17 февраля.

Мат ан.

Домашнее задание №2.

1. $\int \frac{5x-14}{x^3-x^2-4x+4}\,dx \ominus$

$\frac{5x-14}{x^3-x^2-4x+4} = \frac{5x-14}{(x-1)(x^2-4)} =$

$= \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x+2}$

$5x-14 = A(x-2)(x+2) + B(x-1)(x+2) +$

$+ C(x-1)(x-2)$

$x=2: \quad -4 = 4B \Rightarrow B = -1$

$x=1: \quad -9 = -3A \Rightarrow A = 3$

$x=-2: \quad -24 = 12C \Rightarrow C = -2$

$\ominus \int \left(\frac{3}{x-1} - \frac{1}{x-2} - \frac{2}{x+2}\right) dx =$

$= 3\ln|x-1| - \ln|x-2| - 2\ln|x+2| + C.$

2. $\int \frac{x^2+1}{x(x-1)^3}\,dx \ominus \cancel{\int \frac{x}{(x-1)^3}\,dx +}$

$\frac{x^2+1}{x(x-1)^3} = \frac{A}{x} + \frac{B}{(x-1)^3} + \frac{C}{(x-1)^2} + \frac{D}{x-1}$

$x^2+1 = A(x-1)^3 + Bx + Cx(x-1) +$

$+ Dx(x-1)^2$

$x=1: \quad 2 = B$

$x=0: \quad 1 = -A \Rightarrow A = -1$

$x^3: \quad 0 = A + D \Rightarrow D = 1$

$x^2: \quad 1 = -3A + C - 2D \Rightarrow C = 0$

$= -\ln|x| - \frac{1}{\cancel{2}(x-1)^{\cancel{2}}} \cancel{- \frac{5}{(x-1)^2}} + \ln|x-1| + C$

3. $\int \left(\frac{x-1}{x+1}\right)^4 dx = \int \left(1 - \frac{2}{x+1}\right)^4 dx =$

$= \int \left(1 - 4\frac{2}{x+1} + 6\frac{4}{(x+1)^2} - 4\frac{8}{(x+1)^3} + \frac{16}{(x+1)^4}\right) dx =$

$= x - 8\ln|x+1| - \frac{24}{x+1} + \frac{16}{(x+1)^2} - \frac{16}{3(x+1)^3} + C$

4. $\int \frac{2x^2+x+2}{(x^2+1)(x^2-x+1)} dx =$

$\frac{2x^2+x+2}{(x^2+1)(x^2-x+1)} = \frac{Ax+B}{x^2+1} + \frac{Cx+D}{x^2-x+1}$

$2x^2+x+2 = (Ax+B)(x^2-x+1) + (Cx+D)(x^2+1)$

$$\left.\begin{aligned} x^3&: 0 = A + C \\ x^2&: 2 = -A + B + D \\ x&: 1 = A - B + C \\ x^0&: 2 = B + D \end{aligned}\right\} \Rightarrow \begin{aligned} B &= -1 \\ D &= 3 \\ A &= 0 \\ C &= 0 \end{aligned}$$

$= \int -\frac{1}{x^2+1} dx + \int \frac{3}{x^2-x+1} dx =$

$$= -\operatorname{arctg} x + 3\int \frac{1}{\left(x-\frac{1}{2}\right)^2+\frac{3}{4}}\,dx =$$

$$= -\operatorname{arctg} x + 3\,\frac{2}{\sqrt{3}}\,\operatorname{arctg}\frac{2x-1}{\sqrt{3}} + C.$$

5. $$\int \frac{x^{15}+x^7}{x^{16}-256}\,dx = \frac{1}{8}\int \frac{x^8+1}{x^{16}-256}\,dx^8$$

$\exists\, t = x^8$

$$\frac{1}{8}\int \frac{t+1}{t^2-256}\,dt = \frac{1}{8}\int \frac{t+16-15}{(t-16)(t+16)}\,d\overset{t}{\cancel{x}} =$$

$$= \frac{1}{8}\int\left(\frac{1}{t-16} - \frac{15}{(t-16)(t+16)}\right)d\overset{t}{\cancel{x}} =$$

$$= \frac{\ln|t-16|}{8} - \frac{15}{8}\int \frac{1}{t^2-16^2}\,dt =$$

$$= \frac{\ln|t-16|}{8} - \frac{15}{256}\ln\left|\frac{\overset{t}{\cancel{x}}-16}{t+16}\right| + C =$$

$$= \frac{\ln|x^8-16|}{8} - \frac{15}{256}\ln\left|\frac{x^8-16}{x^8+16}\right| + C.$$

