CIS571: Intro to AI

Homework #2 (due 10/23/17)

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Problem 1

Prove the following assertion : For every game tree, the utility obtained by MAX using minimax decisions against a suboptimal MIN will be never be lower than the utility obtained playing against an optimal MIN.

Solution

Proof :

By definition of suboptimal, the min is choosing something that is greater than the true min. If it was always choosing the min then it would be optimal and equal to what minimax utility is with an optimal MIN. Since we take the max of the mins, if the min is suboptimal this can only increase the utility returned by minimax with a suboptimal min, making it always greater than or equal to minimax with an optimal min.

Problem 2

Prove that alpha-beta pruning takes time $O(b^{m/2})$ with optimal move ordering, where m is the maximum depth of the game tree and b is the branch factor.

Solution On an optimal move ordering, alpha-beta pruning will have to expand all nodes in one layer(b), but since all the optimal nodes are expanded first, only one node in the next layer. Following this logic, odd trees will follow O(b*1*b*1.....*1*b) up to m, and even trees will follow O(b*1*b*1...*b*1) up to m, giving us a runtime of $O(b^{m/2})$