# ISOM 2700 Fall 2021

# Operations Management

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## Session 16 **Inventory Management III The Marriage of EOQ and Newsvendor**

## 1 Overall introduction

**EOQ Model**:

- Long lifecycle product(stable demand)
- tradeoff between *setup* and *holding costs*, driven by *frequency of ordering*

$$Q^\star = \sqrt{\frac{2DS}{H}},\ ROP = DL$$

**Newsvendor Model**

- Short lifecycle product(uncertain demand)
- tradeoff between *costs of excess leftover inventory* and *excess demand.*

$$\Pr(D \le Q^\star) = \frac{c_u}{c_u + c_o} \text{ (critical fractile)}$$

In real world, nothing is certain. So how can we cooperate EOQ model with uncertain variables?

## 2 EOQ Model with Uncertain Demand

At Re-Order Point, we always use EOQ Model to decide how much to order. But in this way, uncertain demand may cause *overstocking* and *understocking*, in other words, ROP is no longer exactly a lead time before stock running out.

How to determine ROP?

- $c_o$: cost of overstocking a unit, cost of holding one more unit
- $c_u$: cost of understocking a unit, ..
- Optimal ROP satisfies: $\Pr(\textit{demand during lead time} \leq ROP) = \dfrac{c_u}{c_u + c_o}$

Normal Distribution, still, is a special case:

- *Demand per unit time* is normally distributed with mean $D$ and SD $\sigma$
- Then, *demand during lead time* is normally distributed with mean $DL$ and SD $\sigma\sqrt{L}$ (SD of demand for unit time is $\sigma$, so during lead time $L$: $\sqrt{\sigma^2 + \sigma^2 + \cdots + \sigma^2} = \sigma\sqrt{L}$)
- Then use Newsvendor Model, $ROP = DL + (\sigma\sqrt{L}) \cdot z$, $DL$ is *ROP with deterministic demand*, $\sigma\sqrt{L}$ is *safety stock*, if the demand is less failtile, safety stock is less.

[Example.] Demand is RV with mean 100 per week and SD 20. Orders arrive 0.5 week after ordering, fixed cost = 5000, holding cost = 4 per week. Suppose shop wants to maintain a service level of 85%.

(1) How much to order every time?

$D = 100, \sigma = 20, L = 0.5, S = 5000, H = 4, Q = \sqrt{\dfrac{2DS}{H}} = 500$

(2) When to reorder?

$z = 1.04$

[Example.] Annual demand 15,600 units per year, weekly demand is 300 units with SD 90 units. Lead time is 4 weeks.

(1) ROP to provide 98% service level:

$R = DL + \sigma\sqrt{L} \cdot z = 1200 + 180 * 2.06 = 1570$ units.

(2) If reduce safety stock by 50%, the new service level?

$SS' = 185$, $z = \dfrac{SS'}{\sigma\sqrt{L}} = 1.03$,

# 3 EOQ Model with Quantity Discount

Goal: how can we cooperate EOQ model with quantity discount(when customer buys a lot)

[Example.] Annual demand 40,000. Cost to process an order is 25, inv holding cost rate is 20%, given price schedule for product X.

| Demand | Unit Cost |
|---|---|
| $\leq 1500$ | 2.35 |
| $1500 < D \leq 2500$ | 2.30 |
| $2500 < D \leq 3000$ | 2.25 |
| $D > 3000$ | 2.20 |

- Start with lowest price, find the EOQ
- If the EOQ is not feasible, go to next higher level, solve the EOQ, $\cdots$
- Until you find EOQ falls in that interval
- Calculate total cost at EOQ and *every level breakpoint*, select the one with lowest cost.

At 2.20, EOQ=2,132 units, infeasible. At 2.25, EOQ=2,108 units, infeasible. At 2.30, EOQ=2,085 units, feasible.

How to calculate **total cost**?

$$TC = DC + \frac{DS}{Q} + \frac{QH}{2}$$

The last two terms are exactly the same in EOQ, $DC$: since now unit cost depends on quantity, now we cannot ignore ordering cost anymore, because cost depends on how much you order. Every cycle, $C \cdot Q$ costs, time interval is $Q/D$. $\frac{CQ}{Q/D} = CD$.

