SOLVE TWO-STEP WORD PROBLEMS USING THE FOUR OPERATIONS

3.OA.8

CONTENTS

The types of documents contained in the unit are listed below. Throughout the unit, the documents are arranged by lesson.

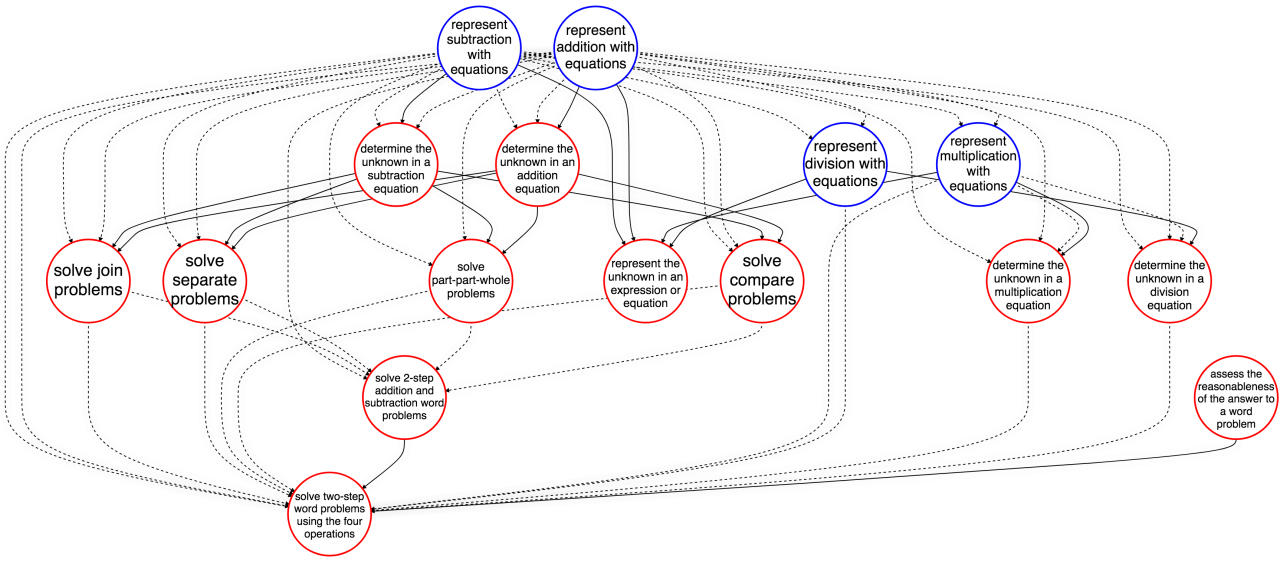
Enhanced Learning Maps developed these materials under a grant from the Department of Education, PR/Award # S368A150013. Contents do not necessarily represent the policy of the Department of Education, and you should not assume endorsement by the Federal Government.

SOLVE TWO-STEP WORD PROBLEMS USING THE FOUR OPERATIONS

LEARNING MAP INFORMATION

STANDARDS

3.OA.8 Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.



**Learning map model of* 3.OA.8

Node Name	Node Description
ASSESS THE REASONABLENESS OF THE ANSWER TO A WORD PROBLEM	Explain the reasonableness of the answer (e.g., "Does your answer make sense?").
DETERMINE THE UNKNOWN IN A DIVISION EQUATION	Given a mathematical equation involving division operation, determine the unknown factor or quotient.
DETERMINE THE UNKNOWN IN A MULTIPLICATION EQUATION	Given a mathematical equation involving multiplication operation, determine the unknown factor or product.
DETERMINE THE UNKNOWN IN A SUBTRACTION EQUATION	Determine the unknown minuend, subtrahend, or difference when given an equation with subtraction operation.
DETERMINE THE UNKNOWN IN AN ADDITION EQUATION	Determine the unknown addend or sum when given an equation with addition operation.
REPRESENT ADDITION WITH EQUATIONS	Use equations to represent addition problems.
REPRESENT DIVISION WITH EQUATIONS	Use an equation to represent a division problem (e.g., $10 \div 2 = 5$).
REPRESENT MULTIPLICATION WITH EQUATIONS	Use an equation to represent a multiplication sentence (e.g., $2 \times 3 = 6$).
REPRESENT SUBTRACTION WITH EQUATIONS	Use equations to represent subtraction problems.
REPRESENT THE UNKNOWN IN AN EXPRESSION OR EQUATION	Through writing or an appropriate assistive technology, represent the unknown in an expression or equation using a symbol or letter.
SOLVE 2-STEP ADDITION AND SUBTRACTION WORD PROBLEMS	Use addition and subtraction to solve two-step word problems including join, separate, part-part-whole, and compare problems.
SOLVE COMPARE PROBLEMS	Solve one-step problems that involve comparing two amounts. Three quantities are involved (smaller amount, larger amount, and the difference), and one of those amounts is unknown. This may include problems that ask "how many more" and "how many fewer".
SOLVE JOIN PROBLEMS	Solve one-step problems where the action is joining with three quantities involved (initial amount, the change amount, and the resulting amount). One of those amounts is unknown, and the resulting amount is the largest.
SOLVE PART-PART-WHOLE PROBLEMS	Solve one-step problems that involve two parts that are combined to make a whole, where one of those amounts is unknown.
SOLVE SEPARATE PROBLEMS	Solve one-step problems where the action is separating with three quantities involved (initial amount, the change amount, and the resulting amount). One of those amounts is unknown, and the initial amount is the largest.
SOLVE 2-STEP WORDS PROBLEMS USING THE 4 OPERATIONS	Use addition, subtraction, multiplication, and division to solve two-step word problems.

SOLVE TWO-STEP WORD PROBLEMS USING THE FOUR OPERATIONS

TEACHER NOTES

This unit includes the following documents:

- Learning Map Information
- Instructional Activity (four lessons)
- Instructional Activity Student Handout (for Lessons 1 – 4)
- Instructional Activity Supplement (for Lessons 1 – 4)
- Student Activity
- Student Activity Solution Guide

In this unit, students will learn how to analyze and solve a two-step word problem using any of the four operations. Students will connect their understanding of solving one-step word problems to the similar process for solving two-step word problems. Additionally, students will explore a four-step problem-solving process that can be applied to solving mathematical word problems.

RESEARCH

After learning basic arithmetic facts, students need to begin to understand the application of the four basic operations. Students start with application of addition and subtraction before moving into the study of applications of multiplication and division. To help students recognize the application of the four operations, students should write their own word problems (Van de Walle et al., 2014). Writing word problems allows students to synthesize their understanding and combine it with their understanding of the world around them. Students naturally progress in their mathematical understanding from one-step to two-step problems (The Common Core Standards Writing Team, 2011). As students move into two-step word problems, their attention should be drawn to the intermediary steps that provide the first information to lead into the second step and final answer (Van de Walle et al., 2014). This skill will later be translated into multi-step problems.

In addition to the mathematical operations and their computations, students need to understand the process for problem solving. In his book *How to Solve It*, George Polya (1957) outlined four basic principles of problem solving. Analysis and use of these steps help students strengthen and develop their problem-solving skills. To unlock the problem-solving process, students need to start with the first step of understanding the problem. Without an understanding of the problem, students are unable to move forward and solve a mathematical word problem. Students need an understanding of the context within a word problem to determine the operation to use in calculation. Talking through the problem-solving process with other students helps develop problem-solving skills that can be transferred between problem situations (Van de Walle et al., 2014). This technique helps English-language learners and other struggling students comprehend and analyze mathematical word problems as part of the problem-solving process.

AN EXAMPLE

The following table outlines the four basic steps of problem solving (Polya, 1957).

Step 1	Understand the problem	Students should understand the problems presented *and* desire to find the solution. This includes: • Student verbally and fluently restating the problem • Student identifying the principle parts of the problem (i.e. what is given and what is unknown) • Student representing the problem situation as a model
Step 2	Make a plan	Once students know what the problem situation is asking and what information is given, they should create a plan based on the operations required to solve the problem. This can be the most difficult part, and teachers should encourage students to develop plans on their own using the following strategies and strategic questions. • Have you solved a problem similar to this before? • Is the unknown similar to the unknown in a problem you have solved before? • Restate the problem in your own words.
Step 3	Carrying out the plan	After creating a plan, students should proceed and follow through with the plan. If a student is working on a plan that came from the "outside" or was given to them by the authority of the teacher, they are more likely to forget the plan and will be less likely to solve the problem. Students should be encouraged to "check each step" of the plan.
Step 4	Look back: Check the work and answer	Once the problem has been solved, students should look back at their work, evaluate their plan, and check the answer to ensure it makes sense. When students reflect on the solution *and* the process, it helps to solidify the understanding of the plan and increases the chances that the student will be able to recall and use this understanding when solving other similar problem situations.

Without a thorough understanding of the problem-solving process, students rush into number crunching when faced with a mathematical word problem. Rather than relying on or emphasizing keyword strategies, students must have the skills to analyze and make sense of a mathematical situation, which is instrumental in the first step of understanding the problem (Van de Walle et al., 2014). To aid in sense making for mathematical application, students need more than just reading skills. Access to physical manipulatives can provide students a way to model a problem situation concretely. This model may help students make connections between the problem situation and the mathematical operation needed for calculating the answer. Students should also develop their understanding of mathematical applications using pictures,

models, and diagrams. These images allow students to visualize and represent the mathematical situation in multiple ways to deepen their understanding of the context given in a problem situation.

