3 апреля.

Мат ан.

Кр № 1.

1) $$\int_3^5 \frac{x^5-4x^4+8x^3-12x^2+16x-13}{x^4-5x^3+11x^2-13x+6}\,dx =$$

$$
\begin{array}{r|l}
x^5-4x^4+8x^3-12x^2+16x-13 & x^4-5x^3+11x^2-13x+6 \\
\cline{2-2}
-\;\underline{x^5-5x^4+11x^3-13x^2+6x} & x+1 \\
-\;x^4-3x^3+x^2+10x-13 & \\
\underline{x^4-5x^3+11x^2-13x+6} & \\
2x^3-10x^2+23x-19 &
\end{array}
$$

$$= \int_3^5 \left(x+1+\frac{2x^3-10x^2+23x-19}{x^4-5x^3+11x^2-13x+6}\right)dx =$$

$$= \frac{1}{2}x^2\Big|_3^5 + x\Big|_3^5 + \int_3^5 \frac{2x^3-10x^2+23x-19}{x^4-5x^3+11x^2-13x+6}\,dx$$ (1)

$\cancel{(x-1)}$

$$x^4-x^3-4x^3+4x^2+7x^2-7x-6x+6 =$$

$$=(x-1)(x^3-4x^2+7x-6) =$$

$$=(x-1)(x^3-2x^2-2x^2+4x+3x-6) =$$

$$=(x-1)(x-2)(x^2-2x+3)$$

$$\frac{2x^3-10x^2+23x-19}{x^4-5x^3+11x^2-13x+6} = \frac{Ax+B}{x^2-2x+3} + \frac{C}{x-1} +$$

$+\frac{D}{x-2}$

$x=2:\ \frac{2\cdot 8-10\cdot 4+23\cdot 2-19}{(2-1)(4-4+3)}=\frac{3}{3}=D=1$

$x=1:\ \frac{2-10+23-19}{(1-2)(1-2+3)}=\frac{-4}{-2}=2=C$

$x^3:\ 2=A+C+D \Rightarrow A=-1$

$x^0:\ -19=2B-6C-3D \Rightarrow B=-2$

$$\int\left(\frac{-x-2}{x^2-2x+3}+\frac{2}{x-1}+\frac{1}{x-2}\right)dx$$

$\frac{2}{x-1}$: $= 2\ln|x-1|$; $\frac{1}{x-2}$: $= \ln|x-2|$

$$-\int\frac{x-1+3}{(x-1)^2+2}dx$$

$$=$$

$$-\frac{1}{2}\int\frac{d(x-1)^2}{(x-1)^2+2}-3\int\frac{1}{(x-1)^2+2}dx=$$

$$=-\frac{1}{2}\ln|(x-1)^2+2|-\frac{3}{\sqrt{2}}\operatorname{arctg}\frac{x-1}{\sqrt{2}}$$

$\cancel{= 8+2-\frac{1}{2}\ln 18-3\sqrt{2}\,\mathrm{arct}}$

$\cancel{= -3\sqrt{2}\,\mathrm{arctg}}$

$$= 8+2-\frac{1}{2}\ln 18-\frac{3}{\sqrt{2}}\operatorname{arctg}2\sqrt{2}+2\ln 4+$$

$$+\ln 3+\frac{1}{2}\ln 6+\frac{3}{\sqrt{2}}\operatorname{arctg}\sqrt{2}-2\ln 2-0$$

② $f(x) = 6x^3 - 60x^2 + 230x - 150$

$g(x) = y = 44x + 30$

$6x^3 - 60x^2 + 230x - 150 = 44x + 30$

$6x^3 - 60x^2 + 230x - 44x - 180 = 0$

$3x^3 - 20x^2 + \cancel{115x - 22x}^{93x} - 90 = 0$

$(x-5)(x-3)(x-2) = 0$

$$\int_2^3 (f(x) - g(x))\, dx =$$

$$= \int (6x^3 - 60x^2 + \cancel{230x - 44x}^{\cancel{93x}\ 186x} - 180)\, dx =$$

$$= \frac{3x^4}{2} - 20x^3 + 93x^2 - 180x + C$$

$$\int_2^3 (f(x) - g(x))\, dx = \frac{3 \cdot 81}{2} - 20 \cdot 27 +$$

$$+ 93 \cdot 9 - 180 \cdot 3 - \frac{3 \cdot 16}{2} + 20 \cdot 8 - 93 \cdot 4 +$$

$$+ 180 \cdot 2 = \frac{5}{2}$$

$$\int_3^5 (f(x) - g(x))\, dx = -16 < 0 \Rightarrow$$

$$\Rightarrow \int_5^3 = 16$$

Ответ: $16 + \frac{5}{2} = \frac{37}{2}$.

