

On the Boltzmann Constant and the redefinition of the kelvin

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In 2005, CIPM recommended that the Boltzmann constant, k_B , should be used to redefine the kelvin degree of temperature. To actualize this, multiple measuring methods have to provide the same value for k_B with a small uncertainty.

This article discusses five of these methods, i.e., Acoustic-Gas Thermometry, Doppler-broadening Thermometry, Dielectric-constant-gas Thermometry, Johnson-noise Thermometry, and Radiation Calorimetry.

Regarding these five methods, Acoustic-Gas Thermometry has achieved the smallest uncertainty concerning k_B , i.e. a value of $1.3806487(14) \times 10^{-23} J/K$ was measured for k_B with a relative uncertainty of 0.91×10^{-6} . Moreover Dielectric and Johnson-noise thermometry are closing in on a 10^{-6} relative uncertainty, and are likely to reach this threshold as well. Therefor it is very likely that the redefinition of the kelvin, using k_B , will take place in the near future.

I. INTRODUCTION

In 2005 the International Committee for Weights and Measures(CIPM)[1], Comit International des Poids et Mesures in French, made the recommendation to redefine the kelvin using the Boltzmann constant, k_B . In this article we will evaluate the progress made recently towards this redefinition, and conclude which experimental methods have the most potential to aid in this progress.

To do this, first the historical and theoretical background of k_B will be sketched. Then we will touch on the subject of the fundamental constant and interpretations of k_B . Subsequently we will give a recap on the definition of temperature and the kelvin. This recap will lead us to the redefinition of the kelvin using k_B , which sparked a new interest to get a more accurate value of k_B using various experimental methods. Subsequently these various methods will be discussed and their progress to an accurate value of k_B will be examined. Finally we will come to a conclusion concerning the future of the kelvin, and which methods have the most potential to aid in the redefinition process.

II. THEORETICAL BACKGROUND

The Boltzmann constant, k_B , made it's first appearance, along with the Planck's constant h , in Planck's article on black body radiation in 1901[2]. They emerge in the Stefan-Boltzmann constant σ employed in the radiation thermal flux density $q_A = \sigma T^4$ emitted by a black body. Here they constitute, together with c , the $\frac{k^4}{h^3 c^2}$ ratio, which determines the scale of q_A .

However k_B is probably best known for its appearance in the equation for entropy, S , as seen in the equation below.

$$S = k_B \ln W \quad (1)$$

where W denotes the number of microstates in a system[3]. Initially entropy was introduced by Boltzmann as $\sigma = \ln(W)$, without k_B , similar to the infor-

mation theory definition of entropy also known as Shannon entropy[4, 5]. However in 1921, Planck introduced k_B to the entropy equation[6] to relate the conventional entropy with thermodynamic temperature. Resulting in the present-day entropy equation seen in eq.1.

Another important use of k_B is the temperature-energy relation, as seen in the Boltzmann distribution in eq.2. One can get the probability for a particle to have a certain energy E_i in a gas with statistical temperature T using this distribution. k_B relates temperature to energy and therefor the ratio $\frac{E_i}{k_B T}$ becomes dimensionless, as required of an exponent, and j goes over all possible energy states of the gas.

$$p_i = \frac{\exp(-\frac{E_i}{k_B T})}{\sum_j \exp(-\frac{E_j}{k_B T})} \quad (2)$$

In reverse, these equation relations can be used to measure the value of k_B . Recent measurements of k_B resulted in the recommended value of k_B by "CODATA"[7] to be:

$$\begin{aligned} k_B &= 1.38064852(79) \times 10^{-23} \text{ JK}^{-1} \\ &= 8.6173303(50) \times 10^{-5} \text{ eVK}^{-1} \end{aligned} \quad (3)$$

In thermodynamics k_B is generally replaced by the gas constant R , with R being k_B times Avogadros number N_A as seen below.

$$R = k_B \times N_A = 8.3144598(48) \text{ Jmol}^{-1} \text{ K}^{-1} \quad (4)$$

This is done for example in the perfect-gas law: $PV = NRT$. Here R relates macroscopic phenomena of the gas, i.e. pressure, thermodynamic temperature, and volume with each other and with the number of particles.

