MAT 102

BASIC CONCEPTS OF MATRICES

INTRODUCTION TO MATRICES

- Definition of Matrix
- Dimension of Matrix
- Classification of Matrix with Examples
- Diagonal and Trace of a Matrix
- Transpose of a Matrix
- Determinant
- Difference between Matrix and Determinant

Advantages:

Matrix algebra has at least two advantages:

- Reduces complicated systems of equations to simple expressions
- Adaptable to systematic method of mathematical treatment and well suited to computers

INTRODUCTION TO MATRICES

- A matrix is a rectangular arrangement of numbers, expressions or symbols in rows and columns enclosed by (   ), [   ] or | | | |.
- Rows run horizontally and columns run vertically.

$$\begin{bmatrix} 1 & -1 \end{bmatrix} \qquad \begin{pmatrix} 4 & 2 \\ -3 & 0 \end{pmatrix} \qquad \begin{Vmatrix} a & b \\ c & d \end{Vmatrix}$$

- The dimensions, or size, of a matrix are:

# of rows X # of columns.

Example:

$$\begin{bmatrix} 1 & 2 & 4 \\ 4 & -1 & 5 \\ 3 & 3 & 3 \end{bmatrix} \qquad \begin{bmatrix} 1 & 1 & 3 & -3 \\ 0 & 0 & 3 & 2 \end{bmatrix} \qquad \begin{bmatrix} 1 & -1 \end{bmatrix}$$

3x3 matrix   2x4 matrix   1x2 matrix

INTRODUCTION TO MATRICES

- A matrix is usually denoted by a capital letter and the elements within the matrix are denoted by lower case letters
- e.g. matrix [A] with elements a_{ij}

$$A_{m\times n} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1j} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2j} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots & & \vdots \\ a_{i1} & a_{i2} & \cdots & a_{ij} & \cdots & a_{in} \\ \vdots & \vdots & & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mj} & \cdots & a_{mn} \end{bmatrix}$$

i goes from 1 to m

j goes from 1 to n

TYPES OF MATRICES (1)

- Row Matrix: A matrix which has only one row is called a row matrix.

Example:

$$\begin{bmatrix} 1 & 1 & 6 \end{bmatrix} \qquad \begin{bmatrix} 0 & 3 & 5 & 2 \end{bmatrix}$$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \cdots & a_{1n} \end{bmatrix}$$

- Column Matrix: A matrix which has only one column is called a column matrix.

Example:

$$\begin{bmatrix} 1 \\ 4 \\ 2 \end{bmatrix} \qquad \begin{bmatrix} 1 \\ -3 \end{bmatrix} \qquad \begin{bmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{bmatrix}$$

FIND THE DIMENSION OF EACH MATRIX

1. $A = \begin{bmatrix} 2 & -1 \\ 0 & 5 \\ -4 & 8 \end{bmatrix}$

Dimension: 3×2

2. $B = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$

Dimension: 4×1

3. $C = \begin{bmatrix} 0 & 5 & 3 & -1 \\ -2 & 0 & 9 & 6 \end{bmatrix}$

Dimension: 2×4

TYPES OF MATRICES (2)

- Rectangular Matrix: A matrix where number of rows is not equal to the number of columns is called rectangular matrix.

Example:

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 2 & 0 & 3 & 3 & 0 \end{bmatrix}$$

- Square Matrix: A matrix which has the same number of rows and columns is called square matrix.

Example:

$$\begin{bmatrix} 1 & 1 \\ 3 & 0 \end{bmatrix} \qquad \begin{bmatrix} 1 & 1 & 1 \\ 9 & 9 & 0 \\ 6 & 6 & 1 \end{bmatrix}$$

TYPES OF MATRICES (3)

- Diagonal Matrix: A square matrix where all the elements are zero except those on the main diagonal.

Example:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 9 \end{bmatrix}$$

- Identity/Unit Matrix: An identity matrix is a square matrix that has 1's along the main diagonal and 0's everywhere else. Notation : I_n for $n \times n$ unit matrix.

