COMPOSING AND DECOMPOSING TO DETERMINE AREA

6.GM.A.1

CONTENTS

The types of documents contained in the unit are listed below. Throughout the unit, the documents are arranged by lesson.

LEARNING MAP INFORMATION	An overview of the standards, the learning map section, and the nodes addressed in this unit
TEACHER NOTES	A brief discussion describing the progression depicted in the learning map section with research-based recommendations for focusing instruction to foster student learning and an introduction to the unit's lessons
OVERVIEW OF INSTRUCTIONAL ACTIVITIES	A table highlighting the lesson goals and nodes addressed in each lesson of this unit
INSTRUCTIONAL ACTIVITY	A detailed walkthrough of the unit
INSTRUCTIONAL ACTIVITY STUDENT HANDOUT	A handout for the guided activity, intended to be paired with the Instructional Activity
INSTRUCTIONAL ACTIVITY SUPPLEMENT	A collection of materials or activities related to the Instructional Activity
STUDENT ACTIVITY	A work-alone activity for students
STUDENT ACTIVITY SOLUTION GUIDE	A solution guide for the work-alone activity with example errors, misconceptions, and links to the learning map section

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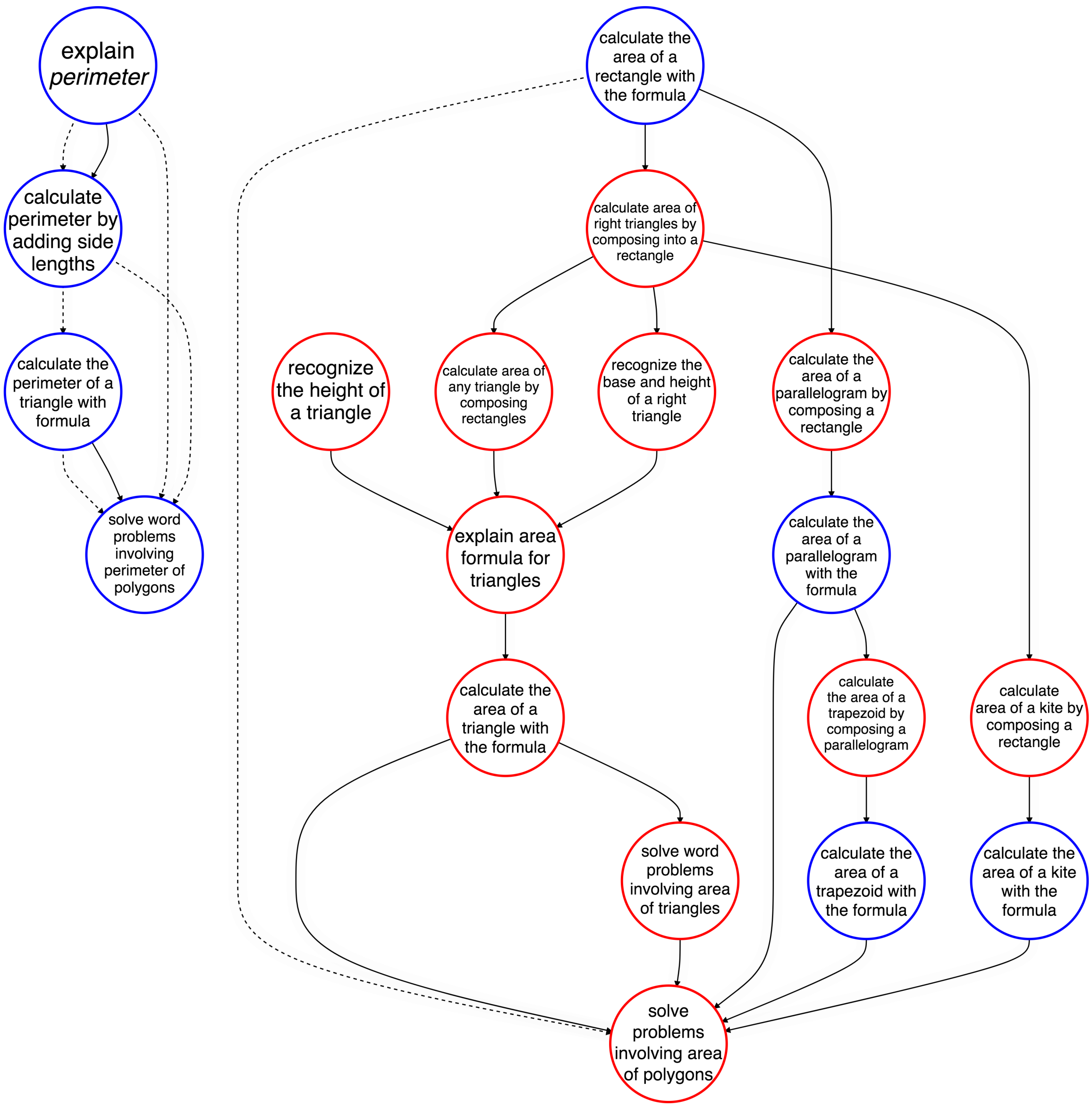
COMPOSING AND DECOMPOSING TO DETERMINE AREA

LEARNING MAP INFORMATION

STANDARDS

6.GM.A.1 Find the area of polygons by composing or decomposing the shapes into rectangles or triangles.



**Learning map model of* 6.G.1

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Node Name	Node Description
CALCULATE AREA OF A KITE BY COMPOSING RECTANGLES	Compose a kite into a rectangle to determine the area of the kite. One method is to construct the diagonals, bisect the kite, and reposition the four interior right triangles to form a rectangle. Determine the area of the rectangle, and show that the area of the original kite equals the area of the rectangle. Another method is to construct the diagonals of the kite and duplicate each of the interior right triangles to form a larger rectangle around the outside the kite. Determine the area of the larger rectangle, and show that the area of the original kite is half the area of the larger rectangle.
CALCULATE AREA OF ANY TRIANGLE BY COMPOSING RECTANGLES	Compose a triangle into two rectangles, one on each side of the altitude of the original triangle. This results in a larger rectangle that is exactly twice the size of the original triangle. Determine the area of the larger rectangle and show that the area of the original triangle is half the area of the larger rectangle.
CALCULATE PERIMETER BY ADDING SIDE LENGTHS	Calculate the perimeter of any polygon by determining the sum of the side lengths.
CALCULATE AREA OF RIGHT TRIANGLES BY COMPOSING INTO RECTANGLE	Compose a right triangle into a rectangle, and find the area of the triangle by calculating one half the area of the rectangle formed.
CALCULATE THE AREA OF A KITE WITH THE FORMULA	Calculate the area of a kite using the formula $A = \frac{1}{2}d_1d_2$ where d_1 is one diagonal of the kite and d_2 is the other diagonal of the kite.
CALCULATE THE AREA OF A PARALLELOGRAM BY COMPOSING A RECTANGLE	Compose a rectangle from a parallelogram by dividing the parallelogram with a segment that is perpendicular to the bases (a vertical line segment when the bases are horizontal) and repositioning the parts to form a rectangle. Determine the area of the rectangle, and show that the area of the composed rectangle equals the area of the parallelogram.
CALCULATE THE AREA OF A PARALLELOGRAM WITH THE FORMULA	Calculate the area of a parallelogram using the formula $A = bh$, where b is the base of the parallelogram and h is the height of the parallelogram.
CALCULATE THE AREA OF A RECTANGLE WITH THE FORMULA	Calculate the area of the rectangle using the formula $A = ab$ where a is the long side length and b is the short side length.
CALCULATE THE AREA OF A TRAPEZOID BY COMPOSING A PARALLELOGRAM	Compose a parallelogram from a trapezoid, where the trapezoid has only one pair of parallel sides, by duplicating the original trapezoid and repositioning the trapezoids to form a parallelogram. Determine the area of the parallelogram, and show that the area of the trapezoid equals half the area of the parallelogram.
CALCULATE THE AREA OF A TRAPEZOID WITH THE FORMULA	Calculate the area of a trapezoid using the formula $A = \frac{1}{2}h(b_1 + b_2)$ where h is the height of the trapezoid, b_1 is one base of the trapezoid, and b_2 is the other base of the trapezoid.
CALCULATE THE AREA OF A TRIANGLE WITH THE FORMULA	Calculate the area of a triangle using the formula $A = (\frac{1}{2})bh$ where b is the length of the base and h is the length of the height in the context of real-world and mathematical problems.
CALCULATE THE PERIMETER OF A TRIANGLE WITH FORMULA	Calculate the perimeter of a triangle by using the formula, $P = a + b + c$ where a, b, c are side lengths of the triangle.

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EXPLAIN AREA FORMULA FOR TRIANGLES	Make known your understanding that the area formula for a triangle is half the length of the base multiplied by the height, and that composing a triangle into rectangles and applying the area formula results in the same answer.
EXPLAIN *PERIMETER*	Make known your understanding that perimeter is a path (length) that surrounds or encloses a plane area.
RECOGNIZE THE BASE AND HEIGHT OF A RIGHT TRIANGLE	Identify or name the base and height (opposite and adjacent sides) of a right triangle as the two sides adjacent to the 90-degree angle.
RECOGNIZE THE HEIGHT OF A TRIANGLE	Identify or name the height of a triangle.
SOLVE PROBLEMS INVOLVING AREA OF POLYGONS	Solve real-world and contextualized mathematical problems involving area of polygons (including special quadrilaterals). For example, compose or decompose a polygon into a collection of simpler shapes, and find the area of the polygon through addition and/or subtraction of the simpler shapes.
SOLVE WORD PROBLEMS INVOLVING AREA OF TRIANGLES	Solve real-world and contextualized mathematical problems involving area of triangles.
SOLVE WORD PROBLEMS INVOLVING PERIMETER OF POLYGONS	Solve word problems involving perimeter of polygons.

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Composing and Decomposing to Determine Area

TEACHER NOTES

This unit includes the following documents:

- Learning Map Information
- Instructional Activity (four lessons)
- Instructional Activity Student Handout (for Lessons 1 – 4)
- Instructional Activity Supplement (for Lessons 1 – 4)
- Student Activity
- Student Activity Solution Guide

In this unit, students will learn how to decompose and compose figures in order to calculate the area. Students will calculate the area of triangles, special quadrilaterals (such as parallelograms, trapezoids, and kites), and irregular figures.

RESEARCH

Before students calculate area, they should first develop an understanding of the concept of area. Such an understanding can reduce the likelihood of students mixing up the formulas for perimeter and area, which is noted by teachers and researchers as a common difficulty. Malloy (1999) states that, "Although many middle grades students can solve (area and perimeter) problems, they may not have fully conceptualized the meaning of the words. They become confused by the formulas and find area when they are asked for perimeter and perimeter when they are asked for area" (p. 88). This lack of conceptual understanding is also portrayed when students are asked for a definition of area and they respond by stating the formula "length times width" (or base times height), instead of giving a definition that explains area as the amount of two-dimensional space inside a boundary (Van de Walle, Bay-Williams, Lovin, & Karp, 2014).

Van de Walle and colleagues (2014) state that, "students should have significant 'cover the surface' experiences with area before learning the formulas" (p. 309). Joram and Oleson (2004) recommend implementing activities that support students' visualization of the connection between direct measurement (counting) and indirect measurement (calculating with a formula). By linking area models and arrays (introduced in previous grades) to the formula for the area of rectangles, students can develop a more conceptual understanding of the formula. The bridge between direct measurement and indirect measurement is scaffolded through repeated addition and the idea of thinking of a rectangle as an array. Students may go from counting each individual unit, to determining how many units are in one row and counting the number of rows in order to repeatedly add the number of units in each row, to multiplying the number of units in one row by the number of rows, and then from that point connecting that conceptual understanding to the formula. Visualizing the area formula for rectangles in connection with an area model or an array also gives students an opportunity to regard variables as something other than simply place holders for numeric values (Hines & Bridges, 2003). For example, when considering an area model or an array, students can visualize the

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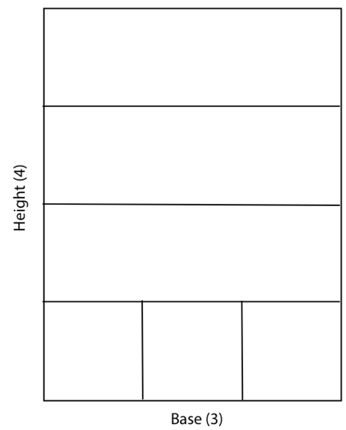
relationship between the base or height of a rectangle and the area of the rectangle. With experience, the variables take on more meaning; that is, the base is not just a number but tells how many units are in each row, and the height is not just a number but is the number of rows.

AN EXAMPLE

The following image illustrates a representation of the connection between an array and the area formula for rectangles, base times height (Joram & Oleson, 2004, p. 452).



Van de Wall et al. (2014) explains that, "the length of one side indicates how many squares will fit on that side. If that set of squares is taken as a unit, then the length of the adjacent side (not a number of squares) will determine how many rows of squares can fit in the rectangle" (p. 313).

Van de Walle et al. (2014) highlights the importance of students developing a strong understanding of the rectangular area formula, especially using the terms *base* and *height*, as opposed to *length* and *width*, because it will be used to develop other formulas for area. "The formula, *base* • *height*, can be generalized to all parallelograms (not just rectangles) and is useful in developing the area formulas for triangles and trapezoids" (Van de Walle et al., 2014, p. 312). It is important that students are familiar with shapes in any rotation. They should recognize that the height will always be a perpendicular segment from the base to the highest point of the figure. Van de Walle et al. (2014) states that, "Students should realize that they can always rotate a shape in order to use more convenient numbers and use any side as the base" (p. 313).

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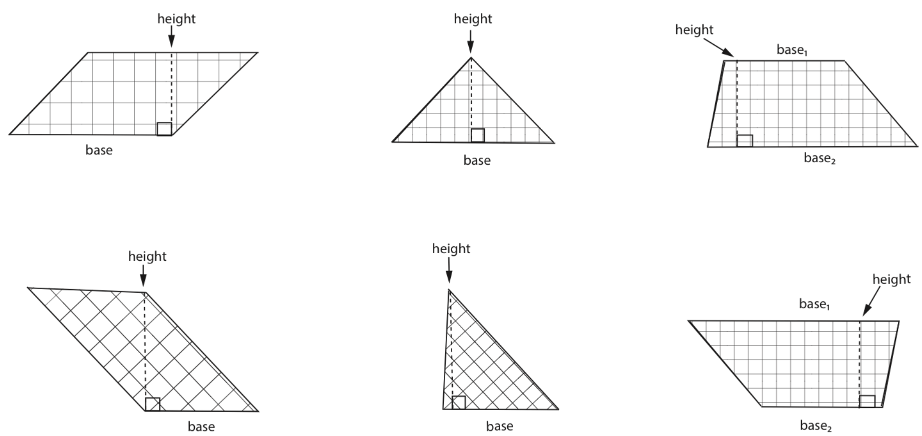
AN EXAMPLE

The following images demonstrate that the base can be any side of the figure with the height as a perpendicular segment.



Hines and Bridges (2003) believe that most students learn best when manipulatives and concrete situations are involved, therefore students should not just be introduced to the area formulas without first exploring situations that would foster conceptual understanding of the formulas. Van de Wall et al. (2014) recommends that once students have a conceptual understanding of the formula for area of a rectangle (*base* • *height*), they should be introduced to a parallelogram and asked how the parallelogram can be broken into pieces and rearranged into a rectangle. A common misconception for students is to misinterpret the length of the slanted side as the height, so it is crucial that students recognize that the height of a figure is always perpendicular to the base of the figure (Van de Walle et al., 2014).

Van de Walle et al. (2014) describes an effective sequence as one that progresses from an understanding of the area formula in regards to parallelograms, then proceeds to an exploration of the area of triangles. Once students have demonstrated understanding of the area formula in regards to a parallelogram, it is appropriate to introduce them to triangles. Beginning with right triangles is a good way for students to visualize the height of the triangle, since one side is a perpendicular segment. Students should understand that "two congruent triangles can always be arranged to form a parallelogram with the same base and height as those of the triangle. The area of the triangle will, therefore, be one-half that of the parallelogram" (Van de Walle et al., 2014, p. 314). Van de Walle and colleagues (2014) continue by stating that, "After developing formulas for parallelograms and triangles, students are ready to develop the formula for area of a trapezoid" and a kite (p. 314). It is important to note that there are several different ways of determining the area of a trapezoid. Van de Walle et al., (2014) lists five strategies and states that there are "at least 10 different methods" to determine the area. Two congruent trapezoids can also be arranged to form a parallelogram, and a kite can be decomposed and recomposed to form a rectangle. These strategies form the basis of the common formulas for area of a trapezoid and kite respectively. Several strategies for determining the area of special

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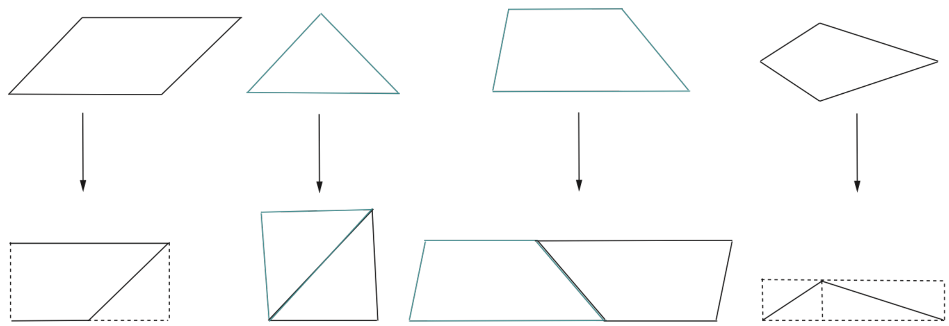
quadrilaterals and triangles include decomposing the figure into rectangles, parallelograms, and triangles, then finding the areas of the parts, which are then added together to determine the area of the original figure.

AN EXAMPLE

The following images model how parallelograms, triangles, trapezoids, and kites can be rearranged or composed into rectangles or parallelograms.



The focus of area instruction should not be on memorization of specific formulas. Instead, students should first develop a secure understanding of the formula for the area of a rectangle and build on this understanding as they learn about the areas of other two-dimensional figures. As students manipulate, decompose, and compose figures to determine their areas, they will develop a conceptual understanding that will endure and serve as a foundation for future work in measurement and geometry. The likelihood of misconceptions and errors occurring is decreased when students are able to problem solve and relate their understanding of area and the shapes to the area formulas (Van de Walle et. al., 2014).

LEARNING MAP INFORMATION

The learning map section for this sequence of activities begins by focusing on the area formula for rectangles. The map then progresses to both triangles and parallelograms. One path begins with determining the area of triangles by composing rectangles prior to calculating with the area formula, and the other path explores determining the area of parallelograms by creating rectangles and then calculating the area with the formula. Rectangles and triangles both precede decomposing polygons into rectangles and triangles to determine the area of trapezoids and kites. The culmination of the learning map section is students' ability to apply their understanding of areas of a variety of shapes to solving word problems involving area of two-dimensional spaces.

NOTE: Although the anticipated path for solving real-world and contextualized mathematical problems involving area of polygons goes through nodes involving calculating area using formulas, students do not

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need to follow this direct path. If a student is comfortable and proficient at determining areas with strategies other than the formulas, then they may use alternate strategies to solve real-world and contextualized math problems.

INSTRUCTIONAL ACTIVITIES

The activities in this unit are designed to scaffold students' conceptual understanding of the area formulas for rectangles, parallelograms, triangles, trapezoids, and kites. Students are presented with real-world situations involving area as they are developing an understanding of the different formulas.

The unit begins with a review of the concept of area and the formula for rectangles with a tiling activity. Students will then apply that understanding to determine how to rearrange a parallelogram to obtain a rectangle. Using paper parallelograms and scissors and later flexible parallelograms, students will explore the area formula for parallelograms and establish the concept of height versus side length (Khoury & Steele, 1997). When manipulating the flexible parallelograms, students will see that the perpendicular segment (the string), which represents the height of the parallelogram, changes as the angles of the parallelogram change, but the length of the edge or side length does not change; it is a fixed length represented by a strip of tag board.

NOTE: By sixth grade, students should have developed an understanding of area for rectangles. However, if scaffolding is needed for students that struggle conceptually, consider providing the images for activities on dot or grid paper for students to better visualize the units.

Students will then transition to triangles, exploring area for right triangles, equilateral triangles, isosceles triangles, and scalene triangles. Students will investigate several different strategies for determining the area of a triangle. For example, students will duplicate a triangle to create a rectangle or parallelogram, in addition to cutting a paper isosceles triangle into two right triangles to be composed as a rectangle.

Once students are familiar with triangles and parallelograms, they will be decomposing and composing trapezoids and kites to determine the area. Again, students will use scissors to decompose paper shapes in order to rearrange the pieces to create a rectangle or a parallelogram.

NOTE: INSTRUCTIONAL ACTIVITIES 3 and 4 and the STUDENT ACTIVITY utilize the definition of a trapezoid as a quadrilateral with *exactly* one set of parallel sides. Reference your local curriculum for the definition your materials provide to make adjustments if needed.

Finally, students will finish the unit by decomposing or composing irregular shapes to determine the area. Students will be presented with problem situations, such as determining areas of a house floor plan, as well as calculating areas for backyard gardens and pools.

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Composing and Decomposing to Determine Area

OVERVIEW OF INSTRUCTIONAL ACTIVITIES

Lesson	Learning Goal	Nodes Addressed
Lesson 1	Students will identify the differences between area and perimeter, then they will apply their understanding of the area formula for a rectangle to determine the area of a parallelogram.	▶ CALCULATE THE AREA OF A RECTANGLE WITH THE FORMULA ▶ CALCULATE THE AREA OF A RECTANGLE BY COMPOSING A RECTANGLE ▶ CALCULATE THE AREA OF A PARALLELOGRAM WITH THE FORMULA
Lesson 2	Students will combine their understanding of the area formula for rectangles and their knowledge of triangles to determine the areas of a variety of triangles.	▶ CALCULATE AREA OF RIGHT TRIANGLES BY COMPOSING INTO A RECTANGLE ▶ RECOGNIZE THE BASE AND HEIGHT OF A RIGHT TRIANGLE ▶ EXPLAIN AREA FORMULA FOR TRIANGLES ▶ RECOGNIZE THE HEIGHT OF A TRIANGLE ▶ CALCULATE AREA OF ANY TRIANGLE BY COMPOSING RECTANGLES ▶ CALCULATE THE AREA OF A TRIANGLE WITH THE FORMULA
Lesson 3	Students will apply their understanding of the area formulas for rectangles and triangles to calculate the areas of trapezoids and kites. Through composing rectangles and parallelograms and decomposing into rectangles and triangles, students will gain a conceptual awareness of both the trapezoid and kite area formulas.	▶ SOLVE PROBLEMS INVOLVING AREA OF POLYGONS ▶ CALCULATE THE AREA OF A TRAPEZOID WITH THE FORMULA ▶ CALCULATE THE AREA OF A KITE WITH THE FORMULA
Lesson 4	Students will determine the area of a given house floor plan by composing the floor plan into a larger rectangle or decomposing the plan into rectangles, parallelograms, triangles, trapezoids, and kites.	▶ CALCULATE THE AREA OF IRREGULAR SHAPES BY PARTITIONING INTO RECTANGLES ▶ SOLVE PROBLEMS INVOLVING AREA OF POLYGONS

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Composing and Decomposing to Determine Area

INSTRUCTIONAL ACTIVITY

Lesson 1

LEARNING GOAL

Students will identify the differences between area and perimeter, then they will apply their understanding of the area formula for a rectangle to determine the area of a parallelogram.

PRIMARY ACTIVITY

Students will explore the differences between area and perimeter by arranging and rearranging square tiles that must share at least one common side. Students will then expand their understanding of the area formula for rectangles to determine the area of parallelograms using a flexible parallelogram.

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

- Quadrilateral
- Square
- Rectangle
- Parallelogram
- Trapezoid
- Base
- Height
- Side
- Area
- Square units
- Right triangle

MATERIALS

- Instructional Activity Student Handout

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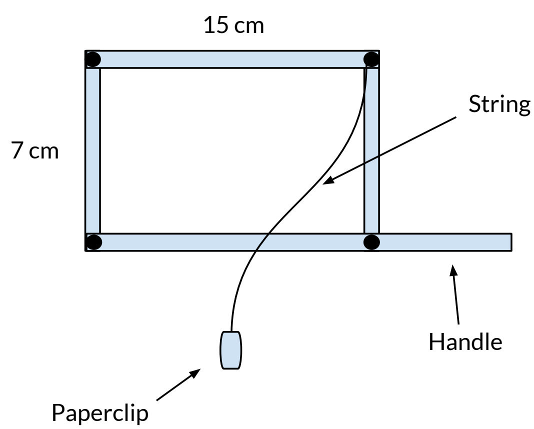
- One-square-inch squares, cut from paper or square pattern blocks (Recommend 12 for every student.)
- One-square-foot square tiles, cut from paper or tiles that can be purchased at a home improvement store (Recommend one for every three to four students.)
- Small rectangular tiles, cut from paper or tiles that can be purchased at a home improvement store (Recommend one for every three to four students.)
- Scissors
- Rulers
- Glue sticks or glue bottles
- INSTRUCTIONAL ACTIVITY SUPPLEMENT
- Flexible parallelograms (Recommend one for every two students.)

IMPLEMENTATION

Before implementing the lesson, construct the flexible parallelograms. For each flexible parallelogram, you will need two seven-centimeter strips of tag board and two 15-centimeter strips of tag board. One 15-centimeter strip can be longer to serve as a handle. Using paper fasteners, attach the four strips together in a rectangle, and add a string weighted with a paper clip to one of the fasteners.



Distribute the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

Require students to complete Question 1 on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

Review the answers to Question 1 by presenting one shape at a time and asking the following questions:

- What is the name of this figure?
- How do you know? What are the attributes that support your answer?

State that you are going to determine the area of several quadrilaterals.

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Ask students, "What is area?"

NOTE: It is a common misconception for students to provide the formula for calculating area when asked to define area, but simply providing a formula is not an adequate definition and does not indicate student understanding of area. Students need to understand that area is the two-dimensional space inside a region.

Distribute square-inch squares to students (12 squares for every student).

Model the following figure using one-inch squares. **Require** students to create the same figure.

Ask students the following questions:

- What is the area of the figure? How do you know?
- What is the perimeter of the figure? How do you know?
- How are area and perimeter different?

Require students to add squares until they reach a perimeter of 16 inches. **Identify** students with different solutions to share their figures with the class. When each student presents, **require** them to state the area of their figure.

Ask students the following questions and **require** them to answer by manipulating their squares:

- What is the smallest number of squares you could add to reach a perimeter of 16 inches?
- What is the greatest number of squares you could add to reach a perimeter of 16 inches?
- What happens to the area when you increase the perimeter?
- [Model the original figure.] Can you increase the area without increasing the perimeter? Explain.
- Where would you place a square to increase the perimeter by one inch? By two inches? By three inches?

