ADDING AND SUBTRACTING FRACTIONS WITH UNLIKE DENOMINATORS

5.NF.1,2

CONTENTS

The types of documents contained in the unit are listed below. Throughout the unit, the documents are arranged by lesson.

Copyright © 2019 by The University of Kansas.

Enhanced Learning Maps developed these materials under a grant from the Department of Education, PR/ Award # S368A150013. Contents do not necessarily represent the policy of the Department of Education, and you should not assume endorsement by the Federal Government.

Learning map materials are freely available for use by educators but may not be used for commercial purposes without written permission.

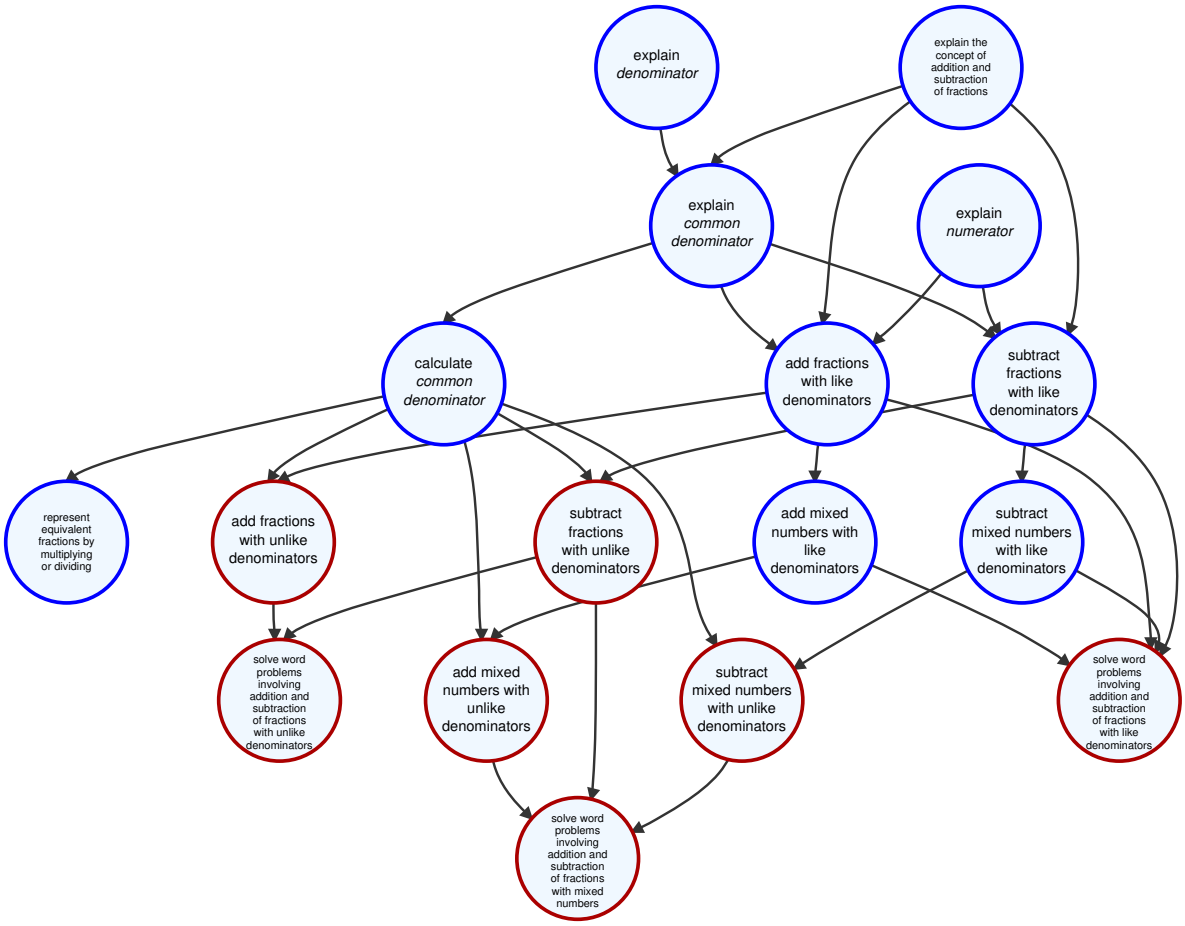
ADDING AND SUBTRACTING FRACTIONS WITH UNLIKE DENOMINATORS

LEARNING MAP INFORMATION

STANDARDS

5.NF.1 Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators.

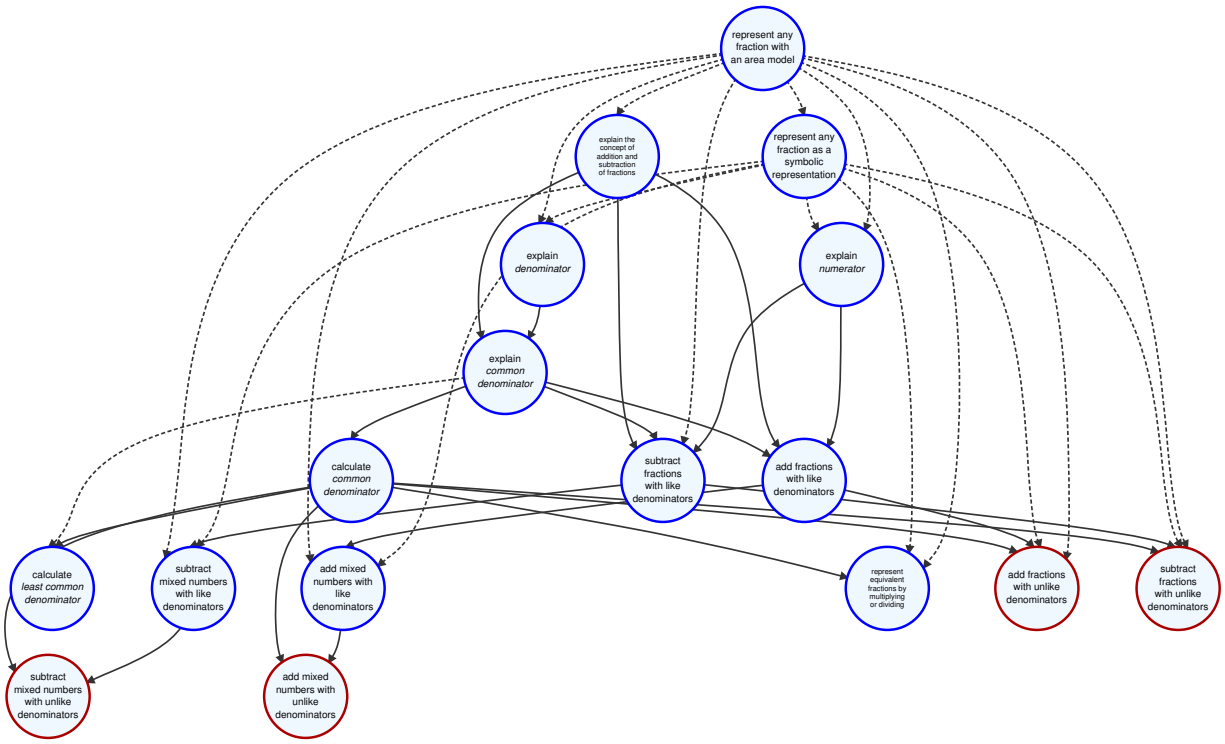
5.NF.2 Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers.



**Learning map model of* 5.NF.1,2

Copyright © 2016 by The University of Kansas

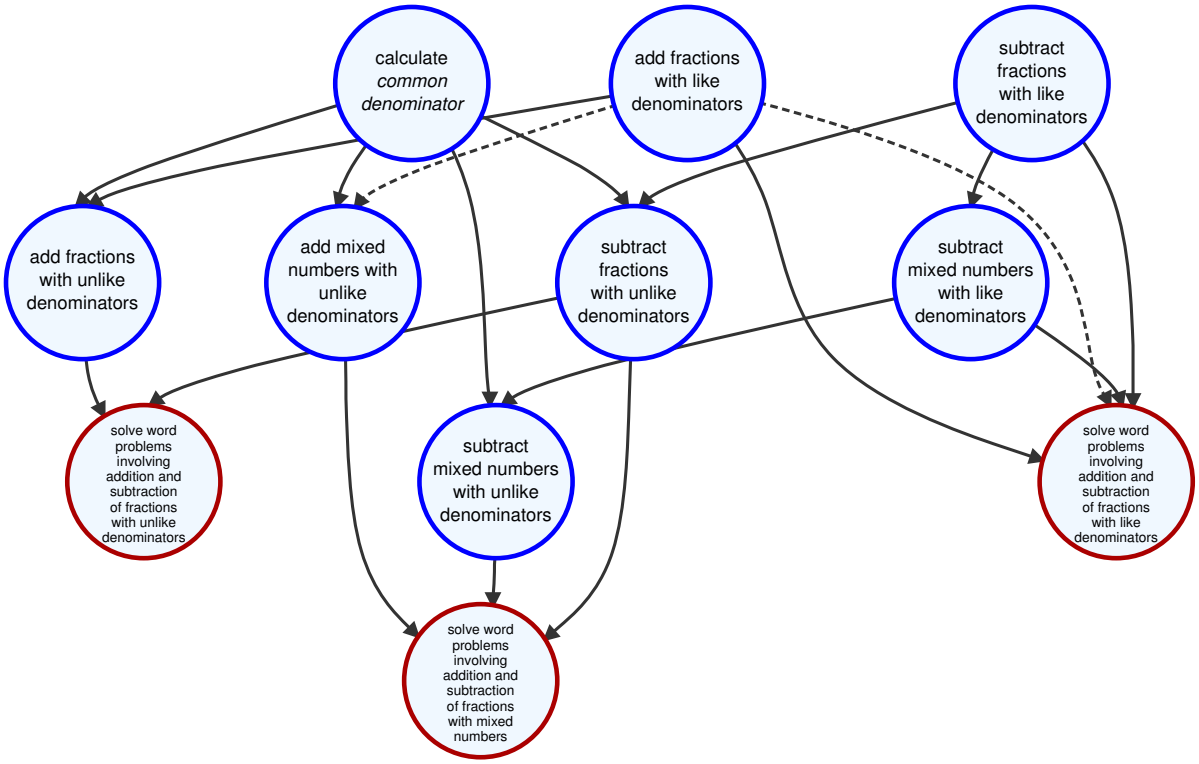
5.NF.1 Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators.



**Learning map model of* 5.NF.1

Copyright © 2016 by The University of Kansas

5.NF.2 Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers.



**Learning map model of* 5.NF.2

Copyright © 2016 by The University of Kansas

Node Name	Node Description
ADD FRACTIONS WITH LIKE DENOMINATORS	Add two fractions with the same denominator.
ADD FRACTIONS WITH UNLIKE DENOMINATORS	Add two fractions with different denominators.
ADD MIXED NUMBERS WITH LIKE DENOMINATORS	Add two mixed numbers with the same denominator.
ADD MIXED NUMBERS WITH UNLIKE DENOMINATORS	Add mixed numbers with unlike denominators.
CALCULATE *COMMON DENOMINATOR*	Calculate a common multiple of the denominator of the fraction. Multiply each fraction (numerator and denominator) by the appropriate factor in order to have the same denominator.
EXPLAIN *COMMON DENOMINATOR*	Make known your understanding that a common denominator among two or more fractions indicates that these fractions represent some number of same-sized parts.
EXPLAIN *DENOMINATOR*	Make known your understanding that the denominator is the number below the fraction bar and represents how many parts make up the whole.
EXPLAIN *NUMERATOR*	Make known your understanding that the numerator is the number above the fraction bar and represents the number of equal parts out of the whole.
EXPLAIN THE CONCEPT OF ADDITION AND SUBTRACTION OF FRACTIONS	Understand addition and subtraction of fractions as joining and separating parts of the same whole. These operations can be used directly on fractional pieces that are the same size, that is, pieces resulting from subdividing wholes into equal numbers of parts, where each part is the same size. For example, thirds can be added together or subtracted from each other, whereas a third and fourth cannot be directly added together or subtracted.
REPRESENT ANY FRACTION AS A SYMBOLIC REPRESENTATION	Through writing or an appropriate assistive technology, represent any fraction as a symbolic representation.
REPRESENT ANY FRACTION WITH AN AREA MODEL	Through writing or an appropriate assistive technology, represent any fraction with an area model.
REPRESENT EQUIVALENT FRACTIONS BY MULTIPLYING OR DIVIDING	Create equivalent fractions by multiplying or dividing the fraction by a fraction form of 1 (i.e., 3/3, 5/5, etc.).
SOLVE WORD PROBLEMS INVOLVING ADDITION AND SUBTRACTION OF FRACTIONS WITH MIXED NUMBERS	Use addition and subtraction to solve word problems involving fractions.
SOLVE WORD PROBLEMS INVOLVING ADDITION AND SUBTRACTION OF FRACTIONS WITH UNLIKE DENOMINATORS	Use addition and subtraction to solve word problems involving fractions with unlike denominators.
SOLVE WORD PROBLEMS INVOLVING ADDITION AND SUBTRACTION OF FRACTIONS WITH LIKE DENOMINATORS	Use addition and subtraction to solve word problems involving fractions with the same denominator.
SUBTRACT FRACTIONS WITH LIKE DENOMINATORS	Subtract two fractions with the same denominator.
SUBTRACT FRACTIONS WITH UNLIKE DENOMINATORS	Subtract two fractions with different denominators.
SUBTRACT MIXED NUMBERS WITH LIKE DENOMINATORS	Subtract two mixed numbers with the same denominator.
SUBTRACT MIXED NUMBERS WITH UNLIKE DENOMINATORS	Subtract mixed numbers with unlike denominators.

Copyright © 2016 by The University of Kansas

Adding and Subtracting Fractions with Unlike Denominators

TEACHER NOTES

This unit includes the following documents:

- Learning Map Information
- Instructional Activity (includes four lessons)
- Instructional Activity Student Handout (for all four lessons)
- Student Activity
- Student Activity Solution Guide

Students need to have robust a understanding of fractions, including recognizing and using fractions as numbers, explaining equivalent fractions, and making sense of the procedures for adding and subtracting fractions. This early knowledge of fractions provides the foundation for and access to algebra and higher-level mathematics (National Mathematics Advisory Panel [NMAP], 2008).

First, students will add and subtract pairs of fractions in which one denominator is a multiple of the other denominator. Then students will add and subtract pairs of fractions in which the denominators do not share a common factor. Students will solve addition and subtraction problems whose sums and differences result in proper fractions and then progress towards sums that yield improper fractions that need to be regrouped into mixed numbers. Students will develop fluency with addition and subtraction of proper fractions before adding and subtracting mixed numbers. Finally, students will add and subtract fractions and mixed numbers that require regrouping.

Cramer, Behr, Post, and Lesh (1997) suggested that it is important for students to develop a conceptual understanding of fractions prior to working with formal fraction symbols and algorithms. Students should work with concrete models over extended periods of time to develop fraction sense, which is strengthened by students' mental images of fraction models. Prior to adding and subtracting fractions, students should have developed mental images to support their understanding of a fraction's size relative to the whole (part-whole relationship) or a fraction's distance from zero (fraction-as-measure). When learning to add and subtract fractions, students will draw on their mental models of part-whole fractions as they use circle models or rectangular fraction bars to visualize the concepts of common denominator and equivalent fractions (Cramer, Wyberg, & Leavitt, 2008). Then, students will draw on their mental models of fractions as measures as they add and subtract fractions using a number line, which will support the visualization of the addition and subtraction of the numerators (Cramer et al., 2008). Lastly, students will relate the traditional algorithm for adding and subtracting fractions to their mental models and use the algorithm to operate with fractions. The same sequence for building a foundational understanding of fractions will be used to create a foundation for mixed numbers, emphasizing the use of concrete models to add and subtract mixed numbers, as well as how to regroup a mixed number in order to subtract. Presenting students with a variety of models prior to the standard algorithm will increase the students' understanding of when and how to use the procedure. This concrete approach to early operations with fractions will prevent the risk identified by Van de Walle, Karp, Lovin, and Bay-Williams (2014) that when students are quickly introduced to only the standard algorithm, it can turn operations with fractions into a meaningless mess.

Copyright © 2016 by The University of Kansas

It is important to reinforce that adding and subtracting fractions is very similar in meaning and underlying purpse to adding and subtracting whole numbers. Addition means putting quantities together, and subtraction means taking away one quantity from another quantity or separating one amount into subgroups. It is, however, important to note that one can combine and separate only similar quantities. Just like apples and oranges can't be added into a new type of fruit, in the context of fractions, it is important to recognize that fraction addends should have the same unit fraction. In other words, one can only combine fifths belonging to the same whole, but cannot combine fifths and sevenths together. Two fractions that have the same denominator can be added by combining the numerators or the number of parts. For example, $\frac{2}{7} + \frac{3}{7}$ can be thought of as two copies of $\frac{1}{7}$ plus three copies of $\frac{1}{7}$, which together equal five copies of $\frac{1}{7}$, i.e. $\frac{5}{7}$.

Two fractions with different denominators, where one denominator is a multiple of the other denominator, can be added or subtracted by converting the the fraction with the larger denominator into an equivalent fraction with the same denominator as the other fraction. Equivalent fractions are two different fractions that represent the same relative amount (Van de Walle et al., 2014). Another way of stating this equivalence is: two fractions are equivalent (e.g., $\frac{3}{4}$ and $\frac{6}{8}$) when there exists the same relationship between the corresponding numerators and denominators (Chval et al., 2013). For example $\frac{3}{4}$ and $\frac{6}{8}$ are equivalent fractions because 3 times 2 is 6 and 4 times 2 is 8. A common method to create a fraction equivalent to a given fraction is by multiplying the numerator and denominator with the same number. For example, a fraction equivalent to $\frac{1}{2}$ would be $\frac{3}{6}$, which is obtained by multiplying the numerator and denominator by 3. The notions of fraction equivalency are important in creating commmon denominators, and in representing fractions in simplest terms.

It has been found that students could obtain correct solutions using concrete models, but that they experience difficulties in transferring their understanding of fractions between concrete representations and symbolic representations (Lesh, Landau, & Hamilton, 1983). As such, the activites in these lessons are designed to support student learning by focusing first on concrete models, then on pictorial representations, and finally on the traditional algorithm applied to symbolic representations of fractions. A noted benefit of this sequence is that operation sense is strengthened when students translate fluently between concrete, pictorial, and symbolical representations of fractions (Huinker, 2002).

Practicing adding and subtracting fractions in the context of real world problems also supports understanding. There are four different types of addition and subtraction problems: joining, separating, part-part-whole, and comparison. Students should be exposed to and required to solve each type of problem in order to build an understanding of what the situation is asking, as well as how to represent and solve the problem (Chval et al., 2013). Mathematical situations involving the combination of two or more parts into a single quantity are known as joining problems. Joining problems require students to add one fraction to another fraction (for example, *Sally has* $\frac{1}{3}$ *lb. of jellybeans and Jessica has* $\frac{3}{4}$ *lb. of jellybeans. How many pounds of jellybeans do Sally and Jessica have together?*). Mathematical situations involving the separation of one part from a discrete set to determine the size of another part are referred to as separating problems. Separating problems require students to take one fraction away from another (for example, *Jake has* $\frac{3}{4}$ *lb. of jellybeans and he gives* $\frac{1}{3}$ *lb. of jellybeans to Johnny. How many pounds of jellybeans does Jake have now?*). Mathematical situations involving the combination of two discrete sets or the splitting of one set into two discrete sets are called change problems

Copyright © 2016 by The University of Kansas

or part-part-whole problems. In a part-part-whole problem, students are given the whole and one part, and the student then needs to solve for the missing part (for example, *For his experiment Matt needs* $1\frac{1}{2}$ *lb. of jellybeans. Matt has* $\frac{2}{3}$ *lb. of jellybeans, how many more pounds of jellybeans does Matt need for his experiment?*). Mathematical situations involving comparisons of two discrete sets of objects (e.g., to identify smaller or larger quantities, differences between quantities, or ratio relationships) are called comparison problems. Comparison problems require students to subtract to find the difference of a comparison (for example, *Jenny has* $\frac{4}{5}$ *lb. of jellybeans and Sophia has* $\frac{3}{4}$ *lb. of jellybeans. How many more pounds of jelly beans does Jenny have then Sophia?*).

In addition to incorporating a variety of problems into their instruction, teachers should be aware of common misconceptions and mistakes students make when adding and subtracting fractions. Seeking and highlighting instances where students make errors, such as not calculating a common denominator or adding both the numerators and denominators, will cause students to attend to these potential pitfalls and improve their success. Teachers should show how such errors appear using models and with symbols. Teachers should keep in mind that just because students are using models to show the operation or to support their solution does not mean that the model and/or work is correct; students may still make mistakes if they do not truly understand the concept of a fraction or the model itself. Generally, students who show mastery over operations with models have a higher likelihood of learning conceptually, which, in turn, will allow them to ally the algorithm meaningfully.

Copyright © 2016 by The University of Kansas

REFERENCES

Chval, K., Lannin, J., & Jones, D. (2013). *Fractions into Practice*. Reston, VA: National Council of Teachers of Mathematics.

Cramer, K., Behr, M., Post, T., & Lesh, R., (1997). *Rational Number Project: Fraction Lessons for the Middle Grades Level 1*. Dubuque, Iowa: Kendall/Hunt Publishing Co.

Cramer, K., Wyberg, T., & Leavitt, S. (2008). The Role of Representations in Fraction Addition and Subtraction. *Mathematics Teaching in the Middle School,* 13 (8), 490-496. doi: 10.2307/41182601.

Huinker, D. (2002). Examining Dimensions of Fraction Operation Sense. In B. Litwiller & G. Bright (Eds.), *Making Sense of Fractions, Ratios, and Proportions* (pp. 72-78). Reston, VA: National Council of Teachers of Mathematics.

Lesh, R., Landau, M., & Hamilton, E. (1983). Conceptual Models in Applied Mathematical Problem Solving Research. In R. Lesh & M. Landau (Eds), *Acquisition of Mathematics Concepts & Processes* (pp. 263-343). NY: Academic Press.

National Mathematics Advisory Panel. (2008). *Foundations for success: The final report of the National Mathematics Advisory Panel*. Retrieved June 20, 2016, from https://www2.ed.gov/about/bdscomm/list/mathpanel/report/final-report.pdf

Van de Walle, J.A., Karp, K.S., Lovin, L.H., & Bay-Williams, J.M. (2014). *Teaching Student-Centered Mathematics*. Upper Saddle River, NJ: Pearson Professional Development.

Copyright © 2016 by The University of Kansas

ADDING AND SUBTRACTING FRACTIONS WITH UNLIKE DENOMINATORS

INSTRUCTIONAL ACTIVITY

Lesson 1

LEARNING GOAL

Students will build on their knowledge of adding and subtracting fractions with like denominators in order to add and subtract fractions with unlike denominators using models (circle fractions or fraction bars).

PRIMARY ACTIVITY

Students will use fraction models to add and subtract fractions with unlike denominators.

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

- Equivalent
- Multiple
- Sum
- Addend
- Difference
- Mixed Number
- Improper fraction
- Numerator
- Denominator

MATERIALS

- INSTRUCTIONAL ACTIVITY STUDENT HANDOUT
- Fraction bars or fraction circles from the INSTRUCTIONAL ACTIVITY SUPPLEMENT, with each fractional part ideally on different colors of paper (two sets of each fractional part for each student, so that students can create mixed numbers if needed):
  - 1 whole (black)
  - 2 halves (yellow)

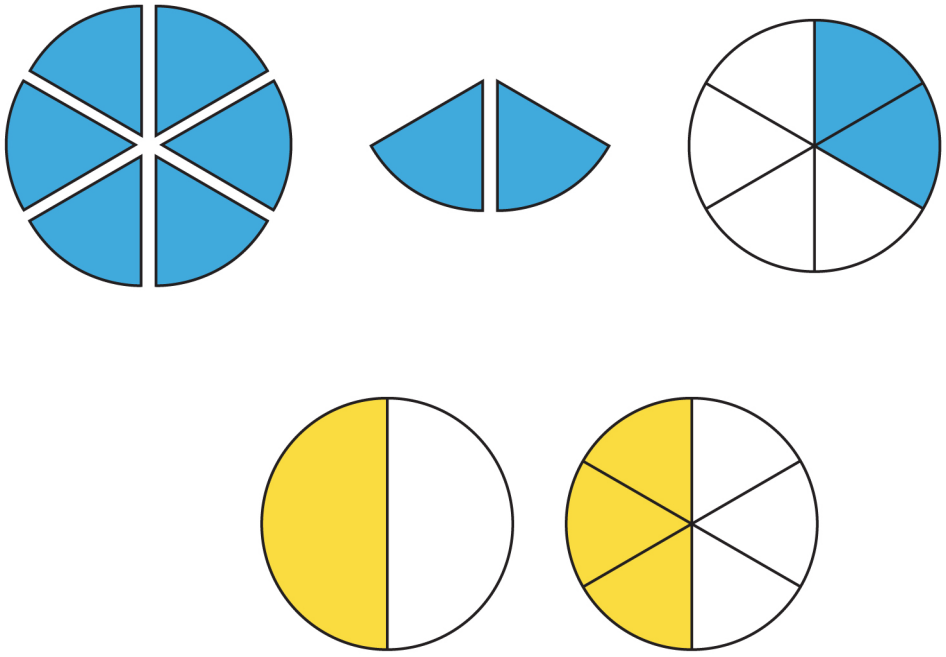
Copyright © 2016 by The University of Kansas

- 3 thirds (brown)
- 4 fourths (blue)
- 5 fifths (orange)
- 6 sixths (pink)
- 7 sevenths (light blue)
- 8 eighths (gray)
- 9 ninths (white)
- 10 tenths (purple)
- 12 twelfths (red)
- 15 fifteenths (green)

IMPLEMENTATION

Have students group all the different fractional parts together in piles so all like pieces are together. Begin the lesson by reviewing what the numerator and denominator represent.

Require students to model $\frac{2}{6}$ with the circle fraction pieces (or the fraction bars) and record the model on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT. Then, require students to repeat the process with $\frac{1}{2}$.

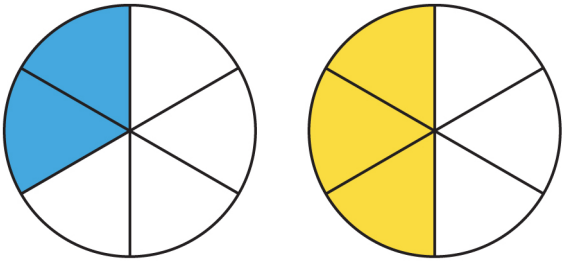


Ask students:

- "What if I wanted to compare this $\frac{2}{6}$ to $\frac{1}{2}$? How can I model $\frac{1}{2}$ so that it can be compared to $\frac{2}{6}$?"

Copyright © 2016 by The University of Kansas

- "Which of these fractions is larger?" (Students should suggest modeling $\frac{1}{2}$ and then lay $\frac{2}{6}$ over the $\frac{1}{2}$ to show that $\frac{1}{2}$ is larger, or modeling $\frac{1}{2}$ and then divide the model into sixths; students should indicate that $\frac{1}{2}$ or $\frac{3}{6}$ is larger than $\frac{2}{6}$.)
- "How much larger than $\frac{2}{6}$ is $\frac{1}{2}$? How do you know that?"



Ask students, "Can I add $\frac{2}{6}$ and $\frac{1}{2}$?" (Students should say that you cannot add $\frac{2}{6}$ and $\frac{1}{2}$ until the fractions have like denominators.) **Remind** students that they were able to represent $\frac{1}{2}$ as $\frac{3}{6}$ in order to compare the two original fractions, and therefore they can replace $\frac{1}{2}$ with $\frac{3}{6}$. **Ask** the students, "Can I add $\frac{2}{6} + \frac{3}{6}$?" (Students should indicate the fractions can be added because the fractional parts are the same size.)

GUIDING QUESTIONS

Determine if the student can EXPLAIN COMMON DENOMINATOR:

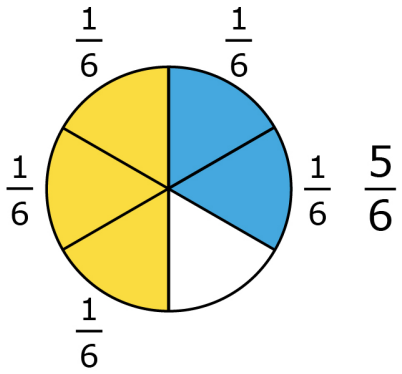
- How would you explain common denominator in your own words?
- How would you determine if two fractions have a common denominator?