A. A fundamental physical constant

In the previous section the rise of k_B was sketched and multiple applications of k_B were introduced. These multiple applications result in multiple interpretations

of k_B , and although k_B is specified as a fundamental physical constant by multiple textbooks[8, 9], some of these interpretations disfavour this specification[10]. For example, the Boltzmann constant can be seen as:

- a fundamental physical constant or universal constant relating temperature to energy[8, 9].
- a numerical coefficient, transforming one unit of energy into another[11, 12].
- a fundamental constant, connecting statistical temperature τ and thermodynamic temperature T with the relation $\tau = k_B T$ [13].
- a signal-to-noise ratio, as Battail[14] does.
- a basic unit of information, i.e, the quantum of information as coined by Brillouin[15].

For further discussion we present a definition of a fundamental constant as proposed by Duff et al. [16]. They state that "a fundamental constant should be independent of frame of reference and basic units used". This way it can be communicated and understood by anyone, no matter their base units or reference frame. When a physical constant is not dimensionless, i.e. it has units, the constant depends on the base units used and therefore should not be classified as a fundamental constant.

When these interpretations and the applications discussed before are compared with the definition of a fundamental constant given by Duff et al, it can be concluded that, because k_B is not dimensionless, k_B is not a fundamental constant[10]. However, this will not hinder the redefinition of the kelvin in any way, because the discussion on being or not being a fundamental constant is mainly a semantic discussion.

B. Temperature and kelvin

Kelvin defined the thermodynamic temperature scale as follow *Construct an engine in which the hot source is at known temperature and the cold sink is the object of interest. The temperature of the latter can be inferred from the measured efficiency of the engine*[17]. Embodied in an equation this definition becomes:

$$T = (1 - \nu)T_{TPW} \quad (5)$$

with ν being the efficiency, T the temperature of the cold sink, and T_{TPW} the temperature of the hot source. When the temperature of the hot source is equal to the triple point of water (TPW), the temperature will acquire the kelvin as base unit. The kelvin, or K , is currently defined as $1/273.16$ of the thermodynamic temperature of the triple point of water[18]. For accurate temperature measurements the ITS-90 provides a way to approximate the thermodynamic temperature as close as

possible on multiple temperature ranges between 0.65K till 1357.77K[18].

Although the previous definition has been used for more than 50 years without issues, another definition of the kelvin is possible. This definition is in terms of the energy unit joule using the energy-temperature relation $E = k_B T$. In 2005 this has been recommended by the CIPM[1]. *For the kelvin, the change would generalize the definition, making it independent of any material substance, technique of realization, and temperature or temperature range. In particular, the new definition would improve temperature measurement at temperatures far away from the triple point of water*[4]. However to realize this the value of the k_B has to be fixed.

Redefinition of the kelvin will support the use of both the thermodynamic and the ITS-90 temperatures, however, Fischer et al.[4] predict that in the long run the thermodynamic temperature will replace the ITS-90.

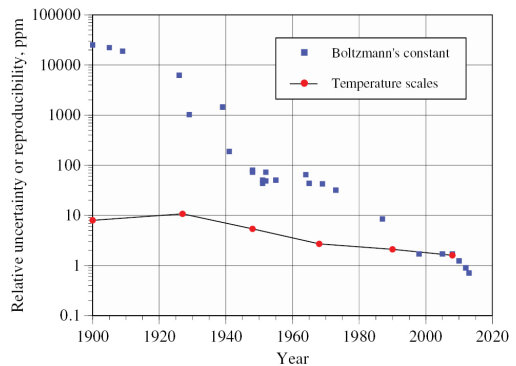


FIG. 1: A constant increase in accuracy of k_B can be clearly distinguished since 2005. Original figure from White et al.[19]

The Task group TG-SI has been appointed to monitor the advances made in measuring k_B more accurately. They will also determine when the accuracy is high enough to use k_B to redefine the kelvin. The Task group would like to see multiple methods reaching an uncertainty at least 1.8×10^{-6} [4]. This uncertainty will not change the value of T_{TPW} , but it does result in a standard uncertainty u_{TPW} of $0.49mK$ of the TPW, whereas currently the T_{TPW} is exact. Furthermore, this uncertainty will have to be taken into account for all historical thermodynamic measurements, due to their dependence on the T_{TPW} .