Ask students to find all the non-congruent rectangles that have an area of 12 square inches. **Require** students to share their solutions with the class; each time a student presents, **require** them to state the length and width of their figure. **Repeat** the process for 16 square inches.

Collect the squares and **ask** students the following questions:

- What is area?
- What is perimeter?

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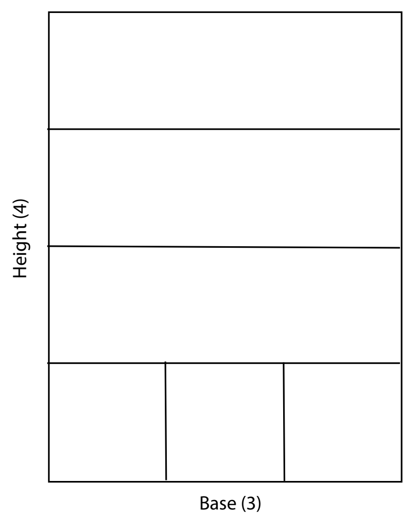
- How are area and perimeter different?
- Thinking about houses, when would area be used in relation to a house? (Ideally, student responses should involve concepts like determining how much carpet would be needed for a room.)
- Thinking about houses, when would perimeter be used in relation to a house? (Ideally, student responses should involve concepts like determining how much trim or baseboard would be needed for a room.)

Group students into groups of three or four. **Provide** each group with a rectangular tile (or paper rectangle) and a smaller square tile (or paper square).

Direct students to determine how many smaller tiles (squares) will fit into the larger tile (rectangle). **Explain** that they may use any method they choose, and that they may use a ruler. This is a good opportunity to see how students respond to fractional parts, because the square tiles will not fit exactly into the rectangle tile.

Require students to record and explain their work on Question 2 of the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

Display the following image:



Ask students, "Do you need to fill in the entire space with tiles to determine the area? How else could you determine the area?" Students should **discuss** why the space does not need to be filled in to determine the area.

Emphasize that the space does not need to be filled in, and identify that the length of one side can be determined by the number of tiles that fit in one row (refer to this as the *base*). Then, the length of the adjacent side could be determined by the number of rows required to fill the area (refer to this as the *height*). Multiply the number in each row by the number of rows (*base* • *height*) to determine the area.

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Arrange students into groups of two or three.

Require students to answer Questions 3 – 5 on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

Use the Guiding Questions to scaffold student understanding.

GUIDING QUESTIONS

Elicit student thinking:

- In what real world situations would you need to determine area?
- [Point to Question 4.] What do you notice about this question?

Determine if the student can CALCULATE THE AREA OF A RECTANGLE WITH THE FORMULA:

- How do you know this shape is a rectangle?
- What is the base of the rectangle?
- What is the height of the rectangle?
- How do you determine the area of a rectangle?
- Can you determine the area of a rectangle without counting the number of squares inside the area? Explain.

Review the answers. **Require** students to share their answers, each time providing a base and a height for the given rectangle.

- 3. Matt will need 234 square feet of carpet.
- 4. The base of the rectangle is eight centimeters.
- 5. The area of the living room is 189 square feet.

Display a parallelogram. **Ask** students, "What is the name of this figure? How do you know?"

Ask students the following questions:

- "You know how to determine the area of what shape?"
- "Is there any way that we can connect what we know about rectangles and what we know about parallelograms to help determine how to calculate the volume of a parallelogram?"

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Arrange students into groups of two or three. **Distribute** the flexible parallelograms.

Provide students about two minutes to explore and experiment with the flexible parallelogram. **Model** how to use the flexible parallelogram by beginning with the flexible parallelogram as a rectangle where the string is in line with the side length.

Identify the base and the sides. **Model** and **require** students to adjust the flexible parallelogram, so that the side length and string are no longer in line, but the side length is now slanted. **Model** holding the string so that it meets the base at a 90° angle.

Ask students what they think the height of the parallelogram is: the slanted side length or the string at a 90° angle. If necessary, refer back to a rectangle to identify that the height is a line segment that meets the base at a 90° angle.

Emphasize that the string represents the height of the parallelogram as opposed to the slanted side length.

Require students to complete the table for Question 6 and then answer Questions 7 – 9 on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT by creating five different parallelograms with the flexible parallelogram. Students should trace their parallelograms on a sheet of graph, printing, or lined paper and attach it to the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

Discuss the responses to Questions 7 – 9 on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

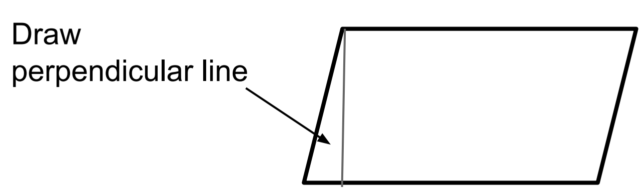
Distribute one parallelogram per student from the INSTRUCTIONAL ACTIVITY SUPPLEMENT and scissors.

Arrange students into groups of two or three. **Ask** students to predict how they are going to calculate the area of the parallelogram. **Require** two or three groups to share their predictions.

Ask students, "Is there any way to change a parallelogram into a rectangle?"

NOTE: Depending on your preference and/or your group of students, the following section can be more exploratory for students as opposed to or before direct instruction.

Using a ruler, **model** drawing a line perpendicular to the base. **Ask** students, "What is significant about the angle measures in a rectangle?" **Emphasize** that because the angles in a rectangle each measure 90°, you drew the line in the parallelogram perpendicular or at a 90° angle with the base.



Require students to draw a line perpendicular to the base.

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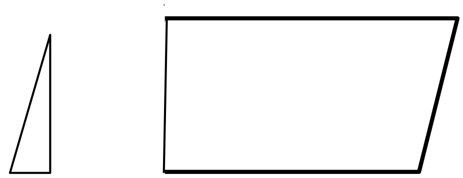
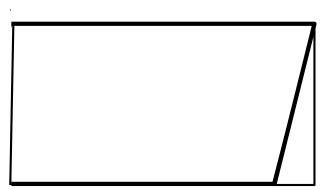
Model cutting the parallelogram along the drawn line to separate the right triangle from the rest of the figure.



Require students to cut along the perpendicular line, separating the right triangle from the parallelogram.

Identify that the right triangle now fits perfectly on the other end of the parallelogram, and **model** moving the triangle over and creating a rectangle.



Require students to create a rectangle by moving the right triangle to the other side of the parallelogram. Students should glue the pieces in the shape of a rectangle on Question 10 of the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

Require students to measure (in inches), the base, the height, and the slanted side length using a ruler. **Ask** students if the slanted side length is the same measure as the height of the figure. Pointing to the slanted side length, **ask** students, "If we multiply base times height to calculate area, is this the height?"

Review the formula for the area of a rectangle (*base* • *height*). **Emphasize** that when determining the area for parallelograms, the height and the side length are not the same measures.

Using their measurements for the base and the height, students should calculate the area for the parallelogram in inches and record all work on Question 10 of the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

Ask students the following questions:

- "What is the area of the rectangle?"
- "When you moved the right triangle, did the area (or inside space) of the shape change?"
- "What is the area of the original parallelogram?"
- "Is the formula for determining the area of a rectangle the same as the formula for determining the area of a parallelogram? Explain."
- "Can we cut and glue a parallelogram into a rectangle every time we want to determine the area? Why or why not?"

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- "How would you determine the height of a parallelogram?"
- "Why is the height the vertical (perpendicular) line and not the slanted side length?"

Arrange students into groups of two or three. **Require** students to complete Questions 11 – 14 on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

GUIDING QUESTIONS

Elicit student thinking:

- How are a parallelogram and a rectangle the same?
- How are a parallelogram and a rectangle different?
- [Point to any question 11 – 14.] What do you notice about this parallelogram?

Determine if the student can CALCULATE THE AREA OF A PARALLELOGRAM BY COMPOSING A RECTANGLE:

- How can you change a parallelogram to create a rectangle?
- When you change a parallelogram to a rectangle, does it change the area? Explain.

Determine if the student can CALCULATE THE AREA OF A PARALLELOGRAM WITH THE FORMULA:

- Which measurement is the height of the parallelogram? How do you know?
- Which measurement is the base of the parallelogram? How do you know?
- How do you determine the area of a parallelogram?
- Are the height and the slanted side length of a parallelogram the same? Explain.

Review the answers. **Require** students to share their answers, each time providing a base and a height for the given parallelogram.

- 11. 22.8 square feet
- 12. 63.44 square meters
- 13. 52 square centimeters

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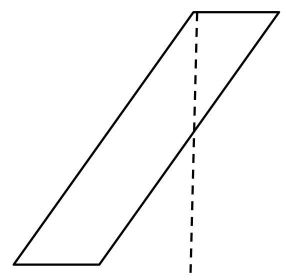
- 14. 207 square inches

Direct student attention to Question 15 on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

Ask students, "Can you cut a right triangle from a perpendicular line with this parallelogram?"

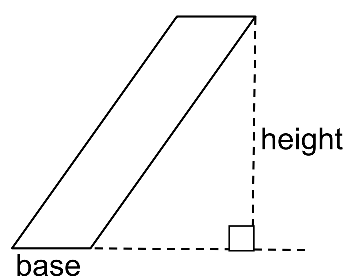
Model that with the parallelogram as it appears, you cannot cut and rearrange the parallelogram like the previous parallelograms. **Emphasize** that the perpendicular line only goes through part of the parallelogram, not the entire shape.

> NOTE: Some students may comment that you can rotate the parallelogram so that you can cut and rearrange the parallelogram. This example is presented to show that the height can be thought of outside of the parallelogram as well as inside.



Ask students, "How could we determine the height of the parallelogram? Would the slanted side length be an appropriate measure for the height of the parallelogram?"

Demonstrate that you can still use a perpendicular line to determine the height of the parallelogram.



Emphasize that the base of the parallelogram does not change; students should extend a line from the base in order to create a perpendicular line to determine the length of the height.

Students should draw the lines to create a perpendicular intersection in order to determine the height of the parallelogram.

Require students to measure the base, height, and slanted side length using a ruler, and then to determine the area of the figure in square inches. **Discuss** their measurements and areas as a class.

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Arrange students into groups of two or three. **Require** students to complete Questions 16 – 18 on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

GUIDING QUESTIONS

Elicit student thinking:

- What do you notice about this parallelogram?
- [Point to Question 16.] How is this parallelogram different from the parallelogram in Question 14?

Determine if the student can CALCULATE THE AREA OF A PARALLELOGRAM WITH THE FORMULA:

- Which measurement is the height of the parallelogram? How do you know?
- Which measurement is the base of the parallelogram? How do you know?
- How do you determine the area of a parallelogram?
- Are the height and the slanted side length of a parallelogram the same? Explain.
- [Point to Question 17.] How do you know this dashed line is perpendicular to the base?

Review the answers. **Require** students to share their answers, each time providing a base and a height for the given parallelogram.

- 16. 189 square feet
- 17. 980.76 square centimeters
- 18. 3.9375 square inches

At the end of the activity, students should answer the following question on a sticky note and stick it to the door to end class:

- What is the formula for the area of rectangles and parallelograms? Why are they the same?

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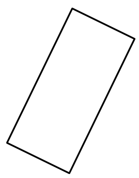
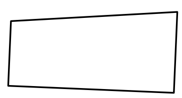
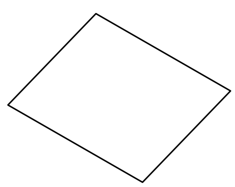
Composing and Decomposing to Determine Area

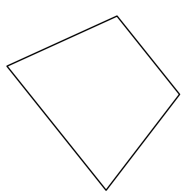
Lesson 1

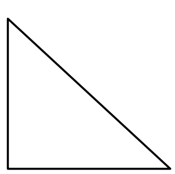
1. Identify the names of the following shapes and explain how you know.











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2. How many small rectangles fit inside the large square? Explain how you determined that amount using words and/or drawings.

3. Matt is replacing the carpet in his bedroom. His bedroom is 13 feet wide and 18 feet long. How many square feet of carpet will Matt need? Draw a diagram indicating the base and the height used to determine the area.

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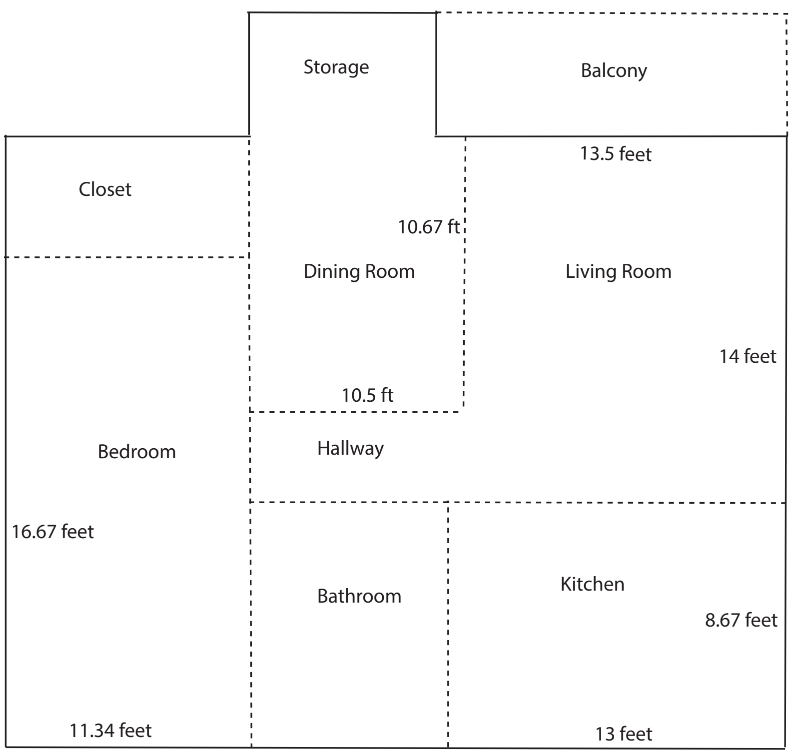
4. The area of a rectangle is 104 centimeters. The height of the rectangle is 13 centimeters. What is the base of the rectangle?

5. Use the following floor plan to determine the area of the living room in square feet.



Base: ______________________

Height: ______________________

Area: ______________________

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6. Create five parallelograms by tracing the flexible parallelogram on a sheet of paper. Measure and label the side lengths in inches. Complete the table. Remember to show all your work.

Parallelogram	Height of the Parallelogram	Length of the Slanted Side Length
1		
2		
3		
4		
5		

7. What stayed the same for the different shapes, and what changed?

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8. As the parallelogram changed, what happened to the side lengths?

9. As the parallelogram changed, what happened to the height length?

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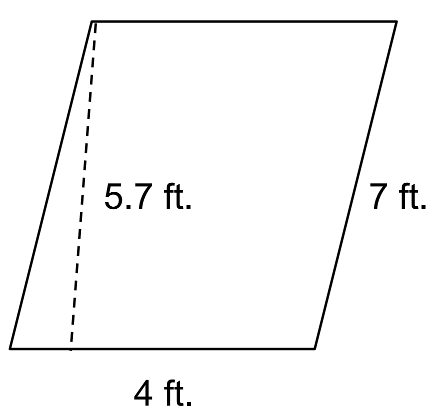
10. Glue and measure the pieces of the parallelogram as a rectangle. Calculate the area of the rectangle in square inches. Remember to show all your work.

11. Determine the area of the parallelogram. Remember to show all your work.



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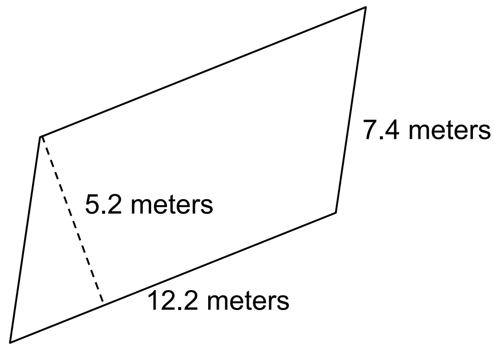
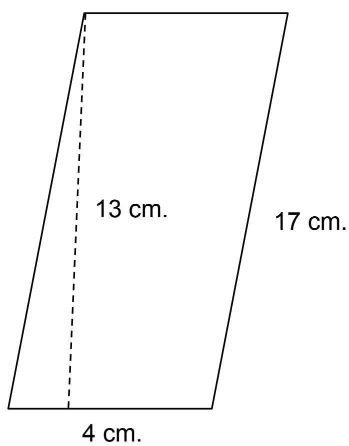
12. Determine the area of the parallelogram. Remember to show all your work.



13. Determine the area of the parallelogram. Remember to show all your work.



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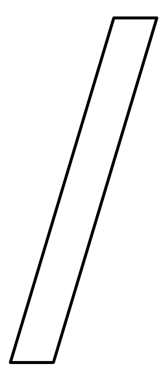
14. Determine the area of the parallelogram. Remember to show all your work.

15. Measure the base, height, and slanted side length in inches. Determine the area of the parallelogram. Remember to show all your work.



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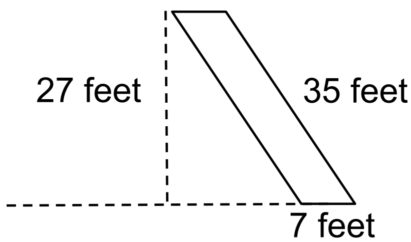
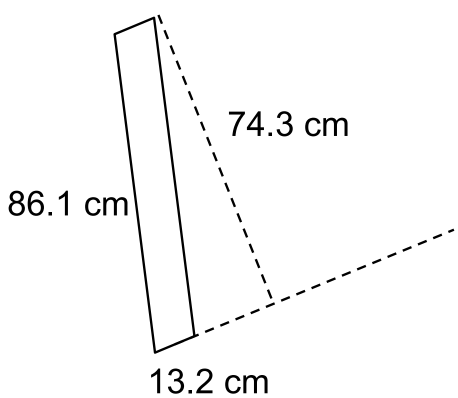
16. Determine the area of the parallelogram. Remember to show all your work.



17. Determine the area of the parallelogram. Remember to show all your work.



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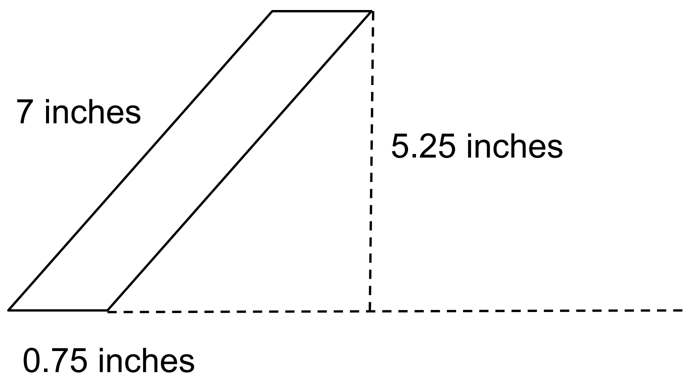
18. Determine the area of the parallelogram. Remember to show all your work.



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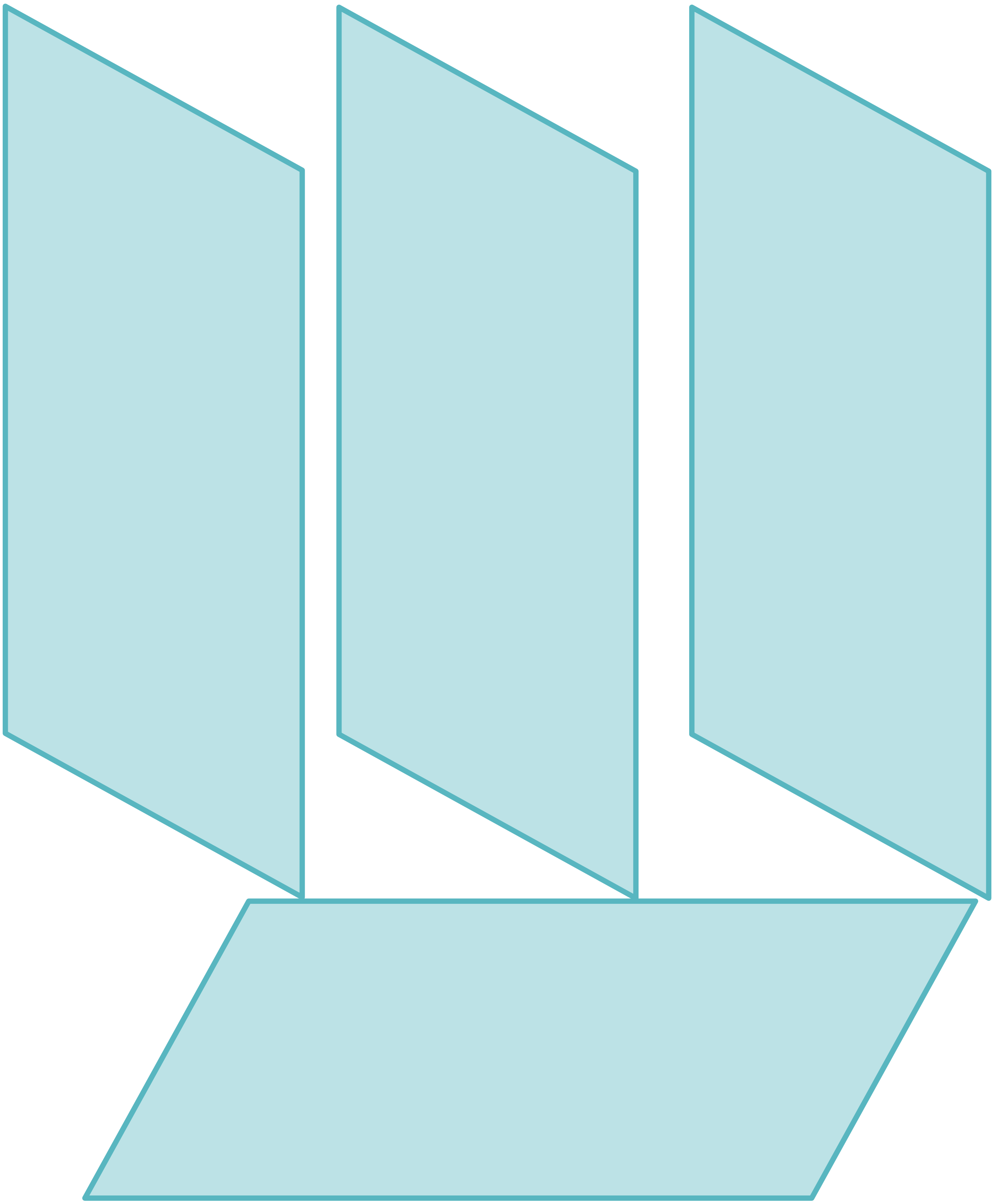
Composing and Decomposing to Determine Area

INSTRUCTIONAL ACTIVITY SUPPLEMENT

Lesson 1



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Composing and Decomposing to Determine Area

INSTRUCTIONAL ACTIVITY

Lesson 2

LEARNING GOAL

Students will combine their understanding of the area formula for rectangles and their knowledge of triangles to determine the areas of a variety of triangles.

PRIMARY ACTIVITY

Students will manipulate and duplicate triangles in order to create rectangles to determine the area of the original triangle. Through these explorations, students will develop a conceptual understanding of the formula to calculate the area of a triangle.

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

- Right triangle
- Obtuse triangle
- Scalene triangle
- Isosceles triangle
- Base
- Height (altitude)
- Side
- Area
- Square units

MATERIALS

- Instructional Activity Student Handout
- Instructional Activity Supplement (Recommend one copy for every eight students.)
- Glue sticks or glue bottles
- Scissors
- Rulers

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IMPLEMENTATION

Distribute the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

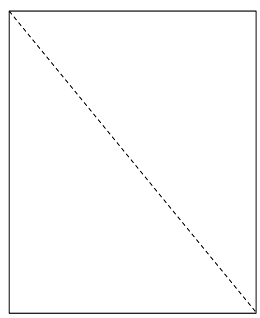
Require students to complete Question 1 on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

Review the answers to Question 1 by presenting one shape at a time and asking the following questions:

- What is the name of this figure?
- How do you know? What are the attributes that support your answer?
- [Show a rectangle.] What is the name of this figure? How do you know?
- How do you think a rectangle and right triangle are similar? How are they different?

Distribute the rectangles from page one of the INSTRUCTIONAL ACTIVITY SUPPLEMENT. Each student will need one pair of scissors and one rectangle.

Model drawing a line bisecting the rectangle into two right triangles. Students should do the same with their rectangle, then cut along the line to create two right triangles.



Ask students, "How many triangles do you have? Before you cut the triangles apart what figure did they make up?"

Arrange students into groups of two or three. **Require** students to complete Question 2 on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

Discuss strategies students used to attempt to determine the area of the triangle. Note if any students tried to relate back to the area formula of a rectangle or parallelogram.

Recall the notion from INSTRUCTIONAL ACTIVITY LESSON 1 that explained why the area formula for a rectangle worked.

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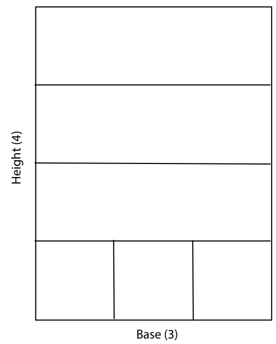
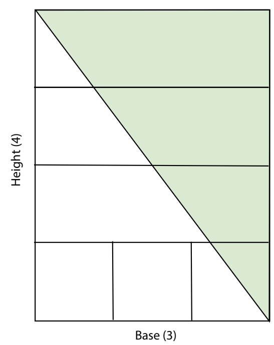


Display the triangle from Question 2 on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT. **Model** that you cannot use the same formula as it is to determine the area of a triangle, because each row does not have the same number of units as the base.



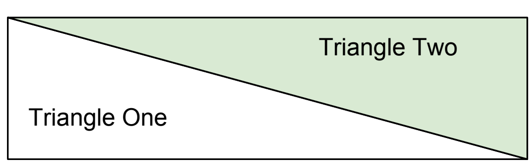
Ask students, "What do you notice about this representation?"

Direct students to the understanding that the area of the right triangle in this image is equal to one-half the area of the rectangle.

Distribute the right triangles from the second and third pages of the INSTRUCTIONAL ACTIVITY SUPPLEMENT, so that each student receives two of the same size and shape.

NOTE: While students do not need to master the concept that the hypotenuse is the side of a right triangle that is opposite the right angle, it would be beneficial to expose them to the definition and use of the word.

Model for students placing the two triangles together, without overlap, along the hypotenuses to create a rectangle.



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Require students to put their two right triangles together to create a rectangle. Students should glue the pieces in the shape of a rectangle on Question 3 of the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

Require students to measure (in inches) the length of the base and the height using a ruler. **Ask** students, "How many triangles make up the rectangle? How should we account for the fact that two triangles make one rectangle when determining the area of a single triangle?"

