3 октября.

Дифуры.

3.1 $(\sin y + y\sin x + \frac{1}{x})\,dx +$

$+ (x\cos y - \cos x - \frac{1}{y})\,dy = 0$ – УПД

Найдём $u(x,y)$:

$u(x,y) = \int(\sin y + y\sin x + \frac{1}{x})\,dx =$

$= x\sin y - y\cos x + \ln|x| + C_y$

$C_y = \int (x\cos y - \cos x - \frac{1}{y}) - (x\sin y -$

$- y\cos x + \ln|x|)'_y\,dy = \int -\frac{1}{y}\,dy =$

$= -\ln|y|$

$C = u(x,y) = x\sin y - y\cos x + \ln|x| - \ln|y|$

3.2 $\frac{2x\,dx}{y^3} + \frac{y^2-3x^2}{y^4}\,dy$

$2xy\,dx + (y^2 - 3x^2)\,dy = 0$

$\frac{dy}{dx} = \frac{2xy}{3x^2 - y^2}$

$x' = \frac{dx}{dy} = \frac{3x}{2y} - \frac{y}{2x}$ $\quad \alpha = -1$

$z = x^{1-\alpha} = x^2 \qquad z' = 2x x'$

$x x' = \frac{3x^2}{2y} - \frac{y}{2} \qquad z' = \frac{\cancel{2}^{3} z}{y} - \frac{y}{\cancel{2}}$

$z = \left(C + \int -\frac{y}{\cancel{2}} \cdot e^{-\int \frac{\cancel{2}^{3}}{y} dy} dy \right) e^{\int \frac{\cancel{2}^{3}}{y} dy} =$

$= \left(C + \int -\frac{y \, \cancel{e^{\ln|y|}}}{\cancel{2} \cdot y^{\cancel{2}3}} dy \right) y^{\cancel{2}3} =$

$= \left(C + \frac{+1}{\cancel{2}\cancel{2} y^{\cancel{4}}} \right) y^{\cancel{6}3} = \cancel{Cy^6 + \frac{y^2}{\cancel{2}\cancel{4}} = x^2}$

$= Cy^{3} + y^2 = x^2$

3.3 $(1 - x^2 y) dx + x^2 (y - x) dy = 0 \quad | \cdot \mu$

$\mu = \mu(x)$

$P'_y = Q'_x$

$0 \cdot P + \mu(-x^2) = \mu' x^2 (y - x) + \mu (2yx - 3x^2)$

$\mu' (y - x)(x^2) = \mu(-x^2 + 3x^2 - 2xy)$

$\mu' (y - x) x^2 = \mu \, 2x (x - y)$

$\mu' x = -2 \cancel{x} \mu \qquad \mu' = -\frac{2}{x} \mu$

$\mu = \cancel{\int} C \cdot e^{\int -\frac{2}{x} dx} = \frac{C}{x^2} = \frac{1}{x^2}$

$\left(\frac{1}{x^2} - y\right) dx + (y - x) dy = 0$ — УПД

$u(x, y) = \int \left(\frac{1}{x^2} - y\right) dx = -\frac{1}{x} - yx + C_y$

$$C_y = \int \left(y - x - \left(-\frac{1}{x} - yx\right)'_y\right) dy =$$

$$= \frac{y^2}{2}$$

$$u(x,y) = -\frac{1}{x} - yx + \frac{y^2}{2} = C$$

3.4 $(2xy^2 - 3y^3)dx + (7 - 3xy^2)dy = 0 \mid \cdot y^{-2}$

$$(2x - 3y)dx + \left(\frac{7}{y^2} - 3x\right) dy = 0$$

$$u(x,y) = \int (2x - 3y)dx = x^2 - 3yx + C_y$$

$$C_y = \int \frac{7}{y^2} - 3x - (x^2 - 3yx)'_y \, dy =$$

$$= \int \frac{7}{y^2} dy = -\frac{7}{y}$$

$$u(x,y) = x^2 - 3yx - \frac{7}{y} = C$$ (или $y = 0$)