Tying student models directly to the activity inherent in the problem scenario helps students observe the various combinations of numbers and operations that can be used. For example, rather than leaping to subtraction to solve a missing-addend problem, students should model the problem with addition of an unknown quantity. Students should think about and explain what is happening with quantities in problems instead of memorizing associations between certain words and operations (e.g., "together" and "addition"); emphasizing keywords does not help students learn to read, interpret, and model the problem (Fuson, 2003).

In addition, it is important to vary the types of problems (i.e. join, separate, part-part-whole, and comparison) students are exposed to. Students are exposed to join and separate problems more frequently, but the other two problem types (part-part-whole and comparison) should not be neglected (Van de Walle et al., 2014). Exposure to a variety of problem types and structures supports computational fluency and competence (Fuson et al., 1997). It is not necessary that students know the different problem structures' names; rather, it is important that they consider and solve a variety of problem types.

ADDITION AND SUBTRACTION PROBLEM TYPES (VAN DE WALLE ET AL., 2014)			
Problem Type	**Result Unknown**	**Change Unknown**	**Start Unknown**
Join (Add To)	Diego has 5 teddy bears. Maria gives him 3 more. How many teddy bears does Diego have altogether?	Diego has 5 teddy bears. Maria gives him some more. Now Diego has 8 teddy bears. How many teddy bears did Maria give him?	Diego has some teddy bears. Maria gives him 3 more. Now Diego has 8 teddy bears. How many teddy bears did Diego have to begin with?
Separate (Take From)	Diego has 8 teddy bears. He gives 3 teddy bears to Maria. How many teddy bears does Diego have now?	Diego has 8 teddy bears. He gives some to Maria. Now he has 5 teddy bears. How many teddy bears did he give to Maria?	Diego has some teddy bears. He gives 3 to Maria. Now Diego has 5 teddy bears. How many teddy bears did Diego have to begin with?
	Whole Unknown	**One Part Unknown**	**Both Parts Unknown**
Part-Part-Whole	Diego has 3 brown teddy bears and 5 black teddy bears. How many teddy bears does he have altogether?	Diego has 8 teddy bears. 3 teddy bears are brown, and the rest are black. How many black teddy bears does Diego have?	Diego has 8 teddy bears. Some of the teddy bears are brown, and some of the teddy bears are black. How many of each could he have? Diego has eight teddy bears. Some of the teddy bears are brown, and some of the teddy bears are black. How many of each could he have?

	Difference Unknown	Bigger Unknown	Smaller Unknown
Comparison	Diego has 8 teddy bears, and Maria has 5 teddy bears. How many more teddy bears does Diego have than Maria? (Or: How many fewer teddy bears does Maria have than Diego?)	Maria has 5 teddy bears. Diego has 3 more teddy bears than Maria. How many teddy bears does Diego have? (Or: Maria has 3 fewer teddy bears than Diego.)	Diego has 8 teddy bears. Diego has 3 more teddy bears than Maria. How many teddy bears does Maria have? (Or: Maria has 3 fewer teddy bears than Diego.)
MULTIPLICATION AND DIVISION PROBLEM TYPES (VAN DE WALLE ET AL., 2014)			
Problem Type	**Product Unknown (Multiplication)**	**Size of Group Unknown (Partitive Division)**	**Number of Groups Unknown (Measurement Division)**
Equal Groups	Jenny has 3 bags of fruit snacks. There are 8 fruit snacks in each bag. How many fruit snacks does Jenny have altogether?	Jose has 36 fruit snacks. He wants to share them equally among his 3 friends. How many fruit snacks will each friend receive?	Jenny and Jose have 36 fruit snacks. They put them into bags containing 9 fruit snacks each. How many bags did Jenny and Jose use?
Comparison	Josie has 7 trading-game cards. Monique has 5 times as many trading-game cards as Josie. How many trading-game cards does Monique have?	Monique won 12 trading-game cards. She won 3 times as many trading-game cards as Josie won. How many trading-game cards did Josie win?	Josie started the game with 18 trading-game cards, and Monique only start with 6 trading-game cards. How many times more trading-game cards did Josie start with than Monique did?
	Product Unknown (Multiplication)	**Side Dimension Unknown (Division)**	
Area	Luke's poster measures 1 foot wide and 2 feet long. What is the area?	A rectangular sandbox has an area of 24 square feet. If one side is 6 feet, how long is the side next to it?	

Lastly, it is possible that students have been trained to look for "keywords" in real-world problems in the past. However, this strategy is dangerous because it is possible that students will connect words to arithmetic operations without considering the context of the meaning of the word (Britton, 2005). For example, a student may see the word "raised" in a problem and believe that addition is appropriate because "raised" means to go higher, without realizing that "raised" does not have anything to do with addition in the context of the problem (Britton, 2005). Because the context of the problem is so critical to determining appropriate operations, it is important to teach students to analyze the relationships or the actions in the problem, rather than to look for keywords (Karp, Bush, & Dougherty, 2014). As is shown in the previous table of problem types, the decision to add, subtract, multiply, or divide depends on the context of the problem and the quantity that is unknown, rather than on isolated words that may appear in the description of the problem.

KEYWORDS TO SOLVE PROBLEMS (KARP ET AL., 2014)		
Keyword	**Commonly Associated Operation**	**Counterexample (Keyword does not match commonly associated operation)**
Altogether	Add	Todd and Jessie have 18 pencils altogether. Todd has 12 pencils. How many pencils does Jessie have?
Left	Subtract	Laurie has 7 pencils in her left hand and 5 pencils in her right hand. How many pencils does Laurie have?
Times	Multiply	Meg has 28 points. She won 4 times as many points as Abbi. How many points does Abbi have?
Each	Division	Becky has 2 bookcases in her room. She has 23 books on each book case. How many books does Becky have altogether?

LEARNING MAP INFORMATION

The learning map section for this sequence of activities begins with writing equations using the four operations of addition, subtraction, multiplication, and division. Then nodes move into determining the unknown in an addition, subtraction, multiplication, or division equation. Different type of applications for addition and subtraction are encountered as one-step word problems before solving two-step word problems in addition and subtraction. Finally, solving two-step word problems using the four operations is reached.

INSTRUCTIONAL ACTIVITIES

The activities in this unit are designed to develop students' understanding of solving two-step word problems. LESSON 1 begins with students writing and modeling one-step word problems from a given expression. The expressions cover all four operations, so students must understand the uses of the different operations. Students will first write a word problem that corresponds to the given numeric expression. Then students will model their word problem using a picture or diagram. Finally, students will describe the thinking and process used to solve their word problem.

LESSON 2 moves students beyond one-step to two-step words problems by identifying the hidden question that connects the first step to the second step. Students will identify the first required step, referred to as the hidden question, and how it connects to the final answer in a two-step word problem.

LESSON 3 takes students through George Polya's four-step problem-solving approach. Students will analyze a word problem through the framework of Polya's four distinct steps. There is a strong emphasis on understanding and analyzing each problem situation because students will be encountering multi-step problems with any combination of the four operations.

LESSON 4 asks students to write an equation using a variable to represent the unknown value. Students will then solve the equations to answer the word problem to decode the answer to a riddle.

REFERENCES

Britton, B. (2005). Solutions to the postage due problem. *Teaching Children Mathematics, 12*(3), 166-172.

Fuson, K. C., Wearne, D., Hiebert, J. C., Murray, H. G., Human, P. G., Olivier, A. I., Carpenter, T. P., & Fennema, E. (1997). Children's conceptual structures for multidigit numbers and methods of multidigit addition and subtraction. *Journal for Research in Mathematics Education*, 130-162.

Karp, K. S., Bush, S. B., & Dougherty, B. J. (2014). 13 rules that expire. *Teaching Children Mathematics, 21*(1), 18-25.

Pólya, G. (1957). *How to solve it: a new aspect of mathematical method.* Princeton: Princeton University Press.

The Common Core Standards Writing Team. (2011). *Progressions for the common core state standards in mathematics (draft).* Retrieved August 13, 2017, from http://ime.math.arizona.edu/progressions/

Van de Walle, J.A., Karp, K.S., Lovin, L.H., & Bay-Williams, J.M. (2014). *Teaching student-centered mathematics.* Upper Saddle River, NJ: Pearson Professional Development.

SOLVE TWO-STEP WORD PROBLEMS USING THE FOUR OPERATIONS

OVERVIEW OF INSTRUCTIONAL ACTIVITIES

Lesson	Learning Goal	Nodes Addressed
Lesson 1	Students will explore and describe the use of mathematical operations in real-world situations.	▶ DETERMINE THE UNKNOWN IN AN ADDITION EQUATION ▶ DETERMINE THE UNKNOWN IN A SUBTRACTION EQUATION
Lesson 2	Students will explore and describe the hidden question that exists in a multi-step word problem.	▶ DETERMINE THE UNKNOWN IN A SUBTRACTION EQUATION ▶ SOLVE 2-STEP WORD PROBLEMS USING THE 4 OPERATIONS
Lesson 3	Students will follow the four-step problem-solving process to solve multi-step word problems.	▶ SOLVE 2-STEP WORD PROBLEMS USING THE 4 OPERATIONS
Lesson 4	Students will write and solve equations to represent two-step operations.	▶ SOLVE 2-STEP WORD PROBLEMS USING THE 4 OPERATIONS

SOLVE TWO-STEP WORD PROBLEMS USING THE FOUR OPERATIONS

INSTRUCTIONAL ACTIVITY

Lesson 1

LEARNING GOAL

Students will explore and describe the use of mathematical operations in real-world situations.

