Мат ан.

Дз № 8.

1. $f(x) = x \ln x, \quad x_0 = 1$

$t = x - 1$

$$f(t) = (t+1)\ln(t+1) = (t+1)\sum_{k=1}^{\infty}(-1)^{k-1}\frac{x^k}{k} =$$

$$= \sum_{k=1}^{\infty}(-1)^{k-1}\frac{t^{k+1}}{k} + \sum_{k=1}^{\infty}(-1)^{k-1}\frac{t^k}{k} =$$

$$= \sum_{k=1}^{\infty}(-1)^{k+1}\frac{(x-1)^{k+1}}{k} + \sum_{k=1}^{\infty}(-1)^{k-1}\frac{(x-1)^k}{k}$$

$t \in (-1, 1) \Rightarrow x \in (0; 2)$

2. $f(x) = \dfrac{x}{(9-x^2)(1+x^2)}, \quad x_0 = 0$

$$f(x) = \frac{x}{10}\left(\frac{1}{9-x^2} + \frac{1}{1+x^2}\right) =$$

$$= \frac{x}{10}\left(\frac{1}{9}\cdot\frac{1}{1-\left(\frac{x}{3}\right)^2_{|x|<3}} + \frac{1}{1+x^2}_{|x|<1}\right) =$$

$$= \frac{x}{10}\left(\frac{1}{9}\sum_{n=0}^{\infty}\frac{x^{2n}}{3^{2n}} + \sum_{n=0}^{\infty}(-1)^n x^{2n}\right) =$$

$$= \cancel{\frac{1}{9}}\ \frac{1}{10}\sum_{n=0}^{\infty}x^{2n+1}\left(\frac{1}{3^{2n+2}} + (-1)^n\right)$$

$x \in (-1; 1)$

3. $f(x) = \sin x \quad \sin 1^\circ ? \quad \delta = 0{,}001$

$|R_n| < \delta$

$$\sin x = \sum_{k=1}^{\infty} (-1)^{k+1} \frac{x^{2k-1}}{(2k-1)!}$$

$|R_n| \leqslant a_{n+1}$, т.к. ряд Лейбница

$$a_{n+1} = \cancel{(-1)} \frac{x^{2n+1}}{(2n+1)!} = \frac{\pi^{2n+1}}{180^{2n+1}(2n+1)!} < \delta = 0{,}001$$

$\sin 1^\circ =$

$$= \sum_{k=1}^{\infty} (-1)^{k+1} \frac{\pi^{2k-1}}{180^{2k-1}(2k-1)!} =$$

$$= \frac{\pi}{180} - \cancel{\frac{\pi^3}{180^3 \cdot 6}} \approx 0{,}017.$$

$< \delta$

4. a)

$$\operatorname{arctg} x = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k+1}}{2k+1}$$

$|R_n| \leqslant a_{n+1}$

$$a_{n+1} = \cancel{\frac{x^{2k+3}}{2k+3}} \ \frac{x^{2n+3}}{2n+3} < \delta = 0{,}001$$

~~$\operatorname{arctg} 1 = 1 =$~~

$$\frac{1}{2n+3} < 10^{-3} \Leftrightarrow 10^3 < 2n+3 \Rightarrow$$

$$\Rightarrow n > \frac{\cancel{3000}\ 2997}{2} = \cancel{1500}\ 1498\tfrac{1}{2}$$

ерпи-

$\sin 1^\circ - 1 \quad 1 \quad 1$

$\cancel{x}\ \pi = 4 \cdot \operatorname{arctg} 1 = 4 \cdot \sum_{k=0}^{1499} (-1)^k \frac{1}{2k+1} \approx$

$\approx 3{,}141$

$\underline{n = 1499}$

б) $\arcsin x = \cancel{\Sigma}\, x + \sum_{n=1}^{\infty} \frac{(2n-1)!!\, x^{2n+1}}{(2n)!!\,(2n+1)}$

5. $$\sum_{n=0}^{\infty} \frac{n^2+1}{2^n n!} x^n = \sum_{n=0}^{\infty} \frac{n^2 x^n}{2^n n!} + \sum_{n=0}^{\infty} \frac{x^n}{2^n n!}$$

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$$\sum_{n=0}^{\infty} \frac{n^2 x^n}{2^n n!} = \sum_{n=1}^{\infty} \frac{n^2 x^n}{2^n n!} = \sum_{n=1}^{\infty} \frac{(n+1-1) x^n}{2^n (n-1)!} =$$

$$= \sum_{n=1}^{\infty} \frac{(n+1) x^n}{2^n (n-1)!} - \sum_{n=1}^{\infty} \frac{x^n}{2^n (n-1)!}$$

$$\sum_{n=1}^{\infty} \frac{(n+1) x^n}{2^n (n-1)!} = \sum_{n=1}^{\infty} \left(\frac{x^{n+1}}{2^n (n-1)!}\right)' =$$

$$= \left(\sum_{n=1}^{\infty} \frac{x^{n+1}}{2^n (n-1)!}\right)' = \left(\frac{x^2}{2} \sum_{n=1}^{\infty} \frac{x^{n-1}}{2^{n-1} (n-1)!}\right)' =$$

$$= \left(\frac{x^2}{2} \sum_{n=0}^{\infty} \left(\frac{x}{2}\right)^n \frac{1}{n!}\right)' = \left(\frac{x^2}{2} e^{\frac{x}{2}}\right)' =$$

$$= x e^{\frac{x}{2}} + \frac{e^{\frac{x}{2}} x^2}{4}$$

$$\sum_{n=1}^{\infty} \frac{x^n}{2^n (n-1)!} = \frac{x}{2} \sum_{n=0}^{\infty} \left(\frac{x}{2}\right)^n \cdot \frac{1}{n!} = \frac{x \cdot e^{\frac{x}{2}}}{2}$$

$$= x e^{\frac{x}{2}} + \frac{e^{\frac{x}{2}} x^2}{4} - \frac{x \cdot e^{\frac{x}{2}}}{2} + e^{\frac{x}{2}}$$

$x \in R$