MULTI-STEP RATIO AND PERCENT PROBLEMS

7.RP.3

CONTENTS

The types of documents contained in the unit are listed below. Throughout the unit, the documents are arranged by lesson.

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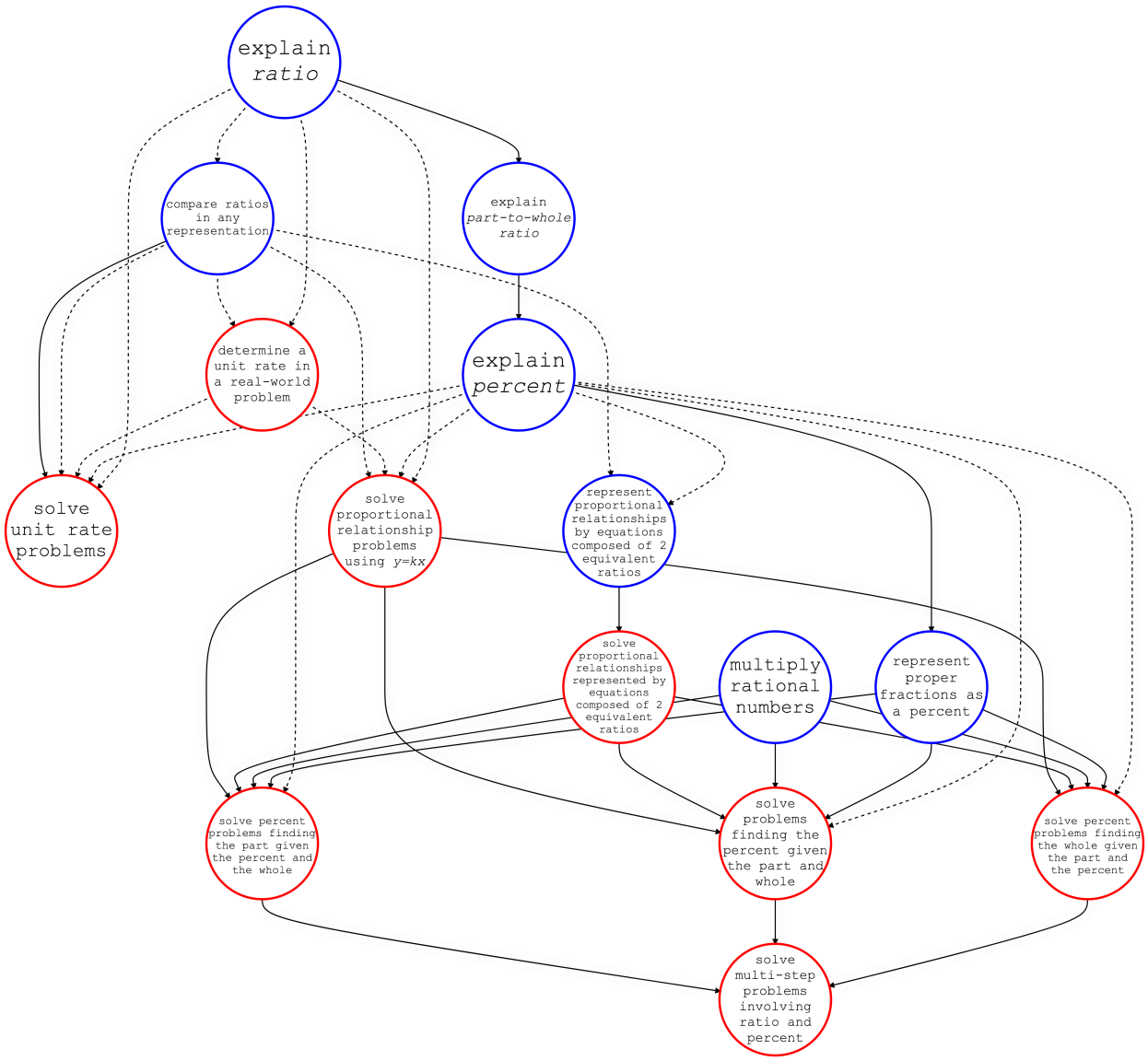
MULTI-STEP RATIO AND PERCENT PROBLEMS

LEARNING MAP INFORMATION

STANDARDS

7.RP.3 Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.*



**Learning map model of* 7.RP.3

Node Name	Node Description
COMPARE RATIOS IN ANY REPRESENTATION	Compare ratios represented in any representation such as tables or fractions.
DETERMINE A UNIT RATE IN A REAL-WORLD PROBLEM	Determine a unit rate in a real-world problem.
EXPLAIN *PART-TO-WHOLE RATIO*	Make known your understanding that ratios can express a comparison between the number of parts to a given whole. These ratios are composed of two quantities measured in the same unit of measurement. For example, in the ratio of zebras to total animals in the zoo, the unit of measurement is one animal.
EXPLAIN *PERCENT*	Make known your understanding that a percent is another way of representing a ratio or a rate per 100.
EXPLAIN *RATIO*	Make known your understanding that a ratio represents a multiplicative comparison of two quantities or the joining of two quantities into a composed unit. For example, the ratio of eyes to nose on a person is 2:1, because for every two eyes there is one nose.
MULTIPLY RATIONAL NUMBERS	Determine the product of two or more rational numbers.
REPRESENT PROPER FRACTIONS AS A PERCENT	Through writing or an appropriate assistive technology, represent common fractions (e.g., $\frac{3}{4}$, $\frac{1}{2}$, $\frac{1}{5}$, $\frac{1}{4}$, $\frac{3}{10}$) as a percent.
REPRESENT PROPORTIONAL RELATIONSHIPS BY EQUATIONS COMPOSED OF 2 EQUIVALENT RATIOS	Through writing or an appropriate assistive technology, represent proportional relationships by equations composed of two equivalent ratios.
SOLVE MULTI-STEP PROBLEMS INVOLVING RATIO AND PERCENT	Apply proportional reasoning to solve real-world problems involving ratio and percent, including simple interest, tax, etc.
SOLVE PERCENT PROBLEMS FINDING THE PART GIVEN THE PERCENT AND THE WHOLE	Given the whole and the percent, find the part.
SOLVE PERCENT PROBLEMS FINDING THE WHOLE GIVEN THE PART AND THE PERCENT	Given the part and the percent, find the whole.
SOLVE PROBLEMS FINDING THE PERCENT GIVEN THE PART AND WHOLE	Given the part and whole, find the percent.
SOLVE PROPORTIONAL RELATIONSHIP PROBLEMS USING *Y=KX*	Solve proportional relationships represented by equations in the form of $y = kx$ where k is the constant of proportionality.
SOLVE PROPORTIONAL RELATIONSHIPS REPRESENTED BY EQUATIONS COMPOSED OF 2 EQUIVALENT RATIOS	Solve proportional relationships represented by equations composed of two equivalent ratios.
SOLVE UNIT RATE PROBLEMS	Solve unit rate problems including unit pricing and constant speed. For example, if five pencils costs $0.25 total, how much does 15 pencils cost?

MULTI-STEP RATIO AND PERCENT PROBLEMS

TEACHER NOTES

This unit includes the following documents:

- Learning Map Information
- Instructional Activity (four lessons)
- Instructional Activity Student Handout (for Lessons 1–4)
- Instructional Activity Supplement (for Lessons 1, 3, & 4)
- Student Activity (Word version)
- Student Activity Solution Guide

In this unit, students will learn how to use proportional reasoning to solve multi-step ratio and percent problems.

RESEARCH

Student understanding of these concepts develops from sixth grade, when students begin exploring ratios and proportionality, to seventh grade, when students further develop their understanding of proportionality and its applications. In order for students to reason proportionally, they must be able to identify contexts where a multiplicative relationship exists, and to distinguish these contexts from situations where there is not a multiplicative relationship among the quantities (Cramer, Post, & Currier, 1993). Students should be familiar with some proportional relationships such as miles per hour, miles per gallon, points per game, yards per carry, etc. Contrasting these scenarios with situations that are not proportional can help a teacher determine whether a student is able to reason proportionally.

AN EXAMPLE

Consider the following scenarios:

1. Jack and Jill are skiing equally fast. Jack started skiing first. When Jack had skied 10 runs, Jill had skied five runs. When Jack has skied 30 runs, how many runs will Jill have skied?
2. Tom drives 50 miles in one hour. If Tom drives at the same rate, how many miles will Tom have driven after four hours?

Students who are not yet able to think proportionally may see these as the same type of problem. Structurally, three values are given and one value is unknown. However, the first scenario does not represent a proportional relationship while the second scenario does. Asking students to analyze and solve each problem and explain why one is proportional and the other is not can shed light on students' understanding of proportionality.

A recent focus in education has been for students to develop mathematical understanding through learning experiences based on real-world mathematics related to aspects of everyday life. The multi-step ratio and percent problems in this unit are related to real-world applications and problem solving situations. Research supports this trend in mathematics, stating students should explore and deepen the concept of proportionality through the use of problem solving and reasoning (NCTM, 2000). Tasks that emphasize the need for new ideas motivate student learning and promote new mathematical knowledge through problem solving (NCTM, 2014). These connections help students see the importance of proportional reasoning. Many parts of our world follow rules of proportionality, therefore proportional reasoning is a life skill that students can apply to real-world experiences (Post, Behr, & Lesh, 1988).

Although the standards for ratio and proportion are only found in the sixth and seventh grades, proportional thinking forms the basis for many advanced topics in mathematics, including linear functions, various algebra concepts, scaling geometric figures, probability, and statistics. Thus, middle school mathematics emphasizes students mastering various applications of ratio and proportions. One such application is problem-solving situations. Proportional thinking from multiple perspectives gives students the flexibility to solve novel problems they may encounter (Post, Behr, & Lesh, 1988). Additionally, students need to see many problem situations that can be modeled and solved through proportional reasoning. These different activities should not be categorized but instead intermixed, as students might see in similar situations outside of textbooks (NCTM, 1989). Furthermore, the techniques to approach and solve all types of percent situations follow the same conceptual use of proportions, rather than distinct strategies (Post, Behr, & Lesh, 1988). When students collaborate and delve into relevant and mathematically sound problems, they will use their problem solving skills in tandem with proportional reasoning to strengthen both skills (Miller & Fey, 2000). Teachers should help students explore and choose different problem solving methods in their classrooms as they work with proportional reasoning (NCTM, 2000).