Ask students, "What do you think would be a formula to determine the area of a triangle?"

Direct students to identify that each triangle is one-half the area of the rectangle, so the formula for the area of a triangle is $\frac{1}{2}b \bullet h$ or $\frac{b \cdot h}{2}$.

Model calculating the area of the triangle that was measured by using the formula. Students should write the work you model on Question 3 of the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

Require students to complete Questions 4 – 6 on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

GUIDING QUESTIONS

Elicit student thinking:

- What do you notice about the triangle?
- How are right triangles and rectangles the same? How are they different?

Determine if the student can CALCULATE AREA OF RIGHT TRIANGLES BY COMPOSING INTO A RECTANGLE:

- How can you compose a right triangle into a rectangle?
- Why is the triangle area formula $\frac{1}{2}$ of the rectangle area formula?
- [Point to Question 4.] Can you show me how this right triangle can become a rectangle?

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Determine if the student can RECOGNIZE THE BASE AND HEIGHT OF A RIGHT TRIANGLE:

- [Point to any Question, 4 – 6.] Which side of this triangle is the base? How do you know?
- [Point to any Question, 4 – 6.] Which side of this triangle is the height? How do you know?
- [Point to Question 5.] Do those values have to be the base and the height, or can the height be $2\frac{2}{3}$? Explain.
- [Point to the hypotenuse.] Can this side be the base or the height? Explain.

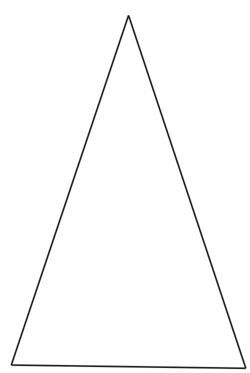
Determine if the student can EXPLAIN AREA FORMULA FOR TRIANGLES:

- When calculating the area of a triangle, why is the area of a triangle one-half the area of a rectangle with the same length base and height?

Review the answers. **Require** students to share their answers, each time providing a base and a height.

- 4. $7\frac{5}{16}$ square feet
- 5. $7\frac{6}{30}$ or $7\frac{2}{10}$ square centimeters
- 6. 75 square inches

Display an isosceles triangle like the following model:



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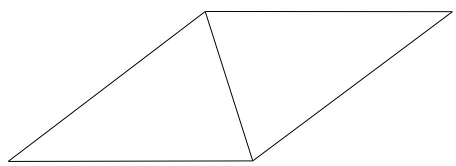
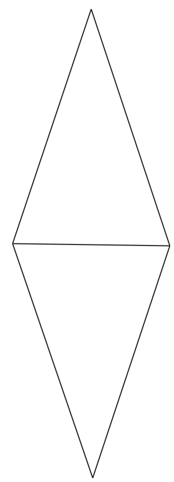
Ask students, "What if the triangle you are given is not a right triangle?" Students should share strategies to determine the area.

Emphasize that there is more than one strategy to manipulate the triangle in order to create a rectangle or parallelogram.

Model the first strategy (there is no particular order). **Demonstrate** that by duplicating the triangle, you can create a parallelogram. Once you create the parallelogram, you should determine the area and then divide by two, because you duplicated the original triangle.

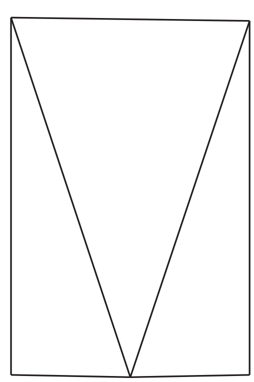




NOTE: Both examples are parallelograms, and students should be exposed to shapes in different directions so that they develop a conceptual understanding of the figures based on the attributes.

If students do not recognize a figure, encourage them to rotate their papers until they can identify a vantage point which they recognize.

Explain that from this figure, the students can relocate a triangle from one side of the parallelogram to the other side in order to create a rectangle. **Note** that from this point, determining the area of the original triangle is similar to the process used with the right triangle previously.



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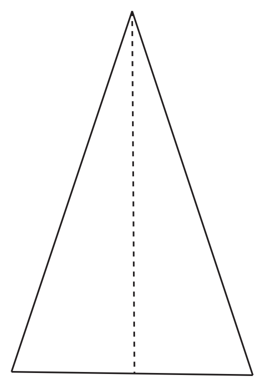
Model the second strategy (there is no particular order). **Ask** students to refer back to LESSON 1: "In order to determine the height, what angle measure must be present?" Students should suggest a 90˚ angle or a right angle.

Ask students how they would create a right angle in an isosceles triangle.

Model bisecting the isosceles triangle. **Explain** that this is the height of the triangle. Similar to parallelograms, the height often will not be the same as the length of a side.

Emphasize that the line segment for the height is perpendicular to the base, like a parallelogram. **Note** that the height can also be referred to as the altitude.

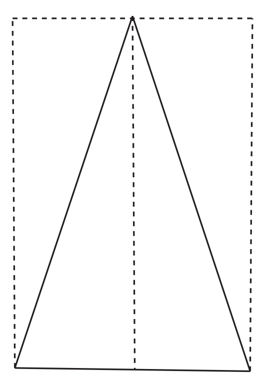


Point out that you have not created any rectangles. **Ask** students for suggestions regarding how to create one or more rectangles to determine the area.

Model creating one rectangle from the line segments that were added. Point out that you can then determine the area of the rectangle and divide by two in order to determine the area of the triangle.

Identify that there is one rectangle and that you divided the original triangle into two right triangles. Therefore, to determine the area of the original triangle, the area of the rectangle must be divided in half.

Note that while it is acceptable that some students may visualize this as two rectangles instead of one, this just requires more work to calculate the area.



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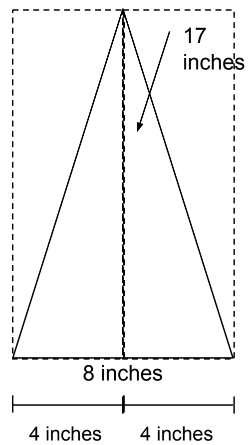
Complete Question 7 on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT with students by creating one or two rectangles. If students are more inclined to create two rectangles, **explain** that when you bisect the triangle into two right triangles, the base must also be divided.



Require students to complete Questions 8 – 10 on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT in groups of two or three by creating one or two rectangles.

GUIDING QUESTIONS

Elicit student thinking:

- What do you notice about the triangle?
- How is this triangle the same as a right triangle? How is it different?

Determine if the student can RECOGNIZE THE HEIGHT OF A TRIANGLE:

- How can you identify the height of a triangle?
- Does the height of a triangle have to be the length of one of the sides? Explain.
- How are the height and the base of a triangle positioned relative to each other?
- [Point to any Question, 8 – 10.] What is the height of this triangle? How do you know?

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Determine if the student is ready to CALCULATE AREA OF ANY TRIANGLE BY COMPOSING RECTANGLES:

- Can you compose this triangle into a rectangle? Explain.
- How would you compose this triangle into a rectangle?

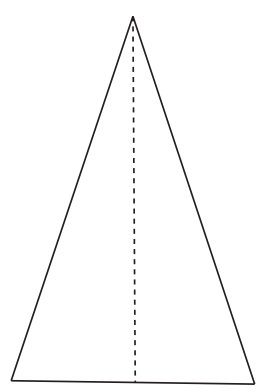
Determine if the student can CALCULATE AREA OF ANY TRIANGLE BY COMPOSING RECTANGLES:

- [Point to any Question, 8 – 10.] What is the area of the rectangle you created? What is the area of the triangle in the rectangle? How do you know?
- [Point to any Question, 8 – 10.] How did you calculate the area of this triangle? Explain.

Review the answers by selecting students to share their answers. Students should explain their process when they share.

- 8. $142\frac{1}{2}$ square feet or 142.5 square feet
- 9. 27. 5 square meters
- 10. 141.24 square feet

Model the third strategy (there is no particular order). **Display** the same isosceles triangle from earlier.



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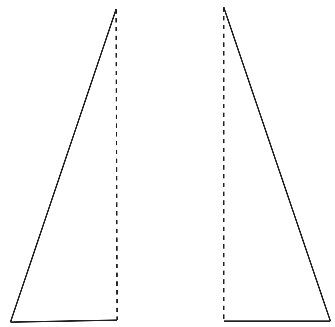
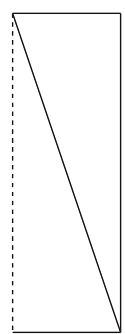
Ask students to recall the activity for determining the area of a parallelogram in LESSON 1, when students cut the right triangle off of one end and moved it to the other side of the parallelogram to make a rectangle.

Distribute the isosceles triangles from the fourth page of the INSTRUCTIONAL ACTIVITY SUPPLEMENT. Students should each have one pair of scissors, one ruler, and one isosceles triangle.

Require students to draw a line segment perpendicular to the base of the triangle. Students should then cut along the line to create two congruent right triangles.



Model rotating one right triangle so that the two right triangles combine to create a rectangle.



Require students to glue the two right triangles together as a rectangle on Question 11 of the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT. Students should measure and label the base and height of the rectangle.

Direct students' attention to the base of the original triangle. **Ask** students, "What happened to the base of the triangle when we bisected it to make a rectangle?" Students should note that the base was divided in half.

Emphasize that in order to divide the base of a triangle exactly in half with a perpendicular segment from the vertex opposite the base, you must have either an isosceles or equilateral triangle.

Point out that because scalene triangles have three different side lengths, when a segment is drawn from one vertex perpendicular to the opposite side, it will not divide the side exactly in half, and therefore scalene triangles cannot be decomposed and rearranged into a rectangle.

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Make clear that the students did not double the area of the original triangle, therefore they do not need to divide the area of the composed rectangle in half. Nothing was added or taken away from the original area; the figure was rearranged into a different shape, but that does not change the amount of area covered by the shape.

Require students to complete Questions 12 – 14 on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT in groups of two or three.

GUIDING QUESTIONS

Elicit student thinking:

- What do you notice about the triangle?
- How is this triangle the same as a right triangle? How is it different?

Determine if the student can RECOGNIZE THE HEIGHT OF A TRIANGLE:

- How can you identify the height of a triangle?
- Does the height of a triangle have to be the length of one of the sides? Explain.
- [Point to any question, 12 – 14.] What is the height of this triangle? How do you know?

Determine if the student can CALCULATE AREA OF ANY TRIANGLE BY COMPOSING RECTANGLES:

- [Point to Question 13.] What is the area of this triangle that has been regrouped into a rectangle? Do you need to divide this rectangle by two in order to determine the area of the original triangle? Why or why not?
- How is this the same or different than creating a rectangle by duplicating a right triangle?

Review the answers by selecting students to share their answers. Students should explain their process when they share.

- 12. *Students should note that you are not adding any additional area (triangles) to the original triangle, and therefore you do not need to divide the total area by two; you are simply rearranging parts of the triangle that already exist.*
- 13. *26 square centimeters*

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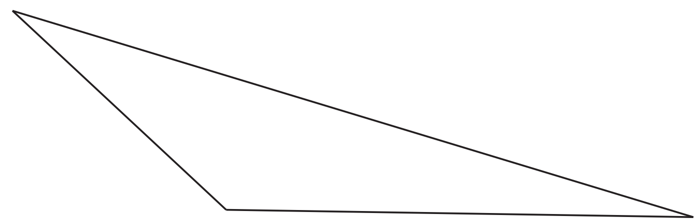
- 14. *Students should explain that you cannot create a single rectangle with the same area as the original triangle by drawing a segment from one vertex perpendicular to the opposite side of the triangle, because it is scalene and when you create a line segment perpendicular to the base, it will not divide the base exactly in half.*

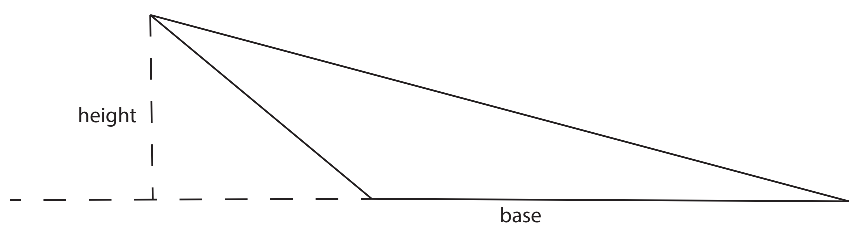
Display an obtuse triangle that looks like the following triangle. **Distribute** two obtuse triangles from the fifth and sixth pages of the INSTRUCTIONAL ACTIVITY SUPPLEMENT to each student.



Ask students if the triangles can be composed into a rectangle or parallelogram. Students should explain why or why not. (Students should identify that duplicating the triangle and aligning the longest sides would make a parallelogram.)

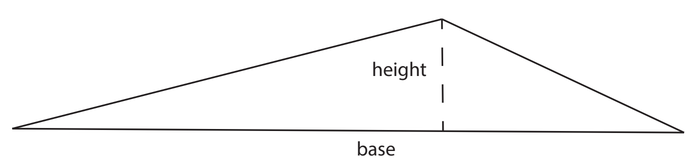
Explain that students can determine the area of the obtuse triangle by creating a parallelogram, or they can use the formula for a triangle using two height options.

Demonstrate that like a parallelogram in Lesson 1, the height of the triangle can be found outside of the figure.



Require students to glue one obtuse triangle on Question 15 of the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT. Students should label the height and base of the triangle as you have demonstrated.

Demonstrate that the triangle can be rotated and the longest side can be used as the base, which would make the segment connecting the opposite vertex the height.



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Require students to glue the second obtuse triangle on Question 15 of the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT. Students should label the height and base of the triangle as you have demonstrated.

Emphasize that the height of the triangle is perpendicular to the base of the triangle.

Review the formula to calculate the area of a triangle that was introduced earlier in the lesson.

$$\frac{1}{2}\, \textit{base} \bullet \textit{height} \text{ or } \frac{\textit{base} \cdot \textit{height}}{2}$$

Students should be required to complete the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT in groups of two or three. Use the guiding questions to support student understanding.

GUIDING QUESTIONS

Elicit student thinking:

- What do you notice about the triangle?
- How is this triangle the same as a right triangle? How is it different?

Determine if the student can RECOGNIZE THE HEIGHT OF A TRIANGLE:

- How can you identify the height of a triangle?
- Does the height of a triangle have to be the length of one of the sides? Explain.
- [Point to Question 16 or 17.] What is the height of this triangle? How do you know?
- Does the height of a triangle have to be identified inside the triangle? Explain.

Determine if the student can CALCULATE AREA OF A TRIANGLE WITH THE FORMULA:

- [Point to Question 16 or 17.] What is the area of this triangle? How do you know?

At the end of the activity, teachers should review student responses to Question 18 on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT to determine student understanding.

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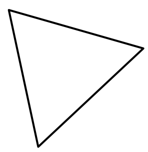
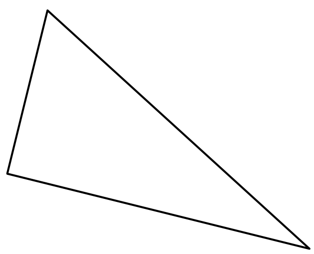
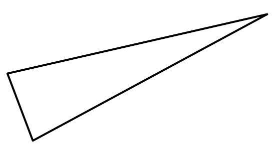
Composing and Decomposing to Determine Area

Lesson 2

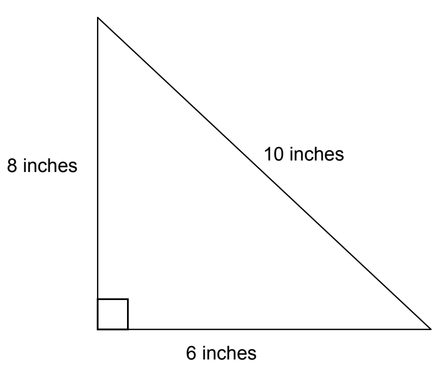
1. Describe what you know about the following shapes.







2. Determine the area of the triangle. Show all your work.



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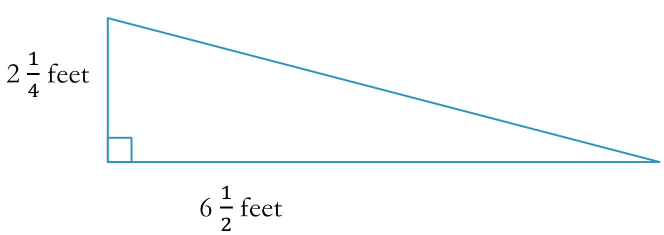
3. Glue both of the triangles together as a rectangle, then measure and label the base and the height of the rectangle. Calculate the area of a single triangle in square inches. Show all your work.

4. Compose the triangle into a rectangle in order to calculate the area of the triangle.



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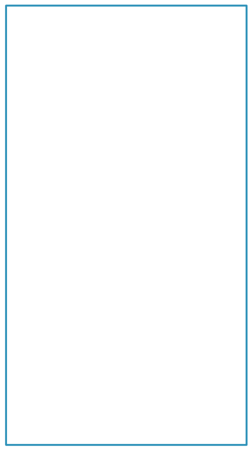
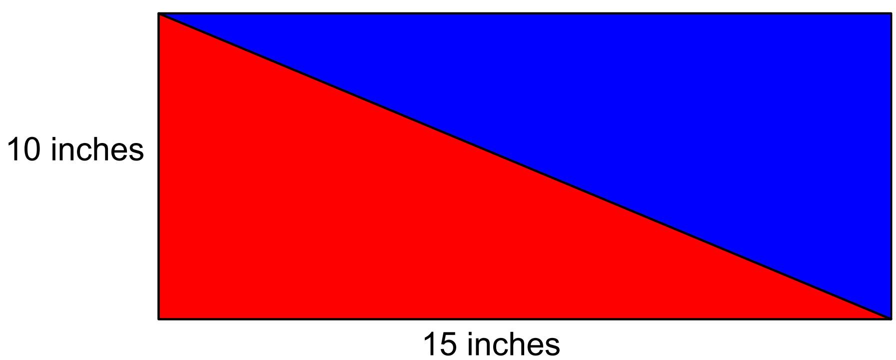
5. Decompose the rectangle into two triangles and determine the area of one triangle.



height = $5\frac{2}{5}$ centimeters

base = $2\frac{2}{3}$ centimeters

6. Dawn is making a cake for her football party. She is going to ice the cake using two colors, one for each team. If she divides the cake into two equal triangles, what is the area she will need to frost for each team in square inches?



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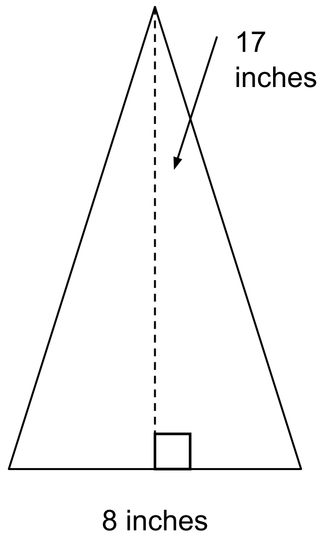
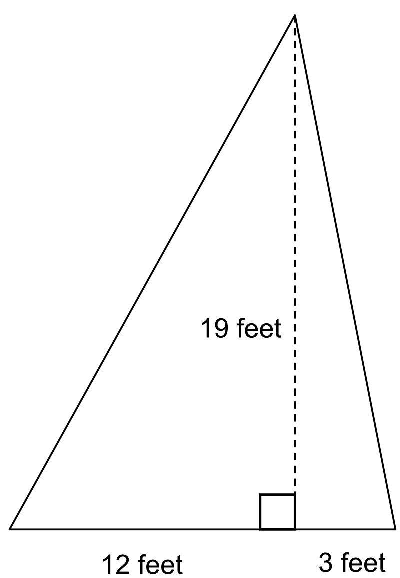
7. Determine the area of the triangle by creating one or two rectangles. Show all your work.



8. Determine the area of the triangle by creating one or two rectangles. Show all your work.



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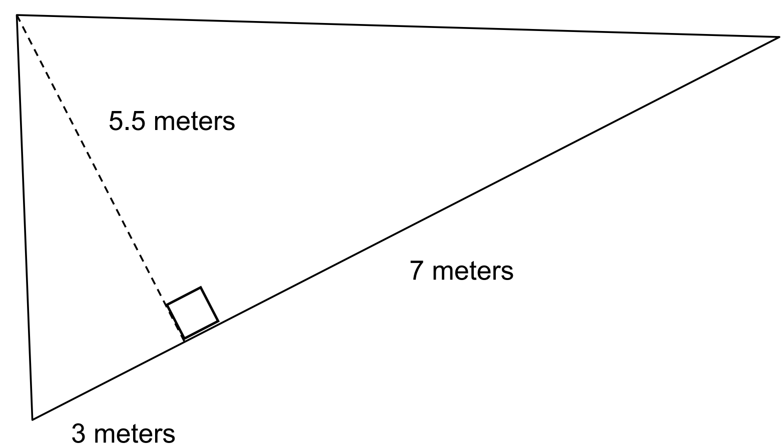
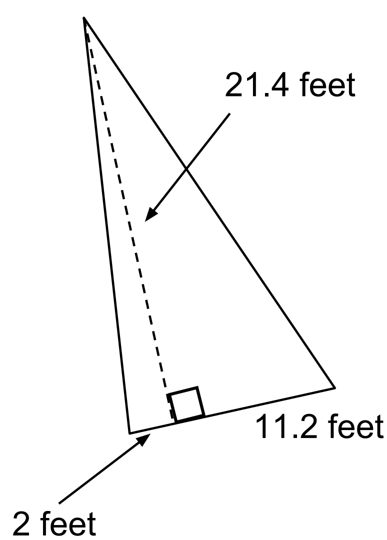
9. Determine the area of the triangle by creating one or two rectangles. Show all your work.



10. Determine the area of the triangle by creating one or two rectangles. Show all your work.



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11. Glue and measure the triangles as a rectangle. Calculate the area in square inches. Show all your work.

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12. Explain why you do not need to multiply by $\frac{1}{2}$ (or divide by 2) when you bisect an isosceles or equilateral triangle and rearrange the pieces to compose a rectangle.

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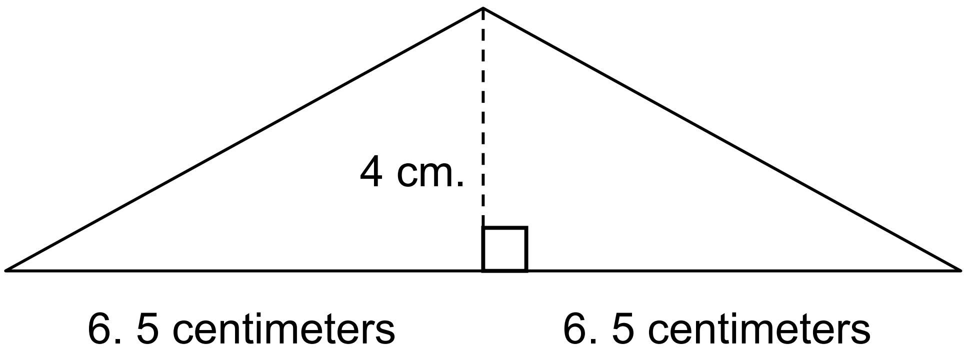
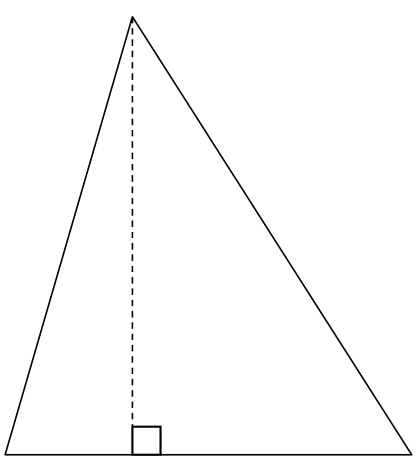
13. Divide the triangle and compose a rectangle to determine the area. Show all your work.



14. Can you divide the following triangle and create a single rectangle that has the same area as the original area? Explain why or why not.



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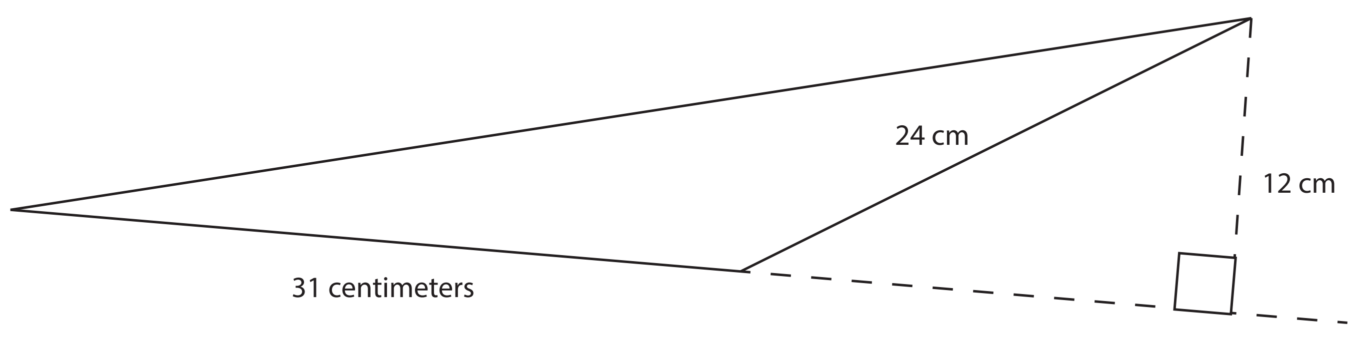
15. Glue and label the bases and heights for each obtuse triangle.

16. Determine the area of the triangle by using the formula. Show all your work.



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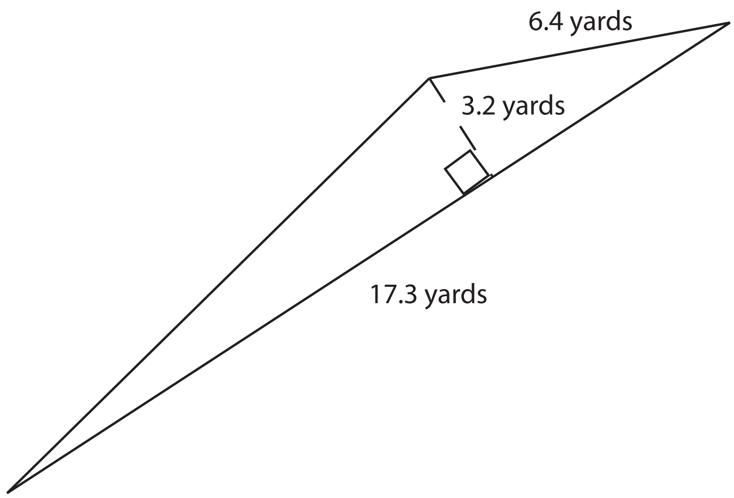
17. Determine the area of the triangle by using the formula. Show all your work.