Although the current value of the k_B recommended by CODATA is accurate enough. This accuracy should be acquired with multiple physical techniques for the reason given by White et al.[19] *There is a possibility of unknown systematic effects that might bias the gas thermometry results, there should also be one or more supporting determinations made using different physical techniques. This will provide assurance that any unrecognised systematic effects must be small.* One can observe in figure 1 a constant increase in the accuracy of the k_B since 2005, when CIPM made their recommendation.

III. METROLOGY

At present multiple research groups are trying to measure k_B utilizing different experimental methods. The methods that will be discussed in this article are the following:

- Acoustic-Gas Thermometry
- Doppler-Broadening Thermometry
- Dielectric-Constant-Gas Thermometry
- Johnson-Noise Thermometry
- Radiation-Calorimetry

All these methods are able to measure temperature, but when the temperature is known beforehand, they can also be used to measure k_B . Since the triple point of water(273.16K) is the only temperature that can be known beforehand by definition, this temperature has to be used to measure k_B . For every method a description will be given, source of uncertainty will be mentioned, and recent values for k_B will be given. At the end of the article in table I is given with all k_B values mentioned in this article.

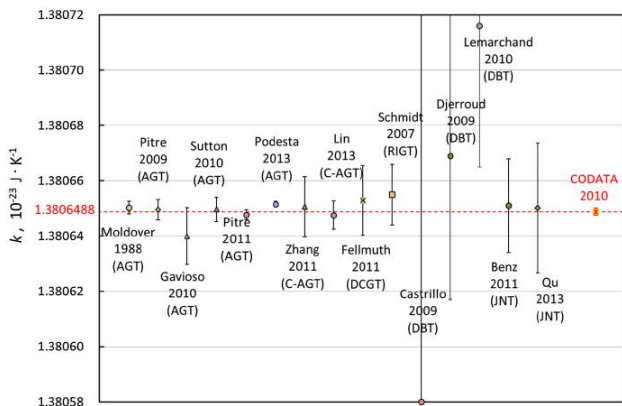


FIG. 2: Results of Boltzmann constant measurements for various thermodynamic thermometers. Bars indicate the combined uncertainties. Original figure from [20].

A. Acoustic-Gas Thermometry

The Acoustic-Gas-Thermometry method (AGT) utilises the velocity of sound in a gas to determine k_B . This velocity can be determined by measuring the acoustic resonance in a perfect-spherical cavity. In practice, a quasi-spherical cavity is often used, because the volume of a quasi-spherical cavity is easier to measure than a perfect-spherical cavity using an electromagnetic resonance method. This is because a cavity will always have



FIG. 3: Sketch of a quasi-spherical resonator, original figure by Pitre et al.[21].

minor defects breaking the symmetry, a quasi-spherical cavity with a minor asymmetry, as seen in fig. 3, is used for a more predictable symmetry breaking in the cavity[21].

For a perfect gas eq. 6 is used to determine k_B with f_{ln}^{ac0} the acoustic resonance frequency for the ln modes, m the mass of the gas particle, T the temperature of the gas, a the radius of the spherical cavity, z_{ln}^{ac} the wavenumber eigen-value k_{ln}^{ac} times the radius a , and the limit is taken to 0 pressure [21].

$$k_B = \left\langle \frac{3m}{5T} \left(\frac{2\pi a}{z_{ln}^{ac}} \right)^2 \lim_{p \rightarrow 0} (f_{ln}^{ac0})^2 \right\rangle \quad (6)$$

However, this expression is only valid if the propagation of sound in the cavity is ideal. Corrections have to be added to get a more realistic expression for k_B . In practice these corrections are introduced as a correction to the acoustic resonance frequency with a Δf_{ln}^{ac} , resulting in eq. 7.

$$k_B = \left\langle \frac{3m}{5T} \left(\frac{2\pi a}{z_{ln}^{ac}} \right)^2 \lim_{p \rightarrow 0} (f_{ln}^{ac} + \Delta f_{ln}^{ac})^2 \right\rangle \quad (7)$$

When we examine this expression, a few major sources of uncertainty can be pinpointed. First we have the uncertainty generated by the uncertainty in Δf_{ln}^{ac} . This uncertainty depends mainly on the accuracy of the model used to determine the corrections due to the non-ideal propagation of sound. A second source of uncertainty depends on the accuracy of the cavities-volume measurement, leading to a value of the parameter a . A third source of error is due to the non-perfect uniformity and thermal stability of the gas and the resonator. And finally, the accuracy of the molar mass m also has influence

on the uncertainty[21]. Nevertheless multiple accurate measurements have been done using Acoustic-Gas Thermometry to determine k_B .