LEARNING INFORMATION

This portion of the learning map model grows out of the area that describes students learning about unit conversions. Experience with unit conversions is where students begin to develop the idea that a rate governs relationships between two measures. These concepts provide students with a concrete foundation for unit rate. The learning map section for this specific sequence of activities begins with an understanding of ratios, which leads to unit rate applications. This concept from the sixth grade standards expands as students explore proportional relationships. The learning map sequence models students' ability to create proportions from real-world situations and use those proportions to solve real-world problems. Proportions are also used as a basis for working with percent, as modeled by the direct links in the learning map model. Students will build upon writing and solving proportions to move into solving percent problems. After one-step percent problems, multi-step ratio and percent problems fall in the learning sequence.

INSTRUCTIONAL ACTIVITIES

The activities in this unit are designed to have students work with ratios and percent in real-world applications. First, students complete one-step and simple two-step application problems with ratios and percent. Next, students apply multi-step ratio and percent problems to a real-world situation similar to what they might encounter in their lives. Students will calculate percentages dealing with shopping and other consumer activities. Then students analyze percent change calculations to find the errors in reasoning and calculation. Finally, students complete two related multi-step ratio and percent problems to determine which outcome is better for a given situation. Application topics included throughout this unit are discounts, sales, markups, tips, taxes, fees, commission, population change, percent change, and simple interest.

REFERENCES

Cramer, K., Post, T., & Currier, S. (1993). Learning and teaching ratio and proportion: Research implications. Research ideas for the classroom: Middle grades mathematics, 159-178.

Miller, J., & Fey, J. (2000). Take Time For Action: Proportional Reasoning. *Mathematics Teaching in the Middle School, 5*(5), 310-313.

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Post, T., Behr, M., & Lesh, R. (1988). Proportionality and the Development of Prealgebra Understandings. In A. F. Coxford & A. P. Shulte (Eds.) *The Ideas of Algebra, K – 12*, 1988 Yearbook of the National Council of Teachers of Mathematics (pp. 78-90). Reston, VA: National Council of Teachers of Mathematics.

MULTI-STEP RATIO AND PERCENT PROBLEMS

OVERVIEW OF INSTRUCTIONAL ACTIVITIES

Lesson	Learning Goal	Nodes Addressed
Lesson 1	Students will solve ratio and percent problems in real-world contexts.	▶ SOLVE PERCENT PROBLEMS FINDING THE PERCENT GIVEN THE PART AND THE WHOLE ▶ SOLVE PERCENT PROBLEMS FINDING THE WHOLE GIVEN THE PART AND THE PERCENT ▶ SOLVE PROPORTIONAL RELATIONSHIPS REPRESENTED BY EQUATIONS COMPOSED OF 2 EQUIVALENT RATIOS ▶ SOLVE MULTI-STEP PROBLEMS INVOLVING RATIO AND PERCENT
Lesson 2	Students will solve percent problems in real-world contexts.	▶ SOLVE PERCENT PROBLEMS FINDING THE PART GIVEN THE PERCENT AND THE WHOLE ▶ SOLVE MULTI-STEP PROBLEMS INVOLVING RATIO AND PERCENT
Lesson 3	Students will analyze problem-solving situations to find and describe the error in computing with percent and percent change.	▶ SOLVE MULTI-STEP PROBLEMS INVOLVING RATIO AND PERCENT
Lesson 4	Students will apply an understanding of solving multi-step ratio and percent problems.	▶ SOLVE MULTI-STEP PROBLEMS INVOLVING RATIO AND PERCENT

MULTI-STEP RATIO AND PERCENT PROBLEMS

INSTRUCTIONAL ACTIVITY

Lesson 1

LEARNING GOAL

Students will solve ratio and percent problems in real-world contexts.

PRIMARY ACTIVITY

Students will complete one-step ratio and percent problems on task cards. These problems are designed to help build students' understanding of ratio and percent problems to lead into working with multi-step ratio and percent problems.

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

- Ratio
- Percent
- Proportion
- Discount

MATERIALS

- Calculator
- Scratch paper
- INSTRUCTIONAL ACTIVITY SUPPLEMENT (Recommended one copy for every group of three to five students.)
- INSTRUCTIONAL ACTIVITY STUDENT HANDOUT

IMPLEMENTATION

Students entering seventh grade should be familiar with percent as a rate per 100; for example, 35% means $\frac{35}{100}$ times the quantity. Students' experiences in sixth grade with percent are strongly related to working with percent as a ratio. Students may have some experience working with percent in an equation and percent in a decimal form. In addition, students should have experience solving problems finding the whole, given a part and the percent. Connections should be made with proportional reasoning as it relates to percent problem-solving situations.

Discuss how to operate with percentages. Students can use equivalent ratios, proportions, or an equation. Ensure students are able to calculate with percentage problems and work with percent problem-solving situations fluently. Students should be able to calculate given part and whole, part and percent, or whole and percent.

Work and **model** the following example as a class to practice finding a percent given a part and a whole. Students should be able to set up a proportion to solve this problem. Students may also choose to create a bar model or number line diagram for this situation. Alternatively, students may use a percent equation to solve the problem.

> Tonia is making a pasta dinner. The recipe calls for a mix of cheeses totaling five cups, recommending at least 50% of the cheese be white, such as mozzarella or provolone. Tonia has three cups of mozzarella, two cups of mild cheddar, and two cups of sharp cheddar. Will this work for her recipe?

Using the example, **discuss** the relationship between part and whole values in a percent situation. Equivalent ratios and proportional reasoning can be used to complete the task after students have set up ratios representing the percent and the part-whole relationship.

> *Example Solution*
>
> Part : whole relationship → three cups mozzarella : five cups total cheese
>
> Using the formulaic proportion $\frac{part}{whole} = \frac{percent}{100}$, set up the proportion $\frac{3\ cups\ mozzarella}{5\ cups\ total\ cheese} = \frac{60}{100} = 60\%$ white cheese. 60% is at least 50%.
>
> Yes, Tonia can use three cups of mozzarella and two cups of either cheddar for her recipe.

GUIDING QUESTIONS

Elicit student thinking:

- What ratio can be set up from this situation?
- Do you need to find the part, the whole, or the percent?
- How do the different values relate to each other in the problem-solving situation?
- What information do you need to use from the problem?

Determine if the student can SOLVE PERCENT PROBLEMS FINDING THE PERCENT GIVEN THE PART AND THE WHOLE:

- What proportion can be set up to represent the unknown percent?
- What ratios are equivalent to 50%?
- Can you draw a model to represent this problem-solving situation?
- What does "at least" mean when comparing ratios and percentages?
- Explain your procedure for solving this problem.

Work and **model** the following example as a class to practice finding the whole given a part and a percent. Again, students should be able to set up a proportion to solve this problem. Students may also choose to create a bar model or number line diagram for this situation. Alternatively, students may use a percent equation to solve the problem.

> Lane correctly answered 34 questions on his quiz. His score was an 85%. How many questions were on the quiz?

Using the example, **discuss** the relationship between part and percent in a situation. Equivalent ratios and proportional reasoning can be used to complete the task after students have set up ratios representing the percent and the part-whole relationship.

> *Example Solution*
>
> Given information: 34 correct questions is the part, 85% score
>
> Unknown information: the whole amount of questions on the quiz

Example Solution (continued)

Using the formulaic proportion $\frac{percent}{100} = \frac{part}{whole}$, set up the proportion $\frac{85}{100} = \frac{34\ correct\ questions}{?\ total\ questions} \rightarrow \frac{17}{20} = \frac{34\ correct\ questions}{40\ total\ questions}$

There were 40 total questions on the quiz.

GUIDING QUESTIONS

Elicit student thinking:

- How do ratios apply to this situation?
- Do you need to find the part, the whole, or the percent?
- How do the different values relate to each other in the problem-solving situation?
- What information do you need to use from the problem?

Determine if the student can SOLVE PERCENT PROBLEMS FINDING THE WHOLE GIVEN THE PART AND THE PERCENT:

- What ratio can be used in this situation?
- What proportion can be set up to represent the unknown whole value?
- What ratios are equivalent to 85%?
- Can you draw a model to represent this problem-solving situation?
- Explain your procedure for solving this problem.

Cut apart the eight Ratio and Percent Task Cards. **Distribute** the INSTRUCTIONAL ACTIVITY SUPPLEMENT to groups of three to five students. Students can complete the task cards individually, but they will share the collection of eight cards.

Distribute the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT to each student. Students will use the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT to record their work and answers.

Monitor students as they work to make sure they are showing work and making connections to previous understanding of ratios and proportions.

GUIDING QUESTIONS

Elicit student thinking:

- How do the different values relate to each other in the problem-solving situation?
- How can you draw a model to represent this problem?
- What information do you know from the situation?
- What information do you need to use from the situation?
- Can you think of more than one way to calculate in this situation?

Determine if the student can SOLVE PROPORTIONAL RELATIONSHIPS REPRESENTED BY EQUATIONS COMPOSED OF 2 EQUIVALENT RATIOS:

- What is the purpose of ratios or percent in this problem-solving situation?
- Describe in words the process you will use to answer the question in this problem.
- How could you use ratios to solve these problems?
- What proportion could be set up to model this problem?
- How would you solve a proportion to complete this task?
- Explain why your solution makes sense.

Determine if the student is ready to SOLVE MULTI-STEP PROBLEMS INVOLVING RATIO AND PERCENT:

- How could the solution be used to continue a problem-solving task?
- What questions could be asked that requires the solution to continue with a calculation?
- What examples of problems might use ratios and percentages like those on the task cards that require two or more steps to solve?

Students should be required to explain their solutions and work procedures. Students may ask each other questions and check each other's work. Students should be able to explain their thinking to others to justify their reasoning. Additionally, students should be able to explain their process to help other students with their calculations.

