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Bibliography Report

Presented by

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Making Flips With Quadrotors In Constrained Environments

Jury

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Abstract

Within the rapidly growing aerial robotics market, one of the most substantial challenges in the quadrotor community is performing aggressive maneuvers, especially multi-flip maneuvers. A proper physical definition of the issue is not addressed by the current approaches in the field and several key aspects of this maneuver are still overlooked. It can be shown, in particular, that making a flip with a quadrotor meanWs crossing the parallel singularity of the dynamic model. The aim of the master thesis is to explore the possibility of defining aggressive trajectories for quadrotors on the basis of their dynamic model degeneracy analysis and to adapt various strategies to control the robot in a closed loop. In addition, the possibility to perform the aggressive maneuver in constrained environments will also be investigated. Therefore, the analysis will be extended from the previous studied to create general feasible trajectories that will allow quadrotors to perform aggressive flip maneuvers while passing through a constrained environment and while guaranteeing a satisfactory degree of robustness to the uncertainties of the dynamic model.

Acknowledgements

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Notations

b	Thrust factor
l	horizontal distance: From the center of the propeller to the CoG
Ω	Spinning speed of a propeller
C_{R_m}	Rolling moment coefficient
C_T	Thrust coefficient
H	Hub force

Abbreviations

IGE	In Ground Effect
OGE	Out of Ground Effect

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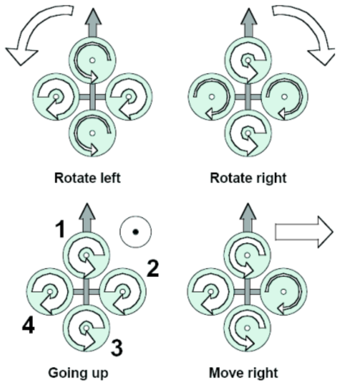
Introduction

The aim of this section is to provide a general summary of the robotic robotic platform that is used for this master thesis and to illustrate the main objective of the research work. In specific, in the sections below, quadrotors and parallel robots are briefly presented.

The quadrotor platform

A quadrotor is a type of unmanned aerial vehicle with four rotors and six degrees of freedom. Typically, drones have a small size and low inertia which allows it to be controller by simple flight control systems. It is typically designed in a cross-configuration such that the electronics are held in the center of the platform and the rotors are placed at the borders. An example of a real quadrotor, namely the DJI Phantom, is shown in fig. 1a. The quadrotor is typically built in a way such that a pair of opposite rotors rotate clockwise, whereas the other pair of rotors rotates in counter-clockwise. The attitude and the position of the drone are controlled by changing the spinning speed of the rotors. An example is shown in figure 1b.

(a) A DJI Phantom quadcopter (UAV)



(b) Typical quadrotor configuration The width of the arrows is proportional to the angular speed of the propellers.[1]

Figure 1: A commercial quadrtotor platform, with a typical quadrotor configuration.

The distinctive mechanical design of the quadrotor permits the actuation system to control all of the six degrees of freedom, even though it is under-actuated. This is due to the fact that the rotational and translational dynamics are tightly coupled. Thus, all the translational and rotational motions can be carried off by properly controlling the magnitude and direction of the spinning speed of the rotors.

Over the last few years, quadrotors have gained a large popularity in academia and in the industry. This is due to several reasons, such as:

1. Quadrotors are very simple to design and they can be easily assembled using relatively cheap components.

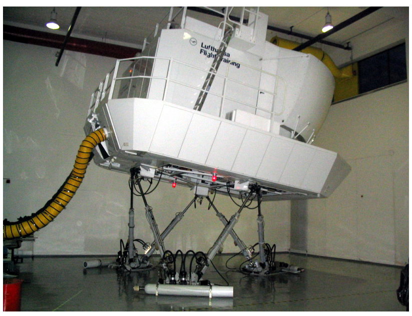
2. As quadrotors became more and more affordable and dependable, the number of quadrotors real-world applications has grown significantly. They are being used for aerial photography, agriculture, surveillance, inspection tasks, in addition to many other uses as well.

3. Quadrotors are quite agile and maneuverable during flight. Especially when compared to other types of unmanned aerial vehicles (UAVs).