PRIMARY ACTIVITY

Students will write story problems, model problem situations, and describe the solution process given expressions and answers.

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

- Expression
- Mathematical operations
- Addition
- Subtraction
- Multiplication
- Division

MATERIALS

- Assorted manipulatives: counters, chips, base ten blocks, small tiles, etc.
- INSTRUCTIONAL ACTIVITY STUDENT HANDOUT
- INSTRUCTIONAL ACTIVITY SUPPLEMENT (one copy recommended to display to the class)

IMPLEMENTATION

Throughout the lesson, **provide** students access to assorted manipulatives such as counters, chips, base-ten blocks, small tiles, etc. These manipulatives can be used in the problem-solving process to help students visualize and manipulate the numbers as they model the expressions. Students are not required to use the manipulatives but can be encouraged to use them as part of their thinking process.

Introduce solving story problems by drawing students' attention to the many ways they may see and encounter mathematics in their daily lives. **Ask** students to share where they have recently used math outside of school.

Provide the following problem for students to solve:

> Annibeth wanted to check out some books from the library. She already has 3 books checked out, but she has found 4 more books she is interested in. If Annibeth checks out the 4 new books, how many books will Annibeth have checked out?

GUIDING QUESTIONS

Elicit student thinking:

- What is happening in this situation?
- What mathematical operation could be used to describe this situation?
- What model can you draw to represent this situation?

Determine if the student can DETERMINE THE UNKNOWN IN AN ADDITION EQUATION:

- What information is known?
- How do these pieces of information relate to each other?
- What expression can be written to model this situation?
- What is unknown?
- Is there enough information given to solve for the unknown information?
- How can the unknown information be found?
- Is there any extra information in the problem?

Display the INSTRUCTIONAL ACTIVITY SUPPLEMENT for students to see. **Explain** and **model** the activity with students. **Encourage** students to use the manipulatives to develop a model or diagram that can be used to represent the situation. **Allow** for multiple story problems to be discussed as possible situations for the expression. Student should realize that there is not one single situation that the expression can represent, but instead multiple situations could be related to a single expression.

GUIDING QUESTIONS

Elicit student thinking:

- What does the mathematical operation in the expression mean to you?
- What situations could fit with the mathematical operation?
- Write a story problem to fit the given expression to your situation.
- What model could you draw to fit the situation?
- How does your model represent the situation?
- Are there other possible ways to model the situation?
- How would you solve your story problem?

Determine if the student can DETERMINE THE UNKNOWN IN A SUBTRACTION EQUATION:

- What is the difference when evaluating this expression?
- What could this difference represent?
- How does the difference fit with the known values?
- How would you find the value of this expression?

Distribute to each student a copy of the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT. Students should work to fill in all three boxes for each expression. **Allow** students to be creative in their situations, as long as the situation matches the given expression. **Encourage** students to use a variety of models to represent their thinking.

After students have completed the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT, **place** students in small groups to share their ideas. Students should explain their thinking and answer

questions from their group. Students can clarify and edit their answers, as needed, after input from their group.

When groups have finished sharing their ideas, each group should present one completed situation from the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT to the class. **Encourage** groups to choose different expressions so the class sees a variety of expressions and situations.

Create a chart listing the four basic operations. **Ask** students to briefly describe different situations that they came up with that fit each operation. **List** those situations on the chart, and **ask** students to compare and contrast the situations for each operation. Students should notice that the situations used for addition and multiplication share some similarities in combining values, whereas subtraction and division share some similarities as they partition or separate values. Students should take note of these generalities in the use of the mathematical operations.

Students should be required to provide a model and situation that fits each given expression. Students should clearly describe the solution process for their story problem.

At the end of the activity, teachers should have students choose one of the expressions from the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT and write a new story problem for the situation. This story problem can be used by the teacher to gauge students' understanding of using the four basic operations.

Name_____

Solve Two-Step Word Problems Using the Four Operations

Lesson 1

On the following pages, students will complete an activity writing story problems, modeling the thinking, and describing the solution process for the problem.

The first four pages give a different expression using each of the four basic arithmetic operations.

The last four pages give the same solution but ask students to write different story problems incorporating each of the four basic arithmetic operations.

Name_____

9 + 8

Use the expression above to complete the following tasks.

- ❑ In Box A, write a story problem that matches the expression.
- ❑ In Box B, draw a model or diagram to represent the expression.
- ❑ In Box C, describe how you would think about and solve the problem you wrote.

Box A

Box B

Box C

Name_____

14 – 2

Use the expression above to complete the following tasks.

- ❑ In Box A, write a story problem that matches the expression.
- ❑ In Box B, draw a model or diagram to represent the expression.
- ❑ In Box C, describe how you would think about and solve the problem you wrote.

Box A

Box B

Box C

Name_____

4×12

Use the expression above to complete the following tasks.

- ❑ In Box A, write a story problem that matches the expression.
- ❑ In Box B, draw a model or diagram to represent the expression.
- ❑ In Box C, describe how you would think about and solve the problem you wrote.

Box A

Box B

Box C

Name_____

15 ÷ 3

Use the expression above to complete the following tasks.

- ❑ In Box A, write a story problem that matches the expression.
- ❑ In Box B, draw a model or diagram to represent the expression.
- ❑ In Box C, describe how you would think about and solve the problem you wrote.

Box A

Box B

Box C

Name_____

3 boxes

Use the expression above to complete the following tasks.

- ❑ In Box A, write an **addition** story problem that matches the expression.
- ❑ In Box B, draw a model or diagram to represent the expression.
- ❑ In Box C, describe how you would think about and solve the problem you wrote.

Box A

Box B

Box C

Name_____

3 boxes

Use the expression above to complete the following tasks.

- ❑ In Box A, write a **subtraction** story problem that matches the expression.
- ❑ In Box B, draw a model or diagram to represent the expression.
- ❑ In Box C, describe how you would think about and solve the problem you wrote.

Box A

Box B

Box C

Name_____

3 boxes

Use the expression above to complete the following tasks.

- ❑ In Box A, write a **multiplication** story problem that matches the expression.
- ❑ In Box B, draw a model or diagram to represent the expression.
- ❑ In Box C, describe how you would think about and solve the problem you wrote.

Box A

Box B

Box C

Name_____

3 boxes

Use the expression above to complete the following tasks.

- ❑ In Box A, write a **division** story problem that matches the expression.
- ❑ In Box B, draw a model or diagram to represent the expression.
- ❑ In Box C, describe how you would think about and solve the problem you wrote.

Box A

Box B

Box C

Solve Two-Step Word Problems Using the Four Operations

INSTRUCTIONAL ACTIVITY SUPPLEMENT

Lesson 1

The following page is to be used in a whole-class discussion to introduce the activity students will complete using the Instructional Activity Student Handout.

17 – 9

Use the expression above to complete the following tasks.

- ❑ In Box A, write a story problem that matches the expression.
- ❑ In Box B, draw a model or diagram to represent the expression.
- ❑ In Box C, describe how you would think about and solve the problem you wrote.

Box A

Box B

Box C

SOLVE TWO-STEP WORD PROBLEMS USING THE FOUR OPERATIONS

INSTRUCTIONAL ACTIVITY

Lesson 2

LEARNING GOAL

Students will explore and describe the hidden question that exists in a multi-step word problem.

PRIMARY ACTIVITY

Students will be given a one-step word problem to solve. Then they will use that answer to ask a second follow-up problem. By combining the first problem with the follow-up problem, students will create a multi-step word problem. Finally, students will write the hidden question within a multi-step problem to illustrate the problem-solving steps for a multi-step word problem.

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

- Expression
- Mathematical operations

MATERIALS

- INSTRUCTIONAL ACTIVITY SUPPLEMENT (Recommend one problem copy for every group of two to three students.)
- INSTRUCTIONAL ACTIVITY STUDENT HANDOUT (Recommend a half page per student.)

IMPLEMENTATION

In LESSON 1 students explored how a single expression may fit into multiple situations. Similarly, in this lesson, students should see how a single value may be used in multiple situations or as part of multiple questions.

Present to students the following fact:

> Cami can walk three miles per hour.

Direct students to discuss the fact and provide different questions that could utilize this fact. Students should also think about the other pieces of information that can be determined using this fact.

GUIDING QUESTIONS

Elicit student thinking:

- Does this seem like a reasonable walking speed?
- What does this walking speed mean in a larger sense?
- In what situations could this walking speed be useful?
- How could this walking speed be used to find other facts?

Discuss with students that many times information will not be explicitly given, but instead a question or problem situation is given. An answer must first be found for the question or problem situation and then built upon to determine further information.

Present to students the following problem situation:

> Nathaniel spent $30 on shoes and $20 on jeans.

Discuss with students the situation and determine what further facts can be found using this information as a starting point.

Expand on the previous information and **present** to students the following problem situation.

> Nathaniel spent $30 on shoes and $20 on jeans. How much did he spend in total?

GUIDING QUESTIONS

Elicit student thinking:

- What is happening in this situation?
- What mathematical operation makes sense with this situation?
- What could this answer be used to tell us?
- What larger situation might this information fit into?
- In what situations could this spending information be useful?
- How could this spending information be used to find more information?

Follow the first problem situation with a second question that requires the first answer to solve:

> Nathaniel started with $70. How much does he have left after purchasing the shoes and jeans?

GUIDING QUESTIONS

Elicit student thinking:

- What mathematical operation makes sense with this new situation?
- How does the first problem situation fit into this second problem situation?
- Can the second problem situation be answered without the first pieces of information?