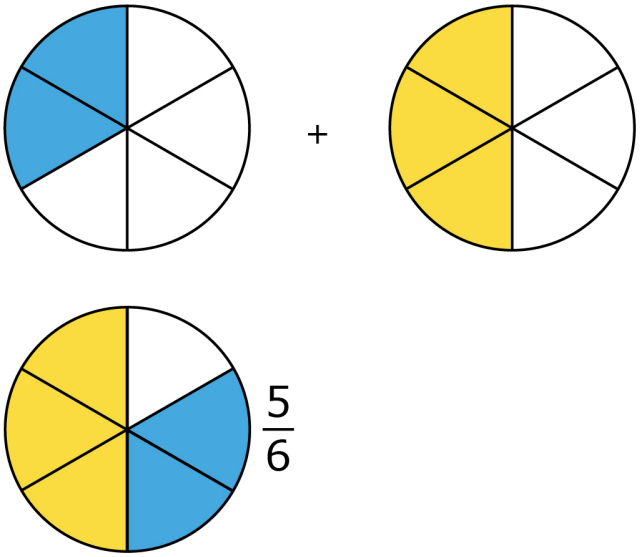
Determine if the student can CALCULATE COMMON DENOMINATOR:

- How would you determine a common denominator for two fractions that have unlike denominators?
- In what situations would you need to determine a common denominator?

Copyright © 2016 by The University of Kansas

Model the addition of $\frac{2}{6} + \frac{3}{6}$ using models and symbols. Be sure to make explicit that $\frac{2}{6}$ is $\frac{1}{6} + \frac{1}{6}$ and that you are adding them together with $\frac{3}{6}$, which is $\frac{1}{6} + \frac{1}{6} + \frac{1}{6}$. Be sure to **emphasize** to the student the misconception of adding the denominators. Make explicit that only the number of equal parts (the numerator) is being added and that the denominators should not change because the size of the equal parts has not changed.





Copyright © 2016 by The University of Kansas

GUIDING QUESTIONS

Determine if the student can ADD FRACTIONS WITH LIKE DENOMINATORS (using a model):

- What can you tell me about adding fractions with your circle models?
- How would you add two fractions with like denominators using a model?

Determine if the student can SUBTRACT FRACTIONS WITH LIKE DENOMINATORS (using a model):

- What can you tell me about subtracting fractions with your circle models?
- How would you subtract two fractions with like denominators using a model?

Determine if the student can DECOMPOSE A FRACTION INTO A SUM OF UNIT FRACTIONS WITH THE SAME DENOMINATOR:

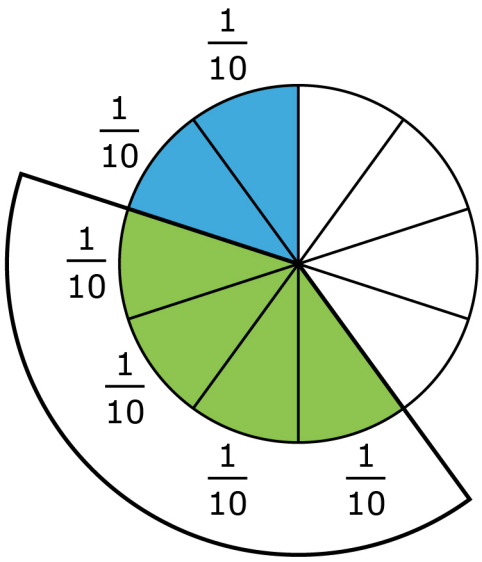
- What is a unit fraction?
- [Model $\frac{6}{10}$] What do you notice about the size of the fractional pieces? How many unit fractions would I need to add to make $\frac{6}{10}$?
- Suppose you have a model of $\frac{5}{6}$ [show a circle model with five of six equal parts shaded]. Can you write an addition sentence that we could use this model to display?

Ask students, "Can I subtract $\frac{2}{10}$ from $\frac{3}{5}$?" (Students should say that you cannot simplify $\frac{3}{5} - \frac{2}{10}$ until they have like denominators.) **Remind** students that when you compared the fractions from the addition problems, they had to make each fraction have the same size fractional parts (common denominator).

Model for students how to determine a common denominator and rewrite $\frac{2}{10}$ and $\frac{3}{5}$ so that they are equivalent fractions that share a common denominator (You can rewrite $\frac{3}{5}$ as $\frac{6}{10}$, or you can rewrite $\frac{2}{10}$ as $\frac{1}{5}$.).

Copyright © 2016 by The University of Kansas

Require students to model $\frac{6}{10} - \frac{2}{10}$ and record the models on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT. Be sure to make explicit that $\frac{6}{10}$ is $\frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10}$ and that you are taking away $\frac{2}{10}$, which is $\frac{1}{10} + \frac{1}{10}$.



Require students to model $\frac{1}{6}$ and $\frac{1}{3}$ with the circle fraction pieces (or the fraction bars). Students should also record the model on Question 1 of the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

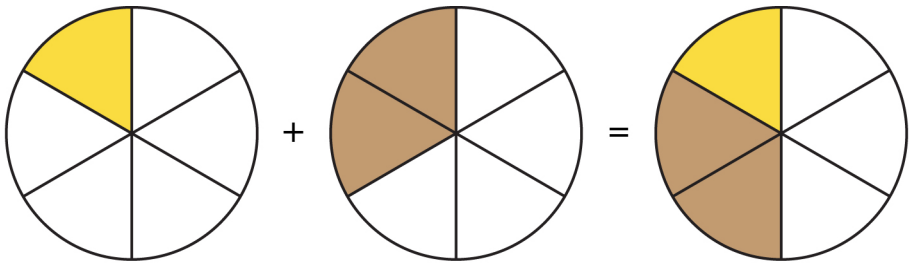
Ask students:

- "How can I add $\frac{1}{6} + \frac{1}{3}$?"
- "What do we need to do to one of these fractions so we can add them together? What are the steps to determine the sum?" (Students should explain that you need a common denominator, and that you need to change $\frac{1}{3}$ to $\frac{2}{6}$.)

Ask students, "Why do you need a common denominator?" each time it is stated that a common denominator is needed. They should repeatedly explain that in order to add fractions, you must add the same size fractional parts.

Require students to record the models on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT using a different color for each addend.

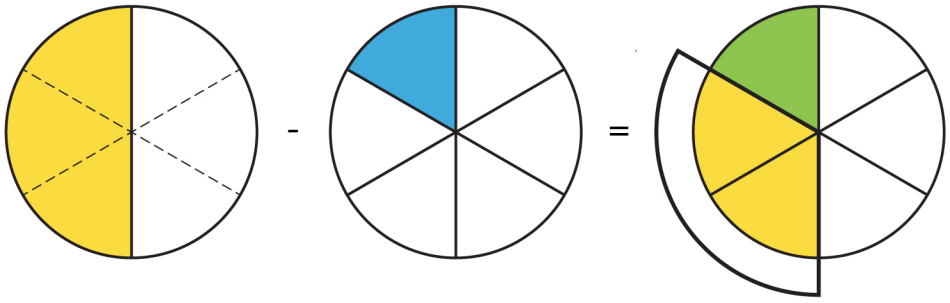
Copyright © 2016 by The University of Kansas



Require students to model $\frac{1}{2}$ and $\frac{1}{6}$ with the circle fraction pieces (or the fraction bars). Students should also record the model on Question 2 of the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

Ask students, "How can I simplify $\frac{1}{2} - \frac{1}{6}$?" (Students should explain that you need a common denominator because the fractional parts must be the same size in order to subtract, and that you need to replace $\frac{1}{2}$ with the equivalent fraction $\frac{3}{6}$.)

Require students to record the models on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT. One method is that students use a different color for each numerator, which means the fractional pieces that are being subtracted should be shaded twice.



Begin the next task with a word problem about the two fractions ($\frac{2}{3}$ and $\frac{2}{6}$). Tell the students, "Josh had a party yesterday. He ordered two large pizzas, a cheese pizza and a veggie pizza. Altogether, he and his friends ate $\frac{2}{3}$ (show the students a model of $\frac{2}{3}$) of the large cheese pizza and $\frac{2}{6}$ (show the students a model of $\frac{2}{6}$) of the large veggie pizza. How much of the two large pizzas did Josh and his friends eat? Show me your work using the models and then record your work on Question 3 of the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT."

Ask students "What is the common denominator of $\frac{2}{3}$ and $\frac{2}{6}$? How did you determine a common denominator? Why are we finding a common denominator?"

Require some students to share with the whole group how they solved the addition problem.

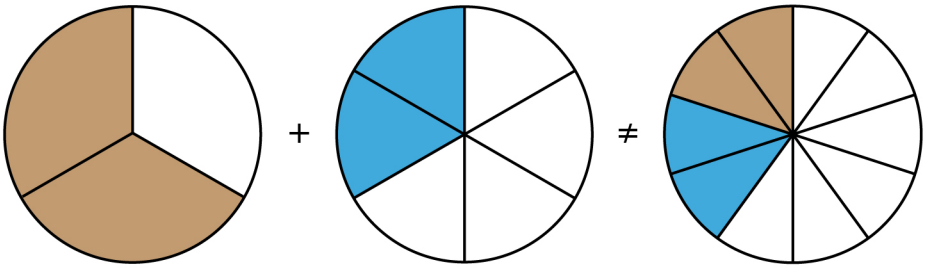
Copyright © 2016 by The University of Kansas

Ask the class as students share their work:

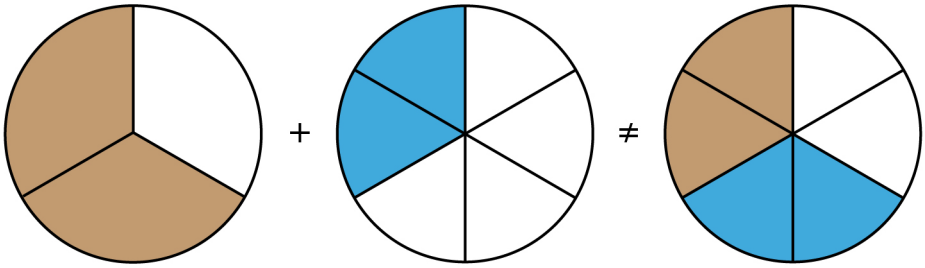
- "What do you think about what [student's name] said? Do you agree or disagree? Why?"
- "Does anyone have the same answer but a different way to explain it?"

Emphasize common student mistakes by displaying the incorrect models/answers and asking, "Can you explain the mistake? How would you generate a plan to correctly solve the problem?" Be sure to relate back to each fraction being a sum of unit fractions, and that this means the denominator would not change because it represents the size of the equal parts.

- Adding the numerators and the denominators: $\frac{2}{3} + \frac{2}{6} \neq \frac{4}{9}$



- Not paying attention to the denominators. For example, only using one of the denominators: $\frac{2}{3} + \frac{2}{6} \neq \frac{4}{6}$



Ask students, "If Josh had ordered one large pizza and one medium sized pizza (model two different sized circle fractions), would you still be able to add the two fractions? Why or why not?" (Students should respond that you cannot add the two fractions because the sizes of the wholes are not equivalent.)

Use the following guiding questions throughout lesson to engage student thinking and to scaffold learning.

Copyright © 2016 by The University of Kansas

Determine if the student can ADD FRACTIONS WITH UNLIKE DENOMINATORS:

- What would happen if you were going to add two fractions with unlike denominators?
- What do you notice about the sizes of the fractional parts?
- What information would you use to solve an addition problem with unlike denominators?
- [Have students represent $\frac{2}{3}$ and $\frac{2}{6}$ with the models] What can you tell me about how you would add these two fractions?

Determine if the student can SUBTRACT FRACTIONS WITH UNLIKE DENOMINATORS:

- What would happen if you were going to subtract two fractions with unlike denominators?
- What do you notice about the sizes of the fractional parts?
- What information would you use to solve a subtraction problem with unlike denominators?
- [Have students represent $\frac{2}{3}$ and $\frac{2}{6}$ with the models] What can you tell me about how you would subtract these two fractions?

Copyright © 2016 by The University of Kansas

Determine if the student can SOLVE WORD PROBLEMS INVOLVING ADDITION AND SUBTRACTION OF FRACTIONS WITH UNLIKE DENOMINATORS:

- Should we add or subtract to determine how much of the two large pizzas Josh and his friends ate? How do you know that?
- What is the problem asking you to do—combine fractions, or take away fractions?
- Does your answer make sense? Does it answer the problem that is being asked?
- If Josh eats the leftovers, is he taking pizza away or adding more pizza? How do you know?

Require students to model $\frac{1}{3}$ and $\frac{2}{9}$ with the circle fraction pieces (or the fraction bars) and record the models on Question 4 of the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

Begin the next task with a word problem about the two fractions ($\frac{1}{3}$ and $\frac{2}{9}$). Tell the students, "The day after the party, Josh had $\frac{1}{3}$ of the large cheese pizza left. He ate $\frac{2}{9}$ of the leftover large cheese pizza for breakfast. What fraction of the leftover large cheese pizza will be left for Josh to eat for lunch?"

Ask students, "How would you determine the common denominator? What is the common denominator of $\frac{1}{3}$ and $\frac{2}{9}$?"

Require students to model how they think the problem would be solved.

Ask students as they work:

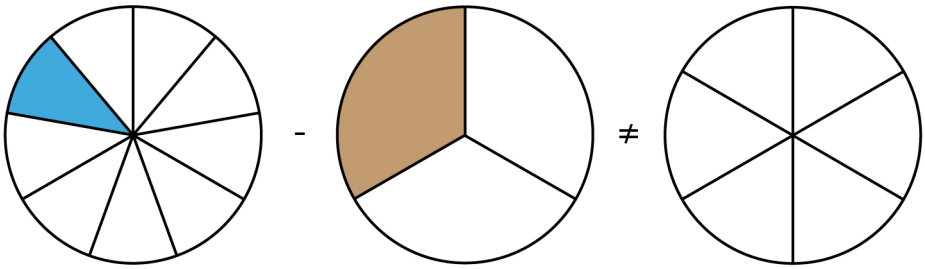
- "What facts can you gather about how to solve this problem?"
- "How would you determine how much pizza was left over?"
- "What steps would you take to solve this problem?"
- "How is $\frac{1}{3} - \frac{2}{9}$ similar to the problem we solved earlier ($\frac{3}{5} - \frac{2}{10}$)?"

Require students to share the solution with the whole class. **Ask** students, "Do you agree or disagree? What would you say to convince the rest of the class that that makes sense?"

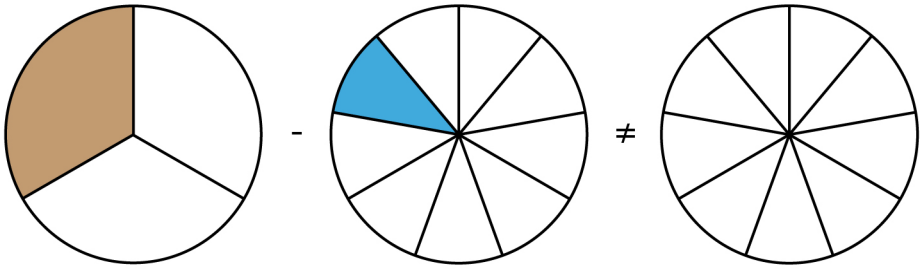
Emphasize common student mistakes by modeling the incorrect models/answers and asking, "Can you identify and explain a mistake shown in this example? How would you generate a plan to correctly solve the same problem?"

Copyright © 2016 by The University of Kansas

- Subtract fractions without calculating like denominators and rearranging the fractions so that the larger denominator is first: $\frac{1}{9} - \frac{1}{3} \neq \frac{0}{6}$



- Not paying attention to the denominators. For example, only using one of the denominators and rearranging the fractions so that the larger numerator is first: $\frac{1}{3} - \frac{1}{9} \neq \frac{0}{9}$



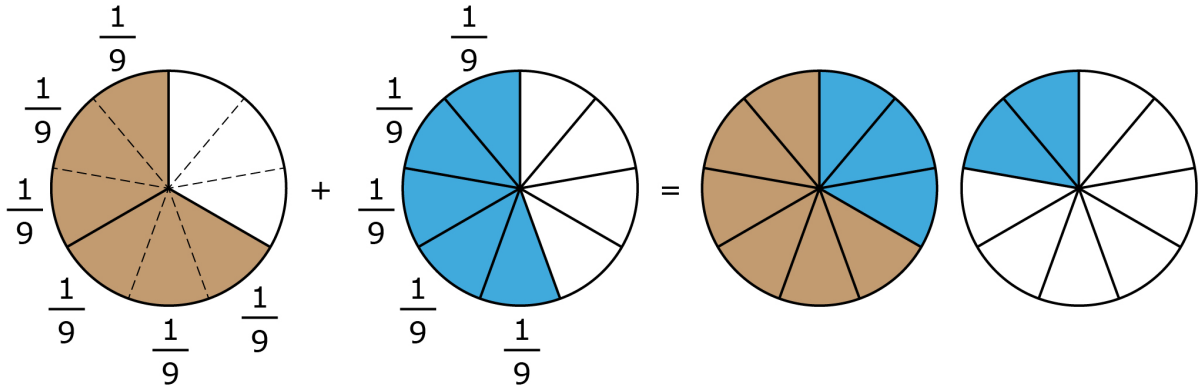
Proceed through four more problems with pairs of fractions, posing each question as a real life example. Be sure to emphasize the fact that the wholes are equivalent in size (for example, two large pizzas or two small trays of brownies). Make sure that you include a variety of problem types: joining, separating, part-part-whole, and comparison (see TEACHER NOTES). For each problem, have students explain their thinking to partners, and then have two or three students share with the whole class the process used to solve the problem. Use the following equations for students to practice on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

- 5. $\frac{1}{8} + \frac{1}{4} = \frac{3}{8}$
- 6. $\frac{2}{3} - \frac{1}{6} = \frac{3}{6} = \frac{1}{2}$
- 7. $\frac{2}{5} + \frac{1}{10} = \frac{5}{10} = \frac{1}{2}$
- 8. $\frac{1}{2} - \frac{2}{5} = \frac{1}{10}$

Copyright © 2016 by The University of Kansas

Require students to model $\frac{2}{3}$ and $\frac{5}{9}$ with the circle fraction pieces (or the fraction bars). Students should also record the model on Question 9 of the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

Model the addition of $\frac{2}{3} + \frac{5}{9}$ using models and symbols. (Be sure to make explicit that $\frac{2}{3}$ written as an equivalent fraction with a like denominator is $\frac{6}{9}$, which is $\frac{1}{9}+\frac{1}{9}+\frac{1}{9}+\frac{1}{9}+\frac{1}{9}+\frac{1}{9}$. Also make explicit that you are adding $\frac{6}{9}$ together with $\frac{5}{9}$, which is $\frac{1}{9}+\frac{1}{9}+\frac{1}{9}+\frac{1}{9}+\frac{1}{9}$.) You should model that the sum is greater than one whole, and be sure to represent the mixed number as both an improper fraction and as a mixed number ($\frac{11}{9} = 1\ \frac{2}{9}$).



Ask students, "How many ninths do I have? How many wholes are there in the sum? How many ninths are left after I make one whole?"

Proceed through three more problems, posing each question as a real life example. Be sure to emphasize the fact that the wholes are equivalent in size, and be sure to draw the connection between the improper fractions and the models of the mixed numbers. For each problem, have students explain their thinking to partners, and then have two or three students share with the whole class the process used to solve the problem. Use the following expressions to have students practice on the ACTIVITY STUDENT HANDOUT.

- 10. $\frac{3}{4}+\frac{1}{3}=\frac{13}{12}=1\ \frac{1}{12}$
- 11. $\frac{11}{12}+\frac{5}{6}=\frac{21}{12}=1\frac{9}{12}=1\ \frac{3}{4}$
- 12. $\frac{7}{16}+\frac{7}{8}=\frac{21}{16}=1\ \frac{5}{16}$

At the end of the activity, assess student learning by reviewing their work on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT and by closing with the following questions:

- Explain the relationship between adding or subtracting fractions and determining a common denominator.
- How would you determine a common denominator for two fractions?

Copyright © 2016 by The University of Kansas

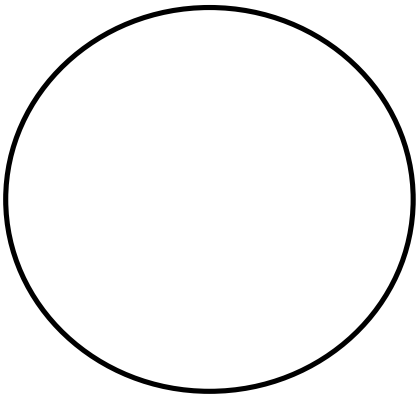
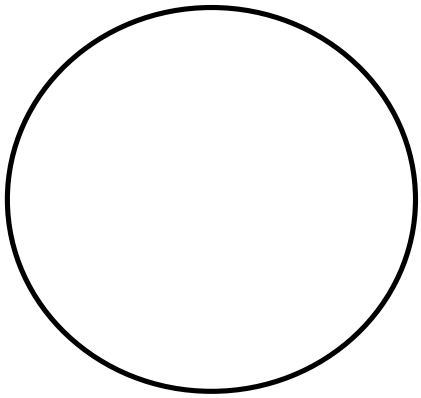
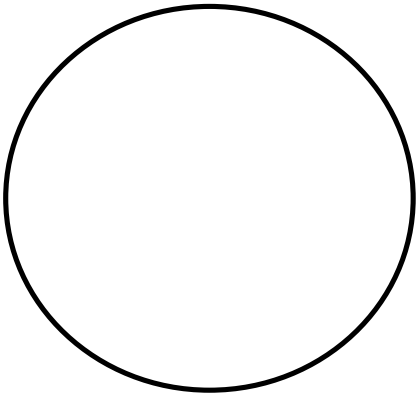
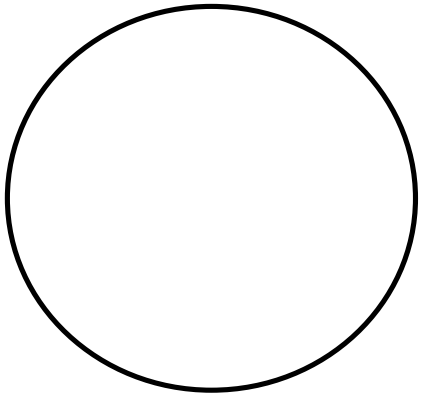
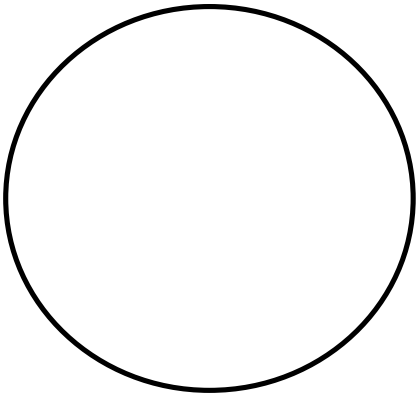
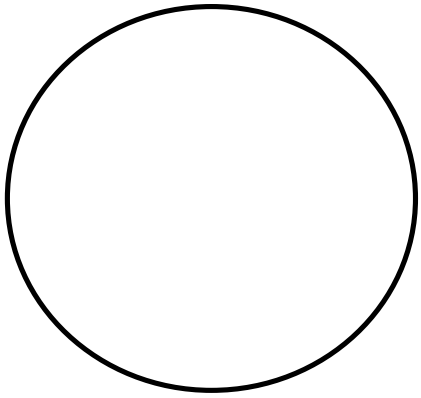
- What are the steps you would use to simplify an addition or subtraction problem with unlike denominators?
- What do you know about adding or subtracting two fractions with unlike denominators?
- What would happen if the sum of the two fractions were greater than one whole? How could you represent a sum that is an improper fraction?
- How would you explain the reason for writing an improper fraction as a mixed number?

Copyright © 2016 by The University of Kansas

Name: _____

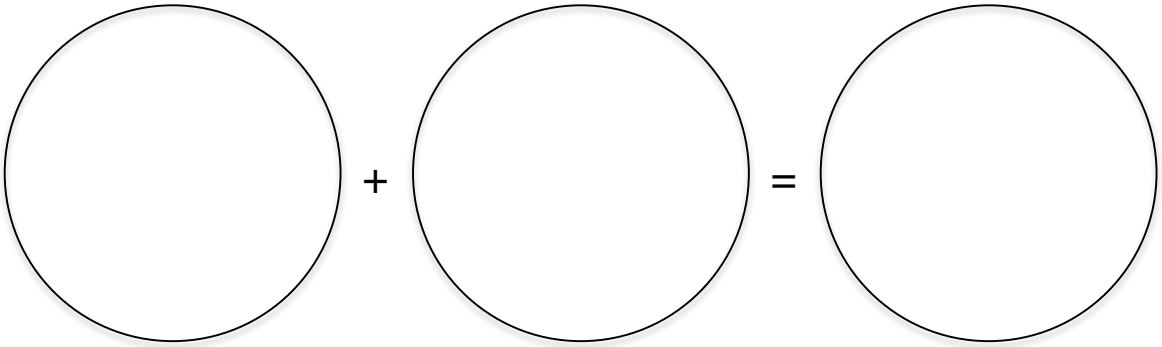
Adding and Subtracting Fractions with Unlike Denominators

Lesson 1

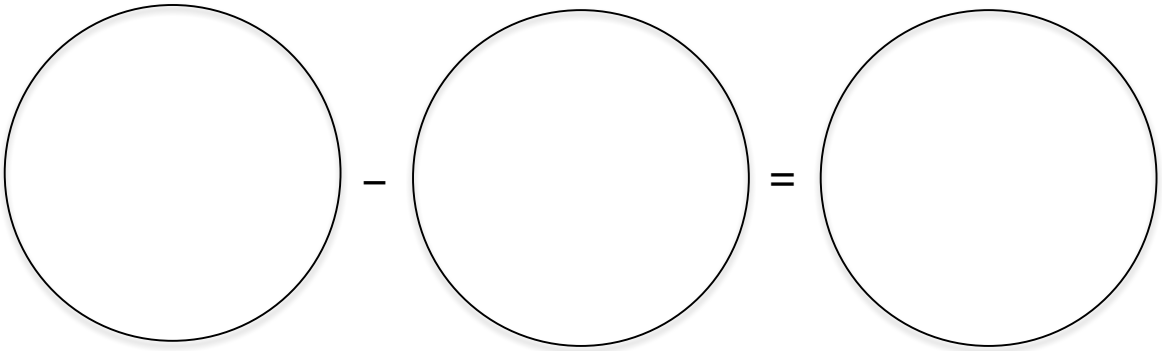







Copyright © 2016 by The University of Kansas

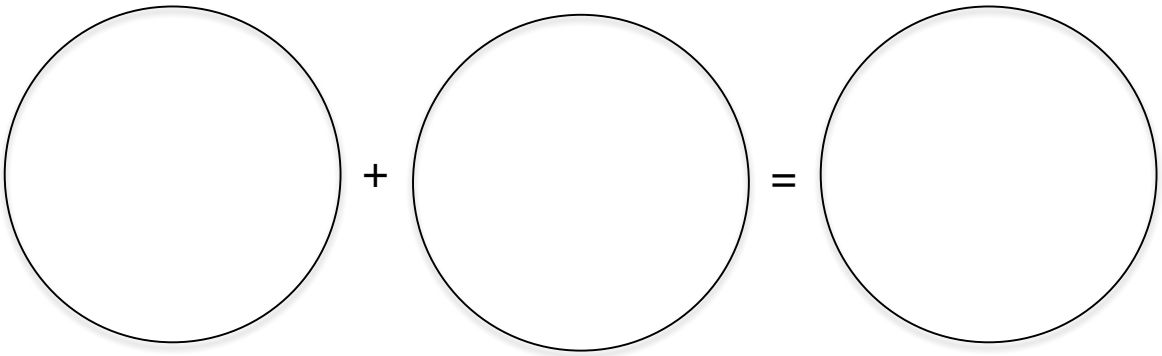
1.



2.

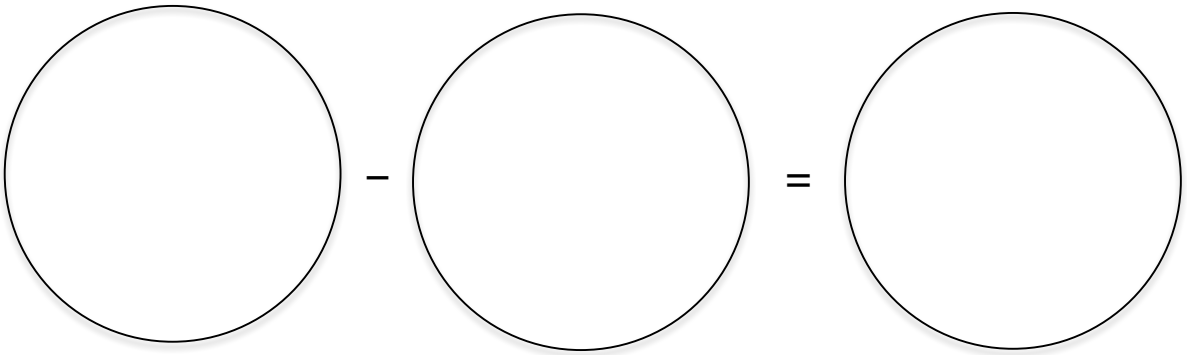


3.

