23 февраля.

Мат ан.

Домашнее задание № 3.

1. $\int \cos 4x \cos 2x \, dx =$

$= \int \frac{1}{2}(\cos 6x + \cos 2x)\, dx =$

$= \frac{\sin 6x}{12} + \frac{\sin 2x}{4} + C$

2. $\int \sin^2 x \cos^2 x = \frac{1}{4}\int \sin^2 2x \, dx =$

$= \frac{1}{8}\int (1 - \cos 4x)\, dx = \frac{1}{8}x - \frac{\sin 4x}{32} + C$

5. $\int \frac{dx}{3 + \sin x} =] \; t = tg\frac{x}{2}$

$= \int \frac{2\,dt}{3(1+t^2) + 2t} = 2\int \frac{dt}{3t^2 + 2t + 3} =$

$= \frac{2}{3}\int \frac{dt}{t^2 + \frac{2}{3}t + \frac{1}{9} + \frac{26}{9}} = \frac{2}{\sqrt{26}} \cdot arctg\frac{3(t+\frac{1}{3})}{\sqrt{26}} + C$

$t = tg\frac{x}{2}$.

7. $\int \frac{\sin 2x \, dx}{\sin^4 x + \cos^4 x} = 2\int \frac{\sin x \cos x}{\sin^4 x + \cos^4 x} dx =$

$= 2\int \frac{tg\, x}{tg^4 x + 1} \cdot \frac{1}{\cos^2 x}\, dx =$

$$= 2\int \frac{tg\,x}{1+tg^4 x}\,d\,tg\,x = \quad \exists\ tg\,x = t$$

$$= 2\int \frac{t\,dt}{1+t^4} = \cancel{2}\int \frac{dt^2}{1+t^4} = arctg\,t^2 =$$

$$= \cancel{x^2} \cdot arctg\,tg^2\,\cancel{t}^{x} + C$$

$$3.\ \int \frac{\cos 2x\,dx}{\sin^6 x} = \frac{1\cdot 2^3}{2}\int \frac{\cos 2x\,d2x}{(1-\cos 2x)^3} =$$

$$\exists\ t = tg\,x$$

$$= 4\int \frac{(1-t^2)\,2dt}{(1+t^2)\left(1-\frac{1-t^2}{1+t^2}\right)^3(1+t^2)} =$$

$$= 8\int \frac{(1-t^2)\,dt}{(1+t^2)^2\left(\frac{1+t^2-1+t^2}{1+t^2}\right)^3} =$$

$$\cancel{= 8\int \frac{(1-t^2)\,dt}{(1+t^2)^2\,2t^2} = 4\int\left(\frac{1}{t^2(t^2+1)} - \frac{1}{t^2+1}\right)dt =}$$

$$\cancel{= 4\int \frac{1}{t^2(t^2+1)}\,dt - 4\,arctg\,t =}$$

$$\cancel{= 4\int\left(\frac{1}{t^2} - \frac{1}{t^2+1}\right)dt - 4\,arctg\,t =}$$

$$\cancel{= -\frac{4}{t} - 4\,arctg\,t - 4\,arctg\,t}$$

$$= 8\int \frac{(1-t^2)(1+t^2)}{8t^6}\,dt = 8\int \frac{1-t^4}{8t^6}\,dt =$$

$$= \int \frac{1}{t^6}\,dt - \int \frac{1}{t^2}\,dt = -\frac{1}{5}\frac{1}{t^5} + \frac{1}{t} + C =$$

$$= \frac{1}{-5tg^5 x} + \frac{1}{tg\, x} + C$$

4. $$\int \frac{dx}{3+\sin^2 x} = \int \frac{dx}{4\sin^2 x + 3\cos^2 x} =$$

$$= \int \frac{d\, tg\, x}{4tg^2 x + 3} = \frac{1}{4} \int \frac{d\, tg\, x}{tg^2 x + \frac{3}{4}} =$$

$$= \frac{1}{2\sqrt{3}} \operatorname{arctg} \frac{tg\, x \cdot \cancel{\sqrt{3}}\, 2}{\sqrt{3}} + C$$

6. $$\int \sqrt{1+\sin x}\, dx =$$

$$\exists\, t = tg\, \frac{x}{2}$$

$$= \int \sqrt{1 + \frac{2t}{t^2+1}}\ \frac{2dt}{t^2+1} = \int \sqrt{\frac{(t+1)^2}{t^2+1}}\ \frac{2dt}{(t^2+1)} =$$

$$= \int \frac{2(t+1)\, dt}{(t^2+1)^{\frac{3}{2}}} = \int \frac{2t}{(t^2+1)^{\frac{3}{2}}} dt + 2\int \frac{dt}{(t^2+1)^{\frac{3}{2}}} =$$

$$= \frac{-2}{\sqrt{t^2+1}} + \frac{2t}{\sqrt{t^2+1}} + C$$

8. $$\int \frac{x\, dx}{1+\cos x} = \int \frac{2\operatorname{arctg} t \cdot 2dt}{(1+t^2)\left(1 + \frac{1-t^2}{1+t^2}\right)} =$$

$$= \int 2\operatorname{arctg} t\, dt = 2t\operatorname{arctg} t -$$

$$- \int t \, d\, 2 \operatorname{arctg} t = 2t \operatorname{arctg} t -$$

$$- 2 \int \frac{t}{t^2+1} = 2t \operatorname{arctg} t - \ln|t^2+1| + C$$

$$t = tg\,\frac{x}{2}$$