18. The formula to calculate the area of a triangle is $\frac{1}{2}$ *base* · *height* or $\frac{base \cdot height}{2}$. Explain why this is so, and be sure to support your explanation with a drawing.

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Rectangle

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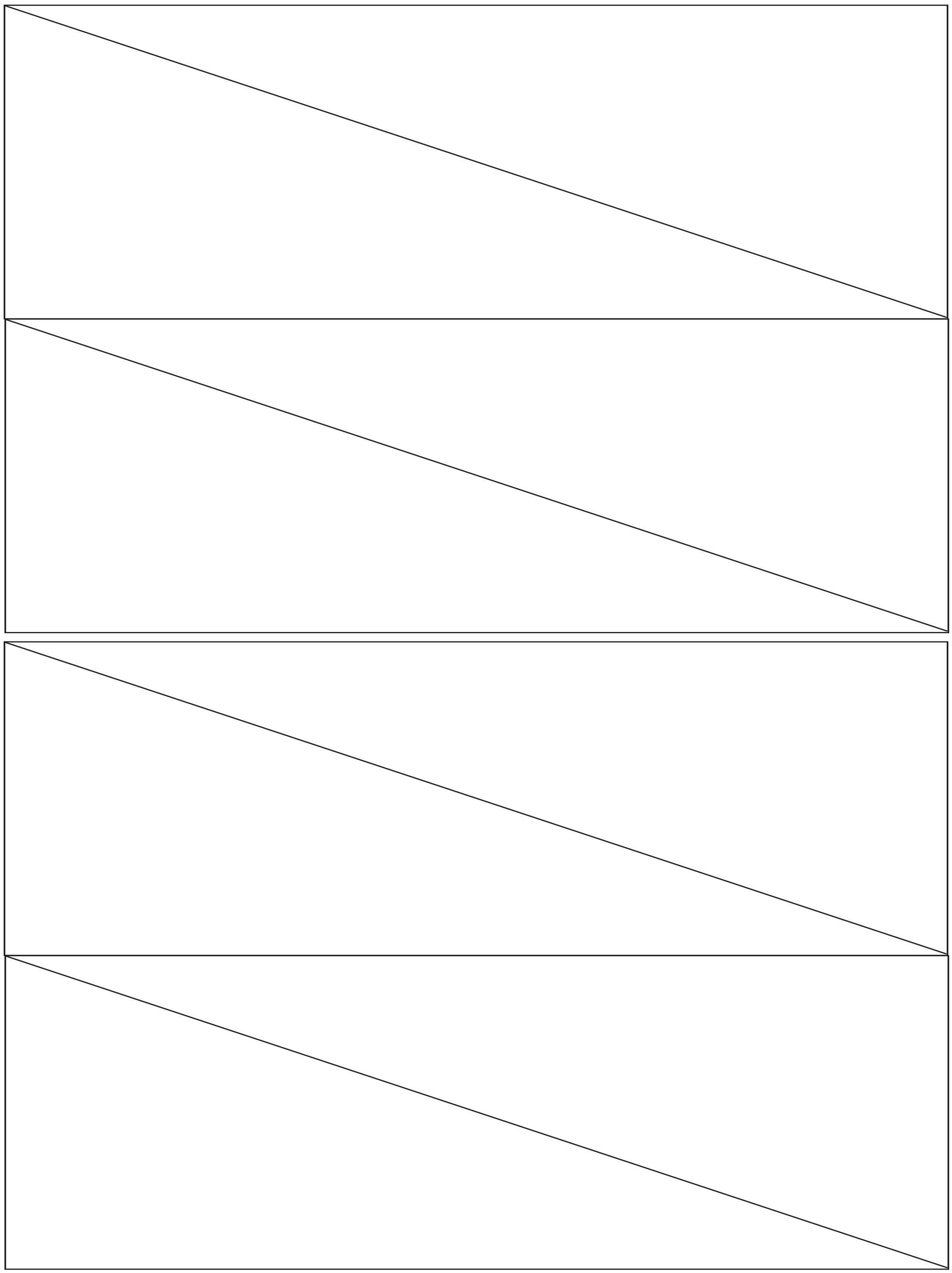
Right triangle



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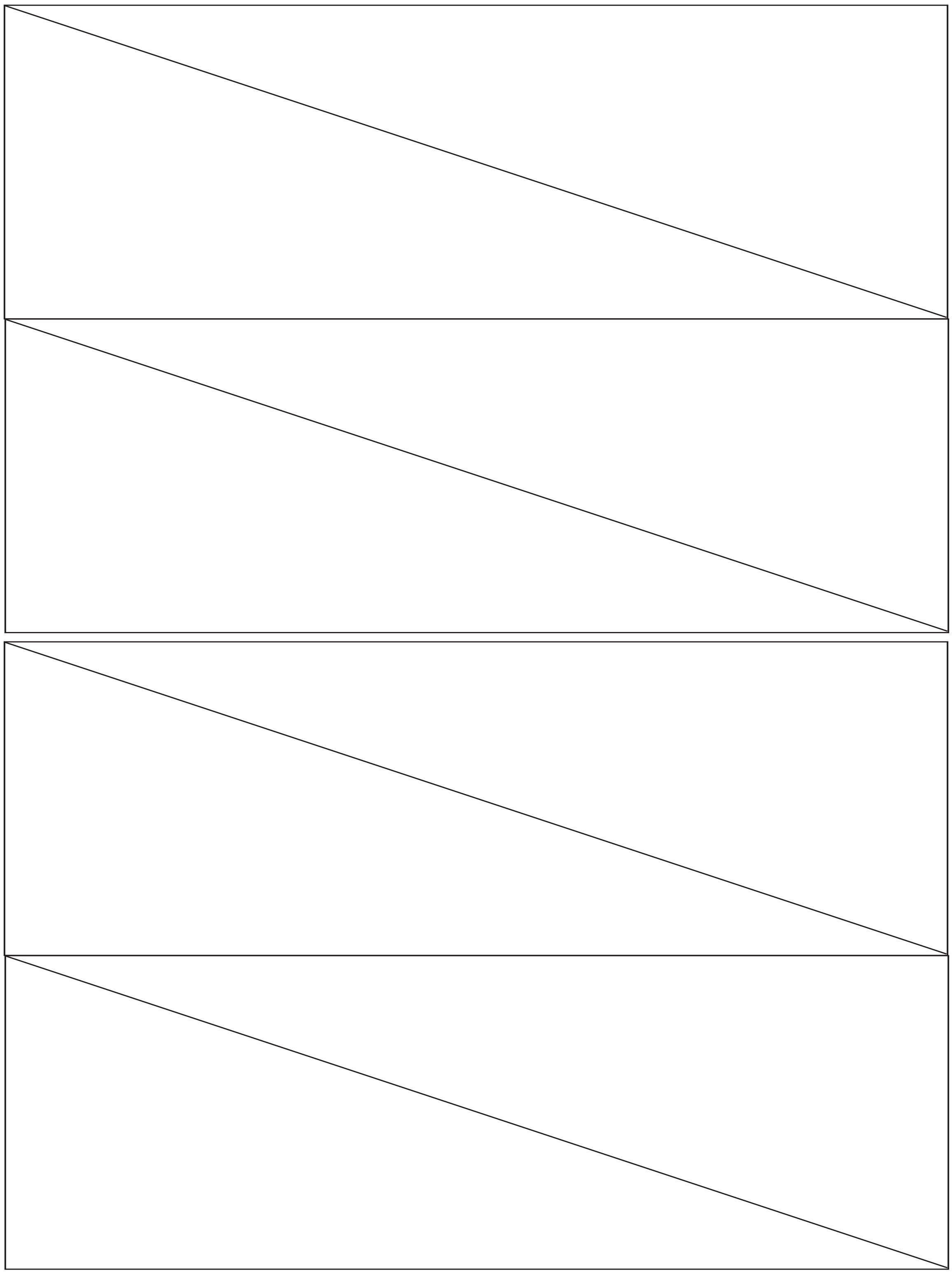
Right triangle



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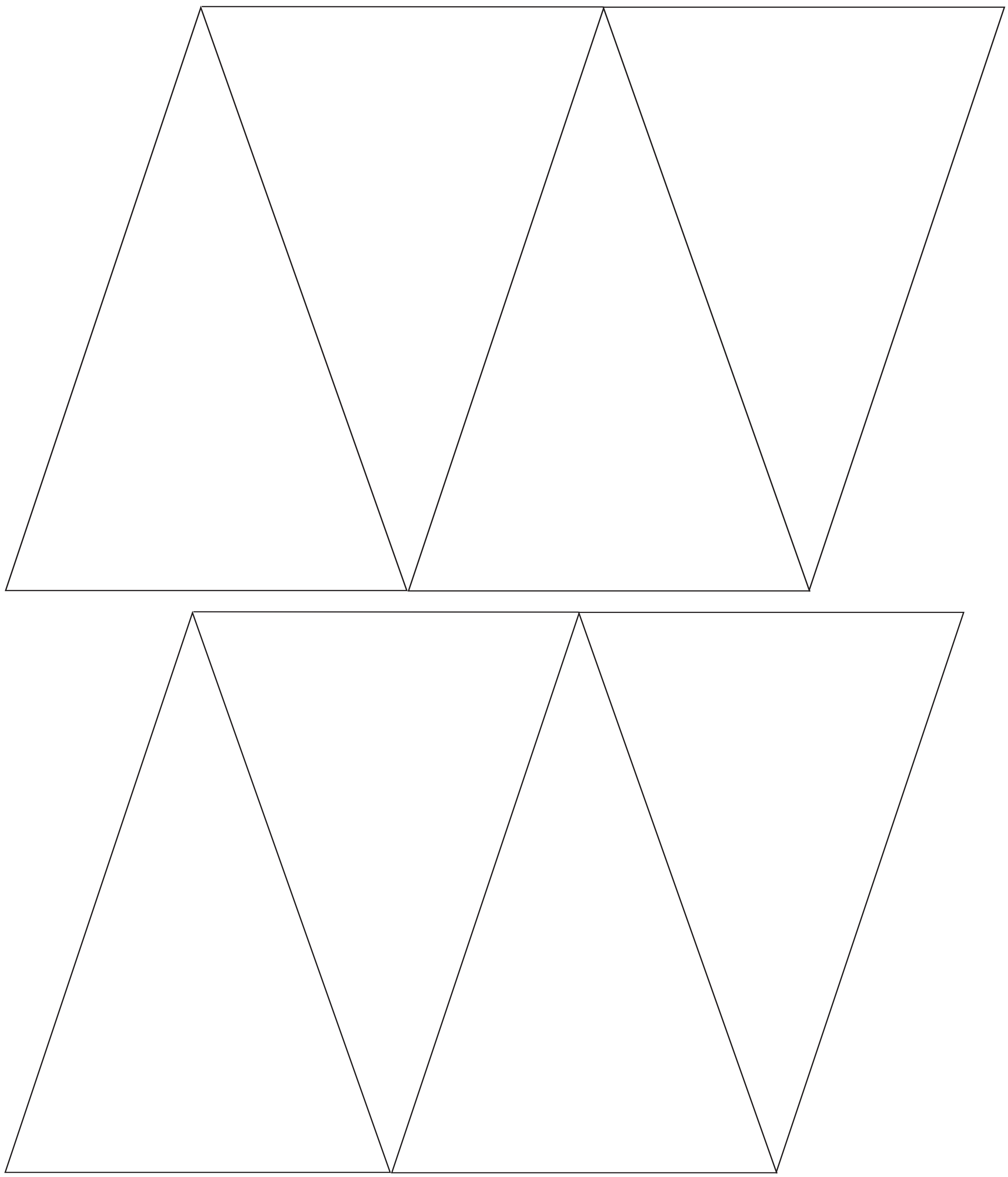
Isosceles triangle



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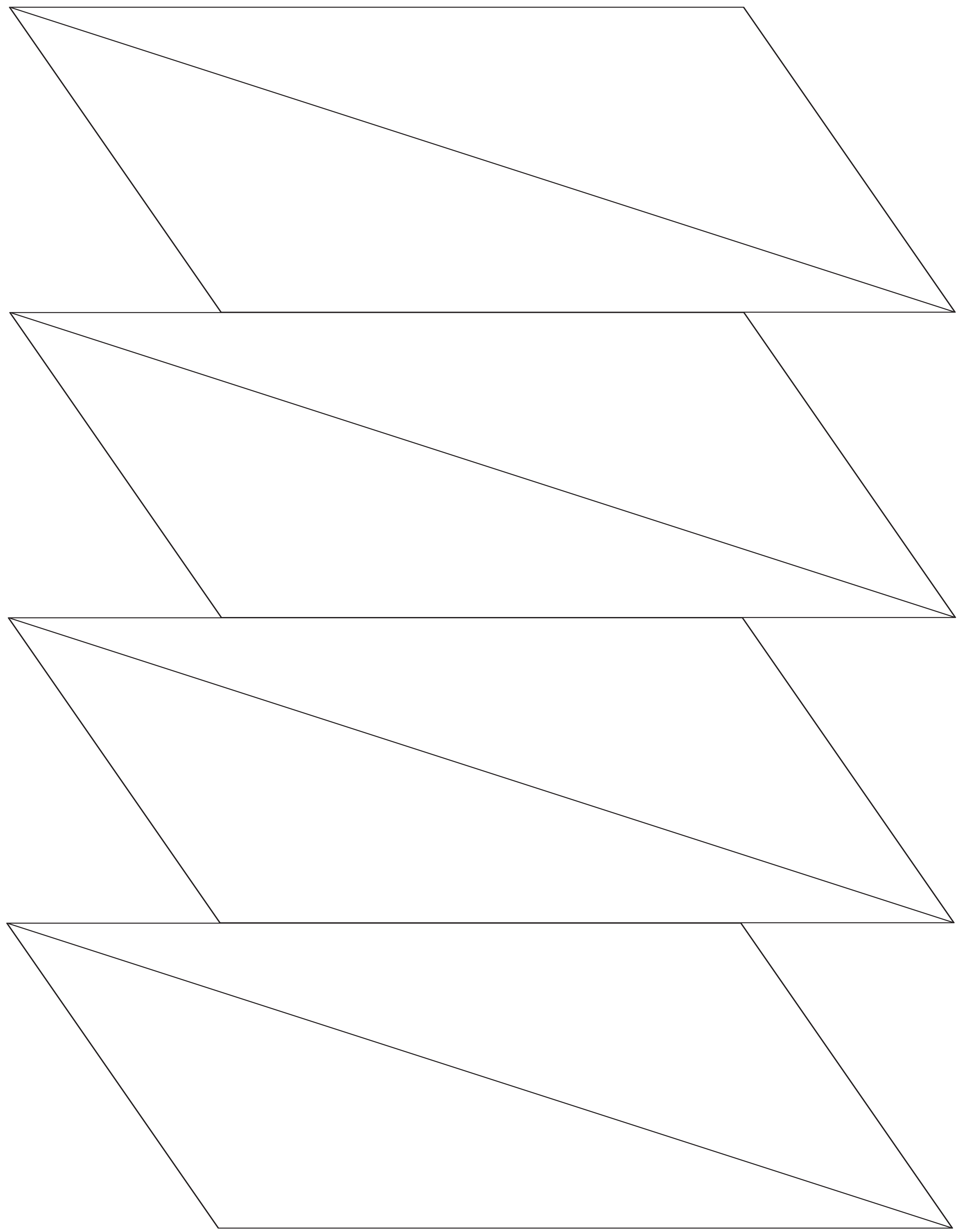
Obtuse triangles



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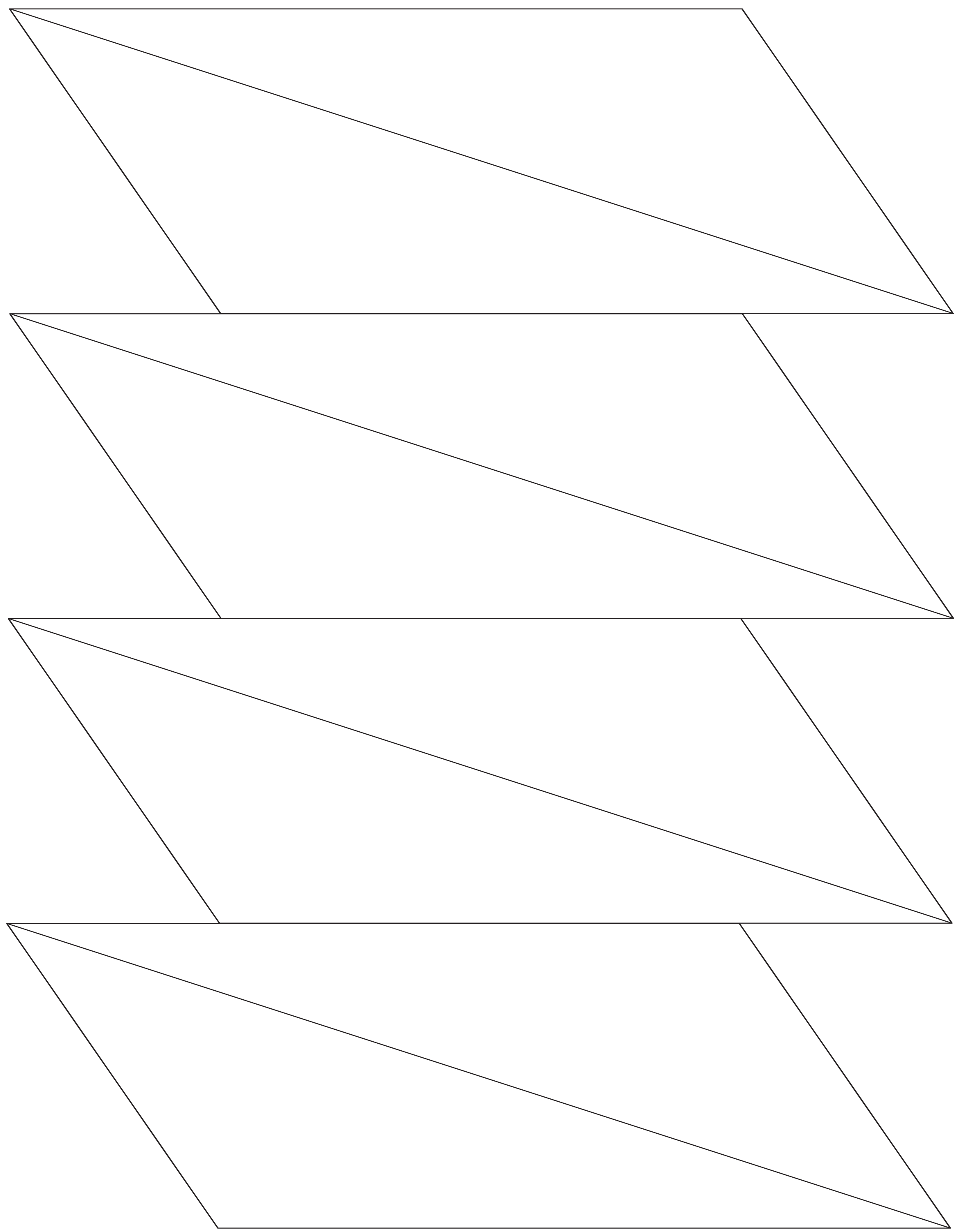
Obtuse triangles



Composing and Decomposing to Determine Area

INSTRUCTIONAL ACTIVITY

Lesson 3

LEARNING GOAL

Students will apply their understanding of the area formulas for rectangles and triangles to calculate the areas of trapezoids and kites. Through composing rectangles and parallelograms and decomposing into rectangles and triangles, students will gain a conceptual awareness of both the trapezoid and kite area formulas.

PRIMARY ACTIVITY

Students will cut apart trapezoids and kites to develop an understanding of how the decomposed shapes can be rearranged in order to compose a parallelogram or a rectangle.

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

- Trapezoid
- Kite
- Base
- Height (altitude)
- Diagonal
- Area
- Square units

MATERIALS

- Instructional Activity Student Handout
- Instructional Activity Supplement (Recommend one for every five to six students, except page one, which you should have one of for the whole class.)
- Scissors
- Glue sticks or glue bottles
- Rulers

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IMPLEMENTATION

Review by asking students the following questions:

- How do you calculate the area of a rectangle or a parallelogram?
- How do you calculate the area of a triangle?
- [Display a trapezoid.] What is the name of this figure? How do you know?
- [Display a trapezoid that looks different than the first trapezoid.] What is the name of this figure? How do you know?

> NOTE: This INSTRUCTIONAL ACTIVITY utilizes the definition of a trapezoid as a quadrilateral with *exactly* one set of parallel sides. Reference your local curriculum for the definition your materials provide to make adjustments if needed.

Distribute the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

Require students to decompose the trapezoid on Question 1 of the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT into parallelograms, rectangles, and/or triangles.

Identify several students with a variety of solutions to share their decompositions with the class, and **discuss** each decomposition as it is presented.

Arrange students into groups of three or four. **Provide** each group with a different trapezoid from page one of the INSTRUCTIONAL ACTIVITY SUPPLEMENT. **Encourage** students to manipulate (e.g., folding or cutting) the trapezoid to decompose it into rectangles, parallelograms, and/or triangles.

Require each group to determine the area of the given trapezoid using the dimensions provided on the shape. **Challenge** each group to create a formula (either formal or informal) to calculate the area of any given trapezoid.

> NOTE: There are more and less challenging trapezoid measurements to allow for differentiation of the student groups.

Require each group to share with the class their trapezoid, the area in square units, and their formula. Each group should be prepared to explain how and why their formula works.

Independently or in groups of two or three, students should complete Questions 2 – 5 on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

- 2. *15.3 + 30.6 + 15.3 = 61.2 square centimeters or 30.6 + 30.6 = 61.2 square centimeters*
- 3. *Yes, Sam decomposed the trapezoid correctly. The total area is 51.525 square inches.*
- 4. $60\frac{1}{16}$ *square feet*
- 5. *Both decompositions are acceptable. Check student work for understanding.*

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Require students to share their work in groups of three or four, and have a shared group solution for each question. **Select** one group to share their solution and thinking for each question.

GUIDING QUESTIONS

Elicit student thinking:

- [Point to a trapezoid.] What do you notice about this shape?
- When have you seen a trapezoid in everyday life?

Determine if the student can SOLVE PROBLEMS INVOLVING AREA OF POLYGONS:

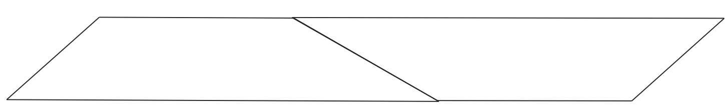
- [Point to any trapezoid.] What shapes could you decompose this trapezoid into?
- If you decompose a trapezoid to determine the area, what do you need to do with the areas of those figures to determine the total area of the trapezoid?
- When you decompose a trapezoid, do you have to have at least one triangle? Explain.
- Can you decompose a trapezoid without creating a rectangle? Explain.

Distribute two trapezoids to each student from page two of the INSTRUCTIONAL ACTIVITY SUPPLEMENT.

Ask students if there is any way to create a rectangle or parallelogram using two trapezoids without cutting either trapezoid, like they did to determine the area of a triangle. Give students a few minutes to manipulate their two trapezoids to attempt to answer the question.

Discuss student strategies, **require** two or three students to share their thoughts.

Model for students how to combine the two trapezoids to create a parallelogram. Students should then glue the trapezoids together as a parallelogram on Question 6 of the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.



Identify that there are two bases, $base_1$ and $base_2$, and **ask** students why they think there are two bases.

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Explain that unlike parallelograms and rectangles, the two opposite, parallel sides of a trapezoid are not the same length. Students should label the height of the trapezoids as four meters, the length of $base_1$ as five meters, and the length of $base_2$ as nine meters.

Ask students how they would determine the area of one of the trapezoids using the area of the composed parallelogram. Use this example to lead students to the formula to determine the area of a trapezoid.

$$A = \frac{1}{2}(base_1 + base_2) \bullet \text{height} \qquad \text{Or} \qquad A = \frac{base_1 + base_2}{2} \bullet \text{height}$$

Require students to determine the area of one of the trapezoids either independently or in groups of two or three, using the formula either formally or informally. (28 square meters)

Emphasize that like they did with the triangles, students should double the trapezoid, therefore the area of one trapezoid is only one-half of the total of the combined parallelogram.

Require students to answer Questions 7 – 10 on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT in groups of two or three. Students do not have to use the standard formula to determine the area of a trapezoid; if they are given enough information, they may decompose the trapezoid using a method they understand and are comfortable implementing.

Use the guiding questions to support student learning.

GUIDING QUESTIONS

Elicit student thinking:

- [Point to a trapezoid.] What do you notice about this shape?
- How is a trapezoid different from a parallelogram?
- When have you seen a trapezoid in everyday life?

Determine if the student can CALCULATE THE AREA OF A TRAPEZOID WITH THE FORMULA:

- [Point to any question 7 – 10.] What is the height of this trapezoid? How long are each of the bases?
- If you double the trapezoid to create a parallelogram, what do you need to do to determine the total area of one trapezoid?

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Determine if the student can SOLVE PROBLEMS INVOLVING AREA OF POLYGONS:

- [Point to any trapezoid.] Into what shapes could you decompose this trapezoid?
- If you decompose a trapezoid to determine the area, what do you need to do with the areas of those figures to determine the total area of the trapezoid?
- [Point to Question 9.] Can you decompose this trapezoid into triangles and/or rectangles? Explain why or why not.

Review student answers, allowing students with alternate formulas or strategies to share their work with the class.

- 7. 4.875 square feet
- 8. 39 square feet
- 9. 50 square centimeters
- 10. 72 square inches

Display a kite. **Ask** students to name the figure and to support their response by explaining the attributes of a kite.

Require students to decompose the kite on Question 11 of the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

Identify students with a variety of solutions to share their decompositions with the class, **discuss** each decomposition as it is presented.

Distribute scissors, a ruler, and two kites from page three of the INSTRUCTIONAL ACTIVITY SUPPLEMENT to each student. Group students into groups of two or three.

Require students to manipulate (cut apart) one of their kites to see if they can rearrange the kite so that they create either a parallelogram or a rectangle.

Ask two or three groups to share their thinking with the class.

Model drawing and measuring both diagonals inside the kite. **Require** students to draw and measure the diagonals on their second kite that has not been cut apart.

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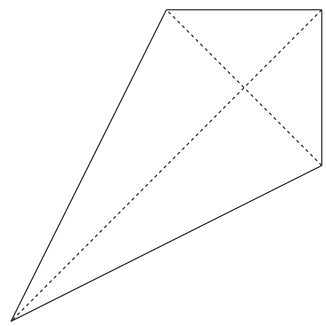
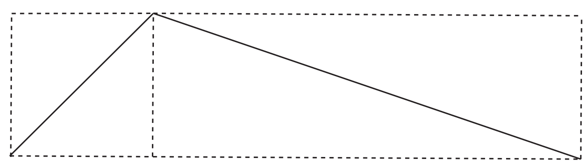


Model and **require** students to cut the kite along the longer diagonal to create two scalene triangles. Students may then glue one-half of the kite onto Question 12 of the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

Next, **cut** the remaining half of the kite into two triangles along the diagonal and **rearrange** the triangles so you create one rectangle.

