

Find all triples  $(x, y, z)$  where  $x, y, z$  are distinct positive integers that satisfy the following equation.

$$\frac{1}{x+1} + \frac{1}{y+2} + \frac{1}{z+3} = \frac{1}{2} \left( \frac{1}{1} + \frac{1}{2} + \frac{1}{3} \right)$$

Let  $a = x+1, b = y+2, c = z+3$  where  $a, b, c \geq 2$

$$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \frac{11}{12} \quad \text{①}$$

WLOG, let  $a \leq b \leq c$  ②

$$\frac{3}{a} = \frac{1}{a} + \frac{1}{a} + \frac{1}{a} \leq \frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \frac{11}{12} \leq \frac{1}{c} + \frac{1}{c} + \frac{1}{c} = \frac{3}{c}$$

$$\frac{11}{12} > \frac{3}{4}$$

$$\therefore c=2$$

$$\frac{1}{a} + \frac{1}{b} = \frac{5}{12}$$

$$12(a+b) = 5ab$$

$$25ab - 60(a+b) \geq 0$$

$$(5a-12)(5b-12) \geq 144$$

144	1	X
12	2	X
48	3	(12, 3)
36	4	X
24	6	X
18	8	(6, 4)
16	9	X
12	12	X

$$\therefore c=3$$

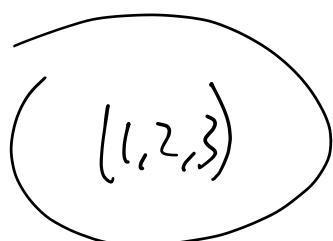
$$\frac{1}{a} + \frac{1}{b} = \frac{7}{12}$$

$$12(a+b) = 7ab$$

$$49ab - 84(a+b) \geq 0$$

$$(12-12)(19-12) = 144$$

144	1	X
12	2	(12, 2)
48	3	X
36	4	X
24	6	X
18	8	X
16	9	(4, 3)
12	12	X



$$(a, b, c)$$

$$(x, y, z)$$

$$(12, 3, 2)$$

$$(12, 1, -1)$$

$$(12, 2, 3)$$

$$(11, 9, 1)$$

$$(3, 2, 12)$$

$$(12, 0, 9)$$

$$(3, 12, 2)$$

$$(12, 10, -1)$$

$$(2, 3, 12)$$

$$(1, 1, 9)$$

$$(2, 12, 8)$$

$$(1, 10, 6)$$

$$(6, 4, 4)$$

$$(1, 2, -1)$$

$$(6, 2, 4)$$

$$(5, 6, 1)$$

$$(4, 6, 2)$$

$$(3, 4, -1)$$

$$(4, 2, 6)$$

$$(3, 0, 3)$$

$$(2, 4, 6)$$

$$(1, 2, 3)$$

$$(2, 4, 9)$$

$$(1, 4, 1)$$

$$(4, 3, 3)$$

$$(3, 9, 3)$$

$$(3, 3, 4)$$

$$(1, 8, 1)$$