COMP2711 Homework5

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Question 1:

(a) There are $5 + 11 = 16$ teachers and students in total. When they sit in a row, they provide 17 spaces(represent by $\circ$):

$$\circ 1 \circ 2 \circ 3 \circ \cdots \circ 15 \circ 16 \circ$$

Firstly, choose 4 spaces from those 17 spaces where guests can seat: $\binom{17}{4}$, then, guests can swap their positions and teachers and students can also swap their positions, this will be $4! \cdot 16!$. So the answer is:

$$\binom{17}{4} \cdot 4! \cdot 16!$$

(b) There are $5 + 11 = 16$ teachers and students in total. When they sit in a row, they provide 16 spaces. Firstly, choose 4 spaces from those 16 spaces where guests can seat: $\binom{16}{4}$, then, guests can swap their positions, which will be $4!$, and teachers and students can also swap their positions, but this time it will be $\frac{16!}{16}$, since arranged in circle. So the answer is:

$$\binom{16}{4} \cdot 4! \cdot \frac{16!}{16} = \binom{16}{4} \cdot 4! \cdot 15!$$

(c) There are $5 + 4 = 9$ teachers and guests in total. When they sit in a row, they provide 10 spaces. Since there are 11 students, it's impossible to let them sit in 10 spaces, i.e., $\binom{10}{11} = 0$. Thus the answer is 0.

Question 2:

Note that the passcode follows non-decreasing order, so as long as we pick out 30 alphabets, there is only one possible order to arrange them. Since $2 + 7 + 1 + 1 = 11$ alphabets have already been chosen, we only need to pick $30 - 11 = 19$ alphabets from A $\sim$ Z.

To solve this problem, we can consider there are 19 balls and 25 bars arranged in a row, such that the 25 bars separate balls into 26 groups, and the number of balls in group i represents the number of the i-th alphabet in the passcode. For example, if we use "$\circ$" to denote balls and use "$|$" to denote bars, then

$$| \circ \circ | \circ | \circ \circ \circ | \cdots$$

means zero A, two B, one C, three D... are chosen.

Thus, we only need to choose 19 positions for balls among the total $19+25=44$ items, which gives $\binom{44}{19}$ ways. That means we have $\binom{44}{19}$ ways to pick 19 alphabets from A $\sim$ Z, and also means there are this number of different passcodes.

The final answer is $\binom{44}{19}$.

Question 3:

Assume that there are $n-3$ apples and $n-2$ pears in total, and you would like to pick 10 fruits to eat. So we would like to calculate how many ways can you do that in two methods:

Method 1:

Since we just want to choose any 10 fruit in $2n-5$ fruites, so there are $\binom{2n-5}{10}$ ways.

Method 2:

We enumerate how many apples to choose: for example, we want to choose k apples and $10-k$ pears, out of $n-3$ apples and $n-2$ pears, so there are $\binom{n-3}{k}\cdot\binom{n-2}{10-k}$ ways. Note that $0\le k\le 10$, so there're $\sum_{k=0}^{10}\binom{n-3}{k}\cdot\binom{n-2}{10-k}$ ways in total.

Therefore, we have:

$$\binom{2n-5}{10}=\sum_{k=0}^{10}\binom{n-3}{k}\cdot\binom{n-2}{10-k}$$

Question 4:

We try to simply consider one team as a whole(as one person), then there'll only be 19 people sitting in a row. Under this assumption, there are $\binom{10}{1}\cdot 19!\cdot 2!$ ways.

However, we find that some situations have been counted more than once. For example, if we denote 10 teams as $1,2,\cdots 10$ and the two people inside one team as A,B, then the situation:

$$1_A1_B2_A2_B3_A3_B4_A4_B\cdots 10_A10_B$$

will be counted when we consider team 1 as a whole, and will also be counted when we consider team 2 as a whole, and also 3, 4, $\cdots$ 10 as a whole.

Thus, we need to use Inclusion-Exclusion Principle, and the final result will be:

$$\sum_{k=1}^{10}(-1)^{(k+1)}\cdot\binom{10}{k}\cdot(20-k)!\cdot(2!)^k$$