Determine if the student can DETERMINE THE UNKNOWN IN A SUBTRACTION EQUATION:

- Why does subtraction fit this situation?
- What subtraction expression would represent this situation?
- How could you model this situation?

Ask students to combine the two situations into a single problem. **Discuss** the hidden question that is incorporated when the two problems situations are combined into one.

Place students into groups of two or three. Each group should be given a one-step problem situation from the INSTRUCTIONAL ACTIVITY SUPPLEMENT. **Give** the students the task of answering the given problem situation and then writing a second question to expand upon their original problem situation. Finally students should combine the two questions into a single multi-step problem situation with a hidden question.

After students have written their problem situation containing a hidden question, **direct** groups to trade with another group. The new task will be to read through the situation and then identify and answer the hidden question in the multi-step problem situation. **Highlight** that a variety of hidden questions may be written. Multiple techniques to solve the multi-step problem could be followed, and different techniques will follow from differently conceived hidden questions.

To assist struggling students or English-language learners, **give** them problem situations to work on with their group; the problem situations should be similar to the ones that were discussed by the whole class. Additionally, continue to **encourage** students to draw models to represent the problem situation and help them visualize how to solve the problem.

GUIDING QUESTIONS

Elicit student thinking:

- What is happening in this situation?
- What mathematical operation makes sense with this situation?
- What could this answer be used to tell us?
- What larger situation might this information fit into?

Determine if the student is ready to SOLVE 2-STEP WORD PROBLEMS USING THE 4 OPERATIONS:

- What hidden question can you write to fit the situation?
- How do you answer the hidden question?
- How does the answer to the hidden question help solve the final question?
- Could another hidden question be found that would help to answer the problem situation?
- Could you work through this problem in more than one way?

Students should be required to connect the two steps in a multi-step word problem using a hidden question. They should then answer the hidden question before moving on to answer the final question asked.

At the end of the activity, teachers should **give** students the exit ticket found in the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT. Students will identify and answer the hidden question in the problem situation. Then students will answer the explicit question from the problem situation.

Name_____

Solve Two-Step Word Problems Using the Four Operations

Lesson 2

The following page is an exit ticket. Each student should receive a half page to complete at the end of the lesson.

Name_____

Exit Ticket

Guy has a $20 bill. If he buys four baseball card packs that cost $4 each, how much money will Craig have left?

Name_____

Exit Ticket

Guy has a $20 bill. If he buys four baseball card packs that cost $4 each, how much money will Craig have left?

Solve Two-Step Word Problems Using the Four Operations

INSTRUCTIONAL ACTIVITY SUPPLEMENT

Lesson 2

The following six pages give various one-step problem situations. In groups of two or three, students will answer the problem situation and write a follow-up question that relies on the answer from the given problem situation. After writing a follow-up question, student will combine the given problem situation with their follow-up question to create a multi-step problem situation that includes a hidden question.

Problem Situation #1

Josey is playing a practice math game online. In Round 1, Josey earns 30 stars. In Round 2, Josey earns 20 stars. How many stars has Josey earned so far?

Problem Situation #2

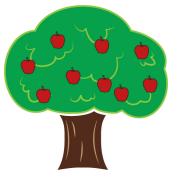
Elsa shops for winter clothes. She buys a pair of boots for $35 and socks for $25. How much does Elsa spend on boots and socks?

Problem Situation #3

Jaylee started the morning with 12 colored pencils. After lunch, Jaylee found 3 more colored pencils in her classroom. How many colored pencils does Jaylee have now?

Problem Situation #4

The apple tree in Johnny's backyard had 24 apples yesterday morning. During the storm last night, 8 apples fell to the ground. How many apples are in the tree today?



Problem Situation #5

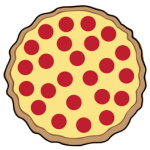
Max has $10 in his savings. Max spends $3 on popcorn. How much money does Max have left in his savings?

Problem Situation #6

A squirrel gathered and buried 40 acorns. Then a chipmunk took 15 acorns. How many acorns does the squirrel have left?

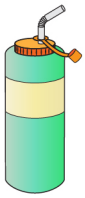
Problem Situation #7

A party-size pizza serves 5 people. How many people can be served with 7 pizzas?



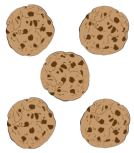
Problem Situation #8

Reusable water bottles are on sale for $7 each. How much money will 6 water bottles cost?



Problem Situation #9

Tory bakes cookies for a party. She thinks each person will eat 5 cookies. How many cookies does Tory need to bake for 8 people?



Problem Situation #10

A jumbo-size package of beads contains 88 beads. One package of beads makes 4 bracelets. How many beads make up each bracelet?

Problem Situation #11

Mrs. Jameson's class brought paper bags to class for a project. Each of the 18 students brought the same amount of bags to class. The class has a total of 54 bags. How many bags did each student bring to class?

Problem Situation #12

Chloe spends $21 on 7 polished rocks. How much money does each polished rock cost?

SOLVE TWO-STEP WORD PROBLEMS USING THE FOUR OPERATIONS

INSTRUCTIONAL ACTIVITY

Lesson 3

LEARNING GOAL

Students will follow the four-step problem-solving process to solve multi-step word problems.

PRIMARY ACTIVITY

Students will explore Polya's four steps to problem solving. Through this exploration, students will break down multi-step words problems to analyze and solve the problem. In the *Understand the Problem* stage, students will focus on making sure they have enough information or eliminating extra information.

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

- Expression
- Four-step problem solving process

MATERIALS

- Assorted manipulatives: counters, chips, base-ten blocks, small tiles, etc.
- Highlighters, colored pencils, or markers
- Dry-erase markers (optional)
- INSTRUCTIONAL ACTIVITY SUPPLEMENT (Recommend one copy to display to the class.)
- INSTRUCTIONAL ACTIVITY STUDENT HANDOUT

IMPLEMENTATION

Students have previously solved one-step word problems in their educational career. In LESSON 2, students focused on the hidden question to create the first step in a two-step word problem. In this lesson, students

will dive into the four-step problem solving process to analyze a two-step word problem for a successful solution.

Throughout the lesson, **make available** different manipulatives for students to use in the problem-solving process. **Encourage** students to represent and model the mathematical situations using the manipulatives, diagrams, or drawings.

Pose the following problem to students. The numbers have been removed to help students avoid the tendency to move straight into number crunching.

> Jaylee has ■ candies to share with her classmates and ■ cookies to share with her family. There are ■ students in Jaylee's class. How many candies will each student receive?

Display the INSTRUCTIONAL ACTIVITY SUPPLEMENT with Polya's four-step problem-solving process. **Walk** students through the four steps, using the problem above to demonstrate the analysis on a one-step word problem. In the first step of the problem-solving process, **encourage** students to highlight or use color to mark important information. Students could use one color to mark information that will be use in calculations and a second color to mark the final question asked. By using color to mark the word problems, students can visually identify the important information and refer back to that information throughout the problem-solving process. Additionally, **encourage** students to cross out unnecessary information. This will help students focus on the information vital to solving the problem.

Next, **give** students the same problem, but this time the numbers are revealed for students. **Prompt** students to use the previous discussion to guide their steps to solving the problem. Students should notice that the analysis of the problem is mostly the same even though the numbers are revealed. Again, students should mark the important information and cross out unnecessary information before operating to solve the problem.

> Jaylee has 72 candies to share with her classmates and 20 cookies to share with her family. There are 24 students in Jaylee's class. How many candies will each student receive?

GUIDING QUESTIONS

Elicit student thinking:

- What vocabulary is used in the problem?
- What does the vocabulary mean?
- What are important terms used in the problem?
- What are you asked to do?
- What information is given in the problem?
- Is there extra information given in the problem?
- Is there enough information given in the problem to solve?
- What do you think the answer will look like or be?
- What model can you draw to represent this problem?
- Does your answer make sense?
- Did you answer the question asked?
- Did you answer all questions asked?

Next, **pose** the following two-step word problem to students. As before, the numbers are removed in the first analysis. Again, **display** the INSTRUCTIONAL ACTIVITY SUPPLEMENT and **walk** students through the four steps of Polya's problem-solving process.

> Walter planted ■ tomato plants and ■ zucchini plants in his garden. Yesterday he picked ■ tomatoes off each of his tomato plants. Today Walter used ■ tomatoes to make a sandwich. How many tomatoes does Walter have left?

After students have initially analyzed the problem, **present** the problem with the numbers revealed. **Ask** students if their plan to solve the problem is any different now that they see the numbers given in the problem.

Walter planted 12 tomato plants and 5 zucchini plants in his garden. Yesterday he picked 3 tomatoes off each of his tomato plants. Today Walter used 2 tomatoes to make a sandwich. How many tomatoes does Walter have left?

GUIDING QUESTIONS

Elicit student thinking:

- What vocabulary is used in the problem?
- What does the vocabulary mean?
- What are important terms used in the problem?
- What are you asked to do?
- What information is given in the problem?
- Is there extra information given in the problem?
- Is there enough information given in the problem to solve?
- What do you think the answer will look like or be?
- What is the hidden question within the two-step problem?
- How would you answer the hidden question?
- How does the hidden question connect to the overall problem?
- What model can you draw to represent this problem?
- Does your answer make sense?
- Did you answer the question asked?
- Did you answer all questions asked?

