21 марта.

Матан.

ДЗ - 4.

1. $\int_0^1 e^x dx = e-1$

$\sigma_\tau = \sum_{i=1}^{n} e^{\frac{i}{n}} \cdot \frac{1}{n}$

$\lim_{n\to\infty} \sum_{i=1}^{n} e^{\frac{i}{n}} \cdot \frac{1}{n} = \lim_{n\to\infty} \frac{1}{n} \frac{e^{\frac{1}{n}}(1-e^{\frac{n}{n}})}{(1-e^{\frac{1}{n}})} =$

$= \lim_{n\to\infty} \frac{e^{\frac{1}{n}}(e-1)}{n \cdot \frac{1}{n}} = e-1.$

2. $\lim_{n\to\infty} \left(\frac{n}{n^2+1} + \frac{n}{n^2+2^2} + \ldots + \frac{n}{2n^2}\right) =$

$\sigma_\tau = \sum_{k=1}^{n} \frac{n}{n^2+k^2} = \sum_{k=1}^{n} \frac{n^2}{n^2+k^2} \cdot \frac{1}{n}$

$f\left(\frac{k}{n}\right) = \frac{n^2}{n^2+k^2} = \frac{1}{1+\left(\frac{k}{n}\right)^2}$

$f(x) = \frac{1}{1+x^2}$

$\int_0^1 \frac{1}{1+x^2} dx = \operatorname{arctg} 1 - \operatorname{arctg} 0 = \frac{\pi}{4}.$

$= \frac{\pi}{4}.$

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4. $\lim\limits_{x \to 0+} \frac{1}{x^3} \int\limits_0^{4x^2} \arcsin\sqrt{t}\, dt =$

$= \lim\limits_{x \to 0+} \dfrac{\left(\int\limits_0^{4x^2} \arcsin\sqrt{t}\, dt \right)'_x}{(x^3)'_x} =$

$= \lim\limits_{x \to 0+} \dfrac{8x \arcsin 2x}{3x^2} =$

$= \lim\limits_{x \to 0+} \dfrac{8 x \cdot x \cdot 2}{3x^2} = \dfrac{16}{3}$

3. $\int\limits_0^{\pi} \dfrac{x \sin x\, dx}{1+\cos^2 x} = -\int\limits_0^{\pi} x\, d\, \mathrm{arctg}(\cos x)) =$

$= -x\, \mathrm{arctg}(\cos x) \Big|_0^{\pi} + \int\limits_0^{\pi} \mathrm{arctg}(\cos x) dx$ (1)

$t = x - \frac{\pi}{2}$

$\int\limits_{-\frac{\pi}{2}}^{\frac{\pi}{2}} -\mathrm{arctg}(\sin(t))\, dt = 0$, т.к. ф-я нечёт.

$(=) - \pi\left(-\frac{\pi}{4}\right) = \frac{\pi^2}{4}$

5. f - непр., $\lim\limits_{x \to +\infty} f(x) = A$

$$\lim_{n \to +\infty} \int_0^1 f(nx)\,dx = \{$$

$t = nx \qquad x = \frac{t}{n} \qquad dx = \frac{dt}{n}$

$$= \lim_{n \to \infty} \int_0^{\frac{1}{n}} \frac{f(t)}{n}\,dt = \lim_{n \to \infty} \frac{\int_0^{\cancel{\frac{1}{n}}\,n} f(t)\,dt}{n} =$$

$$= \lim_{n \to \infty} \frac{f(n)}{n'} = \lim_{n \to \infty} f(n) = A$$

6. $$\int_{-1}^{1} \frac{1+x^2}{1+x^4}\,dx =$$

$$\int \frac{\frac{1}{x^2}+1}{\underbrace{\frac{1}{x^2}+x^2}_{=\left(x-\frac{1}{x}\right)^2+2}}\,dx = \int \frac{dt}{t^2+2} = \frac{1}{\sqrt{2}} \operatorname{arctg} \frac{t}{\sqrt{2}} =$$

$$= \frac{1}{\sqrt{2}} \operatorname{arctg} \frac{x-\frac{1}{x}}{\sqrt{2}}$$

$t = x - \frac{1}{x} \qquad dt = 1 + \frac{1}{x^2}$

$$= 2\int_0^1 \frac{1+x^2}{1+x^4}\,dx = \frac{2}{\sqrt{2}} \operatorname{arctg} \frac{x-\frac{1}{x}}{\sqrt{2}} \Big|_0^1 =$$

$$= \frac{\pi}{\sqrt{2}}$$