At the end of the activity, have students sort and categorize the task cards. Student should analyze the tasks to determine which tasks share similarities and which tasks are different. This requires students to consider the different types of ratio and percent situations, but also to think about the different methods they used to complete the tasks. Students may categorize based on solution method, such as mental math versus paper and pencil or proportions versus equations, solution form such as whole number versus percent, or other logical categories. Students should be able to explain the categories and reasoning they used to sort the task cards.

Name_____

Multi-Step Ratio and Percent Problems

Lesson 1

The following pages contain workspace where students can record their work and answers for the eight Ratio and Percent Task Cards. The cards may be completed in any order, but they should be correlated to the matching numbered workspaces. Also provided with this lesson is an Instructional Activity Supplement containing the eight Ratio and Percent Task Cards.

Name_____

Show all of your work for each task card in the spaces below. Explain each solution.

Ratio and Percent Task Card #1	Ratio and Percent Task Card #2
Ratio and Percent Task Card #3	Ratio and Percent Task Card #4

Name_____

Show all of your work for each task card in the spaces below. Explain each solution.

Ratio and Percent Task Card #5	Ratio and Percent Task Card #6
Ratio and Percent Task Card #7	Ratio and Percent Task Card #8

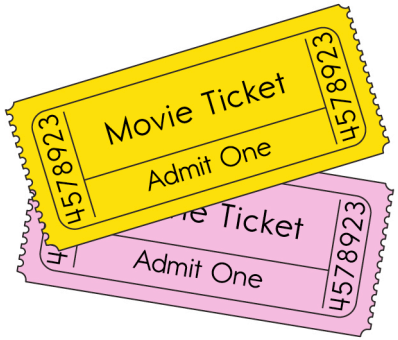
Multi-Step Ratio and Percent Problems

INSTRUCTIONAL ACTIVITY SUPPLEMENT

Lesson 1

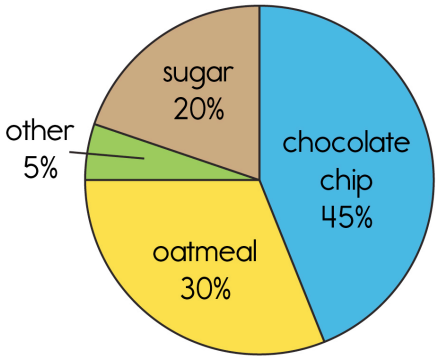
The following pages contain eight Ratio and Percent Task Cards. The pages need to be cut along the lines to yield four cards per page. Students should complete the task for each situation. The cards may be completed in any order. Also provided with this lesson is an Instructional Activity Student Handout where students can record their answers as they complete each card.

1) 24% of students saw the new movie during opening weekend. If there are 725 students, how many saw the movie?



2) The width of a rectangular room is 75% of its length. If the length is 24 inches, what is the perimeter of the room?

3) 27 students chose chocolate chip cookies as their favorite. How many students were surveyed?

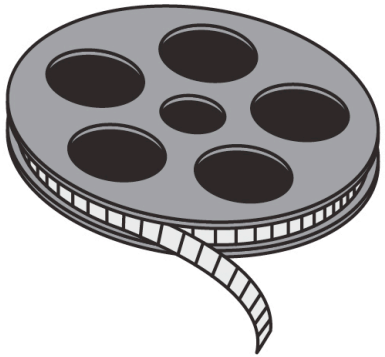


4) The Music and More Store is having a 60% off sale. If you save $36 on an album collection, what was the original cost?

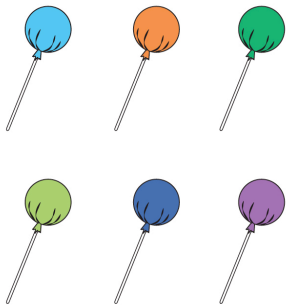
5) Four magazines cost $11.20 each with a yearly 12-issue subscription. How much is the full year subscription?

6) 30% of your monthly budget goes to rent. If you pay $450 in rent, how much is your monthly budget?

7) 105 minutes is what percent of a two-hour, 20-minute-long movie?



8) When six children share one bag of candy, each child gets 12 pieces. If the same bag of candy is shared among nine children, how many pieces does each child get?



MULTI-STEP RATIO AND PERCENT PROBLEMS

INSTRUCTIONAL ACTIVITY

Lesson 2

LEARNING GOAL

Students will solve percent problems in real-world contexts.

PRIMARY ACTIVITY

Students will complete six percent problems in situations that they might encounter on a day trip to the lake.

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

- Percent
- Tax
- Tip
- Discount
- Markup
- Fees
- Profit

MATERIALS

- Calculator
- Scratch paper
- INSTRUCTIONAL ACTIVITY STUDENT HANDOUT

IMPLEMENTATION

Students entering seventh grade should be familiar with some basic percent calculations. Students should have experience working with percentages as a rate per 100 and then applying this rate to equivalent ratios to perform basic calculations. Students should also have experience with some basic applications such as calculating tips, discounts, or markups. However, combining these pieces into multi-step problems will be a new challenge. Connections should continue to be made with proportional reasoning as it relates to percent problem-solving situations.

Discuss how to operate with percentages. Students can use equivalent ratios, proportions, or an equation. Students should have some familiarity with using a bar model or a number line model to use equivalent ratios with percent. Additionally, students should be familiar with using equivalent ratios when working with percent. Ensure students are able to calculate with percentage problems and work with percent problem-solving situations fluently. Students should be able to calculate given part and whole, part and percent, or whole and percent.

Model how to solve a multi-step percent problem in a real-world context with the following problem.

> James has saved $560. He received 30% of the money for his birthday, saved 25% from his allowance, and earned the rest of it by mowing lawns. How much of his savings did James earn by mowing lawns?

Using the example, **explore** the relationship between the given total value and the separate percentages in the situation. **Explore** the multiple steps needed to complete the task. **Discuss** the multiple techniques to solve the problem: 1) find the percentage saved from mowing lawns and then calculate that percent of $560, or 2) calculate the amounts saved from birthday money and allowance and then subtract from the $560. **Emphasize** that multiple techniques may be possible and accurate to solve multi-step problems.

GUIDING QUESTIONS

Elicit student thinking:

- What do you need to determine in this problem?
- How do the different values relate to each other in the problem-solving situation?
- How can you draw a model to represent this problem-solving situation?

Determine if the student can SOLVE PERCENT PROBLEMS FINDING THE PART GIVEN THE PERCENT AND THE WHOLE:

- Explain how to use the given percentages and total amount saved to calculate the separate portions of James' money.
- Did James earn more or less than $560 from mowing lawns? Explain.
- Estimate how much money James earned from mowing lawns.
- How much money did James save from his birthday?
- How much money did James save from his allowance?
- Why is it important to know about the portions James saved from his birthday and allowance?
- What percent of his savings did James earn from mowing the lawn?

Distribute the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT to each student. Students can work in pairs or groups of three to facilitate discussion. Students will benefit from working in small groups as they explore and discuss percent problem situations.

Monitor students as they work to make sure they are showing work and making connections. Students should recall similar problem-solving situations from working with ratios and proportions and apply similar work techniques. Students should make connections between proportional reasoning and percent problem solving.

GUIDING QUESTIONS

Elicit student thinking:

- How do the different values relate to each other in the problem-solving situation?
- What models could be useful in this situation?
- What information do you know from the situation?
- What information do you need to use from the situation?

Determine if the student can SOLVE MULTI-STEP PROBLEMS INVOLVING RATIO AND PERCENT:

- What is the purpose of percent in this problem-solving situation?
- Describe in words the process you will use to answer the question in this situation.
- How could you use ratios to solve these problems?
- Explain why your solution makes sense.
- Can you think of more than one way to calculate in this situation?

Students should be required to explain their solutions and work procedures. When working in small groups, students can explain and justify their findings to their peers. Students should check and verify the work of others.

At the end of the activity, teachers should have students reflect and journal about this question: What connection did you make today that made you say, "I get it!"?

Name_____

Multi-Step Ratio and Percent Problems

Lesson 2

On a day trip to the lake, you notice that percentages are found all around. After your recent studies with ratio and percent, you feel ready to tackle these problem-solving situations. Show your work and explain your answers for each of the following percent situations.

1. When arriving at the lake, you find that your group is large enough to qualify for the group day-use fee. The regular fee for day use is $9 per person, but with your group of 14, a discount of 30% is applied. What is the total cost for your group to visit the lake for the day?

Name_____

2. Once your group has chosen a spot on the sand to relax, you start to unpack your beach bag. That's when you notice you forgot to bring sunscreen. You had made a mental note to bring sunscreen because you know the markup on sunscreen at the Snack Shack at the lake is 50% compared to the stores in town. Yesterday you bought a brand-new bottle of sunscreen in town for $6.68. To the nearest dollar, how much will it cost you to buy sunscreen from the Snack Shack at the lake?

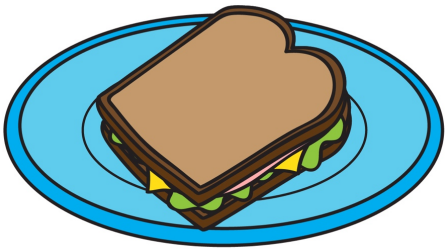
Name_____

3. You and a friend head over to the Snack Shack to find some sunscreen. There you see a stack of sunhats on sale for 45% off. After finding one you like, you check the price tag. If the sunhat was originally $24.99, how much will it cost when discounted?

4. You want to buy both the sunscreen and sunhat, but you know sales tax will also be included. If sales tax is 6.75% at the lake, what will be your total purchase price?

Name_____

5. After enjoying a morning of sun, you are ready for lunch. While some members of your group packed a picnic lunch, you planned to eat at the Lakeside Café. Your small group gets a table for five at the café. Three members of the group order the pizza lunch special for $7.49 each. You and another friend order pasta for $8.49 and a sandwich for $6.99. Everyone orders a soda for $1.99. At the Lakeside Café, tax is included, but tips for the server are not. After a discussion, your group decides to leave an 18% tip. How much money will your group leave as a tip?



