17 сентября.

Диффуры.

Дз №2

2.1) $xy' = y(\ln y - \ln x)$

$\frac{x}{y} y' = \ln \frac{y}{x}$ $\quad \exists\, z = \frac{y}{x}$

$y' = zx = z'x + z$

$z'x + z = z \ln z$

$x \frac{dz}{dx} = z \ln z - z$

$\frac{dz}{z \ln z - z} = \frac{dx}{x}$

$\int \frac{d \ln z}{\ln z - 1} = \ln(\ln(z) - 1) + C_1$

$\ln(\ln(z) - 1) = \ln(x) + C$

$\ln\left(\ln \frac{y}{x} - 1\right) = \ln(x) + C$

2.2) $2x+3y-5+(3x+2y-5)y'=0$

$y' = \frac{2x+3y-5}{5-3x-2y}$

$\begin{cases} 2x+3y-5=0 \\ 5-3x-2y=0 \end{cases} \Rightarrow \begin{cases} x=1 \\ y=1 \end{cases}$

$\begin{cases} u=x-1 \\ v=y-1 \end{cases}$ $\quad y' = \frac{2u+3v}{-3u-2v}$

$dy(-3u-2v)=dx(2u+3v)$

$(2u+3v)\,d\overset{u}{x}+(3u+2v)\,d\overset{v}{y}=0$

$w=\frac{v}{u}$ $\quad dv = (wu)' = dw\cdot u + du\cdot w$

$u(2+3w)du + u(3+2w)(dw\cdot u + du\cdot w)=0$

$\cancel{2u\,du + 3uw\,du + 3u^2\,dw + 3uw\,du +}$

$\cancel{+ 2u^2w\,dw + 2w^2u\,du = 0}$

$(2+3w)du + (3+2w)u\,dw + (3w+2w^2)du=0$

$(2w^2+6w+2)du = -(3+2w)u\,dw$

$\frac{du}{u} = \frac{-(3+2w)}{2w^2+6w+2}dw$

$\frac{dv}{du} = -\frac{2u+3v}{3u+2v} = -\frac{2+3\frac{v}{u}}{3+2\frac{v}{u}}$

$v=tu$ $\quad dv = t\,du + u\,dt$

$$\ln|x| = \frac{1}{\sqrt{z^2+1}+1} + \frac{1}{2}\ln|\sqrt{z^2+1}+1| -$$

$$-\frac{1}{2}\ln|\sqrt{z^2+1}-1| + C$$

2.5) $F = ma \quad a = \frac{dv}{dt}$

$F = kv$

$m\frac{dv}{dt} = kv$

$\ln|v| = \frac{k}{m}t + C_1 \quad \frac{k}{m} = k_0$

$v = C \cdot e^{k_0 t}$

$v(0) = C = 1{,}5$

$\underset{\overset{\parallel}{1}}{v(4)} = 1{,}5 \cdot e^{k_0 t 4} \Rightarrow k_0 = \frac{\ln\frac{2}{3}}{4}$

$v(t) = 1{,}5 \cdot e^{\frac{\ln\frac{2}{3}}{4}t} = \frac{3}{2}\left(\frac{2}{3}\right)^{\frac{t}{4}} =$

$= \left(\frac{2}{3}\right)^{\frac{t}{4}-1}$

$v(t_1) = 0{,}01 = \left(\frac{2}{3}\right)^{\frac{t}{4}-1}$

~~$t = \left(\frac{3}{200}+1\right)\frac{1}{4} = \frac{203}{800}c$~~

$$t_1 = 4\left(1+\frac{\ln 0{,}01}{\ln\frac{2}{3}}\right) \approx 50\text{с}$$

$$v(t) = \frac{ds(t)}{dt}$$

$$ds = \left(\frac{2}{3}\right)^{\frac{t}{4}-1} dt$$

$$s = \frac{4}{\ln\frac{2}{3}}\left(\frac{2}{3}\right)^{\frac{t}{4}-1} + C$$

$$] \; s(0)=0 \Rightarrow C = \cancel{0} = -\frac{4}{\ln\frac{2}{3}}\left(\frac{2}{3}\right)^{-1}$$

$$s(t) = \frac{6}{\ln\frac{2}{3}}\left(\left(\frac{2}{3}\right)^{\frac{t}{4}} - 1\right)$$

$$s_1 = \left|\lim_{t\to\infty} s(t)\right| = \frac{6}{\ln\frac{3}{2}} \approx 15\text{м}$$

<u>Ответ:</u> 50с; 15м.

