MAT1320-LİNEER CEBİR ARASINAV SORULARI	A
Ad Soyad: CEVAP ANAHTARI	**Grup No:**
Öğrenci No:	**Süre:** 90 dakika
Bölüm:	**Tarih:** 27 Kasım 2021, Cumartesi
Öğretim Üyesi:	**İmza:**

UYARI: *YÖK nun 2547 sayılı Kanunun Öğrenci Disiplin Yönetmeliğinin 9. Maddesi olan "Sınavlarda kopya yapmak ve yaptırmak veya buna teşebbüs etmek" fiili işleyenler bir veya iki yarıyıl uzaklaştırma cezası alırlar.*

1. A sütunları toplamı $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ matrisine eşit olan 3×3 tipinde bir matris ve $B = \begin{bmatrix} -1 & 1 & 2 \\ -1 & 1 & 2 \\ -1 & 1 & 2 \end{bmatrix}$ ise AB matrisi aşağıdakilerden hangisidir?

   a) Hiçbiri b) $\begin{bmatrix} -1 & 1 & 2 \\ -1 & 1 & 2 \\ -1 & 1 & 2 \end{bmatrix}$ c) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

   d) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ e) $\begin{bmatrix} -1 & -1 & -1 \\ 1 & 1 & 1 \\ 2 & 2 & 2 \end{bmatrix}$

2. A ve B, $n \times n$ boyutlu iki matris olmak üzere aşağıdakilerden hangileri doğrudur?

   I. $\det\left(A^T B\right) = \det\left(B^T A\right)$.

   II. 3×3 $(n = 3)$ boyutlu bir matrisin determinantı esas köşegen elemenlarının çarpımı ise bu matris ya üst ya da alt üçgensel bir matristir.

   III. 4×4 $(n = 4)$ boyutlu bir B matrisi için her zaman $\det(2B) = 16\det(B)$ dir.

   a) Yalnız I b) I ve II c) II ve III

   d) I ve III e) I, II ve III

3. Aşağıdakilerden hangisi $A = \begin{bmatrix} 0 & 0 & 2 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & -1 & 3 & 0 \\ 2 & 1 & 5 & -3 \end{bmatrix}$ matrisinin tersidir (eğer varsa) ? (İpucu: Elementer satır işlemlerini kullanabilirsiniz.)

   a) $\begin{bmatrix} -4/5 & 3/2 & 1/2 & 4/5 \\ 3/5 & 0 & 0 & 2/5 \\ 1/5 & -1 & 0 & -1/5 \\ 1/5 & 0 & 0 & -1/5 \end{bmatrix}$

   b) $\begin{bmatrix} -4/5 & 3/5 & 1/5 & 1/5 \\ 3/2 & 0 & -1 & 0 \\ 1/2 & 0 & 0 & 0 \\ 4/5 & 2/5 & -1/5 & -1/5 \end{bmatrix}$

   c) $\begin{bmatrix} -4/5 & 3/5 & 3/2 & 4/5 \\ 1/5 & 0 & 0 & 1/5 \\ 2/5 & -1 & 0 & -2/5 \\ 2/5 & 0 & 0 & -2/5 \end{bmatrix}$

   d) $\begin{bmatrix} -4 & 3 & 1 & 4 \\ 3/5 & 0 & 0 & 2/5 \\ 1 & -1 & 0 & -1 \\ 5 & 0 & 0 & -1 \end{bmatrix}$ e) A nın tersi yoktur.

4. Elemanları reel sayılar olan 3×3 boyutlu A ve B matrisleri için $\det(A) = \sqrt{3}$ ve $\det(B) = \frac{1}{2}$ olsun. Aşağıdakilerden hangisi $\det\left(2A^T B^{-3}\right)$ değerine eşittir?

   a) $32\sqrt{3}$ b) $16\sqrt{3}$ c) 48 d) $4\sqrt{3}$ e) $64\sqrt{3}$

5.
$$\begin{bmatrix} 2 & 0 & 0 & -3 & 1 \\ 0 & 0 & 0 & 0 & 7 \\ -3 & 2 & 0 & -1 & -6 \\ 2 & -2 & -1 & 4 & 2 \\ 0 & 0 & 0 & 4 & 3 \end{bmatrix}$$
matrisinin determinantı aşağıdakilerden hangisidir?

a) 0 b) -56 c) 56 d) -112 e) 112

6. k nın hangi değerleri için $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & k \\ 1 & 4 & k^2 \end{bmatrix}$ matrisinin tersi mevcuttur?

a) $\{1, 2\}$ b) $\mathbb{R} - \{1\}$ c) $\mathbb{R} - \{1, 2\}$

d) $\mathbb{R}$ e) Böyle bir k sayısı yoktur.

7. $A = \begin{pmatrix} 1 & x \\ 0 & 1 \end{pmatrix}$ bir involut matris, $B = \begin{pmatrix} y & 0 \\ y & 0 \end{pmatrix}$ sıfırdan farklı bir idempotent matris ve $C = \begin{pmatrix} 1 & -3 \\ -1 & 3 \end{pmatrix}$ bir singüler (tersi olmayan) matris ise $x - y - z$ değeri aşağıdakilerden hangisidir?

a) -1 b) 0 c) 1 d) -2 e) 2

8. $A = \begin{bmatrix} -1 & 1 & 3 \\ 2 & 0 & -2 \\ 1 & 3 & -2 \end{bmatrix}$ terslenebilir bir matris olmak üzere Ek (A^{-1}) matrisi aşağıdakilerden hangisidir?

a) $\begin{bmatrix} 3/7 & 11/14 & -1/7 \\ 1/7 & -1/14 & 2/7 \\ 3/7 & 2/7 & -1/7 \end{bmatrix}$ b) $\begin{bmatrix} -1/14 & 1/14 & 3/14 \\ 1/7 & 0 & -1/7 \\ 1/14 & 3/14 & -1/7 \end{bmatrix}$

c) $\begin{bmatrix} 3/7 & 1/7 & 3/7 \\ 11/14 & -1/14 & 2/7 \\ -1/7 & 2/7 & -1/7 \end{bmatrix}$ d) $\begin{bmatrix} -1/14 & 1/7 & 1/14 \\ 1/14 & 0 & 3/14 \\ 3/14 & -1/7 & -1/7 \end{bmatrix}$

e) Hiçbiri

9. Genişletilmiş katsayılar matrisi
$$\left[\begin{array}{cccc|c} 2 & -1 & 2 & 2 & 1 \\ 1 & 2 & -1 & 1 & -1 \\ 4 & -5 & 0 & k-1 & 2 \end{array}\right]$$
olan bir lineer denklem sistemi k'nın hangi değerleri için çözümsüzdür?

a) 2 b) 4 c) 6 d) $k \in \mathbb{R}$

e) Sistemi çözümsüz yapan bir k değeri yoktur.

