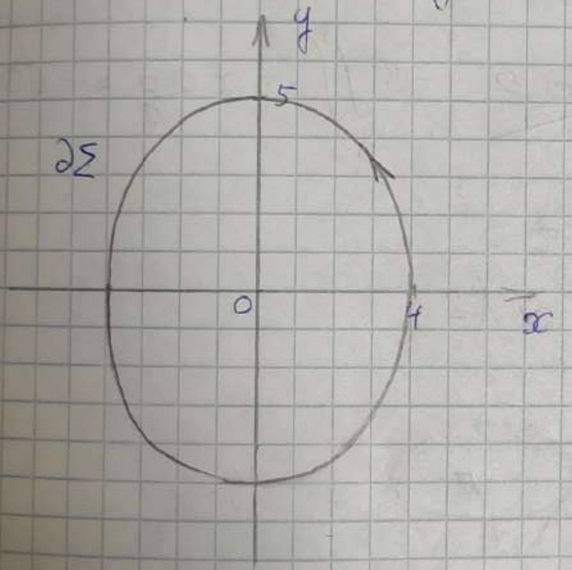
18 декабря.

Матан.

ИДЗ № 2.

2. $\bar{F} = -z\bar{i} - 3x\bar{j} + x\bar{k}$

$\Gamma:\ x = 5\cos t \quad y = 4\sin t \quad z = 4$



$$\oint\limits_{\partial\Sigma} -z\,dx - 3x\,dy + x\,dz =$$

$$= \int_0^{2\pi} \left(20\sin t - \overset{60}{\cancel{15}}\cos^2 t \cancel{+ 5\cos t \cdot 0}\right) dt =$$

$$= -20\cos t\Big|_0^{2\pi} - 60\int_0^{2\pi} \frac{1+\cos 2t}{2}\,dt =$$

$$= -60 \cdot \frac{1}{2} t \Big|_0^{2\pi} - 30 \, \frac{\sin 2t}{2} \Big|_0^{2\pi} =$$

$$= -60\pi$$

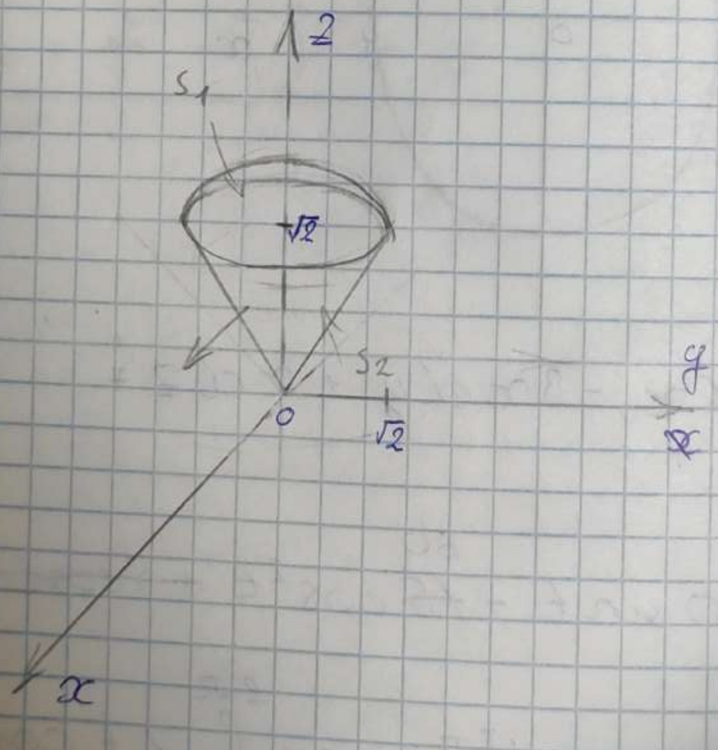
$$C_{\partial\Sigma}(\bar{F}) = \iint\limits_{\Sigma} \underbrace{(-1 + 1)\, dx\, dz}_{0} + \overset{-3\,dx\,dy}{\cancel{Q}} =$$

площадь эллипса

$$= \cancel{-2 \iint\limits_{\Sigma} dx\, dz} - 3 \iint\limits_{S} dx\, dy = -60\pi$$

Ответ: -60π

3.



$$\underset{\Sigma}{\Pi}(\bar F) = \iint_\Sigma \underbrace{x^2 dy dz}_{I_1} + \underbrace{y^2 dx dz}_{I_2} + \underbrace{z^2 dy dz}_{I_3}$$

I_1 при $x > 0$ равен I_1 при $x < 0 \Rightarrow$
$\Rightarrow I_1 = 0$

Аналогично $I_2 = 0$.