1. Measurements

In 2013 a measurement was done by Podesta et al.[22] resulting in a value of $1.38065156 \times 10^{-23} J/K$ with a relative uncertainty of 0.71×10^{-6} using Argon. This is the most accurate measurement found in the literature. However during the measurement, a mistake was made in the measurement of the molar mass. Subsequently, a corrective article was published in 2015[23], in which a new molar mass was measured with a shift of seven times the uncertainty in molar mass measured in 2013. Resulting in a new determination of k_B with a different uncertainty. Sadly the corrective article could not be acquired and consequently their adjusted value for k_B could not be found.

In 2013 Lin et al.[24] measured a value of $1.3806476 \times 10^{-23} J/K$ with a relative standard uncertainty of 3.7×10^{-6} . This was measured in a single cylindrical cavity using Argon. In 2015 they did a new measurement by combining two-argon filled cavities into one virtual acoustic resonator[25]. Sadly, this did not result in the right values of k_B .

In 2015 NMIJ[26] measured a value of $1.3806562 \times 10^{-23} J/K$ with a relative uncertainty of 20×10^{-6} , and Pitre et al.[27] reported a value of $1.3806487(14) \times 10^{-23} J/K$ for k_B employing Helium in a quasi-spherical resonator with a relative uncertainty of 0.91×10^{-6} .

AGT produces accurate values for k_B , but it still faces problems performing temperature measurements in the temperature range of 800K up to 1200K. This makes the AGT-method less useful for accurate temperature measurements at higher temperatures[20].

B. Doppler-Broadening Thermometry

Doppler-Broadening Thermometry(DBT) exploits the dominating effect of the velocity distribution of a gas at low pressure on the Doppler broadening of the absorption line [28, 29]. The velocity distribution is related to k_B as seen in eq. 2. The velocity of a single particle parallel to the propagation of the radiation causes a shift in the lights frequency in the rest frame of this particle. Due to the velocity distribution, this results in a broadening of the absorption line a.k.a. Doppler broadening. An experimental setup consists of a laser shining trough a container with gas. The laser scans a frequency range close to the resonance frequency of the particle, and a transmission spectrum is made, which is shown in fig.4. Due to the relation between the velocity distribution and temperature, k_B can be inferred from this transmission spectrum, as shown below.

$$k_B = \left(\frac{\gamma_D}{\nu} \right)^2 \left(\frac{mc^2}{2 \ln(2)T} \right) \quad (8)$$

with γ_D the Doppler broadening, ν the position of the line, m the mass of the gas particle, c the speed of light in vacuum, and T the temperature of the gas[28].

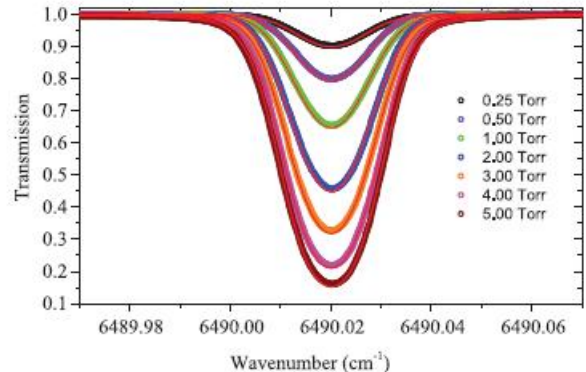


FIG. 4: Doppler broadening in the transmission spectrum of the $p(25)$ line of C_2H_2 gas for multiple pressures and at 295.78 K as measured by Hashemi et al.[28] for the determination of k_B .

The uncertainty in k_B resulting from the Doppler-Broadening method has multiple sources. A source of uncertainty is the uncertainty in frequency. Lasers always have frequency instability, and although this instability is not very large, a laser with a frequency stability smaller than 10 kHz is needed to achieve a 10^{-6} uncertainty in line width for a Doppler width of 1GHz. Secondly an uncertainty in absorbance arises due to the signal-to-noise ratio resulting from the laser power instability. Another uncertainty arises from pressure dependence of the line-shape, at higher pressure the line-shape starts to deviate increasingly from the Guassian profile[29].

