COMP2711 Homework1

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Question 1:

(a) $\neg C(\text{Jan}, \text{Sharon})$

(b) $\neg\forall x\ I(x)$

(c) $\exists x\forall y\,(I(y) \leftrightarrow x \neq y)$

(d) $\forall x\exists y\,(I\,(x) \rightarrow (x \neq y \wedge C(x, y)))$

(e) Apply De Morgan's law: $\forall x\forall y\,(x = y \vee C(x, y))$

(f) There exists 2 students who haven't chatted with each other over the Internet.

Question 2:

(a) **True.** For all n, we can choose $m = n^2+1$, so that m is an integer and $n^2 < n^2+1 = m$.

(b) **True.** Choose $n = 1$, so that for all m, $nm = 1 \cdot m = m$.

(c) **True.** There exists $n = 3, m = 1$ that can make the statement true.

(d) **False.** When $m + n$ is odd, $\frac{m+n}{2}$ is not an integer. For example, pick $m = 5, n = 6$, then $p = \frac{m+n}{2} = \frac{11}{2}$, which is not an integer, then there doesn't exist such integer p.

Question 3:

(a) This statement means that for all different two $x, y \in U$, pick any $z \in U$, there'll be either $z = x$ or $z = y$, or both. Therefore, we pick $U = \{1, 2\}$, so $x = 1, y = 2$ or $x = 2, y = 1$, which are actually equivalent. Then z can be 1 or 2, and in any situation, there'll be either $z = x$ or $z = y$, making the statement true. Overall, $U = \{1, 2\}$ makes the statement true.

(b) Compared to (a), if we want to make the statement false, we only need to guarantee that **not** all $z \in V$ satisfy either $z = x$ or $z = y$. Therefore, we must make sure that z can equal to some values other than x, y, so that we only need to choose a domain that contains more than 2 elements. For example, pick $V = \{0, 1, 2\}$, and x, y will be two values of the three, and when z equals to the remaining element, $(z = x) \vee (z = y)$ is false.

Question 4:

$$\Big(\ ((p \to q) \leftrightarrow (q \to r)) \to (p \to r) \ \Big) \to s$$

$$\equiv \Big(\ ((p \to q) \land (q \to r)) \lor \big(\neg(p \to q) \land \neg(q \to r)\big) \to (p \to r) \ \Big) \to s$$

$$\equiv \Big(\ ((\neg p \lor q) \land (\neg q \lor r)) \lor \big(\neg(\neg p \lor q) \land \neg(\neg q \lor r)\big) \to (\neg p \lor r) \ \Big) \to s$$

$$\equiv \Big(\ ((\neg p \lor q) \land (\neg q \lor r)) \lor \neg\big((\neg p \lor q) \lor (\neg q \lor r)\big) \to (\neg p \lor r) \ \Big) \to s \ \ \text{(by DeMorgan's Law)}$$

$$\equiv \Big(\ ((\neg p \lor q) \land (\neg q \lor r)) \lor \neg\big(\neg p \lor q \lor \neg q \lor r\big) \to (\neg p \lor r) \ \Big) \to s \ \ \text{(by Commutative Laws)}$$

$$\equiv \Big(\ ((\neg p \lor q) \land (\neg q \lor r)) \lor \neg\big(\neg p \lor T \lor r\big) \to (\neg p \lor r) \ \Big) \to s \ \ \text{(by Negation Laws)}$$

$$\equiv \Big(\ ((\neg p \lor q) \land (\neg q \lor r)) \lor F \to (\neg p \lor r) \ \Big) \to s \ \ \text{(by Domination Laws)}$$

$$\equiv \Big(\ ((\neg p \lor q) \land (\neg q \lor r)) \to (\neg p \lor r) \ \Big) \to s \ \ \text{(by Identity Laws)}$$

$$\equiv \Big(\ \neg\big((\neg p \lor q) \land (\neg q \lor r)\big) \lor (\neg p \lor r) \ \Big) \to s$$

$$\equiv \Big(\ \big(\neg(\neg p \lor q) \lor \neg(\neg q \lor r)\big) \lor (\neg p \lor r) \ \Big) \to s \ \ \text{(by DeMorgan's Law)}$$

$$\equiv \Big(\ ((p \land \neg q) \lor (q \land \neg r)) \lor (\neg p \lor r) \ \Big) \to s \ \ \text{(by DeMorgan's Law)}$$

$$\equiv \Big(\ ((p \land \neg q) \lor \neg p) \lor \big((q \land \neg r) \lor r\big) \ \Big) \to s \ \ \text{(by Commutative Laws, Associative Laws)}$$

$$\equiv \Big(\ ((p \lor \neg p) \land (\neg q \lor \neg p)) \lor \big((q \lor r) \land (\neg r \lor r)\big) \ \Big) \to s \ \ \text{(by Distributive Laws)}$$

$$\equiv \Big(\ (T \land (\neg q \lor \neg p)) \lor \big((q \lor r) \land T\big) \ \Big) \to s \ \ \text{(by Negation Laws)}$$

$$\equiv \Big(\ (\neg q \lor \neg p) \lor (q \lor r) \ \Big) \to s \ \ \text{(by Identity Laws)}$$

$$\equiv \big((\neg q \lor q) \lor \neg p \lor r\big) \to s \ \ \text{(by Commutative Laws, Associative Laws)}$$

$$\equiv \big(T \lor \neg p \lor r\big) \to s \ \ \text{(by Negation Laws)}$$

$$\equiv T \to s \ \ \text{(by Domination Laws)}$$

$$\equiv s$$

$$((p \land q) \to p) \land (s \land (r \lor s))$$

$$\equiv (\neg(p \land q) \lor p) \land s \land (r \lor s)$$

$$\equiv (\neg p \lor \neg q \lor p) \land s \land (r \lor s) \ \ \text{(by DeMorgan's Law)}$$

$$\equiv (T \lor \neg q) \land s \land (r \lor s) \ \ \text{(by Commutative Laws, Negation Laws)}$$

$$\equiv s \land (r \lor s)$$

$$\equiv s \quad \text{(by Absorption Laws)}$$

Therefore, the two propositions are logically equivalent.