15 ноября.

Дифуры.

6.1) $y''' - 3y' + 2y = 0$

$\lambda^3 - 3\lambda + 2 = 0$

$(\lambda - 1)^2 (\lambda + 2) = 0$

$\lambda = 1 \quad \lambda = -2$

Ответ: $\underline{y = (C_1 + C_2 x) e^{x} + C_3 e^{-2x}}$

6.2) $y'' + 4y' + 4y = x e^{2x}$

$\lambda^2 + 4\lambda + 4 = 0$

$(\lambda + 2)^2 = 0 \quad \lambda = -2$

$y_0 = \overset{(C_1 x + C_2)}{C_1} e^{-2x}$

$y_1 = \underset{=1}{Q_m}(x) e^{2x} = \overset{(ax+b)}{\cancel{Q_m(x)}} e^{2x}$ не резонанс $= x e^{2x}$

$y_1' = e^{2x} + 2e^{2x} x$

$y_1'' = 2e^{2x} + 2 \cdot 2 e^{2x} x + 2e^{2x}$

$4 + 4x + 4(1 + 2x) + 4x = x$

$x = -\frac{1}{15}$

$y_1' = 2a e^{2x}$

$y_1'' = 4a e^{2x}$

$4a + 8a + 4a = x$

$y_1' = ae^{2x} + 2e^{2x}(ax+b)$

$y_1'' = 2ae^{2x} + 4e^{2x}(ax+b) + a\overset{2}{4}e^{2x}$

$4ax + 4a + 4b + 4a + 8b + 8ax + 4ax + 4b = x$

$\cancel{4a}\begin{cases} 16a = 1 \\ 16b + 8a = 0 \end{cases} \Rightarrow \begin{cases} a = \frac{1}{16} \\ b = -\frac{1}{32} \end{cases}$

$y_1 = \left(\frac{1}{16}x \cancel{+} - \frac{1}{32}\right)e^{2x}$

$y = (C_1x + C_2)e^{-2x} + \left(\frac{x}{16} - \frac{1}{32}\right)e^{2x}$ Ответ

6.3) $y^{(4)} + y'' = 7x - 3\cos x$

$\lambda^4 + \lambda^2 = 0 \qquad \lambda_{1,2} = 0 \qquad \lambda_{3,4} = \pm i$

$y_0 = C_1 + C_2x + C_3\cos x + C_4 \sin x$

резонанс $k=2$

$y_1 = Q_1(x)\cdot e^{0\cdot x}\cdot x^2 = (ax+b)x^2 = ax^3 + bx^2$

$y_1^{(4)} = 0$

$y_1' = 3ax^2 + 2bx$

$y_1'' = 6ax + 2b$

$6ax + 2b = 7x \Rightarrow a = \frac{7}{6}, \; b = 0$

$y_1 = \frac{7}{6}x^3$

$y_2 = x(a\cos x + b\sin x)$

$y_2'' = (-bx - 2a)\sin x + (2b - ax)\cos x$

$y_2^{(4)} = (bx + 4a)\sin x + (ax - 4b)\cos x$

$\begin{cases} 2a = 0 \\ -2b = -3 \end{cases} \Rightarrow \begin{matrix} a = 0 \\ b = \frac{3}{2} \end{matrix}$

$y_2 = \frac{3}{2} x \sin x$

Ответ: $y = C_1 + C_2 x + C_3 \cos x + C_4 \sin x + \frac{7}{6}x^3 +$

$+ \frac{3}{2} x \sin x$

6.4) $y'' + 3y' + 2y = \frac{1}{e^x + 1}$

$\lambda^2 + 3\lambda + 2 = 0 \quad \lambda_2 = -2 \quad \lambda_1 = -1$

$y_0 = C_1 e^{-x} + C_2 e^{-2x}$

$\begin{cases} C_1' y_1 + C_2' y_2 = 0 \\ C_1' y_1' + C_2' y_2' = \frac{1}{e^x + 1} \end{cases}$

$y_1 = e^{-x} \qquad y_1' = -e^{-x}$

$y_2 = e^{-2x} \qquad y_2' = -2e^{-2x}$

$$\begin{cases} C_1' e^{-x} + C_2' e^{-2x} = 0 \\ C_1'(-e^{-x}) + C_2'(-2e^{-2x}) = \frac{1}{e^{x}+1} \end{cases}$$

$$\Delta = -e^{-3x}$$

$$\Delta_1 = \begin{vmatrix} 0 & e^{-2x} \\ \frac{1}{e^{x}+1} & -2e^{-2x} \end{vmatrix} = -\frac{1}{e^{3x}+e^{2x}}$$

$$\Delta_2 = \begin{vmatrix} e^{-x} & 0 \\ -e^{-x} & \frac{1}{e^{x}+1} \end{vmatrix} = \frac{1}{e^{2x}+e^{x}}$$

$$C_1'(x) = \frac{\Delta_1}{\Delta} = +\frac{e^{3x}}{e^{3x}+e^{2x}} = +\frac{e^{2x}}{e^{x}+1}$$

$$C_2'(x) = \frac{\Delta_2}{\Delta} = -\frac{e^{2x}}{e^{x}+1}$$

$$C_1(x) = \ln(1+e^{x}) + C_3$$

$$C_2(x) = \ln(e^{x}+1) - e^{x} + C_4$$

$$y = \frac{\ln(1+e^{x}) + C_3}{e^{x}} + \frac{\ln(e^{x}+1) - e^{x} + C_4}{e^{2x}}$$

