

$$\int_{-\infty}^{+\infty} \sin(\xi t) t e^{-at^2} dt$$

$$\frac{\sin \xi t d e^{-at^2}}{-2a}$$

$$\int_0^{+\infty} t e^{-at^2} \sin(\xi t) dt$$

$$\int_0^{+\infty} t e^{-at^2} \sin(\xi t) dt$$

26 mai.

UDS ~3.

Bapuanm 53.

1.  $f(t) = t e^{-at^2}$ ,  $a > 0$

$$f(x) \sim I = \int_{-\infty}^{+\infty} (a(\xi) \cos(\xi x) + b(\xi) \sin(\xi x)) d\xi$$

$$\sin(\xi x) d\xi$$

$$a(\xi) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} t e^{-at^2} \cos(t\xi) dt = 0$$

III. r. op-e kerin

$$b(\xi) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} t e^{-at^2} \sin(t\xi) dt =$$

$$= \frac{1}{2\pi} \int_{-\infty}^{+\infty} \frac{\sin(t\xi)}{-2a} d e^{-at^2} = \frac{1}{4\pi a} \left[ e^{-at^2} \cos(t\xi) \right]_{-\infty}^{+\infty} = 0$$

$$+ \left( + \frac{1 \cdot \xi}{4\pi a} \right) \cdot \int_{-\infty}^{+\infty} e^{-at^2} \sin(t\xi) dt$$

= I<sub>1</sub>

$$I_1 = \int_{-\infty}^{+\infty} e^{-at^2} \cos(t\xi) dt =$$

$$1 + \frac{1}{1 +}$$

$$\int_0^{\infty} t e^{-at} \sin(\frac{b}{2}t) dt$$

$$\int_{-\infty}^{+\infty} t e^{-at^2} \sin \frac{b}{2}t dt = \int_{-\infty}^{+\infty} \frac{\sin \frac{b}{2}t}{-2a} d$$

$$= \left[ -\frac{1}{2a} \sin \frac{b}{2}t \right]_{-\infty}^{+\infty} - \left( -\frac{1}{2a} \right) \int_{-\infty}^{+\infty} \sin \frac{b}{2}t dt$$

$$= \frac{1}{2} \int_{-\infty}^{\infty} e^{-at^2} \cdot \frac{e^{ibt} + e^{-ibt}}{2} dt =$$

$$= \frac{1}{2} \int_{-\infty}^{\infty} e^{-at^2+ibt} dt + \frac{1}{2} \int_{-\infty}^{\infty} e^{-at^2-ibt} dt =$$

$$= \frac{1}{2} \int_{-\infty}^{\infty} e^{(\sqrt{-a}t + \frac{ib}{2\sqrt{-a}})^2 - \frac{b^2}{4a}} dt +$$

$$+ \frac{1}{2} \int_{-\infty}^{\infty} e^{(\sqrt{-a}t - \frac{ib}{2\sqrt{-a}})^2 - \frac{b^2}{4a}} dt =$$

$$= \frac{e^{-\frac{b^2}{4a}}}{2\sqrt{a}} \int_{-\infty}^{\infty} e^{-(\sqrt{a}t + \frac{b}{2\sqrt{a}})^2} d(\sqrt{a}t + \frac{b}{2\sqrt{a}}) +$$

$$+ \frac{e^{-\frac{b^2}{4a}}}{2\sqrt{a}} \int_{-\infty}^{\infty} e^{-(\sqrt{a}t - \frac{b}{2\sqrt{a}})^2} d(\sqrt{a}t - \frac{b}{2\sqrt{a}}) =$$

$$= \frac{\sqrt{\pi} e^{-\frac{b^2}{4a}}}{2\sqrt{a}} + \frac{\sqrt{\pi} e^{-\frac{b^2}{4a}}}{2\sqrt{a}} = \frac{\sqrt{\pi} e^{-\frac{b^2}{4a}}}{\sqrt{a}}$$

$$b(\frac{b}{2}) = \frac{1 \cdot \frac{b}{2} \sqrt{\pi} e^{-\frac{b^2}{4a}}}{4\pi a \sqrt{a}} = \frac{b a^{-\frac{3}{2}} e^{-\frac{b^2}{4a}}}{4\sqrt{\pi}}$$

$$f(x) \sim \int_{-\infty}^{+\infty} \frac{b a^{-\frac{3}{2}} e^{-\frac{b^2}{4a}}}{4\sqrt{\pi}} \cdot \sin(\frac{b}{2}x) d\frac{b}{2}$$

$$\frac{1}{2n} \quad \frac{1}{2n + \frac{1}{2}}$$



$$\int_{-\infty}^{+\infty} e^{-at^2} dt$$

$$\int_{-\infty}^{+\infty} t e^{-at^2} \sin(\frac{\xi}{2}t) dt$$

$$f(x) = I(x) \quad \forall x \in \mathbb{R}, \text{ m. r.}$$

$$f \in C(\mathbb{R}) \quad (\text{a mod } \omega)$$

2.