2.6)

2. $y' = y(2-y)$

$-\frac{dy}{y(y-2)} = dx$

$-\frac{1}{2}\int\left(\frac{1}{y-2} - \frac{1}{y}\right)dy = -\frac{1}{2}\ln|y-2| + \frac{1}{2}\ln|y| + C_1$

$\parallel$

x

$y=0$ и $y=2$ также являются решениями ур-я.

$y = \frac{2e^{2x-c}}{e^{2x-c}-1} = \frac{2e^{2x}}{e^{2x}-C}$

3. Интегральные кривые для C: 4, 0, −4, −8, −12 (сверху вниз соот-но)

4. $y(0) = \frac{1}{2} = \frac{2}{1-C}$ $\quad 1-C = 4 \Rightarrow C = -3$

6. $h = 0{,}5$: ошибка ~~0,1476~~

0,014763

$h = 0{,}1$: ошибка 0,004518

2.7) $(x^2+2xy-y^2)dx+(y^2+2xy-x^2)dy=$

$$y' = -\frac{x^2+2xy-y^2}{y^2+2xy-x^2}$$

$$z=\frac{y}{x} \qquad y' = z'x+z$$

+4

$$z'x+z = -\frac{x^2+2x^2z-x^2z^2}{x^2z^2+2x^2z-x^2} =$$

$$= \frac{z^2-2z-1}{z^2+2z-1}$$

$$\frac{dz}{\frac{z^2-2z-1}{z^2+2z-1}-z} = \frac{dx}{x}$$

$\parallel$

$$\frac{z^2+2z-1}{(z+1)(z^2+1)} = \frac{A}{z+1}+\frac{Bz+C}{z^2+1} =$$

$$A(z^2+1)+(z+1)(Bz+C) = z^2+2z-1$$

$z=-1$: $2A = 1-2-1=-2 \Rightarrow A=-1$

z^2: $A+B=1 \Rightarrow B=2$

z^0: $A+C=-1 \Rightarrow C=0$

$$= -\frac{1}{z+1}+\frac{2z}{z^2+1}$$

$$\ln(|z+1|)-\ln(z^2+1)+C = \ln|x|$$

$$\ln\frac{|z+1|}{z^2+1} = \ln|x|+C$$

$$\frac{z+1}{z^2+1} = Cx \qquad z+1 = Cxz^2+Cx$$

$$Cxz^2 - z + Cx - 1 = 0$$

$$\cancel{z = \frac{1 \pm \sqrt{5 - 4Cx^2}}{2Cx}}$$

$$z = \frac{1 \pm \sqrt{1 - 4Cx(Cx-1)}}{2Cx} =$$

$$= \frac{1 \pm \sqrt{1 - 4C^2x^2 + 4Cx}}{2Cx}$$

$$y = \frac{1 \pm \sqrt{1 - 4Cx(Cx-1)}}{2C} =$$

