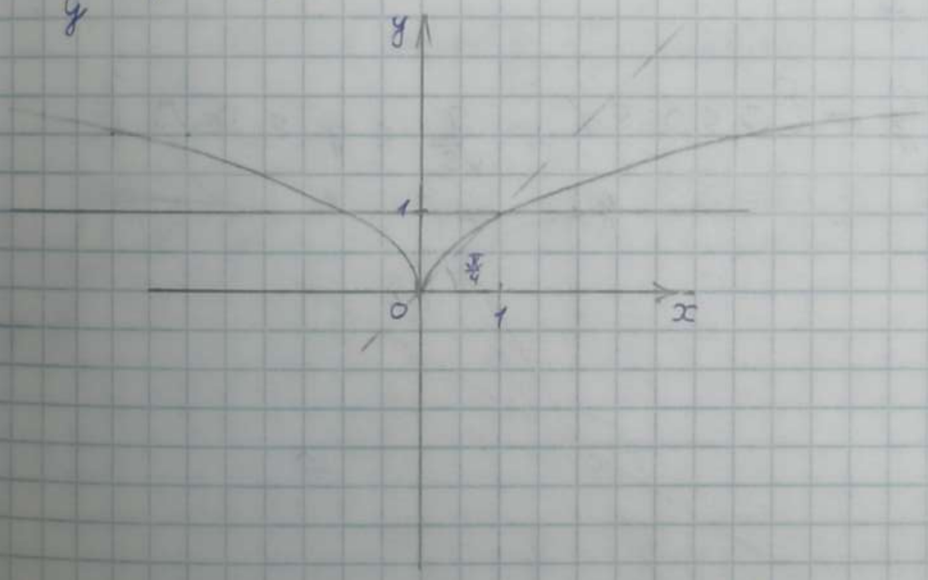
14 ноября.

Матан.

Дз №6

неправильно

1. $\iint\limits_G f\left(\frac{xy}{x^2+y^2}\right)dx\,dy$ $\qquad G = \{\sqrt{|x|} \le y \le 1\}$



$f\left(\frac{xy}{x^2+y^2}\right) = f(\sin\varphi\cos\varphi) = f(\varphi)$

$\iint\limits_G f\,dx\,dy = \iint 2\left(\frac{1}{2} - \int\limits_{\frac{\pi}{4}}^{\frac{\pi}{2}} d\varphi \int\limits_0^{\frac{\cos\varphi}{\sin^2\varphi}} r f(\varphi)\,dr\right) = \iint\limits_{G_1} f\,dx\,dy$

$G_1 = \{x \le y \le \sqrt{x}\}$ $\qquad r^2\sin^2\varphi = r\cos\varphi$

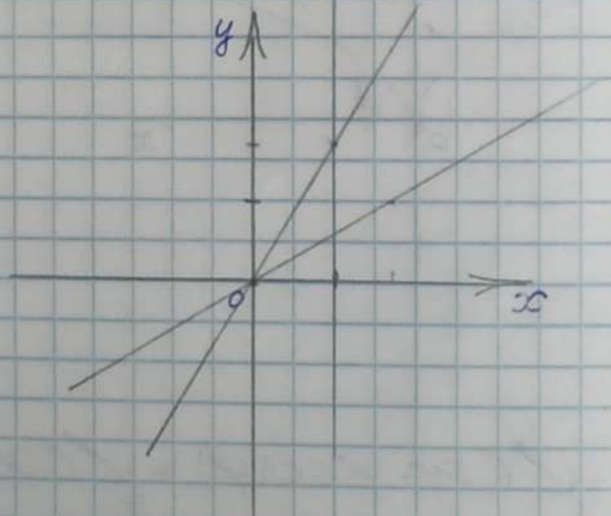
$r = \frac{\cos\varphi}{\sin^2\varphi}$

$$= 2\left(1 - 2\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} f(\varphi)\,\frac{\cos^2\varphi}{2\sin^2\varphi}\,d\varphi\right.$$

Ответ:

2. $$\iint\limits_{G} f(x^2+y^2)\,dx\,dy =$$

$$G = \left\{0 \le x \le 1,\ \frac{x}{\sqrt{3}} \le y \le x\sqrt{3}\right\}$$



$$\frac{\cos\varphi}{\sqrt{3}} = \sin\varphi \Rightarrow \varphi = \operatorname{arctg}\frac{1}{\sqrt{3}}$$