1. Measurements

In 2015 Truong et al.[30] measured a value of $1.380545(98) \times 10^{-23} J/K$ for k_B with a relative uncertainty of 71×10^{-6} using the $6p_{1/2}$ hyperfine splitting of Cs. However the most accurate measurement using Doppler-Broadening Thermometry still remains $1.380631(22) \times 10^{-23} J/K$ with a relative uncertainty of 24×10^{-6} using $H_2^{18}O$ measured by Moretti et al.[31] in 2013.

C. Dielectric-Constant-Gas Thermometry

In Dielectric-Constant-Gas Thermometry(DCGT) the density n/V in the equation of state is replaced by the

dielectric constant ϵ . For the ideal gas law this results in the expression for k_B seen in eq.9, with P the pressure, T the temperature, ϵ_0 the electric constant, and α_0 the static electric dipole polarizability of a particle[32].

$$k_B = P \frac{\alpha_0}{(\epsilon - \epsilon_0)T} \quad (9)$$

A basic DCGT setup consist of: a container, that can adjust the pressure of its content; a capacitor, which can be filled with a measuring gas; and a thermostat[33]. The dielectric constant can be determined via the change in capacitance of the capacitor as a function of the pressure of the gas up to vacuum. The downside of this method is that Helium is the only molecule whose polarizability can be calculated theoretically. Therefor only Helium can be used to measure k_B .

A few major contributions to the uncertainty in k_B using this method are due to: the uncertainty of the dielectric susceptibility, the instability of the capacitance of the measuring capacitor, errors in the calculation of the effective compressibility which can result in deviations of the actual capacitance, and the uncertainty in the pressure measurement[34].

1. Measurements

In 2013 PTB[35] measured $1.3806509 \times 10^{-23} J/K$ with a relative uncertainty of 4.3×10^{-6} . This was an improvement with respect to their measurement done in 2011[34] reducing the uncertainty by 50%. A prediction was made that this uncertainty could be reduced even further to 2×10^{-6} before 2015. However no new article has been published confirming this.

D. Johnson-Noise Thermometry

Johnson-Noise Thermometry(TPW) exploits the noise generated by thermal agitation of carriers within conductors to measure the thermodynamic temperature. Consequently k_B can be obtained from the relation seen in eq.10

$$k_B = \frac{\langle V^2 \rangle}{4TR\Delta f} \quad (10)$$

with $\langle V^2 \rangle$ Johnson noise power squared averaged, R the resistance, Δf the bandwidth, and T the temperature[36–38]. In reality the Johnson-noise voltage is very small and of the same order of the noise introduced by amplification of the signal. Therefore the noise power is often measured by comparing crosscorrelations of noise generated from a reference resistor placed at a known temperature and the noise generated from a sensing resistor placed at the unknown temperature. However, as stated by Yamazawa et al.[20] *conflicting constraints on the values of the two sensing resistors arise*

from the need to match both the bandwidths and the noise powers for the noise power measurements, and with the usual JNT design, this conflict remains unresolved.

To circumvent this restraint a new crosscorrelation method has been introduced, where the noise generated by the reference resistor is replaced by an artificial noise source, for example by a quantum voltage noise source (QVNS). A schematic of such a setup is displayed in fig.5. Both noise sources are connected to the switch, the switch alternately connects one of the two sources to both amplifiers, then the signal is filtered, digitalized, and cross-correlated by the PC. To retrieve k_B from this setup the expression seen in eq.11 is used. Here $\langle V_R^2 \rangle$ Johnson noise, $\langle V_Q^2 \rangle$ is the power of the reference noise, f_S is the clock frequency of the pulse generator, T is the temperature of the TPW, R is the measured resistance, and C is a pre-factor depending on the number of Josephson junctions and the bit length of the code used to drive the chip with pulses[36].

$$k_B = C \frac{\langle V_R^2 \rangle}{\langle V_Q^2 \rangle} \frac{f_S}{4TR} \quad (11)$$

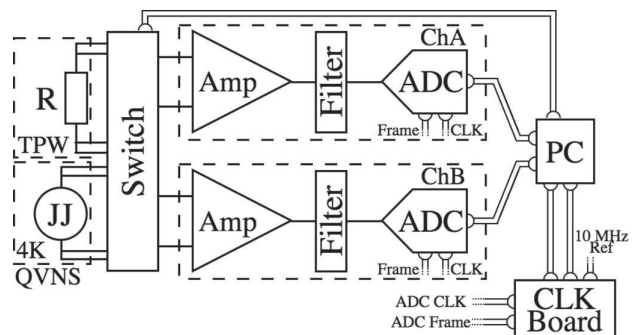


FIG. 5: A schematic of an experimental setup to measure Johnson noise as used by [36]. Where the noise generated by a sensing resistor at the TPW(triple point of water) and the QVNS noise are amplified, filtered, converted, and imported to the PC.