10.
$$4x_1 - 2x_2 + 7x_3 = 0$$
$$8x_1 - 3x_2 + 10x_3 = 0$$
homojen lineer denklem sisteminin çözümü aşağıdakilerden hangisidir?

a) Sadece aşikar (sıfır) çözümü vardır.

b) $x_1 = \frac{1}{4}k,\ x_2 = 4k,\ x_3 = k,\ k \in \mathbb{R}$.

c) $x_1 = 4k,\ x_2 = \frac{1}{4}k,\ x_3 = k,\ k \in \mathbb{R}$.

d) $x_1 = k,\ x_2 = 4k,\ x_3 = k,\ k \in \mathbb{R}$.

e) Sistemin çözümü yoktur.

11. $A = \begin{bmatrix} 0 & 1-a & 2-a \\ 1 & 1 & 1 \\ -1 & 2 & 1-a \end{bmatrix}$ matrisi için $\text{rank}(A) = 2$ ise a nın değerleri aşağıdakilerden hangisidir?

a) $a = -1$ ya da $a = -2$ b) $a = -2$ ya da $a = 2$

c) $a = -1$ ya da $a = 1$ d) $a \neq -1$ ya da $a \neq -2$

e) $a = -1$ ya da $a = 2$

12. Aşağıdakilerden hangileri doğrudur?

I. Satırca eşelon formdaki her matris aynı zamanda satırca indirgenmiş eşelon formdadır.

II. Terslenebilir bir matrisin satırca indirgenmiş eşelon formu birim matristir.

III. Terslenebilir olmayan bir matrisin indirgenmiş eşelon formunda sıfır satırı mevcuttur.

IV. Birim matrise denk olan bir matris terslenebilir değildir (singülerdir).

a) I ve IV b) Yalnız I c) II ve III d) Yalnız IV

e) Hepsi

13. $A = \begin{bmatrix} 0 & -3 & -1 & 1 \\ -2 & 0 & 2 & 5 \\ 3 & -2 & 0 & 0 \\ 1 & -4 & 0 & 0 \end{bmatrix}$ matrisi için aşağıdakilerden hangileri doğrudur?

I. A matrisinin tersi vardır.

II. A matrisinin satırca indirgenmiş eşelon formu I_4 birim matrisidir.

III. $\text{Ek}(A)A = \begin{bmatrix} 70 & 0 & 0 & 0 \\ 0 & 70 & 0 & 0 \\ 0 & 0 & 70 & 0 \\ 0 & 0 & 0 & 70 \end{bmatrix}$

a) Yalnız I b) I ve II c) I ve III

d) II ve III e) I, II ve III

14. $\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = 4$ ise $\begin{vmatrix} a_1 & a_2 & 4a_3 - 2a_2 \\ b_1 & b_2 & 4b_3 - 4b_2 \\ \frac{1}{2}c_1 & \frac{1}{2}c_2 & 2c_3 - c_2 \end{vmatrix}$ değeri aşağıdakilerden hangisidir?

a) 8 b) 6 c) 4 d) 2 e) 1

15. $A = \begin{bmatrix} 3 & 4 \\ 1 & 1 \end{bmatrix}$ ve $AB = \begin{bmatrix} 1 & 0 & -2 & 5 \\ 4 & -1 & 0 & 3 \end{bmatrix}$ ise B matrisinin b_{24} elemanı aşağıdakilerden hangisidir?

a) -2 b) 3 c) 8 d) -4 e) 11

16. Elemanları reel sayılar olan 4×4 boyutlu bir A matrisinin tersi mevcut ise aşağıdakilerden hangisi daima doğrudur?

a) $\text{Rank}(A) = 1$ b) $\det(A) = 1$ c) $\det(A) = 4$
d) $\text{Rank}(A) = 4$ e) $\det(A) = \text{Rank}(A)$

17. 3×3 boyutlu bir A matrisi için $\det(A) = -7$ olsun. $\det\left(2A^{-1} + \text{Ek}(A)\right)$ değeri aşağıdakilerden hangisidir?

a) $\frac{125}{7}$ b) -125 c) $\frac{-125}{7}$ d) $\frac{-1}{7}$ e) $\frac{-7}{125}$

18. $A = \begin{bmatrix} -2 & 3 & 0 \\ 4 & 1 & -3 \\ 2 & 0 & 1 \end{bmatrix}$ matrisi verilsin. $a_{ij} \in A$ elemanının eş çarpanı (kofaktörü) A_{ij} olmak üzere $a_{11}A_{12} + a_{21}A_{22} + a_{31}A_{32}$ değeri aşağıdakilerden hangisidir?

a) 0 b) -32 c) 32 d) 16 e) -16

19.
$$x + y - z = 2$$
$$x + 2y + z = 3$$
$$x + y + (a^2 - 5)z = a$$

lineer denklem sisteminin a'nın hangi değerleri için tek çözümü vardır?

a) $a = 2$ b) $a = -2$ c) $a = \pm 2$ d) $a \neq \pm 2$
e) Böyle bir a sayısı yoktur.

