A model system demonstrating a subcritical Hopf bifurcation can be described in polar coordinates by a system of equations:

$$\begin{cases} \dot{r} = r\left(a - 1 + 2r^2 - r^4\right) \\ \dot{\varphi} = 2\pi \end{cases}$$

r is the radius, φ is the angle, and a is the parameter on which the length of the radius depends.

Tasks:

1. Find all solutions of the model.
2. Carry out a linear analysis and determine the type of each of the solutions found depending on a.
3. Construct a phase-parametric with the indication of all solution types for all parameters a previously found.
4. Construct a phase portrait of the system Cartesian coordinates. Keep in mind that the "limit cycle" solution in Cartesian coordinates is a circle with the center at the origin and the radius determined from the expressions for the stationary states in the system in polar coordinates. Constant solutions are represented by a solid line, and unstable ones by a dashed line.