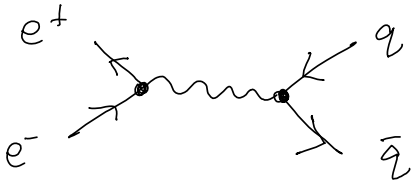


Basic amplitudes

Example 1 : $e^+e^- \rightarrow q\bar{q}$



$$A(p_a, p_b \rightarrow p_1, p_2)$$
$$e^+ e^- \quad q \bar{q}$$

$$\bar{u}(p_1) \gamma^\mu v(p_b) \frac{1}{(p_a + p_b)^2} \bar{u}(p_1) \gamma_\mu v(p_2)$$

$\times Q_q \rightarrow$ "fractional quark charge"

$\times \delta_{ij} \rightarrow$ "QCD charge $i, j = 1, \dots, N_c$ "

$\times g_e \rightarrow$ electromagnetic coupling

$$\Rightarrow \langle |A|^2 \rangle =$$

$$\frac{1}{4} \sum_{\text{Spins}} \left(\begin{array}{c} \text{diagram 1} \\ e \quad q \end{array} \right) \left(\begin{array}{c} \text{diagram 2} \\ e \quad q \end{array} \right)^+$$

$$= \frac{1}{4} \sum_{\text{Spins}} \begin{array}{c} \text{diagram 3} \\ e \quad q \quad e \end{array}$$

$$= \frac{1}{4} \begin{array}{c} \text{diagram 4} \\ p_a \quad e \quad q \end{array}$$

$$= 4 \left(p_a^\mu p_b^\nu + p_b^\mu p_a^\nu - g^{\mu\nu} p_a \cdot p_b \right)$$

$$= \frac{1}{4} \text{tr} \left(p_a \gamma^\mu p_b \gamma^\nu \right) \frac{1}{2 S_{ab}}$$

$$\times \text{tr} \left(p_1 \gamma_\mu p_2 \gamma_\nu \right) \times N_c \times Q_q^2 \times g_e^4$$

$$= 2 g_e^4 Q_q^2 N_c \left(\frac{S_{1a}^2 + S_{1b}^2}{S_{ab}^2} \right)$$

S_{ij} are
"Mandelstam
invariants"

$$= 2 \left(\frac{\alpha}{4\pi} \right)^2 Q_f^2 N_c \left(\frac{S_{1a}^2 + S_{1b}^2}{S_{ab}^2} \right)$$

$$g_e^2 = \frac{\alpha}{4\pi}$$

NB valid below Z-resonance only, above requires evaluation of more complicated diagrams.

Example 2 : $e^+e^- \rightarrow q\bar{q}g$

p_1, p_2 p_1, p_2, p_3

Similar computation of 2 diagrams leads to

$$\mathcal{A} = \text{Diagram 1} + \text{Diagram 2}$$

Diagram 1: A fermion line (electron) with incoming momentum e and outgoing momentum e . It emits a gluon with momentum g and then splits into a quark q and an antiquark \bar{q} . The propagator for the fermion is $1/S_{13}$.

Diagram 2: A fermion line (electron) with incoming momentum e and outgoing momentum e . It emits a gluon with momentum g and then splits into a quark q and an antiquark \bar{q} . The propagator for the fermion is $1/S_{23}$.

The squared amplitude is remarkably compact despite being a lengthy algebraic computation:

$$\langle |A|^2 \rangle = 4 N_c C_F \left(\frac{\alpha_s}{4\pi} \right) \left(\frac{\alpha}{4\pi} \right)^2 Q_q^2$$

$$\times \left(\frac{S_{a1}^2 + S_{a2}^2 + S_{b1}^2 + S_{b2}^2}{S_{ab} S_{13} S_{23}} \right)$$

where $C_F = \frac{N_c^2 - 1}{2N_c} \stackrel{N_c=3}{=} \frac{4}{3}$

[fundamental casimir of $su(N_c)$]