$$\varphi = \operatorname{arctg}\sqrt{3}$$

$$2\cos\varphi = 1 \Rightarrow \varphi = \arccos\frac{1}{2}$$

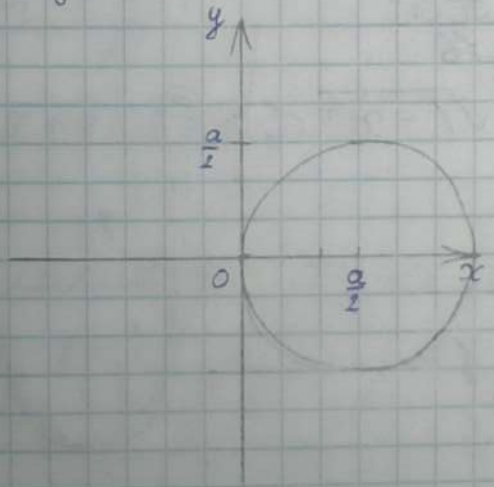
$$f = f(r^2) = f(r)$$

$$= \int_0^{\frac{2}{\sqrt3}} dr \int_{\operatorname{arctg}\frac{1}{\sqrt3}}^{\operatorname{arctg}\sqrt3} f(r)\, r\, d\varphi + \int_{\frac{2}{\sqrt3}}^{2} dr \int_{\arccos\frac12}^{\operatorname{arctg}\sqrt3} f(r)\, r\, d\varphi =$$

$$= \int_0^{\frac{2}{\sqrt3}} f(r)\, r \left(\operatorname{arctg}\sqrt3 - \operatorname{arctg}\frac{1}{\sqrt3}\right) dr +$$

$$+ \int_{\frac{2}{\sqrt3}}^{2} f(r)\, r \left(\operatorname{arctg}\sqrt3 - \arccos\frac{1}{2}\right) dr$$

3. $\iint\limits_G \frac{y^2}{x^2+y^2}\, dx\, dy =$ $\quad G = \{x^2+y^2 \leq ax,\ a>0\}$



$x^2+y^2 = ax \qquad \left(x-\frac{a}{2}\right)^2 + y^2 = \frac{a^2}{4}$

$r^2 \cancel{\cos} = a r \cos\varphi \qquad \frac{y^2}{x^2+y^2} = \sin^2\varphi$

$r = a\cos\varphi$

$$= 2\int_0^{\frac{\pi}{2}} d\varphi \int_0^{a\cos\varphi} \sin^2\varphi \, r \, dr =$$

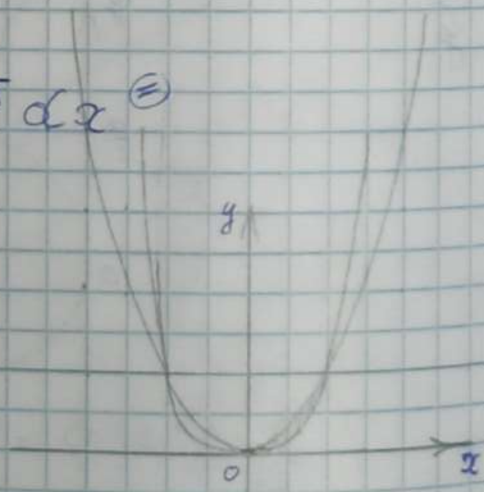
$$= 2\int_0^{\frac{\pi}{2}} \sin^2\varphi \, \frac{a^2\cos^2\varphi}{2} \, d\varphi =$$

$$= a^2\int_0^{\frac{\pi}{2}} \frac{\sin^2 2\varphi}{4} \, d\varphi = a^2 \int_0^{\frac{\pi}{2}} \frac{1-\cos 4\varphi}{8} \, d\varphi =$$

$$= \frac{a^2}{8}\left(\varphi - \frac{\sin(\varphi 4)}{4}\right)\Big|_0^{\frac{\pi}{2}} = \frac{\pi a^2}{16}$$

Ответ: $\frac{\pi a^2}{16}$.

4. $\int_0^1 dy \int_{\sqrt{y}}^{\sqrt[5]{y}} \sqrt{1-x^3}\, dx =$



$0 \le y \le 1$

$\sqrt{y} \le x \le y^{\frac{1}{5}}$

$y = x^2$

$y = x^5$

$0 \le x \le 1$

$G: \{x^5 \le y \le x^2, \ 0 \le x \le 1\}$

$f = \sqrt{1-x^3}$

$$\ominus \iint\limits_{G} \sqrt{1-x^3}\, dx\, dy = \int_0^1 dx \int_{x^5}^{x^2} \sqrt{1-x^3}\, dy =$$

$$= \int_0^1 \sqrt{1-x^3}\,(x^2 - x^5)\, dx =$$

$$= \frac{1}{3}\int_0^1 \sqrt{1-x^3}\,(1-x^3)\, dx^3 = \frac{1}{3}\int_0^1 -\frac{2}{5}\left(\sqrt{1-x^3}\right)^5 =$$

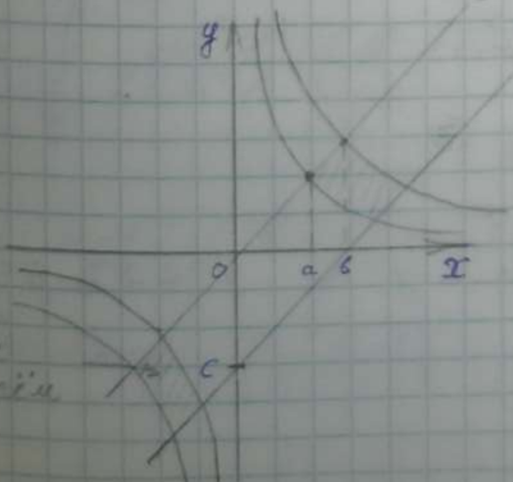
$$= -0 + \frac{2}{15} = \frac{2}{15}$$

<u>Ответ:</u> $\frac{2}{15}$.