However, on the main challenges in the quadrotors community is the capability to design control and planning methods that will allow the quadrotors to carry out aggressive maneuvers. The fast dynamics associated with typically small dimensions of such agile quadrotors, in addition to several aerodynamic effects that will become important during aggressive flight maneuvers, are just a few of the main problems that are faced during the system control design. Moreover, accurate tracking of the provided trajectory is a very big issue in the case of aggressive maneuvers when the rotors are commanded high speeds and accelerations, which will cause rotors to become saturated and may also cause delays.

Parallel manipulators

A parallel manipulator is a mechanical system that consists of two connected platforms, the fixed platform and the moving platform. The latter is linked to the fixed platform thanks to at least two serial chains that are working in parallel. When compared to serial manipulators, parallel manipulators are more accurate and rigid. In addition, the ability to install the motors next to the fixed platform is a very important feature for parallel manipulators. Moreover, parallel manipulators can be used in a wide variety of applications that demand precision and high payload combined with high speed.[2]



(a) Gough-Stewart used for a flight-simulator application.[1]

(b) The "PAR4" 4 degrees of freedom, high-speed, parallel robot prototype.[2]

Figure 2: Two examples of parallel robots.

[1]https://en.wikipedia.org/wiki/Stewart_platform#/media/File:Simulator-flight-compartment.jpeg

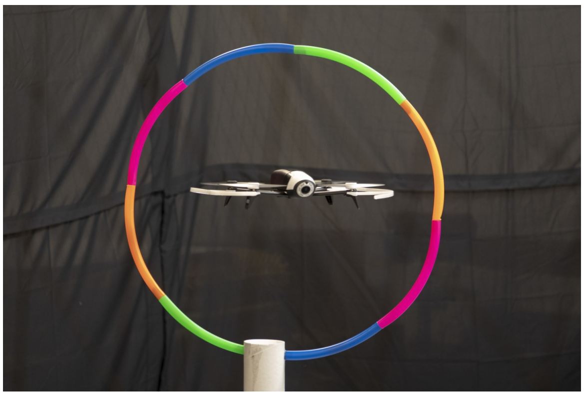
[2]https://en.wikipedia.org/wiki/Parallel_manipulator#/media/File:Prototype_robot_parall%C3%A8le_PAR4.jpg

However, parallel manipulators are subject to singularities, which can lead to big problems in the robot workspace in case they were not handled correctly. Thus, the study of the singular configuartions of parallel manipulators is very important. Because, even just before reaching a singularity, the performance of the parallel manipulator will decrease dramatically. Moreover, the robot may loose the ability of moving in a certain direction, gain uncontrollable motions and it the mechanism could even break. The main difference between serial and parallel manipulators is that singularity configurations may also appear inside the robot workspace (depending on the dimensions of the robot) and not just at the boundaries of the robot workspace, which can significantly decrease the area of the robot workspace. As a result, many works have been developed by robotics researchers in order to allow parallel manipulator manipulators to safely cross these singularities by using trajectory planning and specific control methods.

The goal of this thesis

This master thesis lies at the intersection of parallel robotics and aerial robotics. The two fields may seem very different from each other. However, quadrotors can be seen as a particular case of a parallel manipulator. In fact, a parallel manipulator is made up of a wrench system, applied by the robot limbs on the moving platform. And, this wrench system will define the motion of the moving platform. In the same manner, each propeller in a quadrotor can be considered as limb of a parallel robot and the moving platform to be controlled can be considered as the body of the drone. Specifically, the goal of this master thesis is to study a distinct class of aggressive maneuvers for quadrotors, namely multi-flip maneuvers. By doing multi-flip maneuvers, full rotations around one or more axes of the body of the quadrotor can be done. In addition, the quadrotor must also must also do the flips in a constrained environment.

(a) Quadrotor performing a triple flip.[3]



(b) Quadrotor going though a loop.[1]

Figure 3: Representation of the issues to be tackled in this master thesis.

[1] https://newatlas.com/drones/muscle-signals-drone-control/#gallery:2

Outline of the work

The rest of the bibliography is structured as follows:

Chapter 1 is devoted to introduce the system modeling of quadrotors. Specifically, a simplified dynamic model of the quadrotor will be presented by using Euler-Lagrange formalism. Then, moving on from the simple dynamic model, a more detailed dynamic model will be presented by using the Newton-Euler formalism. Finally, the state-space model of the quadrotor will also be derived.

