5.1 Hypothesis Tests Using Normal Distributions

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Math 261

Outline

- Normal distributions
- Standard normal distribution
- The Central Limit Theorem
- P-values from Normal Distributions
- P-values from Standard Normal Distributions

Normal Distribution

- A **normal distribution** has a *symmetric, single-peaked, and bell-shaped* density curve.
- Normal distributions are completely characterized by their mean μ and standard deviation σ
  - Centered at mean μ
  - Inflection points at $\mu - \sigma$ and $\mu + \sigma$

$$X \sim N(\mu, \sigma)$$

Finding Areas under the Normal Curve

Use StatKey to find areas and endpoints of intervals:

StatKey to accompany *Statistics: Unlocking the Power of Data*
by Lock, Lock, Lock, Lock, and Lock

Descriptive Statistics and Graphs	Bootstrap Confidence Intervals	Randomization Hypothesis Tests
One Quantitative Variable	CI for Single Mean, Median, St.Dev.	Test for Single Mean
One Categorical Variable	CI for Single Proportion	Test for Single Proportion
One Quantitative and One Categorical Variable	CI for Difference In Means	Test for Difference in Means
Two Categorical Variables	CI for Difference In Proportions	Test for Difference In Proportions
Two Quantitative Variables	CI for Slope, Correlation	Test for Slope, Correlation

