VI. Scattering amplitudes

1) Scattering amplitudes in QFT

- 1.1) Input: momenta and (on-shell) phasespace
- 1.2) Examples: $e^+e^- \to qq$ e $e^+e^- \to qqg$
- 1.3) Numerical algorithms

1)

Scattering Amplitudes

$$\mathcal{A} = \sum_i (\text{Feynman diagram})_i \qquad \langle |\mathcal{A}|^2 \rangle = \sum_{\text{spins}} \mathcal{A}^\dagger \mathcal{A}$$

$$\mathcal{A} : (p_i^\mu; p_i^2 = m_i^2, \sum p_i^\mu = 0) \to \mathbb{C}$$

amplitudes are a function of momenta (also other quantum numbers of the particles: helicity, charges etc)

1.1)

Momenta and phase space

(see notes)

$$d\Phi_n = \delta^{(4)}(Q - \sum_{i=1}^{n_{\text{final}}} p_i) \prod_{i=1}^{n_{\text{final}}} d^4 p_i \delta^{(+)}(p_i^2)$$

(modulo factors of 2π, massless only)

need to solve:

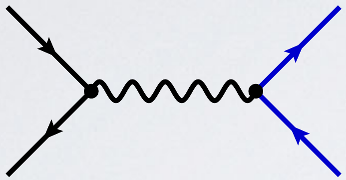
- on-shell ($p_i^2=0$)
- momentum conservation

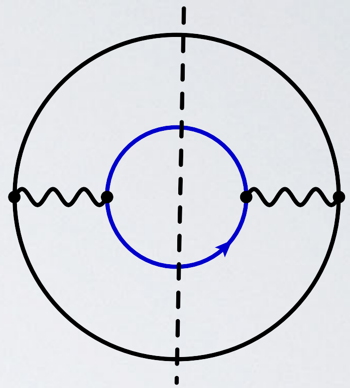
uniform phase space sampling
RAMBO algorithm

1.2)

Example: e⁺e⁻ →qq

(see notes)

$\mathcal{A} =$ 

$$d\hat{\sigma} = \frac{1}{2s_{ab}}\ d\Phi_2$$ 

flux

$$\langle|\mathcal{A}|^2\rangle = \left(\frac{\alpha}{4\pi}\right)^2 N_c Q_q^2 \,\frac{s_{a1}^2 + s_{a2}^2}{s_{ab}^2}$$

$$s_{ab} = (p_a + p_b)^2$$
$$s_{ai} = (p_a - p_i)^2$$

1.2)

Example: $e^+e^- \rightarrow q\bar{q}g$

computation of 2 diagrams leads to:

$$\langle |\mathcal{A}|^2 \rangle = 4 \left(\frac{\alpha}{4\pi}\right)^2 \left(\frac{\alpha_s}{4\pi}\right) N_c C_F Q_q^2 \, \frac{s_{a1}^2 + s_{a2}^2 + s_{b1}^2 + s_{b2}^2}{s_{ab} s_{13} s_{23}}$$

$$C_F = \frac{N_c^2 - 1}{2N_c}$$

very compact - easy to attempt integration of the phase space

1.2)

Monte Carlo integration

$$\hat{\sigma} \sim \int d\Phi_n \langle |\mathcal{A}|^2 \rangle \approx \frac{V}{N} \sum_{i=1}^{N} \langle |\mathcal{A}(p_i)|^2 \rangle$$

V - phase space volume i.e. $$V = \int d\Phi_n$$

For RAMBO, V=1 since we map everything to $\int_0^1 dx_i$

If we apply cuts we must apply it also the the phase space volume: $V = N_{trials}/N_{samples}$

simple implementation is very inefficient - adaptive methods and importance sampling used in most applications

1.3)

Numerical algorithms

There are many efficient methods for computing scattering amplitudes - many at tree-level. New generation of tools able to automate one-loop processes too…

OPENLOOPS, MADLOOP (aMC@NLO), BLACKHAT, NJET, …

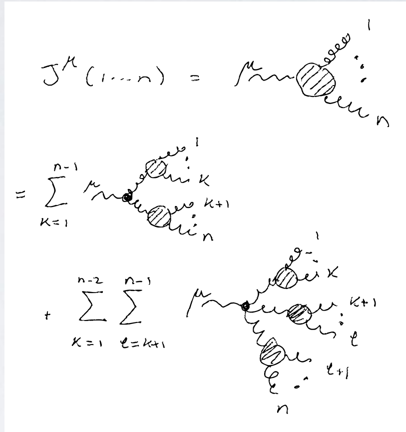
njet-3.1.1-1L.tar.gz

any of these tools would do, I happen to know NJet better than most
[SB, Biedermann, Uwer, Yundin https://arxiv.org/abs/1209.0100] [SB, Moodie (2019-]

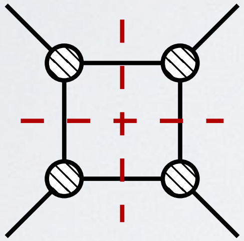
let's try one of these as a source of data for training an efficient MC integration tool

1.3)

What is NJet doing?



recursive tree-level amplitudes
(Berends-Giele)



(generalised) unitarity cuts
(Bern, Dixon, Dunbar, Kosower),
(Britto-Cachazo-Feng-Witten)

integrand reduction
(Ossola, Papadopoulos, Pittau)