Chapter 2 provides an overview of state of the art in quadrotor control in addition to introducing the different potential control methods that can be used during the master thesis in order to properly control the quadrotor.

Chapter 3 provides detailed explanations of how multi-flip maneuvers can be handled. Then, the link between a quadrotor performing a flip and a parallel robot crossing a singularity will be explained. In the end, a literature review is provided in order to show how the problem is tackled by different researches.

Chapter 4 is devoted to trajectory optimization. By using trajectory optimization, it will be possible to create feasible trajectories for quadrotors to perform the aggressive maneuvers in constrained environments.

Chapter 1

System Modeling

The goal of this chapter is to present the dynamic model of the quadrotor. The mathematical notation and the physics of the quadrotor platform are expressed using the Newton-Euler formalism. Then, the state-space model that will be coded on the controller of the quadrotor will be derived.

1.1 Concepts and Generalities

The dynamic model of the quadrotor will be derived based on the following assumptions:

- The quadrotor has a rigid structure.
- The quadrotor has a symmetrical structure.
- The center of gravity (CoG) and the fixed frame at the center of the body are assumed to be coincident.
- The propellers of the quadrotor are assumed to be rigid.
- The thrust and drag forces are assumed to be proportional to the square of the spinning speed of each propeller.

The helicopter is a complex mechanical system, it gathers many physical effects from the domain of mechanics and aerodynamics [4]. Thus, all the significant effects including the gyroscopic effects must be considered in the modeling of the quadrotor. A small list of the most important effects that a helicopter is subject to [5] are briefly described in table 1.1:

Table 1.1: The main physical effects that the helicopter is subject to.

Effect	Source	formulation
Aerodynamic effects	Rotation of propeller Flapping of blades	$C\Omega^2$
Inertial counter torques	Change in propeller spinning speed	$J\dot{\Omega}$
Gravitational effect	Position of the center of mass	
Gyroscopic effects	Orientation change of the rigid body	$I\theta\psi$
	Orientation change of the propeller plane	$J\Omega_r\theta, \phi$
Friction	All helicopter motions	$C\dot{\phi}, \dot{\theta}, \dot{\psi}$

1.2 Modelling with Euler-Lagrange Formalism

The dynamics of the rotation of a simple quadrotor are modeled using the Euler-Lagrange Formalism in this section. A fixed frame E for the world frame and body fixed frame B for the quadrotor are considered as represented in figure 1.1. The orientation of the quadrotor frame in space is provided by a rotation R from B to E, where $R \in SO3$ is a 3×3 rotation matrix.

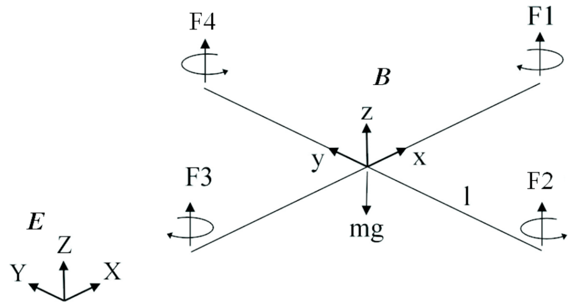


Figure 1.1: Coordinate system of a simple quadrotor. [1]

1.2.1 Kinematics

For any point of the body frame of the quadrotor expressed in the fixed world frame, the following can be written (c: cos, s: sin):

$$\begin{cases} r_X = (c\psi c\theta)x + (c\psi s\theta s\phi - s\psi c\phi)y + (c\psi s\theta c\phi + s\psi s\phi)z \\ r_Y = (s\psi c\theta)x + (s\psi s\theta s\phi + c\psi c\phi)y + (s\psi s\theta c\phi - c\psi s\phi)z \\ r_Z = (-s\theta)x + (c\theta s\phi)y + (c\theta c\phi)z \end{cases} \tag{1.1}$$