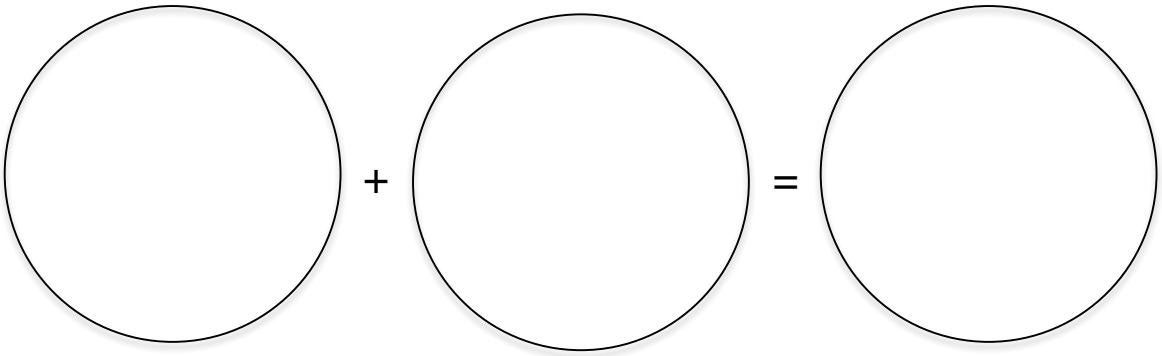


Copyright © 2016 by The University of Kansas

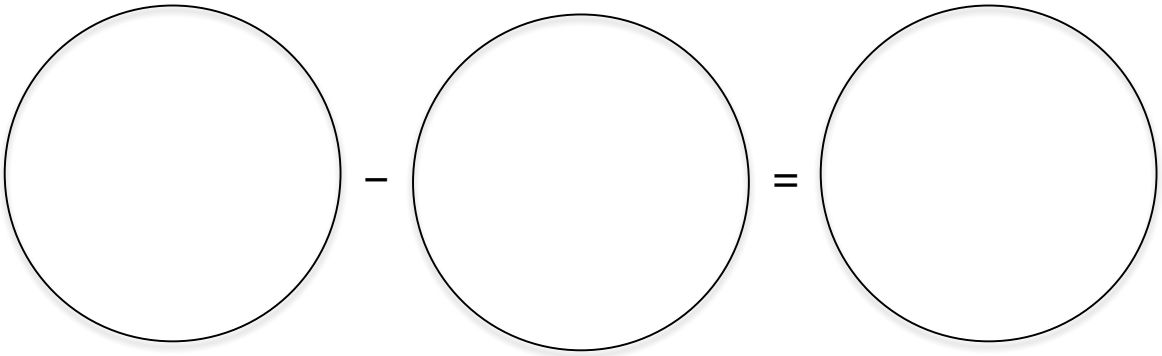
4.



5.

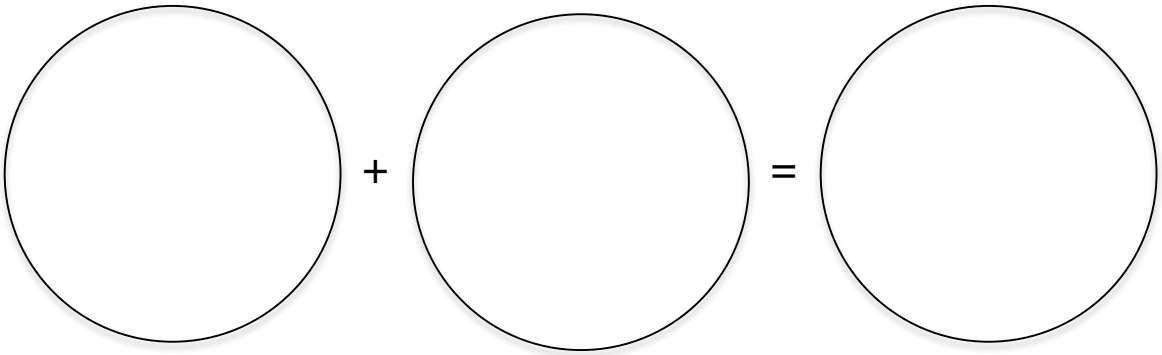


6.

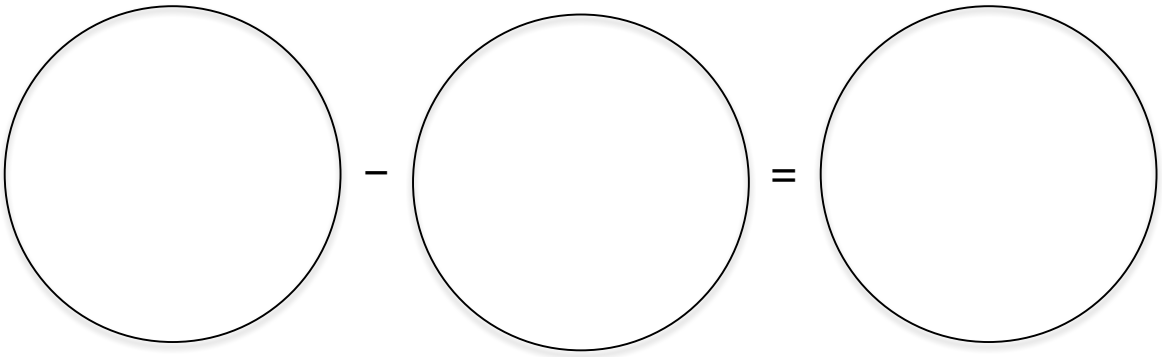


Copyright © 2016 by The University of Kansas

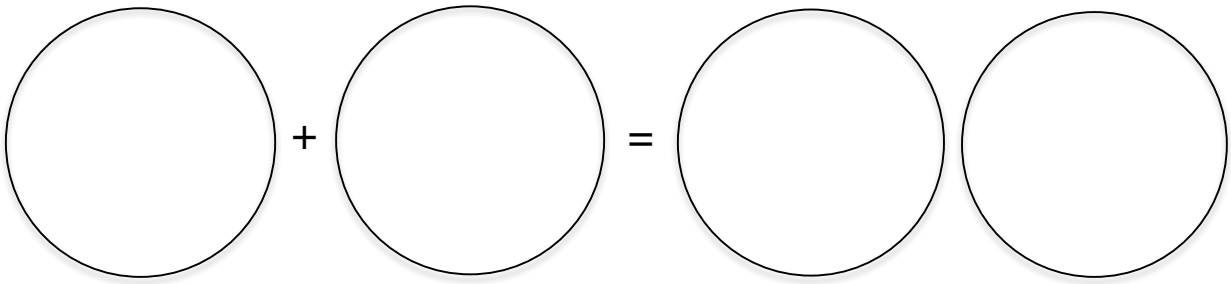
7.



8.

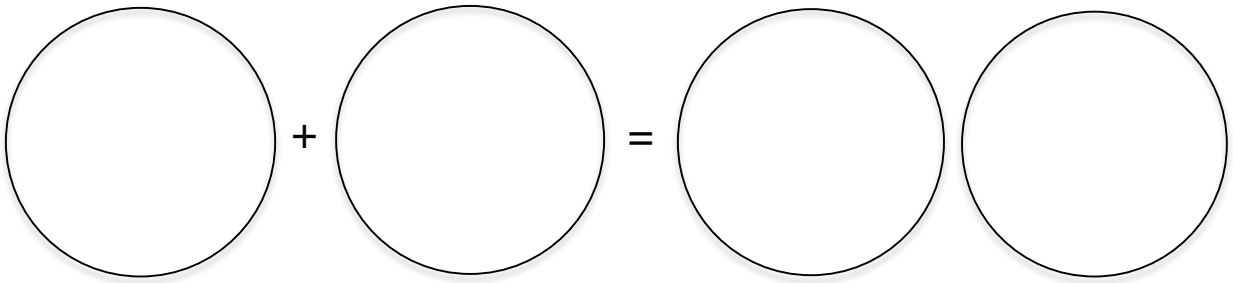


9.

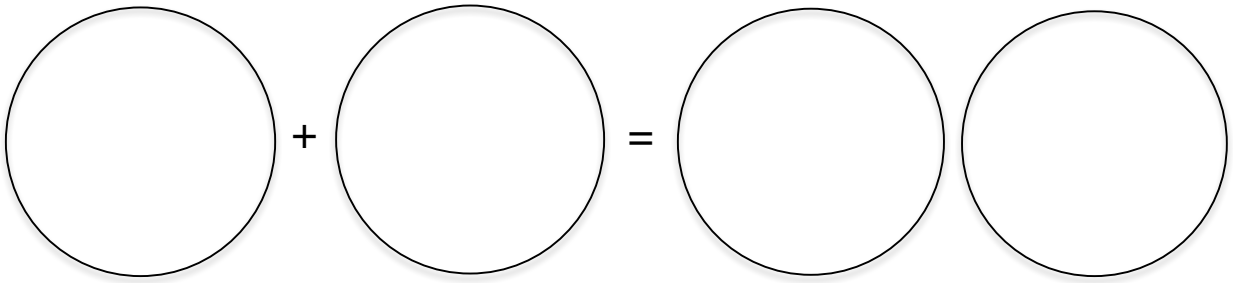


Copyright © 2016 by The University of Kansas

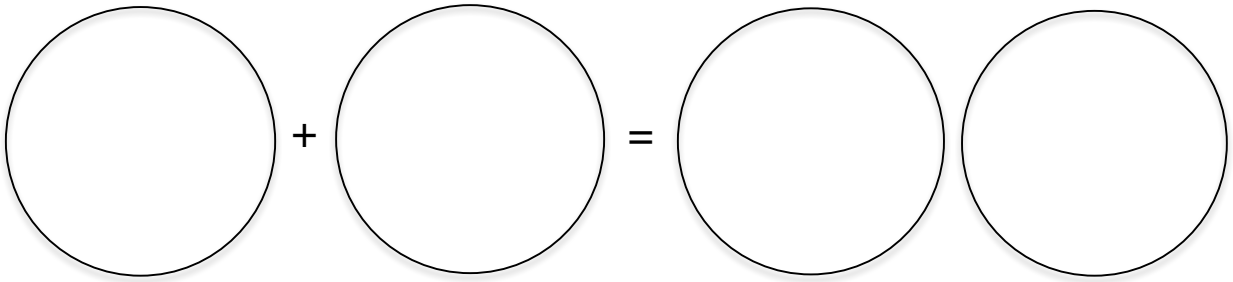
10.



11.



12.

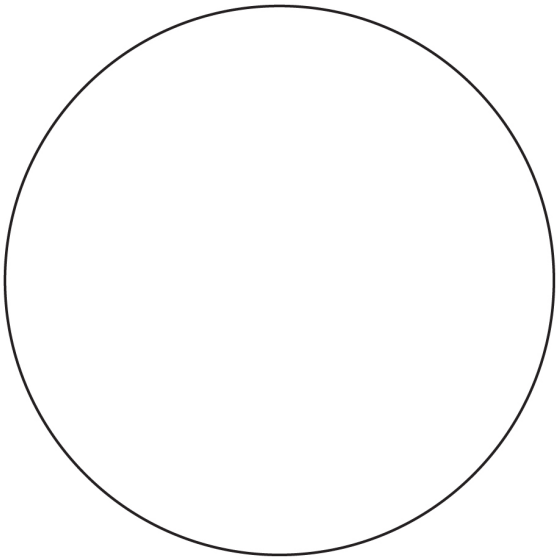


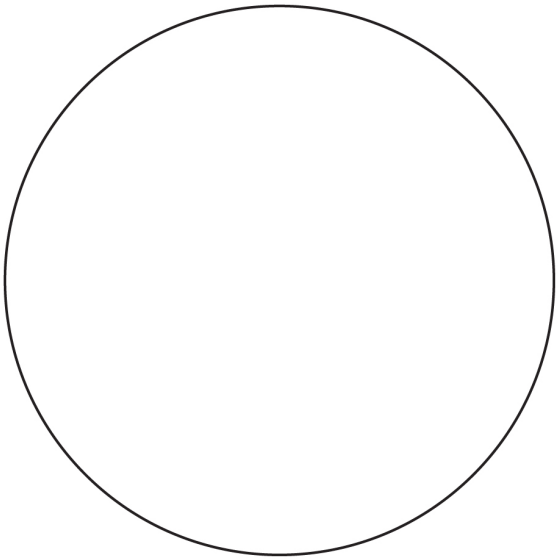
Copyright © 2016 by The University of Kansas

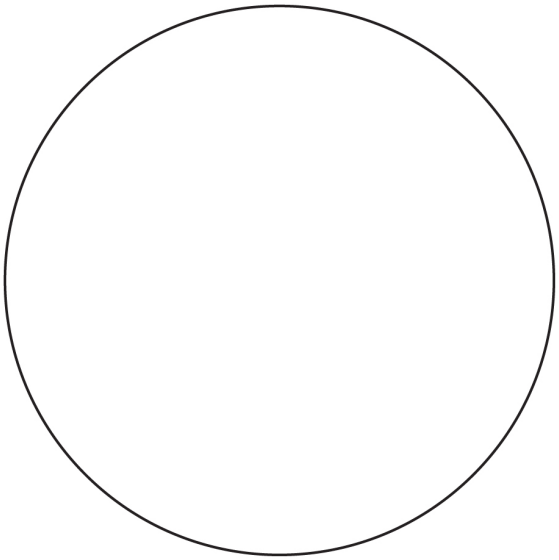
Adding and Subtracting Fractions with Unlike Denominators

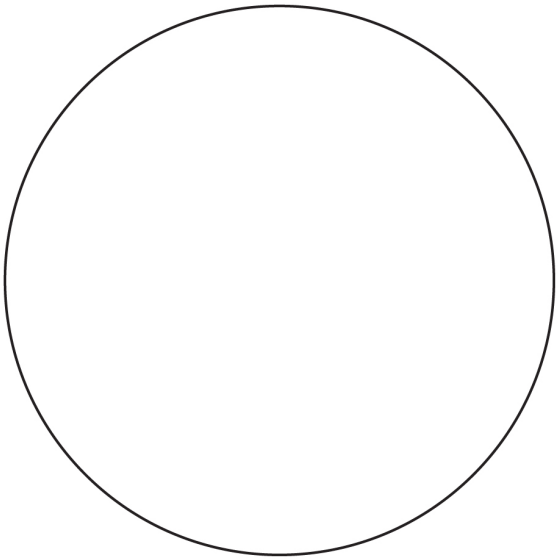
INSTRUCTIONAL ACTIVITY SUPPLEMENT

Lesson 1



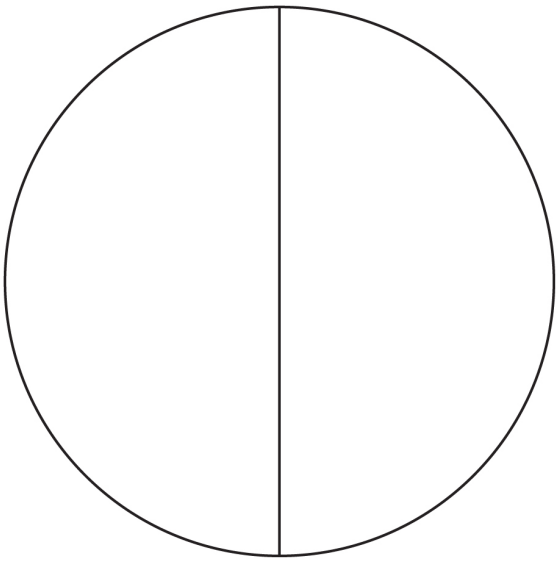


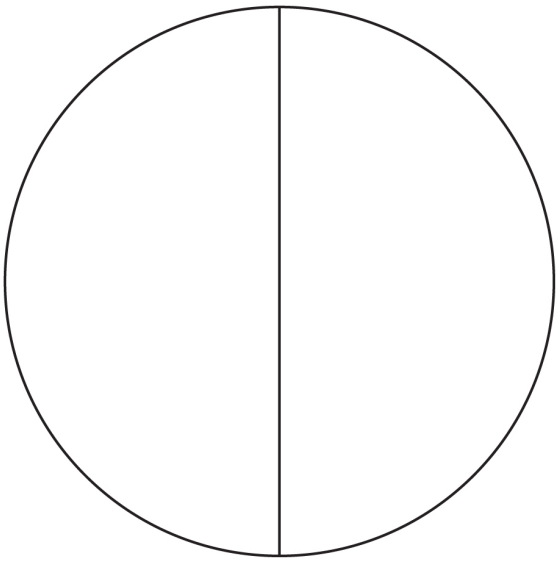


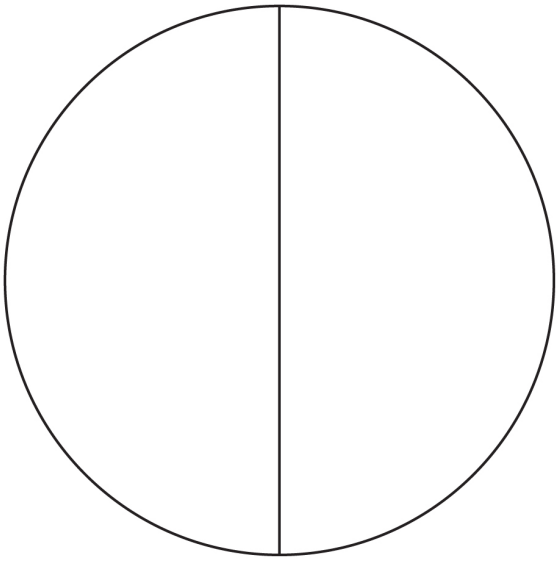


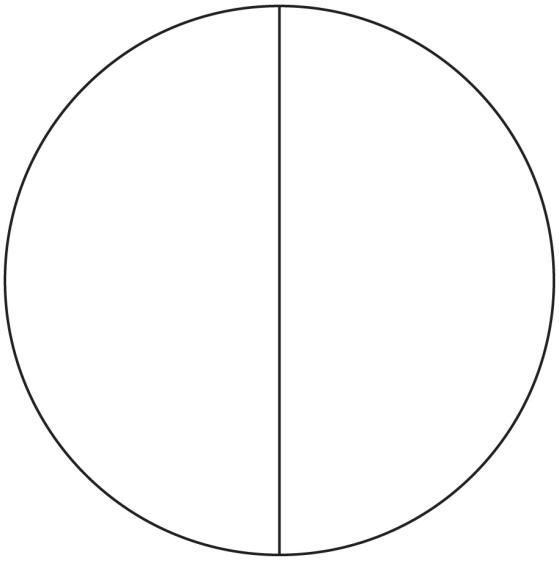
Copyright © 2016 by The University of Kansas

Halves



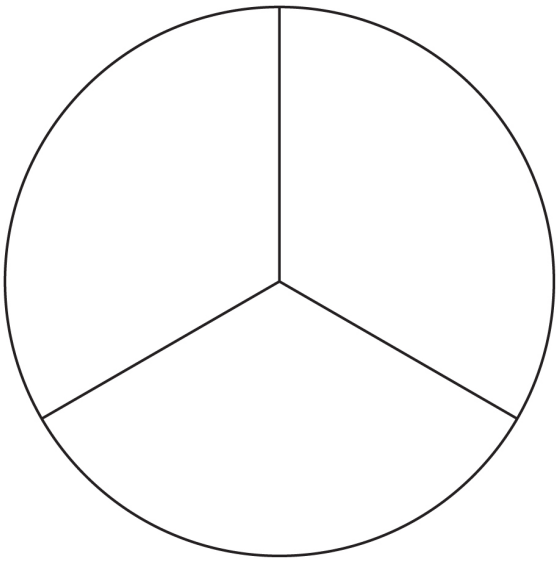


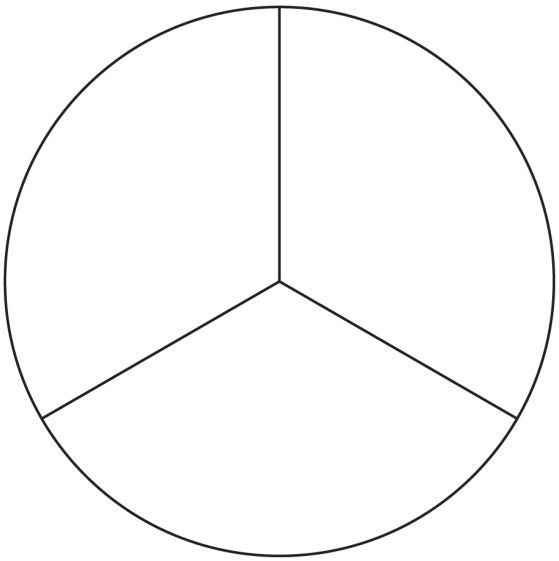


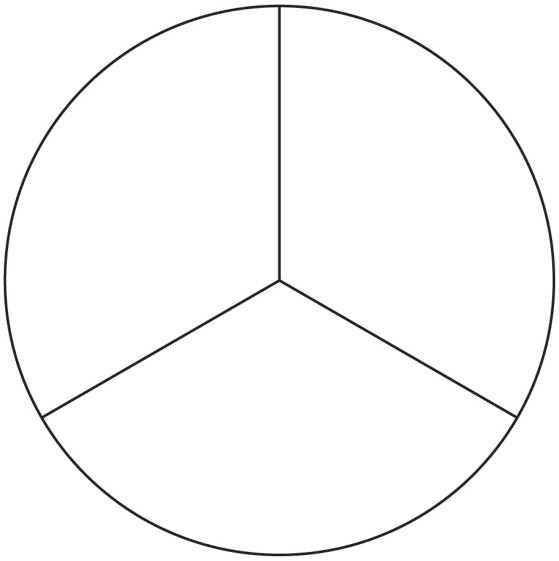


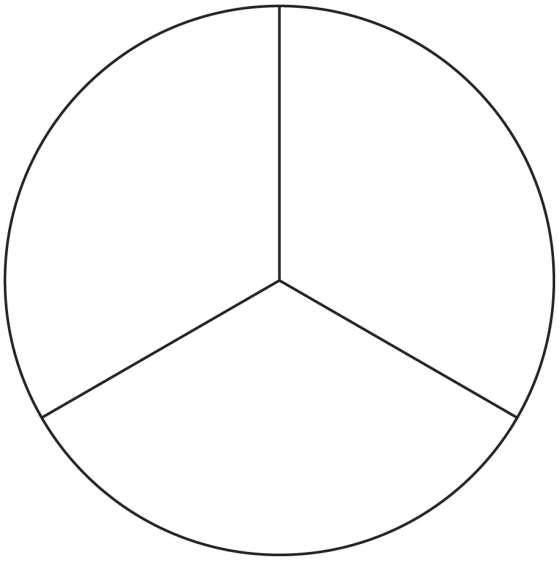
Copyright © 2016 by The University of Kansas

Thirds



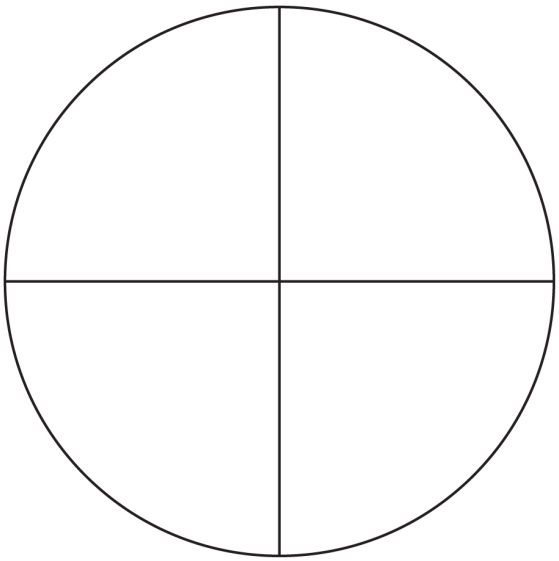


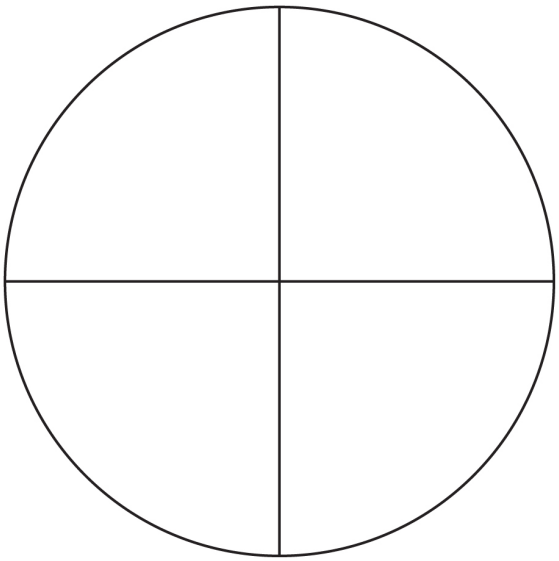


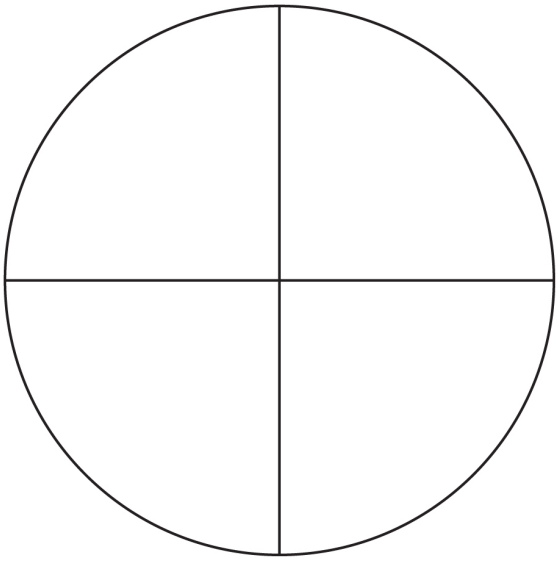


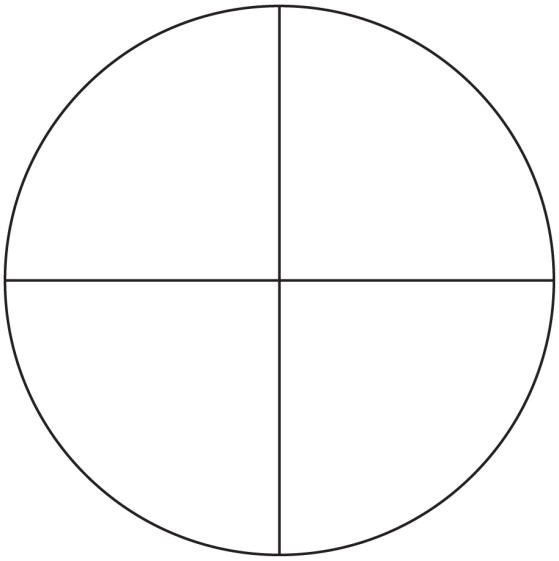
Copyright © 2016 by The University of Kansas

Fourths



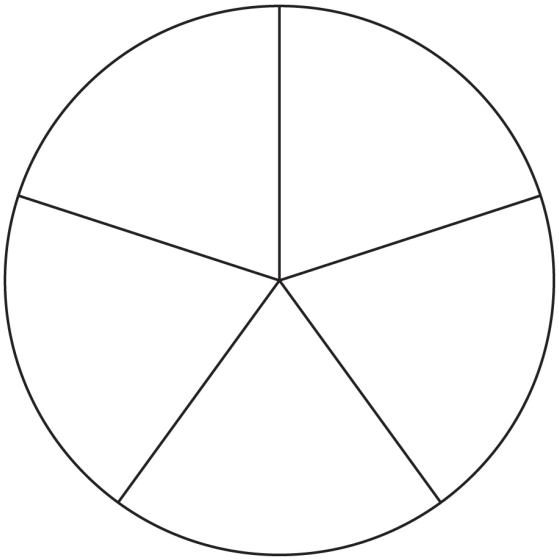


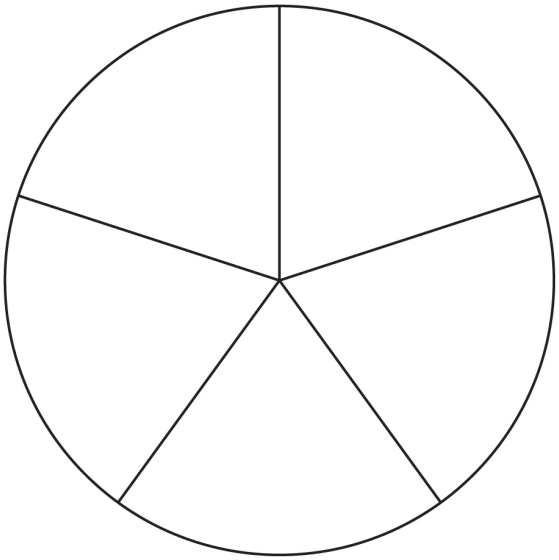


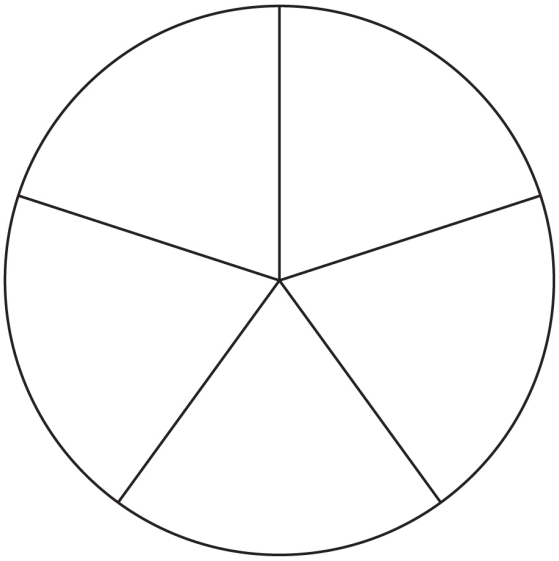


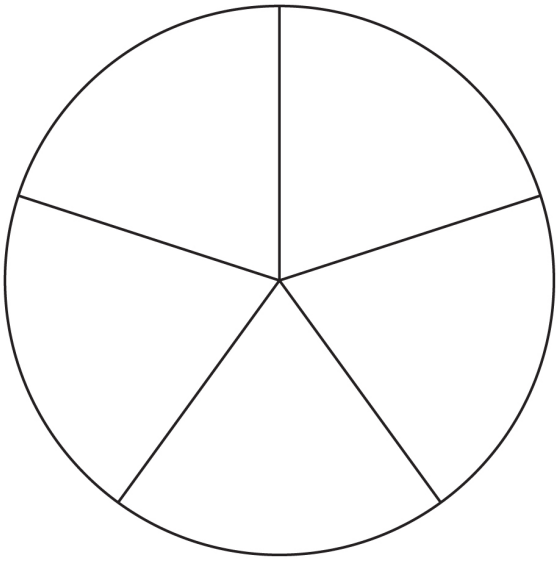
Copyright © 2016 by The University of Kansas