Name_____

6. After an afternoon of sand and surf, you are ready to head home. Before leaving, you treat yourself to an ice cream. Waiting in line, you overhear the ice cream man talking about his profits. He makes a 40% profit from his total sales. Today has been a good day, and he has sold $180 just this afternoon. How much profit has the ice cream man made this afternoon?

MULTI-STEP RATIO AND PERCENT PROBLEMS

INSTRUCTIONAL ACTIVITY

Lesson 3

LEARNING GOAL

Students will analyze problem-solving situations to find and describe the error in computing with percent and percent change.

PRIMARY ACTIVITY

Students will be given cards with problem-solving situations that have been solved with an error. Students will find and describe the error in calculation or reasoning in the given problem-solving situation. Then students will rework the problem to show the correct answer. Finally, students will offer a tip to help prevent the error in future calculations.

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

- Percent
- Part
- Whole
- Percent change
- Percent increase
- Percent decrease
- Original
- Markup
- Discount
- Sale

MATERIALS

- Calculator
- Scratch paper

- INSTRUCTIONAL ACTIVITY STUDENT HANDOUT
- INSTRUCTIONAL ACTIVITY SUPPLEMENT (Recommended one copy for every group of two to three students.)

IMPLEMENTATION

Students entering seventh grade should be familiar with some basic percent calculations. Students should have experience working with percentages as a rate per 100 and then applying this rate to equivalent ratios to perform basic calculations. Students should also have experience with some basic applications such as calculating tips, discounts, or markups. However, students' prior experience would emphasize finding the amount of a tip, discount, or markup. In this lesson, students will be focusing on the percent change from an original value to the new final value. Connections should continue to be made with proportional reasoning as it relates to percent problem-solving situations.

Review how to find percent of a number and calculate percent given a part and a whole. Additionally, students need to be able to convert between decimals and percentages. Students can use equivalent ratios, proportions, or an equation. **Ensure** students are able to calculate with percentage problems and work with percent problem-solving situations fluently.

Discuss how percent change is different from finding the percent of a number. **Review** the pieces of a percent problem: part, whole, percent, original amount, and new amount. Students should understand how finding the percent of a number is based on the part versus the whole, whereas percent change deals with how the value has changed from an original value (sometimes called the *initial* value) to a new value (sometimes called the *final* value). Percent change will identify what percent of the original value the new value has increased or decreased.

Explore the percent change formula.

$$\text{Percent change} = \frac{|\textit{new value} - \textit{original value}|}{\textit{original value}} = \frac{\textit{amount of change}}{\textit{original value}}$$

Students need to be able to distinguish between finding the percent change and the amount of change.

Remind students that amount of change is found from a subtraction problem, but percent change takes a ratio to calculate.

Discuss the meaning of amount of change as the "part" to be used with the "whole" amount of the original value.

Also **point out** to students that amount of change needs to be a positive value with a specification regarding whether it is an increase or a decrease.

Connect the vocabulary options for working with percent change. Students should recognize original, old, regular, initial, beginning, previous, formerly, normal, first, earlier, earliest, and

introductory as some different ways to indicate the original value in the calculation. Similarly, students should recognize new, changed, discounted, sale, markup, increased, decreased, final, closing, end, ending, and concluding as some different ways to indicate the new value in the calculation.

Model how to use reasoning to solve a percent change problem-solving situation with the following example.

> Percy beat his high score on his favorite video game. If his old high score was 140,200 points and his new high school is 189,270 points, what is the percent change? Identify the percent change as a percent increase or a percent decrease. Explain your reasoning.

GUIDING QUESTIONS

Elicit student thinking:

- What do you know about this scenario?
- How are proportions related to percent?
- How could you solve a percent problem?
- What does an increase mean in a percent situation?
- What does a decrease mean in a percent situation?

Determine if the student is ready to SOLVE MULTI-STEP PROBLEMS INVOLVING RATIO AND PERCENT:

- What do the two high scores represent in the percent calculation?
- How can you find the amount of change?
- What proportion could be set up using this situation?
- What ratio can be used to find the percent change?
- Using the new and old values, how do you determine whether the change is an increase or a decrease? Explain.
- Why is the original amount important in a percent change calculation?
- Why is the amount of change important in a percent change calculation?

Model how to use reasoning to solve a multi-step percent problem with the following example.

> A department store is having a "door buster" sale with all jeans on sale for 30% off. In addition, Marie has a $10 off coupon. Marie finds a $34.99 pair of jeans that are exactly what she is looking for. Marie knows that the store will first apply dollar-off coupons and then take the percent off the remainder. To the nearest percent, what will Marie save in total on her jeans purchase?

In multi-step discount problems, it is important for students to realize the order the discounts will be stacked. Dollar-off and other flat-value coupons are simply stacked, but combining dollar-off and percent-off discounts is usually ordered in a specific manner.

Discuss with students the difference between the combination orders and **explore** the store's reasoning. Students can calculate the discounts in both orders to see which provides the smaller discount, which is usually preferred by a place of business.

GUIDING QUESTIONS

Elicit student thinking:

- How is the part of a whole related to a percent?
- How are equivalent ratios used with percent?
- How are proportions related to percent?

Determine if the student is ready to SOLVE MULTI-STEP PROBLEMS INVOLVING RATIO AND PERCENT:

- What is the first step to calculate the final price Marie will pay for her jeans?
- What is the next step to calculate the final price Marie will pay for her jeans?
- What is the overall monetary discount for Marie's purchase?
- What values will be used in the percent change formula?
- What proportion can be used to find the percent change?
- Without changing the order in which the discounts are applied, can you think of another technique to calculate the overall percent change?

Students also need to understand that stacking percentages requires intermediary steps so that the percent is applied to the correct value.

Model how to use reasoning to solve a stacked percent problem with the following example.

> Dave owns a produce stand. He grows his own crops, but sometimes he also buys local products to sell. Dave bought some local honey from a beekeeper for $4 a pound. He then marked up the price 20% for his profit. At the end of the season, Dave discounted the selling price by 25%. Did Dave make a profit on the honey at the end of the season? Explain your reasoning.

In this situation, students need to make sure to step carefully through the problem, using the given percent and current whole value to calculate each change along the way.

Then students will need to compare the final selling price to the very first value when Dave bought the honey.

Confirm that students understand how markups, discounts, and profits work in a real-world situation.

GUIDING QUESTIONS

Elicit student thinking:

- How does percent fit into the problem-solving situation?
- Can equivalent ratios apply in this situation? If so, how? If not, why not?
- Can this problem be solved with only one calculation? If so, how? If not, why not?

Determine if the student is ready to SOLVE MULTI-STEP PROBLEMS INVOLVING RATIO AND PERCENT:

- How do you know whether the price of the honey goes up or down with each percent applied?
- What is the first step to calculate the original selling price of the honey?
- Why is a markup needed for Dave to make a profit selling the local honey?
- What is the next step to calculate the discounted selling price of the honey?
- To what should the final selling price of the honey be compared to determine if Dave made a profit on the honey at the end of the season?
- Without changing the order in which the percentages are applied, can you think of another technique to calculate the overall price change?

The error analysis cards for students contain a mixture of percent change problems and more general percent problem situations including discounts and markups. The cards are numbered for discussion's sake, but they can be completed in any order.

Distribute the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT to each student. Students can work in pairs or groups of three to facilitate discussion. Students will benefit from working in small groups as they explore and discuss percent problem situations. **Distribute** the INSTRUCTIONAL ACTIVITY SUPPLEMENT to each group of students.

With their group members, students will discuss and explore the error analysis cards on the INSTRUCTIONAL ACTIVITY SUPPLEMENT. Students may use calculators or scratch paper to work out the problems. Students have three tasks with each card.

First, students need to identify and describe the error in the worked problem. The errors will be found in various places through the calculations as a whole, but only one error exists on each card.

Then, students will provide a tip to help avoid this error in the future. This step is designed to help students process the correct steps for working through the calculations.

Finally, students will rework the problem correctly. Students should use the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT to record the three tasks for each card.

GUIDING QUESTIONS

Elicit student thinking:

- What are you looking for to solve this problem?
- What information seems important?
- Is there extra information that you don't need?
- Have you encountered a similar situation in your life that you can connect to this problem-solving situation?

Determine if the student can SOLVE MULTI-STEP PROBLEMS INVOLVING RATIO AND PERCENT:

- Can you identify and describe the purpose for each number in the problem situation?
- How do the values relate to each other?
- How do the values relate to a percent?
- What piece of information is missing to be found in the percent problem situation?
- What formula or procedure is needed for this calculation?
- What steps do you see in the percent calculation?
- Can you describe the procedure followed for the percent calculation?
- If you work the problem out, what is your answer? How does that value compare to the problem situation?
- Would you follow the same procedure shown in the percent calculation?
- Does the given solution make sense?
- Do the given calculations follow from each other?

Students should be required to complete all three tasks labeled on the error analysis cards. Students should justify their solutions and explain their reasoning.

At the end of the activity, teachers should have volunteers share a card of their choice with the class. Students should focus on clearly describing the three tasks labeled on the error analysis cards.

Multi-Step Ratio and Percent Problems

Lesson 3

The following pages contain work pages where students can record their answers as they complete each task for the Percent Error Analysis cards. Students have three tasks on each card: identify and describe the error, rework the problem, and provide a tip to prevent the error in the future. The cards may be completed in any order. Also provided with this lesson is an Instructional Activity Supplement containing 16 Percent Error Analysis cards.

Name_____

Percent Error Analysis Problem #1

Notes:

Identify and describe the error.

Rework the problem.

Here's a tip…

Percent Error Analysis Problem #2

Notes:

Identify and describe the error.

Rework the problem.

Here's a tip…

Name_____

Percent Error Analysis Problem #3

Notes:

Identify and describe the error.

Rework the problem.

Here's a tip…

Percent Error Analysis Problem #4

Notes:

Identify and describe the error.

Rework the problem.