Thus, the velocities can be derived by differentiation 1.1, and the squared magnitude of the squared velocity can be expressed as follows for any point:

$$v^2 = v_X^2 + v_Y^2 + v_Z^2 \tag{1.2}$$

1.2.2 Energy

Assuming that the matrix of inertia is diagonal, then from equation 1.2, the expression of the kinetics energy can be calculated:

$$T = \frac{1}{2}I_{xx}(\dot{\phi} - \dot{\psi}s\theta)^2 + \frac{1}{2}I_{yy}(\dot{\theta}c\phi + \dot{\psi}s\phi c\theta)^2 + \frac{1}{2}I_{zz}(\dot{\theta}s\phi - \dot{\psi}c\phi)^2 \tag{1.3}$$

Using the formula of the potential energy, equation 1.3 can be expressed in the fixed world frame as:

$$V = \int xdm(x)(-gs\theta) + \int ydm(y)(gs\phi c\theta) + \int zdm(z)(gc\phi c\theta) \tag{1.4}$$

1.2.3 Equation of Motion

By using the Euler-Lagrange formalism:

$$L = T - V \quad , \quad \Gamma_i = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} \tag{1.5}$$

Where L Γ_i and $\dot{q}_i$ are the Lagrangian, the generalized forces and the generalized coordinates respectively. Thus, the equations of motion can be expressed as follows:

$$\begin{cases} I_{xx}\ddot{\phi} = \dot{\theta}\dot{\psi}(I_{yy} - I_{zz}) \\ \\ I_{yy}\ddot{\theta} = \dot{\phi}\dot{\psi}(I_{zz} - I_{xx}) \\ \\ I_{zz}\ddot{\psi} = \dot{\phi}\dot{\theta}(I_{xx} - I_{yy}) \end{cases} \tag{1.6}$$

Moreover, the torques that are nonconservative and acting on the quadrotor, are due to two different causes. First, it is due to thrust of each rotor pairs in figure 1.1:

$$\begin{cases} \tau_x = bl(\Omega_4^2 - \Omega_2^2) \\ \\ \tau_y = bl(\Omega_3^2 - \Omega_1^2) \\ \\ \tau_z = bl(\Omega_1^2 - \Omega_2^2 + \Omega_3^2 - \Omega_4^2) \end{cases} \tag{1.7}$$

Second, it is also due to the gyroscopic effect which is the result of the rotation of the propellers:

$$\begin{cases} \tau_x' = J_r\omega_y(\Omega_1 + \Omega_3 - \Omega_2 - \Omega_4) \\ \\ \tau_y' = J_r\omega_x(\Omega_2 + \Omega_4 - \Omega_1 - \Omega_3) \end{cases} \tag{1.8}$$

1.2.4 The Derived Dynamic Model

The dynamic model of the quadrotor which describes the rotations of roll, pitch and yaw consists of three terms:

1. The actuator torques.
2. The gyroscopic effects that are due to the rotation of the rigid body.
3. The gyroscopic effects that are due to rotation of the propeller that is coupled with the rotation of the body.

Thus, the dynamic model of the quadrotor is:

$$\begin{cases} I_{xx}\ddot{\phi} = \dot{\theta}\dot{\psi}(I_{yy} - I_{zz}) - J\dot{\theta}\Omega_r + \tau_x \\ \\ I_{yy}\ddot{\theta} = \dot{\theta}\dot{\psi}(I_{zz} - I_{xx}) + J\dot{\phi}\Omega_r + \tau_y \\ \\ I_{zz}\ddot{\psi} = \dot{\phi}\dot{\theta}(I_{xx} - I_{yy}) + \tau_z \end{cases} \tag{1.9}$$

1.2.5 Rotor Dynamics

DC motors are used to drive the rotors of a quadrotor. So, the established equations of a DC motor are the following:

$$\begin{cases} L\frac{di}{dt} = u - R_{mot}i - k_e\omega_m \\ \\ J_m\frac{d\omega_m}{dt} = \tau_m - \tau_d \end{cases} \tag{1.10}$$

Since small motors are used in which they also have little inductance, then the second order equation of the DC motor dynamics is given by:

$$J_m\frac{d\omega_m}{dt} = -\frac{k_m^2}{R_{mot}}\omega_m - \tau_d + \frac{k_m}{R_{mot}}u \tag{1.11}$$

When the gearbox and the propeller models are introduced, then equation 1.12 becomes:

$$\begin{cases} \dot{\omega}_m = -\frac{1}{\tau}\omega_m - \frac{d}{\eta r^3 J_t}\omega_m^2 + \frac{1}{k_m\tau}u \\ \\ \frac{1}{\tau} = \frac{k_m^2}{RJ_t} \end{cases} \tag{1.12}$$

