RA Reading Group: The Financial Accelerator in a Quantitative Business Cycle Framework

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The R.A.'s experience at theory seminars:

1. Hasn't read the papers that the presenter claims should be familiar.
2. Unfamiliar with parts of the model the presenter dismisses as standard in the literature.
3. Few reference points to compare the presenter's results to.

Goals for this session:

1. Expose all of you to one literature standard.
2. Walk through this well-established model carefully so that we can have an easier time working through the next model we are exposed to.
3. Equip ourselves with a frame-work in which to judge the properties of new models presented at seminars or compare empirical results to our newly-developed macro-finance expectations.

The Financial Accelerator in a Quantitative Business Cycle Framework (Bernanke et al. 1999)

- Modify the Dynamic New Keynesian model to allow for financial markets to affect the real economy (Financial Accelerator).

Agents and their roles in the model

Households

Provide labor, consume output, save remaining wages

Retailers

Sell monopolisticly competitive goods, occasionally set prices (Calvo)

Government

Control money supply, consume output

Entrepreneurs

Take out loans to finance investments, invest in capital, produce output

Why retailers?

Want inflation in the model

$\Rightarrow$ We need sticky prices (Calvo)

$\Rightarrow$ Requires fraction of firms to set prices in each period.

$\Rightarrow$ Selling firms exhibit monopolistic competition so that price setters can set prices different from non-price setters.

Differentiated goods will complicate aggregation for entrepreneur sector, so make it a separate agent (retailers).

The equations that generate the financial accelerator are the following:

Asset Price Variability

$$Q_t = \left[\Phi'\left(\frac{I_t}{K_t}\right)\right]^{-1}, \tag{4.3}$$

Investment Financing Supply Curve

$$\mathbb{E}\{R^k_{t+1}\} = s\left(\frac{N_{t+1}}{Q_t K_{t+1}}\right) R_{t+1}, \tag{4.5}$$

Entrepreneurial Equity

$$V_t = R^k_t Q_{t-1} K_t - \left(R_t + \frac{\mu \int_0^{\bar{\omega}} \omega R^k_t Q_{t-1} K_t dF(\omega)}{Q_{t-1} K_t - N_{t-1}}\right)(Q_{t-1} K_t - N_{t-1}). \tag{4.8}$$

Begin with an aggregate capital stock evolution that has increasing marginal adjustment costs,

$$K_{t+1} = \Phi\left(\frac{I_t}{K_t}\right) K_t + (1-\delta)K_t. \tag{4.2}$$

Take derivative with respect to investment,

$$\frac{\partial K_{t+1}}{\partial I_t} = \Phi'\left(\frac{I_t}{K_t}\right) K_t.$$

By definition of price,

$$\Phi'\left(\frac{I_t}{K_t}\right) K_t = \frac{1}{Q_t} K_t,$$

$$Q_t = \left[\Phi'\left(\frac{I_t}{K_t}\right)\right]^{-1}. \tag{4.3}$$

Desired Equation

Capital expenditures, $Q_t K_{t+1}$, is a function of the external finance premium $s_t \equiv \mathbb{E}[R^k_{t+1}]/R_t$ and net worth N_t.

$$Q_t K_{t+1} = \psi(s_t) N_{t+1}, \psi'(\cdot) > 0$$

Since strictly increasing, take inverse to get (4.5).

- Return on lending, R
- Aggregate profits per unit, R^k
- Price of capital, Q
- Idiosyncratic shock ω
  - $\omega \in [0, \infty)$
  - $\mathbb{E}(\omega) = 1$
  - $F(x) = Pr[\omega < x]$, continuous, $F' = f$
- Lender observation cost μ
  - Look at the Lagrangian for optimal finance contract to see why it is needed in this set up.
- Default cutoff μ
  - Same as specifying cost of loan

Set Up

- Borrowing amount $QK - N$
- Total return on capital $\omega R^k QK$
- Monitoring cost $\mu\omega R^k QK$

Either the borrower pays back the loan or the lender audits the borrower and takes everything net audit cost. This results in the following returns,

$$\text{Borrower returns} = \begin{cases} (\omega - \bar{\omega}) R^k QK & \omega \geq \bar{\omega}, \\ 0 & \omega < \bar{\omega}, \end{cases}$$

$$\text{Lender returns} = \begin{cases} \bar{\omega} R^k QK & \omega \geq \bar{\omega}, \\ (1-\mu)\omega R^k QK & \omega < \bar{\omega}. \end{cases}$$

In equilibrium the lender earns the safe rate. Thus (typo in paper),

$$\underbrace{[\bar{\omega}(1 - F(\bar{\omega})) + (1-\mu)\mathbb{E}(\omega|\omega < \bar{\omega})F(\bar{\omega})]}_{\text{Net Share to Lender}} \underbrace{R^k QK}_{\text{Expected Gross Profits}} = R\underbrace{(QK - N)}_{\text{Loan Amount}}. \tag{3.5}$$

Define the following:

$$\Gamma(\omega) \equiv \int_0^{\bar{\omega}} \omega f(\omega) d\omega + \bar{\omega} \int_{\bar{\omega}}^{\infty} f(\omega) d\omega$$

$$\mu G(\omega) \equiv \mu \int_0^{\bar{\omega}} \omega f(\omega) d\omega$$

Let $k \equiv QK/N$ be the capital/wealth ratio and normalize w.r.t. net worth.