$$= \frac{1 \pm \sqrt{1 - 4C^2x^2 + 4Cx}}{2C}$$

$$ky(kx) = \frac{1 \pm \sqrt{1 - 4k^2C^2x^2 + 4Ckx}}{2C}$$

$$\exists\, C_1 = Ck$$

$$y = \frac{1 \pm \sqrt{1 - 4C_1^2x^2 + 4C_1x}}{2C_1} \Rightarrow$$

$\Rightarrow$ мн-во решений переходит в себя при гомотетии

2.4) $4y^6 + x^3 = 6xy^5y'$

$\exists\, z = y^6 \qquad 6y^5y' = (y^6)' = z'$

$4z + x^3 = \cancel{6}x\,z' \qquad 4\frac{z}{x^3} + 1 = x^{-2}z'$

$\cancel{4z} = \quad \exists\, u = \frac{z}{x^3}$

$u' = \frac{z'x^3 - 3x^2z}{x^6} = \frac{z'x - 3z}{x^4}$

$z' = \frac{u'x^4 + 3z}{x} = u'x^3 + 3\frac{z}{x}$

$\frac{z'}{x^2} = u'x + 3\frac{z}{x^2} = u'x + 3u$

$4u + 1 = u'x + 3u$

$u + 1 = \frac{x\,du}{dx}$

$\ln|u+1| = \ln|x| + C$

$u + 1 = C\cdot x$

$\frac{y^6}{x^3} + 1 = C\cdot x \Rightarrow y^6 = Cx^4 - x^3$

<u>Ответ:</u> $y^6 = Cx^4 - x^3$

2.8) $(x+y)dx + (y-x)dy = 0$

$$\frac{x+y}{x-y} = \frac{dy}{dx} \qquad] \; z = \frac{y}{x}$$

$$y' = z'x + z$$

$$\frac{1+z}{1-z} = z'x + z$$

$$\frac{1+z - z(1-z)}{1-z} = z'x$$

$$\frac{1+z}{1-z} = x\frac{dz}{dx}$$

$$\frac{dx}{x} = \frac{1-z}{1+z^2}dz = \cancel{\frac{z+1-2}{1+z}dz}$$

$$\cancel{\ln|x| = -z + 2\ln|1+z| + C}$$

$$\int \frac{1-z}{z^2+1}dz = \operatorname{arctg} z - \frac{1}{2}\ln|z^2+1| + C$$

$$\parallel$$

$$\ln x$$

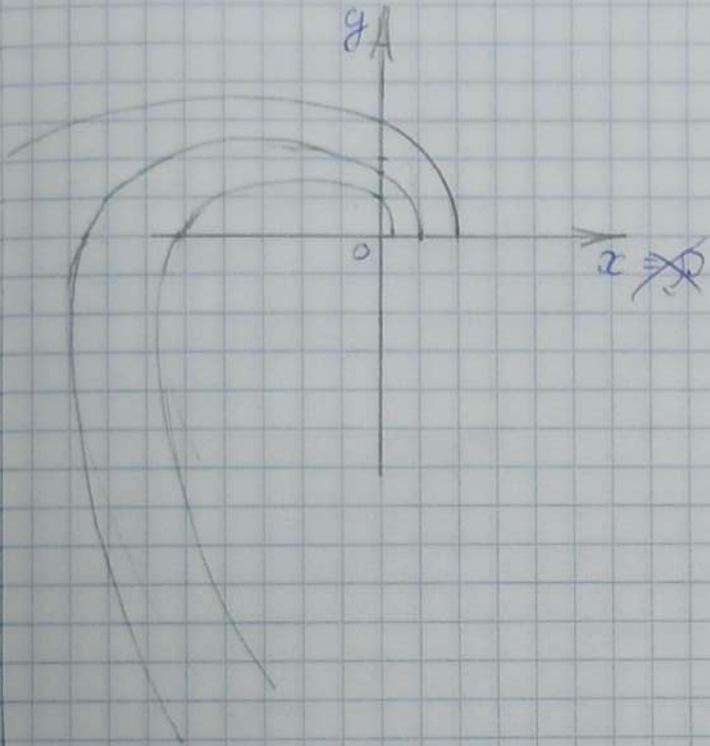
$z = \operatorname{tg}\varphi$

$$\operatorname{arctg}\operatorname{tg}\varphi - \frac{1}{2}\ln|\operatorname{tg}^2\varphi + 1| + C = \ln(\rho\cos\varphi)$$

