21 апреля.

ИДЗ №2.

1. $J(\alpha) = \int\limits_0^{\infty} \frac{x\,dx}{10-4x+x^{\alpha}}, \quad Y = (2, +\infty)$

$f(x,\alpha) = \frac{x}{10-4x+x^{\alpha}}$

$\cancel{|f(x,\alpha)| \le \frac{x}{10-4x+x}}$

Заметим, что 0 – не особая точка, т.к. $10-4x+x^{\alpha} \ge 6$.

$\forall \varepsilon > 0$ рассмотрим сх-ть $J(\alpha)$ на $Y_1 = [2+\varepsilon, +\infty)$

$|f(x,\alpha)| \le g(x)$

$$g(x) = \begin{cases} 1, & x \in [0;1] \\ \frac{x}{10-4x+x^{2+\varepsilon}}, & x \in (1;+\infty) \end{cases}$$

$\int\limits_0^{\infty} g(x)\,dx$ сх-ся. $\Rightarrow$ $J(\alpha)$ сх-ся равн. $\forall \alpha \in [2+\varepsilon, +\infty)$ $\forall \varepsilon$ $\Rightarrow$

$\Rightarrow$ $J(\alpha)$ сх-ся равн. на $(2, +\infty)$

При этом $f(x, \alpha)$ непр. $\Rightarrow$

$\Rightarrow \mathcal{I}(\alpha)$ непр.

2.

$$I = \int_0^{+\infty} \frac{\ln(x)}{\sqrt[3]{x}\,(2+x)}\,dx = \begin{bmatrix} x = 2t \\ dx = 2dt \end{bmatrix} = \int_0^{+\infty} \frac{\ln 2t \; 2dt}{\sqrt[3]{2t} \cdot 2(1+t)} =$$

$$= \frac{1}{\sqrt[3]{2}} \int_0^{+\infty} \frac{\ln 2}{\sqrt[3]{t}(1+t)} + \frac{\ln t}{\sqrt[3]{t}(1+t)}\,dt =$$

$$= \frac{\ln 2}{\sqrt[3]{2}} \underbrace{B\left(\frac{2}{3}, \frac{1}{3}\right)}_{= \frac{\pi}{\sin\frac{2\pi}{3}} = \frac{2\pi}{\sqrt{3}}} + \frac{1}{\sqrt[3]{2}} \underbrace{\int_0^{+\infty} \frac{\ln t}{\sqrt[3]{t}(1+t)}\,dt}_{= I_1}$$

$$I_1 = I_1\left(\frac{1}{3}\right)$$

$$I_1(y) = \int_0^{\infty} \frac{t^y \ln t}{1+t}\,dt = \int_0^{\infty} \frac{\partial}{\partial y} \frac{t^y}{1+t}\,dt =$$

равн непр.

$$= \frac{\partial}{\partial y} \cancel{\int_0^{\infty}} B(y+1, -y) = \frac{\partial}{\partial y} \frac{\pi}{\sin \pi(y+1)} =$$

$$= -\frac{\pi^2 \cos \pi(y+1)}{\sin^2 \pi(y+1)}$$

$$I_1 = \frac{-\pi^2 \cos\frac{4\pi}{3}}{\sin^2\frac{4\pi}{3}} = \frac{\frac{\pi^2}{2}}{\frac{3}{4}} = \frac{2\pi^2}{3}$$

$$I = \frac{\pi^2 \ln 2 \cdot 2^{\frac{2}{3}}}{\sqrt{3}} + \frac{2^{\frac{2}{3}}\pi^2}{9}$$

5. $$\int_0^{+\infty} \frac{\sin\frac{a}{x}}{x(1+x^2)}\,dx = \left[\begin{matrix} t = \frac{1}{x} & dx = -\frac{1}{t^2}\,dt \\ x = \frac{1}{t} & \end{matrix}\right] =$$

$$= +\int_0^{+\infty} \frac{\sin at}{\frac{1}{t}\left(1+\frac{1}{t^2}\right)t^2}\,dt = +\int_0^{+\infty} \frac{t \sin at}{1+t^2}\,dt =$$

$$= \frac{\pi}{2} e^{-|a|} \operatorname{sign} a$$

4. $$\int_0^{+\infty} e^{-b\sqrt{x}} \sin\sqrt{x}\,dx,\ b>0 = \left[\begin{matrix} \sqrt{x} = t \\ x = t^2 \\ dx = 2t\,dt \end{matrix}\right] =$$

огр. первообр.

$$= \int_0^{+\infty} e^{-bt}\, 2t \sin t\,dt = \cancel{\frac{2}{1+b^2}}$$

$$= \frac{\partial}{\partial b} \int_0^{+\infty} -e^{-bt}\, 2\sin t\,dt = \frac{\partial}{\partial b}\left(-\frac{2}{1+b^2}\right) =$$

$$= \frac{4b}{(1+b^2)^2}$$

3. $I'_\alpha = \underbrace{\int\limits_0^\infty \frac{\sin\alpha x \sin\beta x \sin\gamma x}{x^{2}}\,dx}_{\text{сх. равн. на } [\sigma, +\infty)\ \sigma > 0}$ $\cancel{\circledast}$

по Дирихле:

$\sin\alpha x \sin\beta x \sin\gamma x$ первообр. огр.

