

World Coordinate System in LSST

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1. Abstract

This paper describes the implementation of a World Coordinate System for Data Challenge 3 (hereafter DC3). We use the TAN-SIP World Coordinate System convention because it includes a polynomial solution. In this paper the TAN-SIP World Coordinate System is described, example FITS header keywords are given, and an example solution is shown. Along with the TAN-SIP WCS, this document describes the keywords added to the DC3 FITS headers that are required for standard data reduction software. The TAN-SIP World Coordinate System provides astrometric solutions to 0.001 to 0.004 arc-seconds (dispersion) across the focal plane, for a single visit. There is no trend seen in the residuals as a function of chip distance from the focal plane. **For quick reference: Equations (4) and (5) give the relations between pixel coordinates and sky positions in terms of FITS header keywords.**

2. Introduction

One of the many goals of LSST will be high astrometric precision to the level of 10 milli-arc-seconds (Ivezić et al. 2008). This astrometric accuracy is required to study the structure of the Milky Way, where the proper motion of Milky Way stars will be measured. The Congressional NEO mandate requires a similarly high-precision astrometric accuracy. These science goals cannot be met without an accurate World Coordinate System (hereafter WCS).

This document is meant to be a draft. Complaints, comments, and questions are encouraged. It also describes the updates being made to the DC3 FITS headers which correspond to the FITS IAU standard. The FITS header keywords include basic imaging parameters such as readnoise, gain, and saturation levels. The TAN-SIP convention is **not** supported by WCSlib. However, the TAN-SIP convention is supported by StSci HST/ACS, Spitzer, and

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WCSTools which is used by ds9. **If TAN-SIP is not supported by your software, then please email Jim Pizagno.**

This software is planned to be part of the LSST software stack at the University of Washington (hereafter UW). In the current plan, DC3 data will be generated using the Image Simulation software at Purdue, and then sent to UW. At UW, the FITS image headers will have all the relevant information included to allow post-processing. This post-processing includes the generation of noise images, an astrometric solution, magnitude zero points, and time stamp software.

3. World Coordinate System Definition

A WCS is required to convert FITS image pixels to world coordinates, such as right ascension and declination. A description of FITS images can be found at this website¹, and WCS conventions can be found at this website². Many software packages implement FITS WCS using the software package WCSlib³, which is maintained by Mark Calabretta. The TAN-SIP convention is implemented because it is well defined, and has higher order non-linear terms. The TAN-SIP convention is defined by Shupe et al. (2005), where ‘SIP’ stands for Simple Imaging Polynomial. The TAN-SIP convention remains under review by the IAU FITS Working Group, but does work with LSST software.

As described by Shupe et al. (2005) the TAN-SIP convention is defined by:

$$\begin{pmatrix} SKY1 - CRVAL1 \\ SKY2 - CRVAL2 \end{pmatrix} = \begin{pmatrix} CD1.1 & CD1.2 \\ CD2.1 & CD2.2 \end{pmatrix} \begin{pmatrix} u + f(u, v) \\ v + g(u, v) \end{pmatrix}, \quad (1)$$

where u and v are image coordinates (i.e. pixels), and $u = x - CRPIX1$ and $v = y - CRPIX2$. $SKY1$ and $SKY2$ are coordinates on the sky or right ascension and declination, with units of degrees for LSST. The FITS header keywords ‘CTYPE1’ and ‘CTYPE2’ inform the software/user if the $SKY1$ and $SKY2$ values are right ascension or declination, and that this is a TAN-SIP WCS. Notice that the definition of x and y are not the same as in Shupe et al. (2005), who define them as sky coordinates. In this paper, x and y are defined as pixel coordinates.

