10 октября.

Мат.ан.

Дз № 4

2. $f(x,y) = 5x + 10x^2 y + y^5$,

$M(1,2)$, $\bar{l}(4,-3)$

$\frac{\partial f}{\partial x} = 5 + 20x$

$\frac{\partial f}{\partial y} = 10x^2 + 5y^4$

$\text{grad} f = (5+20x)\bar{i} + (10x^2+5y^4)\bar{j}$

а) $\frac{\partial f}{\partial l} = \text{grad}\, u \cdot \bar{l}_0 =$

$= \frac{(5+20x)4 + (10x^2+5y^4)(-3)}{5} =$

$= 4 + 16x - 6x^2 - 3y^4$

$f'_{\bar{l}}(M) = 4 + 16 - 6 - 3 \cdot 16 = -34$

б) $\max\limits_{l} \frac{\partial f}{\partial l} = |\text{grad} f| =$

$= \sqrt{(5+20x)^2 + (10x^2+5y^4)^2}$

$\max\limits_{l} \frac{\partial f}{\partial l}(M) = \sqrt{(25)^2 + (10+80)^2} =$

$= 5\sqrt{349}$

3. $2x^2 + 3y^2 + 4z^2 = 9 \quad A(1,-1,1)$

$f(x,y,z) = 2x^2 + 3y^2 + 4z^2$

$\frac{\partial f}{\partial x} = 4x \quad \frac{\partial f}{\partial y} = 6y \quad \frac{\partial f}{\partial z} = 8z$

$4(x-1) + (-6)(y+1) + 8(z-1) = 0$

$4x - 6y + 8z - 18 = 0$

4. $y\frac{\partial z}{\partial x} - x\frac{\partial z}{\partial y} = (y-x)z$

$z(x,y) \to w(u,v) \quad \begin{cases} u = x^2 + y^2 \\ v = \frac{1}{x} + \frac{1}{y} \\ w = \ln z - x - y \end{cases}$

$\frac{\partial w}{\partial x} = \frac{\partial w}{\partial u}\cdot\frac{\partial u}{\partial x} + \frac{\partial w}{\partial v}\cdot\frac{\partial v}{\partial x} =$

$= \frac{\partial w}{\partial u}\cdot 2x + \frac{\partial w}{\partial v}\cdot\left(-\frac{1}{x^2}\right) = \frac{z'_x}{z} - 1$

$z'_x = 2xz\frac{\partial w}{\partial u} - \frac{z}{x^2}\frac{\partial w}{\partial v} + z$

$\frac{\partial w}{\partial y} = \frac{\partial w}{\partial u}\cdot\frac{\partial u}{\partial y} + \frac{\partial w}{\partial v}\cdot\frac{\partial v}{\partial y} =$

$= \frac{\partial w}{\partial u}\cdot 2y - \frac{1}{y^2}\frac{\partial w}{\partial v} = \frac{z'_y}{z} - 1$

$z'_y = 2yz\frac{\partial w}{\partial u} - \frac{z}{y^2}\frac{\partial w}{\partial v} + z$

$$-\frac{2y}{x^2}\frac{\partial w}{\partial v} + 2y + \frac{2x}{y^2}\frac{\partial w}{\partial v} \mp 2x =$$

$$= (y-x)z$$

$$\frac{\partial w}{\partial v}\left(\frac{2x}{y^2} - \frac{2y}{x^2}\right) = 0 \quad (z = 0 \text{ реш.})$$

$$w = f(u)$$

$$\ln z - x - y = f\left(\cancel{\frac{1}{x} + \frac{1}{y}}\right) \quad x^2 + y^2$$

$$z = e^{f\left(\cancel{\frac{1}{x}+\frac{1}{y}}\right) + x + y} \quad x^2+y^2$$

$$\frac{z'_x}{z} = 1 + 2x \cdot f' \qquad \frac{z'_y}{z} = 1 + 2y \cdot f'$$

$$\cdot 2y \qquad\qquad \cdot 2x$$

Подставим, всё сходится.