Although the QVNS-JNT is rather circuitous, it results in accurate measurements of k_B .

1. Measurements

The NIST,NIM collaboration Qu et al.[39, 40] went from a relative uncertainty of 6×10^{-6} using a 100Ω resistor in 2013, to $k = 1.3806514(48) \times 10^{-23} J/K$ with a relative uncertainty of 3.5×10^{-6} [41] using a 200Ω resistor.

E. Radiation-Calorimetric method using Stefan-Boltzmann constant

The Radiation-Calorimetric method utilises the influence of k_B on Black Body radiation. A black body emits

radiation, and the thermal flux density q_A of this radiation can be measured with Radiation-calorimetry[42, 43]. From this flux density the value of the Stefan-Boltzmann constant σ can be determined by its relation with temperature; $q_A = \sigma T^4$. Consequently the relation seen in eq.12 is used to obtain the value of k_B [44].

$$k_B = \left(\frac{15c^2 h^3}{2\pi^5} \sigma \right)^{\frac{1}{4}} \quad (12)$$

Experimental values for k_B obtained by radiation-calorimetry have not been found in the literature. However an estimation has been made on the relative error in k_B in 2012[44]. The relative error for using radiation-calorimetry is estimated to be 0.95×10^{-6} for Ga molecules and 0.73×10^{-6} for water molecules. However this estimation assumes the radiation radiated by a perfect black body, therefore the most important source of error is not included in the error approximation given above and is due to the uncertainty in blackness coefficient of the radiating body. This coefficient is a measure of how close the real radiator can be approximated by a perfect black body. Fortunately when an adiabatic radiator is used, measurements on the blackness coefficient can be done[44].

IV. CONCLUSION

In this article an attempt was made to examine the redefinition of the kelvin using the Boltzmann constant k_B , and evaluate the methods with the most potential to aid in this process. To do this we started out with a historical- and theoretical background of k_B and temperature. Subsequently we discussed multiple methods

to measure temperature. Due to the connection between temperature and k_B , these methods can be used to measure k_B when the temperature is known.

Five methods were discussed, that is to say, Acoustic-Gas Thermometry, Doppler-Broadening Thermometry, Dielectric-Constant-Gas Thermometry, Johnson-Noise Thermometry, and Radiation Calorimetry. AGT has been the most accurate method up till now, reaching a general uncertainty of 0.96×10^{-6} . However multiple methods have to provide a similar value for k_B , combined with a very small uncertainty, to eradicate the possibility that the value of Boltzmann constant is method dependent. Dielectric and Johnson Noise are close to the 10^{-6} uncertainty at the present moment, and have the potential to reach this uncertainty threshold. Additionally Radiation-calorimetry seems promising on paper, but has not been actualized yet.

With the current accuracy, the large amount of research being done to increase this accuracy, and the progress being made in various thermometric methods, it can be concluded that the redefinition of the kelvin using k_B will just be a matter of time.

Method	k_B value	relative uncertainty	year
CODATA	$1.38064852(79) \times 10^{-23} \text{ J/K}$	0.57×10^{-6}	2014
AGT	$1.3806476 \times 10^{-23} \text{ J/K}$	3.7×10^{-6}	2013
"	$1.3806562 \times 10^{-23} \text{ J/K}$	20×10^{-6}	2015
"	$1.3806487(14) \times 10^{-23} \text{ J/K}$	0.91×10^{-6}	2013
DBT	$1.380631(22) \times 10^{-23} \text{ J/K}$	24×10^{-6}	2013
"	$1.380545(98) \times 10^{-23} \text{ J/K}$	71×10^{-6}	2015
DCGT	$1.3806509 \times 10^{-23} \text{ J/K}$	4.3×10^{-6}	2013
JNT	$1.3806514(48) \times 10^{-23} \text{ J/K}$	3.5×10^{-6}	2015

TABLE I: Table of all values for k_B mentioned in this article.

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