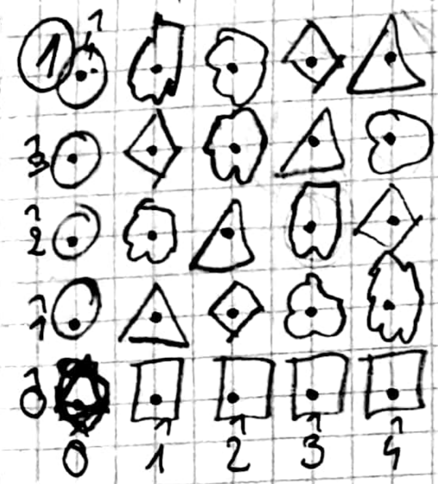
$\Rightarrow$ nem affin ...

Házi feladat 4

$\boxed{4p}$



1-dim. alterek:

~~$W \leq_{\mathbb{Z}_5} \mathbb{Z}_5$~~

$\forall W \leq_{\mathbb{Z}_5} \mathbb{Z}_5^2$ $\dim W = 1 \Rightarrow$

$\Rightarrow \exists w \in \mathbb{Z}_5^2$ ú.h. $W = \langle w \rangle =$

$= \{ kw \mid k \in \mathbb{Z} \}$

$\boxed{1}$: $W = \langle (\hat{0}, \hat{1}) \rangle = \{ \alpha \cdot (\hat{0}, \hat{1}) \mid \alpha \in \{\hat{0}, \hat{1}, \hat{2}, \hat{3}, \hat{4}\} \} =$

$= \{ (\hat{0}, \hat{0}), (\hat{0}, \hat{1}), (\hat{0}, \hat{2}), (\hat{0}, \hat{3}), (\hat{0}, \hat{4}) \} = \langle (\hat{0}, \hat{2}) \rangle = \langle (\hat{0}, \hat{3}) \rangle = \langle (\hat{0}, \hat{4}) \rangle$

2: $W = \langle (\hat{1}, \hat{0}) \rangle = \{ \alpha \cdot (\hat{1}, \hat{0}) \mid \alpha \in \{\hat{0}, \hat{1}, \hat{2}, \hat{3}, \hat{4}\} \} =$

$= \{ (\hat{0}, \hat{0}), (\hat{1}, \hat{0}), (\hat{2}, \hat{0}), (\hat{3}, \hat{0}), (\hat{4}, \hat{0}) \} = \langle (\hat{2}, \hat{0}) \rangle = \langle (\hat{3}, \hat{0}) \rangle = \langle (\hat{4}, \hat{0}) \rangle$

3: $W = \langle (\hat{1}, \hat{1}) \rangle = \{ \alpha \cdot (\hat{1}, \hat{1}) \mid \alpha \in \{\hat{0}, \hat{1}, \hat{2}, \hat{3}, \hat{4}\} \} =$

$= \{ (\hat{0}, \hat{0}), (\hat{1}, \hat{1}), (\hat{2}, \hat{2}), (\hat{3}, \hat{3}), (\hat{4}, \hat{4}) \} = \langle (\hat{2}, \hat{2}) \rangle = \langle (\hat{3}, \hat{3}) \rangle = \langle (\hat{4}, \hat{4}) \rangle$

Scanned with CamScanner

4): $W = \langle \hat{1}, \hat{2} \rangle = \{ \alpha \cdot (\hat{1}, \hat{2}) \mid \alpha \in \{\hat{0}, \hat{1}, \hat{2}, \hat{3}, \hat{4}\} =$

$= \{(\hat{0}, \hat{0}), (\hat{1}, \hat{2}), (\hat{2}, \hat{4}), (\hat{3}, \hat{1}), (\hat{4}, \hat{3})\} = \langle 4, 3 \rangle = \langle 2, 4 \rangle =$

$= \cancel{\langle \hat{2}, \hat{1} \rangle = \langle \hat{4}, \hat{2} \rangle = \langle \hat{1}, \hat{3} \rangle = \langle \hat{3}, \hat{4} \rangle = \langle \hat{5}, \hat{1} \rangle}$

5): $W = \langle \hat{2}, \hat{1} \rangle = \{ \alpha \cdot (\hat{2}, \hat{1}) \mid \alpha \in \{\hat{0}, \hat{1}, \hat{2}, \hat{3}, \hat{4}\} =$

$= \{(\hat{0}, \hat{0}), (\hat{2}, \hat{1}), (\hat{4}, \hat{2}), (\hat{1}, \hat{3}), (\hat{3}, \hat{4})\} =$

$= \langle \hat{2}, \hat{1} \rangle = \langle \hat{3}, \hat{4} \rangle = \langle \hat{4}, \hat{2} \rangle = \langle \hat{1}, \hat{3} \rangle$

6): $W = \langle \hat{1}, \hat{4} \rangle = \{ \alpha \cdot \langle \hat{1}, \hat{4} \rangle \mid \alpha \in \{\hat{0}, \hat{1}, \hat{2}, \hat{3}, \hat{4}\} =$