20. A terslenebilir bir matris ve $A^{-1} = \begin{bmatrix} 4 & 1 \\ 1 & 0 \end{bmatrix}$ olsun. $b = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$ olmak üzere $A^T x = b$ lineer denklem sisteminin çözümü aşağıdakilerden hangisidir?

a) $x = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ b) $x = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$ c) $x = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$
d) $x = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$ e) $x = \begin{bmatrix} -2 \\ -1 \end{bmatrix}$

MAT1320 LINEAR ALGEBRA EXERCISES I

Name Surname:	**Group No:**
Student No:	**Duration:**
Department:	**Date:** 03 October 2022
Lecturer: Dr. Mustafa SARI	**Signature:**

1. Let A be a 3×3 matrix such that the sum of its columns is $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 1 & 2 \\ -1 & 1 & 2 \\ -1 & 1 & 2 \end{bmatrix}$. Then, which of the followings is equal to the matrix AB?

a) $\begin{bmatrix} -1 & 1 & 2 \\ -1 & 1 & 2 \\ -1 & 1 & 2 \end{bmatrix}$ b) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ c) $\begin{bmatrix} -1 & -1 & -1 \\ 1 & 1 & 1 \\ 2 & 2 & 2 \end{bmatrix}$

d) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ e) None of them

3. If $\begin{bmatrix} 1 & x & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 1 \\ 0 & 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ 1 \\ -2 \end{bmatrix} = 0$, then which of the followings is equal to x?

a) -2 b) 2 c) $-\frac{1}{7}$ d) $\frac{7}{8}$ e) $\frac{8}{3}$

2. For the matrices $X_{2\times 3}, Y_{2\times 3}$ and $A_{4\times 3}$, which of the followings is equal to the dimension of the matrix $\left[A\left(X^TY\right)^{-1}A^T\right]^T$?

a) (4×4) b) (4×3) c) (3×4) d) (2×2) e) (4×2)

4. Let A, B, C and D be $n \times n$ matrices having the inverse. If $ACBD = I_n$, then which of the followings is equal to C^{-1} ?

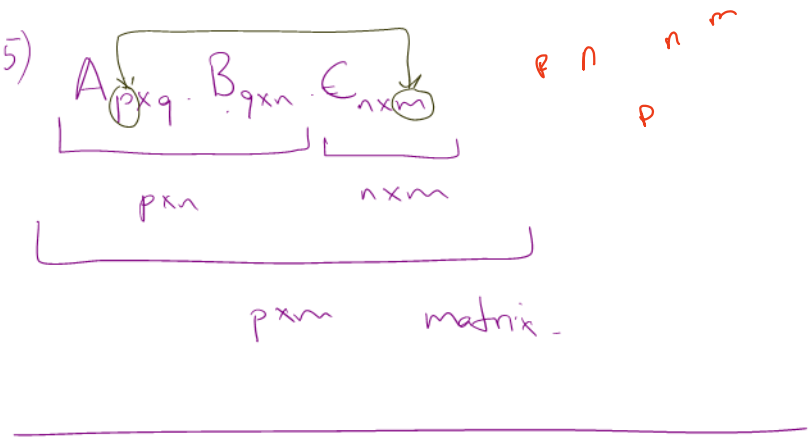
a) BAD b) BDA c) DBA d) $A^{-1}D^{-1}B^{-1}$

e) C^{-1} may not be exist.

5. If $A = [a_{ij}]_{p\times q}$, $B = [b_{jk}]_{q\times n}$ and $C = [c_{kl}]_{n\times m}$, then which of the followings is the dimension of the matrix ABC?

a) $q \times m$ b) $q \times n$ c) $p \times m$

d) $m \times p$ e) $p \times q$



6. If $A = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$, then which of the followings is equal to the matrix $2A^2 + A - 5I_2$?

a) $\begin{bmatrix} -2 & -5 \\ 0 & -2 \end{bmatrix}$ b) $\begin{bmatrix} -2 & -5 \\ 0 & 2 \end{bmatrix}$ c) $\begin{bmatrix} 2 & -5 \\ 0 & -2 \end{bmatrix}$

d) $\begin{bmatrix} -2 & 5 \\ 0 & -2 \end{bmatrix}$ e) $\begin{bmatrix} 2 & 5 \\ 0 & -2 \end{bmatrix}$

7. Let the matrix $A = \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$ and the polynomial $f(x) = 2x^3 - x^2 + 3x + 7$ be given. Then, which of the followings is the matrix $f(A)$?

a) $\begin{bmatrix} 16 & 16 \\ 6 & 9 \end{bmatrix}$ b) $\begin{bmatrix} 17 & 12 \\ 8 & 9 \end{bmatrix}$ c) $\begin{bmatrix} 16 & 16 \\ 9 & 6 \end{bmatrix}$

d) $\begin{bmatrix} 14 & 16 \\ 6 & 10 \end{bmatrix}$ e) $\begin{bmatrix} 8 & 6 \\ 4 & 12 \end{bmatrix}$

8. Let $B = \begin{bmatrix} -1 & 3 \\ -1 & 2 \end{bmatrix}$ and $C = \begin{bmatrix} 3 & 2 \\ -1 & -1 \end{bmatrix}$. If $(AB)^{-1} = (CB)^T$, then which of the following is equal to the matrix A^{-1}?

a) $\begin{bmatrix} -44 & 17 \\ 31 & -12 \end{bmatrix}$ b) $\begin{bmatrix} 44 & -17 \\ 31 & 12 \end{bmatrix}$ c) $\begin{bmatrix} 44 & -17 \\ 31 & -12 \end{bmatrix}$

d) $\begin{bmatrix} -44 & 17 \\ -31 & -12 \end{bmatrix}$ e) $\begin{bmatrix} -44 & -17 \\ 31 & 12 \end{bmatrix}$

9. Let $A = \begin{bmatrix} -2 & 1 & 0 & -2 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 3 & 0 & 0 & 2 \end{bmatrix}$. Which of the following is the $(3,3)$-entry of the inverse matrix A^{-1} ? $\Rightarrow AA^{-1} = I_4$

a) 0 b) -2 c) 1 d) -1 e) 2

$$\begin{bmatrix} -2 & 1 & 0 & -2 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 3 & 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} = ?$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Multiplying the second row of A and the third column of A^{-1}, we get

$0 \cdot a_{13} + 0 \cdot a_{23} + 1 \cdot a_{33} + 0 \cdot a_{43} = 0$

$\Rightarrow \boxed{a_{33} = 0}$

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10) To find $(3,3)$-entry of the matrix AB, we need to calculate the product of third row of A and third column of B.

$$\begin{bmatrix} a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix} \begin{bmatrix} b_{13} \\ b_{23} \\ b_{33} \\ b_{43} \end{bmatrix} = \begin{bmatrix} \\ \\ c_{33} \\ \\ \end{bmatrix}$$

