$$\int_{-\infty}^{+\infty} \sin(\xi t)\, t e^{-at^2} dt$$

$$\frac{\sin \xi t}{-2a}\, d e^{-at^2}$$

$$\int_{0}^{+\infty} t e^{-at^2} \sin(\xi t$$

$$\int^{+\infty} t \cancel{x} e^{-at^2} \sin$$

26 мая.

ИДЗ №3.

Вариант 53.

1. $f(t) = t e^{-at^2}, \quad a > 0$

$$f(x) \sim I = \int_{-\infty}^{+\infty} (a(\xi)\cos(\xi x) + b(\xi)\cdot$$

$$\cdot \sin(\xi x))\, d\xi$$

$$a(\xi) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} t e^{-at^2} \cos(t\xi)\, d\xi = 0$$

т.к. ф-я нечёт.

$$b(\xi) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} t \cancel{x} e^{-at^2} \overset{\sin}{\cancel{\cos}}(t\xi)\, dt =$$

$$= \frac{1}{2\pi} \int_{-\infty}^{+\infty} \frac{\overset{\sin}{\cancel{\cos}}(t\xi)}{-2a}\, d e^{-at^2} = \underbrace{-\frac{1}{4\pi a} e^{-at^2} \overset{\cancel{\cos}}{\sin} t\xi \Big|_{-\infty}^{+\infty}}_{=0}$$

$$+ \left(+\frac{1\cdot\xi}{4\pi a}\right) \cdot \underbrace{\int_{-\infty}^{+\infty} e^{-at^2} \overset{\cos}{\cancel{\sin}}(t\xi)\, dt}_{= I_1}$$

$$I_1 = 2\int_{\cancel{-\infty}\ 0}^{\infty} e^{-at^2} \cos(tb)\, dt =$$

$$\int_0^{+\infty} t e^{-at^2} \sin(\xi t)\, dt$$

$$\int_{-\infty}^{+\infty} t \cancel{\cos} e^{-at^2} \sin \xi t\, dt = \int_{-\infty}^{+\infty} \frac{\sin \xi t}{-2a} d$$

$$= \int_{0-\infty}^{\infty} e^{-at^2} \cdot \frac{e^{ibt} + e^{-ibt}}{2} dt =$$

$$= \frac{1}{2}\int_{0-\infty}^{\infty} e^{-at^2+ibt} dt + \frac{1}{2}\int_{0-\infty}^{\infty} e^{-at^2-ibt} dt =$$

$$= \frac{1}{2}\int_{0-\infty}^{\infty} e^{\left(\sqrt{-a}t + \frac{ib}{2\sqrt{-a}}\right)^2 - \frac{b^2}{4a}} dt +$$

$$+ \frac{1}{2}\int_{0-\infty}^{\infty} e^{\left(\sqrt{-a}t - \frac{ib}{2\sqrt{-a}}\right)^2 - \frac{b^2}{4a}} dt =$$

$$= \frac{e^{-\frac{b^2}{4a}}}{2\sqrt{a}} \int_{0-\infty}^{\infty} e^{-\left(\sqrt{a}t + \frac{b}{2\sqrt{-a}}\right)^2} d\left(\sqrt{a}t + \frac{b}{2\sqrt{-a}}\right) +$$

$$+ \frac{e^{-\frac{b^2}{4a}}}{2\sqrt{a}} \int_{0-\infty}^{\infty} e^{-\left(\sqrt{a}t - \frac{b}{2\sqrt{-a}}\right)^2} d\left(\sqrt{a}t - \frac{b}{2\sqrt{-a}}\right) =$$

$$= \frac{\sqrt{\pi}\, e^{-\frac{b^2}{4a}}}{2\sqrt{a}} + \frac{\sqrt{\pi}\, e^{-\frac{b^2}{4a}}}{2\sqrt{a}} = \frac{\sqrt{\pi}\, e^{-\frac{b^2}{4a}}}{\sqrt{a}}$$

$$b(\xi) = \frac{1 \cdot \xi}{4\pi a \cancel{\xi}} \frac{\sqrt{\pi}\, e^{-\frac{\xi^2}{4a}}}{\sqrt{a}} = \frac{\xi a^{-\frac{3}{2}} \cdot e^{-\frac{\xi^2}{4a}}}{4\sqrt{\pi}\, \cancel{\xi}}$$

$$f(x) \sim \int_{-\infty}^{+\infty} \frac{\xi a^{-\frac{3}{2}} e^{-\frac{\xi^2}{4a}}}{4\sqrt{\pi}\, \cancel{\xi}} \cdot \sin(\xi x)\, d\xi$$

$$\frac{1}{2n} \quad \frac{1}{2n + \frac{1}{2}}$$

$f(x) = I(x)\ \forall x \in \mathbb{R}$, т.к.
$f \in C(\mathbb{R})$ (а любой)