$$\varphi - \ln\sqrt{\frac{1}{\cos^2\varphi}} + C = \ln(\rho\cos\varphi)$$

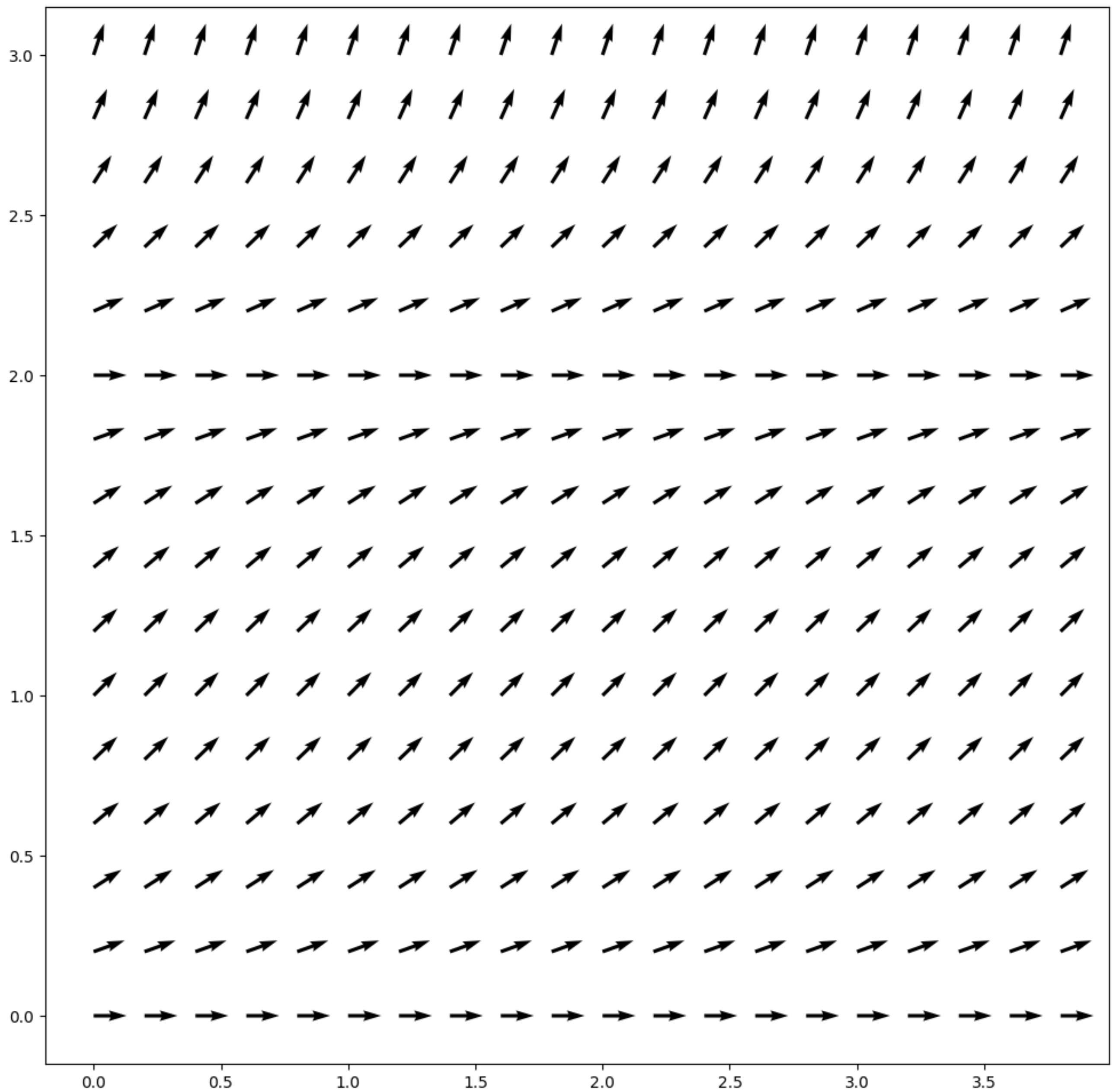
$$\varphi + \ln\cos\varphi + C = \ln(\rho\cos\varphi)$$