3.5 $(3y^2 - x)dx + (2y^3 - 6xy)dy = 0$

$$2\alpha = 1 = 3\alpha + \alpha - 1 = \alpha + 1 + \alpha - 1$$

$\alpha = \frac{1}{2}$] $\cancel{z = y^\alpha}$ $y = z^\alpha = \sqrt{z}$

$$dx(3z - x) + dz(z - 3x) = 0$$

$$\cancel{z' = \frac{3z - x}{3x - z}}$$

$$u = \frac{z}{x} \qquad \cancel{u'_x = \frac{z'x - z}{x^2}}$$

$$u = \frac{z}{x} \qquad dz = d(ux) = x\,du + u\,dx$$

$$dx(3u - 1) + (x\,du + u\,dx)(u - 3) = 0$$

$$3u\,dx - dx + ux\,du + u^2\,dx - 3x\,du -$$

$$- 3u\,dx = 0$$

$$dx(u^2 - 1) + du(ux - 3x) = 0$$

$$dx(u^2 - 1) = du\; x(-u + 3)$$

$$\ln|x| + C = \int \frac{-u+3}{u^2-1}\,du = -\frac{1}{2}\ln|u^2-1| +$$

$$+ \frac{3}{2}\ln\left|\frac{u-1}{u+1}\right| = \ln|x| + C = \ln\frac{(u-1)}{(u+1)^2}$$

$$u = \frac{y^2}{x} \qquad Cx = \ln\frac{\frac{y^2}{x} - 1}{\left(\frac{y^2}{x} + 1\right)^2}$$

3.6 $x\,dx + y\,dy + x(x\,dy - y\,dx) = 0$

$(x - xy)\,dx + (y + x^2)\,dy = 0$

$(\mu P)'_y = (\mu Q)'_x$

$\exists\ \mu(x,y) = \mu(y)$

$\mu'_y P + P'_y \mu = \mu'_x Q + Q'_x \mu$

$\mu'(x - xy) + (-x)\cdot\mu = 0 + 2x\mu$

$\mu'(x - xy) = 3x\mu$

$\mu'_y(1 - y) = 3\mu$

$\mu = C\cdot e^{\int \frac{3}{1-y}dy} = C\cdot\frac{1}{(y-1)^3} = (y-1)^{-3}$

$y \quad -\frac{x}{(y-1)^2}\,dx + \frac{y+x^2}{(y-1)^3}\,dy = 0$

$P'_{xy} = \frac{2x}{(y-1)^3} \qquad Q'_x = \frac{2x}{(y-1)^3} \Rightarrow$ УПД

$u(x,y) = \int -\frac{x}{(y-1)^2}\,dx = -\frac{x^2}{2(y-1)^2} + C_y$

$C_y = \int \frac{y + x^2 \ \cancel{2x}}{(y-1)^3} + \left(\frac{x^2}{2(y-1)^2}\right)'_y dy =$

$= \int\left(\frac{y+x^2}{(y-1)^3} \mp \frac{x^2}{(y-1)^3}\right)dy = \int\frac{y}{(y-1)^3}\,dy =$

$= \int \frac{t+1}{t^3} dt = -\frac{1}{t} - \frac{1}{2t^2} = -\frac{1}{y-1} - \frac{1}{2(y-1)^2}$

$-\frac{x^2}{2(y-1)^2} - \frac{1}{y-1} - \frac{1}{2(y-1)^2} = C$

3.7 $(y + \sqrt{xy})dx = xdy$

$y' = \frac{y+\sqrt{xy}}{x} = \frac{y}{x} + \sqrt{\frac{y}{x}}$

$z = \frac{y}{x}$ $\quad y = zx$

$y' = z'x + z$

$z'x + z = z + \sqrt{z}$

$\frac{dz}{\sqrt{z}} = \frac{dx}{x}$

$2\sqrt{z} = \ln|x| + C$

$2\sqrt{\frac{y}{x}} = \ln|x| + C$

3.8) $2x^2y' = y^3 + xy$

~~$x=0$ – решение~~

$y' = \frac{y}{2x} + \frac{y^3}{2x^2}$

$\alpha = 3$ $\quad z = y^{1-\alpha} = y^{-2}$ $\quad z' = -\frac{2}{y^3}$

$\frac{y'}{y^3} = \frac{y^{-2}}{2x} + \frac{1}{2x^2}$ $\quad y = 0$ – реш.

