MATH370 HW4

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Contents

1 TODO

- **3.1.1abcd** PROBLEM 1a come back to. Not sure, I feel like they only want me to use the definition of $|x - a|$
- **3.1.2ab**
- **3.1.5** This seems so trivial. I think I missing something major. Look at answer key, because this seems stupidly simple.... Oh use the definition 3.1 to prove this.
- **3.1.6**
- **3.1.7** I want to go over this some more. Clean it up, but it I don't get enough time it is good enough to turn in.

2 Problem 3.1.0

(a) For each $n \in \mathbf{N}$, the function $(x-a)^n sin(f(x)(x-a)^{-n}$ has a limit as $x \to a$

(b) Suppose that $\{x_n\}$ is a sequence converging to a with $x_n \neq a$. If $f(x_n) \to L$ as $n \to \infty$, then $f(x) \to L$ as $x \to a$.

2.1 a

2.2 Thoughts

- My initial thought is that there is a limit to $(x-a)^n$.
- My second thought is that $sin(x)$ does not have a limit, but $(x-a)^{-n}$ goes to zero as $n \to \infty$ and $x \to a$.
- Thus $sin((x-a)^{-n})$ would be a limit to zero.
- Now what if was trying to use the defintion of a limit
- Where $|f(x) - z| < \delta$
- What would this be dependent on?

2.3 Proof

2.4 B

Suppose that $\{x_n\}$ is a sequence converging to a with $x_n \neq a$. If $f(x_n) \to L$ as $n \to \infty$, then $f(x) \to L$ as $x \to a$.

2.5 Thoughts

- Okay thinking his outloud. That means that $|x_n - a| < \epsilon$.
- Also that $|f(x_n) - L| < \delta$.
- Cannot think a reason why this would be false. It's by defintion true?

OH SHIT IT IS FALSE!!!! BUT WHY?

2.6 Thoughts on Thoughts

- I see the example in 3.7 where they have two squences that converge to a, but does not mean that the limit exsits.
- I guess what I am wondering then, does that mean for all converging sequences it must be true for the limit to exist?
- I have to think about that for a second and the consequnces of that.

3 Problem 3.1.1

(a) $lim_{x \to 2} x^2 + 2x - 5 = 3$

(b) $lim_{x \to 1} \frac{x^2+x-2}{x-1}$

3.1 a) $lim_{x \to 2} x^2 + 2x - 5 = 3$

- Let $\epsilon > 0$ and set $L = 3$.
- Notice, $f(x) - L = x^2 + 2x - 8 = (x+4)(x-2)$.
- Our inequality then becomes $|x+4||x-2| < \epsilon$
- If $0 < \delta \leq 1$
- Then $|x-2| < \delta \leq 1 \to 1 < x < 3$
- Which implies $|x+4| < 7$
- Consequently $|x-2||x+4| < 7|x-2|$. Now we can say $\delta \leq \epsilon/7$
- Thus we choose $\delta = min\{1, \epsilon/7\}$
- It follows, $|f(x) - L| = |(x+4)(x-2)| = |(x+4)||(x-2)| < 7\delta \leq 7\epsilon/7 = \epsilon$
- So $|f(x) - L| < \epsilon$
- Thus by definition $f(x) \to L$ as $x \to 2$.
- $lim_{x \to 2} f(x) = 3$

3.2 a) $lim_{x \to 1} \frac{x^2+x-2}{x-1}$

- Let $\epsilon > 0$ and set $L = 3$.
- Then $|f(x) - L| = |\frac{x^2+x-2}{x-1} - 3| = |\frac{x^2-2x-1}{x-1}| = |\frac{(x-1)(x-1)}{x-1}| = |x-1|$
- Let $0 < \delta \leq 1$, then $|x-1| < \delta$.
- This means $|x-1| \leq 1$.
- Which means $|f(x) - L| = |x-1| < \delta = \epsilon$.
- Thus by definition, $\lim_{x \to 1} \frac{x^2+x-2}{x-1} = 3$

4 Problem 3.1.2

(a) $lim_{x \to 0} tan(\frac{1}{x})$

(b) $lim_{x \to 0} xcos(\frac{x^2+1}{x^3})$

4.0.1 A) Initial Thoughts $lim_{x \to 0} tan(\frac{1}{x})$

- So I am suppose to show two sequences that approach 0, but create different ranges for $f(x)$.
- For a problem like this, I think the biggest problem I would have on a test is coming up with two sequences for a trig function.
- What is the generic forumla for this because I have seen this in multiple places by now.
- the solution to this problem (or at least one solution) is the same as the example in the book.
- Why does that solution work? Why does those two squences produce different values in $sin(\frac{1}{x})$.