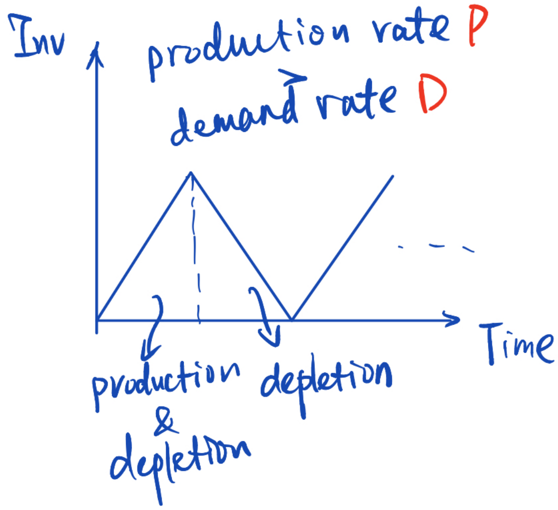
For example, when 2085,

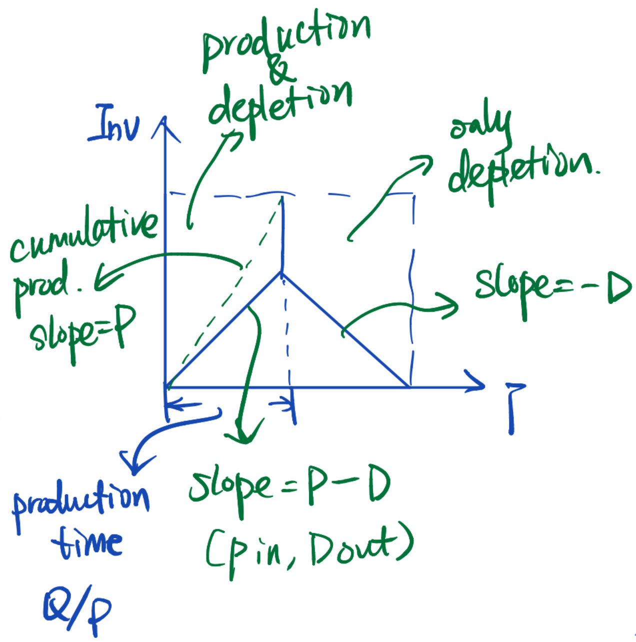
$$TC = DC + \frac{DS}{Q} + \frac{QH}{2} = 40000 \cdot 2.30 + \frac{40000 \cdot 25}{2085} + \frac{2085 \cdot (0.2 \cdot 2.30)}{2} = 92959$$

In the example, $(2085, 92959), (2500, 90963), (3000, 88993)$, thus ordering 3000 is optimal.

# 4 Economic Production Quantity(EPQ) Model

In many cases, we cannot produce everything in one time slot. So how to incoorperate production?




Total Cost $= TC(Q) = \frac{DS}{Q} + \frac{QH}{2}\left(1 - \frac{D}{P}\right)$

Find derivative, $Q^\star = \sqrt{\frac{2DS}{H\left(1-\frac{D}{P}\right)}} > \sqrt{\frac{2DS}{H}}$

EOQ is actually a special case of this model. (when production rate tends to infinity)

[Example.] Demand 1000 per week, produce rate 400 per week, 5000 fixed cost when start production, margin cost 400, cost of capital 1%.

(1) $Q^\star = \sqrt{\frac{2DS}{H\left(1-\frac{D}{P}\right)}} = \sqrt{\frac{2 \cdot 100 \cdot 5000}{4 \cdot (1 - 1/4)}} = 577$

(2) Length of each production run: $\frac{Q^\star}{P} = 1.44$ weeks.

(3) Cycle time for optimal production quantity: $\frac{Q^\star}{D} = 5.77$ weeks.

*This is the end of lecture note. Last modified: Nov 2(during class).*