5. $\iint\limits_{G} (x+y)\, dx\, dy \ominus$

G огр. линиями $xy = a$, $xy = b$, $y = x$, $y = x - c$

$(0 < a < b,\ c > 0)$



$x - c \leq y \leq x$

$\frac{a}{y} \leq x \leq \frac{b}{y}$

$-c \leq y - x \leq 0$

$a \leq xy \leq b$

$0 \leq x - y \leq c$

$xy = u \qquad x - y = v$

$x = v + y \qquad y = \frac{u}{x}$

$\cancel{x^2 = vx + u}$

$\cancel{x = \frac{v \pm \sqrt{v^2 + 4u}}{2}}$

$$J^{-1} = \begin{vmatrix} u'_x & u'_y \\ v'_x & v'_y \end{vmatrix} = \begin{vmatrix} y & x \\ 1 & -1 \end{vmatrix} = -y - x$$

$|J| = +\frac{1}{x+y}$

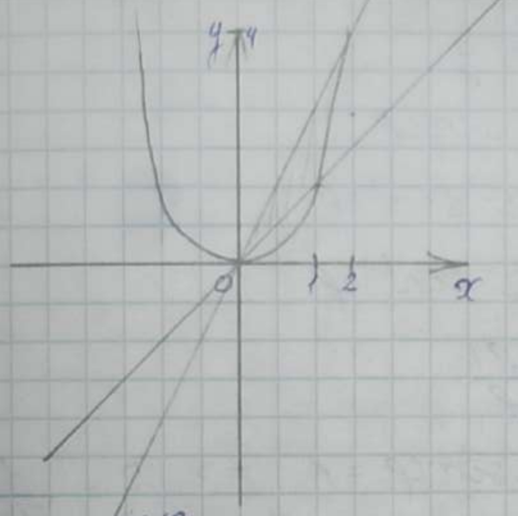
$$\ominus \iint\limits_{G'} du\,dv = \int_0^c dv \int_a^b du =$$

$= (b-a)c$

<u>Ответ:</u> $(b-a)c$.

6. $\iint\limits_G e^{\frac{x^4}{y^2}} dx\,dy \ominus$

G огр. линиями $\quad y = x \quad y = 2x \quad y = x^2$



$$= \int\limits_{\frac{\pi}{4}}^{arctg 2} d\varphi \int\limits_{0}^{\frac{\sin\varphi}{\cos^2\varphi}} e^{\frac{r^2\cos^4\varphi}{\sin^2\varphi}}\, r\, dr =$$

$$= \int\limits_{\frac{\pi}{4}}^{arctg 2} \frac{d\varphi \sin^2\varphi}{\cos^4\varphi\, 2} \left(e^{\frac{r^2\cos^4\varphi}{\sin^2\varphi}} \right|_{0}^{\frac{\sin\varphi}{\cos^2\varphi}} =$$

$$= \int\limits_{\frac{\pi}{4}}^{arctg 2} \frac{\sin^2\varphi}{2\cos^4\varphi}(e-1)\, d\varphi = \frac{(e-1)}{2} \cancel{\int\limits_{\frac{\pi}{4}}^{arctg 2}} \frac{tg^3(\varphi)}{3} \Bigg|_{\frac{\pi}{4}}^{arctg 2} =$$

$$= \frac{e-1}{6}(8-1) = \frac{7(e-1)}{6}$$

Ответ: $\frac{7}{6}(e-1)$

$y^2 = \sqrt{|x|} = y$

$\sqrt{|r\cos\varphi|} = r\sin\varphi$

$|r\cos\varphi| = r^2\sin^2\varphi$

$|\cos\varphi| = r\sin^2\varphi$

$r = \dfrac{|\cos\varphi|}{\sin^2\varphi}$

$y = 1 \qquad r\sin\varphi = 1 \Rightarrow r = \dfrac{1}{\sin\varphi}$

$$\iint\limits_{G} f\,dx\,dy = \int\limits_{\frac{\pi}{4}}^{\frac{3\pi}{4}} d\varphi \int\limits_{\frac{|\cos\varphi|}{\sin^2\varphi}}^{\frac{1}{\sin\varphi}} f(\varphi)\,r\,dr =$$

$$= \int\limits_{\frac{\pi}{4}}^{\frac{3\pi}{4}} f(\sin\varphi\cos\varphi)\left(\frac{1}{2\sin^2\varphi} - \frac{\cos^2\varphi}{2(\sin^2\varphi)^2}\right)d\varphi$$