Example:

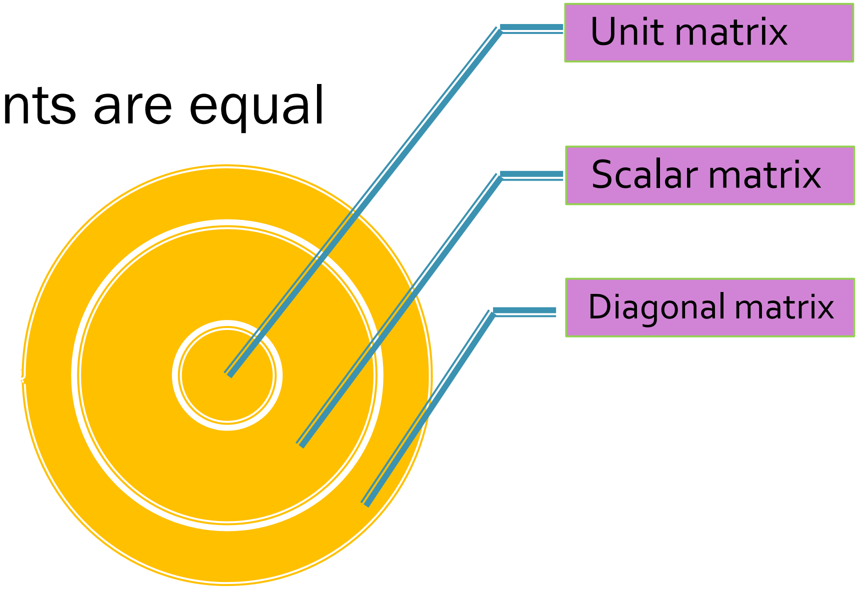
$$I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

TYPES OF MATRICES (4)

- Scalar Matrix: A diagonal matrix whose main diagonal elements are equal

Example:

$$\begin{bmatrix} 6 & 0 & 0 & 0 \\ 0 & 6 & 0 & 0 \\ 0 & 0 & 6 & 0 \\ 0 & 0 & 0 & 6 \end{bmatrix}$$



Diagonal matrix:

$$M = \begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & -5 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}$$

Scalar matrix:

$$M = \begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix}$$

Identity matrix:

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

DIAGONAL AND TRACE

Only a square matrix posses the diagonal & the trace.

- diag(M) = { 3, -2, 2.7, 0, -8.5 }
- *tr* (M) = 3 + (-2) + 2.7 +0 + (-8.5)

  = -4.8

An example (5×5):

$$M = \begin{bmatrix} 3 & 1 & 2 & 0 & -1 \\ 0 & -2 & 1 & -1 & 2 \\ 2 & 1 & 2.7 & 2 & 0 \\ 1 & -1 & 1 & 0 & -2 \\ 6 & 4 & 3 & 2 & -8.5 \end{bmatrix}$$

TYPES OF MATRICES (5)

- Zero/Null/Void matrix :

  each entry is zero(0).

- *Nonzero* matrix :

  at least 1 nonzero ($\neq 0$) entry

- Sparse matrix :

  #(zeroes) > #(nonzeroes)

- Dense matrix :

  #(zeroes) < #(nonzeroes)

N.B. A Sparse/Dense matrix must have *zeroes* and *nonzeroes* as entries.

Example:

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Zero matrix

$$\begin{bmatrix} 0 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

Nonzero matrix

$$\begin{bmatrix} 1 & -4 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Sparse matrix

$$\begin{bmatrix} 1 & 1 & 1 \\ 9 & 9 & 0 \\ 6 & 6 & 1 \end{bmatrix}$$

Dense matrix

TYPES OF MATRICES (6)

- Triangular Matrix: A square matrix whose elements above or below the main diagonal are all zero.

Example:

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 5 & 2 & 3 \end{bmatrix} \qquad \begin{bmatrix} 1 & 8 & 9 \\ 0 & 1 & 6 \\ 0 & 0 & 3 \end{bmatrix}$$

- ➢ Upper Triangular Matrix: A square matrix whose elements below the main diagonal are all zero.

Example:

$$\begin{bmatrix} 1 & 8 & 7 \\ 0 & 1 & 8 \\ 0 & 0 & 3 \end{bmatrix}$$

- ➢ Lower Triangular Matrix: A square matrix whose elements above the main diagonal are all zero.