Provide to students the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT. The first page of INSTRUCTIONAL ACTIVITY STUDENT HANDOUT could be laminated or placed into a page protector for students to write on with a dry-erase marker and then reuse with multiple problems. Alternatively, students could receive multiple copies of this page to use with multiple problems. The

second page provides students two-step word problems to solve using the four-step problem-solving process. **Instruct** students to walk through the four-step problem-solving process carefully for each problem. In the first step of understanding the problem, students should determine if they have extra information, which can be marked out, or missing information that prevents an answer from being found. Some of the problems will have extra information while others may not have enough information to solve.

GUIDING QUESTIONS

Elicit student thinking:

- What information is given in the problem?
- Are you given enough information?
- Is extra information given in the problem?

Determine if the student can SOLVE 2-STEP WORD PROBLEMS USING THE 4 OPERATIONS:

- What situation is being described in the problem?
- What mathematical operation fits with this situation?
- What do you think the answer will look like or be?
- What is the hidden question in the two-step problem?
- How would you solve the hidden question?
- How does the hidden question connect to the overall situation?
- What model could you draw to represent the situation?
- Does your answer make sense?
- Did you answer all questions asked?

Students should be required to explain how each step of the four-step problem solving process is applied to a given two-step word problem.

At the end of the activity, teachers should **direct** students to discuss with a partner how they used the four-step problem-solving process to answer one of the two-step word problems from the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

Name_____

Solve Two-Step Word Problems Using the Four Operations

Lesson 3

Understand the Problem	**Make a Plan**
Carry Out the Plan	**Look Back**

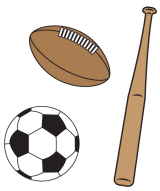
Name_____

1) Lizzie has $100 to shop for new clothes. She buys 1 pair of shoes and 1 pair of jeans. The shoes cost $30, and the jeans cost $24. How much change will Lizzie get back?

2) Sam is helping at his little sister's birthday party. Sam has 40 balloons and 12 spoons to use in a party game. Each of the 12 guests needs 3 balloons for the game. How many extra balloons does Sam have?

Name_____

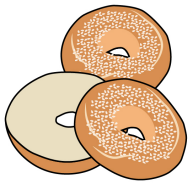
3) Aidan is cleaning his garage. He plans to put all the sports balls into a plastic tub. Aidan finds 4 soccer balls. He finds 2 more footballs than soccer balls. He sees 2 baseball bats hanging on the wall. How many sports balls will Aidan put into his plastic tub?



4) Kenzie needs 28 hangers for her clothes. She finds 2 packages of 10 hangers. How many more hangers should Kenzie look for?

Name_____

5) Tomas is at the grocery store. He buys 2 blueberry bagels, 4 wheat bagels, and some 2-packs of onion bagels. How many bagels does Tomas buy?



6) Trina is collecting leaves for a science project. On Saturday, she collects 13 oak leaves, 5 elm leaves, and 4 maple leaves. On Sunday, Trina collects twice as many leaves. How many total leaves did Trina collect on Sunday?

Name_____

7) Dailon collects comic books. He has 30 total comic books. Half of the comic books are about superheroes. Dailon also has comic books about aliens or monsters. Dailon has 2 fewer comic books about aliens than superheroes. How many alien comic books does Dailon have?

Polya's Four-Step Problem-Solving Process

Step 1: Understand the problem • Begin by reading the problem. Then re-read the problem. • Do you understand all of the vocabulary in the problem? • What are you being asked to do? • Is there enough information to solve the problem? • Is there extra information you don't need?	Step 2: Make a plan • How might you solve this problem? • Would a model, diagram, or picture help? • Is there a simpler problem you could solve? • Do you see a pattern?
Step 3: Carry out the plan • Now that you have a plan, take the steps needed to follow your plan.	Step 4: Look back • Re-read the problem again. • Does your answer make sense? • Double-check your calculations. • Did you answer the question asked? Did you answer all questions asked?

SOLVE TWO-STEP WORD PROBLEMS USING THE FOUR OPERATIONS

INSTRUCTIONAL ACTIVITY

Lesson 4

LEARNING GOAL

Students will write and solve equations to represent two-step operations.

PRIMARY ACTIVITY

Students will move throughout the room, where two-step words problems will be posted. For each word problem, students will first define a variable to represent the unknown information. Then students will write and solve an equation using the defined variable and the given information. Finally, students will look for their answer and the corresponding letter to fill in the blanks to decode the answer to a riddle.

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

- Variable
- Equation
- Hidden question

MATERIALS

- INSTRUCTIONAL ACTIVITY STUDENT HANDOUT
- INSTRUCTIONAL ACTIVITY SUPPLEMENT (Recommend one copy of each problem page to post around the room.)

IMPLEMENTATION

During this activity, students will be moving around the room to read and answer 12 different two-step word problems. The INSTRUCTIONAL ACTIVITY SUPPLEMENT contains the 12 word problems, each on a separate page. It is best to post the pages in order around the room to help students navigate through the pages, but students do not need to work the problems in order. As long as students complete all 12 problems, they will be able to decode the answer to the riddle.

Post or **place** the 12 problem pages around the room where students can easily access the problems. **Allow** enough space between the problems to avoid traffic jams.

A new feature to their problem-solving process in this lesson is the use of a variable to represent the unknown information. This variable will be used to write an equation that can be solved to answer a word problem.

Explain to students that they will be solving word problems using a variable to represent the unknown quantity. **Pose** to students the following word problem.

> Laynia has been saving up her money to buy a new backpack. She saved $15.00 last month and saved $18.00 this month. The backpack she wants to buy is $47.00. How much money does Laynia still need to buy the backpack?

First, **ask** students to identify from the problem the unknown information. **Review**, as needed, the meaning and use of variables. **Define** a variable to represent the unknown information. Next, using the variable, **write** an equation that describes the mathematical situation in the word problem. This step may involve finding and answering the hidden question in the two-step word problems. Finally, **solve** the equation to answer the question.

GUIDING QUESTIONS

Elicit student thinking:

- What information is given in the problem?
- Is enough information given in the problem?
- What are you being asked to find?

Determine if the student can SOLVE TWO-STEP WORD PROBLEMS USING THE FOUR OPERATIONS:

- What situation is being described in the problem?
- What mathematical operation or operations fit with this situation?
- What equation can be written using the variable and the operations for this situation?
- Is there a hidden question in the two-step problem?
- How would you solve the hidden question?
- How does the hidden question connect to the overall situation?
- What model could you draw to represent the situation?
- Does your answer make sense?

Distribute to each student a copy of the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT. Students will use this as their work page to record their work and thinking for the 12 word problems. Additionally, at the top of the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT is the riddle that students will be working to decode.

Instruct students to read each problem carefully. **Remind** students of LESSON 3 where they explored the four-step problem-solving process as they work to solve the two-step word problems. Before solving the word problems, students will first define a variable to use in an equation to represent the situation. Students will write and solve this equation to answer the problem. After finding a solution, students will choose the answer from the three choices given at the bottom of each posted problem page. Students will use the letter beside their answer choice to decode the riddle at the top of the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

Students should be required to complete the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT with variables and equations for the word problems.

At the end of the activity, teachers should have students share and compare with partners their answers, work and thinking, problem solutions, and answer to the riddle.

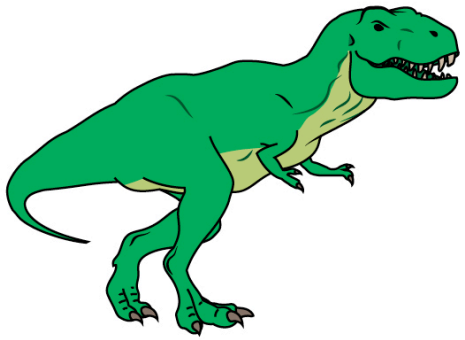
Name_____

Solve Two-Step Word Problems Using the Four Operations

Lesson 4

What do you call two dinosaurs that get into an accident?

5	**12**	**6**	**1**	**4**	**4**	**11**	**2**	**1**	**10**	**6**	**10**	**2**		**3**	**6**	**7**	**8**	**9**	**2**



Name_____

Problem #1

Define a variable for the unknown information.

Write an equation using the variable.

Solve your equation.

Problem #2

Define a variable for the unknown information.

Write an equation using the variable.

Solve your equation.

Name_____

Problem #3

Define a variable for the unknown information.

Write an equation using the variable.

Solve your equation.

Problem #4

Define a variable for the unknown information.

Write an equation using the variable.

Solve your equation.

Name_____

Problem #5

Define a variable for the unknown information.

Write an equation using the variable.

Solve your equation.

Problem #6

Define a variable for the unknown information.

Write an equation using the variable.

Solve your equation.

Name_____

Problem #7

Define a variable for the unknown information.

Write an equation using the variable.

Solve your equation.

Problem #8

Define a variable for the unknown information.

Write an equation using the variable.

Solve your equation.

Name_____

Problem #9

Define a variable for the unknown information.

Write an equation using the variable.

Solve your equation.

Problem #10

Define a variable for the unknown information.

Write an equation using the variable.

Solve your equation.

Name_____

Problem #11

Define a variable for the unknown information.

Write an equation using the variable.

Solve your equation.

Problem #12

Define a variable for the unknown information.

Write an equation using the variable.

Solve your equation.

Solve Two-Step Word Problems Using the Four Operations

INSTRUCTIONAL ACTIVITY SUPPLEMENT

Lesson 4

The following 12 pages are to be posted throughout the room for students.

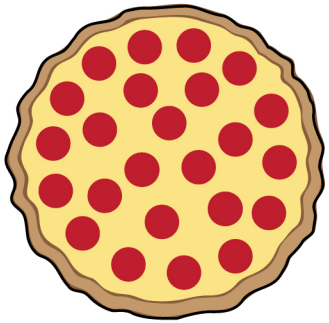
The student work page is provided in the Instructional Activity Student Handout.