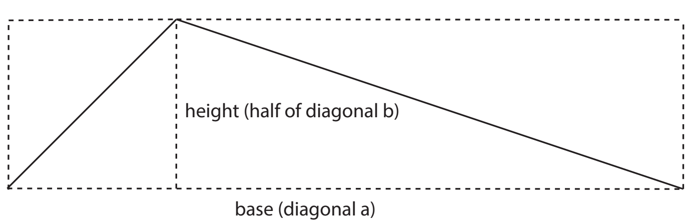
Students should then glue the remaining triangles on to Question 12 of the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT, to create a rectangle.



Ask students the following questions.

- "Have the lengths of any of the sides changed as a result of rearranging the kite?"
- "Have the lengths of either of the diagonals changed as a result of rearranged the kite?"
- "Can we just multiply base times height to determine the area of the original kite? Why or why not?"

Explain that when you cut apart and rearranged the kite, you cut one of the diagonals in half, therefore the height of the rectangle is only one-half the length of that diagonal. Because you divided that diagonal in half, multiply one diagonal length times one-half of the other diagonal length to determine the area.



Students can use the following formula, either formally or informally, to determine the area of kites.

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$$A = \frac{1}{2}(\boldsymbol{diagonal_1} + \boldsymbol{diagonal_2}) \quad \text{Or} \quad A = \frac{\boldsymbol{diagonal_1} + \boldsymbol{diagonal_2}}{2}$$

Display and **model** calculating the area for the kite on Question 13 of the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT. Students should then copy the modeled work on their INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

Ask students if there is another way to decompose or compose the kite in order to determine the area. Although it requires more work, some students may be more comfortable decomposing the kite into rectangles or into two or even four triangles to determine the area.

Require students to complete Questions 14 – 17 on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT in groups of two or three.

Use the guiding questions to support student understanding.

GUIDING QUESTIONS

Elicit student thinking:

- [Point to a kite.] What do you notice about this shape?
- How is a kite different than a rectangle?
- How is a kite the same as a rectangle?
- When do you see a kite in everyday life?

Determine if the student can CALCULATE THE AREA OF A KITE WITH THE FORMULA:

- [Point to any question 14 – 17.] What is the length of one diagonal of this kite? How do you know? What is the length of the second diagonal of this kite? How do you know?
- Do you need to know the side lengths to determine the area of a kite?
- When you use the formula, why do you divide the product of the diagonals by two?

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Determine if the student can SOLVE PROBLEMS INVOLVING AREA OF POLYGONS:

- [Point to a kite.] Into what shapes could you decompose this kite?
- If you decompose a kite into two triangles to determine the area, what do you need to do with the areas of those triangles to determine the total area of the kite? Or four triangles?
- [Point to a kite.] How would you decompose this kite to determine the area?

At the end of the activity, teachers should arrange students into groups of three or four. On a sheet of paper, each student should add a drawing or words summarizing what they learned about determining the area of trapezoids and kites. Each group should then be required to present their paper to the class, explaining the meaning of the words and/or drawings.

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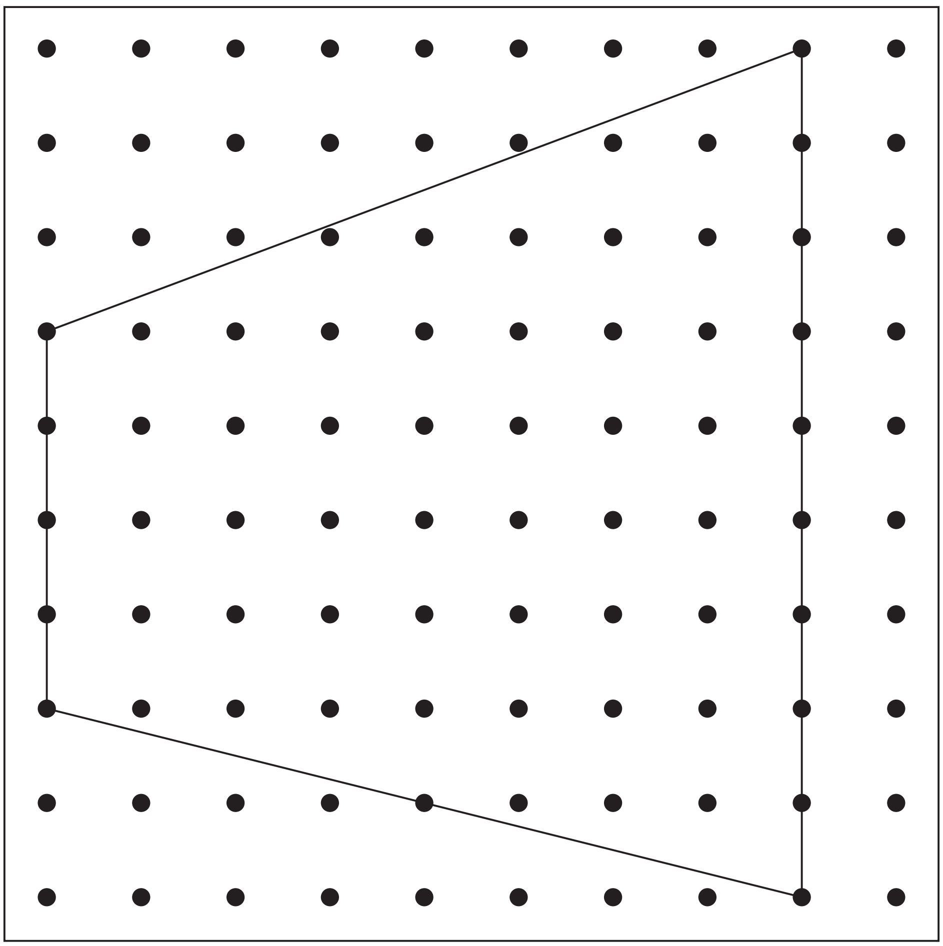
Composing and Decomposing to Determine Area

Lesson 3

1. Decompose the trapezoid into parallelograms, rectangles, and/or triangles.



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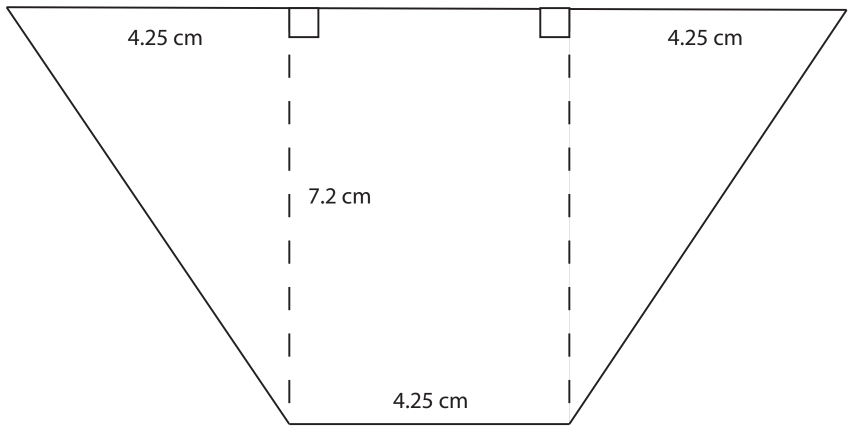
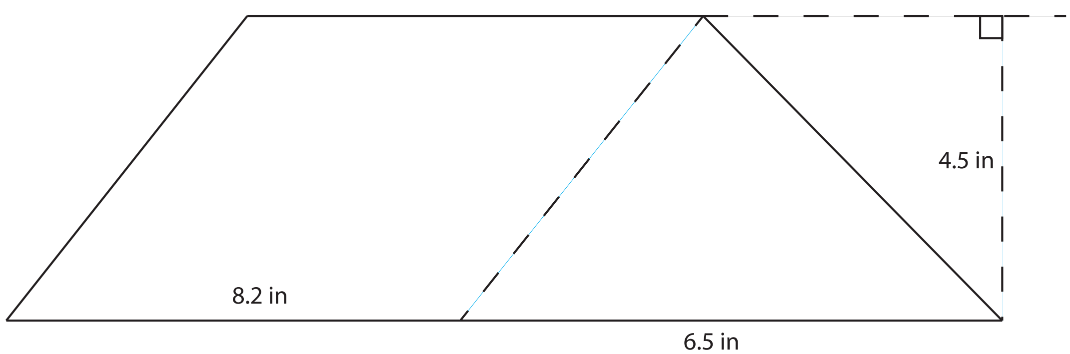
2. Decompose and calculate the area of the trapezoid. Show all your work.



3. Sam decomposed the following trapezoid to determine the area. Did Sam use a correct strategy? Explain why or why not, and provide the total area for the trapezoid. Show all your work.



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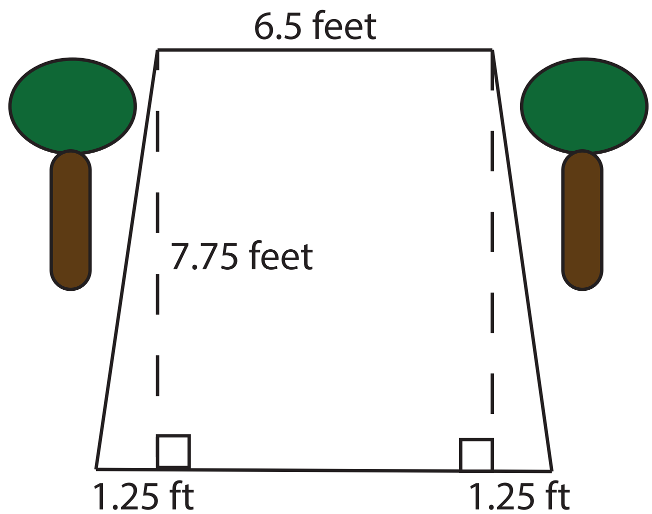
4. Mona is going to spread new soil on top of her garden. She needs to know the area in order to determine how much soil to purchase. Her garden is in the shape of a trapezoid to avoid her two fruit trees. What is the area of Mona's garden? Show all your work.



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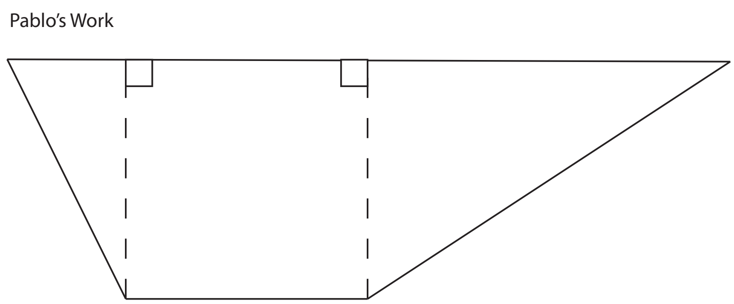
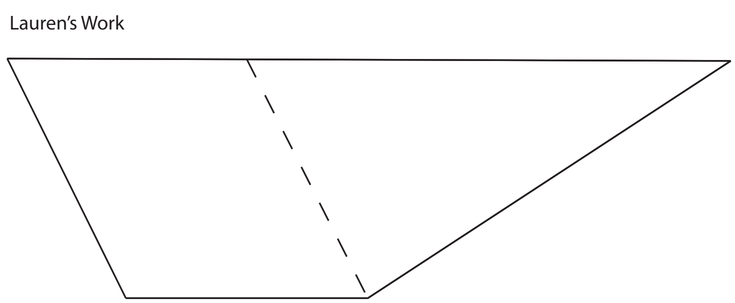
5. Pablo and Lauren decomposed the following trapezoid in order to calculate the area. Do you agree with Pablo's work, Lauren's work, both, or neither? Explain.





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6. Glue two trapezoids together as a parallelogram. Label the lengths for the height and both bases. Determine the area of one trapezoid. Show all your work.

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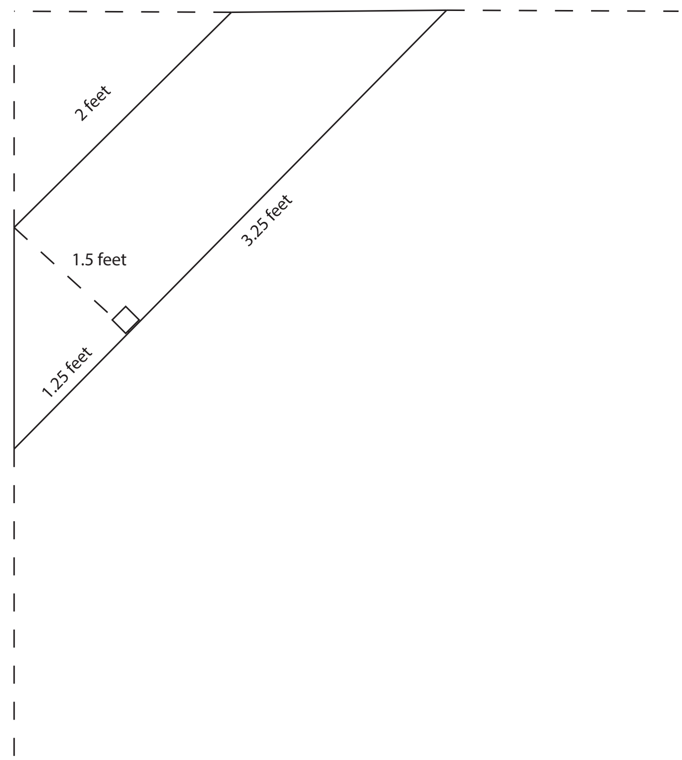
7. Adam made a bookcase to fit in the corner of his living room. From the top, the bookcase looks like a trapezoid. How much space in square feet will Adam need for his bookcase? Show all your work.



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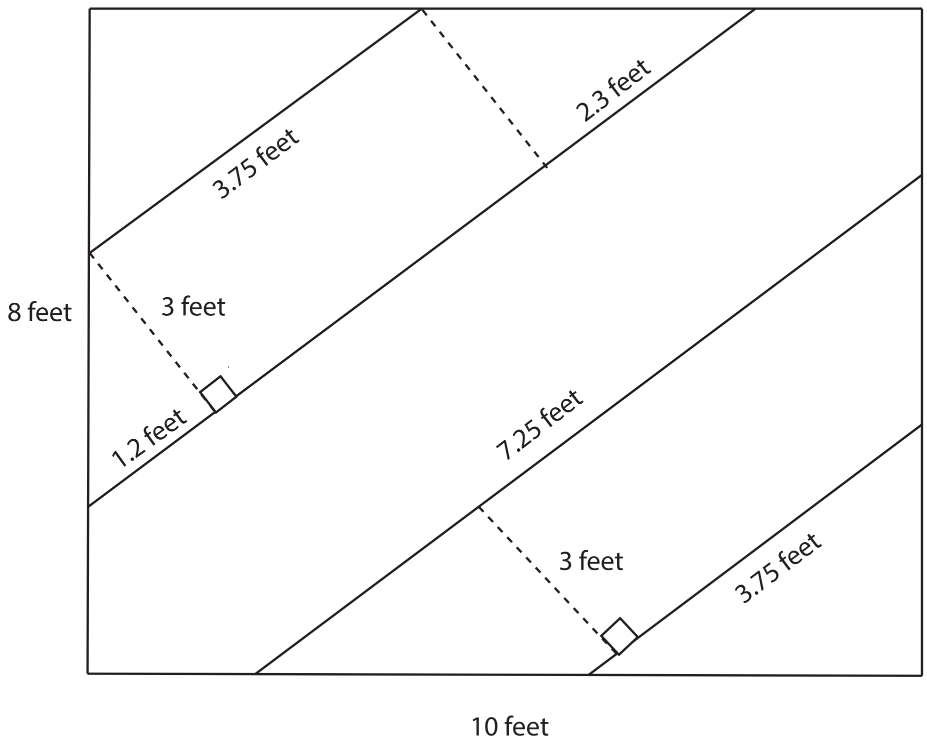
8. Suzanne is redecorating her bedroom. She is going to paint two diagonal stripes of blue across one wall. What is the total area of the wall that will be covered in blue paint? Show all your work.



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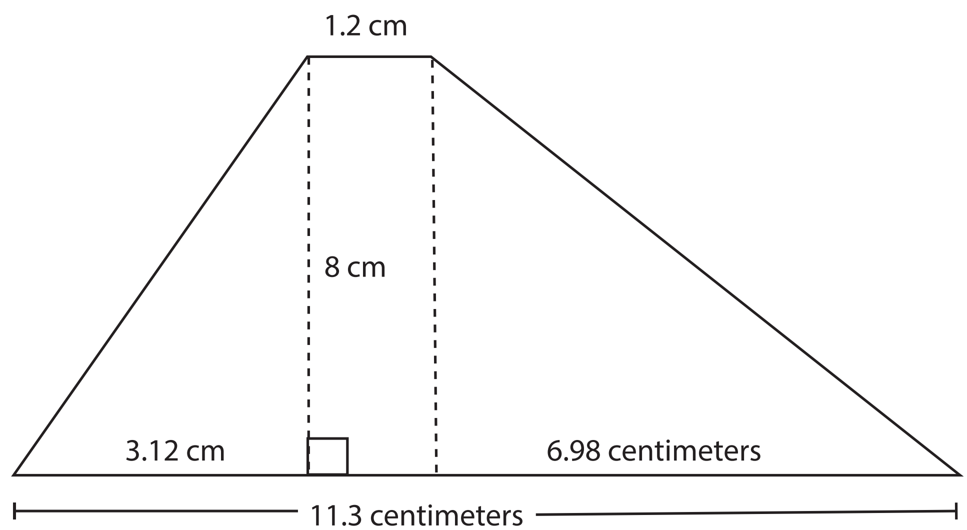
9. Calculate the area of the following trapezoid in square centimeters. Show all your work.



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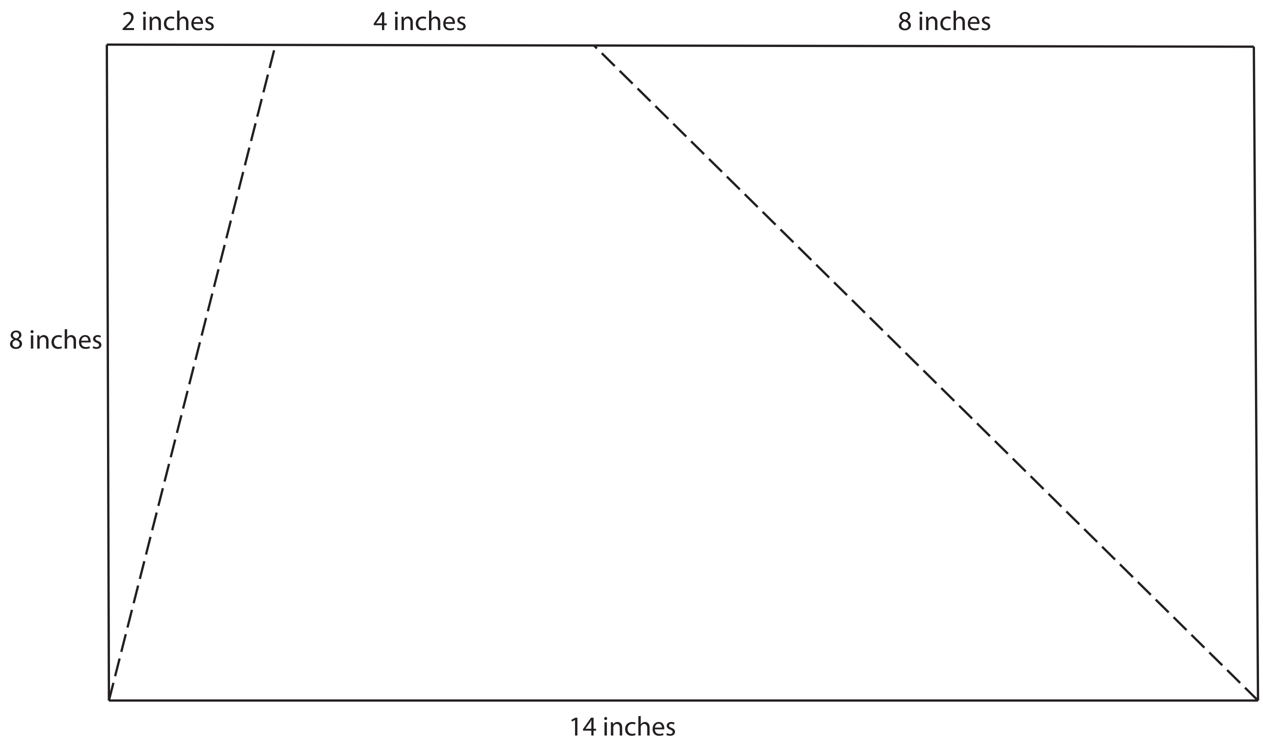
10. During art class, Mandie cut two triangles from her rectangular piece of paper. How much of her original piece of paper will Mandie have left? Show all your work.



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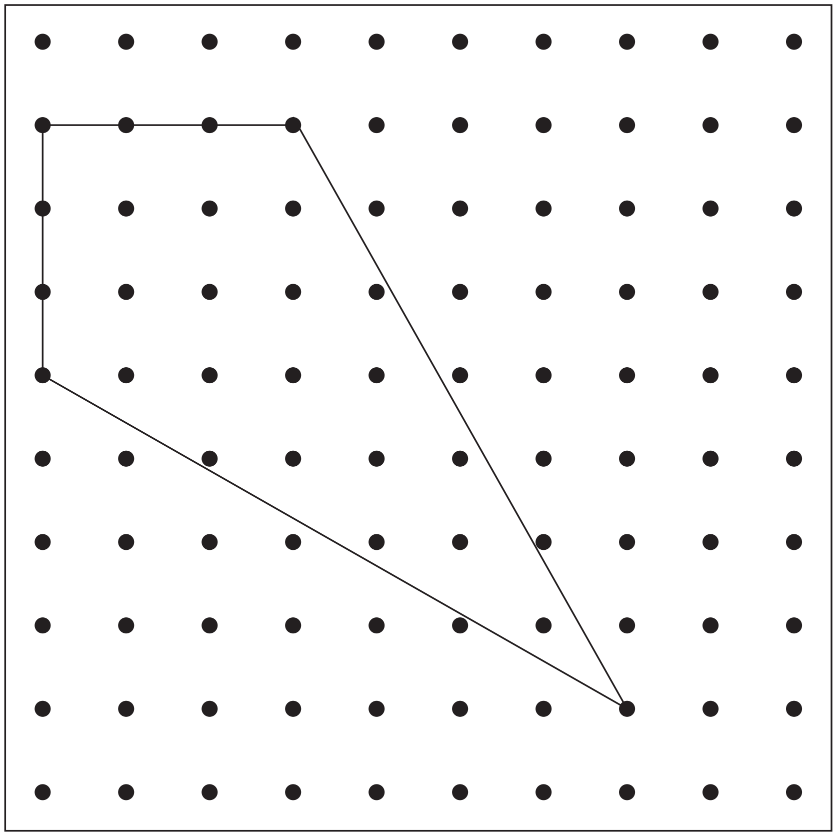
11. Decompose the kite into parallelograms, rectangles, and/or triangles.



12. Glue the pieces of the kite together as a rectangle.

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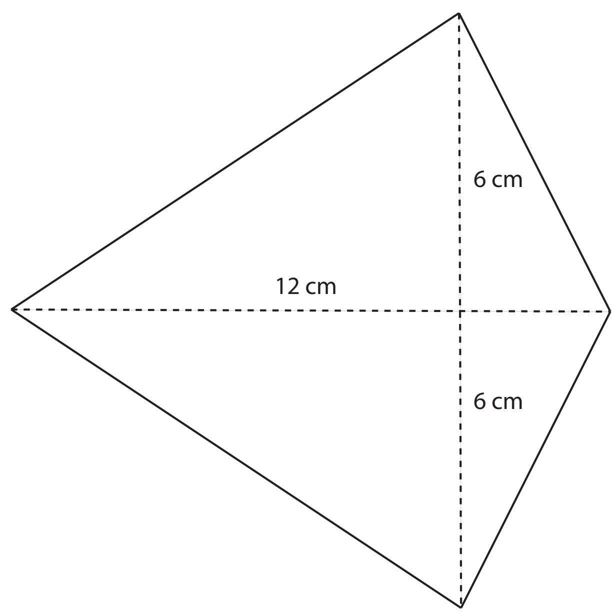
13. Rearrange the kite as a rectangle in order to determine the area of the kite.



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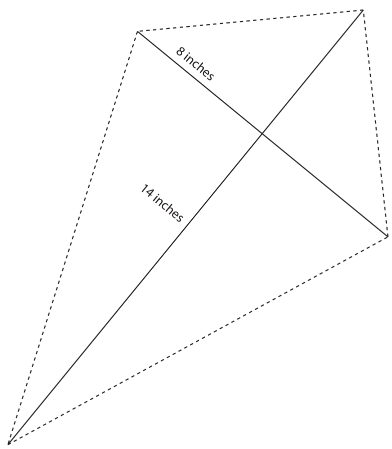
14. Liam is making a kite for his Chinese history project. He wants to cover his kite with fabric. If one of the sticks for his kite is 14 inches and the second stick for his kite is 8 inches, how much fabric in square inches will Liam need for his kite? Decompose and/or rearrange the kite in order to determine the area. Show all your work.



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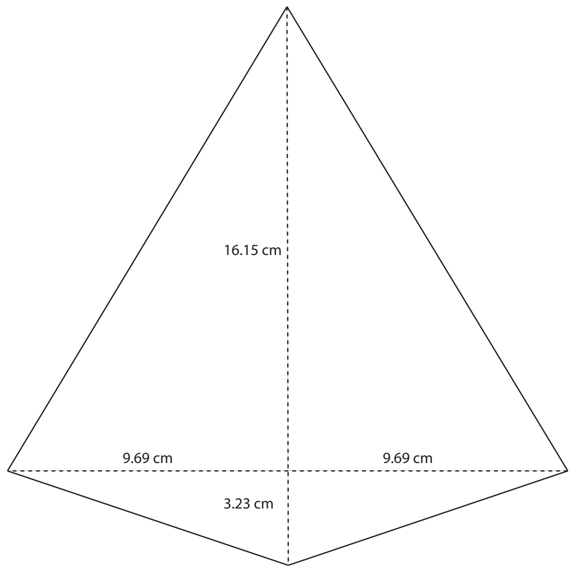
15. Decompose and/or rearrange the kite in order to determine the area. Show all your work.



