25 сентября.

Матан.

Дз №3.

1. $f(x,y) = \operatorname{arctg}\frac{x}{y}$

$$\frac{\partial f}{\partial x} = \frac{1}{y}\cdot\frac{1}{1+\frac{x^2}{y^2}} = \frac{1}{y+\frac{x^2}{y}} = \frac{y}{y^2+x^2}$$

$$\frac{\partial^2 f}{\partial x^2} = -\frac{\frac{2}{y}x}{\left(y+\frac{x^2}{y}\right)^2} = -\frac{2xy}{(x^2+y^2)^2}$$

$$\frac{\partial^2 f}{\partial y} = \frac{1}{1+\frac{x^2}{y^2}}\cdot(-x)\cdot\frac{1}{y^2} = -\frac{x}{y^2+x^2}$$

$$\frac{\partial^2 f}{\partial y^2} = \frac{2xy}{(x^2+y^2)^2}$$

$$\frac{\partial^2 f}{\partial y\,\partial x} = \frac{y^2+x^2-2y^2}{(x^2+y^2)^2} = \frac{x^2-y^2}{(x^2+y^2)^2}$$

$$\frac{\partial^2 f}{\partial x\,\partial y} = \frac{-x^2+-y^2+2x^2}{(x^2+y^2)^2} = \frac{x^2-y^2}{(x^2+y^2)^2}$$

$$df = \frac{y}{y^2+x^2}dx + \left(-\frac{x}{y^2+x^2}\right)dy$$

$$d^2f = -\frac{2xy}{(x^2+y^2)^2}dx^2 + 2\frac{x^2-y^2}{(x^2+y^2)^2}dxdy + \frac{2xy}{(x^2+y^2)^2}dy^2$$

2. $f = u\cos(u\cdot v)$, $u = \sqrt{x^2+y^2}$

$v = \frac{y}{x}$

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial u}\cdot\frac{\partial u}{\partial x} + \frac{\partial f}{\partial v}\cdot\frac{\partial v}{\partial x} =$$

$$= (\cos(u\cdot v) + u\cdot(-\sin(u\cdot v))\cdot v)\,\frac{x}{\sqrt{x^2+y^2}} +$$

$$+ u\cdot(-\sin(uv))\cdot u\cdot\frac{(-y)}{x^2} = \cos\sqrt{y^2+\frac{y^4}{x^2}}\cdot\frac{x}{\sqrt{x^2+y^2}} -$$

$$- y\sin\sqrt{y^2+\frac{y^4}{x^2}} + \frac{(x^2+y^2)\,y}{x^2}\sin\sqrt{y^2+\frac{y^4}{x^2}}$$

$$f = \sqrt{x^2+y^2}\cos\sqrt{y^2+\frac{y^4}{x^2}}$$

$$f'_x = \frac{x}{\sqrt{x^2+y^2}}\cos\sqrt{y^2+\frac{y^4}{x^2}} + \sqrt{x^2+y^2}\cdot\left(-\sin\sqrt{y^2+\frac{y^4}{x^2}}\right)\cdot$$

$$\cdot\frac{1}{2\sqrt{y^2+\frac{y^4}{x^2}}}\;\frac{(-2)\cdot y^4}{x^3}$$

$$f'_y = f'_u\cdot u'_y + f'_v\cdot v'_y =$$

$$= (\cos(u\cdot v) + u\cdot(-\sin(u\cdot v))\,v)\,\frac{y}{\sqrt{x^2+y^2}} +$$

$$+ u\cdot(-\sin(uv))\cdot u\cdot\frac{1}{x} =$$

$$f'_y = \frac{y}{\sqrt{x^2+y^2}}\cos\sqrt{y^2+\frac{y^4}{x^2}} + \sqrt{x^2+y^2}\left(-\sin\sqrt{y^2+\frac{y^4}{x^2}}\right)\cdot$$

$$\cdot\frac{1}{2\sqrt{y^2+\frac{y^4}{x^2}}}\left(2y + \frac{4}{x^2}y^3\right)$$

$= \cos\sqrt{y^2+\cancel{x^2}\frac{y^4}{x^2}} \cdot \frac{y}{x^2+y^2} -$

$- \sin\sqrt{y^2+\frac{y^4}{x^2}}\ \frac{y^2}{x^2} - \sin\sqrt{y^2+\frac{y^4}{x^2}}\ \frac{x^2+y^2}{x}$

3. $f(x,y) = (5x+7y-25)\cdot e^{-(x^2+xy+y^2)}$

$f'_x = 5\cdot e^{-(x^2+xy+y^2)} + e^{-(x^2+xy+y^2)}\cdot$

$\cdot(-2x-y)(5x+7y-25) = 0 \Leftrightarrow$

$f'_y = 7\cdot e^{-(x^2+xy+y^2)} + e^{-(x^2+xy+y^2)}\cdot$

$\cdot(-2y-x)(5x+7y-25) = 0$

$7+(-2y-x)(5x+7y-25)=0$

$7+(-10xy)-14y^2+50y-5x^2-7xy+$

$+25x=0$

$-14y^2-5x^2+50y+25x-17xy+7=0$

$\Leftrightarrow 5+(-10x^2)-14xy+50x-5xy-$

$-7y^2+25y = -7y^2-10x^2+25y+50x-$

$-19xy+5=0$

$$\begin{cases}14y^2+5x^2-50y-25x+17xy-7=0\\ 7y^2+10x^2-25y-50x+19xy-5=0\end{cases}$$

4. $u(x,y,z) = f\left(\frac{x}{y}, x^2+y-z^2\right) = f(u,v)$

$$2xz\frac{\partial u}{\partial x} + 2yz\frac{\partial u}{\partial y} + (2x^2+y)\frac{\partial u}{\partial z} = 0$$

$u'_x = f'_x = f'_u \frac{1}{y} + f'_v \cdot 2x$

$u'_y = f'_y = f'_u \frac{-x}{y^2} + f'_v$

$u'_z = f'_z = f'_u \cdot 0 + f'_v \cdot (-2z)$

$\frac{2xz}{y} f'_u + 4x^2 z f'_v - 2\frac{zx}{y} f'_u + 2yz f'_v +$

$+ (2x^2+y)(-2z) f'_v = 0 \iff 0 = 0$