$= \{(\hat{0}, \hat{0}), (\hat{1}, \hat{4}), (\hat{2}, \hat{3}), (\hat{3}, \hat{2}), (\hat{4}, \hat{1})\} =$

$= \langle \hat{2}, \hat{3} \rangle = \langle 3, 2 \rangle = \langle 4, 1 \rangle$

1p

2) $L = t(W, A)$, ahol $W \leq_{\mathbb{Z}_5} \mathbb{Z}_5$, W 1 dim

$A + W = \{ A + w \mid \text{ahol } w \in W \}$ $\quad A(\hat{2}, \hat{3})$

$W_1: \{(\hat{0}, \hat{0}), (\hat{0}, \hat{1}), (\hat{0}, \hat{2}), (\hat{0}, \hat{3}), (\hat{0}, \hat{4})\}$

$L_1 = t(W_1, A) = \{ A + w_1 \mid w \in W_1 \} =$

$= \{(\hat{2}, \hat{3}), (\hat{2}, \hat{4}), (\hat{2}, \hat{0}), (\hat{2}, \hat{1}), (\hat{2}, \hat{2})\}$

$W_2: \{(\hat{0}, \hat{0}), (\hat{1}, \hat{0}), (\hat{2}, \hat{0}), (\hat{3}, \hat{0}), (\hat{4}, \hat{0})\}$

$L_2 = t(W_2, A) = \{ A + w_2 \mid w \in W_2 \} =$

$= \{(\hat{2}, \hat{3}), (\hat{3}, \hat{3}), (\hat{4}, \hat{3}), (\hat{0}, \hat{3}), (\hat{1}, \hat{3})\}$

$W_3: \{(\hat{0}, \hat{0}), (\hat{1}, \hat{1}), (\hat{2}, \hat{2}), (\hat{3}, \hat{3}), (\hat{4}, \hat{4})\}$

$L_3 = t(W_3, A) = \{ A + w_3 \mid w \in W_3 \} =$

$= \{(\hat{2}, \hat{3}), (\hat{3}, \hat{4}), (\hat{4}, \hat{0}), (\hat{0}, \hat{1}), (\hat{1}, \hat{2})\}$

W_4: $t(W_4, A) = \{A + w_4 \mid w \in W_4\} =$
$= \{(\hat{0},\hat{0}), (\hat{1},\hat{2}), (\hat{2},\hat{4}), (\hat{3},\hat{1}), (\hat{4},\hat{3})\}$

$L_4 = \{(\hat{2},\hat{3}), (\hat{3},\hat{0}), (\hat{4},\hat{2}), (\hat{0},\hat{4}), (\hat{1},\hat{1})\}$

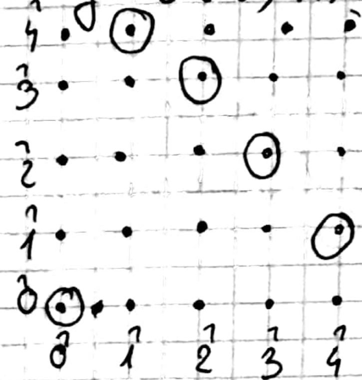
W_5: $t(W_5, A) = \{A + w_5 \mid w \in W_5\} =$
$= \{(\hat{0},\hat{0}), (\hat{2},\hat{1}), (\hat{4},\hat{2}), (\hat{1},\hat{3}), (\hat{3},\hat{4})\}$

$L_5 = \{(\hat{2},\hat{3}), (\hat{4},\hat{4}), (\hat{1},\hat{0}), (\hat{3},\hat{1}), (\hat{0},\hat{2})\}$

W_6: $t(W_6, A) = \{A + w_6 \mid w \in W_6\} =$
$\{(\hat{0},\hat{0}), (\hat{1},\hat{4}), (\hat{2},\hat{3}), (\hat{3},\hat{2}), (\hat{4},\hat{1})\}$

$L_6 = \{(\hat{2},\hat{3}), (\hat{3},\hat{2}), (\hat{4},\hat{1}), (\hat{0},\hat{1}), (\hat{1},\hat{4})\}$ 1p

3. $y = t(W, A)$ egyenes $W = \langle(\hat{3},\hat{2})\rangle \leq \mathbb{Z}_5^2$ $A = (\hat{2},\hat{3})$



$W = \langle(\hat{3},\hat{2})\rangle = \{\alpha \cdot (\hat{3},\hat{2}) \mid \alpha \in \{\hat{0},\hat{1},\hat{2},\hat{3},\hat{4}\}\} =$
$= \{(\hat{0},\hat{0}), (\hat{3},\hat{2}), (\hat{1},\hat{4}), (\hat{4},\hat{1}), (\hat{2},\hat{3})\}$

$y = \{(\hat{2},\hat{3}), (\hat{0},\hat{0}), (\hat{3},\hat{2}), (\hat{1},\hat{4}), (\hat{4},\hat{1})\}$

$L_1 = \{(\hat{4},\hat{0}), (\hat{3},\hat{1}), (\hat{2},\hat{2}), (\hat{1},\hat{3}), (\hat{0},\hat{4})\}$
$L_2 = \{(\hat{3},\hat{0}), (\hat{2},\hat{1}), (\hat{1},\hat{2}), (\hat{0},\hat{3})\}$
$L_3 = \{(\hat{2},\hat{0}), (\hat{1},\hat{1}), (\hat{0},\hat{2})\}$
$L_4 = \{(\hat{1},\hat{0}), (\hat{0},\hat{1})\}$
$L_5 = \{(\hat{4},\hat{2}), (\hat{3},\hat{3}), (\hat{2},\hat{4})\}$
$L_6 = \{(\hat{4},\hat{3}), (\hat{3},\hat{4})\}$

$y = t(W, A)$-val // egyenesek.

