$$104.\ \int \frac{dx}{(x^2+a^2)\sqrt{x^2+b^2}} =$$

$$u = \left(\sqrt{x^2+b^2}\right)' = \frac{x}{\sqrt{x^2+b^2}}$$

$$du = \frac{\sqrt{x^2+b^2} - \frac{x^2}{\sqrt{x^2+b^2}}}{x^2+b^2}\,dx =$$

$$= \frac{b^2}{(x^2+b^2)\sqrt{x^2+b^2}}$$

$$= \int \frac{x^2+b^2}{b^2(x^2+a^2)} \cdot \frac{b^2}{(x^2+b^2)\sqrt{x^2+b^2}}\,dx =$$

$$u^2 = \frac{x^2}{x^2+b^2} \qquad u^2x^2 + u^2b^2 = x^2$$

$$x^2 = \frac{u^2b^2}{1-u^2}$$

$$\frac{x^2+b^2}{b^2(x^2+a^2)} = \frac{\frac{u^2b^2}{1-u^2} + b^2}{b^2\left(\frac{u^2b^2}{1-u^2} + a^2\right)} =$$

$$= \frac{1}{u^2b^2 + a^2 - a^2u^2}$$

$$= \int \frac{1}{u^2(b^2-a^2)+a^2}\,du =$$

$$= \frac{1}{b^2-a^2} \int \frac{1}{u^2 + \frac{a^2}{b^2-a^2}}\,du =$$

$$= \frac{1}{b^2-a^2}\left(\frac{\sqrt{b^2-a^2}}{a} \operatorname{arctg} \frac{u\sqrt{b^2-a^2}}{a}\right)+C$$

$$u = \frac{x}{\sqrt{x^2+b^2}}$$

101. $\int \frac{dx}{x(x^5+6)} = \int \frac{dx^5}{5x^5(x^5+6)} =$

$] \; x^5 = t$

$$= \int \frac{dt}{5t(t+6)} = \frac{1}{5}\int \frac{1}{6}\left(\frac{1}{t} - \frac{1}{t+6}\right)dt =$$

$$= \frac{1}{30}\left(\ln|t| - \ln|t+6|\right)+C$$

102. $\int \frac{dx}{x^{11}+2x^6+x} = \int \frac{dx^5}{5x^5(x^{10}+2x^5+1)} =$

$] \; t = x^5$

$$= \frac{1}{5}\int \frac{dt}{t(t^2+2t+1)} =$$

$$\frac{1}{t(t+1)^2} = \frac{A}{t} + \frac{B}{t+1} + \frac{C}{(t+1)^2}$$

$$1 = A(t+1)^2 + Bt(t+1) + Ct$$

$t=-1$: $\quad C = -1$

$t = 0$: $\quad A = 1$

t^2: $\; 0 = A + B \Rightarrow B = -1$

$$= \frac{1}{5}\int\left(\frac{1}{t} - \frac{1}{(t+1)^2} - \frac{1}{t+1}\right)dt =$$

$$= \frac{1}{5}\left(\ln|t| + \frac{1}{t+1} - \ln|t+1|\right) + C$$

$t = x^5$

103. $\int \frac{x^4+1}{(x-1)(x^4-1)}\,dx =$

$$= \int \frac{x^4+1}{(x-1)(x^2-1)(x^2+1)}\,dx =$$

$$= \int \frac{x^4+1}{(x-1)^2(x+1)(x^2+1)}\,dx =$$

$$= \frac{x^4+1}{(x-1)^2(x+1)(x^2+1)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} +$$

$$+ \frac{C}{x+1} + \frac{Dx+E}{x^2+1}$$

$x^4+1 = A(x-1)(x+1)(x^2+1) + B \cdot$
$\cdot (x+1)(x^2+1) + C(x-1)^2(x^2+1) +$
$+(Dx+E)(x-1)^2(x+1)$

$x = 1: \quad 2 = B \cdot 4 \Rightarrow B = \frac{1}{2}$

$x = -1: \quad 2 = C \cdot 8 \Rightarrow C = \frac{1}{4}$

$x^4: \ 1 = A + C + D$

$x^0: \ 1 = -A + B + C + E$

$x^1: \ 0 = B - 2C + D - E$

~~$1 = A + 2B - C + D$~~ $\quad A = \frac{1}{4} \quad D = \frac{1}{2} \quad E = \frac{1}{2}$

~~$2 = 2A + 2B + 2D$~~

$$= \int\left(\frac{x+1}{2(x^2+1)} + \frac{1}{4(x+1)} + \frac{1}{4(x-1)} + \frac{1}{2(x-1)^2}\right)dx = \frac{1}{4}\int\frac{dx^2}{x^2+1} + \frac{1}{2}\int\frac{dx}{x^2+1} + \frac{1}{4}\ln|x+1| + \frac{1}{4}\ln|x-1| + \frac{1}{2}\,\frac{1}{1-x} =$$

$$= \frac{1}{4}\ln|x^2+1| + \frac{1}{2}\,\mathrm{arctg}\,x + \frac{1}{4}\ln|x+1| + \frac{1}{4}\ln|x-1| + \frac{1}{2}\,\frac{1}{1-x} + C$$

