COMP2711 Homework1

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Question 1:

(a) $\neg C(\text{Jan}, \text{Sharon})$

(b) $\neg \forall x\ I(x)$

(c) $\exists x \forall y\, (I(y) \leftrightarrow x \neq y)$

(d) $\forall x \exists y\, (I\,(x) \rightarrow (x \neq y \wedge C(x, y)))$

(e) Apply De Morgan's law: $\forall x \forall y\, (x = y \vee C(x, y))$

(f) There exists 2 students who haven't chatted with each other over the Internet.

Question 2:

(a) **True.** For all n, we can choose $m = n^2 + 1$, so that m is an integer and $n^2 < n^2 + 1 = m$.

(b) **True.** Choose $n = 1$, so that for all m, $nm = 1 \cdot m = m$.

(c) **True.** There exists $n = 3, m = 1$ that can make the statement true.

(d) **False.** When $m + n$ is odd, $\frac{m+n}{2}$ is not an integer. For example, pick $m = 5, n = 6$, then $p = \frac{m+n}{2} = \frac{11}{2}$, which is not an integer, then there doesn't exist such integer p.

Question 3:

(a) This statement means that for all different two $x, y \in U$, pick any $z \in U$, there'll be either $z = x$ or $z = y$, or both. Therefore, we pick $U = \{1, 2\}$, so $x = 1, y = 2$ or $x = 2, y = 1$, which are actually equivalent. Then z can be 1 or 2, and in any situation, there'll be either $z = x$ or $z = y$, making the statement true. Overall, $U = \{1, 2\}$ makes the statement true.

(b) Compared to (a), if we want to make the statement false, we only need to guarantee that **not** all $z \in V$ satisfy either $z = x$ or $z = y$. Therefore, we must make sure that z can equal to some values other than x, y, so that we only need to choose a domain that contains more than 2 elements. For example, pick $V = \{0, 1, 2\}$, and x, y will be two values of the three, and when z equals to the remaining element, $(z = x) \vee (z = y)$ is false.

Question 4:

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\begin{aligned}
& \Big(\big((p \to q) \leftrightarrow (q \to r)\big) \to (p \to r) \Big) \to s \\
\equiv & \Big(\big((p \to q) \wedge (q \to r)\big) \vee \big(\neg(p \to q) \wedge \neg(q \to r)\big) \to (p \to r) \Big) \to s \\
\equiv & \Big(\big((\neg p \vee q) \wedge (\neg q \vee r)\big) \vee \big(\neg(\neg p \vee q) \wedge \neg(\neg q \vee r)\big) \to (\neg p \vee r) \Big) \to s \\
\equiv & \Big(\big((\neg p \vee q) \wedge (\neg q \vee r)\big) \vee \neg\big((\neg p \vee q) \vee (\neg q \vee r)\big) \to (\neg p \vee r) \Big) \to s \quad \text{(by DeMorgan's Law)} \\
\equiv & \Big(\big((\neg p \vee q) \wedge (\neg q \vee r)\big) \vee \neg\big(\neg p \vee q \vee \neg q \vee r\big) \to (\neg p \vee r) \Big) \to s \quad \text{(by Commutative Laws)} \\
\equiv & \Big(\big((\neg p \vee q) \wedge (\neg q \vee r)\big) \vee \neg\big(\neg p \vee T \vee r\big) \to (\neg p \vee r) \Big) \to s \quad \text{(by Negation Laws)} \\
\equiv & \Big(\big((\neg p \vee q) \wedge (\neg q \vee r)\big) \vee F \to (\neg p \vee r) \Big) \to s \quad \text{(by Domination Laws)} \\
\equiv & \Big(\big((\neg p \vee q) \wedge (\neg q \vee r)\big) \to (\neg p \vee r) \Big) \to s \quad \text{(by Identity Laws)} \\
\equiv & \Big(\neg\big((\neg p \vee q) \wedge (\neg q \vee r)\big) \vee (\neg p \vee r) \Big) \to s \\
\equiv & \Big(\big(\neg(\neg p \vee q) \vee \neg(\neg q \vee r)\big) \vee (\neg p \vee r) \Big) \to s \quad \text{(by DeMorgan's Law)} \\
\equiv & \Big(\big((p \wedge \neg q) \vee (q \wedge \neg r)\big) \vee (\neg p \vee r) \Big) \to s \quad \text{(by DeMorgan's Law)} \\
\equiv & \Big(\big((p \wedge \neg q) \vee \neg p\big) \vee \big((q \wedge \neg r) \vee r\big) \Big) \to s \quad \text{(by Commutative Laws, Associative Laws)} \\
\equiv & \Big(\big((p \vee \neg p) \wedge (\neg q \vee \neg p)\big) \vee \big((q \vee r) \wedge (\neg r \vee r)\big) \Big) \to s \quad \text{(by Distributive Laws)} \\
\equiv & \Big(\big(T \wedge (\neg q \vee \neg p)\big) \vee \big((q \vee r) \wedge T\big) \Big) \to s \quad \text{(by Negation Laws)} \\
\equiv & \Big((\neg q \vee \neg p) \vee (q \vee r) \Big) \to s \quad \text{(by Identity Laws)} \\
\equiv & \big((\neg q \vee q) \vee \neg p \vee r\big) \to s \quad \text{(by Commutative Laws, Associative Laws)} \\
\equiv & \big(T \vee \neg p \vee r\big) \to s \quad \text{(by Negation Laws)} \\
\equiv & \; T \to s \quad \text{(by Domination Laws)} \\
\equiv & \; s
\end{aligned}
$$

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\begin{aligned}
& ((p \wedge q) \to p) \wedge (s \wedge (r \vee s)) \\
\equiv & \; (\neg(p \wedge q) \vee p) \wedge s \wedge (r \vee s) \\
\equiv & \; (\neg p \vee \neg q \vee p) \wedge s \wedge (r \vee s) \quad \text{(by DeMorgan's Law)} \\
\equiv & \; (T \vee \neg q) \wedge s \wedge (r \vee s) \quad \text{(by Commutative Laws, Negation Laws)} \\
\equiv & \; s \wedge (r \vee s) \\
\equiv & \; s \quad \text{(by Absorption Laws)}
\end{aligned}
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Therefore, the two propositions are logically equivalent.