¹(<http://fits.gsfc.nasa.gov/>)

²(<http://fits.gsfc.nasa.gov/fitswcs.html>)

³(<http://www.atnf.csiro.au/people/mcalabre/WCS/>)

The polynomial equations $f(u, v)$ and $g(u, v)$ are defined as:

$$f(u, v) = \sum_{p,q} A_{p,q} u^p v^q, \quad p + q \leq A_ORDER, \quad (2)$$

$$g(u, v) = \sum_{p,q} B_{p,q} u^p v^q, \quad p + q \leq B_ORDER, \quad (3)$$

where $A_{0,1} = 0$, $A_{1,0} = 0$, $B_{0,1} = 0$, $B_{1,0} = 0$, $A_{0,0} = 0$, $B_{0,0} = 0$. To be more explicit, combining the above three equations, using a 2nd order polynomials ($A_ORDER = 2$, $B_ORDER = 2$, etc..), yields:

$$\begin{aligned} SKY1 - CRVAL1 &= CD1_1(u + A2_0u^2 + A0_2v^2 + A1_1uv) \\ &+ CD1_2(v + B2_0u^2 + B0_2v^2 + B1_1uv), \end{aligned} \quad (4)$$

$$\begin{aligned} SKY2 - CRVAL2 &= CD2_1(u + A2_0u^2 + A0_2v^2 + A1_1uv) \\ &+ CD2_2(v + B2_0u^2 + B0_2v^2 + B1_1uv), \end{aligned} \quad (5)$$

The inverse transformation, from RA/DEC (degrees) to x/y (pixels), is required by certain software packages, and is defined by:

$$\begin{pmatrix} U \\ V \end{pmatrix} = CD^{-1} \begin{pmatrix} SKY1 - CRVAL1 \\ SKY2 - CRVAL2 \end{pmatrix}, \quad (6)$$

where pixel coordinates are computed by:

$$u = U + F(U, V) = U + \sum_{p,q} AP_{p,q} U^p V^q, \quad p + q \leq AP_ORDER, \quad (7)$$

and

$$v = V + G(U, V) = V + \sum_{p,q} BP_{p,q} U^p V^q, \quad p + q \leq BP_ORDER, \quad (8)$$

with a similar solution. Notice the 'P' nomenclature after the A and B terms for the inverse transformation.

Please note: the $CD\#-\#$ terms now encode skew, as well as rotation. If your software supports a TAN WCS, but not TAN-SIP WCS, then it may read only the $CD\#-\#$

terms, which will not provide the correction conversion. Mistaking the TAN-SIP $CD\#_#$ values for traditional TAN $CD\#_#$ values may cause systematic astrometric errors as high as 1 arc-second. Also, note that we can provide an arbitrarily large number of polynomial terms, as specified by the FITS header keywords *A_ORDER* and *B_ORDER*. Please note that the $CD\#_#$ values are not required to be the same for each chip. This software is run on each chip, and each solution is independent of the solution of other chips. In other words, the solution in one chip, does not “know” about the solution in another chip.

4. Implementation

This section is meant to provide a specific example of how the TAN-SIP WCS is coded into the FITS header using the appropriate keywords. The above equations relate pixel coordinates to SKY world coordinates using the FITS header keywords for the TAN-SIP convention. Along with the TAN-SIP keywords, keywords required by standard data reduction software are included in the FITS header, such as the gain, readnoise, pixel scale, saturation levels, and date. These FITS header keywords are:

GAIN	gain in electrons per ADU
READNSE	the readnoise
SATURATE	the saturation level
FILTER	LSST filter, a string equal to one of: u,g,r,i,z,y
CCDTYPE	object, bias, or flat
CCD_INFO	position on focal plane, or ChipX/ChipY
DATE	time and date separated by 'T', including 'UWLSST'
AIRMASS	airmass of the observation used in the Image Simulator
EXPTIME	always 15, but needed for data reduction software
EQUINOX	always J2000.0, but may be needed for data reduction software

The *VERSION* keyword refers to the version of the Image Simulation software, maintained by Purdue University. The TAN-SIP convention keywords, and example for image **Wide_0.fits**, are:

