Tuberculosis (TB) Model

In this paper we develop a deterministic mathematical model of tuberculosis (TB). The population under consideration in this study is divided into six compartments based on the epidemiological status of individuals in the population. These are Susceptible $S(t)$, Vaccinated class $V(t)$, Latent class $L(t)$, individuals in latent class are infected with TB with no symptoms of the disease, active TB $J(t)$ are individuals with full symptoms of the disease, treated class $T(t)$ and recovered class $R(t)$. Total population is expressed as $N(t) = S(t) + V(t) + L(t) + J(t) + T(t) + R(t)$.
Recruitment into the susceptible class is at a rate ϕ, the recruitment rate is assumed to be by immigration or by birth. We assume that only individuals in the active TB class can transmit the disease. Hence the force of infection is given as αSJ. We assume that the vaccine was imperfect, thus, vaccinated individuals can be infected with the diseases via an interaction with individuals in the active TB individuals at a reduced rate $(1 - \beta)$. Therefore, the force of infection for vaccinated individuals is expressed as $\alpha(1 - \beta)VJ$. Parameter ρ represents the vaccine wane rate. We assume that natural death rate μ occurs in all the classes while the diseases induced death rate δ only occur at active TB individuals. Parameter ε represent the movement of individuals in the latent class to active TB class while we represent the rate of treatment for individuals in the active TB class. σ represent the movement rate from treated class and θ represents the treatment failure rate. We assume that proportional of treated individuals moved to latent class at a rate $(1 - \theta)\sigma$ due to the remainder of the bacteria in the body system and the rest of $\theta\sigma$ moves to active TB class as a result of treatment failure. The parameter γ represent recovery rate of treated individuals. The above description can be represented by a system of nonlinear differential equations.

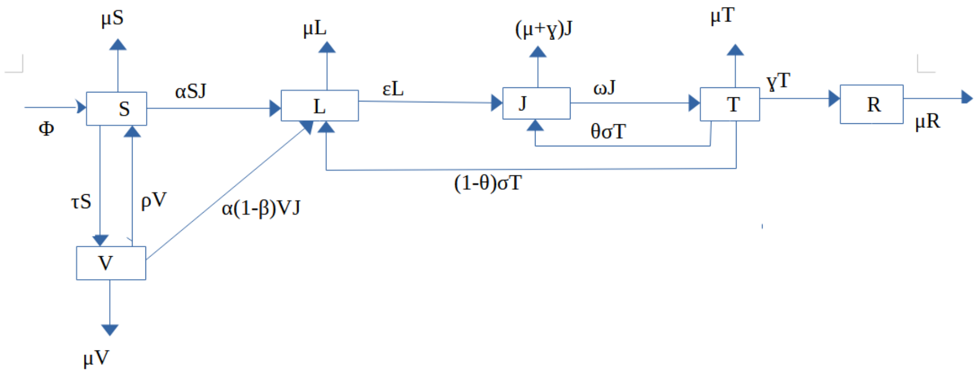

Figure 1: Pictorial diagram of the model

$$\frac{dS}{dt} = \phi + \rho V - \alpha SJ - (\tau + \mu)S$$
$$\frac{dV}{dt} = \tau S - \alpha(1-\beta)VJ - (\rho + \mu)V$$
$$\frac{dL}{dt} = \alpha SJ + \alpha(1-\beta)VJ + (1-\theta)\sigma T - (\varepsilon + \mu)L$$
$$\frac{dJ}{dt} = \varepsilon L + \theta\sigma T - (\mu + \delta + \omega)J$$
$$\frac{dT}{dt} = \omega J - (\mu + \gamma + \sigma)T$$
$$\frac{dR}{dt} = \gamma T - \mu R$$

Table 1: Description of Variables and Parameter

Variables	Description
$S(t)$	Susceptible individuals
$V(t)$	Vaccinated individuals
$L(t)$	Latent individuals
$J(t)$	Active TB individuals
$T(t)$	Treated individuals
$R(t)$	Recovered individuals
Parameters	Description
ϕ	Recruitment rate of individuals in susceptible classes
τ	Vaccine rate
ρ	Vaccine ware rate
α	Contact rate
β	Efficacy of the vaccine
μ	Natural Death rate
δ	Diseases induced death rate
ε	Progression rate from latent to active TB
θ	Treatment failure rate
σ	Movement rate of individuals in the treated class
γ	Recovery rate of treated individuals