$a_{31} = 2,\ a_{32} = 1,\ a_{33} = 6,\ a_{34} = 7$

$b_{13} = -1,\ b_{23} = 1,\ b_{33} = 4,\ b_{43} = 4$

$\Rightarrow c_{33} = 2 \cdot -1 + 1 \cdot 1 + 6 \cdot 4 + 7 \cdot 4 = 51$

10. Let $A = [a_{ij}]_{n \times m}$ and $B = [b_{ij}]_{m \times r}$ be two matrices where $a_{ij} = \begin{cases} i+j, & i \le j \\ i-j & i > j \end{cases}$ and $b_{ij} = \begin{cases} 2i-j, & i < j \\ j+1, & i \ge j \end{cases}$. Then, which of the following is the $(3,3)$-entry of the matrix AB? for $n=4$!

a) 51 b) 54 c) 55 d) 56 e) 57

11. If $A = \begin{bmatrix} 3 & 4 \\ 1 & 1 \end{bmatrix}$ and $AB = \begin{bmatrix} 1 & 0 & -2 & 5 \\ 4 & -1 & 0 & 3 \end{bmatrix}$, then which of the following is the entry b_{24}?

a) -2 b) 3 c) 8 d) -4 e) 11

$A_{2\times 2} \cdot B_{m\times n} = \begin{bmatrix} 1 & 0 & -2 & 5 \\ 4 & -1 & 0 & 3 \end{bmatrix}_{2\times 4} \Rightarrow m=2,\ n=4.$

$$\Rightarrow \begin{bmatrix} 3 & 4 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & b_{23} & b_{24} \end{bmatrix} = \begin{bmatrix} 1 & 0 & -2 & 5 \\ 4 & -1 & 0 & 3 \end{bmatrix}$$

$\Rightarrow 3 \cdot b_{14} + 4 \cdot b_{24} = 5$

$-3 \;/\; 1 \cdot b_{14} + 1 \cdot b_{24} = 3$

$+$

$b_{24} = 5 - 3 \cdot 3 = -4$

12. (D points) Let $B = \begin{bmatrix} -2 & 1 & 0 & -2 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 3 & 0 & 0 & 2 \end{bmatrix}$. Which of the followings is the $(4,4)$-entry of the matrix B^{-1}? $\Rightarrow BB^{-1} = I_4$

a) 0 b) -2 c) 1 d) -1 e) 2

$$\begin{bmatrix} -2 & 1 & 0 & -2 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 3 & 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & b_{23} & b_{24} \\ b_{31} & b_{32} & b_{33} & b_{34} \\ b_{41} & b_{42} & b_{43} & b_{44} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & * & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

→ Second row, fourth column: $\Rightarrow b_{34} = 0$.

→ third row, fourth column: $b_{24} + b_{34} = 0$.

$\Rightarrow b_{24} = 0$.

→ first row, fourth column:

$-2 \cdot b_{14} + 1 \cdot \overbrace{b_{24}}^{0} + 0 \cdot b_{34} - 2 b_{44} = 0$

$\Rightarrow b_{14} + b_{44} = 0 \Rightarrow b_{14} = -b_{44}$

→ fourth row fourth column:

$3 \cdot b_{14} + 2 \cdot b_{44} = 1 \Rightarrow -b_{44} = 1 \Rightarrow \boxed{b_{44} = -1}$

MAT1320 LINEAR ALGEBRA EXERCISES III

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Department:	**Date:** 17 October 2022
Lecturer: Dr. Mustafa SARI	**Signature:**

1. For the matrix $A = \begin{bmatrix} 0 & 0 & 2 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & -1 & 3 & 0 \\ 2 & 1 & 5 & -3 \end{bmatrix}$, which of the followings is the inverse matrix of A (if exists). (Hint: You

2. If $\text{rank}(A) = 2$ for the matrix $A = \begin{bmatrix} a & 1 & 2 \\ 1 & 1 & 1 \\ -1 & 1 & 1-a \end{bmatrix}$, then which of the followings is all the possible values of a?

$= 2$

$\neq -2$

3. (C points) Which of the following matrices are of reduced row echelon form?

$$\mathcal{A} = \begin{bmatrix} 1 & 2 & 0 & 0 & 1 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \mathcal{B} = \begin{bmatrix} 1 & 2 & 0 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 3 \end{bmatrix},$$

$$\mathcal{C} = \begin{bmatrix} 1 & 0 & 3 & 4 \\ 0 & 1 & -2 & 5 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \mathcal{D} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

a) $\mathcal{B}$ and $\mathcal{C}$ b) Only $\mathcal{D}$ c) $\mathcal{A}$ and $\mathcal{D}$ d) Only $\mathcal{A}$ e) Only $\mathcal{B}$

4. Which of the followings is the reduced row echelon form of the matrix $\begin{bmatrix} 2 & 3 & 3 & 5 \\ 6 & 7 & 8 & 9 \\ 1 & 0 & 0 & 4 \end{bmatrix}$?

a) $\begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 7/2 \\ 0 & 0 & 1 & -9/2 \end{bmatrix}$ b) $\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 8 & 9 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

c) $\begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ d) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

e) $\begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & 1 & -8 \end{bmatrix}$

$\begin{bmatrix} 2 & 3 & 3 & 5 \\ 6 & 7 & 8 & 9 \\ 1 & 0 & 0 & 4 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 0 & 0 & 4 \\ 2 & 3 & 3 & 5 \\ 6 & 7 & 8 & 9 \end{bmatrix}$

$r_2 \to r_2 - 2r_1$, $r_3 \to r_3 - 6r_1$: $\begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 3 & 3 & -3 \\ 0 & 7 & 8 & -15 \end{bmatrix}$ $r_2 \to \frac{1}{3}$: $\begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 1 & -1 \\ 0 & 7 & 8 & -15 \end{bmatrix}$

$r_3 \to r_3 - 7r_2$: $\begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 1 & -8 \end{bmatrix}$

$r_2 \to r_2 - r_3$: $\begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & 1 & -8 \end{bmatrix}$