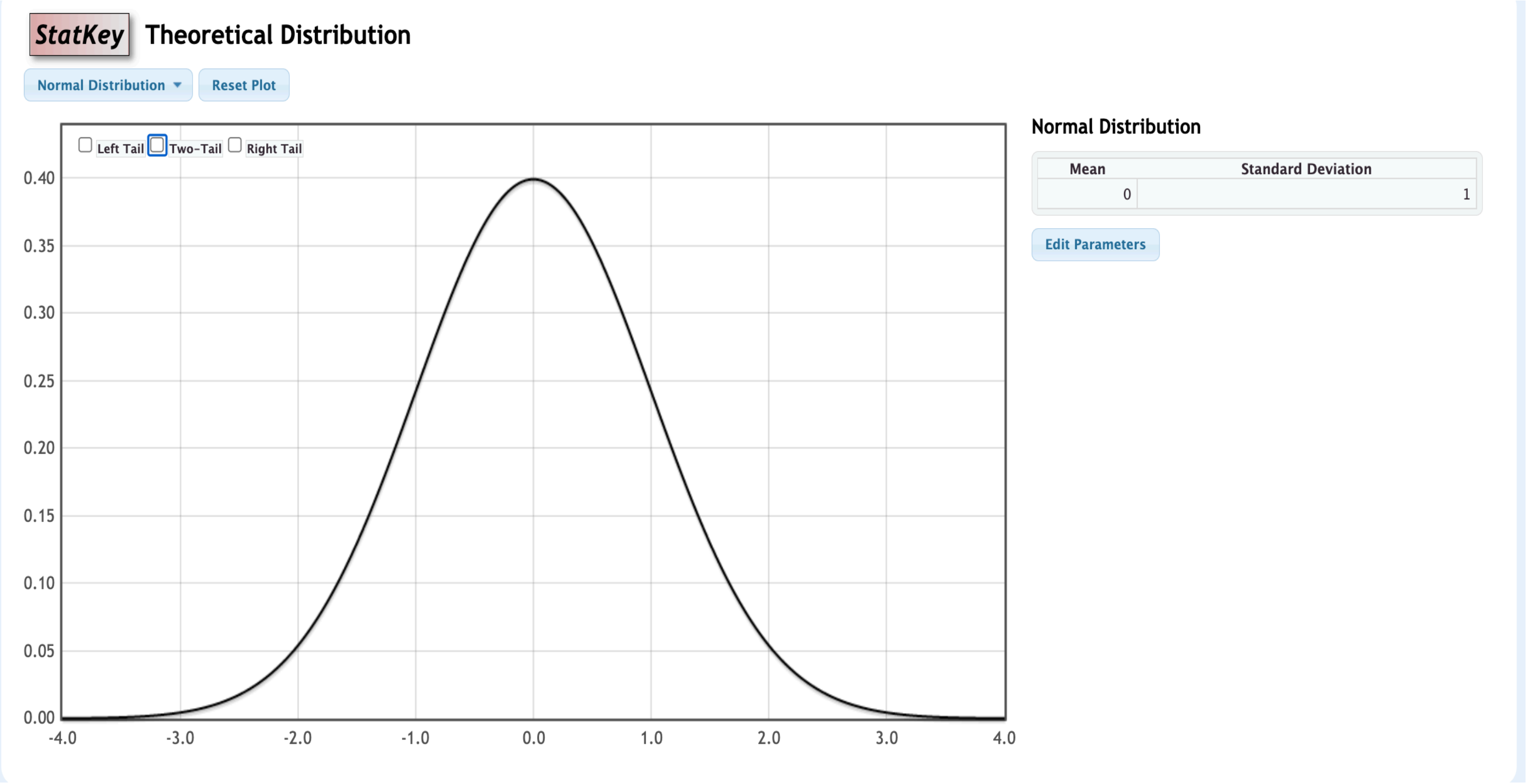
Sampling Distributions	Mean	Proportion

Theoretical Distributions	Normal	t	χ^2	F

More Advanced Randomization Tests	χ^2 Goodness-of-Fit	χ^2 Test for Association	ANOVA for Difference in Means	ANOVA for Regression

Finding Areas under the Normal Curve (cont'd)

Use StatKey to find areas and endpoints of intervals:



Standard Normal Distribution

The standard normal (Z) distribution has

- Mean $\mu = 0$
- Standard deviation $\sigma = 1$
- Every normal distribution can be *standardized* to follow a *standard normal distribution*.

$$Z = \frac{X - \mu}{\sigma}$$

Central Limit Theorem

> For random samples with a *sufficiently large* sample size, the distribution of sample statistics for a mean or a proportion is approximately normal.

Central Limit Theorem

- The central limit theorem holds for **any** original distribution, although "sufficiently large sample size" varies.
  - The more skewed the original distribution is, the larger n has to be for the CLT to work.
- For quantitative variables that are not very skewed, $n \geq 30$ is usually sufficient.
- For categorical variables, counts of at least 10 within each category are usually sufficient.

Connecting the Normal Distribution to Randomization Distributions

If the randomization distribution is normal:

> To calculate a p-value, we just need to find the area in the appropriate tail(s) beyond the observed statistic of the distribution

$$N(\text{null value}, \text{SE})$$

Standardized Test Statistic

We often streamline the hypothesis test process by *standardization*:

> The standardized *test statistic* is the number of standard errors a test statistic is away from the hypothesized null value:

$$z = \frac{\text{Sample statistic} - \text{Null parameter}}{\text{SE}}$$

- Calculating the number of standard errors a statistic is from the null value allows us to assess *extremity* on a common scale.

P-value Using the Standard Normal

If a statistic is normally distributed under H_0, the **p-value** is the probability a standard normal is at or beyond the standardized test statistic.

$$z = \frac{\text{Sample statistic} - \text{Null parameter}}{\text{SE}}$$

- One-tailed p-values
  - $p = P(Z \geq z)$ or $p = P(Z \leq z)$
- Two-tailed p-value
  - Double the one-tailed value

where $P(Z \geq z)$ means the probability *to the right* of the observed test statistic z under the standard normal density represented by Z.

Summary

- If the distribution of a statistic is normal, the standardized test statistic is:

$$z = \frac{\text{Sample statistic} - \text{Null parameter}}{\text{SE}}$$

- Obtain the p-value by comparing z to the **standard normal** $N(0, 1)$ distribution.