Here's a tip…

Name_____

Percent Error Analysis Problem #5 Notes:	Identify and describe the error.
Rework the problem.	Here's a tip…

Percent Error Analysis Problem #6 Notes:	Identify and describe the error.
Rework the problem.	Here's a tip…

Name_____

Percent Error Analysis Problem #7 Notes:	Identify and describe the error.
Rework the problem.	Here's a tip…

Percent Error Analysis Problem #8 Notes:	Identify and describe the error.
Rework the problem.	Here's a tip…

Name_____

Percent Error Analysis Problem #9 Notes:	Identify and describe the error.
Rework the problem.	Here's a tip…

Percent Error Analysis Problem #10 Notes:	Identify and describe the error.
Rework the problem.	Here's a tip…

Name_____

Percent Error Analysis Problem #11 Notes:	Identify and describe the error.
Rework the problem.	Here's a tip…

Percent Error Analysis Problem #12 Notes:	Identify and describe the error.
Rework the problem.	Here's a tip…

Name_____

Percent Error Analysis Problem #13 Notes:	Identify and describe the error.
Rework the problem.	Here's a tip…

Percent Error Analysis Problem #14 Notes:	Identify and describe the error.
Rework the problem.	Here's a tip…

Name_____

Percent Error Analysis Problem #15 Notes:	Identify and describe the error.
Rework the problem.	Here's a tip…

Percent Error Analysis Problem #16 Notes:	Identify and describe the error.
Rework the problem.	Here's a tip…

Multi-Step Ratio and Percent Problems

INSTRUCTIONAL ACTIVITY SUPPLEMENT

Lesson 3

The following pages contain 16 Percent Error Analysis cards. Students have three tasks on each card: identify and describe the error, rework the problem, and provide a tip to prevent the error in the future. The cards may be completed in any order. Also provided with this lesson is an Instructional Activity Student Handout where students can record their answers as they complete each task for each card.

Percent Error Analysis #1

Look at the percent situation below. Identify and describe the error in the worked problem. Then, provide a tip to help the student avoid this error in the future. Finally, correctly rework the problem.

Giselle is shopping for a party dress. Her favorite is $79.99 and on sale for 30% off. Giselle also has a coupon for an additional 20% off. Doing some quick calculations, Giselle decides she will save 50% on her dress. Perfect!

Identify and describe the error.

Rework the problem.

Here's a tip…

Percent Error Analysis #2

Look at the percent situation below. Identify and describe the error in the worked problem. Then, provide a tip to help the student avoid this error in the future. Finally, correctly rework the problem.

Adonis is shopping for a new pair of shorts. He finds a pair of shorts regularly priced $35 that are on sale for 40% off. Adonis calculates that he will pay $14.

Identify and describe the error.

Rework the problem.

Here's a tip…

Percent Error Analysis #3

Look at the percent calculation below. Identify and describe the error in the worked problem. Then, provide a tip to help the student avoid this error in the future. Finally, correctly rework the problem.

Cam is watching a pair of boots on an online auction site. The price of the boots increased from \$55 to \$66. Cam does the following calculation to find the percent of increase.

$$\frac{66}{55} = 1.2 \rightarrow 120\% \text{ increase}$$

Identify and describe the error.

Rework the problem.

Here's a tip…

Percent Error Analysis #4

Look at the percent calculation below. Identify and describe the error in the worked problem. Then, provide a tip to help the student avoid this error in the future. Finally, correctly rework the problem.

Vic ruined his baseball glove in the rain. When looking for a new one, he finds a glove that has decreased from \$54 to \$36.

Vic does the following calculation to find the percent of decrease.

$$\frac{54-36}{36}=\frac{18}{36}=0.50\rightarrow 50\%\text{ decrease}$$

Identify and describe the error.

Rework the problem.

Here's a tip…

Percent Error Analysis #5

Look at the percent calculation below. Identify and describe the error in the worked problem. Then, provide a tip to help the student avoid this error in the future. Finally, correctly rework the problem.

Jay decreased his time in the mile run from 10 minutes to 8 minutes. Jay calculated the following to tweet out his achievement.

$$\frac{8-10}{10} = \frac{-2}{10} = -0.2$$

#-20%decrease!

Identify and describe the error.

Rework the problem.

Here's a tip…

Percent Error Analysis #6

Look at the percent calculation below. Identify and describe the error in the worked problem. Then, provide a tip to help the student avoid this error in the future. Finally, correctly rework the problem.

Ben's weekly salary increased from \$200 to \$260. He is so excited to calculate his percent of change.

$$\frac{200 + 260}{200} = \frac{460}{200} = 2.30$$

230% increase!

Identify and describe the error.

Rework the problem.

Here's a tip…

Percent Error Analysis #7

Look at the percent calculation below. Identify and describe the error in the worked problem. Then, provide a tip to help the student avoid this error in the future. Finally, correctly rework the problem.

Cate had \$2,450.80 in her bank account. Currently, she has \$1,973.40 in her account. In her financial records, she lists a 19% increase based on her calculation.

$$\frac{2450.80-1973.40}{2450.80} = \frac{477.40}{2450.80} = 0.19$$

Identify and describe the error.

Rework the problem.

Here's a tip…

Percent Error Analysis #8

Look at the percent calculation below. Identify and describe the error in the worked problem. Then, provide a tip to help the student avoid this error in the future. Finally, correctly rework the problem.

The jewelry store had a watch sale. The watch Jeremy wants is priced $75 after a 40%-off discount. He calculates the original price below.

$\$75 \times 0.4 = \$30$ discount

$\$75 + \$30 = \$105$ original price

Identify and describe the error.

Rework the problem.

Here's a tip…

Percent Error Analysis #9

Look at the percent situation below. Identify and describe the error in the worked problem. Then, provide a tip to help the student avoid this error in the future. Finally, correctly rework the problem.

A car dealer raised the price of a car from $10,500 to $12,000.

Evie decided that the percent increase is 12.5% because $1500 is 12.5% of $12,000.

Identify and describe the error.

Rework the problem.

Here's a tip…

Percent Error Analysis #10

Look at the percent situation below. Identify and describe the error in the worked problem. Then, provide a tip to help the student avoid this error in the future. Finally, correctly rework the problem.

Steven bought three bags of kiwis. Each bag had a label saying "15% off".

Steven figures that altogether he saves 45%.

Identify and describe the error.

Rework the problem.

Here's a tip…

Percent Error Analysis #11

Look at the percent situation below. Identify and describe the error in the worked problem. Then, provide a tip to help the student avoid this error in the future. Finally, correctly rework the problem.

Marcus earns $280 per week. He has just learned that his boss is giving him a 26% pay raise.

Marcus is already starting to plan how he will spend his $306 per week!

Identify and describe the error.

Here's a tip…

Rework the problem.

Percent Error Analysis #12

Look at the percent situation below. Identify and describe the error in the worked problem. Then, provide a tip to help the student avoid this error in the future. Finally, correctly rework the problem.

A newly released movie cost $24.99. Jaina waited to buy it until the price dropped. Now it is $19.99.

Jaina now says she won't buy the movie yet because it has only decreased in price by 5%.

Identify and describe the error.

Rework the problem.

Here's a tip…

Percent Error Analysis #13

Look at the percent situation below. Identify and describe the error in the worked problem. Then, provide a tip to help the student avoid this error in the future. Finally, correctly rework the problem.

Dylan is saving to buy the newest video game. His dad says he needs to increase his lawn mowing business by 20% to afford the video game.

"That's 20 more customers!" exclaims Dylan. Then his dad corrects him.

Identify and describe the error.

Rework the problem.

Here's a tip…

Percent Error Analysis #14

Look at the percent situation below. Identify and describe the error in the worked problem. Then, provide a tip to help the student avoid this error in the future. Finally, correctly rework the problem.

Calia is examining population data for a school report. She finds that two different states both had population increases of 13%. Calia then reads that the first state increased by 500,000 people, so she decides the other state must have also.

Identify and describe the error.

Rework the problem.

Here's a tip…

Percent Error Analysis #15

Look at the percent situation below. Identify and describe the error in the worked problem. Then, provide a tip to help the student avoid this error in the future. Finally, correctly rework the problem.

Imari works at an electronics store. She knows that most items at her store are marked up by 40%. So, she decides she can discount items by 40% without losing money for the store.

Identify and describe the error.

Rework the problem.

Here's a tip…

Percent Error Analysis #16

Look at the percent situation below. Identify and describe the error in the worked problem. Then, provide a tip to help the student avoid this error in the future. Finally, correctly rework the problem.

Ethan bought a collectible trading card for $4. He wants to sell it and earn a 100% profit. To attract customers Ethan prices it at $12 and then marks a 50% discount.

Identify and describe the error.

Rework the problem.

Here's a tip…

MULTI-STEP RATIO AND PERCENT PROBLEMS

INSTRUCTIONAL ACTIVITY

Lesson 4

LEARNING GOAL

Students will apply an understanding of solving multi-step ratio and percent problems.

PRIMARY ACTIVITY

When given two problem situations, students will decide which situation is a better option for them. Students will need to calculate with percent and provide mathematical reasoning to support their preferred problem situation.

OTHER VOCABULARY

Students will need to know the meaning of the following terms:

- Percent
- Simple interest
- Markup
- Discount
- Sale
- Tip
- Tax
- Fees
- Commission
- Percent increase

MATERIALS

- Calculator
- Scratch paper
- INSTRUCTIONAL ACTIVITY STUDENT HANDOUT

- INSTRUCTIONAL ACTIVITY SUPPLEMENT (Recommended one copy for every group of four to five students.)

IMPLEMENTATION

This activity combines multiple concepts and problem situations with percent. Students should be familiar with a variety of percent problem types from the previous lessons in this unit.

Begin with a collaborative problem to remind students how to work with percent problem-solving situations. Using the following problem, have students decide whether they agree with Layna's conclusion. Students should discuss problem solving techniques and justification for agreement or disagreement.

> Layna received a 15% pay raise. Her brother received a 10% pay raise. Layna determined that her pay raise is 5% greater than her brother's pay raise, so her raise must be better.