5. Which of the followings is the rank of the matrix

$$\begin{bmatrix} 1 & -1 & -2 & 0 & 2 \\ 2 & 2 & -4 & 0 & 1 \\ 3 & 3 & -6 & 0 & -7 \\ 4 & -2 & -8 & 0 & 17 \\ 5 & 4 & -10 & 0 & -4 \end{bmatrix} ?$$

a) 3 b) 4 c) 5 d) 2 e) 1

Since rank is the number of nonzero rows in R.R.E.F or E.F, zero column doesn't change the rank. So, we don't have to write it.

$$\begin{bmatrix} 1 & -1 & -2 & 2 \\ 2 & 2 & -4 & 1 \\ 3 & 3 & -6 & -7 \\ 4 & -2 & -8 & 17 \\ 5 & 4 & -10 & -4 \end{bmatrix}$$

Also, recall that $\text{rank}(A) = \text{rank}(A^T)$.

$$\Rightarrow \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ -1 & 2 & 3 & -2 & 4 \\ -2 & -4 & -6 & -8 & -10 \\ 2 & 1 & -7 & 17 & -4 \end{bmatrix} \xrightarrow[r_3 \to r_3+2r_1,\ r_4 \to r_4-2r_1]{r_2 \to r_2+r_1} \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 0 & 4 & 6 & 2 & 9 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -3 & -13 & 9 & -14 \end{bmatrix} \xrightarrow{r_2 \to r_2+r_4} \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 0 & 1 & -7 & 11 & -5 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -3 & -13 & 9 & -14 \end{bmatrix}$$

$$\xrightarrow{r_4 \leftrightarrow r_3} \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 0 & 1 & -7 & 11 & -5 \\ 0 & -3 & -13 & 9 & -14 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow[r_3 \to r_3+3r_2]{r_1 \to r_1-2r_2} \begin{bmatrix} 1 & 0 & 17 & -18 & 15 \\ 0 & 1 & -7 & 11 & -5 \\ 0 & 0 & -34 & 42 & -29 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow{r_3 \to \frac{1}{34} r_3} \begin{bmatrix} 1 & 0 & 17 & -18 & 15 \\ 0 & 1 & -7 & 11 & -5 \\ 0 & 0 & 1 & -42/34 & 29/34 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

In echelon form.

$\Rightarrow$ The number of the nonzero rows is 3. $\Rightarrow \text{rank}(A) = 3$.

6. Let the matrix $B = [b_{ij}]$ be given as $1 \le i, j \le n$, $n \ge 3$ and $b_{ij} = i.j$. Then, which of the followings is the rank of the matrix B?

a) 0 b) 1 c) 2 d) 3 e) 4

Write some rows of B.

$$B = \begin{bmatrix} 1 & 2 & 3 & 4 & \dots & n \\ 2 & 4 & 6 & 8 & \dots & 2n \\ 3 & 6 & 9 & 12 & \dots & 3n \\ & & \vdots & & & \end{bmatrix}$$

$$\begin{matrix} r_2 \to r_2 - 2r_1 \\ r_3 \to r_3 - 3r_1 \\ \vdots \\ r_n \to r_n - nr_1 \end{matrix} \begin{bmatrix} 1 & 2 & 3 & 4 & \dots & n \\ 0 & 0 & 0 & 0 & \dots & 0 \\ & & \vdots & & & \\ 0 & 0 & 0 & 0 & \dots & 0 \end{bmatrix}$$

$\Rightarrow \text{rank}(B) = 1$.

MAT1320 LINEAR ALGEBRA EXERCISES II

Name Surname:	**Group No:**
Student No:	**Duration:**
Department:	**Date:** 10 October 2022
Lecturer: Dr. Mustafa SARI	**Signature:**

1. Let $A = \begin{pmatrix} 1 & x \\ 0 & 1 \end{pmatrix}$ be an involut matrix, let $B = \begin{pmatrix} y & 0 \\ y & 0 \end{pmatrix}$ be a nonzero idempotent matrix and let $C = \begin{pmatrix} 1 & -3 \\ z & 3 \end{pmatrix}$ be a singular matrix (not invertible). Then, which of the followings is the value $x - y - z$?

a) -1 b) 0 c) 1 d) -2 e) 2

2. If $A = \begin{bmatrix} 0 & a+b & c+2 \\ a & 2 & c \\ 4 & a+b & 4 \end{bmatrix}$ is a symmetric matrix, then which of the followings is true for the matrix $B = \begin{bmatrix} 0 & a & -2 \\ b-a & 0 & 1 \\ c & -1 & b \end{bmatrix}$?

I. B is a skew-symmetric matrix.
II. B^2 is a symmetric matrix.
III. $\mathrm{Tr}(B) = \mathrm{Tr}(A)$.

a) Only I b) I and II c) II and III
d) I and III e) All of them

3. Let A be $n \times n$ skew-symmetric matrix and let x be a vector with n components. Which of the followings is equal to $x^T Ax$ for all $x \in \mathbb{R}^n$?

a) $\mathbf{0} \in \mathbb{R}^n$ b) x^T c) $0 \in \mathbb{R}$
d) x e) $-x^T$

4. If $A = \begin{bmatrix} 0 & a-2 & 2-c \\ d & b & a+b \\ -1 & e & a+3 \end{bmatrix}$ is a skew-symmetric matrix, then which of the followings is true for the matrix $B = \begin{bmatrix} a & 5 & e \\ d & b & b-2 \\ 3 & a+c & c+2 \end{bmatrix}$?

I. B is a symmetric matrix.
II. B^2 is a symmetric matrix.
III. $\mathrm{Tr}(B) = \mathrm{Tr}(A)$.

a) Only I b) I and II c) II and III
d) I and III e) All of them

5. Let $A = \begin{bmatrix} 1 & x \\ 0 & 1 \end{bmatrix}$ be an involut matrix, let $B = \begin{bmatrix} y & 0 \\ y & 0 \end{bmatrix}$ be a nonzero idempotent matrix and let $C = \begin{bmatrix} 1 & -2 \\ z & 2 \end{bmatrix}$ be a noninvertible matrix. Then, which of the followings is equal to the value $x + y + z$?

a) -1 b) 1 c) 0 d) -2 e) 2

$A^2 = I \Rightarrow \begin{pmatrix} 1 & x \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & x \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 2x \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$\Rightarrow x = 0.$

$B^2 = B \Rightarrow \begin{pmatrix} y & 0 \\ y & 0 \end{pmatrix}\begin{pmatrix} y & 0 \\ y & 0 \end{pmatrix} = \begin{pmatrix} y^2 & 0 \\ y^2 & 0 \end{pmatrix} = \begin{pmatrix} y & 0 \\ y & 0 \end{pmatrix}$

$\Rightarrow y^2 = y \Rightarrow y^2 - y = y(y-1) = 0 \Rightarrow y = 0, 1$

Since B is nonzero, $y = 1$.

