9 февраля.

Домашнее задание № 1.

1. $\int 5^{2x-1}\,dx = \frac{1}{5}\int 25^{x}\,dx =$

$= \frac{25^{x}}{5\cdot \ln 25} + C$

2. $\int \frac{dx}{x^2-3x+1} \circleddash$

$\frac{1}{x^2-3x+1} = \frac{1}{\left(x-\frac{3-\sqrt{5}}{2}\right)\left(x \pm \frac{3+\sqrt{5}}{2}\right)} =$

$= \frac{A}{\left(x-\frac{3-\sqrt{5}}{2}\right)} + \frac{B}{\left(x-\frac{3+\sqrt{5}}{2}\right)}$

$1 = Ax - \frac{3+\sqrt{5}}{2}A + Bx - \frac{3-\sqrt{5}}{2}B$

$\begin{cases} A+B=0 \\ 2=(-\sqrt{5}-3)A+(\sqrt{5}-3)B \end{cases}$

$\begin{cases} A=-B \\ 2=(\sqrt{5}+3)B+(\sqrt{5}-3)B \end{cases}$

$B=\frac{1}{\sqrt{5}} \quad A=-\frac{1}{\sqrt{5}}$

$\circleddash \int\left(\frac{\frac{1}{\sqrt{5}}}{x-\frac{3+\sqrt{5}}{2}} - \frac{\frac{1}{\sqrt{5}}}{x-\frac{3-\sqrt{5}}{2}}\right)dx =$

$$= \frac{1}{\sqrt{5}} \ln\left|x - \frac{3+\sqrt{5}}{2}\right| - \frac{1}{\sqrt{5}} \ln\left|x - \frac{3-\sqrt{5}}{2}\right| + C$$

3. $$\int \frac{dx}{1-3x-x^2} = -\int \frac{dx}{x^2+3x-1} =$$

$$\frac{1}{x^2+3x-1} = \frac{A}{x - \frac{-3-\sqrt{13}}{2}} + \frac{B}{x - \frac{-3+\sqrt{13}}{2}} =$$

$=$

$$1 = Ax - A \cdot \frac{-3+\sqrt{13}}{2} + Bx - B \cdot \frac{-3-\sqrt{13}}{2}$$

$$\begin{cases} A = -B \\ -2 = A(-3+\sqrt{13}) + B(-3-\sqrt{13}) \end{cases}$$

$$B = \frac{1}{\sqrt{13}} \qquad A = -\frac{1}{\sqrt{13}}$$

$$= -\frac{1}{\sqrt{13}} \int \left(\cancel{\frac{1}{\sqrt{13}}} \frac{1}{x - \frac{-3+\sqrt{13}}{2}} + -\frac{1}{x - \frac{-3-\sqrt{13}}{2}} \right) \cdot$$

$$\cdot dx = -\frac{\ln\left|x - \frac{-3+\sqrt{13}}{2}\right| - \ln\left|x - \frac{-3-\sqrt{13}}{2}\right|}{\sqrt{13}} +$$

$+ C$

4. см10.

5. $$\int x^2 \sqrt[3]{x^3-1}\, dx = \frac{1}{3} \int \sqrt[3]{x^3-1}\, dx^3 =$$

$$= \frac{3}{4} \frac{1}{3} (x^3-1)^{\frac{4}{3}} = \frac{1}{4} (x^3-1)^{\frac{4}{3}} + C$$

6. $\int \cos x \, e^{\sin x} dx = \int e^{\sin x} d \sin x =$

$= e^{\sin x} + C$

7. $\int \frac{dx}{x \ln x} = \int \frac{d \ln x}{\ln x} = \ln \ln x + C$

8. $\int \frac{dx}{x(x+5)} \ominus$

I $\cancel{= \int \frac{d \ln(x)}{x+5} = \frac{\ln(x)}{x+5} + \int \ln(x)}$

$\frac{A}{x} + \frac{B}{x+5} = 1 \Rightarrow A = \frac{1}{5} \; ; \; B = -\frac{1}{5}$

$\ominus \frac{1}{5} \int \left(\frac{1}{x} - \frac{1}{x+5}\right) dx = \frac{\ln|x| - \ln|x+5|}{5} + C$

II $\ominus \int \frac{dx}{(x+2,5)^2 - 2,5^2} = \int \frac{d(x+2,5)}{(x+2,5)^2 - 2,5^2} =$

$= \cancel{\int 1} \; \cancel{\frac{1}{5} \ln \left| \frac{x-2,5}{x+2,5} \right|} + C$

