11 сентября.

Мат ан.

1. $\rho(x, y) = \frac{|x-y|}{xy}$ $(x; y \in N)$

$\left.\begin{array}{l} x = y \Rightarrow \rho(x, y) = 0 \\ \rho(x, y) = 0 \Rightarrow x = y \end{array}\right\}$ 1) вып.

Очевидно, 2) вып.

$\rho(x, z) \leq \rho(x, y) + \rho(y, z)$

$\frac{|x-z|}{xz} \leq \frac{|x-y|}{xy} + \frac{|y-z|}{yz}$

$y|x-z| \leq \underbrace{|x-y|z + |y-z|x}_{\geq}$

$|zx - yz + yx - zx| = y|x-z|$

2. а) да

б) да

в) да

3. $\ln(1+\rho(x,y))=0 \Leftrightarrow \rho(x,y)=0 \Leftrightarrow$

$\Leftrightarrow x=y$, т.к. ρ метрика

$\ln(1+\rho(x,y))=\ln(1+\rho(y,x)) \Leftrightarrow$

$\Leftrightarrow \rho(x,y)=\rho(y,x)$ — верно, т.к. ρ метрика

$\ln(1+\rho(x,z)) \leq \ln(1+\rho(x,y))+$

$+\ln(1+\rho(y,z)) \Leftrightarrow 1+\rho(x,z) \leq$

$\leq (1+\rho(x,y))(1+\rho(y,z)) \Leftrightarrow$

$\Leftrightarrow \underbrace{\rho(x,z) \leq \rho(x,y)+\rho(y,z)}_{\text{выполн., т.к. } \rho \text{ метрика}}+$

$+\underbrace{\rho(x,y)}_{\geq 0}\cdot\rho(y,z) \Rightarrow$ верно

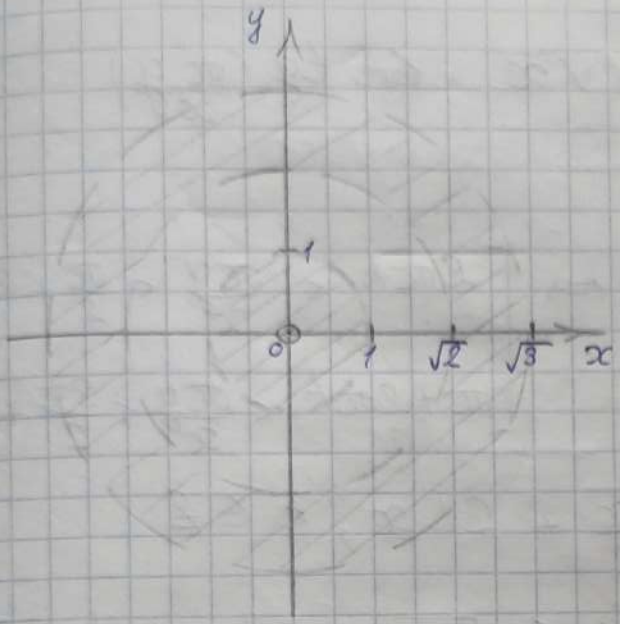
Ответ: верно.