C is noninvertible, then second row must be a multiple of the first row.

$\Rightarrow \begin{pmatrix} 1 & 2 \\ z & -2 \end{pmatrix} \Rightarrow (z \ -2) = -1(1 \ \ 2)$

$\Rightarrow z = -1$

$\Rightarrow x + y + z = 0 + 1 - 1 = 0.$

$\begin{pmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{pmatrix}^2 = \begin{pmatrix} 0 & 0 & 0 \\ 3 & 3 & 9 \\ -1 & -1 & 3 \end{pmatrix} = A^2$

$A^3 = A^2 A = \begin{pmatrix} 0 & 0 & 0 \\ 3 & 3 & 9 \\ -1 & -1 & 3 \end{pmatrix}\begin{pmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{pmatrix} =$

$= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

Since $A^k = 0$ for $k \geq 3$, A is nilpotent.

6. Which of the followings is true for the matrix $A = \begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix}$?

a) Idempotent b) Involut c) Orthogonal
d) Nilpotent e) None of them

7. Which of the followings is true for the matrix $B = \begin{bmatrix} ab & -a^2 \\ b^2 & -ab \end{bmatrix}$?

$B^2 = I$ $B^T = B^{-1}$ $B^2 = B$

a) Involut b) Orthogonal c) Idempotent
d) Hermitian e) Nilpotent

$B^H = B$ $B^k = 0$

$B^2 = \begin{bmatrix} ab & -a^2 \\ b^2 & -ab \end{bmatrix}\begin{bmatrix} ab & -a^2 \\ b^2 & -ab \end{bmatrix}$

$= \begin{bmatrix} a^2b^2 + -a^2b^2 & -a^3b + a^3b \\ ab^3 - ab^3 & -a^2b^2 + a^2b^2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$\bar{X} = \begin{pmatrix} -i & 1+i & 2 \\ a_1 - a_2 i & -3i & b_1 - b_2 i \\ c_1 - c_2 i & -i & 0 \end{pmatrix}$ $-(\bar{X})^T = \begin{pmatrix} -i & a_1 - a_2 i & c_1 - c_1 i \\ 1+i & -3i & -i \\ 2 & b_1 - b_2 i & 0 \end{pmatrix}$

$= -X$

$a_1 - a_2 i = -1 + i \Rightarrow a_1 = -1, \ a_2 = -1$ $\boxed{a = -1 - i}$

$c_1 - c_2 i = -2 \Rightarrow c_1 = -2, \ c_2 = 0 \Rightarrow \boxed{c = -2}$

$a + b + c = -1 - i + i - 2 = -3$

$a_1 + a_2 i$ $(\bar{X})^T = X^H = -X$

8. If $X = \begin{bmatrix} i & 1-i & 2 \\ a & 3i & b \\ c & i & 0 \end{bmatrix}$ is a skew-Hermitian matrix, then which of the followings is the value $a + b + c$?

$b_1 - b_2 i = -i \Rightarrow b_1 = 0, \ b_2 = 1$

a) $3 - 2i$ b) -3 c) $1 - i$
d) 2 e) $-3 + i$

$b = 0 + 1 \cdot i = i$

9. Let A and B be 3×3 real nonzero matrices and $(AB)^T + B^{-1}A = 0$. If B is an orthogonal matrix, then which of the followings is true for the matrix A?

a) Symmetric b) Idempotent c) Nilpotent d) Orthogonal e) Skew-symmetric

$(AB)^T + B^{-1}A = 0$, $B^T = B^{-1}$

$\Downarrow$

$B^T A^T + B^{-1}A = 0$) Since $B^T = B^{-1}$

$\Rightarrow B^{-1}A^T + B^{-1}A = 0$

$\Rightarrow B^{-1}(A^T + A) = 0$) Since B is invertible

$\Rightarrow A^T + A = 0$

$\Rightarrow A^T = -A \Rightarrow A$ is skew-symmetric.

MAT1320 LINEAR ALGEBRA EXERCISES IV-V

Name Surname:	**Group No:**
Student No:	**Duration:**
Department:	**Date:** 31 October 2022
Lecturer: Dr. Mustafa SARI	**Signature:**

1. What is the determinant of the following matrix?

$$\begin{bmatrix} 2 & 0 & 0 & -3 & 1 \\ 0 & 0 & 0 & 0 & -7 \\ -3 & 2 & 0 & -1 & -6 \\ 2 & -2 & -1 & 1 & 4 \\ 0 & 0 & 0 & 4 & 3 \end{bmatrix}$$

a) 112 b) -56 c) -112 d) -28 e) 28

2. If the following matrix A is invertible, which of the followings is all possible values of k?

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & k \\ 1 & 4 & k^2 \end{bmatrix}$$

a) $\{1, 2\}$ b) $\mathbb{R} - \{1\}$ c) $\mathbb{R} - \{1, 2\}$
d) $\mathbb{R}$ e) There is no such k.

