17 октября.

Матан.

Дз 5.

1. $u^3 + xu + y^2 = 0$ $M = (-2, 1)$ $u(M) = 1$

du ? du^2 ?

Усл-я т-мы вып-ны => F задаёт $u = f(x,y)$ и дифф-ма в нек. окр. (·) M.

~~u'_x~~ $=$

$3\cancel{2}u^2u'_x + u + u'_x x = 0$

$u'_x = -\dfrac{u}{3\cancel{2}u^2 + x}\Big|_M = -\dfrac{1}{3 + (-2)} = -1$

$3u^2u'_{xy} + xu'_y + 2y = 0$

$u'_y = -\dfrac{2y}{3u^2 + x}\Big|_M = -\dfrac{2}{1} = -2$

$u''_{xx} = -\left(\dfrac{u}{3u^2 + x}\right)'_x = -\dfrac{u'_x(3u^2 + x) - u(6u\overset{u'_x *}{\cancel{x}} + 1)}{(3u^2 + x)^2}$

$u''_{xx}\Big|_M = -\dfrac{-1(3 + (-2)) - 1(6 \cdot (-1) + 1)}{(3 + (-2))^2} =$

$= -\dfrac{-1 + 5}{1} = -4$

$$u''_{yy} = -\frac{2(3u^2+x) - 2y(6uu'_y)}{(3u^2+x)^2}\Bigg|_M =$$

$$= -\frac{2(1) - 2(-12)}{1} = -26$$

$$u''_{xy} = -\frac{u'_y(3u^2+x) - u(6uu'_y)}{(3u^2+x)^2}\Bigg|_M =$$

$$= -\frac{-2(1) - (-12)}{1} = -10$$

$$du = \begin{pmatrix} u'_x & u'_y \end{pmatrix} \begin{pmatrix} dx \\ dy \end{pmatrix}\Bigg|_M = -dx - 2dy$$

$$du^2 = u''_{xx}\,dx^2 + 2u''_{xy}\,dx\,dy + u''_{yy}\,dy^2 =$$

$$= -4dx^2 - 20\,dx\,dy - 26\,dy^2$$

8

2. $u(x,y)$ $v(x,y)$:

$$\begin{cases} u+v = x+y \\ \dfrac{\sin u}{\sin v} = \dfrac{x}{y} \end{cases} \Leftrightarrow \begin{cases} u+v-x-y=0 \\ \dfrac{\sin u}{\sin v} - \dfrac{x}{y} = 0 \end{cases}$$

$A: (x,y) = \left(0, \frac{\pi}{2}\right) \quad (u,v) = \left(\pi, -\frac{\pi}{2}\right)$

$$\cancel{F'_{(u,v)} = \begin{pmatrix} 1 & 1 \\ \frac{\cos u}{\sin v} & -\frac{\sin u \cos v}{\sin^2 v} \end{pmatrix}}$$

$$\begin{cases} u+v-x-y=0 \\ \sin u \, \overset{y}{\cancel{\sin v}} - x\sin v = 0 \end{cases}$$

$$F'_{(u\,v)} = \begin{pmatrix} 1 & 1 \\ y\cos u & -x\cos v \end{pmatrix}\Bigg|_A = \begin{pmatrix} 1 & 1 \\ -\frac{\pi}{2} & 0 \end{pmatrix}$$

$$F'_{(x\,y)} = \begin{pmatrix} -1 & -1 \\ -\sin v & \sin u \end{pmatrix}\Bigg|_A = \begin{pmatrix} -1 & -1 \\ 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} u \\ v \end{pmatrix}'_{(x,y)} = -\left(F'_{(u,v)}\right)^{-1} \cdot F'_{(x,y)} \circledcirc$$

$$\left(F'_{(u\,v)}\right)^{-1} = +\frac{2}{\pi} \begin{pmatrix} 0 & +\frac{\pi}{2} \\ -1 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -\frac{2}{\pi} & +\frac{2}{\pi} \end{pmatrix}^T =$$

$$= \begin{pmatrix} 0 & -\frac{2}{\pi} \\ 1 & \frac{2}{\pi} \end{pmatrix}$$

$$\textcircled{=} \begin{pmatrix} 0 & \frac{2}{\pi} \\ -1 & -\frac{2}{\pi} \end{pmatrix} \begin{pmatrix} -1 & -1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} \frac{2}{\pi} & 0 \\ 1-\frac{2}{\pi} & 1 \end{pmatrix}$$

$$du = \frac{\partial u}{\partial x} dx + \frac{\partial u}{\partial y} dy \Big|_{M} = \frac{2}{\pi} dx$$

$$dv\Big|_{M} = \left(1 - \frac{2}{\pi}\right) dx + dy$$

$$\cancel{F'_x}, \quad (F'_{(u,v)})^{-1} = \frac{1}{-x\cos v - y\cos u} \cdot \begin{pmatrix} -x\cos v & -1 \\ -y\cos u & 1 \end{pmatrix}$$

$$\cancel{u'_x} \quad u'_y = \frac{1}{-x\cos v - y\cos u} \cdot (y\cos u - \sin v)$$

$$u'_{yy} = (x\cos v + y\cos u)^2 = \alpha \frac{\pi^2}{4}$$

[illegible]
[illegible]

