

Compute $(19! + 19, 20! + 19)$.

Wilson's Theorem

$\forall p \in \mathbb{N}$, p is a prime iff $p \mid (p-1)! + 1$

$$(a, b) = (a, an \pm b) \quad \text{①}$$

let $d \mid a, d \mid b \Rightarrow d \mid an \pm b \Rightarrow (a, an \pm b)$ is a subset of (a, b)

let $d \mid a$ (us)
 $d \mid an \pm b$ (us)
 $\therefore d \mid b$ (induced)

Because 19 is a prime, $19 \mid 18! + 1$.

$$(19! + 19, 20! + 19) = (19! + 19, 20! + 20 \cdot 19 - 20! - 19)$$

$$= (19! + 19, 19 \cdot 19) \quad \text{②}$$

$$= (19(18! + 1), 19 \cdot 19)$$

$$\text{let } d = (19! + 19, 20! + 19).$$

$$19 \mid d,$$

$$361 \mid d.$$

$$361$$