1. Given an directed weighted graph G=(V,E) with V = {s, 1, 2, 3, 4, t} and edges with capacities E={e_1(s;1;10), e_2(s;2;8), e_3(s;3;5), e_4(2;1;3), e_5(1;4;3), e_6(2;4;10), e_7(2;3;3), e_8(3;4;3), e_9(1;t;5), e_{10}(4;t;8), e_{11}(3;t;10)**}.**

a. List all possible available min-cuts of this graph.

ANSWER: Cuts 2 points each: (S = {s, 1, 2, 3}, T = {4, t})
(S = {s, 1}, T = {2, 3, 4, t})

b. What is the cost of the minimum cut this graph.

ANSWER: 21

c. Consider this graph represents a microscale internet backbone infrastructure, and node 4 becomes unavailable due to power outage. What would be the maximum flow value?

ANSWER: 13

d. Which edge(s) should be upgraded to preserve the same max flow value as before?
e_7, e_9

2. You have 11 hours **(H)** of study time today. You have 5 homeworks **(n)** h1,..,h5, the ith homework takes wi>=0 hours and it has a distinct value vi>=0.

hi	1	2	3	4	5
wi	2	3	3	1	5
vi	11	12	13	14	15

You can not divide the time wi, i.e. once you start doing a homework, you have to complete it and only then you can start another homework. You need to select a set of homeworks S which is a subset of {1,..,5}. Your goal is to:

maximize the sum of the values you gain by doing the homeworks in S: $\sum_{i \in S} v_i$,

subject to the constraint that you do not exceed the 11 hours of time that you have,

i.e. $\sum_{i \in S} w_i \leq 11$

a) Compute the optimal solution (i.e. set of homeworks selected) for this problem.

{__________} (write the homeworks in the optimal solution as h1,h2,h3 separating by commas)

(correct answer: h1,h3,h4,h5)

b) What is the value of the optimal solution? _________ (integer value)

(correct answer: 53)

c) If, h5 was not there, what would be the value of the optimal solution? _________ (integer value)

(correct answer: 50)

d) What is the complexity of the solution in the theta notation?

Θ(______) (by using **H, n, w, v** – whichever necessary) (You should represent 2^i as 2^i. Do not use any character to represent multiplication sign. For logn, use parentheses like log(n))

(correct answer: nH or Hn)