Стукников Александр М3235

Кр 2

1. $\bar{a} = \overset{P}{(y-4x)}\bar{i} + \overset{Q}{(x+2y+z)}\bar{j} + \overset{R}{(y+2z)}\bar{k}$

$rot(\bar{a}) = \left(\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z}\right)\bar{i} + \left(\frac{\partial P}{\partial z} - \frac{\partial R}{\partial x}\right)\bar{j} + \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)\bar{k} =$

$= 0 \Rightarrow \bar{a}$ потенц.

$div(\bar{a}) = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z} = -4+2+2=0 \Rightarrow$

$\Rightarrow \bar{a}$ соленоид.

$$u(\tilde{x},\tilde{y}) = \int_{A(0,0,0)}^{B(\tilde{x},\tilde{y},\tilde{z})} (y-4x)\,\cancel{dy\,dz}\,dx + (x+2y+z)\,dy +$$

$$+ (y+2z)\,dz = \int_{A(0,0,0)}^{C(\tilde{x},0,0)} -4x\,dx + \int_{C(\tilde{x},0,0)}^{D(\tilde{x},\tilde{y},0)} (\tilde{x}+2y)\,dy +$$

$$+ \int_{D(\tilde{x},\tilde{y},0)}^{B(\tilde{x},\tilde{y},\tilde{z})} (\tilde{y}+2z)\,dz = -2x^2\Big|_0^{\tilde{x}} + (\tilde{x}y + y^2)\Big|_0^{\tilde{y}} +$$

$$+ (\tilde{y}z + z^2)\Big|_0^{\tilde{z}} = -2\tilde{x}^2\cancel{\tilde{x}} + \tilde{x}\tilde{y} + \tilde{y}^2 + \tilde{y}\tilde{z} + \tilde{z}^2$$

$grad\,\bar{u} = \bar{a} \Rightarrow \bar{u}$ - потенциал

Ответ: $-2x^2 + xy + y^2 + yz + z^2$

2. $\bar{a} = (3x^2y + 2y^4)\bar{i} + (12y^2 + x^3 + 8xy^3)\bar{j}$

$L: \ y = (\sqrt{x}+1)^{-1}$

$$\int_1^0 \left(\left(\frac{3x^2}{\sqrt{x}+1} + \frac{2}{(\sqrt{x}+1)^4}\right)\cancel{dx} + \left(\frac{12}{(\sqrt{x}+1)^2} + x^3 + \frac{8x}{(\sqrt{x}+1)^3}\right) \cdot \right.$$

$$\left. \cdot \underbrace{\cancel{(3x^2 + 8y^3)}}_{-\frac{1}{2(\sqrt{x}+1)^2 \cdot \sqrt{x}}}\right) dx = \frac{23}{8}$$

Ответ: $\frac{23}{8}$.

3. $L: x^2 + y^2 = 25 \qquad \rho(x,y) = \cancel{\frac{x^2y^2}{x^2}} + y^4$

$$m = \int_L (x^2y^2 + y^4)\, dl = \int_L 25\sin^2\varphi \cdot 25\, dl =$$

$x = 2\cos\varphi$
$y = 2\sin\varphi$
$0 \le \varphi \le 2\pi$

$$= 625 \int_0^{2\pi} \sqrt{2^2\cos^2\varphi + 2^2\sin^2\varphi} \cdot \sin^2\varphi\, d\varphi =$$

$$= 625 \int_0^{2\pi} 5\sin^2\varphi\, d\varphi = \cancel{625}\,3125\,\pi$$

Ответ: ~~625π~~ 3125π

4. $x^2 + z^2 = y, \quad 0 \le y \le 1$

$\rho(x, y, z) = y$

$$\iint\limits_{\Sigma} \rho(x, y, z)\, d\sigma = \iint\limits_{D} (x^2 + z^2)\sqrt{1 + {y'_x}^2 + {y'_z}^2}\, dx\, dz =$$

$$= \iint\limits_{D} (x^2 + z^2)\sqrt{1 + 4x^2 + 4z^2}\, dx\, dz =$$

$$= \cancel{\iint} \iint\limits_{D} r\, r^2 \sqrt{1 + 4r^2}\, dr\, d\varphi =$$

$$= \int_0^1 r^3 \sqrt{1 + 4r^2}\, dr \cdot \int_0^{2\pi} d\varphi = 2\pi\left(\frac{5^{\frac{3}{2}}}{24} + \frac{1}{120}\right)$$

<u>Ответ:</u> $2\pi\left(\frac{5^{\frac{3}{2}}}{24} + \frac{1}{120}\right)$

5. $\bar{a} = y\bar{i} + x\bar{j} + 2z\bar{k}$

т.к. сторона внутр.

$x^2 + y^2 = 1, \quad z = 0, \quad z = 1$

$$-\Pi_{\Sigma}(\bar{a}) = \iiint\limits_{\Omega} 2\, dx\, dy\, dz = 2\, V_{\text{цилиндра}} = 2\pi$$

по О.-Г.

$\Pi_{\Sigma}(\bar{a}) = -2\pi$

<u>Ответ:</u> -2π

6. $\bar{a} = 0\cdot\bar{i} + x^2\cdot\bar{j} + y^2\bar{k}$

$x^2 + z^2 = y^2, \quad 0 \le y \le 1$

$$\underset{\Sigma}{\Pi}(\bar{a}) = \iint\limits_{\Sigma} \underbrace{x^2\,dx\,dz}_{I_1} + \underbrace{y^2\,dy\,dz}_{I_2}$$

нормаль в другую сторону

$I_2 = 0$, т.к. при $\overset{x}{\cancel{y}} < 0$ $I_{21} = -I_{22}$ при $\overset{x}{\cancel{y}} > 0$.

~~Аналогично для I_2~~

~~Ответ: 0~~.

$$-I_1 = \iint\limits_{D} x^2\,dx\,dz = \int_0^{2\pi} d\varphi \int_0^1 r^3\cos^2\varphi\,dr =$$

окр-ть $\to D$

$$= \frac{1}{4}\int_0^{2\pi} \cos^2\varphi\,d\varphi = \frac{\pi}{4}$$

$I_1 = -\frac{\pi}{4}$

Ответ: $-\frac{\pi}{4}$.