Fifths



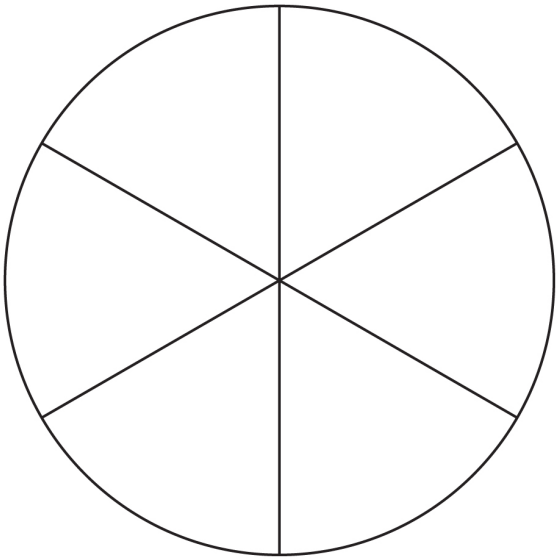


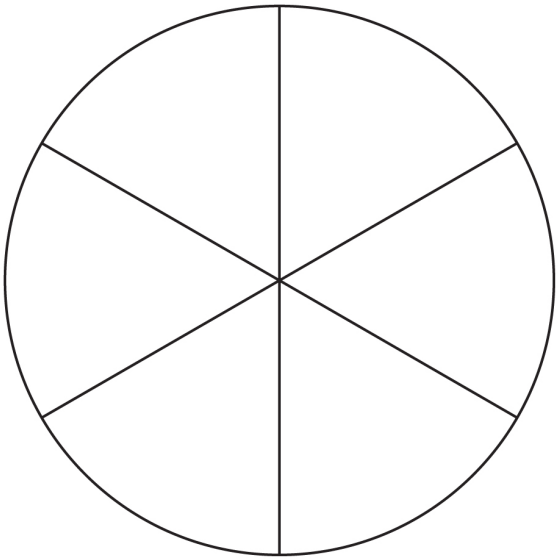


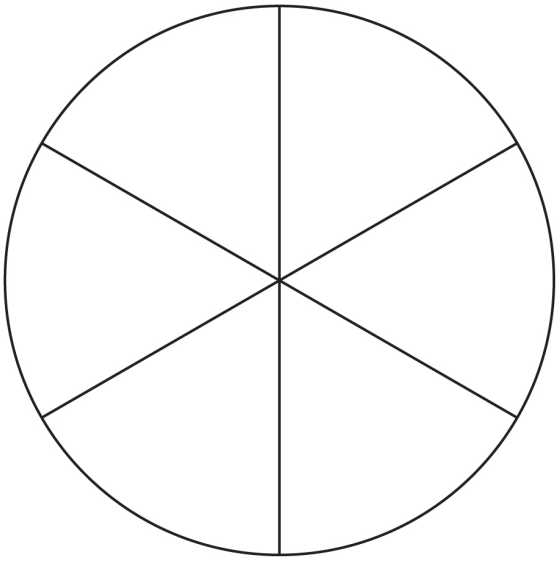


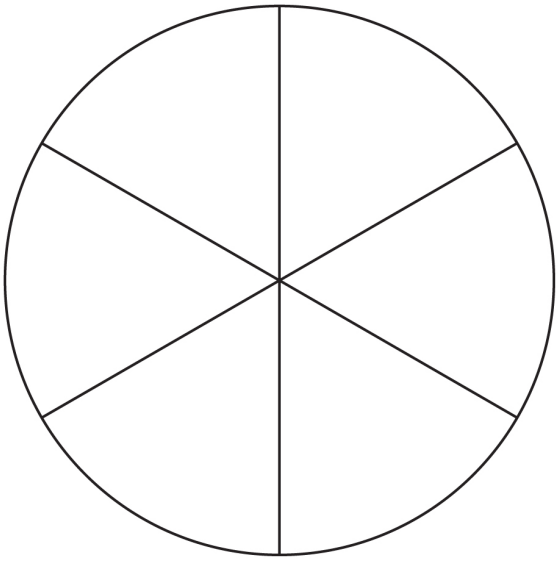
Copyright © 2016 by The University of Kansas

Sixths



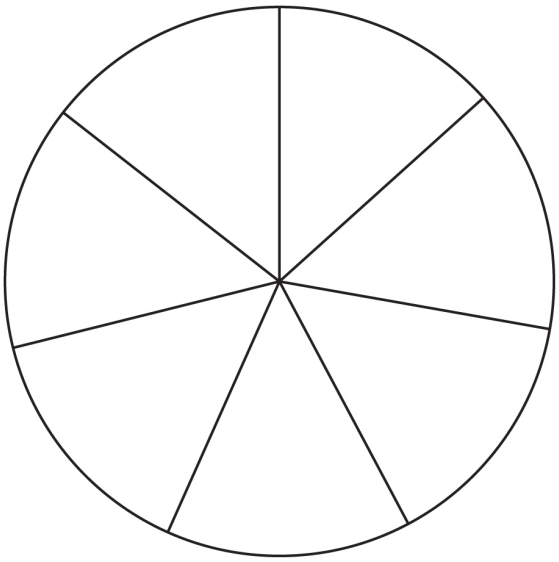


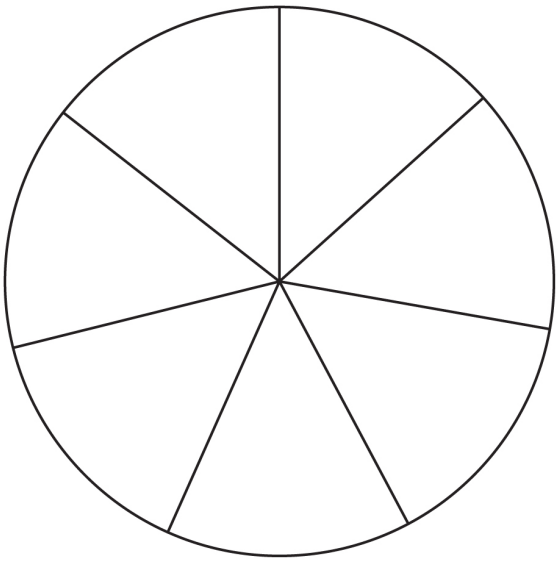


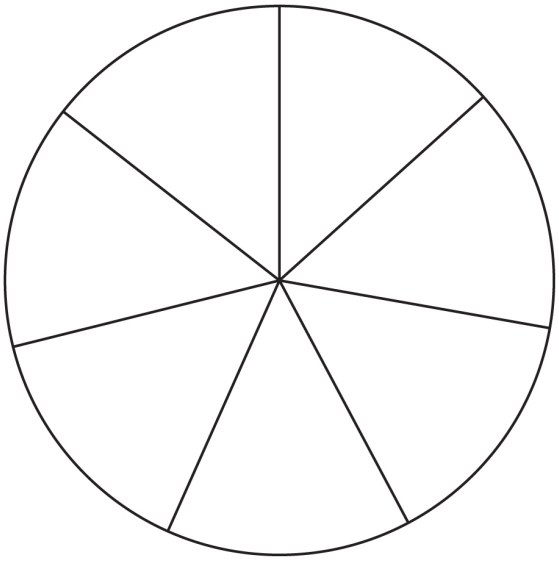


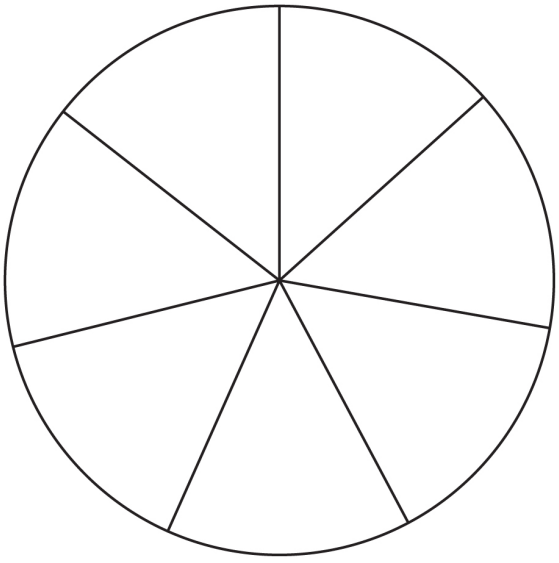
Copyright © 2016 by The University of Kansas

Sevenths



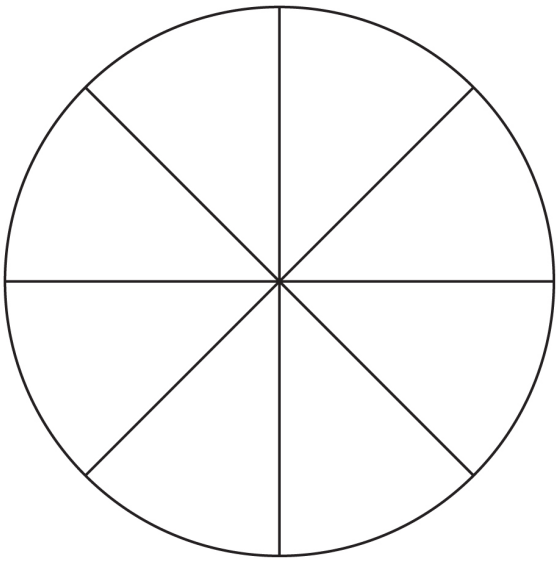


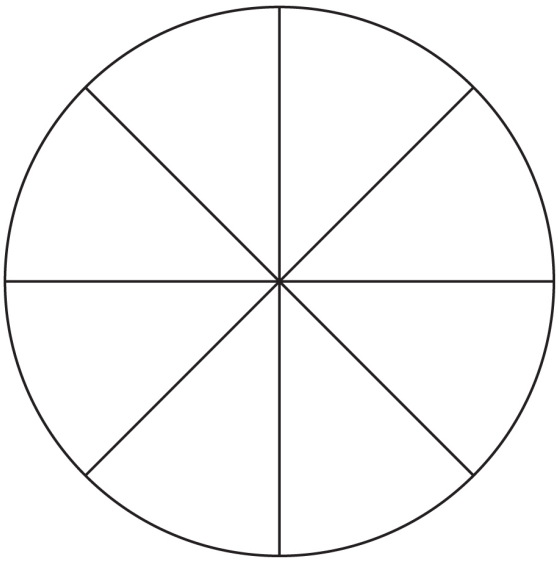


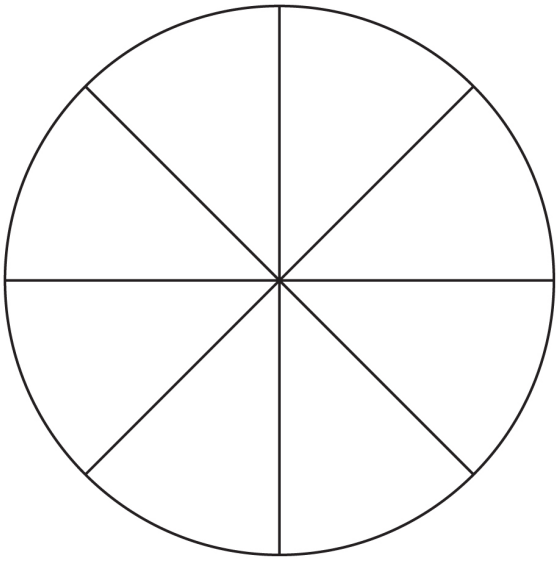


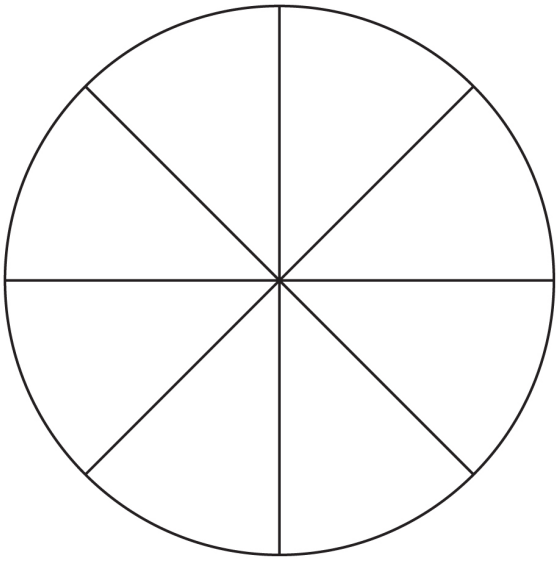
Copyright © 2016 by The University of Kansas

Eighths



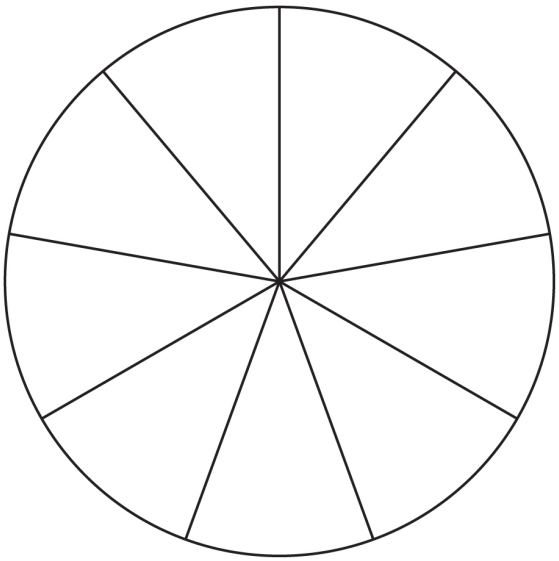


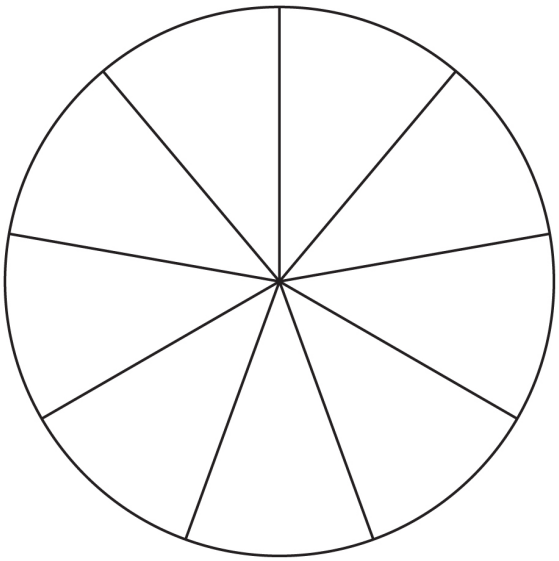


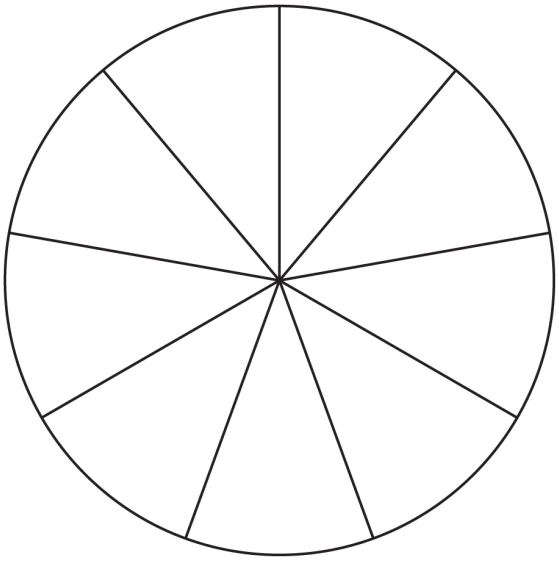


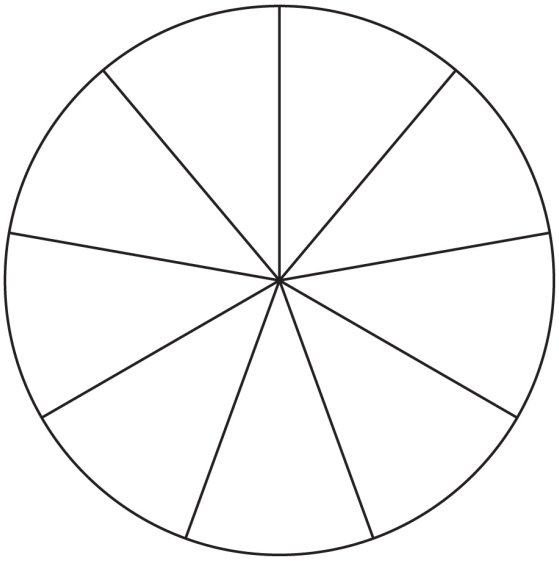
Copyright © 2016 by The University of Kansas

Ninths



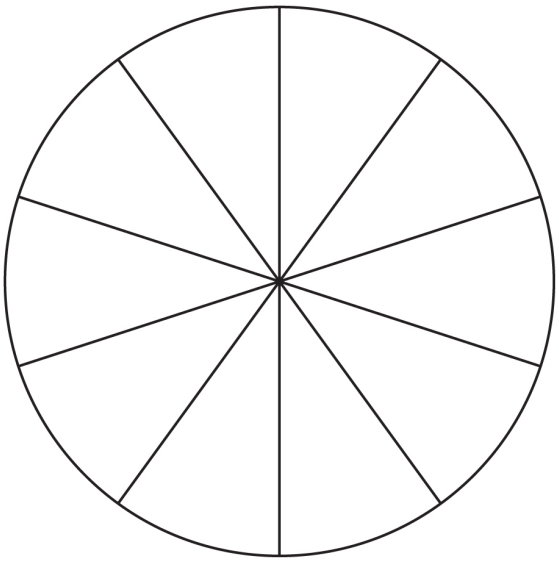


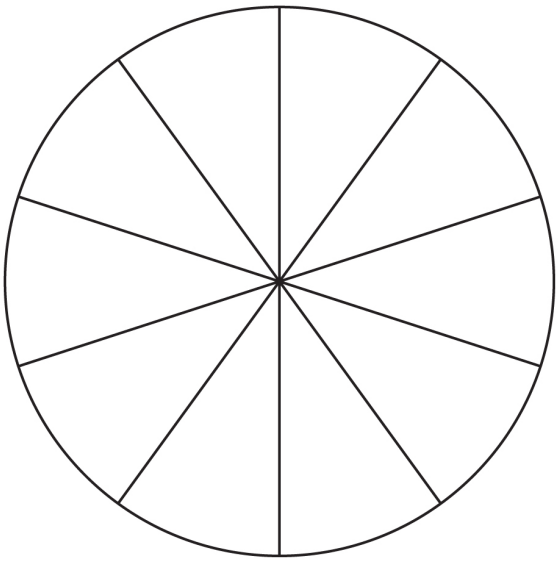


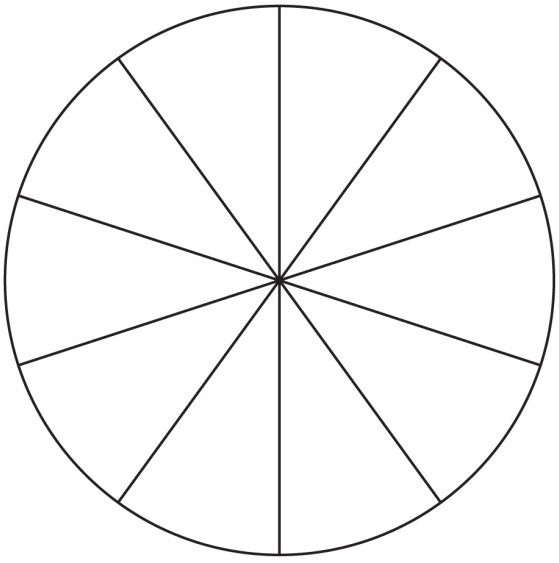


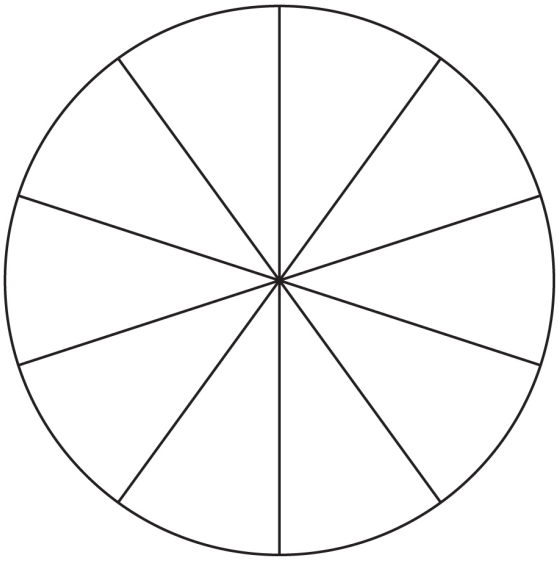
Copyright © 2016 by The University of Kansas

Tenths



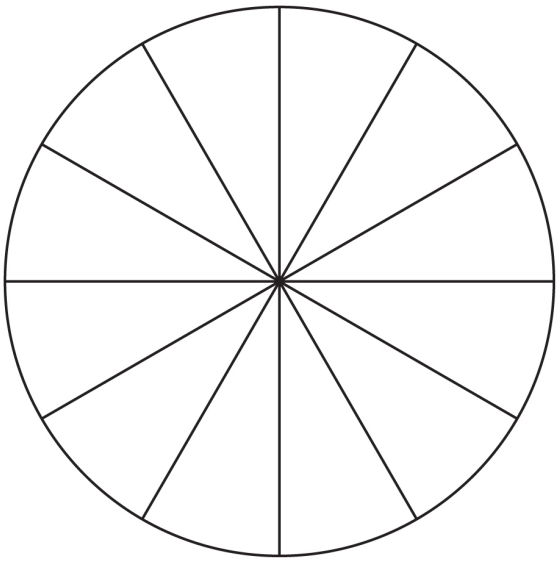


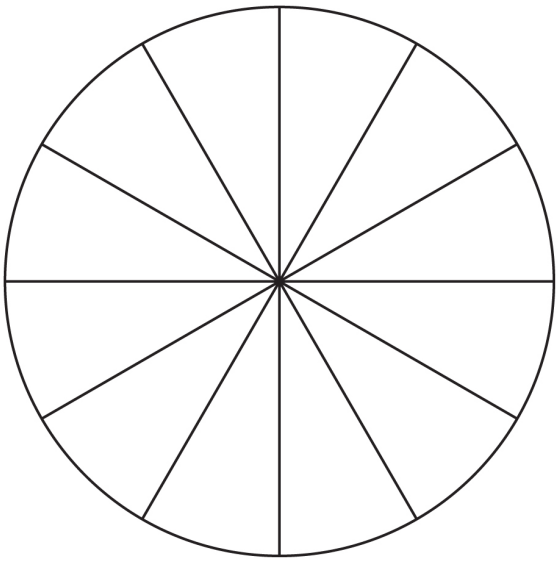


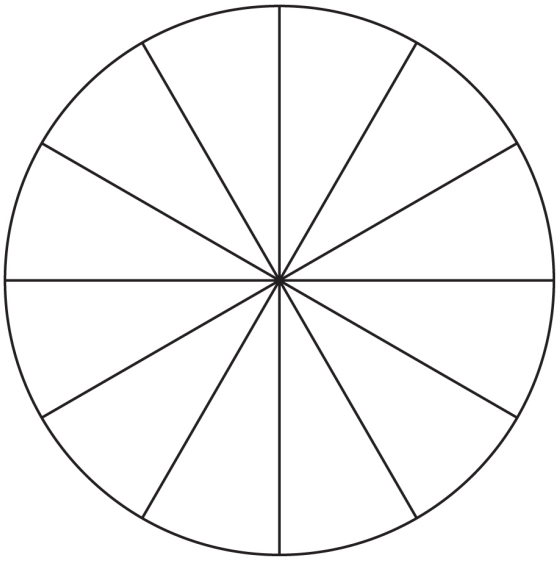


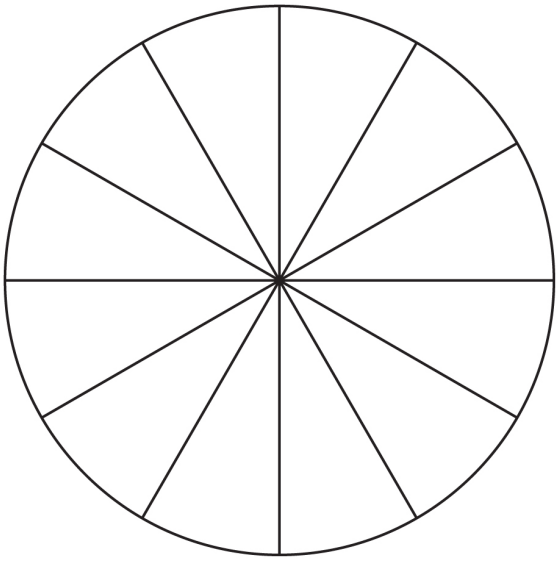
Copyright © 2016 by The University of Kansas

Twelfths



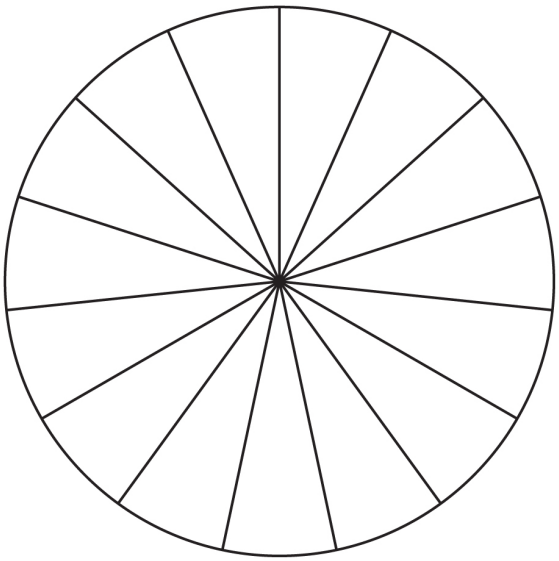


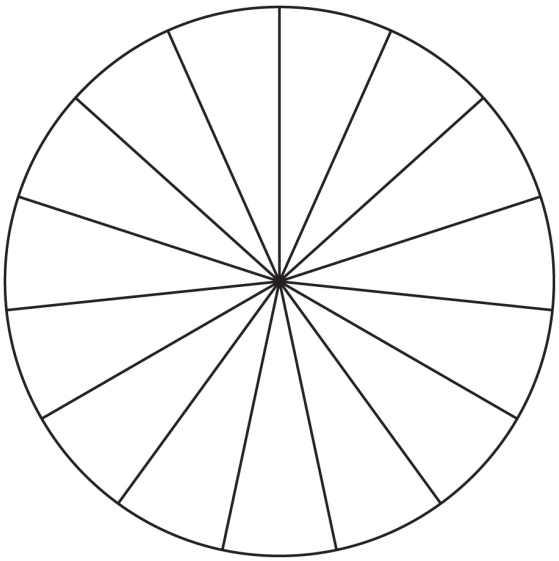


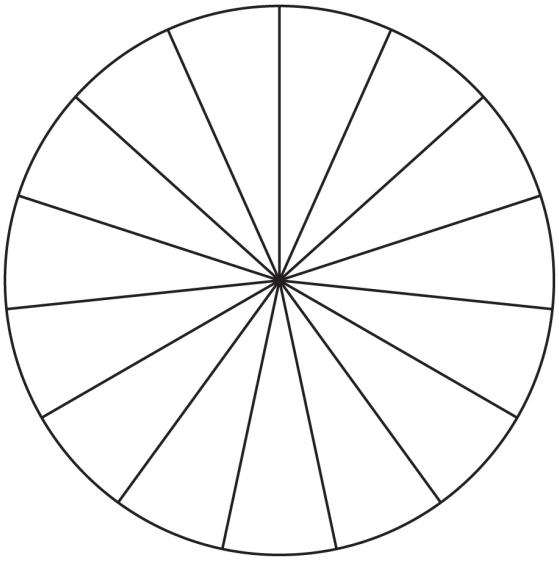


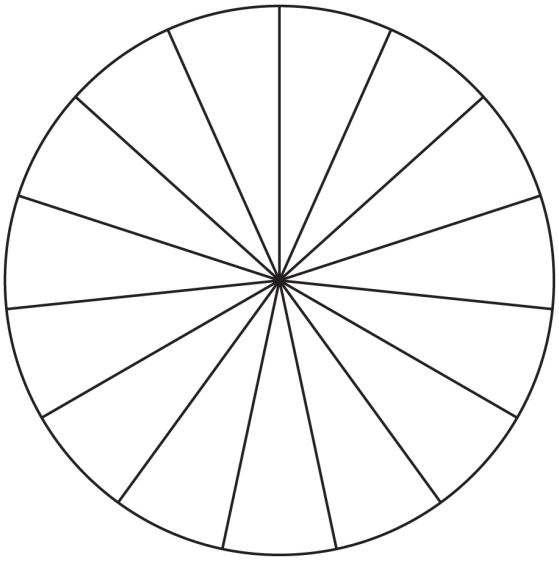
Copyright © 2016 by The University of Kansas

Fifteenths









Copyright © 2016 by The University of Kansas

ADDING AND SUBTRACTING FRACTIONS WITH UNLIKE DENOMINATORS

INSTRUCTIONAL ACTIVITY

Lesson 2

LEARNING GOAL

Students will build on their knowledge of adding and subtracting fractions with like and unlike denominators in order to add and subtract fractions with unlike denominators using number lines.