SIMPLE =	T	/ file does conform to FITS standard
BITPIX =	-32	/ number of bits per data pixel
NAXIS =	2	/ number of data axes
NAXIS1 =	4096	/ length of data axis 1
NAXIS2 =	4096	/ length of data axis 2
EXTEND =	T	/ FITS dataset may contain extensions
COMMENT	FITS (Flexible Image Transport System)	
COMMENT	format is defined in 'Astronomy	
COMMENT	and Astrophysics', volume 376,	
COMMENT	page 359; bibcode: 2001A&A...376..359H	
CTYPE1 =	'RA—TAN-SIP'	
CRPIX1 =	0.0	
CRVAL1 =	22.34148392932257	
CTYPE2 =	'DEC—TAN-SIP'	
CRPIX2 =	0.0	
CRVAL2 =	-0.3404765496109723	
CD1_1 =	-5.555963143569605E-05	
CD1_2 =	1.706850157982605E-08	
CD2_1 =	4.341248182813349E-09	
CD2_2 =	5.555841416621764E-05	
RADESYS =	'FK5 '	
EQUINOX =	2000.0	
CREATOR =	'LSST IMAGE SIMULATOR'	
VERSION =	161	

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DATE =      '2009-2-4T9:42:7'      / updated at UWLSSST on YYYY-MO-DAYThh:mm:ss
A_ORDER =   2                      / polynomial order, axis 1
A_0_2 =     5.921223900517382E-09   / distortion coefficient
A_1_1 =     1.311413044322747E-08   / distortion coefficient
A_2_0 =     1.7786949135529E-08     / distortion coefficient