$\frac{1}{x^2} \rightrightarrows 0$

Заметим, что $I(\alpha, \beta, \gamma)$ непр. по $\alpha$ в (·) 0:

$|I| \leqslant \frac{1}{x^3}$ – сумм на $[A, +\infty)$ $(\angle\, \alpha)$

$I''_{\alpha\alpha} = \underbrace{\int\limits_0^\infty \frac{\sin\alpha x \sin\beta x \sin\gamma x}{x}\,dx}_{\text{сх. равн на } [\sigma, +\infty),\ \sigma > 0}$ $\circleddash$

аналогично

$I'_\alpha(\alpha, \beta, \gamma)$ непр. по $\alpha$ в (·) 0 аналогично.

$$3.\ I(\alpha,\beta,\gamma) = \int_0^{+\infty} \frac{\sin\alpha x \sin\beta x \sin\gamma x}{x^3}\,dx =$$

$I''_{\alpha\alpha}$

$$= \frac{1}{2}\int_0^{+\infty} \frac{(\cos((\alpha-\beta)x) - \cos((\alpha+\beta)x))\sin\gamma x}{x^{\cancel{3}}}\,dx =$$

$$= \frac{1}{2}\int_0^{+\infty} \frac{\sin\gamma x\cos((\alpha-\beta)x) - \sin\gamma x\cos((\alpha+\beta)x)}{x^{\cancel{3}}}\,dx$$

$$= \frac{1}{2}\int_0^{+\infty} \frac{\frac{1}{2}(\sin(\gamma+\alpha-\beta)x + \sin(\gamma-\alpha+\beta)x) - \frac{1}{2}(\sin(\gamma+\alpha+\beta)x + \sin(\gamma-\alpha-\beta)x)}{x^{\cancel{3}}}\,dx =$$

$$= \frac{1}{4}\int_0^{+\infty} \frac{\sin(\gamma+\alpha-\beta)x}{x^{\cancel{3}}}\,dx + \frac{1}{4}\int_0^{+\infty} \frac{\sin(\gamma-\alpha+\beta)x}{x^{\cancel{3}}}\,dx +$$

$$+ \frac{1}{4}\int_0^{+\infty} \frac{\sin(\gamma+\alpha+\beta)x}{x^{\cancel{3}}}\,dx - \frac{1}{4}\int_0^{+\infty} \frac{\sin(\gamma-\alpha-\beta)x}{x^{\cancel{3}}}\,dx \ (*)$$

$$\text{Д} \int_0^{+\infty} \frac{\sin a x}{x^3}\,dx = I(a)$$

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$$I'(a) = \int_0^{+\infty} \frac{\cos a x}{x^2}\,dx$$

сх-ся равн. на

$x \in [\sigma, +\infty)$, $\sigma > 0$ по Дирихле :

$$I''_{\alpha\alpha} = \frac{1}{4}\left(\frac{\pi}{2}\operatorname{sign}(\gamma+\alpha-\beta) + \frac{\pi}{2}\operatorname{sign}(\gamma-\alpha+\beta) + \right.$$

$$\left. + \frac{\pi}{2}\operatorname{sign}(\gamma+\alpha+\beta) + \frac{\pi}{2}\operatorname{sign}(\gamma-\alpha-\beta)\right)$$

$$I'_{\alpha} = \frac{\pi}{8}\left(|\gamma+\alpha-\beta| + |\gamma-\alpha+\beta| + |\gamma+\alpha+\beta| + \right.$$

$$\left. + |\gamma-\alpha-\beta|\right) + C_1$$

$$\cancel{I'_{\alpha}(0)=0} \quad I'_{\alpha}(0,0,0) = 0 \Rightarrow C_1 = 0$$

$$I = \frac{\pi}{8}\left(\frac{(\gamma+\alpha-\beta)^2}{2}\operatorname{sign}(\gamma+\alpha-\beta) + \frac{(\gamma-\alpha+\beta)^2}{2} \cdot \right.$$

$$\cdot \operatorname{sign}\cancel{\alpha}(\gamma-\alpha+\beta) + \frac{(\gamma+\alpha+\beta)^2}{2}\operatorname{sign}(\alpha+\beta+\gamma) +$$

$$\left. + \frac{(\gamma-\alpha-\beta)^2}{2}\operatorname{sign}(\gamma-\alpha-\beta)\right) + C_2$$

$$I(0,0,0) = 0 \Rightarrow C_2 = 0$$