PRIMARY ACTIVITY

Students will use number lines to add and subtract fractions with unlike denominators.

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

- Equivalent
- Multiple
- Sum
- Addend
- Difference
- Numerator
- Denominator
- Mixed number
- Improper fraction

MATERIALS

- Fraction circles (from LESSON 1) or fraction tiles
- Ruler (one per student)
- Strips of paper or register paper take with a blank number line drawn on the paper (two per student)
- INSTRUCTIONAL ACTIVITY STUDENT HANDOUT

Copyright © 2016 by The University of Kansas

IMPLEMENTATION

Distribute the fraction pieces and ask students to group all the different fractional parts together in piles so all like pieces are together. Begin the lesson by reviewing what the numerator and denominator represent.

Emphasize that the denominator is the total number of equal parts into which one whole has been divided. This will help prepare students to divide a number line into equal parts.

Emphasize that the numerator is the given number of equal parts being used.

Ask students the following guiding questions to elicit thinking about parts of a fraction, number lines, and adding and subtracting fractions with unlike denominators.

GUIDING QUESTIONS

Elicit student thinking:

- What are other representations you can use to model fractions?

Determine if the student can EXPLAIN *NUMBER LINE*:

- What can you tell me about number lines?
- Describe a number line for me.
- What are examples of number lines in real life?

Determine if the student can EXPLAIN THE POSITION OF FRACTIONS ON THE NUMBER LINE:

- What do you know about the relationship between fractions and a number line?
- Explain how you would represent the denominator of a fraction on a number line. For example, if you were to identify $\frac{3}{4}$ on a number line, how are the 4 and the number of equal divisions on the number line similar?
- What can you tell me about the numerator of a fraction and a number line?
- How would you locate ____ [use a specific fraction] on a number line?

Copyright © 2016 by The University of Kansas

Determine if the student can RECOGNIZE EQUIVALENT FRACTIONS WITH NUMBER LINE MODELS:

- What can you tell me about equivalent fractions on the number line?
- How would you know if two fractions plotted on a number line were equivalent?

Determine if the student can USE NUMBER LINE TO MODEL ADDITION AND SUBTRACTION:

- What do you know about adding/subtracting numbers on the number line?
- How would you model adding fractions on a number line?
- How would you model subtracting fractions on a number line?

Model a number line on the board with a fraction circle. Hand out a paper number line (the strip of paper or register paper) and require students to label the end on the left with marked with a zero. Explain to students, "This is zero. This is the starting point for measuring distances on the number line." Then have students label the right end with a one.

Ask students:

- "Look at the space between zero and one. What number belongs between zero and one?" $(\frac{1}{2})$
- "Where between zero and one would $\frac{1}{2}$ belong? How far from zero should we mark $\frac{1}{2}$? How far from one should we mark $\frac{1}{2}$?" (the same distance or right in the middle)

Require students to fold the paper strip number line into two equal parts and label the middle line $\frac{1}{2}$. Compare the paper strip number line with the circle model. Ask students, "Do you notice how the distance from zero to $\frac{1}{2}$ is equal to the distance from $\frac{1}{2}$ to one? Look at the circle model. The two $\frac{1}{2}$ pieces are the same size, just as the distances on the number line are the same length."

Ask students:

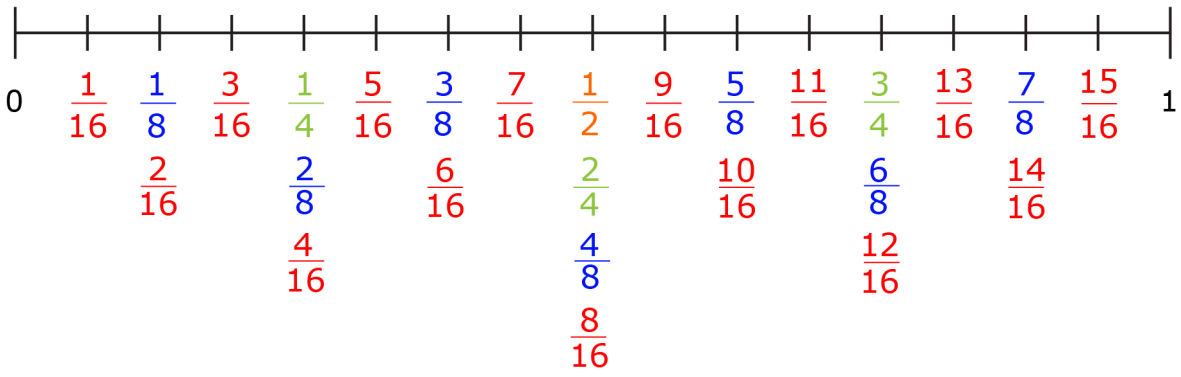
- "What are some other numbers we could add to our number line? Can you think of a number that would go between zero and $\frac{1}{2}$?"
- "If you divide one of the $\frac{1}{2}$ fraction circles into two equal parts, what is one of the fraction parts you now have?" $(\frac{1}{4})$

Copyright © 2016 by The University of Kansas

- "Where do you think $\frac{1}{4}$ should be added between zero and $\frac{1}{2}$?"
- "Fold your paper strip number line halfway between zero and $\frac{1}{2}$ and mark that line $\frac{1}{4}$. If you had two $\frac{1}{4}$ fraction circle parts, what fraction is that equivalent?" $(\frac{1}{2})$

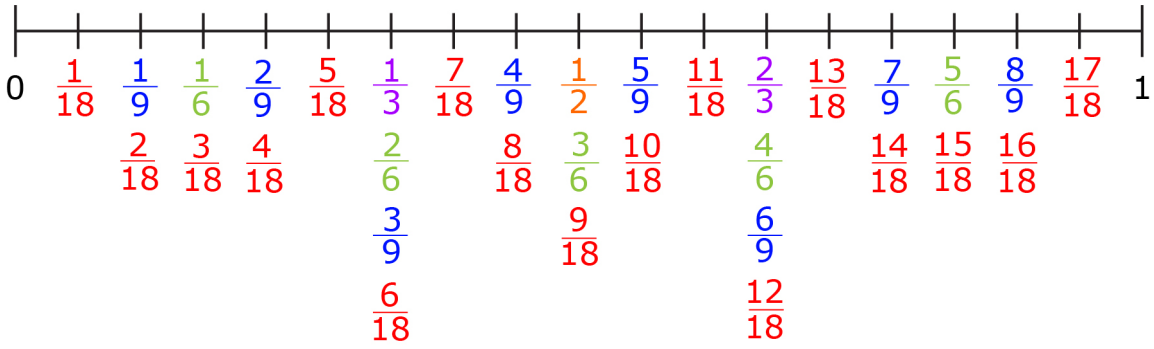
Explain that if you have two – $\frac{1}{4}$ pieces, that is also equivalent to $\frac{1}{2}$. Write $\frac{2}{4}$ underneath the $\frac{1}{2}$ on the paper strip number line, since $\frac{2}{4}$ is equivalent to $\frac{1}{2}$. Continue this questioning and emphasize the equivalent fractions until you have divided the number line by multiples of two all the way to sixteenths. Each times, make sure to refer to a new fraction to connect the fraction with its distance from zero on the number line.

Require students to divide the number line on Example 1 of the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.



Ask students, using the same questioning as above, to divide the second paper strip number line into thirds all the way to eighteenths.

Require students to copy the paper strip number line onto the number line on Example 2 of the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

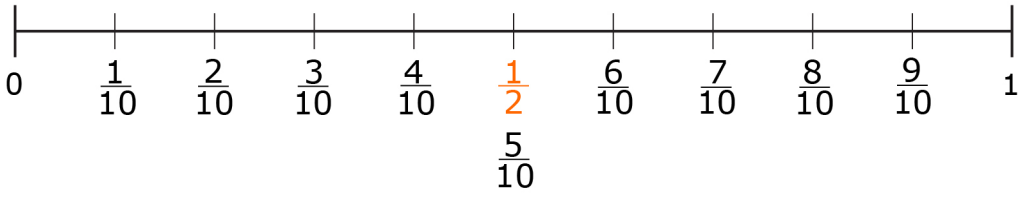


Once students have successfully divided number lines into equal parts, proceed to adding and subtracting with the number line. **Emphasize** that each fraction is a sum of unit fractions.

Copyright © 2016 by The University of Kansas

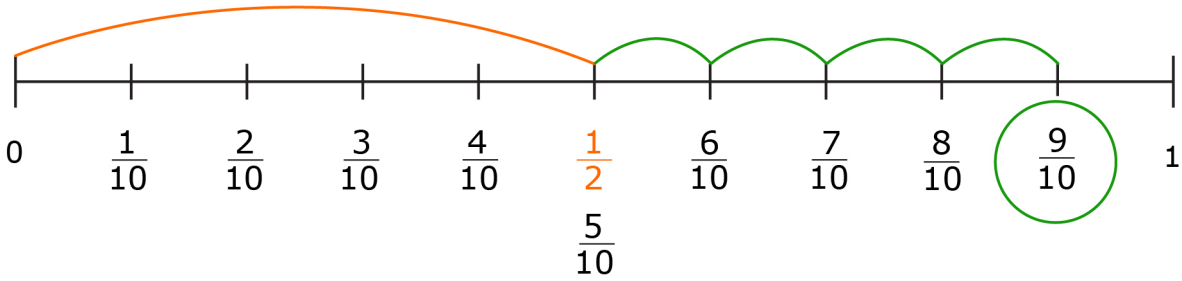
Ask students, "How would you determine a common denominator for the expression $\frac{1}{2}+\frac{4}{10}$?"

Require students to identify the common denominator and then divide the number line accordingly on Question 1 of the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT into equally spaced fractional parts based on the common denominator for the problem (tenths).



Model this addition on the number line and **require** students to do the same on Question 1 of the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT. Use a different color for each fraction. After students have modeled $\frac{1}{2}+\frac{4}{10}$ on the number line, have them write symbolically the equivalent expression for the modeled problem, $\frac{5}{10}+\frac{4}{10}$ as well as the sum ($\frac{9}{10}$).

Ask students, "How would you simplify the expression $\frac{1}{2}+\frac{4}{10}$ using what we have modeled on the number line?"



Proceed through similar steps to simplify the expression $\frac{4}{5}-\frac{1}{3}$.

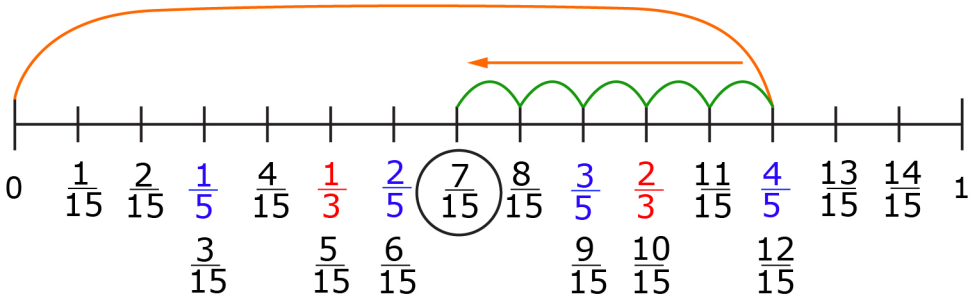
Ask students, "How would you determine a common denominator for the expression $\frac{4}{5}-\frac{1}{3}$? How does the common denominator affect the structure of the number line needed to model this subtraction?"

Require students to divide the number line on Question 2 of the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT into the like denominator for the expression $\frac{4}{5}-\frac{1}{3}$.

Copyright © 2016 by The University of Kansas

Model this subtraction on the number line and **require** students to do the same on Question 2 of the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT to simplify the expression. Use a different color for each fraction. After students have modeled $\frac{4}{5} - \frac{1}{3}$ on the number line using equivalent fractions ($\frac{12}{15} - \frac{5}{15}$), have them write the expression and difference symbolically.

Ask students, "How would you simplify the expression $\frac{4}{5} - \frac{1}{3}$ using what we have modeled on the number line?"



Require students to simplify $\frac{1}{4} + \frac{1}{3}$ on Question 3 of the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT with a partner. Students should use a different color for each fraction.

Require one or two partner groups to present to the whole class how they determined the sum.

Ask student partners as they present, "How did you reach that sum? Why do you think that is correct?"

Ask students when partners present, "Do you agree or disagree? Why or why not? Does anyone have a different way to explain finding the sum?"

Proceed through the same steps for the subtraction expression $\frac{3}{4} - \frac{2}{6}$.

Require students to simplify $\frac{3}{4} - \frac{2}{6}$ on Question 4 of the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT with a partner. Students should use a different color for each fraction.

Require one or two partner groups to present how they determined the difference to the whole class.

Ask student partners as they present, "How did you determine the difference?"

Ask students when partners present:

- "Do you agree or disagree? Why or why not? Does anyone have a different way to explain finding the solution?"
- "What do two equivalent fractions look like on a circle model?"

Copyright © 2016 by The University of Kansas

- "How do you know if two fractions are equivalent on a number line?"
- "How are equivalent fractions on a number line similar to equivalent fractions on a circle model?"
- "If two equivalent fractions on a circle model are the same size, how do you know if fractions on a number line are equivalent?"
- "How does recognizing the distance a fraction is plotted from zero on a number line help you identify equivalent fractions?"
- "How do you know if two fractions are equivalent without a model? If you were given two fractions in written fraction form, how would you know if they were equivalent?"

Proceed through the next four problems, requiring students to complete the addition/subtraction on the number lines in the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT. **Pose** each question as a real life problem using a variety of types of questions; see the TEACHER NOTES for more explanation regarding this recommendation. Be sure to have students think about what the question is asking, and then check their answers with the questions to make sure that the answer is reasonable.

GUIDING QUESTIONS

Elicit student thinking:

- What does it mean to add?
- What does it mean to subtract?

Determine if the student can SOLVE WORD PROBLEMS INVOLVING ADDITION AND SUBTRACTION OF FRACTIONS WITH UNLIKE DENOMINATORS:

- How do you know when a problem is asking you to add?
- When you add these two numbers together, is the sum going to be larger or smaller than the addends? Why?
- How do you know when a problem is asking you to subtract?
- Look at your answer and the question. Does your answer make sense?

As in the previous lesson, when there is an improper fraction/mixed number for the sum, point out that the numerator is greater than the denominator, which means there are more fractional parts than are needed to make one whole.

- 5. $\frac{1}{2}+\frac{4}{5}=\frac{13}{10}=1\frac{3}{10}$
- 6. $\frac{4}{6}-\frac{1}{4}=\frac{5}{12}$

Copyright © 2016 by The University of Kansas

- 7. $\frac{2}{3}+\frac{1}{2}=\frac{7}{6}=1\frac{1}{6}$
- 8. $\frac{4}{5}-\frac{1}{3}=\frac{7}{15}$

At the end of the activity, assess student learning by reviewing their work on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT and by closing with the following questions:

- What are the steps you would take to add or subtract fractions using a number line?
- What happens when you add two fractions that are each greater than $\frac{1}{2}$? How would you represent the sum on a number line?
- When adding and subtracting fractions, which method did you prefer: the fraction circle model or the number line? Why did you prefer this method?
- Explain how adding and subtracting fractions using a number line is similar to and different from adding and subtracting fractions with models.

Copyright © 2016 by The University of Kansas

Name: _____

Adding and Subtracting Fractions with Unlike Denominators

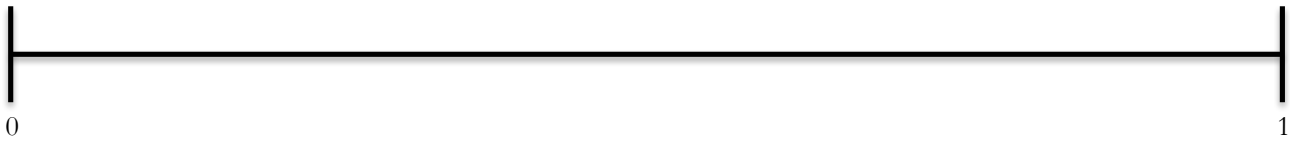
Lesson 2

Example 1

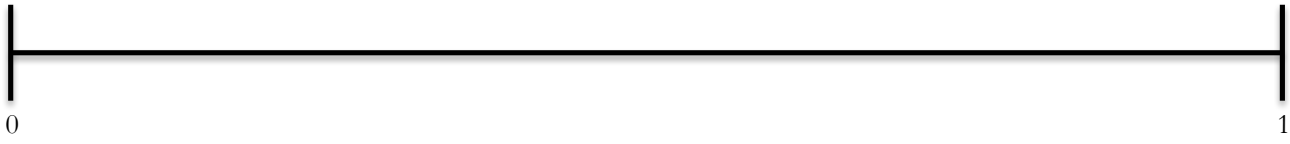
Example 2

Copyright © 2016 by The University of Kansas

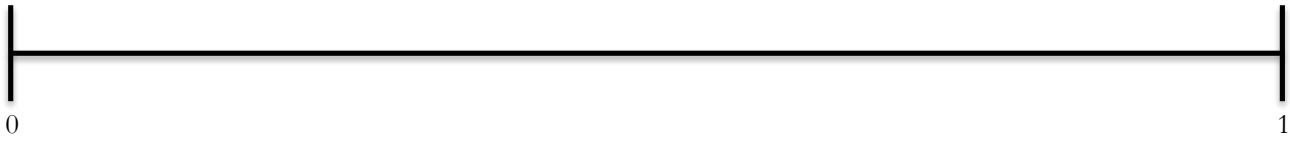
1.



2.



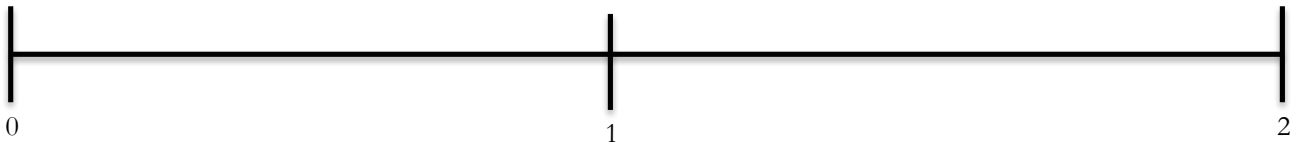
3.



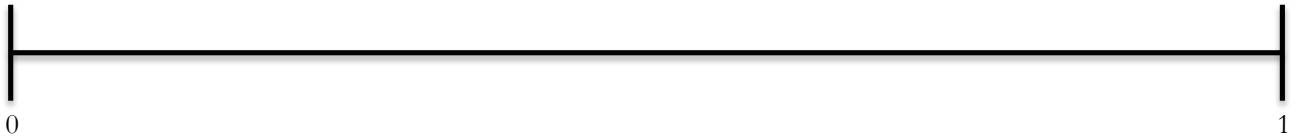
4.

Copyright © 2016 by The University of Kansas

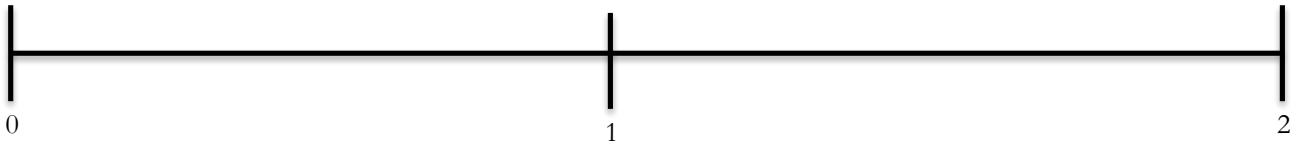
5.



6.



7.



8.

Copyright © 2016 by The University of Kansas

ADDING AND SUBTRACTING FRACTIONS WITH UNLIKE DENOMINATORS

INSTRUCTIONAL ACTIVITY

Lesson 3

LEARNING GOAL

Students will build on their knowledge of adding and subtracting fractions with unlike denominators using models in order to add and subtract fractions with unlike denominators using the standard algorithm.

PRIMARY ACTIVITY

Students will calculate a common denominator and create equivalent fractions in order to add and subtract fractions with unlike denominators.

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

- Equivalent
- Multiple
- Sum
- Addend
- Numerator
- Denominator
- Difference
- Mixed number
- Improper fraction

MATERIALS

- INSTRUCTIONAL ACTIVITY STUDENT HANDOUT

Copyright © 2016 by The University of Kansas

IMPLEMENTATION

Begin the lesson by reviewing adding and subtracting fractions with unlike denominators using fraction circles and number lines.

Ask students, "What would you do if you had to simplify an expression without a model? We are going to take what we know about adding and subtracting fractions with models and simplify expressions in written form. What is the first step to simplifying a fraction expression with unlike denominators?" (The first step is finding equivalent fractions to represent common denominators.)

Ask students:

- "Suppose you needed to find the common denominator of six and four. How did you find a common denominator using circle models?" (They divided each circle model until there were the same number of equal parts.)
- "How did you find a common denominator using a number line?" (They divided each number line into equal parts until there were the same number of equal parts.)

Explain to students, "Now we are going to take what we know about multiplication and division and explore a method to calculate a common denominator. Suppose you have the fractions $\frac{1}{6}$ and $\frac{3}{4}$. You need to find a common denominator."

Ask students first to determine whether one denominator is a multiple of the other (in this case, whether 6 is a multiple of 4). Because it is not, the next step is to consider the multiples of 6 and 4.

Require students to list several multiples of 6 and 4, and then to compare the multiples of 6 to the multiples of 4. Students should notice that 12 and 24 (and any other multiple of 12) are multiples of both six and four.

- List the multiples of each denominator until you find at least one value in common.
  4: 4, 8, 12, 16, 20, 24
  6: 6, 12, 18, 24

Ask students whether these numbers could be used as a common denominator in order to add or subtract the fractions. Students should indicate that either (or all) values could be used as a common denominator.

> Using the smallest value, or the least common denominator, is the simplest for computation, though it is not necessary to use the least common denominator to add and subtract fractions

Explain that another way to calculate a common multiple is to multiply the two denominators together:

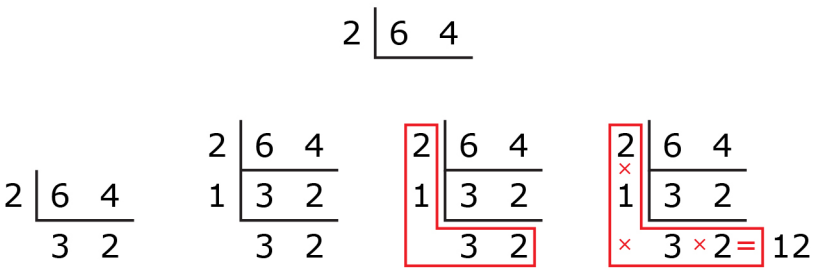
Copyright © 2016 by The University of Kansas

- "Suppose you needed to find the common denominator of $\frac{1}{6}$ and $\frac{3}{4}$, and instead of listing out the multiples, you multiply the two denominators. Will that create a common multiple? If you multiply 6 and 4, what is the product? (24) Did you determine using the first method that 24 is a common multiple of 6 and 4? Therefore, 24 could be a common denominator of $\frac{1}{6}$ and $\frac{3}{4}$."
  - Multiply the two denominators together: $4 \times 6 = 24$

Explain that the following is one method that is useful for finding the least common multiple:

- "Another way to find a common denominator of $\frac{1}{6}$ and $\frac{3}{4}$ is to divide both numbers by common factors until you reach a factor of 1 for one or both of the denominators."
- [Write the ladder on the board with 6 and 4 inside the ladder.] "6 and 4 can both be divided by what factor? (2) Yes, 2." [Write 2 outside the ladder.]
- "6 divided by 2 is 3." [Write 3 under the 6.] "And 4 divided by 2 is 2." [Write 2 under the 4.]
- "Do 3 and 2 share a common factor? (1) The only factor that 3 and 2 share is 1." [Repeat the steps for dividing by 2 using 1.]
- "Because I have reached a factor of 1, I cannot divide my denominators by any other common factors. Now outline all the factors outside the ladder in an 'L' shape, because we are finding the least common multiple." [Outline all the factors in an 'L' shape, as shown in the following images.]
- "Next, multiply all the factors you outlined: 2 times 1 is 2, 2 times 3 is 6, 6 times 2 is 12. Could you use 12 as a common denominator of $\frac{1}{6}$ and $\frac{3}{4}$?"

This method is one possibility to determine a common denominator; it is a suggestion, but it is not the only method available. This method works by finding the denominators' shared factors and multiplying them by the unshared factors of each denominator.



Explain to students, "Now that we have calculated a common denominator, we need to create equivalent fractions in order to add $\frac{1}{6} + \frac{3}{4}$. How did you create equivalent fractions using circle models?"

Copyright © 2016 by The University of Kansas

Ask, "How did you calculate equivalent fractions using the number line? Each equivalent fraction was the same size, but each was represented with a different written fraction."

Explain to students:

- "Now, use what you know about multiplication to represent equivalent fractions of $\frac{1}{6}$ and $\frac{3}{4}$ in written fraction form."
- "Do you know the identity property of multiplication? What is the product of any number times one?" (the original number)
- "So, 6 times 1 is 6, and 4 times 1 is 4. How do you know if you have one whole using a fraction model?" (You have all the possible pieces.)
- "If you had a fraction circle divided into two equal parts, how many pieces would you need to make one whole?" (You would need two pieces.) "Could you then say that $\frac{2}{2}$ is equal to one whole?" (yes)
- "If you change the denominator to 12, does the size of your equal parts on a circle model change?" (yes) "Therefore, you need to change how many equal parts you have in order to maintain the same fraction. So in order to represent $\frac{1}{6}$ with a denominator of 12, you have to multiply 6 times 2."
- "Remember that you cannot only change the size of the parts; you have to change the number of parts as well. So, we need to multiply the fraction $\frac{1}{6}$ times one whole to keep the fraction equivalent, but we also need the 6 to change to a 12. Therefore, you are going to multiply $\frac{1}{6} \times \frac{2}{2}$, which is $\frac{2}{12}$. Is $\frac{2}{2}$ equivalent to one whole?" (yes) "So, when you multiply, you are not changing the number of pieces you have in the fraction. You are only changing the size of the equal parts in the fraction."
- "Now, let's represent $\frac{3}{4}$ as an equivalent fraction with a denominator of 12. What would you multiply by 4 in order to represent 12 equal parts on a circle fraction?" (3) "Therefore, to get a denominator of 12, we need to multiply 4 times 3. In order to create an equivalent fraction, we need to multiple the fraction times one whole. One whole as a written fraction with a denominator of 3 is $\frac{3}{3}$. Now you can multiply $\frac{3}{4} \times \frac{3}{3}$, which is $\frac{9}{12}$. Can you now add $\frac{2}{12} + \frac{9}{12}$?" (Yes, and the sum is $\frac{11}{12}$.)

Model the expression $\frac{1}{4} + \frac{1}{3}$ on the board using fraction circles (or fraction tiles) or a number line. **Require** students to write the expression using symbols.

