Exercise Sheet 2

Linear Algebra

Deadline: 16.11.2023 23:59

Guidelines: You are expected to work in a group of 2-3 students. While submitting the assignments, please make sure to include the following information for all our teammates in your PDF/python script:

Name:

Student ID (matriculation number):

Email:

Your submissions should be zipped as **Name1_id1_Name2_id2_Name3_id3.zip** when you have multiple files. For assignments where you are submitting a single file, use the **same naming convention** without creating a zip. For any clarification, please reach out to us on the **CMS Forum**.
Note that the above instructions are mandatory. If you are not following them, tutors can decide not to correct your exercise.

Exercise 2.1 - Linear Algebra   (1+1+1 points)

a) Use that every matrix $A \in \mathbb{R}^{m \times n}$ can be decomposed into $A = UDV^T$, where V are the eigenvectors of $A^T A$, to show that the singular values of A (i.e. the values along the diagonal of D) are the square roots of the eigenvalues of $A^T A$.

b) Compute the eigenvalues and eigenvectors of the matrix $A = \begin{bmatrix} 3 & 1 \\ 0 & 4 \end{bmatrix}$

c) Show that if λ is an eigenvalue of AB, then it is also an eigenvalue of BA where $A \in \mathbb{R}^{n \times n}, B \in \mathbb{R}^{n \times n}$.

Exercise 2.2 - Matrix Calculus   (0.5+1+1+1 points)

In this lecture we will often compute the derivatives of multivariate functions and matrix valued functions. Let $f : \mathbb{R}^n \to \mathbb{R}$; $w, x, c \in \mathbb{R}^n$, $A \in \mathbb{R}^{n \times n}$ and $B \in \mathbb{R}^{n \times n}$. Prove that the following rules hold.

a) $f(x) = w^T x$, then $\nabla_x f(x) = w$

b) $f(x) = x^T A x$, then $\nabla_x f(x) = Ax + A^T x$

c) $f(x) = \|Bx\|_2^2$, then $\nabla_x f(x) = 2B^T Bx$

d) $f(x) = \|Bx - c\|_2^2$, then $\nabla_x f(x) = 2B^T(Bx - c)$

Exercise 2.3 - Eigenfaces (3.5 points)

See the accompanying jupyter notebook.