

## Assignment Sheet 1.

Submit on Tuesday, February 15, 2022, 10:00.

**Exercise 1.** (Random experiments)

The disjoint events  $A$ ,  $B$ , and  $C$  are defined in a sample space  $\Omega$ . Find expressions for the following probabilities in terms of sums the probabilities  $P(\emptyset), P(A), P(B), P(A \cup B), P(A \cup C), P(B \cup C), P(A \cup B \cup C)$ :

(Example: the probability that  $A$  or  $B$  occurs:  $P(A) + P(B)$ )

- a) the probability that exactly two of  $A$ ;  $B$ ;  $C$  occur (*at the same time!*);
- b) the probability that exactly one of these events occurs;

(4 Points)

**Exercise 2.** We consider the chance experiment of four tosses of a fair coin.

- a) Give the sample space  $\Omega$  for this experiment and define a discrete random variable  $X$  to describe the number of heads obtained in four tosses of a fair coin.
- b) Find the PMF and CDF of the defined random variable and plot the CDF.
- c) Compute the probability of the event  $X$  IS BIGGER THAN 1
- d) Compute the mean of the random variable.

(4 Points)

**Exercise 3.** Let the function  $\rho$  with  $\rho(x) = \begin{cases} c|x|(1+x)(1-x) & \text{if } -1 \leq x \leq 1, \\ 0 & \text{else,} \end{cases}$  be given.

- a) Compute the constant  $c$ , such that  $\rho$  becomes a density of some random variable.
- b) Consider a random variable  $X$  with the just computed density  $\rho$ . Find its CDF and plot it.
- c) Use the CDF to compute

$$P(X < -0.5), \quad P(X > 0.5), \quad P(-0.5 < X < 0.5).$$

(4 Points)

**Programming Exercise 1.** In this first programming exercise, we would like to familiarize us with typical linear algebra computing tasks. To this end, we define the vectors / matrices

$$\mathbf{a} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} -1 \\ 2 \\ 5 \end{pmatrix}, \quad M = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}.$$

Students are free to choose between the programming languages Python and C/C++. In case of Python, the module *NumPy* should be used for the linear algebra tasks. For C++, there exists a convenient numerics header-only library called *Eigen*. Classical C programmers can go for *LAPACK*. (Reference solutions will only be provided in Python.) The submission format for Python is a Jupyter notebook. The submission format for C/C++ is standard source files.

Use one of the listed programming languages with the respective library to implement the following tasks:

- a) Compute and print the inner product between  $\mathbf{a}$  and  $\mathbf{b}$ .
- b) Compute and print the matrix-vector product between  $M$  and  $\mathbf{b}$ .
- c) Compute and print the  $l_2$  norm of  $\mathbf{b}$ .
- d) Compute and print the the solution  $\mathbf{x}$  of the linear system of equations  $M\mathbf{x} = \mathbf{b}$  using the LU factorization.
- e) Compute and print the the solution  $x$  of the linear system of equations  $M\mathbf{x} = \mathbf{b}$  using the Cholesky factorization.

(4 Points)