B_ORDER =   2                      / polynomial order, axis 2
B_0_2 =     1.950611554118362E-08   / distortion coefficient
B_1_1 =     1.273296676431013E-08   / distortion coefficient
B_2_0 =     9.277794484196512E-09   / distortion coefficient
CDELTA1 =   5.555555555555556E-05   / plate scale in degrees per pixel
CDELTA2 =   5.555555555555556E-05   / plate scale in degrees per pixel
AP_ORDER=   2                      / polynomial order, axis 1
AP_0_2 =    -5.917670337638511E-09   / distortion coefficient
AP_1_1 =    -1.310686262312195E-08   / distortion coefficient
AP_2_0 =    -1.778093944402193E-08   / distortion coefficient
BP_ORDER=   2                      / polynomial order, axis 2
BP_0_2 =    -1.949916078363988E-08   / distortion coefficient
BP_1_1 =    -1.272452633694242E-08   / distortion coefficient
BP_2_0 =    -9.273540184318227E-09   / distortion coefficient
GAIN =      1.0                    / gain electrons per ADU
READNSE =   0.0                    / readnoise
SATURATE=   100000.0               / saturation level
FILTER =    'r '                   / LSST
CCDTYPE =   'object '               / object, bias, flat
CCD_INFO=    '-1/-1 '               / position on focal plane: ChipX/ChipY
BINNING =   1                      / pixel binning
DATEOBS =   '2009-2-4T9:42:7'      / same as DATE for now
AIRMASS =   1.0                    / airmass of observation
EXPTIME =   15.0                    / seconds
END

```

5. Results for DC3 Run

The TAN-SIP WCS solution is implemented by comparing the known positions of stars to those measured by Source-Extractor(Bertin & Arnouts 1996). Source-Extractor is run on each simulated image with default values, except the analysis threshold is set to analyze stars 20 times above the noise. Input catalog stars are matched to the stars measured by Source-

Extractor by comparing stars of similar apparent brightness levels ($\Delta\text{mag} < 0.1$ magnitudes) which are also within a 50 pixel radius of each other. Stars with multiple matches, and stars with Source-Extractor flags, are excluded in the WCS solution. There are typically 50 stars per CCD used to calculate the solution. The matching between star catalog sky coordinates (Right Ascension & Declination) and coordinates measured