$$I_3 = \overbrace{\iint_{S_1} z^2 dx dy}^{I_{31}} - \overbrace{\iint_{S_2} z^2 dxy}^{I_{32}}$$

$$I_{31} = \iint_{S_1} 4 - x^2 - y^2 dx dy =$$

$$I_{32} = \iint_{S_1} x^2 + y^2$$

$$I_3 = \iint_{S_1} 4 - 2x^2 - 2y^2 dx dy = \int_0^{2\pi} d\varphi \int_0^{\sqrt2} (4 - 2r^2) \cdot r dr =$$

$$= 2\pi \left(2r^2 - \frac{r^4}{2}\right)\Big|_0^{\sqrt2} = 4\pi$$

$$\underset{\Sigma}{\Pi}(\bar a) = \iiint_\Omega (2x + 2y + 2z) dx dy dz =$$

$$= \cancel{2} \int_0^{\frac{\pi}{4}} d\Theta \int_0^{2\pi} d\varphi \int_0^{2} 2r^4 (\cos\varphi \sin\Theta + \sin\varphi \sin\Theta + \cos\Theta) dr =$$

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$$= \frac{16}{3} 8 \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} d\Theta \int_{0}^{2\pi} (\cos\varphi \sin^2\Theta + \sin\varphi \sin^2\Theta + \sin\Theta\cos\Theta) d\varphi =$$

$$= \frac{16}{3} 8 \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin\varphi \sin^2\Theta - \cos\varphi \sin^2\Theta + \varphi \sin\Theta \cos\Theta \Big|_0^{2\pi} d\Theta =$$

$$= \frac{16}{3} 8 \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (\cancel{2\sin^2\Theta} + 2\pi \sin\Theta\cos\Theta) d\Theta =$$

$$= 8\frac{16}{3} \left(\cancel{\Theta - \frac{\sin 2\Theta}{2}} + 2\pi \frac{\sin^2\Theta}{2} \right) \Bigg|_{\frac{\pi}{4}}^{\frac{\pi}{2}} =$$

$$= \cancel{\frac{16}{3}\left(\frac{\pi}{2} - \frac{1}{2} + \frac{\pi}{2}\right) = \frac{16\pi}{3} - \frac{8}{3}} \quad 4\pi$$

<u>Ответ:</u> 4π

1. $(a^2x^2+b^2y^2+c^2z^2)^2 = cz(x^2+y^2)$

$$x = \frac{r\cos\varphi\sin\theta}{a}$$

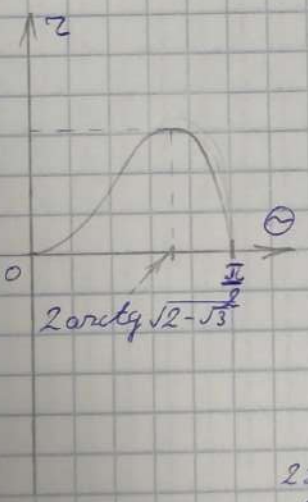
$$y = \frac{r\sin\varphi\sin\theta}{b}$$

$$z = \frac{r\cos\theta}{c}$$

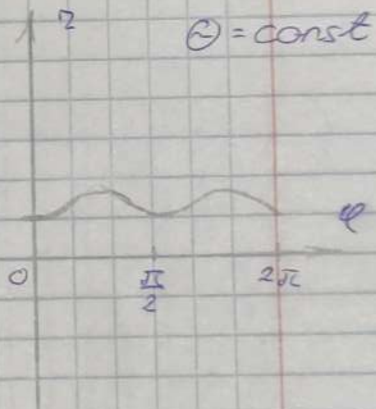
$$r^4 = r\cos\theta \, r^2\sin^2\theta\left(\frac{\cos^2\varphi}{a^2}+\frac{\sin^2\varphi}{b^2}\right)$$

$$r = \cos\theta\sin^2\theta\left(\frac{\cos^2\varphi}{a^2}+\frac{\sin^2\varphi}{b^2}\right), \quad \varphi\in[0;2\pi), \ \theta\in[0;\tfrac{\pi}{2}]$$

$\varphi = const$



$\theta = const$



$$V = \iiint\limits_{\Omega} \frac{r^2\sin\theta}{abc}\, d\varphi\, d\theta\, dr = \int_0^{2\pi} d\varphi \int_0^{\frac{\pi}{2}} d\theta \int_0^{\cos\theta\sin^2\theta\left(\frac{\cos^2\varphi}{a^2}+\frac{\sin^2\varphi}{b^2}\right)} \cancel{dr}\, r^2\sin\theta\, dr =$$

$$= \frac{1}{3}\cdot\frac{1}{abc}\cdot\int_0^{2\pi} d\varphi \int_0^{\frac{\pi}{2}} \cos^3\theta\sin^7\theta\left(\frac{\cos^2\varphi}{a^2}+\frac{\sin^2\varphi}{b^2}\right)^3 d\theta =$$

$$= \frac{1}{3} \cdot \frac{1}{abc} \cdot \int_0^{2\pi} \left(\frac{\cos^2\varphi}{a^2} + \frac{\sin^2\varphi}{b^2} \right)^3 \cdot \int_0^{\frac{\pi}{2}} (\sin^2\Theta - 1)\sin^7\Theta \, d\sin\Theta =$$

$$= \frac{1}{abc\,3} \, \frac{1}{a^6 b^6} \, \frac{\pi(5b^6 + 3a^2b^4 + 3a^4b^2 + 5a^6)}{8} \cdot \frac{1}{40}$$