$$z' = -\frac{z}{x} - \frac{1}{x^2}$$

$$z = \left(C + \int -\frac{1}{x^2} e^{-\int -\frac{1}{x} dx}\right) e^{\int -\frac{1}{x} dx} =$$

$$= \left(C + \int -\frac{1}{x} dx\right) \cdot \frac{1}{x} = \frac{C}{x} - \frac{\ln|x|}{x}$$

3.9 $x^2 y' + xy + 1 = 0$

$$y' = -\frac{y}{x} - \frac{1}{x^2}$$

$$y = \left(C + \int -\frac{1}{x^2} e^{-\int -\frac{1}{x} dx}\right) e^{\int -\frac{1}{x} dx} =$$

$$= \frac{C}{x} - \frac{\ln|x|}{x}$$

3.10) $xy^2 y' = x^2 + y^3$

$$y' = \frac{y}{x} + \frac{x}{y^2}$$

$$\alpha = -2 \quad z = y^{1-\alpha} = y^3$$

$$y' y^2 = \frac{y^3}{x} + x$$

$$\frac{1}{3} z' = \frac{z}{x} + x$$

$$z' = \frac{3z}{x} + 3x$$

$$z = \left(C + \int 3x e^{-\int \frac{3}{x} dx}\right) e^{\int \frac{3}{x} dx} =$$

$= \left(C + 3\int \frac{1}{x^2}\,dx\right)x^3 = \left(C - 3\cdot\frac{1}{x}\right)x^3 =$

$= Cx^3 - 3x^2$

$y^3 = Cx^3 - 3x^2$

3.11) $(1 + y^2 \sin 2x)\,dx - 2y\cos^2 x\,dy = 0$

$P'_y = 2\sin 2x\, y \qquad Q'_x = -2y \cdot 2\cos x \cdot (-\sin x)$

$= 2y \sin 2x$

УПД

$u(x,y) = \int (1 + y^2 \sin 2x)\,dx =$

$= x - \frac{y^2 \cos 2x}{2} + C_y$

$C_y = \int Q - \left(x - \frac{y^2\cos 2x}{2}\right)'_y\,dy =$

$= \int (-2y\cos^2 x + \cos 2x\, y)\,dy =$

$= \int (-y - y\cos 2x + y\cos 2x)\,dy =$

$= -\frac{y^2}{2}$

$x - \frac{y^2\cos 2x}{2} - \frac{y^2}{2} = C$

3.12) $xy\,dx = (y^3 + x^2 y + x^2)\,dy$

$(\mu P)'_y = (\mu Q)'_x$

$\mu'_y P + P'_y \mu = \mu'_x Q + Q'_x \mu$

$\mu(x,y) = \mu(y)$

$\mu' xy + x\mu = (-2xy - 2x)\mu$

$\mu' y + \mu = -2\mu y - 2\mu$

$\mu' = -2\mu - \frac{3\mu}{y}$

$\frac{d\mu}{\mu} = dy\left(-2 - \frac{3}{y}\right)$

$\ln|\mu| = -2y - 3\ln|y|$

$\mu = e^{-2y} \cdot y^{-3} = \frac{1}{y^3 e^{2y}}$

$\frac{x}{y^2 e^{2y}}\,dx - \left(\frac{1}{e^{2y}} + \frac{x^2}{y^2 e^{2y}} + \frac{x^2}{y^3 e^{2y}}\right)dy = 0$

$P'_y = x\left(-2y^{-3}e^{-2y} + (-2)e^{-2y}y^{-2}\right) =$

$= Q'_x \Rightarrow$ УПД

$u(x,y) = \int \frac{x}{y^2 e^{2y}}\,dx = \frac{x^2}{2y^2 e^{2y}} + C_y$

$C_y = \int -\left(\frac{1}{e^{2y}} + \frac{x^2}{y^2 e^{2y}} + \frac{x^2}{y^3 e^{2y}}\right) - \left(\frac{x^2}{2y^2 e^{2y}}\right)'_y dy$

$= \int -\frac{1}{e^{2y}}\, dy = \frac{e^{-2y}}{2}$

$\frac{x^2}{2y^2 e^{2y}} + \frac{e^{-2y}}{2} = C$

$dy =$

$= 0$

$dy =$