GUIDING QUESTIONS

Elicit student thinking:

- What information do you need from the problem-solving situation to make a conclusion?
- Do ratios or proportions apply to this situation? Explain.
- How do the different values relate to each other in the problem-solving situation?
- How can you compare percentages?

Determine if the student can SOLVE MULTI-STEP PROBLEMS INVOLVING RATIO AND PERCENT:

- What assumptions did Layna make?
- What mistake in reasoning about percent did Layna make?
- How did Layna come up with the number 5%?
- What procedure would you follow to check Layna's conclusion?
- Can you provide an example to test Layna's conclusion?
- How could Layna's conclusion be correct? Explain.
- How could Layna's conclusion be incorrect? Explain.

Briefly **discuss** the difference between compound interest versus simple interest. Students may have experience with either or both types of interest, but at this time they only need to be able to calculate simple interest.

Model how to use reasoning to solve a simple interest problem with the following example.

Polly wants a new bike. She has been saving money and has $150 to spend. The bike she wants is $245. Her uncle is loaning her the money she needs at a simple interest rate of 6%.

How much will Polly need to pay back her uncle in a year?

How much would Polly need to pay back her uncle in 6 months?

How much would Polly need to pay back her uncle in 3 months?

GUIDING QUESTIONS

Elicit student thinking:

- What information do you need from the problem-solving situation for your calculation?
- Do ratios or proportions apply to this situation? Explain.
- How do the different values relate to each other in the problem-solving situation?
- How does interest work on borrowed money?
- How does interest work on invested money?
- How is an interest rate connected to the value of the money?

Determine if the student can SOLVE MULTI-STEP PROBLEMS INVOLVING RATIO AND PERCENT:

- How much money is Polly borrowing from her uncle?
- What does the 6% interest rate mean?
- Will Polly pay her uncle back more or less than she borrows? Explain.
- How much in interest will Polly accrue for each of the given time frames?
- How do you find the total amount of money Polly will have to pay back?
- Does Polly pay her uncle back more or less money if she pays him back faster? Explain your reasoning.

Distribute the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT to each student. **Distribute** the INSTRUCTIONAL ACTIVITY SUPPLEMENT to each group of students.

Individually, students should consider each problem-solving situation on the "Would You Rather..." cards. Students may use calculators or scratch paper to work out the problems. **Check** that students are supporting their decisions with mathematical reasoning. This reasoning may include information found from calculations completed for each option given. Students should record their initial decision with reasoning on the INSTRUCTIONAL ACTIVITY STUDENT HANDOUT.

Next, **require** students to discuss their decisions in small groups. Students benefit not only from discussing their personal justification, but also from hearing the justification provided by others. Students may choose to change their decisions based on the justification provided by members of their group, but students need to be able to explain why they changed their mind.

GUIDING QUESTIONS

Elicit student thinking:

- Can you identify what information you need to make your choice?
- How do the values relate to each other?
- How do the values relate to a ratio or percent?

Determine if the student can SOLVE MULTI-STEP PROBLEMS INVOLVING RATIO AND PERCENT:

- Can you identify and describe the purpose for each number in the problem situation?
- What formula or procedure is needed for this calculation?
- What is the purpose of percent in this problem situation?
- How does a ratio apply in the problem situation?
- How do the two situations compare to each other?
- Which would be a better outcome for this situation?

Students should be required to justify their decision with mathematics. Students should practice using precise vocabulary as they construct viable arguments and critique the reasoning of others.

At the end of the activity, teachers should give students the following task as closure.

> Would you rather use one coupon for 70% off or three discounts: first a 50% discount followed by two 10%-off coupons? Explain your choice with mathematics.

Multi-Step Ratio and Percent Problems

Lesson 4

The following pages contain work pages where students can record their reasoning as they make decisions for the "Would You Rather…" cards. The cards may be completed in any order, but they should be correlated to the matching numbered work spaces. Also provided with this lesson is an Instructional Activity Supplement containing 10 "Would You Rather…" cards.

Name_____

Would You Rather… **#1**

Explain your choice with mathematics.

I would rather

because

Would You Rather… **#2**

Explain your choice with mathematics.

I would rather

because

Name_____

Would You Rather… **#3**

Explain your choice with mathematics.

I would rather

because

Would You Rather… **#4**

Explain your choice with mathematics.

I would rather

because

Name_____

Would You Rather… **#5**

Explain your choice with mathematics.

I would rather

because

Would You Rather… **#6**

Explain your choice with mathematics.

I would rather

because

Name_____

Would You Rather… #7

Explain your choice with mathematics.

I would rather

because

Would You Rather… #8

Explain your choice with mathematics.

I would rather

because

Name_____

Would You Rather… **#9**

Explain your choice with mathematics.

I would rather

because

Would You Rather… **#10**

Explain your choice with mathematics.

I would rather

because

Multi-Step Ratio and Percent Problems

INSTRUCTIONAL ACTIVITY SUPPLEMENT

Lesson 4

The following pages contain 10 "Would You Rather…" cards. Students have two options to choose from in a percent problem-solving situation. Students will need to consider and calculate for each of the two options as they are instructed to explain their choice using mathematics. The cards may be completed in any order. Also provided with this lesson is an Instructional Activity Student Handout where students can record their answers as they make a decision for each card.

Would You Rather…

#1

have the interest earned on $100 at a simple interest rate of 4% for 10 years?	**OR**	have the interest earned on $90 at a simple interest rate of 10% for four years?

Explain your choice with mathematics.

I would rather

because

Would You Rather…

#2

buy a used $50 video system marked up by 80%?

OR

buy a used $80 video system marked up by 50%?

Explain your choice with mathematics.

I would rather

because

Would You Rather...

#3

buy a $136 bike on sale for 30% off?

OR

buy a $136 bike on sale for $50 off?

Explain your choice with mathematics.

I would rather

because

Would You Rather…

#4

be a server earning an 18% tip on a $60 bill?

OR

be a server earning a 20% tip on a $55 bill?

Explain your choice with mathematics.

I would rather

because

Would You Rather…

#5

pay 9.5% taxes on $250? **OR** pay 8.8% taxes on $280?

Explain your choice with mathematics.

I would rather

because

Would You Rather…

#6

pay 4% ATM fees on $250?

OR

pay 3% ATM fees on $375?

Explain your choice with mathematics.

I would rather

because

Would You Rather…

#7

be a salesperson earning 2% commission on a sale of $2200?	**OR**	be a salesperson earning 10% commission on a sale of $560?

Explain your choice with mathematics.

I would rather

because

Would You Rather…

#8

own a flock of 25 chickens that increases by 160%?

OR

own a flock of 50 chickens that increases by $\frac{1}{5}$?

Explain your choice with mathematics.

I would rather

because

Would You Rather…

#9

buy t-shirts at a "Buy One Get One Free" sale?

OR

buy t-shirts at a 60%-off sale?

Explain your choice with mathematics.

I would rather

because

Would You Rather...

#10

make a purchase with a $5-off coupon and a 10% discount?

OR

make a purchase with a 15%-off coupon?

Explain your choice with mathematics.

I would rather

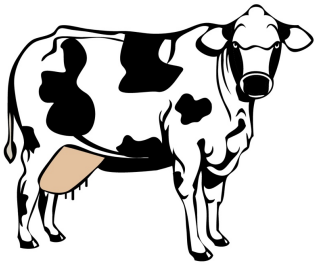
because

Name_____

Multi-Step Ratio and Percent Problems

Lessons 1 – 4

1. A rancher's herd of dairy cattle is 10% smaller than it was the previous year. The rancher figured out that if each cow can increase milk production by 10%, then milk production will be the same as last year.



1.a. Do you agree with the rancher? Why or why not?

1.b. Test the rancher's reasoning using two example dairy cattle herd sizes. Assume each cow produces six gallons of milk per day.

Name_____

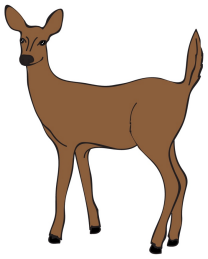
1.c. Do you still agree with your answer from 1.a.? Why or why not?

1.d. Describe which value you would use when calculating the 10% herd size decrease from the previous year.

1.e. Describe which values you would use when calculating the 10% increase in milk production.

Name_____

2. Mario read that the population of deer in a nature reserve increases by 20% each year. He declared that the population would, therefore, double after five years.



2.a. What mistake in reasoning about percentages did Mario make?

2.b. Using two example deer populations, explain why Mario's reasoning is incorrect.

2.c. What does it mean for a population to "double" in terms of percent?

2.d. How do you think Mario came up with this percent based on what he read?

Name_____

3. An increase of 36 orchestra students represents a 90% increase to the orchestra size.

3.a. Should the original value be more or less than 36? Explain your reasoning.

3.b. What is the original orchestra size? Explain your answer.

3.c. What is the final orchestra size? Explain your answer.

4. Describe and correct the error in finding the percent increase from an initial value of 14 to a new value of 26.

$$\frac{26 - 14}{26} \approx 0.46 = 46\% \text{ increase}$$

Name_____

5. Amelia cut a wood plank into three parts. Part A is 71 centimeters long, and Part B is 41 centimeters long. Part C is the remaining length.

5.a. How do Parts A and B compare to the original wood plank? Describe in words and a ratio.

5.b. If the total length of A and B is 70% of the length of the original wood plank, what is the length of Part C? Explain your answer.

Name_____

6. A $45 sweater is on sale with a 15% discount. Sales tax is 6%.

6.a. How much, in dollars, is the discount on the sweater?

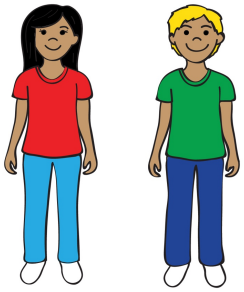
6.b. What is the price of the sweater after the discount?

6.c. How much sales tax will be charged on the discounted sweater?

6.d. What will be the total cost, including tax? Explain your answer.