$$\max_{k,\bar{\omega}} (1 - \Gamma(\bar{\omega})) R^k k$$

$$\text{s.t. } [\Gamma(\bar{\omega}) - \mu G(\bar{\omega})] R^k k = R(k-1)$$

First order conditions are the following,

$$\bar{\omega} : \Gamma'(\bar{\omega}) - \lambda[\Gamma'(\bar{\omega}) - \mu G'(\bar{\omega})] = 0, \tag{1}$$

$$k : [(1 - \Gamma(\bar{\omega}) + \lambda(\Gamma(\bar{\omega}) - \mu G(\bar{\omega}))]s - \lambda = 0, \tag{2}$$

$$\lambda : [\Gamma(\bar{\omega}) - \mu G(\bar{\omega})]sk - (k - 1) = 0. \tag{3}$$

Thus, in the interior, we can write k as a function of s by combining the three equations and writing ω as a function of s (1-1 relation in interior).
Interior requires the following assumption,

$$q'(\bar{\omega}) > 0,$$

$$q(\bar{\omega}) \equiv \bar{\omega}(f(\bar{\omega})/(1 - F(\bar{\omega}))$$

Derive the entrepreneurial equity starting with contract assumptions.

$$V_t = \underbrace{R_t^k Q_{t-1} K_t}_{\text{Gross Returns}} - \underbrace{[1 - F(\bar{\omega})]\,\bar{\omega} R_t^k Q_{t-1} K_t}_{\text{Loan Repayment for Non-Default}} - \underbrace{\int_0^{\bar{\omega}} \omega f(\omega) d\omega R_t^k Q_{t-1} K_t}_{\text{Loan Repayment for Default}}$$

$$V_t = R_t^k Q_{t-1} K_t - [1 - F(\bar{\omega})] \bar{\omega} R_t^k Q_{t-1} K_t - \int_0^{\bar{\omega}} \omega f(\omega) d\omega R_t^k Q_{t-1} K_t$$

$$V_t = R_t^k Q_{t-1} K_t - [1 - F(\bar{\omega})] \bar{\omega} R_t^k Q_{t-1} K_t - (1 - \mu) \int_0^{\bar{\omega}} \omega f(\omega) d\omega R_t^k Q_{t-1} K_t$$
$$- \mu \int_0^{\bar{\omega}} \omega f(\omega) d\omega R_t^k Q_{t-1} K_t$$

Use equation 3.5.

$$V_t = R_t^k Q_{t-1} K_t - R_t (Q_{t-1} K_t - N_t) - \mu \int_0^{\bar{\omega}} \omega f(\omega) d\omega R_t^k Q_{t-1} K_t$$

$$V_t = R_t^k Q_{t-1} K_t - \left(R_t + \frac{\mu \int_0^{\bar{\omega}} \omega f(\omega) d\omega R_t^k Q_{t-1} K_t}{Q_{t-1} K_t - N_t} \right) (Q_{t-1} K_t - N_t) \tag{4.8}$$

Premium on risk-free rate is consequently,

$$\frac{\mu \int_0^{\bar{\omega}} \omega f(\omega) d\omega R_t^k Q_{t-1} K_t}{Q_{t-1} K_t - N_t}$$

Unexpected Shift in Gross Return to Capital

$$U_t^{rk} \equiv R_t^k - \mathbb{E}_{t-1}\{R_t^k\}$$

Unexpected Shift in conditional default cost (typo in paper)

$$U_t^{dp} \equiv \int_0^{\bar{\omega}} \omega f(\omega) d\omega - \mathbb{E}_{t-1}\{\int_0^{\bar{\omega}} \omega f(\omega) d\omega\} \geq 1$$

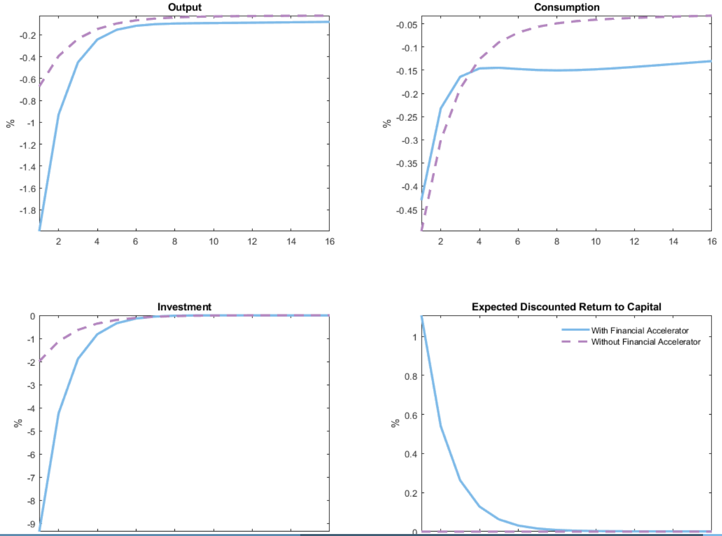
Equity can be rewritten as the following,

$$V_t = [U_t^{rk}(1 - \mu U_t^{dp})]Q_{t-1} + E_{t-1}\{V_t\} \tag{4.9}$$

Elasticity w.r.t unexpected shift in return to capital,

$$\frac{\partial V_t / E_{t-1}\{V_t\}}{\partial U_t^{rk} / E_{t-1}\{U_t^{rk}\}} = \frac{E_{t-1}\{R_t^k\}Q_{t-1}K_t}{E_{t-1}\{V_t\}} \geq 1$$

- Use Cesa-Bianchi 2012 replication code
- Shock the model by an unexpected 25 basis increase in risk-free rate


Output
Consumption
Investment
Expected Discounted Return to Capital
With Financial Accelerator
Without Financial Accelerator
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Discussion

- How can we use this model to analyze current financial conditions and what can we expect from the real economy in the coming quarters (assuming lagged investment)?
- What assumptions (constant returns to scale, no economies of scale for audit, no competing industries for financing, etc.) in the model should be challenged?
- What further empirical results would you like to see to confirm this theory?