by Source Extractor (x/y pixels) is done using a Python linear least-squares minimization routine (Python module: `numpy.linalg.lstsq`). The Right Ascension & Declination of the stars are known from the star input catalog provided to the Image Simulation software (i.e. `StarCatalogFormatTrim.dat`).

Figures 1 and 2 show the residuals for the image `Wide_0.fits`, after a 2nd order polynomial was used to solve the TAN-SIP WCS. Figures 1 and 2 show that the solution is good to 0.004 arc-seconds for the Right Ascension and 0.001 for the Declination per CCD, with a few outliers. There are no higher order trends seen in the residuals. Figures 1 and 2 show that the TAN-SIP WCS provides sky positions to better than 10mas, which meets current requirements by the Data Management group. Figure 3 shows the residuals across part of the focal plane, which covers Chip positions between $-1 < \text{ChipY} < 1$ and $-1 < \text{ChipX} < 7$. This figure also shows that the TAN-SIP WCS provides a solution to better than 10 milli-arcseconds on average.

Figures 4 and 5 show the residuals for image `Wide_431.fits`, after a 2nd order polynomial was used to solve the TAN-SIP WCS. This image was selected because it is at the upper corner of one raft in the DC3 run, or at $\text{ChipX}/\text{ChipY} = 7/1$. Comparing the linear WCS to the WCS with a 2nd order polynomial shows the necessity for a non-linear polynomial solution. This image, at the edge of a raft, is not an average solution, but displays an image with one of the largest WCS residuals. Note, that the residuals in this image are still mostly within the 10 milli-arcsecond solution required by LSST.

REFERENCES

- Bertin, E. & Arnouts, S. (1996), *A&AS*, 117, 393
- Calabretta, M. R., Valdes, F. G., & Allen, S. L. (2006), *A&A*, 446, 744
- Ivezić, Z., et al. (2008) <http://xxx.lanl.gov/abs/0805.2366>
- Shupe D. L., Moshir, M., Li. J., Makovoz, D., Narron, R., & Hook R. N. (2005) *Astronomical Data Analysis Software and Systems XIV*, ASP Conference, p 3.2.18

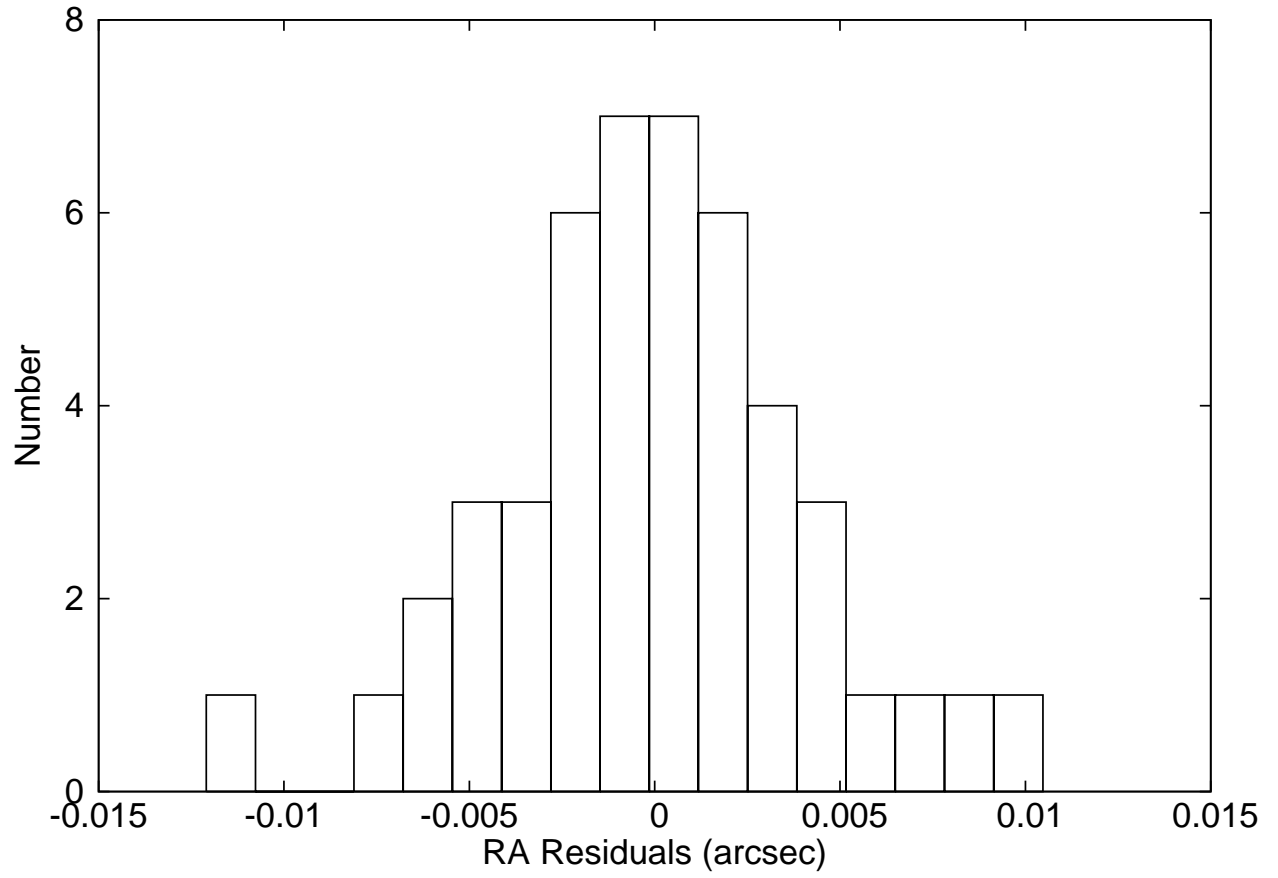


Fig. 1.— A histogram of the difference between measured Right Ascension and true/input Right Ascension. These results are for the image Wide_0.fits.

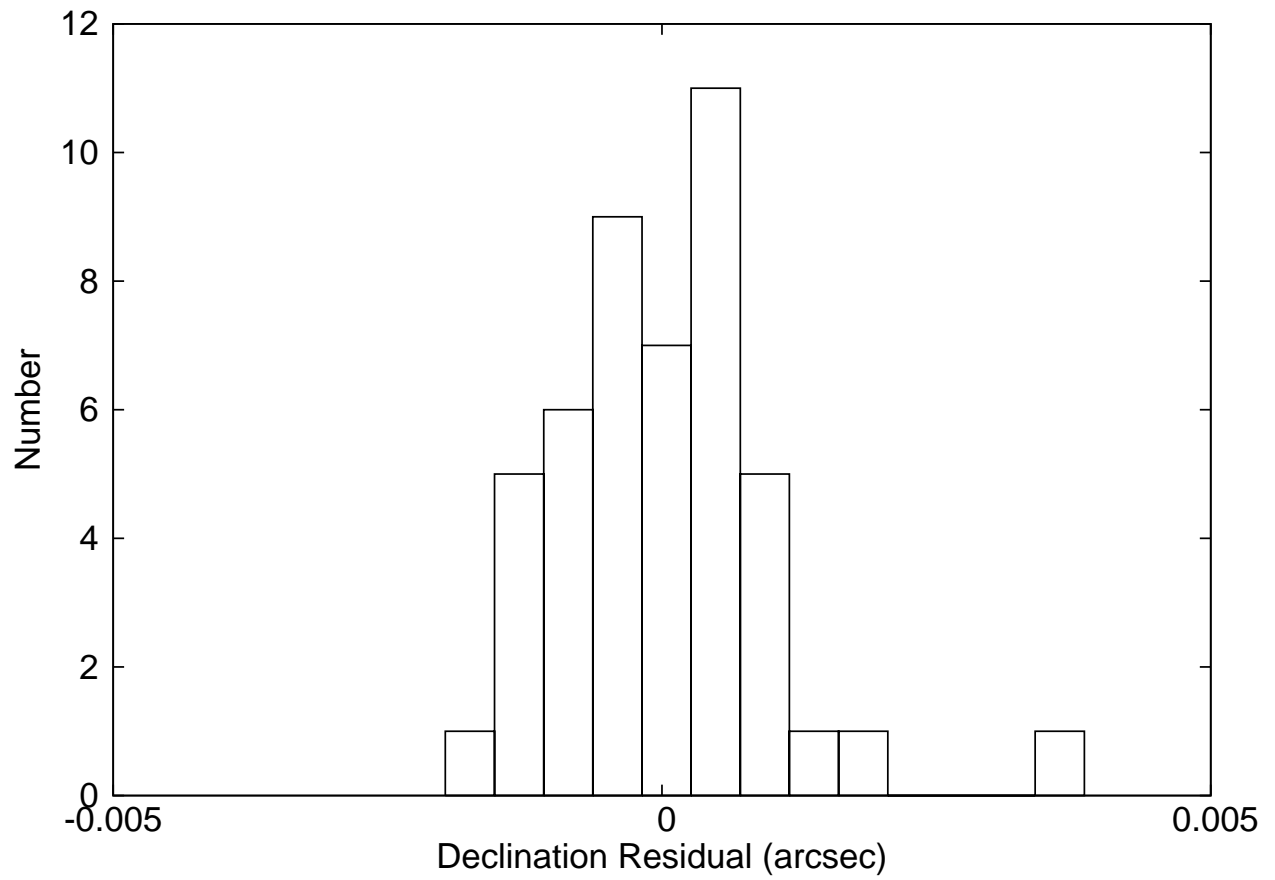