Moreover, linearization of equation 1.12 can be done around an operation point $\dot{\omega}_0$ to the form $\dot{\omega}_m = -A\omega_m + Bu + C$ with:

$$A = \left(\frac{1}{\tau} + \frac{2d\omega_0}{\eta r^3 J_t}\right),\ B = \left(\frac{1}{k_m\tau}\right),\ C = \left(\frac{d\omega_0^2}{\eta r^3 J_t}\right) \tag{1.13}$$

1.3 Modeling with Newton-Euler Formalism

The model above was derived in succession as shown in papers [6, 7, 8] . The dynamic equations below includes contain rolling moments R_m, hub forces H and various aerodynamic effects. Thus, this is a more realistic dynamic model, especially when the quadrotor flies in a forward manner. With the previous versions of the dynamic model, it was required to to moderately tune the control parameters in order to have experiments that are successful.

The dynamic model expressed in the Newton-Euler formalism of a rigid body that is subject to external forces acting on the center of mass is expressed as follows [9]:

$$\begin{bmatrix} mI_{3\times3} & 0 \\ 0 & I \end{bmatrix}\begin{bmatrix} \dot{V} \\ \dot{\omega} \end{bmatrix} + \begin{bmatrix} \omega \times mV \\ \omega \times I\omega \end{bmatrix} = \begin{bmatrix} F \\ \tau \end{bmatrix} \tag{1.14}$$

Considering a fixed world frame E and a fixed body frame B on the quadrotor as shown in figure 1.2. Then, by the use of the Euler angles, the orientation of the rigid body of the quadrotor in space is expressed by a rotation R from B to E, where $R \in SO3$ is a rotation matrix.

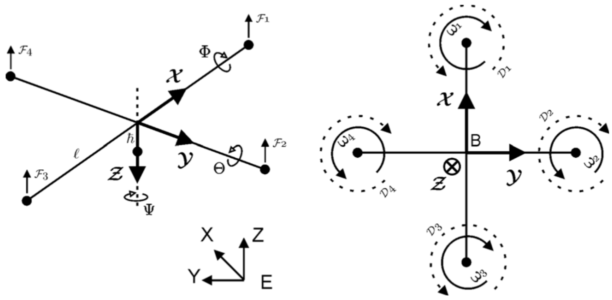


Figure 1.2: Coordinate system of a simple quadrotor. [1]

1.3.1 Aerodynamic Forces and Moments

The aerodynamic forces and moments are computed using a mix of blade element and momentum theory [10]. This is based off of the work of Gary Fay during the project of the Mesicopter [11]. For a simpler readings of the equations provided below, some symbols are recalled:

σ: solidity ration	λ: inflow ratio
a: lift slope	υ: induced velocity
μ: rotor advance ration	ρ: air density

Thrust Force

The thrust force is due to all the vertical forces that the blade elements are subject to.

$$\begin{cases} T = C_T \rho A (\Omega R_{rad})^2 \\ \\ \frac{C_T}{\sigma a} = (\frac{1}{6} + \frac{1}{4}\mu^2)\theta_0 - (1+\mu^2)\frac{\theta_{tw}}{8} - \frac{1}{5}\lambda \end{cases} \tag{1.15}$$

Hub Force

The hub force is due to all the horizontal forces that the blade elements are subject to.

$$\begin{cases} H = C_H \rho A (\Omega R_{rad})^2 \\ \\ \frac{C_H}{\sigma a} = (\frac{1}{4a}\mu\overline{C_d} + \frac{1}{4}\lambda\mu(\theta_0 - \frac{\theta_{tw}}{2}) \end{cases} \tag{1.16}$$

Drag Moment

The drag moment about the rotor shaft is due to the aerodynamic forces that the blade elements are subject to. The horizontal forces that are acting on the rotor are multiplied by the moment arm and integrated over the rotor. The drag moment gives the required power to spin the rotor.

$$\begin{cases} Q = C_Q \rho A (\Omega R_{rad})^2 R_{rad} \\ \\ \frac{C_Q}{\sigma a} = \frac{1}{8a}(1+\mu^2)\overline{C_d} + \lambda(\frac{1}{6}\theta_0 - \frac{1}{8}\theta_{tw} - \frac{1}{4}\lambda) \end{cases} \tag{1.17}$$