Example:

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 5 & 2 & 3 \end{bmatrix}$$

PERMUTATION MATRIX

A permutation matrix (P) looks very similar to an identity Matrix. A permutation Matrix (P):

- Is a square matrix
- Consists of only 1's and 0's
- Each row must consist of a single 1
- Each column must consist of a single 1

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \qquad \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Matrix P_1 Matrix P_2 Matrix P_3

TRANSPOSE OF A MATRIX

- Matrix Transpose: Transpose of a matrix A is a matrix obtained by interchanging the rows and columns of A.
- Denoted by: $A^T \; or, \; A'$

$$\underset{4x1}{X} = \begin{bmatrix} 12 \\ 9 \\ -4 \\ 0 \end{bmatrix} \qquad \underset{1x4}{X'} = \begin{bmatrix} 12 & 9 & -4 & 0 \end{bmatrix}$$

$$\underset{3x4}{A} = \begin{bmatrix} 21 & 62 & 33 & 93 \\ 44 & 95 & 66 & 13 \\ 77 & 38 & 79 & 33 \end{bmatrix} \qquad \underset{4x3}{A'} = \begin{bmatrix} 21 & 44 & 77 \\ 62 & 95 & 38 \\ 33 & 66 & 79 \\ 93 & 13 & 33 \end{bmatrix}$$

DETERMINANT (1)

- A Determinant is a number associated with a matrix. Only SQUARE matrices have a determinant.
- The symbol for a determinant can be the phrase “det” in front of a matrix variable, det(A); or vertical bars around a matrix, |A|

$$\underset{1x1}{C} = \begin{bmatrix} 4 \end{bmatrix} \qquad |C| = 4$$

$$\underset{2x2}{C} = \begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix} \qquad |C| = (a_1 * b_2) - (b_1 * a_2)$$

$$\underset{2x2}{C} = \begin{bmatrix} 3 & 2 \\ 5 & 1 \end{bmatrix} \qquad |C| = (3*1) - (2*5) = 3 - 10 = -7$$

DIFFERENCE BETWEEN MATRIX AND DETERMINANT

Matrix	Determinant
A matrix cannot be reduced to a single number.	A determinant can be reduced to a single number
In a matrix, the number of rows may not be equal to the number of columns.	In a determinant, the number of rows must be equal to the number of columns.
An interchange of rows or columns gives a different matrix.	An interchange of rows or columns gives the same determinant with +ve or –ve sign.
Examples: $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ $\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$	Examples: $\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}$ $\begin{vmatrix} 1 & 2 & 3 \\ -2 & 1 & 0 \\ 2 & 3 & 4 \end{vmatrix}$

DETERMINANT (2)

Let M be a matrix & $|M|$ be the corresponding determinant.

Rectangular	$\longleftrightarrow$	$\mid M \mid$ undefined
Singular	$\longleftrightarrow$	$\mid M \mid = 0$
Nonsingular	$\longleftrightarrow$	$\mid M \mid \neq 0$

PRACTICE ⇢ PERFECTION

You can justify your understanding by answering the following questions

- Write 1 difference between Matrix and Determinant.
- Why is it impossible to write a determinant with 33 entries?
- What is the trace of $-5I_{132}$?
- Every diagonal matrix is a scalar matrix: **True or False?**
- Every scalar matrix is a diagonal matrix: **True or False?**
- What will be the type of transpose of a 55 × 55 diagonal matrix?
- What is the trace of a *zero* matrix ?
- What is the trace of a square *zero* matrix ?
- $17\,\mathrm{tr}\,(I_{23}) - \mathrm{tr}\,(22\,I_{17}) = ?$

Give examples:

- Upper triangular matrix (6×6)
- Lower triangular matrix (6×7)
- Unit matrix (6×6)
- Unit matrix (5×7)
- Permutation matrix (5×5)
- Permutation matrix (5×6)
- Scalar matrix (5×6)
- Diagonal matrix (6×6)
- Column matrix with 11 entries
- Row matrix with 13 entries
- 7×2 column matrix

ANY QUESTION???