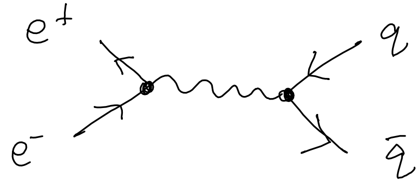
Basic amplitudes

Example 1 : $e^+e^- \to q\bar{q}$



$$\mathcal{A}\left(\underset{e^+}{p_a} \, \underset{e^-}{p_b} \to \underset{q}{p_1} \, \underset{\bar{q}}{p_2} \right)$$

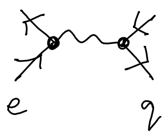
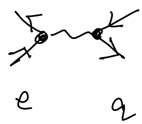
$$\bar{u}(p_a)\gamma^\mu v(p_b) \frac{1}{(p_a+p_b)^2} \bar{u}(p_1)\gamma_\mu v(p_2)$$

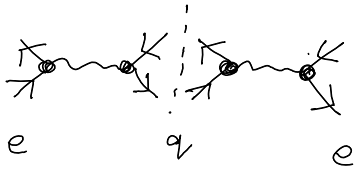
$\times\ Q_q \longrightarrow$ "fractional quark charge"

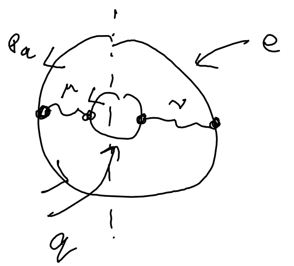
$\times\ \delta_{ij} \longrightarrow$ "QCD charge $i,j = 1, \dots, N_c$"

$\times\ g_e \longrightarrow$ electromagnetic coupling

$$\Rightarrow \langle |A|^2 \rangle =$$

$$\frac{1}{4}\sum_{\text{spins}} \Big($$  $$\Big)\Big($$  $$\Big)^\dagger$$

$$= \frac{1}{4}\sum_{\text{spins}}$$ 

$$= \frac{1}{4}$$ 

$$= 4\left(p_a^\mu p_b^\nu + p_b^\mu p_a^\nu - g^{\mu\nu}\, p_a\cdot p_b\right)$$

$$= \frac{1}{4}\,\mathrm{tr}\left(\not p_a \gamma^\mu \not p_b \gamma^\nu\right) \frac{1}{s_{ab}^2}$$

$$\times\, \mathrm{tr}\left(\not p_1 \gamma_\mu \not p_2 \gamma_\nu\right) \times N_c \times Q_q^2 \times g_e^4$$

$$= 2 g_e^4 Q_q^2 N_c \left(\frac{s_{1a}^2 + s_{1b}^2}{s_{ab}^2}\right)$$

s_{ij} are "Mandestam invariants"

$$= 2\left(\frac{\alpha}{4\pi}\right)^2 Q_q^2 N_c \left(\frac{s_{1a}^2 + s_{1b}^2}{s_{ab}^2} \right)$$

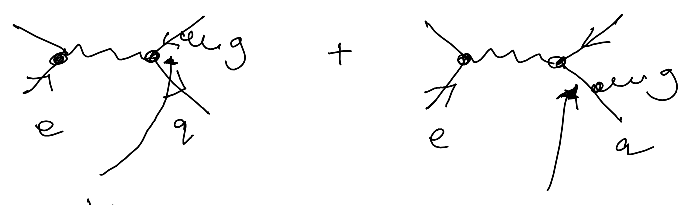
$$g_e^2 = \frac{\alpha}{4\pi} .$$

$\boxed{\text{NB}}$ valid below Z-resonance only, above requires evaluation of more complicated diagrams.

<u>Example 2</u> : $e^+ e^- \to q \bar{q} g$

$p_a \, p_b \qquad p_1 p_2 p_3$

Similar computation of 2 diagrams leads to

$\mathcal{A} =$ 

$1/s_{13} \qquad 1/s_{23}$

the squared amplitude is remarkably compact despite being a lenghty algebraic computation :

$$\langle |A|^2 \rangle = 4 N_c C_F \left(\frac{\alpha_s}{4\pi}\right)\left(\frac{\alpha}{4\pi}\right)^2 Q_q^2$$

$$\times \left(\frac{S_{a1}^2 + S_{a2}^2 + S_{b1}^2 + S_{b2}^2}{S_{ab}\, S_{13}\, S_{23}} \right)$$

where $$C_F = \frac{N_c^2 - 1}{2N_c} \overset{N_c=3}{=} \frac{4}{3}$$

[fundamental casimir of $SU(N_c)$]