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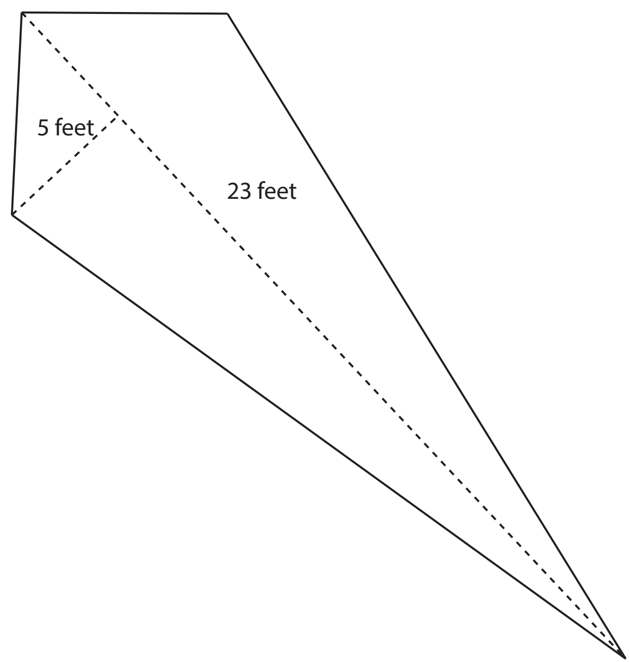
16. Julia thinks that she can determine the area of the following kite with the information given. Zoe does not think there is enough information provided to determine the area of the kite. Who do you think is correct? Why? If you think Julia is correct, calculate the area of the kite. If you think Zoe is correct, what information is still needed to calculate the area of the kite?



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17. Use a ruler to draw a kite that has one diagonal that is 11 inches long and a second diagonal that is $5\frac{1}{2}$ inches long. Decompose and/or rearrange the kite in order to determine the area. Show all your work.

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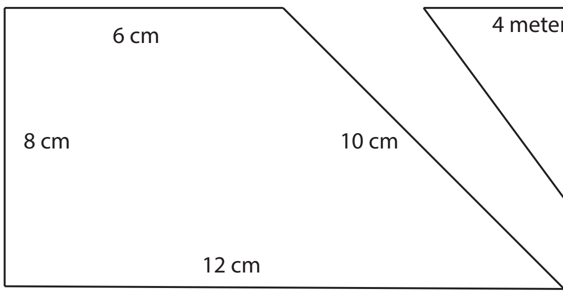
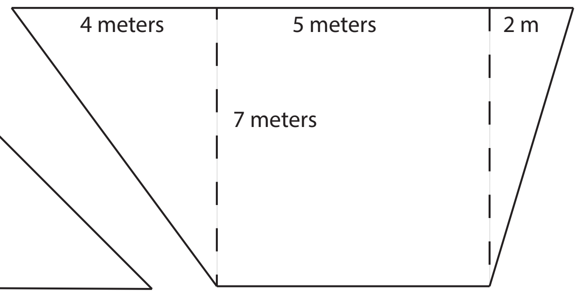
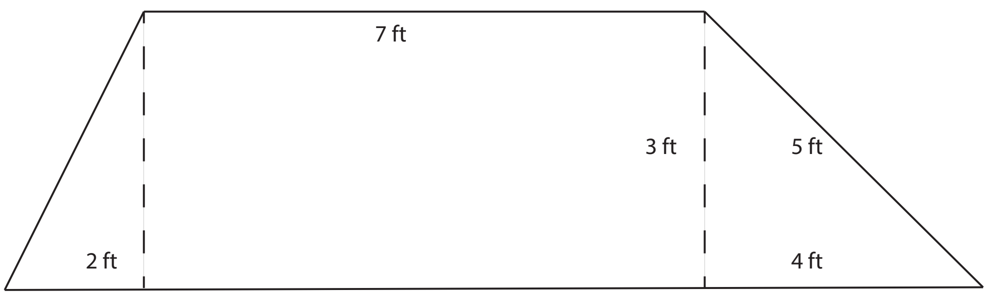
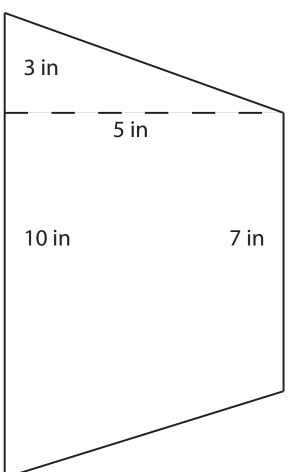
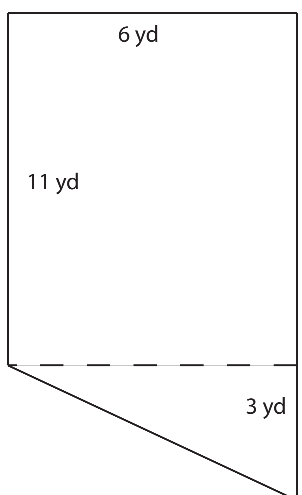
Trapezoids (1)

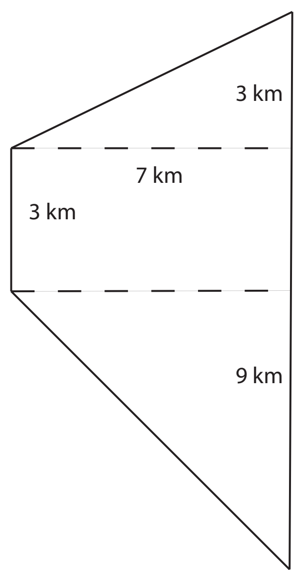


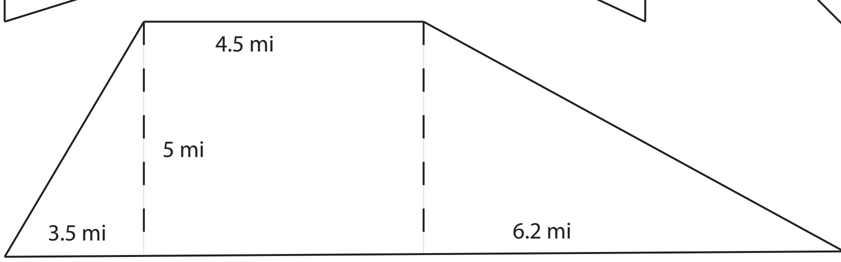












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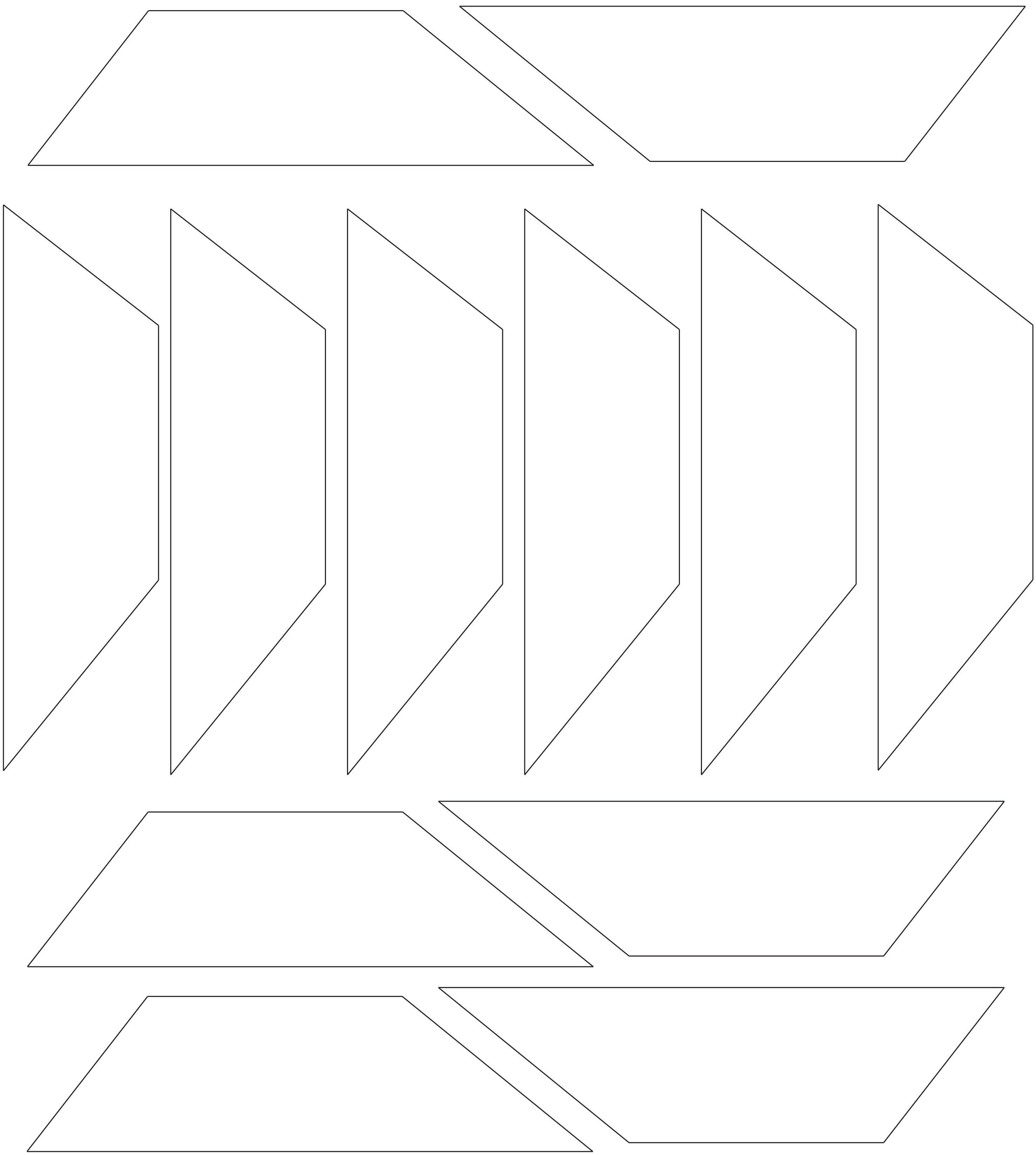
Trapezoids (2)



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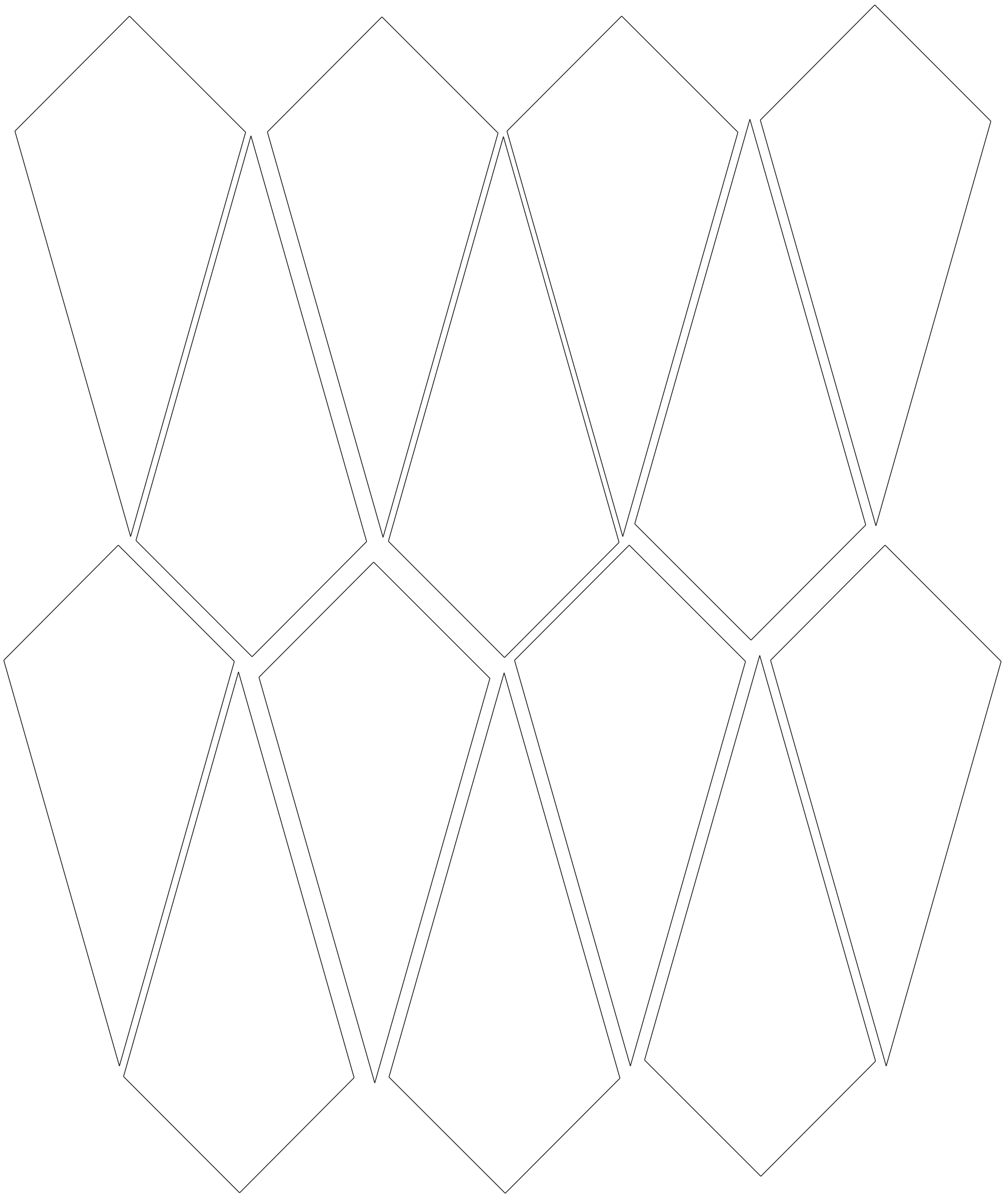
Kite



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Composing and Decomposing to Determine Area

INSTRUCTIONAL ACTIVITY

Lesson 4

LEARNING GOAL

Students will use their understanding of area formulas for rectangles, parallelograms, triangles, trapezoids, and kites to decompose irregular figures and/or compose irregular figures into rectangles in order to determine the area of the irregular figure.

PRIMARY ACTIVITY

Students will determine the area of a given house floor plan by composing the floor plan into a larger rectangle or decomposing the plan into rectangles, parallelograms, triangles, trapezoids, and kites.

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

- Base
- Height (altitude)
- Area
- Square units
- Composition
- Decomposition

MATERIALS

- Instructional Activity Student Handout
- Instructional Activity Supplement (Recommend one copy for pages one and two, and one copy for every two students for page three.)

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IMPLEMENTATION

Display the floor plan on page one of the INSTRUCTIONAL ACTIVITY SUPPLEMENT.

Ask students the following questions:

- "How would you decompose this floor plan to determine the area?"
- "After you decompose the plan and determine the area of each figure, what do you need to do to determine the total area of the house?"
- "What if you only wanted to know the area of the house and the porch, but not the garage? How would you determine that area?"
- "What if you only wanted to know the area of the garage and the porch? How would you determine that area?"

Bring attention to the fact that not all the sides are labeled.

Ask students if they can determine how long the missing side length is using other information from the floor plan. Students should identify that you can add the 12 feet from the largest rectangle to the 24 feet from the garage, because those two lengths combined are equal to the missing side length.

Write "36 feet" inside the floor plan along the missing side length, so that it is similar to the other measurements.

Model cutting apart the floor plan by cutting the garage and the porch off. You should then have three rectangles.

Require students to determine the area of each rectangle. **Select** a student to share each of the areas, and **write** the area inside the rectangle.

Model putting the three rectangles back together to create a visual image which shows that the three areas need to be added together to find the total area of the floor plan.

Add the three areas for a total area.

Require students to complete Questions 1 – 3 on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT in groups of two or three. To scaffold student awareness of decomposition, students can either draw lines separating the figures or redraw the figures as separate shapes.

Use the guiding questions to support student understanding.

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GUIDING QUESTIONS

Elicit student thinking:

- What is a floor plan?
- How is a floor plan related to area?

Determine if the student can CALCULATE THE AREA OF IRREGULAR SHAPES BY PARTITIONING INTO RECTANGLES:

- How is decomposing an irregular figure the same as decomposing a parallelogram or trapezoid to determine the area? How is it different?
- [Point to the floor plan for Questions 1 – 3.] Into what shapes can you decompose these figures?
- Once you have calculated the area of each shape, how do you determine the total area?
- Can the figure only be decomposed into rectangles? Why or why not?
- [Point to a "room" on the floor plan for Questions 1 – 3.] What is the area of this figure? How do you know?

Determine if the student can SOLVE PROBLEMS INVOLVING AREA OF POLYGONS:

- [Point to any question, 1 – 3.] What is the problem asking you? How do you know?
- Are you determining the area for the entire figure or a fraction of the figure? How do you know?

Review the answers for Questions 1 – 3 on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT by having one or two groups explain/show how they determined the area.

- 1. 356.475 square feet
- 2. 127.65 square feet
- 3. 781.125 square feet

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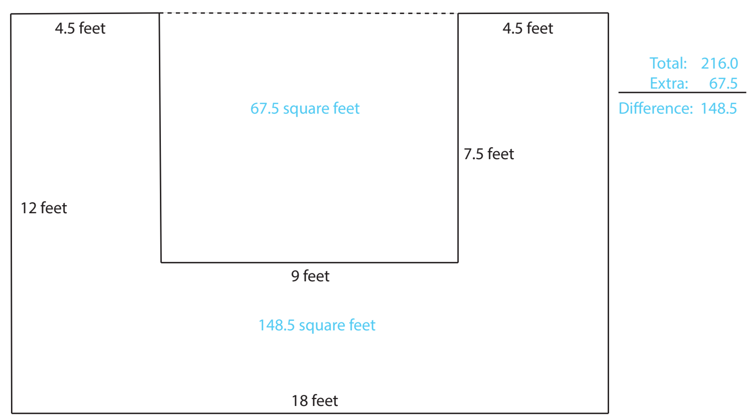
Display the table plan on page one of the INSTRUCTIONAL ACTIVITY SUPPLEMENT. Tell students that this is a table arrangement for a party. The party planners need to know how much space the table will occupy.

Explain that the figure could be decomposed into three rectangles to determine the total area, however it would be more efficient to compose the figure into a single rectangle and then subtract the area of the extra space that was enclosed.

Model composing the figure into a rectangle. **Require** students to calculate the area of the total rectangle and then to write the area to the side of the figure.

Require students to calculate the area of the extra enclosed space. **Write** this area inside the extra space *and* beneath the total area.

Subtract the extra enclosed space from the total area. The difference is the area of the table arrangement.



Explain that deciding whether it is more efficient to compose or decompose an irregular figure to determine the area is not always so apparent. Sometimes it is equally efficient to compose or decompose, and sometimes it depends on the task or the question being asked.

Model how composing or decomposing an irregular figure can be equally efficient with the obstacle course figure from page two of the INSTRUCTIONAL ACTIVITY SUPPLEMENT.

Tell students that the school is hosting an obstacle course as a fundraiser. The image is an outline of the amount of space that is needed for the course. The school needs to know how much space, in square feet, is needed to set up the obstacle course.

Decompose the obstacle course, explaining how to determine the lengths that are not provided by utilizing the lengths you are given.

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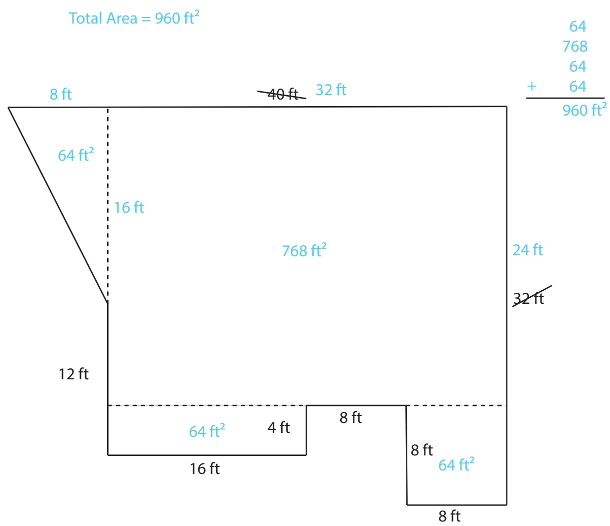
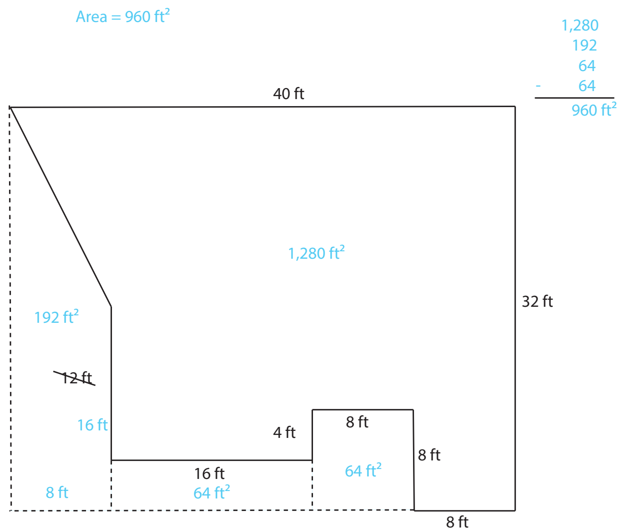


Compose the obstacle course into a rectangle, explaining how to determine the lengths that are not provided by utilizing the lengths you are given.



Bring students' attention to the fact that both the decomposition and composition strategies involved determining four areas to calculate the total area of the irregular figure. One strategy (decomposition) required the four areas to be added, while the second strategy (composition) required three of the areas to be subtracted from a larger total area.

Arrange students into groups of two and distribute one copy of page three of the INSTRUCTIONAL ACTIVITY SUPPLEMENT to each group. Students should sit with the Composition/Decomposition paper between them, so that one student is looking at the Composition half and one student is looking at the Decomposition half.

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The student that is looking at the Decomposition half of the paper will calculate the area of the figure by decomposing it, and the student looking at the Composition half of the paper will calculate the area of the figure by composing a rectangle.

Students should switch strategies for each question by turning the Composition/Decomposition paper, so that each student answers two questions using each strategy.

Require students to complete Questions 4 – 7 on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

Use the guiding questions to support student understanding.

GUIDING QUESTIONS

Determine if the student can CALCULATE THE AREA OF IRREGULAR SHAPES BY PARTITIONING INTO RECTANGLES:

- Once you have calculated the area of each shape, how do you determine the total area?
- How do you calculate the area of one rectangle?
- Can any of the figures in Questions 4 – 7 be partitioned into rectangles only? If so, which ones? How do you know?
- Can the figure only be decomposed into rectangles? Why or why not?

Determine if the student can SOLVE PROBLEMS INVOLVING AREA OF POLYGONS:

- [Point to any question 4 – 7.] How did you decompose this figure?
- Did you use any shapes other than rectangles and triangles? Why or why not?
- Could you decompose this figure into rectangles and triangles? If so, how?
- [Point to any question, 4 – 8.] What is the problem asking? How do you know?
- Are you determining the area for the entire figure or a fraction of the figure? How do you know?
- [Point to any question, 4 – 8.] What strategy are you using to solve this problem? Would you use this approach if you had to solve the same problem (or a similar problem) again? Why or why not?

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Review the answers for Questions 4 – 8 on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

After each answer is identified, poll the class to see if composition or decomposition was the most efficient way to determine the area. **Discuss** why students chose either composition or decomposition as the most efficient strategy.

- 4. *6.5 square kilometers (Composition is the most efficient strategy.)*
- 5. *38,537 square feet (Decomposition is the most efficient strategy.)*
- 6. *270 square feet (Both strategies are equally efficient.)*
- 7. *The area of the pool is less than Mark's backyard: 632 square feet (Composition is the most efficient strategy.)*
- 8. *No, the pool will not fit in Mark's backyard, because no matter what direction he builds the pool, one of the lengths of the pool is longer than the length of his back yard. For example, 22 feet > 20 feet, and 32 feet > 20 feet.*

At the end of the activity, students should answer Question 9 on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT as an exit ticket.

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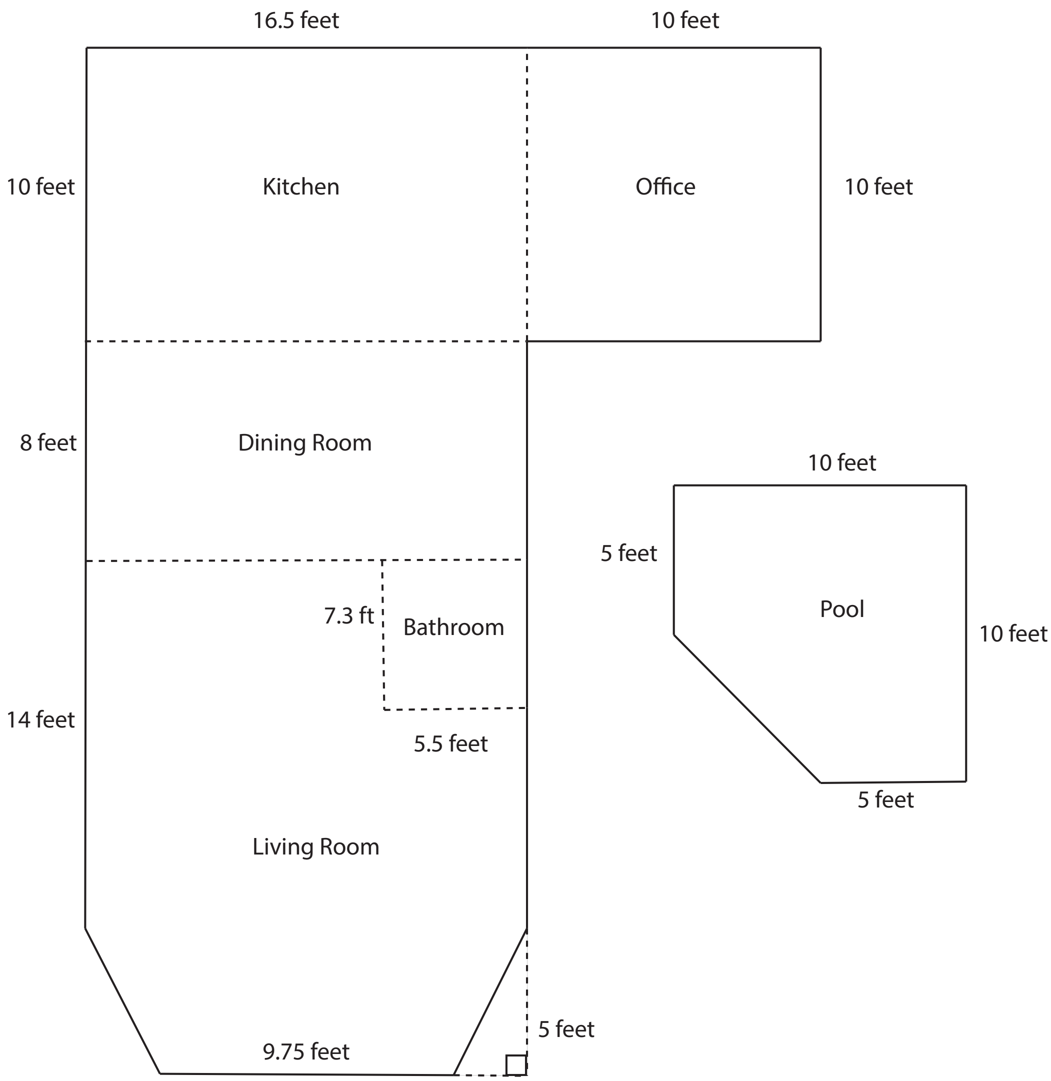
COMPOSING AND DECOMPOSING TO DETERMINE AREA

Lesson 4

Use the following floor plan to answer Questions 1 – 3.



