18 октября.

Дифффуры.

Дз № 4.

4.2) $y'^2 - y^2 = 0 \qquad (y'-y)(y'+y)=0$

$$\left[\begin{array}{l} y' = y \\ y' = -y \end{array}\right.$$

$y' = y \qquad \ln y + C_1 = x \qquad y = e^{x-C_1}$

$y' = -y \qquad \ln y + C_2 = -x \qquad y = e^{-x-C_2}$

Ответ: $y = C_1 e^{x}$; $y = \frac{C_2}{e^{x}}$

4.3) $y' = e^{\frac{xy'}{y}} \qquad \ln y' = \frac{xy'}{y}$

$y = \frac{xy'}{\ln y'} \qquad (y' \neq 1)$

$y' = p \qquad y = \frac{xp}{\ln p}$

$dy = y'dx = y'_x dx + y'_p dp = \frac{p\,dx}{\ln p} + \frac{x\ln p - x}{\ln^2 p} dp$

$\quad \| $

$\frac{p}{\ln p} dx$

$\left(\frac{-p}{\ln p} + p\right) dx = \frac{x\ln p - x}{\ln^2 p} dp$

$(p + p\ln p)\,dx = \dfrac{x(\ln p - 1)}{\ln p}\,dp$

$\dfrac{+p\ln p}{x} = \dfrac{dp}{dx}$

$+\ln x + C = \ln \ln p$

$\cancel{\ln p = \dfrac{1}{x} + C}$

$\cancel{p = e^{\frac{1}{x}} + C}$

$\cancel{y' = e^{\frac{1}{x}} + C}$

$\ln p = Cx$

$p = e^{Cx}$

$y' = e^{Cx}$

$y = \dfrac{e^{Cx}}{C} + C_1$

Ответ: $y = \dfrac{e^{Cx}}{C} + C_1$

$\cancel{4.4)\ xy' - y = \ln y}$

$\cancel{x = \dfrac{\ln y + y}{y'}}$ $\qquad y' = p$

$\cancel{x = \dfrac{\ln y + y}{p}}$

4.4) $xy' - y = \ln y$

$$\frac{dy}{\ln y + y} = \frac{dx}{x}$$

$$\int \frac{1}{\ln y + y} = \ln|x|$$

4.5) $y'^4 + y^2 = y^4$

$$y' = \pm\sqrt[4]{y^4 - y^2}$$

$$\frac{dy}{\sqrt[4]{y^4 - y^2}} = \pm dx$$

$$\pm x = \frac{\ln\left|\frac{\sqrt[4]{y^2-1}}{\sqrt{y}} + 1\right| - \ln\left|\frac{\sqrt[4]{y^2-1}}{\sqrt{y}} - 1\right|}{2}$$

$$- \operatorname{arctg} \frac{\sqrt[4]{y^2-1}}{\sqrt{y}} + C$$

4.6) $y'^2 - y'^3 = y^2$

$y' = p \qquad y = \pm\sqrt{p^2 - p^3} = \pm p\sqrt{1-p}$

1) $y = p\sqrt{1-p}$

$$dy = \sqrt{1-p}\,dp + \frac{(-1)\cdot p\,dp}{2\sqrt{1-p}}$$

$$dy = p\,dx$$

$$p\,dx = \left(\frac{1}{\sqrt{1-p}} - \frac{3p}{2\sqrt{1-p}}\right)dp$$

$$dx = \frac{3\sqrt{1-p}}{2(p-1)}\left(\frac{1}{p\sqrt{1-p}} - \frac{3}{2\sqrt{1-p}}\right)dp$$

$$x = \left(3\sqrt{1-p} + \ln\left(\frac{\sqrt{1-p}-1}{\sqrt{1-p}+1}\right)\right) + C$$

2) $y = -p\sqrt{1-p}$

Аналогично

$$x = -\left(3\sqrt{1-p} + \ln\left(\frac{\sqrt{1-p}-1}{\sqrt{1-p}+1}\right)\right) + C$$

4.7) $y = 2y'^2 - 2y'^3$

$y' = p$ $\quad y = 2p^2 - 2p^3$

$dy = (4p - 6p^2)\,dp$

$dy = y'\,dx = p\,dx$

$dx = (4 - 6p)\,dp$

$x = 4p - 3p^2 + C$

4.8) $y'^2 - yy' + e^x = 0$

$y' = p \quad p^2 - yp + e^x = 0 \quad y = \frac{p^2 + e^x}{p}$

$dy = dp + \frac{e^x \cancel{\ln}}{p^2} \cdot (-1)\,dp = p\,dx$

$\frac{dp}{p^3} = \frac{dx}{1+e^x}$

$$\cancel{\frac{1}{\sqrt{2p^2}} = \int \frac{de^x}{e^x+e^{2x}} = -\ln\left(\frac{1}{e^x}+1\right)+C}$$

$$dy = dp + \frac{e^x dx p - dp e^x}{p^2} = p dx$$

$$p^2 dp + e^x dx p - dp e^x = p^3 dx$$

$$(p^2 - e^x) dp = p(p^2 - e^x) dx$$

$$C + \ln|p| = x \qquad e^x = e^C \cdot p$$

$$y = \cancel{p^2+} \; p + e^C$$

4.1) $\rho = \dfrac{k}{\cos\alpha}$

$$\rho = \frac{(1+y'^2)^{\frac{3}{2}}}{|y''|}, \quad \alpha = \operatorname{arctg} y'$$

$$(1+y'^2)^{\frac{3}{2}} = k|y''|(1+y'^2)^{\frac{1}{2}}$$

$$ky'' = 1 + y'^2 \qquad \frac{y''}{1+y'^2} = \frac{1}{k} \qquad (\operatorname{arctg} y')' = \frac{1}{k}$$

$$\operatorname{arctg} y' = \frac{x}{k} + C_1 \qquad y' = \operatorname{tg}\left(\frac{x}{k}+C_1\right)$$

$$y = -k\ln\left|\cos\left(\frac{x}{k}+C_1\right)\right| + C_2$$