Repeat the steps listed previously with students to adjust each fraction as necessary to create equivalent fractions and rewrite fractions with like denominators. **Require** students to record each of the steps needed to solve $\frac{1}{4} + \frac{1}{3}$ on Question 1 of the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

Emphasize during each problem that you are multiplying each fraction by one whole—which does not change the value of the fraction—in order to create an equivalent fraction that shares a like

Copyright © 2016 by The University of Kansas

denominator with the other fraction. During each problem, bring students back to the model (circle or number line) understanding that the size of the whole and the amount of space the pieces take up does not change. It is simply the size of the pieces themselves that change, so you will need more pieces to take up the same amount of space. Therefore $\frac{1}{4} \times \frac{3}{3} = \frac{3}{12}$, and $\frac{1}{3} \times \frac{4}{4} = \frac{4}{12}$.

Emphasize each time you rewrite the fractions as equivalent fractions that share a common denominator that you are then able to add them because you are now adding together pieces of the same size fractional parts. The value of the fraction does not change; both the numerator and denominator are being adjusted to represent the same fraction. Therefore, the same fraction is being represented in smaller or larger fractional pieces. Notice that each fraction was multiplied by one whole. Ask students, "What is the product of any whole number multiplied by one (e.g., 3 · 1)?" (the original number) "Therefore, the 'new' fractions are still equivalent to the original fractions."

Ask students, "If you were to add $\frac{1}{6} + \frac{2}{5}$, would it be easier to use a model or the standard algorithm? Why? What would be the common denominator for the two fractions?" (30) "What can you tell me about the relationship between the denominator and the size of the fractional parts?"

Explain that modeling larger denominators using fraction circles and number lines would be difficult due to the actual size of the fractional parts. Therefore, when students encounter larger common denominators, it makes more sense for them to use symbolic expressions.

Model what thirtieths would look like on a number line, using a fraction circle, or using a fraction bar. **Draw attention** to the size of the fractional parts and make note of how difficult it would be to add or subtract fractions with large like denominators using models. The model representations are used for students to understand *why* the procedure works and serve as a mental reference for students to make sense of symbolic procedures. Students should transition to applying standard algorithms to increase their accuracy or efficiency with a range of problem types.

Require students to simplify the expression on Question 2 of the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT, showing how they calculated the common denominator, rewrote the fractions as equivalent fractions, and determined the sum.

$$\frac{1}{6} + \frac{2}{5}$$

$$\downarrow$$

$$\frac{5}{30} + \frac{12}{30} = \frac{17}{30}$$

LCD=30

$$\frac{1}{6} \times \boxed{\frac{5}{5}} = \frac{5}{30}$$

$$\frac{2}{5} \times \boxed{\frac{6}{6}} = \frac{12}{30}$$

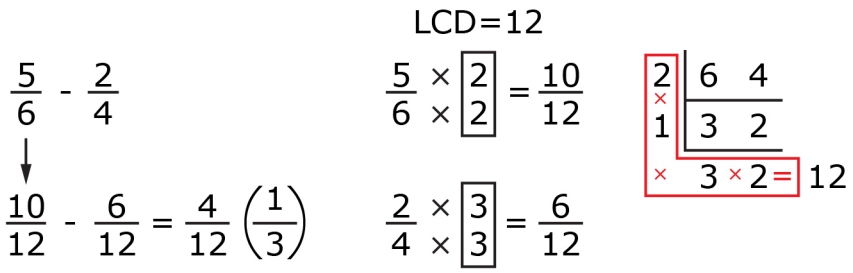
Require students to determine the common denominator for the problem $\frac{5}{6} - \frac{2}{4}$.

Ask students, "How would I find a common denominator for these two fractions? Why do I need a common denominator to subtract these fractions?"

Copyright © 2016 by The University of Kansas

Explain that if you multiply the two denominators together (24), it does produce a common denominator. However, 24 is not the *least* common denominator. Using the listing method or the ladder method results in a denominator of 12, which *is* the least common denominator. Some students may struggle with this, and as long as they find a common denominator to solve the problem, any common denominator is acceptable. The focus is on calculating a common denominator in order to add or subtract fractions.

Require students to simplify the expression on Question 3 of the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT, showing how they calculated the common denominator, rewrote the fractions as equivalent fractions, and found the difference.



Copyright © 2016 by The University of Kansas

GUIDING QUESTIONS

Elicit student thinking:

- What would be the benefit of simplifying an expression with the standard algorithm instead of a model?

Determine if the student can CALCULATE A COMMON DENOMINATOR:

- What can you tell me about multiples?
- What are strategies you can use to find common multiples of two numbers?
- How are common multiples and common denominators related?
- How would you determine an equivalent fraction to make a denominator a common multiple of two fractions?
- Would it be possible for a pair of fractions to have more than one common denominator? How is this possible? Can you provide an example?

Determine if the student can CALCULATE LEAST COMMON DENOMINATOR:

- How do we use multiples when we look for common denominators?
- Would it be possible for a pair of fractions to have more than one least common denominator? Why or why not?

Require students to calculate a common denominator for the fractions in the expression $\frac{4}{6} - \frac{1}{8}$. Instruct one or two students to share with the class how they calculated the common denominator (24 or 48). **Ask** students to share how they have calculated both a common denominator and the *least* common denominator.

Ask students:

- "Do you agree with their answer?"
- "How would you calculate the common denominator differently?"
- "Does it matter if I use the least common denominator versus a different common denominator? Can you explain why or why not?"

Require students to solve $\frac{4}{6} - \frac{1}{8}$ on Question 4 of the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT. Be sure that students show how they calculated the common denominator, created equivalent fractions, and found the difference.

Copyright © 2016 by The University of Kansas

Require students to find the common denominator for the expression $\frac{4}{5} + \frac{1}{3}$ on Question 5 of the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT. Then have students create equivalent fractions and add the two fractions.

Ask students:

- "What do you notice about the sum?" (The sum is $\frac{17}{15}$; students should note that this sum is greater than one whole.)
- "How would I represent this improper fraction as a mixed number? Think back to using the models and the number line." (Students should identify that there is one whole and two fifteenths left over for a mixed number of $1\frac{2}{15}$.)

Require students to simplify Questions 6-9 on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT. For each question, read the word problem aloud, then require students to solve the problem in partnered groups, and then call on one or two groups to share their method or solution with the whole class. This will give you an opportunity to move around the room and support students using the following guiding questions.

Copyright © 2016 by The University of Kansas

GUIDING QUESTIONS

Determine if the student can REPRESENT EQUIVALENT FRACTIONS BY MULTIPLYING OR DIVIDING:

- If the common denominator is 20, how do you represent $\frac{3}{5}$ so that it is an equivalent fraction with 20 as the denominator?
- How would you create an equivalent fraction for $\frac{3}{4}$ that has a common denominator of 20?

Determine if the student can SOLVE WORD PROBLEMS INVOLVING ADDITION AND SUBTRACTION OF FRACTIONS WITH UNLIKE DENOMINATORS:

- How do you know if you should add or subtract to find how tall Maggie's plant was on Friday?
- Once you have your answer, look at the problem again. Does your answer make sense? Is it a reasonable answer? Why or why not?
- If you are solving how tall a plant has grown, is it reasonable to determine an answer that is less than the original height? Why or why not?
- How are adding and subtracting fraction word problems similar to adding and subtracting whole number word problems? Would the word problem change if the fractions were whole numbers? Explain.

- 6. Maggie's science class was doing an experiment with plants. When Maggie measured her plant on Monday, it was $\frac{3}{5}$ centimeter tall. By Friday, her plant had grown another $\frac{3}{4}$ centimeter. How many centimeters tall was her plant on Friday? ($\frac{27}{20} = 1\frac{7}{20}$ centimeters)
- 7. Sam was also in Maggie's science class. When Sam measured his plant on Monday, it was $\frac{1}{4}$ centimeter tall. On Friday, Sam's plant was $\frac{9}{10}$ centimeter tall. How many centimeters did Sam's plant grow? ($\frac{13}{20}$ centimeters)
- 8. Molly took a package of cookies to work. Of the entire package, $\frac{3}{4}$ of the cookies had chocolate chips. Of the chocolate chip cookies, $\frac{7}{12}$ were oatmeal chocolate chip, and the rest were peanut butter with chocolate chips. What fraction of the chocolate chip cookies were peanut butter with chocolate chips? ($\frac{2}{12} = \frac{1}{6}$ of the cookies)

Copyright © 2016 by The University of Kansas

- 9. After school, Matt walked to Liam's house to work on homework. Liam lived $\frac{8}{12}$ mile from school. When they finished their homework, Matt walked $\frac{3}{8}$ mile from Liam's house to his own house. How many miles did Matt walk in all after school? ($\frac{25}{24} = 1\frac{1}{24}$ miles)

At the end of the activity, assess student learning by reviewing their work on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT and by closing with the following questions:

- What are the steps you would take to add or subtract fractions?
- How would you calculate a common denominator? What is a least common denominator?
- What does it mean if the sum is an improper fraction?
- Explain the relationship between improper fractions and mixed numbers.

Copyright © 2016 by The University of Kansas

Name: _____

Adding and Subtracting Fractions with Unlike Denominators

Lesson 3

1) $\frac{1}{4} + \frac{1}{3}$

2) $\frac{1}{6} + \frac{2}{5}$

3) $\frac{5}{6} - \frac{2}{4}$

4) $\frac{4}{6} - \frac{1}{8}$

5) $\frac{4}{5} + \frac{1}{3}$

Copyright © 2016 by The University of Kansas

6) Maggie's science class was doing an experiment with plants. When Maggie measured her plant on Monday, it was $\frac{3}{5}$ centimeter tall. By Friday, her plant had grown another $\frac{3}{4}$ centimeter. How many centimeters tall was her plant on Friday?

7) Sam was also in Maggie's science class. When Sam measured his plant on Monday, it was $\frac{1}{4}$ centimeter tall. On Friday, Sam's plant was $\frac{9}{10}$ centimeter tall. How many centimeters did Sam's plant grow?

8) Molly took a package of cookies to work. Of the entire package, $\frac{3}{4}$ of the cookies had chocolate chips. Of the chocolate chip cookies, $\frac{7}{12}$ were oatmeal chocolate chip, and the rest were peanut butter with chocolate chips. What fraction of the chocolate chip cookies were peanut butter with chocolate chips?

9) After school, Matt walked to Liam's house to work on homework. Liam lived $\frac{8}{12}$ mile from school. When they finished their homework, Matt walked $\frac{3}{8}$ mile from Liam's house to his own house. How many miles did Matt walk in all after school?

Copyright © 2016 by The University of Kansas

ADDING AND SUBTRACTING FRACTIONS WITH UNLIKE DENOMINATORS

INSTRUCTIONAL ACTIVITY

Lesson 4

LEARNING GOAL

Students will build on their knowledge of adding and subtracting fractions with unlike denominators in order to add and subtract mixed numbers with unlike denominators using models and the standard algorithm.

PRIMARY ACTIVITY

Students will use models, number lines, and the standard algorithm to add and subtract mixed numbers with unlike denominators. Students will need to regroup mixed numbers in order to solve some expressions.

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

- Equivalent
- Multiple
- Sum
- Addend
- Numerator
- Denominator
- Difference
- Mixed number
- Improper fraction
- Regroup

MATERIALS

- Fraction circles (from LESSON 1) or fraction tiles
- INSTRUCTIONAL ACTIVITY STUDENT HANDOUT

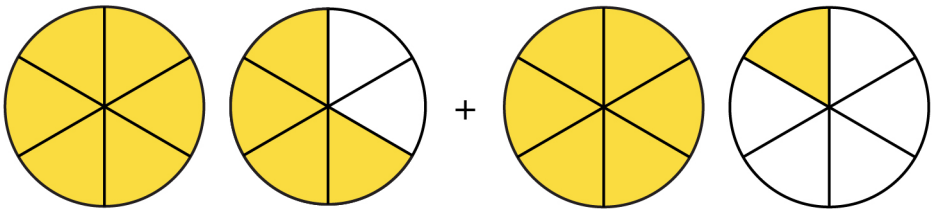
Copyright © 2016 by The University of Kansas

IMPLEMENTATION

Begin the lesson by displaying the expression, fraction circle model or fraction bar, *and* the number line required to add or subtract the expressions. Use the Guiding Questions to get students thinking. Following are a few examples, but more are available in the previous lessons.

Model $1\frac{4}{6} + 1\frac{1}{6}$ on the board using fraction circles (or fraction tiles), a number line, and the numeric expression.

Ask students, "What can you tell me about mixed numbers? What can you tell me about adding mixed numbers?"



Ask students:

- "What information would be necessary to simplify this expression?"
- "What is happening in the model? What is happening on the number line?"
- "What information can you gather about the expression from the model? From the number line?"
- "What is the simplified form of the expression?"

Require students to consider subtraction using the same fractions, $1\frac{4}{6} - 1\frac{1}{6}$.

Ask students:

- "What do you know about subtracting fractions that would help you subtract these mixed numbers?"
- "How would you determine the difference?"

Require students to show the difference using a model or number line." ($1\frac{4}{6} - 1\frac{1}{6} = \frac{3}{6}$ or $\frac{1}{2}$)

Model $2\frac{3}{5} + 1\frac{3}{10}$ on the board using fraction circles (or fraction tiles), a number line, and the numeric expression.

Require students to simplify the expression using a model or a number line.

Then, **ask** students to consider the expression $2\frac{3}{5} - 1\frac{3}{10}$ and simplify using models or a number line.

Copyright © 2016 by The University of Kansas

Model the expression $3\frac{2}{3}+1\frac{4}{9}$ using only models or a number line. **Require** students to identify and write the expression and the sum. ($3\frac{2}{3}+1\frac{4}{9}=4\frac{10}{9}$, which is $4+1\frac{1}{9}=5\frac{1}{9}$)

Ask students:

- "How did you think about solving the problem?"
- "How does this relate to adding and subtracting fractions? How is this different from adding and subtracting fractions?"
- "What do you notice about the sum?"
- "How would I regroup the improper fraction so that I have a proper fraction? Once you have regrouped the mixed number, what is the sum?" [Show students the relationship between the model and the mixed number with the improper fraction.]

Emphasize the relationship between addition of unit fractions to make a fraction and regrouping an improper fraction into a mixed number. (You need nine one-ninths to create one whole, therefore the remaining unit fraction is represented as the proper fraction portion of the mixed number.)

Proceed to model four more addition and subtraction problems with circle fractions, fraction bars, or a number line, and require students to write the expression and the solution using symbols.

Require students to complete Questions 1-4 on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT with a partner. Be sure to read each question aloud and have a partner group share the answer each time. **Ask** students after each partner group shares, "Why do you think that? How did you reach that solution?" **Ask** other students, "Do you agree or disagree? Why or why not? Does anyone have the same answer but a different way to explain it?"

- 1. Mason was driving from Kansas City to Dallas. On the first day, he drove for $4\frac{2}{3}$ hours, stopped to eat and get gas, and then drove for $5\frac{3}{4}$ hours. How many hours did Mason drive on the first day of his trip? ($4\frac{2}{3}+5\frac{3}{4}=9\frac{17}{12}=10\frac{5}{12}$ hours)
- 2. Olivia was baking bread using her grandmother's recipe. She needed $3\frac{3}{4}$ cups of flour total. The recipe said to mix $2\frac{2}{3}$ cups of flour with the other dry ingredients and put the rest of the flour aside to mix in later. How many cups of flour will Olivia put aside to mix in later? ($3\frac{3}{4}-2\frac{2}{3}=1\frac{1}{12}$ cups)
- 3. On Saturday, Emily played at the park with her friend Harper for $4\frac{1}{2}$ hours. On Sunday, Emily played at the same park with Ethan for $3\frac{3}{4}$ hours. How many hours did Emily spend at the park over the weekend? ($4\frac{1}{2}+3\frac{3}{4}=7\frac{5}{4}=8\frac{1}{4}$ hours)
- 4. Michael gets $6\frac{5}{6}$ hours of computer/TV time a week. By Wednesday, Michael has already used $3\frac{2}{3}$ hours. How many more hours of computer/TV time does Michael have left this week? ($6\frac{5}{6}-3\frac{2}{3}=3\frac{1}{6}$ hours)

Copyright © 2016 by The University of Kansas

GUIDING QUESTIONS

Determine if the student can ADD MIXED NUMBERS WITH UNLIKE DENOMINATORS:

- How is adding mixed numbers with unlike denominators similar to adding fractions with unlike denominators? How is it different?
- Do you see a relationship between adding fractions and adding mixed numbers?
- Have we ever solved a problem like this before?

Determine if the student can EXPLAIN MIXED NUMBERS:

- Can you create a model of that mixed number?
- What does the whole number represent?
- What do the fractional parts represent?

Determine if the student can SOLVE WORD PROBLEMS INVOLVING ADDITION AND SUBTRACTION OF FRACTIONS WITH MIXED NUMBERS:

- When is addition appropriate in a word problem?
- What does it mean to add two mixed numbers?
- When is subtraction appropriate in a word problem?
- What does it mean to subtract two mixed numbers?
- Is the question asking you to add or subtract mixed numbers? How do you know?
- How would you find the total number of hours Mason drove on the first day? Explain.
- What is the total amount of flour Olivia needs? How would you find the amount of flour Oliva needs to put aside? Explain.

Continue the lesson by modeling $3\frac{1}{5} - 1\frac{8}{10}$ using fraction circles, fraction bars, or a number line.

Copyright © 2016 by The University of Kansas

Ask students, "What can you tell me about adding and subtracting fractions?" How are adding and subtracting numbers different?" (Use addition to add together, and use subtraction to take away.)

Remind students that number order matters in subtraction. **Explain** to students, "When we model subtraction with positive values, we do not add on to the circle model since we are taking pieces away. Therefore, the way we model subtraction is different from the way we model addition. With a number line, instead of moving away from zero, we start at the first mixed number or fraction and move towards zero, taking pieces away from our starting point."

Ask students, "You need to find the difference of the two mixed numbers. Explain the steps for subtracting with unlike denominators. What should you do first?" (Students should indicate they first need to find a common denominator.)

Require students to find the common denominator.

Ask students, "What is the next step now that we know what the common denominator is?" (Students should suggest creating equivalent fractions by multiplying by a fraction equivalent to one whole.)

Require students to create the equivalent fraction for $3\frac{2}{10}$.

Emphasize that the whole number did not change—only the fractional parts were rewritten.

Ask students:

- "What is the next step?" (Students should suggest subtracting mixed numbers.)
- "Can you subtract $\frac{8}{10}$ from $\frac{2}{10}$ and get a positive result?" (no)
- "Have you ever solved a subtraction problem where you could not subtract two numbers and get a positive result? How does this relate to that problem?"
- "What can you tell me about regrouping?"
- "Explain how you would regroup an improper fraction into a mixed number."
- "What would happen if I needed to regroup a whole number to make an improper fraction? How would you do that? Why would you do that?"

Model $3\frac{2}{10}$ for students using fraction circles or a number line. **Ask** students, "Using what I know about unit fractions, how many tenth-size pieces do I have?" [Count out all of the pieces in the whole numbers to show that one whole is 10 equal pieces.] (Students should indicate that you have 32 equal pieces.)

Emphasize that each whole has 10 equal pieces. Therefore, there are three wholes, and two $\frac{1}{10}$ pieces from a fourth whole.

Remind students that when regrouping with whole numbers, they "borrow" from the next largest place value; with a mixed number, the next largest place value is the ones place. Therefore, it is necessary to "borrow" one whole from the ones place in order to regroup and subtract.

Copyright © 2016 by The University of Kansas

Ask students, "How many equal fraction pieces are there in one whole?" (There are 10 equal pieces.)

Remind students that when they regroup with whole numbers, they add what is "borrowed" to the existing number; when regrouping with fractions, the logic is the same. In this example, students should add one whole to the proper fraction, creating an improper fraction that will be greater than the value of the fraction that is to be subtracted.

Use the model to show taking one whole and grouping it together with the fraction. **Move** the fraction circles so that the two wholes are slightly separated from the one whole and $\frac{2}{10}$, then count how many pieces there are with the one whole and $\frac{2}{10}$.

Now the model can be perceived to contain two wholes and 12 equal-size tenth pieces ($3\frac{2}{10} = \frac{10}{10} + \frac{10}{10} + \frac{10}{10} + \frac{2}{10}$). [Write the mixed number using symbols next to the model.] **Explain** that you are regrouping one of the $\frac{10}{10}$ with the $\frac{2}{10}$ to make $2\frac{12}{10}$.

Emphasize that, as with equivalent fractions, there are still the same number of pieces—they are just represented with different numbers. Therefore, the value is the same ($3\frac{2}{10} = 2\frac{12}{10}$).

Ask students, "Now can I subtract eight tenths from 12 tenths and get a positive result?" (Students should indicate that this is now possible.)

Emphasize again how the model conveys $3\frac{2}{10}$ and $2\frac{12}{10}$ are equivalent mixed numbers and why rewriting the mixed number is useful when subtracting (one whole has been regrouped to make an improper fraction in order to subtract).

Require students to simplify the expression (the model should be on the board).

Require students to model and solve Questions 5-8 on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT as a class, proceeding through each problem as you did for $3\frac{1}{5} - 1\frac{8}{10}$. Read each question aloud before requiring students to solve.

- 5. Jasmine was walking in a 7-mile race. After $4\frac{1}{2}$ miles, Jasmine stopped to get a drink and use the restroom. How much farther does Jasmine have left to walk? ($7 - 4\frac{1}{2} = 2\frac{1}{2}$ miles)
- 6. Jose was building a raised garden bed in his backyard. He bought 10 boards that were $5\frac{3}{16}$ feet long, but he realized they were too long. Jose only needed boards that were $2\frac{5}{8}$ feet long. How much does Jose need to cut off each board? ($5\frac{3}{16} - 2\frac{5}{8} = 2\frac{9}{16}$ feet)
- 7. James and Shantel went to see a play at the local theater. The play was $2\frac{1}{2}$ hours long with one intermission (break). After $1\frac{2}{3}$, hours the curtain closed for the intermission. How many hours were remaining in the play after the intermission? ($2\frac{1}{2} - 1\frac{2}{3} = \frac{5}{6}$ hour)

Copyright © 2016 by The University of Kansas

- 8. Sam was preparing the punch for the end of the school year party. To make sure that there was enough punch for everyone, Sam needed a total of $4\frac{1}{3}$ gallons. The punch was a combination of juice and sparking soda. Sam added $1\frac{3}{4}$ gallons of juice. How many gallons of sparkling soda does Sam need to add to the punch? ($4\frac{1}{3} - 1\frac{3}{4} = 2\frac{7}{12}$ gallons)

Require students to model and simplify Questions 9-12 on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT with a partner or independently. This will give you an opportunity to move around the room or work with a small group and support students using the guiding questions.

- 9. Mr. Rodriguez, the science teacher, needed $4\frac{3}{8}$ ounces of rubbing alcohol for an experiment testing liquid density. He had $3\frac{5}{6}$ ounces left over from a previous experiment. How many more ounces of rubbing alcohol does Mr. Rodriguez need? ($4\frac{3}{8} - 3\frac{5}{6} = \frac{13}{24}$ ounce)
- 10. Sienna and Joel went for a 6-mile bike ride. Sienna wanted to stop about halfway for a lunch and water break. After lunch, Sienna and Joel still had to bike $3\frac{5}{7}$ miles. How many miles did they bike before they stopped for lunch? ($6 - 3\frac{5}{7} = 2\frac{2}{7}$ miles)
- 11. Jackson solved this problem in math class: $2\frac{6}{9} + \frac{3}{4} = 2\frac{15}{36}$. He simplified the solution as $2\frac{5}{12}$. Is Jackson correct? Why or why not? What is the correct answer? (No, Jackson is not correct. He did not regroup the one whole from $2\frac{51}{36}$. The correct answer is $3\frac{15}{36} = 3\frac{5}{12}$.)
- 12. The Smith family is traveling west on the Oregon Trail with their wagon and one horse. On Tuesday, after the Smith family walks for $1\frac{7}{9}$ miles, a wagon wheel breaks. The Smith family wants to walk $5\frac{1}{5}$ miles each day. How much farther will the Smith family have to walk after the wheel is fixed to meet their goal on Tuesday? ($5\frac{1}{5} - 1\frac{7}{9} = 3\frac{19}{45}$ miles)

Copyright © 2016 by The University of Kansas

GUIDING QUESTIONS

Elicit student thinking:

- What can you tell me about regrouping?
- Why is regrouping necessary?
- Can you give me an example of a time when you have had to regroup before?

Determine if the student can DECOMPOSE A FRACTION INTO A SUM OF UNIT FRACTIONS WITH THE SAME DENOMINATOR:

- What can you tell me about this mixed number?
- How would I determine how many unit fractions are in this mixed number?
- How would decomposing a fraction or mixed number help you group that mixed number?
- What would a model of this mixed number look like as a sum of unit fractions?

Determine if the student can SUBTRACT MIXED NUMBERS WITH UNLIKE DENOMINATORS:

- How is subtracting mixed numbers with unlike denominators similar to subtracting fractions with unlike denominators? How is it different?
- Do you see a relationship between subtracting fractions and subtracting mixed numbers?
- Have we ever solved a problem like this before? How does subtracting mixed numbers relate to subtracting fractions?

Copyright © 2016 by The University of Kansas

GUIDING QUESTIONS

Determine if the student can SOLVE WORD PROBLEMS INVOLVING ADDITION AND SUBTRACTION OF FRACTIONS WITH MIXED NUMBERS:

- How do you know if the word problem is asking you to add or subtract?
- How are adding and subtracting mixed number word problems the same as adding and subtracting fraction word problems, or whole number word problems? How are they different? Explain.
- Have we ever solved a problem like this before? How does adding and subtracting mixed numbers relate to adding and subtracting fractions?

At the end of the activity, assess student learning by reviewing their work on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

Copyright © 2016 by The University of Kansas

Name: _____

Adding and Subtracting Fractions with Unlike Denominators

Lesson 4

$1\frac{4}{6} + 1\frac{1}{6}$

$1\frac{4}{6} - 1\frac{1}{6}$

$2\frac{3}{5} + 1\frac{3}{10}$

$2\frac{3}{5} - 1\frac{3}{10}$

$3\frac{2}{3} + 1\frac{1}{9}$

Copyright © 2016 by The University of Kansas

1. Mason was driving from Kansas City to Dallas. On the first day, he drove for $4\frac{2}{3}$ hours, stopped to eat and get gas, and then drove for $5\frac{3}{4}$ hours. How many hours did Mason drive on the first day of his trip?

2. Olivia was baking bread using her grandmother's recipe. She needed $3\frac{3}{4}$ cups of flour total. The recipe said to mix $2\frac{2}{3}$ cups of flour with the other dry ingredients and put the rest of the flour aside to mix in later. How many cups of flour will Olivia put aside to mix in later?

3. On Saturday, Emily played at the park with her friend Harper for $4\frac{1}{2}$ hours. On Sunday, Emily played at the same park with Ethan for $3\frac{3}{4}$ hours. How many hours did Emily spend at the park over the weekend?

Copyright © 2016 by The University of Kansas

4. Michael gets $6\frac{5}{6}$ hours of computer/TV time a week. By Wednesday, Michael has already used $3\frac{2}{3}$ hours. How many more hours of computer/TV time does Michael have left this week?

5. Jasmine was walking in a 7-mile race. After $4\frac{1}{2}$ miles, Jasmine stopped to get a drink and use the restroom. How much farther does Jasmine have left to walk?

6. Jose was building a raised garden bed in his backyard. He bought 10 boards that were $5\frac{3}{16}$ feet long, but he realized they were too long. Jose only needed boards that were $2\frac{5}{8}$ feet long. How much does Jose need to cut off each board?

Copyright © 2016 by The University of Kansas

7. James and Shantel went to see a play at the local theater. The play was $2\frac{1}{2}$ hours long with one intermission (break). After $1\frac{2}{3}$, hours the curtain closed for the intermission. How many hours were remaining in the play after the intermission?

8. Sam was preparing the punch for the end of the school year party. To make sure that there was enough punch for everyone, Sam needed a total of $4\frac{1}{3}$ gallons. The punch was a combination of juice and sparking soda. Sam added $1\frac{3}{4}$ gallons of juice. How many gallons of sparkling soda does Sam need to add to the punch?

9. Mr. Rodriguez, the science teacher, needed $4\frac{3}{8}$ ounces of rubbing alcohol for an experiment testing liquid density. He had $3\frac{5}{6}$ ounces left over from a previous experiment. How many more ounces of rubbing alcohol does Mr. Rodriguez need?

Copyright © 2016 by The University of Kansas

10. Sienna and Joel went for a 6-mile bike ride. Sienna wanted to stop about halfway for a lunch and water break. After lunch, Sienna and Joel still had to bike $3\frac{5}{7}$ miles. How many miles did they bike before they stopped for lunch?

11. Jackson solved this problem in math class: $2\frac{6}{9} + \frac{3}{4} = 2\frac{15}{36}$. He simplified the solution as $2\frac{5}{12}$. Is Jackson correct? Why or why not? What is the correct answer?

12. The Smith family is traveling west on the Oregon Trail with their wagon and one horse. On Tuesday, after the Smith family walks for $1\frac{7}{9}$ miles, a wagon wheel breaks. The Smith family wants to walk $5\frac{1}{5}$ miles each day. How much farther will the Smith family have to walk after the wheel is fixed to meet their goal on Tuesday?