$$f(t) = \begin{cases} e^t \cos t, & t \in (-\infty; 0] \\ e^{-2t}, & t \in (0; +\infty) \end{cases}$$

$$F[f](\xi) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} f(x) e^{-i\xi x} dx =$$

$$= \frac{1}{\sqrt{2\pi}} \left( \int_{-\infty}^0 e^t \cos t e^{-i\xi t} dt + \int_0^{+\infty} e^{-2t} e^{-i\xi t} dt \right)$$

$$= \frac{1}{\sqrt{2\pi}} \left( I_1(\xi) + I_2(\xi) \right)$$

$$I_1(\xi) = \int_{-\infty}^0 \cos t e^{t(1-i\xi)} dt =$$

$$= + \int_0^{+\infty} \cos(-t) e^{-(1-i\xi)t} dt =$$

$$= \frac{1-i\xi}{1+(1-i\xi)^2} = \frac{1-i\xi}{2-2\xi(\frac{\xi}{2}+2i)}$$

$$\int_{-\infty}^{+\infty} \frac{3\xi+10}{\xi^3+2\xi+4i} d\xi$$

$$1 + \frac{1}{1+i}$$

$$t e^{-at^2} dt$$

$$\frac{t}{a} d e^{-at^2}$$

$$L \{ e^{-at^2} \}$$

$$\int_0^{+\infty} t e^{-at^2} \sin(\frac{1}{2}t) dt$$

$$\int_{-\infty}^{+\infty} t e^{-at^2} \cos \frac{1}{2}t dt =$$

$$I_2(\frac{1}{2}) = \int_0^{+\infty} e^{(-2-i\frac{1}{2})t} dt = \frac{e^{(-2-i\frac{1}{2})t}}{-2-i\frac{1}{2}} \Big|_0^{+\infty} =$$

$$= \frac{e^{-2t} (\cos(-\frac{1}{2}t) + i \sin(-\frac{1}{2}t))}{-2-i\frac{1}{2}} \Big|_0^{+\infty} =$$

$$= -\frac{1}{-2-i\frac{1}{2}} = \frac{1}{i\frac{1}{2}+2}$$

$$f(t) \sim \frac{1}{2\pi} \int_{-\infty}^{+\infty} \left( \frac{1-i\frac{1}{2}}{2-\frac{1}{2}(\frac{1}{2}+2i)} + \frac{1}{i\frac{1}{2}+2} \right) e^{it\frac{1}{2}} d\frac{1}{2}$$

$$= \frac{1}{2\pi} \int_{-\infty}^{+\infty} \frac{3\frac{1}{2}+4i}{(\frac{1}{2}^3+2\frac{1}{2}+4i)} e^{it\frac{1}{2}} d\frac{1}{2} = I(t)$$

$$I(2) \approx 0,018 \dots \quad f(2) \approx 0,018 \dots$$

$$I(-13) = f(-13) \approx 2,051 \dots \cdot 10^{-6}$$

$$3. \quad f(x) = \begin{cases} 0, & x=0 \\ x \operatorname{sign}(\sin \frac{\pi}{2x}), & 0 < x \leq 1 \end{cases}$$

$$B(-) \frac{1}{n} \quad \forall n=1 \dots \infty \quad f(x)=0$$

Заметим, что  $|f(\frac{1}{n+\frac{1}{2}}) - f(\frac{1}{n+\frac{3}{2}})| =$   
(но график выше)

$$= \frac{1 - \frac{1}{1+(1+i\frac{1}{2})^2}}{1+(1+i\frac{1}{2})^2} = \frac{1}{2} -$$

$$\frac{1}{2n} \quad \frac{1}{2n+}$$



$$\int_{-\infty}^{+\infty} \sin(\frac{1}{2}t) t e^{-at^2} dt$$

$$\frac{\sin \frac{1}{2}t}{-\frac{1}{2}a} d e^{-at^2}$$

$$e^{-i \frac{1}{2}x} \cdot t e^{-at^2}$$

$$\int_0^{+\infty} t e^{-at^2}$$

$$= \frac{1}{n + \frac{3}{2}} + \frac{1}{n + \frac{1}{2}} \quad \forall n = 1 \dots +\infty$$

$$V_{-\infty}^{+\infty}(f) \geq \sum_{n=1}^{\infty} \left( \frac{1}{n + \frac{3}{2}} + \frac{1}{n + \frac{1}{2}} \right) = +\infty \Rightarrow V_{-\infty}^{+\infty}(f) = +\infty$$

