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1. Найти повт. пределы и предел.

$f(x,y) = \dfrac{x^2y^2}{x^2y^2+(x-y)^2}$, $(0,0)$

$\lim\limits_{x\to0}\lim\limits_{y\to0} \dfrac{x^2y^2}{x^2y^2+(x-y)^2} = \lim\limits_{x\to0} \dfrac{\cancel{x^2}\,0}{\cancel{x^2}+1} = 0 =$

$= \lim\limits_{y\to0}\lim\limits_{x\to0} f(x,y)$

$f(0,\frac{1}{n}) \xrightarrow[n\to\infty]{} 0$

$\Rightarrow$ предела $f(x,y)$ в точке $(0,0)$ не сущ.

$f(\frac{1}{n},\frac{1}{n}) \xrightarrow[n\to\infty]{} 1$

2. $dz = ?$ $d^2z = ?$ в точке $(1,1) = (\cdot)M$

$z = z(x,y)$ $2z^3 - xz^2 + xyz + 2y^2 = 2$, $z(1,1)=0$

$dz = \dfrac{\partial z}{\partial x}dx + \dfrac{\partial z}{\partial y}dy$

$\dfrac{\partial z}{\partial x}$: ~~$6z^2z'_x - 2xzz'_x + xyz'_x = 0 \Rightarrow$~~

~~$\Rightarrow z'_x =$~~

$6z^2z'_x - (z^2 + 2zz'_x x) + yz + z'_x xy = 0$

$z'_x = \dfrac{z^2 - yz}{6z^2 - \cancel{z^2}\,2zx + xy}\Big|_M = 0$

$$\frac{\partial z}{\partial y}: \quad 6z^2 z'_y - x2zz'_y + xz + z'_y xy + 4y = 0$$

$$z'_y = -\frac{xz + 4y}{6z^2 - 2xz + xy} = -4$$

$$dz(M) = -4dy$$

$$d^2 z = \frac{\partial^2 z}{\partial x^2} dx^2 + 2\frac{\partial^2 z}{\partial x \partial y} dx dy + \frac{\partial^2 z}{\partial y^2} dy^2$$

$$\frac{\partial^2 z}{\partial x^2} = (z'_x)'_x = \frac{(2zz'_x - y\overset{=0}{z'_x})(6z^2 - 2z\overset{=1}{x} + xy) - (12zz'_x - (2z'_x \overset{=1}{x} + 2z) + y)\cdot(\overset{=0}{z^2 - yz})}{\underset{=1}{(6z^2 - 2zx + xy)^2}}\Bigg|_M = 0$$

$$\frac{\partial^2 z}{\partial x \partial y}\Bigg|_M = \frac{(2zz'_y - (z + z'_y y))\cdot 1 - (12zz'_y - 2x\overset{=9}{z'_y} + x)(\overset{=0}{z^2 - yz})}{1}\Bigg|_M = 4$$

$$\frac{\partial^2 z}{\partial y^2} = \frac{(-x\underset{=0}{z'_y} - 4)\cdot 1 - 9(-x\overset{=-4}{z} - 4y)}{1} = 36$$

$$d^2 z|_M = 8dxdy + 36dy^2$$

3. Сост. ур-е кас. пл-ти

$z(x,y) = 4x^2y^2 - x^4 - y^4 + 3$ в $(\cdot)\, M = (-1,-1,5)$

$z'_x = 8xy^2 - 4x^3 \Big|_M = -4$

$z'_y = 8x^2y - 4y^3 \Big|_M = -4$

$z - 5 = z'_x(x+1) + z'_y(y+1)$

$z - 5 = -4(x+1) - 4(y+1)$

$4x + 4y + z = -3$

4. $z = x^2 - xy + y^2 \quad (\cdot)\, M = (-1,2)$

$\cancel{u(x,y,z) =}$

$\operatorname{grad} z = z'_x \bar{i} + z'_y \bar{j} = (2x-y)\bar{i} + (2y-x)\bar{j}$

$\max\limits_{l} \dfrac{\partial z}{\partial l} = |\operatorname{grad} z| = \sqrt{(2x-y)^2 + (2y-x)^2} =$

$= \sqrt{5x^2 - 8xy + 5y^2}\Big|_M = \sqrt{5+16+20} = \sqrt{41}$

$\cancel{y = kx \Rightarrow z = x^2 - kx^2 + k^2x^2 \Big|_M =}$

$\bar{l}_0 = (x,y)$

$\dfrac{\partial z}{\partial l}\Big|_M = \lim\limits_{t \to 0+} \dfrac{z(M + (x,y)t) - 7}{t} =$

$= \lim\limits_{t \to 0+} \dfrac{(-1+tx)^2 - (-1+tx)(2+ty) + (ty+2)^2 - 7}{t} = \cancel{-2tx} \lim\limits_{t \to 0+} \dfrac{-2tx + t^2x^2 - 2tx + ty + t^2xy + t^2y^2 + 4ty}{t} =$

$= \cancel{ti} \quad -4x + 5y$

$\cancel{|z| = \sqrt{16+25} \rightarrow}$

$$\begin{cases} -4x + 5y = \sqrt{41} \\ x^2 + y^2 = 1 \end{cases} \Rightarrow \quad x = -\frac{4}{\sqrt{41}}$$

$$y = \frac{5}{\sqrt{41}}$$

$$l_0 = \left(-\frac{4}{\sqrt{41}}, \quad \frac{5}{\sqrt{41}}\right)$$

5. $x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = \frac{x}{z}$

$u = 2x - z^2 \qquad v = -\frac{y}{z}$

$z(x,y) \rightarrow \cancel{(z} \; z(u, v)$

$$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial u} \cdot \frac{\partial u}{\partial x} + \frac{\partial z}{\partial v} \cdot \frac{\partial v}{\partial x} =$$

$$= \frac{\partial z}{\partial u}\left(2 - 2z \frac{\partial z}{\partial x}\right) + \frac{\partial z}{\partial v}\left(+ \frac{y}{z^2} z'_x\right)$$

$$\frac{\partial z}{\partial x} = \frac{2 \frac{\partial z}{\partial u}}{2z \frac{\partial z}{\partial u} + - \frac{y}{z^2} \frac{\partial z}{\partial v} + 1}$$

$$\frac{\partial z}{\partial y} = \frac{\partial z}{\partial u} \cdot \frac{\partial u}{\partial y} + \frac{\partial z}{\partial v} \cdot \frac{\partial v}{\partial y} =$$

$$= \frac{\partial z}{\partial u} \cdot \left(-2z z'_y\right) + \frac{\partial z}{\partial v} \cdot \left(-\frac{z - z'_y y}{z^2}\right)$$

$$\frac{\partial z}{\partial y} = \frac{\frac{\partial z}{\partial u} - \frac{1}{z}\frac{\partial z}{\partial v}}{1 + 2z\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v}\cdot\frac{y}{z^2}}$$

$$x = \frac{u + z^2}{2}$$

Подставим всё в формулу.