105. $$\int\frac{\ln x\,dx}{\sqrt{x+a}} = 2\int \ln x\,d\sqrt{x+a} =$$

$$= 2\ln x\sqrt{x+a} - 2\underbrace{\int\frac{\sqrt{x+a}}{x}\,dx}_{=} =$$

$\exists\ t = \sqrt{x+a}$

$$\int\frac{t}{t^2-a}\,d(t^2-a) =$$

$$= \int\frac{2t^2\,dt}{t^2-a} = \int\left(2 + \frac{2a}{t^2-a}\right)dt = 2t + 2a\,\frac{1}{2\sqrt{a}}\ln\left|\frac{t-\sqrt{a}}{t+\sqrt{a}}\right| + C$$

$$= 2\ln x\sqrt{x+a} - 4\sqrt{x+a} - 2\sqrt{a}\cdot\ln\left|\frac{\sqrt{x+a}-\sqrt{a}}{\sqrt{x+a}+\sqrt{a}}\right| + C$$

108. $\int \frac{dx}{x^4\sqrt{1+x^2}} =$

$m = 4 \quad p = \frac{1}{2} \quad n = 2$

$t^2 = x^{-2} + 1$

$x^2 = \frac{1}{t^2-1} \qquad x^2+1 = \frac{t^2}{t^2-1}$

$dx = d(t^2-1)^{-\frac{1}{2}} = -\frac{t}{(t^2-1)^{\frac{3}{2}}}\,dt$

$= -\int \frac{(t^2-1)^2\sqrt{t^2-1}\cdot t}{t\,(t^2-1)^{\frac{3}{2}}}\,dt =$

$= -\int (t^2-1)\,dt = -\frac{1}{3}t^3 + t + C$

$t = \sqrt{\frac{x^2+1}{x^2}}$

109. $\int \frac{x^3\,dx}{(x^8+1)^2} = \int \frac{dx^4}{(x^8+1)^2} =$

~~m = 3~~ $]\; t = x^4$

$= \int \frac{dt}{(t^2+1)^2} = \int \frac{d(t^2+1)^{-1}}{-2t} =$

$= -\frac{1}{2t(t^2+1)} \mp \frac{1}{2}\int \frac{dt}{(t^2+1)\,t^2} =$

$= -\frac{1}{2t(t^2+1)} - \frac{1}{2}\int\left(\frac{1}{t^2} - \frac{1}{t^2+1}\right)dt =$

$$= -\frac{1}{2t(t^2+1)} - \frac{1}{2}\int\left(-\frac{1}{t}\right) + \frac{1}{2}\operatorname{arctg} t + C$$

$t = x^4$

110. $$\int \frac{dx}{x^8+7x} = \int \frac{dx^7}{7x^7(x^7+7)} =$$

$\exists\ t = x^7$

$$= \int \frac{dt}{7t(t+7)} = \frac{1}{7}\cdot\frac{1}{7}\int\left(\frac{1}{t} - \frac{1}{t+7}\right)dt =$$

$$= \frac{1}{49}\left(\ln|t| - \ln|t+7|\right) + C$$

~~113.~~ $$\cancel{\int \frac{dx}{\sqrt{2}+\sqrt{1+x}+\sqrt{1-x}}}$$

114. $$\int \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}\,dx = \int \frac{1-\sqrt{x}}{\sqrt{1-x}}\,dx =$$

$$\cancel{= \int \frac{1}{\sqrt{1-x}}}\,dx \quad \exists\ t = \sqrt{1-x}$$

$$= \int \frac{1-\sqrt{-t^2+1}}{t}\,d(-t^2+1) = \int\left(-2+2\sqrt{-t^2+1}\right)dt =$$

$$= -2t + 2\int\sqrt{1-t^2}\,dt \ominus$$

$$\parallel$$

$$t\sqrt{1-t^2} - \int \frac{t^2}{\sqrt{1-t^2}}\,dt$$

$\exists\ t = \sin u$

$$\int \frac{\sin^2 u \cos u}{\cos u}\,du =$$

$$= \int \frac{1 - \cos 2u}{2} du = \frac{1}{2} u - \frac{\sin 2u}{4} + C$$

$$= \frac{1}{2} \arcsin t - \frac{\sin(2 \cdot \arcsin t)}{4} + C$$

$$\ominus -2t + 2t\sqrt{1-t^2} + \left(+ \frac{1}{2} \arcsin t + \right.$$

$$\left. + \frac{1}{2} t \sqrt{1-t^2} \right) \cdot 2 + C$$

$$t = \sqrt{1-x}$$

115. $\int \frac{dx}{x^8 + 8x^6 + 16x^4}$

116. $\int \frac{dx}{x(3+x^6)^2} = \frac{1}{6} \int \frac{dx^6}{x^6(3+x^6)^2} =$

$$\text{Г } t = x^6$$

$$= \frac{1}{6} \int \frac{dt}{t(t+3)^2} \ominus$$

$$\frac{1}{t(t+3)^2} = \frac{A}{t} + \frac{B}{t+3} + \frac{C}{(t+3)^2} =$$

$$1 = (t+3)^2 A + t(t+3)B + tC$$

$$t = -3: \quad 1 = -3C \Rightarrow C = -\frac{1}{3}$$

$$t = 0: \quad 1 = 9A \Rightarrow A = \frac{1}{9}$$

$$t^2: \quad \cancel{1 = 9A +} \; 0 = A + B \Rightarrow B = -\frac{1}{9}$$

$$\Leftrightarrow \frac{1}{6}\left(\frac{1}{9}\ln|t| - \frac{1}{9}\ln|t+3| + \frac{1}{3(t+3)}\right) + C$$