4.1 A) Proof $lim_{x \to 0} tan(\frac{1}{x})$

- A stated in the text, we need ti that function does not exist as $x \to a$ if we can find two sequences that converge to 'a' whose image under f have different limits.
- We can choose $a_n = \frac{1}{2\pi n}$ and $b_n = \frac{1}{2\pi n + \frac{\pi}{2}}$
- While $f(a_n)$ tends to 0 and $f(b_n)$ tends to $-\infty$.
- Thus $lim_{x \to 0} tan(\frac{1}{x})$ does not exist.

4.2 B) Proof $lim_{x \to 0} xcos(\frac{x^2+1}{x^3})$

- Let $f(x) = x$ and $g(x) = cos(\frac{x^2+1}{x^3})$.
- We can see that $cos(\frac{x^2+1}{x^3}) \leq 1$
- We also can see that $lim_{x \to 0} f(x) = 0$
- Note that multiplication rule for limits says that the product of the limits is the same as the limit of the product of two functions.
- Therefore $lim_{x \to 0} f(x)g(x) = lim_{x \to 0} f(x) \times lim_{x \to 0} g(x) = 0$
- Thus, $lim_{x \to 0} xcos(\frac{x^2+1}{x^3}) = 0$

5 Problem 3.1.5

Thoughts

- This doesn't seem that hard to prove.
- Given $f(x) \leq g(x)$ for all $x \in I$
- And both have limits as $x \to a$
- How would I answer this using the defitnion of a limit?

Thoughts

- Aww gosh I totally wrong on this. I am suppose to use the defintion of a proof.

5.1 Proof

- Since $f(x) \leq g(x)$, then we can make a new $h(x) = g(x) - f(x)$
- Taking the limit of $h(x)$ as $x \to a$ we know that $h(x) \geq 0$

6 Problem 3.1.6

A

- We can see this is not far from the definition of a limit.
- $0 < |x - a| < \delta$ implies $|f(x) - L| < \epsilon$
- Notice $||f(x)| - |L|| \leq |f(x) - L| < \epsilon$ and that $||f(x)| - |L||$
- $|f(x) - L| \leq |f(x)| - |L| < \epsilon$
- This implies $lim_{x \to a} |f(x)| = |L|$ by defintion of a limit.
- Then $|f(x)| \to |L|$ as $x \to a$

B

- Note that we can take advantage of the fact that $|f(x)| \to |L|$, by making the left side approach 1 and the right side approach -1, with the absolute of both being equal.
- $$f(x) = \begin{cases} -1 & x < a \\ 1 & x > a \end{cases}$$

7 Problem 3.1.7

7.0.1 A

- We can see $|f(x)| + f(x) = 0$ if $f(x) < 0$, otherwise $|f(x)| + f(x) = 2f(x)$
- Therefore for all $x \in Dom(f)$, $f^+(x) = \frac{|f(x)|+f(x)}{2} \geq 0$.
- The same logic can be used for the case of $f^-(x)$

for $f(x) = f^+(x) - f^-(x)$ **and** $|f(x)| = f^+(x) + f^-(x)$

- Since both $f^+(x)$ and $f^-(x)$ both hold for every $x \in Dom(f)$ then the addition and subtraction of two also hold the same fact.

7.0.2 B

- Note that $L = lim_{x \to a} f(x)$
- Then $0 < |x - a| < \delta$ and $|f(x) - L|\epsilon$.
- This implies that $-\delta < x - a < \delta$.
- If we take that inequality from $0 \leq x - a < \delta$.
- We can see that still $|f(x) - L| < \epsilon$.
- And thus $lim_{x \to a} f^+(x) = L$.
- the same logic can be applied to $f^-(x)$ using the interval of $-\delta < x - a < 0$