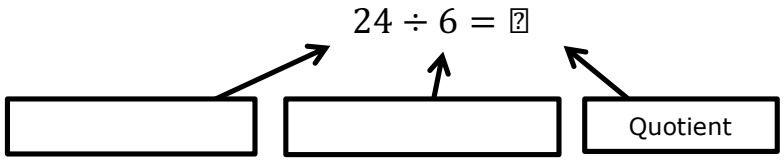
CORRECT ANSWER

There will be six plants in each row.

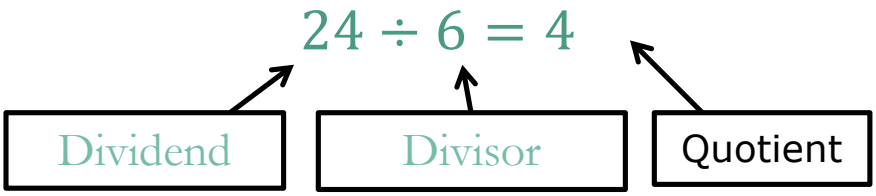
ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
There will be four plants in each row.	does not pay attention to detail, repeats the information given	DETERMINE THE UNKNOWN IN A DIVISION EQUATION
There will be 24 plants in each row.	does not focus on the size of the individual rows, believes the question is asking how many total plants there are	DETERMINE THE UNKNOWN IN A DIVISION EQUATION

6.b. Label and fill in all the missing components of the division problem representing this situation.



CORRECT ANSWER



ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

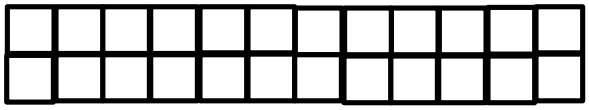
Example Error	Misconception	Missing Knowledge
Student switches the dividend and divisor.	does not know the definition of dividend or divisor	EXPLAIN DIVIDEND, EXPLAIN DIVISOR
Student gives the quotient as the same incorrect answer from part (a).	cannot find the missing factor of 24	DETERMINE THE UNKNOWN IN A DIVISION EQUATION

6.c. Mr. McGregor realized that his garden does not have room for six rows. Draw and explain another rectangular arrangement besides six rows for planting his 24 vegetables.

CORRECT ANSWER

Check student work for understanding. The following is an example of a possible response:

Mr. McGregor could plant his vegetables in two rows, with twelve vegetable plants in each row.



ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Student draws an figure with unequal rows or columns.	does not understand that an array must have equal-size rows and columns	SOLVE ARRAY PROBLEMS
Student draws an array that does not show a total of 24.	does not demonstrate understanding that the total area is the same but the dimensions are different	SOLVE ARRAY PROBLEMS
Student draws the an array with four rows and six columns.	does not find different dimensions/factors of the array	SOLVE ARRAY PROBLEMS