2.

$$f(t) = \begin{cases} e^{t} \cos t, & t \in (-\infty; 0] \\ e^{-2t}, & t \in (0; +\infty) \end{cases}$$

$$F[f](\xi) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} f(x) e^{-i\xi x} dx =$$

$$= \frac{1}{\sqrt{2\pi}} \underbrace{\int_{-\infty}^{0} e^{t} \cos t \, e^{-i\xi t} dt}_{I_1(\xi)} +$$

$$+ \frac{1}{\sqrt{2\pi}} \underbrace{\int_{0}^{+\infty} e^{-2t} e^{-i\xi t} dt}_{I_2(\xi)}$$

$$I_1(\xi) = \int_{-\infty}^{0} \cos t \, e^{t(1-i\xi)} dt =$$

$$= + \int_{0}^{+\infty} \cos(-t) e^{-(1-i\xi)t} dt =$$

$$= \frac{1-i\xi}{1+(1-i\xi)^2} = \frac{1-i\xi}{2-\xi(\xi+2i)}$$

$$I_2(\xi) = \int_0^{+\infty} e^{(-2-i\xi)t}\,dt = \frac{e^{(-2-i\xi)t}}{-2-i\xi}\Bigg|_0^{+\infty} =$$

$$= \frac{e^{-2t}\left(\cos(-\xi t) + i\sin(-\xi t)\right)}{-2-i\xi}\Bigg|_0^{+\infty} =$$

$$= -\frac{1}{-2-i\xi} = \frac{1}{i\xi+2}$$

$$f(t) \sim \frac{1}{2\pi}\int_{-\infty}^{+\infty}\left(\frac{1-i\xi}{2-\xi(\xi+2i)} + \frac{1}{i\xi+2}\right) e^{it\xi}\, d\xi$$

$$= \frac{1}{2\pi}\int_{-\infty}^{+\infty}\frac{3\xi+4i}{(\xi^3+2\xi+4i)}\, e^{it\xi}\, d\xi \neq = I(t)$$

$I(2) \approx 0{,}018\ldots \qquad f(2) \approx 0{,}018\ldots$

$I(-13) = f(-13) \approx 2{,}051\ldots \cdot 10^{-6}$

3. $$f(x) = \begin{cases} 0 & , x = 0 \\ x\,\mathrm{sign}\left(\sin\frac{\pi}{x}\right) & , 0 < x \leq 1 \end{cases}$$

В $(\cdot)$ $\frac{1}{n}$ $\forall n = 1\ldots\infty$ $f(x) = 0$

Заметим, что $\left|f\left(\frac{1}{n+\frac{1}{2}}\right) - f\left(\frac{1}{n+\frac{3}{2}}\right)\right| =$
~~(по графику видно)~~

$=$

$$\int_{-\infty}^{+\infty} \sin(\xi t)\, t\, e^{-at^2}\, dt$$

$=1$

$=31$

$$\frac{\frac{\sin \xi t}{-2a}\, d\, e^{-at^2}}{e^{-i\xi x} \cdot t\, e^{-at^2}}$$

$=1$

$=3$

$$\int_0^{+\infty} t\, e^{-at^2}$$

$$\int_{-\infty}^{+\infty} t \cancel{\,dx}$$

$$= \frac{1}{n+\frac{3}{2}} + \frac{1}{n+\frac{1}{2}} \qquad \forall n = 1 \ldots +\infty$$

$$V_{-\infty}^{+\infty}(f) \geq \sum_{n=1}^{\infty} \left(\frac{1}{n+\frac{3}{2}} + \frac{1}{n+\frac{1}{2}} \right) = +\infty \Rightarrow V_{-\infty}^{+\infty}(f) = +\infty$$