3. $f(x,y)=\dfrac{1}{x-y}$ $\quad$ $(\cdot)\ M=(2,1)$

$\|h\|=\sqrt{x^2+y^2}$

$\cancel{f(x,y)=}$ $\qquad\qquad\qquad -x+y$

$f(2+x,1+y)=f(2,1)+df(2,1)+$

$+\dfrac{1}{2}\underbrace{d^2f(2,1)}_{2x^2-4xy+2y^2}+o(\|h^8\|^2)$

$f'_x=-\dfrac{1}{(x-y)^2}\Big|_M=-1$

$f'_y=\dfrac{1}{(x-y)^2}\Big|_M=1$

$f''_{xx}=\dfrac{2}{(x-y)^3}\Big|_{(2,1)}=2$

$f''_{yy}=\dfrac{2}{(x-y)^3}\Big|_{(2,1)}=2$

$f''_{xy}=-\dfrac{2}{(x-y)^2}\Big|_{(2,1)}=-2$

$f(2+x,1+y)=1+x+y+x^2-2xy+y^2+$

$+o(\|h^8\|^2)$

4. $f(x,y) = \sqrt{1-x^2-y^2}$ в $(\cdot)$ $(0,0)$ до $o(\|h^4\|)$

$$f(x,y) = f(0,0) + df(0,0) + \frac{1}{2} d^2 f(0,0) +$$

$$+ \frac{1}{6} d^3 f(0,0) + \frac{1}{24} d^4 f(0,0) + o(\|h^4\|)$$

$$f'_x = \frac{-x}{\sqrt{1-x^2-y^2}}\Big|_{(0,0)} = 0 \qquad f'_y = \frac{-y}{\sqrt{1-x^2-y^2}}\Big|_{(0,0)} = 0$$

$$f''_{xx} = \frac{-\sqrt{1-x^2-y^2} - \frac{x^2}{\sqrt{1-x^2-y^2}}}{1-x^2-y^2}\Bigg|_{(0,0)} = -1$$

$$f''_{yy} = \frac{-\sqrt{1-x^2-y^2} - \frac{y^2}{\sqrt{1-x^2-y^2}}}{1-x^2-y^2}\Bigg|_{(0,0)} = -1$$

$$f''_{xx} = -(1-x^2-y^2+x^2)\sqrt{1-x^2-y^2} = (y^2-1)\sqrt{1-x^2-y^2}$$

$$f''_{yy} = -(1-x^2-y^2+y^2)\sqrt{1-x^2-y^2} = (x^2-1)\sqrt{1-x^2-y^2}$$

$$f''_{xy} = -\frac{+\frac{xy}{\sqrt{1-x^2-y^2}}}{1-x^2-y^2} = -xy\sqrt{1-x^2-y^2}\Big|_{(0,0)} = 0$$

$$f'''_{xxx} = \frac{(y^2-1)(-\cancel{2}x)}{\sqrt{1-x^2-y^2}}\Big|_{(0,0)} = 0 \qquad f'''_{yyy} = \frac{(x^2-1)\cdot(-y)}{\sqrt{1-x^2-y^2}}\Big|_{(0,0)} = 0$$

$$f'''_{x^2y} = 2y\sqrt{1-x^2-y^2} - \frac{y(y^2-1)}{\sqrt{1-x^2-y^2}}\Big|_{(0,0)} = 0$$

$$f'''_{xy^2} = 2x\sqrt{1-x^2-y^2} - \frac{x(x^2-1)}{\sqrt{1-x^2-y^2}}\Big|_{(0,0)} = 0$$

$f''''_{xxxx} = (y^2-1)(f'_x)'_x = (y^2-1)^2\sqrt{1-x^2-y^2}\Big|_{(0,0)} =$

$= 1$

$f''''_{yyyy} = (x^2-1)\, f''_{yy} = (x^2-1)^2\sqrt{1-x^2-y^2}\Big|_{(0,0)} = 1$

$f''''_{x^3y} = ((y^2-1)f'_x)'_y = 2y f'_x + (y^2+1) f''_{xy}\Big|_{(0,0)} =$

$= 0$

$\cancel{f''''_{x^2y^2} =}$

$f'''_{x^2y} = \dfrac{2y\,(1-x^2-y^2) - y\,(y^2-1)}{\sqrt{1-x^2-y^2}} =$

$= \dfrac{y\,(2-2x^2-2y^2-y^2+1)}{\sqrt{1-x^2-y^2}} =$

$= \dfrac{-3y^3-2x^2y+3y}{\sqrt{1-x^2-y^2}}$

$f''''_{x^2y^2}\Big|_{(0,0)} = 0$

$f''''_{xy^3} = ((x^2-1)f'_y)'_x = 2x f'_y + (x^2-1) f''_{yx}\Big|_{(0,0)} =$

$= 0$

$d^2 f = -x^2 - y^2$

$d f = 0$

$d^3 f = 0$

$d^4 f = x^4 - y^4$

$$f(x,y) = 1 + \frac{1}{2}(-x^2 - y^2) + \frac{1}{24}(x^4 - y^4) + o(\|h^4\|)$$