6. $$\int e^{-x}\operatorname{arctg} e^x\,dx = -\int \operatorname{arctg} e^x\,de^{-x} =$$

$$= -e^{-x}\cdot\operatorname{arctg} e^x + \int e^{-x}\,d\operatorname{arctg} e^x =$$

$$= -e^{-x}\operatorname{arctg} e^x + \int \frac{de^x}{e^x(1+e^{2x})} =$$

$\exists\, t = e^x$

$$\int \frac{1}{t(t^2+1)}\,dt$$

$$\frac{1}{t(t^2+1)} = \frac{A}{t} + \frac{B\overset{t}{\cancel{x}}+C}{t^2+1}$$

$$1 = A(t^2+1) + (B\overset{t}{\cancel{x}}+C)t$$

$t=0: \quad A=1$

$t^1: \quad 0=C$

$t^2: \quad 0 = A+B \Rightarrow B=-1$

$$\int \frac{1}{t(t^2+1)}\,dt = \ln|t| - \frac{1}{2}\ln|t^2+1| + C$$

$$= -e^{-x}\operatorname{arctg} e^{x} + \ln|e^{x}| - \frac{1}{2}\ln|e^{2x}+1| + C =$$

$$= -e^{-x}\cdot\operatorname{arctg} e^{x} + x - \frac{1}{2}\ln|e^{2x}+1| + C.$$

7. $$\int\left(\frac{\cos x}{e^x}\right)^2 dx = \int \frac{1+\cos 2x}{2\cdot e^{2x}}\,dx =$$

$]\ t = 2x$

$$= \int \frac{1+\cos t}{4e^t}\,d\overset{t}{\cancel{x}} = \frac{1}{4}\int e^{-t}\,dt + \frac{1}{4}\int \frac{\cos t}{e^t}\,dt =$$

$$= -\frac{e^{-t}}{4} + \frac{1}{4}\underbrace{\int e^{-t}\,d\sin t}_{\parallel}$$

$$e^{-t}\sin t - \int \sin t\,de^{-t} =$$

$= e^{-t}\sin t - \int e^{-t}\,d\cos t =$

$= e^{-t}\sin t - e^{-t}\cos t + \int \cos t\,d e^{-t} =$

$= e^{-t}\sin t - e^{-t}\cos t - \int e^{-t}\cos t\,dt$

$\cancel{-\frac{e^{-t}}{4}} + \cancel{\frac{1}{4}}\int e^{-t}\cos t\,dt = \cancel{\frac{1}{4}}(e^{-t}\sin t -$

$- e^{-t}\cos t - \int e^{-t}\cos t\,dt)$

$\int e^{-t}\cos t\,dt = \frac{e^{-t}\sin t \cancel{+ e^{-t}} - e^{-t}\cos t}{2}$

$\int \left(\frac{\cos x}{e^{x}}\right)^2 dx = -\frac{e^{-2x}}{4} +$

$+ \frac{e^{-t}\sin t \cancel{+ e^{-t}} - e^{-t}\cos t}{8} =$

$= \frac{e^{-2x}\sin 2x - 2e^{-2x} - e^{-2x}\cos 2x}{8} + C.$

8. $\int \cos x \cdot \ln(1+\sin^2 x)\,dx =$

$= \int \ln(1+\sin^2 x)\,d\sin x =$

] $\sin x = t$

$= \int \ln(1+t^2)\,dt = t\ln(1+t^2) -$

$- \int t\,d\ln(1+t^2) = t\ln(1+t^2) -$

$$-\int \frac{t\cdot 2t}{1+t^2}\,dt =$$

$$\int \frac{2t^2}{1+t^2}\,dt = \int\left(2 - \frac{2}{t^2+1}\right)dt = 2t -$$

$$-2\,\mathrm{arctg}\,\overset{t}{\cancel{x}} + C$$

$$= t\ln(1+t^2) - 2t + 2\,\mathrm{arctg}\,t + C =$$

$$= \sin x \ln(1+\sin^2 x) - 2\sin x +$$

$$+ 2\,\mathrm{arctg}(\sin x) + C.$$