3. Let A and B be two real matrices of the size 3×3. If $\det(A) = \sqrt{3}$ and $\det(B) = \frac{1}{2}$, then what is $\det\left(2A^T B^{-3}\right)$?

a) $32\sqrt{3}$ b) $16\sqrt{3}$ c) 48 d) $4\sqrt{3}$ e) $64\sqrt{3}$

4. Let $A = \begin{bmatrix} -1 & 1 & 3 \\ 2 & 0 & -2 \\ 1 & 3 & -2 \end{bmatrix}$ be an invertible matrix. What is $\text{Adj}\left(A^{-1}\right)$?

a) $\begin{bmatrix} 3/7 & 11/14 & -1/7 \\ 1/7 & -1/14 & 2/7 \\ 3/7 & 2/7 & -1/7 \end{bmatrix}$ b) $\begin{bmatrix} -1/14 & 1/14 & 3/14 \\ 1/7 & 0 & -1/7 \\ 1/14 & 3/14 & -1/7 \end{bmatrix}$

c) $\begin{bmatrix} 3/7 & 1/7 & 3/7 \\ 11/14 & -1/14 & 2/7 \\ -1/7 & 2/7 & -1/7 \end{bmatrix}$ d) $\begin{bmatrix} -1/14 & 1/7 & 1/14 \\ 1/14 & 0 & 3/14 \\ 3/14 & -1/7 & -1/7 \end{bmatrix}$

e) None of them

5. Which of the followings is true for the matrix $A = \begin{pmatrix} 0 & -3 & -1 & 1 \\ -2 & 0 & 2 & 5 \\ 3 & -2 & 0 & 0 \\ 1 & -4 & 0 & 0 \end{pmatrix}$?

I. A is invertible.

II. The reduced row echelon form of A is I_4.

III. $\text{Adj}(A)A = \begin{pmatrix} 60 & 0 & 0 & 0 \\ 0 & 60 & 0 & 0 \\ 0 & 0 & 60 & 0 \\ 0 & 0 & 0 & 60 \end{pmatrix}$

a) Onyl I b) I and II c) I and III
d) II and III e) I, II and III

6. If $\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = 4$, then what is $\begin{vmatrix} a_1 & a_2 & 4a_3 - 2a_2 \\ b_1 & b_2 & 4b_3 - 2b_2 \\ \frac{1}{2}c_1 & \frac{1}{2}c_2 & 2c_3 - c_2 \end{vmatrix}$?

a) 8 b) 6 c) 4 d) 2 e) 1

7. Let $A = \begin{bmatrix} -2 & 3 & 0 \\ 4 & 1 & -3 \\ 2 & 0 & 1 \end{bmatrix}$. Recall that A_{ij} is the cofactor of the component a_{ij}. Then, what is $a_{11}A_{12} + a_{21}A_{22} + a_{31}A_{32}$?

a) 0 b) -32 c) 32 d) 16 e) -16

MAT1320 LINEAR ALGEBRA EXERCISES IV-V

Name Surname:	**Group No:**
Student No:	**Duration:**
Department:	**Date:** 31 October 2022
Lecturer: Dr. Mustafa SARI	**Signature:**

$A_{n\times n}$ matris $\Rightarrow |kA| = k^n |A|$

$|AB| = |A|\cdot|B|$

$|A^T| = |A|$

1.
$$\begin{bmatrix} 2 & 0 & 0 & -3 & 1 \\ 0 & 0 & 0 & 0 & 7 \\ -3 & 2 & 0 & -1 & -6 \\ 2 & -2 & -1 & 1 & 4 \\ 0 & 0 & 0 & 4 & 3 \end{bmatrix}$$
matrisinin determinantı aşağıdakilerden hangisidir?

a) 112 b) -56 c) -112 d) -28 e) 28

$= 7\cdot(-1)^{2+5}\begin{vmatrix} 2 & 0 & 0 & -3 \\ -3 & 2 & 0 & -1 \\ 2 & -2 & -1 & 1 \\ 0 & 0 & 0 & 4 \end{vmatrix}$ (Laplace expansion for 2nd row)

$= -7\cdot 4\cdot(-1)^{4+4}\begin{vmatrix} 2 & 0 & 0 \\ -3 & 2 & 0 \\ 2 & -2 & -1 \end{vmatrix}$ (Laplace expansion for 4th row)

$= -28\cdot 2\,(-1)^{1+1}\begin{vmatrix} 2 & 0 \\ -2 & -1 \end{vmatrix} = -56\cdot -2 = 112$

Note: A^{-1} exist $\Leftrightarrow |A|\neq 0 \Leftrightarrow$ RREF of $A = I_n \Leftrightarrow \operatorname{Rank} A = n$.

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & k \\ 1 & 4 & k^2 \end{vmatrix} \overset{\substack{r_2\to r_2-r_1 \\ = \\ r_3\to r_3-r_1}}{\begin{vmatrix} 1 & 0 & 0 \\ 1 & 1 & k-1 \\ 1 & 3 & k^2-1 \end{vmatrix}} = 1\cdot A_{11}$$

$= (-1)^{1+1}\begin{vmatrix} 1 & k-1 \\ 3 & k^2-1 \end{vmatrix} = k^2-1-3(k-1)$

$= k^2-3k+2 = (k-2)(k-1) = 0$

$\Rightarrow$ If $k\neq 1$, $k\neq 2$, $|A|\neq 0$ and then A^{-1} exist.

$\Rightarrow \mathbb{R}-\{1,2\}$

2. A terslenebilir bir matris ise k nın tüm değerleri aşağıdakilerden hangisidir?
$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & k \\ 1 & 4 & k^2 \end{bmatrix}$$

a) $\{1,2\}$ b) $\mathbb{R}-\{1\}$ c) $\mathbb{R}-\{1,2\}$

d) $\mathbb{R}$ e) Böyle bir k sayısı yoktur.

3. A ve B, 3×3 boyutlu ve reel bileşenli iki matris olmak üzere $\det(A) = \sqrt{3}$ ve $\det(B) = \frac{1}{2}$ ise $\det\left(2A^TB^{-3}\right)$ değeri aşağıdakilerden hangisidir?

a) $32\sqrt{3}$ b) $16\sqrt{3}$ c) 48 d) $4\sqrt{3}$ e) $64\sqrt{3}$

$|2A^T\cdot B^{-3}| = 2^3\cdot|A^TB^{-3}|$

$= 8\cdot|A^T|\cdot|B^{-3}|$, $|B^{-3}| = |B^{-1}B^{-1}B^{-1}|$

$= 8\cdot|A|\cdot\left(\frac{1}{|B|}\right)^3$ / $= |B^{-1}|^3 = \left(\frac{1}{|B|}\right)^3$

$= 8\cdot\sqrt{3}\cdot\left(\frac{1}{\frac{1}{2}}\right)^3 = 64\sqrt{3}$

4) Recall that $A^{-1} = \frac{Adj(A)}{|A|}$

Replacing A by A^{-1}, we get

$(A^{-1})^{-1} = \frac{Adj(A^{-1})}{|A^{-1}|} \Rightarrow Adj(A^{-1}) = |A^{-1}|\cdot A = \frac{1}{|A|}\cdot A$