4. а) $u(x,y)=\ln\sin\pi(x^2+y^2)$

$\sin(\pi(x^2+y^2))>0$

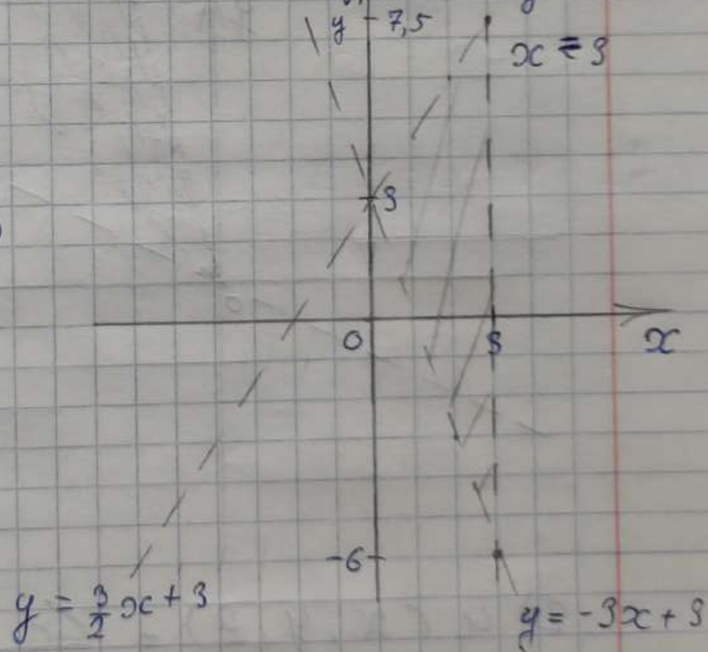
$2\pi k<\pi(x^2+y^2)<\pi+2\pi k,\ k\in\mathbb{Z}$

$2k<x^2+y^2<1+2k,\ k\in\mathbb{Z}$



б) $u(x,y) = \ln(3x+y-3) + \dfrac{\ln(3-x)}{\sqrt{3x-2y+6}}$

$$\begin{cases} 3x+y-3>0 \\ 3-x>0 \\ 3x-2y+6>0 \end{cases}$$

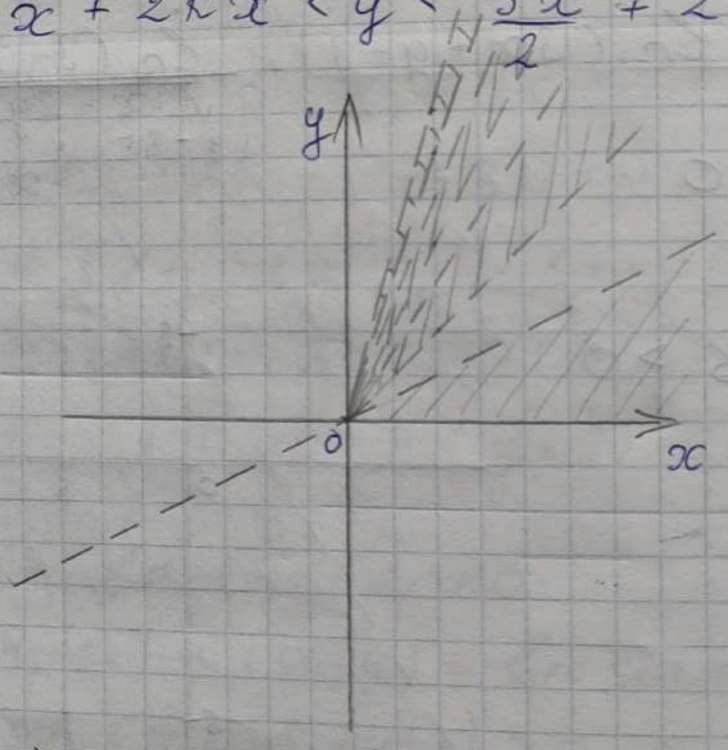


в) $u(x,y) = \sqrt{x} \cdot \ln tg\left(\frac{\pi y}{x}\right)$

$x > 0$

$tg \frac{\pi y}{x} > 0 \Leftrightarrow \left[\begin{array}{l} 2\pi k < \frac{\pi y}{x} < \frac{\pi}{2} + 2\pi k, \ k \in Z \\ \pi + 2\pi k < \frac{\pi y}{x} < \frac{3\pi}{2} + 2\pi k, \ k \in Z \end{array}\right.$

$\left[\begin{array}{l} 2kx < y < \frac{x}{2} + 2kx, \ k \in Z \\ x + 2kx < y < \frac{3x}{2} + 2kx, \ k \in Z \end{array}\right.$



2) $u(x,y,z) = \frac{\ln(z^2 - x^2 - y^2)}{\sqrt{1 - x^2 - y^2 - z^2}}$

$\begin{cases} z^2 - x^2 - y^2 > 0 \\ 1 - x^2 - y^2 - z^2 > 0 \end{cases}$

$k,$

$\in Z$

$\pi k,$

$\in Z$