$\varphi + C = \ln \rho \qquad \rho = C_1 \cdot e^{\varphi}$



$\cancel{\rho = 0} \quad \rho(0) = 1 \Rightarrow 1 = C_1 \cdot e^{0} \Rightarrow C_1 = 1$

$\rho = e^{\varphi} \quad \rho(\pi) = e^{\pi} \approx 23,14\ldots$



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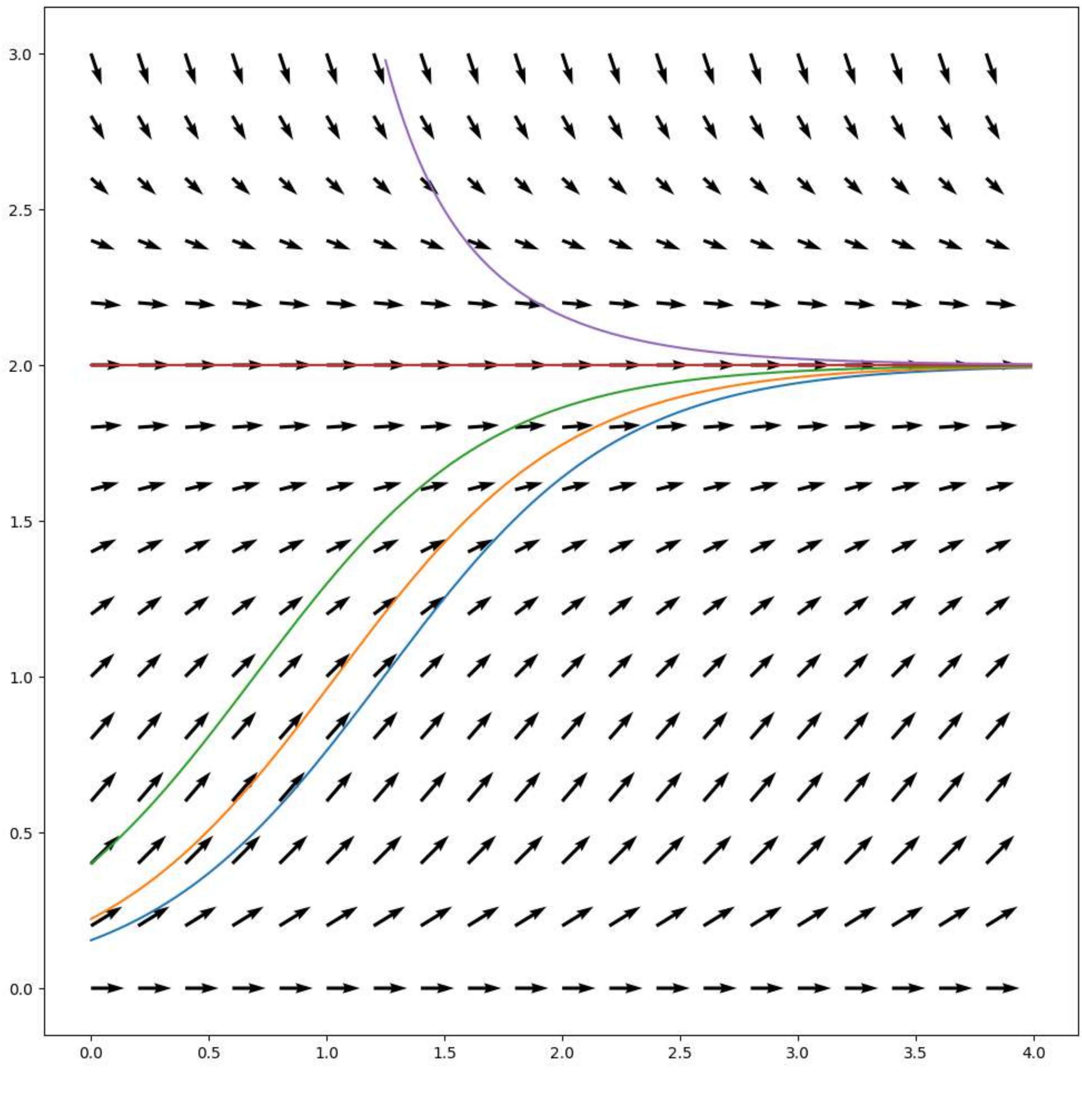
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