Rolling moment

The rolling moment of a propeller occurs when the blade that is advancing is producing more lift than the blade that is retreating in forwarding flight. It is the integration of the lift of every single section that is acting at a given radius over the entire rotor. The reader should notice that the rolling moment is not the same as the propeller radius, or the overall rolling moment which is due to other effects or the rotation matrix R. So, there should not be any confusion.

$$\begin{cases} R_m = C_{R_m}\rho A(\Omega R_{rad})^2 R_{rad} \\ \frac{C_{Rm}}{\sigma a} = -\mu(\frac{1}{6}\theta_0 - \frac{1}{8}\theta_{tw} - \frac{1}{8}\lambda) \end{cases} \tag{1.18}$$

Ground Effect

When operating near the ground (at a height equivalent to half the diameter of the rotor), helicopters experience thrust augmentation which is caused by greater efficiency of the rotor.This is linked to a decrease in the velocity of induced airflow. Moreover, this is called Ground Effect. Different approaches to deal with this effect can be found in literature, for example, adaptive techniques can be used [12]. However, the objective is to find a model that is simple and mainly captures the change in the velocity of the induced inflow. Cheeseman [13] states (reached from the images method [14]) that if the power is constant ($T_{OGE}v_{i,OGE} = T_{IGE}v_{i,IGE}$), the generated vecolity at the center of the rotor by its imageis $\delta v_i = Av_i/16\pi z^2$. Cheesman acquired the relation (1.19) by using the assumption that both v_i and δv_i are constant over disk, which results in $v_{i,IGE} = v_i - \delta v_i$.

$$\frac{T_{IGE}}{T_{OGE}} = \frac{1}{1 - \frac{R_{rad}^2}{16z^2}} \tag{1.19}$$

An alternative way to move forward is to consider that the inflow ration of the inflow ratio is $\lambda_{IGE} = (v_{i,OGE} - \delta v_i - \dot{z})/\Omega R_{rad}$, where the change of the velocity of the induced inflow is $\delta v_i = v_i/(4z/R_{rad})^2$.Then, the thrust coefficient (1.15) IGE can be rewritten as:

$$\begin{cases} T_{IGE} = C_T^{IGE}\rho A(\Omega R_{rad})^2 \\ \frac{C_T^{IGE}}{\sigma a} = \frac{C_T^{OGE}}{\sigma a} + \frac{\delta v_i}{4\Omega R_{rad}} \end{cases} \tag{1.20}$$

1.3.2 General Moments and Forces

The motion of the quadrotor is the result of several forces and moments that are originating from different physical effects [1]. In this model, the following effects are considered (with c: cos, s:sin).

Rolling Moments

body gyro effect	$\dot{\theta}\dot{\psi}(I_{yy} - I_{zz})$
propeller gyro effect	$J_r\dot{\theta}\Omega_r$
pitch actuators action	$l(-T_2 + T_4)$
hub moment due to forward flight	$h(\sum_{i=1}^{4} H_{yi})$
rolling moment due to forward flight	$(-1)^{i+1}\sum_{i=1}^{4} R_{mxi}$

Pitching Moments

body gyro effect	$\dot{\phi}\dot{\psi}(I_{zz} - I_{xx})$
propeller gyro effect	$J_r\dot{\phi}\Omega_r$
pitch actuators action	$l(T_1 - T_3)$
hub moment due to forward flight	$h(\sum_{i=1}^{4} H_{xi})$
rolling moment due to side-ward flight	$(-1)^{i+1}\sum_{i=1}^{4} R_{myi}$

Yawing Moments

body gyro effect	$\dot{\theta}\dot{\phi}(I_{xx} - I_{yy})$
inertial counter-torque	$J_r\dot{\Omega}_r$
counter-torque unbalance	$(-1)^i\sum_{i=1}^{4} Q_i$
hub force unbalance in forward flight	$l(H_{x2} - H_{x4})$
hub force unbalance in sideward flight	$l(-H_{y1} + H_{y3})$

