Find all functions $f : \mathbb{R} \to \mathbb{R}$ so that

$$f(f(x) + x + y) = f(x + y) + yf(y)$$

for all real numbers x, y.

$x = y = 0$

$f(f(0)) = f(0)$

$x = 0$

$f(f(0) + y) = f(0) + y f(0)$

$y = 0$

$f(f(x) + x) = f(x)$

$y = f(x)$

$f(f(x) + x + f(x)) = f(x + f(x)) + f(x)f(f(x))$

let $a = x + f(x)$

$f(x) = f(a) = f(f(a) + a) = f(f(x) + x + f(x))$

$= f(a) + f(x) f(f(x))$

$\cancel{f(x)} = \cancel{f(x)} + f(x) f(f(x))$

$f(x) f(f(x)) = 0$

$f(0) f(f(0)) = 0$

$\therefore \underline{f(0) = 0}$

$f(y) = f(y) + y f(y)$

$y f(y) = 0$

$y = 0$ or $\underline{f(y) = 0} \quad \forall y \ne 0, \, y \in \mathbb{R}$

$\boxed{f(x) = 0} \quad \forall x \in \mathbb{R}$