Then, we need $|A|$.

$$\begin{vmatrix} -1 & 1 & 3 \\ 2 & 0 & -2 \\ 1 & 3 & -2 \end{vmatrix} = \begin{vmatrix} -1 & 1 & 3 \\ 2 & 0 & -2 \\ 4 & 0 & -11 \end{vmatrix} = 1\cdot(-1)^{1+2}\begin{vmatrix} 2 & -2 \\ 4 & -11 \end{vmatrix}$$

$= -(-22+8) = 14 \Rightarrow Adj(A^{-1}) = \frac{1}{14}\cdot A$

4. $A = \begin{bmatrix} -1 & 1 & 3 \\ 2 & 0 & -2 \\ 1 & 3 & -2 \end{bmatrix}$ terslenebilir bir matris olmak üzere Ek (A^{-1}) matrisi aşağıdakilerden hangisidir?

a) $\begin{bmatrix} 3/7 & 11/14 & -1/7 \\ 1/7 & -1/14 & 2/7 \\ 3/7 & 2/7 & -1/7 \end{bmatrix}$ b) $\begin{bmatrix} -1/14 & 1/14 & 3/14 \\ 1/7 & 0 & -1/7 \\ 1/14 & 3/14 & -1/7 \end{bmatrix}$

c) $\begin{bmatrix} 3/7 & 1/7 & 3/7 \\ 11/14 & -1/14 & 2/7 \\ -1/7 & 2/7 & -1/7 \end{bmatrix}$ d) $\begin{bmatrix} -1/14 & 1/7 & 1/14 \\ 1/14 & 0 & 3/14 \\ 3/14 & -1/7 & -1/7 \end{bmatrix}$

e) Hiçbiri

5. $A = \begin{pmatrix} 0 & -3 & -1 & 1 \\ -2 & 0 & 2 & 5 \\ 3 & -2 & 0 & 0 \\ 1 & -4 & 0 & 0 \end{pmatrix}$ ise aşağıdakilerden hangileri doğrudur?

I. A matrisinin tersi vardır.

II. A matrisinin satırca indirgenmiş eşelon formu I_4 birim matrisidir.

III. $\text{Adj}(A)A = \begin{pmatrix} 60 & 0 & 0 & 0 \\ 0 & 60 & 0 & 0 \\ 0 & 0 & 60 & 0 \\ 0 & 0 & 0 & 60 \end{pmatrix}$

a) Yalnız I b) I ve II c) I ve III
d) II ve III e) I, II ve III

$$|A| = \begin{vmatrix} 0 & -3 & -1 & 1 \\ -2 & 0 & 2 & 5 \\ 3 & -2 & 0 & 0 \\ 1 & -4 & 0 & 0 \end{vmatrix} = \begin{vmatrix} 0 & -3 & -1 & 1 \\ -2 & -6 & 0 & 7 \\ 3 & -2 & 0 & 0 \\ 1 & -4 & 0 & 0 \end{vmatrix}$$

$r_2 \to r_2 + 2r_1$

$$= (-1)\cdot(-1)^{1+3} \begin{vmatrix} -2 & -6 & 7 \\ 3 & -2 & 0 \\ 1 & -4 & 0 \end{vmatrix} = (-1)\cdot 7(-1)^{1+3} \begin{vmatrix} 3 & -2 \\ 1 & -4 \end{vmatrix}$$

$$= -7\cdot(-12+2) = 70.$$

Since $|A| = 70 \neq 0$, A^{-1} exists.

Recall that

A^{-1} exists $\Leftrightarrow |A| \neq 0 \Leftrightarrow$ R.R.E.F. is $I_n \Leftrightarrow \text{Rank}\, A = n$

Recall that $A \cdot \text{adj}\, A = |A| \cdot I_n$

$$\Rightarrow A \cdot \text{adj}\, A = \begin{vmatrix} 70 & 0 & 0 & 0 \\ 0 & 70 & 0 & 0 \\ 0 & 0 & 70 & 0 \\ 0 & 0 & 0 & 70 \end{vmatrix}$$

6. $\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = 4$ ise $\begin{vmatrix} a_1 & a_2 & 4a_3 - 2a_2 \\ b_1 & b_2 & 4b_3 - 2b_2 \\ \frac{1}{2}c_1 & \frac{1}{2}c_2 & 2c_3 - c_2 \end{vmatrix}$ değeri aşağıdakilerden hangisidir?

a) 8 b) 6 c) 4 d) 2 e) 1

$$6) \begin{vmatrix} a_1 & a_2 & 4a_3 - 2a_2 \\ b_1 & b_2 & 4b_3 - 2b_2 \\ \frac{1}{2}c_1 & \frac{1}{2}c_2 & 2c_3 - c_2 \end{vmatrix}$$

$$= \frac{1}{2} \cdot \begin{vmatrix} a_1 & a_2 & 4a_3 - 2a_2 \\ b_1 & b_2 & 4b_3 - 2b_2 \\ c_1 & c_2 & 4c_3 - 2c_2 \end{vmatrix}$$

$c_3 \to c_3 + 2c_2$

$$= \frac{1}{2} \cdot \begin{vmatrix} a_1 & a_2 & 4a_3 \\ b_1 & b_2 & 4b_3 \\ c_1 & c_2 & 4c_3 \end{vmatrix} =$$

$$= \frac{4}{2} \cdot \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = 2 \cdot 4 = 8$$

7. $A = \begin{bmatrix} -2 & 3 & 0 \\ 4 & 1 & -3 \\ 2 & 0 & 1 \end{bmatrix}$ matrisi verilsin. A_{ij}, a_{ij} bileşeninin kofaktörü olmak üzere $a_{11}A_{12} + a_{21}A_{22} + a_{31}A_{32}$ değeri aşağıdakilerden hangisidir?

a) 0 b) -32 c) 32 d) 16 e) -16

a_{11}, a_{21}, a_{31}: the elements of the first column

A_{12}, A_{22}, A_{32}: the cofactors of the elements of the second column

Since they are different, this sum is equal to 0.