Copyright © 2016 by The University of Kansas

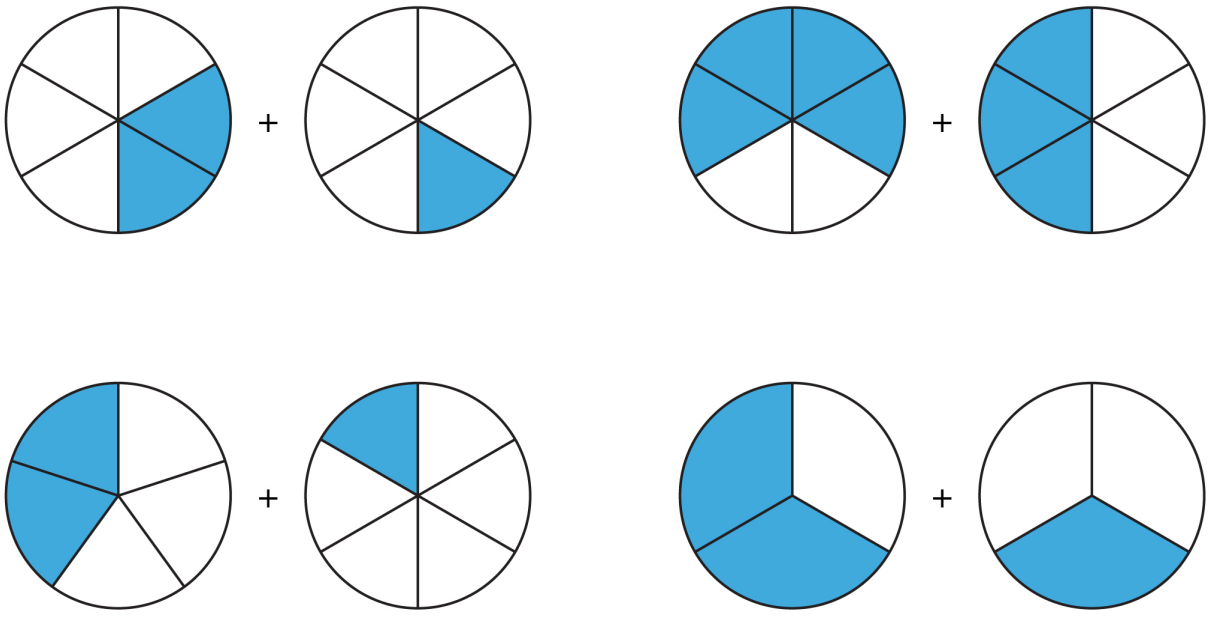
Name: _____

Adding and Subtracting Fractions with Unlike Denominators

Lessons 1 – 4

1. Explain the steps, in order, necessary to add or subtract two fractions with unlike denominators. Create an example to support your response.

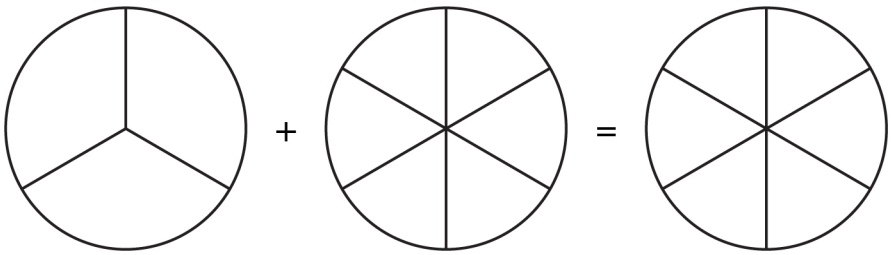
2. Circle the model that best represents the expression $\frac{2}{3} + \frac{1}{2}$.



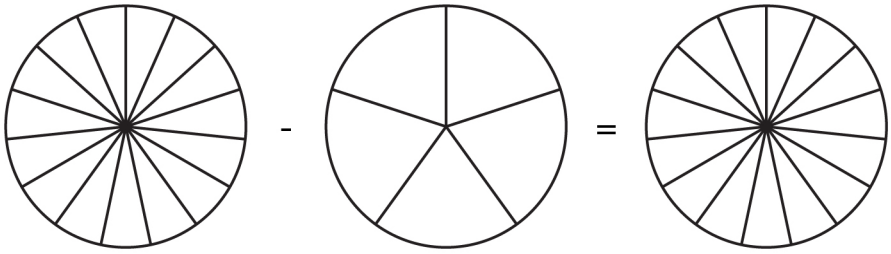
Copyright © 2016 by The University of Kansas

3. Evaluate the following expressions using the models provided.

3.a. $\frac{1}{3}+\frac{2}{6}$

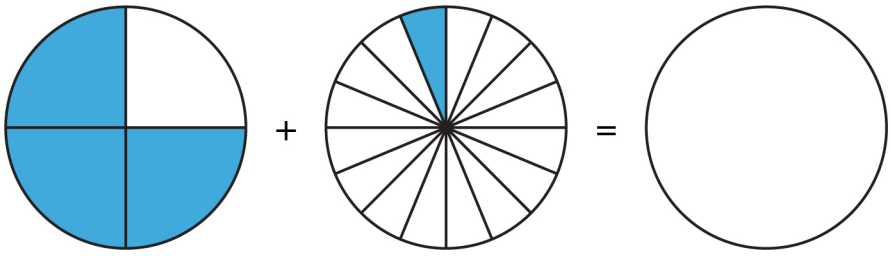


3.b. $\frac{10}{15}-\frac{3}{5}$



4. Evaluate. Write the expression modeled, then write the equation with common denominators symbolically. Be sure to include the sum or difference.

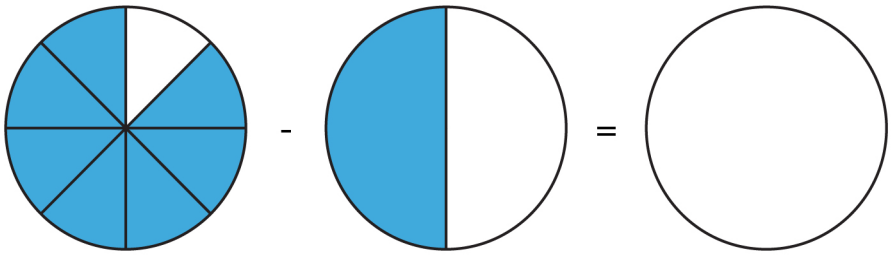
4.a.



Copyright © 2016 by The University of Kansas

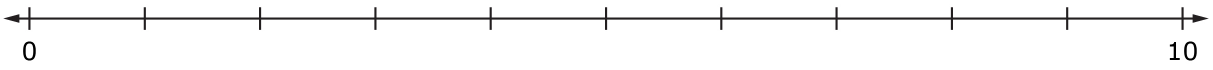
Name: _____

4.b.

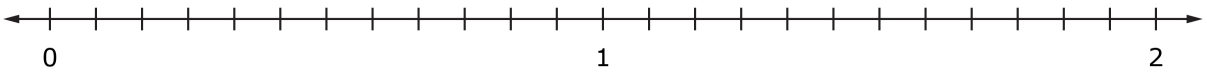


5. Evaluate the following expressions using a number line.

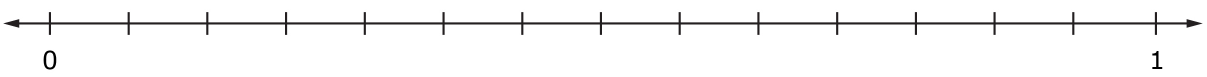
5.a. $\frac{1}{2}+\frac{4}{5}$



5.b. $\frac{5}{6}+\frac{3}{4}$

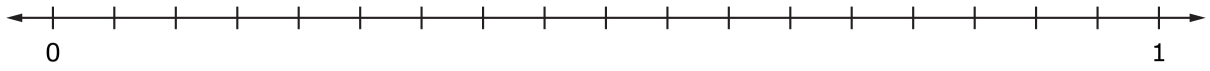


5.c. $\frac{5}{7}-\frac{1}{2}$

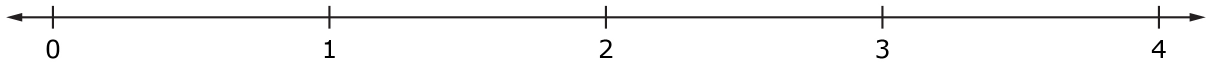


Copyright © 2016 by The University of Kansas

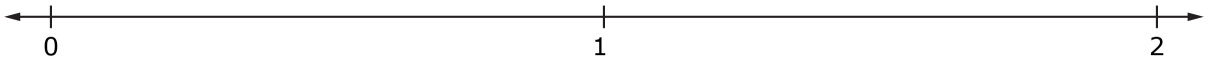
5.d. $\frac{4}{6} - \frac{5}{9}$



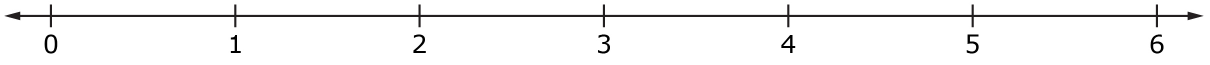
5.e. $2\frac{1}{4} + 1\frac{1}{2}$



5.f. $1\frac{1}{6} - \frac{3}{4}$



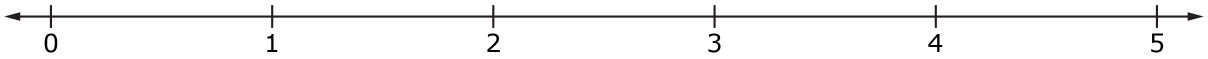
5.g. $3\frac{1}{2} + 2\frac{2}{3}$



Copyright © 2016 by The University of Kansas

Name: _____

5.h. $4\frac{2}{3} - 1\frac{1}{4}$



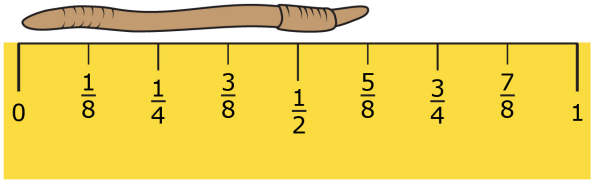
6. Mark and Sally each made small pizzas. Mark ate $\frac{2}{5}$ of his pizza, and Sally ate $\frac{1}{3}$ of her pizza.

6.a. How much more of his small pizza did Mark eat than Sally? Show your work to support your answer.

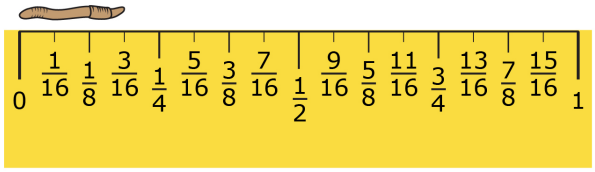
6.b. How would you determine how much pizza Mark and Sally ate altogether? How much pizza did Mark and Sally eat altogether?

Copyright © 2016 by The University of Kansas

7. Use the worms and rulers to answer the following questions.



Worm A



Worm B

Worm C

7.a. Explain the relationship between the divisions on a ruler and the denominator of a fraction. How would you determine a common denominator to compare the two worms?

7.b. How much longer is Worm A than Worm B? Use symbols to support your answer.

7.c. Worm C is the same length as Worm A and Worm B combined. How long is Worm C? Use models or symbols to support your answer.

8. Megan made a mistake when she simplified this equation: $\frac{9}{10} - \frac{2}{5} = \frac{7}{5}$

8.a. What was Megan's mistake?

Copyright © 2016 by The University of Kansas

Name: _____

8.b. What is the correct solution to the expression $\frac{9}{10} - \frac{2}{5}$? Use models or symbols to defend your answer.

9. James wants to put a new bookcase in his room. His wall is $3\frac{2}{6}$ feet long, and the bookcase is $2\frac{3}{4}$ feet long. Will James have enough wall space left over to add a chair that is $1\frac{1}{2}$ feet long next to the bookcase? Support your answer with models or symbols.

10. Use this list of ingredients for cookies made during the American Revolution to answer the following questions:

$\frac{1}{2}$ cup molasses

$\frac{1}{4}$ cup brown sugar

$\frac{1}{4}$ cup lard

$1\frac{1}{2}$ cup flour

$\frac{1}{2}$ teaspoon cinnamon

$\frac{1}{4}$ teaspoon baking soda

$\frac{1}{4}$ teaspoon ginger

10.a. How many combined cups of molasses and brown sugar are needed to make the cookies?

Copyright © 2016 by The University of Kansas

10.b. How many more cups of flour than lard are needed to make the cookies?

10.c. After the molasses, brown sugar, lard, and flour are mixed together, how many cups of dough will be in the bowl?

11. Harry walked $1\frac{3}{8}$ miles to the store. On his way home, Harry took a different route through the park. The route home was $\frac{2}{3}$ mile longer than his trip to the store.

11.a. How many miles did Harry walk on his way home? Use models or symbols to support your answer.

11.b. How many miles did Harry walk altogether? Use models or symbols to support your answer.

Copyright © 2016 by The University of Kansas

Name: _____

Adding and Subtracting Fractions with Unlike Denominators

Lessons 1 – 4

1. Explain the steps, in order, necessary to add or subtract two fractions with unlike denominators. Create an example to support your response.

CORRECT ANSWER

First, determine a value that will serve as the common denominator. Then, create equivalent fractions by multiplying one or both fractions by a form of one whole so that the value of the fractions does not change but the fractions now have common denominators. Next, simplify the expression either by adding or subtracting the numerators only. Because the size of the fractional parts does not change, the denominator remains the same. Simplify the solution to lowest terms (as an optional but not necessary step).

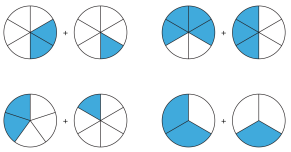
(Check student examples for accuracy.)

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

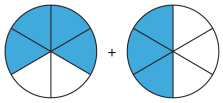
Example Error	Misconception	Missing Knowledge
The student does not describe determining a common denominator and does not determine a common denominator in the example they provide.	does not understand how to determine a common denominator or why it is important to establish a common denominator before adding or subtraction fractions with unlike denominators	CALCULATE *COMMON DENOMINATOR* and EXPLAIN *COMMON DENOMINATOR*
The student describes adding or subtracting the numerators before mentioning a common denominator. In their example, the student uses the first denominator, the last denominator, or the sum/difference of the denominators.	ignores the denominators when solving, only adding or subtracting the numerators	CALCULATE *COMMON DENOMINATOR* and EXPLAIN *COMMON DENOMINATOR*
The student calculates a common denominator but does not change the numerators.	understands the need to calculate a common denominator but does not create equivalent fractions; uses existing numerators	REPRESENT EQUIVALENT FRACTIONS BY MULTIPLYING OR DIVIDING

Copyright © 2016 by The University of Kansas

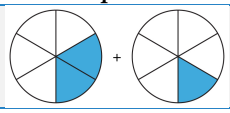
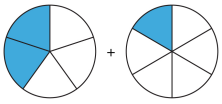
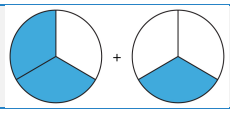
2. Circle the model that best represents the expression $\frac{2}{3} + \frac{1}{2}$.



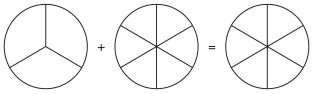
CORRECT ANSWER



ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
	determines the common denominator to be 6 but does not create equivalent fractions; uses existing numerators	REPRESENT EQUIVALENT FRACTIONS BY MULTIPLYING OR DIVIDING
	does not use equivalent fractions to create a common denominator; adds the two denominators to create a common denominator; does not create equivalent fractions; uses existing numerators	CALCULATE *COMMON DENOMINATOR* and/or REPRESENT EQUIVALENT FRACTIONS BY MULTIPLYING OR DIVIDING
	uses the first denominator and ignores the second denominator	CALCULATE *COMMON DENOMINATOR*

3. Evaluate the following expressions using the models provided.

3.a. $\frac{1}{3} + \frac{2}{6}$ 

CORRECT ANSWER

Copyright © 2016 by The University of Kansas

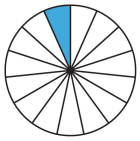
Name: _____

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student only shades in $\frac{3}{6}$ as the sum.	only adds the numerators and ignores the denominators	CALCULATE *COMMON DENOMINATOR* and/or REPRESENT EQUIVALENT FRACTIONS BY MULTIPLYING OR DIVIDING
The student creates ninths in the sum circle and shades in $\frac{3}{9}$.	adds the numerators and denominators without calculating a common denominator	CALCULATE *COMMON DENOMINATOR* or ADD FRACTIONS WITH UNLIKE DENOMINATORS

3.b. $\frac{10}{15} - \frac{3}{5}$ 

CORRECT ANSWER



ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

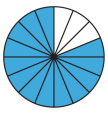
Example Error	Misconception	Missing Knowledge
The student shades in $\frac{7}{15}$.	subtracts the numerators and ignores the denominators	CALCULATE *COMMON DENOMINATOR*, REPRESENT EQUIVALENT FRACTIONS BY MULTIPLYING OR DIVIDING, and SUBTRACT FRACTIONS WITH UNLIKE DENOMINATORS

4. Evaluate. Write the expression modeled, then write the equation with common denominators symbolically. Be sure to include the sum or difference.

4.a. 

Copyright © 2016 by The University of Kansas

CORRECT ANSWER



$\frac{3}{4} + \frac{1}{16}$

$\frac{12}{16} + \frac{1}{16} = \frac{13}{16}$

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student creates 20 equal parts and shades in 4 parts, then writes $\frac{3}{4} + \frac{1}{16} = \frac{4}{20}$.	adds the numerators and denominators without calculating a common denominator	CALCULATE *COMMON DENOMINATOR* and ADD FRACTIONS WITH UNLIKE DENOMINATORS
The student creates 16 equal parts and shades in 4 parts, then writes $\frac{3}{4} + \frac{1}{16} = \frac{4}{16}$.	does not create an equivalent fraction to ensure the fractions have a common denominator when adding; adds the numerators and uses the larger of the existing denominators	CALCULATE *COMMON DENOMINATOR* and ADD FRACTIONS WITH UNLIKE DENOMINATORS
The student creates 4 equal parts and shades in all 4 parts, then writes $\frac{3}{4} + \frac{1}{16} = \frac{4}{4}$.	does not understand that fractions are numbers, and that if the numerator or denominator of the fraction changes disproportionately to the other, the value changes; adds the numerators and uses the smaller of the existing denominators	CALCULATE *COMMON DENOMINATOR* and ADD FRACTIONS WITH UNLIKE DENOMINATORS
The student creates 16 equal parts and shades in 4 parts, then writes $\frac{3}{16} + \frac{1}{16} = \frac{4}{16}$.	does not understand that fractions are numbers, and that if the numerator or denominator of the fraction changes disproportionately to the other, the value changes; multiplies the denominator of the first addend by four but does not multiply the numerator of the first addend by four	REPRESENT EQUIVALENT FRACTIONS BY MULTIPLYING OR DIVIDING and ADD FRACTIONS WITH UNLIKE DENOMINATORS

4.b. 

Copyright © 2016 by The University of Kansas

Name: _____

CORRECT ANSWER

$\frac{7}{8} - \frac{1}{2}$

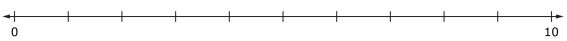
$\frac{7}{8} - \frac{4}{8} = \frac{3}{8}$

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student creates 6 equal parts and shades in all 6 parts, then writes $\frac{7}{8} - \frac{1}{2} = \frac{6}{6}$.	subtracts the numerators and denominators without calculating a common denominator	CALCULATE *COMMON DENOMINATOR* and SUBTRACT FRACTIONS WITH UNLIKE DENOMINATORS
The student creates 6 equal parts and shades in 2 parts. Or the student just shades in the whole circle, then writes $\frac{7}{8} - \frac{1}{2} = \frac{6}{2}$.	does not understand that fractions are numbers, and that if the numerator or denominator of the fraction changes disproportionately to the other, the value changes; subtracts the numerators and either subtracts the denominators or uses the smaller of the existing denominators	CALCULATE *COMMON DENOMINATOR* and SUBTRACT FRACTIONS WITH UNLIKE DENOMINATORS
The student creates 8 equal parts and shades in 6 parts, then writes $\frac{7}{8} - \frac{1}{2} = \frac{6}{8}$.	does not understand that fractions are numbers, and that if the numerator or denominator of the fraction changes disproportionately to the other, the value changes; subtracts the numerators and uses the larger of the existing denominators	CALCULATE *COMMON DENOMINATOR* and SUBTRACT FRACTIONS WITH UNLIKE DENOMINATORS
The student creates 8 equal parts and shades in 6 parts, then writes $\frac{7}{8} - \frac{1}{8} = \frac{6}{8}$.	does not understand that fractions are numbers, and that if the numerator or denominator of the fraction changes disproportionately to the other, the value changes; multiplies the denominator of the subtrahend by four but does not multiply the numerator of the subtrahend by four	REPRESENT EQUIVALENT FRACTIONS BY MULTIPLYING OR DIVIDING and SUBTRACT FRACTIONS WITH UNLIKE DENOMINATORS

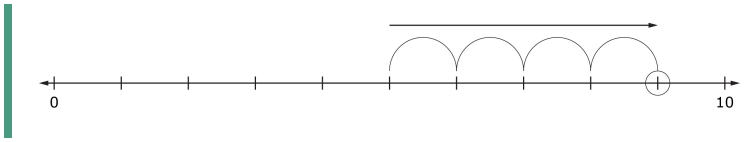
5. Evaluate the following expressions using a number line.

5.a. $\frac{1}{2} + \frac{4}{5}$



Copyright © 2016 by The University of Kansas

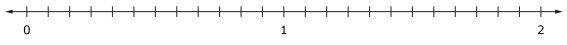
CORRECT ANSWER



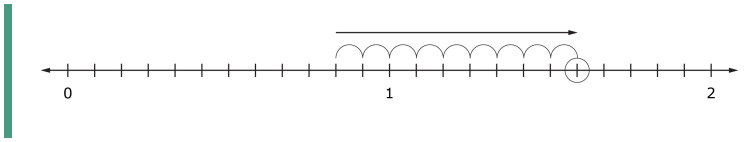
ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student does not create equivalent fractions by multiplying by one whole to create fractions with common denominators. The student starts at 0 and moves the sum of the numerators, 5, modeling $\frac{1}{10}+\frac{4}{10}=\frac{5}{10}$.	does not understand that fractions are numbers, and that if the numerator or denominator of the fraction changes disproportionately to the other, the value changes	REPRESENT EQUIVALENT FRACTIONS BY MULTIPLYING OR DIVIDING and ADD FRACTIONS WITH UNLIKE DENOMINATORS

5.b. $\frac{5}{6}+\frac{3}{4}$



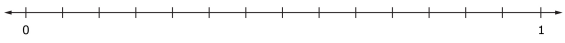
CORRECT ANSWER



ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student does not create equivalent fractions by multiplying by one whole when creating fractions with common denominators. The student starts at 0 and only moves the sum of the numerators, 8, modeling $\frac{5}{6}+\frac{3}{4}=\frac{8}{12}$.	does not understand that fractions are numbers, and that if the numerator or denominator of the fraction changes disproportionately to the other, the value changes	REPRESENT EQUIVALENT FRACTIONS BY MULTIPLYING OR DIVIDING and ADD FRACTIONS WITH UNLIKE DENOMINATORS

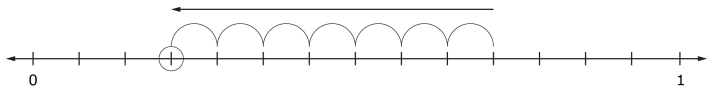
5.c. $\frac{5}{7}-\frac{1}{2}$



Copyright © 2016 by The University of Kansas

Name: _____

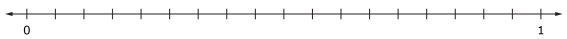
CORRECT ANSWER



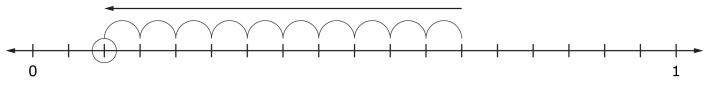
ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student does not create equivalent fractions by multiplying by one whole when creating fractions with common denominators. The student starts at $\frac{5}{14}$ and then subtracts $\frac{1}{14}$ to stop at $\frac{4}{14}$.	does not understand that fractions are numbers, and that if the numerator or denominator of the fraction changes disproportionately to the other, the value changes	REPRESENT EQUIVALENT FRACTIONS BY MULTIPLYING OR DIVIDING and SUBTRACT FRACTIONS WITH UNLIKE DENOMINATORS
The student starts at $\frac{5}{14}$ and then adds $\frac{1}{14}$ to stop at $\frac{6}{14}$.	misinterprets the subtraction expression for an addition expression; does not understand that fractions are a number, and that if the numerator or denominator of the fraction changes disproportionately to the other, the value changes	SUBTRACT FRACTIONS WITH UNLIKE DENOMINATORS and REPRESENT EQUIVALENT FRACTIONS BY MULTIPLYING OR DIVIDING

5.d. $\frac{4}{6} - \frac{5}{9}$



CORRECT ANSWER

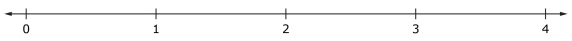


Copyright © 2016 by The University of Kansas

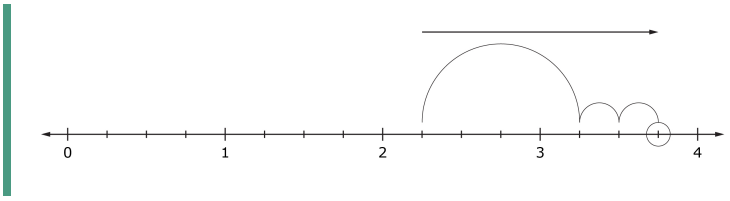
ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student does not create equivalent fractions by multiplying by one whole when creating fractions with common denominators. The student switches the numerators in order to subtract. The student starts at $\frac{5}{18}$ and then subtracts $\frac{4}{18}$ to stop at $\frac{1}{18}$.	does not understand that fractions are numbers, and that if the numerator or denominator of the fraction changes disproportionately to the other, the value changes	REPRESENT EQUIVALENT FRACTIONS BY MULTIPLYING OR DIVIDING and SUBTRACT FRACTIONS WITH UNLIKE DENOMINATORS
The student starts at $\frac{4}{18}$ and then adds $\frac{5}{18}$ to stop at $\frac{9}{18}$.	misinterprets the subtraction expression for an addition expression; does not understand that fractions are a number, and that if the numerator or denominator of the fraction changes disproportionately to the other, the value changes	SUBTRACT FRACTIONS WITH UNLIKE DENOMINATORS and REPRESENT EQUIVALENT FRACTIONS BY MULTIPLYING OR DIVIDING

5.e. $2\frac{1}{4} + 1\frac{1}{2}$



CORRECT ANSWER



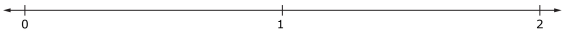
Copyright © 2016 by The University of Kansas

Name: _____

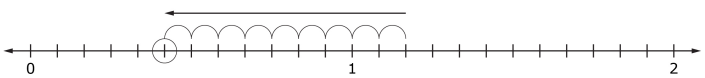
ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student divides the number line into sixths and adds the numerators and denominators, modeling $2\frac{1}{4} + 1\frac{1}{2} = 3\frac{2}{6}$.	adds the numerators and denominators without calculating a common denominator	CALCULATE *COMMON DENOMINATOR* and ADD MIXED NUMBERS WITH UNLIKE DENOMINATORS
The student divides the number line into halves using the smaller denominator, adds the numerators, and models $2\frac{1}{4} + 1\frac{1}{2} = 3\frac{2}{2}$.	ignores the denominators when adding, using the second addend's denominator as the denominator of the sum	CALCULATE *COMMON DENOMINATOR* and ADD MIXED NUMBERS WITH UNLIKE DENOMINATORS
The student divides the number line into fourths using the larger denominator, adds the numerators, and models $2\frac{1}{4} + 1\frac{1}{2} = 3\frac{2}{4}$.	ignores the denominators when solving, using the larger denominator as the denominator of the sum	CALCULATE *COMMON DENOMINATOR* and ADD MIXED NUMBERS WITH UNLIKE DENOMINATORS
The student divides the number line into fourths, does not properly create an equivalent fraction in the second addend, and models $2\frac{1}{4} + 1\frac{1}{4} = 3\frac{2}{4}$.	does not understand that fractions are numbers, and that if the numerator or denominator of the fraction changes disproportionately to the other, the value changes	REPRESENT EQUIVALENT FRACTIONS BY MULTIPLYING OR DIVIDING and ADD MIXED NUMBERS WITH UNLIKE DENOMINATORS
The student marks the fractions only between 0 and 1, ignores the whole numbers, and models $2\frac{1}{4} + 1\frac{2}{4} = \frac{3}{4}$.	does not recognize that a mixed number is a single value rather than two separate values (the whole number separate from the fraction)	ADD MIXED NUMBERS WITH LIKE DENOMINATORS
The student marks only the whole numbers and models $2\frac{1}{4} + 1\frac{2}{4} = 3$.	does not recognize that a mixed number is a single value, not two separate values (the whole number separate from the fraction)	ADD MIXED NUMBERS WITH LIKE DENOMINATORS