Problem #1

Read the problem below.

1) **Using your work page, identify the unknown quantity you are asked to find. Then define a variable to represent the unknown quantity.**
2) **Write an equation using the given information and the variable defined.**
3) **Solve the equation.**
4) **Find your answer at the bottom of this page. Write the letter of the answer in the box corresponding to the problem number on your work page to decode the riddle.**

Your class is celebrating Pi Day! For a special lunch, your teacher is buying pizza. Your class needs 5 pizzas for everyone to have lunch. Pizzas cost $13 each. How much change will your teacher get back if he pays with $80?



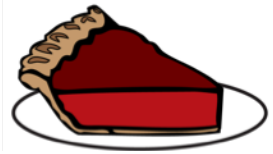
A: $15 in change N: $65 in change F: $25

Problem #2

Read the problem below.

1) **Using your work page, identify the unknown quantity you are asked to find. Then define a variable to represent the unknown quantity.**
2) **Write an equation using the given information and the variable defined.**
3) **Solve the equation.**
4) **Find your answer at the bottom of this page. Write the letter of the answer in the box corresponding to the problem number on your work page to decode the riddle.**

After pizza, the class will have pie for dessert. Several students volunteered to bring pie. Sally is bringing 2 pies. Joey and Fryna are each bringing 1 pie. If each pie is cut into 8 pieces, how many pieces of pie will there be?



E: 8 pieces I: 24 pieces S: 32 pieces

Problem #3

Read the problem below.

1) **Using your work page, identify the unknown quantity you are asked to find. Then define a variable to represent the unknown quantity.**
2) **Write an equation using the given information and the variable defined.**
3) **Solve the equation.**
4) **Find your answer at the bottom of this page. Write the letter of the answer in the box corresponding to the problem number on your work page to decode the riddle.**

To eat the pizza and pie, everyone needs a plate. Your class has 5 tables with 4 students each. A package of plates contains 16 plates. How many more plates will be needed for the class?

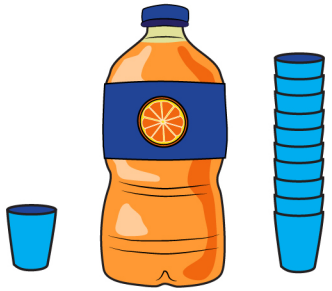
T: 16 more plates W: 4 more plates H: 5 more plates

Problem #4

Read the problem below.

1) **Using your work page, identify the unknown quantity you are asked to find. Then define a variable to represent the unknown quantity.**
2) **Write an equation using the given information and the variable defined.**
3) **Solve the equation.**
4) **Find your answer at the bottom of this page. Write the letter of the answer in the box corresponding to the problem number on your work page to decode the riddle.**

Everyone will get a cup of juice. The class needs 21 cups. Johanna brought 10 red cups, and Tyler brought 7 black cups, how many more cups are needed?



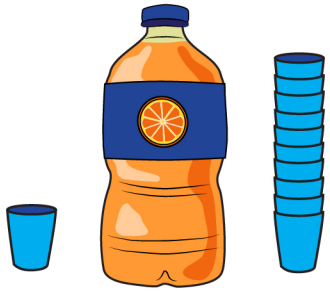
N: 4 more cups M: 7 more cups D: 18 more cups

Problem #5

Read the problem below.

1) **Using your work page, identify the unknown quantity you are asked to find. Then define a variable to represent the unknown quantity.**
2) **Write an equation using the given information and the variable defined.**
3) **Solve the equation.**
4) **Find your answer at the bottom of this page. Write the letter of the answer in the box corresponding to the problem number on your work page to decode the riddle.**

Suzie has a 64-ounce juice bottle to pour evenly into 6 cups. When pouring the first cup, Suzie spills 4 ounces. If she doesn't spill anymore, how much juice can go evenly into each remaining cup?



D: 6 ounces I: 9 ounces T: 10 ounces

Problem #6

Read the problem below.

1) **Using your work page, identify the unknown quantity you are asked to find. Then define a variable to represent the unknown quantity.**
2) **Write an equation using the given information and the variable defined.**
3) **Solve the equation.**
4) **Find your answer at the bottom of this page. Write the letter of the answer in the box corresponding to the problem number on your work page to decode the riddle.**

To celebrate on Pi Day, your class plays basketball. During his game, Jaymes makes 6 two-point shots and 3 three-point shots. How many points does Jaymes score in total?

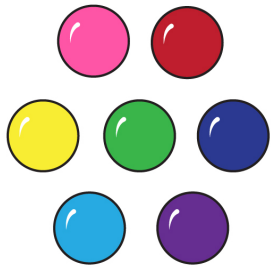
W: 24 points R: 21 points K: 9 points

Problem #7

Read the problem below.

1) **Using your work page, identify the unknown quantity you are asked to find. Then define a variable to represent the unknown quantity.**
2) **Write an equation using the given information and the variable defined.**
3) **Solve the equation.**
4) **Find your answer at the bottom of this page. Write the letter of the answer in the box corresponding to the problem number on your work page to decode the riddle.**

Brynn has 2 packs of gumballs to share with her table. Each pack has 10 gumballs. If 4 people share the gumballs evenly, how many gumballs does each person get?



A: 20 gumballs E: 5 gumballs O: 2 gumballs

Problem #8

Read the problem below.

1) **Using your work page, identify the unknown quantity you are asked to find. Then define a variable to represent the unknown quantity.**
2) **Write an equation using the given information and the variable defined.**
3) **Solve the equation.**
4) **Find your answer at the bottom of this page. Write the letter of the answer in the box corresponding to the problem number on your work page to decode the riddle.**

Because Pi Day is also Albert Einstein's birthday, each student has an Einstein coloring page. On the counter is a tub of colored pencils. The tub filled with 9 boxes of 12 colored pencils, but 3 colored pencils have been lost. How many colored pencils are in the tub now?

A: 111 colored pencils

E: 87 colored pencils

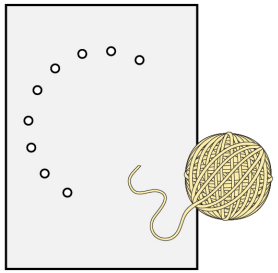
C: 105 colored pencils

Problem #9

Read the problem below.

1) **Using your work page, identify the unknown quantity you are asked to find. Then define a variable to represent the unknown quantity.**
2) **Write an equation using the given information and the variable defined.**
3) **Solve the equation.**
4) **Find your answer at the bottom of this page. Write the letter of the answer in the box corresponding to the problem number on your work page to decode the riddle.**

During art class, you work on some string art. The circle outline needs 72 holes. First you punch 20 holes. Then you punch 18 more holes. How many holes do you have left to punch?



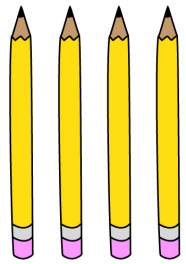
K: 34 more holes L: 32 more holes M: 36 more holes

Problem #10

Read the problem below.

1) **Using your work page, identify the unknown quantity you are asked to find. Then define a variable to represent the unknown quantity.**
2) **Write an equation using the given information and the variable defined.**
3) **Solve the equation.**
4) **Find your answer at the bottom of this page. Write the letter of the answer in the box corresponding to the problem number on your work page to decode the riddle.**

Your teacher bought special pencils with the digits of pi on them. He has 3 packs of 15 pencils. He gives each of the 20 students 2 pencils. How many pencils does your teacher have left over?



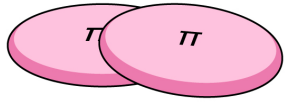
A: 0 pencils left over	U: 5 pencils left over	Y: 20 pencils left over

Problem #11

Read the problem below.

1) **Using your work page, identify the unknown quantity you are asked to find. Then define a variable to represent the unknown quantity.**
2) **Write an equation using the given information and the variable defined.**
3) **Solve the equation.**
4) **Find your answer at the bottom of this page. Write the letter of the answer in the box corresponding to the problem number on your work page to decode the riddle.**

The principal provided circular erasers for everyone. She gave your class a bag of 12 erasers, a pack of 24, and 4 loose erasers. How many erasers will each of the 20 students in your class get?



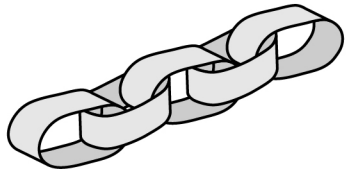
T: 3 erasers P: 1 eraser O: 2 erasers

Problem #12

Read the problem below.

1) **Using your work page, identify the unknown quantity you are asked to find. Then define a variable to represent the unknown quantity.**
2) **Write an equation using the given information and the variable defined.**
3) **Solve the equation.**
4) **Find your answer at the bottom of this page. Write the letter of the answer in the box corresponding to the problem number on your work page to decode the riddle.**

To decorate, your class writes the digits of pi on a paper chain and hangs it around the room. The ceiling is 120 inches high. How many 3-inch rings will it take to reach from the floor to the ceiling 5 times?



Y: 200 rings S: 72 rings G: 1800 rings

Name_____

Solve Two-Step Word Problems Using the Four Operations

Lessons 1 – 4

1. Gerry brought 48 cookies to school to celebrate National Cookie Day. He gave a cookie to the 18 students in his third-grade class other than himself. He also gave a cookie to each of the 17 students in the other third-grade class. Then Gerry gave 2 cookies to all 3 of the second-grade teachers and both third-grade teachers.