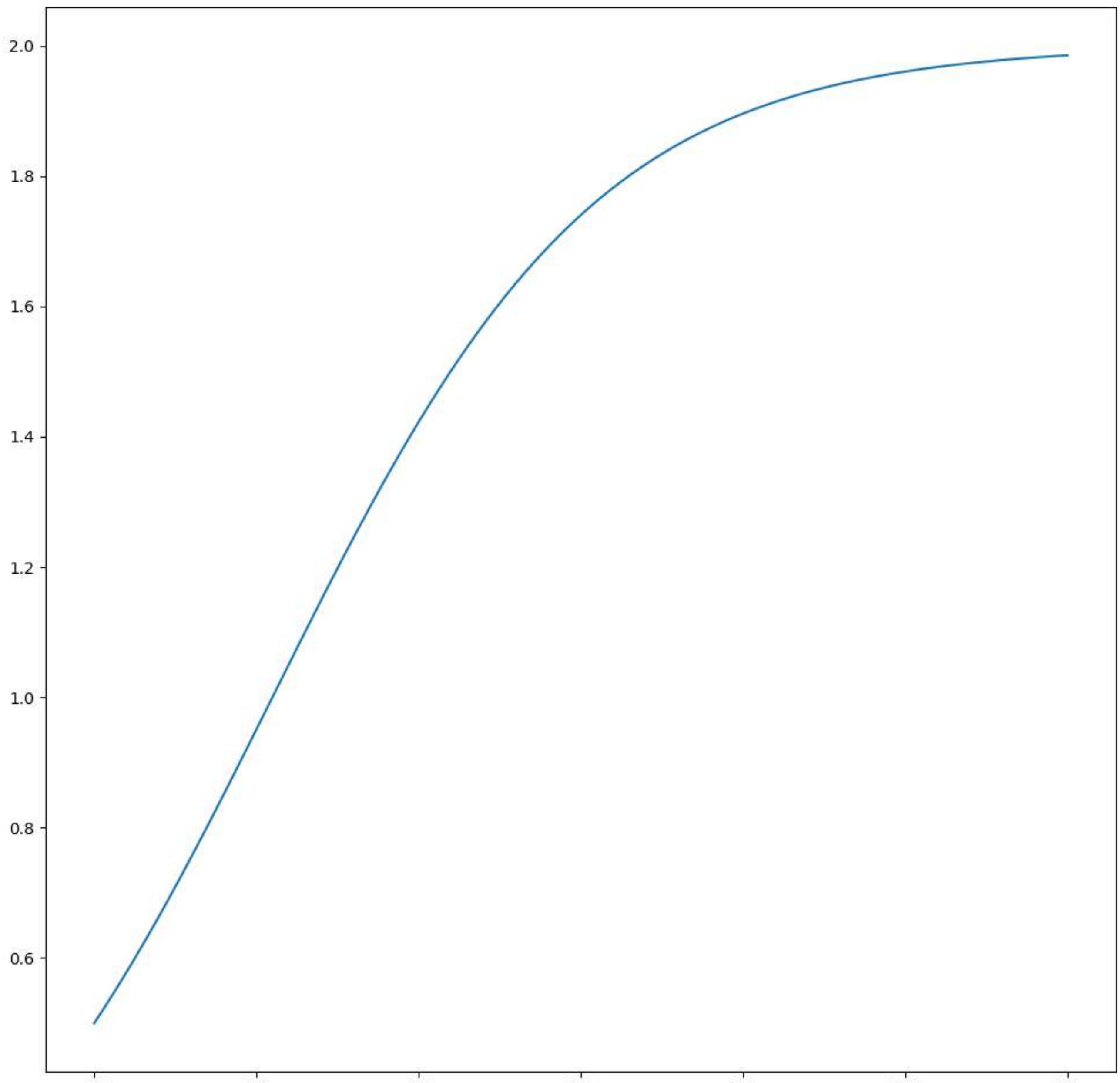
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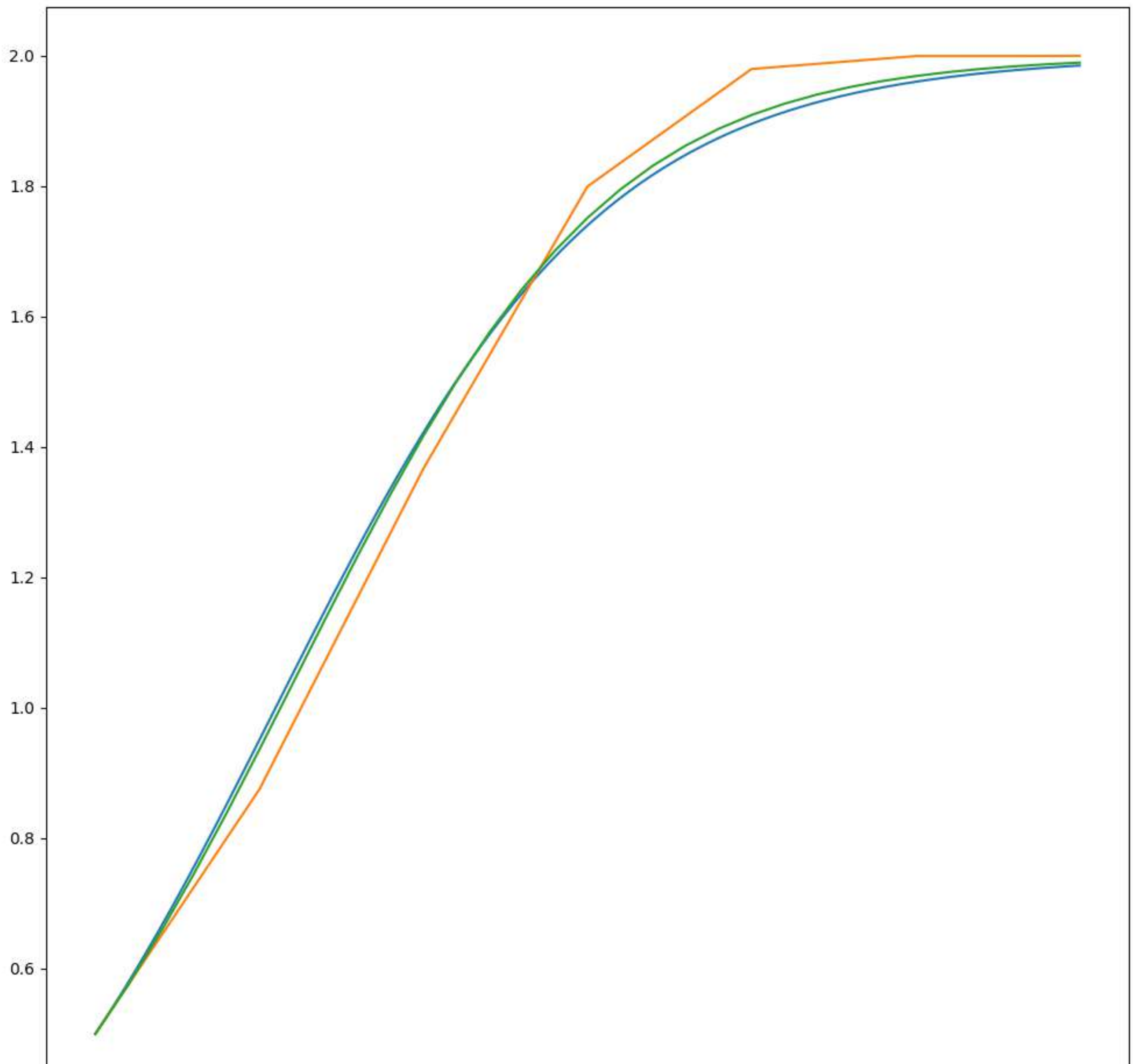
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