Minden egyenesen 5 pont kell legyen. 0,5p

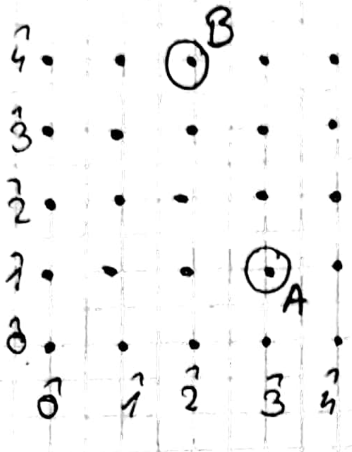
4. $A=(\hat{3},\hat{1})$ $\quad B=(\hat{2},\hat{4})$

$L=t(\vec{L},A) \Rightarrow |L|=|\vec{L}|=\{\lambda v \mid \lambda\in K\}=|K|$ $\quad v\in\vec{L},\ v\neq 0$

$\vec{AB}=(\hat{2}-\hat{3},\ \hat{4}-\hat{1}) \Leftrightarrow \vec{AB}(-\hat{1},\ \hat{3}) \Leftrightarrow \vec{AB}(\hat{4},\hat{3})$

$AB=t(\langle\vec{AB}\rangle, A)=\{A+\lambda\vec{AB} \mid \lambda\in\mathbb{Z}_5\}$

- ha $\lambda=0$, megkapjuk az A-t.
- ha $\lambda=1 \Rightarrow$ megkapjuk a B-t



∀ AB egyenes 5 pontot tartalmaz

ha $\lambda=2 \Rightarrow A+2\vec{AB}=(\hat{3},\hat{1})+(\hat{3},\hat{1})=(\hat{1},\hat{2})$

ha $\lambda=\hat{3} \Rightarrow A+3\vec{AB}=(\hat{3},\hat{1})+(\hat{2},\hat{4})=(\hat{0},\hat{0})$

ha $\lambda=\hat{4} \Rightarrow A+4\vec{AB}=(\hat{3},\hat{1})+(\hat{1},\hat{2})=(\hat{4},\hat{3})$

$$\frac{x-x_A}{x_B-x_A}=\frac{y-y_A}{y_B-y_A} \Leftrightarrow \frac{x-\hat{3}}{\hat{4}}=\frac{y-\hat{1}}{\hat{3}}$$

0,7p

5. $\mathbb{Z}_5$-ben 5 pont van.
$\mathbb{Z}_5^2$-ben 25 pont van.

$\mathbb{Z}_5$-ben összesen 6 egyenes lehet párhuzamos egyszerre. ∀ egyenessel 5 párhuzamos egyenes húzható.

Vagyis:
$$\frac{C_{25}^2\cdot C_{20}^1}{C_3^2}=\frac{\frac{25!}{2!\cdot 23!}\cdot\frac{20!}{1!\cdot 19!}}{\frac{3!}{2!\cdot 1!}}=$$

$$=\frac{\frac{\cancel{23!}\cdot \overset{12}{\cancel{24}}\cdot 25}{\cancel{2!}\cdot\cancel{23!}}\cdot 20}{3}=\frac{\overset{6}{\cancel{12}}\cdot \overset{25}{15}\cdot 20}{\cancel{3}}=\overset{4}{6}\cdot \overset{5}{3}00=1800$$

1800 △ létezik

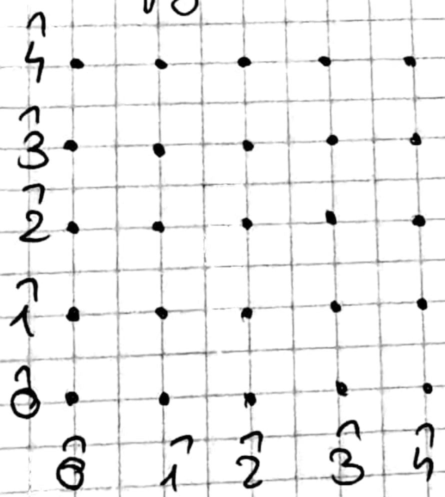
2000

0,8p

6. $\mathbb{Z}_5$-ben összesen 5 pont

$\mathbb{Z}_5^2$-ben összesen 25 pont.

Max. 5 egyenes lehet párhuzamos egy adott egyenessel.



0p