1.a. How many cookies did Gerry give to third-grade students?

1.b. How many cookies did Gerry give to the teachers?

Name_____

1.c. How many cookies did Gerry give away altogether?

1.d. How many cookies does Gerry have left over?

2. Kanzee Elementary School is collecting box tops to raise money to fix the slide on the playground. To start, the principal puts 50 box tops in the bin.

2.a. In the first week, three classes all place 30 box tops into the bin. How many box tops are in the bin at the end of the first week?

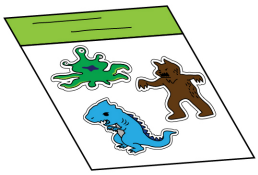
Name_____

2.b. In the second week, Mr. Prince's class adds 56 box tops, Mrs. Lowe's class adds 34 box tops, and Ms. Waller's class adds 86 box tops. How many box tops are in the bin at the end of the second week?

2.c. The school's goal is to collect 500 box tops. How many more box tops does the school need?

Name_____

3. Makio has 140 monster stickers in his collection. First, Makio gives half of the monster stickers to his brother. Then he trades 13 monster stickers to his friend Satha for a book.



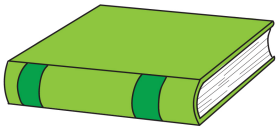
3.a. How many monster stickers does Makio have left?

3.b. Satha has been collecting monster stickers for 3 weeks. Each week, she adds 7 new monster stickers to her collection. How many monster stickers does Satha have after trading with Makio?

Name_____

3.c. Who has more monster stickers after the trade, Makio or Satha? How many more stickers does that person have compared to the other?

4. Victor is reading a book that has 350 pages. He reads 35 pages on both Sunday and Monday.



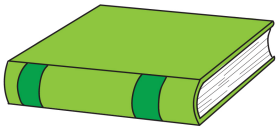
4.a. Write an equation to represent how many pages Victor has left to read. Use *a* to stand for the amount of pages Victor has left to read.

Name_____

4.b. Solve for *a*.

4.c. Victor needs to finish the book in 5 more days. How many pages does Victor need to read each day?

5. Johnny wants to buy a book that costs $26. He already has $5 from doing yard work. Johnny plans to save the rest of the money for the book from his weekly allowance of $3.



5.a. How many weeks will Johnny need to save money to have enough to buy the book? Explain your answer.

Name_____

5.b. Johnny earns an extra $5 for mowing his neighbor's lawn. Now how many weeks will Johnny need to save his allowance to buy the book? Explain your answer.

5.c. After doing yard work, mowing his neighbor's yard, and saving his allowance, will Johnny have the exact amount of money needed to buy the book or some extra? If Johnny will have extra money, how much extra will he have?

Name_____

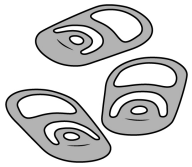
6. Leigha has 14 stuffed animals. Kylee has twice as many stuffed animals as Leigha. How many stuffed animals do Leigha and Kylee have altogether?

6.a. What is the first step needed to answer the question?

6.b. Find how many stuffed animals Leigha and Kylee have altogether.

Name_____

7. The following table shows the pounds of pop tabs collected by third, fourth, and fifth-grade classes at South Elementary School, Trails Elementary School, and Frontier School.



	South Elementary School	**Trails Elementary School**	**Frontier School**
Third Grade	10 pounds	9 pounds	14 pounds
Fourth Grade	8 pounds	12 pounds	10 pounds
Fifth Grade	12 pounds	11 pounds	9 pounds

7.a. Which school collected the most pop tabs? How much did that school collect?

7.b. Which grade collected the most pop tabs? How much did that grade collect?

Name_____

7.c. South Elementary School finds an extra 2 pounds of pop tabs in a lost collection box. Does this amount change the answer to 7.a? Explain your answer.

SOLVE TWO-STEP WORD PROBLEMS USING THE FOUR OPERATIONS

STUDENT ACTIVITY SOLUTION GUIDE

Lessons 1 – 4

1. Gerry brought 48 cookies to school to celebrate National Cookie Day. He gave a cookie to each of the 18 students in his third-grade class other than himself. He also gave a cookie to each of the 17 students in the other third-grade class. Then Gerry gave 2 cookies to all 3 of the second-grade teachers and both third-grade teachers.

1.a. How many cookies did Gerry give to third-grade students?

CORRECT ANSWER

Gerry gave 35 cookies to third-grade students.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Gerry gave 36 cookies to the third-grade students.	adds 2 classes of 18 students each instead of a class of 18 and a class of 17	SOLVE ADDITION WORD PROBLEMS WITHIN 100
Gerry gave 34 cookies to the third-grade students.	adds 2 classes of 17 students each instead of a class of 18 and a class of 17	SOLVE ADDITION WORD PROBLEMS WITHIN 100
Gerry gave 25 cookies to the third-grade students.	adds incorrectly the class of 18 with the class of 17	ADD WITHIN 100
Gerry gave 83 cookies to the third-grade students.	adds the students in the classes with the number of cookies brought	SOLVE ADDITION WORD PROBLEMS WITHIN 100

1.b. How many cookies did Gerry give to the teachers?

CORRECT ANSWER

Gerry gave 10 cookies to the teachers.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Gerry gave 5 cookies to the teachers.	does not calculate 2 cookies given to each teacher	MATHEMATIZE CONTEXTUAL SITUATIONS INVOLVING MULTIPLICATION
Gerry gave 6 cookies to the teachers.	only calculates giving 2 cookies to each of the 3 second-grade teachers	MATHEMATIZE CONTEXTUAL SITUATIONS INVOLVING MULTIPLICATION
Gerry gave 8 cookies to the teachers.	does not count the number of teachers correctly	MATHEMATIZE CONTEXTUAL SITUATIONS INVOLVING MULTIPLICATION

1.c. How many cookies did Gerry give away altogether?

CORRECT ANSWER

> Gerry gave away 45 cookies altogether.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Gerry gave away 35 cookies altogether.	does not calculate cookies given to both students and teachers	SOLVE ADDITION WORD PROBLEMS WITHIN 100
Gerry gave away 25 cookies altogether.	does not add the cookies given to the students and teachers but instead subtracted	SOLVE ADDITION WORD PROBLEMS WITHIN 100
Gerry gave away 48 cookies altogether.	adds incorrectly	ADD WITHIN 100

1.d. How many cookies does Gerry have left over?

CORRECT ANSWER

> Gerry has 3 cookies left over.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Gerry had 45 cookies left over.	does not calculate leftover cookies but instead the cookies given away	MATHEMATIZE CONTEXTUAL SITUATIONS INVOLVING SUBTRACTION
Gerry had 2 cookies left over.	subtracts incorrectly	SUBTRACT WITHIN 100
Gerry had no cookies left over.	subtracts incorrectly	SUBTRACT WITHIN 100

2. Kanzee Elementary School is collecting box tops to raise money to fix the slide on the playground. To start, the principal puts 50 box tops in the bin.

2.a. In the first week, three classes all place 30 box tops into the bin. How many box tops are in the bin at the end of the first week?

CORRECT ANSWER

> At the end of the first week, 140 box tops are in the bin.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
At the end of the first week, 90 box tops are in the bin.	only calculates the box tops placed in the bin by the 3 classes	SOLVE ADDITION WORD PROBLEMS WITHIN 100
At the end of the first week, 80 box tops are in the bin.	adds the box tops placed in the bin by the principal and only 1 class	SOLVE ADDITION WORD PROBLEMS WITHIN 100
At the end of the first week, 150 box tops are in the bin.	adds incorrectly	ADD WITHIN 1,000

2.b. In the second week, Mr. Prince's class adds 56 box tops, Mrs. Lowe's class adds 34 box tops, and Ms. Waller's class adds 86 box tops. How many box tops are in the bin at the end of the second week?

CORRECT ANSWER

> At the end of the second week, 316 box tops are in the bin.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
At the end of the second week, 176 box tops are in the bin.	only calculates the box tops brought in the second week	MATHEMATIZE CONTEXTUAL SITUATIONS INVOLVING ADDITION
At the end of the second week, 140 box tops are in the bin.	does not add the box tops brought in the second week	MATHEMATIZE CONTEXTUAL SITUATIONS INVOLVING ADDITION
At the end of the second week, 306 box tops are in the bin.	adds incorrectly	ADD WITHIN 1,000

2.c. The school's goal is to collect 500 box tops. How many more box tops does the school need?

CORRECT ANSWER

> The school needs to collect 184 more box tops.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The school needs to collect 316 more box tops.	does not subtract the collected box tops from the 500 needed	MATHEMATIZE CONTEXTUAL SITUATIONS INVOLVING SUBTRACTION
The school needs to collects 174 more box tops.	subtracts incorrectly	SUBTRACT WITHIN 1,000
The school needs to collects 816 more box tops.	adds the collected box tops to the required 500 box tops	MATHEMATIZE CONTEXTUAL SITUATIONS INVOLVING SUBTRACTION

3. Makio has 140 monster stickers in his collection. First, Makio gives half of the monster stickers to his brother. Then he trades 13 to his friend Satha for a book.

3.a. How many monster stickers does Makio have left?

CORRECT ANSWER

Makio has 57 monster stickers left.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Makio has 70 monster stickers left.	does not subtract the stickers traded to Satha	SOLVE 2-STEP ADDITION AND SUBTRACTION WORD PROBLEMS
Makio has 127 monster stickers left.	does not subtract the stickers given to brother	SOLVE 2-STEP ADDITION AND SUBTRACTION WORD PROBLEMS

3.b. Satha has been collecting monster stickers for 3 weeks. Each week, she adds 7 new monster stickers to her collection. How many monster stickers does Satha have after trading with Makio?

