12 декабря.

Матан.

Дз № 8

1\. $\Sigma: \ x^2+y^2 = \dfrac{R^2 z^2}{h^2} \qquad z = \dfrac{h}{R}\sqrt{x^2+y^2}, \ z \le h$

$S_\Sigma = \iint\limits_\Sigma d\sigma \qquad \dfrac{\partial z}{\partial x} = \dfrac{hx}{R\sqrt{x^2+y^2}}$

$\dfrac{\partial z}{\partial y} = \dfrac{hy}{R\sqrt{x^2+y^2}}$

$d\sigma = \sqrt{\dfrac{R^2(x^2+y^2) + h^2x^2 + h^2y^2}{R^2(x^2+y^2)}} =$

$= \sqrt{\dfrac{(R^2+h^2)(x^2+y^2)}{R^2(x^2+y^2)}} = \dfrac{\sqrt{R^2+h^2}}{R}$

$S_\Sigma = \iint\limits_D \dfrac{\sqrt{R^2+h^2}}{R}\,dx\,dy = \pi R\sqrt{R^2+h^2}$

$D: \ x^2+y^2 \le R^2$

<u>Ответ:</u> $\pi R\sqrt{R^2+h^2}$

2.

$S: \ x^2+y^2=1, \ y \geq 0, \ 0 \leq z \leq 3$

$\rho(x,y,z) = yz$

~~$x = \cos\varphi \sin\theta$~~

~~$y =$~~

$$\begin{cases} x = \cos\varphi \\ y = \sin\varphi \\ z = z \end{cases} \qquad r'_\varphi = \begin{pmatrix} -\sin\varphi \\ \cos\varphi \\ 0 \end{pmatrix} \quad r'_z = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$E = 1 \quad G = 1 \quad F = 0$

$$m = \iint_S \rho \, d\sigma = \iint_D z \sin\varphi \, 1 \, d\varphi \, dz =$$

$D: \ 0 \leq y \leq 1 \quad 0 \leq z \leq 3$

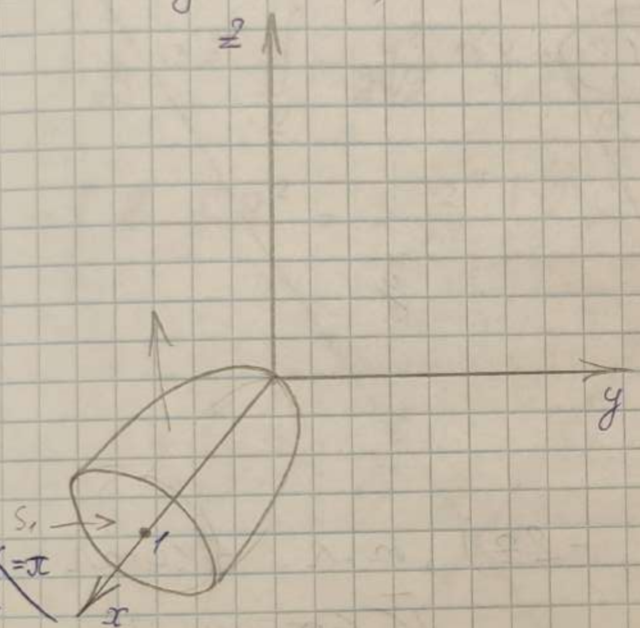
$$= \int_0^{\cancel{2}\pi} d\varphi \int_0^3 z \sin\varphi \, dz = -\cos\varphi \Big|_0^\pi \cdot \frac{z^2}{2}\Big|_0^3 =$$

$$= 2 \cdot \frac{9}{2} = 9$$

Ответ: 9

3. $I=\iint\limits_S x^2\,dy\,dz + z^2\,dx\,dy$

$S:\quad x = y^2+z^2,\quad 0\le x\le 1$



$+\iint\limits_{S_1} = \pi$

$$\cancel{I = \iiint (2x+2z)\,dx\,dy\,dz =}$$

$$\cancel{= \int_0^1 2x\,dx \int_{-\sqrt{x}}^{\sqrt{x}} 2z\,dz \int_{-\sqrt{x-z^2}}^{\sqrt{x-z^2}} dy = \int_0^1 2x\cdot 2\sqrt{x}\cdot 2x\,dx =}$$

$$\cancel{= 8\int_0^1 x^{\frac{5}{2}}\,dx = 8\,\frac{2}{7}\,x^{\frac{7}{2}}\Big|_0^1 = \frac{16}{7}}.$$

~~Ответ: $\frac{16}{7}$~~.

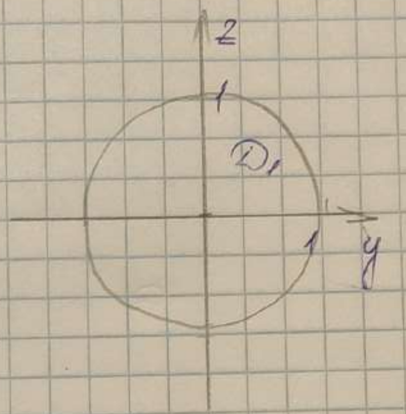
$$= \int_0^1 2x\,dx \int_{-\sqrt{x}}^{\sqrt{x}} 4z\sqrt{x-z^2}\,dz = -4\int_0^1 x\left((x-z^2)^{\frac{3}{2}}\cdot\frac{2}{3}\right)\Big|_{-\sqrt{x}}^{\sqrt{x}}\,dx = 0$$