Name_____

7. There were 28 boys and 25 girls in a math club last year. This year the number of boys increased by 25%, but the number of girls decreased by 20%. Was there an increase or decrease in overall membership? Explain your reasoning.



MULTI-STEP RATIO AND PERCENT PROBLEMS

STUDENT ACTIVITY SOLUTION GUIDE

Lessons 1 – 4

1. A rancher's herd of dairy cattle is 10% smaller than it was the previous year. The rancher figured out that if each cow can increase milk production by 10%, then milk production will be the same as last year.

   1.a. Do you agree with the rancher? Why or why not?

CORRECT ANSWER

Student reasoning is the most important part of this answer. Two possible answers are shown below.

Yes, I agree with the rancher because decreasing the herd by 10% means less milk, but increasing the milk production by 10% raises milk production back up.

OR

No, I don't agree with the rancher because decreasing the herd by 10% will decrease the herd population, but increasing by the same amount on a smaller herd size will not regain the lost milk production.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Student agrees or disagrees without any justification.	does not explain answer	EXPLAIN *PERCENT*
Student agrees or disagrees with invalid justification.	incorrectly calculates using percent and whole to provide invalid justification	SOLVE PERCENT PROBLEMS FINDING THE PART GIVEN THE PERCENT AND THE WHOLE

1.b. Test the rancher's reasoning using two example dairy cattle herd sizes. Assume each cow produces six gallons of milk per day.

CORRECT ANSWER

1) If the rancher had 100 cows last year and each cow produced 6 gallons a day, the rancher got 600 gallons of milk per day last year.

10% fewer cows would be 10 less cows, or a total of 90 cows. If each cow increase its six gallons of milk by 10%, an additional 0.6 gallons is added for a daily total of 6.6 gallons per cow. The total herd of 90 cows would then produce 594 gallons of milk per day this year.

2) If the rancher had 20 cows last year and each cow produced six gallons a day, the rancher got 120 gallons of milk per day last year.

10% fewer cows would be two less cows, or a total of 18 cows. If each cow increase its six gallons of milk by 10%, an additional 0.6 gallons is added for a daily total of 6.6 gallons per cow. The total herd of 18 cows would then produce 118.8 gallons of milk per day this year.

The rancher's reasoning is incorrect.

A variety of possible herd sizes could be used. Check to make sure students are calculating correctly with the chosen herd size.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
If the rancher had 100 cows last year, 10% fewer cows would be 10 less cows, or a total of 90 cows. A 10% increase would then be nine more cows for a new total of 99 cows.	does not calculate and compare milk production with the 10% increase in milk production	SOLVE MULTI-STEP PROBLEMS INVOLVING RATIO AND PERCENT
If the rancher had 100 cows last year and each cow produced six gallons a day, the rancher got 600 gallons of milk per day. 10% fewer cows would be 10 cows, or a total of 110 cows. If each cow increases its six gallons of milk by 10%, an additional 0.6 gallons is added for a daily total of 6.6 gallons per cow. The total herd of 110 cows would then produce 726 gallons of milk per day this year.	calculates an increase rather than a decrease for the herd size	SOLVE MULTI-STEP PROBLEMS INVOLVING RATIO AND PERCENT

1.c. Do you still agree with your answer from 1.a.? Why or why not?

CORRECT ANSWER

Student reasoning is the most important part of this answer. Two possible answers are shown below.

Yes, because my tests from 1.b. support my statement from 1.a.

No, in 1.b. I found that the opposite happened from what I had expected.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Student agrees or disagrees without any justification.	does not explain answer	EXPLAIN *PERCENT*
Student misinterprets the results of calculations from 1.b. to determine agreement or disagreement.	does not understand the results from trial percent calculations	EXPLAIN *PERCENT*
Student does not consider the results from 1.b. to reevaluate conclusion in 1.a.	does not understand how the trial calculations help analyze the situation	COMPARE RATIOS IN ANY REPRESENTATION

1.d. Describe which value you would use when calculating the 10% herd size decrease from the previous year.

CORRECT ANSWER

> I need to use the amount of the whole herd from last year to calculate the 10% herd size decrease.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
I would use the amount of milk from last year to calculate the 10% decrease.	does not understand the whole value in the percent situation	SOLVE PERCENT PROBLEMS FINDING THE PART GIVEN THE PERCENT AND THE WHOLE
I would use the amount of cows in the herd this year to calculate the 10% decrease.	uses the value of the part to calculate instead of the whole value	SOLVE PERCENT PROBLEMS FINDING THE PART GIVEN THE PERCENT AND THE WHOLE

1.e. Describe which values you would use when calculating the 10% increase in milk production.

CORRECT ANSWER

> I need to use the six gallons of daily milk production and increase it by 10%. Then I would take that increased amount times the new amount of the whole herd from this year to find the new total milk production.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
I would use the amount of cows from last year to calculate the 10% increase in milk production.	does not correctly identify the whole value related to the percent calculation	SOLVE PERCENT PROBLEMS FINDING THE PART GIVEN THE PERCENT AND THE WHOLE
I would use the amount of cows in the herd this year to calculate the 10% increase in milk production.	does not correctly identify the whole value related to the percent calculation	SOLVE PERCENT PROBLEMS FINDING THE PART GIVEN THE PERCENT AND THE WHOLE

2. Mario read that the population of deer in a nature reserve increases by 20% each year. He declared that the population would, therefore, double after five years.

2.a. What mistake in reasoning about percentages did Mario make?

CORRECT ANSWER

> Mario forgot that the 20% increase each year would be calculated based on the new population for each year.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Mario said five years would double the poplation when double means two. Mario should have said the population will double in two years.	does not apply the percent change and strictly correlates the word "double" with "two", incorrectly stating it will take two years to double the population	SOLVE MULTI-STEP PROBLEMS INVOLVING RATIO AND PERCENT
Mario made no mistake, 20% times five would be an additional 100% to double the population.	does not apply the percent to the correct whole value	SOLVE PERCENT PROBLEMS FINDING THE PART GIVEN THE PERCENT AND THE WHOLE

2.b. Using two example deer populations, explain why Mario's reasoning is incorrect.

CORRECT ANSWER

1) If the deer population is 100 in the first year, a 20% increase would add 20 deer for a new population of 120 in the second year. A 20% increase in the second year would add 24 deer for a new population of 144 in the third year. A 20% increase in the third year would add about 29 deer for a new population of 173 in the fourth year. A 20% increase in the fourth year would add about 35 deer for a new population of 208 in the fifth year. The deer population would be doubled *before* the fifth year.
2) If the deer population is 20 in the first year, a 20% increase would add four deer for a new population of 24 in the second year. A 20% increase in the second year would add five deer for a new population of 29 in the third year. A 20% increase in the third year would add about six deer for a new population of 35 in the fourth year. A 20% increase in the fourth year would add about seven deer for a new population of 43 in the fifth year. The deer population would be doubled *before* the fifth year.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Mario is correct because 20% + 20% + 20% + 20% + 20% = 100%. 100% increase doubles the population.	adds the percentages instead of calculating the percent increase	SOLVE MULTI-STEP PROBLEMS INVOLVING RATIO AND PERCENT
If the deer population is 100 in the first year, a 20% increase would add 20 deer for a new population of 120 in the second year. Another 20% increase in the second year would add 20 deer for a new population of 140 in the third year. A 20% increase in the third year would add about 20 deer for a new population of 160 in the fourth year. A 20% increase in the fourth year would add about 20 deer for a new population of 200 in the fifth year. The deer populuation would be doubled in the fifth year.	does not apply the percent to each increased population	SOLVE PERCENT PROBLEMS FINDING THE PART GIVEN THE PERCENT AND THE WHOLE

2.c. What does it mean for a population to "double" in terms of percent?

CORRECT ANSWER

If a population is doubled, there is 200% of the original population.

OR

If a population is doubled there is 100% more than before.

OR

A doubled population has a percent increase of 100%.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
If a population is doubled, there is 50% of the original whole.	confuses doubling a population with halving a population	REPRESENT PROPER FRACTIONS AS A PERCENT
If a population is doubled, there is 200% more than before.	confuses "200% more" with a value (part) that is 200% of the original whole value	SOLVE PERCENT PROBLEMS FINDING THE PERCENT GIVEN THE WHOLE AND THE PART
A doubled population has a percent increase of 200%.	confuses a "percent increase of 200%" with a value (part) that is 200% of the original whole value	SOLVE PERCENT PROBLEMS FINDING THE PERCENT GIVEN THE WHOLE AND THE PART

2.d. How do you think Mario came up with this percent based on what he read?

CORRECT ANSWER

Mario added 20% repeatedly until he found an additional 100% population change to double the population.

OR

20% goes into 100% five times, so Mario decided the five times meant five years until the population is doubled.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Does not recognize that 20% times five equals 100%, which is related to doubling a population.	multiplication fact error	MULTIPLY RATIONAL NUMBERS

3. An increase of 36 orchestra students represents a 90% increase to the orchestra size.

3.a. Should the original value be more or less than 36? Explain your reasoning.

CORRECT ANSWER

The original value should be more than 36 because a 90% increase is less than doubling, so the 36 is not quite as large as the original whole value.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The original value should be less than 36 because the value increases.	confuses a whole value with a percent increase part	SOLVE MULTI-STEP PROBLEMS INVOLVING RATIO AND PERCENT

3.b. What is the original orchestra size? Explain your answer.

CORRECT ANSWER

The original orchestra size is 40 students. 90% of 40 is 36, so the orchestra was originally 40 students.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The original orchestra size is 32 students. 90% of 36 is about 32, so the orchestra was originally 32 students.	switches the part for the whole value in calculation	SOLVE PERCENT PROBLEMS FINDING THE WHOLE GIVEN THE PART AND THE PERCENT
The original orchestra size is 76 students. 90% of 40 is 36, so the orchestra was originally 76 students or 40 + 36.	confuses the original value with the new total value	REPRESENT PROPORTIONAL RELATIONSHIPS BY EQUATIONS COMPOSED OF 2 EQUIVALENT RATIOS

3.c. What is the final orchestra size? Explain your answer.