Forces Along z Axis

actuators action	$c\psi c\phi(\sum_{i=1}^{4} T_i$
weight	mg

Forces Along x Axis

actuators action	$(s\psi s\phi + c\psi s\theta c\phi)(\sum_{i=1}^{4} T_i)$
hub force in x axis	$-\sum_{i=1}^{4} H_{xi}$
friction	$\frac{1}{2} C_x A_c \rho \dot{x} \lvert \dot{x} \rvert$

Forces Along y Axis

actuators action	$(-c\psi s\phi + s\psi s\theta c\phi)(\sum_{i=1}^{4} T_i)$
hub force in y axis	$-\sum_{i=1}^{4} H_{yi}$
friction	$\frac{1}{2} C_y A_c \rho \dot{y} \lvert \dot{y} \rvert$

1.3.3 Equations of Motion

The equations of motion are derived from (1.14) in addition to all the forces and the moments that were listed in subsection 1.3.2.

$$\begin{cases} I_{xx}\ddot{\phi} = \dot{\theta}\dot{\psi}(I_{yy} - I_{zz}) + J_r\dot{\theta}\Omega_r + l(-T_2 + T_4) - h(\sum_{i=1}^{4} H_{yi}) + (-1)^{i+1} \sum_{i=1}^{4} R_{mxi} \\ I_{yy}\ddot{\theta} = \dot{\phi}\dot{\psi}(I_{zz} - I_{xx}) - J_r\dot{\phi}\Omega_r + l(T_1 - T_3) + h(\sum_{i=1}^{4} H_{xi}) + (-1)^{i+1} \sum_{i=1}^{4} R_{mxi} \\ I_{zz}\ddot{\psi} = \dot{\phi}\dot{\psi}(I_{xx} - I_{yy}) + J_r\dot{\Omega}_r + (-1)^{i} \sum_{i=1}^{4} Q_i + l(H_{x2} - H_{x4}) + l(-H_{y1} + H_{y3}) \\ m\ddot{z} = mg - (c\psi c\phi) \sum_{i=1}^{4} T_i \\ m\ddot{x} = (s\psi s\phi + c\psi s\theta c\phi) \sum_{i=1}^{4} T_i - \sum_{i=1}^{4} H_{xi} - \frac{1}{2} C_x A_c \rho \dot{x} \lvert \dot{x} \rvert \\ m\ddot{y} = (-c\psi s\phi + s\psi s\theta c\phi) \sum_{i=1}^{4} T_i - \sum_{i=1}^{4} H_{yi} - \frac{1}{2} C_y A_c \rho \dot{y} \lvert \dot{y} \rvert \end{cases} \tag{1.21}$$

1.4 State-Space Model

The model 1.21 that was developed in subsecton 1.3.3 expresses the differential equations of the system. However, for the purpose of control design, it is desirable to reduce the complexity and simplify the model to satisfy the real-time limitations of the embedded control loop. Thus, the thrust and the drag coefficients are assumed to be constant and the hub forces and rolling moments are neglected. As a result, the system can be expressed in state-space form $\dot{X} = f(X, U)$ with X the state vector and U the control input vector.

The state vector has the following form:

$$X = \begin{bmatrix} \phi & \dot{\phi} & \theta & \dot{\theta} & \psi & \dot{\psi} & z & \dot{z} & x & \dot{x} & y & \dot{y} \end{bmatrix}^\intercal \tag{1.22}$$

With,

$$\begin{array}{r|r} \begin{aligned} x_1 &= \phi \\ x_2 = \dot{x}_1 &= \dot{\phi} \\ x_3 &= \theta \\ x_4 = \dot{x}_3 &= \dot{\theta} \\ x_5 &= \psi \\ x_6 = \dot{x}_5 &= \dot{\psi} \end{aligned} & \begin{aligned} x_7 &= z \\ x_8 = \dot{x}_7 &= \dot{z} \\ x_9 &= x \\ x_{10} = \dot{x}_9 &= \dot{x} \\ x_{11} &= y \\ x_{12} = \dot{x}_{11} &= \dot{y} \end{aligned} \end{array} \tag{1.23}$$