5.f. $1\frac{1}{6} - \frac{3}{4}$



CORRECT ANSWER

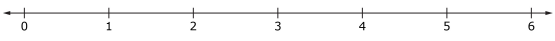


Copyright © 2016 by The University of Kansas

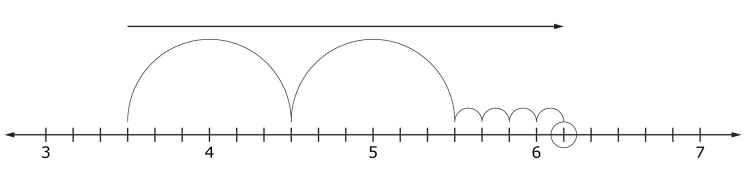
ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student divides the number line into sixths, then subtracts $1\frac{1}{6} - \frac{3}{6}$ and stops at $\frac{4}{6}$.	subtracts the numerators without calculating a common denominator; uses the larger denominator or the first denominator that appears in the expression	CALCULATE *COMMON DENOMINATOR* and SUBTRACT MIXED NUMBERS WITH UNLIKE DENOMINATORS
The student divides the number line into sixths, then subtracts $\frac{3}{6} - \frac{1}{6}$, ignoring the whole number, and stops at $\frac{2}{6}$.	does not recognize that a mixed number is a single value, not two separate values (the whole number separate from the fraction)	SUBTRACT MIXED NUMBERS WITH UNLIKE DENOMINATORS
The student divides the number line into fourths, then subtracts $1\frac{1}{4} - \frac{3}{4}$ and stops at $\frac{2}{4}$.	ignores the denominators when solving, using the second fraction's denominator, which is 4	CALCULATE *COMMON DENOMINATOR* and SUBTRACT MIXED NUMBERS WITH UNLIKE DENOMINATORS
The student divides the number line into twelfths and then subtracts $1\frac{1}{12} - \frac{3}{12}$ and stops at $\frac{10}{12}$.	does not understand that fractions are numbers, and that if the numerator or denominator of the fraction changes disproportionately to the other, the value changes	REPRESENT EQUIVALENT FRACTIONS BY MULTIPLYING OR DIVIDING and SUBTRACT MIXED NUMBERS WITH UNLIKE DENOMINATORS
The student divides the number line into twelfths and then subtracts $\frac{12}{12} - \frac{3}{12}$ and stops at $\frac{9}{12}$.	does not recognize that a mixed number is a single value, not two separate values (the whole number separate from the fraction); does not understand that the mixed number regrouped is equivalent to an improper fraction; does not create equivalent fractions by multiplying by one whole when creating fractions with common denominators	SUBTRACT MIXED NUMBERS WITH UNLIKE DENOMINATORS and REPRESENT EQUIVALENT FRACTIONS BY MULTIPLYING OR DIVIDING

5.g. $3\frac{1}{2} + 2\frac{2}{3}$



CORRECT ANSWER



Copyright © 2016 by The University of Kansas

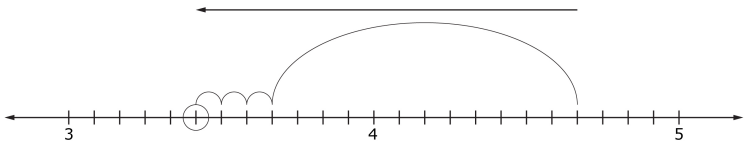
Name: _____

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student divides the number line into fifths and adds the numerators and denominators, stopping at $5\frac{3}{5}$.	adds the numerators and denominators without calculating a common denominator	CALCULATE *COMMON DENOMINATOR* and ADD MIXED NUMBERS WITH UNLIKE DENOMINATORS
The student divides the number line into halves and adds the numerators, ending at $6\frac{1}{2}$. OR The student divides the number line into thirds and adds the numerators, ending at 6.	ignores the denominators when solving; uses one of the existing denominators	CALCULATE *COMMON DENOMINATOR* and ADD MIXED NUMBERS WITH UNLIKE DENOMINATORS
The student divides the number line into sixths and adds $3\frac{1}{6} + 2\frac{2}{6}$, ending at $5\frac{3}{6}$.	does not understand that fractions are numbers, and that if the numerator or denominator of the fraction changes disproportionately to the other, the value changes	REPRESENT EQUIVALENT FRACTIONS BY MULTIPLYING OR DIVIDING and ADD MIXED NUMBERS WITH UNLIKE DENOMINATORS
The student adds only the fractional parts after creating equivalent fractions $\frac{3}{6} + \frac{4}{6}$, ending at $1\frac{1}{6}$.	doesn't recognize that a mixed number is a single value, not two separate values (the whole number separate from the fraction)	ADD MIXED NUMBERS WITH UNLIKE DENOMINATORS
The student adds only the whole numbers 3 + 2, ending at 5.	doesn't recognize that a mixed number is a single value, not two separate values (the whole number separate from the fraction)	ADD MIXED NUMBERS WITH UNLIKE DENOMINATORS

5.h. $4\frac{2}{3} - 1\frac{1}{4}$

CORRECT ANSWER



Copyright © 2016 by The University of Kansas

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student divides the number line into thirds and subtracts $4\frac{2}{3} - 1\frac{1}{3}$, ending at $3\frac{1}{3}$.	subtracts the numerators without calculating a common denominator; uses the denominator that appears first in the expression	CALCULATE *COMMON DENOMINATOR* and SUBTRACT MIXED NUMBERS WITH UNLIKE DENOMINATORS
The student divides the number line into fourths and subtracts $4\frac{2}{4} - 1\frac{1}{4}$, ending at $3\frac{1}{4}$.	subtracts the numerators without calculating a common denominator; uses the denominator that appears last in the expression	CALCULATE *COMMON DENOMINATOR* and SUBTRACT MIXED NUMBERS WITH UNLIKE DENOMINATORS
The student divides the number line into twelfths and subtracts $4\frac{2}{12} - 1\frac{1}{12}$, ending at $3\frac{1}{12}$.	does not understand that fractions are numbers, and that if the numerator or denominator of the fraction changes disproportionately to the other, the value changes	REPRESENT EQUIVALENT FRACTIONS BY MULTIPLYING OR DIVIDING and SUBTRACT MIXED NUMBERS WITH UNLIKE DENOMINATORS

6. Mark and Sally each made small pizzas. Mark ate $\frac{2}{5}$ of his pizza, and Sally ate $\frac{1}{3}$ of her pizza.

6.a. How much more of his small pizza did Mark eat than Sally? Show your work to support your answer.

CORRECT ANSWER

Mark ate $\frac{1}{15}$ more of his pizza more than Sally.

$\frac{2}{5} - \frac{1}{3}$

$\frac{6}{15} - \frac{5}{15} = \frac{1}{15}$

Copyright © 2016 by The University of Kansas

Name: _____

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
$\frac{2}{5}-\frac{1}{3}=\frac{1}{2}$	subtracts the numerators and denominators	CALCULATE *COMMON DENOMINATOR* and SUBTRACT FRACTIONS WITH UNLIKE DENOMINATORS
$\frac{2}{5}-\frac{1}{5}=\frac{1}{5}$	replaces one denominator with another denominator, using the first/larger denominator in the expression	CALCULATE *COMMON DENOMINATOR* and SUBTRACT FRACTIONS WITH UNLIKE DENOMINATORS
$\frac{2}{3}-\frac{1}{3}=\frac{1}{3}$	replaces one denominator with another denominator, using the last denominator in the expression	CALCULATE *COMMON DENOMINATOR* and SUBTRACT FRACTIONS WITH UNLIKE DENOMINATORS
$\frac{2}{15}-\frac{1}{15}=\frac{1}{15}$	does not understand that fractions are numbers, and that if the numerator or denominator of the fraction changes disproportionately to the other, the value changes	REPRESENT EQUIVALENT FRACTIONS BY MULTIPLYING OR DIVIDING and SUBTRACT FRACTIONS WITH UNLIKE DENOMINATORS
The student adds the fractions instead of subtracting the fractions.	is unclear about the action the word problem is describing	SOLVE WORD PROBLEMS INVOLVING ADDITION AND SUBTRACTION OF FRACTIONS WITH UNLIKE DENOMINATORS

6.b. How would you determine how much pizza Mark and Sally ate altogether? How much pizza did Mark and Sally eat altogether?

CORRECT ANSWER

To determine how much pizza Mark and Sally ate altogether, you would need to add together the two fractions of how much of the pizzas they both ate. Because you had to find like denominators to compare the two fractions, you would just add the new equivalent fractions together to find how much Mark and Sally ate altogether $(\frac{6}{15}+\frac{5}{15}=\frac{11}{15})$.

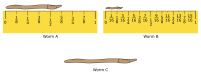
Altogether, Mark and Sally ate $\frac{11}{15}$ of the two pizzas.

Copyright © 2016 by The University of Kansas

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
$\frac{2}{5}+\frac{1}{3}=\frac{3}{8}$	adds the numerators and denominators	CALCULATE *COMMON DENOMINATOR* and ADD FRACTIONS WITH UNLIKE DENOMINATORS
$\frac{2}{5}+\frac{1}{5}=\frac{3}{5}$	replaces one denominator with another denominator, using the first/larger denominator in the expression	CALCULATE *COMMON DENOMINATOR* and ADD FRACTIONS WITH UNLIKE DENOMINATORS
$\frac{2}{3}+\frac{1}{3}=\frac{3}{3}$	replaces one denominator with another denominator, using the last denominator in the expression	CALCULATE *COMMON DENOMINATOR* and ADD FRACTIONS WITH UNLIKE DENOMINATORS
$\frac{2}{15}+\frac{1}{15}=\frac{3}{15}$	does not understand that fractions are numbers, and that if the numerator or denominator of the fraction changes disproportionately to the other, the value changes	REPRESENT EQUIVALENT FRACTIONS BY MULTIPLYING OR DIVIDING and ADD FRACTIONS WITH UNLIKE DENOMINATORS
The student subtracts the fractions instead of adding the fractions.	is unclear about the action the word problem is describing	SOLVE WORD PROBLEMS INVOLVING ADDITION AND SUBTRACTION OF FRACTIONS WITH UNLIKE DENOMINATORS
The student solves the addition but does not provide an explanation on *how* the sum was determined	is unable to explain the process of adding fractions with unlike denominators to determine the sum	EXPLAIN THE CONCEPT OF ADDITION AND SUBTRACTION OF FRACTIONS

7. Use the worms and rulers to answer the following questions.



7.a. Explain the relationship between the divisions on a ruler and the denominator of a fraction. How would you determine a common denominator to compare the two worms?

CORRECT ANSWER

The denominator of a fraction represents the number of equal parts in a whole. The divisions on a ruler represent the number of equal parts in which a length has been divided. They both represent the number of equal parts in one whole.

I would find a common denominator by determining how many equal parts are on each ruler, then use one of the following methods:

- List out the multiples of both numbers
- Multiply the two numbers together
- Use the ladder method dividing by common factors

Copyright © 2016 by The University of Kansas

Name: _____

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student states something other than the relationship between ruler divisions and the denominator of a fraction is the number of equal parts in one whole.	misunderstanding that a ruler is a number line, and that the denominator represents the equal number of parts in one whole	EXPLAIN *DENOMINATOR*
The student incorrectly identifies a method to calculate a common denominator.	does not understand a strategy to identify a common denominator	CALCULATE *COMMON DENOMINATOR*

7.b. How much longer is Worm A than Worm B? Use symbols to support your answer.

CORRECT ANSWER

Worm A is $\frac{7}{16}$ inch longer than Worm B.

$\frac{5}{8} - \frac{3}{16}$

$\frac{10}{16} - \frac{3}{16} = \frac{7}{16}$

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
$\frac{5}{16} - \frac{3}{8} = \frac{2}{8}$	tries to subtract numerators and denominators but cannot, therefore switches the denominators so the larger denominator appears first	CALCULATE *COMMON DENOMINATOR* and SUBTRACT FRACTIONS WITH UNLIKE DENOMINATORS
$\frac{5}{8} - \frac{3}{8} = \frac{2}{8}$	ignores the denominators when solving; uses the denominator that appears first in the expression	CALCULATE *COMMON DENOMINATOR* and SUBTRACT FRACTIONS WITH UNLIKE DENOMINATORS
$\frac{5}{16} - \frac{3}{16} = \frac{2}{16}$	ignores the denominators when solving; uses the second or larger denominator	CALCULATE *COMMON DENOMINATOR* and SUBTRACT FRACTIONS WITH UNLIKE DENOMINATORS
$\frac{5}{16} - \frac{3}{16} = \frac{2}{16}$	does not understand that fractions are numbers, and that if the numerator or denominator of the fraction changes disproportionately to the other, the value changes	REPRESENT EQUIVALENT FRACTIONS BY MULTIPLYING OR DIVIDING and SUBTRACT FRACTIONS WITH UNLIKE DENOMINATORS
The student adds the fractions instead of subtracting the fractions.	student is unclear about the action the word problem is describing	SOLVE WORD PROBLEMS INVOLVING ADDITION AND SUBTRACTION OF FRACTIONS WITH UNLIKE DENOMINATORS

Copyright © 2016 by The University of Kansas

7.c. Worm C is the same length as Worm A and Worm B combined. How long is Worm C? Use models or symbols to support your answer.

CORRECT ANSWER

Worm C is $\frac{13}{16}$ inches.

$\frac{5}{8} + \frac{3}{16}$

$\frac{10}{16} + \frac{3}{16} = \frac{13}{16}$

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
$\frac{5}{8} + \frac{3}{16} = \frac{8}{24}$	adds the numerators and denominators	CALCULATE *COMMON DENOMINATOR* and ADD FRACTIONS WITH UNLIKE DENOMINATORS
$\frac{5}{8} + \frac{3}{8} = \frac{8}{8}$	ignores the denominators when solving; uses the denominator that appears first in the expression	CALCULATE *COMMON DENOMINATOR* and ADD FRACTIONS WITH UNLIKE DENOMINATORS
$\frac{5}{16} + \frac{3}{16} = \frac{8}{16}$	ignores the denominators when solving; uses the second or larger denominator	CALCULATE *COMMON DENOMINATOR* and ADD FRACTIONS WITH UNLIKE DENOMINATORS
$\frac{5}{16} + \frac{3}{16} = \frac{8}{16}$	does not understand that fractions are numbers, and that if the numerator or denominator of the fraction changes disproportionately to the other, the value changes	REPRESENT EQUIVALENT FRACTIONS BY MULTIPLYING OR DIVIDING and ADD FRACTIONS WITH UNLIKE DENOMINATORS
The student subtracts the fractions instead of adding the fractions.	student is unclear in the action the word problem is describing	SOLVE WORD PROBLEMS INVOLVING ADDITION AND SUBTRACTION OF FRACTIONS WITH UNLIKE DENOMINATORS

8. Megan made a mistake when she simplified this equation: $\frac{9}{10} - \frac{2}{5} = \frac{7}{5}$

8.a. What was Megan's mistake?

CORRECT ANSWER

Megan did not calculate a common denominator, and she subtracted both the numerators *and* the denominators.

Copyright © 2016 by The University of Kansas

Name: _____

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Megan did not make a mistake—she is correct.	student thinks that subtracting the numerators and denominators is the correct method to simplify the expression	CALCULATE *COMMON DENOMINATOR* and SUBTRACT FRACTIONS WITH UNLIKE DENOMINATORS
$\frac{9}{10}-\frac{2}{10}=\frac{7}{10}$	The student may have chosen to just use the larger denominator, not understanding the need to calculate a common denominator. The student may also have calculated a common denominator but did not create an equivalent fraction, instead just leaving the numerator alone.	REPRESENT EQUIVALENT FRACTIONS BY MULTIPLYING OR DIVIDING and CALCULATE COMMON DENOMINATOR

8.b. What is the correct solution to the expression $\frac{9}{10}-\frac{2}{5}$? Use models or symbols to defend your answer.

CORRECT ANSWER

$\frac{9}{10}-\frac{2}{5}$

$\frac{9}{10}-\frac{4}{10}=\frac{5}{10}=\frac{1}{2}$

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
$\frac{9}{10}-\frac{2}{5}=\frac{7}{5}$	subtracts the numerators and denominators	CALCULATE *COMMON DENOMINATOR* and SUBTRACT FRACTIONS WITH UNLIKE DENOMINATORS
$\frac{9}{10}-\frac{2}{10}=\frac{7}{10}$	ignores the denominators when solving; uses the denominator that appears first in the expression	CALCULATE *COMMON DENOMINATOR* and SUBTRACT FRACTIONS WITH UNLIKE DENOMINATORS
$\frac{9}{5}-\frac{2}{5}=\frac{7}{5}$	ignores the denominators when solving; uses the second denominator	CALCULATE *COMMON DENOMINATOR* and SUBTRACT FRACTIONS WITH UNLIKE DENOMINATORS
$\frac{9}{10}-\frac{2}{10}=\frac{7}{10}$	does not understand that fractions are numbers, and that if the numerator or denominator of the fraction changes disproportionately to the other, the value changes	REPRESENT EQUIVALENT FRACTIONS BY MULTIPLYING OR DIVIDING and SUBTRACT FRACTIONS WITH UNLIKE DENOMINATORS

Copyright © 2016 by The University of Kansas

9. James wants to put a new bookcase in his room. His wall is $3\frac{2}{6}$ feet long, and the bookcase is $2\frac{3}{4}$ feet long. Will James have enough wall space left over to add a chair that is $1\frac{1}{2}$ feet long next to the bookcase? Support your answer with models or symbols.

CORRECT ANSWER

No, James will not have enough wall space left over to add a chair next to the bookcase. James has less than one foot left over, and the chair is more than one foot.

$3\frac{2}{6} - 2\frac{3}{4} = 3\frac{4}{12} - 2\frac{9}{12}$

$2\frac{16}{12} - 2\frac{9}{12} = \frac{7}{12}$

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student does not regroup the first mixed number. The student cannot subtract the numerators, so the student switches the numerators.	may not understand that a mixed number is a number, and that changing one part of the mixed number changes the number, or does not understand how to regroup a mixed number to create an improper fraction	CONVERT MIXED NUMBERS TO IMPROPER FRACTIONS
The student ignores the denominators when subtracting and uses one of the denominator that appears in the expression.	understands the need for a common denominator, but may not know how to calculate a common denominator and uses an existing denominator instead	CALCULATE *COMMON DENOMINATOR* and SUBTRACT MIXED NUMBERS WITH UNLIKE DENOMINATORS
The student regroups $3\frac{3}{6}$ as $2\frac{6}{6}$.	recognizes the need to regroup the mixed number, but may not understand that when regrouping, the one whole needs to be added to the proper fraction	CONVERT MIXED NUMBERS TO IMPROPER FRACTIONS
The student calculates a common denominator but does not create equivalent fractions.	does not understand that fractions are numbers, and that if the numerator or denominator of the fraction changes disproportionately to the other, the value changes	REPRESENT EQUIVALENT FRACTIONS BY MULTIPLYING OR DIVIDING and SUBTRACT MIXED NUMBERS WITH UNLIKE DENOMINATORS
The student adds the fractions instead of subtracting the fractions.	unclear about the action the word problem is describing	SOLVE WORD PROBLEMS INVOLVING ADDITION AND SUBTRACTION OF FRACTIONS WITH MIXED NUMBERS
The student says that James will have enough space left over for a chair.	does not recognize the resulting fraction as being less than one foot and that the chair requires more than one foot of space	COMPARE FRACTIONS TO BENCHMARKS 0, $\frac{1}{2}$, AND 1

Copyright © 2016 by The University of Kansas

Name: _____

10. Use this list of ingredients for cookies made during the American Revolution to answer the following questions:

$\frac{1}{2}$ cup molasses
$\frac{1}{4}$ cup brown sugar
$\frac{1}{4}$ cup lard
$1\frac{1}{2}$ cup flour
$\frac{1}{2}$ teaspoon cinnamon
$\frac{1}{4}$ teaspoon baking soda
$\frac{1}{4}$ teaspoon ginger

10.a. How many combined cups of molasses and brown sugar are needed to make the cookies?

CORRECT ANSWER

$\frac{3}{4}$ cup of molasses and brown sugar are needed to make the cookies.

$\frac{1}{2} + \frac{1}{4}$

$\frac{2}{4} + \frac{1}{4} = \frac{3}{4}$

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student adds both the numerators and denominators, writing $\frac{1}{2} + \frac{1}{4} = \frac{2}{6}$.	does not understand the need or know how to calculate a common denominator	CALCULATE *COMMON DENOMINATOR* and ADD FRACTIONS WITH UNLIKE DENOMINATORS
The student ignores the denominators when subtracting and uses one of the denominators that appears in the expression.	understands the need for a common denominator, but may not know how to calculate a common denominator and uses an existing denominator	CALCULATE *COMMON DENOMINATOR* and ADD FRACTIONS WITH UNLIKE DENOMINATORS
The student calculates a common denominator but does not create equivalent fractions by multiplying by one whole when creating fractions with common denominators. ($\frac{1}{4} + \frac{1}{4} = \frac{2}{4}$)	does not understand that fractions are numbers, and that if the numerator or denominator of the fraction changes disproportionately to the other, the value changes	REPRESENT EQUIVALENT FRACTIONS BY MULTIPLYING OR DIVIDING and ADD FRACTIONS WITH UNLIKE DENOMINATORS
The student subtracts the fractions instead of adding the fractions.	is unclear in the action the word problem is describing	SOLVE WORD PROBLEMS INVOLVING ADDITION AND SUBTRACTION OF FRACTIONS WITH UNLIKE DENOMINATORS

Copyright © 2016 by The University of Kansas

10.b. How many more cups of flour than lard are needed to make the cookies?

CORRECT ANSWER

$1\frac{1}{4}$ cups more flour than lard is needed.

$1\frac{1}{2} - \frac{1}{4}$

$1\frac{2}{4} - \frac{1}{4} = 1\frac{1}{4}$

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student switches the fractions in order to subtract the numerators and denominators. ($\mathbf{1\frac{1}{4} - \frac{1}{2} = 1\frac{0}{2}}$)	does not understand the need or know how to calculate a common denominator	CALCULATE *COMMON DENOMINATOR* and SUBTRACT MIXED NUMBERS WITH UNLIKE DENOMINATORS
The student ignores the denominators when subtracting and uses one of the denominator that appears in the expression.	understands the need for a common denominator, but may not know how to calculate a common denominator and uses an existing denominator	CALCULATE *COMMON DENOMINATOR* and SUBTRACT MIXED NUMBERS WITH UNLIKE DENOMINATORS
The student calculates a common denominator but does not create equivalent fractions by multiplying by one whole when creating fractions with common denominators. ($\mathbf{1\frac{1}{4} - \frac{1}{4} = 1\frac{0}{4}}$)	does not understand that fractions are numbers, and that if the numerator or denominator of the fraction changes disproportionately to the other, the value changes	REPRESENT EQUIVALENT FRACTIONS BY MULTIPLYING OR DIVIDING and SUBTRACT MIXED NUMBERS WITH UNLIKE DENOMINATORS
The student ignores or leaves out the 1 in $\mathbf{1\frac{1}{2}}$, using just $\frac{1}{2}$.	does not understand that a mixed number is a number, not two separate parts	EXPLAIN *MIXED NUMBER*
The student adds the fractions instead of subtracting the fractions.	is unclear about the action the word problem is describing	SOLVE WORD PROBLEMS INVOLVING ADDITION AND SUBTRACTION OF FRACTIONS WITH MIXED NUMBERS

10.c. After the molasses, brown sugar, lard, and flour are mixed together, how many cups of dough will be in the bowl?

Copyright © 2016 by The University of Kansas

Name: _____

CORRECT ANSWER

$2\frac{1}{2}$ cups of dough will be in the bowl.

$\frac{1}{2}+\frac{1}{4}+\frac{1}{4}+1\frac{1}{2}$

$\frac{2}{4}+\frac{1}{4}+\frac{1}{4}+1\frac{2}{4}$

$1\frac{6}{4}=2\frac{2}{4}=2\frac{1}{2}$

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student adds both the numerators and denominators.	does not understand the need or know how to calculate a common denominator	CALCULATE *COMMON DENOMINATOR* and ADD MIXED NUMBERS WITH UNLIKE DENOMINATORS
The student ignores the denominators when subtracting and uses one of the denominators that appears in the expression.	understands the need for a common denominator, but may not know how to calculate a common denominator and uses an existing denominator	CALCULATE *COMMON DENOMINATOR* and ADD MIXED NUMBERS WITH UNLIKE DENOMINATORS
The student calculates a common denominator but does not create equivalent fractions by multiplying by one whole when creating fractions with common denominators. ($\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\mathbf{1}\frac{1}{4}$)	does not understand that fractions are numbers, and that if the numerator or denominator of the fraction changes disproportionately to the other, the value changes	REPRESENT EQUIVALENT FRACTIONS BY MULTIPLYING OR DIVIDING and ADD MIXED NUMBERS WITH UNLIKE DENOMINATORS
The student ignores or leaves out the 1 in $\mathbf{1}\frac{1}{2}$, using just $\frac{1}{2}$.	does not understand that a mixed number is a number, not two separate parts	EXPLAIN *MIXED NUMBER*
The student subtracts the fractions instead of adding the fractions.	is unclear about the action the word problem is describing	SOLVE WORD PROBLEMS INVOLVING ADDITION AND SUBTRACTION OF FRACTIONS WITH MIXED NUMBERS

11. Harry walked $\mathbf{1}\frac{3}{8}$ miles to the store. On his way home, Harry took a different route through the park. The route home was $\frac{2}{3}$ mile longer than his trip to the store.

11.a. How many miles did Harry walk on his way home? Use models or symbols to support your answer.

Copyright © 2016 by The University of Kansas

CORRECT ANSWER

Harry walked $2\frac{1}{24}$ miles on his way home.

$1\frac{3}{8}+\frac{2}{3}$

$1\frac{9}{24}+\frac{16}{24}=1\frac{25}{24}=2\frac{1}{24}$

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student adds the numerators and denominators. ($\mathbf{1}\frac{3}{8}+\frac{2}{3}=\mathbf{1}\frac{5}{11}$)	does not understand the need or know how to calculate a common denominator	CALCULATE *COMMON DENOMINATOR* and ADD MIXED NUMBERS WITH UNLIKE DENOMINATORS
The student ignores the denominators when subtracting and uses one of the denominator that appears in the expression.	understands the need for a common denominator, but may not know how to calculate a common denominator and uses an existing denominator	CALCULATE *COMMON DENOMINATOR* and ADD MIXED NUMBERS WITH UNLIKE DENOMINATORS
The student calculates a common denominator but does not create equivalent fractions by multiplying by one whole when creating fractions with common denominators. ($\mathbf{1}\frac{3}{24}+\frac{2}{24}=\mathbf{1}\frac{5}{24}$)	does not understand that fractions are numbers, and that if the numerator or denominator of the fraction changes disproportionately to the other, the value changes	REPRESENT EQUIVALENT FRACTIONS BY MULTIPLYING OR DIVIDING and ADD MIXED NUMBERS WITH UNLIKE DENOMINATORS
The student ignores or left out the 1 in $\mathbf{1}\frac{3}{8}$, using just $\frac{3}{8}$.	does not understand that a mixed number is a number, not two separate parts	EXPLAIN *MIXED NUMBER*
The student subtracts the fractions instead of adding the fractions.	is unclear in the action the word problem is describing	SOLVE WORD PROBLEMS INVOLVING ADDITION AND SUBTRACTION OF FRACTIONS WITH MIXED NUMBERS

11.b. How many miles did Harry walk altogether? Use models or symbols to support your answer.

CORRECT ANSWER

Altogether, Harry walked $3\frac{10}{24}$ or $3\frac{5}{12}$ miles.

$2\frac{1}{24}+1\frac{3}{8}$

$2\frac{1}{24}+1\frac{9}{24}=3\frac{10}{24}=3\frac{5}{12}$

Copyright © 2016 by The University of Kansas

Name: _____

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The student adds the numerators and denominators. $(2\frac{1}{24} + 1\frac{9}{24} \neq 3\frac{10}{48})$	does not understand the need or know how to calculate a common denominator	CALCULATE *COMMON DENOMINATOR* and ADD MIXED NUMBERS WITH UNLIKE DENOMINATORS
The student ignores or leaves out the whole numbers. $(\frac{1}{24} + \frac{9}{24} = \frac{10}{24})$	does not understand that a mixed number is a number, not two separate parts	EXPLAIN *MIXED NUMBER*
The student subtracts the fractions instead of adding the fractions.	is unclear about the action the word problem is describing	SOLVE WORD PROBLEMS INVOLVING ADDITION AND SUBTRACTION OF FRACTIONS WITH MIXED NUMBERS

Copyright © 2016 by The University of Kansas