1. What is the area of the living room and the office in square feet? Show all your work.

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2. What is the area of the pool and the bathroom in square feet? Show all your work.

3. What is the total area of the floor plan, including the pool, in square feet? Show all your work.

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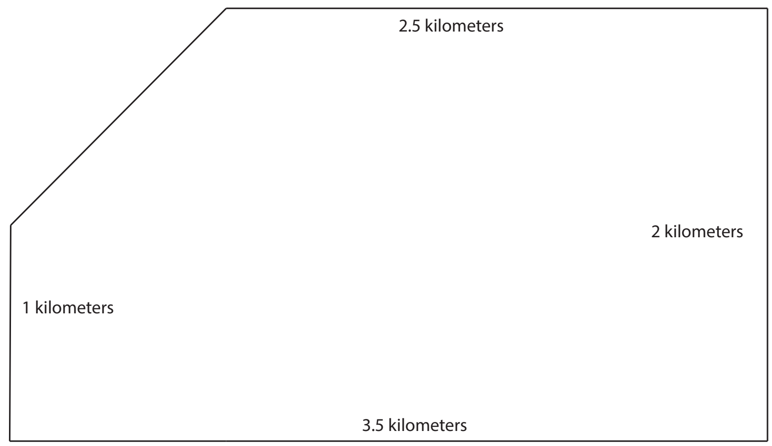
4. The city is clearing land for a new park. How much area in square kilometers will they need to clear to build the park? Identify whether you used composition or decomposition. Show all your work.



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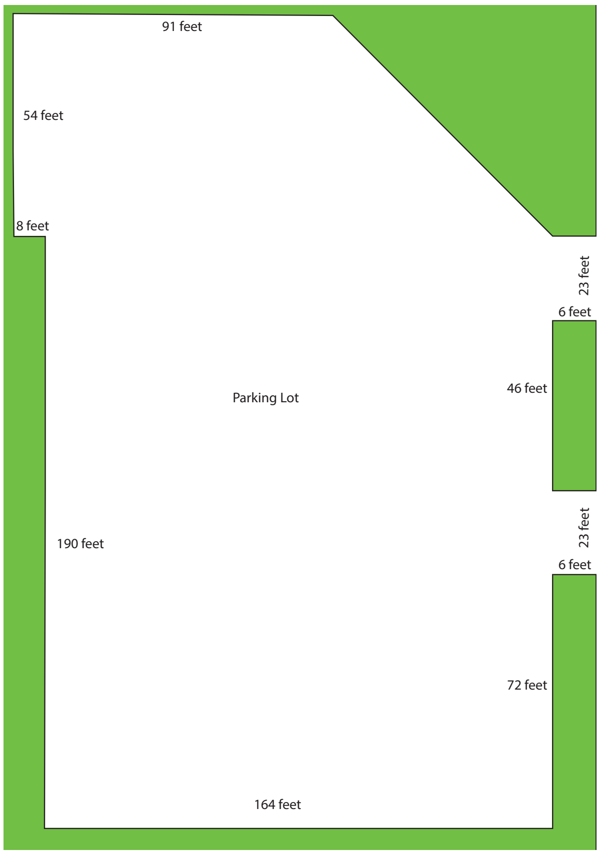
5. The school is planning to repave the parking lot during the school break. The construction workers need to know the area of the parking lot in order to ensure they have enough supplies. What is the area of the parking lot in square feet? Identify whether you used composition or decomposition. Show all your work.



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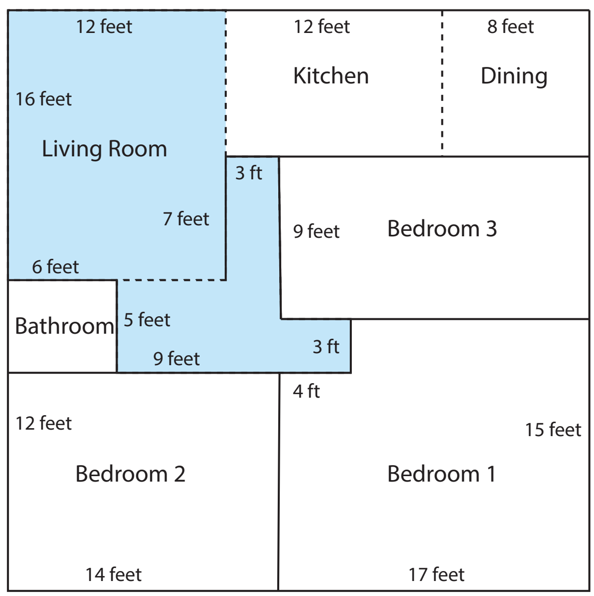
6. Maggie wants to change the carpet in her living room and hallways. How much carpet will Maggie need in square feet? Identify whether you used composition or decomposition. Show all your work.



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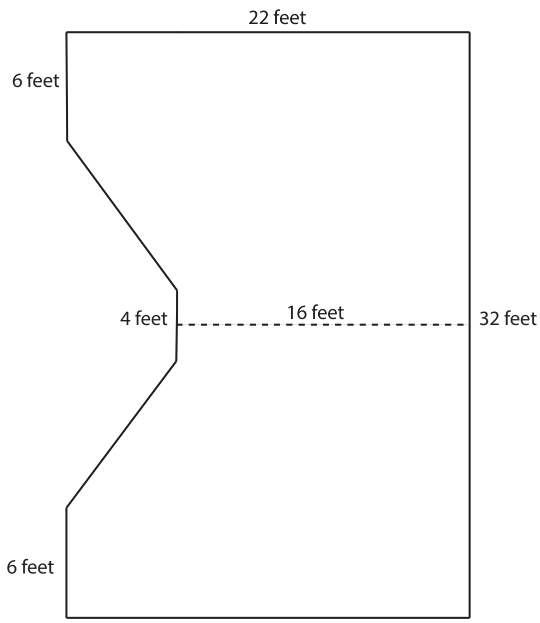
7. Mark wants to put a pool in his backyard. His backyard is 1,000 square feet. Is the size of the swimming pool less than, equal to, or greater than the size of his backyard? What is the area of the pool in square feet? Identify whether you used composition or decomposition. Show all your work.



8. If Mark's backyard is 20 feet long and 50 feet wide, will the pool fit in his backyard? Why or why not?

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9. Which strategy – composition or decomposition – do you find easier to use to determine the area of an irregular figure? Why?

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Floor plan

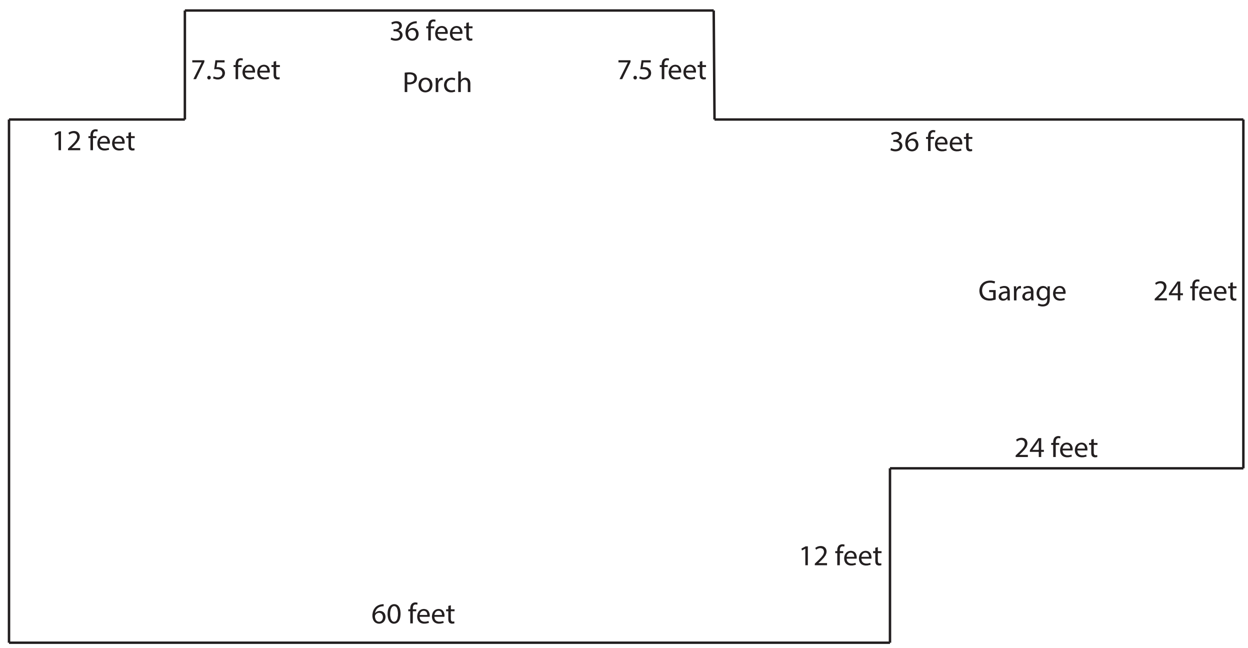
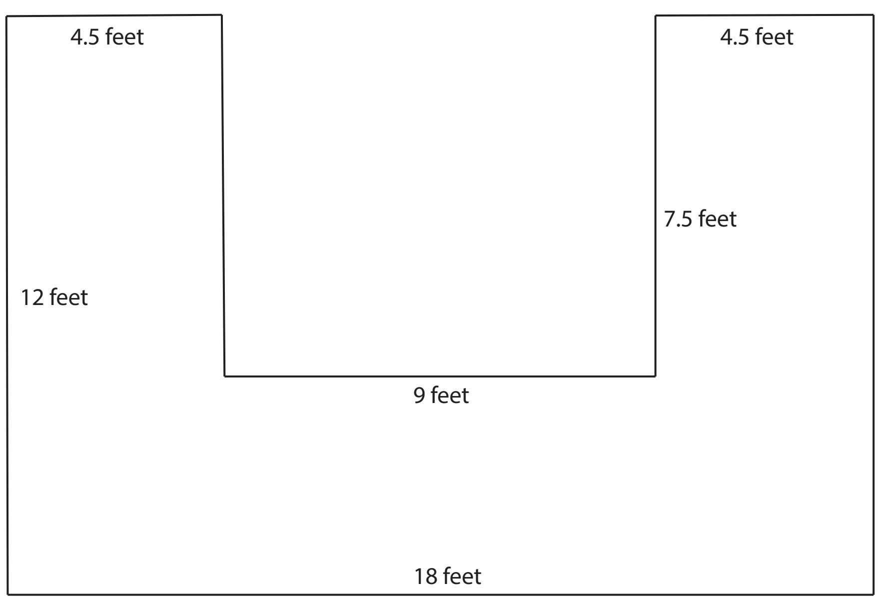


Table plan



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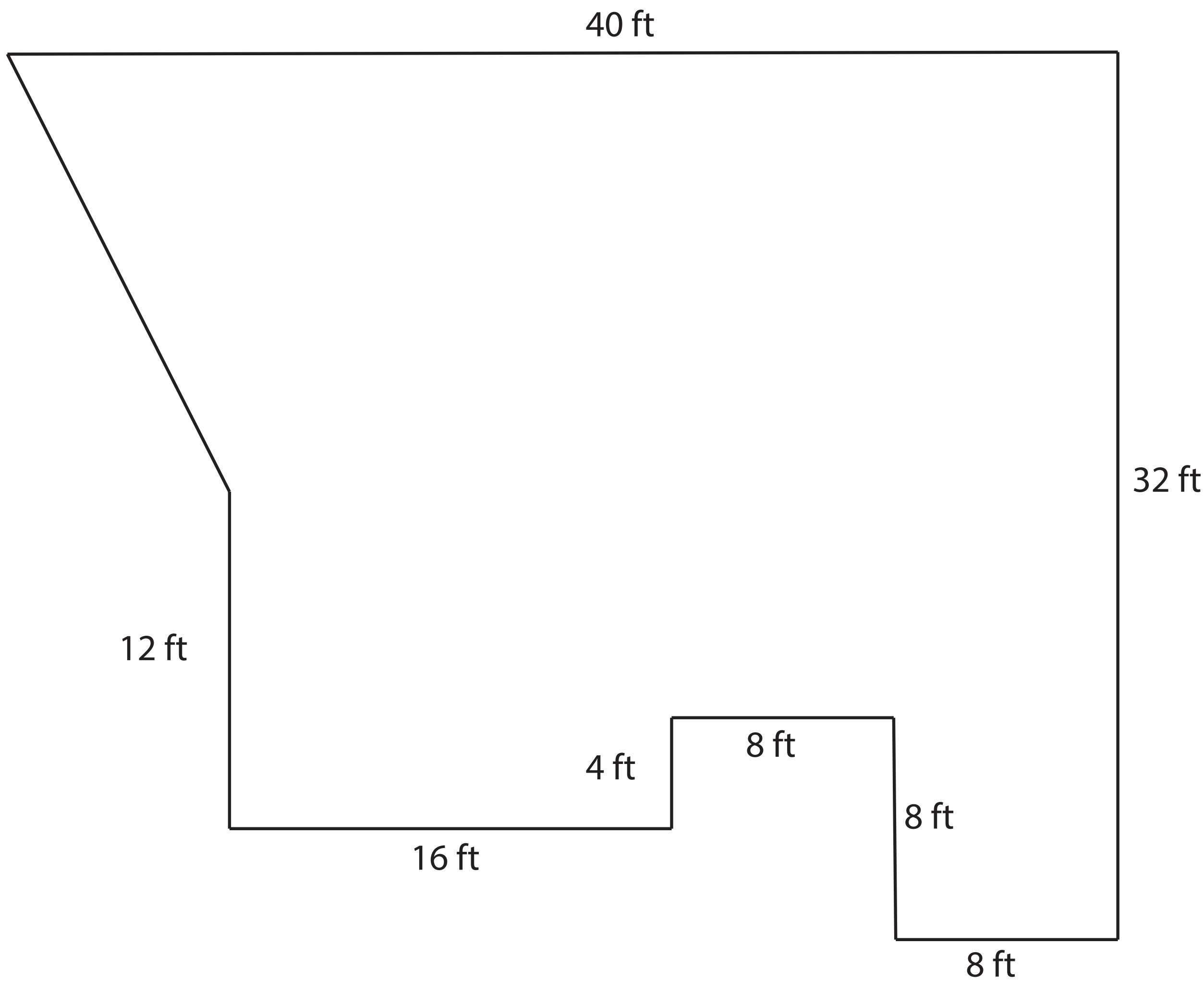
Obstacle course



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Decomposition

Composition

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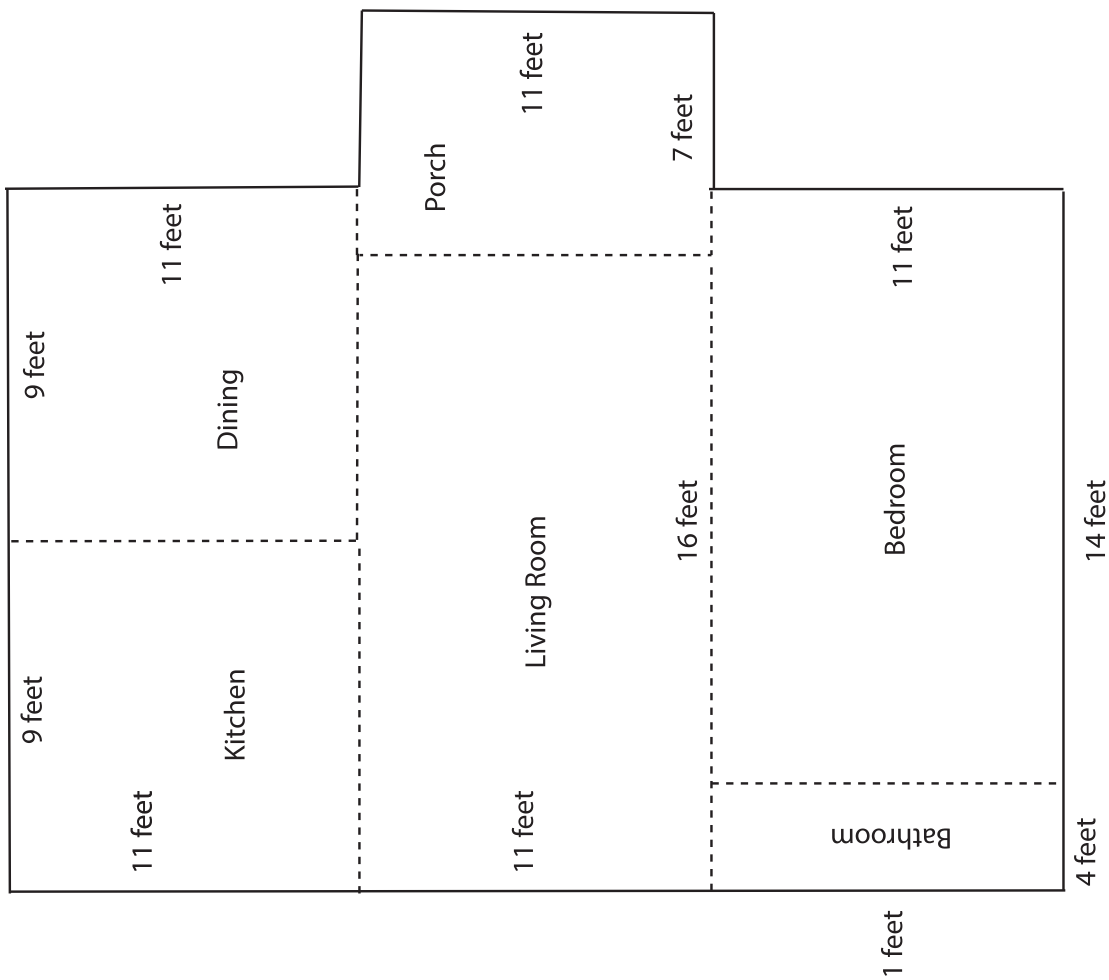
COMPOSING AND DECOMPOSING TO DETERMINE AREA

Lessons 1 – 4

1. Answer the following questions using the floor plan. Show all your work.



1.a. What is the area of the bathroom in square feet?

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1.b. What is the perimeter of the bathroom in feet?

1.c. What is the combined area of the kitchen and dining room in square feet?

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1.d. Sophia says that the area of the porch is 36 square feet. What mistake did Sophia make, and what is the correct area of the porch in square feet?

1.e. Is the combined area of the bathroom and bedroom greater than, less than, or equal to the combined area of the kitchen and dining room? How do you know?

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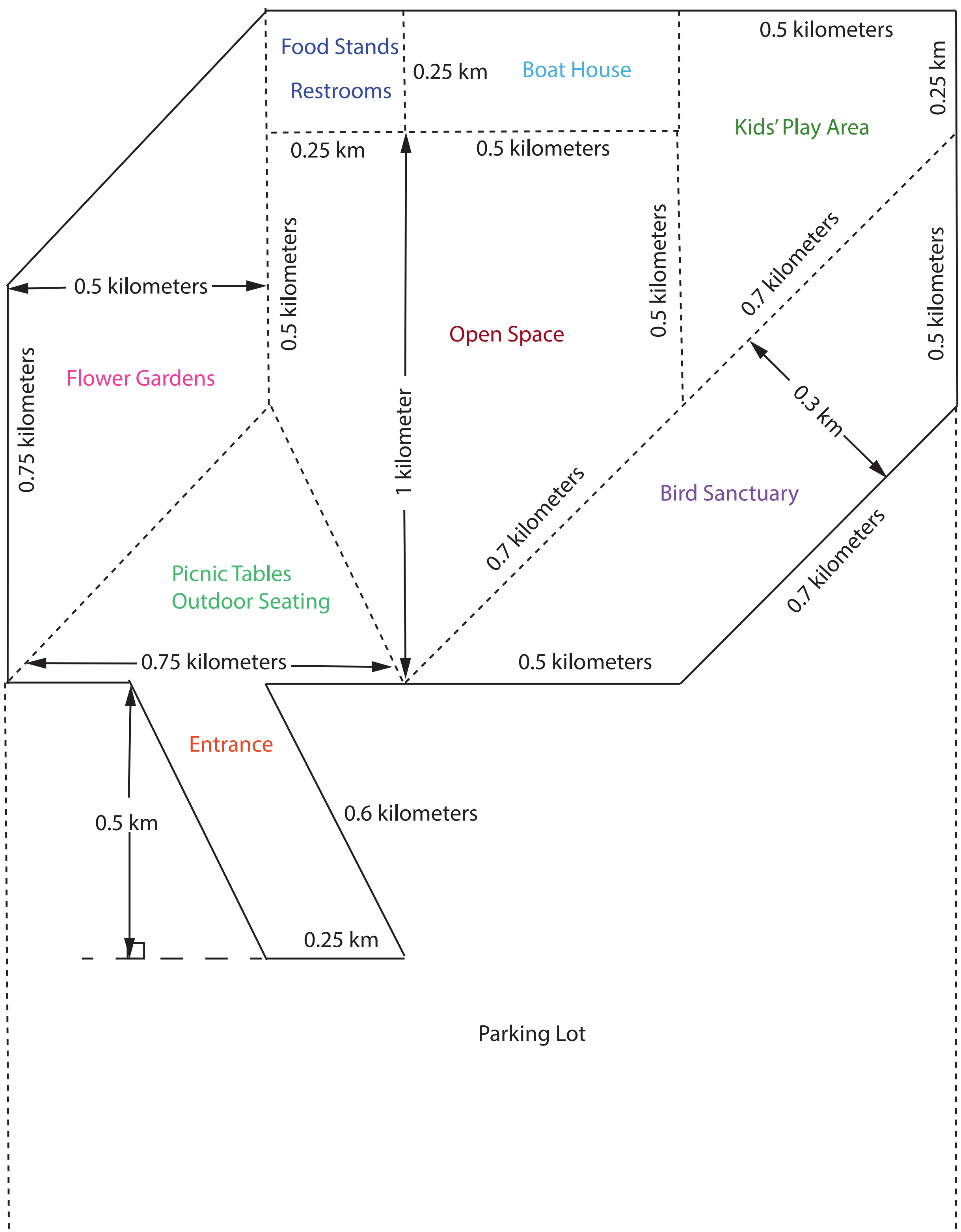
Name____

2. Answer the following questions using the park map. Show all your work.



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Name____

2.a. What is the area of the flower gardens in square kilometers?

2.b. The park has designed a new play structure for the kids' play area. If the amount of area needed for the new play structure is 0.5 square kilometers, will the kids' play area be large enough for the new structure? Why or why not?

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Name____

2.c. The park is replacing the fence around the bird sanctuary. How many kilometers of fencing will they need? How do you know?

2.d. Tyler says that he can calculate the area of the picnic tables and outdoor seating area. Jayden says that there is not enough information to calculate the area of the picnic tables and outdoor seating area. Who is correct? How do you know?

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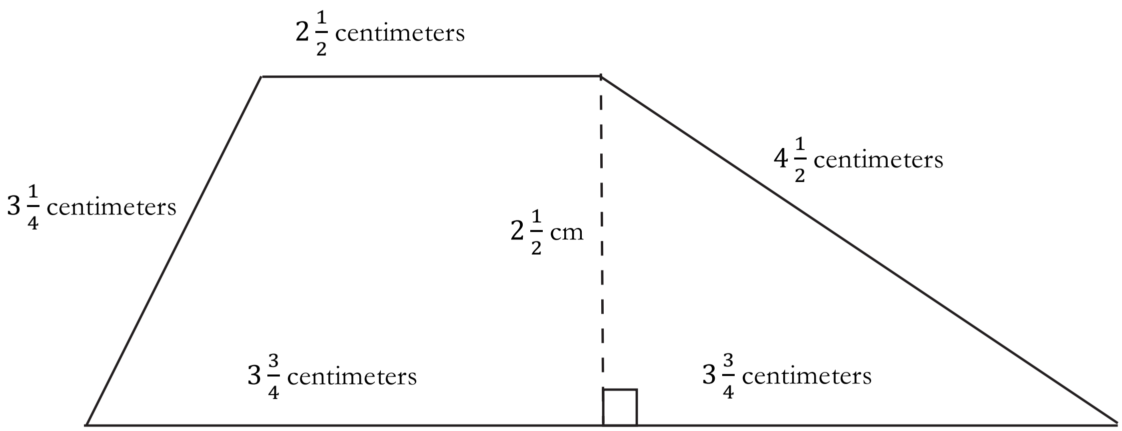
Name____

3. Decompose or compose the following figures to determine the area.

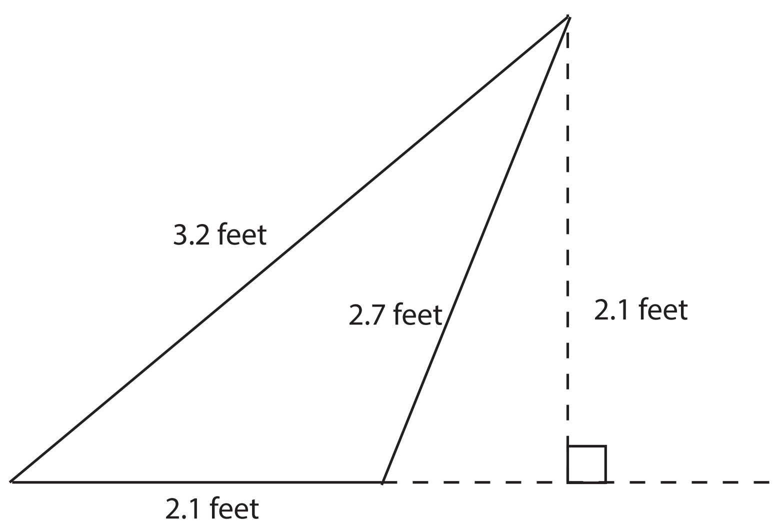
3.a. Trapezoid

________________________________ square centimeters



3.b. Triangle

________________________________ square feet



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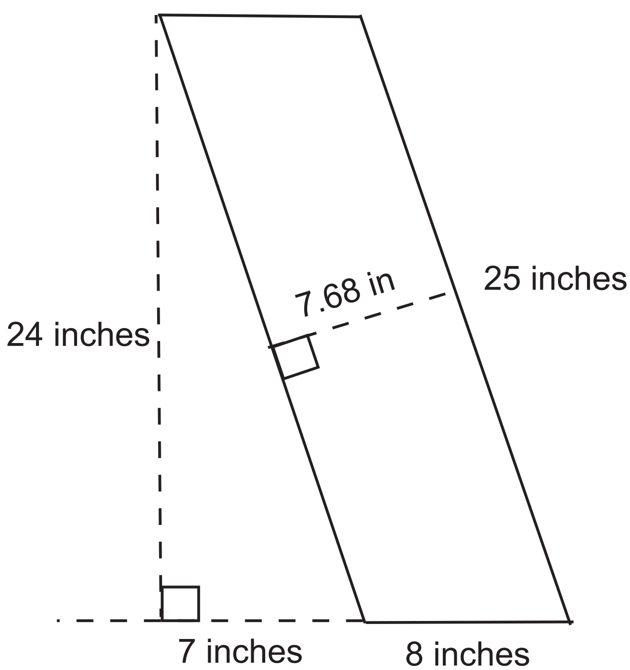
Name______________________________________

3.c. Parallelogram

________________________________ square inches



4. Brenna wants to replace the material on her kite. She measures the perpendicular poles to determine the amount of fabric she will need to purchase. If one pole is 2.25 feet and the second pole is 1.8 feet, what is the area of fabric Brenna will need? Show all your work.