Moreover, the control input vector has the following form:

$$X = \begin{bmatrix} U_1 & U_2 & U_3 & U_4 \end{bmatrix}^\intercal \tag{1.24}$$

Where the control inputs are mapped by:

$$\begin{cases} U_1 = b(\Omega_1^2 + \Omega_2^2 + \Omega_3^2 + \Omega_4^2) \\ \\ U_2 = b(-\Omega_2^2 + \Omega_4^2) \\ \\ U_3 = b(\Omega_1^2 - \Omega^3) \\ \\ U_4 = d(-\Omega_1^2 + \Omega_2^2 - \Omega_3^2 + \Omega_4^2) \end{cases} \tag{1.25}$$

The transformation matrix between the rate change of the attitude angles $(\dot{\phi}, \dot{\theta}, \dot{\psi})$ and the angular velocities of the body(p, q, r) can be regarded as the identity matrix if the disturbances due to hover flight are small. As a result, the following can be written:

$$(\dot{\phi}, \dot{\theta}, \dot{\psi}) \approx (p, q, r) \tag{1.26}$$

Simulation tests have demonstrated that this assumption is reasonable [1].

From equations (1.21),(1.22),(1.24), the following expression is obtained after simplification:

$$f(X,U) = \begin{pmatrix} \dot{\phi} \\ \dot{\theta}\dot{\psi}a_1 + \dot{\theta}a_2\Omega_r + b_1U_2 \\ \dot{\theta} \\ \dot{\phi}\dot{\psi}a_3 - \dot{\phi}a_4\Omega_r + b_2U_3 \\ \dot{\psi} \\ \dot{\theta}\dot{\psi}a_5 + b_3U_4 \\ \dot{z} \\ g - (\cos\phi\cos\theta)\frac{1}{m}U_1 \\ \dot{x} \\ u_x\frac{1}{m}U_1 \\ \dot{y} \\ u_y\frac{1}{m}U_1 \end{pmatrix} \tag{1.27}$$

With,

$$\left.\begin{aligned} a_1 &= (I_{yy} - I_{zz})/I_{xx} \\ a_2 &= J_r/I_{xx} \\ a_3 &= (I_{zz} - I_{xx})/I_{yy} \\ a_4 &= J_r/I_{yy} \\ a_5 &= (I_{xx} - I_{yy})/I_{zz} \end{aligned}\right| \quad \begin{aligned} b_1 &= l/I_{xx} \\ b_2 &= l/I_{yy} \\ b_3 &= l/I_{zz} \end{aligned} \tag{1.28}$$

$$\begin{aligned} u_x &= (\cos\phi\sin\theta\cos\psi + \sin\phi\sin\psi) \\ u_y &= (\cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi) \end{aligned} \tag{1.29}$$

It is important to note that that the angles and the derivatives of the angles do not depend on the components of the translation in the system represented by equation (1.27). Contrarily, the translation components depend of the angles. So, the system represented by equation(1.27) can be depicted as two subsystems, the angle subsystem and the translation subsystem as shown in figure

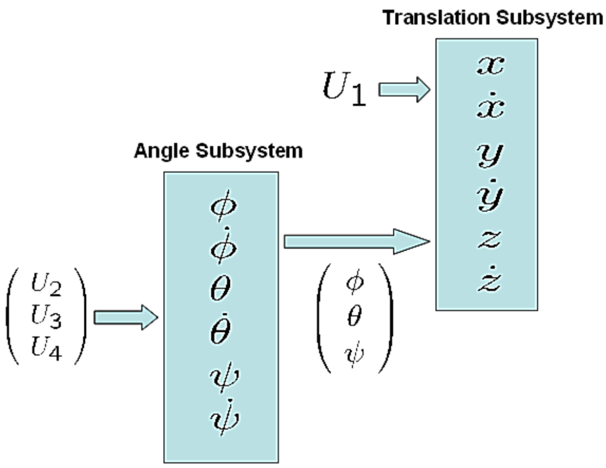


Figure 1.3: Link between the rotation and the translation subsystems.[1]

Chapter 2

Control of quadrotors

2.1 General control architecture

2.2 Model Predictive control

2.3 Lyapunov Stability Analysis

2.4 Sliding mode control

2.5 Other types of control

Chapter 3

Multi-flips maneuver with quadrotors

3.0.1 Quadrotor flip physics

3.0.2 Link to parallel robots

3.0.3 Control approaches for multi-flip maneuvers

Chapter 4

Trajectory optimization

Conclusion

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