

includes asymptomatic infected patients who play a role in the infection cycle without displaying any symptoms, causing a new disease possible. In the new model developed in this section, the notation $P(t)$ indicates the total number of population with seven sub-categories, *i.e.*, susceptible people $S(t)$, vulnerable individuals $V(t)$ who do not show the disease symptoms, clinically symptomatic infected people $I(t)$, asymptomatic infected persons $A(t)$ (with hardly any clinical signs), quarantined people $K(t)$, under treatment individuals $U(t)$, and people who are recuperated $R(t)$. The seafood market $M(t)$ is mainly responsible for this infection because people, who visit the markets and purchase the food, get infected and spread it throughout the environment. As a result, an ordinary differential equations system, which describes the disease, could be derived by applying the above assumptions as below [18]

$$\begin{aligned}\frac{dS(t)}{dt} &= \beta - \zeta S(t) - v(t)S(t), \\ \frac{dV(t)}{dt} &= -((1 - \omega)\sigma_1 + \omega\sigma_2 + \zeta + \kappa_v)V(t) + v(t)S(t), \\ \frac{dI(t)}{dt} &= -(\rho_i + \zeta + \xi_i + \eta_i)I(t) + (1 - \omega)\sigma_1 V(t), \\ \frac{dA(t)}{dt} &= -(\rho_a + \zeta)A(t) + \omega\sigma_2 V(t), \\ \frac{dK(t)}{dt} &= -(\zeta + \rho_k + \eta_k)K(t) + \kappa_v V(t), \\ \frac{dU(t)}{dt} &= -(\zeta + \rho_u + \xi_u)U(t) + \eta_i I(t) + \eta_k K(t), \\ \frac{dR(t)}{dt} &= -\zeta R(t) + \rho_i I(t) + \rho_a A(t) + \rho_k K(t) + \rho_u U(t), \\ \frac{dM(t)}{dt} &= -\tau_m M(t) + \tau_a A(t) + \tau_i I(t),\end{aligned}\quad (15)$$

in which

$$v(t) = \frac{\theta_1(I(t) + \phi A(t))}{P(t)} + \theta_2 M(t). \quad (16)$$

The infected people with clinically symptomatic or without clinical symptoms, as well as the infected individuals of seafood markets ($I(t)$, $A(t)$, $M(t)$) can infect the susceptible people to the disease, which is shown as $v(t)$. For susceptible individuals, the corresponding birth rate is β , and the human mortality rate can be measured by ζ . Healthy people would become infectious after exposure to the infected and asymptotically infected individuals at the rate θ_1 , whereas the transmissibility factor is denoted by ϕ . The number of people who are affected as a result of visiting seafood markets is increased by θ_2 . The parameter ω generates the asymptomatic infection. The periods of incubation are represented by σ_1 and σ_2 . The parameter κ_v represents the number of individuals who have been exposed to the infection and are being quarantined. The parameters ρ_a and ρ_i refer to the percentage recuperation of asymptomatic infection and infection, respectively. Also, the parameters ρ_u and ρ_k are, respectively, the recuperation of hospitalized and quarantined people. The disease-related death rates of hospitalized and infected people are represented by ξ_u and ξ_i , respectively. In addition, the hospitalization rate for the infected and quarantine individuals has been calculated with η_i and η_k , respectively. Not only are the parameters τ_i and

τ_a , respectively, shown the infections came from the infected and asymptotically infected people at seafood markets, but also the parameter τ_m represents the removed infected individuals from the markets.

3.2. Non-negativity of the solution

Lemma 3.1. Let $\Delta(t)$ be the state vector of the model (15), *i.e.*,

$$\Delta(t) = (S(t), V(t), I(t), A(t), K(t), U(t), R(t), M(t)),$$

and $\Delta(0) = (S_0, V_0, I_0, A_0, K_0, U_0, R_0, M_0) \geq 0$ be the non-negative initial state vector. Then the model (15) has a non-negative solution for any time $t > 0$. Moreover, we have $\lim_{t \rightarrow \infty} P(t) \leq \frac{\beta}{\zeta}$ where $P(t)$ would be the total population computed by $P(t) = S(t) + V(t) + I(t) + A(t) + K(t) + U(t) + R(t)$.

Proof. Let us take into account

$$t_{sup} = \sup\{t > 0 : \Delta(t) > 0\}.$$

Thus, $t_{sup} > 0$. The first equation of the model (15) leads to the following relation

$$\frac{dS(t)}{dt} = \beta - \zeta S(t) - v(t)S(t) = \beta - S(t)(\zeta + v(t)), \quad (17)$$

where $v(t) = \frac{\theta_1(I(t) + \phi A(t))}{P(t)} + \theta_2 M(t)$. Thus, the Eq. (17) can be written as

$$\begin{aligned}\frac{d}{dt} \left\{ S(t) \exp \left(\zeta t + \int_0^{t_{sup}} v(s) ds \right) \right\} \\ = \beta \exp \left(\zeta t + \int_0^{t_{sup}} v(s) ds \right).\end{aligned}\quad (18)$$

Therefore,

$$\begin{aligned}S(t_{sup}) \exp \left(\zeta t_{sup} + \int_0^{t_{sup}} v(s) ds \right) - S(0) \\ = \int_0^{t_{sup}} \beta \exp \left(\zeta y + \int_0^y v(\zeta) d\zeta \right) dy,\end{aligned}\quad (19)$$

such that

$$\begin{aligned}S(t_{sup}) &= S(0) \exp \left\{ -(\zeta t_{sup} + \int_0^{t_{sup}} v(s) ds) \right\} \\ &\quad + \exp \left\{ -(\zeta t_{sup} + \int_0^{t_{sup}} v(s) ds) \right\} \\ &\quad \times \int_0^{t_{sup}} \beta \exp(\zeta y + \int_0^y v(\zeta) d\zeta) dy > 0.\end{aligned}\quad (20)$$

We can follow a similar strategy for the remaining equations in the system (15) to prove $\Delta(t) > 0$ for all $t > 0$. For further assertion, consider that the initial conditions satisfy $0 < S_0, V_0, I_0, A_0, K_0, U_0, R_0 \leq P(t)$. In the system (15), all the equations except the last one are added together; thus, we obtain

$$\frac{dP(t)}{dt} = \beta - \zeta P(t) - \zeta_i I(t) - \zeta_u U(t) \leq \beta - \zeta P(t).$$

Consequently, we have

$$\lim_{t \rightarrow \infty} P(t) \leq \frac{\beta}{\zeta}.$$

□