CORRECT ANSWER

After trading with Makio, Satha has 34 monster stickers.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
After trading with Makio, Satha has 13 monster stickers.	does not add the weekly collected stickers	SOLVE 2-STEP WORD PROBLEMS USING THE 4 OPERATIONS
Satha has 21 monster stickers.	only calculates the weekly collected stickers without adding the stickers traded with Makio	SOLVE 2-STEP WORD PROBLEMS USING THE 4 OPERATIONS
After trading with Makio, Satha has 23 monster stickers.	incorrectly calculates using the 3 given numbers	SOLVE 2-STEP WORD PROBLEMS USING THE 4 OPERATIONS

3.c. Who has more monster stickers after the trade, Makio or Satha? How many more stickers does that person have compared to the other?

CORRECT ANSWER

Makio has 23 more monster stickers than Satha.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Satha has 23 more monster stickers than Makio.	incorrectly compares the 2 values	SOLVE COMPARE PROBLEMS
Makio has 25 more monster stickers than Satha.	incorrectly finds the difference between Makio's and Satha's monster stickers	SUBTRACT WITHIN 100

4. Victor is reading a book that has 350 pages. He reads 35 pages each on both Sunday and Monday.

4.a. Write an equation to represent how many pages Victor has left to read. Use a to stand for the amount of pages Victor has left to read.

CORRECT ANSWER

$350 - 35 \times 2 = a$ *or* $a = 350 - 35 \times 2$

Either arrangement of the equation is correct as long as the expressions on each side are written correctly.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
$350 + 35 \times 2 = a$	adds instead of subtracts the pages Victor has read	MATHEMATIZE CONTEXTUAL SITUATIONS INVOLVING SUBTRACTION
$350 - 35 = a$	does not calculate 35 pages read for **both** Sunday and Monday	MATHEMATIZE CONTEXTUAL SITUATIONS INVOLVING MULTIPLICATION
$350 - a = 35$	misapplies the variable a to the pages Victor has read	REPRESENT EXPRESSIONS WITH NUMBERS AND/OR VARIABLES
$350 - 35 \times 2$	does not write an equation	EXPLAIN *EQUATION*

4.b. Solve for *a*.

CORRECT ANSWER

$a = 280$

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
$a = 420$	adds instead of subtracts the pages Victor has read	MATHEMATIZE CONTEXTUAL SITUATIONS INVOLVING SUBTRACTION
$a = 315$	does not calculate 35 pages read for **both** Sunday and Monday	MATHEMATIZE CONTEXTUAL SITUATIONS INVOLVING MULTIPLICATION

4.c. Victor needs to finish the book in 5 more days. How many pages does Victor need to read each day?

CORRECT ANSWER

Victor needs to read 56 pages each day.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Victor needs to read 70 pages each day.	divides the original 350 pages over the 5 days left to read	SOLVE 2-STEP WORD PROBLEMS USING THE FOUR OPERATIONS
Victor needs to read 40 pages each day.	divides the 280 pages left over 7 total days	SOLVE 2-STEP WORD PROBLEMS USING THE 4 OPERATIONS
Victor needs to read 60 pages each day.	divides incorrectly	DIVIDE BY 5

5. Johnny wants to buy a book that costs $26. He already has $5 from doing yard work. Johnny plans to save the rest of the money for the book from his weekly allowance of $3.

5.a. How many weeks will Johnny need to save money to have enough to buy the book? Explain your answer.

CORRECT ANSWER

> Johnny needs to save for 7 weeks, because $26 - $5 is $21, and $21 ÷ $3 is 7 weeks.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Johnny needs to save for 8 weeks, because $26 - $5 is $21, and $21 ÷ $3 is 8 weeks.	divides incorrectly	DIVIDE BY 3
Johnny needs to save for 9 weeks, because 9 weeks of $3 allowance is 9 × $3 =$27, which is enough for his $26 book.	does not subtract the $5 earned for yard work before dividing using $3 weekly allowance	SOLVE 2-STEP WORD PROBLEMS USING THE 4 OPERATIONS
Johnny needs to save for 18 weeks, because $26 – $5 is $21, and $21 - $3 is 18 weeks.	does not divide by the weekly allowance to determine how many weeks are needed to save	MATHEMATIZE CONTEXTUAL SITUATIONS INVOLVING DIVISION

5.b. Johnny earns an extra $5 for mowing his neighbor's lawn. Now how many weeks will Johnny need to save his allowance to buy the book? Explain your answer.

CORRECT ANSWER

> Johnny needs to save for 6 weeks, because Johnny earns $10 total for yard work, and $26 – $10 is $16 left. A $3 weekly allowance for 6 weeks gives Johnny $18, which is enough to cover the remaining $16.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Johnny needs to save for 5 weeks, because Johnny earns $10 total for yard work, and $26 – $10 is $16, and $16 ÷ $3 is 5 weeks.	divides incorrectly	DIVIDE BY 3
Johnny needs to save for 9 weeks, because 9 weeks of $3 allowance is 9 × $3 = $27, which is enough for his $26 book.	does not subtract the $10 earned for yard work before dividing using $3 weekly allowance	SOLVE 2-STEP WORD PROBLEMS USING THE 4 OPERATIONS
Johnny needs to save for 13 weeks, because $26 – $10 is $16, and $16 - $3 is 13 weeks.	does not divide by the weekly allowance to determine how many weeks are needed to save	MATHEMATIZE CONTEXTUAL SITUATIONS INVOLVING DIVISION

5.c. After doing yard work, mowing his neighbor's yard, and saving his allowance, will Johnny have the exact amount of money need to buy the book or some extra? If Johnny will have extra money, how much extra will he have?

CORRECT ANSWER

> Johnny will have $2 extra.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Johnny will have the exact amount.	divides incorrectly	DIVIDE BY 3
Johnny will have $1 extra.	subtracts incorrectly	SUBTRACT WITHIN 10

6. Leigha has 14 stuffed animals. Kylee has twice as many stuffed animals as Leigha. How many stuffed animals do Leigha and Kylee have altogether?

6.a. What is the first step needed to answer the question?

CORRECT ANSWER

First, determine how many stuffed animals each girl has. Leigha's amount is given, but Kylee's must be found by multiplying 14 times 2 to find that Kylee has 28 stuffed animals.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
First, add 14 and 14 for both girls stuffed animals.	does not calculate Kylee's stuffed animals to be twice as many as Leigha's	MATHEMATIZE CONTEXTUAL SITUATIONS INVOLVING MULTIPLICATION
First, determine how many stuffed animals each girl has. Leigha's amount is given, but Kylee's must be found by multiplying 14 times 3 to find that Kylee has 42 stuffed animals.	incorrectly calculates Kylee's stuffed animals as triple the amount of Leigha's	MATHEMATIZE CONTEXTUAL SITUATIONS INVOLVING MULTIPLICATION

6.b. Find how many stuffed animals Leigha and Kylee have altogether.

CORRECT ANSWER

Altogether Leigha and Kylee have 42 stuffed animals.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Altogether Leigha and Kylee have 14 stuffed animals.	does not add Kylee's stuffed animals to Leigha's	MATHEMATIZE CONTEXTUAL SITUATIONS INVOLVING ADDITION
Altogether Leigha and Kylee have 28 stuffed animals.	does not calculate Kylee's stuffed animals to be twice as many as Leigha's	MATHEMATIZE CONTEXTUAL SITUATIONS INVOLVING MULTIPLICATION

7. The following table shows the pounds of pop tabs collected by third, fourth, and fifth-grade classes at South Elementary School, Trails Elementary School, and Frontier School.

	South Elementary School	Trails Elementary School	Frontier School
Third Grade	10 pounds	9 pounds	14 pounds
Fourth Grade	8 pounds	12 pounds	10 pounds
Fifth Grade	12 pounds	11 pounds	9 pounds

7.a. Which school collected the most pop tabs? How much did that school collect?

CORRECT ANSWER

Frontier School collected the most pop tabs at 33 pounds.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
South Elementary School collected the most pop tabs at 30 pounds.	chooses the school with the least pop tabs collected	SOLVE COMPARE PROBLEMS
Trails Elementary School collected the most pop tabs at 34 pounds.	adds incorrectly	ADD WITHIN 100

7.b. Which grade collected the most pop tabs? How much did that grade collect?

CORRECT ANSWER

The third grade collected the most pop tabs at 33 pounds.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The fourth grade collected the most pop tabs at 30 pounds.	chooses the school with the least pop tabs collected	SOLVE COMPARE PROBLEMS
The fifth grade collected the most pop tabs at 34 pounds.	adds incorrectly	ADD WITHIN 100

7.c. South Elementary School finds an extra 2 pounds of pop tabs in a lost collection box. Does this amount change the answer to 7.a? Explain your answer.

CORRECT ANSWER

> No, South Elementary School will now have 32 pounds of pop tabs, but that is still less than Frontier School's 33 pounds of pop tabs.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Yes, South Elementary School will now have 32 pounds of pop tabs.	chooses a school that did not have the most pop tabs collected	SOLVE COMPARE PROBLEMS
No, South Elementary School will now have 30 pounds of pop tabs, but that is still less than Frontier School's 33 pounds of pop tabs.	does not add the additional 2 pounds of pop tabs found	MATHEMATIZE CONTEXTUAL SITUATIONS INVOLVING ADDITION