Fig. 2.— A histogram of the difference between measured Declination and true/input Declination. These results are for the image Wide_0.fits.

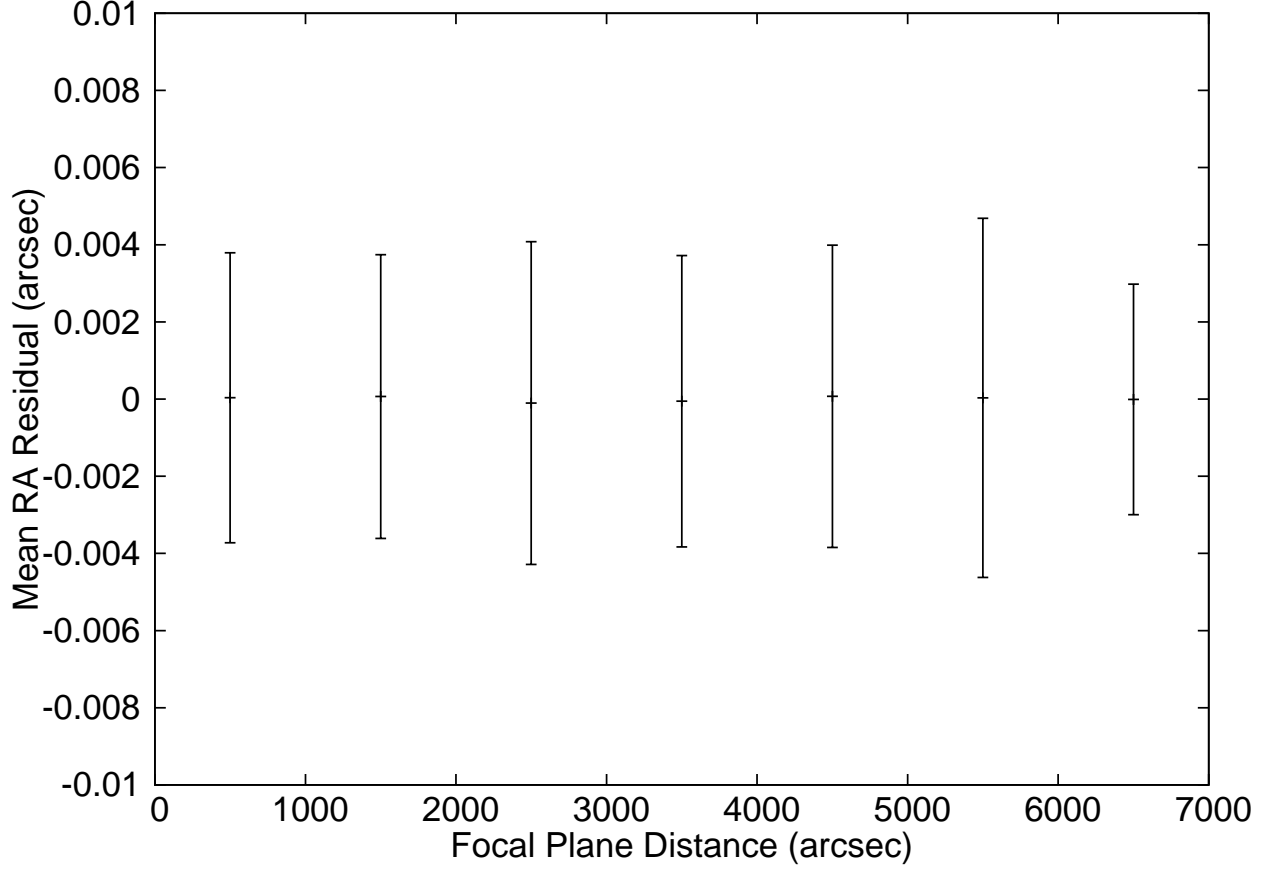


Fig. 3.— The mean Right Ascension residual as a function of distance from the center of the focal plane. Each data point is a mean of the data within ± 500 pixels of the data point focal plane distance. The "error bars" indicate the standard deviation. A similar trend is seen for the Declination residuals. The LSST goal is 0.010 arc-second astrometric accuracy (Ivezić et al. 2008).

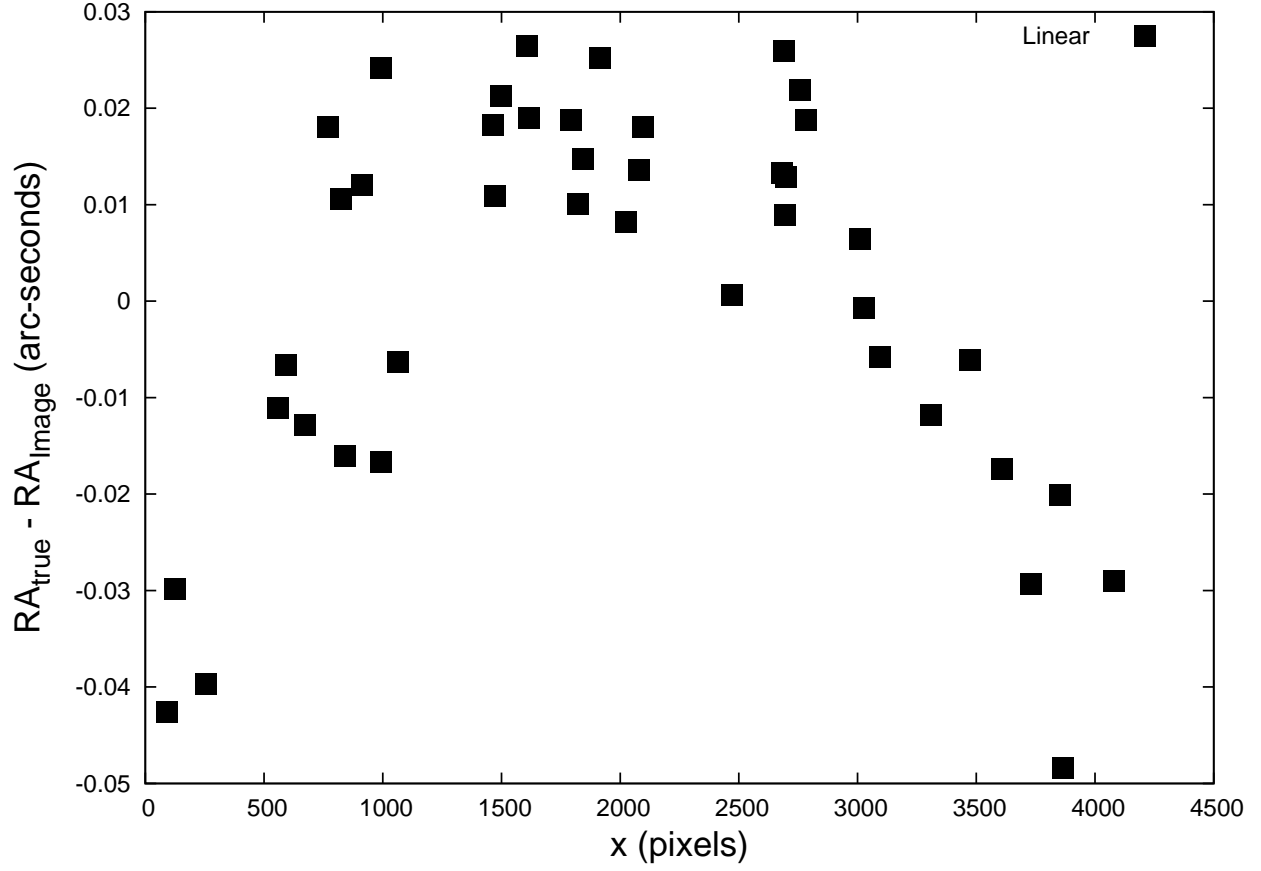


Fig. 4.— The Right Ascension residuals for the image Wide_431.fits when using a linear WCS, or without the polynomial terms in the TAN-SIP. Each data point is a star, which was matched between Source Extractor (measured/image) and the input catalog (true answer).

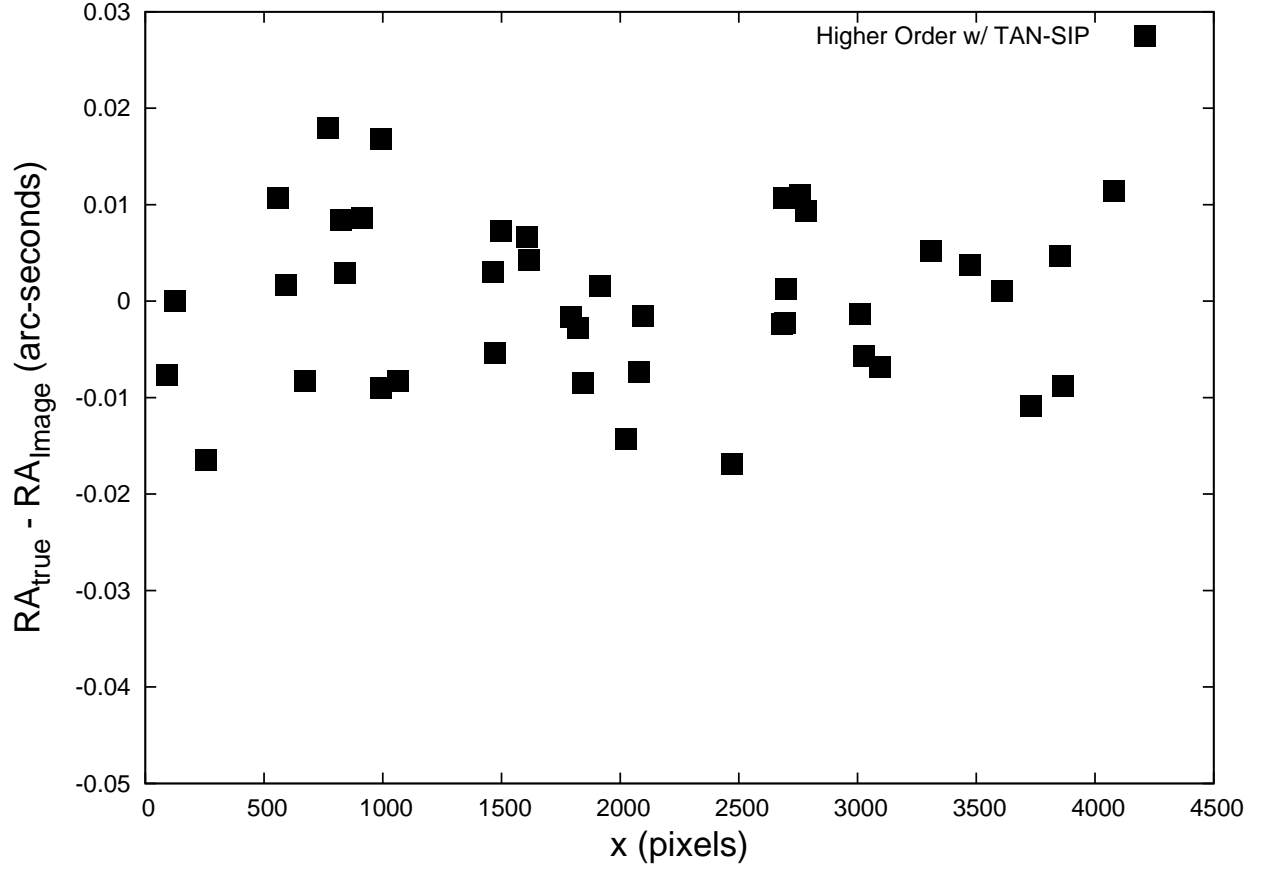


Fig. 5.— The Right Ascension residuals for the image Wide_431.fits when using a 2nd order polynomial WCS. Each data point is a star, which was matched between Source Extractor (measured/image) and the input catalog (true answer).