③ $x(t) = -6t\cos(6t) + \sin 6t$

$y(t) = 6t\sin(6t) + \cos(6t)$

$x'_t = -6\cos 6t + 6t \cdot 6\sin 6t \cancel{+\sin 6t} = \cancel{+6\cos 6t} =$

$= 36t\sin 6t$

$y'_t = 6\sin 6t + 36t\cos 6t - 6\sin 6t =$

$= 36t\cos 6t$

$L = \int_{-9}^{-7} \cancel{t}\ 36|t|\, dt = \int_{7}^{9} 36t\,dt = 18t^2 \Big|_{7}^{9} = -18\cdot 49 + 18\cdot 81 =$

$= 576$

Ответ: 576.

④ $\int_0^{\pi} (7x^2 + 9x + 4)\cos\frac{x}{2}\,dx$

$\int_0^{\pi}(7x^2+9x+4)\,2\,d\sin\frac{x}{2} = 2\sin\frac{x}{2}(7x^2+9x+4)\Big|_0^{\pi} -$

$-2\int_0^{\pi}\sin\left(\frac{x}{2}\right)(14x+9)\,dx =$

$= 14\pi^2 + 18\pi + 8 + \cancel{2}^{4}\int_0^{\pi}(14x+9)\,d\cos\frac{x}{2} =$

$$= 14\pi^2 + 18\pi + 8 + 4(14x+9)\cos\frac{x}{2}\Big|_0^{\pi} -$$

$$- 4\int_0^{\pi} 14\cos\frac{x}{2}\,dx = 14\pi^2 + 18\pi - 28 -$$

$$- 56\, 2\sin\frac{x}{2}\Big|_0^{\pi} = 14\pi^2 + 18\pi - 140$$

<u>Ответ:</u> $14\pi^2 + 18\pi - 140$

(5) $$\int_{13}^{\infty} \left|\ln\frac{18x^2+9}{14x^3+1}\right|^{-\alpha} \frac{1}{x}\,dx$$

~~$\alpha \to 0$:~~

$$\left|\ln\frac{18x^2+9}{14x^3+1}\right|^{-\alpha} \sim \left|\ln\frac{9}{7x}\right|^{-\alpha}$$

$$\int_{13}^{\infty} \left|\ln\frac{9}{7x}\right|^{-\alpha} \frac{1}{x}\,dx \ominus$$

$$\exists\, t = \frac{1}{x} \quad dx = -\frac{1}{t^2}\,dt$$

$$\ominus -\int_{\frac{1}{13}}^{0} \frac{\left|\ln\frac{9}{7}t\right|^{-\alpha}}{t^{\cancel{2}}}\,dt = \int_0^{\frac{1}{13}} \frac{1}{\left|\ln\frac{9}{7}t\right|^{\alpha} t}\,dt =$$

$$= -\int_0^{\frac{1}{13}} \frac{1}{\cancel{(}\left(\ln\frac{9}{7}t\right)^{\alpha}}\, d\,\ln\frac{9}{7}t \overset{\alpha \neq 1}{=}$$

$$= -\,\cancel{(-\alpha+1)}\,\frac{\left(\ln\frac{9}{7}t\right)^{-\alpha+1}}{-\alpha+1}\Bigg|_0^{\frac{1}{13}} =$$

$$= \frac{\left(\ln \frac{9}{7} t\right)^{-\alpha+1}}{\alpha-1} \Bigg|_{0}^{\frac{1}{13}}$$

$-\alpha+1>0 \Leftrightarrow \alpha<1 \Rightarrow$ сх.

$-\alpha+1<0 \Leftrightarrow \alpha>1 \Rightarrow$ расх.

$\alpha=1: \ -\ln \ln\left(\frac{9}{7} t\right) \Big|_{0}^{\frac{1}{13}} \Rightarrow$ расх

<u>Ответ:</u> $(-\infty; 1)$