$= \frac{1}{5} \ln \left| \frac{x}{x+5} \right| + C.$

9. $\int \arcsin 2x\,dx = x\arcsin 2x -$

$- \int x\,d\arcsin 2x = \arcsin 2x -$

$- \int \frac{x \cdot 2\,dx}{\sqrt{1-4x^2}} = x\arcsin 2x -$

$- \frac{\cancel{1}}{\cancel{2}}\int \frac{dx^2}{\sqrt{1-4x^2}} = x\arcsin 2x + \frac{\cancel{1}}{\cancel{2}} \cdot 2 \cdot \frac{1}{2}$

$\cdot \sqrt{1-4x^2} = \cancel{x\arcsin 2x + \sqrt{1-x^2}}$

$= x\arcsin 2x + \frac{1}{2}\sqrt{1-4x^2}$

10. $\int \frac{x\,dx}{x^2-3x+1} =$

$\frac{x}{x^2-3x+1} = \frac{A}{x - \frac{3-\sqrt{5}}{2}} + \frac{B}{x - \frac{3+\sqrt{5}}{2}}$

$Ax - \frac{3+\sqrt{5}}{2}A + Bx - \frac{3-\sqrt{5}}{2}B = x$

$$\begin{cases} A + B = 1 \\ \frac{3+\sqrt{5}}{2}A + \frac{3-\sqrt{5}}{2}B = 0 \end{cases}$$

$$\begin{cases} A = 1 - B \\ (3+\sqrt{5})(1-B) = (-3+\sqrt{5})B \end{cases}$$

$B = \frac{3+\sqrt{5}}{\cancel{6}2\sqrt{5}}$ $A = \frac{3-\sqrt{5}}{\cancel{6}2\sqrt{5}}$

$$= \int \left(\frac{\frac{3-\sqrt{5}}{6}}{x - \frac{3-\sqrt{5}}{2}} + \frac{\frac{3+\sqrt{5}}{6}}{x - \frac{3+\sqrt{5}}{2}} \right) dx =$$

$$= \frac{3-\sqrt{5}}{\cancel{6}2\sqrt{5}} \ln\left| x - \frac{3-\sqrt{5}}{2} \right| + \frac{3+\sqrt{5}}{\cancel{6}2\sqrt{5}} \cdot$$

$$\cdot \ln\left| x - \frac{3+\sqrt{5}}{2} \right| + C$$

11. $$\int (x^2 - 6x + 3) \cos 3x \, dx =$$

$$= \frac{1}{3} \int (x^2 - 6x + 3) \, d \sin 3x =$$

$$= \frac{1}{3} (x^2 - 6x + 3) \cdot \sin 3x -$$

$$- \frac{1}{3} \int \sin 3x \, d(x^2 - 6x + 3) =$$

$$= \frac{1}{3} (x^2 - 6x + 3) \cdot \sin 3x -$$

$$- \frac{1}{3} \int (2x - 6) \sin 3x \, dx =$$

$$= \frac{1}{3} (x^2 - 6x + 3) \sin 3x + \frac{1}{9} \int (2x - 6) \, d \cos 3x =$$

$$= \frac{1}{3} (x^2 - 6x + 3) \sin 3x + \frac{1}{9} \cancel{\int} (2x - 6) \cdot \cos 3x -$$

$$- \frac{1}{9} \int \cos 3x \, d(2x - 6) =$$

$= \frac{1}{3}(x^2-6x+3)\sin 3x + \frac{1}{9}(2x\mp 6)\cos 3x -$

$- \frac{2}{9}\cdot\frac{1}{3}\sin 3x$

12. $\int x^3 e^{-x^2} dx = \int \frac{x^3}{-2x} de^{-x^2} =$

$= -\frac{1}{2}\int x^2 de^{-x^2} = -\frac{1}{2}(x^2 \cdot e^{-x^2} -$

$- \int e^{-x^2} dx^2) = -\frac{1}{2} x^2 \cdot e^{-x^2} + e^{-x^2}.$

13. $\int \sqrt{1+x^2}\, dx = x\sqrt{1+x^2} -$

$- \int x\, d\sqrt{1+x^2} = x\sqrt{1+x^2} - \int \frac{x^2}{\sqrt{1+x^2}} dx =$

$\cancel{= x\sqrt{1+x^2} - \frac{1}{2}\int \frac{dx^2}{\sqrt{1+x^2}} =}$

$\cancel{= x\sqrt{1+x^2} - \sqrt{1+x^2}}$

$= x\sqrt{1+x^2} - \int \left(\sqrt{x^2+1} - \frac{1}{\sqrt{1+x^2}}\right) dx =$

$= x\sqrt{1+x^2} + \ln|x+\sqrt{x^2+1}| -$

$- \int \sqrt{x^2+1}\, dx + C$

$\int \sqrt{x^2+1}\, dx = \frac{x\sqrt{1+x^2} + \ln|x+\sqrt{x^2+1}|}{2} + C$