

Compute $(19! + 19, 20! + 19)$.

Wilson's Theorem

If $p \in \mathbb{N}$, p is a prime iff $p | (p-1)! + 1$

$$(a, b) = (a, an+b) \quad \text{①}$$

Let $d | a, d | b \Rightarrow d | an+b \Rightarrow (a, an+b) \text{ is a } \text{subset of } (a, b)$

Let $d | a$
 $a \equiv 1 \pmod{d}$
 $a \equiv -1 \pmod{d}$
 $\therefore a^2 \equiv 1 \pmod{d}$

Because 19 is a prime, $19 | 18! + 1$.

$$\begin{aligned} (19! + 19, 20! + 19) &= (19! + 19, 20! + 20 \cdot 19 - 20! - 19) \\ &= (19! + 19, 19 \cdot 19) \\ &= (19(18! + 1), 19 \cdot 19) \end{aligned}$$

let $d = (19! + 19, 19 \cdot 19)$.

$$\begin{array}{c} 19 \mid d, \\ 361 \mid d. \end{array}$$

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