6.5) $x^2y'' - 4xy' + 6y = 0$

$\lambda(\lambda-1) - 4\lambda + 6 = 0$

$\lambda^2 - 5\lambda + 6 = 0$

$y'' - 5y' + 6y = 0$

$\lambda_1 = 3 \qquad \lambda_2 = 2$

$y_0 = C_1 e^{3t} + C_2 e^{2t} \qquad t = \ln x$

$y = y_0 = C_1 x^3 + C_2 x^2$

6.6) $x^2 y''' - 2y' = 0$

$y' = z$

$x^2 z'' = 2z$

$(\lambda-1)\lambda - 2 = 0$

$\lambda^2 - \lambda - 2 = 0$

$z^2 - z - 2 = 0$

$z = C_1 e^{2t} + C_2 e^{-t} = \cancel{t}\, C_1 x^2 + C_2 x^{-1}$

$y' = C_1 x^2 + C_2 x^{-1}$

$y = C_1 \frac{x^3}{3} + C_2 \ln(x) + C_3$

6.7) $x^3 y'' - 2xy = \ln x$

$x^2 y'' - 2y = \frac{\ln x}{x}$

$$\lambda(\lambda-1)-2=\frac{t}{e^t}$$

$$y''-y'-2=te^{-t}$$

$$y_0=C_1e^{2t}+C_2e^{-t}$$

$$y_1=t(at+b)e^{-t}$$

$$y_1'=(2at+b)e^{-t}+(-1)e^{-t}(at^2+bt)$$

$$y_1''=(at^2+(b-4a)t-2b+2a)e^{-t}$$

$$-6At-3B+2A=t$$

$$\begin{cases}-6A=1\\-3B+2A=0\end{cases}\Rightarrow\begin{cases}A=-\frac{1}{6}\\B=-\frac{1}{9}\end{cases}$$

$$y_1=t\left(-\frac{1}{6}t-\frac{1}{9}\right)e^{-t}$$

$$y=C_1x^2+C_2x^{-1}+\ln(x)\left(-\frac{1}{6}\ln(x)-\frac{1}{9}\right)x^{-1}$$

6.8) $y''+2y'+y=\cos(ix)=\frac{e^{-x}}{2}+\frac{e^{x}}{2}$

$\lambda^2+2\lambda+1=0 \qquad \lambda_{1,2}=-1$

$y_0=(C_1x+C_2)e^{-x}$

$y_1=x^2ae^{-x}$

$y_1'=2axe^{-x}-e^{-x}x^2a$

$y_1'' = ax^2e^{-x} - 4axe^{-x} + 2ae^{-x}$

$2a = \frac{1}{2} \Rightarrow a = \frac{1}{4}$

$y_1 = \frac{x^2e^{-x}}{4}$

$y_2 = ae^{x}$

$y_2' = ae^{x}$

$y_2'' = ae^{x}$

$4ae^{x} = \frac{e^{x}}{2} \Rightarrow a = \frac{1}{8}$

$y_2 = \frac{e^{x}}{8}$

$y = (C_1x + C_2)e^{-x} + \frac{x^2e^{-x}}{4} + \frac{e^{x}}{8}$

6.9) $x^2y'' - 2y = \frac{3x^2}{x+1}$

$\lambda(\lambda-1) - 2 = 0$

$y^2 - y - 2 = 0 \qquad \lambda_1 = 2 \qquad \lambda_2 = -1$

$y = C_1e^{2t} + C_2e^{-t} = C_1x^2 + C_2x^{-1}$

$f(t) = \frac{3e^{2t}}{e^{t}+1} \qquad a_0 = 1$

$y_1 = e^{2t} \qquad y_2 = e^{-t} \qquad y_1' = 2e^{2t} \quad y_2' = -e^{-t}$

$$\begin{cases} C_1' e^{2t} + C_2' e^{-t} = 0 \\ C_1' 2e^{2t} + C_2'(-1)e^{-t} = \dfrac{3e^{2t}}{e^t+1} \end{cases}$$

$$\Delta = -3e^t$$

$$\Delta_1 = \begin{vmatrix} 0 & e^{-t} \\ \dfrac{3e^{2t}}{e^t+1} & -e^{-t} \end{vmatrix} = -\frac{3e^t}{e^t+1}$$

$$\Delta_2 = \begin{vmatrix} e^{2t} & 0 \\ 2e^{2t} & \dfrac{3e^{2t}}{e^t+1} \end{vmatrix} = \frac{3e^{4t}}{e^t+1}$$

$$C_1' = \frac{\Delta 1}{\Delta} = \frac{1}{e^t+1}$$

$$C_2' = \frac{\Delta 2}{\Delta} = -\frac{3e^{3t}}{e^t+1}$$

$$C_1(x) = -\ln(e^t+1) + t + C_3$$

$$C_2(x) = -\ln(e^t+1) - \frac{e^{2t}}{2} + e^t + C_4$$

$$y = (-\ln(x+1) + \ln(x) + C_3)x^2 +$$

$$+ \cancel{C_2}(-\ln(x+1) - \frac{x^2}{2} + x + C_4)x^{-1}$$