5. $x\frac{\partial u}{\partial y} - y\frac{\partial u}{\partial x} = 0$

$u(x,y) \to u(r,\varphi)$

$r = \sqrt{x^2+y^2}$

$\cancel{\varphi = \operatorname{arctg}\frac{y}{x}}$

$$\begin{cases} x = r\cos\varphi \\ y = r\sin\varphi \end{cases}$$

$$\frac{\partial u}{\partial r} = \frac{\partial u}{\partial x}\cdot\frac{\partial x}{\partial r} + \frac{\partial u}{\partial y}\cdot\frac{\partial y}{\partial r} =$$

$$= \frac{\partial u}{\partial x}\cdot\cos\varphi + \frac{\partial u}{\partial y}\cdot\sin\varphi$$

$$\frac{\partial u}{\partial \varphi} = \frac{\partial u}{\partial x}(-r\sin\varphi) + \frac{\partial u}{\partial y}\,r\cos\varphi$$

$$\Delta = \begin{vmatrix} \cos\varphi & \sin\varphi \\ -r\sin\varphi & r\cos\varphi \end{vmatrix} = r\cos^2\varphi + r\sin^2\varphi = r$$

$$\Delta_1 = \begin{vmatrix} u'_r & \sin\varphi \\ u'_\varphi & r\cos\varphi \end{vmatrix} = u'_r\cdot r\cos\varphi - u'_\varphi\sin\varphi$$

$$\Delta_2 = \begin{vmatrix} \cos\varphi & u'_r \\ -r\sin\varphi & u'_\varphi \end{vmatrix} = u'_\varphi\cos\varphi + u'_r\,r\sin\varphi$$

$$\frac{\partial u}{\partial x} = \frac{\Delta_1}{\Delta} = \frac{u'_\varphi\cos\varphi}{r} + u'_r\sin\varphi$$

$$\frac{\partial u}{\partial y} = \frac{\Delta_2}{\Delta} = u'_r\cos\varphi - \frac{u'_\varphi\sin\varphi}{r}$$

$\overset{r\cos\varphi}{x}\left(u'_r\cos\varphi - \frac{u'_\varphi\sin\varphi}{r}\right) -$

$- \overset{r\sin\varphi}{y}\left(\frac{u'_\varphi\cos\varphi}{r} + u'_r\sin\varphi\right) = 0$

$r u'_r(\cos^2\varphi - \sin^2\varphi) - \frac{u'_\varphi}{\cancel{r}}(2\sin\varphi\cos\varphi) =$

$= 0$

? $r\cos 2\varphi\,\frac{\partial u}{\partial r} - \cancel{r}\sin 2\varphi\,\frac{\partial u}{\partial\varphi} = 0$

$\overset{r\cos\varphi}{x}\left(\frac{u'_\varphi\cos\varphi}{r} + u'_r\sin\varphi\right) - \overset{r\sin\varphi}{y}\left(u'_r\cos\varphi -\right.$

$\left.\frac{u'_\varphi\sin\varphi}{r}\right) = 0$

$u'_\varphi\cos^2\varphi + u'_r r\sin\varphi\cos\varphi - u'_r r\sin\varphi\cdot$

$\cos\varphi + u'_\varphi\sin^2\varphi = 0$

$\varphi'_\varphi = 0 \Rightarrow u = f(r) = f(\sqrt{x^2+y^2})$

Подставим:

$x\cdot\frac{\cancel{2}yf'}{\sqrt{x^2+y^2}} - y\cdot\frac{xf'}{\sqrt{x^2+y^2}} = 0$

Всё верно.

1. $f(x,y) = x|y| + y|x|$

f дифф-ма по дост. условию во всех точках кроме: $(0,0)$, $(0,\overset{y}{\cancel{x}})$, $(x,0)$.

$$\frac{\partial f}{\partial x} = \begin{cases} |y| + y, & x \geq 0 \\ |y| - y, & x \leq 0 \end{cases}$$

$$\frac{\partial f}{\partial y} = \begin{cases} |x| + x, & y \geq 0 \\ |x| - x, & y \leq 0 \end{cases}$$

$(0,0)$: $A = (0,0)$

$$\Delta f = f(\Delta x, \Delta y) = \cancel{\Delta x + \Delta y} + o(\sqrt{\Delta x^2 + \Delta y^2}) =$$

$$= \Delta x |\Delta y| + \Delta y |\Delta x|$$

$$|\Delta f| \leq 2|\Delta x \Delta y| \leq \Delta x^2 + \Delta y^2$$

$$\frac{\Delta x|\Delta y| + \Delta y|\Delta x|}{\sqrt{\Delta x^2 + \Delta y^2}} \leq \sqrt{\Delta x^2 + \Delta y^2} \xrightarrow[(\Delta x, \Delta y) \to (0,0)]{} 0$$

$\Rightarrow f$ дифф-ма в $(0,0)$