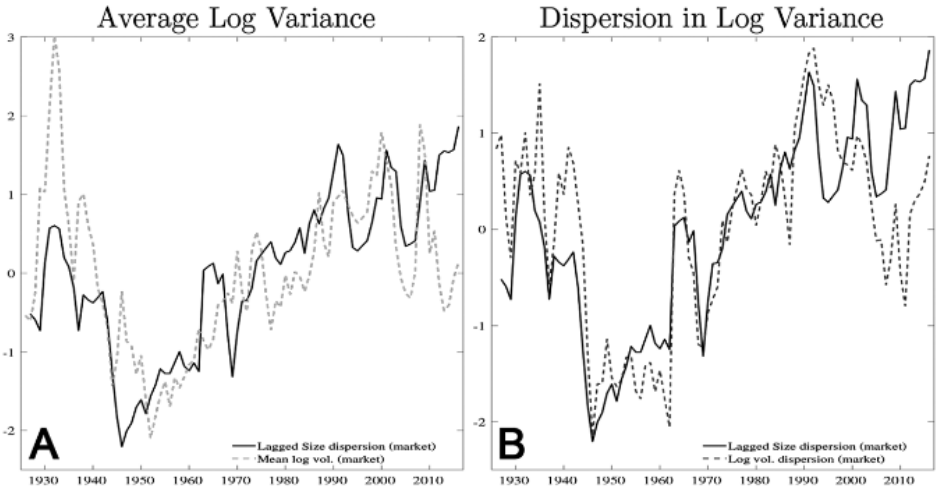
Endogenous Networks, Market Concentration, and Volatility

Ethan Nourbash

October 13, 2025

Introduction

Herskovic et al 2020 find firm size dispersion positivley comoves with production variance and dispersion in variance. That is, the larger the dispersion in firm size. They explain the comovement using a network model of production.



Variance and variance dispersion have risen alongside dispersion of firm size since the 1940s. The goal of this paper is to explain this trend by endogenizing production networks.
Model Features:

- Consumer Risk Aversion
- Delivery uncertainty
- Heterogeneous productivity
- Minimum order size (or relation fixed cost)
- No Loops???
- Markets Clear ex-ante???

Risk aversion is adopted by the suppliers and makes them care about the volatility of their output. Differences in productivity creates firm size heterogeneity. Minimum order size makes optimal number of suppliers comove with firm size. No loops (possibly) to make firm supplier decisions tractable and solve identification issue empirically.
Transactions occur ex-ante on expected quantities. Prices will capture volatility.
The results are as follows:

- An aggregate shock increases size of large firms, further increasing volatility.
- In certain situations, a shock to a large firm increases its size.

Model

Real GDP

$$Y = \max_{\mathbf{c}} \mathbb{E}\, \mathcal{D}(\mathbf{c})$$

$\mathcal{D}' > 0$, $\mathcal{D}'' < 0$.
Subject to

$$\sum_{i}^{N} p_i \,\mathbb{E}\, c_i = \sum_{i=1}^{N} \pi_i.$$

CRS firm production

$$y_i = A_i F(\mathbf{x_i}).$$

Profits determined by marginal costs of inputs and fixed cost as a function of nonzero inputs. Networks are chosen before volatility is realized

$$\pi_i = p_i \,\mathbb{E}\, y_i - \mathbf{p}\,\mathbb{E}\,\mathbf{x_i}$$

subject to minimum *expected* (due to uncertainty in delivery) order

$$\mathbb{E}\,\mathbf{x_i} \geq c\mathbf{1}_N$$

$c \geq 0$.
Productivity is log normal with idiosyncratic volatility with a common and firm-specific component

$$\log A_i \sim N(0, \sigma + \sigma_i).$$

Firms compete in bertrand competition.
Firms total volatility is a function of its own and that of its suppliers.
Orders are placed in expected values and recieved after A_i is realized.

Equilibrium

An equilibrium is define when 1) markets clear 2) consumer is maximizing welfare given prices 3) firms maximize profits given other prices.

Issues with model:

- Constraints on producer maximization problem lead to analytic difficulties.
- Relations determined simultaneuously (rather than loops) makes model complicated (or maybe impossible idk).