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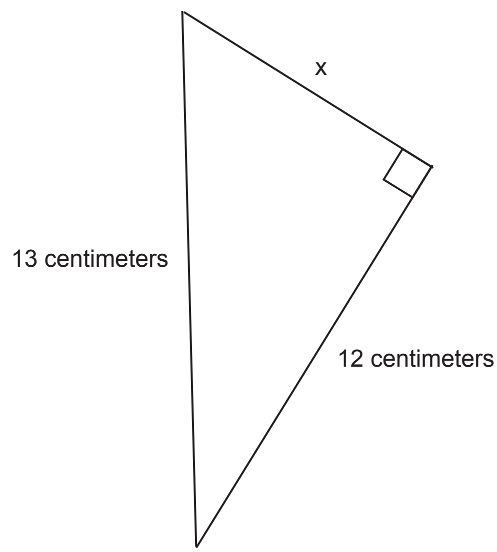
Name____

5. The perimeter of the following triangle is 30 centimeters. Determine the value of *x* and calculate the area of the triangle. Show all your work.



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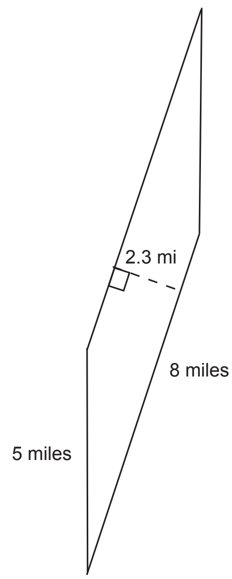
Name____

6. Matthew is working on his math homework. One question asks him to calculate the area of the following parallelogram. Matthew thinks the answer is 20 square miles. Is Matthew correct? If so, explain how you know he is correct. If not, determine the correct area and explain what Matthew did incorrectly. Show all your work.



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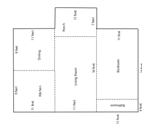
COMPOSING AND DECOMPOSING TO DETERMINE AREA

STUDENT ACTIVITY SOLUTION GUIDE

Lessons 1 – 4

1. Answer the following questions using the floor plan. Show all your work.

   1.a. What is the area of the bathroom in square feet?



CORRECT ANSWER

The area of the bathroom is 44 square feet (feet^2).

$11 \bullet 4 = 44$

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student adds together all the side lengths for a total of 30 feet.	calculates the perimeter of the figure, not the area	EXPLAIN *AREA* and CALCULATE THE AREA OF A RECTANGLE WITH THE FORMULA
The student adds together the base and the height for a total of 15 feet.	recognizes the need to use the base and height of the figure but adds instead of multiplies	EXPLAIN THE AREA FORMULA FOR RECTANGLES and CALCULATE THE AREA OF A RECTANGLE WITH THE FORMULA
The student adds together the base and the height for a total of 15 feet and then multiples by two for a total of 30 feet.	recognizes the need to use the base and the height and understands that multiplication is involved but still adds the base and height together, or calculates the perimeter of the figure, not the area	EXPLAIN THE AREA FORMULA FOR RECTANGLES, CALCULATE THE AREA OF A RECTANGLE WITH THE FORMULA, and EXPLAIN *AREA*

1.b. What is the perimeter of the bathroom in feet?

CORRECT ANSWER

The perimeter of the bathroom is 30 feet.

$11 + 4 + 11 + 4 = 30$ or $(11 \bullet 2) + (4 \bullet 2) = 30$ or $2(11 + 4) = 30$

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ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student multiples the base and the height for a total of 44 feet.	calculates the area of the figure, not the perimeter	EXPLAIN *PERIMETER* and CALCULATE THE PERIMETER BY ADDING SIDE LENGTHS
The student multiples all the side lengths for a total of 1,936 feet.	recognizes that all the side lengths are involved but multiples instead of adds the side lengths	CALCULATE THE PERIMETER BY ADDING SIDE LENGTHS
The student adds together the base and the height for a total of 15 feet.	recognizes the need to add but only adds the base and the height instead of adding all side lengths	CALCULATE THE PERIMETER BY ADDING SIDE LENGTHS

1.c. What is the combined area of the kitchen and dining room in square feet?

CORRECT ANSWER

The area of the kitchen and dining room is 198 square feet (feet^2).

$2(9 \bullet 11) = 2 \bullet 99 = 198$

or

$18 \bullet 11 = 198$

or

$(9 \bullet 11) + (9 \bullet 11) = 99 + 99 = 198$

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student only determines the area of the kitchen or the dining room, 99 square feet.	does not combine the areas of both the rooms listed	SOLVE PROBLEMS INVOLVING AREA OF POLYGONS
The student adds together all the side lengths for a total of 58 feet.	calculates the perimeter of the figure, not the area	EXPLAIN *AREA* and CALCULATE THE AREA OF A RECTANGLE WITH THE FORMULA
The student adds together the base and the height for a total of 18 feet and then multiples by two for a total of 36 feet.	recognizes the need to use the base and the height but adds them together; multiplies by two because there are two figures	EXPLAIN THE AREA FORMULA FOR RECTANGLES and CALCULATE THE AREA OF A RECTANGLE WITH THE FORMULA

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1.d. Sophia says that the area of the porch is 36 square feet. What mistake did Sophia make, and what is the correct area of the porch in square feet?

CORRECT ANSWER

> Sophia found the perimeter of the porch instead of the area. The correct area of the porch is 77 square feet (feet^2).
> 7 • 11 = 77

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student identifies that Sophia calculated the perimeter but does not provide a correct area, or provides an incorrect "correct" area.	realizes that Sophia found the perimeter but does not know how to calculate the correct area	CALCULATE THE AREA OF A RECTANGLE WITH THE FORMULA
The student cannot identify what Sophia did wrong, but identifies the correct area of the porch.	unsure of the mistake that Sophia made which resulted in her incorrect answer	CALCULATE THE PERIMETER BY ADDING SIDE LENGTHS

1.e. Is the combined area of the bathroom and bedroom greater than, less than, or equal to the combined area of the kitchen and dining room? How do you know?

CORRECT ANSWER

> The area of the bathroom and bedroom are equal to the area of the kitchen and dining room. The area is 198 square feet (feet^2).

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student states that the area of the bathroom and bedroom are greater than the kitchen and the bathroom.	does not calculate either or both of the areas correctly	CALCULATE THE AREA OF A RECTANGLE WITH THE FORMULA
The student states that the area of the bathroom and bedroom are less than the kitchen and the bathroom.	does not calculate either or both of the areas correctly	CALCULATE THE AREA OF A RECTANGLE WITH THE FORMULA
The student states that the areas are equal but does not calcuate the area correctly, and identifies the area as 58 square feet (feet^2) or another incorrect area.	recognizes that the areas are the same size, but does not calculate the area correctly, and provides the perimeter or an incorrect response	EXPLAIN *AREA* and CALCULATE THE AREA OF A RECTANGLE WITH THE FORMULA

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2. Answer the following questions using the park map. Show all your work.

2.a. What is the area of the flower gardens in square kilometers?

CORRECT ANSWER

The area of the flower gardens is 0.375 square kilometers (kilometers^2).
$0.75 \bullet 0.5 = 0.375$

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student calculates the area by multiplying the base times the height, then dividing by two. $(0.75 \cdot 0.5) \div 2 = 0.1875$ square kilometers	mistakes the formula for area of a triangle as the formula for area of a parallelogram	CALCULATE THE AREA OF A PARALLELOGRAM WITH THE FORMULA
The student adds the base and the height and multiplies by two. $2(0.75 + 0.5) = 2.5$ square kilometers	recognizes the need to use the base and the height but adds them together, and multiplies by two because there are two figures	CALCULATE THE AREA OF A PARALLELOGRAM WITH THE FORMULA

2.b. The park has designed a new play structure for the kids' play area. If the amount of area needed for the new play structure is 0.5 square kilometers, will the kids' play area be large enough for the new structure? Why or why not?

CORRECT ANSWER

No, there will not be enough room from the new play structure, because the new play structure will use 0.5 square kilometers but the area of the kids' play area is 0.25 square kilometers. 0.5 square kilometers is larger than 0.25 square kilometers.

$(0.5 \bullet 0.25) + \frac{0.5 \cdot 0.5}{2} = 0.125 + 0.125 = 0.25$

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ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student calculates the area of the rectangle correctly but does not divide by two when determining the area of the triangle. $(0.5 \bullet 0.25) + (0.5 \bullet 0.5) = 0.375$	does not recognize that the area of a triangle is only one-half the area of a rectangle with the same length base and height, and therefore they must divide the product of the base and the height by two	CALCULATE THE AREA OF A TRIANGLE WITH THE FORMULA
The student calculates the area of the rectangle correctly but does not use the correct base and height of the triangle. $(0.5 \bullet 0.25) + \frac{0.5 \cdot 0.7}{2} = 0.3$	does not recognize that the height must be perpendicular to the base	RECOGNIZE THE BASE AND HEIGHT OF A RIGHT TRIANGLE
The student adds together all the side lengths. $0.25 + 0.7 + 0.5 + 0.25 + 0.5 = 2.2$ Or, the student only adds the marked side lengths. $0.25 + 0.7 + 0.5 + 0.5 = 1.95$	calculates the perimeter of the figure, not the area	EXPLAIN *AREA* and SOLVE PROBLEMS INVOLVING AREA OF POLYGONS
The student composes a rectangle and divides the total by two. $(0.75 \bullet 0.5) \div 2 = 0.1875$ Or, the student composes a rectangle but only uses the marked side lengths to multiply. $(0.5 \bullet 0.5) \div 2 = 0.125$	composes a rectangle but divides the product by two instead of subtracting the “extra” space	SOLVE PROBLEMS INVOLVING AREA OF POLYGONS

2.c. The park is replacing the fence around the bird sanctuary. How many kilometers of fencing will they need? How do you know?

CORRECT ANSWER

> They will need 3.1 kilometers of fencing to go around the bird sanctuary. 3.1 kilometers is the perimeter of the bird sanctuary.
>
> $0.5 + 0.7 + 0.7 + 0.5 + 0.7 = 3.1$

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ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student calculates the area of the bird sanctuary for a total of 0.315 square kilometers. $\frac{1.4+0.7}{2}$ • 0.3 = 1.05 • 0.3 = 0.315	calculates the area of the trapezoid instead of the perimeter of the trapezoid	CALCULATE PERIMETER BY ADDING ALL THE SIDE LENGTHS and SOLVE WORD PROBLEMS INVOLVING PERIMETER OF POLYGONS
The student adds the two bases together, divides the product by two, then multiplies the quotient by the side length for a total of 0.525 square kilometers. $\frac{1.4+0.7}{2}$ • 0.5 = 1.05 · 0.5 = 0.525	incorrectly calculates the area of the trapezoid instead of the perimeter of the trapezoid by multiplying by the side length of the trapezoid and not the height of the trapezoid	CALCULATE PERIMETER BY ADDING SIDE LENGTHS, SOLVE WORD PROBLEMS INVOLVING PERIMETER OF POLYGONS, and CALCULATE THE AREA OF A TRAPEZOID WITH THE FORMULA
The student adds the two bases and the height of the trapezoid for a total of 2.4 kilometers. 0.7 + 0.7 + 0.7 + 0.3 = 2.4	recognizes that addition is required but mixes the formulas for area and perimeter together, and adds the bases and the height instead of adding all side lengths	CALCULATE PERIMETER BY ADDING SIDE LENGTHS

2.d. Tyler says that he can calculate the area of the picnic tables and outdoor seating area. Jayden says that there is not enough information to calculate the area of the picnic tables and outdoor seating area. Who is correct? How do you know?

CORRECT ANSWER

Tyler is correct, because the height of the open space is equal to the sum of the height of the picnic tables and outdoor seating area and a side length of the open space. If you subtract the side length of the open space (0.5 kilometers) from the height of the open space (1 kilometer), that will be the height of the triangle.

Height of triangle: 1 kilometer – 0.5 kilometers = 0.5 kilometers

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student identifies Jayden's as the correct answer: that there is not enough information to determine the area of the triangle.	does not realize that the height of the triangle can be determined by utilizing other lengths within the park map	SOLVE WORD PROBLEMS INVOLVING AREA OF TRIANGLES

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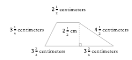
3. Decompose or compose the following figures to determine the area.

3.a. Trapezoid 

CORRECT ANSWER

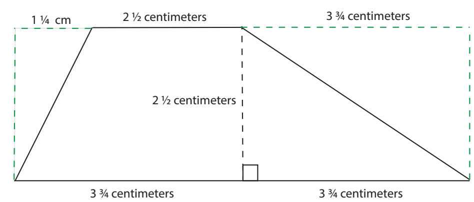
$A = 12\frac{1}{2}$ square centimeters (centimeters^2)

There are a variety of solutions students could represent.

$3\frac{3}{4} \bullet 3\frac{3}{4} = 7\frac{1}{2}$

$7\frac{1}{2} \bullet 2\frac{1}{2} = 18\frac{3}{4}$

$(18\frac{3}{4} - 1\frac{9}{16}) - 4\frac{11}{16} = 12\frac{1}{2}\text{ cm}^2$

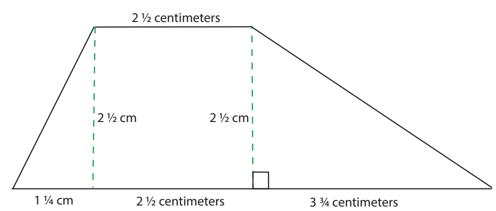


$(1\frac{1}{4} \bullet 2\frac{1}{2}) \div 2 = 1\frac{9}{16}$

$2\frac{1}{2} \bullet 2\frac{1}{2} = 6\frac{1}{4}$

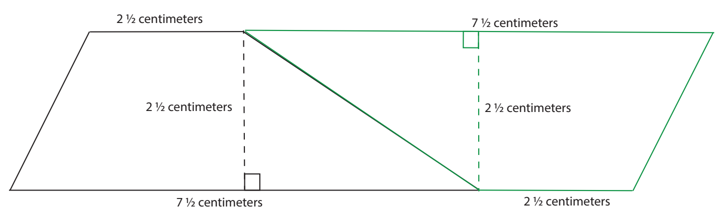
$(3\frac{3}{4} - 2\frac{1}{2}) \div 2 = 4\frac{11}{16}$

$1\frac{9}{16} + 6\frac{1}{4} + 4\frac{11}{16} = 12\frac{1}{2}\text{ cm}^2$



$(7\frac{1}{2} + 2\frac{1}{2}) \div 2 = 5$

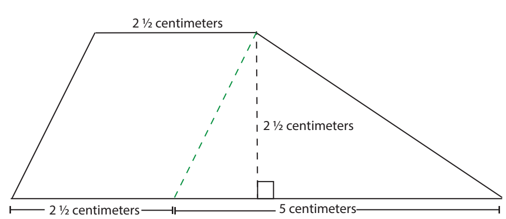
$5 \bullet 2\frac{1}{2} = 12\frac{1}{2}\text{ cm}^2$



$2\frac{1}{2} \bullet 2\frac{1}{2} = 6\frac{1}{4}$

$(5 \bullet 2\frac{1}{2}) \div 2 = 6\frac{1}{4}$

$6\frac{1}{4} + 6\frac{1}{4} = 12\frac{1}{2}\text{ cm}^2$



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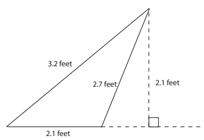
ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student solves for the area but does not compose or decompose the trapezoid.	does not conceptually understand determining area of a trapezoid, or does not follow the directions and finds the formula to be the simplest	CALCULATE THE AREA OF A TRAPEZOID BY COMPOSING A PARALLELOGRAM or SOLVE PROBLEMS INVOLVING AREA OF POLYGONS
The student composes the trapezoid into a rectangle but does not subtract the "extra" space for an area of 18 $\frac{3}{4}$ square centimeters.	does not recognize that the area that was added when composing the rectangle also needs to be subtracted from the total area	CALCULATE THE AREA OF A TRAPEZOID BY COMPOSING A PARALLELOGRAM

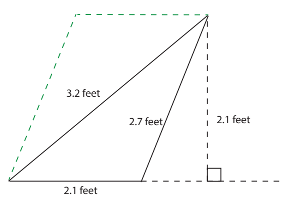
3.b. Triangle



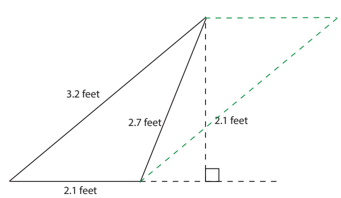
CORRECT ANSWER

A = 2.205 square feet (feet^2)

There are a variety of solutions students could represent.



$(2.1 \bullet 2.1) \div 2 = 2.205 \text{ feet}^2$



$(2.1 \bullet 2.1) \div 2 = 2.205 \text{ feet}^2$

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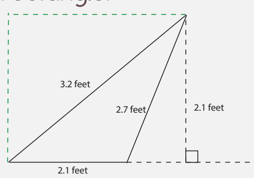
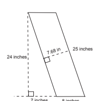
ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student calculates the area either correctly or incorrectly and composes a rectangle. 	recognizes they can compose a rectangle, but does not realize there is not enough information to determine the area	SOLVE PROBLEMS INVOLVING AREA OF POLYGONS
When calculating the area, the student does not divide the composed area in half and provides a total area of 4.41 square feet.	does not recognize that when composed, the triangle was duplicated, therefore to determine the area of one, the area of two must be divided by two	SOLVE PROBLEMS INVOLVING AREA OF POLYGONS
The student multiplies the base times either side length. $(2.1 \bullet 2.7) \div 2 = 2.835$ feet^2 or $(2.1 \bullet 3.2) \div 2 = 3.36$ feet^2	does recognize the difference between the height of the triangle and the side length	RECOGNIZE THE HEIGHT OF A TRIANGLE

3.c. Parallelogram 

CORRECT ANSWER

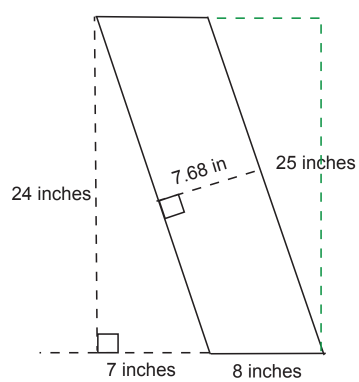
A = 192 square inches (inches^2)

There are a variety of solutions students could represent.

$15 \bullet 24 = 360$

$(7 \bullet 24) \div 2 = 84$

$(360 - 84) - 84 = 192$ inches^2



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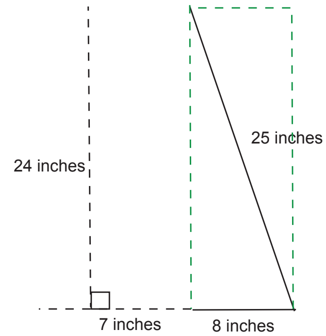
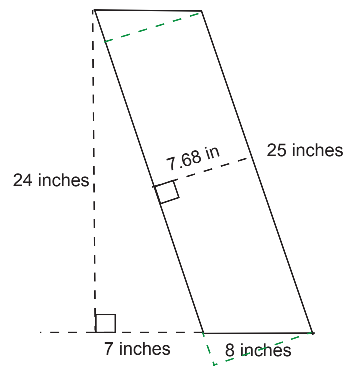
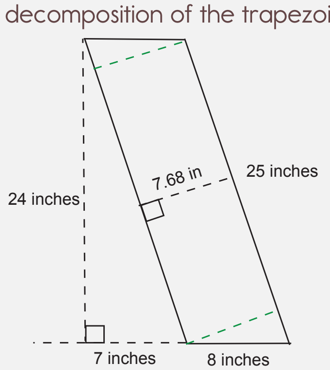
$8 \bullet 24 = 192$ inches^2



$7.68 \bullet 25 = 192$ inches^2



ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student composes a rectangle, multiplies base times height, then divides by two for a total area of 96 square inches.	does not recognize that a parallelogram composed as a rectangle has the same area; no area was added to the existing figure – the area was just rearranged as a rectangle	CALCULATE AREA OF ANY TRIANGLE BY COMPOSING RECTANGLES
The student multiplies the base times the side length for a total area of 200 square inches.	does not recognize the difference between the height of the parallelogram and the side length	CALCULATE THE AREA OF A PARALLELOGRAM WITH THE FORMULA
The student's computation is inconsistent with or does not relate to their decomposition of the trapezoid. 	recognizes they can decompose a parallelogram into a rectangle and two triangles, but does not realize there is not enough information provided to determine the area using this method	CALCULATE AREA OF ANY TRIANGLE BY COMPOSING RECTANGLES

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4. Brenna wants to replace the material on her kite. She measures the perpendicular poles to determine the amount of fabric she will need to purchase. If one pole is 2.25 feet and the second pole is 1.8 feet, how much fabric Brenna will need to cover her kite? Show all your work.

CORRECT ANSWER

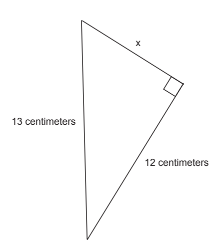
Brenna will need 2.025 square feet (feet^2) of fabric to cover her kite.

$\frac{2.25 \cdot 1.8}{2} = \frac{4.05}{2} = 2.025$

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
2.25 • 1.8 = 4.05 square feet	does not recognize the need to divide the product of the perpendicular segments by two	CALCULATE THE AREA OF A KITE WITH THE FORMULA
The student does not provide a response.	does not understand what the question is asking	SOLVE PROBLEMS INVOLVING AREA OF POLYGONS
The student adds together the perpendicular segments and does or does not divide by two. 2.25 + 1.8 = 4.05 or (2.25 + 1.8) ÷ 2 = 2.025 NOTE: the area is the same as the correct area when divided by two. Be sure to check student work.	does not recognize that the perpendicular segments need to be multiplied rather than added	CALCULATE THE AREA OF A KITE WITH THE FORMULA

5. The perimeter of the following triangle is 30 centimeters. Determine the value of x and calculate the area of the triangle. Show all your work.



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CORRECT ANSWER

The value of x is 5 centimeters, because 13 + 12 = 25 and 30 – 25 = 5.

A = 12 • 5 ÷ 2

= 60 ÷ 2

= 30 square centimeters (centimeters^2)

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student does not correctly determine the value of x and provides a number other than five.	does not recognize that the sum of the side lengths is the perimeter	CALCULATE THE PERIMETER OF A TRIANGLE WITH FORMULA
The student multiplies 13 • 12 to calculate the area and does or does not divide the product by two. 13 • 12 = 156 square centimeters or 13 • 12 ÷ 2 = 78 square centimeters	does not recognize the difference between the height of the triangle and the side length	RECOGNIZE THE BASE AND HEIGHT OF A RIGHT TRIANGLE and CALCULATE THE AREA OF A TRIANGLE WITH THE FORMULA
The student adds 13 + 12, then multiplies by two to determine the area. (13 + 12) • 2 = 50 square centimeters	does not understand the formula to calculate the area of a triangle but knows that multiplication and the number 2 are involved	CALCULATE THE AREA OF A TRIANGLE WITH THE FORMULA and RECOGNIZE THE BASE AND HEIGHT OF A RIGHT TRIANGLE
The student adds 12 + 5, then multiplies by two to determine the area. (12 + 5) • 2 = 34 square centimeters	does not understand the formula to calculate the area of a triangle but knows that multiplication and the number 2 are involved	CALCULATE THE AREA OF A TRIANGLE WITH THE FORMULA
The student multiplies 5 • 12 to determine the area but does not divide by two or multiply by one-half. 5 • 12 = 50 square centimeters	does not understand the area formula for triangles (that when base and height are multiplied, that is the area for a rectangle), and that to determine the area of the triangle, the area of the rectangle needs to be divided by two	CALCULATE THE AREA OF A TRIANGLE WITH THE FORMULA
The student does not provide an answer.	is unsure what the question is asking or how to solve the word problem	SOLVE WORD PROBLEMS INVOLVING AREA OF TRIANGLES and CALCULATE THE PERIMETER OF A TRIANGLE WITH FORMULA

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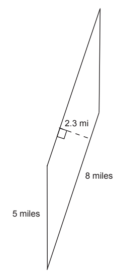
6. Matthew is working on his math homework. One question asks him to calculate the area of the following parallelogram. Matthew thinks the answer is 40 square miles. Is Matthew correct? If so, explain how you know he is correct. If not, determine the correct area and explain what Matthew did incorrectly. Show all your work.



CORRECT ANSWER

Matthew is not correct. He multiplied the two side lengths (rather than the base and the height).

5 • 8 = 40

The correct area is 18.4 square miles.

2.3 • 8 = 18.4

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student identifies Matthew's work as correct.	does not recognize the difference between the height of the parallelogram and the side length	CALCULATE THE AREA OF A PARALLELOGRAM WITH THE FORMULA
The student identifies Matthew's work as incorrect, but when providing the correct answer, the student multiplies the base and height but divides by two for an area of 9.2 square miles.	does not recognize that a parallelogram composed as a rectangle has the same area; does not add any figures to the existing figure, meaning the area was just rearranged as a rectangle	CALCULATE THE AREA OF A PARALLELOGRAM BY COMPOSING A RECTANGLE and CALCULATE THE AREA OF A PARALLELOGRAM WITH THE FORMULA

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