

$\forall a_1, b_1, \dots, a_n, b_n \in \mathbb{R}^+$,

$$(a_1 + \dots + a_n)^{\lambda_1} (b_1 + \dots + b_n)^{\lambda_2} \dots (z_1 + \dots + z_n)^{\lambda_n} \geq a_1^{\lambda_1} b_1^{\lambda_2} \dots z_1^{\lambda_n} + \dots + a_n^{\lambda_n} b_n^{\lambda_1} \dots z_n^{\lambda_2}$$

for all $\lambda_i \geq 0$ s.t. $\lambda_1 + \lambda_2 + \dots + \lambda_n = 1$

$$(x_1^2 + \dots + x_n^2)(y_1^2 + \dots + y_n^2) \geq (x_1 y_1 + \dots + x_n y_n)^2$$

proof $\lambda_1 = \lambda_2 = \frac{1}{2}$

$$(a_1 + \dots + a_n)^{\frac{1}{2}} (b_1 + \dots + b_n)^{\frac{1}{2}} \geq a_1^{\frac{1}{2}} b_1^{\frac{1}{2}} + \dots + a_n^{\frac{1}{2}} b_n^{\frac{1}{2}}$$

$$x_i = \sqrt{a_i}, \quad y_i = \sqrt{b_i}$$

$$(x_1^2 + \dots + x_n^2)^{\frac{1}{2}} (y_1^2 + \dots + y_n^2)^{\frac{1}{2}} \geq x_1 y_1 + \dots + x_n y_n$$

$$(x_1^2 + \dots + x_n^2)(y_1^2 + \dots + y_n^2) \geq (x_1 y_1 + \dots + x_n y_n)^2$$