CORRECT ANSWER

> The final orchestra size is 76 students. 90% of 40 is 36, so the orchestra was originally 40 students. Then add 40 and 36 for a final total of 76 students.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The final orchestra size is 40 students. 90% of 40 is 36, so the final orchestra size is 40 students.	confuses the original value with the new total value	SOLVE MULTI-STEP PROBLEMS INVOLVING RATIO AND PERCENT
The final orchestra size is 68 students. 90% of 36 is about 32, so the final orchestra size is 36 + 32 or 68 students.	interchanges the part for the whole value in calculation	REPRESENT PROPORTIONAL RELATIONSHIPS BY EQUATIONS COMPOSED OF 2 EQUIVALENT RATIOS

4. Describe and correct the error in finding the percent increase from an initial value of 14 to a new value of 26.

$$\frac{26 - 14}{26} \approx 0.46 = 46\% \text{ increase}$$

CORRECT ANSWER

The correct formula is $\frac{amount\ of\ change}{initial\ value}$, but the problem was worked using $\frac{amount\ of\ change}{new\ value}$.

The problem should be worked $\frac{26 - 14}{14} = \frac{12}{14} \approx 0.86 = 86\%$ increase.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The problem was worked correctly, but the final answer should be labeled as a decrease because 26 is larger than 14. So the answer should be a 46% decrease from 26 to 14.	interchanges the initial value and the new value	REPRESENT PROPORTIONAL RELATIONSHIPS BY EQUATIONS COMPOSED OF 2 EQUIVALENT RATIOS
The amount of change was miscalculated. Because 14 is the initial value, amount of change should be 14 – 26 = -12. Then the formula $\frac{amount\ of\ change}{initial\ value}$ gives $\frac{14 - 26}{26} = \frac{-12}{26} \approx -0.46 = -46\%$ decrease.	does not represent amount of change as a positive value and uses new value instead of initial value in the formula	SOLVE MULTI-STEP PROBLEMS INVOLVING RATIO AND PERCENT
The amount of change was miscalculated. Because 14 is the initial value, amount of change should be 14 – 26 = -12. And the new value was used instead of the initial value. Then the formula $\frac{amount\ of\ change}{initial\ value}$ gives $\frac{14 - 26}{14} = \frac{-12}{14} \approx -0.86 = -86\%$ decrease.	does not represent amount of change as a positive value	SOLVE MULTI-STEP PROBLEMS INVOLVING RATIO AND PERCENT

5. Amelia cut a wood plank into three parts. Part A is 71 centimeters long, and Part B is 41 centimeters long. Part C is the remaining length.

5.a. How do Parts A and B compare to the original wood plank? Describe in words and a ratio.

CORRECT ANSWER

Part A and Part B are smaller sections of the original wood plank. Together their lengths equal less than the total original length.

Three ratios could be used: 1) length of Part A : original length, 2) length of Part B : original length, and 3) length of Part A + length of Part B : original length.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Part A and Part B are sections of the original wood plank equaling the total original length. Three ratios could be used: 1) length of Part A : original length, 2) length of Part B : original length, and 3) length of Part A + length of Part B : original length , which equals a 1 : 1 ratio.	does not account for Part C in the length of the original plank	EXPLAIN *PART-TO-WHOLE RATIO*

5.b. If the total length of A and B is 70% of the length of the original wood plank, what is the length of Part C? Explain your answer.

CORRECT ANSWER

Length of Part A + length of Part B = 71 + 41 = 112

112 is 70% of the original length. Using a proportion $\frac{112}{original\ length} = \frac{70}{100}$ or $\frac{112}{original\ length} = \frac{7}{10}$.

The original length is 160 centimeters.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Length of Part A + length of Part B = 71 + 41 = 112 The original length is 112 centimeters.	does not account for Part C in the length of the original plank	EXPLAIN *PART-TO-WHOLE RATIO*
Length of Part A - length of Part B = 71 - 41 = 30 30 is 70% of the original length. Using a proportion $\frac{30}{original\ length} = \frac{70}{100}$ or $\frac{30}{original\ length} = \frac{7}{10}$. The original length is $42\frac{6}{7}$ centimeters.	subtracts the lengths of Part A and Part B instead of adding them	SOLVE MULTI-STEP PROBLEMS INVOLVING RATIO AND PERCENT

6. A $45 sweater is on sale with a 15% discount. Sales tax is 6%.

6.a. How much, in dollars, is the discount on the sweater?

CORRECT ANSWER

15% of $45 is $6.75.

The discount on the sweater is $6.75.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
6% of $45 is $2.70. The discount on the sweater is $2.70.	interchanges the percent discount and the sales tax percent	SOLVE PERCENT PROBLEMS FINDING THE PART GIVEN THE PERCENT AND THE WHOLE
15% of $45 is $675. The discount on the sweater is $675.	does not use percent as a ratio out of 100	EXPLAIN *PERCENT*
The discount on the sweater is $15.	does not use the percent to calculate the part which would be the discount	SOLVE PERCENT PROBLEMS FINDING THE PART GIVEN THE PERCENT AND THE WHOLE

6.b. What is the price of the sweater after the discount?

CORRECT ANSWER

$45 – $6.75 = $38.25.

The price of the sweater after the discount is $38.25.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
$45 – $2.70 = $42.30. The price of the sweater after the discount is $42.30.	interchanges the percent discount and the sales tax percent	SOLVE PERCENT PROBLEMS FINDING THE PART GIVEN THE PERCENT AND THE WHOLE
$45 – $15 = $30. The price of the sweater after the discount is $30.	does not use the percent to calculate the part which would be the discount	SOLVE PERCENT PROBLEMS FINDING THE PART GIVEN THE PERCENT AND THE WHOLE
$45 + $6.75 = $51.75. The price of the sweater after the discount is $51.75.	adds the discount when they should subtract the discount	SOLVE MULTI-STEP PROBLEMS INVOLVING RATIO AND PERCENT

6.c. How much sales tax will be charged on the discounted sweater?

CORRECT ANSWER

6% of $38.25 is $2.295.

The sales tax on the discounted sweater is $2.30.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
6% of $45 is $2.70. The sales tax on the discounted sweater is $2.70.	did not calculate a percent of the discounted price	SOLVE PERCENT PROBLEMS FINDING THE PART GIVEN THE PERCENT AND THE WHOLE
6% of $38.25 is $229.5. The sales tax on the discounted sweater is $229.50.	does not use percent as a ratio out of 100	EXPLAIN *PERCENT*
The sales tax on the discounted sweater is $6.	does not use the percent to calculate the part which would be the sales tax	SOLVE PERCENT PROBLEMS FINDING THE PART GIVEN THE PERCENT AND THE WHOLE

6.d. What will be the total cost, including tax? Explain your answer.

CORRECT ANSWER

The discount on the sweater is $6.75.

The price of the sweater after the discount is $38.25.

The sales tax on the discounted sweater is $2.30.

$38.25 + $2.30 = $40.55

The total cost of the discounted sweater, including tax, will be $40.55.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
The discount on the sweater is $2.70. The price of the sweater after the discount is $42.30. The sales tax on the discounted sweater is $2.54. $42.30 + $2.54 = $44.84 The total cost of the discounted sweater, including tax, will be $44.84.	uses the sales tax percent instead of the discount percent	SOLVE MULTI-STEP PROBLEMS INVOLVING RATIO AND PERCENT
The discount on the sweater is $15. The price of the sweater after the discount is $30. The sales tax on the discounted sweater is $1.80. $30 + $1.80 = $31.80 The total cost of the discounted sweater, including tax, will be $31.80.	does not use discount percent as a ratio out of 100	EXPLAIN *PERCENT*
The discount on the sweater is $6.75. The price of the sweater after the discount is $51.75. The sales tax on the discounted sweater is $3.11. $51.75 + $3.11 = $54.86 The total cost of the discounted sweater, including tax, will be $54.86.	adds the discount when they should subtract the discount	SOLVE MULTI-STEP PROBLEMS INVOLVING RATIO AND PERCENT
The discount on the sweater is $6.75. The price of the sweater after the discount is $38.25. The sales tax on the discounted sweater is $2.30. $38.25 - $2.30 = $35.95 The total cost of the discounted sweater, including tax, will be $35.95.	subtracts the sales tax when they should add the sales tax	SOLVE MULTI-STEP PROBLEMS INVOLVING RATIO AND PERCENT

7. There were 28 boys and 25 girls in a math club last year. This year the number of boys increased by 25%, but the number of girls decreased by 20%. Was there an increase or decrease in overall membership? Explain your reasoning. 

CORRECT ANSWER

25% of 28 is 7, so the number of boys in math club this year is 28 + 7 or 35 boys.

20% of 25 is 5, so the number of girls in math club this year is 25 – 5 or 20 girls.

Last year the total membership in math club was 28 + 25 or 53 students.

This year the total membership in math club is 35 + 20 or 55 students.

55 students > 53 students, so the overall membership of math club has increased this year.

ERRORS, MISCONCEPTIONS, AND MISSING KNOWLEDGE

Example Error	Misconception	Missing Knowledge
Because a 25% increase is greater than a 20% decrease, the overall membership of math club has increased this year.	incorrect reasoning to support the conclusion	EXPLAIN *PERCENT*
25% of 28 is 7, so the number of boys in math club this year is 28 - 7 or 21 boys. 20% of 25 is 5, so the number of girls in math club this year is 25 – 5 or 20 girls. Last year the total membership in math club was 28 + 25 or 53 students. This year the total membership in math club is 21 + 20 or 41 students. 53 students > 41 students, so the overall membership of math club has decreased this year.	calculates percent decrease for boys instead of percent increase	SOLVE MULTI-STEP PROBLEMS INVOLVING RATIO AND PERCENT
Student incorrectly calculates within the percent increase or decrease to give a total sum less than last year's membership, thus reasoning that the overall membership of math club has decreased this year.	errors calculating part in percent change	SOLVE PERCENT PROBLEMS FINDING THE PART GIVEN THE PERCENT AND THE WHOLE