Ответ: $-\pi$

$$\underset{z>0}{I_2} = \underset{z<0}{I_2} \Rightarrow I_2 = 0$$

$$+I_1 = +\iint\limits_{D_1} (y^2+z^2)^2\, dy\, dz = \int_0^{2\pi} d\varphi \int_0^1 r^5\, dr = \frac{2\pi}{6} = \frac{\pi}{3}$$

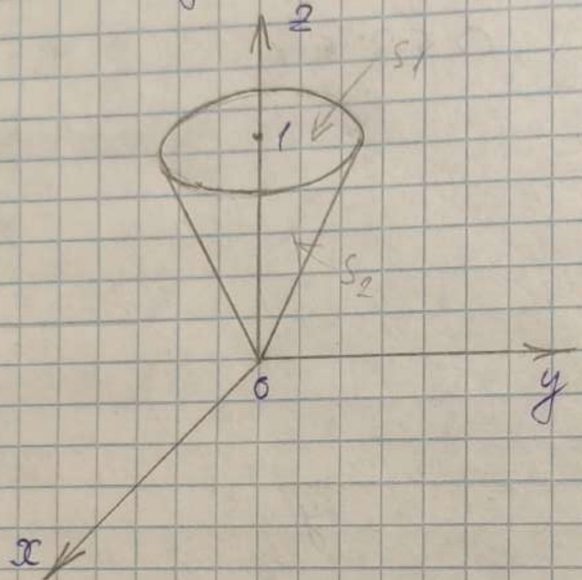
$I_1 = +\frac{\pi}{3}$ $\qquad$ $I = I_1 + I_2 = +\frac{\pi}{3}$



Ответ: $+\frac{\pi}{3}$

4. $\bar{a} = x\bar{i} + y\bar{j} + z\bar{k}$

$z^2 = x^2 + y^2, \quad 0 \leq z \leq 1$



$\bar{N}(2x, 2y, -2z)$, т.к. внутр

$\bar{N}(-x, -y, z)$

$$\bar{n}\left(\frac{-x}{\sqrt{x^2+y^2+z^2}}, \frac{-y}{\sqrt{x^2+y^2+z^2}}, \frac{z}{\sqrt{x^2+y^2+z^2}}\right)$$

$$a_n = \frac{-x^2 - y^2 + z^2}{\sqrt{x^2+y^2+z^2}}$$

$$\Pi = \iint\limits_{\Sigma} \underbrace{\frac{-x^2}{\sqrt{x^2+y^2+z^2}}\,dy\,dz}_{I_1} + \underbrace{\frac{-y^2}{\sqrt{x^2+y^2+z^2}}\,dx\,dz}_{I_2} +$$

$$+ \underbrace{\frac{z^2}{\sqrt{x^2+y^2+z^2}}\,dx\,dy}_{I_3} =$$

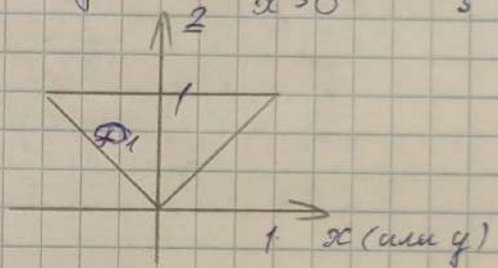
$$\Pi_{\Sigma} = \iint_{\Sigma} x\,dy\,dz + y\,dx\,dz + z\,dx\,dy$$

S_2:

$$I_{1\,x>0} = \cancel{+}-\iint_{D_1} x\,dy\,dz = -\int_0^1 dz \int_{-z}^{z} \sqrt{z^2-y^2}\,dy =$$

$$= -\int_0^1 \pi z^2\,dz = \frac{\pi z^3}{3}\Big|_0^1 = -\frac{\pi}{3}$$

$$I_{1\,x<0} = +\iint_{D_1} -\sqrt{z^2-y^2}\,dy\,dz = I_{1\,x>0} = -\frac{\pi}{3}$$

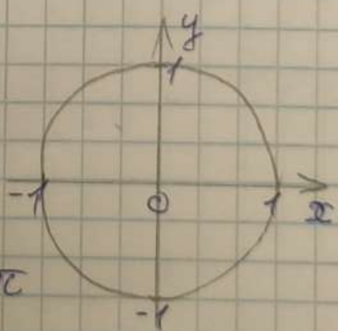


$$I_1 = -\frac{2\pi}{3}$$

$$I_2 = I_1 = -\frac{2\pi}{3}$$

$$I_3 = \iint_{D_2} \sqrt{x^2+y^2}\,dx\,dy = \int_0^{2\pi} d\varphi \int_0^1 z^2\,dz =$$

$$= \frac{2\pi}{3}$$



S_2: $\Pi_{\Sigma_{S_2}} = 0$

S_1: $\Pi_{\Sigma_{S_1}} = -\iint_{S_1} dx\,dy = -\pi$

$$\Pi_{\Sigma} = \Pi_{S_1} + \Pi_{S_2} = \cancel{0} - \pi$$

$$\Pi_{\Sigma} = -\iiint_{\Omega} 3\,dx\,dy\,dz = -3V_{\Omega} = -\pi$$

Ответ: $-\pi$.